UNIT-II

(CP set)
ite sets then the
B is
$b \in B\}$

* Binary Relations:-

* Set:-

- A set is a collection of objects or members.

Ex: IT $A = \{1, 2, 3, \ldots 65\}$

sets then the
is

* Types of sets:-

1.) Finite set:
- A set 'A' consists of limited elements is said to be finite set.

2.) Infinite set:-
- A set 'A' consists of unlimited elements is said to be Infinite set.

3.) Null set:
- A set 'A' is said to be null set when, it contain no elements.

Ex: $A = \{\}$ (or) ϕ

4.) Power set: (P)

A set 'A' is finite set, the powerset of 'A' is $P(A)$ the set of all elements with null set.

Ex: $A = \{1, 2, 3\} = 2^3 = 8$

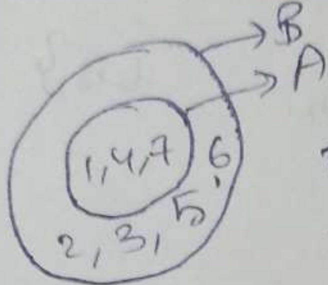
$P(A) = \{\{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, \{1,2,3\}, \{\}\}$

$\sim p$ is true :: are finite sets then every
therefore A' is subset

S-3:- $\sim q, \sim p \{4, 7\}$
$\sim q - \{2, 3, 4, 5, 6, 7\}$

Contradicti

S-1:- p: n :
q.



$\boxed{A \subset B}$

6.) Union set:-

- Let A, B both are finite sets then A∪B

$$\boxed{A \cup B = \{x \mid x \in A \text{ (or) } x \in B\}}$$

7.) Intersection set:-

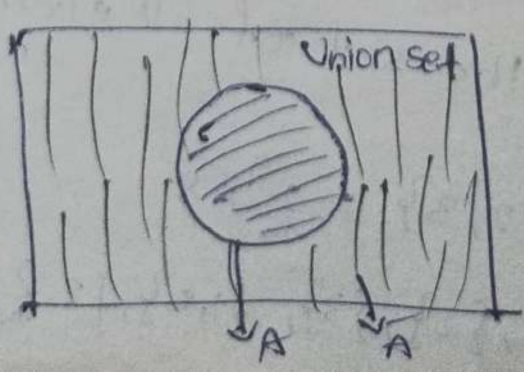
- Let A, B both are finite sets the A∩B

$$\boxed{A \cap B = \{x \mid x \in A \text{ and } x \in B\}}$$

8.) Complement of set:-

- Let A' is a finite set then A complement is

$$\boxed{\bar{A} = \{x \mid x \in \text{Union set} - x \in A\}}$$



* 9) Cartesian product set :- (CP set)

- Let A, B both are finite sets then the cartesian product of A & B is

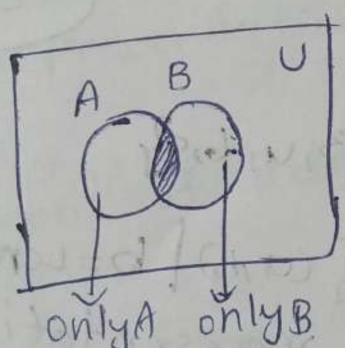
$$\boxed{A \times B = \{ (a, b) \mid a \in A \text{ and } b \in B \}}$$

* 10) Symmetric set :- (Δ)

- Let A, B both are finite sets then the symmetric set of A & B is

$$A \Delta B = |A \cup B| - |A \cap B|$$

$$= \text{only } A + \text{only } B.$$



* Matrix representation of Set :-

- Let 'A' is finite set then the matrix representation is

$$\underset{\text{row}}{M}_{i \atop \text{column}\, j} = \begin{cases} 1 & \text{A relation from a to b} \\ 0 & \text{no relation from a to b} \end{cases}$$

Ex: Let $A = \{1, 2, 3, 4\}$ the relation r is $(a < b)$ find the above set of matrix representation.

Sol:- S-1:- $A = \{1, 2, 3, 4\}$

Relation R is $(a<b)$

S-2:- Relation set $R = \{(1,2), (1,3), (1,4), (2,3), (2,4), (3,4)\}$

$a<b$	1	2	3	4
1	0	1	1	1
2	0	0	1	1
3	0	0	0	1
4	0	0	0	0

21/9/24 Saturday

*Problem①:- Let $A = \{1, 2, 3, 4, 6, 9\}$

Relation R is $R = \{(a,b) \mid b = LCM(a,b), a,b \in A\}$

Find the matrix representation of set A

Sol:- S-1:- $A = \{1, 2, 3, 4, 6, 9\}$

$R = \{(a,b) \mid b = LCM(a,b), a,b \in A\}$

S-2:- Relation set $R = \{(1,1), (1,2)\ (1,3)\ (1,4)\ (1,6)\ (1,9)\ (2,2)\ (2,4)\ (2,6)\ (3,3)\ (3,6)\ (3,9)\ (4,4)\ (6,6)\ (9,9)\}$

S-3 b=LCM(a,b)

b=LCM(a,b)	1	2	3	4	6	9
1	1	1	1	1	1	1
2	0	1	0	1	1	0
3	0	0	1	0	1	1
4	0	0	0	1	0	0
6	0	0	0	0	1	0
9	0	0	0	0	0	1

* Problem ②:- Let A = {LET, EGG, DOG, BED, CAT}

Relation R is R = {(a,b) | some common letters, a,b∈A}

i.) Find the relation set R

ii.) Matrix representation of set A

iii.) Diagraph

iv.) Each element of indegree and outdegree

Sol:- i.)

S-1:- A = {LET, EGG, DOG, BED, CAT}

Relation R = {(a,b) | some common letters, a,b∈A}

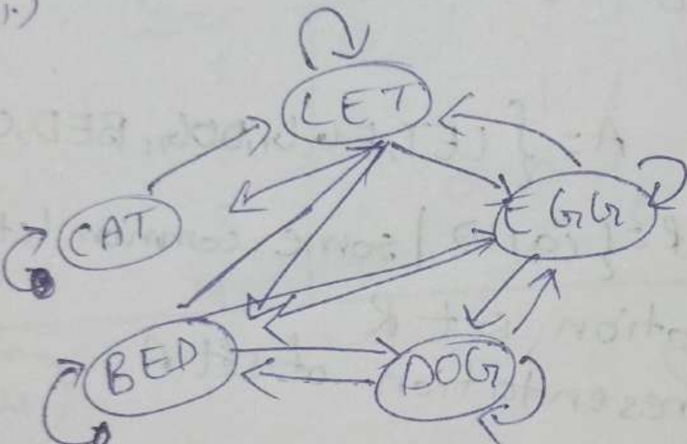
S-2:-

{(LET, EGG) (LET, EGG) (LET, BED) (LET, CAT)
(EGG, EGG) (EGG, LET) (EGG, DOG) (EGG, BED)
(DOG, DOG) (DOG, EGG) (DOG, BED) (BED, BED)
(BED, LET) (BED, EGG) (BED, DOG) (CAT, CAT)
(CAT, LET)} ~~CAT~~

S-3 ii.)

	LET	EGG	DOG	BED	CAT
LET	1	1	0	1	1
EGG	1	1	1	1	0
DOG	0	1	1	1	0
BED	1	1	1	1	0
CAT	1	0	0	0	1

S-4 iii.)



S5 iv)

	Indegree	outdegree
LET	4	4
EGG	4	4
DOG	3	3
BED	4	4
CAT	2	2
	17	17

*Problem 3 :- Let $A = \{1, 2, 3, 4, 6\}$.

Relation R is $R = \{(a, b) \mid a$ is divisor of $b, a, b \in A\}$

i.) Find the relation

ii.) Matrix representation of set A

iii) Diagraph

iv) Each element of indegree & out degree

Sol: $A = \{1, 2, 3, 4, 6\}$

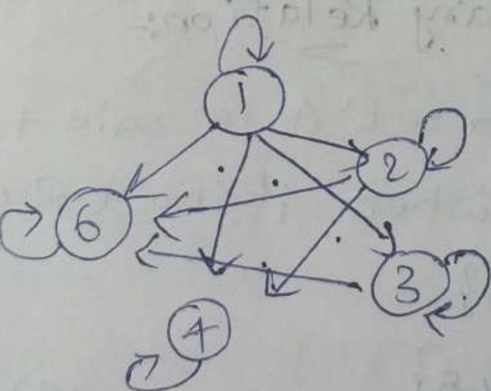
S-1: $R = \{(a,b) \mid a$ is divisor of $b, a, b \in A\}$

S-2: i) $R = \{(1,1)(2,2)(3,3)(4,4)(6,6)(2,4)(1,2)(1,3)(1,4)(1,6)(2,6)(3,6)\}$

S-3: ii)

	1	2	3	4	6
1	1	1	1	1	1
2	0	1	0	1	1
3	0	0	1	0	1
4	0	0	0	1	0
6	0	0	0	0	1

S-4 iii)



S-5 iv)

	Indegree	outdegree
1	1	5
2	2	3
3	2	2
4	3	1
6	4	1
	12	12

5/09/24 Wednesday

*Binary relation:-

1) Reflexive Binary Relation:-

• A relation 'R' on a set 'A' is said to be reflexive binary relation, when if each element $(a,a) \in R, a \in A$.

$$R_3 = \{(1,1),(2,2),(3,3),(4,4),(5,5)\}$$

2) Irreflexive Binary relation:-

• A relation 'R' on a set 'A' is said to be irreflexive Binary relation when if each element not belongs to $(a,a) \notin R; \{a \in A\}$

$$R = \{(1,1),(2,2),(3,3)\}$$

3) Symmetric Binary Relation:-

• A relation 'R' on a set 'A' is said to be binary Relation when if $(b,a) \in R$ where $(a,b) \in R$, $\{a,b \in A\}$

$$A = \{1,2,3,4,5\}$$

$(1,5)$ $(5,1)$ $(5,5)$

$$R = \{(1,1),(2,2),(3,3),(3,4),(4,3),(4,4)\}$$

Symmetric reflexive relation

4) Asymetric Binary Relation:-

• A Relation 'R' on a set 'A' is said to be assymetric binary relation when $(b,a) \notin R$ where $(a,b) \in R$, $\{a,b \in A\}$

5) Anti-Symmetric binary Relation:-

- When $(a,b) \in R$ & $(b,a) \in R$ then $a=b$, $\{a,b \in A\}$ or when $(a,b) \in R$ then $(a<b)$, $\{a,b \in A\}$

$R = \{(1,1)(2,2), (3,3), (3,4)\}$

6) Transitive Binary Relation:-

- A Relation 'R' on a set 'A' is said to be Transitive binary relation when $(a,b) \in R$ & $(b,c) \in R$ then $(a,c) \in R$, $\{a,b,c \in A\}$

$R = \{(1,1)(1,2)(2,1)(2,2)(1,3)(3,1)(3,2)(3,3)(2,3)\}$

7) Compatability binary relation:-

- A relation 'R' on a set 'A' is said to be compatability binary relation when if "R" is reflexive and symmetrix binary relation

$R = \{(1,1)(2,2)(3,3)(4,4)\}$

8) Equivalence:-

- A relation 'R' on a set 'A' is said to be equivalence when if 'R' reflexive, symmetric, transitive binary relation.

9) Poset:- (Partially ordered set) $\langle A, R \rangle$

- A relation 'R' is on a set 'A' non-emety is said to be partially ordered set when 'R' is reflexive, antisymmetric and transitive binary relation.

relationset ($\leq$)

10.) Hasse diagram:-

The graphical representation of poset is called Hasse diagram.

Step 1:- Ex:- Let $A = \{1,2,3,4\}$ to the relation

$$R = \{(a,b) \mid a < b,\ a,b \in A\}$$

Step 2:- $R = \{(1,2),(1,3),(1,4)\ (2,3)\ (2,4)\ (3,4)\}$

Step 3:- Hasse diagram not possible R is not a reflexive binary relation

2.)

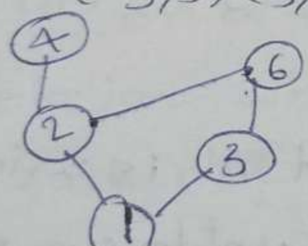
S-1:- $A = \{1,2,3,4,6\}$

R is "a is divisor of b").

Draw the Hasse Digaram

S-2:- $R = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,6)\ (2,2)\ (2,4)\ (2,6)$
$(3,3)\ (3,6)\ (4,4)\ (6,6)\}$

S-3:-

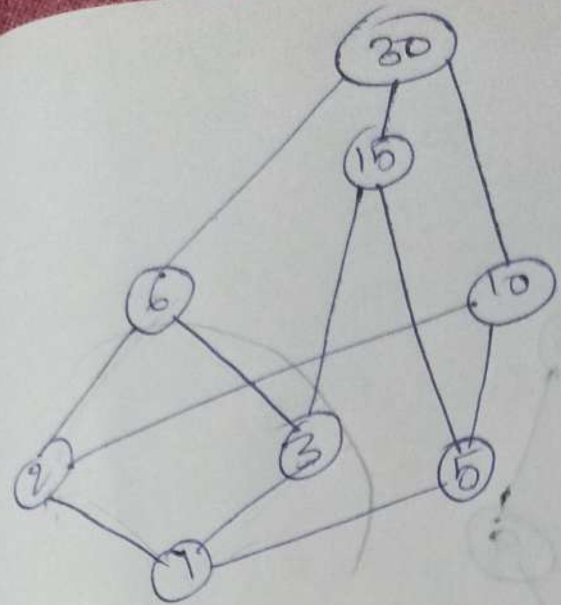


3.) Let $A = \{1,2,3,5,6,10,15,30\}$

S-1:- Relation R is "a is divisor of b"

Draw the Hasse Diagram

S-2:- $R = \{(1,1)\ (1,2)\ (1,3)\ (1,5)\ (1,6)\ (1,10)\ (1,15)\ (1,30)$
$(2,2)\ (2,6)\ (2,10)\ (2,30)\ (3,3)\ (3,6)\ (3,15)\ (3,30)$
$(5,5)\ (5,10)\ (5,15)\ (5,30)\ (6,6)\ (6,30),\ (10,10)\ (10,30)$
$(15,15)\ (15,30)\ (30,30)\}$

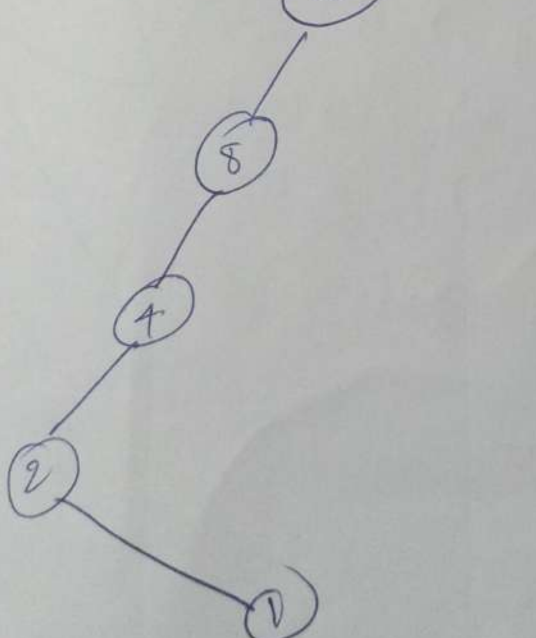


4.) Draw the D_{32} and D_{36} Hasse Diagram

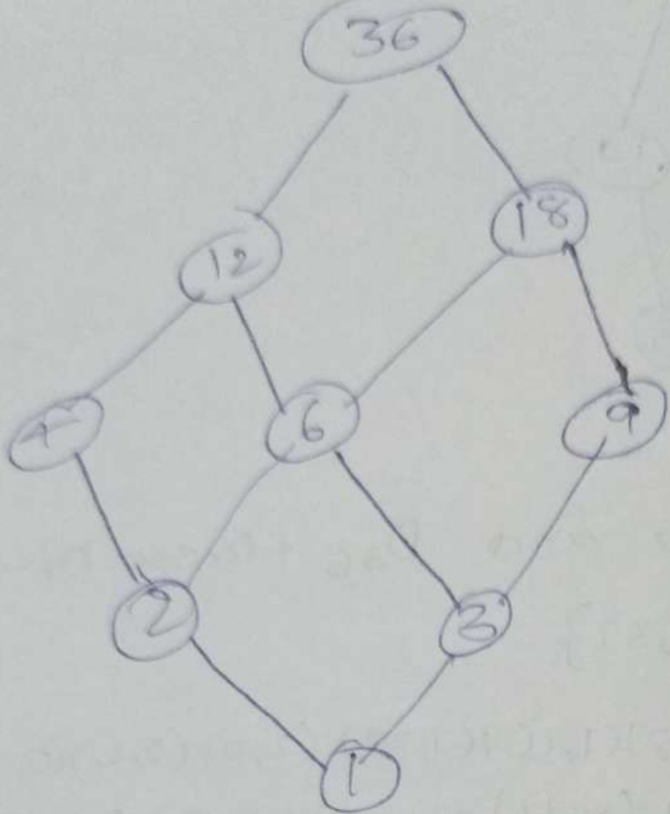
Sol: $D = \{1, 2, 4, 8, 16, 32\}$

$= \{(1,1)(1,2)(1,4)(1,8)(1,16)(1,32)(2,2)(2,4)(2,8)(2,16)$
$(2,32)(4,4)(4,8)(4,16)(4,32)(8,8)(8,16)(8,32)$
$(16,16)(16,32)(32,32)\}$

D_{32}



$\underline{\underline{D_{36}}}$



26/09/24

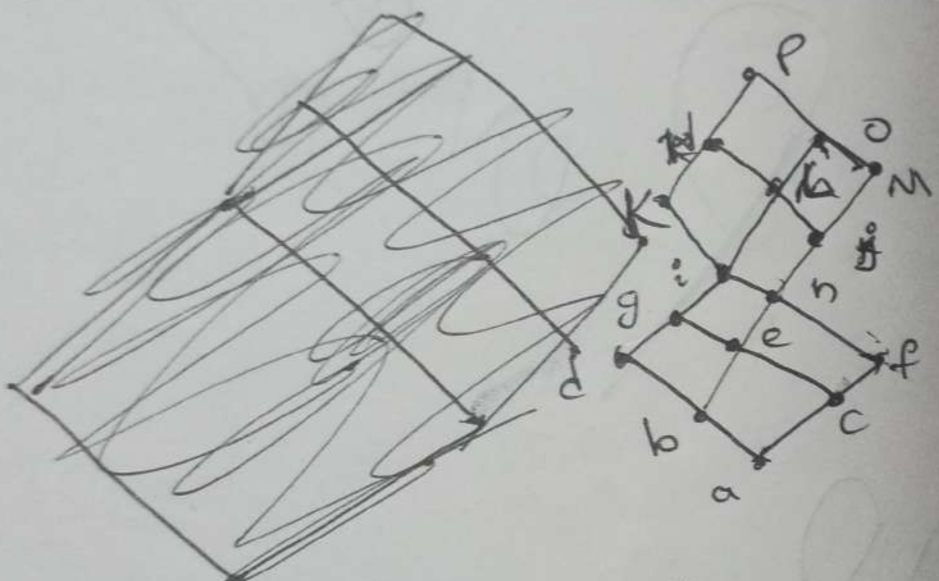
Thursday

* Lattice:-

• Let <A,R> is a poset then above poset is called "Lattice" when, "if every every two elements have (i) LUB (Least upper Bound: "a∨b" "a join b" (ii) GLB (Greater lower Bound): "a∧b" "a meet b".

(nonempty → Relation order (≤))

* Solve below diagram of LUB & GLB's

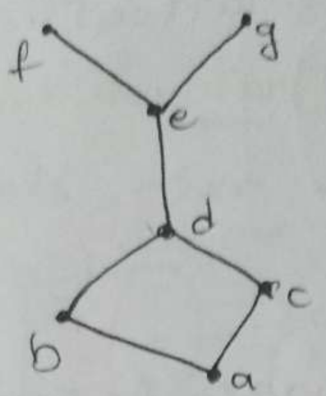


i.) LUB(i,M) ii.) GLB(g,J) iii.) GLB(d,h)
iv.) LUB(d,M) v.) GLB(K,M) vi.) LUB(d,O)
vii.) GLB(e,h) viii.) LUB(e,i)

Sol:- i.) O ii.) e iii.) b iv.) O v.) h vi.) O vii.) e
viii.) i

Problem(1): See whether they are lattice or not.

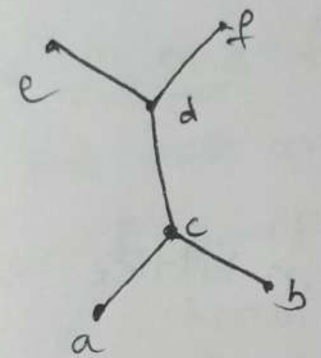
i.)



(Incomplete diagram)

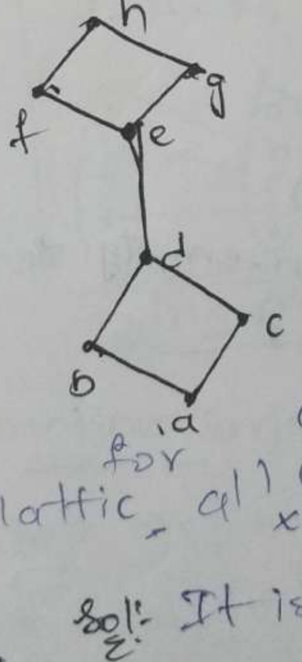
sol: It is not lattic because $LUB(f, g)$ is not possible

ii.)



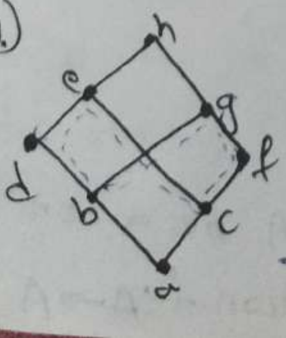
sol: It is not lattic because $GLB(a, b)$ is not possible

iii.)



sol: It is lattic, for all elements GLB, LUB is Possible

iv.)



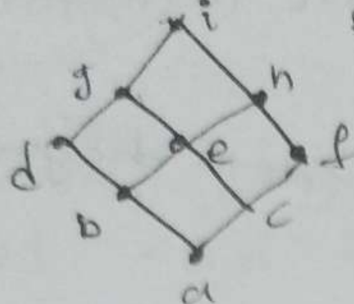
sol: It is not lattic because

$LUB(b, c) = e = g$

e or g

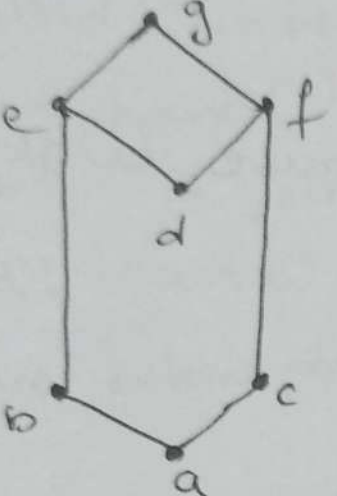
There should only be one

(v)



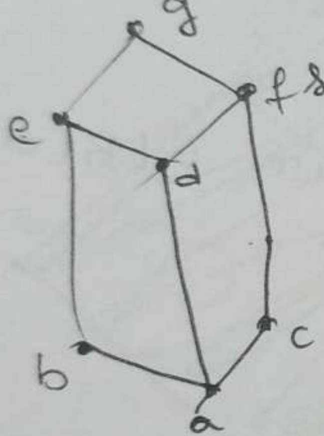
Sol:- It is lattic

(vi)



Sol:- LUB(a,d) = e (or) f
so not possible

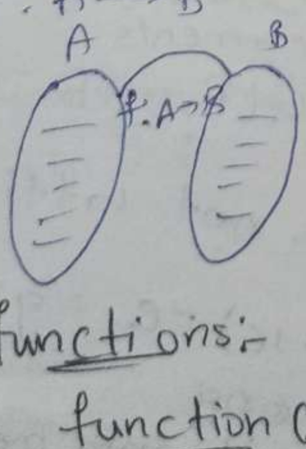
(v)



Sol:- It is in lattice

* Functions:- (mapping)

• Let A,B both are non-empty sets then a function $f: A \rightarrow B$

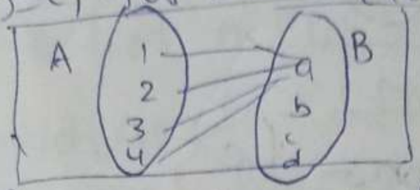


* Types of functions:-

i) Identity function (I_A):- Let A be a non empty set then a function $f: A \rightarrow A$

such that $f(a) = a$ is said to be "Identity functions" (I_A).

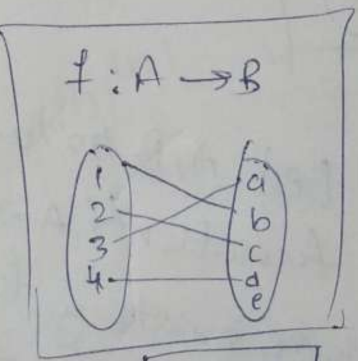
2) Constant function (I_c):- Let A & B are non-empty sets then a function $f: A \to B$ such that $f(a) = c$, for every element $a \in A$ and c



is the fixed element of B then a function is said to be a constant function.

3) One to one function (Injection):-

- Let A, B both are non-empty sets then a function $f: A \to B$ is said to be one to one function when, if different elements of A have different images in B.

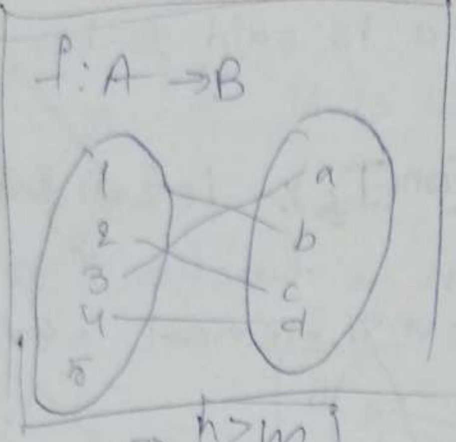


Note:- No. of elements in A always less than or equal to no. of elements in B.

$\Rightarrow \boxed{n \leq m}$

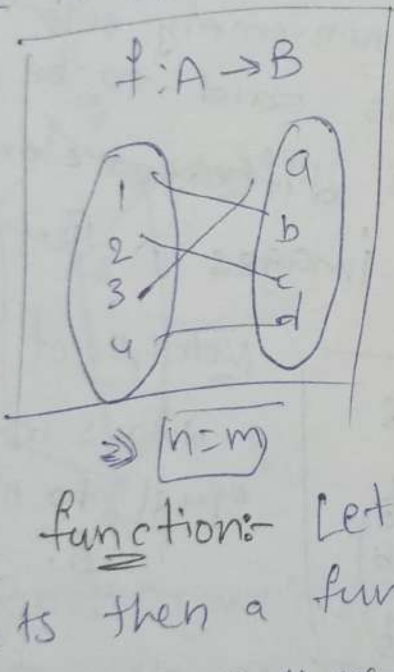
4) Onto function (Surjection):-

- Let A, B both are non-empty sets then a function $f: A \to B$ is said to be onto function when, if every element of B has a different image in A.



$\Rightarrow \boxed{n \geq m}$

5) **Bijection function:-** Let A, B both are non-empty sets then a function $f : A \to B$ is said to be Bijection function when, if f is one to one function and onto function



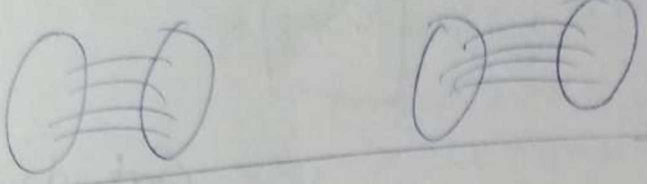
$\Rightarrow \boxed{n = m}$

6.) **Inverse function:-** Let A, B both are non-empty sets then a function $f : A \to B$ is said to be Inverse if there exists a function $f : A \to B$ function under the
$g : B \to A$

7.) **Composite function:-** Let A, B and C be non-empty sets then the function $f : A \to B$ and function $g : B \to C$ then the composite function

$\boxed{f \circ g : A \to C}$ $\quad f \circ g = f(g(x))$

$f: A \to B$ $\quad$ $g: B \to C$



*problem ②:- Let $A = \{1,2,3,4\}$, $B = \{a,b,c,d,e,f\}$

i) Find the no. of functions from A to A

ii) find the no. of functions from A to B 6^4

iii) Find the no. of functions from B to A 4^6

iv) find the no. of one to one functions from A to B

v) Find the no. of one to one functions from B to A

vi) find the no. of onto functions from A to B.

vii) Find the no. of onto functions from B to B

sol:- $A = \{1,2,3,4\} \Rightarrow |n| = 4$

$B = \{a,b,c,d,e,f\} \Rightarrow |m| = 6$

(i) $|A|^{|A|} = |n|^{|n|} = 4^4 = 256$

(ii) $|B|^{|A|} = 6^4 = 1296$

(iii) $|A|^{|B|} = 4^6 = 4096$

(iv) condition $(n \le m)$ $= \dfrac{m!}{(m-n)!} = \dfrac{6!}{(6-4)!}$
$4 \le 6$

$= 6 \times 5 \times 4 \times 3 = 360$

(v) condition $(n \le m)$ (not possible condition
$6 \le 4$ does not satisfied)

(vi) condition $(n \ge m)$ (not satisfied)
$4 \ge 6$

(vii) condition $(m \geq n)$ $= {}^6P_4$
$6 \geq 4$

$\Rightarrow \boxed{{}^6P_4}$

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Saturday

* Algebraic Systems:-

- Let "s" be a non-empty set then "S*S", set of all order pairs on S under the operation "*" is denoted by $\boxed{< S, * >}$

* Algebraic System properties:-

1. Commutotive property
2. Associative property
3. Identity element
4. Inverse element

1. Commutative property:-

- Let 'S' be a non-empty set, * be a binary operation $a * b = b * a, \{a, b \in S\}$

Ex: $A = \{1, 2, 3, 4\}$, * be a binary operation, follow +, &, × check whether which one is a by commutative property or not.

Sol:- Step 1:- $A = \{1, 2, 3, 4\}$

* be a binary operation '+'

Step 2:- $a * b = b * a$, $(a, b \in S)$

$$\left.\begin{array}{l} 1+2 = 2+1 \\ 3+2 = 2+3 \\ 3+4 = 4+3 \end{array}\right\} \begin{array}{l} <S,+> \\ \text{commutative property} \\ \text{satisfies} \end{array}$$

step 2: $A = \{1, 2, 3, 4\}$

$*$ be a binary operation '$-$'

$a * b = b * a$, $(a, b \in S)$

$1 - 2 \neq 2 - 1$

$\langle S, - \rangle$ commutative property not satisfy

step 3: $A = \{1, 2, 3, 4\}$

$*$ be a binary operation '$/$'

$1/2 \neq 2/2$

$\langle S, / \rangle$ commutative property not satisfy

step 4: $A = \{1, 2, 3, 4\}$

$*$ be a binary operation '$\times$'

$\left.\begin{array}{l} 1 * 2 = 2 \times 1 \\ 3 \times 4 = 4 \times 3 \end{array}\right\}$ $\langle S, \times \rangle$ commutative property satisfies

2.) <u>Associative property</u>:-

• Let 'S' be a non-empty set, $*$ be a binary operation $(a * b) * c = a * (b * c)$ $\forall \{a, b, c \in S\}$

Ex: $A = \{1, 2, 3, 4\}$, $*$ be a binary operation $+, -, /, \times$ check whether which one is a followed by associative property or not.

Sol: step 1: $A = \{1, 2, 3, 4\}$

$*$ be a binary operation '$+$'

$$\left.\begin{array}{l} (1+2)+3 = 1+(2+3) \\ 3+3 = 1+5 \\ 6 = 6 \end{array}\right\} \langle S, + \rangle \text{ associative property satisfies}$$

step 2: $A = \{1, 2, 3, 4\}$

$*$ be a binary operation '$-$'

$$(1-2)-3 = 1-(2-3)$$
$$-1-3 = 1+1$$
$$-4 \neq 2$$

$\langle S, - \rangle$ not satisfy

Step 3:- $A = \{1, 2, 3, 4\}$

* be a binary operation '/'

$$(1/2)/3 \neq 1/(2/3)$$

$\langle S, / \rangle$ (not satisfy)

Step 4:- $A = \{1, 2, 3, 4\}$

* be a binary operation '×'

$$(1 \times 2) \times 3 = 1 \times (2 \times 3)$$
$$6 = 6$$

$\langle S, \times \rangle$ satisfies

3) Identity element (e):-

- Let 'S' be a non-empty set, & there is a identity element e and at the binary operation * then $\boxed{a * e = a}$ $\{a, e \in S\}$ for each element

4) Inverse element:-

- Let 'S' be a non-empty set & there is a identity element e and at the binary operation * then $\boxed{a * a^{-1} = e}$ for each element $\{a, a^{-1}, e \in S\}$

Semi group:-	Monoid	Ring/Group/abelian Group
• An algebraic system $<s, *>$ consists of a non-empty set 'S' and * be a binary operation which satisfies → commutative & → associative property is said to be semigroup under the operation *.	• Let $<s, *>$ is a semigroup this semigroup is called Monoid when if 'S' contain identity element 'e' under the binary operation *.	• Let 'S' be a non-empty set, * be a binary operation then $<s, *>$ is a Ring (or) Group (or) abelian group when if (i) $a*b = b*a, \{a,b \in s\}$ (ii) $(a*b)*c = a*(b*c), \{a,b,c \in s\}$ (iii) $a*e = a$, for each element $\{a, e \in s\}$ (iv) $a*a^{-1} = e$ for each element $\{a, a^{-1}, e \in s\}$

3/10/24 Thursday

*Problem ①:- Check whether above representation of Matrix representation of Graph $<s,*>$ is a semigroup (or) not?

*	a	b
a	a	b
b	b	a

sol:

Step 1:-

*	a	b
a	a	b
b	b	a

Step 2:- $S = \{a, b\}$, * be a binary

Step 3:- Check $<s, *>$ is a commutative property (or) not?

$$\boxed{\begin{aligned} a*b &= b*a, \{a,b \in s\} \\ b &= b \end{aligned}}$$

Step 4:- Check $<s,*>$ is a associative property (or) not

$a * (b * c) = (a * b) * c \; \{a, b, c \in S\}$

$$\boxed{\begin{aligned} a * (b * b) &= (a * b) * b \\ a * a &= b * b \\ a &= a \end{aligned}}$$

step 5:- $\boxed{\therefore \text{ Semigroup}}$

*Problem ②:- Let $S = \{1, 2, 3, 6\}$, $*$ derived by LCM of (a, b) check whether $\langle S, * \rangle$ is a monoid or not.

sol:- Step 1:- $S = \{1, 2, 3, 6\}$

$*$ derived by LCM(a, b)

step 2:-

LCM (a,b)	1	2	3	6
1	1	2	3	6
2	2	2	6	6
3	3	6	3	6
6	6	6	6	6

step 3:- check commutative property

$a * b = b * a \; \{a, b \in S\}$

$1 * 3 = 3 * 1$ $3 = 3$	$2 * 3 = 3 * 2$ $6 = 6$

Satisfies

step 4:- check associative property

$(a * b) * c = a * (b * c)$

$(1 * 2) * 3 = 1 * (2 * 3)$

$2 * 3 = 1 * 6$

$6 = 6$

satisfies

step 5: check Identity element

$a * e = a$ for each element $\{a, e \in S\}$

$1 * 1 = 1$
$2 * 1 = 2$
$3 * 1 = 3$
$6 * 1 = 6$

$\boxed{e = 1 \in S}$ for each element

step 6:- $\boxed{\therefore \text{Monoid}}$

*Problem ③:- Let $S = \{0, 1, 2, 3, 4\}$, * derived by Modula Addition 5 (Mod+5). check whether <S, *> is a group or not.

sol:- step 1:- $S = \{0, 1, 2, 3, 4\}$

* derived by Mod +5

step 2:-

Mod+5	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

step 3:- Check commutative property

$a * b = b * a \quad \{a, b \in S\}$

$2 * 3 = 3 * 2$	$1 * 4 = 4 * 1$
$0 = 0$	$0 = 0$

satisfies

∴ step 4 :- check associative property

$a*(b*c) = (a*b)*c \quad \{a,b,c \in S\}$

∴ satisfied

$1*(2*3) = (1*2)*3$

$1*0 = 3*3$

$1 = 1$

step 5 :- check Identity element

$a*e = a$ for each element $\{a, e \in S\}$

∴ satisfied

$0*0 = 0$

$1*0 = 1$

$2*0 = 2$

$3*0 = 3$

$4*0 = 4$

$e = 0 \in S$

step 6 :- check Inverse element

$a*a^{-1} = e, \quad \{a, a^{-1}, e \in S\}$

∴ satisfied

$0*0 = 0$

$1*4 = 0$

$2*3 = 0$

$3*2 = 0$

$4*1 = 0$

step 7 :- $\boxed{\therefore \text{group}}$

*Problem ④ :- Let S = { 1, 2, 3, 4, 5, 6 }, * derived by Modula Multiplication 7 (mod×7). Check whether <S, *> is a group or not?

Sol:- $S = \{1, 2, 3, 4, 5, 6\}$

Step 1 :- * derived by modula multiplication 7 (mod ×7)

Step 2 :-

mod×7	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	4
4	4	1	5	2	6	3
5	5	3	1	6	4	2
6	6	5	4	3	2	1

Step 3 :- check commutative property

$a * b = b * a \quad \{a, b \in S\}$

$1 * 5 = 5 * 1$ | $6 * 4 = 4 * 6$

$5 = 5$ | $3 = 3$

∴ satisfied

Step 4 :- check associative property

$(a * b) * c = a * (b * c) \quad \{a, b, c \in S\}$

$(1 * 3) * 6 = 1 * (3 * 6)$

$3 * 6 = 1 * 4$

$4 = 4$

∴ satisfied

step 5:- Check Identity element.

$$\boxed{\begin{array}{l} 1 * 1 = 1 \\ 2 * 1 = 2 \\ 3 * 1 = 3 \\ 4 * 1 = 4 \\ 5 * 1 = 5 \\ 6 * 1 = 6 \\ e = 1 \in S \end{array}}$$

∴ satisfied

step 6:- check Inverse element

$$\boxed{\begin{array}{l} 1 * 1 = 1 \\ 2 * 4 = 1 \\ 3 * 5 = 1 \\ 4 * 2 = 1 \\ 5 * 3 = 1 \\ 6 * 6 = 1 \end{array}}$$

∴ satisfied

step 7:- $\boxed{\text{∴ group}}$

* Problem 6:- Let set S is the i^{th} roots, * derived by 'x' i) check whether $\langle S, * \rangle$ is a group or not ii) for each element of degree

Sol:- i)

step 1:- Set S is the i^{th} roots

$S = \{1, -1, i, -i\}$ $\boxed{i^2 = -1}$

* is derived by "x"

Step 2:

$\times$	1	-1	i	$-i$
1	1	-1	i	$-i$
-1	-1	1	$-i$	i
i	i	$-i$	-1	1
$-i$	$-i$	i	1	-1

Step 3:- check commutative property

$a * b = b * a \quad \{a, b \in S\}$

$1 * -1 = -1 * 1$

$-1 = -1$

$\therefore$ satisfied

Step 4:- check associative property

$(a*b)*c = a*(b*c)$

$(1*-1)*i = 1*(-1*i)$

$-1*i = 1*-i$

$-i = -i$

$\therefore$ satisfied

Step 5:- check Identity element

$1 * 1 = 1$

$-1 * 1 = -1$

$i * 1 = i$

$-i * 1 = -i$

$e = 1 \in S$

$\therefore$ satisfied

Step 6: check Inverse element

$1 * 1 = 1$

$-1 * -1 = 1$

$i * -i = 1$

$-i * i = 1$

$\therefore$ satisfied

Step 7: $\boxed{\therefore \text{Group}}$

Step 8: check each element of degree

$(x^n) = e$

x → set of elements; n → degree of above element; e → Identity element

(ii) 1 degree $(1)^1 = 1 \Rightarrow$ degree 1

-1 degree $(-1)^n = 1 \Rightarrow$ degree 2

i degree $(i)^4 = 1 \Rightarrow$ degree 4

$-i$ degree $(-i)^4 = 1 \Rightarrow$ degree 4

Problem (6):- Let set S is the "ω" cubes, $*$ derived by '$\times$' i) check whether $<S, *>$ is a group or not ii) Find the each element of degree.

Sol:- step 1:- set S is the "ω" cubes

$S = \{1, \omega, \omega^2\}$

$*$ is derived by '$\times$'

step 2:-

$\times$	1	ω	ω^2
1	1	ω	ω^2
ω	ω	ω^2	1
ω^2	ω^2	1	ω

step 3:- check commutative property

$a * b = b * a$

$1 * \omega^2 = \omega^2 * 1$

$\omega^2 = \omega^2$

$\therefore$ satisfied

step 4: check assoc

$(a*b)*c$

$(1*\omega^2)$ homomorphism function

$\omega^2*\omega$ groups $<R^+,\times>$ & $<R,+>$ then

$\therefore$ satisfied

$\therefore f(a)=\log_e x$ check

$1=1$

step 5: check Identity element

$1*1=1$

$\omega*1=\omega$

$\omega^2*1=\omega^2$

$e=1\in S$

$\therefore$ satisfied

step 6: check Inverse element

$1*1=1$

$\omega*\omega^2=1$

$\omega^2*\omega=1$

$\therefore$ satisfied

step 7: $\boxed{\therefore \text{group}}$

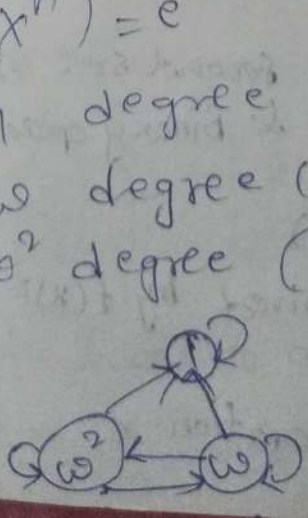
step 8: check each element of degree

ii.) $(x^n)=e$

1 degree $(1)^1=1 \Rightarrow$ degree 1

ω degree $(\omega)^3=1 \Rightarrow$ degree 3

ω^2 degree $(\omega^2)^3=1 \Rightarrow$ degree 3



Step 7: $\boxed{\therefore \text{Group}}$ omorphism

Step 8: check each elem > be a two semigroup
degree of $(x^n) = e$; $S_1 \to S_2$ is said to be
set of elem (n) ; Identity tion when ~~f(a*b)~~
$\boxed{= f(a) *_2 f(b), \{a, b \in S_1\}}$

• Let $\langle S_1, *_1 \rangle$ & $\langle S_2, *_2 \rangle$ two semigroups then a function $f: S_1 \to S_2$ is said to be Isomorphism function when if

i) $f(a *_1 b) = f(a) *_2 f(b), \{a, b \in S_1\}$

ii) f is one to one function

iii) f is onto function

*Note:- $\boxed{H \subset I}$ ($\subset$ → subset)

5/10/24

Saturday

Problem ①:- Let two semigroups $\langle R^+, \times \rangle$ & $\langle Z, + \rangle$ function $f: R^+ \to Z$ derived by $f(x) = 2^x$. Check whether function f is homomorphism (or) not?

Sol:- S-1:- Two semigroups $\langle R^+, \times \rangle$ & $\langle Z, + \rangle$

S-2:- first set $S_1 = R^+$ & Binary operation $*_1 \to \times$ | Second set $S_2 = Z$ & Binary operation $*_2 \to +$

S-3:- $f(a *_1 b) = f(a) *_2 f(b)$

function $f: R^+ \to Z$ derived by $f(x) = 2^x$

S-4:- $f(x) = 2^x$

$*_1$ = Binary operation $\times$

put $x = a \times b$

$f(a \cdot b) = 2^{(a \cdot b)} = (2^a)^b$

$f(a *_1 b) = (2^a)^b \neq 2^a + 2^b$

$f: R^+ \to Z$ is not a homomorphism function

*Problem (2): Let two semigroups $\langle R^+, \times \rangle$ & $\langle R, + \rangle$ then a function $f: R^+ \to R$ derived by $f(a) = \log_e x$. Check whether function f is Isomorphism or not?

Sol: S-1: Two semigroups $\langle R^+, \times \rangle$ & $\langle R, + \rangle$

S-2: first set $S_1 = R$ & Binary operation $*_1 = \times$ | second set $S_2 = R$ & Binary operation $*_2 = +$

S-3: $f(a *_1 b) = f(a) *_2 f(b)$

function $f: R^+ \to R$ derived by $f(x) = \log_e x$

S-4: $f(x) = \log_e x$

put $*_1 \to$ Binary operation $\times$

$x = a \times b$

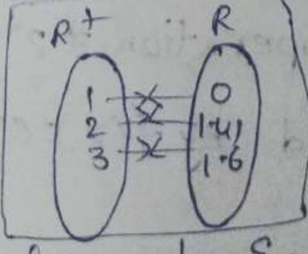
$f(a \cdot b) = \log_e (a \cdot b)$

$f(a \cdot b) = \log_e a + \log_e b = f(a) + f(b)$

$f(a *_1 b) = f(a) *_2 f(b) = \log_e a + \log_e b$

S-5: function $f: R^+ \to R$ is homomorphism function

S-6: Check whether function f is Isomorphism function or not



$f(x) = \log_e x$

f is onto & one to one function

$\boxed{\therefore f: R^+ \to R \text{ is Isomorphism function}}$

*Problem ③:- Let semigroups $<N,+>$ & $<Z,\times>$ function $f: N \to Z$ derived by $f(a) = x^2$. Check whether function f is Isomorphism or not.

f is Isomorphism $<N,+>$ & $<Z,\times>$

Sol:- S-1:- Two semigroups

S-2:- first set $S_1 = N$ & Binary operation $*_1 = +$ | second set $S_2 = Z$ & Binary operation $*_2 = \times$

S-3:- $f(a *_1 b) = f(a) *_2 f(b)$

function $f: N \to Z$ derived by $f(x) = x^2$

S-4:- $f(x) = x^2$

put $*_1 \to$ Binary operation +

$x = a + b$

$f(a+b) = (a+b)^2$

$f(a+b) = a^2 + b^2 + 2ab$

$$\boxed{f(a+b) \neq a^2 + b^2 + 2ab \neq f(a) \times f(b)}$$

$f: N \to Z$ is not a Homomorphism or Isomorphism function.

*Problem ④:- Let two semigroups $<N,+>$ & $<Z,\times>$ function $f: N \to Z$ derived by $f(x) = e^x$. Check whether function f is Isomorphism or not.

Sol:- S-1:- Two semigroups $<N,+>$ & $<Z,\times>$

S-2:- first set $S_1 = N$ & Binary operation $*_1 = +$ | second set $S_2 = Z$ & Binary operation $*_2 = \times$

S-3:- function $f: N \to Z$ is derived by $f(x) = e^x$

S-4:- $f(x) = e^x$

$*_1 \to$ Binary operation +

put $x = a + b$

$f(a+b) = e^{a+b} = e^a \cdot e^b$

$$f(a+b) = f(a) + f(b) = e^a \cdot e^b$$

5:- function $f: N \to Z$ is homomorphism function

6:- check whether function is isomorphism or not

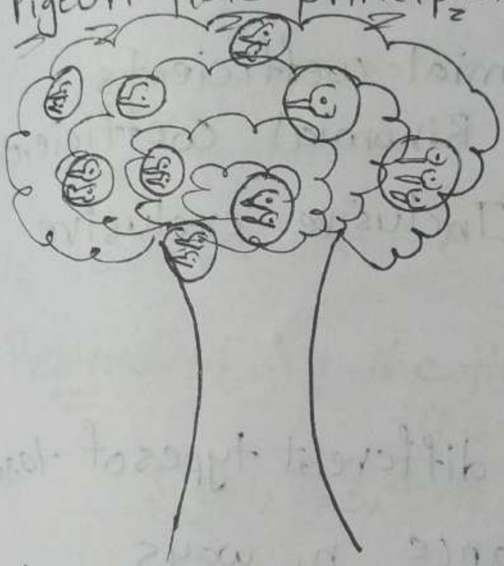
N	Z
1	2.71
2	7.38
3	20.08

$f(x) = e^x$

f is one to one and onto

$\boxed{f: N \to Z \text{ is isomorphism function}}$

* Pigeon Hole principle:-



 → pigeons(m)

O → pigeon Holes(n)

$m \geq n$

Let m pigeons and n pigeon holes ($m \geq n$) then two (or) more than pigeons occupy pigeon holes is

$$\boxed{P = \left|\frac{m-1}{n}\right| + 1}$$

*Problem ⑤:- How many students atleast 5 of them will have birthdays same month of the calender?

Sol:- → pigeons ⇒ m

O → O (Jan) - - - O (Dec) ⇒ $n = 12$

$p = 5$

$5 = \frac{m-1}{12} + 1 \Rightarrow m-1 = 48$

$$\boxed{m \geq 49}$$