

UNIT-V

- 1 Fit a straight line to the following data in least square sense

X : 6 7 7 8 8 8 9 9 10

Y : 5 5 4 5 4 3 4 3 3

error

Find the Mean square

- 2 Expand $f(x) = (1 - x)^{\frac{5}{2}}$ as Taylor's series.

- 3 Obtain McLaurin's series expansion of $f(x) = \sin(x)$

- 4 Show that $\sqrt{x} = 1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \dots$ for $0 < x < 2$

- 5 Write Taylor's series approximation of $6x^2 + 2x + 5$ in powers of $(x - 2)$.

- 6 Fit a parabola of the form $y = a + bx + cx^2$ to the following data.

(i)

x	1	2	3	4	5	6	7
y	23	5.2	9.7	16.5	29.4	35.5	54.4

- 7 Fit a least square curve of the form $y = a + bx^2$ for the following data

x	1	2.5	3.5	4.0
y	3.8	15.0	26.0	33.0

- 8 Fit a curve of the form $y = ae^{bx}$ to the following data

(i)

x	1	5	7	9	12
y	10	15	12	15	21

(ii)

x	77	100	185	239	285
y	2.4	3.4	7	11.1	19.6

- 9 Fit a curve of the form $y = ab^x$ for the following data

(i)

x	2	3	4	5	6
y	8.3	15.4	33.1	65.2	127.4

(ii)

x	1	5	7	9	12
y	10	15	12	15	21

- 10 Fit a power function of the form $y = ax^b$ to the following data and estimate y at $x = 12$.

Price x	20	16	10	11	14
Demand y	22	41	120	89	56

- 11 Fit a curve of the form $y = a + \frac{b}{x}$ for the following data

x	1	5	7	9	12
y	10	15	12	15	21

- 12 Fit a curve of the form $y = ax + \frac{b}{x^2}$ for the following data

x	1	2	3	4	5
y	4	3	6	7	11

- 13 Fit a curve of the form $y = a + bx + \frac{c}{x}$ for the following data

x	1	5	7	9	12
y	10	15	12	15	21

14 Write the Chebyshev differential equation. Show that $T_n(x) = \cos(\cos^{-1} x)$ satisfies the Chebyshev differential equation

15 Derive expressions for the Chebyshev polynomials of first kind - T_0, T_1, T_2, T_3, T_4 .

16 Express the polynomials in terms of Chebyshev polynomials of first kind.

(i) x^2 . (ii) $x^2 - 3x + 5$. (iii) $x^3 - 2x^2 + 4x - 3$. (iv) $x^4 - 2x^2 + 2x - 1$.

17 Prove that $T_n(x)$ Satisfies the recurrence relation

(i). $T_{n+1}(x) - 2xT_n(x) + T_{n-1}(x) = 0$. (ii). $(1-x^2)T'_n(x) = -nT_n(x) + nT_{n-1}(x)$.

18 Approximating the function $f(x) = \sin^{-1}x$ with a Chebyshev series. Deduce the result at $x=1$.

19 Prove the orthogonality of the Chebyshev polynomials of first kind $T_n(x)$

$$\int_{-1}^1 \frac{T_m(x)T_n(x)}{\sqrt{1-x^2}} dx = 0, \quad m \neq n;$$

$$= \pi, \quad m = n = 0; = \pi/2, \quad m = n \neq 0.$$

20 Express the following polynomials as Fourier-Chebyshev series.

(i) $x^2 + x - 3$. (ii) $x^3 - 3x^2 - 1$.