15.S50 - Poker Theory and Analytics

Basic Strategy





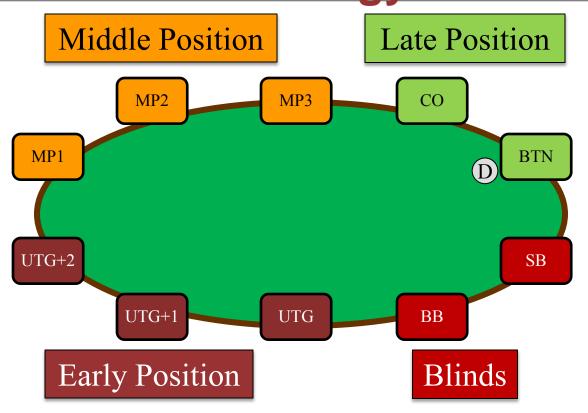
Basic Strategy

- Terminology Position
- Pot Odds
- Implied Odds
- Fold Equity and Semi-Bluffing





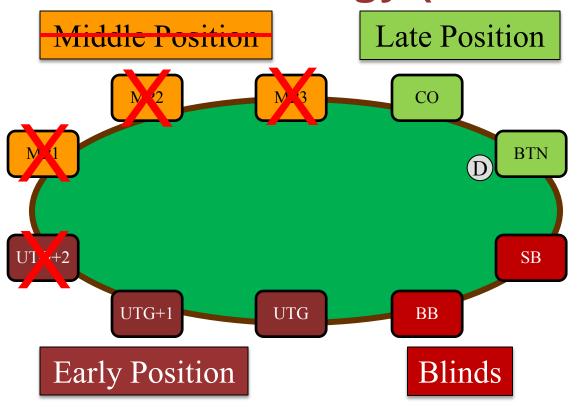
Position Terminology







Position Terminology (6-handed)







Position Basics

- In general, later position is preferred since you get more information before acting
- Playable hands are wider for later positions
- Blinds get a discount to see flops, but are in the worst position for every round thereafter
- Early position offers more opportunity for aggression, and is preferred in some low-M situations
 - e.g. in the "Game of Chicken" situation, first actor gets to
 "throw the steering wheel out the window"



Basic Strategy

- Terminology Position
- Pot Odds
- Implied Odds
- Fold Equity





Why do odds come into play?

- Common situation is weak made hand vs drawing hand
 - i.e. pair or two pair on flop vs straight or flush draw
 - Or pocket pair vs anything else pre flop
- Drawer has to balance chance of hitting draw vs how much each addition card costs
- Made hand wants to
 - Bet enough for the drawer to not have a +EV call
 - Bet an amount that bad players might mistake as good odds





Pot Odds









Pot Odds

John_VH925 (UTG+1): \$500 Blinds 20/40 + 10

Hero (MP1): \$500

Pre Flop: (\$140) Hero is MP1 with A♥ T♥ 1 fold, John_VH925 raises to \$120, Hero calls \$120, 5 folds

Flop: (\$380) 8♥ 3♥ K♣ (2 players)
John_VH925 bets \$370 all in, Hero...

Should the hero call?





Pot Odds

What is the maximum bet the hero should call?







Concept – Expected Value (EV)

- Expected Value is the probability-weighted average of possible results
- EV = Win% * WinAmt Lose% * LoseAmt
- For example,
 - If Win% = 25% and you are facing a \$60 bet into a pot of \$100
 - EV = 25% * (100+60) 75% * 30 = 17.5
- In general, decision rules will be made based on Expected Value
- In Scenario A,
 - our Hero is facing a bet into a pot of \$380
 - EV = W% * (380 + x) L% * x
 - Calling threshold is at EV = 0





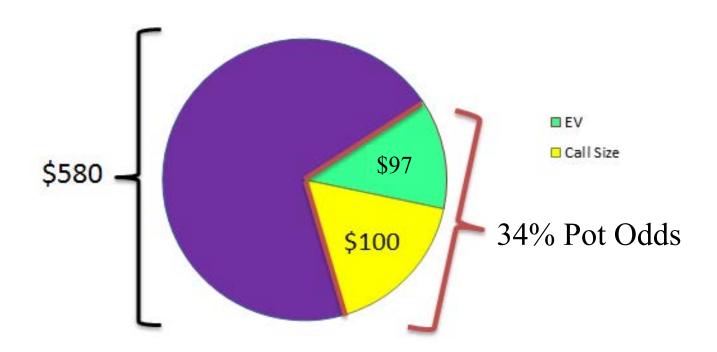
Concept - Pot Odds

- Pot Odds are the relationship of the call amount to the size of the pot
- In general, a call will be +EV if Win% > CallAmt/(PotAfterCall)
- For example in our scenario,
 - If the bet were \$100 into pot of \$380
 - Pot Odds would be 100/580, where 580 = (Pot + Bet + Call)
 - Hero's call contributes ~17% of the pot
 - He can profitably call if Win% > 17% of the time
- Win% is based on "Outs" (cards that result in a win)
- Outs are 9 hearts to hit flush
- Win% = $1 (40/49 * 39/48) \approx 34\%$. This gives us the odds to call
- EV = 34% * \$480 \$100 * 66% = \$97.2





Concept - Pot Odds

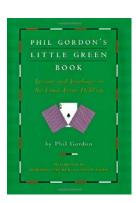






Pot Odds – Gordon's Rule of 2 or 4

- Phil Gordon
 - Fourth Place in 2001 WSOP ME
 - One WPT title
 - Win Two North American Bridge Championships
 - Head Referee World Series of Rock Paper Scissors
 - Author of Phil Gordon's Little Green Book



Teachings in No Limit Texas Hold'em. Simon Spotlight, 2005.





Pot Odds – Gordon's Rule of 2 or 4

- Phil Gordon
 - Author of *Phil Gordon's Little Green Book*
- Each Out is worth about 2% equity per card
- If you get to see both turn and river, use 4% per card



- For example, if have a low pair on the flop and are drawing to three-of-a-kind, you have 2 outs or about 4% to make your hand on each card.
- Other common examples include:
 - Flush Draw (9 outs) gives you odds of $9/47 \approx 18\% = 9*2\%$
 - Inside Straight Draw (4 outs) gives you odds of $4/47 \approx 8\% = 4*2\%$





Pot Odds – Gordon's Rule of 2 or 4

- Phil Gordon
 - Author of *Phil Gordon's Little Green Book*
- Each Out is worth about 2% equity per card
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- For example, if have a low pair on the flop and are drawing to three-of-a-kind, you have 2 outs or about 4% to make your hand on each card.
- Other common examples include:
 - Flush Draw (9 outs) gives you odds of $9/47 \approx 18\%$
 - Inside Straight Draw (4 outs) gives you odds of $4/47 \approx 8\%$





Concept – Pot Odds

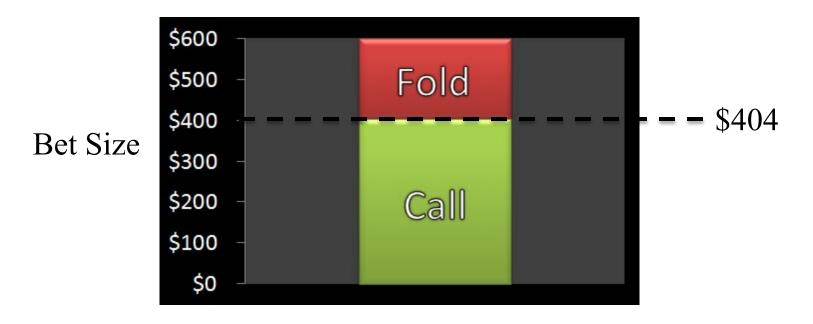
- Breakeven is when EV = 0
- Bet is *x* into a pot of \$380
- Chance of hitting flush is 9 Outs * 4% (since we get both cards)
- Win\% $\approx 36\%$
- Exact Win% = $1 (40/49 * 39/48) \approx 34\%$.
- EV = 34% * (\$380+x) 66% * x = 0 at x = \$404
- So the maximum bet we should call is \$404
- Check with $404 / (404*2 + 380) \approx 0.34$





Solution Set

• Our Hero should call any bet **up to \$404** and fold to anything larger







Practical Solution

John_VH925 (UTG+1): \$500 Blinds 20/40 + 10

Hero (MP1): \$500

Pre Flop: (\$140) Hero is MP1 with A♥ T♥ 1 fold, John_VH925 raises to \$120, Hero calls \$120, 5 folds

Flop: (\$380) 8♥ 3♥ K♣ (2 players)
John_VH925 bets \$370 all in, Hero...

Should the hero call?





Practical Solution

• In real time: Our Hero knows he will hit the flush about 36% of the time, so he can profitably call up to 36% of the new pot. In the case of a \$370 bet, the Hero will decide to call since the new pot will be 380 + 370 + 370 = 1120 and his contribution is 370/1120 (33%), which is less than his chance of winning (36%)





Villain (MP): \$250 Blinds 20/40 + 10

Hero (BTN): \$1000

Pre Flop: (\$140) Hero is BTN with 6 7 7

Villain raises to \$90, Hero calls \$90

Flop: (\$320) 8♠ 5♥ K♣ (2 players)

Villain bets \$150 all in, Hero...





- 1. What are we drawing to?
 - Straight (open-ended)
- 2. What are our outs?
 - Any 9, any 4 (8 cards total)
- 3. Chance of hitting draw?
 - -8*4% = 32%
- 4. Correct play?
 - Call, since call is 150 of 620 or 24%
- 5. EV of decision?
 - 32% * 470 68% * 150 = 48.4





Villain (MP): \$3000 Blinds 100/200

Hero (BTN): \$3000

Pre Flop: (\$300) Hero is BTN with 5♦ 5♥

Villain raises to \$400, 2 calls, Hero calls \$400

Flop: (\$1900) 5 • A • 6 • (2 players)

Villain bets \$200, 2 folds, Hero...





- 1. What are we drawing to?
 - Full House or 4-of-a-kind
- 2. What are our outs?
 - 3x A or 6, 1x 5 (7 cards total)
- 3. Chance of hitting draw?

$$-7*2\% = 14\%$$

- 4. Correct play?
 - Call, since call is 200 of 2300 or 9%
- 5. EV of decision?
 - -14% * 2100 86% * 200 = 122





Villain (BB): \$200 Blinds 100/200

Hero (SB): \$1000

Pre Flop: (\$300) Hero is SB with 5♣ 7♥

Hero...





- 1. What are we drawing to?
 - Anything
- 2. Chance of hitting draw?
 - 570 vs ATC $\approx 40\%$ [320 vs ATC $\approx 32\%$]
- 3. Correct play?
 - Call, since call is 100 of 400 or 25%
- 4. EV of decision?
 - -40% * 300 60% * 100 = 60



Basic Strategy

- Terminology Position
- Pot Odds
- Implied Odds
- Fold Equity





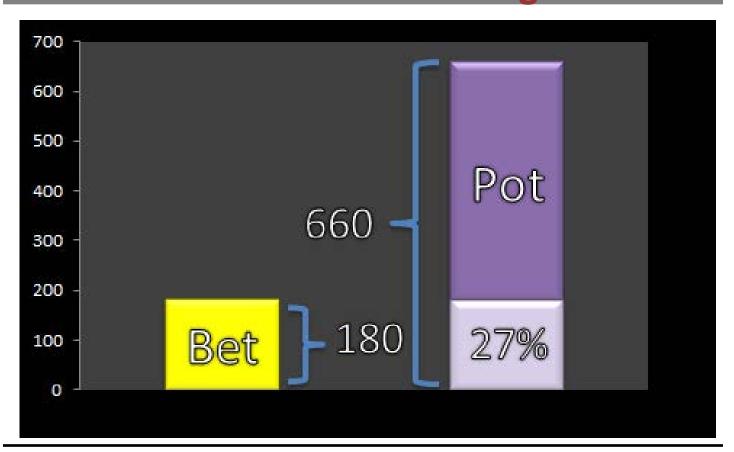
Implied Odds - Hand Rules

- We are trying to find the amount of chips we need to win after hitting our draw to make the bet we are facing a good call
- We do this by figuring out what the pot would have to be after our call to make our x% chance of winning equal to the x% of the pot for the call
- For example, if we have a flush draw (18% to hit), and we are facing a bet of \$180 into a pot of \$300, then our call represents \$180/\$660 = 27% of the pot (i.e. too expensive to call)
- This would be a good call if we contributed 18% of the pot, or \$180/\$1000. So we need to find \$1000 \$660 = \$340 in dead money
- The additional \$340 after the draw makes our \$180 bet worth 18% of a \$1000 pot





We need 18% for this to be a good call







We make our call 18% by adding \$340 of dead money













Villain (MP): \$3000 Blinds 25/50

Hero (BTN): \$3000

Pre Flop: (\$75) Hero is BTN with K♦ T♥

Villain raises to \$150, 2 folds, Hero calls \$150, 2 folds

Flop: (\$375) **T**♣ **A**♥ **6**♦ (2 players)

Villain bets \$100, Hero...





- 1. What are we drawing to?
 - Two pair or 3-of-a-kind
- 2. What are our outs?
 - 3x K, 2x T (5 cards total)
- 3. Chance of hitting draw?
 - 5 * 2% = 10%
- 4. Pot odds?
 - \$100 of \$575, or about 19%, too expensive
- 5. Additional bets after draw to breakeven?
 - \$100/10% = \$1000 \$575 = \$425 more











Villain (MP): \$3000 Blinds 25/50

Hero (BTN): \$3000

Pre Flop: (\$75) Hero is BTN with K♣ Q♣

Villain raises to \$100, 2 folds, Hero calls \$100, 2 folds

Flop: (\$275) **T**♣ **J**♣ **3**♥ (2 players)

Villain bets \$600, Hero...





Implied Odds Examples

- 1. What are we drawing to?
 - Straight or Flush
- 2. What are our outs?
 - Any A, any 9, 7 other ♣ (15 cards total)
- 3. Chance of hitting draw?
 - -15*2% = 30%
- 4. Pot odds?
 - \$600 of \$1475, or about 41%, too expensive
- 5. Additional bets after draw to breakeven?
 - -\$600/30% = \$2000 \$1475 = \$525 more





Drawing Formulas

- EV (Marginal Value of Any Decision)
 - -x = Win%*WinAmt Lose%*LoseAmt
- Rule of 2 or 4 (Chance of Hitting Draw)
 - -x = 2% * #Outs * #FreeCards
- Pot Odds (Decision Rule to Call Bet)
 - Win% > CallAmt/(Pot + BetAmt + CallAmt)
- Implied Odds (Additional Chips After Draw Hits Needed to Call)
 - -x = (BetAmt / Win%) (Pot + BetAmt + CallAmt)





Drawing Formulas (Example)

- EV (Marginal Value of Decision)
 - Calling a \$150 bet into a \$320 pot to have a 32% chance of winning
 - \$48.4 = 32%*(\$320+\$150) 68%*\$150
- Rule of 2 or 4 (Chance of Hitting Draw)
 - You have 9 Outs to a Flush and get to see Turn (not River)
 - -18% = 2% * 9 * 1
- Pot Odds (Decision Rule to Call Bet)
 - You are facing a \$370 all-in bet for a \$380 pot with a flush draw
 - -36% > \$370/(\$380 + \$370 + \$370) =**TRUE**
- Implied Odds (Additional Chips After Draw Hits Needed to Call)
 - \$100 bet into pot of \$375 with 2-pair/3-o-a-k draw on Turn
 - \$425 = (\$100 / 10%) (\$375 + \$100 + \$100)







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YouTube video:

ManiaOFpoker. "World Series Of Poker 2014 Main Event Episode 14 HD 720p." November 11, 2017. *YouTube*. Accessed May 1, 2014.

https://youtube/Q1HkLjq-GGQ?t=23m24s











Hero (UTG): 22,450k 150k/300k Blinds + 50k

Tonking (SB): 6,775k

Pre Flop: (950k) Hero is UTG with A♣ J♣

Hero calls 300k, 7 folds, Tonking calls 150k, Sindelar checks

Flop: (1,400k) 7♣ 8♥ T♣ (3 players)

Tonking checks, Sindelar bets 500k, Hero raises to 1,750k, Tonking raises 4,525k to 6,275k all in, Sindelar folds

Hero...





- What are we drawing to?
 - Flush, maybe straight
- What are our outs?
 - 2,3,4,5,6,8,9,Q,K of ♣ (9 cards) and maybe 9 of ♠ ♦ ♥ (3 cards)*50%
 - Count this as 10.5 outs
- Chance of hitting draw?
 - -10.5*4% = 42%
- Correct play?
 - Pot will be 1400k + 500k + 2*6,275k = 14,450k. Call amount is 4,525k or $\approx 31\%$. So call.
- EV of Decision?
 - EV = 42% * 9925k 58% * 4,525k = 1544k





<u>Drawing – Be careful about</u>

- Drawing to a hand that might not win at showdown
 - i.e. a Q-high or lower flush
 - Or the low end of a straight
 - Or a flush/straight on a paired board
- Assuming you will get to see turn and river for one bet
 - This very rarely happens unless the aggressor is all-in
 - A lot of players will bet on flop with a draw to get this
- Overestimating how easy it is to extract additional chips
 - Flush draws hitting on turn/river are very easy to spot
 - Straight draws are less easy, hitting sets is difficult to see
- Betting too little and letting other players make +EV calls
 - Most flop, turn bets should be around half to 2/3rds of the pot



Basic Strategy

- Terminology Position
- Pot Odds
- Implied Odds
- Fold Equity and Semi-Bluffing





Fold Equity







Fold Equity

Turkito694 (UTG): \$2098.00

Blinds \$5/\$10

Hero (CO): \$990.00

Pre Flop: (\$15.00) Hero is CO with 6♠ 7♠

River: (\$350.00) A♦ 5♥ 8♦ Q♠ 2♦ (2 players)

Turkito694 checks

Hero bets \$150...

How often does this bluff have to work to be profitable?





Concept – Fold Equity

- Fold Equity is the value a player gains from the likelihood that the other player will fold to his bet, assuming a call will result in a loss
- Fold Equity = Current Pot * Fold% Bet * (1-Fold%)
 - If SD-Win% = 0
- Fold Equity = Current Pot * Fold% + (1-Fold%) * EV-if-Called
 - If SD-Win% > 0
- SD Value = (1-Fold%) * EV-if-Called
- **Bluffing** is a bet that is +EV because Fold Equity > 0
- **Semi-bluffing** is a bet that is +EV with negative Fold Equity offset by sufficiently high Showdown-Win%





Fold Equity

River: (\$350.00) A♦ 5♥ 8♦ Q♠ 2♦ (2 players)

Turkito694 checks

Hero bets \$150...

How often does this bluff have to work to be profitable?

Bet is 150 into pot of 350. Showdown-Win\% = 0.

$$EV = 350 * Fold\% - 150 * (1 - Fold\%)$$

$$EV > 0$$
 when Fold% $> 150/(350+150) = 30\%$

Check with EV =
$$30\% * 350 - 70\% * 150 = 0$$

This seems +EV, given that Hero is representing a flush





Fold Equity

Turkito694 (UTG): \$2098.00

Blinds \$5/\$10

Hero (CO): \$990.00

Pre Flop: (\$15.00) Hero is CO with 6♠ 7♠

River: (\$350.00) A♦ 5♥ 8♦ Q♠ 2♦ (2 players)

Turkito694 checks

Hero bets \$150...

How often does this bluff have to work to be profitable?



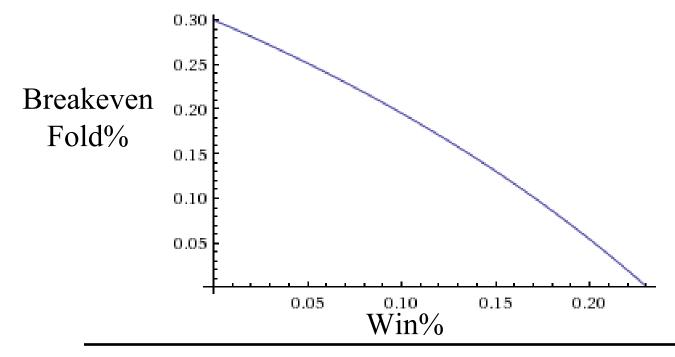


- Using our example:
 - BetAmt = 150
 - Pot = 350
 - $Fold\% = \frac{-2*BetAmt*Win\%+BetAmt-Pot*Win\%}{-2*BetAmt*Win\%+BetAmt-Pot*Win\%+Pot}$
 - Fold% = (13W% 3) / (13W% 10)





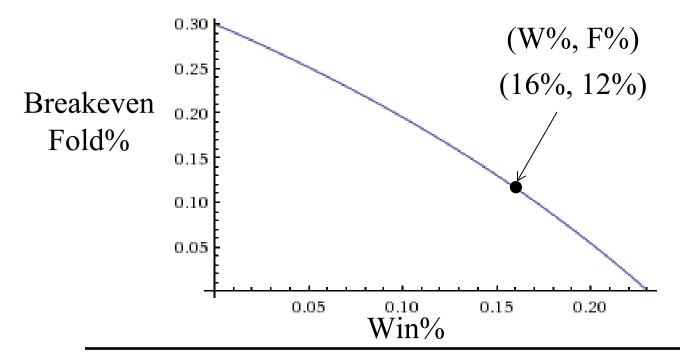
BetAmt =
$$150$$
 Pot = 350
Fold% = $(13Win\% - 3) / (13Win\% - 10)$







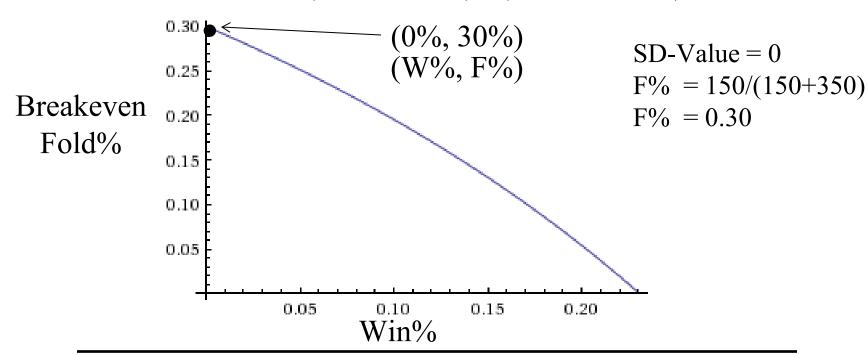
BetAmt =
$$150$$
 Pot = 350
Fold% = $(13Win\% - 3) / (13Win\% - 10)$







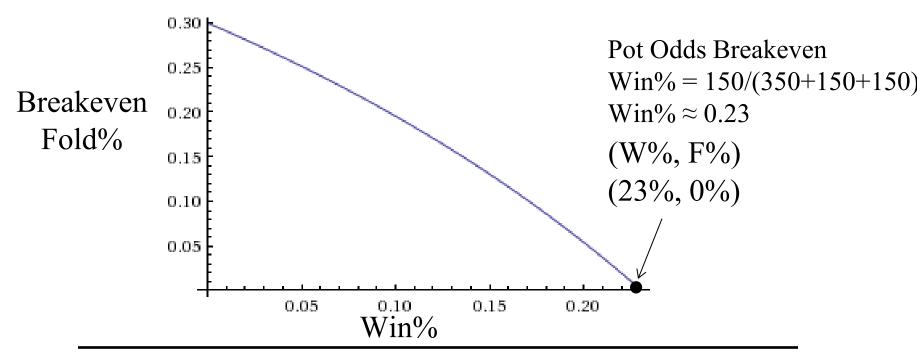
BetAmt =
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 Pot = 350
Fold% = $(13Win\% - 3) / (13Win\% - 10)$







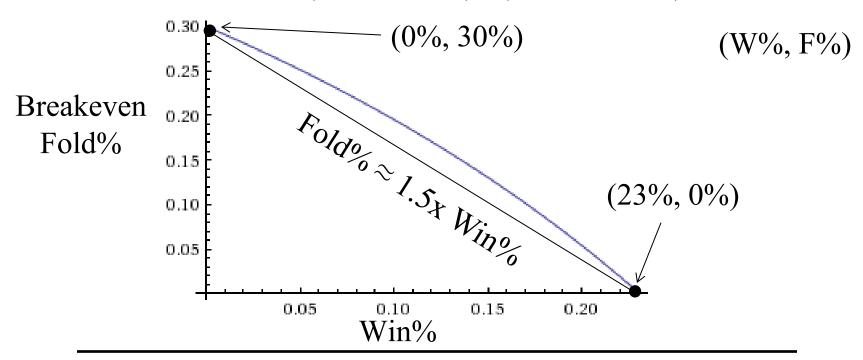
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 Pot = 350
Fold% = $(13Win\% - 3) / (13Win\% - 10)$







Sensitivity to Bet Size – Impact of Win%

• A pot sized bet would mean a 1% increase in Win% leads to a 1.5% decrease in breakeven Fold%

$$\lim_{Bet\to\infty} \frac{\partial F\%}{\partial W\%} = 2$$

• A higher bet increases the sensitivity, but it is bound by the interval (1,2)

$$\lim_{Bet \to Pot} \frac{\partial F\%}{\partial W\%} = 1.5$$

$$\lim_{Bet\to 0} \frac{\partial F\%}{\partial W\%} = 1$$





Fold Equity – Real Time

- When SDValue = 0
 - F% needed = bet / (pot + bet)
 - Pot sized bet needs to win 50% of time
 - Scales approximately linearly down to zero
 - i.e. a half pot size bet needs to win about 25% of the time
 - actual fold rate needed is .5 / 1.5 = 33%
- When SDValue > 0
 - This is difficult to develop quick rules
 - In general, your value is much higher if you have a real draw
 - A good assumption is your SD-Win% decreases the Fold% 1.5x to 1
 - Preflop is basically always semi-bluffing





Massachusetts Institute of Technology



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Fold Equity Examples

Villain (MP): \$3000 Blinds 25/50

Hero (BTN): \$3000

Pre Flop: (\$75) Hero is BTN with T♦ 5♠

Villain raises to \$150, 2 folds, Hero calls \$150, 2 folds

River: (\$375) 4♣ 8♥ 9♦ 6♥ 6♦ (2 players)

Villain checks, Hero bets \$250...





Fold Equity Examples

- 1. Bluff or semi-bluff?
 - Bluff
- 2. What is our Showdown Win%?
 - 0
- 3. What is our breakeven Fold%?
 - \$250 / \$625 = 40%
- 4. Is this a good bet if Villain calls 25% of the time?
 - Yes, 75% > 40%
- 5. What is our Fold Equity if Villain calls 25%?
 - \$375 * 75% \$250 * 25% = 218.75











Fold Equity Examples

Villain (MP): \$800 Blinds 25/50

Hero (BTN): \$1500

Pre Flop: (\$75) Hero is BTN with 9 TV

Villain raises to \$150, 2 folds, Hero calls \$150, 2 folds

Turn: (\$775) 4♣ 8♠ 7♦ 2♥ (2 players)

Villain checks, Hero bets \$450 ...





Fold Equity Examples

- 1. Bluff or semi-bluff?
 - Semi-bluff, SD Win% = 16%
- 2. What is our Showdown Value if Villain calls 80%?
 - -80% * [16% * \$1225 84% * \$450] = -\$145.6
- 3. What is our breakeven Fold%?
 - \$450 / \$1225 = 37% 16% *1.5 = 13%
- 4. Is this a good bet if Villain calls 80% of the time?
 - Yes, 20% > 13%
- 5. What is our Fold Equity if Villain calls 80%?
 - \$775*.20 + .80*(16%*\$1225 84%*\$450) = 9.4







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YouTube video:

ManiaOFpoker. "World Series Of Poker 2014 Main Event Episode 14 HD 720p." November 11, 2013. *YouTube*. Accessed May 1, 2015.

https://youtu.be/Q1HkLjq-GGQ?t=30m5s











Jacobson (MP2): 22,000k 150k/300k Blind + 50k

Hero (CO): 18,000k

Pre Flop: (950k) Hero is CO with A♣ Q♥ 4 folds, Jacobson raises to 650k, 1 fold, Hero raises to 1,425k, 3 folds, A♥ exposed, Jacobson calls 775k

Flop: (3,800k) K♦ J♥ 3♣ (2 players) Jacobson checks, Hero bets 1,800k

Is this a good bet?





Pre Flop: (950k) Hero is CO with A♣ Q♥

Flop: (3,800k) K♦ J♥ 3♣ (2 players) Jacobson checks, Hero bets 1,800k

Is this a good bet?

If SD-Win% = 0, the bet is +EV at F%>1800/(3800+1800) = 33%

If only the inside straight draw is good, Win% = 8%, making the breakeven Fold% closer to 21%





Pre Flop: (950k) Hero is CO with A♣ Q♥

Flop: (3,800k) K♦ J♥ 3♣ (2 players) Jacobson checks, Hero bets 1,800k

Is this a good bet?

If we assume any T wins (4 card) and any A wins sometimes (2 cards * .5) then chance to make draw is about 5 * 2 = 10%

Full solution is

$$EV = 3800 * F\% - [90\% * 1800 * (1-F\%)] + [10\% * (1-F\%) * 5600] = 0 \text{ at } F\% = 21.8\%$$

This is profitable if the Villain folds more than 22% of the time.





Live Example (Result)



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Bluffing Formulas

- Fold% (Minimum Fold Rate if SDWin% = 0)
 - -x = BetAmt / (Pot + BetAmt)
- Fold Equity (EV of Bluff, assuming SDWin% = 0)
 - -x = Fold% * Pot (1-Fold%) * BetAmt
- Showdown-Value (EV Contribution of Being Called)
 - -x = (1-Fold%) * (Win%*WinAmt Lose%*LoseAmt)
- Fold Equity (EV of Semi-Bluff, if SDWin% > 0)
 - -x = Fold% * Pot + (1-Fold%) * (Win%*WinAmt Lose%*LoseAmt)
- Semi-Bluff Fold% (Quick Rule for Breakeven Semi-Bluff Fold%)
 - -x = BetAmt / (Pot + BetAmt) 1.5x Win%





Bluffing Formulas (Example)

- Fold% (Minimum Fold Rate if SDWin% = 0)
 - Making a \$150 bluff into a \$350 pot
 - **30%** = \$150 / (\$350 + \$150)
- Fold Equity (EV of Bluff, assuming SDWin% = 0)
 - Making a \$250 bluff into a pot of \$625 against a 25% call rate
 - **\$218.75** = 75% * \$625 (1-75%) * \$250
- Showdown-Value (EV Contribution of Being Called)
 - Making a \$450 bluff into a pot of \$775 with a 16% WinRate against an 80% call rate
 - **-\$145.6** = (1-20%) * (16%*\$1225 84%*\$450)
- Fold Equity (EV of Semi-Bluff, if SDWin% > 0)
 - \$9.4 = 20% * \$1225 + (1-20%) * (16% * 1225 84% * 450)
- Semi-Bluff Fold% (Quick Rule for Breakeven Semi-Bluff Fold%)
 - 13% = \$450 / (\$775 + \$450) 1.5*16%





Bluffing – Be careful about

- Betting too little on a bluff
 - If you had a real hand, you wouldn't bet 1/3rd of the pot
 - Or at least you shouldn't, but we'll get to that
 - Bet enough to make a draw -EV
- Betting too much on a bluff
 - Pot overbets are basically never a good idea (unless you are pot committed on a normal sized bet)
 - If you are short-stacked, don't bluff an amount that would require you to call a raise (i.e. you would have the odds to call a raise)
- Being afraid of being caught bluffing or showing down bad cards
 - This is really common, especially live
- Semi-bluffing when a free card is offered
- Bluffing calling stations





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