DIP Assignment - 2

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(1) Given $W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$; $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

pad with 0's to retain same size.

$$\Rightarrow W' = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad ; I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

now do convolution with filter (I) by stride = 1.

i.e., move filter window by one step each time and multiply with corresponding elements & calculate value.

$0\times1 + 0\times0 + 0\times0 + 1\times1 = 1.$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & - & - \\ & - & & \\ & & & \end{bmatrix}$$

by doing these procedure.

$$\Rightarrow W' * I = \begin{bmatrix} (1\times0+0\times0+0\times0+1\times1) & (1\times0+0\times0+0\times1+1\times0) & (1\times0+0\times0+0\times0+1\times0) & (0\times1+0\times0+0\times0+1\times0) \\ (1\times0+0\times1+0\times0+1\times0) & (1\times1+0\times0+0\times0+0\times1) & (0\times1+0\times0+0\times0+0\times1) & 0 \\ 0 & 0 & (0\times1+0\times0+0\times0+-1\times1) & (0+0+-1\times0+0) \\ 0 & 0 & (0+0+-1\times0+0) & (-1\times1+0+0+0) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$\Rightarrow W' * I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

~~3 b~~

4 Inverse transform

$x(w) : \delta(w - kw_0) + \delta(w + kw_0)$

$x(t) = ?$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} [\delta(w - kw_0) + \delta(w + kw_0)] e^{jwt} dw$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} [\delta(w - kw_0)] e^{jwt} dw + \frac{1}{2\pi} \int_{-\infty}^{\infty} \delta(w + kw_0) e^{jwt} dw$$

$$= \frac{e^{jkw_0 t}}{2\pi} + \frac{e^{-jw_0 tk}}{2\pi}$$

$$= \frac{1}{2\pi} \left[e^{jkw_0 t} + e^{-jkw_0 t} \right]$$

$$= \frac{1}{\pi} \cos(kw_0 t) = \frac{\cos(kw_0 t)}{\pi}.$$

$$\rightarrow x(t) = \frac{\cos(kw_0 t)}{\pi}$$

(3b) .. convolution equation

$$G(x,y) = \underbrace{f(w,y)}_{\substack{\text{Image} \\ \text{i/p}}} \underbrace{*}_{\text{Convolution}} \underbrace{w(x,y)}_{\text{Filter.}}$$

Equation of Unsharp masking

$$U(x,y) = f(x,y) + G_{mask}$$

$$= f(x,y) + f(x,y) - f(x,y) * w(x,y) \rightarrow ①.$$

Now as we know, $f(x,y) . \delta(x,y) = f(x,y) \rightarrow ②$

where $\delta(x,y)$ is Impulse function/matrix which is the convolution Identity.

⇒ Substitute ② in ①.

$$= f(x,y) * \delta(x,y) + f(x,y) \cdot \delta(x,y) - f(x,y) * w(x,y)$$

$$= f(x,y) * [\delta(x,y) + \delta(x,y) - w(x,y)]$$

$$= f(x,y) * w'(x,y)$$

where $w'(x,y) = \delta(x,y) + \delta(x,y) - w(x,y)$

⇒ Unsharp mask = (original − Blur) + original

= 2 original − Blur Image

$$= 2 f(x,y) * \delta(x,y) - f(x,y) * \underbrace{\text{Box filter.}}_{7 \times 7}$$

$$= 2 f(x,y) * \delta(x,y) - f(x,y) * \frac{1}{49}\left[\sum_{i=3}^{3}\sum_{j=3}^{3} \delta(x-i, y-j)\right]$$

$$= f(x,y) * \left[2\delta(x,y) - \frac{1}{49}\left[\sum_i \sum_j \delta(x-i, y-j)\right]\right]$$

$$= f(x,y) * \frac{1}{49}\left[98\,\delta(x,y) - \sum_i \sum_j \delta(x-i, y-j)\right]$$

i.e., $\frac{1}{49}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & & & & & & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{7\times7} - \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ \vdots & & & & & & \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}_{7\times7}$

$$w(x,y) = \frac{1}{49}\begin{bmatrix} -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & 97 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 \end{bmatrix}_{7\times7}$$ filter.