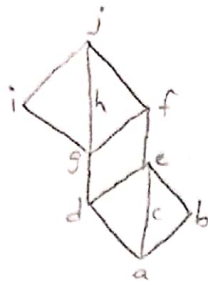5) b) Prove that the given Hasse diagram Lattice and also verify, it is distributive Lattice or not?

Sol:

V	a	b	c	d	e	f	g	h	i	j
a	a	b	c	d	e	f	g	h	i	j
b	b	b	e	e	de/fj	fj	f	j	j	j
c	c	e	c	e	e	j	f	j	j	j
d	d	e	e	d	e	j	g	j	j	j
e	e	e	e	e	e	f	f	j	j	j
f	f	j	j	j	f	f	f	j	j	j
g	g	f	f	g	f	f	g	h	j	j
h	h	j	j	j	j	j	h	h	j	j
i	i	j	j	j	j	j	j	j	i	j
j	j	j	j	j	j	j	j	j	j	j

∧	a	b	c	d	e	f	g	h	i	j
a	a	a	a	a	a	a	a	a	a	a
b	a	b	a	a	b	a	a	a	a	b
c	a	a	c	a	c	c	a	a	a	c
d	a	a	a	d	d	a	d	d	d	d
e	a	b	c	d	e	e	d	d	d	e
f	a	a	c	a	e	f	g	g	g	f
g	a	a	a	d	d	g	g	g	g	g
h	a	a	a	d	d	g	g	h	g	h
i	a	a	a	d	d	g	g	g	i	i
j	a	b	c	d	e	f	g	h	i	j

∴ The given Hasse diagram is a lattice

To proove, it is a distributive lattice

$e \lor (g \land i) = (e \lor g) \land (e \lor i)$

$e \lor g \qquad = f \land j$

$\qquad f \quad = \quad f$

It is a distributive lattice

$e \lor (f \land g) = (e \lor f) \land (e \lor g)$

$e \lor g \qquad = f \land f$

$\qquad f \quad = \quad f$

It is a distributive lattice

$d \lor (g \land h) = (d \lor g) \land (d \lor h)$

$d \lor g \qquad = g \land j$

$\qquad g \quad = \quad g$

It is a distributive lattice

∴ The given Hasse diagram is a distributive lattice