DERIVATIONS IN APPLIED PHYSICS

* Schrodinger's 1D wave equation :

$\rightarrow y = a \sin \frac{2\pi}{\lambda}(x - vt)$; $y \rightarrow$ displacement of wave in y-direction.
$x \rightarrow$ displacement along x-axis at any time 't'.

$\Rightarrow \frac{dy}{dx} = a \cos \frac{2\pi}{\lambda}(x-vt) \cdot \frac{2\pi}{\lambda}$

$\Rightarrow \frac{d^2y}{dx^2} = -\left(\frac{2\pi}{\lambda}\right)^2 a \sin\left(\frac{2\pi}{\lambda}\right)(x-vt)$

$\Rightarrow \frac{d^2y}{dx^2} = \left(\frac{2\pi}{\lambda}\right)^2(-y)$

$y \xrightarrow{\text{replace}} \psi$

$\lambda \longrightarrow \lambda = \frac{h}{mv}$

$\Rightarrow \frac{d^2\psi}{dx^2} + \left(\frac{2\pi}{\lambda}\right)^2 \psi = 0$

$\Rightarrow \frac{d^2\psi}{dx^2} + \frac{4\pi^2 m^2 v^2}{h^2} \psi = 0$

$E = U + V$

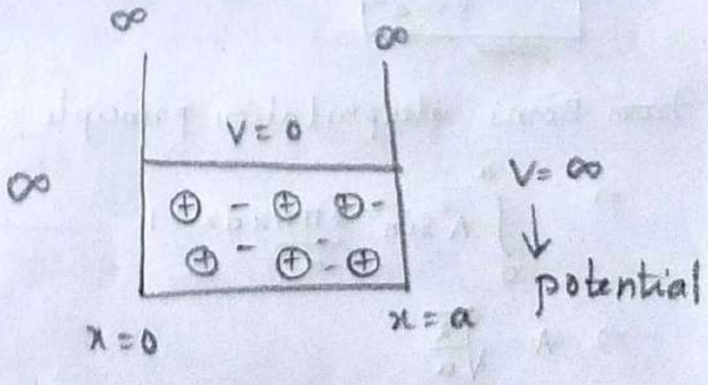
$E \rightarrow$ total energy
$U \rightarrow$ kinetic energy
$V \rightarrow$ potential energy

$\Rightarrow \frac{d^2\psi}{dx^2} + \frac{4\pi^2 2m(E-V)}{h^2}\psi = 0$

$$\Rightarrow \boxed{\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m(E-V)}{h^2}\psi = 0}$$

* Particle in 1D box :

$\rightarrow V(x) = 0$; $0 < x < a$ & $\psi(x) = 1$
$V(x) = \infty$; $0 \geq x \geq a$ & $\psi(x) = 0$



$\psi(x) \rightarrow$ probability of finding the particle.

$\therefore V = 0$ inside the box ;

$\Rightarrow \frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E-0)\psi = 0$

$$\Rightarrow \frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2} E\psi = 0$$

$$\Rightarrow \frac{d^2\psi}{dx^2} + k^2\psi = 0 \quad ; \text{ where } \quad k^2 = \frac{8\pi^2 m E}{h^2}$$

$k \rightarrow$ propagation vector.

Let general solution of the above equation be :

$$\psi(x) = A\sin kx + B\cos kx$$

At $x = 0 \Rightarrow \psi(x) = 0$

$$\Rightarrow 0 = A\sin k(0) + B\cos k(0)$$

$$\Rightarrow B = 0$$

At $x = a \Rightarrow \psi(x) = 0$

$$\Rightarrow 0 = A\sin ka + 0$$

$$\Rightarrow A = 0 \text{ (or) } \sin ka = 0$$

$\Rightarrow$ if $A = 0$ our assumption may contradict

$$\Rightarrow \sin ka = 0$$

$$\Rightarrow ka = n\pi \qquad n = 1, 2, \ldots$$

$$\Rightarrow k = \frac{n\pi}{a}$$

$$\because k^2 = \frac{8\pi^2 m E}{h^2}$$

$\Rightarrow \boxed{E_n = \frac{n^2 h^2}{8ma^2}}$ is the discrete energy level.

From Born's interpretation principle;

$$\Rightarrow \int_0^a A^2 \sin^2 \frac{2\pi n x}{a} dx = 1 \qquad \because \sin^2 \frac{n\pi x}{a} = 1 - \cos \frac{2\pi n x}{a}$$

$$\Rightarrow A = \sqrt{\frac{2}{a}}$$

$\therefore$ Normalised wave function is $\boxed{\psi_n = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a}}$

* Electron density in conduction band for intrinsic semi-conductor : (3)

→ dn → no. of e^-s whose energy lies in the energy interval E & E+dE in the conduction band.

$\Rightarrow$ $dn = Z(E) f(E) dE$

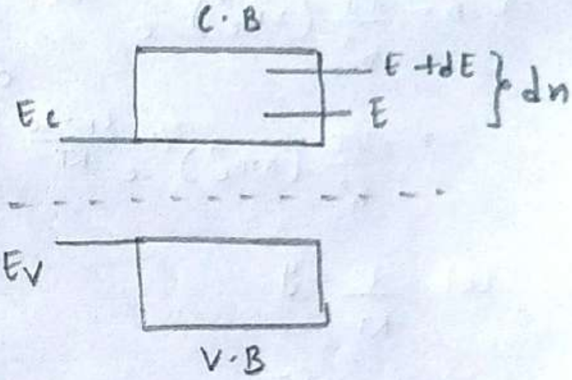
$Z(E) \rightarrow$ density of states

$f(E) \rightarrow$ probability of occupation of e^- in C.B.

where $Z(E)dE = \frac{4\pi}{h^3}(2m_e^*)^{3/2} E^{1/2} dE$

$m_e^* \rightarrow$ rest mass of e^-

$E \xrightarrow{\text{replace}} E - E_c$



$$\Rightarrow Z(E)dE = \frac{4\pi}{h^3}(2m_e^*)^{3/2}(E-E_c)^{1/2} dE$$

$$\& \quad f(E) = \frac{1}{1+\exp\left(\frac{E-E_F}{KT}\right)}$$

in this case $E - E_F >> KT$

$$\Rightarrow f(E) = \frac{1}{\exp\left(\frac{E-E_F}{KT}\right)} = e^{\left(\frac{E_F-E}{KT}\right)}$$

$$\therefore dn = Z(E) f(E) dE$$

$$= \frac{4\pi}{h^3}(2m_e^*)^{3/2}(E-E_c)^{1/2} \cdot \exp\left(\frac{E_F-E}{KT}\right) dE$$

$$\Rightarrow n = \int_{E_c}^{\infty} dn = \frac{4\pi}{h^3}(2m_e^*)^{3/2} \int_{E_c}^{\infty} (E-E_c)^{1/2} \cdot \exp\left(\frac{E_F-E}{KT}\right) dE$$

$$= \frac{4\pi}{h^3}(2m_e^*)^{3/2} \int_{E_c}^{\infty} (E-E_c)^{1/2} \frac{e^{\frac{E_F}{KT}}}{e^{\frac{E}{KT}}} dE$$

$$= \frac{4\pi}{h^3}(2m_e^*)^{3/2} e^{E_F/KT} \int_{E_c}^{\infty} (E-E_c)^{1/2} e^{-E/KT} dE$$

$\Rightarrow$ let $E - E_c = t$

$\Rightarrow E = t + E_c$

$\Rightarrow dE = dt$

at $E = E_c \Rightarrow t = 0$

at $E = \infty \Rightarrow t = \infty$

$$\Rightarrow n = \frac{4\pi}{h^3}(2m_e^*)^{3/2} e^{E_F/KT} \int_0^\infty t^{1/2} e^{-\frac{(t+E_c)}{KT}} dt$$

$$= \frac{4\pi}{h^3}(2m_e^*)^{3/2} e^{\frac{E_F - E_c}{KT}} \int_0^\infty t^{1/2} . e^{-t/KT} dt$$

Let $\frac{t}{KT} = y$

$\Rightarrow t = yKT$

$\Rightarrow dt = KTdy$

at $t=0 \Rightarrow y = 0$

at $t = \infty \Rightarrow y = \infty$

$$\Rightarrow n = \frac{4\pi}{h^3}(2m_e^*)^{3/2} e^{\left(\frac{E_F - E_c}{KT}\right)} \int_0^\infty (y\,KT)^{1/2} . e^{-y} KTdy$$

$$= \frac{4\pi}{h^3}(2m_e^* KT)^{3/2} . e^{\left(\frac{E_F - E_c}{KT}\right)} \int_0^\infty y^{1/2} . e^{-y} dy$$

$$= \frac{4\pi}{h^3}(2m_e^* KT)^{3/2} e^{\left(\frac{E_F - E_c}{KT}\right)} \Gamma 3/2$$

$\because \int_0^\infty y^{1/2} e^{-y} dy = \Gamma 3/2$

gama function

$\Gamma 3/2 = \frac{\sqrt{\pi}}{2}$

$$= \frac{4\pi}{h^3}(2m_e^* KT)^{3/2} e^{\left(\frac{E_F - E_c}{KT}\right)} \frac{\sqrt{\pi}}{2}$$

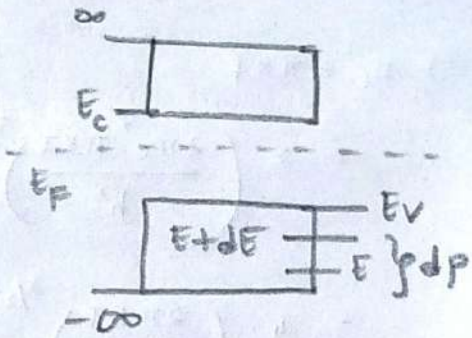
$$\therefore \boxed{n = 2\left(\frac{2m_e^* \pi KT}{h^2}\right)^{3/2} \exp\left(\frac{E_F - E_c}{KT}\right)}$$

* Hole density in valency band of intrinsic Semiconductor :

→ Similar to electron density

dp → no. of holes whose energy lies in $E \to E+dE$ interval.

$$dp = Z(E)\,dE\,(1-f(E))$$



$$\because\ 1-f(E) = 1-\frac{1}{1+\exp\left(\frac{E-E_F}{KT}\right)}$$

$\because\ E-E_F << KT$

$$\Rightarrow \frac{E-E_F}{KT} << 1$$

$$\Rightarrow\ 1-f(E) = \frac{\cancel{1}+\exp\left(\frac{E-E_F}{KT}\right)-\cancel{1}}{1+\exp\left(\frac{E-E_F}{KT}\right)} = \exp\left(\frac{E-E_F}{KT}\right)$$

$$\&\ Z(E)\,dE = \frac{4\pi}{h^3}(2m_h^*)^{3/2}E^{1/2}\,dE$$

$m_h^* \to$ rest mass of hole

$$= \frac{4\pi}{h^3}(2m_h^*)^{3/2}(E_V-E)^{1/2}\,dE$$

$E \xrightarrow{\text{replace}} E_V - E$

$$\therefore\ dp = Z(E)(1-f(E))\,dE$$

$$P = \int_{-\infty}^{E_V} dp = \frac{4\pi}{h^3}(2m_h^*)^{3/2}\int_{-\infty}^{E_V}(E_V-E)^{1/2}\exp\left(\frac{E-E_F}{KT}\right)dE$$

$$\because\ \boxed{P = 2\left(\frac{2m_h^*\pi KT}{h^2}\right)^{3/2}\exp\left(\frac{E_V-E_F}{2}\right)}$$

* Intrinsic concentration :

$n_i \rightarrow$ intrinsic density

$n_i = n = p$

$$\Rightarrow n_i^2 = n \times p$$

$$= 2\left(\frac{2m_e^* \pi KT}{h^2}\right)^{3/2} e^{\left(\frac{E_F - E_C}{KT}\right)} \cdot 2\left(\frac{2m_h^* \pi KT}{h^2}\right)^{3/2} e^{\left(\frac{E_V - E_F}{KT}\right)}$$

$$= 4\left(\frac{2\pi KT}{h^2}\right)^{3/2 \times 2} (m_e^* m_h^*)^{3/2} e^{\left(\frac{E_F - E_C + E_V - E_F}{KT}\right)}$$

$$= 4\left(\frac{2\pi KT}{h^2}\right)^{3} (m_e^* m_h^*)^{3/2} e^{\left(\frac{E_V - E_C}{KT}\right)}$$

$$= 4\left(\frac{2\pi KT}{h^2}\right)^{3} (m_e^* m_h^*)^{3/2} e^{\left(\frac{-E_g}{KT}\right)} \qquad \because E_C - E_V = E_g$$

(forbidden energy gap).

$$\Rightarrow \boxed{n_i = 2\left(\frac{2\pi KT}{h^2}\right)^{3/2} (m_e^* m_h^*)^{3/4} e^{\left(\frac{-E_g}{2KT}\right)}}$$

* Intrinsic concentration at Fermi level :

$\rightarrow$ Assume $m_e^* = m_h^*$

$\rightarrow$ at Fermi level $n = p$

$$\Rightarrow \cancel{2}\left(\frac{2\pi m_e^* \cancel{KT}}{\cancel{h^2}}\right)^{3/2} e^{\left(\frac{E_F - E_C}{KT}\right)} = \cancel{2}\left(\frac{2\pi \cancel{m_h^*} KT}{\cancel{h^2}}\right)^{3/2} e^{\left(\frac{E_V - E_F}{KT}\right)}$$

$$\Rightarrow E_F - E_C = E_V - E_F$$

$$\Rightarrow \boxed{E_F = \frac{E_V + E_C}{2}}$$

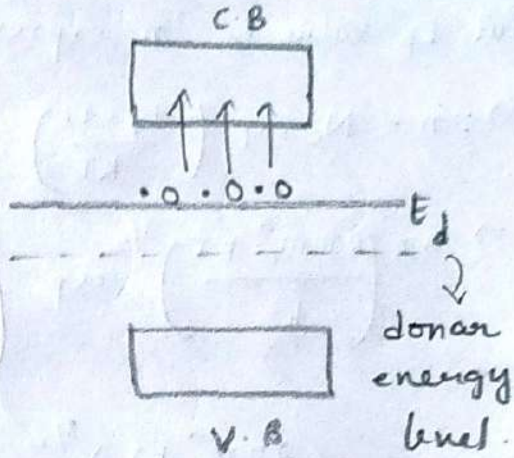
* Extrinsic carrier concentration in n-type s.c :

→ Let N_d → conc of donors in the material.

Assuming e^-s conc in C.B = no. of donors

$$n = N_d^+$$

& the no. of e^-s being transfered to C.B = no. of holes /+ve charge carriers being created at E_d



$$\Rightarrow N_d^+ = N_d\,(1 - f(E_d))$$

no. of donar atoms × prob of holes occupancy

$$\Rightarrow N_d^+ = N_d \exp\left(\frac{E_d - E_F}{KT}\right)$$

$$\therefore n = N_d^+$$

$$\Rightarrow 2\left(\frac{2\pi m_e^* KT}{h^2}\right)^{3/2} \exp\left(\frac{E_F - E_C}{KT}\right) = N_d \exp\left(\frac{E_d - E_F}{KT}\right)$$

$$\Rightarrow 2\left(\frac{2m_e^*\pi KT}{h^2}\right)^{3/2} \exp\left(\frac{2E_F - (E_d + E_C)}{KT}\right) = N_d$$

Let $2\left(\frac{2\pi m_e^* KT}{h^2}\right) = N_c$

$$\Rightarrow N_c \cdot \exp\left(\frac{2E_F - (E_d + E_C)}{KT}\right) = N_d$$

$$\Rightarrow \frac{2E_F - (E_d + E_C)}{KT} = \ln\left(\frac{N_d}{N_c}\right)$$

$$\Rightarrow 2E_F - (E_d + E_C) = KT \ln\left(\frac{N_d}{N_c}\right)$$

$$\Rightarrow \boxed{E_F = \frac{KT}{2}\ln\left(\frac{N_d}{N_c}\right) + \left(\frac{E_d + E_C}{2}\right)}.$$

(8)

At $T = 0K$

$$\Rightarrow E_F = \frac{E_d + E_c}{2}$$

Put E_F value in 'n' expression

$$\Rightarrow \because n = N_c \exp\left(\frac{E_F - E_c}{KT}\right)$$

$$\Rightarrow n = \left(\frac{2\pi m_e^* KT}{h^2}\right)^{3/2} \exp\left(\frac{\frac{KT}{2}\ln\left(\frac{N_d}{N_c}\right) + \left(\frac{E_c + E_d}{2}\right) - E_c}{KT}\right)$$

$$= 2\left(\frac{2\pi m_e^* KT}{h^2}\right)^{3/2} e^{\left(\frac{1}{2}\ln\left(\frac{N_d}{N_c}\right) + \frac{E_d - E_c}{2KT}\right)}$$

$$= 2\left(\frac{2\pi m_e^* KT}{h^2}\right)^{3/2} \sqrt{\frac{N_d}{N_c}} \cdot e^{\left(\frac{E_d - E_c}{2KT}\right)}$$

$$= 2\left(\frac{2\pi m_e^* KT}{h^2}\right)^{3/2} \frac{(N_d)^{1/2} \cdot e^{\left(\frac{E_d - E_c}{2KT}\right)}}{\left[2\left(\frac{2 m_e^* \pi KT}{h^2}\right)^{3/2}\right]^{1/2}} \qquad \because N_c = 2\left(\frac{2 m_e^* \pi KT}{h^2}\right)^{3/2}$$

$$= 2^{1/2}\left(\frac{2\pi m_e^* KT}{h^2}\right)^{3/4} (N_d)^{1/2} \exp\left(\frac{E_d - E_c}{2KT}\right)$$

$$\therefore \boxed{n = (2N_d)^{1/2}\left(\frac{2 m_e^* \pi KT}{h^2}\right)^{3/4} \exp\left(\frac{E_d - E_c}{2KT}\right)}$$

* Extrinsic carrier concentration in p-type semiconductor : (9)

→ Let N_a → conc of acceptors in the material.

Assuming hole conc in the V.B = no. of acceptors

$$\Rightarrow p = N_a^-$$

$$\Rightarrow p = 2\left(\frac{2\pi m_h^* KT}{h^2}\right)^{3/2} \exp\left(\frac{E_v - E_F}{KT}\right)$$

Let $2\left(\frac{2\pi m_h^* KT}{h^2}\right)^{3/2} = N_v$

$$\Rightarrow p = N_v \exp\left(\frac{E_v - E_F}{KT}\right)$$

Also $N_a^- = N_a \exp\left(\frac{E_F - E_a}{KT}\right)$ $\qquad$ E_a → acceptor energy level

$$\because p = N_a^-$$

$$\Rightarrow N_v \exp\left(\frac{E_v - E_F}{KT}\right) = N_a \exp\left(\frac{E_F - E_a}{KT}\right)$$

$$\Rightarrow \left(\frac{E_F - E_a}{KT}\right) - \left(\frac{E_v - E_F}{KT}\right) = \ln\left(\frac{N_v}{N_a}\right)$$

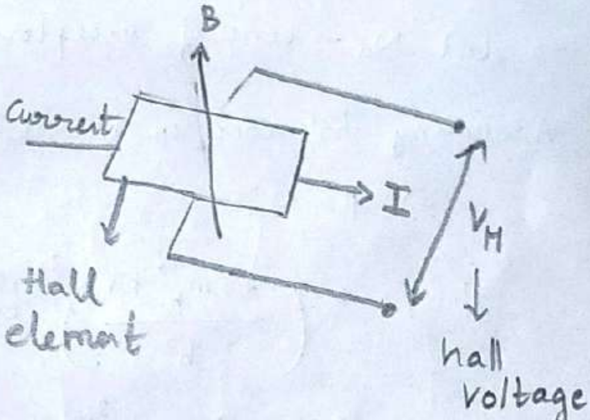
$$\Rightarrow \boxed{E_F = \left(\frac{E_v + E_a}{2}\right) - \left(\frac{KT}{2}\right)\ln\left(\frac{N_a}{N_v}\right)}$$

At $T = 0\,K$; $E_F = \frac{E_v + E_a}{2}$

put value of E_F in 'p' expression

we get $\boxed{p = (2N_a)^{1/2}\left(\frac{2\pi m_h^* KT}{h^2}\right)^{3/4} \exp\left(\frac{E_v - E_a}{2KT}\right)}$

* Hall effect :



$v \rightarrow$ velocity of $\bar{e}s$ in n-type s.c

$B \rightarrow$ transverse applied magnetic field

Force experienced by $\bar{e}s$ $F = Bev$

At equillibrium

$\Rightarrow eE_H = Bev$

$\Rightarrow E_H = Bv$ — ①

$\because J = nev$

$\Rightarrow v = \frac{J}{ne}$ — ②

from ① & ②

$\Rightarrow E_H = \frac{BJ}{ne}$

$\because R_H = \frac{1}{ne}$

$\Rightarrow E_H = BJR_H$

or $\Rightarrow \boxed{R_H = \frac{1}{ne} = \frac{E_H}{BJ}}$

$E_H \rightarrow$ hall electric field

$V_H \rightarrow$ hall voltage

$J \rightarrow$ current density

$n \rightarrow$ no. of charge carriers

$R_H \rightarrow$ Hall coefficient.

(i) Calculation of carrier concentration :

$\because R_H = \frac{1}{ne}$

$\Rightarrow \boxed{n = \frac{1}{eR_H}}$ (no. of $\bar{e}s$)

(ii) Determination of Mobility:

$\Rightarrow \sigma = ne\mu$

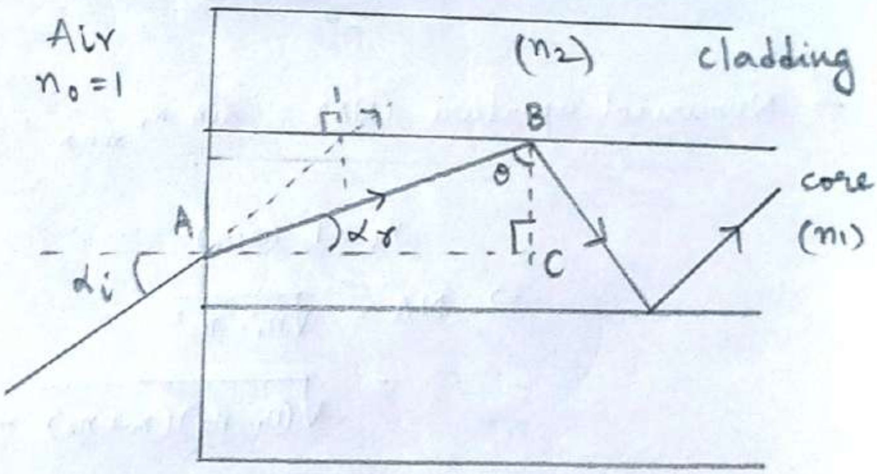
$\Rightarrow \boxed{\mu = \frac{\sigma}{ne} = \sigma R_H}$

$\sigma \rightarrow$ conductivity

$\mu \rightarrow$ mobility

* Acceptance angle in optical fibre transmission & Numerical Aperture: (11)

$\alpha_i \rightarrow$ acceptance angle
$n_0 \rightarrow$ ref. index of air
$n_1 \rightarrow$ ref. index of core
$n_2 \rightarrow$ ref. index of cladding
$\alpha_r \rightarrow$ angle of refraction



→ From ΔABC;

$$\Rightarrow \alpha_r + \theta = 90^\circ$$

$$\Rightarrow \alpha_r = 90^\circ - \theta \quad \text{— (1)}$$

→ From Snell's law;

$$\Rightarrow n_0 \sin\alpha_i = n_1 \sin\alpha_r$$

$$\Rightarrow \sin\alpha_i = \frac{n_1 \sin(90^\circ - \theta)}{n_0} \quad (\text{from (1)})$$

$$\Rightarrow \sin\alpha_i = \frac{n_1 \cos\theta}{n_0} \quad \text{— (2)}$$

Limiting condition $\theta = \theta_c$ for TIR

$$\Rightarrow \alpha_i = \alpha_{i_{max}}$$

$$\Rightarrow \sin\alpha_{i_{max}} = \frac{n_1 \cos\theta}{n_0}$$

$\because$ if $\theta = \theta_c$
$\sin\theta_c = \frac{n_2}{n_1}$ from Snell's law

$$= \frac{n_1}{n_0}\sqrt{1-\sin^2\theta_c}$$

$$= \frac{n_1}{n_0}\sqrt{1-\left(\frac{n_2}{n_1}\right)^2}$$

$$= \frac{\cancel{n_1}}{n_0}\sqrt{\frac{n_1^2-n_2^2}{\cancel{n_1^2}}}$$

$$= \sqrt{\frac{n_1^2-n_2^2}{n_0}}$$

∴ Acceptance angle $\boxed{\alpha i_{max} = \sin^{-1}\left(\frac{\sqrt{n_1^2 - n_2^2}}{n_0}\right)}$ (12)

→ Numerical aperture (NA) = $\sin \alpha i_{max} = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$

$n_0 = 1$ (air)

$\Rightarrow$ $NA = \sqrt{n_1^2 - n_2^2}$

$= \sqrt{(n_1 - n_2)(n_1 + n_2)}$ — (3)

∵ $\Delta = \frac{n_1 - n_2}{n_1}$ $\Delta \rightarrow$ fractional difference in ref. ind.

$\Rightarrow$ $\Delta n_1 = n_1 - n_2$ — (4)

put (4) in (3)

$\Rightarrow$ $NA = \sqrt{(\Delta n_1)(n_1 + n_2)}$

For most fibres ; $n_1 \approx n_2$

$\Rightarrow$ $NA = \sqrt{(\Delta n_1)(2n_1)}$

$\Rightarrow$ $\boxed{NA = n_1\sqrt{2\Delta}}$

* Calculation of electronic polarisibility : (α_e)

→ Without electric field

charge of nucleus = $+Ze$

charge of electron cloud = $-Ze$

$Z \rightarrow$ atomic no.

$R \rightarrow$ radius of atom

$$\text{Charge density } (\rho) = \frac{\text{charge}}{\text{volume}} = \frac{-Ze}{\frac{4}{3}\pi R^3}$$

$$\Rightarrow \rho = -\frac{3}{4}\frac{Ze}{\pi R^3}$$

→ When external field is applied ;

Lorentz force will seperate the nucleus & electron cloud

Columbs force will try to get them closer

∴ At equillibrium ; L.F = C.F along with distance of seperation (x)

Lorentz force $L.F = -ZeE$

Columbs force $C.F = +Ze \dfrac{Q}{4\pi\epsilon_0 x^2}$

$E \rightarrow$ electric field

$Q \rightarrow$ charge enclosed in sphere of radius x

Q = charge density of electron × Vol of sphere of radius x

$$= -\frac{\cancel{3}}{\cancel{4}}\frac{Ze}{\cancel{\pi} R^2} \times \frac{\cancel{4}}{\cancel{3}}\cancel{\pi} x^3$$

$$= \frac{-Zex^3}{R^3}$$

$$\therefore \text{Columbs force} = +Ze\left(\frac{-Zex^3}{R^3}\right)\frac{1}{4\pi\epsilon_0 x^2} = -\frac{Z^2e^2x}{4\pi\epsilon_0 R^3}$$

At eql; $LF = CF$

$$\Rightarrow -\cancel{Z}\cancel{e}E = -\frac{Z^{\cancel{2}}e^{\cancel{2}}x}{4\pi\epsilon_0 R^3}$$

$$\Rightarrow x = \frac{4\pi\epsilon_0 R^3 E}{Ze}$$

$\therefore$ Dipole moment $\mu = |\cancel{Z}e| \times \dfrac{4\pi\epsilon_0 R^3 E}{\cancel{Ze}} = 4\pi\epsilon_0 R^3 E$ (14)

also $\mu = \alpha_e E = 4\pi\epsilon_0 R^3 E$

$\alpha_e \rightarrow$ electronic polarisibility

$$\Rightarrow \boxed{\alpha_e = 4\pi\epsilon_0 R^3}$$

* Ionic polarisibility (α_i) :

$\rightarrow$ Dipole moment $\mu = e(x_1 + x_2)$

$x_1, x_2 \rightarrow$ distance of seperations of both ions

$\rightarrow$ Restoring force acting on ions $F = \beta_1 x_1 = \beta_2 x_2$

$F \rightarrow$ force exp by ions due to electric field.

$\beta_1, \beta_2 \rightarrow$ proportionatity constants

$\beta_2, \beta_1 \propto$ mass & ang. frequency of respective ions

$\because$ if m, M are mass of +ve & -ve ions

$\omega_0 \rightarrow$ ang. freq

$E \rightarrow$ electric field

$\alpha_i \rightarrow$ ionic polarisibility

$$\Rightarrow \beta_1 = m\omega_0^2 \quad ; \quad \beta_2 = M\omega_0^2$$

$$\because F = \beta_1 x_1 = \beta_2 x_2$$

$$\Rightarrow x_1 = \frac{F}{\beta_1} \quad ; \quad x_2 = \frac{F}{\beta_2}$$

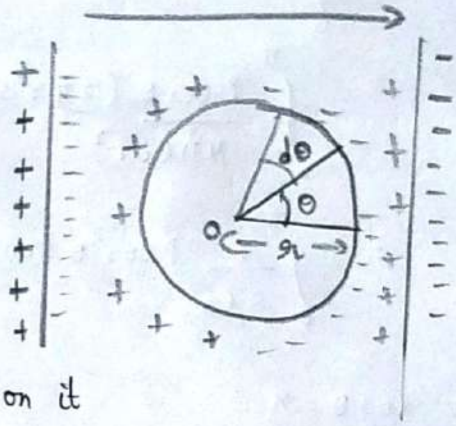
$$\Rightarrow x_1 = \frac{eE}{m\omega_0^2} \quad ; \quad x_2 = \frac{eE}{M\omega_0^2}$$

$$\Rightarrow \mu = e\left[\frac{eE}{m\omega_0^2} + \frac{eE}{M\omega_0^2}\right] = \alpha_i E$$

$$= \frac{e^2 E}{\omega_0^2}\left(\frac{1}{m} + \frac{1}{M}\right) = \alpha_i E$$

$$\Rightarrow \boxed{\alpha_i = \frac{e^2}{\omega_0^2}\left(\frac{1}{m} + \frac{1}{M}\right)}$$

* Internal electric field : (E_i)



→ Consider a dielectric slab in a uniform electric field E.

→ Let a molecule be at point 'O' & be surrounded by a cavity of radius 'r'.

→ The molecule experiences 3 electric fields on it

(i) external electric field E

(ii) electric field E_1 due to induced charges on the surface of cavity.

(iii) field E_2 due to molecular dipoles. But due to symmetry, they cancel out & $E_2 = 0$

∴ Internal field $E_i = E + E_1 + E_2 = E + E_1$

→ At each point of sphere, surface charge density

$$\sigma = P\cos\theta$$

P → polarisation vector

θ → angle b/w radius vector 'r' & direction of E.

→ Charge on element ds on the surface

$$dq = \sigma\, ds$$
$$= P\cos\theta\, ds$$

Also $ds = 2\pi r^2 \sin\theta\, d\theta$

$$\Rightarrow dq = P\cos\theta\,(2\pi r^2 \sin\theta\, d\theta)$$

→ This charge will produce an electric field dE_1 at centre of sphere

$$dE_1 = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}$$

(i) $dE_1\cos\theta$ parallel direction of E

(ii) $dE_1 \sin\theta$ ⊥lar to E, cancel out

∴ only parallel field component contributes to internal field.

$$\Rightarrow E_1 = \int_0^{\pi} dE_1 \cos\theta \, ds$$

$$= \int_0^{\pi} \frac{p\cos\theta \, (2\pi r^2 \sin\theta \, d\theta)\cos\theta}{4\pi\epsilon_0 r^2}$$

$$= \int_0^{\pi} \frac{1}{2\epsilon_0} p \cos^2\theta \sin\theta \, d\theta$$

let $\cos\theta = x$

$\Rightarrow -\sin\theta \, d\theta = dx$

at $\theta = 0^\circ \Rightarrow x = 1$

at $\theta = \pi \Rightarrow x = -1$

$$\Rightarrow E_1 = \frac{p}{2\epsilon_0} \int_{+1}^{-1} -x^2 \, dx$$

$$\Rightarrow E_1 = \frac{p}{3\epsilon_0}$$

$$\therefore \boxed{E_i = E + \frac{p}{3\epsilon_0}}$$

* Clausius Masoti Equation :

$\rightarrow$ let $\alpha_0 = \alpha_i = 0$

$\rightarrow$ Polarisation vector $P = N\alpha E$

$= N\alpha_e E_i$

$\alpha_0 \rightarrow$ orientation polarizibility

$E_i \rightarrow$ internal electric field

$\alpha_e \rightarrow$ electronic polarisibility

$$\because E_i = E + \frac{P}{3\epsilon_0}$$

$$\Rightarrow P = N\alpha_e \left(E + \frac{P}{3\epsilon_0} \right)$$

$$\Rightarrow P = \frac{N\alpha_e E}{1 - \frac{N\alpha_e}{3\epsilon_0}} \quad \text{— (1)}$$

→ Displacement vector $D = P + \epsilon_0 E$

$$\Rightarrow P = D - \epsilon_0 E \qquad ; \quad D = \epsilon_0 \epsilon_r E$$

$$\Rightarrow P = \epsilon_0 \epsilon_r E - \epsilon_0 E$$

$\epsilon_0 \rightarrow$ permitivity in free space

$$\Rightarrow P = \epsilon_0 E(\epsilon_r - 1) \quad \text{—} \; (2)$$

$\epsilon_r \rightarrow$ relative permitivity

From (1) & (2)

$$\Rightarrow \epsilon_0 \cancel{E}(\epsilon_r - 1) = \frac{N\alpha_e \cancel{E}}{\left(1 - \frac{N\alpha_e}{3\epsilon_0}\right)}$$

$$\Rightarrow 1 - \frac{N\alpha_e}{3\epsilon_0} = \frac{N\alpha_e}{\epsilon_0(\epsilon_r - 1)}$$

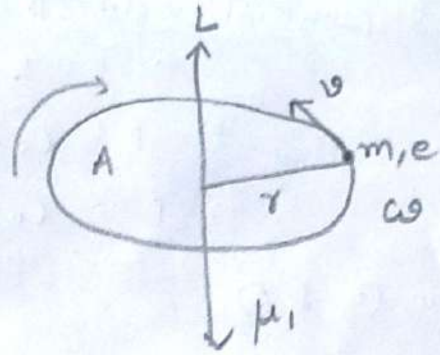
$$\Rightarrow 1 = \frac{N\alpha_e}{3\epsilon_0} + \frac{N\alpha_e}{\epsilon_0(\epsilon_r - 1)}$$

$$\Rightarrow 1 = \frac{N\alpha_e}{\epsilon_0}\left(\frac{1}{3} + \frac{1}{\cancel{\epsilon_0}(\epsilon_r - 1)}\right)$$

$$\Rightarrow 1 = \frac{N\alpha_e}{3\epsilon_0}\left(\frac{\epsilon_r + 2}{\epsilon_r - 1}\right)$$

$$\Rightarrow \boxed{\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N\alpha_e}{3\epsilon_0}}$$

* Orbital magnetic moment of e^-



Current $I = -\frac{\text{charge flow}}{\text{unit time}} = -\frac{e}{T}$

$T \rightarrow$ time taken for one revolution

$T = \frac{2\pi}{\omega}$ $\qquad \omega \rightarrow$ angular speed

$\Rightarrow \quad I = -\frac{e\omega}{2\pi}$ — ①

$\rightarrow$ Magnitude of magnetic moment $\mu_m = I \cdot A$

$$= -\frac{e\omega}{2\cancel{\pi}}(\cancel{\pi} r^2)$$

$$= -\frac{e\omega r^2}{2}$$

$$= \frac{-e}{2m}(m\omega r^2)$$

$\because L = m\omega r^2$ $\qquad = -\left(\frac{e}{2m}\right)L$ — ②

$\hookrightarrow$ orbital angular momentum

$\Rightarrow$ Orientation of L when placed in external magnetic field

$$L_{\ell, B} = m_\ell \frac{h}{2\pi} \quad — ③$$

put ③ in ②

$\Rightarrow \quad \mu_m = -\left(\frac{e}{2m}\right) m_\ell \frac{h}{2\pi}$

$$= -\left(\frac{eh}{4\pi m}\right) m_\ell$$

$$\mu_m = -\mu_B m_\ell \quad — ④$$

where $\boxed{\mu_B = \frac{eh}{4\pi m} = 9.27 \times 10^{-24} Am^2}$ is called Bohr magneton.