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AML Assignment - 1

1 (a) $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 4 \\ 2 & 3 & -1 \end{bmatrix}$; Rank(A) = ? Inverse(A) = ?

Rank(A) :-

$A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 4 \\ 2 & 3 & -1 \end{bmatrix} \xrightarrow[R_3 = R_3 - 2R_1]{R_2 = R_2 - R_1} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{bmatrix} \xrightarrow{R_3 = R_3 + R_2} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \text{Rank}(A) = 2$

Inverse (A) :-

Here $|A| = 0 \rightarrow$ singular matrix ($\because$ Rank(A) $\neq 3$)

$\Rightarrow$ Inverse of A is not exists.

(b) $A = \begin{bmatrix} 0 & -2 & 4 \\ 1 & 1 & -1 \\ 2 & 4 & -5 \end{bmatrix}$ Rank(A) = ? , Inverse (A) = ?

Rank (A) :-

$A = \begin{bmatrix} 0 & -2 & 4 \\ 1 & 1 & -1 \\ 2 & 4 & -5 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_1} \begin{bmatrix} 1 & 1 & -1 \\ 0 & -2 & 4 \\ 2 & 4 & -5 \end{bmatrix} \xrightarrow{R_3 = R_3 - 2R_1} \begin{bmatrix} 1 & 1 & -1 \\ 0 & -2 & 4 \\ 0 & 2 & -3 \end{bmatrix} \xrightarrow{R_3 = R_3 + R_1} \begin{bmatrix} 1 & 1 & -1 \\ 0 & -2 & 4 \\ 0 & 0 & 1 \end{bmatrix}$ $|A| \neq 0$ $\Rightarrow$ Rank (A) = 3

Inverse (A) :

Here Rank(A) = 3 $\Rightarrow |A| \neq 0 \rightarrow$ non-singular matrix

$|A| = -1(10 - 16) + 2(2 - 4) = 2$

$A^{-1} = \frac{\text{adj } A}{|A|}$; adj A = B^T Here B = Cofactor matrix

$\Rightarrow$ B :-

cofactors of I row = $(-1)^{0+0} \begin{vmatrix} 1 & -1 \\ 4 & -5 \end{vmatrix}, (-1)^{0+1} \begin{vmatrix} 1 & -1 \\ 2 & -5 \end{vmatrix}, (-1)^{0+2} \begin{vmatrix} 1 & 1 \\ 2 & 4 \end{vmatrix}$

$= -1, 3, 2$

• Cofactors of II row = $-\begin{vmatrix} -2 & 4 \\ 4 & -5 \end{vmatrix}, \begin{vmatrix} 0 & 4 \\ 2 & -5 \end{vmatrix}, -\begin{vmatrix} 0 & -2 \\ 2 & 4 \end{vmatrix}$

$= 6, -8, -4$

• Cofactors of III row = $-2, 4, 2$

$\Rightarrow B = \begin{bmatrix} -1 & 3 & 2 \\ 6 & -8 & -4 \\ -2 & 4 & 2 \end{bmatrix}$ $\Rightarrow B^T = \begin{bmatrix} -1 & 6 & -2 \\ 3 & -8 & 4 \\ 2 & -4 & 2 \end{bmatrix}$

$$\text{adj} A = B^T = \begin{bmatrix} -1 & 6 & -2 \\ 3 & -8 & 4 \\ 2 & -4 & 2 \end{bmatrix} \quad ; |A| = 2.$$

$$\Rightarrow A^{-1} = \frac{\text{adj} A}{|A|} = \begin{bmatrix} -0.5 & 3 & -1 \\ 1.5 & -4 & 2 \\ 1 & -2 & 1 \end{bmatrix}$$

(c). $A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 2 & 5 & 5 & 1 \\ -2 & -3 & 0 & 3 \\ 3 & 4 & -2 & -3 \end{bmatrix}$; Rank(A) = ? Inverse [A] = ?

Rank (A) :-

$$A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 2 & 5 & 5 & 1 \\ -2 & -3 & 0 & 3 \\ 3 & 4 & -2 & -3 \end{bmatrix} \xRightarrow{R_3 = R_3 + R_2} \begin{bmatrix} 1 & 2 & 1 & 0 \\ 2 & 5 & 5 & 1 \\ 0 & 2 & 5 & 4 \\ 3 & 4 & -2 & -3 \end{bmatrix} \xRightarrow[R_4 = R_4 - 3R_1]{R_2 = R_2 - 2R_1} \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 3 & 1 \\ 0 & 2 & 5 & 4 \\ 0 & -2 & -4 & -3 \end{bmatrix} \xRightarrow[R_4 = R_4 + 2R_2]{R_3 = R_3 - 2R_2} \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 2 & -1 \end{bmatrix}$$

$$\Downarrow R_4 = R_4 - 2R_3.$$

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & -5 \end{bmatrix}$$

$|A| \neq 0.$

Rank(A) = 4.

Inverse (A) :-

$$|A| = \begin{vmatrix} 1 & 2 & 1 & 0 \\ 2 & 5 & 5 & 1 \\ -2 & -3 & 0 & 3 \\ 3 & 4 & -2 & -3 \end{vmatrix} \xRightarrow[R_4 = R_4 - 3R_1]{R_2 = R_2 - 2R_1,\ R_3 = R_3 + 2R_1} \begin{vmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 3 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & -2 & -5 & -3 \end{vmatrix} \xRightarrow[R_4 = R_4 + 2R_2]{R_3 = R_3 - R_2} \begin{vmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 1 & -1 \end{vmatrix} \xRightarrow{R_4 = R_4 + R_3} \begin{vmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{vmatrix} = -1.$$

$$A^{-1} = \frac{\text{adj} A}{|A|}$$

Cofactors of I row $= 1\begin{vmatrix} 5 & 5 & 1 \\ -3 & 0 & 3 \\ 4 & -2 & -3 \end{vmatrix}, (-1)\begin{vmatrix} 2 & 5 & 1 \\ -2 & 0 & 3 \\ 3 & -2 & -3 \end{vmatrix}, \begin{vmatrix} 2 & 5 & 1 \\ -2 & -3 & 3 \\ 3 & 4 & -3 \end{vmatrix}, -\begin{vmatrix} 2 & 5 & 5 \\ -2 & -3 & 0 \\ 3 & 4 & -2 \end{vmatrix}.$

$= 51, -31, 10, +3.$

Cofactors of II row $= -\begin{vmatrix} 2 & 1 & 0 \\ -3 & 0 & 3 \\ 4 & -2 & -3 \end{vmatrix}, \begin{vmatrix} 1 & 1 & 0 \\ -2 & 0 & 3 \\ 3 & -2 & -3 \end{vmatrix}, -\begin{vmatrix} 1 & 2 & 0 \\ -2 & -3 & 3 \\ 3 & 4 & -3 \end{vmatrix}, \begin{vmatrix} 1 & 2 & 1 \\ -2 & -3 & 0 \\ 3 & 4 & -2 \end{vmatrix}$

$= -15, 9, -3, -1.$

Cofactors of III row $= \begin{vmatrix} 2 & 1 & 0 \\ 5 & 5 & 1 \\ 4 & -2 & -3 \end{vmatrix}, -\begin{vmatrix} 1 & 1 & 0 \\ 2 & 5 & 1 \\ 3 & -2 & 3 \end{vmatrix}, \begin{vmatrix} 1 & 2 & 0 \\ 2 & 5 & 1 \\ 3 & 4 & -3 \end{vmatrix}, -\begin{vmatrix} 1 & 2 & 1 \\ 2 & 5 & 5 \\ 3 & 4 & -2 \end{vmatrix}$

$= -7, 4, -1, -1.$

IV row. $= -\begin{vmatrix} 2 & 1 & 0 \\ 5 & 5 & 1 \\ -3 & 0 & 3 \end{vmatrix}, \begin{vmatrix} 1 & 1 & 0 \\ 2 & 5 & 1 \\ -2 & 0 & 3 \end{vmatrix}, -\begin{vmatrix} 1 & 2 & 0 \\ 2 & 5 & 1 \\ -2 & -3 & 3 \end{vmatrix}, \begin{vmatrix} 1 & 2 & 1 \\ 2 & 5 & 5 \\ -2 & -3 & 0 \end{vmatrix}$

$= -12, 7, -2, -1$

$\Rightarrow B = \begin{bmatrix} 51 & -31 & 10 & 3 \\ -15 & 9 & -3 & -1 \\ -7 & 4 & -1 & -1 \\ -12 & 7 & -2 & -1 \end{bmatrix} \Rightarrow B^T = \begin{bmatrix} 51 & -15 & -7 & 12 \\ -31 & 9 & 4 & 7 \\ 10 & -3 & -1 & -2 \\ 3 & -1 & -1 & -1 \end{bmatrix}$, $|A| = -1$.

$A^{-1} = \frac{\text{adj } A}{|A|} = \begin{bmatrix} -51 & 15 & 7 & 12 \\ 31 & -9 & -4 & -7 \\ -10 & 3 & 1 & 2 \\ -3 & 1 & 1 & 1 \end{bmatrix}$

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(3). let 5 variables $= a, b, c, d, e$

given mean (μ) of 5 variables $= \frac{a+b+c+d+e}{5} = 3$.

$a^2+b^2+c^2 = 14 \rightarrow$ (i)

$c^2+d^2+e^2 = 50 \rightarrow$ (ii)

$a^2+b^2 = 5 \rightarrow$ (iii)

from (i), (iii)

$c^2 = 14-5 = 9 \Rightarrow c = \sqrt{9} = \pm 3 \rightarrow$ (iv)

$\Rightarrow a^2+b^2 = 14-9 = 5$ $\Rightarrow$ let $a=1, b=2$ (possible) (or) $a=2, b=1$.

from (ii), (iii)

$d^2+e^2 = 50-9 = 41 \Rightarrow d=4, e=5$ (or) $d=5, e=4$

$\Rightarrow$ mean $= \frac{a+b+c+d+e}{5} = 3$.

$\Rightarrow a+b+c+d+e = 15 \Rightarrow 1+2+3+4+5 = 3$ ✓

(Variance) $\sigma^2 = \frac{1}{n} \Sigma \left[(a-\mu)^2 + (b-\mu)^2 + (c-\mu)^2 + (d-\mu)^2 + (e-\mu)^2\right]$

$= \frac{1}{5}(4+1+0+1+4) = \frac{10}{5} = 2$

$\Rightarrow$ Standard Deviation $(\sigma) = \sqrt{\text{variance}} = \sqrt{2}$.

$P(A=0 \cap B=0) = \ldots = 0.8 \times 0.4$ | $0.48 = 0.48$

3 a True

because – in Decision tree, to select a node it will calculate gain & grows. If it is fully grown then noisy data will also be at some bud → It will also get fit (It depends on depth). Here given fully grown → Noise also get fit.

A

b True

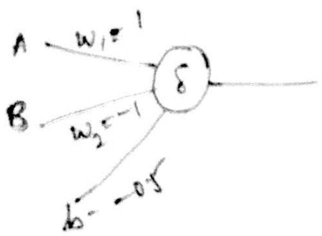
because – In most of the cases, like SVM, neural n/w etc ~~we~~ this will get overfit easily. (If Feature space is larger then there is a possibility that a single feature can distinguish with other points) → overfit.

→ It also depends on other factors like Dataset size (where it can also get not overfit).

(In some cases we can't say) ↗

c Yes,

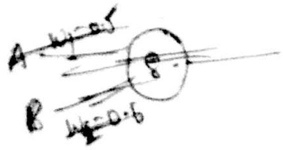
with Bias:



A	B	
0	0	$1(0)+(-1)(0)-0.5 = -0.5 < 0 \to 0$
0	1	$1(0)+(-1)(1)-0.5 = -1.5 < 0 \to 0$
1	0	$1(1)+(-1)(0)-0.5 = 0.5 > 0 \to 1$
1	1	$1(1)+(-1)(1)-0.5 = -0.5 < 0 \to 0$

Threshold = 0.

~~without Bias~~ :-



A	B	
0	0	$0 < 0.6 \to 0$
0	1	$0.6 \geq 0.6 \to 1$
1	0	$0.5 < 0.6$
1	1	$1.1 \geq 0.6$

d

A	B	p(A,B)
0	0	0.32
0	1	0.48
1	0	0.08
1	1	0.12

	B=0	B=1	
A=0	0.32	0.48	= 0.8
A=1	0.08	0.12	= 0.2
	= 0.4	= 0.6	

(i)

→ $P(A=0) = P(A=0, B=0) + P(A=0, B=1) = 0.32 + 0.48 = 0.8$

$P(A=1) = 0.08 + 0.12 = 0.2$

$P(B=0) = 0.32 + 0.08 = 0.4$

$P(B=1) = 0.48 + 0.12 = 0.6$

(ii) A, B independent?

$p(A=0 \cap B=0) = p(A=0)\ p(B=0)$

$0.32 = 0.8 \times 0.4$

$= 0.32$

$p(A=1, B=0) = p(A=1) \times p(B=0)$

$= 0.2 \times 0.4$

$0.08 = 0.08$

$p(A=0, B=1) = p(A=0) \cdot p(B=1)$

$= 0.8 \times 0.6$

$0.48 = 0.48$

$p(A=1, B=1) = p(A=1)\ p(B=1)$

$= 0.2 \times 0.6$

$0.12 = 0.12$

→ All cases are satisfied ⇒ A, B are independent.

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⑤ Ⓐ ~~(i)~~ Nearest Neighbors

(i) Reasons of overfit :-

→ using low value for 'k' results in overfitting.

→ overfit due to curse-of-dimensionality problem

→ If the dataset is ~~not~~ large enough (training data) ⇒ It may have ~~…~~ less test data.

(ii) To reduce overfit :-

→ choose appropriate 'k' value.

→ If overfitting is due to curse-of-dimensionality then do dimensionality reduction or use feature selection algorithms.

Ⓑ Decision Trees :-

(i) Overfit reasons :-

→ fully grown / full depth

→ caused due to noisy data

→ Even though no noise, overfitting will be caused by small no of data points associated with leaf nodes.

(ii) To reduce overfitting :-

→ restrict the depth of the tree.

→ stop growing tree earlier, before it reaches the point where it perfectly classifies the training data. (pruning)

→ Remove noisy data (outliers, missing values, etc.)

(6) Given data points

$$X = \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array}$$
$$X = \begin{bmatrix} 252 & 4 & 155 & 175 \\ 175 & 10 & 186 & 200 \\ 82 & 131 & 230 & 100 \\ 115 & 138 & 80 & 88 \end{bmatrix}$$

$$y = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \rightarrow \text{one-hot-encoding} = \left[\begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right].$$

Initial weights

$$W = \left[\underbrace{\begin{bmatrix} -0.00256 & 0.00889 \\ 0.00146 & 0.00322 \\ 0.00816 & 0.00258 \\ -0.00597 & -0.00876 \end{bmatrix}_{4\times 2}}_{W_0}, \underbrace{\begin{bmatrix} 0.00647 & 0.00540 \\ 0.00374 & -0.00005 \end{bmatrix}_{2\times 2}}_{W_1} \right]$$

(bias) $$b = \left[\underbrace{\begin{bmatrix} -0.00469 \\ 0.00797 \end{bmatrix}}_{b_0}, \underbrace{\begin{bmatrix} -0.00374 \\ -0.00232 \end{bmatrix}}_{b_1} \right]$$

Forward propagation:-

$\Rightarrow$ Used Sigmoid activation for Hidden layers:- $\sigma(z) = \left(\frac{1}{1+e^{-z}}\right)$.

$$\left[z = W^T \cdot x + b\right].$$

For final layer used Softmax function.

$$\text{softmax}(z) = \left(\frac{e^z}{\sum e^z}\right) \Rightarrow \text{gives probability of each class.}$$

Backward propogation:-

used derivatives

$$\sigma(x) = \frac{1}{1+e^{-z}} \Rightarrow \frac{\partial \sigma}{\partial x} = \sigma \cdot (1-\sigma)$$

For softmax:- $= \frac{(z-y)}{m}$ $\quad z = W^T x + b$. $\quad y$ = actual o/p.

$\Rightarrow$ $a_0 = X$

b_1: bias at layer 1.

W_1: ~~b~~ weights at layer 1.

~~$z = \sigma(x)$~~; $z = W^T x + b$; activation $a_i = \sigma(z)$

for Hidden layers

$a_l = \text{softmax}(z)$

|-3
Ite

$$\begin{vmatrix} 0 & 0 & 0 & -2 & -1 \\ 0 & 0 & 0 & 2 & 2 \end{vmatrix}$$

(7)

(6)

Iteration 1 :- Forward propagation :-

$a_0 = X^T \rightarrow 4\times4$ dim.

$z_1 = w_0^T * X + b_0 = \begin{bmatrix} -0.00256 & 0.00146 & 0.00816 & -0.00597 \\ 0.00889 & 0.00322 & 0.00258 & -0.00876 \end{bmatrix} \begin{bmatrix} 252 & 175 & 82 & 115 \\ 4 & 10 & 131 & 18 \\ 155 & 186 & 280 & 80 \\ 175 & 200 & 100 & 88 \end{bmatrix}$ matrix

$+ \begin{bmatrix} -0.00469 \\ 0.00797 \end{bmatrix}$

$z_1 = \begin{bmatrix} -0.42392 & -0.11433 & 1.25645 & 0.02983 \\ 1.12803 & 0.3238 & 0.87677 & 0.9102 \end{bmatrix}$

ts.

~~$a_1 = w_1^T x +$~~ $a_1 = \sigma(z_1) = \begin{bmatrix} 0.0137588 & -0.00206002 \\ -0.0185448 & 0.0198 \\ 0.0128 & 0.00381 \\ 0.00803 & -0.0174 \end{bmatrix}$

$z_2 = \underset{2\times2}{w_1^T} . \underset{2\times4}{a_1} + \underset{(2\times1)}{b_1} = \begin{bmatrix} -0.0056 & -0.00676 & -0.0082 & -0.0064 \\ -0.00022 & 0.0049 & 0.0018 & 0.0003 \end{bmatrix}$

$\sigma(z_2) = a_2 = \begin{bmatrix} 0.498 & 0.498 & 0.497 & 0.498 \\ 0.501 & 0.501 & 0.502 & 0.5017 \end{bmatrix}$

Backpropagation :-

Loss at layer 3 (loss 3) = Cross-entropy loss (a_2, y-actual).

Other layer losses at loss(i) = $(w_i * loss(i+1)) \cdot a_i (1 - a_i)$.

$loss(1) = \begin{bmatrix} -3.53 & -3.68 & 2.57 & 3.72 \\ 8.72 & 1.14 & -9.88 & -9.72 \end{bmatrix} \cdot \frac{1}{10000}$

$= \begin{bmatrix} -0.000353 & -0.000368 & 0.000257 & 0.000372 \\ 0.000872 & 0.000114 & -0.000988 & 0.000972 \end{bmatrix}$

For updating weights & Bias.

$weight[i] = w_i - \alpha * \underset{\text{layer 'i' derivative of w.}}{(L_i\, dw)}$

$w = \begin{bmatrix} -0.00057629 & 0.00831 \\ -0.00053 & 0.00384 \\ 0.00920 & 0.00246 \\ -0.00404 & -0.0092 \end{bmatrix}$, $w_1 = \begin{bmatrix} -0.00515 & 0.00406 \\ 0.0040 & -0.0003 \end{bmatrix}$

First update

[redacted]

to update bias$(b_i) = b_i - \alpha \dfrac{\partial S}{\partial b_i}$

derivative wrt bias.

$$b_0 = \begin{bmatrix} -0.0046 \\ 0.00796 \end{bmatrix}, \quad b_1 = \begin{bmatrix} -0.00581 \\ -0.0023 \end{bmatrix}$$

After execution :-

$$\text{Final weights} = \underbrace{\begin{bmatrix} 0.0137588 & -0.00206 \\ -0.018514 & 0.01987 \\ 0.0128 & 0.0038 \\ 0.0080 & -0.01742 \end{bmatrix}}_{W_0}, \quad \underbrace{\begin{bmatrix} -0.00476 & 0.003 \\ 0.0189 & -0.0152 \end{bmatrix}}_{W_1}$$

$$\text{bias } (b) = \underbrace{\begin{bmatrix} -0.00469408 \\ 0.00798 \end{bmatrix}}_{b_0}, \quad \underbrace{\begin{bmatrix} -0.00534 \\ -0.0028 \end{bmatrix}}_{b_1}$$

∴ Thus, trained till convergence.

(9)

4 B) $A = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ -3 & -3 & -3 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & -1 & -1 \end{bmatrix}$

Singular value decompostion (A).

$SVD(A) = U \cdot \Sigma \cdot V^T$ → diagonal matrix

orthogonal matrix

U = eigenvector $(A \cdot A^T)$

Σ = $\sqrt{\text{eigen values}(A \cdot A^T)}$ ⇒ diagonal elements.

V = eigenvectors $(A^T \cdot A)$.

$$A \cdot A^T = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ -3 & -3 & -3 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -3 & 2 & 0 & 0 & 0 \\ 1 & -3 & 2 & 0 & 0 & 0 \\ 1 & -3 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 3 & -9 & 6 & 0 & 0 & 0 \\ -9 & 27 & -18 & 0 & 0 & 0 \\ 6 & -18 & 12 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & -4 & 2 \\ 0 & 0 & 0 & -4 & 8 & -4 \\ 0 & 0 & 0 & 2 & -4 & 2 \end{bmatrix}$$

$|A A^T - \lambda I| = 0.$

$$\Rightarrow \begin{vmatrix} 3-\lambda & -9 & 6 & 0 & 0 & 0 \\ -9 & 27-\lambda & -18 & 0 & 0 & 0 \\ 6 & -18 & 12-\lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 2-\lambda & -4 & 2 \\ 0 & 0 & 0 & -4 & 8-\lambda & -4 \\ 0 & 0 & 0 & 2 & -4 & 2-\lambda \end{vmatrix} = 0.$$

for U =

$$A^T A = \begin{bmatrix} 1 & -3 & 2 & 0 & 0 & 0 \\ 1 & -3 & 2 & 0 & 0 & 0 \\ 1 & -3 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ -3 & -3 & -3 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 14 & 14 & 14 & 0 & 0 \\ 14 & 14 & 14 & 0 & 0 \\ 14 & 14 & 14 & 0 & 0 \\ 0 & 0 & 0 & 6 & 6 \\ 0 & 0 & 0 & 6 & 6 \end{bmatrix}$$

for $V = (A^T A - \lambda I) = \begin{bmatrix} 14-\lambda & 14 & 14 & 0 & 0 \\ 14 & 14-\lambda & 14 & 0 & 0 \\ 14 & 14 & 14-\lambda & 0 & 0 \\ 0 & 0 & 0 & 6-\lambda & 6 \\ 0 & 0 & 0 & 6 & 6-\lambda \end{bmatrix}$

for U :-

Eigenvalues of $(A \cdot A^T - \lambda I)$

$= \begin{bmatrix} 3-\lambda & -9 & 6 & 0 & 0 & 0 \\ -9 & 27-\lambda & -18 & 0 & 0 & 0 \\ 6 & -18 & 12-\lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 2-\lambda & -4 & 2 \\ 0 & 0 & 0 & -4 & 8-\lambda & -4 \\ 0 & 0 & 0 & 2 & -4 & 2-\lambda \end{bmatrix} = 0.$

$\Rightarrow$ after solving for the eigen values.

$(\lambda - 42)(\lambda - 12)(\lambda) = 0.$

$\lambda = 42, 12, 0.$

write in Decreasing order.

$\Rightarrow \lambda_1 = 42, \lambda_2 = 12, \lambda_3 = 0, \lambda_4 = 0, \lambda_5 = 0, \lambda_6 = 0.$

for. $\Sigma \Rightarrow \sigma_1 = \sqrt{42}, \sigma_2 = \sqrt{12}, \sigma_3 = \sqrt{0}, \sigma_4 = \sqrt{0}, \sigma_5 = \sqrt{0}, \sigma_6 = \sqrt{0}.$

$= \begin{bmatrix} \sigma_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_6 \end{bmatrix} = \begin{bmatrix} \sqrt{42} & 0 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

to calculate U = Eigenvector $(A \cdot A^T - \lambda I)$ with $\lambda_1 = 42, \lambda_2 = 12,$
$\lambda_3 = \lambda_4 = \lambda_5 = \lambda_6 = 0.$

$\Rightarrow$ for $\lambda = 0.$

$(A \cdot A^T - \lambda I)X = \begin{bmatrix} 3 & -9 & 6 & 0 & 0 & 0 \\ -9 & 27 & -18 & 0 & 0 & 0 \\ 6 & -18 & 12 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & -4 & 2 \\ 0 & 0 & 0 & -4 & 8 & -4 \\ 0 & 0 & 0 & 2 & -4 & 2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = 0.$

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$$\left[\begin{array}{cccccc|c} 3 & -9 & 6 & 0 & 0 & 0 & 0 \\ -9 & 27 & -18 & 0 & 0 & 0 & 0 \\ 6 & -18 & 12 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & -4 & 2 & 0 \\ 0 & 0 & 0 & -4 & 8 & -4 & 0 \\ 0 & 0 & 0 & 2 & -4 & 2 & 0 \end{array}\right]$$

after reduction.

$x_1 = 3x_2 - 2x_3$
$x_2 = x_2$
$x_3 = x_3$
$x_4 = 2x_5 - x_6$
$x_5 = x_5$
$x_6 = x_6$.

$$\Rightarrow X = \begin{bmatrix} 3x_2 - 2x_3 \\ x_2 \\ x_3 \\ 2x_5 - x_6 \\ x_5 \\ x_6 \end{bmatrix}.$$

let $x_2 = 1, x_3 = 0, x_5 = 0, x_6 = 0$. $\Rightarrow \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ → Here we will get many such vectors.

final. U = orthogonal matrix.

is.

$$\begin{bmatrix} -0.2673 & 0 & -0.9482 & 0 & -0.1719 & 0 \\ 0.8018 & 0 & -0.3178 & 0 & 0.5061 & 0 \\ -0.5345 & 0 & -0.0026 & 0 & 0.8452 & 0 \\ 0 & -0.4082 & 0 & -0.9129 & 0 & 0.4472 \\ 0 & 0.8165 & 0 & -0.3651 & 0 & 0.8944 \\ 0 & -0.4082 & 0 & 0.1826 & 0 & \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} \sqrt{42} & 0 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

For $\underline{V}$:- Eigenvectors of $(A^T A - \lambda I)$.

$\lambda = 42, 12, 0, 0, 0, 0.$

$\Rightarrow$ after calculation. $V = \begin{bmatrix} -0.5774 & 0 & 0.8165 & 0 & 0 \\ -0.5774 & 0 & -0.4082 & 0 & -0.7071 \\ -0.5774 & 0 & -0.4082 & 0 & 0.7071 \\ 0 & 0.7071 & 0 & -0.7071 & 0 \\ 0 & 0.7071 & 0 & 0.7071 & 0 \end{bmatrix}$

($\because$ V from solved by library)

$U \Sigma V^T$

$$\begin{bmatrix} -0.2673 & 0 & -0.9482 & 0 & -0.1719 & 0 \\ 0.8018 & 0 & 0.1176 & 0 & 0.5061 & 0 \\ 0.5345 & 0 & -0.0026 & 0 & 0.8452 & 0 \\ 0 & -0.4082 & 0 & -0.9129 & 0 & 0 \\ 0 & .8165 & 0 & -0.3651 & 0 & 0.4472 \\ 0 & -.4082 & 0 & -0.1826 & 0 & 0.8944 \end{bmatrix}_{6\times 6} \cdot$$

$$\begin{bmatrix} \sqrt{42} & 0 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{6\times 6} \cdot \begin{bmatrix} -0.5774 & 0 & 0.8165 & 0 & 0 \\ -0.5774 & 0 & -0.4082 & 0 & -0.7071 \\ -0.5774 & 0 & -0.4082 & 0 & 0.7071 \\ 0 & 0.7071 & 0 & -0.7071 & 0 \\ 0 & 0.7071 & 0 & 0.7071 & 0 \end{bmatrix}_{5\times 5}$$

rincipal component of original data point $= U\Sigma$.

$-0.2673 \quad 0 \quad -0.9482 \quad 0 \quad -0.1719 \quad 0$