Multiple linear regression

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Grading the professor

Many college courses conclude by giving students the opportunity to evaluate the course and the instructor anonymously. However, the use of these student evaluations as an indicator of course quality and teaching effectiveness is often criticized because these measures may reflect the influence of non-teaching related characteristics, such as the physical appearance of the instructor. The article titled, "Beauty in the classroom: instructors' pulchritude and putative pedagogical productivity" (Hamermesh and Parker, 2005) found that instructors who are viewed to be better looking receive higher instructional ratings. (Daniel S. Hamermesh, Amy Parker, Beauty in the classroom: instructors pulchritude and putative pedagogical productivity, *Economics of Education Review*, Volume 24, Issue 4, August 2005, Pages 369-376, ISSN 0272-7757, 10.1016/j.econedurev.2004.07.013. http://www.sciencedirect.com/science/article/pii/S0272775704001165.)

In this lab we will analyze the data from this study in order to learn what goes into a positive professor evaluation.

The data

The data were gathered from end of semester student evaluations for a large sample of professors from the University of Texas at Austin. In addition, six students rated the professors' physical appearance. (This is aslightly modified version of the original data set that was released as part of the replication data for *Data Analysis Using Regression and Multilevel/Hierarchical Models* (Gelman and Hill, 2007).) The result is a data frame where each row contains a different course and columns represent variables about the courses and professors.

```r
load("more/evals.RData")
```

variable	description
`score`	average professor evaluation score: (1) very unsatisfactory - (5) excellent.
`rank`	rank of professor: teaching, tenure track, tenured.
`ethnicity`	ethnicity of professor: not minority, minority.
`gender`	gender of professor: female, male.
`language`	language of school where professor received education: english or non-english.
`age`	age of professor.
`cls_perc_eval`	percent of students in class who completed evaluation.
`cls_did_eval`	number of students in class who completed evaluation.

variable	description
cls_students	total number of students in class.
cls_level	class level: lower, upper.
cls_profs	number of professors teaching sections in course in sample: single, multiple.
cls_credits	number of credits of class: one credit (lab, PE, etc.), multi credit.
bty_f1lower	beauty rating of professor from lower level female: (1) lowest - (10) highest.
bty_f1upper	beauty rating of professor from upper level female: (1) lowest - (10) highest.
bty_f2upper	beauty rating of professor from second upper level female: (1) lowest - (10) highest.
bty_m1lower	beauty rating of professor from lower level male: (1) lowest - (10) highest.
bty_m1upper	beauty rating of professor from upper level male: (1) lowest - (10) highest.
bty_m2upper	beauty rating of professor from second upper level male: (1) lowest - (10) highest.
bty_avg	average beauty rating of professor.
pic_outfit	outfit of professor in picture: not formal, formal.
pic_color	color of professor's picture: color, black & white.

Exploring the data

1. Is this an observational study or an experiment? The original research question posed in the paper is whether beauty leads directly to the differences in course evaluations. Given the study design, is it possible to answer this question as it is phrased? If not, rephrase the question.

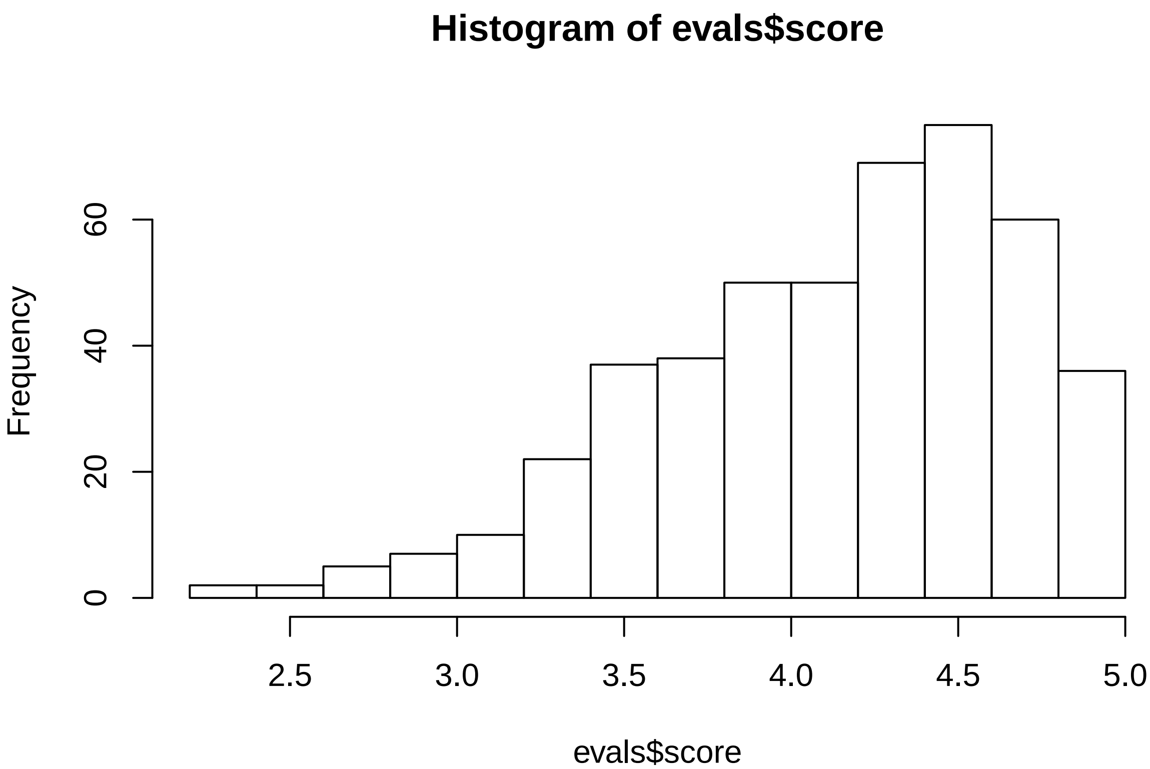
Answer

This is an observational study. I would rephrase the question like "Is there association between beauty and course evaluation?"

2. Describe the distribution of `score`. Is the distribution skewed? What does that tell you about how students rate courses? Is this what you expected to see? Why, or why not?

Answer

```r
hist(evals$score)
```




```r
summary(evals$score)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   2.300   3.800   4.300   4.175   4.600   5.000
```

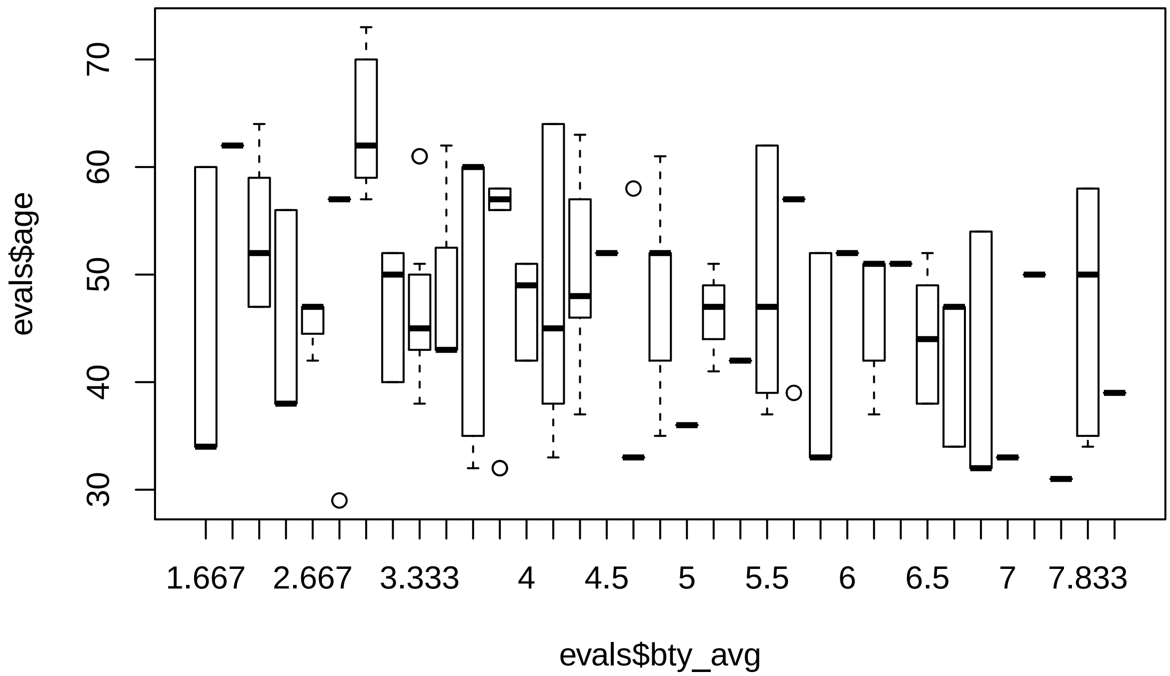
Above histogram shows distribution of `score`, it is seems to little left skewed. This shows most student like their professors. I would have assumed less skew and more normal distribution but looks like more students like their professors.

3. Excluding `score`, select two other variables and describe their relationship using an appropriate visualization (scatterplot, side-by-side boxplots, or mosaic plot).

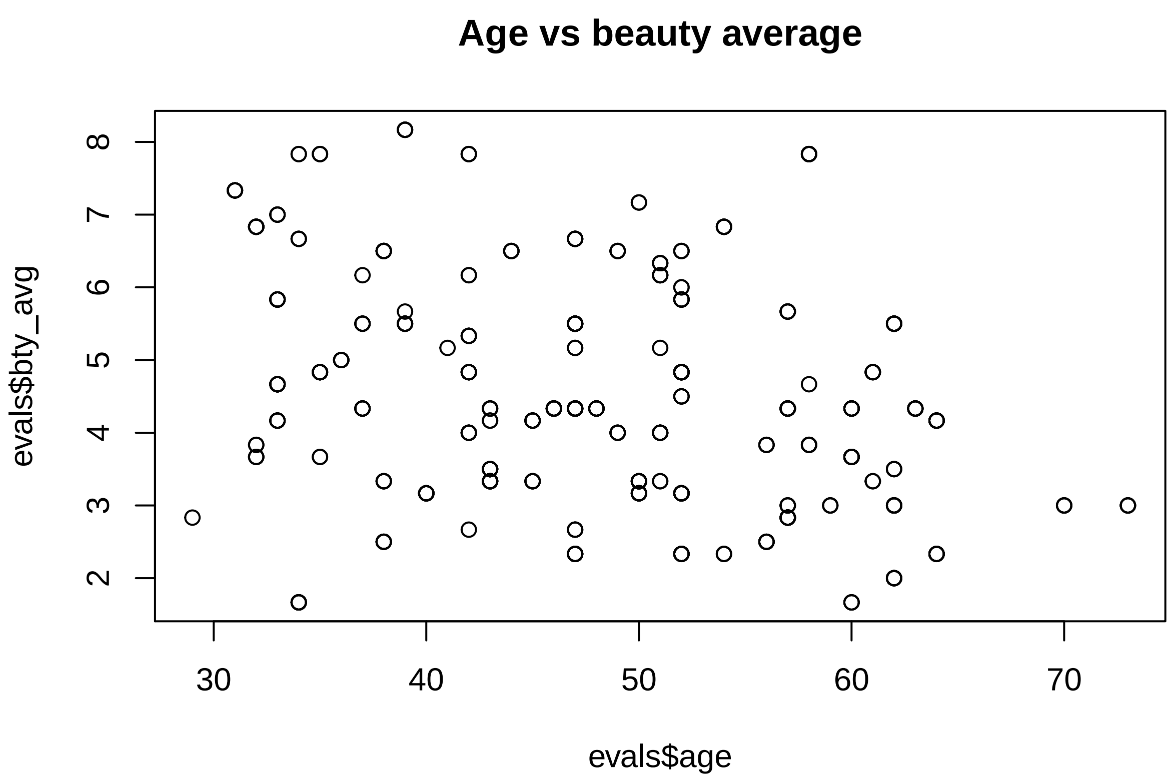
Answer

Select `age` and `bty_avg` to see the relationship between them.

```r
boxplot(evals$age ~ evals$bty_avg)
```




```r
plot( evals$age, evals$bty_avg, main = "Age vs beauty average")
```



Based on scatter plot visualization, seems to be a negative relationship in between the two variables as the instructor gets older the beauty avg reduce.

Simple linear regression

The fundamental phenomenon suggested by the study is that better looking teachers are evaluated more favorably. Let's create a scatterplot to see if this appears to be the case:

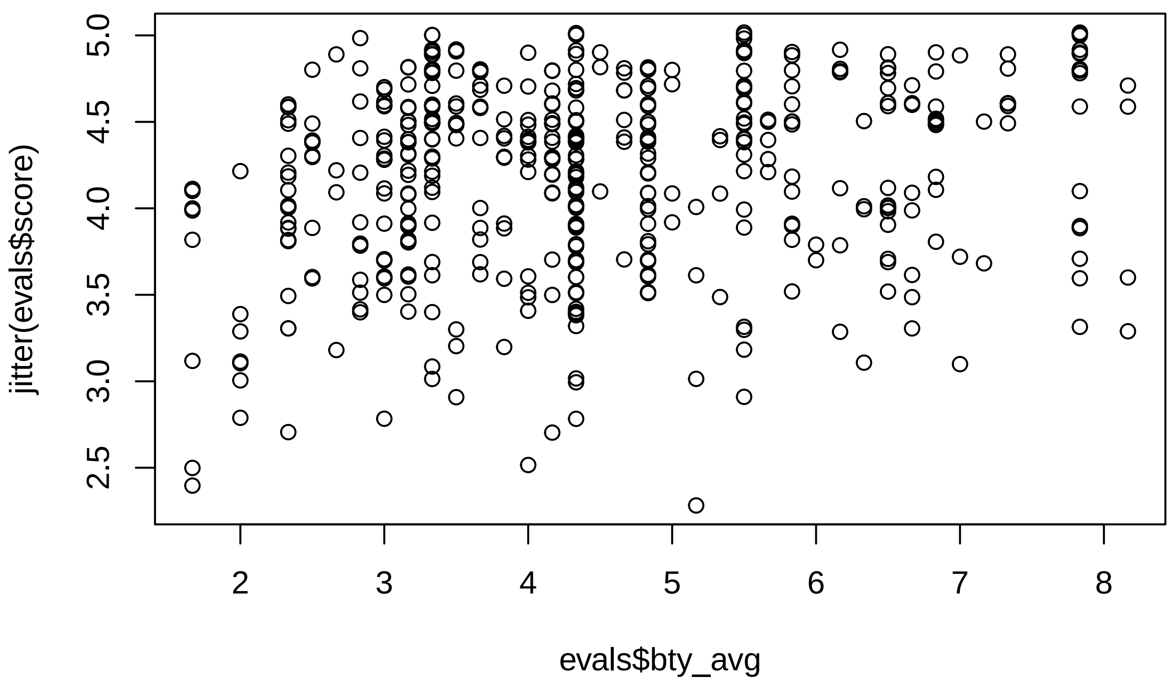
```
plot(evals$score ~ evals$bty_avg)
```

Before we draw conclusions about the trend, compare the number of observations in the data frame with the approximate number of points on the scatterplot. Is anything awry?

4. Replot the scatterplot, but this time use the function `jitter()` on the y- or the x-coordinate. (Use `?jitter` to learn more.) What was misleading about the initial scatterplot?

Answer

```
plot(jitter(evals$score) ~ evals$bty_avg)
```

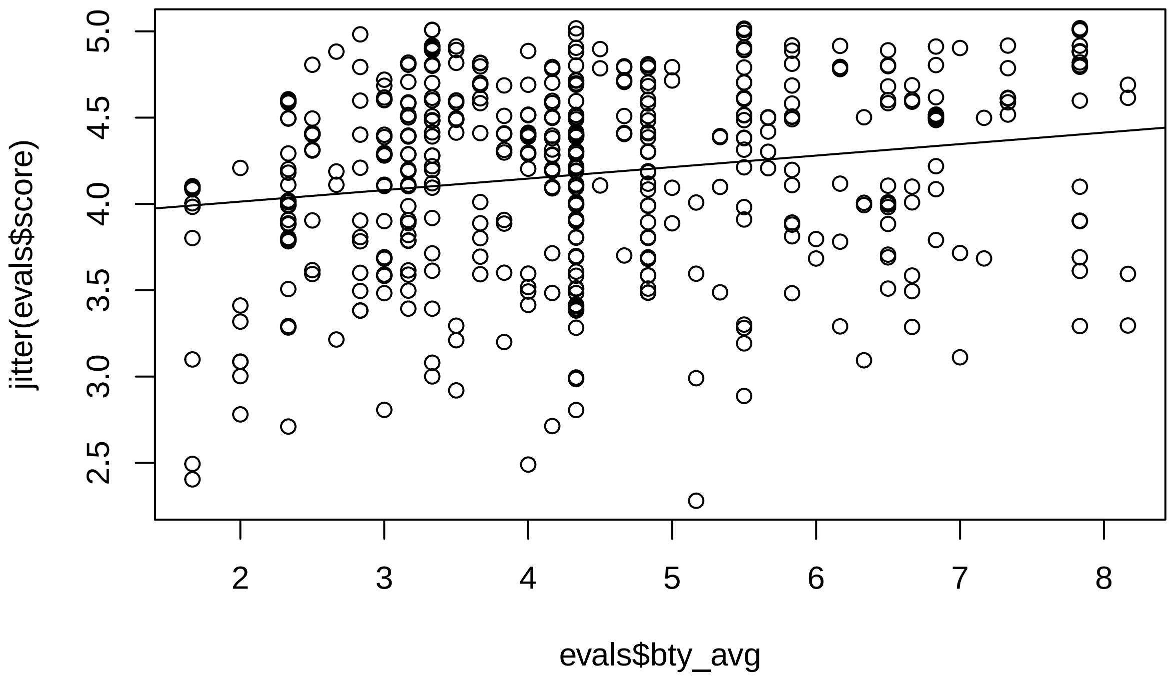


The first scatterplot displays significantly less points of observations. By usiing jitter() it is shown that this was due to multiple observations having the same bty_avg and score values, so points were plotted on top of one another.

5. Let's see if the apparent trend in the plot is something more than natural variation. Fit a linear model called `m_bty` to predict average professor score by average beauty rating and add the line to your plot using `abline(m_bty)`. Write out the equation for the linear model and interpret the slope. Is average beauty score a statistically significant predictor? Does it appear to be a practically significant predictor?

Answer

```r
m_bty <- lm(evals$score ~ evals$bty_avg)
plot(jitter(evals$score) ~ evals$bty_avg)
abline(m_bty)
```




```r
summary(m_bty)
```

```
##
## Call:
## lm(formula = evals$score ~ evals$bty_avg)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.9246 -0.3690  0.1420  0.3977  0.9309
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)    3.88034    0.07614   50.96  < 2e-16 ***
## evals$bty_avg  0.06664    0.01629    4.09 5.08e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5348 on 461 degrees of freedom
## Multiple R-squared:  0.03502,    Adjusted R-squared:  0.03293
## F-statistic: 16.73 on 1 and 461 DF,  p-value: 5.083e-05
```

Linear model equation :

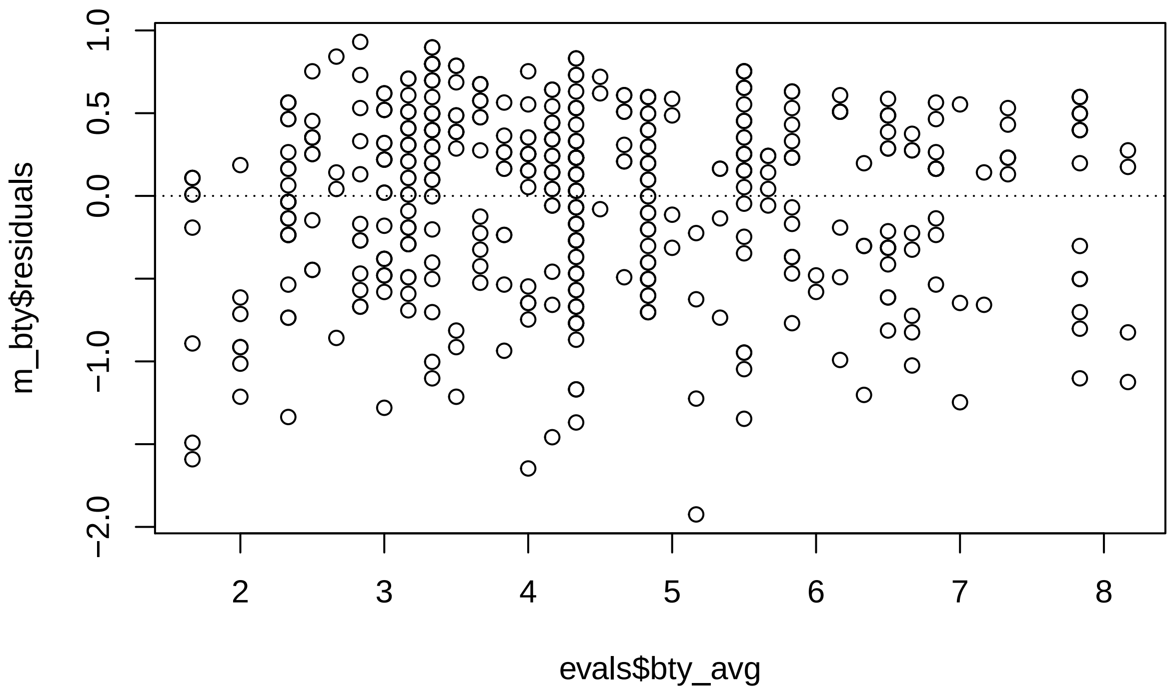
$$\widehat{score} = 3.88 + 0.067 \times bty\_avg$$

Since the p-value is almost 0, we'd say bty_avg is statistically significant. Not the BEST predictors as R2 and slope is pretty low.

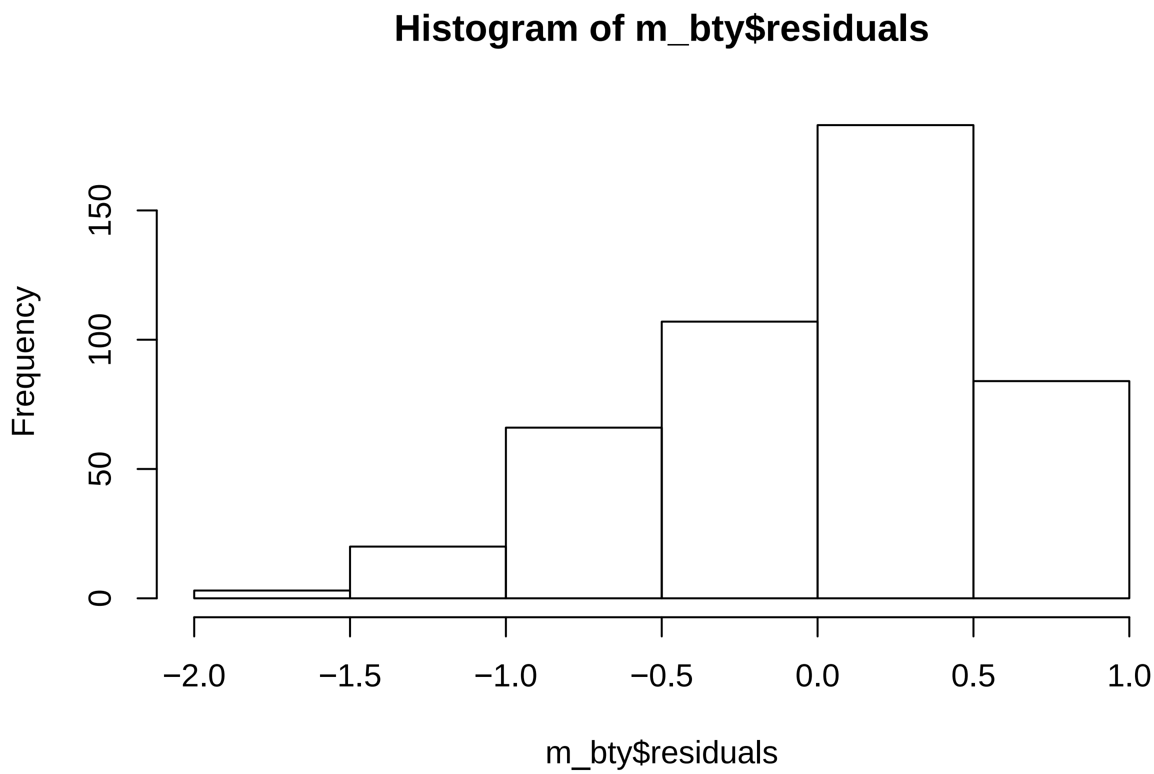
6. Use residual plots to evaluate whether the conditions of least squares regression are reasonable. Provide plots and comments for each one (see the Simple Regression Lab for a reminder of how to make these).

Answer

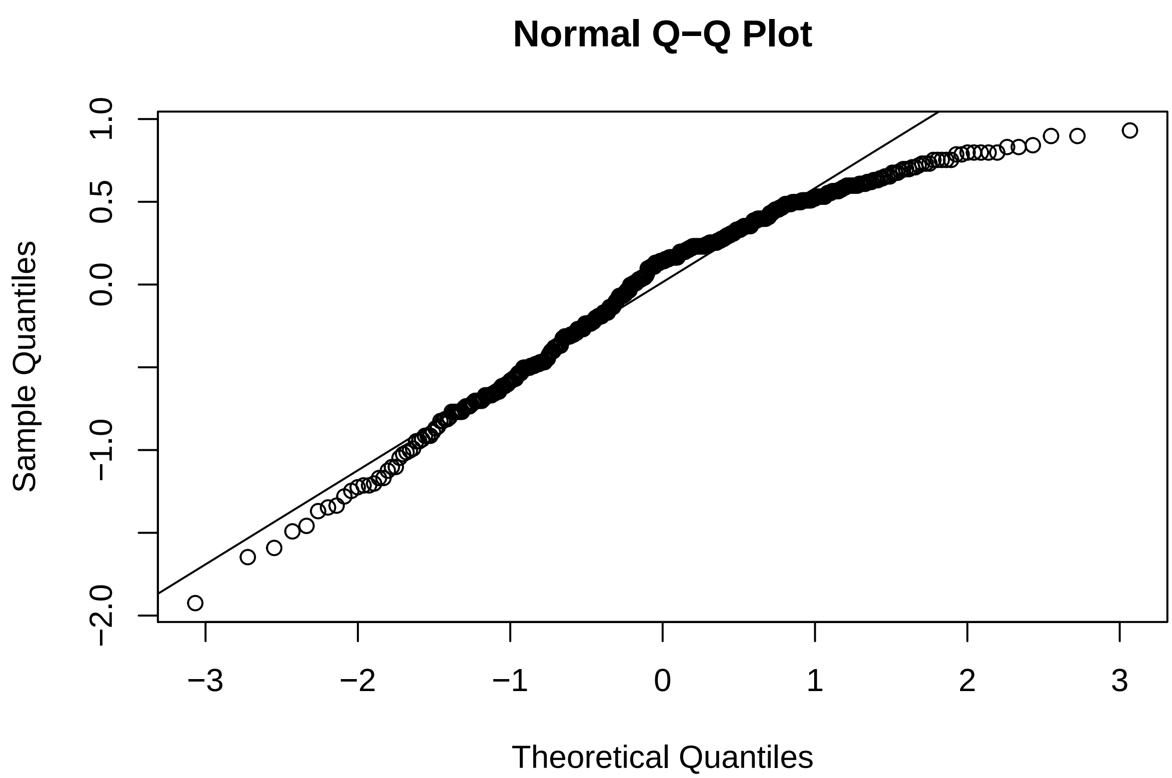
```
plot(m_bty$residuals ~ evals$bty_avg)
abline(h = 0, lty = 3)
```




```
hist(m_bty$residuals)
```




```r
qqnorm(m_bty$residuals)
qqline(m_bty$residuals)
```



The scatter plot of residuals shows that the relationship looks linear.

The histogram of residuals appears to be little left-skewed. So it is not follow some sort of normality

And by looking at the Q-Q Plot, we can observe how the distribution is not following around a straight line. There is some distraction at the end of the lines.

Hence we can conclude that this model does not satisfies the nearly normal residuals condition.

Multiple linear regression

The data set contains several variables on the beauty score of the professor: individual ratings from each of the six students who were asked to score the physical appearance of the professors and the average of these six scores. Let's take a look at the relationship between one of these scores and the average beauty score.

```
plot(evals$bty_avg ~ evals$bty_f1lower)
cor(evals$bty_avg, evals$bty_f1lower)
```

As expected the relationship is quite strong - after all, the average score is calculated using the individual scores. We can actually take a look at the relationships between all beauty variables (columns 13 through 19) using the following command:

```
plot(evals[,13:19])
```

These variables are collinear (correlated), and adding more than one of these variables to the model would not add much value to the model. In this application and with these highly-correlated predictors, it is reasonable to use the average beauty score as the single representative of these variables.

In order to see if beauty is still a significant predictor of professor score after we've accounted for the gender of the professor, we can add the gender term into the model.

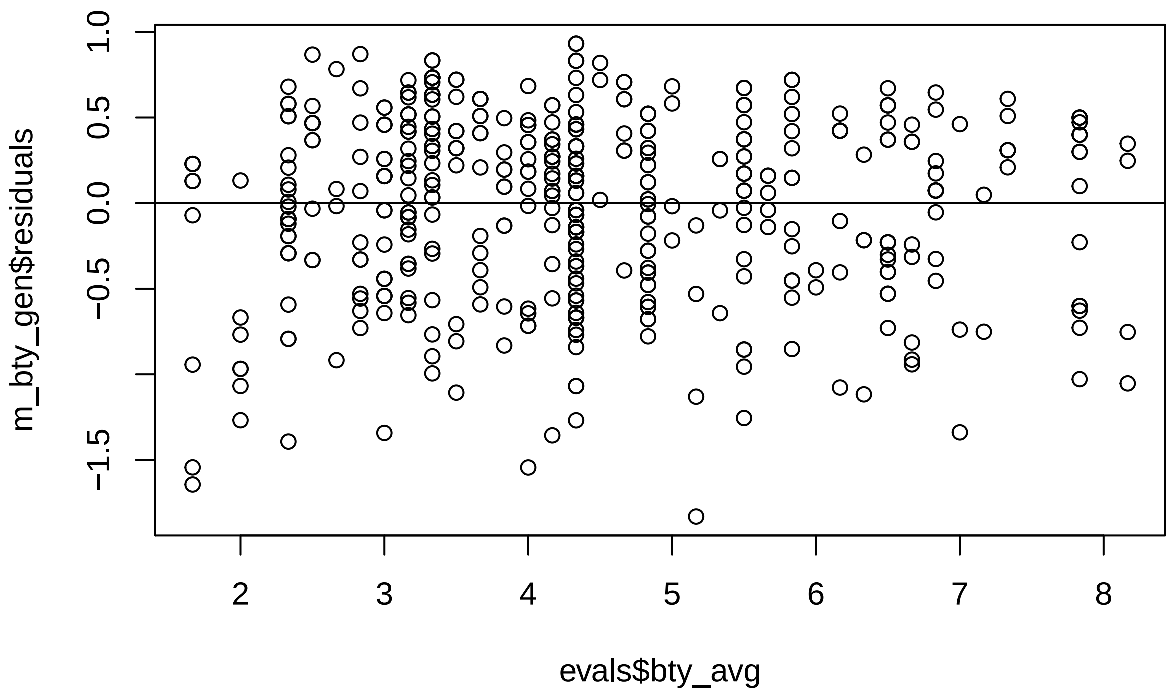
```
m_bty_gen <- lm(score ~ bty_avg + gender, data = evals)
summary(m_bty_gen)
```

```
##
## Call:
## lm(formula = score ~ bty_avg + gender, data = evals)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.8305 -0.3625  0.1055  0.4213  0.9314
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  3.74734    0.08466  44.266  < 2e-16 ***
## bty_avg      0.07416    0.01625   4.563 6.48e-06 ***
## gendermale   0.17239    0.05022   3.433 0.000652 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5287 on 460 degrees of freedom
## Multiple R-squared:  0.05912,    Adjusted R-squared:  0.05503
## F-statistic: 14.45 on 2 and 460 DF,  p-value: 8.177e-07
```

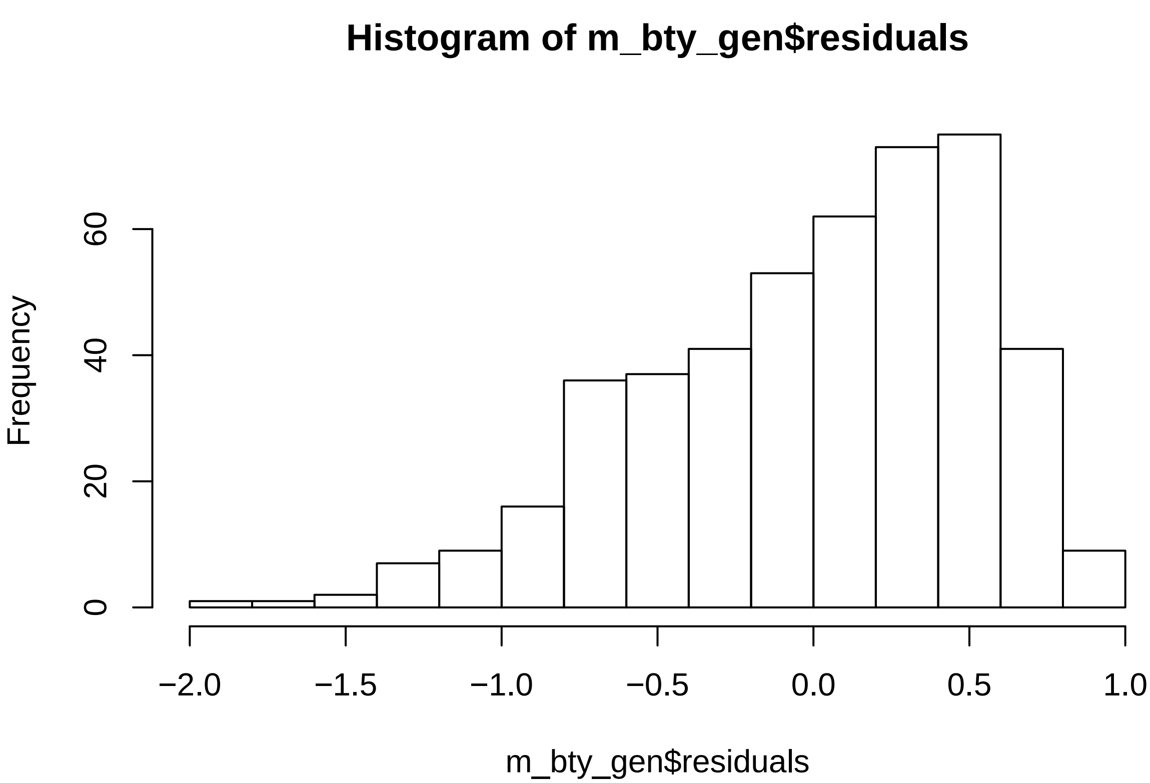
7. P-values and parameter estimates should only be trusted if the conditions for the regression are reasonable. Verify that the conditions for this model are reasonable using diagnostic plots.

Answer

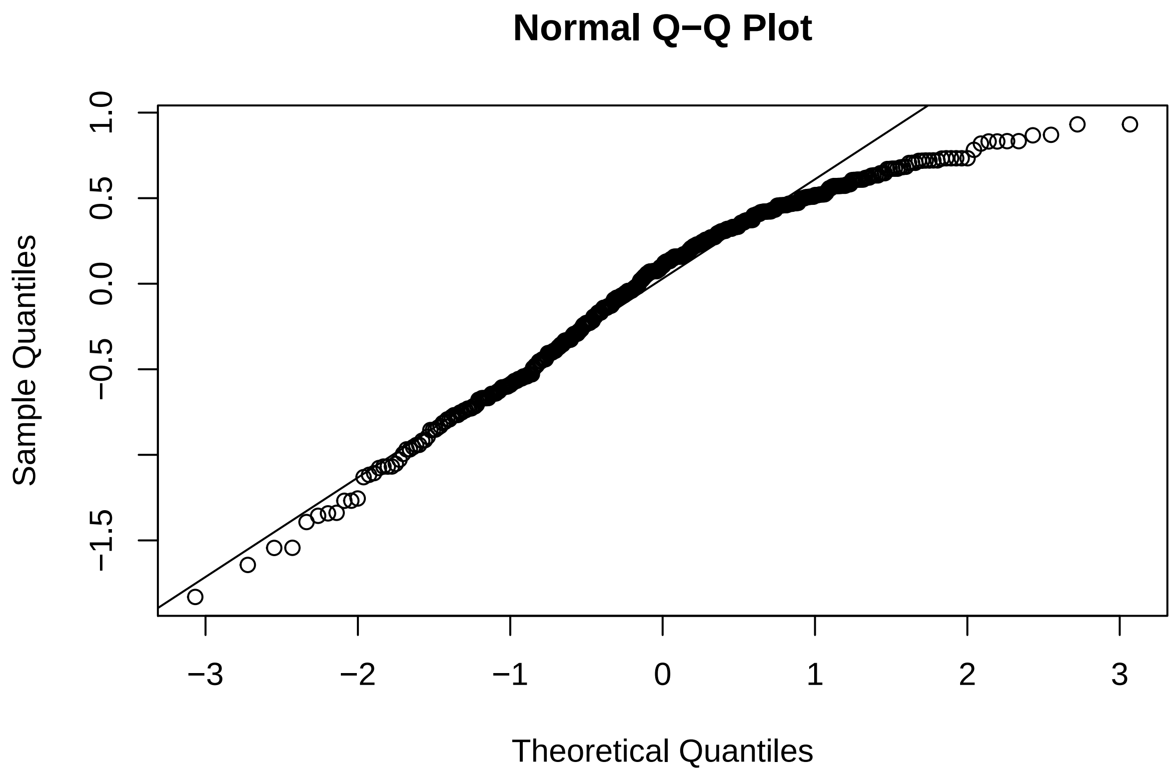
```
plot(m_bty_gen$residuals ~ evals$bty_avg)
abline(h = 0)
```




```
hist(m_bty_gen$residuals)
```



```r
qqnorm(m_bty_gen$residuals)
qqline(m_bty_gen$residuals)
```



Linearity: The residual plot is randomly dispersed, but satisfied, linearity is met

Normal residuals: Histogram shows left skewed rather than normal distribution.

Constant variability: It is fairly constant

Independence of observations: We can assume this to be met since is this observational study represents less than 10% of the population.

8. Is `bty_avg` still a significant predictor of `score`? Has the addition of `gender` to the model changed the parameter estimate for `bty_avg`?

Answer

```r
lm(score ~ bty_avg + gender, evals, y = TRUE)
```

```
##
## Call:
## lm(formula = score ~ bty_avg + gender, data = evals, y = TRUE)
##
## Coefficients:
## (Intercept)      bty_avg   gendermale
##     3.74734      0.07416      0.17239
```

Yes, bty_avg is still significant. The addition of gender has changed the estimate only slightly, from 0.067 to 0.074.

Note that the estimate for `gender` is now called `gendermale`. You'll see this name change whenever you introduce a categorical variable. The reason is that R recodes `gender` from having the values of `female` and `male` to being an indicator variable called `gendermale` that takes a value of 0 for females and a value of 1 for males. (Such variables are often referred to as "dummy" variables.)

As a result, for females, the parameter estimate is multiplied by zero, leaving the intercept and slope form familiar from simple regression.

$$\begin{aligned} \widehat{score} &= \hat{\beta}_0 + \hat{\beta}_1 \times bty\_avg + \hat{\beta}_2 \times (0) \\ &= \hat{\beta}_0 + \hat{\beta}_1 \times bty\_avg \end{aligned}$$

We can plot this line and the line corresponding to males with the following custom function.

```r
multiLines(m_bty_gen)
```

9. What is the equation of the line corresponding to males? (*Hint:* For males, the parameter estimate is multiplied by 1.) For two professors who received the same beauty rating, which gender tends to have the higher course evaluation score?

Answer

```r
summary(m_bty_gen)
```

```
##
## Call:
## lm(formula = score ~ bty_avg + gender, data = evals)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.8305 -0.3625  0.1055  0.4213  0.9314
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  3.74734    0.08466  44.266  < 2e-16 ***
## bty_avg      0.07416    0.01625   4.563 6.48e-06 ***
## gendermale   0.17239    0.05022   3.433 0.000652 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5287 on 460 degrees of freedom
## Multiple R-squared:  0.05912,    Adjusted R-squared:  0.05503
## F-statistic: 14.45 on 2 and 460 DF,  p-value: 8.177e-07
```

Since the gender male can be represented with 1

$$\begin{aligned} \widehat{score} &= \hat{\beta}_0 + \hat{\beta}_1 \times bty\_avg + \hat{\beta}_2 \times male \\ \widehat{score} &= 3.74734 + 0.07416 \times bty\_avg + 0.17239 \times 1 \\ \widehat{score} &= 3.91973 + 0.07416 \times bty\_avg \end{aligned}$$

For 2 professors of the same beauty rating, the model predicts that the male professor will have a score that is 0.1723 points higher.

The decision to call the indicator variable `gendermale` instead of`genderfemale` has no deeper meaning. R simply codes the category that comes first alphabetically as a 0. (You can change the reference level of a categorical variable, which is the level that is coded as a 0, using the`relevel` function. Use `?relevel` to learn more.)

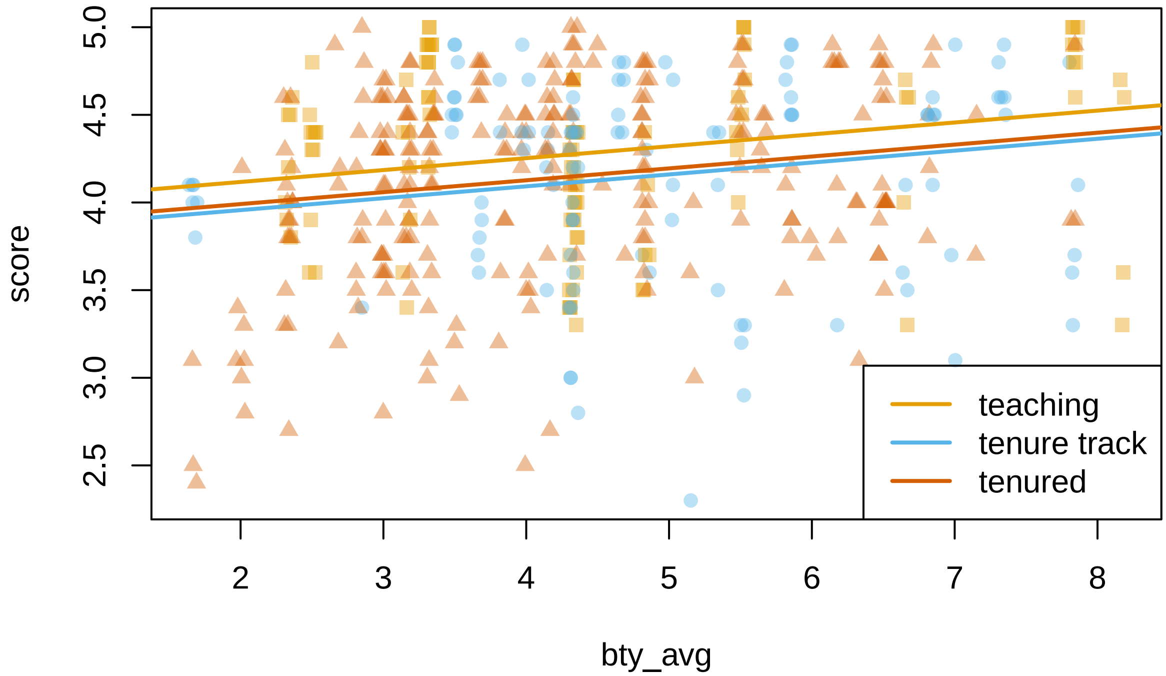
10. Create a new model called `m_bty_rank` with `gender` removed and `rank` added in. How does R appear to handle categorical variables that have more than two levels? Note that the rank variable has three levels: `teaching`, `tenure track`, `tenured`.

Answer

```
m_bty_rank <- lm(score ~ bty_avg + rank, evals)
summary(m_bty_rank)
```

```
##
## Call:
## lm(formula = score ~ bty_avg + rank, data = evals)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.8713 -0.3642  0.1489  0.4103  0.9525
##
## Coefficients:
##                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)       3.98155    0.09078  43.860  < 2e-16 ***
## bty_avg           0.06783    0.01655   4.098 4.92e-05 ***
## ranktenure track -0.16070    0.07395  -2.173   0.0303 *
## ranktenured      -0.12623    0.06266  -2.014   0.0445 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5328 on 459 degrees of freedom
## Multiple R-squared:  0.04652,    Adjusted R-squared:  0.04029
## F-statistic: 7.465 on 3 and 459 DF,  p-value: 6.88e-05
```

```
multiLines(m_bty_rank)
```



R splits the `rank` variable with three levels i.e `ranktenure track`, `tenured` and `teaching`

The interpretation of the coefficients in multiple regression is slightly different from that of simple regression. The estimate for `bty_avg` reflects how much higher a group of professors is expected to score if they have a beauty rating that is one point higher *while holding all other variables constant*. In this case, that translates into considering only professors of the same rank with `bty_avg` scores that are one point apart.

The search for the best model

We will start with a full model that predicts professor score based on rank, ethnicity, gender, language of the university where they got their degree, age, proportion of students that filled out evaluations, class size, course level, number of professors, number of credits, average beauty rating, outfit, and picture color.

11. Which variable would you expect to have the highest p-value in this model? Why? *Hint:* Think about which variable would you expect to not have any association with the professor score.

Answer

I would expect the `cls_profs` to have the highest p-value because I don't believe this affects the professor's score.

Let's run the model…

```
m_full <- lm(score ~ rank + ethnicity + gender + language + age + cls_perc_eval
             + cls_students + cls_level + cls_profs + cls_credits + bty_avg
             + pic_outfit + pic_color, data = evals)
summary(m_full)
```

12. Check your suspicions from the previous exercise. Include the model output in your response.

Answer

In the previous question my assumption was correct. The model out put shows `cls_profs` has highest p-value.

13. Interpret the coefficient associated with the ethnicity variable.

Answer

`ethnicitynot minority` variable has a coefficent of 0.1234929.

14. Drop the variable with the highest p-value and re-fit the model. Did the coefficients and significance of the other explanatory variables change? (One of the things that makes multiple regression interesting is that coefficient estimates depend on the other variables that are included in the model.) If not, what does this say about whether or not the dropped variable was collinear with the other explanatory variables?

Answer

Highest p-value has `cls_profs`. Lets drop this variable from the model.

```
m_full1_new <- lm(score ~ rank + ethnicity + gender + language + age + cls_perc_eval
             + cls_students + cls_level + cls_credits + bty_avg
             + pic_outfit + pic_color, data = evals)
summary(m_full1_new)
```

```
##
## Call:
## lm(formula = score ~ rank + ethnicity + gender + language + age +
##     cls_perc_eval + cls_students + cls_level + cls_credits +
##     bty_avg + pic_outfit + pic_color, data = evals)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.7836 -0.3257  0.0859  0.3513  0.9551
##
## Coefficients:
##                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)             4.0872523  0.2888562  14.150  < 2e-16 ***
## ranktenure track       -0.1476746  0.0819824  -1.801 0.072327 .
## ranktenured            -0.0973829  0.0662614  -1.470 0.142349
## ethnicitynot minority   0.1274458  0.0772887   1.649 0.099856 .
## gendermale              0.2101231  0.0516873   4.065 5.66e-05 ***
## languagenon-english    -0.2282894  0.1111305  -2.054 0.040530 *
## age                    -0.0089992  0.0031326  -2.873 0.004262 **
## cls_perc_eval           0.0052888  0.0015317   3.453 0.000607 ***
## cls_students            0.0004687  0.0003737   1.254 0.210384
## cls_levelupper          0.0606374  0.0575010   1.055 0.292200
## cls_creditsone credit   0.5061196  0.1149163   4.404 1.33e-05 ***
## bty_avg                 0.0398629  0.0174780   2.281 0.023032 *
## pic_outfitnot formal   -0.1083227  0.0721711  -1.501 0.134080
## pic_colorcolor         -0.2190527  0.0711469  -3.079 0.002205 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4974 on 449 degrees of freedom
## Multiple R-squared:  0.187,  Adjusted R-squared:  0.1634
## F-statistic: 7.943 on 13 and 449 DF,  p-value: 2.336e-14
```

After remove `clf_profs` their is slightly change in coefficients, R^2 and p-values.

15. Using backward-selection and p-value as the selection criterion, determine the best model. You do not need to show all steps in your answer, just the output for the final model. Also, write out the linear model for predicting score based on the final model you settle on.

Answer

```
m_full_backward <- lm(score ~ gender + language + age + cls_perc_eval
             + cls_credits + bty_avg + pic_color, data = evals)
summary(m_full_backward)
```

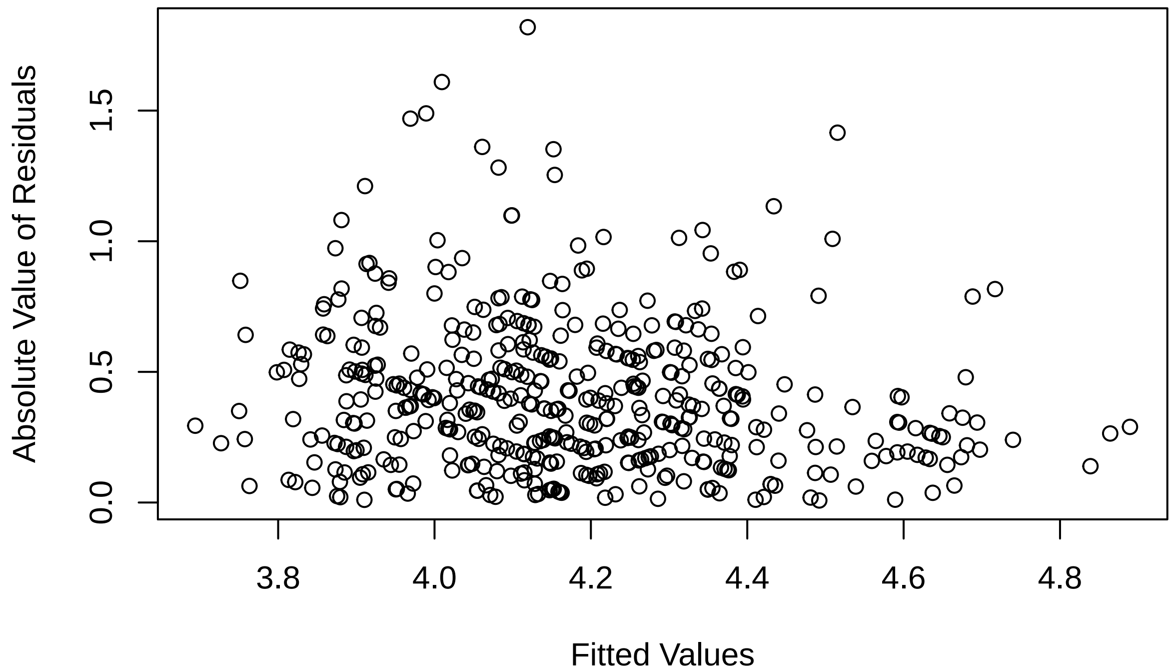
```
##
## Call:
## lm(formula = score ~ gender + language + age + cls_perc_eval +
##     cls_credits + bty_avg + pic_color, data = evals)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.81919 -0.32035  0.09272  0.38526  0.88213
##
## Coefficients:
##                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)              3.967255   0.215824  18.382  < 2e-16 ***
## gendermale               0.221457   0.049937   4.435 1.16e-05 ***
## languagenon-english     -0.281933   0.098341  -2.867  0.00434 **
## age                     -0.005877   0.002622  -2.241  0.02551 *
## cls_perc_eval            0.004295   0.001432   2.999  0.00286 **
## cls_creditsone credit    0.444392   0.100910   4.404 1.33e-05 ***
## bty_avg                  0.048679   0.016974   2.868  0.00432 **
## pic_colorcolor          -0.216556   0.066625  -3.250  0.00124 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5014 on 455 degrees of freedom
## Multiple R-squared:  0.1631, Adjusted R-squared:  0.1502
## F-statistic: 12.67 on 7 and 455 DF,  p-value: 6.996e-15
```

$$\widehat{score} = 3.967255+0.221457\times gender+(-0.281933)\times language+(-0.005877)\times age+(0.004295)\times cls\_perc\_eval+(0.444392)\times c$$

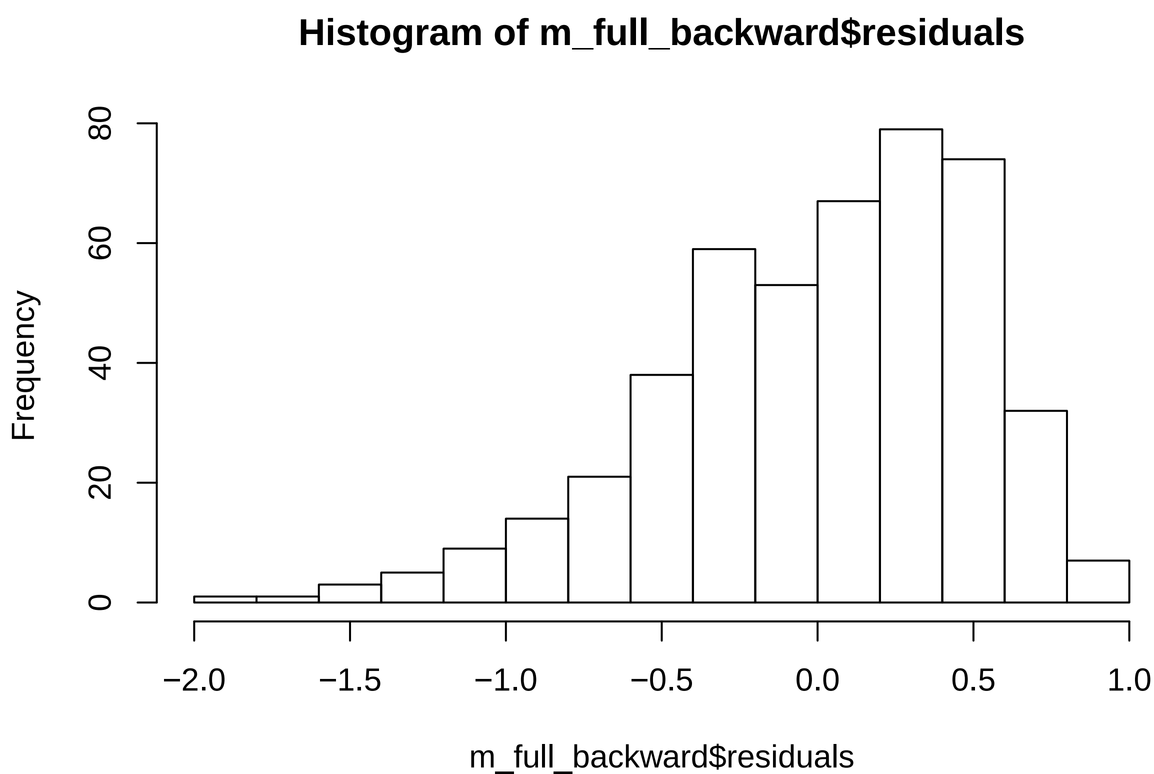
16. Verify that the conditions for this model are reasonable using diagnostic plots.

Answer

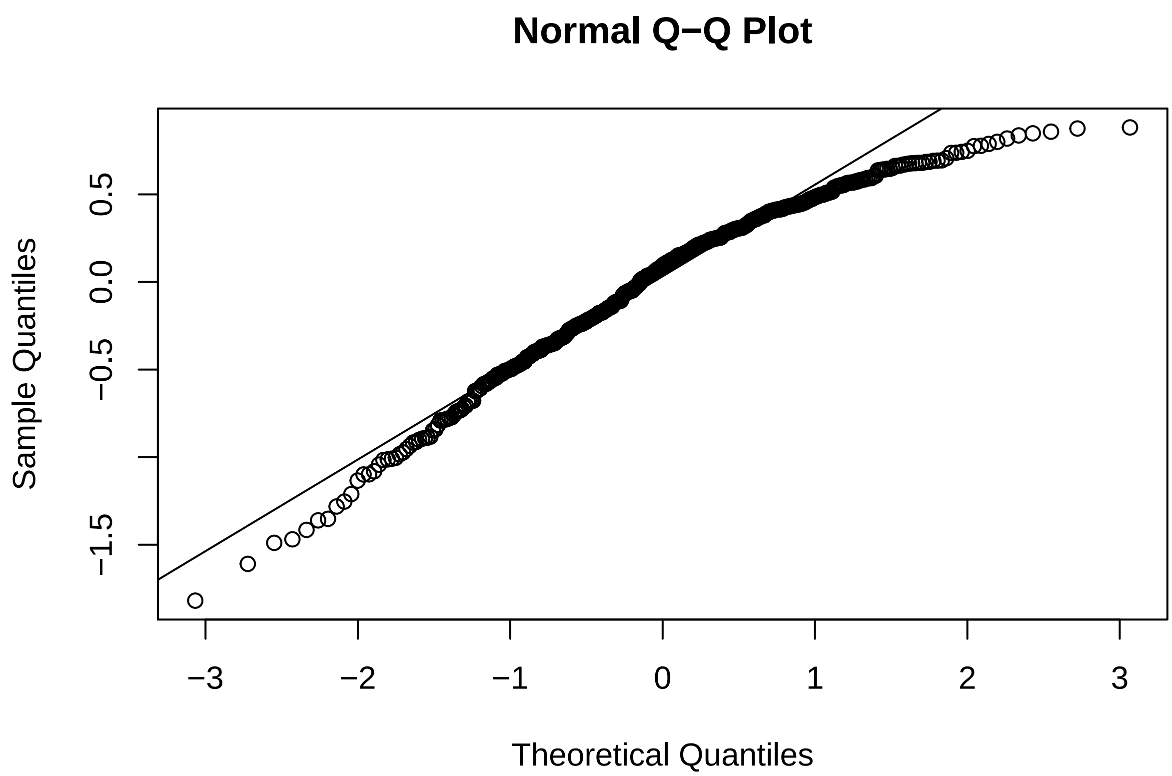
```
plot(abs(m_full_backward$residuals) ~ m_full_backward$fitted.values,
     xlab = "Fitted Values", ylab = "Absolute Value of Residuals")
```




```r
hist(m_full_backward$residuals)
```




```r
qqnorm(m_full_backward$residuals)
qqline(m_full_backward$residuals)
```



Linearity: The residual plot mets linearity.

Normal residuals: Histogram shows left skewed rather than normal distribution.

17. The original paper describes how these data were gathered by taking a sample of professors from the University of Texas at Austin and including all courses that they have taught. Considering that each row represents a course, could this new information have an impact on any of the conditions of linear regression?

Answer

Basically, class courses are not dependent with each other. Due to condition of independence, evaluation scores from one course is independent of the other because if an instructor teaches more than one course it should not affect, but if the same student takes two or more classes with the same instructor this may affect the outcome since independence will not be satisfied.

18. Based on your final model, describe the characteristics of a professor and course at University of Texas at Austin that would be associated with a high evaluation score.

Answer

Based on the model, the highest score will be associated with professors with below quality

- Who is male
- Who is not part of minority group
- Who taught in English
- Who is younger and looks good

19. Would you be comfortable generalizing your conclusions to apply to professors generally (at any university)? Why or why not?

Answer

I would prefer to not generalize due to regional difference each university has different culture.

And this is an observational study and focus on beauty and age, which will change over time.