CS 5990 (Advanced Data Mining) - Assignment #2
Maximum Points: 100 pts.

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Answers:

1) Binary Classification Problem:

a) The entropy of the collection of training examples with respect to the class attribute

$$Entropy = -\sum_{i=0}^{c-1} p_i(t) log_2 p_i(t)$$

$$p(+) = \frac{4}{9} = 0.44$$
$$p(-) = \frac{5}{9} = 0.55$$
$$\therefore \; E = \; -0.44 \, log_2 \, 0.44 - 0.55 \, log_2 \, 0.55$$
$$= -0.44(-1.1844) - 0.55(-0.8625)$$
$$= 0.52 + 0.47$$
$$= 0.9911$$

b) The information gains of $a1$ and $a2$ relative to the training examples is
Information Gain = P-M

Information Gain:

$$Gain_{split} = Entropy(p) - \sum_{i=1}^{k} \frac{n_i}{n} Entropy(i)$$

Parent node, p is split into k partitions (children)
n_i is number of records in child node i

$$Entropy(p) = 0.9911$$
$$E(a_1) = \frac{4}{9}\left[-\frac{3}{4} log_2 \left(\frac{3}{4}\right) - \frac{1}{4} log_2 \left(\frac{1}{4}\right)\right] + \frac{5}{9}\left[-\frac{1}{5} log_2 \left(\frac{1}{5}\right) - \frac{4}{5} log_2 \left(\frac{4}{5}\right)\right]$$
$$= 0.7616$$
$$\therefore \; Information\ Gain(a_1) = Entropy(p) - E(a_1)$$
$$= 0.9911 - 0.7616 = 0.2294$$

$$E(a_2) = \frac{4}{9}\left[-\frac{2}{4}log_2\left(\frac{2}{4}\right) - \frac{2}{4}log_2\left(\frac{2}{4}\right)\right] + \frac{5}{9}\left[-\frac{2}{5}log_2\left(\frac{2}{5}\right) - \frac{3}{5}log_2\left(\frac{3}{5}\right)\right]$$
$$= 0.9839$$
$$\therefore\ Information\ Gain(a_2) = Entropy(p) - E(a_2)$$
$$= 0.9911 - 0.9839 = 0.0072$$

c) For a_3, the information gain for every possible split using the efficient computation technique which includes sorting, splitting, filling class distribution

$For\ a_3$	$+$	$-$	$+$	$-\ -$	$+$	$-\ +$	$-$
$Sorted\ Values$	1.0	3.0	4.0	5.0 5.0	6.0	7.0 7.0	8.0
$Split\ Values$	0.5	2.0	3.5	4.5	5.5	6.5	7.5
$+$		1 3	1 3	2 2	2 2	3 1	4 0
$-$		0 5	1 4	1 4	3 2	3 2	4 1
$Entropy(a_3)$		0.8484	0.9885	0.9183	0.9838	0.9728	0.8889
$Information\ Gain(a_3)$		0.1427	0.0026	0.0728	0.0072	0.0183	0.1022

$\therefore$ The best split for a_3 occurs at split value 2.0 with an information gain of 0.1427

d) According to the information gain a_1 produces the best split.

e)

$$Gain\ Ratio = \frac{Gain_{split}}{Split\ Info} \qquad Split\ Info = -\sum_{i=1}^{k}\frac{n_i}{n}log_2\frac{n_i}{n}$$

$$SI(a_1) = -\frac{4}{9}log_2\frac{4}{9} - \frac{5}{9}log_2\frac{5}{9}$$
$$= -0.4444(-1.17) - 0.5555(-0.848)$$
$$= 0.991$$
$$SI(a_2) = 0.991$$
$$SI(a_3) = -\frac{1}{9}log_2\frac{1}{9} - \frac{8}{9}log_2\frac{8}{9}$$
$$= -0.1111(-3.17) - 0.8888(-0.16993)$$
$$0.503$$
$$GR(a_1) = \frac{0.2294}{0.991} = 0.231$$
$$GR(a_2) = \frac{0.0072}{0.991} = 0.0072$$

$$GR(a_3) = \frac{0.1427}{0.991} = 0.2836$$

$\therefore\ a_3$ *has best split according to gain ratio as per information gain.*

f)

$$\mathbf{Gini\ Index} = \mathbf{1} - \sum_{i=0}^{c-1} p_i(t)^2$$

$$GI(a_1) = \frac{4}{9}\left[1 - \left(\frac{3}{4}\right)^2 - \left(\frac{1}{4}\right)^2\right] + \frac{5}{9}\ [1 - \left(\frac{1}{5}\right)^2 - \left(\frac{4}{5}\right)^2 = 0.3444$$

$$GI(a_2) = \frac{5}{9}\left[1 - \left(\frac{2}{5}\right)^2 - \left(\frac{3}{5}\right)^2\right] + \frac{4}{9}\ [1 - \left(\frac{2}{4}\right)^2 - \left(\frac{2}{4}\right)^2 = 0.4889$$

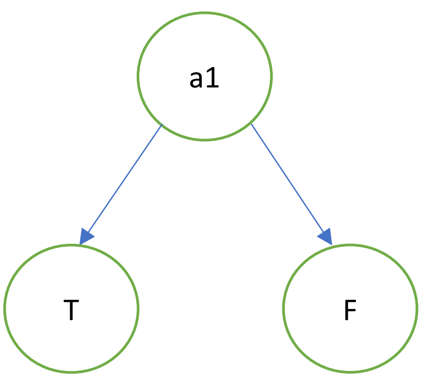
$\therefore$ *Gini Index of* a_1 *is smaller & it produces better split.*

g)

Classification Error Rate

$$Classification\ error = 1 - \max[p_i(t)]$$

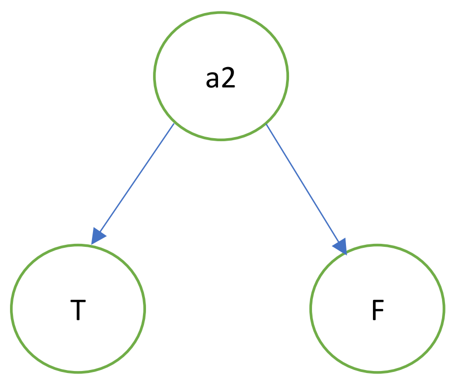
But using shortcut method:



+	-		+	-
3	1		1	4

Minority = 1+1 =2

$E(t)a_1 = \frac{2}{9} = 0.222$



+	-		+	-
2	3		2	2

Minority = 1+1 =2

$E(t)a_2 = \frac{4}{9} = 0.444$

Since $E(t)$ *of* a_1 *is less than* a_2

$\therefore\ a_1$ *will provide best split according to the classification error rate.*

2) Two-level decision tree:

a)

X	C_1	C_2
0	60	60
1	40	40

$$Class_{err}(X) = \frac{40 + 40}{200} = 0.4$$

Y	C_1	C_2
0	40	60
1	60	40

$$Class_{err}(Y) = \frac{40 + 40}{200} = 0.4$$

Z	C_1	C_2
0	30	70
1	70	30

$$Class_{err}(Z) = \frac{30 + 30}{200} = 0.3$$

Z is the lowest in terms of classification error and is chosen as the splitting attribute at level 1.

Now, **Z** = 0

X	C_1	C_2
0	15	45
1	25	25

Y	C_1	C_2
0	15	45
1	15	25

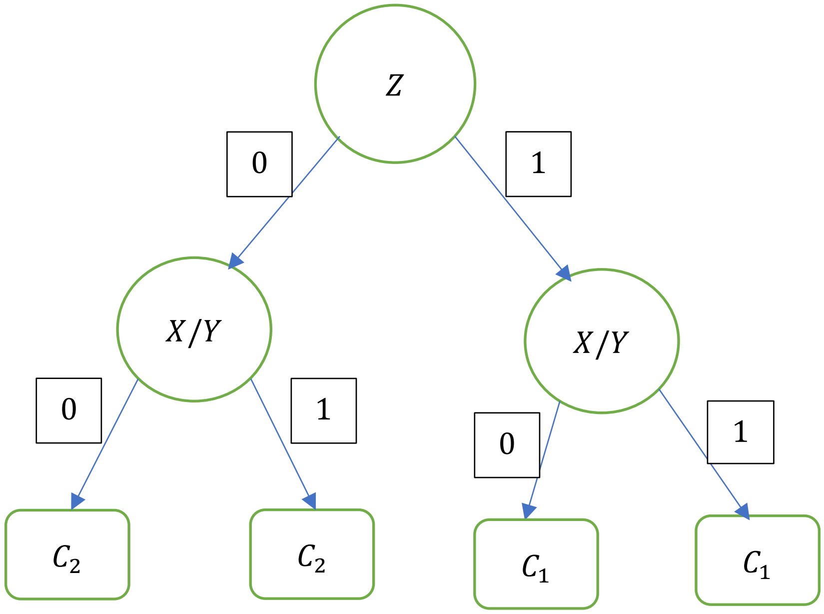
$$Class_{err}Z_0(X, Y) = \frac{15 + 15}{100} = 0.3$$

Now, **Z** = 1

X	C_1	C_2
0	45	15
1	25	15

Y	C_1	C_2
0	25	15
1	45	15

$$Class_{err}Z_1(X, Y) = \frac{15 + 15}{100} = 0.3$$



b) Overall error rate of the induced tree is $\frac{(15+15+15+15)}{200} = 0.3$

3) Model decision tree:

Ground Truth	$B+$		$B-$		$C+$		$C-$	
Predicted	$+$	$-$	$+$	$-$	$+$	$-$	$+$	$-$
Training - ID	$1,2$	NA	3	4	$5,6$	$7,9,10$	8	NA
Validation - ID	11	NA	12	NA	$13,15$	NA	NA	14
Test - ID	23	21	16	NA	$17,22$	NA	19	$18,20$

a) The generalization error rate of the tree using the optimistic approach

$$err_{tr} = \frac{5}{10} = 0.5$$

b) The generalization error rate of the tree using the pessimistic approach with factor of 0.5

$$err_{gen}(T) = err(T) + \Omega \times \frac{k}{N_{train}}$$

$$err_{gen}(T) = \frac{5}{10} + 0.5\left(\frac{4}{10}\right) = 0.7$$

c) The generalization error rate of the tree using the validation set

$$err_{gen}(V) = \frac{1}{5} = 0.2$$

d) Accuracy of the model on the test set

	ID	$Total$
$True\ Positive\ (TP)$	17,22,23	3
$True\ Negative\ (TN)$	18,20	2
$False\ Positive\ (FP)$	16,19	2
$False\ Negative\ (FN)$	21	1

$$Accuracy = \frac{TP + TN}{TP + TN + FP + FN} = \frac{3 + 2}{3 + 2 + 2 + 1} = 0.625$$

e)

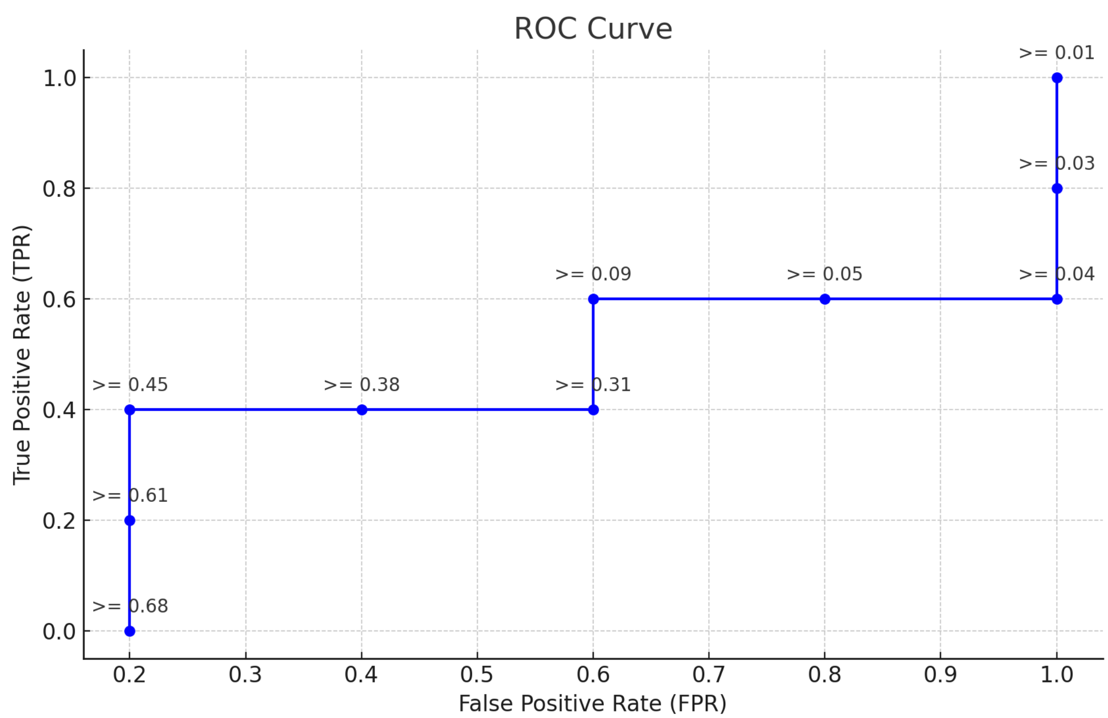
$$Precision\ (P) = \frac{TP}{TP + FN} = \frac{3}{3 + 2} = 0.6$$

$$Recall\ (R) = \frac{TP}{TP + FN} = \frac{3}{3 + 1} = \frac{3}{4} = 0.75$$

$$F1\ Score = 2 * \frac{PR}{P + R} = \frac{2(0.6)(0.75)}{0.6 + 0.75} = \frac{0.9}{1.35} = 0.667$$

4) ROC Curve:

Class	+	+	−	−	+	−	−	+	+	−
Threshold >=	**0.01**	**0.03**	**0.04**	**0.05**	**0.09**	**0.31**	**0.38**	**0.45**	**0.61**	**0.68**
TP	5	4	3	3	3	2	2	2	1	0
FP	5	5	5	4	3	3	2	1	1	1
TN	0	0	0	1	2	2	3	4	4	4
FN	0	1	2	2	2	3	3	3	4	5
TPR	1.0	0.8	0.6	0.6	0.6	0.4	0.4	0.4	0.2	0.0
FPR	1.0	1.0	1.0	0.8	0.6	0.6	0.4	0.2	0.2	0.2



5) Python program (decision_tree.py)

6) Python program (roc_curve.py)