SRout Assign 14

11/25/2020

This week, we'll work out some Taylor Series expansions of popular functions.

- **1. f(x) = $\frac{1}{1-x}$**

Solution

$$f(x) = \frac{1}{1-x}$$

Formula for a Taylor Series Expansion:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \frac{f''''(a)}{4!}(x-a)^4 + ...$$

Derivatives of first order, second order, third order and fourth order of f(x) shown below:

$$f(x) = \frac{1}{1-x}, f(0) = \frac{1}{1-0} = 1$$

$$f'(x) = \frac{1}{(1-x)^2}, f'(0) = \frac{1}{(1-0)^2} = 1$$

$$f''(x) = \frac{2}{(1-x)^3}, f''(0) = \frac{2}{(1-0)^3} = 2$$

$$f'''(x) = \frac{6}{(1-x)^4}, f'''(0) = \frac{6}{(1-0)^4} = 6$$

$$f''''(x) = \frac{24}{(1-x)^5}, f''''(0) = \frac{24}{(1-0)^5} = 24$$

if we do till,

$$f^n(x) = \frac{n!}{(1-x)^{(n+1)}}$$

Substitute expressions into Taylor Series expansion:

$$f(x) = 1 + \frac{1}{1!}x^1 + \frac{2}{2!}x^2 + \frac{6}{3!}x^3 + \frac{24}{4!}x^4 + ... + \frac{n!}{n!}x^n$$

$$f(x) = \sum_{n=0}^{\infty} x^n$$

- **2. f(x) = e^x**

Solution

$$f(x) = e^x$$
$$f'(x) = e^x, f'(0) = e^0 = 1$$
$$f''(x) = e^x, f''(0) = e^0 = 1$$
$$f'''(x) = e^x, f'''(0) = e^0 = 1$$
$$.....$$

$$f^{(n)}(x) = e^x, f^n(0) = e^0 = 1$$

Substitute expressions into Taylor Series expansion:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$f(x) = e^0 + e^0(x-0) + e^0(x-0)^2 + e^0(x-0)^3 + ... + e^0(x-0)^n$$

$$f(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + ... + \frac{x^n}{n!}$$

$$f(x) = \sum_{n=0}^{\infty} \frac{1}{n!} x^n$$

$$f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

- **3. f(x) = ln(1 + x)**

Solution

$$f(x) = ln(1+x), f(0) = 0$$
$$f'(x) = \frac{1}{1+x}, f'(0) = 1$$

$$f''(x) = \frac{-1}{(1+x)^2}, f''(0) = -1$$
$$f'''(x) = \frac{2}{(1+x)^3}, f'''(0) = 2$$

$$f''''(x) = \frac{-6}{(1+x)^4}, f'''(0) = -6$$
$$......$$

$$f^n(x) = \frac{(-1)^{(n-1)}(n-1)!}{(x+1)^n}, f^n(0) = (-1)^{(n-1)}(n-1)!$$

Substitute expressions into Taylor Series expansion:

$$f(x) = f(0) + f'(0)(x-0) + \frac{f''(0)}{2!}(x-0)^2 + \frac{f'''(0)}{3!}(x-0)^3 + \frac{f''''(0)}{4!}(x-0)^4 + ...$$

$$f(x) = 0 + x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + ...$$

$$f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{(n-1)}(n-1)!}{n!}(x)^n$$