1)

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Output of full convolution is 4X4

$I \rightarrow$ zero padded to 4X4

$$\begin{matrix} (-1,-1)\ 0 & 0 & 0 & 0 \\ 0 & \overset{(0,0)}{1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$$

$W \rightarrow$ rotated by $180°$

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Response at ~~(-1)~~ $(-1,-1)$

$-1 \cdot 0 + 0 \cdot 0 + \cdots + 1 \cdot 1 = 1$

Response at $(0,0)$

$-1 \cdot 0 + 0 \cdot 1 + 0 \cdot 0 + \cdots + 1 \cdot 1 = 1$

at $(1,1)$

$-1 \cdot 1 + 0 \cdot 0 + \cdots + 0 \cdot 1 + \cancel{0 \cdot 0}\, 1 \cdot 0 = -1$

at $(2,2)$ again response is -1

Output

$$\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{matrix}$$

If required →
In general, if you want to blur image in, say photoshop, you would want output image to be of same size as that of image. Here, we know that input image starts at (0,0). We can pick up size of 2x2 starting at (0,0) i.e

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Q 3-b) Unsharp masking

$$f(x,y) + g_{mask}(x,y)$$

$$g_{mask}(x,y) = f(x,y) - f(x,y) * \bar{\omega}(x,y)$$

$\bar{\omega}(x,y) \rightarrow$ blur filter

then, $f(x,y) + f(x,y) - f(x,y) * \bar{\omega}(x,y)$

$$\delta(x,y) * 2f(x,y) - f(x,y) * \bar{\omega}(x,y)$$

$$f(x,y) * \underbrace{\left(2\delta(x,y) - \bar{\omega}(x,y)\right)}_{\omega(x,y)}$$

$$\omega(x,y) = \begin{bmatrix} 0 & \cdots & \cdot \\ \vdots & 2 & \vdots \\ 0 & \cdots & 0 \end{bmatrix} - \frac{1}{49}\begin{bmatrix} 1 & \cdots & 1 \\ \vdots & 1 & \vdots \\ 1 & \cdots & 1 \end{bmatrix}$$

only 2 at origin

all ones

Q4) $\frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\delta(\omega-K\omega_0)+\delta(\omega+K\omega_0)\right]e^{j\omega t}\,d\omega$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\delta(\omega-K\omega_0)\,e^{jK\omega_0 t}\,d\omega +$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\delta(\omega+K\omega_0)\,e^{-jK\omega_0 t}\,d\omega$$

$$= \frac{1}{2\pi}e^{jK\omega_0 t}\int_{-\infty}^{\infty}\delta(\omega-K\omega_0)\,d\omega$$

$$+\frac{1}{2\pi}e^{-jK\omega_0 t}\int_{-\infty}^{\infty}\delta(\omega+K\omega_0)\,d\omega$$

$$= \frac{1}{2\pi}\,2\cos K\omega_0 t$$