GaussianP

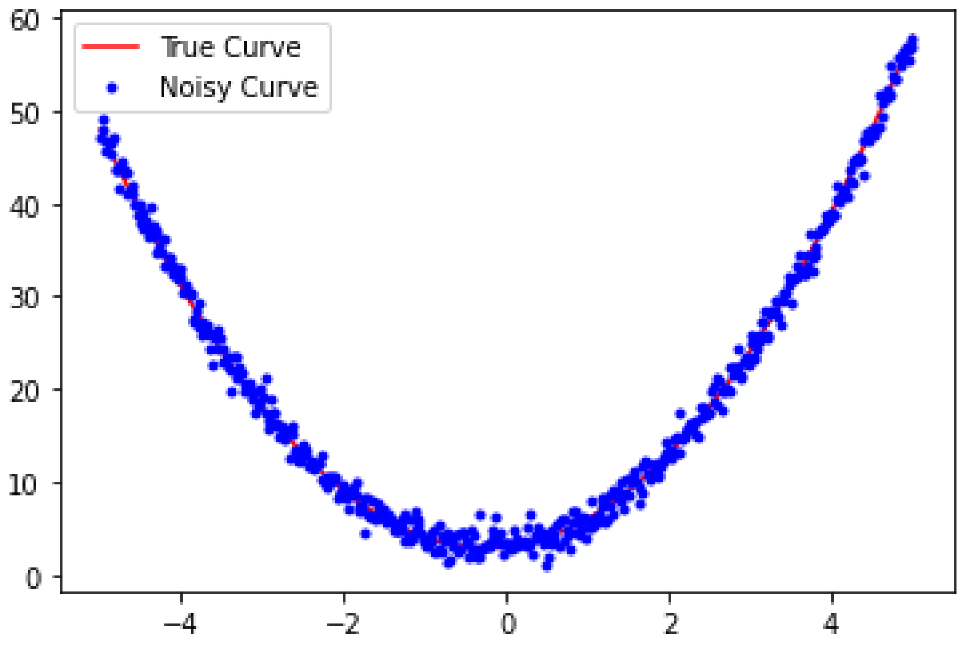
April 16, 2023

All the libraries required throughtout the notebook are imported below

[1]:
```python
# Importing libraries
import numpy as np #Mathematical operation
import matplotlib.pyplot as plt #Plotting library
from sklearn.gaussian_process import GaussianProcessRegressor
from sklearn.gaussian_process.kernels import RBF
from tqdm.notebook import tqdm,trange
import time
```

1 Polynomial curve with normal noise.

[2]:
```python
# Define the true polynomial coefficients
a = 2
b = 1
c = 3

# Define the range of x values
x = np.linspace(-5, 5, 501)

# Generate random normal noise with mean 0 and standard deviation 1
noise = np.random.normal(0, 1, len(x))

# Calculate the y values with added noise
y = a*x**2 + b*x + c + noise

# Plot the true curve and the noisy curve
plt.plot(x, a*x**2 + b*x + c, 'r-', label='True Curve')
plt.plot(x, y, 'b.', label='Noisy Curve')
plt.legend()
plt.show()
```

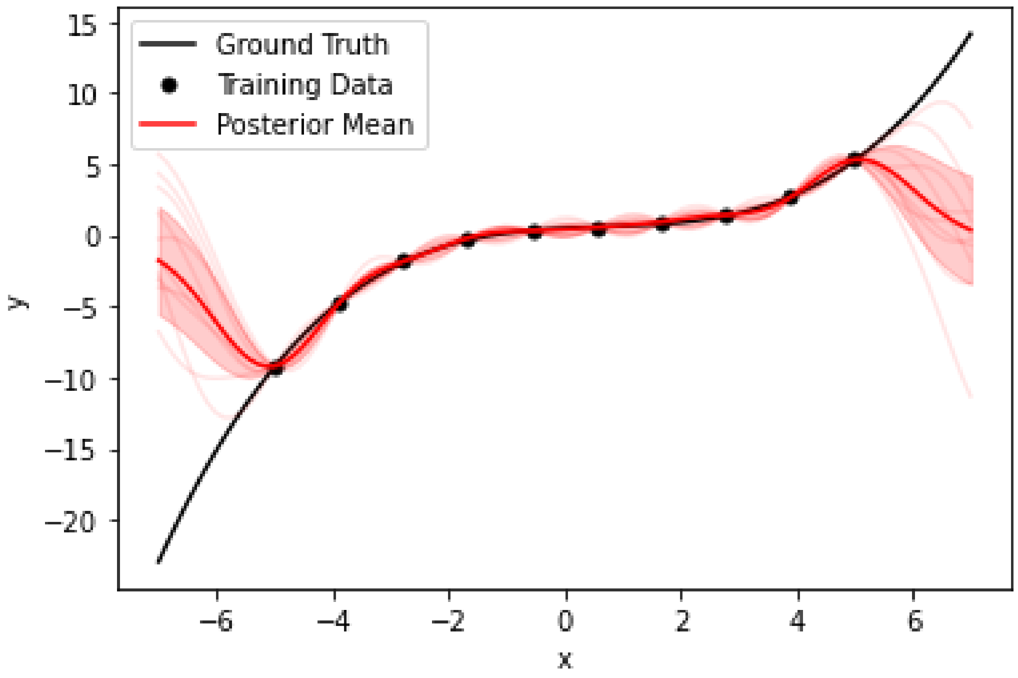


2 GaussianP

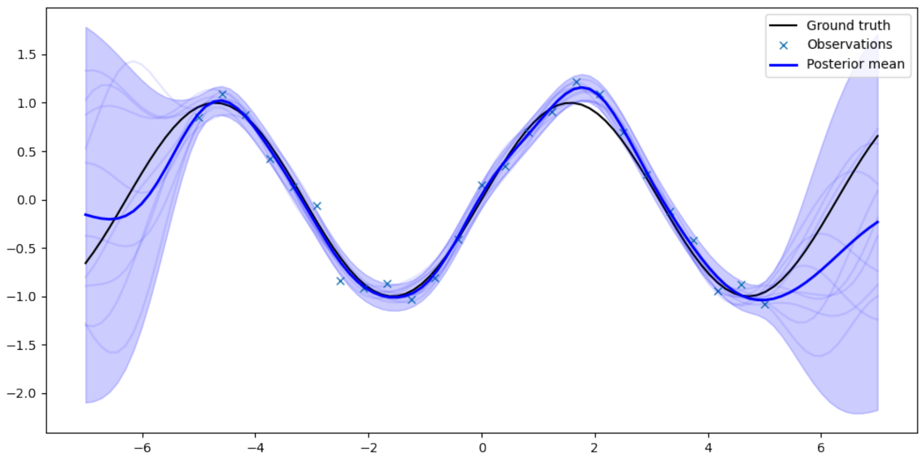
A random 3rd order polynomial is provided as ground truth and the GaussianP algorithm is carried out using sklearn package. Radial basis function kernel (aka squared-exponential kernel) is used from sklearn library and a zero mean GaussianP prior is selected.

[37]:
```
# Set seed for reproducibility
np.random.seed(42)

# Define ground truth polynomial curve
def ground_truth(x):
    return 0.5 + 0.2*x - 0.1*x**2 + 0.05*x**3

# Generate training data
n = 10 # number of training points
x_train = np.linspace(-5, 5, n)
y_train = ground_truth(x_train) + np.random.normal(scale=0.1, size=n)

# Define test points
x_test = np.linspace(-7, 7, 100)

# Define Gaussian process kernel
kernel = RBF(length_scale=1.0)

# Define Gaussian process prior
```

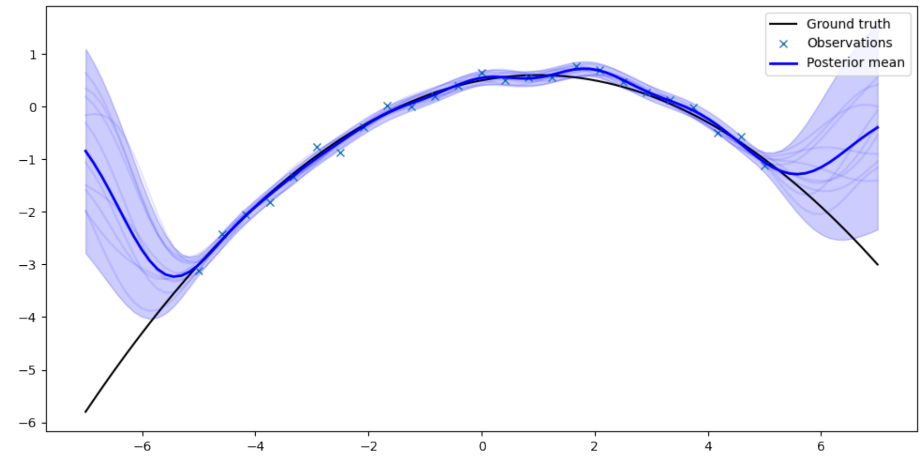
```
gp_prior = GaussianProcessRegressor(kernel=kernel, alpha=0,normalize_y=True)

# Fit Gaussian process to training data
gp_posterior = gp_prior.fit(x_train.reshape(-1,1), y_train)

# Predict mean and standard deviation of test points
y_mean, y_std = gp_posterior.predict(x_test.reshape(-1,1), return_std=True)

# Plot results
plt.figure()
plt.plot(x_test, ground_truth(x_test), 'k-', label='Ground Truth')
plt.plot(x_train, y_train, 'k.', markersize=10, label='Training Data')
plt.plot(x_test, y_mean, 'r-', label='Posterior Mean')
plt.fill_between(x_test, y_mean - y_std, y_mean + y_std, alpha=0.2, color='r')
y_samples = gp_posterior.sample_y(x_test.reshape(-1,1), 10) # To fill the color␣
 ↪between the mean and the cov value
plt.plot(x_test, y_samples, 'r-', alpha=0.1)
plt.xlabel('x')
plt.ylabel('y')
plt.legend()
plt.show()
```



Similar task like above is performed but this time without using any external libraries like sklearn or gflow. This process totally fails at predicting the correct function. I tried more conventional functions like sin(x), and it performs well in those cases. But I tried with 2nd and 3rd order polynomials it fails completely.



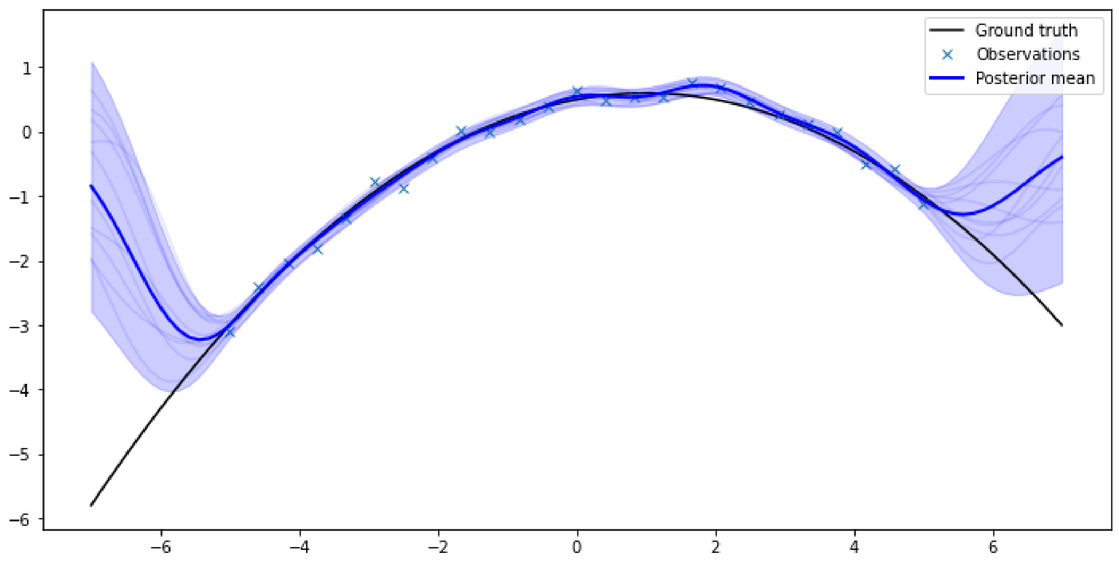
Sin x fitting



Polynomial order 2 (0.5 + 0.2x - *0.1*x**2) fitting

[36]:
```
import numpy as np
import matplotlib.pyplot as plt

# Define the ground truth polynomial curve
def true_curve(x):
    return 0.5 + 0.2*x - 0.1*x**2 + 0.05*x**3

# Generate noisy observations of the curve
np.random.seed(123)
x_obs = np.linspace(-5, 5, 25)
```

```
y_obs = true_curve(x_obs) + np.random.normal(loc=0, scale=0.1, size=len(x_obs))

# Define the kernel function
def squared_exponential_kernel(x1, x2, l=1.0, sigma=1.0):
    dist_sq = np.sum(x1**2, 1).reshape(-1, 1) + np.sum(x2**2, 1) - 2*np.dot(x1,␣
 ↪x2.T)
    return sigma**2 * np.exp(-0.5 / l**2 * dist_sq)

# Define the Gaussian process model
def gp_regression(x_train, y_train, x_test, kernel, sigma_n=0.1):
    K = kernel(x_train, x_train)
    K_star = kernel(x_train, x_test)
    K_star_star = kernel(x_test, x_test)
    K_inv = np.linalg.inv(K + sigma_n**2 * np.eye(len(x_train)))
    mu = np.dot(K_star.T, np.dot(K_inv, y_train))
    cov = K_star_star - np.dot(K_star.T, np.dot(K_inv, K_star))
    return mu, cov

# Define test points
x_test = np.linspace(-7, 7, 100)[:, None]

# Compute posterior mean and variance
mu, cov = gp_regression(x_obs[:, None], y_obs, x_test,␣
 ↪squared_exponential_kernel, sigma_n=0.1)

# Sample from the posterior
samples = np.random.multivariate_normal(mu.ravel(), cov, size=10)

# Plot the results
plt.figure(figsize=(12, 6))
plt.plot(x_test, true_curve(x_test), 'k', label='Ground truth')
plt.plot(x_obs, y_obs, 'x', label='Observations')
plt.plot(x_test, mu, 'b', lw=2, label='Posterior mean')
plt.fill_between(x_test[:, 0], mu - 2*np.sqrt(np.diag(cov)),
                 mu + 2*np.sqrt(np.diag(cov)), color='b', alpha=0.2)
for i in range(samples.shape[0]):
    plt.plot(x_test, samples[i], 'b', alpha=0.1)
plt.legend()
plt.savefig("pol2.png",dpi=100)
plt.show()
```



3 Gaussian Mixture

1000 Datapoints are generated using Ancestral Sampling Scheming. First five generated data are printed out.

[4]:
```
# Set the parameters
M = 3
alpha = np.ones(M) / M
means = np.array([[-1, 0], [1, 0], [0, 1]])
variance = 0.25**2
num_data_points = 1000

# Sample mixture weights from a Dirichlet distribution
mixing_weights = np.random.dirichlet(alpha)

# Sample component labels and data points for each data point
component_labels = np.zeros(num_data_points, dtype=int) #To store the choices␣
 ↪of the draw
data_points = np.zeros((num_data_points, 2)) # To store the datapoints
for n in range(num_data_points):
    # Sample a component label from a categorical distribution
    component_labels[n] = np.random.choice(M, p=mixing_weights)

    # Sample a data point from a Gaussian distribution with the mean of the␣
 ↪selected component
    data_points[n] = np.random.normal(loc=means[component_labels[n]], scale=np.
 ↪sqrt(variance))
```

```
# Print the first few data points
print(data_points[:5])
```

```
[[-0.10701152  0.81439829]
 [ 0.36641219  0.94355592]
 [ 1.63987201  0.09855826]
 [-1.28774839  0.0939245 ]
 [ 0.84993654  0.23685996]]
```

Gibbs sampler to draw posterior samples

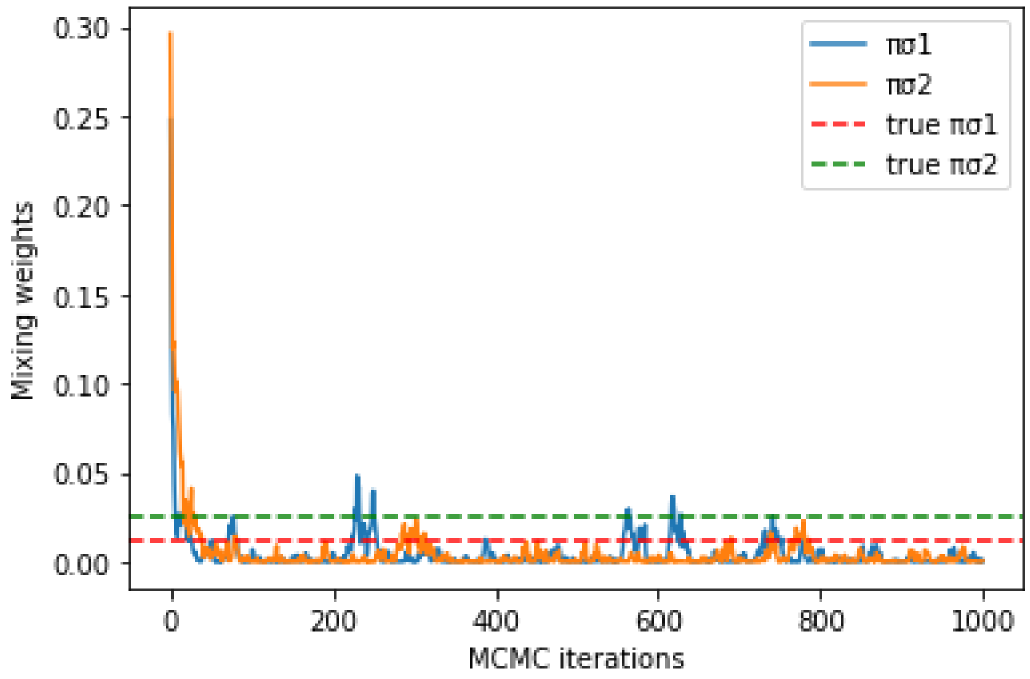
[5]:
```
# Set the parameters
M = 3
alpha = np.ones(M) / M
means = np.array([[-1, 0], [1, 0], [0, 1]])
variance = 0.252
num_data_points = 1000

# Generate synthetic data
mixing_weights_true = np.random.dirichlet(alpha)
component_labels_true = np.zeros(num_data_points, dtype=int)
data_points = np.zeros((num_data_points, 2))
for n in range(num_data_points):
    component_labels_true[n] = np.random.choice(M, p=mixing_weights_true)
    data_points[n] = np.random.normal(loc=means[component_labels_true[n]],␣
  ↪scale=np.sqrt(variance))

# Initialize the values of s1:N and  1:M
component_labels = np.zeros(num_data_points, dtype=int)
mixing_weights = np.ones(M) / M

# Set the number of iterations
num_iterations = 1000

# Run the Gibbs sampler
for t in trange(num_iterations):
    # Sample the values of s1:N from the conditional distribution p(s1:N | w1:
  ↪N,  1:M)
    for n in range(num_data_points):
        posterior_probs = np.zeros(M) #To store the intermediate prosterior␣
  ↪values
        for k in range(M):
            posterior_probs[k] = np.exp(-np.sum((data_points[n] - means[k]) **␣
  ↪2) / (2 * variance)) * mixing_weights[k]
        posterior_probs /= np.sum(posterior_probs)
        component_labels[n] = np.random.choice(M, p=posterior_probs)
    # Sample the value of  1:M from the conditional distribution p( 1:M | s1:N,␣
↪w1:N)
    mixing_weights = np.random.dirichlet(alpha + np.bincount(component_labels,␣
↪minlength=M))

# Print the true and inferred mixture weights
print("True mixing weights:", mixing_weights_true)
print("Inferred mixing weights:", mixing_weights)
print("Error in inferred weights :",np.
↪abs(mixing_weights_true-mixing_weights)*100/mixing_weights_true,"%")
```

```
  0%|          | 0/1000 [00:00<?, ?it/s]

True mixing weights: [0.12442482 0.85240826 0.02316692]
Inferred mixing weights: [0.12509537 0.84819123 0.0267134 ]
Error in inferred weights : [ 0.53891627  0.49471908 15.30838517] %
```

1:2 are potted below along with the ground truth as the horizontal line.

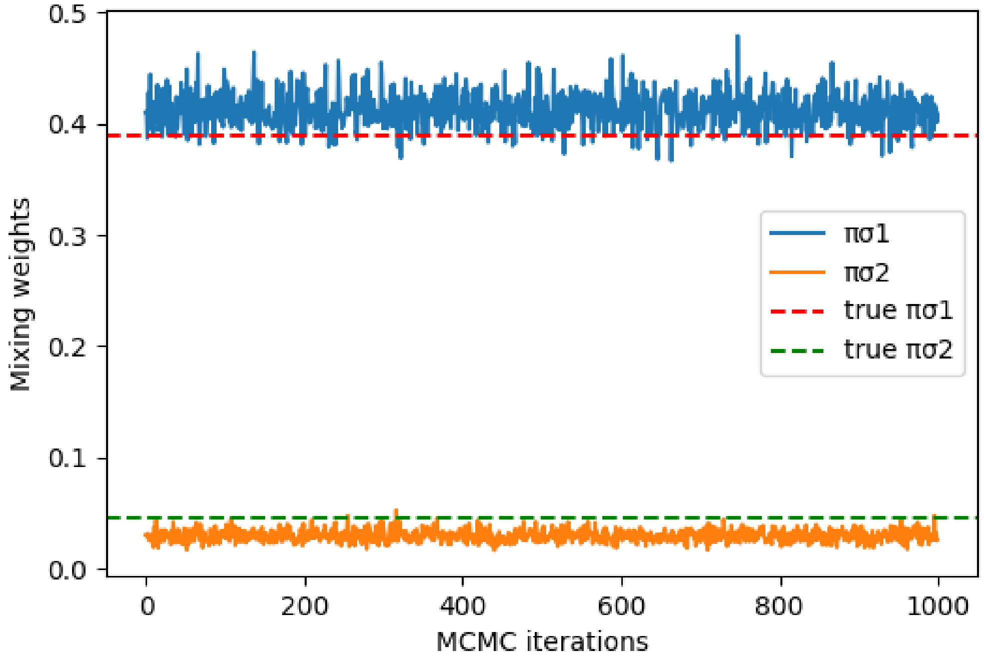
[6]:
```
# Set the parameters
M = 3
alpha = np.ones(M) / M
means = np.array([[-1, 0], [1, 0], [0, 1]])
variance = 1
num_data_points = 1000

# Generate synthetic data
mixing_weights_true = np.random.dirichlet(alpha)
component_labels_true = np.zeros(num_data_points, dtype=int)
data_points = np.zeros((num_data_points, 2))
for n in range(num_data_points):
    component_labels_true[n] = np.random.choice(M, p=mixing_weights_true)
    data_points[n] = np.random.normal(loc=means[component_labels_true[n]],␣
↪scale=np.sqrt(variance))

# Initialize the values of s1:N and  1:M
component_labels = np.zeros(num_data_points, dtype=int)
mixing_weights = np.ones(M) / M

# Set the number of iterations
num_iterations = 1000

# Create arrays to store the mixing weights
mixing_weights_history = np.zeros((num_iterations, 2))

# Run the Gibbs sampler
for t in trange(num_iterations):
    # Sample the values of s1:N from the conditional distribution p(s1:N | w1:
 ↪N,  1:M)
    for n in range(num_data_points):
        posterior_probs = np.zeros(M)
        for k in range(M):
            posterior_probs[k] = np.exp(-np.sum((data_points[n] - means[k]) **␣
 ↪2) / (2 * variance)) * mixing_weights[k]
        posterior_probs /= np.sum(posterior_probs)
        component_labels[n] = np.random.choice(M, p=posterior_probs)

    # Sample the value of  1:M from the conditional distribution p( 1:M | s1:N,␣
 ↪w1:N)
    mixing_weights = np.random.dirichlet(alpha + np.bincount(component_labels,␣
 ↪minlength=M))

    # Store the mixing weights
    mixing_weights_history[t] = mixing_weights[:2]

# Print the true and inferred mixture weights
print("True mixing weights:", mixing_weights_true)
print("Inferred mixing weights:", mixing_weights)
print("Error in inferred weights :",np.
 ↪abs(mixing_weights_true-mixing_weights)*100/mixing_weights_true,"%")

# Plot the mixing weights as a function of iterations
plt.plot(mixing_weights_history[:, 0], label=" 1") #Plotting the first weight
plt.plot(mixing_weights_history[:, 1], label=" 2") #Plotting the 2nd weight
plt.axhline(y=mixing_weights_true[:2][0], color='r', linestyle='--',␣
 ↪label="true  1") #Ground truth weight 1
plt.axhline(y=mixing_weights_true[:2][1], color='g', linestyle='--',␣
 ↪label="true  2") # Ground truth weight 2
# plt.ylim(mixing_weights_true[:2][1]-0.01, mixing_weights_true[:2][0]+0.01)␣
 ↪#Setting the limit of the y axis
plt.legend() #Show the legends
plt.xlabel("MCMC iterations") #xlabels
plt.ylabel("Mixing weights") #ylabels
# plt.savefig("1.png",dpi=100)
plt.show()
```

```
  0%|          | 0/1000 [00:00<?, ?it/s]

True mixing weights: [0.01229262 0.02471209 0.96299529]
Inferred mixing weights: [2.92735386e-05 4.10406636e-04 9.99560320e-01]
Error in inferred weights : [99.76186084 98.33924751  3.79701002] %
```

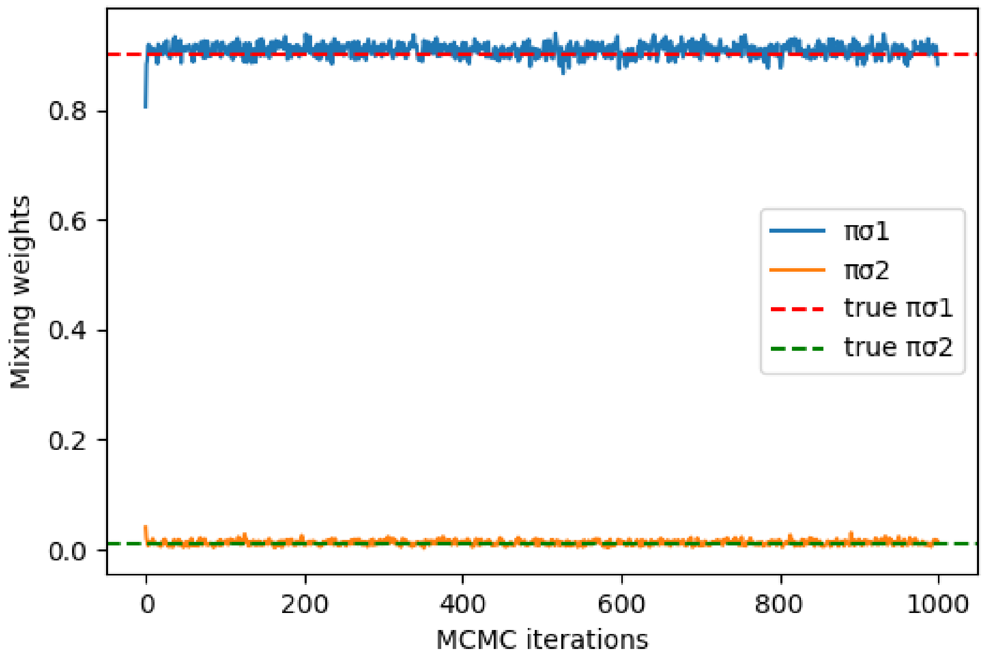


Weights and the plot of the 1:2 is shown below. The error in estimation of the weights follows more or less the same pattern irrespective of the change in the variations.

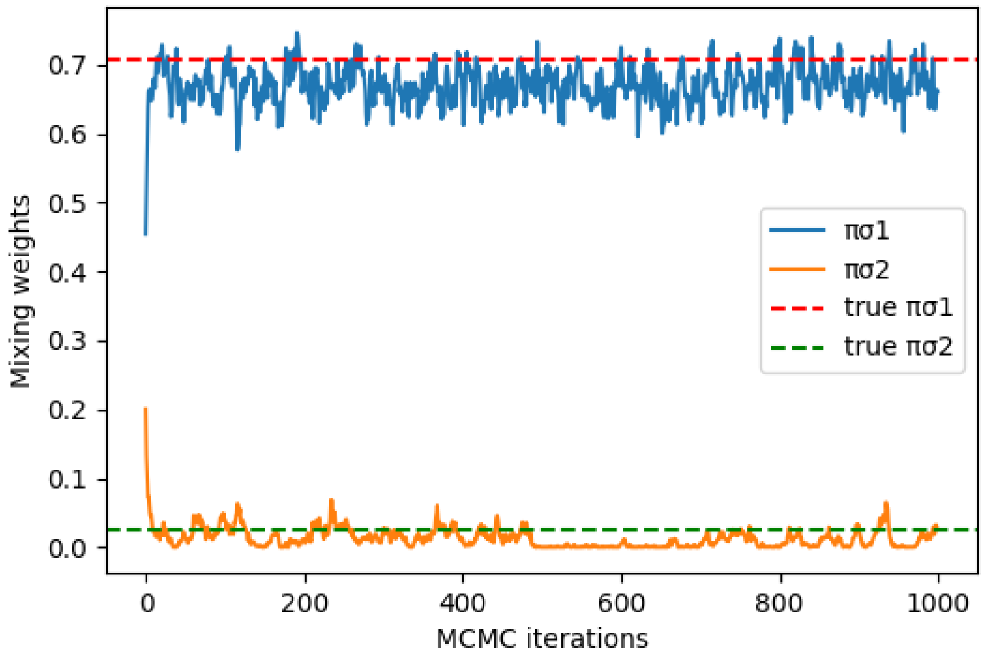
Variance	w1 true	w1 estimated	w2 true	w2 estimated	w3 true	w3 estimated	%Err w1	%Err w2	%Err w3
0.25^2	0.3897611	0.40868218	0.0449455	0.02576964	0.5652933	0.56554818	4.85%	42.66%	0.05%
0.5^2	0.901094	0.8826052	0.0104222	0.01508606	0.0884838	0.10230874	2.05%	44.75%	15.62
1	0.7082429	0.6610483	0.0250292	0.02648702	0.2667278	0.31246468	6.66%	5.82%	17.14



Variance = 0.25^2



Variance = 0.5^2


Mixing weights
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
200
400
600
800
1000
MCMC iterations
πσ1
πσ2
true πσ1
true πσ2

Variance = 1