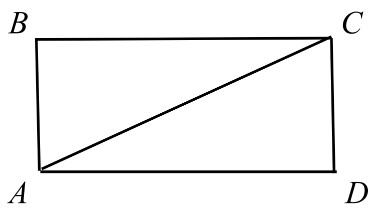
HINDUSTAN
INSTITUTE OF TECHNOLOGY & SCIENCE
(DEEMED TO BE UNIVERSITY)

EMA51005 – Discrete Mathematics (Question Bank)

Part – A

1. Define Tautology with an example.
2. Write a truth table for biconditional proposition of two given propositions.
3. Show that $(P \to Q) \leftrightarrow (\neg P \vee Q)$ is a tautology.
4. Define a rule of universal specification.
5. Assuming that repetitions are not permitted, how many four-digit numbers can be formed from the six digits 1, 2, 3, 5, 7, 8?
6. Show that if seven colours are used to paint 50 bicycles, at least 8 bicycles will be the same colour.
7. How many 7 letter words can be formed using the letters of the word BENZENE?
8. State pigeonhole principle.
9. If $A = \{a, b, c\}$, find the power set $p(A)$.
10. The relation R on the set A= {1,2,3,4,5} and R = {(a, b) |a-b is divisible by 3}, find R and R^{-1}.
11. Define Characteristic function.
12. Define Permutation function.
13. Define hashing function.
14. Define a cyclic group with an example.
15. If * is the binary operations on the set R of real numbers defined by a*b = a+b+2b, then find the identity element if it exists.
16. Find the identity element of the group of integers with the binary operation defined by $a * b = a + b + 2$, for all $a, b \in \mathbb{N}$.
17. Define normal subgroup
18. If * is the binary operations on the set R of real numbers defined by a*b = a+b+2ab, then find the inverse element if it exists.
19. Define code and decode.
20. Define a pseudograph.
21. Check Hamilton circuit is existing or not for the following graph.



22. What is complete graph? Draw a complete graph on 4 vertices.
23. Draw a graph which contains Eulerian path and circuit.
24. Define a 3 – regular graphs with an example.
25. What you mean by bipartite graph and give an example.

Part - B

1. Prove that $(\neg p \vee q) \wedge (p \wedge (p \wedge q)) \equiv p \wedge q$.
2. Prove that P$\longrightarrow$ (Q $\longrightarrow R) \Longrightarrow (P \longrightarrow Q) \longrightarrow (P \longrightarrow R)$ using truth table.
3. Check whether the compound proposition $(\sim q \wedge (p \longrightarrow q)) \longrightarrow \sim p$ is tautology or not?
4. Show that R is a valid inference from the premises $P \rightarrow Q, Q \rightarrow R$ and P.
5. Show that the conclusion $\forall x(P(x) \rightarrow \neg Q(x))$ follows from the premises $\exists x(P(x) \wedge Q(x)) \rightarrow \forall y(R(y) \rightarrow S(y))$ and $\exists y(R(y) \wedge \neg S(y))$.
6. Show that $\forall x(P(x) \vee Q(x)) \Rightarrow \forall x P(x) \vee \exists x Q(x)$ using the indirect method of proof.
7. Use mathematical induction to prove that $n^3 + 2n$ is divisible by 3, for $n \geq 1$.
8. A club consisting of 6 men and 7 women, in how many ways can we select a committee of 4 persons which has at least one woman?
9. There are 250 students in an engineering college. Of these 188 have taken a course in Fortran, 100 have taken a course in C and 35 have taken courses in Java. Further 88 have taken courses in both Fortran and C. 23 have taken course in both C and Java and 29 have taken courses in both Fortran and Java. If 19 of these students have taken all the three courses, how many of these 250 students have not taken a course in any of these three programming languages?
10. Suppose there are 6 boys and 5 girls.
    (i) In how many ways can they sit in a row.
    (ii) In how many ways can they sit in row if the boys and girls each sit together.
    (iii) In how many ways they can sit in a row if the girls are to sit together and the boys do not sit together.
11. Solve the recurrence relation $a_{n+2} - 6a_{n+1} + 9a_n = 32^n + 73^n, n \geq 0$, given that $a_0 = 1 \; and \; a_1 = 4$.
12. Prove that the relation $\subseteq$ of set inclusion is a partial ordering on any collection of sets.
13. Draw the Hasse diagram for the posets $(\{1,2,3,4,5\}, \leq)$, where $(x, y) \in R$ if and only if $x \leq y$.
14. Draw the HASSE diagram for $D_{36}(\, set\ of\ all\ divisor\ of\ 36)$, if and only if the relation defined by $a \leq b \; if \; a|b$.
15. Give an example of a function $\mathbb{N} \times \mathbb{N}$ as a set of ordered pairs which is
    (i) one-to-one but not onto,
    (ii) onto but not one-to-one,
    (iii) neither one-to-one nor onto.
16. If f(a,b) is defined recursively by f(a,b) = $\begin{cases} 5 & ; if \; a < b \\ f(a-b, b+2) + a & ; if \; a \geq b \end{cases}$ where a and b are non-negative integers, find f(2,7), f(10,3) and f(15,2).
17. Using characteristic function prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.
18. Show that the intersection of two subgroups of G is also a subgroup of G.
19. Show that the set Q^+ of all positive rational numbers forms an abelian group under the operation * defined by $a * b = \frac{ab}{2}\,; a, b \in Q^+$.

20. If *{G, *}* is an abelian group, show that $(a * b)^n = a^n * b^n$ for all a, b $\in G$, where n is a positive integer.
21. If the binary operation defined as $a * b = \frac{ab}{2}$ for all $a, b \in \mathbb{R}$, where $\mathbb{R}$ is the set of non-zero real numbers. Find the identity and inverse of a in $\mathbb{R}$.
22. Prove that the number of vertices of odd degree in an undirected graph is even.
23. Draw a graph which contains,
    (i) an Eulerian circuit that is also a Hamiltonian circuit,
    (ii) an Eulerian circuit, but not a Hamiltonian circuit.
    (iii) A Hamiltonian circuit, but not an Eulerian circuit.
24. State and prove the Handshaking theorem.
25. Define graph isomorphism and give an example for isomorphism and non-isomorphism with justification.

Part – C

1. Prove the following
    (i) $p \to (q \to p) \equiv \neg p \to (p \to q)$
    (ii) $\neg p \to (q \to r) \equiv q \to (p \vee r)$.
2. Using indirect method of proof, derive $p \to \neg s$ from the premises $p \to (q \vee r), q \to \neg p, s \to \neg r, p$.
3 Show that the following set of premises inconsistent:
    If Rama gets his degree, he will go for a job,
    If he goes for a job, he will get married soon,
    If he goes for higher study, he will not get married,
    Rama gets his degree and goes for higher study.
4. Prove that the following set of premises is inconsistent.

    If Raj misses many classes through illness, then he fails high school.

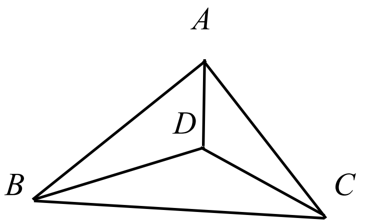
    If Raj fails high school, then he is uneducated.

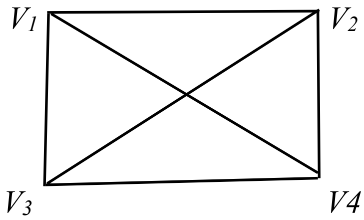
    If Raj reads a lot of books, then he is not uneducated.

    Raj misses many classes through illness and reads a lot of books.
5. Prove the equivalence without using truth table
    $\neg((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q) \equiv p$.
6. Prove that the premises $p \to q, q \to r, s \to \neg r \; and \; p \wedge s$ are inconsistent.
7. Show that the premises "one student in this class knows how to write programs in JAVA" and "Everyone who knows how to write programs in JAVA can get a high-paying job" imply the conclusion "Someone in this class can get a high-paying job".
8. Solve the recurrence relation $a_{n+1} - a_n = 3n^2 - n; n \geq 0, a_0 = 3$.
9. Use mathematical induction to prove that

    $1^2 + 3^2 + 5^2 + \cdots + (2n-1)^2 = \frac{1}{3}n(2n-1)(2n+1)$.
10. Find a formula for the general term a_n of the Fibonacci sequence 0,1,1,2,3,5,8, 13,.
11. Prove by mathematical induction that $1^2 + 3^2 + 5^2 + \cdots + (2n-1)^2 = \frac{1}{3}n(2n-1)(2n+1)$.

12. Find the number of integers between 1 and 250 both inclusive that are not divisible by any of the integers 2, 3, 5 and 7.
13. Determine whether the relation R on the set of all integers is reflexive, symmetric, anti-symmetric and/or transitive, where aRb if and only if
    i. $a \neq b$
    ii. a is a multiple of b.
14. Define a relation on the set A, and list the properties of relations with an example.
15. If R is the relation on the set of positive integers such that aRb if and only if a^2+b is even, prove that R is an equivalence relation.
16. Using the hashing function $h(x) = x^2 (mod\ 11)$, show that the following data would be insert in order given initially empty cells and cells are indexed from 0 to 10. *Data(x)*: 53, 13, 281, 743, 377, 20, 10, 796.
17. Let H be a non-empty subset of {G, *}, then H is a subgroup of G if and only if $\forall a, b \in H \Rightarrow a * b^{-1} \in H$.
18. If * is the binary operation on the set G of real numbers defined by a*b=a+b+2ab. Show that (G, *) is an abelian group.
19. Prove that the order of a subgroup of a finite group divides the order of the group.
20. Show that $\{\mathbb{R} - \{1\}, *\}$is an abelian group, where $*$ is defined as $a * b = a + b + ab$ for all $a, b \in \mathbb{R} - \{1\}$.
21. State and prove Lagrange's theorem of a group *G*
22. Prove that the maximum number of edges in a simple disconnected graph with 'n' vertices and k components is $(n-k)(n-k+1)/2$
23. (i) Define isomorphism of a graph *G*.
    (ii) Using circuits, check whether the following pairs of graphs G_1 and G_2 are isomorphic or not and justify your answer.





G_1 G_2

24. (i) If *G= (V, E)* is an undirected graph with *e* edges, then prove that $\sum_i \deg(v_i) = 2e$.
    (ii) Prove that the number of vertices of odd degree in an undirected graph is even
25. If all the vertices of an undirected graph are each of odd degree *k*, show that the number of edges of the graph is a multiple of *k*.