28/08/2024 Wednesday

UNIT-1

Statement:-

- A statement is a declarative sentence which is either true or false but not both

* Notations *

1. **Negation**:- (NOT) {¬ (or) ~}

- Let p is a statement then the opposite of the statement p is called the negation of p. Denoted by ~p (or) ¬p.

Truth table for Negation:-

P	~P
T	F
F	T

2. **Disjunction** (or) "∨":-

Let p & q are two different statements then the compound of (P∨Q) is true. When any the statements is true. else it is false.

Truth table for Disjunction:-

P	q	(P∨q)
F	F	F
F	T	T
T	F	T
T	T	T

3) Conjunction (and) "$\wedge$":-

Let p, q are two different statements then the compound statement $(p \wedge q)$ is true when both are true else false

Truth table for conjunction:-

p	q	$p \wedge q$
F	F	F
F	T	F
T	F	F
T	T	T

4) Condition (if) "$\rightarrow$":-

Let p, q are two different statements then the compound statement $(p \rightarrow q)$ is false when false is followed by true else false.

Truth table for condition:-

p	q	$p \rightarrow q$
T	F	F
F	F	T
F	T	T
T	T	T

5) Bicondition (iff) "$\leftrightarrow$":-

Let p, q be two different statements then the compound statement $(p \leftrightarrow q)$ is true when both statements either true or false.

Truth table for Bicondition

p	q	$p \leftrightarrow q$
F	F	T
T	F	F

F	T	F
T	T	T

6.) Exclusive or "$\veebar$":-

Let p,q be two different statements then the compound statement $(p \veebar q)$ is true when any statement is true or false but not both

Truth table for exclusive or:-

p	q	$p \veebar q$
T	F	T
F	T	T
F	F	F
T	T	F

* Note:- $p \leftrightarrow q = \sim(p \veebar q)$

7.) NAND ($\uparrow$):-

Let p,q be two different statements then the compound statement $(p \uparrow q)$ is ~~conjunction~~ Negation of $(p \wedge q)$

Truth table for NAND:-

p	q	$p \wedge q$	$\sim(p \wedge q)$
F	F	F	T
F	T	F	T
T	F	F	T
T	T	T	F

8.) NOR ($\downarrow$):-

Let p,q be two different statements then the compound statement $(P \downarrow Q)$ is equal to $\sim(P \vee Q)$

Truth table for NOR:

P	q	$P \vee q$	$\sim(P \vee q)$
F	F	F	T
T	F	T	F
F	T	T	F
T	T	T	F

*LAWS OF LOGICS:-

1.) Double negation:- $(\sim(\sim P) = P)$

2.) Idempotent law:- $(P \vee P) = P$ (or) $(P \wedge P) = P$

3.) Identity Law:- $(P \vee F_0) = P$ (or) $(P \wedge T_0) = P$

4.) Domination law:- $(P \vee T_0) = T_0$ (or) $(P \wedge F_0) = F_0$

5.) Inverse law:- $(P \vee \sim P) = T_0$ (or) $(P \wedge \sim P) = F_0$

6.) Demorgan's law:- $\sim(P \vee q) = \sim P \wedge \sim q$ (or) $\sim(P \wedge q) = \sim P \vee \sim q$

7.) Commutative law:- $(P \vee q) = (q \vee P)$ (or) $(P \wedge q) = (q \wedge P)$

8.) Associative law:- $(P \vee q) \vee R = P \vee (q \vee R)$ (or) $(P \wedge q) \wedge r = P \wedge (q \wedge r)$

9.) Condition law:- $P \rightarrow q = \sim P \vee q$

10.) Absorption law:- $(P \vee q) \wedge P = P$ (or) $(P \wedge q) \vee P = P$

11.) Distributive law:- $P \vee (q \wedge r) = (P \vee q) \wedge (P \vee R)$

12.) Bi-conditional law:- $(P \leftrightarrow q) = \sim(P \veebar q)$

$(P \rightarrow q) \wedge (q \rightarrow P)$

*problem(1):- let

P : x is an integer

Q : x is Irrational number

R : 7 is divisible by 3

Find the above compound sentence in terms of statements: $P \wedge \sim q$, $(P \rightarrow q) \veebar r$, $(P \rightarrow q) \rightarrow r$,

$(p \veebar q) \to r, (p \leftrightarrow \sim q) \to r, (p \wedge \sim q) \veebar \sim r$

Sol:- $p \wedge \sim q$: x is an integer and not an irrational number

$(p \to q) \veebar r$: Either if x is an integer then x is irrational number or 7 is divisible by 3

$(p \to q) \to r$:- If x is an integer then x is an irrational number then 7 is divisible by 3.

$(p \veebar q) \to r$:- Either x is an integer or x is irrational number but not both then 7 is divisible by 3.

$(p \leftrightarrow \sim q) \to r$:- If x is an integer if and only if x is an irrational number then 7 is divisible by 3.

$(p \wedge \sim q) \veebar \sim r$:- Either if x is an integer and x is irrational number but not both than 7 is not divisible by 3.

*Problem ②:-

Let p, q be the primitive statement $p \to q$ is false find the above compound statement of truth values: $(p \to \sim q)$, $(p \veebar q)$, $(p \leftrightarrow q) \veebar (q \to \sim p)$

Sol:- 1.) $(p \to \sim q) = T \to T = T$

2.) $(p \veebar q) = T \veebar F = T$

3.) $(p \leftrightarrow q) \veebar (q \to \sim p) = (T \leftrightarrow F) \veebar (F \to F)$

$= F \veebar T = T$

*Problem ③:-

Let p, q and r are three different statements then the primitive statement $p \wedge (q \to r)$ is true Find the truth values of p, q & r

Sol:- $p \wedge (q \to r) = T$

$p = T$

$q \to r = T = ?$

$F \to F = T$

$F \to T = T$

$T \to T = T$

p	q	r
T	F	F
T	F	T
T	T	T

$x \veebar r$

$x \wedge y = T$

$\left.\begin{array}{l} x = T \\ y = T \end{array}\right\}$

31/8/24 Saturday

*Problem ①:-
Let x is a conic circle then $\sqrt{5}$ is a prime number. Find the above compound statement of negation in terms of statements.

Sol:- step 1:- x is a conic circle then $\sqrt{5}$ is a prime number.

step 2:- p: x is a conic circle

q: $\sqrt{5}$ is a prime number

step 3:- Above compound statement in terms of notation

step 4:- $(p \to Q)$

step 5:- apply negation of above compound statement: $\sim(p \to q)$

step 6:- $\sim(\sim p \vee q) = (p \wedge \sim q)$ ($\because$ condition law)

step 7:- x is a conic circle and $\sqrt{5}$ is not a prime number.

*Problem ②:- If x is either even number or odd number but not both then 7 is divisible by 3. Find the above compound statement of negation in terms of statements.

Sol:- step 1:- If x is either even number or odd number but not both then 7 is divisible by 3.

step 2:- p: x is even number

q: x is odd number
r: 7 is divisible by 3.

step 3:- Above compound statement in terms of notation.

step 4:- $(P \vee Q) \rightarrow R$

step 5:- apply negation of above compound statement : $\sim\{(P \vee Q) \rightarrow R\}$

step 6:- $\sim\{\sim(P \vee Q) \vee R\} = ((P \vee Q) \wedge \sim R)$

step 7:- If x is either even number or odd number and 7 is not divisible by 3.

* Truth tables:-

→ To (Tautology)
→ Fo (contradiction)
→ {T (or) FO} (contingency)

→ T_0 (Tautology):-

- Let p, q be two different statements then the compound statement of all truth values is equal to true then above compound statement is "Tautology"

→ F_0 (contradiction):-

- Let p, q be two different statements then above compound statement of all truth values is equal to false then above compound statement is "contradiction".

→ **Contingency:-**

• Let p, q be two different statements then the above compound statement of all possible truth values is equal to either true or flase then above compound statement is "contingency".

* Problem ③:- Find the above compound statement of truth table

$$(p \veebar q) \longleftrightarrow (\sim p \rightarrow q)$$

Sol:-

p	q	$p \veebar q$	$\sim p$	$\sim p \rightarrow q$	$(p \veebar q) \leftrightarrow (\sim p \rightarrow q)$
F	F	F	T	F	T
F	T	T	T	T	T
T	F	T	F	T	T
T	T	F	F	T	F

* Problem ④:- Find the above compound statement of truth table

$$\{(p \rightarrow q) \wedge (q \rightarrow R)\} \vee \{(q \rightarrow p) \wedge (r \rightarrow q)\}$$

Sol:-

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$q \rightarrow p$	$r \rightarrow q$	$(p \rightarrow q) \wedge (q \rightarrow R)$	$(q \rightarrow p) \wedge (r \rightarrow q)$
F	F	F	T	T	T	T	T	T
F	T	F	T	F	F	T	F	F
F	F	T	T	T	T	F	T	F
T	F	F	F	T	T	T	F	T
T	T	F	T	F	T	T	F	T
F	T	T	T	T	F	T	T	F
T	F	T	F	T	T	F	F	F
T	T	T						

T	T	T	T	T	T	T	T	T

$\{(p \to q) \wedge (q \to R)\} \vee \{(q \to p) \wedge (r \to q)\}$
T
F
T
T
T
T
F
T

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*Problem ①: Find the above compound statement of truth table

$$(P \longleftrightarrow q) \veebar (R \veebar S)$$

Sol:-

p	q	r	s	$R \leftrightarrow q$	$R \veebar S$	$(P \leftrightarrow q) \veebar (R \veebar S)$
F	F	F	F	T	F	T
F	T	F	F	F	F	F
T	F	F	F	F	F	F
F	F	T	F	T	T	F
F	F	F	T	T	T	F
T	T	F	F	T	F	T
T	F	T	F	F	T	T
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	T	F	T	F	T	T
F	F	T	T	T	F	T
						T

Contingency

T	T	T	F	T	T	F
F	T	T	T	F	F	F
T	F	T	T	F	F	F
T	T	F	T	T	T	F
T	T	T	T	T	F	T

*Problem ②:- Find the above compound statement of truth table

$$(p \veebar q) \wedge (q \leftrightarrow p)$$

Sol:-

p	q	$p \veebar q$	$q \leftrightarrow p$	$(p \veebar q) \wedge (q \leftrightarrow p)$
T	T	F	T	F
T	F	T	F	F
F	T	T	F	F
F	F	F	T	F

contradiction

* Problem ③:-

$$(p \veebar q) \vee (q \leftrightarrow p)$$

Sol:-

p	q	$p \veebar q$	$q \leftrightarrow p$	$(p \veebar q) \vee (q \leftrightarrow p)$
F	F	F	T	T
T	F	T	F	T
F	T	T	F	T
T	T	F	T	T

Tautology

*Problem ④:- Find the below compound statements in terms of only 'NAND' (or) only 'NOR'

Sol:-

① $\sim p$

$\sim p = \sim(p \wedge p)$ $\quad$ ($\because p = p \wedge p$) Idempotent

$= p \uparrow p$

$= \boxed{\text{only NAND}}$

($\because \sim(p \wedge q) = p \uparrow q$)
(NOT + AND) $\quad$ (NAND)

② $\sim p$

$\sim p = \sim(p \vee p)$ $\quad$ (same)

($\because \sim(p \vee q) = p \downarrow q$)
(NOT + OR) $\quad$ (NOR)

$= p \downarrow p$

$= \boxed{\text{only NOR}}$

③ $(p \vee q)$

$(p \vee q) = \sim\{\sim(p \vee q)\}$ $\quad$ ($\because \sim(\sim p) = p$ double negation)

$= \sim(\sim(p \vee q) \vee \sim(p \vee q))$ $\quad$ ($\because x = x \vee x$ Idempotent law)

$= \sim(p \vee q) \downarrow \sim(p \vee q)$

$= (p \downarrow q) \downarrow (p \downarrow q)$

$= \boxed{\text{only nor}}$

$\therefore p \vee q = \sim\{\sim(p \vee q)\}$

$= \sim\{\sim p \wedge \sim q\} = \sim p \uparrow \sim q$

$= (p \uparrow p) \uparrow (q \uparrow q) \quad (\because \sim p = p \uparrow p)$

$= \boxed{\text{only NAND}}$

(4.) $p \wedge q$

Sol: $\Rightarrow \sim(\sim(p \wedge q))$

$= \sim(\sim(p \wedge q) \wedge \sim(p \wedge q))$

$= \sim(p \wedge q) \uparrow \sim(p \wedge q)$

$= (p \uparrow q) \uparrow (p \uparrow q)$

$= \boxed{\text{only NAND}}$

$(p \wedge q)$

Sol: $\Rightarrow \sim(\sim p \vee \sim q)$

$= \sim p \downarrow \sim q$

$= (p \downarrow p) \downarrow (q \downarrow q)$

$= \boxed{\text{only NOR}}$

(5.) $p \to q$ Let $\sim p = x$

$= \sim p \vee q$ $= \sim(\sim(x \vee q))$

$= \sim\{\sim(x \vee q) \vee \sim(x \vee q)\}$

$= \sim(x \vee q) \downarrow \sim(x \vee q)$

$= (x \downarrow q) \downarrow (x \downarrow q)$

$= ((p \downarrow p) \downarrow q) \downarrow ((p \downarrow p) \downarrow q)$

$= \boxed{\text{only NOR}}$

$p \to q$

$= \sim p \vee q$ let $\sim p$ be x

$\cancel{\sim(\sim(\sim p \wedge \sim q))}$

$\cancel{= \sim(\sim p \uparrow \sim q)}$

$= x \vee q$

$= \sim(\sim \cancel{x} \vee \sim(\sim q))$

$= \sim(\sim x \wedge \sim q)$

$= \sim x \uparrow \sim q$

$= (x \uparrow x) \uparrow (q \uparrow q)$

$= ((p \uparrow p) \uparrow (p \uparrow p)) \uparrow (q \uparrow q)$

$=$ $\boxed{\text{only NAND}}$

(6) $p \leftrightarrow q$

$= (p \to q) \wedge (q \to p)$

let $p \to q$ be x & $q \to p$ be y

$= x \wedge y$

$= \sim(\sim(x \wedge y))$

$= \sim(\sim(x \wedge y) \wedge \sim(x \wedge y))$

$= \sim(x \wedge y) \uparrow \sim(x \wedge y)$

$= (x \uparrow y) \uparrow (x \uparrow y)$

$= (((p \uparrow p) \uparrow (p \uparrow p)) \uparrow (q \uparrow q) \uparrow (((q \uparrow q) \uparrow (q \uparrow q)) \uparrow (p \uparrow p))) \uparrow$

$(((p \uparrow p) \uparrow (p \uparrow p)) \uparrow (q \uparrow q)) \uparrow (((q \uparrow q) \uparrow (q \uparrow q)) \uparrow (p \uparrow p)))$

* Problem ⑤:- Simplify the following compound statements by using laws of logics

(1.) $p \vee \{p \vee (p \wedge q)\}$

step 1:- $p \vee \{p \vee (p \wedge q)\}$

($\because X \vee (X \wedge Y) = X$)
(Absorption law)

step 2:- $p \vee p$ ($\because$ Idempotent law $p \vee p = p$)

step 3:- p

(2.) $(p \vee q) \wedge \sim\{\sim p \vee q\}$

Step 1:-

Step 2:-

Step 3:-

Step 4:-

(3.) $(p \vee q) \vee [\sim p \wedge \sim q \wedge \sim r] \Longleftrightarrow (p \vee q \vee \sim r)$

show that above compound statement is equal to $(p \vee q \vee \sim r)$

Sol:-

($\because$ Demorgan's law)

step 1:- $(p \vee q) \vee [\sim(p \vee q) \wedge \sim r]$

step 2:- $[(p \vee q) \vee \sim(p \vee q)] \wedge [(p \vee q) \vee \sim r]$

($\because$ Distributive law)

$[X \vee (Y \wedge r)]$

$= (X \vee Y) \wedge (X \vee Z)$

step 3:- $T_0 \wedge [p \vee q \vee \sim r]$ ($\because$ Inverse law)

Step 4:- $p \vee q \vee \sim r$ ($\because$ Identity law)

Hence proved

*Problem (6): $(p \to q) \wedge [\sim q \wedge (r \vee \sim q)]$. Simplify the following compound statement by using law of logics

Sol:- Step 1:- $(p \to q) \wedge [\sim q \wedge (r \vee \sim q)]$

Step 2:- $(p \to q) \wedge (\sim q)$ ($\because$ Absorbtion law)

Step 3:- $(\sim p \vee q) \wedge \sim q$ ($\because$ condition law)

Step 4:- $(\sim p \wedge \sim q) \vee (\sim q \wedge q)$ ($\because$ Distributive law)

($\because$ Inverse law)

Step 5:- $(\sim p \wedge \sim q) \vee F_0$ ($\because$ Identity law)

Step 6:- $\sim p \wedge \sim q$

($\because$ Demorgan's law)

Step 7:- $\sim (p \vee q)$

*Problem (7): $[\sim p \wedge (\sim q \wedge r)] \vee (q \wedge r) \vee (p \wedge r) \Leftrightarrow r$

Show that above compound statement is equal to 'r'.

Sol:- Step 1:- $[\sim p \wedge (\sim q \wedge r)] \vee (q \wedge r) \vee (p \wedge r)$

Step 2:- $[\sim p \wedge (\sim q \wedge r)] \vee [r \wedge (q \vee p)]$

($\because$ reverse Distribution law)

Step 3:- $[(\sim p \wedge \sim q) \wedge r] \vee [(q \vee p) \wedge r]$

($\because$ Associative law)

Step 4:- $[\sim (p \vee q) \wedge r] \vee [(q \vee p) \wedge r]$

($\because$ Demorgan's law)

step 5:- $[\sim(p \vee q) \vee (p \vee q)] \wedge r$

(∵ reverse distribution law)

step 6:-

$T_0 \wedge r$ (∵ Inverse law)

step 7:- r (∵ Identity law)

10/9/24

Tuesday

* Duality:-

- Let U & V, two different types of compound statements then the duality of U, $\boxed{U^d = V}$ when if

1. conjunction replace disjunction
2. disjunction replace conjunction
3. Tautology replace contradiction
4. contradiction replace Tautology

* Problem ①:- Find the above compound statement of duality

$$\{(p \leftrightarrow q) \rightarrow (p \rightarrow q)\}$$

sol:- step 1:- $\{(p \leftrightarrow q) \rightarrow (p \rightarrow q)\}$

$p \rightarrow q = \sim p \vee q$

$p \leftrightarrow q = (p \rightarrow q) \wedge (q \rightarrow p)$

step 2:- $\{\sim(p \leftrightarrow q) \vee (p \rightarrow q)\}$

step 3:- $\{\sim(p \leftrightarrow q) \vee (\sim p \vee q)\}$

step 4:- $\{\sim[((p \rightarrow q) \vee (q \rightarrow p)) \vee (\sim p \vee q)]\}$

$= \{((p \wedge \sim q) \wedge (q \wedge \sim p)) \vee (\sim p \vee q)\}$

step 5:- $\boxed{U^d = ((p \vee \sim q) \vee (q \vee \sim p)) \wedge (\sim p \wedge q)}$

* Problem ②:- $\{\{p \leftrightarrow (q \rightarrow r)\} \vee (p \rightarrow q)\}$ find duality

so different compo

Sol:- when V consists

step 1:- $\{\{p\to(q\to r)\}\ \underline{\vee}\ (p\to q$ ... is said to be

step 2:- $\sim\{\{p\to(q\to r)\}\leftrightarrow(p$ form".

step 3:- $\sim\{[\sim p\vee(\sim q\vee r)]\leftrightarrow(\sim p\vee$

step 4:- $\sim\{[\sim p\vee(\sim q\vee r)]\to(\sim p\vee q)\wedge(\sim$ ... $x)$

...und state

step 5:- $\sim\{[\sim\{\tilde{p}\vee(\sim q\vee r)\}]\vee(\sim p\vee q)]\wedge[\sim$

step 6:- $[\sim p\vee(\sim q\vee r)\wedge p\wedge\sim q]\vee[\sim p\vee q\wedge p\wedge(q\wedge\sim r)]$

step 7:- $\boxed{U^d=[\sim p\wedge(\sim q\wedge r)\vee p\vee\sim q]\wedge[\sim p\wedge q\vee p\vee(q\vee\sim r)]}$

*Logical equivalance:-

- Let p, q be two different statements then implication $p\to q$. then
  1. Converse: $q\to p$ is converse of $p\to q$
  2. Inverse:- $\sim p\to\sim q$ is inverse of $p\to q$
  3. Contrapositive:- $\sim q\to\sim p$ is contrapositive of $p\to q$

*Normal forms:- $(\equiv)$

- Let U, V two different types of compound statements $U\equiv V$ the
  1. DNF (Disjunction normal form)
  2. CNF (conjunction normal form)
  3. PDNF (principle disjunction normal form)
  4. PCNF (principle conjunction normal form)

step 5:- [~(p∨q

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step 6:-

step 7:-
Tally equal statement of compoun
: which is disjunction of statem
...e conjunctions of the statement i
called "Disjunction Normal form". (SOP)

$(\ ) \vee (\ )$

which is disjunction which are conjunction

*CNF:-

• The logically equal statement of compound statement which is conjunction of statement which are disjunctions of the statement is called " Conjunction Normal form". (POS)

$(\ ) \wedge (\ )$

which is conjunction which are disjunctions

*PDNF:-

• Let p, q two different statements and $p \wedge q$, $p \wedge \sim q$, $\sim p \wedge q$, $\sim p \wedge \sim q$ are "min Terms" of p, q statements. Let U, V are two different compound statements then $U \equiv V$ when V consists of disjunctions of min terms is said to be "principle disjunction normal form.

(min) ∨ (min) ∨ (min) ∨ (min)

*PCNF:-

• Let p, q two different statements and $p \vee q$, $p \vee \sim q$, $\sim p \vee q$, $\sim p \vee \sim q$ are "max terms" of p, q statements. Let U, V are two different compound statements then U ≡ V when V consists of conjunctions of max terms is said to be "principle conjunction normal form".

(max) $\wedge$ (max) $\wedge$ (max) $\wedge$ (max)

*problem ①:- find the above compound statement of DNF

$(p \to q) \wedge (p \vee q)$

sol:- step 1:- $(p \to q) \wedge (p \vee q)$

step 2:- $(\sim p \vee q) \wedge (p \vee q)$ = CNF

step 3:- $[\sim p \wedge (p \vee q)] \vee [q \wedge (p \vee q)]$ } Distributive law

step 4:- $[(\sim p \wedge p) \vee (\sim p \wedge q)\} \vee q$ → absorption law

F_0 ↳ Inverse law, Identity law

step 5:- $(\sim p \wedge q) \vee q$.

step 6:- $(\sim p \wedge q) \vee (q \wedge q)$ (Idempotent law)

step 7:- DNF

*problem ①:- find the above compound statement of DNF and CNF

$p \wedge (p \to q)$

sol:- step 1:- $p \wedge (p \to q)$

step 2:- $p \wedge (\sim p \vee q)$ → condition law

step 3:- $(p \wedge \sim p) \vee (q \wedge p)$

step 4:- DNF

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step 1:- $p \wedge (p \to q)$

step 2:- $p \wedge (\sim p \vee q)$

step 3: $(p \vee p) \wedge (\sim p \vee q)$ → Idempotent law

step 4: CNF

*Problem ③: find the below compound statement of CNF and DNF

$$p \leftrightarrow q$$

sol: step 1: $p \leftrightarrow q$

step 2: $(p \rightarrow q) \wedge (q \rightarrow p)$

step 3: $(\sim p \vee q) \wedge (\sim q \vee p)$

(∵ Distributive law)

step 4: CNF

step 5: $[(\sim p \vee q) \wedge \sim q] \vee [(\sim p \vee q) \wedge p]$

step 6: $[(\sim p \wedge \sim q) \vee (q \wedge \sim q)] \vee [(\sim p \wedge p) \vee (q \wedge p)]$

step 7: $(\sim p \wedge \sim q) \vee (q \wedge p)$

step 8: DNF

*Problem ④: find the below compound statement of PDNF.

$$\sim(p \wedge q)$$

sol: step 1: $\sim(p \wedge q)$

step 2: $\sim p \vee \sim q$

step 3: $(\sim p \wedge T_0) \vee (\sim q \wedge T_0)$ → Identity law

→ Inverse law

step 4: $[\sim p \wedge (q \vee \sim q)] \vee [\sim q \wedge (p \vee \sim p)]$

→ Distributive law

step 5: $(\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (\sim q \wedge p) \vee (\sim q \wedge p)$

step 6: PDNF

*problem ⑤: Find the below compound statement of PCNF $\sim(p \vee q)$

sol:- step 1:- $\sim(p \vee q)$

step 2:- $\sim p \wedge \sim q$ → Identity law

step 3:- $(\sim p \vee F_0) \wedge (\sim q \vee F_0)$ → Inverse law

step 4:- $(\sim p \vee (q \wedge \sim q)) \wedge (\sim q \vee (p \wedge \sim p))$ → Distributive law

step 5:- $(\sim p \vee q) \wedge (\sim p \vee \sim q) \wedge (\sim q \vee p) \wedge (\sim q \vee \sim p)$

step 6:- PCNF

*Problem ⑥:- Find the below compound statement PCNF of $(\sim p \rightarrow r) \wedge (q \leftrightarrow p)$

sol:- step 1:- $(\sim p \rightarrow r) \wedge (q \leftrightarrow p)$

step 2:- $(p \vee r) \wedge [(\sim q \vee p) \wedge (\sim p \vee q)]$

step 3:- $[(p \vee r) \vee F_0] \wedge \{[(\sim q \vee p) \vee F_0] \wedge [(\sim p \vee q) \vee F_0]\}$

step 4:- $[(p \vee r) \vee (q \wedge \sim q)] \wedge \{[(\sim q \vee p) \vee (r \wedge \sim r)] \wedge [(\sim p \vee q) \vee (r \wedge \sim r)]$

step 5:- $(p \vee r \vee q) \wedge (p \vee r \vee \sim q) \wedge (\sim q \vee p \vee r) \wedge (\sim q \vee p \vee \sim r)$
$\wedge (\sim p \vee q \vee r) \wedge (\sim p \vee q \vee \sim r)$

* Predicate Logic:-

- The logic based upon the "analaysis of any statement" is called "Predicate Logic".

Ex:- Ramu is Intelligent $(R(r))$

Here "is Intelligent" is above statement of predicate logic. Ramu is above statement of element or member or object

* Multiple predicate logic:-

Ex:- i) Ramu is taller than Rakash ($R(r_1, r_2)$)

Here "is taller than" is, above statemen of "multiple predicate logic". Here Ramu and Rakesh above statement of members or elements or objects

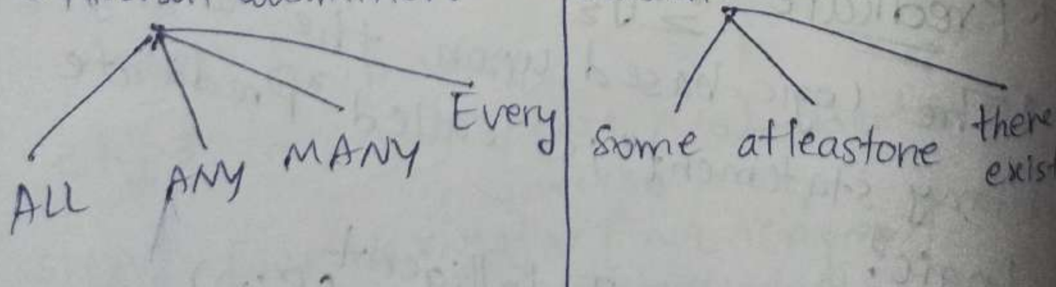
2) Ramu sits between raju and rakesh ($R(r_1, r_2, r_3)$)

Here "sits between" is, above statement of "multiple predicate logic", Here ramu, raju and rakesh are above statement of elements (or) members (or) objects.

* Quatifier predicate logic:-

- In discrete mathematics supporting two types of quantifiers:-
  1.) Universal quantifiers ($\forall x$)
  2.) Existential Quantifiers ($\exists x$)

Universal Quantifiers | Existential Quantifiers



* Example ①:- Write the below compound statements in terms of symbolic logical predicate
① There exists a positive integer is even number

2.) Some even integers are divisible by 3

3.) Every integer is either even number or odd number, but not both

4.) If x is even number and a complex number then x is not divisible by 3

Sol:

$P(x)$: x is positive integer

$Q(x)$: x is even number

$R(x)$: x is divisible by 3

$S(x)$: x is complex number

1.) $$\exists_x, \{P(x) \wedge Q(x)\}$$

2.) $$\exists_x, \{Q(x) \wedge R(x)\}$$

3.) $$\forall_x, \{Q(x) \underline{\vee} \sim Q(x)\}$$

4.) $$\forall_x, \{[Q(x) \wedge S(x)] \rightarrow \sim R(x)\}$$

* Example ②:

i.) $\exists_x, [\sim P(x) \leftrightarrow Q(x)] \underline{\vee} R(x)$

ii.) $\forall x, [S(x) \underline{\vee} Q(x)] \rightarrow \sim P(x)$

iii.) $\forall x; [Q(x) \underline{\vee} \sim Q(x)]$

iv.) $\forall x, \{[Q(x) \wedge S(x)] \rightarrow \sim R(x)\}$

Find the above predicate compound statement convert them into statements.

Sol:- i.) There exists, either x is negative number if and only if x is even number or x is divisible by 3 but not both

2.) Every x, either x is complex number or x is even number but not both then x is negat[ive] number

* Negations of Quantifiers:-

1. $\sim\{\forall_x, P(x)\} = \exists_x, \sim P(x)$
2. $\sim\{\exists_x, P(x)\} = \forall_x, \sim P(x)$
3. $\sim\{\forall_x \neg P(x)\} = \exists_x, P(x)$
4. $\sim\{\exists_x \neg P(x)\} = \forall_x, P(x)$
5. $\sim\{\forall_x, [P(x) \vee Q(x)]\} = \exists_x, [\sim P(x) \wedge \sim Q(x)]$
6. $\sim\{\exists_x, [P(x) \wedge Q(x)]\} = \forall_x, [\sim P(x) \vee \sim Q(x)]$
7. $\sim\{\forall_x, (P(x) \wedge Q(x))\} = \exists_x, [\sim P(x) \vee \sim Q(x)]$
8. $\sim\{\exists_x, (P(x) \vee Q(x))\} = \forall_x, [\sim P(x) \wedge \sim Q(x)]$

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* Problem ①:- "Some even numbers are prime numbers then x is irrational number."
write the following compound statement of negation in terms of symbolic predicate logic

Sol:- Step 1:- Some even numbers are prime numbers then x is irrational number

Step 2:- $P(x) = x$ is even number
$Q(x) = x$ is prime number

$R(x) = x$ is irrational number

Step 3:- Above compound statement interms of Notations

$$\exists_{x}, \{P(x) \wedge Q(x)\} \rightarrow R(x)$$

Step 4:- apply negation of above compound statement

Step 5:- $\sim\{\exists_{x}, \{P(x) \wedge Q(x)\} \rightarrow R(x)\}$

$= \sim\{\exists_{x}, \{\sim[P(x) \wedge Q(x)] \vee R(x)\}$

Step 6:- $\forall x \{[P(x) \wedge Q(x)] \wedge \sim R(x)\}$

* Problem ②:- "If every x is either even number or odd number but not both then x is not a perfect square".

Find the following compound statement of negation in terms of symbolic predicate logic

Sol:- Step 1:- "If every x is either even number or odd number but not both then x is not a perfect square".

Step 2:- $P(x)$ = x is even number.
$Q(x)$ = x is odd number
$R(x)$ = x is a perfect square

Step 3:- Above compound statement in terms of Notations.

$$\forall_{x}, \{[P(x) \veebar Q(x)] \rightarrow \sim R(x)\}$$

Step 4:- apply negation of above compound statement step 5: $\sim\{\forall x, [P(x) \underline{\vee} Q(x)] \rightarrow \sim R(x)\}$

$= \sim\{\forall x, \sim[P(x) \underline{\vee} Q(x)] \vee \sim R(x)\}$

step 6:

$\Rightarrow \exists x \{(P(x) \underline{\vee} Q(x)) \wedge R(x)\}$

*Rules of inferences:-

- Let $c_1, c_2, c_3, c_4 \cdots c_n$ different types of compound statements then

c_1 is true

c_2 is true

c_3 is true

$\vdots$

c_n is true

$\therefore c$ is true

$$\boxed{c_1 \wedge c_2 \wedge c_3 \cdots \wedge c_n = c}$$

this rule is called "Rule of Inferences".

*Types of rules of Inferences:-

1.) Conjunctive Simplication:-

- Let p, q two different types of statements here $p \wedge q$ is true then p is true

$$\boxed{\begin{array}{c} p \wedge q \\ \hline \therefore p \end{array}}$$

2.) Disjunctive Amplication:- (Addition of)

- Let p, q two different types of statements here p is true then $p \vee q$ is true

$$\boxed{\begin{array}{c} p \\ \hline \therefore p \vee q \end{array}}$$

3.) Rule of syllogism:-

- Let p, q and r three types of statements here $p \to q$ is true and $q \to r$ is true then $p \to r$ is true

$$\boxed{\begin{array}{c} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}}$$

4.) Rule of Modus pones:-

- Let p, q two different types of statements here p is true and $p \to q$ is true then q is true

$$\boxed{\begin{array}{c} p \\ p \to q \\ \hline \therefore q \end{array} \quad \begin{array}{c} p \to q \\ p \\ \hline \therefore q \end{array}}$$

5.) Modus Tollens:-

- Let p, q two types of different statements here $p \to q$ is true and $\sim q$ is true then $\sim p$ is true

$$\boxed{\begin{array}{c} p \to q \\ \sim q \\ \hline \therefore \sim p \end{array}}$$

6.) Disjunctive syllogism:-

- Let p, q two different types of statements here $p \vee q$ is true & $\sim p$ is true then q is true.

$$\boxed{\begin{array}{c} p \vee q \\ \sim p \\ \hline \therefore q \end{array}}$$

*Problem ③: Q: If kohli hits a century then he get a free car.

Sol: Q: Kohli hits a century

∴ He get a free car

Check whether above compound statement valid or not

Sol: Step 1: If kohli hits a century then he get a free car

kohli hits a century

∴ He get a free car

Step 2: $P(x)$: Kohli hits a century

$Q(x)$: He get a free car

Step 3: Convert into Notations for above statements

Step 4:

$$\begin{array}{l} P(x) \rightarrow Q(x) \\ P(x) \\ \hline \therefore Q(x) \end{array}$$

Step 5:

$$\left.\begin{array}{l} p \rightarrow q \\ p \\ \hline \therefore q \end{array}\right\}$$ Equal to modus pones

Step 6: Valid

*Problem ④: If kohli hits a century then he get a free car

kohli does not get a century

∴ kohli does not get a free car

sol: Step 1: If kohli hits a century then he get a free car
Kohli does not get a century

∴ Kohli does not get a free car

Step 2: $P(x)$: Kohli hits a century
$Q(x)$: He get a free car

Step 3: Convert into notations

Step 4:
$$\frac{\begin{array}{c} P(x) \longrightarrow Q(x) \\ \sim P(x) \end{array}}{\therefore \sim Q(x)}$$

Step 5:
$$\left.\frac{\begin{array}{c} P \rightarrow q \\ \sim P \end{array}}{\sim q}\right\} \text{No rule}$$

Step 6: Not valid / Invalid

Problem (5): C_1: If I study I will not fail in the examination
C_2: If I do not watch TV in the eevening then I will study
C_3: I failed in the examination

I must have watch TV in the evening

Check whether above compound statement valid or not

P, q, r

sol:
$$\frac{\begin{array}{c} p(x) \rightarrow \sim r(x) \\ \sim q(x) \rightarrow p(x) \\ r(x) \end{array}}{q(x)}$$

step 1:- if I study, I will not fail in the exam

I do not watch TV in the evening than I will study

I fail in the examination

I must watch TV in the evening

step 2:- $p(x)$: I will study

$q(x)$: I will not fail in the exam

$r(x)$ = I must watch TV in the evening

step 3:- Above compound statement convert into notations

step 4:-

$$p(x) \rightarrow q(x)$$
$$\sim r(x) \rightarrow p(x)$$
$$\sim q(x)$$
$$\overline{\therefore r(x)}$$

step 5:-

$p \rightarrow q \longrightarrow c_1$

$\longrightarrow c_2$

$\sim r \rightarrow p \longrightarrow c_3$

$\sim q$

$\therefore r \longrightarrow \therefore c$

step 6:- $(p \rightarrow q) \wedge (\sim r \rightarrow p) \wedge \sim q$

step 7:- $(\sim r \rightarrow p) \wedge (p \rightarrow q) \wedge \sim q$

step 8:- $(\sim r \rightarrow q) \wedge \sim q$

Modus Tollens $\left\{ \begin{array}{l} \sim r \rightarrow q \\ \sim q \\ \hline \therefore \sim(\sim r) = r \end{array} \right.$

step 9: R

step 10:- Valid

*Problem ⑥:- Check whether below compound statement valid or not

$$p \rightarrow q$$
$$r \rightarrow s$$
$$\underline{p \vee r}$$
$$\therefore q \vee s$$

sol:- step 1:-

$$p \rightarrow q$$
$$r \rightarrow s$$
$$\underline{p \vee r}$$
$$\therefore q \vee s$$

step 2:- $(p \rightarrow q) \wedge (r \rightarrow s) \wedge (p \vee r)$

step 3:- $(p \rightarrow q) \wedge (r \rightarrow s) \wedge (\sim p \rightarrow r)$

step 4:- $(p \rightarrow q) \wedge (\sim p \rightarrow r) \wedge (r \rightarrow s)$

step 5:- $(p \rightarrow q) \wedge (\sim p \rightarrow s)$ $(\because p \rightarrow q = \sim q \rightarrow \sim p$ contrapositive)

step 6:- $(\sim q \rightarrow \sim p) \wedge (\sim p \rightarrow s)$

step 7:- $\sim q \rightarrow s$

step 8:- $q \vee s$

step 9:- valid

*Problem ⑦:-

$$p \rightarrow q$$
$$r \rightarrow s$$
$$\underline{\sim q \vee \sim s}$$
$$\therefore \sim (p \wedge r)$$

Check whether above compound statement vaild or not

sol:- step 1:-

$$p \rightarrow q$$
$$r \rightarrow s$$
$$\sim q \vee \sim s$$

Step2: $(p \to q) \wedge (r \to s) \wedge (\sim q \vee \sim s)$

step3: $(p \to q) \wedge (r \to s) \wedge (q \to \sim s)$

step4: $(p \to q) \wedge (q \to \sim s) \wedge (r \to s)$

Step5: $(p \to \sim s) \wedge (r \to s)$

step6: $(p \to \sim s) \wedge (\sim s \to \sim r)$

step7: $(p \to \sim r)$

step8: $\sim p \vee \sim r = \sim (p \wedge r)$

step9: valid

Problem 8: $(\sim p \vee \sim q) \to (r \wedge s)$

$r \to t$

$\sim t$

$\therefore p$

$\sim p \vee \sim q \to r \wedge s \wedge r \to t \wedge \sim t$

Check whether compound statement is valid or not

sol: Step 1: $(\sim p \vee \sim q) \to (r \wedge s)$

$(r \to t)$

$\sim t$

Step 2: $[(\sim p \vee \sim q) \to (r \wedge s)] \wedge (r \to t) \wedge \sim t$

step 3: $[(\sim p \vee \sim q) \to (r \wedge s)] \wedge \sim r$ ($\because$ Modus Tollens)

Here $\sim r, \sim s$ two statements $\sim r$ is true

$\therefore \sim r \vee \sim s$

Step4: $[(\sim p \vee \sim q) \to (r \wedge s)] \wedge (\sim r \vee \sim s)$

step 5: $[(\sim p \vee \sim q) \to (r \wedge s)] \wedge \sim (r \wedge s)$

Step 6: $\sim(\sim p \vee \sim q)$

Step 7: $p \wedge q$

Step 8: p ($\because$ Conjuctive simplication).

Step 9: valid

* Problem (2):-

$$(\sim p \vee q) \to r$$
$$r \to (s \vee t)$$
$$\sim s \wedge \sim u$$
$$\sim u \to \sim t$$
$$\therefore p$$

check valid or not

Sol: Step 1: $(\sim p \vee q) \to r$

$r \to (s \vee t)$

$\sim s \wedge \sim u$

$\sim u \to \sim t$

Step 2: $((\sim p \vee q) \to r) \wedge [r \to (s \vee t)) \wedge (\sim s \wedge \sim u) \wedge (\sim u \to \sim t)$

$\sim s \wedge [\sim u \wedge (\sim u \to \sim t)]$

Step 3: $[(\sim p \vee q) \to (s \vee t)] \wedge$ ($\because$ Associative law, Modus pones)

($\because$ syllogism)

Step 4: $[(\sim p \vee q) \to (s \vee t)] \wedge (\sim s \wedge \sim t)$

Step 5: $[(\sim p \vee q) \to (s \vee t)] \wedge \sim(s \vee t)$

Step 6: $\sim(\sim p \vee q)$ ($\because$ Modus tollens)

Step 7: $p \wedge \sim q$ ($\because$ De morgans law)

Here $p, \sim q$ two different statements

$p \wedge \sim q$ is true

$\therefore p$ is true

Step 8: p

Step 9: valid

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*Proof of contradiction:- Let p, q be two different statements the implication $p \to q$

1.) Direct 2.) In Direct 3.) Contradiction

1.) Direct:-

step 1:-

Analysis:-

Let p, q be two different statements

p is true

q is true

step 2:-

Hypothesis:- p is true

q is true

Identify the p, q values

step 3:-

Conclusion:- p is true, q is true

then $p \to q$ is true

2.) InDirect:-

step 1:-

Analysis:- Let p, q are two different statements

p is true

q is true

step 2:-

Hyphothesis:- $\sim q$ is true

$\sim p$ is true

Identify the $\sim q, \sim p$ values.

step 3:-

Conclusion:- $\sim q \to \sim p$ is true

:) Contradiction:-

Step 1:
Analysis:- Let p, q are two different statement
p is true
q is true

Step 2:-
Hyphothesis:- $\sim q$ is true
:) Identify the $\sim q$ value
then p is false

Step 3:-
Conclusion:- Contradiction of p is true

* Problem ①:- Check whether the following compound statement satisfies direct method, indirect method or not
If K and L both are odd integers then K+L is even integer

Sol:- p : K and L both are odd integers
q : K+L is even integer

1:) Direct:-
S-1:- p : K & L both are odd integers
q : K+L is even integer
p is true & q is true

S-2:- p is true when : p : K & L

$$\begin{bmatrix} K = 2m+1 \\ L = 2n+1 \end{bmatrix}$$ odd numbers

q is true when : q : K+L
$= 2m+1+2n+1 = 2m+2n+2$

S-3:- $p \to q$ is true

2.) Indirect:- $K \wedge L = \sim(K \wedge L) = \sim K \vee \sim L$

S-1:- p : K&L both are odd integers

q : K+L is even integer

p is true and q is true

S-2:- $\sim q$ is true when : K+L is odd

$\sim p$ is true when : $\sim p$: K, L is odd or even Integers

$K = 2m+1$, $L = 2n$ } $K+L = 2m+2n+1$

S-3:- $\sim q$, $\sim p$ is true

$\sim q \rightarrow \sim p$ is true

*Problem(2):- If n^2 is odd integer then n is odd integer

Check whether above compound statement satisfy or not contradiction method

Sol:- Contradiction:-

S-1:- p: n^2 is odd integer

q: n is odd integer

p is true, q is true

S-2:- $\sim q$ is true when: $\sim q$: n is even integer

$n = 2m$

then p is false : p : n^2 is odd integer

$: (2*m)^2 = 4m^2$ = even (false)

S-3:- Contradiction of p is true

*Problem(3):- If $x+y >= 100$ then $x >= 50$ or $y >= 50$

Check whether the contradiction method satisfy

Sol:- Contradiction

S-1:- p : $x+y>=100$ ~~is odd integer~~

q : $x>=50$ or $y>=50$

p, q is true

S-2:- $\sim q$ is true when: $x \leq 50$ and $y<50$

then p is false: $x+y \geq 100$ (false)

S-3:- Contradiction of p is true

* Problem(4):- Check whether below compound statement direct, indirect, contradiction methods satisfy or not

If n is even number then n+3 is odd integer

Sol:- Direct

S-1:- p : n is even number

q : n+3 is odd integer

p is true, q is true

S-2:- p is true : p : $n=2m$

q is true : q : $n=2m+3$ is odd

S-3:- p is true, q is true

$p \rightarrow q$ is true

Indirect

S-1:- p : n is even number

q : n+3 is odd integer

p, q is true

S-2:- $\sim q$ is true : $\sim q$: $n=2m+1$

$\sim p$ is true : $\sim p$: $n = 2m+1+3$

$n = 2m+4$ is even

S-3:-

$\sim q, \sim p$ is true

$\sim q \to \sim p$ is true

Contradiction:-

S-1:- p: n is even number

q: n+3 is odd number.

p, q is true

S-2:-

$\sim q$ is true : $\sim q$: n +3 n is always odd

p is false : p: n is even number (false)

S-3:-

Contradiction of p is true