MALLA REDDY UNIVERSITY

MR22-1CS0104

ADVANCED DATA STRUCTURES

II YEAR B.TECH. (CSE) / II – SEM

Unit-1

Introduction - List ADT

Array List: Dynamic Arrays – Implementation and Amortized Analysis.

Positional List: Positions — Positional List ADT Singly Linked List Implementation.

Iterators: The Iterable interface and java's For-Each Loop – Implementing Iterators.

Applications of Stack – Infix to Prefix and infix to Postfix Conversions.

Array

- Array is a collection of elements of same data type referred to by a common name.
- Arrays may be stored in contiguous memory.
- Locations within an array are an integer index.
- the first element of a sequence has index 0, and the last has index n-1.
- 'n' denotes the total number of elements.
- Arrays are objects in Java, we can find their length using the object property "length".

Array

Declare an array:

```
int[] num;
int num[];
```

Initialize an array:

```
int[] num = \{10, 20, 30, 40, 50\};
```

Array Length:

System.out.println(num.length);

Access the Elements of an Array:

```
num[1];
```

```
for (int i = 0; i < num.length; i++)
System.out.println("Element : "+ arr[i]);
```

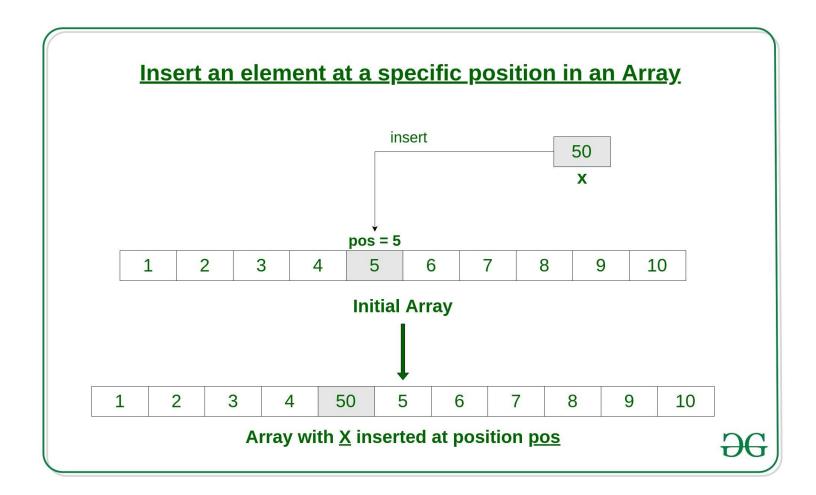
Array

Example:

```
import java.io.*;
class Arr {
      public static void main (String[] args) {
      int [] array1=new int [4];
       array1[0]=10;
       array1[1]=20;
       array1[2]=30;
       array1[3]=40;
       for (int i = 0; i < arr.length; i++)
       System.out.println("Element: "+array1[i]);
```

Array - Insert

Insert at any position:



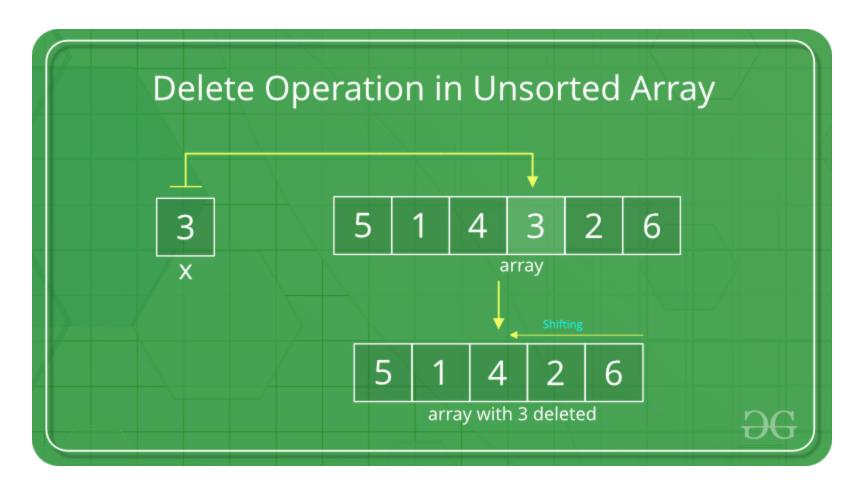
Array - Insert

Example:

```
import java.io.*;
class ArrayInsert {
           static void insertElement(int arr[], int n, int x, int pos)
                      for (int i = n - 1; i >= pos; i--)
                      arr[i + 1] = arr[i];
                      arr[pos] = x;
           public static void main(String[] args)
                      int arr[] = new int[15];
                      arr[0] = 2;
                      arr[1] = 4;
                      arr[2] = 1:
                      arr[3] = 8:
                      arr[4] = 5;
                      int n = 5;
                      int x = 10, pos = 2;
                      insertElement(arr, n, x, pos);
                      n += 1;
                      System.out.print("\n\nAfter Insertion: ");
                      for (int i = 0; i < n; i++)
                                 System.out.print(arr[i] + " "); }
```

Array - Delete

Delete at any position:



Array - Delete

```
Example:
class Main {
            static int findElement(int arr[], int n, int key)
                         int i;
                         for (i = 0; i < n; i++)
                         if (arr[i] == key)
                         return i;
                         return -1;
            static int deleteElement(int arr[], int n, int key)
                         int pos = findElement(arr, n, key);
                         if (pos == -1) {
                                      System.out.println("Element not found");
                                      return n;
                         for (int i = pos; i < n - 1; i++)
                                      arr[i] = arr[i + 1];
                         return n - 1;
            public static void main(String args[])
                         int i;
                         int arr[] = \{10, 50, 30, 40, 20\};
                         int n = arr.length;
                         int key = 30;
                         n = deleteElement(arr, n, key);
                         System.out.println("\n\nArray after deletion");
                         for (i = 0; i < n; i++)
                                      System.out.print(arr[i] + " ");
```

List ADT

- Java defines a general interface, java.util.List, that includes the following index-based methods,
 - size()
 - isEmpty()
 - get(i)
 - set(i, e)
 - add(i, e)
 - remove(i)

List ADT

Example:

Method	Return Value	List Contents
add(0, A)	-	(A)
add(0, B)	_	(B, A)
get(1)	Α	(B, A)
set(2, C)	"error"	(B, A)
add(2, C)	_	(B, A, C)
add(4, D)	"error"	(B, A, C)
remove(1)	Α	(B, C)
add(1, D)	_	(B, D, C)
add(1, E)	_	(B, E, D, C)
get(4)	"error"	(B, E, D, C)
add(4, F)	_	(B, E, D, C, F)
set(2, G)	D	(B, E, G, C, F)
get(2)	G	(B, E, G, C, F)

List ADT

A simplified version of the java.util.List interface

```
public interface List<E>
int size();
boolean isEmpty();
E get(int i) throws IndexOutOfBoundsException;
E set(int i, E e) throws IndexOutOfBoundsException;
void add(int i, E e) throws IndexOutOfBoundsException;
E remove(int i) throws IndexOutOfBoundsException;
```

List ADT - Example

```
import java.util.*;
class ListAdtExample {
public static void main(String args[])
    List<String> adt = new ArrayList<>();
     adt.add("Data");
     adt.add("Structures");
     adt.add(1, "Using");
     adt.set(1,"Java");
     System.out.println("Size of the List:" +adt.size());
     System.out.println("Elements of List:" +adt);
     adt.remove(1);
     System.out.println("Elements after removing:" +adt);
    System.out.println("Element in the List at position 1: "+adt.get(1));
```

List ADT - Example

Output:

Size of the List:3

Elements of List: [Data, Java, Structures]

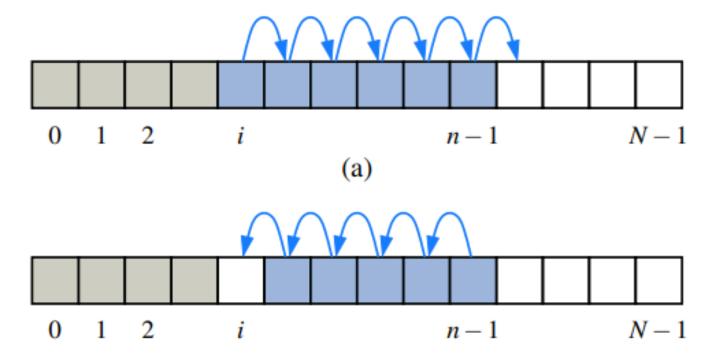
Elements after removing: [Data, Structures]

Element in the List at position 1 : Structures

ArrayList

With a representation based on an array A,

- get(i) and set(i, e) methods are easy to implement
- add(i, e) and remove(i) are more time consuming



ArrayList

The Performance of a Simple Array-Based Implementation:

Method	Running Time
size()	O(1)
isEmpty()	O(1)
get(i)	O(1)
set(i, e)	O(1)
add(i, e)	O(n)
remove(i)	O(n)

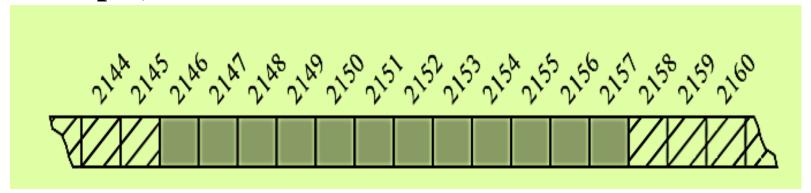
Dynamic Arrays

- An user is unsure of the size of a collection
 - causing an inefficient waste of memory, or
 - causing a fatal error when exhausting that capacity.
- Java's ArrayList class provides a more robust abstraction,
- allows a user to add elements to the list with no limit on the capacity.
- **Dynamic array** to provide this abstraction.

Dynamic Arrays

- In reality, elements of an ArrayList are stored in a traditional array,
- The precise size of that traditional array must be internally declared.

For example,



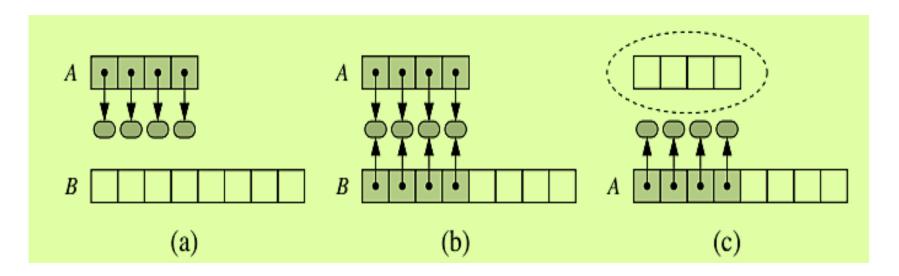
• an array with 12 cells that might be stored in memory locations 2146 through 2157 on a computer system.

Dynamic Arrays

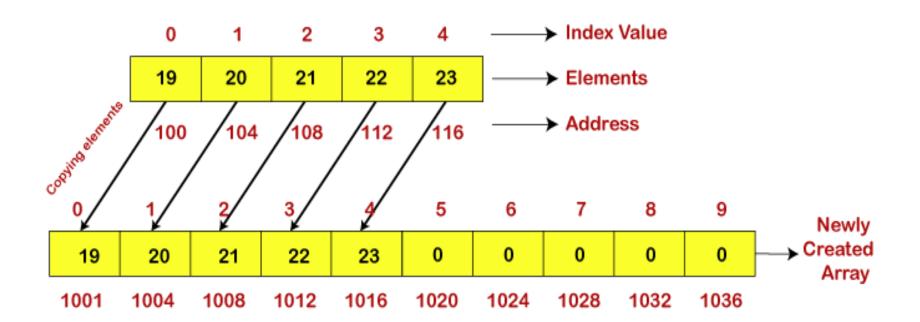
- array list instance maintains an **internal array** that often has greater capacity than the current length of the list.
- if all reserved capacity is exhausted,
 - the class requests a new, larger array from the system
 - copies all references from the smaller array into the new array
- the old array can be reclaimed by the system.

Implementing a Dynamic Array

- 1. Allocate a new array B with larger capacity.
- 2. Set B[k] = A[k], for k = 0,...,n-1, where n denotes current number of items.
- 3. Set A = B, that is, we henceforth use the new array to support the list.
- 4. Insert the new element in the new array.

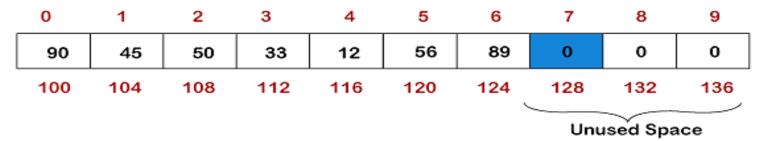


Add Element in a Dynamic Array

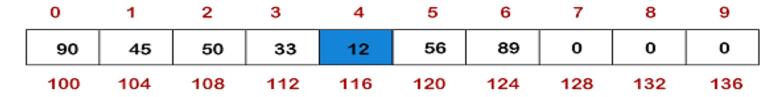


Delete an element in a Dynamic Array

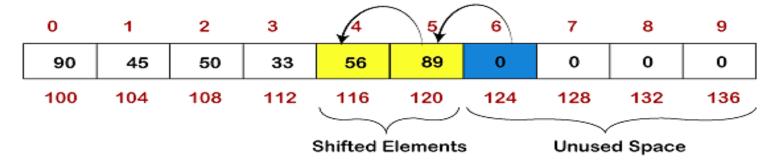
Using remove() method to delete an element



Using removeAt(4) method to delete an element



After deleting the element stored at 4th index

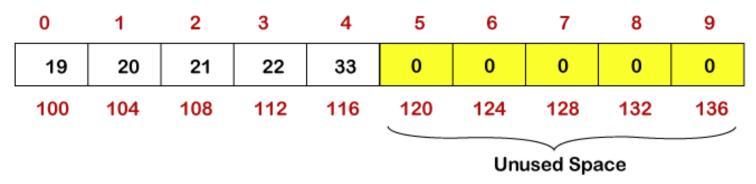


We need to resize an array in two scenarios if,

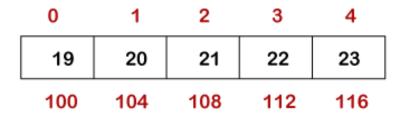
- The array uses extra memory than required.
- The array occupies all the memory and we need to add elements.
 - srinkSize()
 - growSize()

srinkSize()



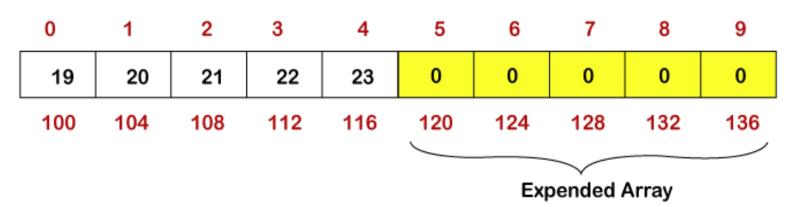


After resizing the array



• growSize()

Using growSize() method to resize the array



After inserting the elements

0	1	2	3	4	5	6	7	8	9
19	20	21	22	23	24	25	26	27	28
100	104	108	112	116	120	124	128	132	136

Dynamic Array

Creating a Dynamic Array:

ArrayList<data-type> arr=new ArrayList<>();

Add Element in a Dynamic Array:

arr.add(elem);

Example:

```
import java.util.ArrayList;
public class Main {
  public static void main(String[] args) {
    ArrayList<Integer> arr = new ArrayList<>();
    arr.add(89);
    arr.add(15);
    System.out.println("The dynamic array is: " + arr);
    System.out.println("Size " + arr.size());  } }
```

Internal implementation using array:

```
public void resize() {
   if (arr.length == capacity) {
          int[] narr = new int[2 * capacity];
   for (int i = 0; i < \text{capacity}; i++) {
           narr[i] = arr[i];
   arr = narr;
```

Dynamic Array

```
import java.util.*;
public class ArrayListExample {
 public static void main(String[] args) {
  ArrayList<Integer> arr = new ArrayList<>();
  System.out.println("Initial Size: " + arr.size());
  arr.add(10);
  arr.add(15);
  arr.add(20);
  arr.add(25);
  arr.add(30);
  System.out.println("Size after addition: " + arr.size());
  System.out.println("The dynamic array is: " + arr);
  System.out.println("First element: " + arr.get(0));
  arr.remove(1);
  System.out.println("The dynamic array is: " + arr);
  System.out.println("Size after deletion: " + arr.size()); } }
```

Dynamic Array

Output:

Initial Size: 0

Size after addition: 5

The dynamic array is: [10, 15, 20, 25, 30]

First element: 10

The dynamic array is: [10, 20, 25, 30]

Size after deletion: 4

- detailed analysis of the running time of operations on dynamic arrays.
- refer to the insertion of an element in an array list.

There are two possible behaviors,

- There is still room in the array and we can simply add the value into the array.
- There is no more room in the array so we need to make a new, larger array, copy over all the elements into the new array, and then finally add the new value.

- a single insert operation may require O(n) time to perform.
- amortized order of appending to a dynamic array will depend on how often we have to resize.

When we resize, we have two options for large to make the new internal array:

- We could increase the size of the internal array by a constant k
- We could increase the size of the internal array by a constant factor k

Increase by a Constant k:

- Start with an array of size k
- Try to add the k+1th element to the list.
- Need to resize.
- Creating a new array of size 2k and copy the k elements over into the new array.
- Add the 2k+1th element, we'll resize again and copy over 2k elements.
- Add up how many copies we'll need to make in the process of adding N+1 elements to the array.

Let's assume that worst case:

Number of copies
$$=k+2k+3k+\ldots+N=k\left(1+2+3+\ldots+rac{N}{k}
ight)$$

• By factoring out the k,

$$k\left(1+2+3+\ldots+rac{N}{k}
ight)=k\left(rac{(rac{N}{k})(rac{N}{k}+1)}{2}
ight)pprox O(N^2)$$

- the process of copying over all of the elements while resizing takes O(N2) time.
- Dividing this by the N+1 insertions, the amortized cost for each call to add() would be: $\frac{O(N^2)}{N+1} \approx O(N)$

Increase by a Constant Factor k:

Generalized to Any Factor (k>1)

$$k^0 + k^1 + k^2 + \ldots + N = \sum_{i=0}^{\log_k N} k^i$$

• This is a geometric sum and the formula for a partial geometric sum is:

$$\sum_{i=0}^{\log_k N} k^i = rac{k^{\log_k N+1}-1}{k-1} = rac{k^{\log_k N}k+1}{k-1} = rac{Nk+1}{k-1} pprox O(N)$$

• Thus, resizing by any factor k also results in O(N) time for resizing.

Positional Lists

- indices are not a good abstraction for describing a more local view of a position in a sequence.
- because the index of an entry changes over time due to insertions or deletions.
- goal is to design an abstract data type that provides a user a way to refer to elements anywhere in a sequence.
- Example, a text document.
- develop our own abstract data type that we denote as a positional list.

Positional Lists

- Positional list is an abstraction, and need not rely on a linked list for its implementation.
- In defining the positional list ADT, the concept of a **position** is introduced.
- **Position** formalizes the intuitive notion of the "location" of an element relative to others in the list.

Positions

- **getElement():** Returns the element stored at this position.
- A position p, associated with some element e in a list L, does not change, even if the index of e changes in L due to insertions or deletions elsewhere in the list.
- The only way in which a position becomes invalid is if that position and its element are explicitly removed from the list.

- The accessor methods includes,
 - first(): Returns the position of the first element of L or null
 - last(): Returns the position of the last element of L or null
 - before(p): Returns the position of L immediately before position
 - after(p): Returns the position of L immediately after position p
 - isEmpty(): Returns true if list L does not contain any elements.
 - size(): Returns the number of elements in list L.
 - An error occurs if a position p is not a valid position for the list.

- first() and last() methods return the associated positions, not the elements.
- The first element of a positional list can be determined by,

first().getElement;

A typical traversal of a positional list

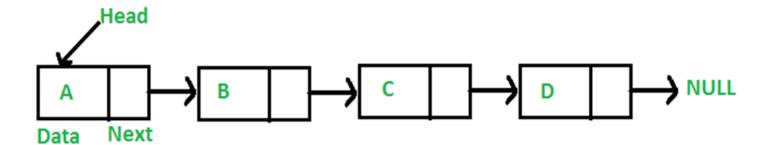
```
Position<String> cursor = guests.first();
while (cursor != null)
{
    System.out.println(cursor.getElement());
    cursor = guests.after(cursor);
}
```

Updated Methods of a Positional List,

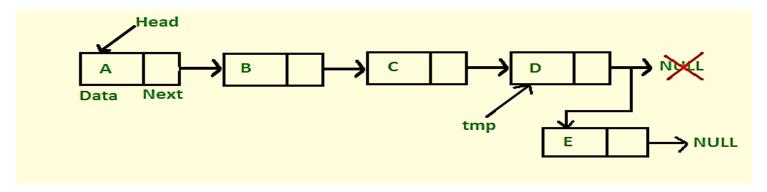
- addFirst(e)
- addLast(e)
- addBefore(p, e)
- addAfter(p, e)
- set(p, e)
- remove(p)

Method	Return Value	List Contents
addLast(8)	p	(8p)
first()	p	(8p)
addAfter(p, 5)	\boldsymbol{q}	(8p, 5q)
before(q)	p	(8p, 5q)
addBefore $(q, 3)$	r	(8p, 3r, 5q)
r.getElement()	3	$(8_p, 3_r, 5_q)$
after(p)	r	$(8_p, 3_r, 5_q)$
before(p)	null	$(8_p, 3_r, 5_q)$
addFirst(9)	S	$(9_s, 8_p, 3_r, 5_q)$
remove(last())	5	$(9_s, 8_p, 3_r)$
set(p, 7)	8	$(9_s, 7_p, 3_r)$
remove(q)	"error"	$(9_s, 7_p, 3_r)$

- linear data structure in which each element of the list contains a pointer which points to the next element in the list.
- Each element in the singly linked list is called a node.



Creation and Insertion:



```
class Node {
    int data;
    Node next;
    public Node(int d)
    {       this.data = d;
            this.next=null;
    }
}
```

Insertion:

```
public void insert(int data) {
    Node newNode = new Node(data);
    if (head == null) {
      head = newNode;
    } else {
      Node current = head;
      while (current.next != null) {
         current = current.next;
      current.next = newNode;
```

Traversal:

For traversal, below is a general-purpose function printList() that prints any given list by traversing the list from head node to the last.

```
public void traverse() {
    Node current = head;
    while (current != null) {
        System.out.print(current.data + " ");
        current = current.next;
    }
    System.out.println();
    }
}
```

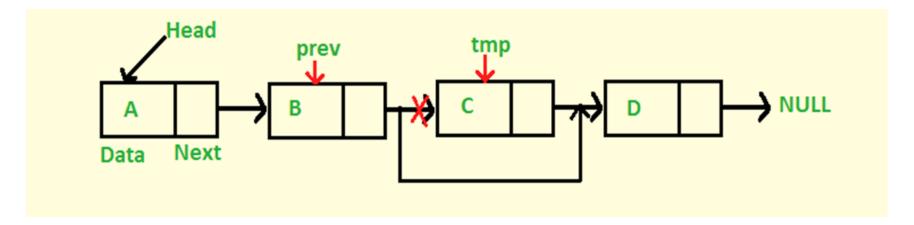
Deletion By KEY:

- 1. Search the key for its first occurrence in the list
- 2. Now, Any of the 3 conditions can be there:

Case 1: The key is found at the head

Case 2: The key is found in the middle or last, except at the head

Case 3: The key is not found in the list



Deletion By KEY:

```
public void delete(int data) {
       if (head == null) {
         return;
       if (head.data == data) {
       head = head.next;
       return;
       Node current = head;
        while (current.next != null && current.next.data != data) {
       current = current.next;
       if (current.next != null) {
       current.next = current.next.next;
```

Iterators

- the process of scanning through a sequence of elements, one element at a time.
- Java defines the **java.util.Iterator** interface with the following methods:
 - hasNext()
 - **next()**
 - remove()
- If the next() method of an iterator is called, when no further elements are available, a "NoSuchElementException" is thrown.
- hasNext() method can be used to detect that condition before calling next().
- The combination of these two methods allows a general loop construct for processing elements of the iterator.

Iterators

Example:

```
while (iter.hasNext( ))
{
String value = iter.next( );
System.out.println(value);
}
```

The Iterable Interface and Java's For-Each Loop

- A single iterator instance supports only one pass through a collection
- there is no way to "reset" the iterator back to the beginning of the sequence.
- Java defines another parameterized interface, named Iterable.
 iterator()
- Java's Iterable class also supports the "for-each" loop syntax.

```
for (ElementType variable : collection) {
    loopBody // may refer to "variable" }
```

Syntax is shorthand for,

```
Iterator<ElementType> iter = collection.iterator();
while (iter.hasNext()) {
ElementType variable = iter.next();
loopBody // may refer to "variable" }
```

The Iterable Interface and Java's For-Each Loop

• Iterator's remove method cannot be invoked when using the for-each loop syntax. Instead, we must explicitly use an iterator.

```
ArrayList<Double> data;

Iterator<Double> walk = data.iterator();

while (walk.hasNext())

if (walk.next() < 0.0)

walk.remove();
```

Implementing Iterators

- Two general styles for implementing iterators,
- A snapshot iterator maintains copy of the sequence of elements, which is constructed at the time the iterator object is created
- It requires O(n) time and O(n) auxiliary space.
- A lazy iterator does not make an upfront copy, instead performing a piecewise traversal only when the next() method is called.
- It requires only O(1) space and O(1) construction time.

Iterations with the ArrayList class

Iterate through ArrayList using for-each loop:

```
import java.util.ArrayList;
class Main {
 public static void main(String[] args) {
  ArrayList<String> languages = new ArrayList<>();
  languages.add("Java");
  languages.add("JavaScript");
  languages.add("Python");
  System.out.println("ArrayList: " + languages);
  System.out.println("Iterating over ArrayList using
                                                           for-each
loop:");
  for(String language : languages) {
   System.out.print(language);
   System.out.print(", "); } } }
```

Iterations with the ArrayList class

Iterate through ArrayList using listIterator(): import java.util.ArrayList; import java.util.ListIterator; class Main { public static void main(String[] args) { ArrayList<Integer> numbers = new ArrayList<>(); numbers.add(1); numbers.add(3); numbers.add(2); System.out.println("ArrayList: " + numbers); ListIterator<Integer> iterate = numbers.listIterator(); System.out.println("Iterating over ArrayList:");

System.out.print(iterate.next() + ", "); } }

while(iterate.hasNext()) {

Iterations with the LinkedPositionalList class

- standard iterator() method return an iterator of the elements of the list.
- positions() to iterate through the positions of a list.

for (Position<String> p : waitlist.positions())

- We define three new inner classes.
- PositionIterator, providing the core functionality of our list iterations.
- PositionIterable inner class
- ElementIterator class

Iterations with the LinkedPositionalList class

Methods:

- public int size()
- public boolean isEmpty()
- public <u>Position</u><<u>E</u>> first()
- public <u>Position</u><<u>E</u>> last()
- public $\underline{Position} < \underline{E} > before(\underline{Position} < \underline{E} > p)$ throws $\underline{IllegalArgumentException}$
- public $\underline{Position} < \underline{E} > after(\underline{Position} < \underline{E} > p)$ throws $\underline{IllegalArgumentException}$
- public <u>Position</u><<u>E</u>> addFirst(<u>E</u> e)
- public $\underline{Position} < \underline{E} > addLast(\underline{E} e)$
- public $\underline{Position} < \underline{E} > \text{ addBefore}(\underline{Position} < \underline{E} > \text{ p, } \underline{E} \text{ e)}$ throws $\underline{IllegalArgumentException}$
- public $\underline{Position} < \underline{E} > \text{ addAfter}(\underline{Position} < \underline{E} > p, \underline{E} e)$ throws $\underline{IllegalArgumentException}$
- public <u>E</u> remove(<u>Position</u><<u>E</u>> p) throws <u>IllegalArgumentException</u>
- public <u>Iterable</u><<u>Position</u><<u>E</u>>> positions()
- public <u>Iterator</u><<u>E</u>> iterator()

Applications of Stack

- Stack is a simple linear data structure used for storing data.
- Stack follows the LIFO
- It can be implemented through an array or linked lists.
- Some of its main operations are: push(), pop(), top(), isEmpty(), size(), etc.

Applications:

- Expression conversion
- Function calls and recursion
- Undo/Redo operations
- Expression evaluation
- Browser history
- Balanced Parentheses

Applications of Stack

Expression Conversion:

- Infix to Postfix conversion
- Infix to Prefix conversion

Notations for Arithmetic Expression:

There are three notations to represent an arithmetic expression:

- Infix Notation A + B, (C D)
- Prefix Notation +AB, -CD
- Postfix Notation **AB+**, **CD+**

Conversion of Arithmetic Expression into various Notations:

Infix Notation	Prefix Notation	Postfix Notation
A * B	* A B	AB*
(A+B)/C	/+ ABC	AB+C/
(A*B) + (D-C)	+*AB - DC	AB*DC-+

- Scan the infix expression from left to right.
- If the scanned character is an operand, put it in the postfix expression.
- Otherwise, do the following
 - If the precedence of the new operator is greater than the precedence of the operator in stack then push the new operator into stack.
 - Else, Pop all the operators from the stack which are greater than or equal to in precedence than that of the new operator.
 - After doing that Push the scanned operator to the stack.
- If the scanned character is a '(', push it to the stack.
- If the scanned character is a ')', pop the stack and output it until a '(' is encountered
- Repeat the steps until the infix expression is scanned.
- Once the scanning is over, Pop the stack and add the operators in the postfix expression until it is not empty.
- Finally, print the postfix expression.

Infix Expression Stack Postfix Expression A + B / C + D * (E - F) ^ G) A A - B / C + D * (E - F) ^ G) ii) A iii) A + B / C + D * (E - F) ^ G) AB iv) A + B / C + D * (E - F) ^ G) ABC ŧ A + B / C + D * (E - F) ^ G) ABC/+ A + B / C + D * (E - F) ^ G) vi) ABC/+D vii) A + B / C + D * (E - F) ^ G) ABC/+D A + B / C + D * (E - F) ^ G) ABC/+D A + B / C + D * (E - F) ^ G) ABC/+D A + B / C + D * (E - F) ^ G) \mathbf{x} ABC/+DE $A + B / C + D * (E_{F} F) ^ G)$ xi) ABC/+DE xii) A + B / C + D * (E - F) ^ G) ABC/+DEF $A + B / C + D * (E - F) \land G$ xiii) ABC/+DEF-<u>^</u> A + B / C + D * (E - F) A G) xiv) ABC/+DEF-A + B / C + D * (E - F) ^ G) Ŷ. XV) ABC/+DEF-G

A + B / C + D * (E - F) ^ G)

xvi)

ABC/+DEF-G^*+

	Stack	postfix
(A+B)/C		
	<u> </u>	Α
	(+	A
	(+	A B
	X+X	AB
. 6.		AB+
		AB+
*	/	AB+C
		AB+C/

(A*B)+ (D-C)		7
	Stack	postfix
(19-6-1	5 , I V
A		Α
*	(*	A
B	· · · · · · · · · · · · · · · · · · ·	AB
)	(*)	AB
		AB*
+	+	AB *
C	+(AB*
P	1 C	AB* D
	+ (-	AB* D
	+(-	ABXDC
)	+(-)	AAXAC
		AB* DC-+

Example:

Input: A + B * C + D

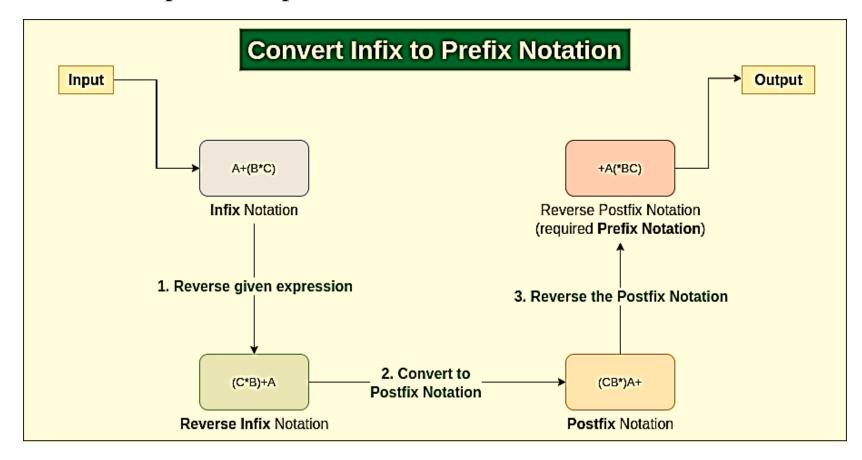
Output: ABC*+D+

Input: ((A + B) - C * (D / E)) + F

Output: AB+CDE/*-F+

```
public static String infixToPostfix(String infixExpression) {
     StringBuilder postfix = new StringBuilder();
     Stack<Character> operatorStack = new Stack<>();
     for (char ch : infixExpression.toCharArray()) {
       if (Character.isLetterOrDigit(ch)) {
         postfix.append(ch);
       } else if (ch == '(') {
         operatorStack.push(ch);
       } else if (ch == ')') {
          while (!operatorStack.isEmpty() && operatorStack.peek() != '(') {
            postfix.append(operatorStack.pop());
         operatorStack.pop();
         else if (isOperator(ch)) {
          while (!operatorStack.isEmpty() && getPrecedence(ch) <=</pre>
getPrecedence(operatorStack.peek())) {
            postfix.append(operatorStack.pop());
         operatorStack.push(ch);
while (!operatorStack.isEmpty()) {
       postfix.append(operatorStack.pop());
    return postfix.toString();
```

- Reverse the infix expression. Note while reversing each '(' will become ')' and each ')' becomes '('.
- Convert the reversed infix expression to "nearly" postfix expression
- Reverse the postfix expression.



	Prefix (A*B)+(A-c)	Page No
Step	$1 \Rightarrow (c-D)$.	+ (B * A)	Gapsession
	Scan	Stack	the same
Steps	· => (C	P 0
	С		С
		(–	C
	Q	(–	CB
)	(-)	CD -
	+	_	CD -
	(B	+(CD-B
	*	+ (*	CD -B
	A	+(*	CD-BA
	, , , , , , , , , , , , , , , , , , ,	+(*)	CO -BA
	/		CD-BAX+
	kehîx	=> [+* A-B-	- DC
	1 0		

Example:

Input: A * B + C / D

Output: + * A B / C D

Input: (A - B/C) * (A/K-L)

Output: *-A/BC-/AKL

```
public static String infixToPrefix(String infixExpression) {
     StringBuilder prefix = new StringBuilder();
     Stack<Character> operatorStack = new Stack<>();
     String reversedInfix = reverse(infixExpression);
     for (char ch : reversedInfix.toCharArray()) {
       if (Character.isLetterOrDigit(ch)) {
          prefix.append(ch); // Append operands directly to the result
       } else if (ch == ')') {
         operatorStack.push(ch);
       } else if (ch == '(') {
     while (!operatorStack.isEmpty() && operatorStack.peek() != ')') {
            prefix.append(operatorStack.pop());
          operatorStack.pop(); } else if (isOperator(ch)) {
     while (!operatorStack.isEmpty() && getPrecedence(ch) <
getPrecedence(operatorStack.peek())) {
            prefix.append(operatorStack.pop());
          operatorStack.push(ch); }
     while (!operatorStack.isEmpty()) {
       prefix.append(operatorStack.pop());
return reverse(prefix.toString());
```