# Malla Reddy University

## School of Engineering

## B.Tech II year II semester Department of CS&IOT

## Subject: PROBABILITY AND STATISTICS

## Subject Code: MR22–1BS0108

## Question Bank

| S.no | Unit-1<br>(Probability Distributions) | Max Marks |
|---|---|---|
| 1 | a) Define Probability and write axioms of probability<br>b) A class consists of 6 girls and 10 boys. If a committee of 3 is chosen at random from the class, find the probability that (a) 3 boys are selected (ii) exactly 2 girls are selected. | 8 |
| 2 | A sample space consists of 7 events namely: $E_i$,i=1,2,3,4,5,6,7 with the assignment of probabilities 0.05, 0.20, 0.20, 0.25, 0.15, 0.10 and 0.05 respectively. Let $A=\{E_1,E_4,E_6\}$, $B=\{E_2,E_4,E_7\}$ and $C=\{E_2,E_3,E_5,E_7\}$. Then find the following : $P(A)$, $P(B)$, $P(C)$, $P(A \cap B)$ and $P(A \cup B)$ | 8 |
| 3 | a) Define Conditional probability of A given B and B given A<br>b) Fifteen numbered cards are there from 1 to 15, and two cards a chosen at random such that the sum of the numbers on both the cards is even. Find the probability that the chosen cards are odd-numbered. | 8 |
| 4 | In a certain assembly plant, three machines $B_1$, $B_2$ and $B_3$ make 30%, 45% and 25% respectively of the products. It is known from the past experience that 2%, 3% and 2% of the products made by each machine, respectively are defective. What is the probability that the product is defective? If the product was chosen randomly and found to be defective, what is the probability that it was made by machine: (i)$B_1$ (ii)$B_2$ (iii)$B_3$ | 8 |
| 5 | A random variable X has the following probability function<br><table><tr><td>Values of X, x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr><tr><td>P(X=x)</td><td>0</td><td>k</td><td>2k</td><td>2k</td><td>3k</td><td>$k^2$</td><td>$2k^2$</td><td>$7k^2+k$</td></tr></table>i) Find the value of k<br>ii) Evaluate: P(X<6), P(X>6) and P(O<X<5)<br>iii) Calculate the mean, variance and standard deviation of X | 8 |
| 6 | The amount of bread (in hundreds of pounds) x that a certain bakery is able to sell in a day is found to be a numerical valued random phenomena, with a probability function as specified by f(x) and is given by: | 8 |

<table>
<tr><td></td><td>$$f(x)=\begin{cases} kx, & 0 \le x < 5, \\ k(10-x), & 5 \le x < 10\,, \\ 0, \text{ else where} & \end{cases}$$<br>Find the value of k such that f(x) is a probability density function<br>What is the probability that the number of pounds of bread that will be sold tomorrow is<br>(i) more than 500 pounds<br>(ii) less than 500 pounds<br>(iii) between 250 and 750 pounds?</td><td></td></tr>
<tr><td>7</td><td>Out of 800 families with 5 children each, how many would you expect to have (i) 3 boys (ii) 5 girls (iii) either 2 or 3 boys (iv) at least one boy ? Assume equal probability for boys and girls.</td><td>8</td></tr>
<tr><td>8</td><td>Fit a binomial distribution for the following data<br><table><tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>F</td><td>13</td><td>25</td><td>52</td><td>58</td><td>32</td><td>16</td><td>4</td></tr></table></td><td>8</td></tr>
<tr><td>9</td><td>If a poisson distribution is such that P(x=1). $\frac{3}{2}$ = P(x = 3), find the probabilities that (i) P(x≥ 1) (iii) P( x≤3) (iii) P( 2 ≤ x ≤ 5)</td><td>8</td></tr>
<tr><td>10</td><td>The number of breakdowns of a computer is a random variable having Poisson distribution with a mean of 1.8 per month. Find the probability that the computer will function for a month<br>a) without any breakdowns<br>b) with only one breakdown<br>c) with at least 2 breakdowns.</td><td>8</td></tr>
<tr><td></td><td>Unit-2<br>(Continuous Probability distribution and fundamentals of Sampling)</td><td>8</td></tr>
<tr><td>1</td><td>Given a standard normal distribution, find the area under the curve that lies:<br>a) to the left of z = -1.39;<br>b) to the right of z = -0.89<br>c) between z = -0.48 and z = 1.74</td><td>8</td></tr>
<tr><td>2</td><td>If a random variable has the standard normal distribution, find the probability that it will take on a value:<br>a) less than 1.65;<br>b) greater than -1.95;<br>c) lies in between -1.75 and -1.04</td><td>8</td></tr>
<tr><td>3</td><td>Given a standard normal distribution, find the value of k such that<br>a) P (Z > k) = 0.2946;<br>b) P (Z < k) = 0.0427;<br>c) P (k < Z < −0.18) = 0.4197.<br>d) P (−0.93 < Z < k) = 0.7235.</td><td>8</td></tr>
<tr><td>4</td><td>In a normal distribution 7 % of the items are under 35 and 89% of the items are under 63. Determine the mean and variance of the distribution.</td><td>8</td></tr>
<tr><td>5</td><td>A population consists of observations 2,3,6,8 and 11. Consider all samples of size 2 which can be drawn with replacement from this population find<br>a) The population mean<br>b) The population standard deviation<br>c) The mean of the sampling distribution of means<br>d) The standard deviation of the sampling distribution of means</td><td>8</td></tr>
</table>

| | | |
|---|---|---|
| 6 | A population consists of 6 numbers 4,8,12,16,20,24. Consider all samples of size 2 which can be drawn without replacement from this population find.<br>a) The population mean<br>b) The population standard deviation.<br>c) The mean of sampling distribution means.<br>d) The standard deviation of the sampling distribution of the means | 8 |
| 7 | A population consists of 4 observations 10, 20, 30, 40. Determine the mean and variance of the population. Write all the possible samples of size 2 (with replacement and without replacement). Construct the sampling distribution about mean. Show that the mean of sample means is equal to the population mean | 8 |
| 8 | . (i) A random sample of size 100 has a standard deviation of 5. What can you say maximum error with 95% confidence.<br>(ii) A random sample of size 100 is taken from a population with s.d 5.1. Given that the sample mean is 21.6. Construct 95% confidence limits | 8 |
| 9 | Find 95 % confidence limits for the mean of a normality distributed population from the following samples was taken 15, 17, 10, 18, 16, 9, 7, 11, 13, 14. | 8 |
| 10 | A random sample of 20 fuses subjected to overload has mean time for blow of 10.63min. with S.D. of 2.48 min. What can we assert with 95% confidence about the maximum error if we use $\bar{x}$ = 10.63 min. as a point estimate of true average it takes such fuses for blow when subjected to overload. | 8 |
| | Unit-III<br>(Testing of Hypothesis) | 8 |
| 1 | a) Define Null hypothesis and Alternative hypothesis<br>b) An electrical firm manufactures light bulbs that have a lifetime that is approximately normally distributed with a mean of 800 hours and a standard deviation of 40 hours. Test the hypothesis that $\mu = 800$ hours against the alternative $\mu \neq 800$ hours, if a random sample of 30 bulbs has an average life of 788 hours. Use a 0.05 level of significance. | 8 |
| 2 | An investigation of two kinds of photocopying equipment showed that 71 failures of the first kind of equipment took on the average 83.2 minutes to repair with a standard deviation of 19.3 minutes, while 75 failures of the second kind of equipment took on the average 90.8 minutes to repair with a standard deviation of 21.4 minutes. Test the null hypothesis $\mu_1 - \mu_2 = 0$ (the hypothesis that on the average it takes an equal amount of time to repair either kind of equipment) against the alternative hypothesis $\mu_1 - \mu_2 \neq 0$ at the 0.05 level of significance. | 8 |
| 3 | A manufacturer claims that the average tensile strength of Thread-A exceeds the average tensile strength of Thread-B by at least 12 kilograms. To test this claim, 50 pieces of each type of thread were tested under similar conditions. Type A thread had an average strength of 86.7 kilograms with a standard deviation of 6.28 kilograms, while Type B thread had an average tensile strength of 77.8 kilograms with a standard deviation of 5.61kilograms. Test the manufacturer's claim using a 0.05 level of significance. | 8 |
| 4 | Ten individuals are chosen at random from a normal population and their heights are found to be: 63, 63, 66, 67, 68, 69, 70, 70, 71, 71 in inches. Test if the sample belongs to the population | 8 |

| | | |
|---|---|---|
| | whose mean height is 66 inches? | |
| 5 | A manufacturer of gunpowder had developed a new powder which is designed to produce a muzzle velocity equal to 3000ft/sec. Seven shells are loaded with the charge and the muzzle velocities are measured. The resulting velocities are: 3005, 2935, 2965, 2995, 3905, 2935 and 2905. Do these data present sufficient evidence to indicate that the average velocity differs from 3000ft/sec? | 8 |
| 6 | The gain in weight of two random samples of patients fed on two different Diets: A and B are given below. Examine whether the difference in mean increase in weight is significant? | 8 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Diet-A | 13 | 14 | 10 | 11 | 2 | 16 | 10 | 8 | |
| Diet-B | 7 | 10 | 12 | 8 | 10 | 11 | 9 | 10 | 11 |

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| 7 | Two independent samples of 8 and 7 items respectively had the following values of the variable: | 8 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Sample – I | 9 | 11 | 13 | 11 | 15 | 9 | 12 | 14 |
| Sample – II | 10 | 12 | 10 | 14 | 9 | 8 | 10 | |

Do the estimates of population variances differ significantly?

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|---|---|---|
| 8 | The following table shows the number of aircraft accidents that occurred during the six days of a week. Find whether the accidents are uniformly distributed over the week? | 8 |

| Day | Mon | Tue | Wed | Thu | Fri | Sat | Sun |
|---|---|---|---|---|---|---|---|
| No of accidents | 14 | 18 | 12 | 11 | 15 | 14 | 84 |

| | | |
|---|---|---|
| 9 | 200 digits were chosen at random from a set of tables. The frequency of the digits were: | 8 |

| Digits | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| Frequency | 18 | 19 | 23 | 21 | 16 | 25 | 22 | 20 | 21 | 15 |

Use Chi-Square test, to assess the correctness of hypothesis that the digits were distributed in equal number in the table at the level of significance 0.05

| | | |
|---|---|---|
| 10 | To determine whether there really is a relationship between an employee's performances in the company's training programme and his or her ultimate success in the job, the company takes a sample of 400 cases from its very extensive files and obtained the results shown in the following table: | 8 |

| | | Performance in Training Programme | | |
|---|---|---|---|---|
| | | Below Average | Average | Above Average |
| Success in Job (Employers Rating) | Poor | 23 | 60 | 29 |
| | Average | 28 | 79 | 60 |
| | Very Good | 9 | 49 | 63 |

| | | |
|---|---|---|
| | Unit-IV<br>(Correlation, Regression & Curve Fitting) | 8 |
| 1 | Calculate the correlation coefficient between X and Y from the following data: | 8 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| X | 65 | 66 | 67 | 67 | 69 | 68 | 70 | 72 |
| Y | 67 | 68 | 65 | 68 | 72 | 72 | 69 | 71 |

| | | |
|---|---|---|
| 2 | The marks obtained by 10 students in Mathematics (X) and in Statistics (Y) | 8 |

are given below. Compute the correlation coefficient between X and Y.

| Roll No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| X | 75 | 30 | 60 | 80 | 53 | 35 | 15 | 40 | 38 | 48 |
| Y | 85 | 45 | 54 | 91 | 58 | 63 | 35 | 43 | 45 | 44 |

**3** The marks secured by recruits in Selection Test (X) and in the Proficiency Test (Y) are given below. Use rank correlation method to determine the relationship between X and Y. **8**

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| X | 10 | 15 | 12 | 17 | 13 | 16 | 24 | 14 | 22 | 20 |
| Y | 30 | 42 | 45 | 46 | 33 | 34 | 40 | 35 | 39 | 38 |

**4** Given the following Aptitude and I.Q. Scores for a group of students. Compute the rank correlation coefficient between them. **8**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Aptitude Score | 57 | 58 | 59 | 59 | 60 | 61 | 60 | 64 |
| I.Q Score | 97 | 108 | 95 | 106 | 120 | 126 | 113 | 110 |

**5** A Chemical company wishing to study the effect of extraction time(X) on the efficient of extraction operation(Y), obtained the following data: **8**

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| X | 27 | 45 | 41 | 19 | 35 | 39 | 19 | 49 | 15 | 31 |
| Y | 57 | 64 | 80 | 46 | 62 | 72 | 52 | 77 | 57 | 68 |

Obtain the two regression lines. Also determine the extraction efficiency one can expect when the extraction time is 35 minutes.

**6** The following data gives the experience of the machine operators and their performance ratings as given by the number of good parts turned out per 100 pieces. **8**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Experience(X) | 16 | 12 | 18 | 4 | 3 | 10 | 5 | 12 |
| Performance Ratings (Y) | 88 | 87 | 89 | 68 | 78 | 80 | 75 | 83 |

Obtain the regression line of performance ratings on experience and estimate the probable performance if the operator has 7 years of experience.

**7** For the following bivariate data obtain the two lines of regression. Determine the value of Y when X=3.5 **8**

| | | | | | | |
|---|---|---|---|---|---|---|
| X | 1 | 2 | 3 | 4 | 5 | 6 |
| Y | 14 | 33 | 40 | 63 | 76 | 85 |

**8** Fit a Straight line of the form: Y = a + bX for the following data
For 8 randomly selected observations, the following data were recorded: **8**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| X | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Y | 1.0 | 1.2 | 1.8 | 2.5 | 3.6 | 4.7 | 6.6 | 9.1 |

**9** Fit a Second-degree parabola of the form: $Y = a + bX + cX^2$ for the following data For 10 randomly selected observations, the following data were recorded: **8**

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Over time (X) | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 5 | 6 | 7 |
| Additional Units (Y) | 2 | 7 | 7 | 10 | 8 | 12 | 10 | 14 | 11 | 14 |

**10** Fitting of an exponential curve for the following data **8**

| | | | | | |
|---|---|---|---|---|---|
| x | 1 | 5 | 7 | 8 | 12 |
| y | 10 | 15 | 12 | 15 | 21 |

## UNIT-V
## (Queuing Theory)
**8**

**1** A television repairman finds that the time spent on his jobs has an exponential distribution with mean of 30 minutes. If he repairs sets in the order in which they came in, and if the arrival of sets follows a Poisson distribution approximately with an average rate of 10 per 8-hour day, what is the **8**

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| | repairman's expected idle time each day? How many jobs are ahead of the average set just brought in? | |
| 2 | Students arrive at the head office according to a Poisson input process with a mean rate of 40 per hour. The time required to serve a student has an exponential distribution with a mean of 50 per hour. Assume that the students are served by a single individual, find the average waiting time of a student. | 8 |
| 3 | New Delhi Railway Station has a single ticket counter. During the rush hours, customers arrive at the rate of 10 per hour. The average number of customers that can be served is 12 per hour. Find out the following: (i) Probability that the ticket counter is free. (ii) Average number of customers in the queue. | 8 |
| 4 | There is congestion on the platform of a railway station. The trains arrive at a rate of 30/days. The service time for any train is ED with an average of 36mins. Calculate: (a) Mean queue size (b) Probability that there are more than 10 trains in the system. | 8 |
| 5 | At a one-man barber shop customers arrive according to P.D with a mean arrival rate of 5/hr. The hair cutting time is ED with a haircut taking 10 min on an average assuming that the customers are always willing to wait find:<br>a) Average number of customers in the shop<br>b) Average waiting time of a customer<br>c) The percent of time an arrival Can walk right without having to wait<br>d) The probability of a customer waiting more than 5mins. | 8 |
| 6 | Consider a single server queuing system with Poisson input, exponential service times. Suppose the mean arrival rate is 3 calling units per hour, the expected service time is 0.25 hour and the maximum permissible calling units in the system is two. Derive the steady-state probability distribution of the number of calling units in the system, and then calculate the expected number in the system. | 8 |
| 7 | At a railway station, only one train is handled at a time. The railway yard is sufficient only for two trains to wait while the other is given signal to leave the station. Trains arrive at the station at an average rate of 6 per hour and the railway station can handle them on an average of 12 per hour. Assuming Poisson arrivals and exponential service distribution, find the steady-state probabilities for the various number of trains in the system. Also find the average waiting time of a new train coming into the yard. | 8 |
| 8 | If for a period of 2 hours in the day (8 to 10 a.m.) trains arrive at the yard every 20 minutes but the service time continues to remain 36 minutes, then calculate for this period<br>a) the probability that the yard is empty, and<br>b) the average number of trains in the system, on the assumption that the line capacity of the yard is limited to 4 trains only. | 8 |
| 9 | A one – person barbershop has six chairs to accommodate people waiting for haircut. Assume customers who arrive when all six chairs are full, leave without entering the barbershop. Customers arrive at the average rate of 3 per hour and spend on average of 15 minutes in the barbershop. Then find the<br>a) the probability a customer can get directly into the barber chair upon arrival.<br>b) Expected number of customers waiting for haircut.<br>c) Effective arrival rate. | 8 |

| | | |
|---|---|---|
| | d) The time a customer can expect to spend in the barbershop. | |
| 10 | Trains arrive at the yard every 15 minutes and the service rate is 33 minutes.<br>If the line capacity at the yard is limited to 4 trains, find<br>a) The probability that the yard is empty.<br>b) The average number of trains in the system. | 8 |