

Assignment 4

I have used the following simplification:

- $y_{2n} = 1 - y_{1n}$
- $\pi_1 = \frac{\sum(y_{1n})}{\sum(y_{1n} + y_{2n})} = \frac{\sum(y_{1n})}{\sum(y_{1n} + 1 - y_{1n})} = \frac{\sum(y_{1n})}{n}$
- Took my initial guess of μ as mean of the data generated for faster convergence.
- Similarly took the initial variance as the variance of the data.

Sample output:

True Parameters: $\pi_1 = 0.7$, $\mu_1 = 1.0$, $\mu_2 = 4.0$, $v = 0.5$

Estimated Parameters: $\pi_1 = 0.7036487636639586$, $\mu_1 = 1.0110600112242416$,
 $\mu_2 = 4.02350646764783$, $v = 0.5017586475461915$

%Error in estimation : $\pi_1 = 0.5212519519940871\%$, $\mu_1 = 1.1060011224241606\%$,
 $\mu_2 = 0.5876616911957511\%$, $v = 0.3517295092382966\%$

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#Importing library
import numpy as np; #For mathematical operations
from tqdm import tqdm; # Progress bar

def generate_data(n, pi_1, mu_1, mu_2, v): #Generating gaussian random data.
    data = np.zeros(n) # Store the random data here
    for i in range(n): # loop from 0 to n-1 to generate n random gaussian mixture data.
        if np.random.rand() < pi_1: # Binary choice
            data[i] = np.random.normal(mu_1, np.sqrt(v)) # Generate the data for first gaussian
        else: # if 1-pi_1 generate the second gaussian
            data[i] = np.random.normal(mu_2, np.sqrt(v)) # Generate the 2nd gaussian
    return data # Return the data generated

# Initial guess
pi_1 = 0.7
mu_1 = 1.0
mu_2 = 4.0
v = 0.5
n=10000;

#Generate random data
data = generate_data(n, pi_1, mu_1, mu_2, v)

pi_1_hat = 0.5; #some random guess
mu_1_hat = np.mean(data) #taking mu_1_old close to the mean so that we get the convergence faster.
mu_2_hat = np.mean(data)+np.random.rand() #adding some random number from the mean for faster convergence.
v_hat = np.var(data) # obtaining the variance of the data again for the faster convergence.

for i in tqdm(range(100)): #Looping through 100 steps
    # E step
    # Calculating the y1_old as z using equation from example 3.7
    z = pi_1_hat * np.exp(-0.5 * (data - mu_1_hat)**2 / v_hat) / np.sqrt(2*np.pi*v_hat) \
        / ((1-pi_1_hat) * np.exp(-0.5 * (data - mu_2_hat)**2 / v_hat) / np.sqrt(2*np.pi*v_hat) \
            + pi_1_hat * np.exp(-0.5 * (data - mu_1_hat)**2 / v_hat) / np.sqrt(2*np.pi*v_hat))
    # M step
    pi_1_hat = np.sum(z) / n # Calculating new p1
    mu_1_hat = np.sum(z * data) / np.sum(z) # Calculating new mu_1
    mu_2_hat = np.sum((1 - z) * data) / np.sum(1 - z) # Calculating new mu_2
    v_hat = np.sum(z * (data - mu_1_hat)**2) / np.sum(z) #Calculating new v

print("True Parameters: pi_1 = {}, mu_1 = {}, mu_2 = {}, v = {}".format(pi_1, mu_1, mu_2, v)) #Print original
print("Estimated Parameters: pi_1 = {}, mu_1 = {}, mu_2 = {}, v = {}".format(pi_1_hat, mu_1_hat, mu_2_hat, v_hat)) #Print obtained values
print("%Error in estimation : pi_1 ={} %, mu_1 = {} %, mu_2 = {} %, v ={} %".format(np.abs(pi_1-pi_1_hat)*100/pi_1, np.abs(mu_1-mu_1_hat)*100/mu_1, np.abs(mu_2-mu_2_hat)*100/mu_2, np.abs(v-v_hat)*100/v))

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