$$= \frac{n+1}{2}\left[\frac{2(2n+1)-3(n+1)}{6}\right]$$

$$= \frac{n+1}{2}\left[\frac{4n+2-3n-3}{6}\right]$$

$$= \frac{n+1}{2}\cdot\left(\frac{n-1}{6}\right) = \frac{(n+1)(n-1)}{12} = \frac{n^2-1}{12}$$

Continuous Random Variable:

R.V X takes the values continuously then it is called C.R.V

probability Density Function:

For a C.R.V 'X' a fn $f(x)$ is said to be probability density fn if

(i) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

(ii) $f(x) \geq 0,\ x \in R$

Cummulative Distribution Function:

The C.D.F of C.R.V 'X' denoted by

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)\,dx$$

Mean of a C.R.V. 'X':

$$\mu = E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$$

If x is defined from a to b then

$$\mu = \int_{a}^{b} x f(x) dx$$

The mean of any fn $\phi(x)$ is $\mu = \int_{-\infty}^{\infty} \phi(x) f(x) dx$

Variance: $V(x) = \int_{-\infty}^{\infty} x^2 f(x) dx - \mu^2$

$$= E(x^2) - \mu^2$$

or

$$\sigma^2 = V(x) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x) dx$$

Mode: Mode is a point at which fn have maximum value.

To find Maximum value $f'(x) = 0$

$$f''(x) < 0$$

Medium: ~~If~~ It is the pt which divides the entire distribution into two equal parts

→ Medium is a point which divides the total into two equal parts.

→ Denoted by M

$$\int_a^M f(x)\,dx = \int_M^b f(x)\,dx = \frac{1}{2}$$

Note:

D.R.V	C.R.V
Σ	$\int$
$P(X=x_i)$	$f(x)\,dx$
x_i	x

① A C.R.V has the probability density fn

$f(x) = kxe^{-\lambda x}$, $x \geq 0$, $\lambda > 0$; $f(x) = 0$ otherwise

find ⓐ k ⓑ mean ⓒ Variance.

Soln: To find k

W.K.T $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\Rightarrow \int_{-\infty}^{0} f(x)\,dx + \int_0^{\infty} f(x)\,dx = 1$$

$$0 + \int_0^{\infty} kxe^{-\lambda x}\,dx = 1$$

$$\Rightarrow k \int_0^{\infty} xe^{-\lambda x}\,dx = 1$$

$\int uv\,dx = u\int v\,dx - \int u'\int v\,dx$
$= uv_1 - u'v_2 + u''v_3$

$$\Rightarrow k\left[x\frac{e^{-\lambda x}}{-\lambda} - \frac{e^{-\lambda x}}{\lambda^2}\right]_0^\infty = 1$$

$$\Rightarrow k\left[(0-0) - \left(0 - \frac{1}{\lambda^2}\right)\right] = 1$$

$$\frac{k}{\lambda^2} = 1 \Rightarrow \boxed{k = \lambda^2}$$

b) Mean: $E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$

$$= \int_{-\infty}^{0} x f(x)\,dx + \int_0^\infty x f(x)\,dx$$

$$= 0 + \int_0^\infty x \cdot k \cdot x e^{-\lambda x}\,dx$$

$$= k\int_0^\infty x^2 e^{-\lambda x}\,dx$$

$u = x^2 \quad v = e^{-\lambda x}$
$u_1' = 2x \quad v_1 = \frac{e^{-\lambda x}}{-\lambda}$
$u_2'' = 2 \quad v_2 = \frac{e^{-\lambda x}}{\lambda^2}$
$u''' = 0 \quad v_3 = \frac{e^{-\lambda x}}{-\lambda^3}$

$$= k\left[-\frac{x^2 e^{-\lambda x}}{\lambda} - \frac{2x e^{-\lambda x}}{\lambda^2} - \frac{2e^{-\lambda x}}{\lambda^3}\right]_0^\infty$$

$$= k\left[0 - \left(-0 - 0 - \frac{2}{\lambda^3}\right)\right]$$

$$= \frac{2k}{\lambda^3} = \frac{2}{\lambda^3}\cdot\lambda^2 = \frac{2}{\lambda}$$

$\therefore$ Mean $= \frac{2}{\lambda}$

(c) Variance $V(x) = E(x^2) - [E(x)]^2$

i.e $V(x) = E(x^2) - \mu^2$

Consider $E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$

$$= \int_{-\infty}^{0} x^2 f(x)\,dx + \int_{0}^{\infty} x^2 f(x)\,dx$$

$$= 0 + \int_{0}^{\infty} x^2 \cdot kx\, e^{-\lambda x}\,dx \qquad (\because k = \lambda^2)$$

$$= \lambda^2 \int_{0}^{\infty} x^3 e^{-\lambda x}\,dx$$

$$\begin{array}{ll} x^3 & e^{-\lambda x} \\ 3x^2 & \dfrac{e^{-\lambda x}}{-\lambda} \\ 6x & \dfrac{e^{-\lambda x}}{\lambda^2} \\ 6 & \dfrac{e^{-\lambda x}}{-\lambda^3} \\ & \dfrac{e^{-\lambda x}}{\lambda^4} \end{array}$$

$$= \lambda^2 \left[\frac{x^3 e^{-\lambda x}}{-\lambda} - \frac{3x^2 e^{-\lambda x}}{\lambda^2} + 6x\left(\frac{e^{-\lambda x}}{-\lambda^3}\right) - 6 \cdot \frac{e^{-\lambda x}}{\lambda^4} \right]_0^{\infty}$$

$$= \lambda^2 \left[0 - \left(\frac{-6}{\lambda^4} \right) \right] = \frac{6}{\lambda^2}.$$

Sub in (I)

$$V(x) = \frac{6}{\lambda^2} - \frac{4}{\lambda^2} = \frac{2}{\lambda^2}$$

S.D $\sigma = \sqrt{\text{Variance}} \Rightarrow \sigma = \sqrt{\frac{2}{\lambda^2}} = \frac{\sqrt{2}}{\lambda}$

(2) The total no. of hours measured in units of 100 hrs that a family runs a vaccum cleaner over a period of 1 year is continuous random variable X that has the following density fn

$$f(x) = \begin{cases} x ; & 0 < x < 1 \\ 2-x ; & 1 < x < 2 \\ 0 ; & \text{otherwise} \end{cases}$$

Find the probability that over a period of 1 year a family runs their vaccum cleaner

(i) < 120 hrs

(ii) b/w 50 to 100 hrs

iii) calculate mean, variance and S.D

soln:

i) $P(x < 120 \text{ hrs}) = P(x < 1.2)$ $\left(\because \text{measured } 100 \text{ hrs per unit} \quad \frac{120}{100} = 1.2 \right)$

$$= \int_{-\infty}^{1.2} f(x)\,dx$$

$$= \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{1.2} f(x)\,dx$$

$$= 0 + \int_{0}^{1} x\,dx + \int_{1}^{1.2} (2-x)\,dx$$

$$= \left(\frac{x^2}{2}\right)_0^1 + \left(2x - \frac{x^2}{2}\right)_1^{1.2}$$

$$= \frac{1}{2} + \left[\left(2(1.2) - \frac{(1.2)^2}{2}\right) - \left(2 - \frac{1}{2}\right)\right]$$

$$= \frac{1}{2} + \left[2.4 - \frac{1.44}{2}\right] - \left(2 - \frac{1}{2}\right)$$

$$= \frac{1}{2} + [2.4 - 0.72] - \frac{3}{2}$$

$$= \frac{1}{2} + 1.68 - 1.5 = 0.68$$

$||^{ly}$ $P(50 < x < 100) = P(0.5 < x < 1)$

$$= \int_{0.5}^{1} f(x)\,dx = \int_{0.5}^{1} x\,dx = \left(\frac{x^2}{2}\right)_{0.5}^{1}$$

$$= \frac{1}{2}\left[1 - (0.5)^2\right] = \frac{1}{2}\left[1 - 0.25\right] = \frac{0.75}{2} = 0.37$$

<u>Mean</u>: $E(x) = \int_{-\infty}^{\infty} x f(x)\,dx = \int_{-\infty}^{0} x \cdot f(x)\,dx + \int_{0}^{1} x f(x)\,dx + \int_{1}^{2} x f(x)$

$$+ \int_{2}^{\infty} x \cdot f(x)\,dx$$

$$= 0 + \int_{0}^{1} x f(x)\,dx + \int_{1}^{2} x f(x)\,dx + 0$$

$$= \int_{0}^{1} x \cdot x\,dx + \int_{1}^{2} x(2-x)\,dx$$

$$= \int_{0}^{1} x^2\,dx + \int_{1}^{2} (2x - x^2)\,dx = \left(\frac{x^3}{3}\right)_0^1 + \left(\frac{2x^2}{2} - \frac{x^3}{3}\right)_1^2$$

$$= \frac{1}{3} + \left(4 - \frac{8}{3}\right) - \left(1 - \frac{1}{3}\right)$$

$$= \frac{1}{3} + \frac{4}{3} - \frac{2}{3} = \frac{5-2}{3} = \frac{3}{3} = 1$$

Variance: $V(x) = \int_{-\infty}^{\infty} x^2 f(x)\,dx - (\text{mean})^2$

$= \int_0^1 x^2 f(x)\,dx + \int_1^2 x^2 f(x)\,dx - 1$

$= \int_0^1 x^3\,dx + \int_1^2 x^2(2-x)\,dx - 1$

$= \left(\frac{x^4}{4}\right)_0^1 + 2\left[\frac{x^3}{3}\right]_1^2 - \left[\frac{x^4}{4}\right]_1^2 - 1$

$= \frac{1}{4} + 2\left(\frac{8}{3} - \frac{1}{3}\right) - \left(\frac{16}{4} - \frac{1}{4}\right) - 1$

$= \frac{1}{4} + 2 \times \frac{7}{3} - \left(\frac{15}{4}\right) - 1$

$= \frac{1}{4} + \frac{14}{3} - \frac{15}{4} - 1$

$= \frac{3 + 56 - 45 - 12}{12} = \frac{2}{12} = \frac{1}{6}$

S.D: $\sigma = \sqrt{V(x)} = \sqrt{\frac{1}{6}} = 0.408$

$\int uv\,dx = uv_1 - u'v_2 + u''v_3 - u'''v_4 + \cdots$

③ Measurement of Scientific Systems are always subject to variation some more than others. They are many structures for measuring errors and statiscian spent a great deal of time modelling these errors. The measurement error X (R.v) of certain quantity is decided by the density fn

$$f(x) = \begin{cases} k(3-x^2) & ; -1 \le x \le 1 \\ 0 & ; \text{otherwise} \end{cases}$$

determine (i) k

(ii) Find proby that a random error in measurement is < 0.5 for this particular measurement

It is undesirable if the magnitude of the error |x| exceeds 0.8. What is the proba that this occurs.

(i) To find k: $\int_{-\infty}^{\infty} f(x)dx = 1$

$$\Rightarrow \int_{-\infty}^{-1} f(x)\,dx + \int_{-1}^{1} f(x)\,dx + \int_{1}^{\infty} f(x)\,dx = 1$$

$$\Rightarrow 0 + \int_{-1}^{1} k(3-x^2)\,dx + 0 = 1$$

$$\Rightarrow k\int_{-1}^{1}(3-x^2)\,dx = 1$$

$\Rightarrow k\left[3x - \frac{x^3}{3}\right]_{-1}^{1} = 1$

$\Rightarrow k\left[\left(3 - \frac{1}{3}\right) - \left(-3 + \frac{1}{3}\right)\right] = 1$

$\Rightarrow k\left(\frac{8}{3} + \frac{8}{3}\right) = 1$

$\Rightarrow \frac{16k}{3} = 1 \Rightarrow \boxed{k = \frac{3}{16}}$

(ii) $p(x < 0.5) = \int_{-\infty}^{0.5} f(x)dx$

$= \int_{-\infty}^{-1} f(x)dx + \int_{-1}^{0.5} f(x)dx + \int_{0.5}^{\infty} f(x)dx$

$= 0 + \int_{-1}^{0.5} k(3 - x^2)dx + 0$

$= k\left[3x - \frac{x^3}{3}\right]_{-1}^{0.5} = 0.77$

(iii) $p(|x| > 0.8) \Rightarrow x > 0.8\ ,\ x < -0.8$

$-0.8 < x < 0.8$

$= \int_{-\infty}^{\infty} f(x)dx + \int_{-1}^{-0.8} f(x)dx + \int_{-0.8}^{1} f(x)dx$

$= \int_{-1}^{-0.8} k(3 - x^2)dx + \int_{-0.8}^{1} k(3 - x^2)dx$

$= k\left[3x - \frac{x^3}{3}\right]_{-1}^{-0.8} + k\left[3x - \frac{x^3}{3}\right]_{-0.8}^{1}$

$= 1.03$

17-W

① The amount of bread (in hundreds of pounds) x that a certain bakery is able to sell in a day is found to be numerical valued random phenomena, with probability fn as specified by f(x) and is given by

$$f(x) = \begin{cases} kx & ; 0 \le x \le 5 \\ k(10-x) & ; 5 \le x \le 10 \\ 0 & ; \text{otherwise} \end{cases}$$

(i) find the value of k ∋ f(x) is a p.d.f

What is the probability that the no. of pounds of bread that will be sold tomorrow is

① more than 500 pounds

② less than 500 pounds

③ b/w 250 and 750 pounds

② The following is the distribution fn of the R.V x

value of x	−3	−1	0	1	2	3	5	8
Cumulative F(x)	0.10	0.3	0.45	0.65	0.75	0.9	0.95	1.00

(a) Find the probability distribution of x

(b) Find P(x is even) and P(x is lies b/w 1, 8 incl

$\Downarrow$ $P(2)+P(8)$

$\Downarrow$ $P(1 \le x \le 8)$

③ Let x be a R.V with the following prob. distribution

x	-2	3	5
$f(x)$	0.2	0.2	0.5

Verify whether it can be treated as P.D of the R.V "X". Also determine its

(i) expected value (ii) Variance (iii) S.D