Assignment 1b: Intro Simulink

1DT059: Model-based Design of Embedded Software
Uppsala University

September 21, 2022

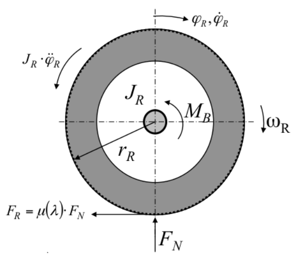
In this assignment you produce Simulink models. Each exercise should be implemented in a single .slx-file that is named after the exercise (the solution to assignment 1b, exercise 1 should be in file `a1be1.slx`). Also include a file `a1b.pdf` with a short high-level description of the ideas of your solutions (you do not need to include obvious things). Include your name at the top of all submitted files.

Assignments are to be solved by students individually. You can discuss ideas and concepts with fellow students, but it is absolutely forbidden to share or copy (even parts of) solutions, lines of code, or similar.

Problem 3. Car Braking, without and with ABS (40p)
The aim of this problem is to model the braking behavior of a car, without and with ABS control, and use the model to calculate the time and distance needed to bring the car to a full stop. We introduce the below quantities with respective values.

variable	value	description
$v_{F,0}$	30 m/s	Initial car speed when braking starts.
m	$1,500$ kg	mass of car
r_R	0.3 m	wheel radius
J_R	0.8 kg m^2	Wheel inertia
b	0.36 kg/m^2	coefficient for air friction
g	9.81 m/s^2	acceleration constant
c_1	0.86	coefficient for calculating friction
c_2	33.82	coefficient for calculating friction
c_3	0.36	coefficient for calculating friction

The model has two parts: One part considers the forces on the rotating wheel. The other part considers the forces on the car. They are connected by the friction force F_R of the wheels against the road. For simplicity, we let all wheels of the car be represented by only one wheel. The model of the rotating wheel uses the notation in the below figure: We let φ_R denote the



angular position of the wheel. The wheel rotates clockwise with an angular velocity of ω_R, where ω_R is another notation for $\dot{\varphi_R}$, the derivative of φ_R. When the wheel rotates clockwise and a braking force is applied, this results in an angular moment M_B which acts counterclockwise around the wheel axis (note that M_B is relevant only as long as the wheel is still rotating). When the wheel rotation becomes slower due to the force M_B, then the velocity of the wheel where it meets the road is slower than the velocity of the car, and the wheel starts to slip. This causes a friction between the wheel and the road, modeled by a friction force F_R at the edge of the wheel which gives rise to an angular moment $F_R \cdot r_R$ acting clockwise on the wheel, thereby counteracting M_B. Thus, the total angular moment around the the wheel axis is $F_R \cdot r_R - M_B$. The effect on the rotation of the wheel is given by Newtons second law:

$$J_R \cdot \ddot{\varphi_R} = F_R \cdot r_R - M_B$$

It says that the angular moment $F_R \cdot r_R - M_B$ on the wheel is the same as the angular acceleration $\ddot{\varphi_R}$ times the inertia J_R of the wheel.

Let us next consider the relation between the friction force F_R and the velocity of the car. Let v_F be the velocity of the car, and let v_R be the velocity of the rotating wheel at the edge, i.e., $v_R = r_R \cdot \dot{\varphi_R}$. Define the *slip* λ by

$$\lambda = \frac{v_F - v_R}{v_F} \quad .$$

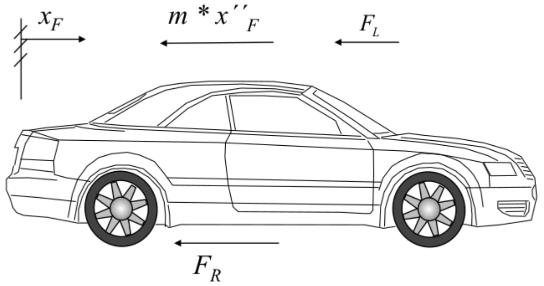
Intuitively, the slip is a measure of the difference between the velocity of the car v_F and the corresponding velocity of the wheel v_R. If the wheel rotates along with the car (as it would do if no braking or engine force is applied), then $\lambda = 0$. If the wheel is completely locked (as it would be if a very strong braking force is applied), then $\lambda = 1$. On dry asphalt, the friction coefficient μ can be calculated from the slip λ by the formula

$$\mu(\lambda) = c_1 \cdot (1 - e^{-c_2 \cdot \lambda}) - c_3 \cdot \lambda \quad ,$$

where the coefficients are as defined in the table at the beginning of this problem description (note that if $\lambda = 1$, then $\mu(\lambda) = 0$). The friction force F_R can then be obtained from the mass m of the car and the friction coefficient through

$$F_R = \mu(\lambda) \cdot 1.5 \cdot m \cdot g$$

We can now set up equations for the speed of the car. See the below figure. Let x_F be the position of the car, i.e., $\dot{x_F}$ is the speed (which is sometimes



also denoted v_F), and $\ddot{x_F}$ is the acceleration. The sum of forces on the car is $-F_R - F_L$, where F_L is the air resistance, which can be calculated by the formula

$$F_L = b \cdot (\dot{x_F})^2 \quad ,$$

where b is as in the table. The acceleration $\ddot{x}_F$ is then related to the forces acting on the car $-F_R - F_L$ by Newtons second law.

$$m \cdot \ddot{x_F} = -F_R - F_L \quad .$$

The first two tasks of this exercise are as follows.

a) Make a model in Simulink of the braking procedure of the car, based on the above equations. A small detail to consider is that the modeled forces on the wheel should not cause the wheel to rotate backwards (when the car stops, then clearly the braking cannot anymore be represented by a force M_B). You can see to this, e.g., by ensuring that the angular velocity of the wheel cannot become negative.

b) Simulate the model assuming a high braking moment M_B amounting to $5,500$ Nm and an initial speed of 30 m/s. How many seconds does it take for the car to stop? How many meters are needed to bring the car to a stop?

A problem with applying a high braking moment M_B is that it can possibly lock the wheels, so that $v_R = 0$ and $\lambda = 1$ throughout the braking period. It turns out that the the friction coefficient $\mu(\lambda)$ has its maximal value for

λ somewhere between 0.1 and 0.3 and decreases for larger λ. This makes braking less efficient. It would be desirable to adjust the braking moment so that λ is close to its maximal value. This is what is done in ABS brakes. Your next tasks are as follows:

c) Plot the function $\mu(\lambda)$ for $0 \leq \lambda \leq 1$, and find the value λ_{max} of λ which maximizes $\mu(\lambda)$.

d) Thereafter, extend your model in task a) by a feedback-controller which aims to keep λ close to its maximizing value λ_{max}. This controller takes v_F and v_R as input and outputs a correction signal $\Delta(t)$ which is added to the braking moment M_B. The correction signal $\Delta(t)$ should be generated so that its value $\Delta(t)$ at time t is

$$\Delta(t) = \int_0^t K_I \cdot (\lambda_{max} - \lambda(\tau)) \, d\tau \quad ,$$

where $\lambda(\tau)$ is the value of λ at time τ, and K_I is a suitable constant which you can determine by trial-and error (or tuning, if you like). Moreover, the total braking force $M_B + \Delta(t)$ cannot exceed $10,000$ Nm (and of course not be negative). You are not required to find the "optimal" value of K_I (although you may find it interesting to find it), but your value should at least improve the braking.

e) Simulate the model with a braking moment $M_B + \Delta(t)$ and an initial speed of 30 m/s. How many seconds does it take for the car to stop? How many meters are needed to bring the car to a stop?

What to hand in: As your solution, submit

- a model and simulation for tasks a) and b),
- a plot of $\mu(\lambda)$ for task c)
- a model and simulation for tasks d) and e).

Submission

Feedback on ideas and preliminary solutions can be obtained on **Mon-Wed, September 26-28, 2022**. Please sign up for a slot.

Solutions (all files) to this assignment are to be submitted via the Student Portal by **Friday, September 30, 2022**