21/11/24

UNIT - V.

Thursday

Graphs

- Types of graphs
  - planar graph
  - non-planar graph
  - colour graph
  - Bipartite graph
  - Complete Bipartite graph
  - Eular and hamiltanian graph
  - Isomorphism graph

Trees

- Tree
- Spanning Tree
- Minimum spawaning tree
- DFS
- BFS

* Graph:

. A graph is a pair G(V,E) where 'V' is a non-empty set and 'E' is set of unordered pairs of elements taken from the set 'V'

* Types of graphs:

1) Simple graph:-

- A graph 'G' which does not contain loops and multiple edges is called simple graph.

2) Multiple graph:-

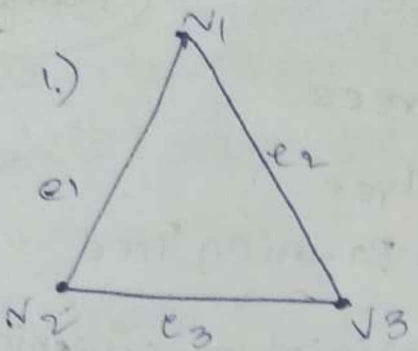
- A graph 'G' contains multiple edges but no loops is called Multiple graph.

3) General graph:-

- A graph 'G' which contains multiple edges and loops is called general graph.

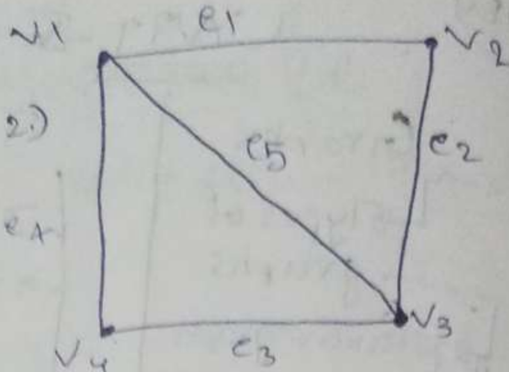
Ex:-

1.)



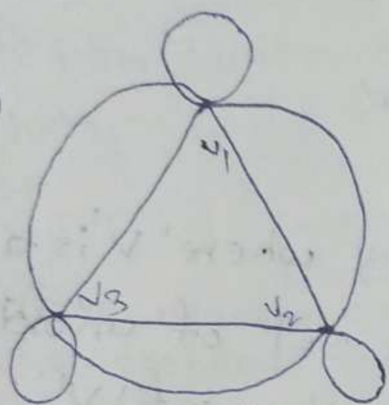
Simple graph

2.)



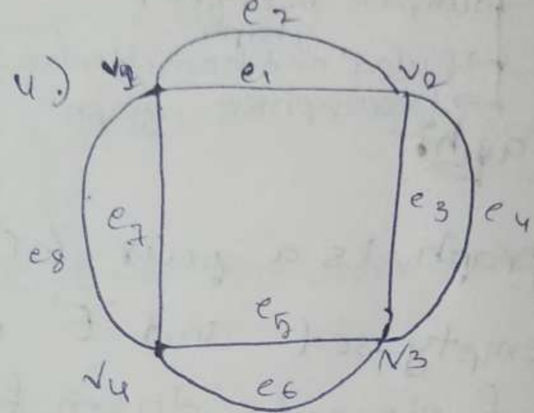
sim
[~~Mult~~]ple graph

3)



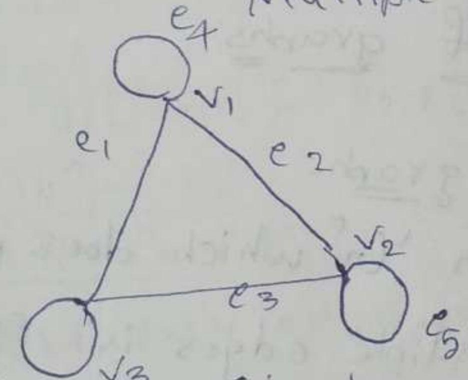
General graph

4.)



Multiple graph

5.)



simple graph

*Planar Graph:-

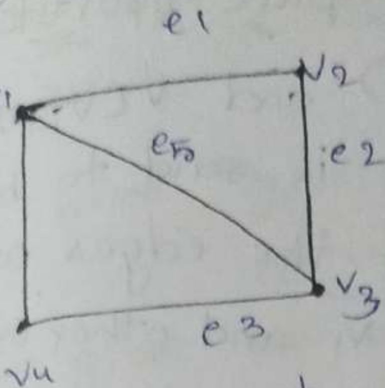
- A graph 'G', If no two edges of graph intersect then the graph is called planar graph.

(or)

- The no. of vertices 'V', the no. of edges 'E' and the no. of region's 'R' then the above eular's formula = $\boxed{V-E+R=2}$

graph is called planar graph.

Ex: 1)

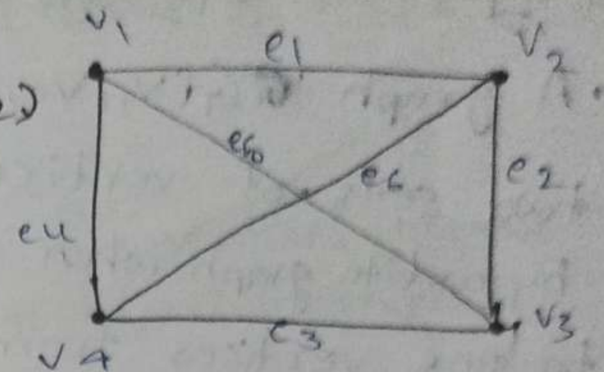


planar graph

$V-E+R=2$

$4-5+3=-1+3$
$=2$ ✓

2)

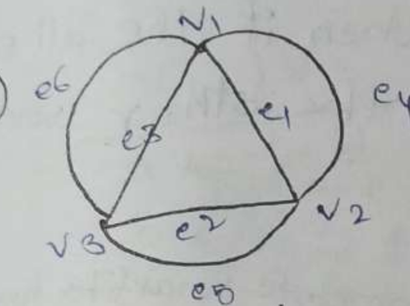


Non-planar graph

$V-E+R=2$

$4-6+5=-2+5$
$=3$ ✗

3)

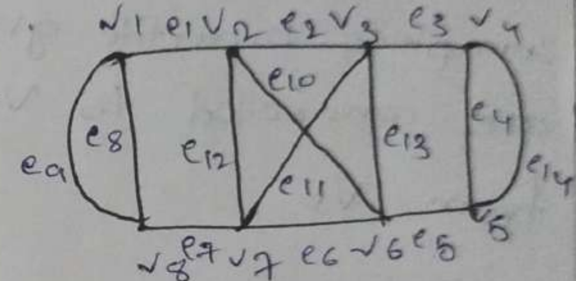


planar graph

$V-E+R=2$

$3-6+5=-3+5$
$=2$ ✓

4)

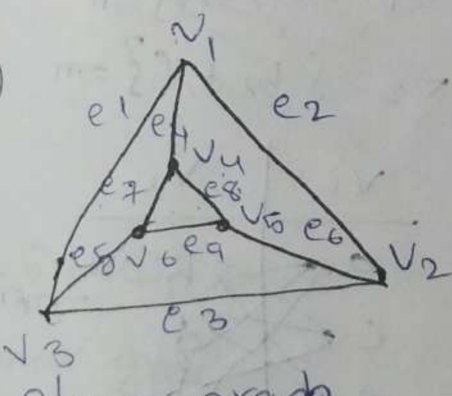


non-planar graph

$V-E+R=2$

$8-14+9=-6+9$
$=3$ ✗

5)

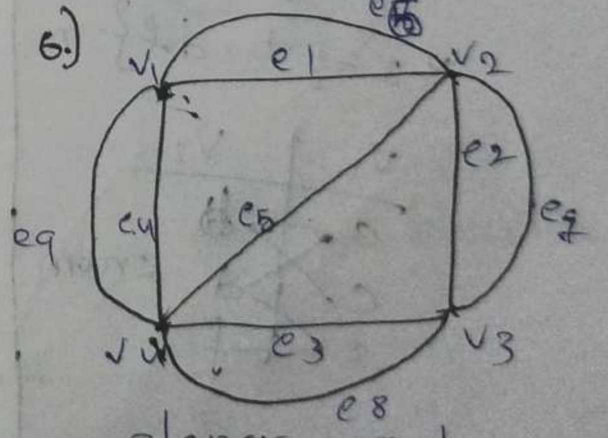


planar graph

$V-E+R=2$

$6-9+5=-3+5$
$=2$ ✓

6)



planar graph

$V-E+R=2$

$4-9+7=-5+7$
$=2$ ✓

* Bipartite Graph and Complete Bipartite graph:-

• A graph $G(V_1, V_2, E)$ and $V(V_1, V_2)$ are two sets of vertices is said to be bipartite graph when if the edges are connected to one vertices from V_1 and other vertices from V_2.

* Complete bipartite graph:-

• A bipartite graph $G(V_1, V_2, E)$ is called complete bipartite graph when if the all edges are connected to V_1 and the other vertices from V_2.

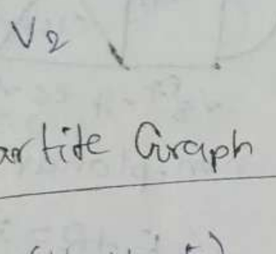
Bipartite Graph

$G(V_1, V_2, E)$.

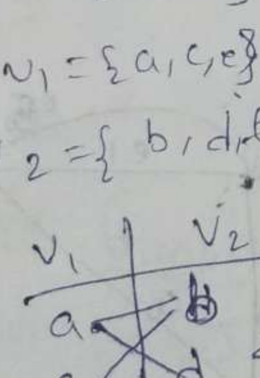
$V = \{V_1, V_2\}$

$V_1 = \{a, c, e\} = n$

$V_2 = \{b, d, f\} = m$



$< n*m$



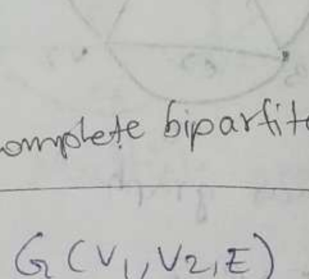
Complete bipartite Graph

$G(V_1, V_2, E)$

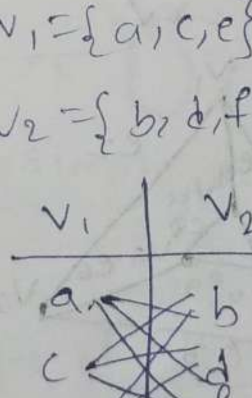
$V = \{V_1, V_2\}$

$V_1 = \{a, c, e\} = n$

$V_2 = \{b, d, f\} = m$



$= n*m$



*Problem ①:- Draw the $[K_{3,4}]$ complete bipartite graph

Sol:- here given

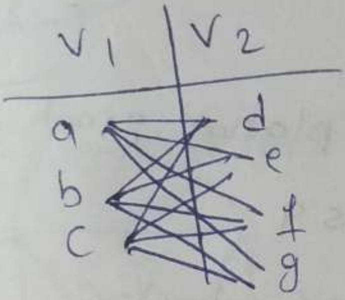
$n=3, m=4 \rightarrow$ vertices $= m+n = 7$

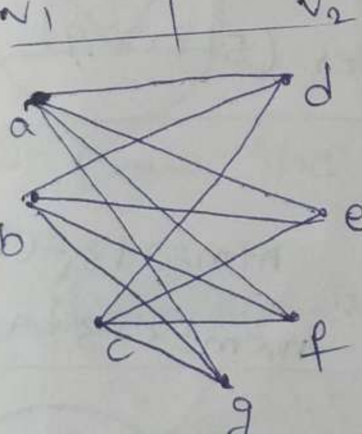
also give complete bipartite graph so

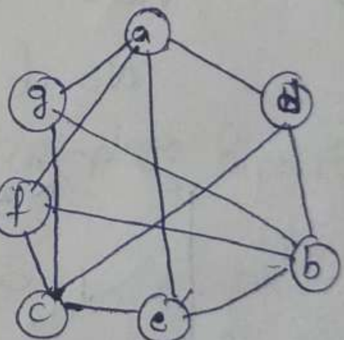
edges $= n * m = 3 \times 4$

$\boxed{\text{edges} = 12}$

$V_1 = \{a, b, c\}, V_2 = \{d, e, f, g\}$







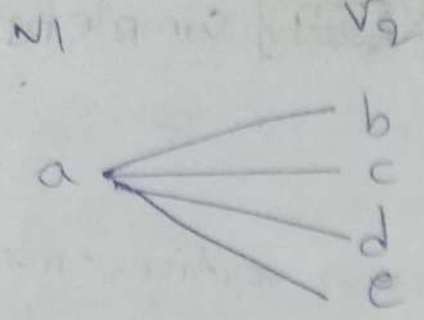
*Problem ②:- Draw the $K_{1,4}$ complete bipartite planar graph.

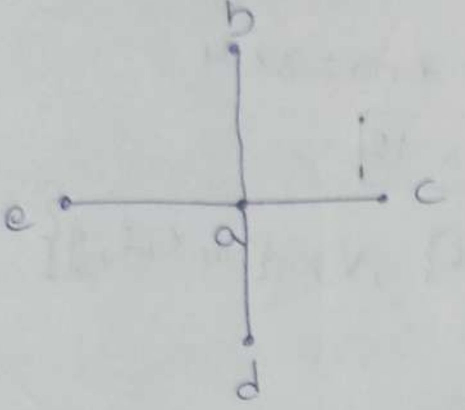
Sol:- here $n=1, m=4$

vertices $n+m = 1+4 = 5$

edges $n \times m = 1 \times 4 = 4$

$V_1 = \{a\}, V_2 = \{b, c, d, e\}$





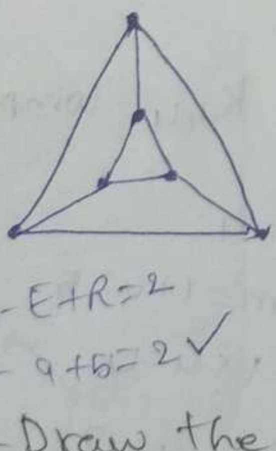
$V - E + R = 5 - 4 + 1$
$= 2$ ✓

***Problem③**:- Draw the planar graph of order is 6 and size is 9

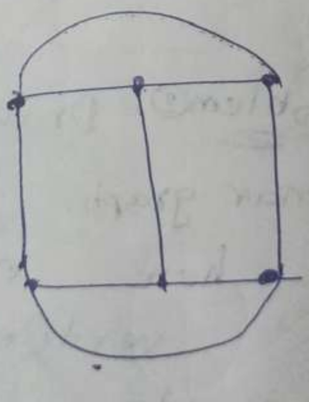
Sol:- no of vertices (order) = 6
No. of edges (size) = 9

$K_{3,3}$

because $n + m = 3 + 3 = 6$ (vertices)
$n \times m = 3 \times 3 = 9$ (edges)



(or)



$V - E + R = 2$
$6 - 9 + 5 = 2$ ✓

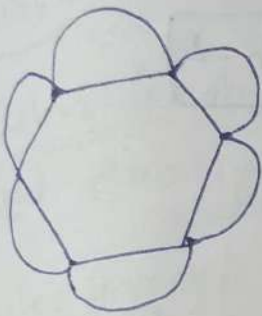
***Problem④**:- Draw the planar graph of order is 6 and size is 12.

Sol: Given

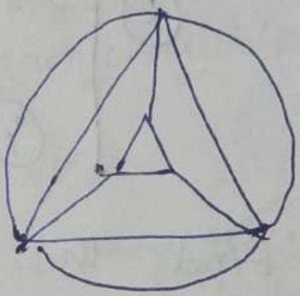
no. of vertices = 6

no. of edges = 12

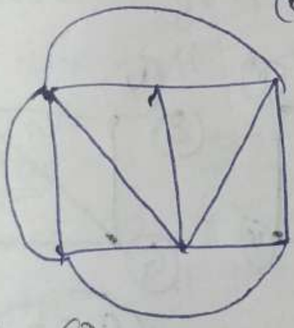
$$V - E + R = 2$$
$$6 - 12 + R = 2$$
$$-6 + R = 2$$
$$R = 8$$



(or)

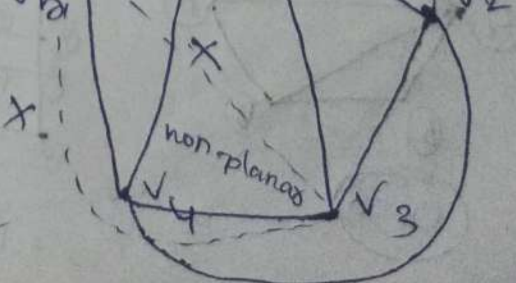


(or)



*Problem ⑥:- Draw the K_5 complete Bipartite planar graph.

Sol: no. of vertices = 5



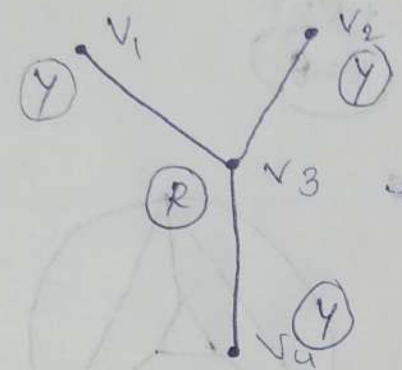
So, K_5 is complete bipartite but non-planar

* Color Graph (or) chromotic number (C_n):-

• If a graph 'G' planar or non-planar, if no two adjacent vertices are not same colour.

Ex:- Find the below graph of chromotic number

Sol:-

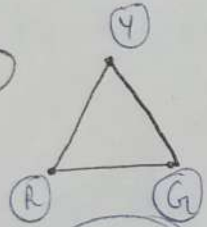


$\boxed{C_n = 2}$

* Problem (6):- Find the below graph of chromotic number

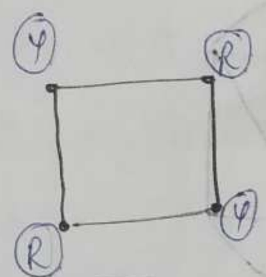
Sol:-

(1.)



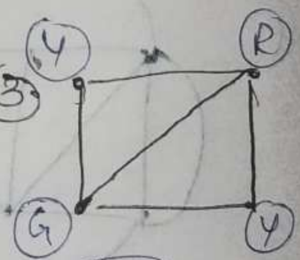
$C_n = 3$

(2.)



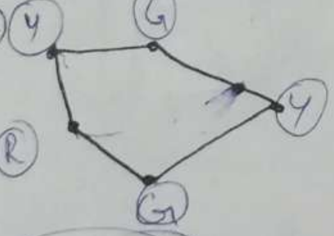
$C_n = 2$

(3.)



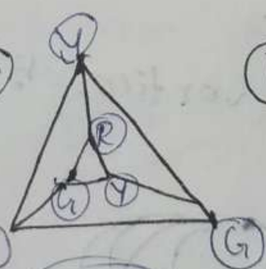
$C_n = 3$

(4.)



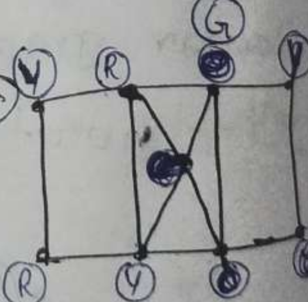
$C_n = 3$

(5.)



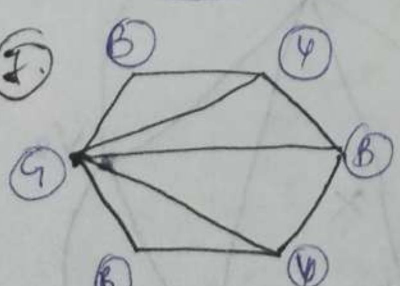
$C_n = 3$

(6.)



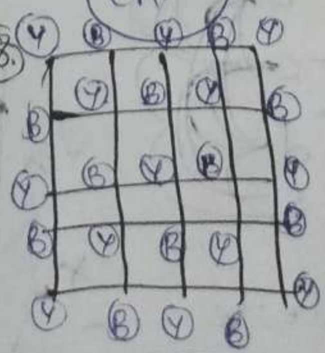
$C_n = 3$

(7.)



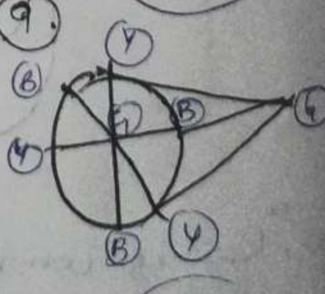
$C_n = 3$

(8.)



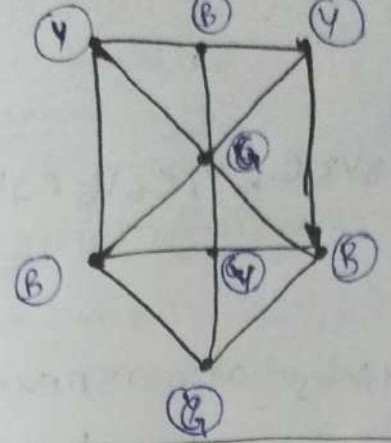
$C_n = 2$

(9.)



$C_n = 3$

(10)



$C_n = 3$

28/11/24

Thursday

* Euler's Graph / circuit / path and Hamiltanian Graph / circuit / path:-

Euler Graph:-

. A connected graph 'G' if there is circuit in 'G' that contain all edges is said to be Euler! Graph/ circuit/path.

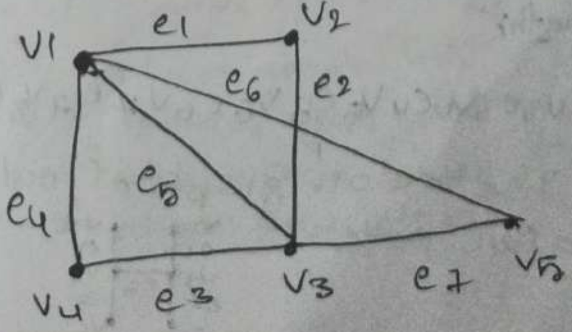
Hamiltanian Graph:-

. A connected graph 'G' if there is circuit in 'G' that contain all the vertices is said to be hamiltanian Graph/ circuit/path.

Note:- 1) Euler graph:- All the edges visit exactly one but may repeat the vertices.

2) Hamiltanian Graph:- All the vertices visit exactly one but may repeat edges

*Problem (1):- Find the below graph of eular and hamiltanian path



Sol:-

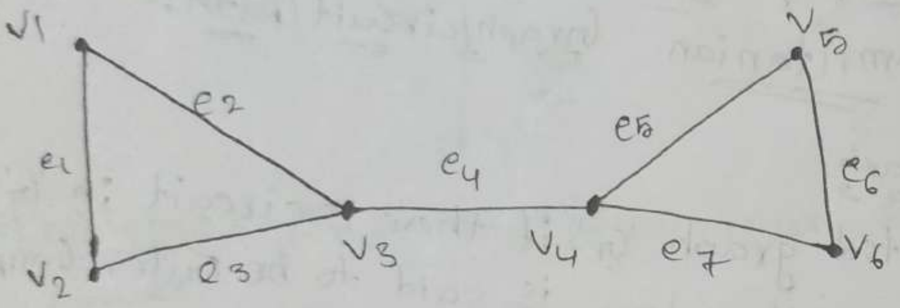
eulars path

- $v_3 e_2 v_2 e_1 v_1 e_4 v_4 e_3 v_3 e_5 v_1 e_6 v_5 e_7 v_3$

hamiltanian path

- × no circuit (vertices are repeating)

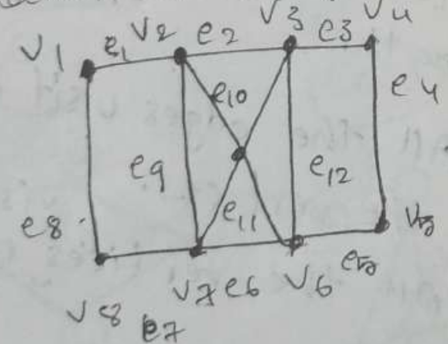
*Problem②:- find the below graph of eular and hamiltanian path



Sol:-

no eular and hamiltanian path exists

*Problem③:- find the below graph of eular and hamiltanian path



Sol:-

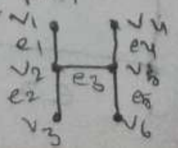
eular path:-

- $v_1 e_1 v_2 e_2 v_3 e_3 v_4 e_4 v_5 e_5 v_6 e_{12} v_3 e_{11} v_7 e_9 v_2 e_{10} v_6 e_6 v_7$ $e_7 v_8 e_8 v_1$

hamiltanian path:-

- $v_1 e_1 v_2 e_2 v_3 e_3 v_4 e_4 v_5 e_5 v_6 e_6 v_7 e_7 v_8 e_8 v_1$

*Problem④:- find the below graph of eular and hamiltaian path



eular path:-

sol:- not possible

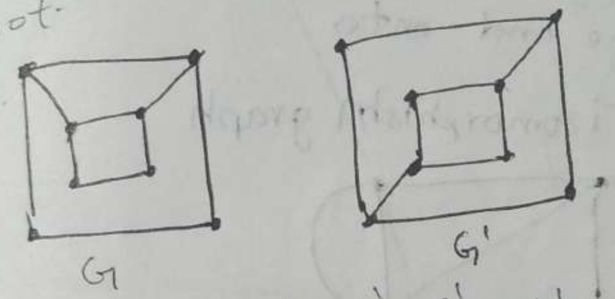
Hamiltanian path:-
not possible

* Isomorphism graphs:-

• Let two graphs $G(V,E)$ & $G'(V',E')$ then a function $f: G \to G'$ is said to be Isomorphism graph when if 'f' is 1) one to one function 2) onto function 3) the no of equal vertices G & G' 4) the no of equal edges G & G' 5) the sum of vertices G & G' is equal 6) the sum of edges G & G' is equal 7) the sumation of degree G & G' is equal.

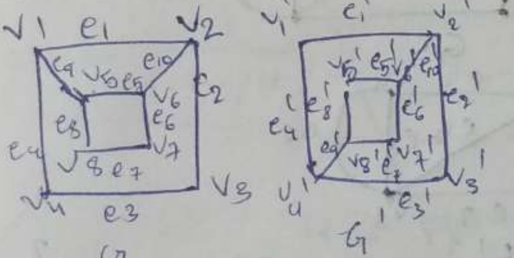
* Problem (5):- Find the below graphs isomorphism graphs or not.

1)



sol:-

S-1:-



S-2:- Let $G(V,E)$

the no of vertices $= \{v_1 --- v_8\}$

the no of edges $= \{e_1 --- e_{10}\}$

S-3:- Let $G'(V',E')$

the no of vertices $= \{v_1' --- v_8'\}$

the no of edges $= \{e_1' --- e_{10}'\}$

S-4:- the no of vertices graph $G = 8$

the no of vertices graph $G' = 8$

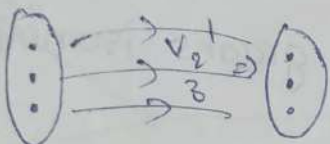
S-5:- the no of edges graph $G = 10$

the no of edges graph $G' = 10$

S-6: Graph G | Graph G'

Graph G	Graph G'
v_1 - 3	v_1' - 2
v_2 - 3	v_2' - 3
v_3 - 2	v_3' - 2
2	v_4' - 3
v_4 - 3	v_5' - 2
v_5 -	
3	v_6' - 3
v_6 -	
v_7 - 2	v_7' - 2
v_8 - 2	v_8' - 3
20	20

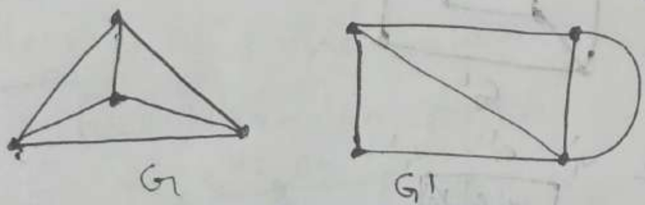
S-7: $v_1 \Rightarrow$ 3 $v_2' \Rightarrow$ 3



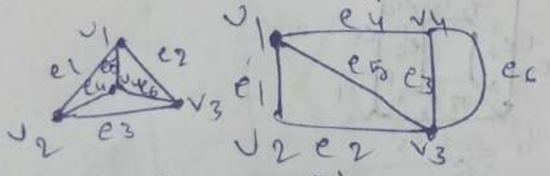
f is one to one and onto

S-8: $f: G \to G'$ is isomorphism graph

2)



sol: S-1:



S-2: Let $G(V,E)$
the no.of vertices $V = \{v_1, v_2, v_3, v_4\}$
no.of edges $E = \{e_1, e_2 --- e_6\}$

S-3: Let $G'(V', E')$
the no.of vertices $V' = \{v_1', v_2', v_3', v_4'\}$
the no.of edges $E_1' = \{e_1', e_2' --- e_6'\}$

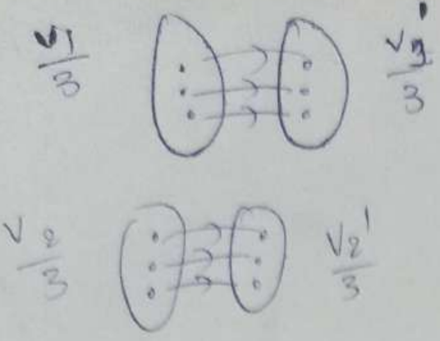
S-4: the no.of vertices in graph G = 4
the no.of vertices in graph G' = 4

S-5: the no.of edges in graph G = 6
the no.of edges in graph G' = 6

S-6:

Graph G	Graph G'
v_1 - 3	v_1' - 3
v_2 - 3	v_2' - 3
v_3 - 3	v_3' - 4
v_4 - 3 /12	v_4' - 2 /12

S-7/2:



f is not one to one and onto function

S-8: $f: G \to G'$

not a isomorphism graph

30/11/24 — Saturday

*Tree:

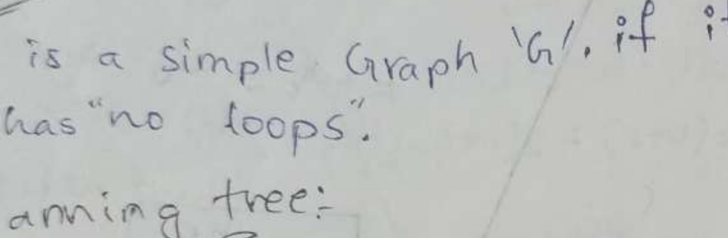
- It is a simple Graph 'G', if it is connected and has "no loops".

*Spanning tree:

- A subgraph 'G'' is called spanning tree when i) it is a tree ii) sub-graph contain all the vertices of graph 'G' iii) sub graph contain $\leq n-1$ edges (or) $< n$ edges

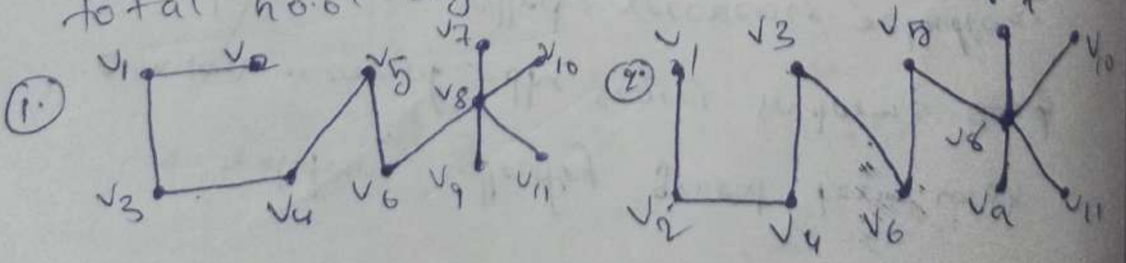
Ex: Find the below graph of spanning tree

1)

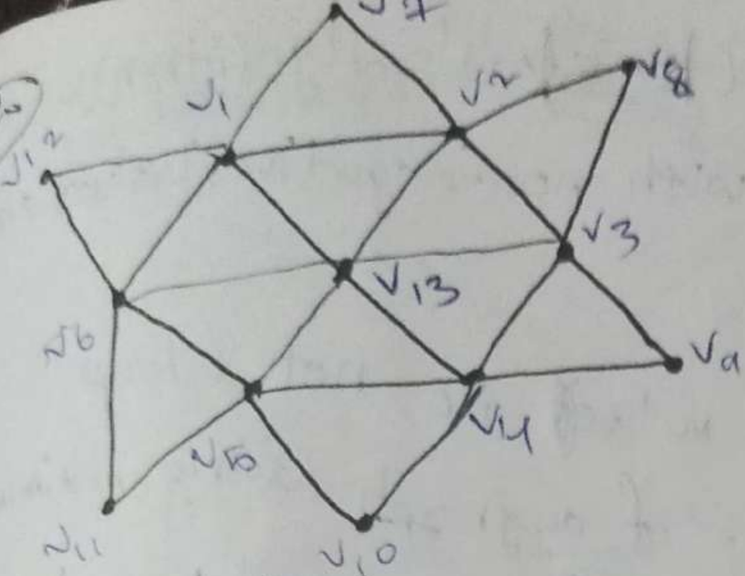


Sol: total no. of vertices = 11

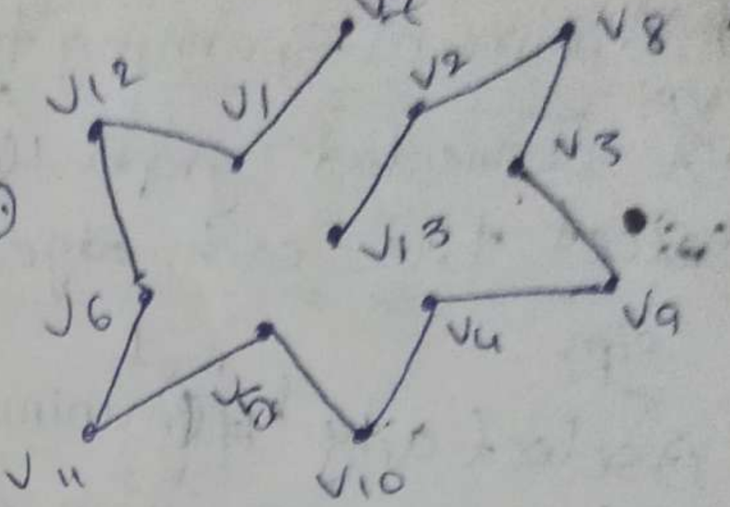
total no. of edges = 13



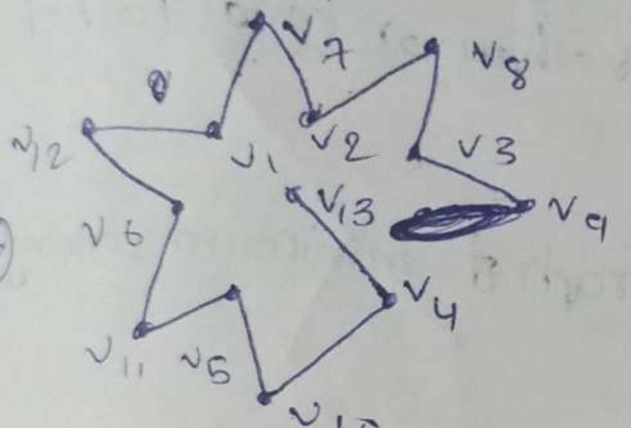
9.


v7
v1
v2
v8
v12
v3
v13
v6
v9
v14
v5
v11
v10

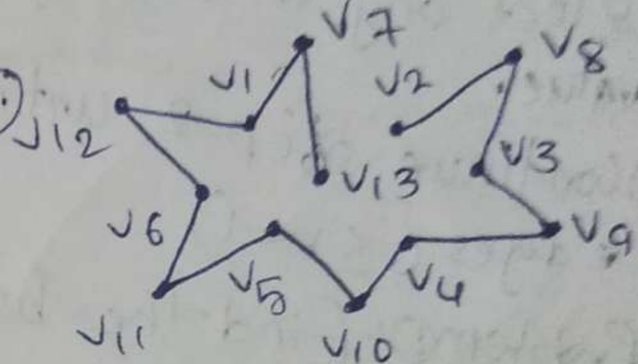
1.


v7
v8
v12
v1
v2
v3
v13
v6
v9
v4
v5
v11
v10

2


v7
v8
v12
v1
v2
v3
v13
v9
v6
v4
v11
v5
v10

3.


v7
v8
v1
v2
v12
v13
v3
v6
v9
v5
v4
v11
v10

*Minimum Spanning tree (kruskal's algorithm):-

i/p:-

· A connected graph 'G' with non-negative values on select the each edge

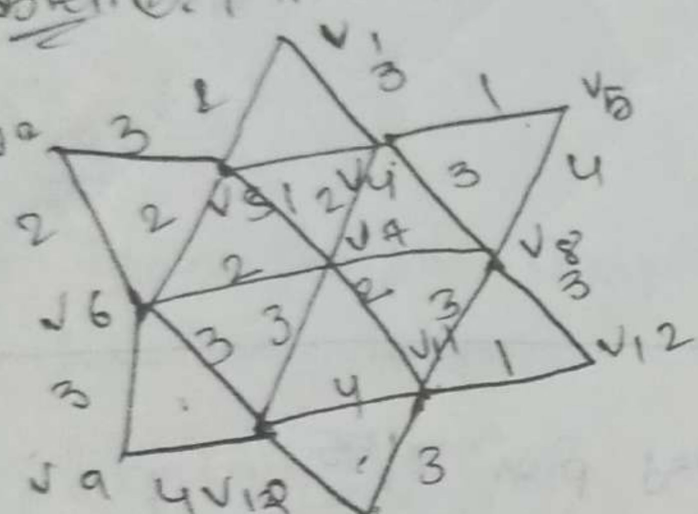
o/p:-

1.) select any 1 of minimum value i.e, not a loop

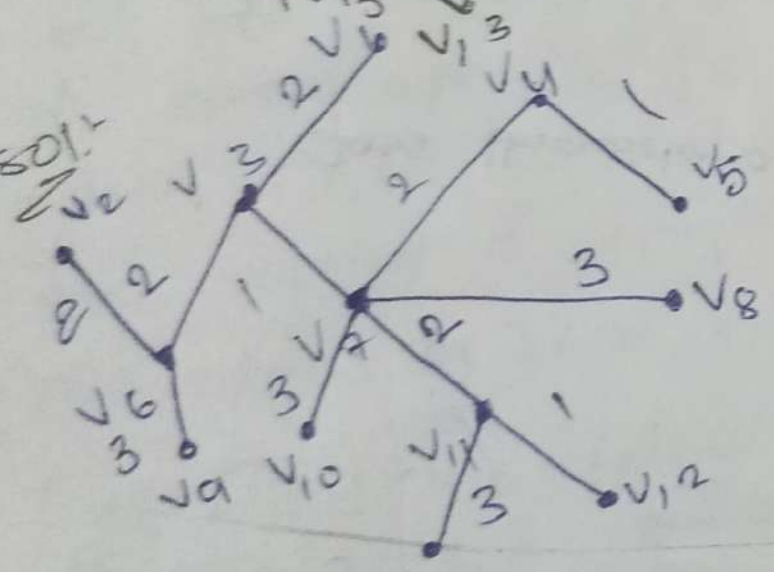
2.) select any remaining edge of a graph having minimum value.

3.) continue step-2 until less than or equal to n-1 edges ($\leq n-1$)

* Problem①:- Find the below graph of minimum spanning tree

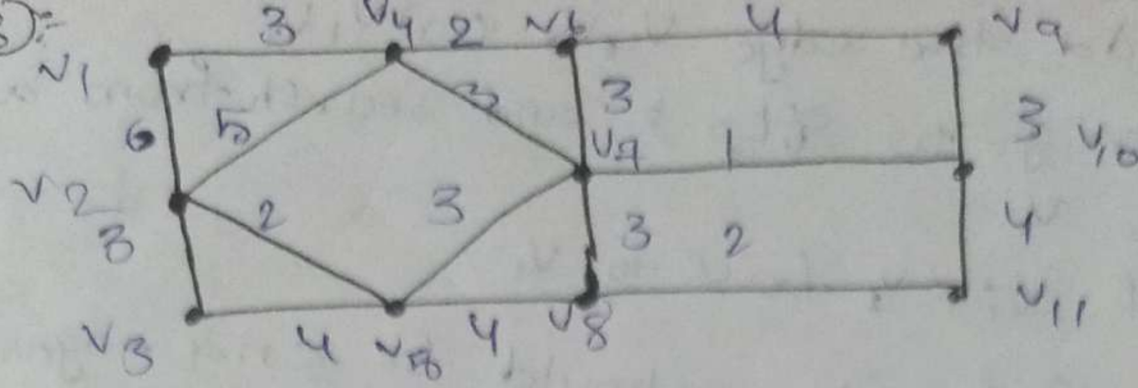


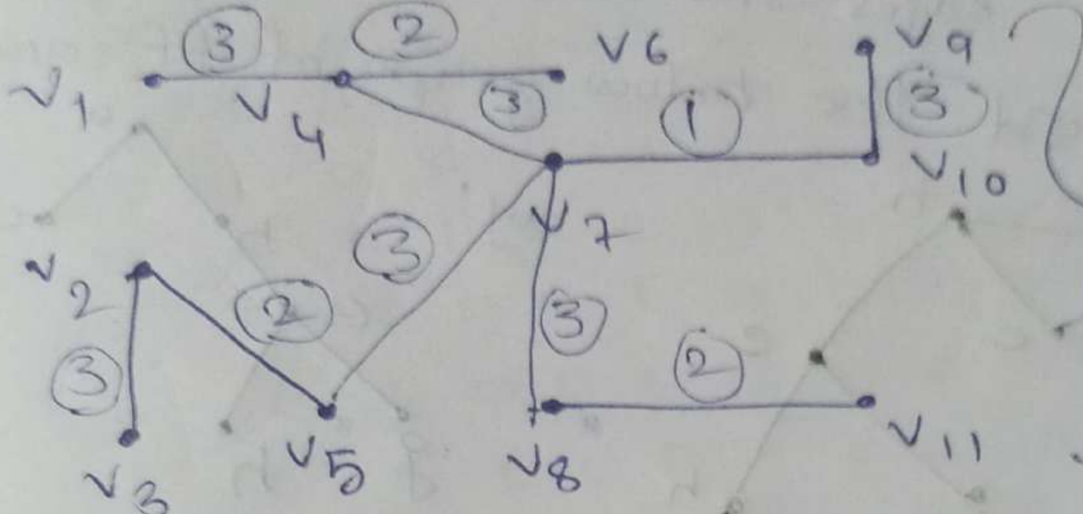
sol:-



minimum path = 25

Problem(3):





minimum path=25

3/12/24 Monday

* BFS / DFS :-

Step 1 :- Select the minimum vertex V_1 and choose V_1 as the root of the vertex.
Step 2 :- Select the next vertex V_2, if V_1 and V_2 is an edge, otherwise V_1 & V_i.

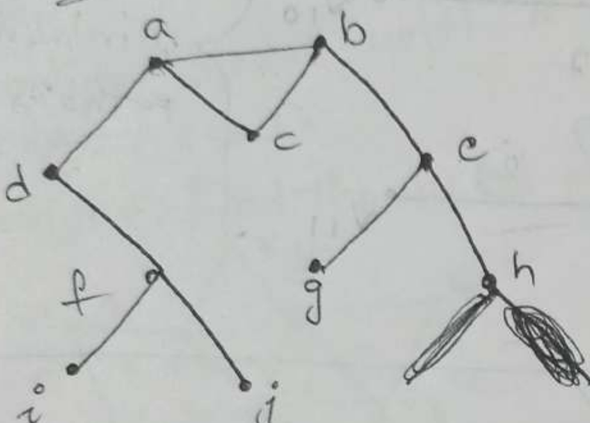
Step3:- add the edge $V_1 V_2$ (or) $V_1 V_i$

Step4: Then go to Step 2 and search from another vertices V_i

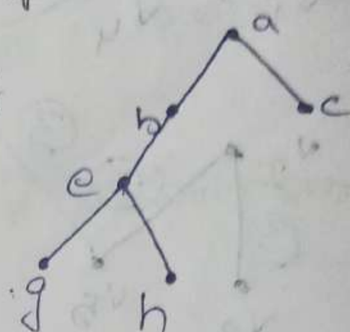
Step5:- If $V_i \neq V_1$ back to V_1

Note:- the no. of edges should be $\leq n-1$ edges

*Problem①:- find the below graph of BFS and DFS



sol:-

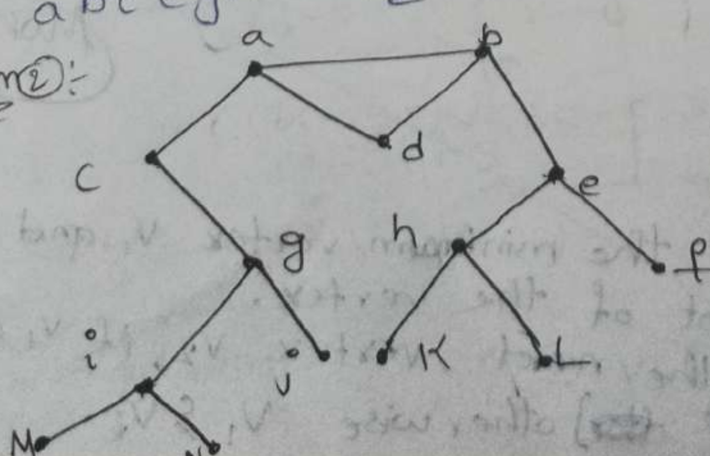


sol:-

	a	b	c	d	e	f	g	h	i	j
→ a	0	1	1	1	0	0	0	0	0	0
→ b	1	0	1	0	1	0	0	0	0	0
→ c	1	1	0	0	0	0	0	0	0	0
X d	1	0	0	0	0	1	0	0	0	0
→ e	0	1	0	0	0	0	1	1	0	0
X f	0	0	0	1	0	0	0	0	1	1
→ g	0	0	0	0	1	0	0	0	0	0
→ h	0	0	0	0	1	0	0	0	0	0
X i	0	0	0	0	0	1	0	0	0	0
X j	0	0	0	0	0	1	0	0	0	0

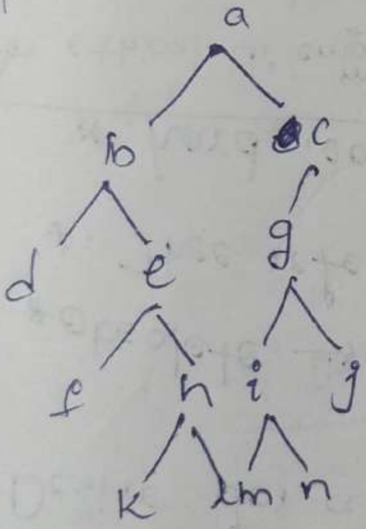
BFS:- abcegh DFS:- abeghc

*Problem②:-



sol:-

	a	b	c	d	e	f	g	h	i	j	k	l	m	n
a	0	1	1	1	0	0	0	0	0	0	0	0	0	0
b	1	0	0	1	1	0	0	0	0	0	0	0	0	0
c	1	0	0	0	0	0	1	0	0	0	0	0	0	0
d	1	1	0	0	0	0	0	0	0	0	0	0	0	0
e	0	1	0	0	0	1	0	1	0	0	0	0	0	0
f	0	0	0	0	1	0	0	0	0	0	0	0	0	0
g	0	0	1	0	0	0	0	1	1	0	0	0	0	0
h	0	0	0	0	1	0	0	0	0	0	1	1	0	0
i	0	0	0	0	1	0	0	0	0	0	0	0	1	1
j	0	0	0	0	0	0	1	0	0	0	0	0	0	0
k	0	0	0	0	0	0	0	1	0	0	0	0	0	0
l	0	0	0	0	0	0	0	1	0	0	0	0	0	0
m	0	0	0	0	0	0	0	0	1	0	0	0	0	0
n	0	0	0	0	0	0	0	0	1	0	0	0	0	0



BFS:- abcdegfhij klmn

DFS:- abdefhklcgimnj