Matrices and Differential Equations Practice Questions for Minor-2

1. a. State Newton's Law of Cooling.
   b. Define Orthogonal Trajectory.
   c. State Convolution theorem.
   d. State First Shifting theorem for Laplace Transforms.
2. A body cools from 60^0C to 50^0C in 10 minutes when kept in air at 30^0C in the next 10 minutes what is the temperature of the body.
3. The number of bacteria culture grows at the rate proportional to N, the value of N was initially 100 and it increases to 332 in one hr. What would be the value of N after $1\frac{1}{2}hr$.
4. If 30% of radioactive substance disappear in 10 days. How long will it take for 90% of it to disappear?
5. Show that the family of con-focal conics $\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}=1$ is self orthogonal.
6. Find the orthogonal trajectories of the family of curves $r^n = a^n \cos n\theta$
7. Suppose that an object is heated to 300F and allowed to cool is a room whose air temperature 20F, it after 10 min, the temperature of the object is 250F, what will be its temperature after 20 min?
8. Solve $\left(D^2-2D+1\right)y = xe^x \sin x$
9. Solve $\left(D^2-2D\right)y = e^x \sin x$ by the method of variation of parameters.
10. Solve $\frac{d^2y}{dx^2}-\frac{1}{x}\frac{dy}{dx}+\frac{y}{x^2}=\frac{2\log x}{x^2}$
11. Solve $\left(D^2-4D+4\right)y = 8x^2e^{2x}\sin 2x$
12. Solve $y''-6y'+9y=\frac{e^{3x}}{x^2}$ by the method of variation of parameters.
13. Solve $\left(D^2-2D+2\right)y = e^x \tan x$

14. Find the Laplace transform of the following functions

a.sin5t cos2t b. e^{2t} sin3t sin5t c. $\frac{1-\cos at}{t}$ d. t e^{t} sint

15. Evaluate

a. $\int_0^\infty te^{-2t}\sin 5t dt$ b. $\int_0^\infty \frac{e^{-t}\sin^2 t}{t}dt$ c. $\int_0^\infty te^{-2t}\sin 5t dt$ d. $\int_0^\infty e^{-3t}t\sin t dt$

16. Using convolution theorem, Find $L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}$

17. Using convolution theorem, Find $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$

18. Apply Laplace transform to find the solution of the D.E $y''+2y'-3y=\sin t,\ y(o)=y'(o)=0$

19. Apply Laplace transform to find the solution of the D.E. $y''+2y'+5y=e^{-t}\sin t,\ y(o)=y'(o)=0$

20. Apply Laplace transform to find the solution of the D.E. $y''+4y'+3y=e^{-t},\ y(0)=1=y'(0)$