Question Bank

Engineering Mathematics –I

Unit-I

MATRICES

1. Reduce the matrix to Echelon form and find its rank Where A = $\begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$.

2. Find the rank of $\begin{bmatrix} 2 & -4 & 3 & -1 & 0 \\ 1 & -2 & -1 & -4 & 2 \\ 0 & 1 & -1 & 3 & 1 \\ 4 & -7 & 4 & -4 & 5 \end{bmatrix}$ using Echelon form.

3. Find the value of k such that the rank of $\begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & k & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$ is 2.

4. Find the rank of $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ by reducing it to the normal form

5. Find the rank of $\begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & -4 & 6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}$ by reducing it to normal form

6. Discuss for what values of $\lambda\, and\, \mu$ the simultaneous system of equations $x+y+z=6, x+2y+3z=10, x+2y+\lambda z=\mu$ have (i) no solution (ii) a unique solution and (iii) an infinite number of solutions.

7. Find whether the following system of equations is consistent. If so solve them. $x+2y-z=3, 3x-y+2z=-1, 2x-2y+3z=2, x-y+z=-1.$

8. Show that the only real number λ for which the system $x+2y+3z=\lambda x, 3x+y+2z=\lambda y, 2x+3y+z=\lambda z$ has non-zero solution is 6 and solve them when $\lambda=6$.
9. Determine whether the following equations will have a non-trivial solution if so solve them. $4x+2y+z+3w=0, 6x+3y+4z+7w=0, 2x+y+w=0$.
10. Solve the system of equations x + y -z = 4, x-2y +3z = -6, 2x+3y +z = 7 by using LU Decomposition method
11. Solve the system of equations x + y +z = 1, 4x+3y -z = 6, 3x+5y +3z = 7 by using LU Decomposition method
12. Find the values of 'a' and 'b' for which the equations, $x+y+z=3, x+2y+2z=6, x+9y+az=b$ have

i) No solution.

ii) A unique solution.

iii) Infinite number of solutions.

13. Discuss for all values of λ the system of equations $x+y+4z=6,\ x+2y-2z=6$ $\lambda x+y+z=6$ with regard to consistence.

14. Solve the system of equations $x+3y-2z=0,\ 2x-y+4z=0,\ x-11y+14z=0$.
15. Show that the system of equations $2x_1-2x_2+x_3=\lambda x_1, 2x_1-3x_2+2x_3=\lambda x_2$ $-x_1+2x_2=\lambda x_3$ composes a non-trivial solution only if $\lambda=1, \lambda=-3$.
16. If $a+b+c\neq 0$ show that the system of equations: $-2x+y+z=a, x-2y+z=b,$ $x+y-2z=c$ has no solution If $a+b+c=0$, show that it has infinitely many solutions.

Unit-II

EIGEN VALUES AND EIGEN VECTORS

17. Find the Eigen values and the corresponding Eigen vectors of the matrix

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

18. Find the Eigen values and the corresponding Eigen vectors of $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

19. Find the Eigen values and the corresponding Eigen vectors of $\begin{bmatrix} 2 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

20. Read all the properties of Eigen values and Eigen vectors

21. If $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ verify Cayley-Hamilton theorem. Find A^{-1} and A^4.

22. If 2, 3, 5 are the Eigen values of a matrix A then find the Eigen values of $2A^3 + 3A^2 + 5A + 3I$.

23. Find the characteristic equation of the matrix, $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence compute A^{-1}. Also find the matrix represented by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$

24. Diagonalize the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{bmatrix}$ find a) A^8 b) A^4

25. Diagonalize the matrix by an orthogonal transformation $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 6 & 0 \\ 4 & 0 & 2 \end{bmatrix}$

26. Determine the modal matrix P of $A=\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ verify that $P^{-1}AP$ is a diagonal matrix.

27. Reduce the quadratic form $3x^2+5y^2+3z^2-2yz+2zx-2xy$ to canonical form by an orthogonal reduction.

28. Reduce the quadratic form $3x^2+2y^2+3z^2-2yz-2xy$ to canonical form by an orthogonal reduction.

29. Identify the nature of the quadratic form $3x^2+3y^2+3z^2-2yz+2zx+2xy$

UNIT-III

FIRST ORDER FIRST DEGREE ORDINARY DIFFERENTIAL EQUATIONS

1. A) Solve $(\cos x - x\cos y)dy - (\sin y + y\sin x)dx = 0$

   B) A body cools from 60^0C to 50^0C in 10 minutes when kept in air at 30^0C in the next 10 minutes what is the temperature of the body.

2. A) Solve $\left(1+e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy = 0$

   B) The number of bacteria culture grows at the rate proportional to N, the value of N was initially 100 and it increases to 332 in one hr. What would be the value of N after $1\frac{1}{2}hr$

3. A) Solve $\frac{dy}{dx} + y\sec^2 x = \tan x \sec^2 x$

   B) If 30% of radioactive substance disappear in 10 days. How long will it take for 90% of it to disappear.

4. A) Solve $\frac{dy}{dx} + \frac{3x^2}{1+x^3}y = \frac{\sin^2 x}{1+x^3}$.

   B) Show that the family of con-focal conics $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$ is self orthogonal.

5. A) Solve $\frac{dy}{dx} + y\tan x = x^m \cos x$

   B) If radioactive carbon-14 has a half-life of 5750 years, what will remain of 1 gram after 3000 years?

6. A) Solve $x\frac{dy}{dx} + y = x^2 + 3x + 2$

B) Find the orthogonal trajectories of the family of curves $r^n = a^n \cos n\theta$

7. A) Solve $\left(x+2y^3\right)\frac{dy}{dx} - y = 0$

B) Suppose that an object is heated to 300F and allowed to cool is a room whose air temperature 20F, it after 10 min, the temperature of the object is 250F, what will be its temperature after 20 min?

UNIT-IV

HIGHER ORDER ORDINARY DIFFERENTIAL EQUATIONS

1. A) Solve $\left(D^2 - 2D + 1\right) y = xe^x \sin x$

   B) Solve $\left(D^2 - 2D\right) y = e^x \sin x$ by the method of variation of parameters.

2. A) Solve $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = -2\cosh x$ *Also find when* $y = 0, \frac{dy}{dx} = 1 \, at \, x = 0$

   B) Solve $\frac{d^2y}{dx^2} - \frac{1}{x}\frac{dy}{dx} + \frac{y}{x^2} = \frac{2\log x}{x^2}$.

3. A) Solve $\left(D^2 - 4D + 4\right) y = 8x^2 e^{2x} \sin 2x$

   B) Solve $y'' - 6y' + 9y = \frac{e^{3x}}{x^2}$ by the method of variation of parameters.

4. A) Solve $\frac{d^2x}{dt^2} + n^2 x = k\cos(nt + \alpha)$

   B) Solve $(2x-1)^2 \frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx} - 2y = 8x^2 - 2x + 3$

5. A) Solve $\frac{d^4x}{dt^4} + 2\frac{d^2x}{dt^2} + x = t^2 \cos t$

   B) Determine the charge on the capacitor in an *LRC* series circuit at when *inductance* 1 H, resistance 7Ω, capacitance 0.1 F, $E(t) = e^t$ V, $q(0) = 2$ C, and $i(0) = 0$ A.

6. A) Solve $\left(D^2 + 1\right) y = x^2 e^{3x}$

   B) Solve by the method of variation of parameters $\frac{d^2y}{dx^2} - y = \frac{2}{\left(1+e^x\right)}$.

7. A) Solve $\left(D^2 - 2D + 2\right) y = e^x \tan x$.

   B) Determine charge q and current i in the LRC circuit with inductance 0.5H, resistance 6 ohms, *capacitance* (1/16)F, E(t) = sinht, and the initial conditions are q(0)=0, i(0)=1.

8. A) *Solve* $\left(D^2-3D+2\right)y=\sin\left(e^{-x}\right)$

B) Solve $x^3\dfrac{d^3y}{dx^3}+2x^2\dfrac{d^2y}{dx^2}+2y=10\left(x+\dfrac{1}{x}\right)$

UNIT-V

LAPLACE TRANSFORMS

1. Find the Laplace transform of $(i)e^{2t}+4t^3-2\sin 3t+3\cos 3t$ $(ii)\sin(\omega t+\alpha)$ (iii) $\sin^3 2t$ (iv) $\sin 2t.\cos 3t$
2. Find the Laplace transform of $(i)\ e^{-3t}(2\cos 5t-3\sin 5t)$ $(ii)\cosh at\cos at$ $(iii)e^{3t}\sin^2 t$ (iv) $L\left\{e^t\left(\cos 2t+\dfrac{1}{2}\sinh 2t\right)\right\}$
3. Find the Laplace transforms of $(i)t^2\cos at$ $(ii)\ te^{-t}\sin 3t$ $(iii)L\left\{t^2\sin 2t\right\}$
4. Find $(i)\ L\left\{\int_0^t \dfrac{e^{-t}\sin t}{t}dt\right\}$ $(ii)\ L\left\{\int_0^t \dfrac{1-e^{-t}}{t}dt\right\}$ $(iii)\ L\left\{\dfrac{1-\cos t}{t}\right\}$ (iv) $\dfrac{\sin t\sin 5t}{t}$
5. Evaluate $(i)\int_0^\infty te^{-2t}\cos 3t\,dt$ $(ii)\int_0^\infty t^2e^{-4t}\sin 2t\,dt$
6. Find inverse Laplace transform of (i) $\dfrac{s+1}{s^2+6s+25}$ (ii) $\dfrac{s+2}{s^2(s+3)}$ (iii) $\tan^{-1}\left(\dfrac{a}{s}\right)$ (iv) $\dfrac{1}{s\left(s^2+a^2\right)}$
7. Apply convolution theorem to evaluate

$(i)L^{-1}\left\{\dfrac{s}{(s^2-a)^2}\right\}$ $(ii)\ L^{-1}\left\{\dfrac{s^2}{(s^2+a)^2(s^2+b)^2}\right\}$ $(iii)\ L^{-1}\left\{\dfrac{1}{(s+1)(s^2+1)}\right\}$ $(iv)\ L^{-1}\left\{\dfrac{1}{(s+2)^2(s-2)}\right\}$ $(v)L^{-1}\left\{\dfrac{s}{(s^2+a^2)^2}\right\}$

8. Use Laplace transform to solve(i) $y''-3y'+2y=4t+e^{3t},\ y(0)=1, y'(0)=1$

(ii) $\dfrac{d^2y}{dt^2}+2\dfrac{dy}{dt}-3y=\sin t$, $y=\dfrac{dy}{dt}=0$ when $t=0$

(iii) $\frac{d^2x}{dt^2}+2\frac{dx}{dt}+x=3te^{-t}$ given $x=4$ and $\frac{dx}{dt}=0$ at $t=0$

(iv) $\left(D^2+5D-6\right)y=x^2e^{-x}, y(0)=a, y^{'}(0)=b$

(v) $\frac{d^2y}{dt^2}+2\frac{dy}{dt}+5y=e^{-t}\sin t$, $y(0)=0, y^{'}(0)=1$