UNIT-III

- Basics of counting
  - sum rule
  - product rule
- Permutation
  - with repetition
  - without repetition
- Combination
  - with repetition
  - without repetition
- coefficients
  - Binomial coefficients
  - multiple Binomial coefficients
- Principle of Inclusive-exclusive

* Basics of counting:-

Sum rule:-

- Let $T_1, T_2, T_3, T_4 --- T_n$ different types of tasks then T_1 task performance n_1 ways

  T_2 Task performance n_2 ways

  $\vdots$

  T_n task performance n_n ways

$$\boxed{\text{Sum rule} = n_1 + n_2 + n_3 + --- + n_n}$$

Product rule:-

- Let $T_1, T_2, T_3 --- T_n$ different types of tasks then T_1 task performance n_1 ways

  T_2 task performance n_2 ways

  $\vdots$

  T_n task performance n_n ways

$\therefore$ Product rule $= n_1 \times n_2 \times n_3 \times \cdots \times n_n$

*Problem ①:- Find the three digit odd integer (no repetitions)

$\{0,1,2,3,4,5,6,7,8,9\}$

T_1 T_2 T_3 T_4 T_5

Sol:

__ __ __

T_1 task performance $\underline{8}$ $\underline{8}$ $\underline{①}$ $= 8\times8 = 64$ $+$

$\times 0$

T_2 task performance $\underline{8}$ $\underline{8}$ ③ $= 8\times8 = 64$ $+$

T_3 task performance $\underline{8}$ $\underline{8}$ ⑤ $= 8\times8 = 64$ $+$

T_4 task performance $\underline{8}$ $\underline{8}$ $\underline{⑦}$ $= 8\times8 = 64$ $+$

T_5 task performance $\underline{8}$ $\underline{8}$ $\underline{⑨}$ $= 8\times8 = 64$

320

*Problem②:- Find the three digit even integer (no repetition)

Sol:

$\{\underline{0},1,\underline{2},3,\underline{4},5,\underline{6},7,\underline{8},9\}$

$\underline{9}$ $\underline{8}$ $\underline{⓪}$ $= 72$ $+$

$\underline{8}$ $\underline{8}$ $\underline{①}$ $= 64$ $+$

$\underline{8}$ $\underline{8}$ ④ $= 64$ $+$

$\underline{8}$ $\underline{8}$ $\underline{⑥}$ $= 64$ $+$

$\underline{8}$ $\underline{8}$ $\underline{⑧}$ $= 64$

$\boxed{328}$

*Problem③:- Find the 5 digit number with "7 digit number" exactly once

Sol:- 8 9 9 9 7

8 9 9 7 9

8 9 7 9 9

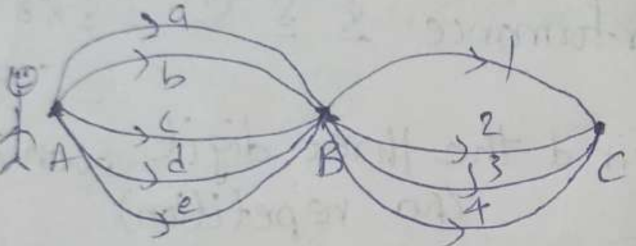
8 7 9 9 9

7 9 9 9 9 $= 8^4 + 9^6$

$= 29889$

*Problem ④: There are 5 Bus routes between A and B & 4 bus routes between B & C. In how many ways a student make a round trip A to C via B (repetitions are not allowed).

Sol:-



$5 \times 4 \times 3 \times 4 = 240$

$(m \times n \times (n-1) \times (m-1))$

16/10/24

Wednesday

*Problem ①:- There are 35 students and 4 teachers in how many ways a student shake hands with other students and all teachers.

Sol:- Students | Teachers

1) | 1) $\Rightarrow 34 + 4 = 38$

2) | 2) $\Rightarrow 33 + 4 = 37$

⋮ | 3) $\Rightarrow 32 + 4 = 36$

35) | 4) $\Rightarrow 0 + 4 = 4$

$38 + 37 + 36 + \cdots + 4$

*Problem ②:- Find the proper division of 4,41,000

Sol:-

$$\begin{array}{r|l} 2 & 4,41,000 \\ 2 & 2,20500 \\ 2 & 110250 \\ 5 & 55125 \\ 5 & 11025 \\ 5 & 2205 \\ 7 & 441 \\ 7 & 63 \\ 3 & 9 \\ 3 & 3 \\ & 1 \end{array}$$

$4,41,000 = 2^3 \times 3^2 \times 5^3 \times 7^2$

Task1 Task2 Task3 Task4

$2^0 2^1 2^2 2^3$ $3^0 3^1 3^2$ $5^0 5^1 5^2 5^3$ $7^0 7^1 7^2$

$4 \times 3 \times 4 \times 3 = 144$

*Problem ③:- In how many ways $10 \leq n \leq 9999$

Sol:- Task $1 \rightarrow$ 2 digits $\underset{\times 0}{\underline{9}}\ \underline{10} = 90$

Task $2 \rightarrow$ 3 digits $\underset{\times 0}{\underline{9}}\ \underline{10}\ \underline{10} = 900$

Task $3 \rightarrow$ 4 digits $\underset{\times 0}{\underline{9}}\ \underline{10}\ \underline{10}\ \underline{10} = 9000$

$\overline{9990}$

*Problem ④:- A palindrome word reads forward & backward "In how many ways 9 letter palindrome word possible"?

Sol:-

i) $\underline{}_{26}\ \underline{}_{26}\ \underline{}_{26}\ \underline{}_{26}\ \underline{}_{26}\ \underline{}_{1}\ \underline{}_{1}\ \underline{}_{1}\ \underline{}_{1}$

$= 26 \times 26 \times 26 \times 26 \times 26 = 26^5$

ii) In how many ways five digit number palindrome is possible

$\underline{}_{9}\ \underline{}_{10}\ \underline{}_{10}\ \underline{}_{1}\ \underline{}_{1}$

$= 9 \times 10^2 = 900$

*Problem (5): In how many ways 6 digit telephone number

(1) only even numbers are used

(2) only 2,4,7,8,9 used

(3) first digit start "2" and end with zero

(4) only odd number close with "prime number"

Sol:- (1)

$$\underset{5}{\_}\ \underset{5}{\_}\ \underset{5}{\_}\ \underset{5}{\_}\ \underset{5}{\_}\ \underset{5}{\_} = 5^6$$

(2)

$$\underset{5}{\_}\ \underset{5}{\_}\ \underset{5}{\_}\ \underset{5}{\_}\ \underset{5}{\_}\ \underset{5}{\_} = 5^6$$

(3)

$$\underset{1}{\_}\ \underset{10}{\_}\ \underset{10}{\_}\ \underset{10}{\_}\ \underset{10}{\_}\ \underset{1}{\_} = 10^4$$

(4)

$$\underline{5}\ \underline{5}\ \underline{5}\ \underline{5}\ \underline{5}\ \underset{3,5,7}{\underline{3}} = 5^5 \times 3$$

* Permutations and Combinations:-

* Permutations:-

- There are 'n' objects we have to arrange 'r' objects (without order) therefore

$$\boxed{{}^nP_r = \frac{n!}{(n-r)!}}$$

* Combinations:-

- There are 'n' objects we have to select 'r' with order

$$\boxed{{}^nC_r = \frac{n!}{(n-r)!\,r!}}$$

*Problem①: Find the permutation of "DIFFICULTY" word.

Sol:- 10 letters $\Rightarrow \dfrac{n!}{r_1!\,r_2!\cdots r_n!} = \dfrac{10!}{2!\,2!}$.

$\downarrow$ repetitions letters

*Problem②:- Find the permutations of with digits 3, 4, 4, 5, 5, 6, 7 "n" exceeds 50,00,000

Sol:- $n > 50,00,000$ 3, 4, 4, 5, 5, 6, 7

5 _ _ _ _ _ _ $= \dfrac{6!}{2!}$

5 _ _ _ _ _ _ $= \dfrac{6!}{2!}$

6 _ _ _ _ _ _ $= \dfrac{6!}{2!\,2!}$

7 _ _ _ _ _ _ $= \dfrac{6!}{2!\,2!}$

$= 2\times\dfrac{6!}{2!} + 2\times\dfrac{6!}{2!\,2!}$

*Problem③:- There are 20 females and 15 males in a juniour class room and 30 females and 20 males in a seniour class room in how many ways a comittiee of 10 students with juniours & 3 females

Sol:-

J		S		Comittiee (10)
				5 J 3 Fe
				10
F (20)	M (15)	F (30)	M (20)	
$^{20}C_3$	$^{15}C_2$	$^{30}C_0$	$^{20}C_5$	$^{20}C_3 \times {}^{15}C_2 \times {}^{30}C_0 \times {}^{20}C_5 +$
$^{20}C_2$	$^{15}C_3$	$^{30}C_1$	$^{20}C_4$	$^{20}C_2 \times {}^{15}C_3 \times {}^{30}C_1 \times {}^{20}C_4 +$
$^{20}C_1$	$^{15}C_4$	$^{30}C_2$	$^{20}C_3$	$^{20}C_1 \times {}^{15}C_4 \times {}^{30}C_2 \times {}^{20}C_3 +$

$^{20}C_0$	$^{15}C_5$	$^{30}C_3$	$^{20}C_2$	$^{20}C_0 \times {}^{15}C_5 \times {}^{30}C_3 \times {}^{20}C_2$

*Problem ④:- There are "15 Multiple choice problems" in how many ways a student can answer only four problems are correct

Sol:-

$$\left.\begin{array}{l} 1) \_\ \_\ \_\ \_ \\ 2) \_\ \_\ \_\ \_ \\ \vdots \\ 15) \_\ \_\ \_\ \_ \end{array}\right\} {}^{15}C_4$$

$$= {}^{15}C_4 \times 3^{11}$$

*Problem ⑤:- There are 50 football games in how many ways only 22 games are winning choices

Sol:-

$$\left.\begin{array}{l} 1) \_\ \_\ \_ \\ 2) \_\ \_\ \_ \\ \vdots \\ 50) \_\ \_\ \_ \end{array}\right\} {}^{50}C_{22} \times 2^{28}$$

*Problem ⑥:- A Gokaraju shoe store maintain 30 different styles of shoes, 20 different types of colours, 15 different types of length 20 different types of width in how many ways gokaraju shoe store kept in a stall?

Sol:-

Gokaraju

Styles (30)	Colour (20)	length (15)	width (20)
30 ×	20 ×	15 ×	20

$$= 30 \times 20 \times 15 \times 20$$

17/10/24 Thursday

* A poker Game (52 cards)

- 4 suits
  - Kalaver (13)
  - Spade (13)
  - Diamond (13)
  - Heart (13)
- 13 kinds
  - Joker (4)
  - 2 (4)
  - 3 (4)
  - ... K (4)
  - $\boxed{= 13 \times 4 = 52}$
- colours
  - Red (26)
  - Black (26)
  - 52 cards

***Problem ①:-** In how many ways a poker game consists of "5 cards hands"

1. 3 aces, 2 kings...
2. 5 cards from one suit
3. 2 cards from one suit & 3 cards from another suit
4. 3 cards from one suit & 2 cards from different suit
5. 3 cards from one kind and 2 cards from another kind
6. 3 cards from one kind & 3 cards from different kinds
6. 2 cards from one kind & 3 cards from different kinds
7. 5 cards from different kinds

Sol:-
1.) $^4C_3 \times {}^4C_2$
2.) $^{13}C_5 \times {}^4C_1$
3.) $^4C_1 \times {}^{13}C_2 \times {}^{39}C_3$
4.) $^4C_1 \times {}^{13}C_3 \times {}^3C_1 \times {}^{13}C_1 \times {}^2C_1 \times {}^{13}C_1$
5.) $^{13}C_1 \times {}^4C_3 \times {}^{48}C_2$
6.) $^{13}C_1 \times {}^4C_2 \times {}^{12}C_1 \times {}^4C_1 \times {}^{11}C_1 \times {}^4C_1 \times {}^{10}C_1 \times {}^4C_1$
7.) $^{13}C_1 \times {}^4C_1 \times {}^{12}C_1 \times {}^4C_1 \times {}^{11}C_1 \times {}^4C_1 \times {}^{10}C_1 \times {}^4C_1 \times {}^9C_1 \times {}^4C_1$

***Problem ②:-** In how many ways a poker game consists of "5 cards hands"

1. 3 kings
2. 2 cards from single kind

③ Exactly 2 pairs

④ 3 cards from singde suit

⑤ 3 cards from single kind, 2 card from different kind

Sol:-

① $4C_3 \times 48C_2$

② $13C_1 \times 4C_2 \times 48C_3$

③ $13C_1 \times 4C_2 \times 12C_1 \times 4C_2 \times 44C_1$ (52-4-4)

④ $4C_1 \times 13C_3 \times 39C_2$

⑤ $13C_1 \times 4C_3 \times 12C_1 \times 4C_1 \times 11C_1 \times 4C_1$

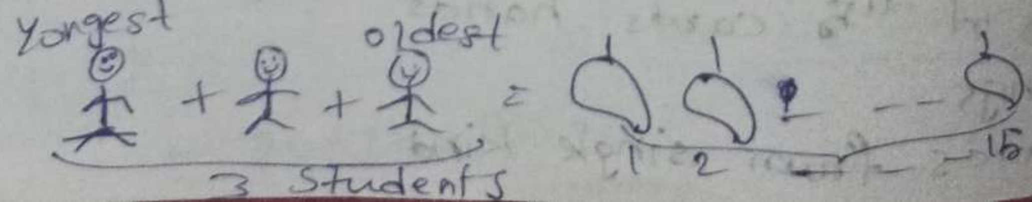
*Problem ③:- A new borna baby can be given 1 (or) 2 (or) 3 names. In how many ways can a child be given names we can choose from 300 names.

Sol:- $300C_1 + 300C_1 \times 299C_1 + 300C_1 \times 299C_1 \times 298C_1$

*Problem ④:- A total 15 mangoes distributing to 3 Students

① Every student get equal mangoes

② Yongest student get 5 mangoes and oldest student get 3 mangoes.

Sol:-


(1) 3 students = equal mangoes $\Big\} \dfrac{n!}{n_1!\, n_2!\, n_3!}$

$\Rightarrow n = n_1 + n_2 + n_3$

$$= \frac{15!}{5!\,5!\,5!}$$

(2) $\left.\begin{array}{l} n = 15 \\ n_1 = 5 \\ n_2 = 7 \\ n_3 = 3 \end{array}\right\}$ $n = n_1 + n_2 + n_3$

$$= \frac{15!}{5!\,7!\,3!}$$

*Problem (6) :- A classroom consists of 26 girls and 30 male students. In how many ways

(1) 2 girls or 3 boys

(2) 2 boys and 3 girls

(3) 10 boys and 10 girls

Sol:-

1.) $26C_2 + 30C_3$

2.) $30C_2 \times 26C_3$

3.) $30C_{10} \times 26C_{10}$

* Permutation & Combinations with repetitions:-

- There are 'n' objects we have to select 'r' objects (with order) $= n^r$
- There are 'n' objects we have to arrange 'r' objects (without order) $= {}^{(n-1+r)}C_r$

(or)

- The no. of non negative integral solutions

$$x_1 + x_2 + x_3 + \cdots + x_n = r$$

(or)

'r' similar balls distributing into 'n' different boxes

$$\underbrace{\square \quad \square \quad \square \quad \square \quad \square}_{n} = r$$

19/10/24

Saturday

*Problem ①: In how many ways 20 similar balls distributing into 5 different boxes where each box is non-empty.

Sol:-

$$\boxed{}\ \boxed{}\ \boxed{}\ \boxed{}\ \boxed{} = 20$$

$$x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5$$

$$x_1 + x_2 + x_3 + x_4 + x_5 = 20$$

here $n=5,\ r=20$

$$^{5-1+20}C_{20} = {}^{24}C_{20} \text{ (or) } {}^{24}C_4$$

*Problem ②:- In how many ways $x_1+x_2+x_3+x_4 = 23$ where $x_i \geq i$

Sol:-

$$\boxed{1}\ \boxed{2}\ \boxed{3}\ \boxed{4} = 23$$

$$x_1 \quad x_2 \quad x_3 \quad x_4$$

$i = 1 \Rightarrow x_1 \geq 1$

$i = 2 \Rightarrow x_2 \geq 2$

$i = 3 \Rightarrow x_3 \geq 3$

$i = 4 \Rightarrow x_4 \geq 4$

$$(x_1+1)+(x_2+2)+(x_3+3)+(x_4+4) = 23$$

$$x_1+x_2+x_3+x_4 = 13$$

$$n=4,\ r=13$$

$$^{4-1+13}C_{13} = {}^{16}C_{13} \text{ or } {}^{16}C_3$$

*Problem ③:- In how many ways $x_1+x_2+x_3+x_4+x_5 = 35$ where $x_1 \geq 3,\ x_2 \geq -4,\ x_3 \geq 5,\ x_4 \geq \cancel{6}\ -2,\ x_5 \geq 6$

Sol:-

$$\boxed{3}\ \boxed{-4}\ \boxed{5}\ \boxed{-2}\ \boxed{6} = 35$$

$$x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5$$

$(x_1+3)+(x_2-4)+(x_3+5)+(x_4-2)+(x_5+6)=35$

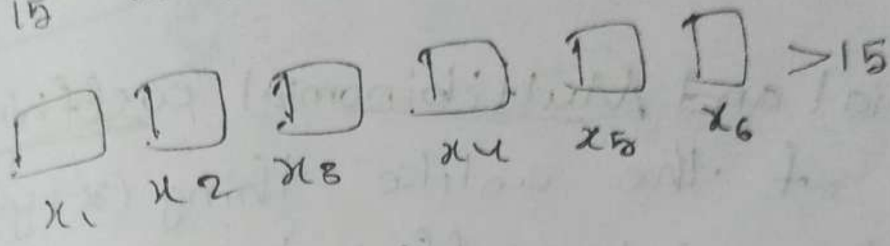
$x_1+x_2+x_3+x_4+x_5=27$

$n=5\ ,\ r=27$

$^{5-1+27}C_{27}={}^{31}C_{27}$

Problem (4):- In how many ways $x_1+x_2+x_3+x_4+x_5+x_6>15$ where each box is non-empty.

Sol:-



*Note $x_1+x_2+x_3+\text{---}+x_n>m$

$$\boxed{x_1+x_2+x_3+\text{---}+x_{n+1}=m+1}$$

$x_1+x_2+x_3+x_4+x_5+x_6+x_7=16$

$n=7,\ r=16$

$^{7-1+16}C_{16}={}^{22}C_{16}={}^{22}C_6$

Problem (5):- A total amount of 1500 rupees distributing into three persons A, B and C (in multiple of 100 rupees)

i) Every one must get atleast 300 rupees

ii) 'A' must get at least ₹300 and B, C must get atleast ₹400

Sol:-

$A+B+C=1500$

$A+B+C=15$

i) $A\geq 3,\ B\geq 3,\ C\geq 3\}\ (A+3)+(B+3)+(C+3)=15$

$A+B+C=6$ $\qquad$ $^{3-1+6}C_6={}^8C_6$

$n=3,\ r=6$

(2) $\left.\begin{matrix} A \geq 3, B \geq 4, C \geq 4 \end{matrix}\right\}$ $(A+3)+(B+4)+(C+4)=15$

$\boxed{A+B+C=4}$

$n=3, r=4$

${}^{3-1+4}C_4 = {}^{6}C_4$

Zukoba

Thursday

*Binomial and Multibinomial coefficient:

- The sum of the unlike thing $(x+y)^n$ is called binomial coefficient
- The sum of the unlike things $(x+y+z)^n$ is called ~~[illegible]~~ trinomial coefficient.
- The sum of the unlike things $(x_1+x_2+x_3+ \cdots x_n)^n$ is called multibinomial coefficient

Binomial coefficients	Multi Binomial coefficients
$(x+y)^0 = 1$ $(x+y)^1 = x+y$ $(x+y)^2 = x^2+y^2+2xy$ $(x+y)^3 = x^3+y^3+2x^2y+2xy^2$ $\vdots$ $(x+y)^n = {}^nC_0 x^n y^0 + \cdots {}^nC_n x^0 y^n$	Let n, t both are positive numbers the coefficient of $(x_1+x_2+x_3+ \cdots x_t)^n$ in the expansion of $\boxed{\dfrac{n!}{n_1! n_2! \cdots n_t!} (x_1)^{n_1}(x_2)^{n_2} \cdots}$ here $\boxed{n = n_1+n_2+ \cdots n_t}$

- *Problem: Find the co-efficient of $x^2 y^2$ in the expansion of $(3x^2-y)^3$?

Sol:- Binomial $(3x-y)^3$

$= (3x)^3 + (-y)^3 + 3(3x)^2(-y) + 3(3x)(-y)^2$

$= 27x^3 - y^3 - 9x^2y + 9xy^2$

$\therefore$ The coefficient of xy^2 is $+9$

Multibinomial:

$(3x-y)^3 \Rightarrow xy^2$

$n = 3,\ n_1 = 1,\ n_2 = 2$

$x_1 = 3x,\ x_2 = -y$

$= \frac{3!}{2!}(3x)^1(-y)^2$

$= 3 \times 3x \times y^2 = 9xy^2$

Problem ②: Find the coefficient of $x^3y^3z^2$ in the expansion of $(x+3y-z)^8$

Sol: Multibinomial:-

$(x+3y-z)^8 \Rightarrow x^3y^3z^2$

$n = 8,\ n_1 = 3,\ n_2 = 3,\ n_3 = 2$

$n = n_1 + n_2 + n_3$

$\frac{8!}{3!3!2!}(x)^3(3y)^3(-z)^2$

$= \frac{8!}{3!3!2!} x^3\, 27\, y^3 z^2 =$

$= \boxed{\frac{8!}{3!3!2!}\, 27}\, x^3y^3z^2$

Problem ③: Find the coefficient of $a^3b^2c^3d^4$ in the expansion of $(a+2b+3c+4d+z)^{16}$

sol:-

$$a^3 b^2 c^3 d^4 \leftarrow (a+2b+3c+4d+z)^{16}$$

$$n=16,\ n_1=3,\ n_2=2,\ n_3=3,\ n_4=4$$

$$n_5+n_1+n_2+n_3+n_4$$

$$16 \qquad\qquad \Rightarrow 12$$

$$n_5=4\ (16-12)$$

$$= \frac{16!}{2!3!3!4!4!} (a)^3 (2b)^2 (3c)^3 (4d)^4 (z)^4$$

$$\boxed{= \frac{16!}{2!3!3!4!4!}\ 4\times 27\times 256 \times z^4} \ a^3 b^2 c^3 d^4$$

*Problem 4:- Find the coefficient of $x^{11} y^4$ in the expansion of $(3x^3 - 2xy^2 + z)^6$

sol:-

$$(3x^3 - 2xy^2 + z)^6$$

$$n=6$$

$$\frac{6!}{n_1!\,n_2!\,n_3!}\left((3x^3)^{n_1} (-2xy^2)^{n_2} (z)^{n_3}\right.$$

$$n=6 \Rightarrow \boxed{n_1+n_2+n_3=n=6}$$

$$x^{11} y^4 = (3x^3)^{n_1} (-2xy^2)^{n_2} (z)^{n_3}$$

$$x^{11} = (x^3)^{n_1} (x)^{n_2}$$

$$\boxed{3n_1+n_2=11}$$

$$y^4 = (y^2)^{n_2}$$

$$2n_2 = 4 \Rightarrow \boxed{n_2=2}$$

$3n_1 = 9$

$\boxed{n_1 = 3}$

$n_1 + n_2 + n_3 = 6$

$n_3 = 6 - 5 \Rightarrow \boxed{n_3 = 1}$

$= \dfrac{6!}{3!\,2!\,1!} (3x^3)^3 (-2xy^2)^2 (z)^1$

$$\boxed{= \frac{6!}{3!\,2!\,1!} \times 27 \times 4 \times z} \; (x^{11} y^4)$$

*Problem (5):- Find the x variable in the expansion of below problems

(1) $\binom{12}{3,5,x}$ (2) $\binom{15}{2\;3\;5\;x}$

Sol:-

(1) $n = 12$

$n_1 + n_2 + n_3 = 12$

$x = 12 - 8$

$x = 4$

(2) $n = 15$

$n_1 + n_2 + n_3 + n_4 = 15$

$x = 15 - 10$

$x = 5$

26/10/24 Saturday

*Principle of Inclusion - Exclusion:-

Inclusion: $+$ Exclusion: $-$

Two variable $+$ $-$

three variables $+$ $-$ $+$

four variables $+$ $-$ $+$ $-$

⋮

n variables $+$ $-$ $+$ $-$ $+$ - - - $-$ $+$

• Let $A_1, A_2, A_3 - - - A_n$ be non-empty sets then

$$\boxed{|A_1 \cup A_2 \cup A_3 \cup - - - A_n| = \sum_{i=1}^{n} A_i - \sum_{i=1, j=1}^{n} (A_i \cap A_j) - - - (1) + \sum_{i=1, j=1 - - n}^{i,j - n} (A_i \cap A_j - - - A_n)}$$

* **Problem ①**: In how many ways in between 1 and 300
i) divisible by 3 or 5 or 8
ii) Not divisible by 3 and 5 and 8

Sol:
⇒ Step 1 :- $A = \{1, 2, 3 - - - 300\}$

Step-2 :- A_1 = divisible by 3
A_2 = divisible by 5
A_3 = divisible by 8

Step 3 :- $|A_1|$ = divisible by 3 = $\left|\frac{300}{3}\right| = 100$
$|A_2|$ = divisible by 5 = $\left|\frac{300}{5}\right| = 60$
$|A_3|$ = divisible by 8 = $\left|\frac{300}{8}\right| = 35$

Step 4 :- $|A_1 \cap A_2|$ = divisible by (3 & 5) = $\left|\frac{300}{3 \times 5}\right| =$
$|A_1 \cap A_3|$ = divisible by (3 & 8) = $\left(\frac{300}{3 \times 8}\right) = 12$
$|A_2 \cap A_3|$ = divisible by (5 & 8) = $\left(\frac{300}{5 \times 8}\right) = 7$

Step 5 :- $|A_1 \cap A_2 \cap A_3|$ = divisible by (3, 5, 8) = $\left(\frac{300}{3 \times 5 \times 8}\right)$
$= 2$

Step 6 :- Divisible by 3 (or) 5 (or) 8 = $|A_1 \cup A_2 \cup A_3|$
Step 7 :- $|A_1 \cup A_2 \cup A_3| = \{A_1 + A_2 + A_3\} - \{(A_1 \cap A_2) + (A_1 \cap A_3)$

$+(A_1 \cap A_3)\} - \{(A_1 \cap A_2 \cap A_3)\}$

step = $(100+60+35) - (20+12+7)+2$

$= 195 - 39 + 2$

$= 197 - 39$

$= 158$

step 8:- not divisible by 3 & 5 & 8

$A_1 \cap A_2 \cap A_3 = |A| - \{(A_1 \cup A_2 \cup A_3)\} = 300 - 158$

$= 142$

*Problem ②:- In how many ways

1. Hearts (or) spades
2. Red cards (or) Diamonds
3. Kings (or) Kalavar's
4. A card from 2 to 10 by using principle of inclusion and exclusion.

sol:-

1. Hearts (or) spades

$|A_1| =$ Hearts $= 13$

$|A_2| =$ spades $= 13$

$|A_1 \cap A_2| = 0$

$|A_1 \cup A_2| = \{(A_1)+(A_2)\} - \{(A_1 \cap A_2)\}$

$= (13+13) - 0$

$= 26$

2. Red cards (or) Diamonds

$|A_1| =$ Red cards $= 26$

$|A_2| =$ Diamonds $= 13$

$|A_1 \cap A_2| = 13$

$|A_1 \cup A_2| = (26+13) - 13$
$= 26$

(3.) Kings or kalavar's

$|A_1| = 4$

$|A_2| = 13$

$|A_1 \cap A_2| = 1$

$|A_1 \cup A_2| = (13+4) - 1$
$= 16$

(4.) A card from 2 to 10

$$\left.\begin{array}{l} |A_1| = \text{A card from } 2 = 4 \\ |A_2| = \text{A card form } 3 = 4 \\ |A_3| = \text{A card from } 4 = 4 \\ \vdots \\ |A_9| = \text{A card from } 10 = 4 \end{array}\right\}$$

$4 \times 9 = 36$

* Problem (3):- In how many ways $\{5\ a's, 4\ b's, 3\ c's\}$ so that all identical letters not a single block by using principle of inclusion and exclusion

sol:- Step 1:- set $A = \{5\ a's, 4\ b's, 3\ c's\}$

$$\boxed{|A| = \frac{12!}{5!\,4!\,3!}}$$

$\boxed{aaaaa}$

Step 2:- $|A_1|$ = single block of 5 a's $= \frac{(12-5+1)!}{4!\,3!}$

$= \frac{8!}{4!\,3!}$

$|A_2| =$ single block of 4 b's $= \frac{(12 - \boxed{bbbb} + 1)!}{5!\,3!}$

$= \frac{9!}{5!\,3!}$

$|A_3| =$ single block of 3 c's $= \frac{(12 - \boxed{ccc} + 1)!}{5!\,4!}$

$= \frac{10!}{5!\,4!}$

Step 3: single block of 5 a's & 4 b's =

$\frac{(12 - \boxed{aaaaa} - \boxed{bbbb} + 1)!}{3!}$

$= \frac{4!}{3!} = 4$

single block of 5a's & 3c's

$= \frac{(12 - \boxed{aaaaa} - \boxed{ccc} + 1)}{4!} = \frac{5!}{4!} = 5$

single block of 4b's & 3c's

$= \frac{12 - \boxed{bbbb} - \boxed{ccc} + 1}{5!} = \frac{6!}{5!} = 6$

Step 4: single block of 5a's & 4b's & 3 c's

$= (12 - 5 - 4 - 3 + 1)! = 1!$

Step 5: Not a single block of {5a's, 4b's, 3c's}

$= |A| - \{(|A_1| + |A_2| + |A_3|) + \{(A_1 \cap A_2) + (A_2 \cap A_3) + (A_1 \cap A_3)\}$
$+ \{A_1 \cap A_2 \cap A_3\}$

step 6:- $\frac{12!}{5!\,4!\,3!} - \left\{ \left(\frac{8!}{4!\,3!} + \frac{9!}{5!\,3!} + \frac{10!}{5!\,4!} \right) + \left((4+5+6)+1 \right) \right\}$

*Problem (4):- In how many ways $x_1+x_2+x_3=20$ where $x_i \geq 8$ by using principle of inclusion and exclusion

sol:- step 1:- $x_1+x_2+x_3=20$

step 2:- $\underset{x_1}{\boxed{\ }} + \underset{x_2}{\boxed{\ }} + \underset{x_3}{\boxed{\ }} = 20$

step 3:- $|A_1|$ = box $x_1 \Rightarrow$ where $x_i \geq 8$
$x_1 \geq 8$

$\underset{x_1}{\boxed{\ \boxed{8}}} \quad \underset{x_2}{\boxed{\ }} \quad \underset{x_3}{\boxed{\ }} = 20$

$A_1 = 1,\ n = 3,\ r = 20-8$
$\boxed{r = 12}$

$|A_1| = {}^{3-1+12}C_{12} = {}^{14}C_{12}$

step 4:- $|A_2|$ = box $x_2 \Rightarrow x_2 \geq 8$

$\underset{x_1}{\boxed{\ }} \quad \underset{x_2}{\boxed{\ \boxed{8}}} \quad \underset{x_3}{\boxed{\ }} = 20$

$n = 3,\ r = 20-8$
$r = 12$

$|A_2| = {}^{3-1+12}C_{12} = {}^{14}C_{12}$

step 5:- $|A_3|$ = box $x_3 \Rightarrow x_3 \geq 8$

$\underset{x_1}{\boxed{\ }} \quad \underset{x_2}{\boxed{\ }} \quad \underset{x_3}{\boxed{\ \boxed{8}}}$ $\quad n = 3,\ r = 20-8$
$r = 12$
$|A_3| = {}^{14}C_{12}$

Step 6: $|A_1 \cap A_2| \Rightarrow x_1 \geq 8, x_2 \geq 8$

$$\boxed{8}\ \boxed{8}\ \boxed{\ } = 20$$

$x_1 \quad x_2 \quad x_3$

$n = 3, \quad r = 20 - 16 = 4$

$$|A_1 \cap A_2| = {}^{3-1+4}C_4 = {}^{6}C_4$$

Step 7: $|A_1 \cap A_3| \Rightarrow x_1 \geq 8, x_3 \geq 8$

$$\boxed{8}\ \boxed{\ }\ \boxed{8}$$

$x_1 \quad x_2 \quad x_3$

$n = 3, \quad r = 20 - 16 = 4$

$$\boxed{|A_1 \cap A_3| = {}^{6}C_4}$$

Step 8: $|A_2 \cap A_3| \Rightarrow x_2 \geq 8, x_3 \geq 8$

$$\boxed{\ }\ \boxed{8}\ \boxed{8}$$

$x_1 \quad x_2 \quad x_3$

$n = 3, \quad r = 20 - 16 = 4$

$$\boxed{|A_2 \cap A_3| = {}^{6}C_4}$$

Step 9: $|A_1 \cap A_2 \cap A_3| \Rightarrow x_1 \geq 8, x_2 \geq 8, x_3 \geq 8$

$$\boxed{|A_1 \cap A_2 \cap A_3| = 0}$$

Step 10: $|A_1 \cup A_2 \cup A_3| = \{({}^{14}C_2 + {}^{14}C_{12} + {}^{14}C_{12}) - ({}^{6}C_4 + {}^{6}C_4 + {}^{6}C_4) + 0\}$

$$\boxed{|A_1 \cup A_2 \cup A_3| = 3 \times {}^{14}C_{12} - 3 \times {}^{6}C_4}$$