SRout Assign 8

(11) **A company buys 100 lightbulbs, each of which has an exponential lifetime of 1000 hours. What is the expected time for the first of these bulbs to burn out? (See Exercise 10.)**

Answer

$$E(X) = \mu/n = 1000/100 = 10hours$$

$Z(14)$ **Assume that X1 and X2 are independent random variables, each having an exponential density with parameter λ. Show that Z = X1 - X2 has density**

$$fz(z) = (1/2)\lambda e^{-\lambda|z|}$$

Answer The convolution formula of W = X + Y, we know the probability density would be:

$$fw(W) = \int_{-\infty}^{\infty} fx(x)fy(w-x)dx$$

X1 and X2 are independent random variables. Z = X1 - X2 or X1 + (-X2) X2 = Z - X1 (To keep it simple, X1 = X, X2 = -Y)

$$fz(z) = \int_{-\infty}^{\infty} fx(x)f(-y)(z-x)dx$$

$$fz(z) = \int_{-\infty}^{\infty} fx(x)f(y)(x-z)dx$$

For Z < 0

$$fz(z) = \int_{0}^{\infty} \lambda e^{-\lambda x}\lambda e^{-\lambda(x-z)}dx$$

$$fz(z) = \lambda e^{-\lambda z}\int_{0}^{\infty} \lambda e^{-2\lambda x}dx = (\lambda/2)e^{\lambda z}$$

Similarly, for z >= 0, we will have to take the negative the exponent

$$fz(z) = (\lambda/2)e^{-\lambda z}$$

Hence, combing both limit Z<0 and Z >= 0 we can write:

$$fz = (\frac{1}{2})\lambda e^{-\lambda|z|}$$

Reference: https://www.youtube.com/watch?v=f8Nli1AfygM

(1) **Let X be a continuous random variable with mean μ = 10 and variance σ^2 = 100/3. Using Chebyshev's Inequality, Find an upper bound for the following probabilities. Answer**

$$P(|X-\mu| >= k\sigma) =< \frac{\sigma^2}{k^2\sigma^2} = \frac{1}{k^2}$$

(a) P(|X - 10| >= 2)

```r
mu <- 10
variance <- 100/3
sigma <- sqrt(variance)
# k * sigma = 2
k <- 2 / sigma
1 / k ^ 2
```

```
## [1] 8.333333
```

Probability cannot be greater than 1 so, upper bound is 1. **(b) P(|X - 10| >= 5)**

```r
k <- 5 / sigma
1 / k ^ 2
```

```
## [1] 1.333333
```

Probability cannot be greater than 1 so, upper bound is 1. **(c) P(|X - 10| >= 9)**

```r
k <- 9 / sigma
1 / k ^ 2
```

```
## [1] 0.4115226
```

Upper bound is : 0.4115226 **(d) P(|X - 10| >= 20)**

```r
k <- 20 / sigma
1 / k ^ 2
```

```
## [1] 0.08333333
```

Upper bound is: 0.08333333