UNIT-I

1 Which of the following are unitary, Hermitian, skew Hermitian matrices

(a) $\begin{bmatrix} 0 & 1+i & 2-3i \\ -1+i & 4i & 4+5i \\ -2-3i & -4+5i & -3i \end{bmatrix}$. (b) $\begin{bmatrix} \frac{1+i}{2} & \frac{-1+i}{2} \\ \frac{1+i}{2} & \frac{1-i}{2} \end{bmatrix}$.

(c) $\begin{bmatrix} 3 & 7-4i & -2+5i \\ 7+4i & -2 & 3+i \\ -2-5i & 3-i & 4 \end{bmatrix}$ d) $\begin{bmatrix} 1+i & 3+i & 2+i \\ 3+i & 2+i & 1+i \\ 2+i & 1+i & 3+i \end{bmatrix}$

2 Examine whether the following vectors are linearly independent. (a) $\{v_1 = (1, 1, 3, 2),\ v_2 = (2, 3, 4, 5),\ v_3 = (5, 7, 11, 12)\}$.

(b) $\{v_1 = (1, 0, 3, 2),\ v_2 = (2, 2, 4, 4),\ v_3 = (4, 7, 11, 2)\}$.

(c) $\{v_1 = (-1, 2, -4),\ v_2 = (5, -10, 20),\ v_3 = (4, -8, 16)\}$.

(d) (2,i,-1), (1,-3,i), (2i,-1,5) ‘

(e) (1,3,4) (1,1,0),(1,4,2), (1,-2,1)

3 Which of the following matrices are orthogonal? If orthogonal Find a,b,c values

(a) $\begin{bmatrix} \cos\phi & o & \sin\phi \\ \sin\theta\cos\phi & \cos\theta & \sin\theta\cos\phi \\ -\sin\theta\sin\phi & \sin\theta & \cos\theta\cos\phi \end{bmatrix}$. (b) $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$.

(c) $\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$

4 Let $\bar{V}_1 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \bar{V}_2 = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$ *and* $\bar{V}_3 = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}, \bar{V}_4 = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}$ be elements of $\mathbb{R}^3$. Show that the set of vectors $\{\bar{V}_1, \bar{V}_2, \bar{V}_3, \bar{V}_4\}$ is linearly dependent.

5 Find *p, q* so that the following equations have (i) no solution,

(ii) unique solution, (iii) infinite number of solutions.

$2x + 3y + 5z = 9,\ 7x + 3y + 2z = 8,\ 2x + 3y + pz = q.$

6 For what values of *k* the system of equations will have a non-trivial solution, and solve them for those values of *k*.

$2x + 3ky + (3k + 4)z = 0;\quad x + (k + 4)y + (4k + 2)z = 0$

$x + 2(k + 1)y + (3k + 4)z = 0.$

7 Show that the only real value of λ for which the following equations have non-trivial solution is 6 and solve them, when $\lambda = 6$.

$x + 2y + 3z = \lambda x;\ 3x + y + 2z = \lambda y;\ 2x + 3y + z = \lambda z.$

8 Determine the values of λ for which the following equations have non-trivial solution and solve them in each case.

$3x_1 + x_2 - \lambda x_3 = 0,\ 4x_1 - 2x_2 - 3x_3 = 0,\ 2\lambda x_1 + 4x_2 + \lambda x_3 = 0.$

9 Solve $3x + 4y - z - 6w = 0,\ 2x + 3y + 2z - 3w = 0,\ 2x + y - 14z - 9w = 0,$

x + 3y + 13z + 3w = 0.

10 Solve $2x_1 + x_2 + 2x_3 + x_4 = 6,\ 6x_1 - 6x_2 + 6x_3 + 12x_4 = 36,\ 4x_1 + 3x_2 + 3x_3 - 3x_4 = -1,$
$2x_1 + 2x_2 - x_3 + x_4 = 10.$

11 Solve $x_1 + 2x_2 + x_3 = 2,\ 3x_1 + x_2 - 2x_3 = 1,\ 4x_1 - 3x_2 - x_3 = 3,\ 2x_1 + 4x_2 + 2x_3 = 4.$

12 Solve 3x + 3y + 2z = 1, x + 2y = 4, 10 y + 3z = –2, 2x – 3y – z = 5

13 Solve $x_1 + x_2 + x_3 + x_4 = 0,\ x_1 + x_2 + x_3 - x_4 = 4,\ x_1 + x_2 - x_3 + x_4 = -4,\ x_1 - x_2 + x_3 + x_4 = 2.$

14 For what value of *k*, the matrix A has rank 3 for

(i) $A = \begin{bmatrix} 4 & 4 & -3 & 1 \\ 1 & 1 & -1 & 0 \\ k & 2 & 2 & 2 \\ 9 & 9 & k & 3 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 2 & -1 & 3 \\ 4 & 1 & 2 & 1 \\ 3 & -1 & 1 & 2 \\ -1 & 2 & 0 & k \end{bmatrix}$

15 Show that if the matrix A is orthogonal, then A^T and A^{-1} are also orthogonal

16 For what value of *k*, the matrix A has rank 2 for $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & k & 0 \end{bmatrix}$

17 Show that Every Square matrix can be Express as sum of its symmetric and Skew symmetric matrices

18 Show that Every Square matrix can be Express as sum of its Hermitian and Skew Hermitian matrices

19 Identify the values of λ and μ so that $2x+3y+5z=9$, $7x+3y-2z=8$, $2x+3y+\lambda z=\mu$, have (i) no solution, (ii) a unique solution and (iii) infinite number of solutions.

20 Identify the values of k for which the system of equations $(3k-8)x+3y+3z=0$, $3x+(3k-8)y+3z=0$, $3x+3y+(3k-8)z=0$ has a non-trivial solution