DIP Assignment-4

Name :- Subhani Shaik
Roll No. :- MT18117

1 (a) $\hat{F} = W \cdot G$, $G = (HF + N)$ (N: noise) $\Rightarrow \hat{F} = W(HF+N) = FHW + WN$

(∵ from lecture slide)

given error (e) $= \sum_{k,l} |(F - \hat{F})|^2 + \lambda |\hat{F}.L|^2$

$$= \sum_{k,l} |F(k,l) - \hat{F}(k,l)|^2 + \lambda |\hat{F}(k,l).L|^2$$

$$= \sum_{k,l} |F - \hat{F}|^2 + \lambda |\hat{F}.L|^2$$

$$= \sum_{k,l} |F - FHW - WN|^2 + \lambda |(FHW + WN).L|^2$$

$$= \sum_{k,l} \{|F|^2|1-HW|^2 + |WN|^2\} + \lambda (|FHW.L|^2 + |WNL|^2)$$

$$= \sum_{k,l} |F|^2 (1-HW)(1-HW)^* + |N|^2 W.W^* + \lambda (|FHL + NL|^2 |W|^2)$$

$\sum |z|^2 = \sum z.z^*$

$$e = \sum_{k,l} |F|^2 (1-HW)(1-HW)^* + |N|^2 . W.W^* + \lambda (|FHL + NL|^2 . W.W^*)$$

$$\frac{\partial e}{\partial W} = |F|^2 \cancel{2} (1-HW)^* (-H) + |N|^2 . \cancel{2} W^* + \lambda |FHL + NL|^2 . \cancel{2} W^* = 0.$$

$\Rightarrow |F|^2 (1-HW)^* (-H) + |N|^2 W^* + \lambda |FHL + NL|^2 . W^* = 0.$

$\Rightarrow \underline{|F|^2 (-H)} + |F|^2 |H|^2 W^* + |N|^2 W^* + |FHL + NL|^2 . W^* = 0.$

$\Rightarrow |F|^2 |H|^2 W^* + |N|^2 W^* + \lambda |FHL + NL|^2 = |F|^2 . H$

$$\Rightarrow W^* = \frac{H . |F|^2}{|F|^2 . |H|^2 + |N|^2 + \lambda |FHL + NL|^2}$$

divide by $|F|^2$.

$$\Rightarrow W^* = \frac{H}{|H|^2 + \frac{|N|^2}{|F|^2} + \frac{\lambda |FHL + NL|^2}{|F|^2}}$$

$$W^* = \frac{H}{|H|^2 + \frac{|N|^2}{|F|^2} + \lambda \frac{|FHL + NL|^2}{|F|^2}}$$

$$(W^*)^* = W = \frac{H^*}{|H|^2 + \frac{|N|^2}{|F|^2} + \lambda \frac{|FHL + NL|^2}{|F|^2}}$$

$$\Downarrow$$

$$\frac{\cancel{|F|^2}|H|^2|L|^2}{\cancel{|F|^2}} + \frac{|N|^2|L|^2}{|F|^2}$$

$$= |H|^2|L|^2 + \frac{|N|^2|L|^2}{|F|^2}$$

$$\Rightarrow W = \frac{H^*}{|H|^2 + \frac{|N|^2}{|F|^2} + \lambda |H|^2|L|^2 + \lambda\frac{|N|^2|L|^2}{|F|^2}}$$

let $C = \frac{\overset{\text{noise}}{|N|^2}}{|F|^2}$ constant

$$= \frac{H^*}{|H|^2 + C + \lambda |H|^2|L|^2 + \lambda C \cdot |L|^2}$$

$$= \frac{H^*}{|H|^2 + C + \lambda |L|^2 (|H|^2 + C)}$$

$$\Rightarrow W = \frac{H^*}{|H|^2 + C + \lambda |L|^2 (|H|^2 + C)}$$

(2) (a) given $f(x,y) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 100 & 2 \\ 1 & 2 & 2 \end{bmatrix}$

$\theta = \{0, \pi/4, \pi/2\}, \quad \rho = \{0, +1, -1\}$

(from text book)

$$g(\rho,\theta) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\,\delta(x\cos\theta + y\sin\theta - \rho)$$

$\theta = 0, \rho = \{0, +1, -1\}$

$$g(\rho, 0) = \sum_{x,y} f(x,y)\,\delta(x - \rho).$$

$$\rightarrow g(1,0) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\,\delta(x-1)$$

$$g(0,0) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\,\delta(x)$$

$$g(-1,0) = \sum_{x,y} f(x,y)\,\delta(x+1)$$

$\theta = \frac{\pi}{4}$..

$$g(\rho, \pi/4) = \sum_{x,y} f(x,y)\,\delta\left(\frac{x}{\sqrt2} + \frac{y}{\sqrt2} - \rho\right)$$

$$g(-1, \pi/4) = \sum_{x,y} f(x,y)\,\delta\left(\frac{x}{\sqrt2} + \frac{y}{\sqrt2} + 1\right)$$

$$g(0, \pi/4) = \sum_{x,y} f(x,y)\,\delta\left(\frac{x}{\sqrt2} + \frac{y}{\sqrt2}\right)$$

$$g(1, \pi/4) = \sum_{x,y} f(x,y)\,\delta\left(\frac{x}{\sqrt2} + \frac{y}{\sqrt2} - 1\right)$$

$\cos\frac{\pi}{4} = 1/\sqrt2$
$\sin\pi/4 = 1/\sqrt2$
$\cos\pi/2 = 0$
$\sin\pi/2 = 1$

$\theta = \pi/2$.

$$\left.\begin{aligned} g(-1, \pi/2) &= \sum_{x,y} f(x,y)\,\delta(y+1) \\ g(0, \pi/2) &= \sum_{x,y} f(x,y)\,\delta(y) \\ g(1, \pi/2) &= \sum_{x,y} f(x,y)\,\delta(y-1). \end{aligned}\right\} \text{one column.}$$