**CS 5990 (Advanced Data Mining) - Assignment #3**
**Maximum Points: 100 pts.**

**Bronco ID: 017314921**
**Last Name: Panda**
**First Name: Subham**

# Answers:

1.
Building the Euclidean distance table between samples:

$$d(x, y) = \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2}$$

| $Sl.No.$ | $X$ | $Y$ | $Class$ | $dis_1$ | $dis_2$ | $dis_3$ | $dis_4$ | $dis_5$ | $dis_6$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 3 | − | 0 | 1 | 1.41 | 2 | 4.47 | 4.47 |
| 2 | 1 | 4 | − | 1 | 0 | 1 | 2.23 | 5 | 4.12 |
| 3 | 2 | 4 | + | 1.41 | 1 | 0 | 1.41 | 4.24 | 3.16 |
| 4 | 3 | 3 | + | 2 | 2.23 | 1.41 | 0 | 2.82 | 2.82 |
| 5 | 5 | 1 | − | 4.47 | 5 | 4.24 | 2.82 | 0 | 4 |
| 6 | 5 | 5 | − | 4.47 | 4.12 | 3.16 | 2.82 | 4 | 0 |

a)
Now, building the (LOO-CV) table for 1NN

| $Sl.No.$ | $X$ | $Y$ | $Class$ | $1NN$ | $Neighbors$ |
|---|---|---|---|---|---|
| 1 | 1 | 3 | − | − | (1,4) |
| 2 | 1 | 4 | − | + | (1,3) |
| 3 | 2 | 4 | + | − | (1,4) |
| 4 | 3 | 3 | + | + | (2,4) |
| 5 | 5 | 1 | − | + | (3,3) |
| 6 | 5 | 5 | − | + | (3,3) |

**The Error Rate is** $\frac{4}{6} = 0.667$

b)
Now, building the (LOO-CV) table for 3NN

| $Sl.No.$ | $X$ | $Y$ | $Class$ | $3NN$ | $Neighbors$ |
|---|---|---|---|---|---|
| 1 | 1 | 3 | $-$ | $+$ | $[(1,3),(2,4),(3,3)]$ |
| 2 | 1 | 4 | $-$ | $+$ | $[(1,3),(2,4),(3,3)]$ |
| 3 | 2 | 4 | $+$ | $-$ | $[(1,4),(1,3),(3,3)]$ |
| 4 | 3 | 3 | $+$ | $-$ | $[(2,4),(1,4),(1,3)]$ |
| 5 | 5 | 1 | $-$ | $+$ | $[(3,3),(5,5),(2,4)]$ |
| 6 | 5 | 5 | $-$ | $+$ | $[(3,3),(2,4),(5,1)]$ |

**The Error Rate is** $\boldsymbol{\frac{6}{6} = 1}$

c)
Distance weights:

For $P_1$:
Nearest 3 neighbors-

$$\frac{1}{1^2} = \mathbf{1}\ (-)$$
$$\frac{1}{1.41^2} + \frac{1}{2^2} = 0.752\ (+)$$

Therefore, this will be $(+)$

For $P_2$:
Nearest 3 neighbors-

$$\frac{1}{1^2} = \mathbf{1}\ (-)$$
$$\frac{1}{1^2} + \frac{1}{2.23^2} = 1.201\ (+)$$

Therefore, this will be $(+)$

For $P_3$:
Nearest 3 neighbors-

$$\frac{1}{1^2} + \frac{1}{1.41^2} = \mathbf{1.5}\ (-)$$
$$\frac{1}{1.41^2} = 0.50\ (+)$$

Therefore, this will be $(-)$

For $P_4$:
Nearest 3 neighbors-

$$\frac{1}{2^2} + \frac{1}{2.23^2} = 0.451\ (-)$$
$$\frac{1}{1.41^2} = \mathbf{0.50}\ (+)$$

Therefore, this will be $(-)$

For $P_5$:
Nearest 3 neighbors-

$$\frac{1}{2.82^2} + \frac{1}{4.24^2} = \mathbf{0.181}\ (+)$$
$$\frac{1}{4^2} = 0.0625\ (-)$$

Therefore, this will be $(+)$

For $P_6$:
Nearest 3 neighbors-

$$\frac{1}{2.82^2} + \frac{1}{3.16^2} = \mathbf{0.225}\ (+)$$
$$\frac{1}{4^2} = 0.0625\ (-)$$

Therefore, this will be $(+)$

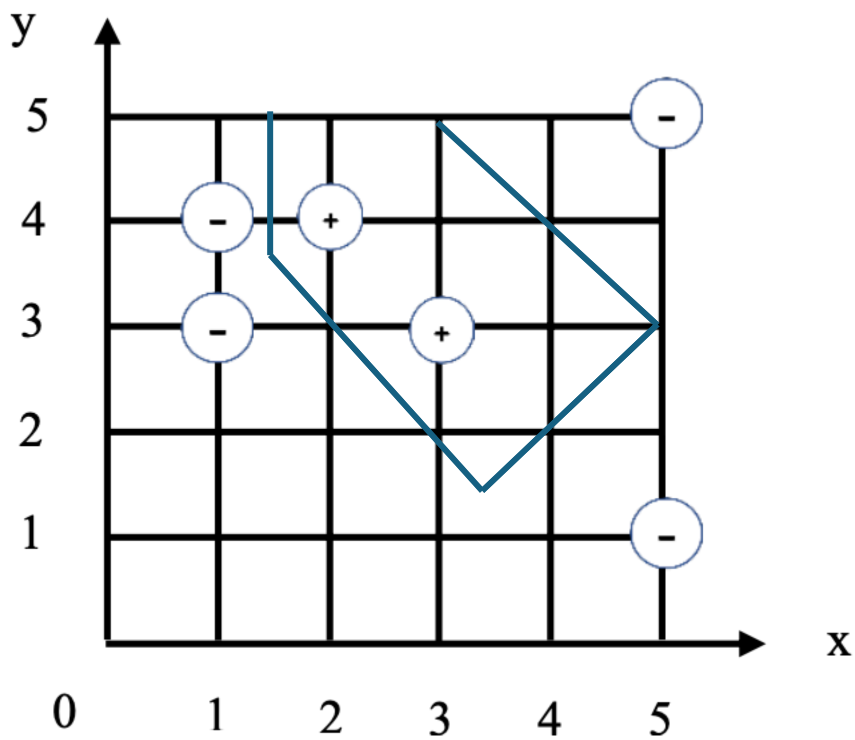
Now, building the (LOO-CV) table for 3NN(DW)

| $\boldsymbol{Sl. No.}$ | $\boldsymbol{X}$ | $\boldsymbol{Y}$ | $\boldsymbol{Class}$ | $\boldsymbol{3NN(DW)}$ | $\boldsymbol{Neighbors}$ |
|---|---|---|---|---|---|
| 1 | 1 | 3 | $-$ | $-$ | $[(1,3), (2,4), (3,3)]$ |
| 2 | 1 | 4 | $-$ | $+$ | $[(1,3), (2,4), (3,3)]$ |
| 3 | 2 | 4 | $+$ | $-$ | $[(1,4), (1,3), (3,3)]$ |
| 4 | 3 | 3 | $+$ | $+$ | $[(2,4), (1,4), (1,3)]$ |
| 5 | 5 | 1 | $-$ | $+$ | $[(3,3), (5,5), (2,4)]$ |
| 6 | 5 | 5 | $-$ | $+$ | $[(3,3), (2,4), (5,1)]$ |

**The Error Rate is** $\boldsymbol{\frac{4}{6} = 0.667}$

d)

The decision boundary learned by 1NN algorithm (pseudo-model) is



2.
Using KNN with $K = 3$ to classify test sample $(t1 = 1, t2 = 2, t3 = 3, t4 = 4, t5 = 4)$ with $L1$ norm for distance computations:

Let, $t = (t1 = 1, t2 = 2, t3 = 3, t4 = 4, t5 = 4)$

L1 norm distances between test sample and training dataset would be:

$$P_{t1} = |1-2| + |2-3| + |3-4| + |4-5| + |4-5| = 5$$

$$P_{t2} = |1-0| + |2-1| + |3-2| + |4-3| + |4-5| = 5$$

$$P_{t3} = |1-2| + |2-2| + |3-2| + |4-2| + |4-4| = 4$$

$$P_{t4} = |1-0| + |2-1| + |3-2| + |4-3| + |4-5| = 5$$

$$P_{t5} = |1-4| + |2-2| + |3-4| + |4-4| + |4-4| = 4$$

Therefore, the **test sample for $\boldsymbol{K = 3}$ would be classified as $\boldsymbol{Sell}$** with the 3 nearest neighbors being $P_3 = Sell, P_4 = Sell, P_5 = Sell$

3.

[KNN Program GitHub Link]

4.
Naïve Bayes Approach

$$P(+\mid A=0, B=1, C=0) = P(A=0, B=1, C=0 \mid +) * P(+)$$

$$\therefore P(A=0\ |+) * P(B=1\ |+) * P(C=0\ |+) * P(+)$$

$$= (2/5) * (1/5) * (1/5) * (5/10) = 0.008$$

$$P(-\mid A=0, B=1, C=0) = P(A=0, B=1, C=0\ |-) * P(-)$$

$$\therefore P(A=0\ |-) * P(B=1\ |-) * P(C=0\ |-) * P(-)$$

$$= (3/5) * (2/5) * (0/5) * (5/10) = 0$$

Therefore, $(A=0, B=1, C=0)$ would be classified as $+$.

Now, estimating the probabilities using m-estimate with $p = \frac{1}{2}$ and $m = 4$

$$\mathbf{m\text{−}estimate:}\boldsymbol{P(A_i|C) = \frac{N_{ic} + mp}{N_c + m}}$$

$$P(+|A=0) = \frac{2+2}{5+4} = \frac{4}{9}$$

$$P(+|B=1) = \frac{1+2}{5+4} = \frac{3}{9}$$

$$P(+|C=0) = \frac{1+2}{5+4} = \frac{3}{9}$$

$$P(-|A=0) = \frac{3+2}{5+4} = \frac{5}{9}$$

$$P(-|B=1) = \frac{2+2}{5+4} = \frac{4}{9}$$

$$P(-|C=0) = \frac{0+2}{5+4} = \frac{2}{9}$$

Now using the m-estimate probabilities calculating the naïve bayes classifier for the test sample:

$$P(+ \mid A = 0, B = 1, C = 0)$$

$$= (4/9) * (3/9) * (3/9) * (5/10) = 0.0247$$

$$P(- \mid A = 0, B = 1, C = 0)$$

$$= (5/9) * (4/9) * (2/9) * (5/10) = 0.0274$$

Now normalizing the scores:

For $+$,

$$\frac{0.0247}{0.0247 + 0.0274} = 0.474$$

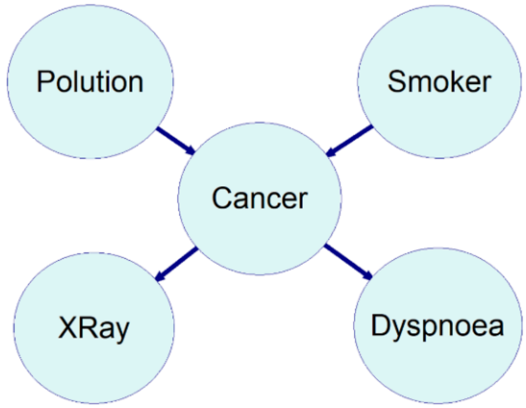
For $-$,

$$\frac{0.0274}{0.0247 + 0.0274} = \mathbf{0.525}$$

After using the m-estimate values for naïve bayes classification for the sample test the most probable classification is –

Therefore, $(A = 0, B = 1, C = 0)$ would be classified as $-$.

5.




a) The conditional probability tables for each variable are:

| $Pollution$ |
| --- |
| $L = 11/20 = 0.55$ |
| $H = 9/20 = 0.45$ |

| $Smoker$ |
| --- |
| $False = 13/20 = 0.65$ |
| $True = 7/20 = 0.35$ |

| $Cancer$ | | | | |
| --- | --- | --- | --- | --- |
| | $P = L$ | $P = L$ | $P = H$ | $P = H$ |
| | $S = False$ | $S = True$ | $S = False$ | $S = True$ |
| $False$ | $7/8 = 0.875$ | $2/3 = 0.667$ | $3/5 = 0.6$ | $2/4 = 0.5$ |
| $True$ | $1/8 = 0.125$ | $1/3 = 0.333$ | $2/5 = 0.4$ | $2/4 = 0.5$ |

| $Xray$ | | |
| --- | --- | --- |
| | $C = False$ | $C = True$ |
| $Neg$ | $10/14 = 0.714$ | $2/6 = 0.333$ |
| $Pos$ | $4/14 = 0.285$ | $4/6 = 0.666$ |

| $Dyspnoea$ | | |
| --- | --- | --- |
| | $C = False$ | $C = True$ |
| $False$ | $10/14 = 0.714$ | $2/6 = 0.333$ |
| $True$ | $4/14 = 0.285$ | $4/6 = 0.666$ |

b)
Given,

$(\mathrm{P} = \mathrm{L}, \mathrm{S} = \mathrm{True}, \mathrm{C} = \mathrm{True}, \mathrm{X} = \mathrm{pos})$

$P(P, S, C, X, D) = P(P) * P(S) * P(C|P, S) * P(X|C) * P(D|C)$

For:

$$\Rightarrow P(P = L) * P\ (S = True) * P(C = True|P = L, S = True) * P(X = Pos|C = True) * P(D|\ C = True)$$

Now for $D = False$:

$$\Rightarrow 0.55 * 0.35 * 0.333 * 0.666 * P(D = False|C = True)$$

$$\Rightarrow\ 0.0426 * 0.333 = 0.014$$

Now for $D = True$:

$$\Rightarrow 0.55 * 0.35 * 0.333 * 0.666 * P(D = True|C = True)$$

$$\Rightarrow\ 0.0426 * 0.666 = 0.0283$$

Now Normalizing,

For False:

$$\frac{0.014}{0.014 + 0.0283} = 0.3309$$

**For True:**

$$\boldsymbol{\frac{0.0283}{0.014 + 0.0283} = 0.6690}$$

Therefore, the patient is likely to have Dyspnoea.

6.

[Naive Bayes Program GitHub Link]