UNIT-IV

1 Locate the stationary points and examine their nature (a) $f(x,y)=x^4+y^4-2x^2+4xy-2y^2$ (b) $x^3+3xy^2-15x^2-15y^2+72x$

2 Find the stationary points of $u(x,y)=\sin x\sin y\sin(x+y), 0<x,y<\pi$ and find the maximum value of u

3 Examine the following functions for extreme values: (a). $xye^{-(2x+3y)}$.

(b) $(x^2+y^2)e^{6x+2x^2}$

4 Examine the following functions for extreme values (a) f (x , y) = 6 x + 8y $-x^2$ $-2y^2$+ 50
(b) f (x , y) = x^2+2y^2 – 4 x -16 y + 30

5 Find the maximum and minimum values of the function (a) $f(x,y)=2(x^2-y^2)-x^4+y^4$ (b) $x^3y^2(1-x-y)$

6 (a) Minimize $x^2+y^2+z^2$ subject to the constraint $x+y+z=3a$.
(b) Maximum of xy^2z^3 subject to the constrain x+y+z= 12

7 Find the volume of the largest rectangular box that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

8 A rectangular box open at the top has a constant surface area 108 sq ft. Find its dimensions such that its volume is a maximum.

9 Given $x+y+z=a$, find the maximum value of $x^m y^n z^p$.

10 Find the points on the surface $z^2=1+xy$ that are nearest to the origin.

11 Find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid $4x^2+4y^2+9z^2=36$.

12 Show that the functions $u=x+y+z,\ v=x^2+y^2+z^2-2xy-2yz-2zx$ and

$w=x^3+y^3+z^3-3xyz$ are functionally

13 Given the transformations $u=\tan^{-1}x+\tan^{-1}y$ and $v=\frac{x+y}{1-xy}$ show that u and v are

functionally related and hence find the relation between them

14 Expand $f(x,y)=e^y\log(1+x)$ in powers of x *and* y.

15 Find the Taylor's expansion of $f(x,y)=\cot^{-1}(xy)$ in powers of $(x+0.5)$ *and* $(y-2)$ up to second degree terms.

16 If $u=x+y+z,\ uv=y+z,\ uvw=z$ then prove that $\frac{\partial(x,y,z)}{\partial(u,v,w)}=u^2v$

17 If $x=\sqrt{vw}$, $y=\sqrt{wu}x=\sqrt{uv}$ and$u=\rho\,Sin\Theta Sin\varphi$, $v=\rho\,Sin\Theta Cos\varphi$, $w==\rho$

$Cos\Theta$ then find $J\left(\frac{x,y,z}{\rho,\theta,\varphi}\right)$

18 Using polar co-ordinates, show that $u_{xx}+u_{yy}=0$ transforms into $u_{rr}+\frac{1}{r}u_r+\frac{1}{r^2}u_{\theta\theta}=0$

19 i). If $u=e^{x+y}/(e^x+e^y)$, show that $u_x+u_y=u$.

ii). If $U=\ln(x^3+y^3-x^2y-xy^2)$ show that $U_{xx}+U_{yy}+2U_{xy}=-4(x+y)^{-2}$

20 Find d^2y/dx^2 given $x^5+y^5=5a^3x^2$.