

Write True/False clearly and provide precise justification wherever applicable. Any instance of cheating is considered academic dishonesty. (**Question 01-07 and 09: 1 Marks Each and Questions 08: 2 Marks**)

Given the input dataset  $X = [x_1, \dots, x_K], x_i \in \mathbb{R}^d$ , a dictionary  $D \in \mathbb{R}^{d \times n}$  and representation  $R = [r_1, \dots, r_K], r_i \in \mathbb{R}^n$

1. The dictionary  $D$  can be "under complete" if  $n < d$  or "overcomplete" in case  $n > d$ . **True**
2. Under complete dictionaries represent the setup in which the actual input data lies in a lower-dimensional space. **True** (Undercomplete dictionaries represent the setup in which the actual input data lies in a lower-dimensional space. This case is strongly related to [dimensionality reduction](#) and techniques like [principal component analysis](#) which require atoms  $d_1, \dots, d_n$  to be orthogonal.)
3. Overcomplete dictionaries do not require the atoms to be orthogonal. **True** (Overcomplete dictionaries, however, do not require the atoms to be orthogonal (they will never be a [basis](#) anyway) thus allowing for more flexible dictionaries and richer data representations.)
4. The objective function of dictionary learning is  $\min_{D,Z} \|X - DZ\|_F^2, s.t. \|Z\|_0 \geq \tau$ ? **False** It should be  $\min_{D,Z} \|X - DZ\|_F^2, s.t. \|Z\|_0 \leq \tau$
5. The optimization function of dictionary learning has been used only for Synthesis purpose not for Analysis purposes? **True**
6. Dictionary learning is a representation learning technique? **True**
7. Can dictionary learning algorithm be used for supervised and unsupervised tasks? **True**
8. What are the two steps of the K-SVD algorithm?

**K-SVD** is an algorithm that performs **SVD** at its core to update the atoms of the dictionary one by one and basically is a generalization of **K-means**. It enforces that each element of the input data

$\{x_i\}$  is encoded by a linear combination of not more than  $\{T_0\}$  elements in a way identical to the MOD approach:

$$(i) \min_{D,R} \|X - DR\|_F^2 \text{ s.t. } \forall i \|r_i\|_0 \leq T_0$$

This algorithm's essence is to first fix the dictionary, find the best possible

$\{R\}$  under the above constraint (using [Orthogonal Matching Pursuit](#)) and then iteratively update the atoms of

$$(ii) \text{ dictionary } \{D\} \text{ in the following manner: } \|X - DR\|_F^2 = \|X - \sum_{i=1}^K d_i x_T^i\|_F^2$$

In Brief: K-SVD Algorithm: two steps

**(i) Sparse Coding:** Producing sparse representations matrix  $X$ , given the current dictionary  $D$

**(ii) Dictionary Update:** (the main innovation) Updating dictionary atoms, given the current sparse representations

9. Out of the following which strategy is used by Dictionary Learning to shape the energy function?
  - a. Use a regularizer that limits the volume of space that has low energy (**Correct**)
  - b. Minimize the gradient and maximize the curvature around the data points
  - c. Build a mechanism so that the volume of low energy stuff is constant
  - d. Train the dynamical system so that the dynamics go to the data manifold.