SRout Assignment 13

1. Use integration by substitution to solve the integral below.

$$\int 4e^{-7x}dx$$

Solution

Note: $\int e^u du = e^u$

$$\int 4e^{-7x}dx$$

Lets v = -7x dv = dx substitute this below:

$$\int 4e^{-7x}dx = 4\int e^v dv = 4e^v + C = -\frac{4}{7}e^{-7x} + C$$

2. Biologists are treating a pond contaminated with bacteria. The level of contamination is changing at a rate of $\frac{dN}{dt} = -\frac{3150}{t^4} - 220$ bacteria per cubic centimeter per day, where t is the number of days since treatment began. Find a function N(t) to estimate the level of contamination if the level after 1 day was 6530 bacteria per cubic centimeter.

Solution

Given,

t = 1

N(1) = 6530

$$\frac{dN}{dt} = -\frac{3150}{t^4} - 220 = N(t) = -\int \frac{3150}{t^4}dt - 200\int dt + C$$

$$N(t) = -3150 * t^{-4} - 200\int dt + C = -\frac{3150}{-3}t^{-3} - 200(t) + C$$

Substitue value of t and N(t) :

$$N(1) = \frac{1050}{(1)^3} - 220(1) + C$$

$$6530 = 1050 - 220 + C$$

$$C = 6530 - 1050 + 220$$

$$C = 5700$$

Substitute C value:

$$N(t) = 1050t^{-3} - 200t + 5700$$

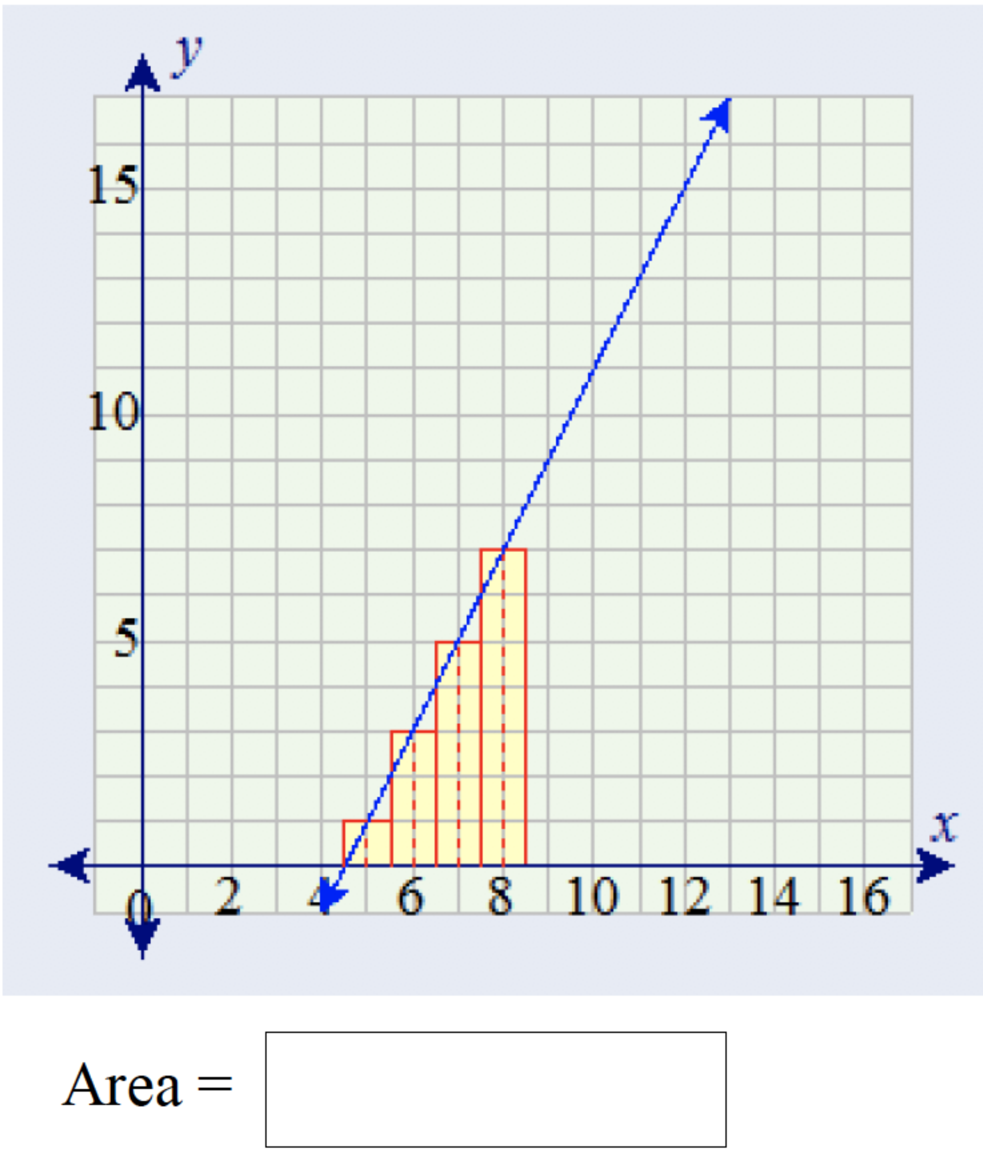


Figure 1: Figure 1

3. Find the total area of the red rectangles in the figure below, where the equation of the line is f (x) = 2x - 9.

Solution

Above plot shows limit (lower = 4.5, upper = 8.5)

```
area_fun <- function(x)
  {
  return(2 * x - 9)
}

value <- integrate(area_fun, lower = 4.5, upper = 8.5)$value

print(value)
```

```
## [1] 16
```

4. Find the area of the region bounded by the graphs of the given equations.

$$y = x^2 - 2x - 2, y = x + 2$$

Enter your answer below.

Solution

value of x

$$x^2 - 2x - 2 = x + 2$$

$$x^2 - 3x - 4 = 0$$

$$x^2 - 4x + x - 4 = 0$$

$$x(x - 4) + 1(x - 4) = 0$$

$$(x - 4) * (x + 1) = 0$$

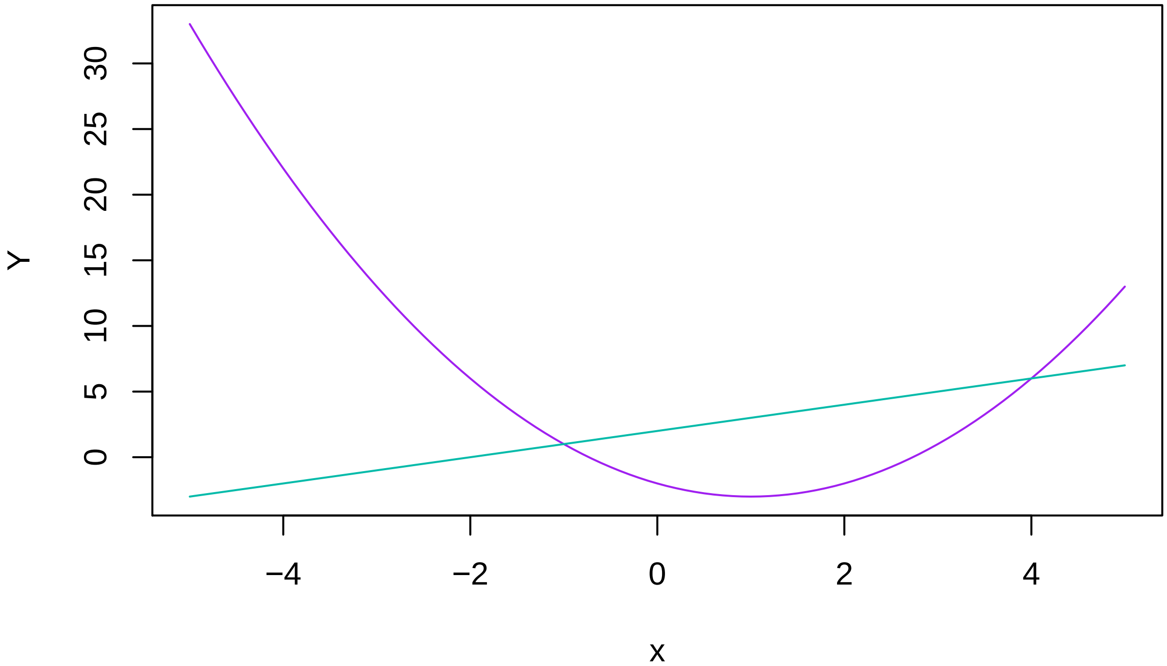
x = (-1, 4)

Create 2 functions for 2 equations.

```
graph_1 <- function(x){
  return(x^2 - 2*x -2)
}

graph_2 <- function(x){
  return(x +2)
}
```

```r
plot(graph_1, -5, 5, col = 'purple', ylab = "Y")
plot(graph_2, -5, 5, col = '#05bbaa', add = TRUE)
```



Plot graphs.

```r
func_intersect <- function(x)
  {
    return((x+2) - (x^2 - 2*x -2))
}

area <- integrate(func_intersect, lower = -1, upper = 4)$value
print(area)
```

Area of intersection

```
## [1] 20.83333
```

5. A beauty supply store expects to sell 110 flat irons during the next year. It costs $3.75 to store one flat iron for one year. There is a fixed cost of $8.25 for each order. Find the lot size and the number of orders per year that will minimize inventory costs.

Solution

Suppose half of inventory keeps stock, let x = size of iron

storage cost = 3.75 * x/2

order cost = 8.25 * 110/x

Total Cost = 1.875x + 907.5/x

First order derivate 1.875 - 907.5/x^2 = 0

$$1.875 * x^2 - 907.5 = 0$$
$$1.875 * x^2 = 907.5$$
$$x^2 = \frac{907.5}{1.875}$$
$$x = \sqrt{\frac{907.5}{1.875}}$$

```
x <- sqrt(907.5/1.875)
print(x)
```

```
## [1] 22
```

```
times <- 110/x
print(times)
```

```
## [1] 5
```

Each year iron can order 5 times size of 22.

6. Use integration by parts to solve the integral below.

$$\int ln(9x)\mathring{u}x^6dx$$

Solution

Integration by parts formula:

$$\int u\ v\ dx = u\int v\ dx - \int u'\ (\int v\ dx)\ dx$$

u = ln(9x)

du = $\frac{1}{x}dx$

v = xˆ6

dv = $\frac{1}{7}x^7$

$$\int (uv)dx = ln(9x) * \frac{1}{7}x^7 - \int \frac{1}{x}\frac{1}{7}x^7dx$$
$$\int (uv)dx = ln(9x) * \frac{1}{7}x^7 - \frac{1}{7}\int x^6dx$$
$$\int (uv)dx = ln(9x) * \frac{1}{7}x^7 - \frac{1}{49}x^7 + C$$

7. Determine whether f (x) is a probability density function on the interval [1, e^6] . If not, determine the value of the definite integral.

$$f(x) = \frac{1}{6x}$$

Solution

$$f(x) = \frac{1}{6x}$$

$$\int_1^{e^6} \frac{1}{6x} dx = \frac{1}{6} * lnx|_1^{e^6} = \frac{1}{6} ln(e^6) - \frac{1}{6} ln(1) = \frac{1}{6}[6 - 0] = 1$$