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UNIT-II

1 Verify that the sum of eigen values is equal to the trace of A for the matrix $A=\begin{bmatrix}3 & -1 & 1\\ -1 & 5 & -1\\ 1 & -1 & 3\end{bmatrix}$ and find the corresponding eigen vectors.

2 Let λ be an eigen value of a square matrix A and x be its corresponding eigenvector. Then, verify the following properties of A (a) $(1/\lambda)$ is an eigen value of A^{-1}.

(b) A–k I has the eigen value $(\lambda-k)$ and the corresponding eigen vector is x for any scalar k.

(c) $(1/\lambda)$ is an eigen value of A^{-1}.

(d) The determinant of A is the product of all its eigen values.

(e) The Trace of A is the sum of its eigen values.

3 Determine the eigen values and eigen vectors of $B=2A^2-(1/2)A+3I$, where $A=\begin{bmatrix}8 & -4\\ 2 & 2\end{bmatrix}$.

4 Show that the matrix are Hermitian and its eigen values are real then find eigen vectors

a) $A=\begin{bmatrix}2 & 3+4i\\ 3-4i & 2\end{bmatrix}$ b) $\begin{pmatrix}2 & 1+i\\ 1-i & 1\end{pmatrix}$

5 (a) Prove that two eigen vectors of a real symmetric matrix are orthogonal.

(b) Prove that the eigen values of a real symmetric matrix are real.

(c) Prove that the eigen values of a real skew symmetric matrix are either zero or purely imaginary.

6 (a) Prove that the two eigen vectors corresponding to the two different eigen values are linearly independent.

(b) Show that the eigen values of an unitary matrix are of unit modulus.

(c) Prove that the inverse of a unitary matrix is unitary.

7 If A and B are n rowed square matrices and if A is invertible show that $A^{-1}B$ and BA^{-1} have the same eigen values.

8 Let A and B be two square matrices of the same order n. (i) Show that AB and BA have the same eigen values. (ii) Show also that A and $B^{-1}AB$ have the same eigen values. Verify these properties when

$$A=\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix},\quad B=\begin{bmatrix}8 & -4\\ 2 & 2\end{bmatrix}.$$

9 Diagonalize the following matrices.

(i) $A=\begin{bmatrix}11 & -4 & -7\\ 7 & -2 & -5\\ 10 & -4 & -6\end{bmatrix}$ and find A^5. (ii) $A=\begin{bmatrix}1 & 1 & 1\\ 0 & 2 & 1\\ -4 & 4 & 3\end{bmatrix}$ find A^4.

10 Diagonalize the following matrices.

(iii) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{bmatrix}$ (iv) $\begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$

11 Diagonalize the following matrices.

(v) $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ (vi) $\begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$

12 Find the nature of the quadratic form and reduce it to canonical form. Find rank, index and signature $4x^2 + y^2 - 8z^2 + 8yz - 4zx + 4xy$.

13 By the use of an orthogonal transformation, reduce the quadratic forms into their canonical forms and find their nature, index and signature. (a) $3x^2 - 2y^2 - z^2 + 12yz + 8zx - 4xy$.

(b) $8x^2 + 7y^2 + 3z^2 - 12xy - 8yz + 4zx$.

14 Find the nature of the quadratic form and reduce it to canonical form. Find rank, index and signature $2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx$.

15 Show that the matrix $A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$ is not diagonalizable.

16 a) If A = $\begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$, then find A^{50}. (b) If A = $\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$, then find A^{40}.

17 *Find an orthogonal matrix that will diagonalise the real symmetric matrix*

$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$ *and also find the resulting diagonal matrix.*

18 Orthogonally diagonalize the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

19 Find the eigen values and Eigen Vectors of $A = \begin{bmatrix} -i & 0 & 0 \\ 0 & 0 & -i \\ 0 & -i & 0 \end{bmatrix}$

20 Reduce the quadratic form $3{x_1}^2 + 3{x_2}^2 + 3{x_3}^2 + 2x_1x_2 + 2x_1x_3 - 2x_2x_3$ into sum of squares form by an orthogonal transformation