ELL 205: Signals & Systems
Tutorial Sheet-1
2024-25(1st Semester)

Q1. Check if the following signals are periodic. If so, what is the fundamental time period?

a) $y(t) = \sum_{n=-\infty}^{\infty} x(t - nT)$ where $x(t)$ is an arbitrary signal.

b) $y(t) = 1 + \sum_{n=1}^{4} \sin(n\omega_1 t)$.

c) $y[n] = \cos\left(\frac{\pi n^2}{8}\right)$,

d) $x[n] = \exp(j2n)$,

e) $x[n] = y[2n]$ with $y[n]$ is periodic with periodicity 3.

Q2. Determine if the following systems are invertible or not. If so, then construct the inverse system, otherwise specify two inputs that result in the same output.

(a) $y[n] = \begin{cases} x[n-1], & n \geq 1 \\ 0, & n = 0 \\ x[n], & n \leq -1 \end{cases}$

(b) $y(t) = \int_{-\infty}^{t} x(\tau)e^{-(t-\tau)}d\tau$

(c) $y[n] = \sum_{k=-\infty}^{n} \left(\frac{1}{2}\right)^{n-k} x[k]$

(d) $y[n] = \begin{cases} x[n+1], & n \geq 0 \\ x[n], & n \leq -1 \end{cases}$

(e) $y[n] = \begin{cases} x[n/2], & n = \text{even} \\ 0, & n = \text{odd} \end{cases}$

Q3. Consider a discrete-time system with input $x[n]$ and output $y[n]$ with the input-output relationship given as:

$$y[n] = x[n]x[n-2].$$

(a) Is the system memoryless?
(b) Determine the output when the input is $A\delta[n]$ where A is a constant.
(c) Is the system invertible?

Q4. Determine if the following systems are causal or non-causal.
(a) $y(t) = \cos(3t)x(t)$
(b) $y(t) = \int_{-\infty}^{2t} x(\tau)d\tau$
(c) $y(t) = x\left(\frac{t}{3}\right)$

Q5. Determine if the following systems are linear or not.

(a) $y[n] = x[n^2]$

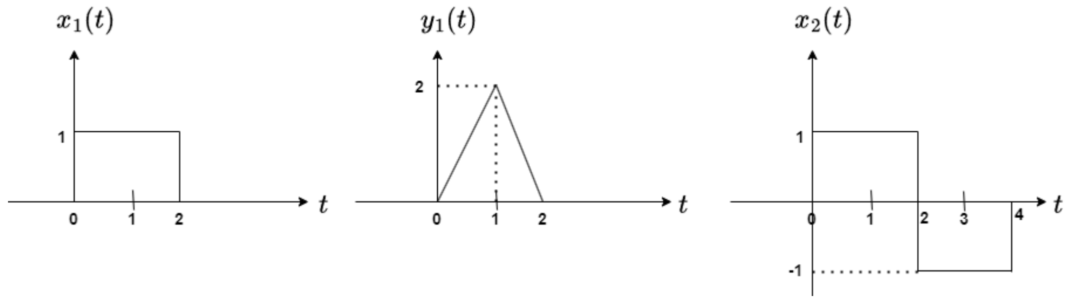
(b) $y[n] = e^{x[n]}$

Q6. Determine if the following systems are time-invariant or variant.

(a) $y(t) = t^2 x(t-1)$

(b) $y[n] = \mathcal{O}\{x[n]\}$, where $\mathcal{O}\{\cdot\}$ denotes the odd part of the signal.

Q7. Consider a linear and time-invariant system whose response to input $x_1(t)$ is $y_1(t)$ as shown in the following figure. Determine its response to input $x_2(t)$, also shown in the following figure.



Q8. Show that $\delta(2t) = \frac{1}{2}\delta(t)$, where $\delta(t)$ denotes the continuous-time unit impulse function. What can you say about $u(2t)$?

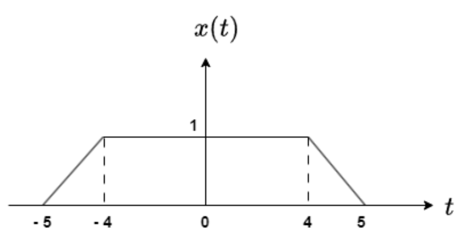
Q9. A system with input $x(t)$ and output $y(t)$ is described by the input-output relation:

$$y(t) = \int_t^{\infty} x(\tau) e^{t-\tau} d\tau$$

(a) Is this system stable?

(b) Is this system time-invariant?

Q10. The following signal $x(t)$ is applied to a differentiator, defined by $y(t) = \frac{d}{dt}x(t)$.



(a) Determine the resulting output $y(t)$ of the differentiator.

(b) Determine the total energy of $y(t)$.

(c) Sketch $x(5t)$ and $x(t/5)$.

Q11. Consider the continuous-time signal $x(t)$ applied to a differentiator to obtain output $y(t)$.

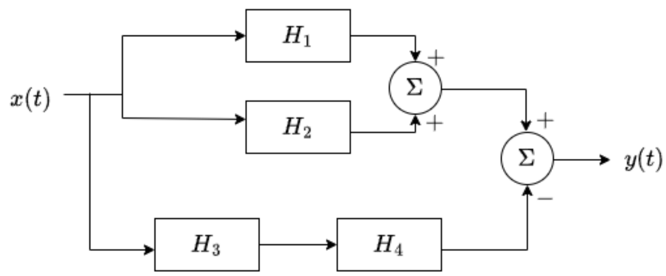
$$x(t) = \begin{cases} t/T + 0.5, & -T/2 \le t \le T/2 \\ 1, & t > T/2 \\ 0, & t < -T/2 \end{cases}$$

Determine $x(t)$ and $y(t)$ as T approaches to zero.

Q12. A system consists of several subsystems, as shown below.

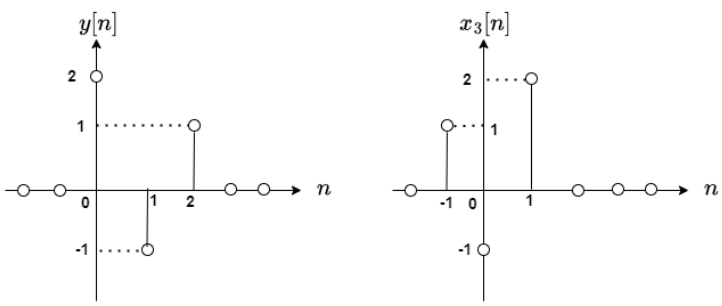
$$H_1 : y(t) = x(t)x(t-1)$$
$$H_2 : y(t) = |x(t)|$$
$$H_3 : y(t) = 1 + 2x(t)$$
$$H_4 : y(t) = cos(x(t))$$

Determine the input-output relation between $x(t)$ and $y(t)$.



Q13. A discrete-time system is both linear and time-invariant. Suppose the output due to an input $x[n] = \delta[n]$ is as shown below.

(a) Find the output due to input $x_1[n] = \delta[n-1]$.
(b) Find the output due to input $x_2[n] = 2\delta[n]$.
(c) Find the output due to input $x_3[n]$, also shown below.



Q14. The sinusoidal signal $x(t) = A_1 \cos(\omega_1 t + \phi_1) + A_2 \cos(\omega_2 t + \phi_2)$ is passed through a square-law device defined by input-output relation $y(t) = x^2(t)$. Specify the dc component and the amplitudes and frequencies of the sinusoidal components in the output of $y(t)$.

Q15. Determine the even and odd parts of the signal $x(t) = (2 + \sin(t))^2$.

Q16. State whether the following signals are energy or power signals, and also compute their energy/power.

(a) $x[n] = A\cos(\Omega n)$ for $\Omega = \pi/4$.

(b) $x(t) = \begin{cases} \frac{1}{2}[\cos(\omega t) + 1], & -\pi/\omega \leq t \leq \pi/\omega \\ 0, & \text{otherwise} \end{cases}$