Assignment 1a: Intro Simulink

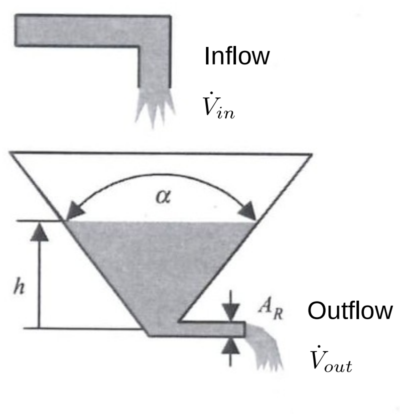
1DT059: Model-based Design of Embedded Software
Uppsala University

September 9, 2022

In this assignment you produce Simulink models. Each exercise should be implemented in a single .slx-file that is named after the exercise (the solution to assignment 1a, exercise 2 should be in file `a1ae2.slx`). Also include a file `a1a.pdf` with a short high-level description of the ideas of your solutions (you do not need to include obvious things). Include your name at the top of all submitted files.

Assignments are to be solved by students individually. You can discuss ideas and concepts with fellow students, but it is absolutely forbidden to share or copy (even parts of) solutions, lines of code, or similar.

Problem 1 Filling a Conical Container (30p)*:*



Consider a conical container, with angle α between opposite walls. The container has an outflow pipe with a cross-section area of A_R. From above, a liquid is poured into the container (which is initially empty) with a speed of $\dot{V}_{in}$. The level of liquid in the container increases until a steady-state is reached (steady-state is when liquid is poured into the container at the same rate as it flows out through the outflow pipe). We introduce the below quantities with respective values.

variable	value	description
α	60 degrees	opening angle of the container
$\dot{V}_{in}$	20 m^3/h	liquid inflow
A_R	12 cm^3	cross-section area of outflow pipe opening
g	9.81 m/s^2	Gravity acceleration

To set up equations for the container, let $V(t)$ be the volume of liquid in the contained at time t, and let $\dot{V}_{out}$ be the rate at which liquid flows out of the container. The volume V follows the equation

$$\dot{V}_{in} - \dot{V}_{out} = \frac{dV}{dt}$$

i.e., the change of volume V is the inflow minus the outflow.

The outflow $\dot{V}_{out}$ is obtained as $\dot{V}_{out} = v_{out} \cdot A_R$, where v_{out} is the speed at which liquid passes through the outlow pipe. By Torricelli's law (you might want to google), we have $v_{out} = \sqrt{2gh}$. The volume V can be calculated by standard geometry as a function of h using the following formulas. (1) The radius r of the liquid surface at height h is $r = h\tan(\frac{\alpha}{2})$. (2) The surface area A is $A = \pi r^2$. (3) The volume V is $V = \frac{1}{3}hA$.

Your tasks are as follows:

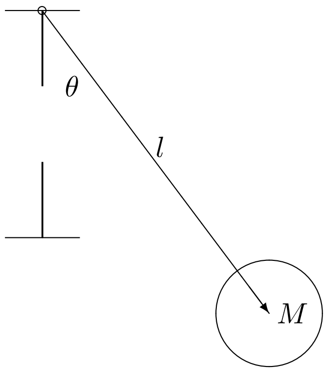
1. From the above information, derive the differential equation(s) that governs the height $h(t)$ as a function of time.

2. From the differential equation, derive the liquid height when it reaches equilibrium (this is obtained by simply setting $\frac{dh}{dt}$ to 0 and solving for h).

3. Model and simulate time-dependent behavior, starting from initial conditions with $h(0) = 0$, to see how fast equilibrium is reached. Construct a Simulink model which represents the system from the equations you developed in task 1, and simulate it. How long does it take for the level to reach within 1% of the equilibrium level.

In all tasks you should derive the equations using letters (i.e., names of variables) for the quantities in the above table (not actual values). Only thereafter, insert the actual values to derive a requested number (such as height at equilibrium). In the same way, the Simulink model should use the quantities as parameters, which can easily changed (e.g., from Matlab). Carefully construct a good layout of the Simulink model for readability and so that you can fix problems in it. Be careful with units (note that we sometimes use h (hours) for time, etc, which does not match, e.g., the units for gravity: Use SI-units). throughout.

Problem 2 Pendulum. (30p)*:*
In this exercise, you model and simulate a simple pendulum, and use simulations to make some simple analysis. The pendulum is anchored in the ceiling. It has a mass of M at the end of a rod of length l (ignore the mass of the rod). See the picture below.



Let us set up equations that govern the pendulum dynamics. We let the state of the system be represented by the angle θ of the pendulum (where $\theta = 0$) represents the pendulum hanging vertically) and its derivative $\dot{\theta}$ (the angular velocity). To derive a model for the dynamics of the pendulum, we use Newton's second law of motion. Let us consider forces in the direction of the current movement of the mass (i.e., perpendicular to the rod). We let positive direction be in the direction of increasing θ (i.e., right-upwards in the above picture). The acceleration in this direction is $l\ddot{\theta}$ (the length of the rod times the angular acceleration). According to Newton's second law, $Ml\ddot{\theta}$ should be equal to the sum of the forces acting on the pendulum in the direction of movement. These are:

- Gravity, which in the downward direction is Mg. In the direction of movement it has magnitude $Mg\sin(\theta)$ and is directed in the direction of negative θ.

- Friction, which we just approximate as b times the velocity for some friction coefficient b. I.e., it is $-bl\dot{\theta}$ (note the sign)

Thus the sum of forces acting on the pendulum is $-Mg\sin(\theta) - bl\dot{\theta}$. As values for parameters, we can take

- $M = 1$ kg,
- $g = 9.82\ m/s^2$,
- $l = 1$ m,
- $b = 0.001$ kg$/s$.

The assignment consists of the following sub-problems

a) Make two Simulink models of the pendulum.

- One model, which exactly follows the equations above
- An approximate model, obtained by linearization around $\theta = 0$. You get such a model by approximating $\sin(\theta)$ by θ and approximating $\cos(\theta)$ by 1.

Make compact and well-structured models. Let the constants M, l, and b be parameters of the model. Check that they work by simulating them.

b) Now compare the two models to see how much the linearization around 0 affects the behavior of the model, i.e., understand how much error is introduced by the linearization. In this case, we assume that the coefficient of friction b is 0. Compare the frequencies (or periods) of the two models when starting at rest from a maximal angle of $\theta = \pi/4$ (i.e., you can have initial conditions $\theta = \pi/4$ and $\dot{\theta} = 0$). How much do the frequencies or periods differ (in %)? You should instrument your model by suitable blocks, so that the difference can be seen or calculated (by post-processing) from some output signal of some block (i.e., it is not satisfactory to just simulate each model and inspect the outputs "by hand"). There are of course many ways to do this. One possibility is to add a component which observes the pendulum and counts the number of completed periods, and after some time compares the number of completed periods in both models.

c) At what maximal starting angle (with $\dot{\theta}$) do the frequencies (or periods) differ by only 1%? A satisfactory solution should not find this by "manual trial and error", but programmatically by performing a sequence of simulations to derive the answer.

As solution, you should hand in models, a report briefly describing how they are constructed, and selected simulation output.

Submission

Feedback on ideas and preliminary solutions can be obtained on **Tue-Fri, September 13-16, 2022 (tentative)**. Please sign up for a slot.

Solutions (all files) to this assignment are to be submitted via the Student Portal by **Wednesday, September 21, 2022**