AML Assignment 2 **Due Date: Sept , 2019** **Total Marks: 90**

Instructions: You are allowed to discuss but the final answer should be your own. Any instance of cheating will be considered as academic dishonesty and penalty will be applied.

Q1 [20 Marks]. Data points are: Negative: (-1, 0) (2, -2) Positive: (1, 0)
Recall that for SVMs, the negative class is represented by a desired output of -1 and the positive class by a desired output of 1.

(i) For each of the following separators (for the data shown), indicate whether they satisfy all the conditions required for a support vector machine, assuming a linear kernel. Justify your answers very briefly.

a) x1 + x2 = 0
b) x1 + 1.5x2 = 0
c) x1 + 2x2 = 0
d) 2x1 + 3x2 = 0

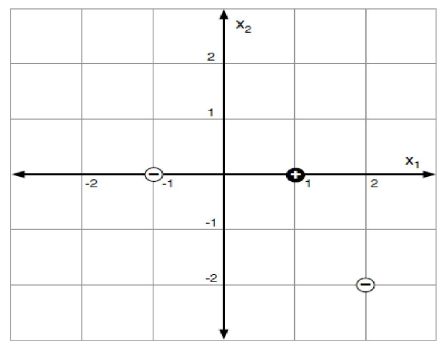


Figure Data points diagram

Write a clear and brief justification for each of the points listed in the question.

Solutions:

a) Goes through the (2,2) point so obviously not maximal margin
b) Yes. All three points are support vectors, with margin = 1
c) No. Three points are needed to define a line, with two support vectors there is no unique maximal margin line
d) No. The margin for the points is 2, not 1

(ii) For each of the kernel choices below, find the decision boundary diagram that best matches. In these diagrams, the brightness of a point represents the magnitude of the SVM output; red means positive output and blue means negative. The black circles are the negative training points and the white circles are the positive training points.

a) Polynomial kernel, degree 2
b) Polynomial kernel, degree 3
c) Radial basis kernel, sigma = 0.5
d) (d) Radial basis kernel, sigma = 1.0

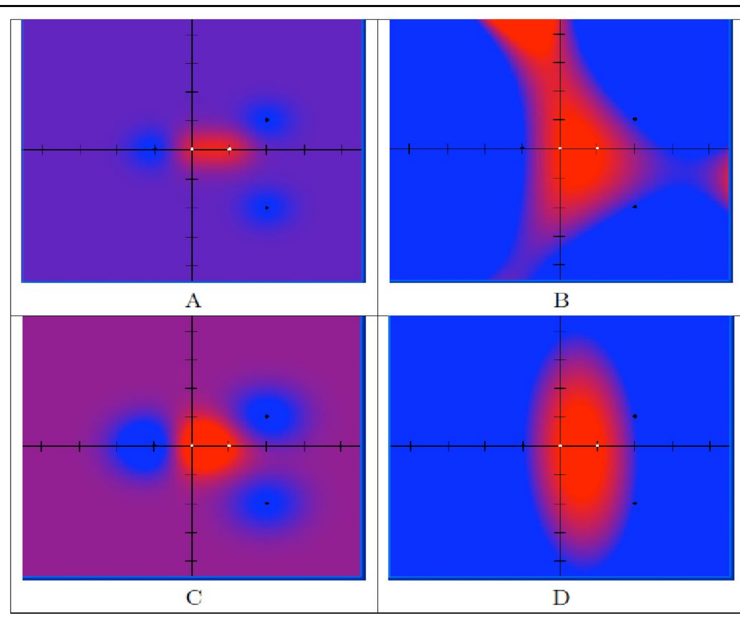


Figure Decision boundary diagram

Along with the chosen diagram, write the concise justification of your answer as well.

Solutions:

a) D
b) B
c) A
d) C

Q2 [40 Marks]. Use breast cancer classification dataset for multi-class classification. Train the SVM using following kernel functions and for each of the kernel learn the SVM using OVA and AVA techniques (https://archive.ics.uci.edu/ml/datasets/Breast+Cancer+Wisconsin+(Diagnostic)):

- Linear
- Gaussian (radial basis) Kernel for each of the following values of the parameter $\gamma \in \{1, 10, 100, 1000, 10000\}$

a) Plot the decision boundary on top of a scatterplot of the data. Please make the boundary lines easy to see.
b) State-based on the plot—whether the model overfits, underfits, or performs well
c) [For OVA Linear Kernel SVM only] Perform 10-fold cross validation to pick the appropriate γ for this dataset. Show how the testing and training errors averaged across folds change with γ. What's the value of γ you would choose and what are its corresponding test/training errors?

You need to submit the code along with the 1-2 page analysis report regarding the experimentations you have performed. Decision boundary plots are also required as mentioned in (a). Do not forget to mention the accuracy computed in each step.

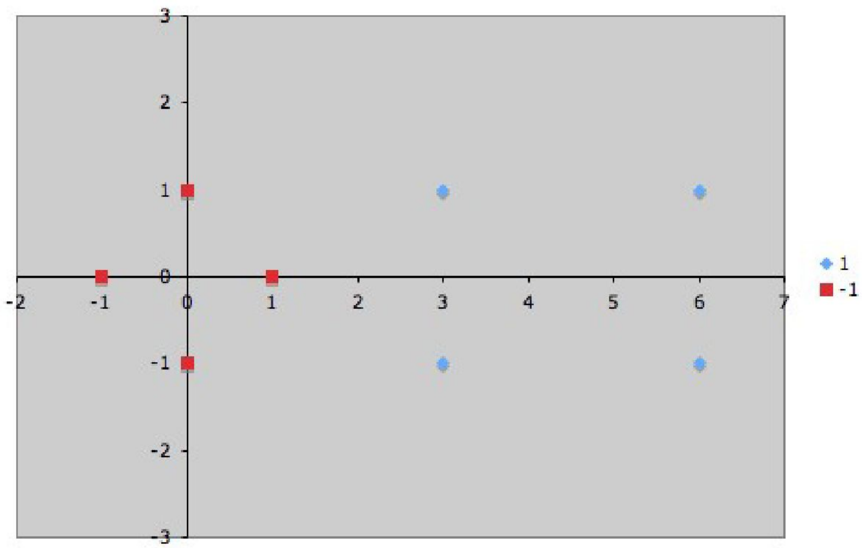
Q3 [20 Marks]. Perform online SVM learning on the above database. Perform the detailed analysis including the benefits of online SVM learning compared to traditional kernel learning.

You need to submit the code along with the 1-2 page analysis report regarding the experimentations. Also breif description of the algorithm used.

Hints (reading material of some online SVM learning):
http://digitalimaginggroup.ca/members/Shuo/Incremental%20Support%20Vector%20Learning%20for%20Ordinal%20Regression.pdf
http://web.mit.edu/seyda/www/Papers/GHC06_ACMSRC_abstract.pdf

https://www.semanticscholar.org/paper/Online-SVM-learning%3A-from-classification-to-data-Tax-Laskov/fc5664f83c49f88b079d3da355609db8b03aa706

Q4 [10 Marks]. Suppose we are given the following positively and negatively labeled data points in R^2. Find the SVM that can accurately discriminates the two classes (i.e., discriminating hyperplane)



<u>Show the weight vector and bias and the steps used in computation.</u>

Solutions:

By inspection, it should be obvious that there are three support vectors (see Figure below):
s_1 = (1, 0), s_2 =(3, 1), and s_3 = (3, -1)

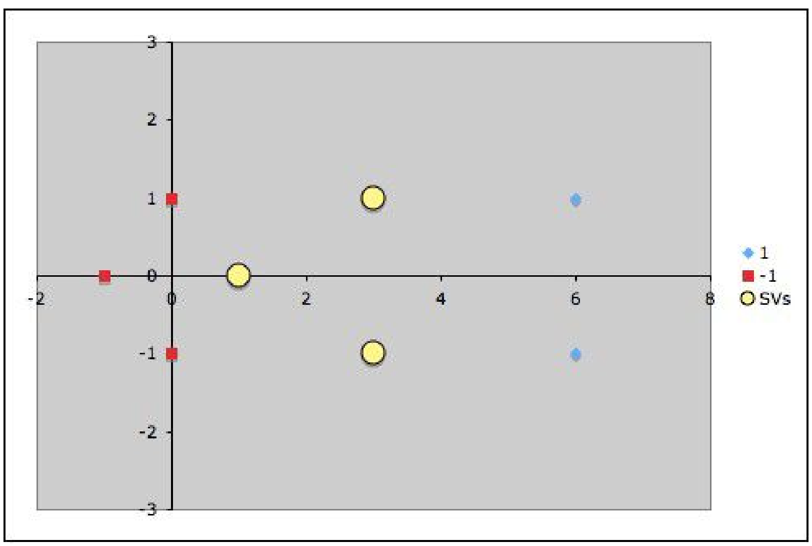


Figure: The three support vectors are marked as yellow circles.

In what follows we will use vectors augmented with a 1 as a bias input, and for clarity we will di erentiate these with an over-tilde. So, if s_1 = (10), then $\tilde{s_1}$ = (101).

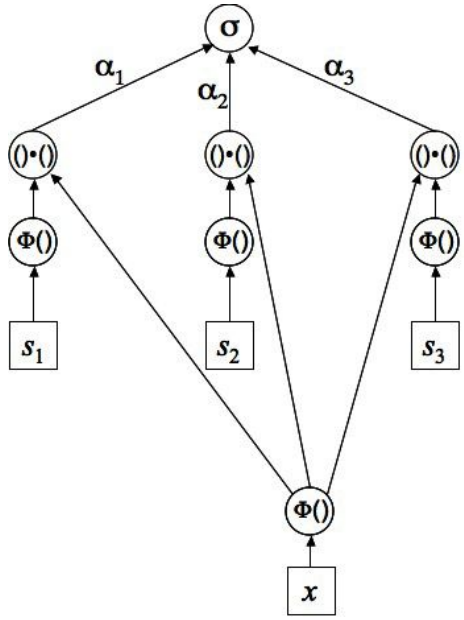


Figure: The SVM architecture.

$$2 * \alpha_1 + 4 * \alpha_2 + 4 * \alpha_3 = -1$$

$$4 * \alpha_1 + 11 * \alpha_2 + 9 * \alpha_3 = +1$$

$$4 * \alpha_1 + 9 * \alpha_2 + 11 * \alpha_3 = +1$$

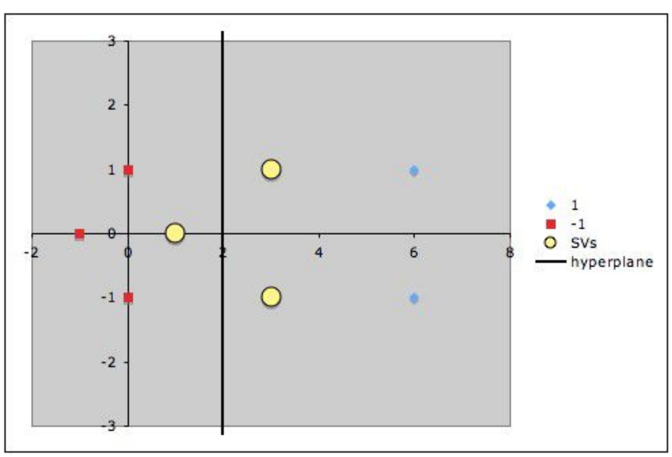
A little algebra reveals that the solution to this system of equations is α_1 = -3.5; α_2 = 0.75 and α_3 = 0.75.

Now, we can look at how these values relate to the discriminating hyperplane; or, in other words, now that we have the α_i, how do we nd the hyperplane that discriminates the positive from the negative examples? It turns out that

$$\tilde{w} = \sum_i \alpha_i \tilde{s}_i$$

= -3.5 (1 0 1) + 0.75 (3 1 1) + 0.75 (3 -1 1) = (1 0 -2)

Finally, remembering that our vectors are augmented with a bias, we can equate the last entry in $\tilde{w}$ as the hyperplane o set b and write the separating hyperplane equation y = wx+b with w = (1 0) and b = -2. Plotting the line gives the expected decision surface (see Figure below).



Q5 [10 Marks]. What are the limitations of SVM when applied for large scale database? What is the computation complexity of SVM and how it limits its implementation for large scale database. Provide one deatiled solutions/algorithm (along with pseudo code) which can be applied to solve the problem large scale data classification using SVM.

Complexity of SVM:

It's **O**(max(n,d) min (n,d)^2), where n is the number of points and d is the number of dimensions, according to:

Chapelle, Olivier. "Training a support vector machine in the primal." *Neural Computation* 19.5 (2007): 1155-1178.

"Olivier Chapelle" talks extensively about complexity of SVM in this paper. and find an optimization. He made an arugument that as "Support Vector Machines

(SVMs) first state the primal optimization problem, and then go directly to the

dual formulation" and one should solve either the primal or the dual optimization problem depending on whether n is larger or smaller than d, resulting in an O(max(n, d) min(n, d)^2) complexity. where

Given a matrix(dataset) X ∈ R^(n×d) representing the coordinates of n points in d dimensions

Training complexity of *nonlinear* SVM is generally between O(n^2) and O(n^3) with n the amount of training instances. The following papers are good references:

- Support Vector Machine Solvers by Bottou and Lin
- SVM-optimization and steepest-descent line search by List and Simon