1)

(a) * Probability - Axiomatic Approach :-

Let S be a finite sample space. A real valued function P from the power set of S into R is called a probability function on S if the following three axioms are satisfied

Axioms of Probability :-

(i) Axiom of positivity :- $\boxed{P(E) \geq 0}$ for every subset E of S

(ii) Axiom of certainty :- $\boxed{P(S) = 1}$

(iii) Axiom of union :- If E_1, E_2 are disjoint subsets of S, then

$$\boxed{P(E_1 \cup E_2) = P(E_1) + P(E_2)}$$

The image P(E) of E is called Probability of event E.

Note :- If $E_1, E_2, \ldots, E_n$ are disjoint subsets of S, then

$$\boxed{P(E_1 \cup E_2 \cup \ldots \cup E_n) = P(E_1) + P(E_2) + \ldots + P(E_n)}$$

This is the generalization of axiom (iii)

(b) No. of girls = 6

No. of boys = 10

Total = 16

The no. of ways of selecting 3 students among 16 = $^{16}C_3 = 560$ ways

(i) The no. of ways of selecting 3 boys & 0 girls is $= {}^{10}C_3 \cdot {}^{6}C_0 = 120$

$\Rightarrow P(\text{3 boys are selected}) = \frac{120}{560} = 0.2142857143$

(ii) The no. of ways of selecting 1 boy & 2 girls is

$= {}^{10}C_1 \cdot {}^{6}C_2 = 150$

$\Rightarrow P(\text{exactly 2 girls are selected}) = \frac{150}{560} = 0.2678571429$

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2) $S = \{E_1, E_2, E_3, E_4, E_5, E_6, E_7\}$

$P(E_1) = 0.05$

$P(E_2) = 0.20$

$P(E_3) = 0.20$

$P(E_4) = 0.25$

$P(E_5) = 0.15$

$P(E_6) = 0.10$

$P(E_7) = 0.05$

$A = \{E_1, E_4, E_6\} = E_1 \cup E_4 \cup E_6$

$\Rightarrow P(A) = P(E_1) + P(E_4) + P(E_6)$

$= 0.05 + 0.25 + 0.10 = 0.40$

$B = \{E_2, E_4, E_7\} = E_2 \cup E_4 \cup E_7$

$\Rightarrow P(B) = P(E_2) + P(E_4) + P(E_7)$

$= 0.20 + 0.25 + 0.05$

$C = \{E_2, E_3, E_5, E_7\} = E_2 \cup E_3 \cup E_5 \cup E_7$

$\Rightarrow P(C) = P(E_2) + P(E_3) + P(E_5) + P(E_7) = 0.20 + 0.20 + 0.15 + 0.05$

$= 0.60$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.4 + 0.5 - P(E_4)$

$= 0.4 + 0.5 - 0.25 = 0.65$

$P(A \cap B) = P(E_4) = 0.25$

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3)

(a) Conditional Probability:

If A and B are two events in a sample space S & $P(A) \neq 0$, then the probability of B, after the event A has occured is called the conditional probability of the event of B given A & is denoted by $P\left(\frac{B}{A}\right)$ or $P(B|A)$ & we define $\boxed{P(B|A) = \frac{P(A \cap B)}{P(A)}}$

Similarly we define $\boxed{P(A|B) = \frac{P(A \cap B)}{P(B)}}$

(b) The no. of ways of drawing 2 cards among 15 is ${}^{15}C_2 = 105$ ways.

Let A: Sum of the numbers is even

$$\Rightarrow P(A) = \frac{\overset{\text{even}}{\overset{\Uparrow}{{}^{7}C_2}} + \overset{\text{odd}}{\overset{\Uparrow}{{}^{8}C_2}}}{105} = \frac{21+28}{105} = 0.4666$$

Let B: Odd numbered cards whose sum is even

$\Rightarrow$ $P(B|A) = \dfrac{P(A \cap B)}{P(A)}$

Now, $P(A \cap B) = \dfrac{{}^8C_2}{105} = \dfrac{4}{15} = 0.2666$

$\Rightarrow P(B|A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{\frac{4}{15}}{\frac{49}{105}} = \dfrac{4}{7} = 0.5714285714$

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4) Let E_1, E_2, E_3 be the events that the product is being made by machines B_1, B_2, B_3 respectively

$P(E_1) = \dfrac{30}{100} = 0.3$, $P(E_2) = \dfrac{45}{100} = 0.45$, $P(E_3) = \dfrac{25}{100} = 0.25$

Let A: Product is defective

$P(A|E_1) = \dfrac{2}{100} = 0.02$, $P(A|E_2) = \dfrac{3}{100} = 0.03$, $P(A|E_3) = \dfrac{2}{100} = 0.02$

As per Baye's Theorem

$$\boxed{P(E_i|A) = \frac{P(E_i) \cdot P(A|E_i)}{\sum_{i=1}^{k} P(E_i) \cdot P(A|E_i)}}$$

(i) $P(E_1|A) = \dfrac{P(E_1) \cdot P(A|E_i)}{\sum_{i=1}^{3} P(E_i) \cdot P(A|E_i)}$

$= \dfrac{(0.3)(0.02)}{(0.3)(0.02) + (0.45)(0.03) + (0.25)(0.02)}$

$= \frac{12}{49} = 0.2448$

(ii) $P(E_2|A) = \frac{P(E_2) \cdot P(A|E_2)}{\sum_{i=1}^{3} P(E_i) \cdot P(A|E_i)}$

$= \frac{(0.45)(0.03)}{(0.3)(0.02) + (0.45)(0.03) + (0.25)(0.02)}$

$= \frac{27}{49} = 0.5510$

(iii) $P(E_3|A) = \frac{P(E_3) \cdot P(A|E_3)}{\sum_{i=1}^{3} P(E_i) \cdot P(A|E_i)}$

$= \frac{(0.25)(0.02)}{(0.3)(0.02) + (0.45)(0.03) + (0.25)(0.02)}$

$= \frac{10}{49} = 0.2040$

5)

Values of x, x	0	1	2	3	4	5	6	7
$P(X=x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

(i) We know that $\boxed{\sum P_i = 1}$

$\Rightarrow 0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$\Rightarrow 10k^2 + 9k = 1$

$\Rightarrow 10k^2 + 9k - 1 = 0$

$\Rightarrow k = 0.1, -1$

$k > 0$

$\Rightarrow \boxed{k = 0.1 = \frac{1}{10}}$

(ii) $P(x<6) = P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4)$
$+ P(x=5)$

$= 0 + k + 2k + 2k + 3k + k^2$

$= k^2 + 8k$

$= \left(\frac{1}{10}\right)^2 + 8\left(\frac{1}{10}\right)$

$= \frac{1}{100} + \frac{8}{10} = \frac{81}{100} = 0.81$

$P(x>6) = P(x=7) = 7k^2 + k$

$= 7\left(\frac{1}{10}\right)^2 + \frac{1}{10}$

$= \frac{17}{100} = 0.17$

$P(0<x<5) = P(x=1) + P(x=2) + P(x=3) + P(x=4)$

$= k + 2k + 2k + 3k$

$= 8k$

$= 8\left(\frac{1}{10}\right) = 0.8$

(iii) Mean $(\mu) \Rightarrow \boxed{E(x) = \sum x_i P(x_i)}$

$\Rightarrow E(x) = 0\times 0 + 1\times k + 2\times 2k + 3\times 2k + 4\times 3k + 5\times k^2 + 6\times 2k^2 +$
$7\times(7k^2 + k)$

$= 0 + k + 4k + 6k + 12k + 5k^2 + 12k^2 + 49k^2 + 7k$

$= 66k^2 + 30k$

$= 66\left(\frac{1}{10}\right)^2 + 30\left(\frac{1}{10}\right)$

$= \frac{66}{100} + 3 = 3.66$

$$\boxed{\text{Variance } (\sigma^2) = E(x^2) - [E(x)]^2} \rightarrow ①$$

$$\boxed{E(X^2) = \Sigma\,(x_i^2)\,P(x_i)}$$

$\Rightarrow E(x^2) = 0^2 \times 0 + 1^2 \times k + 2^2 \times 2k + 3^2 \times 2k + 4^2 \times 3k + 5^2 \times k^2 + 6^2 \times 2k^2 + 7^2 \times (7k^2 + k)$

$= 0 + k + 8k + 18k + 48k + 25k^2 + 72k^2 + 343k^2 + 49k$

$= 440k^2 + 124k$

$= 440\left(\frac{1}{10}\right)^2 + 124\left(\frac{1}{10}\right) = 16.8$

Sub in ①

$\Rightarrow \sigma^2 = 16.8 - 3.66 = 3.4044$

Standard deviation $(\sigma) = \sqrt{\text{Variance}} = \sqrt{3.4044}$

$= 1.8451$

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6)

$$f(x) = \begin{cases} kx & , 0 \le x < 5 \\ k(10 - x) & , 5 \le x < 10 \\ 0 & , \text{elsewhere} \end{cases}$$

(ii) We know that $\boxed{\int_{-\infty}^{\infty} f(x)\,dx = 1}$

$$\Rightarrow \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{5} f(x)\,dx + \int_{5}^{10} f(x)\,dx + \int_{10}^{\infty} f(x)\,dx$$

$$\Rightarrow 0 + \int_{0}^{5} kx\,dx + \int_{5}^{10} k(10-x)\,dx + 0 = 1$$

$$\Rightarrow k\left[\frac{x^2}{2}\right]_0^5 + k\left[10x - \frac{x^2}{2}\right]_5^{10} = 1$$

$$\Rightarrow k\left[\frac{25}{2}\right] + k\left[\frac{25}{2}\right] = 1$$

$$\Rightarrow \cancel{2}k\left[\frac{25}{\cancel{2}}\right] = 1$$

$$\Rightarrow \boxed{k = \frac{1}{25}}$$

$$\boxed{\begin{gathered}\int kx\,dx = k\int x\,dx \\ \int x\,dx = \frac{x^2}{2} + c\end{gathered}}$$

(ii) ~~$P(x>5)$~~ $P(x > 500\text{ pounds}) = \int_{5}^{\infty} f(x)\,dx$

$$= \int_{5}^{10} f(x)\,dx + \int_{10}^{\infty} f(x)\,dx$$

$$= \int_{5}^{10} k(10-x)\,dx + 0$$

$$= k\int_{5}^{10} (10-x)\,dx$$

$$= \left(\frac{1}{25}\right)\left[10x - \frac{x^2}{2}\right]_5^{10}$$

$$= \left(\frac{1}{\cancel{25}}\right)\left[\frac{\cancel{25}}{2}\right] = \frac{1}{2} = 0.5$$

(ii) $P(x < 500 \text{ pounds}) = \int_{-\infty}^{5} f(x)\,dx$

$$= \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{5} f(x)\,dx$$

$$= 0 + \int_{0}^{5} kx\,dx$$

$$= 0 + k\int_{0}^{5} x\,dx$$

$$= 0 + \left(\frac{1}{25}\right)\left[\frac{x^2}{2}\right]_0^5$$

$$= 0 + \left(\frac{1}{\cancel{25}}\right)\left[\frac{\cancel{25}}{2}\right] = \frac{1}{2} = 0.5$$

(iii) $P(250 < x < 750) = \int_{2.5}^{7.5} f(x)\,dx$

$$= \int_{2.5}^{5} f(x)\,dx + \int_{5}^{7.5} f(x)\,dx$$

$$= \int_{2.5}^{5} kx\,dx + \int_{5}^{7.5} k(10-x)\,dx$$

$$= k\int_{2.5}^{5} x\,dx + k\int_{5}^{7.5} (10-x)\,dx$$

$$= \left(\frac{1}{25}\right)\left[\frac{x^2}{2}\right]_{2.5}^{5} + \left(\frac{1}{25}\right)\left[10x - \frac{x^2}{2}\right]_5^{7.5}$$

$$= \left(\frac{1}{\cancel{25}}\right)\left(\frac{\overset{3}{\cancel{75}}}{8}\right) + \left(\frac{1}{\cancel{25}}\right)\left(\frac{\overset{3}{\cancel{75}}}{8}\right)$$

$$= \frac{3}{8} + \frac{3}{8} = \frac{6}{8} = 0.75$$

7) No. of families $(N) = 800$

No. of children $(n) = 5$

Probability of boy $(p) = \frac{1}{2}$ [$\because$ Assume equal probabilities for boys & girls]

Probability of girl $(q) = \frac{1}{2}$

By binomial distribution, $\boxed{P(X=r) = {}^nC_r\, p^r\, q^{n-r}}$

(i) $P(X = 3 \text{ boys}) = {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{5-3}$

$= \frac{5}{16} = 0.3125$

(ii) $P(X = 5 \text{ girls}) = {}^5C_5 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{5-0}$

$= \frac{1}{32} = 0.03125$

(iii) $P(X = 2 \text{ or } 3 \text{ boys}) = P(X=2) + P(X=3)$

$= {}^5C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{5-2} + {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{5-3}$

$= \frac{5}{16} + \frac{5}{16}$

$= \frac{10}{16} = 0.625$

(iv) $P(X = \text{atleast one boy}) = P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$

$= {}^5C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^4 + {}^5C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^3 + {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2 +$

${}^5C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^1 + {}^5C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^0$

$= \frac{5}{32} + \frac{5}{16} + \frac{5}{16} + \frac{5}{32} + \frac{1}{32}$

$= \frac{5+10+10+5+1}{32} = \frac{31}{32} = 0.96875$

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2)

X	0	1	2	3	4	5	6
F	13	25	52	58	32	16	4

We know that $\boxed{N = \sum f_i} = 13+25+52+58+32+16+4$

$= 200$

$\Rightarrow \boxed{N = 200}$

$$\boxed{\text{Mean } (\mu) = np = \frac{\sum x_i f_i}{N}}$$

$\Rightarrow \text{Mean} = \frac{0\times13 + 1\times25 + 2\times52 + 3\times58 + 4\times32 + 5\times16 + 6\times4}{200}$

$= \frac{535}{200} = 2.67$

Here, $\boxed{n = 6}$

$\Rightarrow 6p = 2.67$

$\Rightarrow p = \frac{2.67}{6} = 0.38 \simeq 0.4 \Rightarrow \boxed{p = 0.4}$

We know that $\boxed{p+q=1} \Rightarrow q = 1 - 0.4 = 0.6 \Rightarrow \boxed{q = 0.6}$

By binomial distribution $\boxed{P(X=r) = {}^nC_r\, p^r q^{n-r}}$

$P(x=0) = 6C_0 (0.4)^0 (0.6)^6 = 0.0466$

$P(x=1) = 6C_1 (0.4)^1 (0.6)^5 = 0.186$

$P(x=2) = 6C_2 (0.4)^2 (0.6)^4 = 0.311$

$P(x=3) = 6C_3 (0.4)^3 (0.6)^3 = 0.276$

$P(x=4) = 6C_4 (0.4)^4 (0.6)^2 = 0.138$

$P(x=5) = 6C_5 (0.4)^5 (0.6)^1 = 0.036$

$P(x=6) = 6C_6 (0.4)^6 (0.6)^0 = 0.004$

Expected frequencies are

$f(0) = N \cdot P(x=0) = 200 \times 0.0466 = 9.32$

$f(1) = N \cdot P(x=1) = 200 \times 0.186 = 37.2$

$f(2) = N \cdot P(x=2) = 200 \times 0.311 = 62.2$

$f(3) = N \cdot P(x=3) = 200 \times 0.276 = 55.2$

$f(4) = N \cdot P(x=4) = 200 \times 0.138 = 27.6$

$f(5) = N \cdot P(x=5) = 200 \times 0.036 = 7.2$

$f(6) = N \cdot P(x=6) = 200 \times 0.004 = 0.8$

$199.52 \simeq 200 = N$

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9) We know that $\boxed{P(x) = \frac{e^{-\lambda} \lambda^x}{x!}}$

Given $P(1) \cdot \frac{3}{2} = P(3)$

$\Rightarrow \frac{e^{-\lambda} \lambda^1}{1!} \times \frac{3}{2} = \frac{e^{-\lambda} \lambda^3}{3!}$

$\Rightarrow \cancel{\frac{3\lambda}{2} = \frac{\lambda^2}{6}}$ $\quad \frac{3}{2}\lambda = \frac{\lambda^3}{6}$

$\cancel{\Rightarrow 9\lambda = \lambda^3}$

$\cancel{\Rightarrow \lambda^3 - 9\lambda = 0}$

$\cancel{\Rightarrow \lambda = 0, 3, -3}$

$\Rightarrow \frac{18}{2} = \lambda^2$

$\Rightarrow \lambda^2 = 9$

$\Rightarrow \lambda = -3, 3$

$\because$ In a poisson distribution, λ is always +ve

$\Rightarrow \boxed{\lambda = 3}$

(i) $P(X \geq 1) = 1 - P(X < 1)$

$= 1 - P(X=0)$

$= 1 - \frac{e^{-\lambda}\lambda^0}{0!} = 1 - \frac{e^{-3}(3)^0}{0!} = 1 - e^{-3}$

$= 0.950$

(ii) $P(X \leq 3) = P(0) + P(1) + P(2) + P(3)$

$= \frac{e^{-3}(3)^0}{0!} + \frac{e^{-3}(3)^1}{1!} + \frac{e^{-3}(3)^2}{2!} + \frac{e^{-3}(3)^3}{3!}$

$= e^{-3}\left[1 + 3 + \frac{9}{2} + \frac{27}{6}\right]$

$= 0.647$

(iii) $P(2 \leq X \leq 5) = P(2) + P(3) + P(4) + P(5)$

$= \frac{e^{-3}(3)^2}{2!} + \frac{e^{-3}(3)^3}{3!} + \frac{e^{-3}(3)^4}{4!} + \frac{e^{-3}(3)^5}{5!}$

$= e^{-3}\left[\frac{9}{2} + \frac{27}{6} + \frac{81}{24} + \frac{243}{120}\right]$

$= 0.716$

10) Given $\lambda = 1.8$

Let x be the number of breakdowns.

(i) $P(x=0) = \dfrac{e^{-\lambda}\lambda^{x}}{x!} = \dfrac{e^{-1.8}(1.8)^{0}}{0!} = e^{-1.8} = 0.165$

(ii) $P(x=1) = \dfrac{e^{-1.8}(1.8)^{1}}{1!} = e^{-1.8}(1.8) = 0.297$

(iii) P (atleast 2 breakdowns) $= P(x \geq 2) = \cancel{(P}$

$= 1 - P(x<2)$

$= 1 - [P(x=0) + P(x=1)]$

$= 1 - \left[\dfrac{e^{-1.8}(1.8)^{0}}{0!} + \dfrac{e^{-1.8}(1.8)^{1}}{1!}\right]$

$= 0.537$

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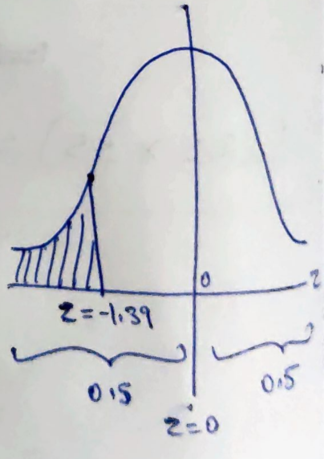
11) (i) to the left of $z = -1.39$ $\boxed{\text{z - table}}$

Required Area (A) = 0.5 − Area (0 to −1.39)

= 0.5 − Area (0 to 1.39)

$= 0.5 - \underbrace{0.4177}$

= 0.0823



0.5 − 0.4177
⇓
table

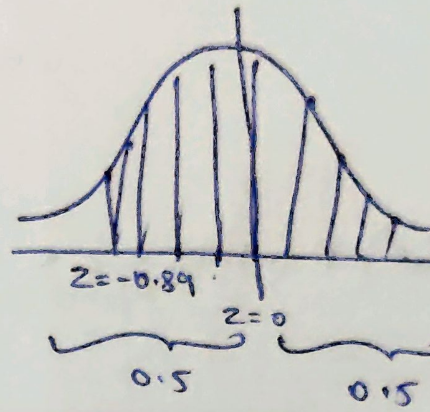
(ii) To the right of $z = -0.89$.

$$\text{Required Area }(A) = 0.5 + \text{Area }(0 \text{ to } -0.89)$$

$$= 0.5 + \text{Area }(0 \text{ to } 0.89)$$

$$= 0.5 + \underbrace{0.3132}_{0.8132 - 0.5 \Downarrow \text{ table}}$$

$$= 0.8132$$



(iii) Between $z = -0.48$ and $z = 1.74$

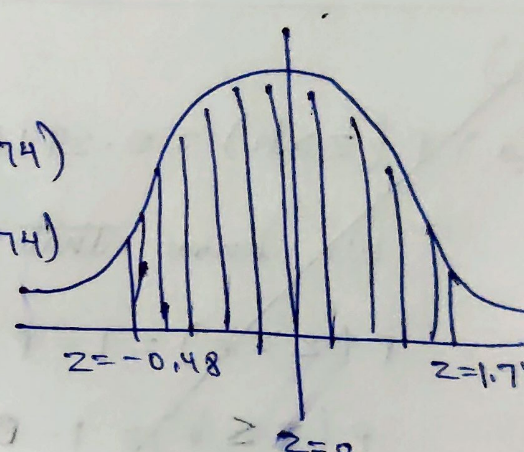
Required area (A)

$$= \text{Area }(0 \text{ to } -0.48) + \text{Area }(0 \text{ to } 1.74)$$

$$= \text{Area }(0 \text{ to } 0.48) + \text{Area }(0 \text{ to } 1.74)$$

$$= \underbrace{0.1843}_{0.6843 - 0.5} + \underbrace{0.4581}_{0.9581 - 0.5}$$

$$= 0.6424$$

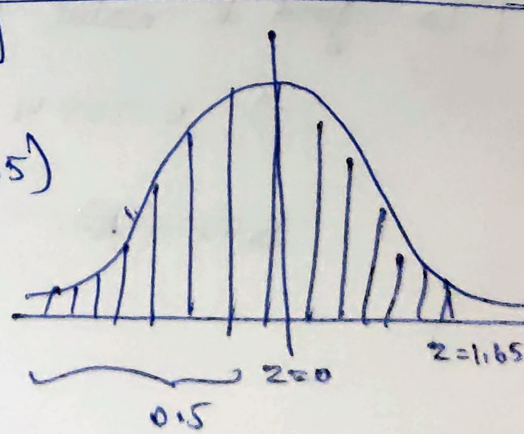


12) (a) $P(Z < 1.65)$ $\boxed{\text{Z-Table}}$

$$P(Z<1.65) = 0.5 + \text{Area }(0 \text{ to } 1.65)$$

$$= 0.5 + 0.4505$$

$$= 0.9505$$

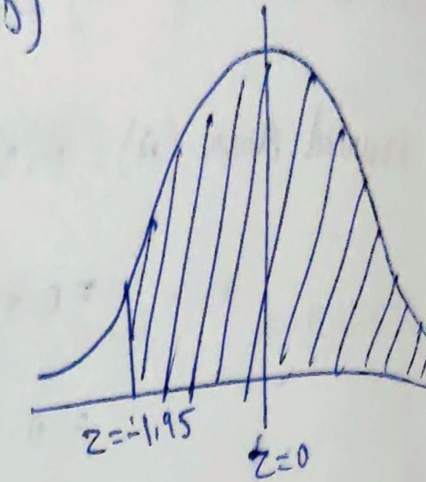


(b) $P(z > -1.95) = 0.5 + \text{Area}(0 \text{ to } -1.95)$

$= 0.5 + \text{Area}(0 \text{ to } 1.95)$

$= 0.5 + 0.4744$

$= 0.9744$



(c) $P(-1.75 < z < -1.04) = \text{Area}(0 \text{ to } -1.75) - \text{Area}(0 \text{ to } -1.04)$

$= \text{Area}(0 \text{ to } 1.75) - \text{Area}(0 \text{ to } 1.04)$

$= 0.4599 - 0.3508$

$= 0.1091$

$P(-0.18 < Z < 0)$ -

$\Rightarrow$

14) Let μ be the mean $(Z=0)$ & σ be the SD of the normal curve

7% of items < 35

$\Rightarrow P(X<35) = \frac{7}{100} = 0.07$

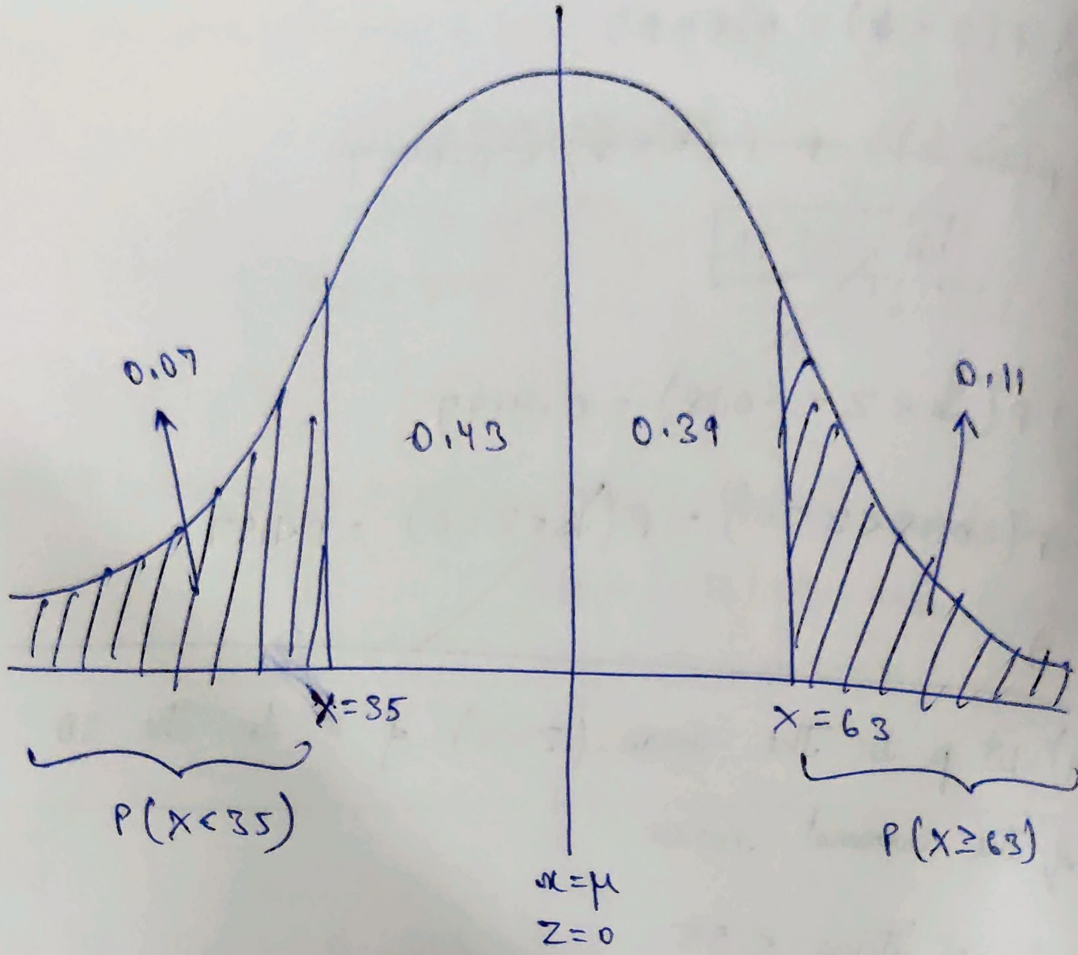
89% items < 63

$\Rightarrow P(X<63) = \frac{89}{100} = 0.89$

$\Rightarrow$

$\Rightarrow P(X \geq 63) = 1 - P(X<63)$

$= 1 - 0.89 = 0.11$



We need to find μ & σ^2

When $x = 35$, $Z = \frac{x-\mu}{\sigma} = -Z$ (left side)

$\Rightarrow Z_1 = \frac{35-\mu}{\sigma} \rightarrow (1)$

$\because Z = -\infty$ to 0, Area is 0.5

$A_1 = P(-Z_1 < Z < 0) = P(0 < Z < Z_1)$

$\boxed{A = 0.43}$

$\Rightarrow \boxed{\text{table}(Z_1) = 1.48}$

When $x = 63$, $Z_2 = \frac{63-\mu}{\sigma} \Rightarrow Z_2 = 0.39 \rightarrow (2)$

$A_2 = P(0 < Z < Z_2) = 0.39$

$\Rightarrow \boxed{\text{Table}(z_2) = 1.23}$

$\because z_1$ is on left side $\Rightarrow z_1 = -1.48$

$\Rightarrow z_2 = 1.23$

Sub in ① & ②

$\Rightarrow -1.48 = \frac{35-\mu}{\sigma}$ & $1.23 = \frac{63-\mu}{\sigma}$

$\Rightarrow -1.48\sigma = 35-\mu \rightarrow$ ③

$(-)\quad 1.23\sigma = 63-\mu$

$-2.71\sigma = -28$

$\Rightarrow \sigma = \frac{28}{2.71} = 10.332 \Rightarrow \boxed{\sigma = 10.332}$

Sub in ③ $\Rightarrow -1.48(10.332) = 35-\mu$

$\Rightarrow \mu = 35 + 1.48(10.332)$

$= 35 + 15.3 = 50.3$

$\Rightarrow \boxed{\mu = 50.3}$

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15) $N = 5$

(a) $\boxed{\mu = \frac{\Sigma x_i}{N}} = \frac{2+3+6+8+11}{5} = 6$

(b) $\sigma^2 = \frac{\Sigma(x_i-\mu)^2}{N} = \frac{(2-6)^2 + (3-6)^2 + (6-6)^2 + (8-6)^2 + (11-6)^2}{5}$

$\sigma^2 = 10.8 \Rightarrow \sigma = \sqrt{10.8} = 3.28$

(c) The no. of samples we can select from the population with replacement $= N^n = N_1$

$n = 2, \ N = 5$

The no. of samples of size 2 are $5^2 = 25 = N_1$

{ (2,2), (2,3), (2,6), (2,8), (2,11)
(3,2), (3,3), (3,6), (3,8), (3,11)
(6,2), (6,3), (6,6), (6,8), (6,11)
(8,2), (8,3), (8,6), (8,8), (8,11)
(11,2), (11,3), (11,6), (11,8), (11,11) }

Sample means are

$$\left.\begin{matrix} \{\ 2, & 2.5, & 4, & 5, & 6.5 \\ 2.5, & 3, & 4.5, & 5.5, & 7 \\ 4, & 4.5, & 6, & 7, & 8.5 \\ 5, & 5.5, & 7, & 8, & 9.5 \\ 6.5, & 7, & 8.5, & 9.5, & 11\ \} \end{matrix}\right\} 25 = N_1$$

Mean of sample means are $= \dfrac{150}{25} = 6$

(d) $\sigma^2 = \dfrac{(x_i - \mu)^2}{N_1}$

$$= \frac{(2-6)^2 + (2.5-6)^2 + (4-6)^2 + \dots + (11-6)^2}{25}$$

$= 2.385$

16) $N=6,\ n=2$

(a) $\mu = \frac{\sum x_i}{N} \Rightarrow \mu = \frac{4+8+12+16+20}{6} = 14$

(b) $\sigma^2 = \frac{\sum (x_i-\mu)^2}{N}$

$= \frac{(4-14)^2+(8-14)^2+(12-14)^2+(16-14)^2+(20-14)^2}{6}$

$= \frac{100+36+4+4+36+100}{6} = \frac{280}{6} = 46.6$

$\simeq 47$

$\sigma = \sqrt{47} = 6.831 \simeq 7$

$\boxed{\sigma = 7}$

(c) Total no. of samples we can select from the population without replacement $= {}^{N}C_{n} = N_1$

$= {}^{6}C_{2} = 15$

{(4,8), (4,12), (4,16), (4,20), (4,24)

(8,12), (8,16), (8,20), (8,24)

(12,16), (12,20), (12,24)

(16,20), (16,24)

(20,24)} $= 15 = N_1$

Sample means are

~~{2, 2.5, 4,~~ $\{6, 8, 10, 12, 14$

10, 12, 14, 16

14, 16, 18

18, 20

22 $\}$

Mean of the sampling means $= \frac{210}{15} = 14$

(d) $\sigma^2 = \frac{\Sigma (x_i - \mu)^2}{N_1}$

$= \frac{(6-14)^2 + (8-14)^2 + \dots + (22-14)^2}{15}$

$= \frac{328}{15} = 21.86 \simeq 22$

$\sigma^2 = 22 \Rightarrow \sigma = \sqrt{22} = 4.690$

---

17) ① Mean of the population

$\mu = \frac{\Sigma x_i}{N} = \frac{10+20+30+40}{4} = \frac{100}{4} = 25$

② Variance of the population

$\sigma^2 = \frac{\Sigma (x_i - \mu)^2}{N} = \frac{(10-25)^2 + (20-25)^2 + (30-25)^2 + (40-25)^2}{4}$

$= \frac{500}{4} = 125$

With Replacement

(3) The samples of size 2 with replacement $= N^n = 4^2 = 16$
$= N_1$

$$\left.\begin{matrix} \{(10,10) & (10,20) & (10,30) & (10,40) \\ (20,10) & (20,20) & (20,30) & (20,40) \\ (30,10) & (30,20) & (30,30) & (30,40) \\ (40,10) & (40,20) & (40,30) & (40,40)\} \end{matrix}\right\} N_1 = 16$$

Sample means are

$$\left.\begin{matrix} \{10 & 15 & 20 & 25 \\ 15 & 20 & 25 & 30 \\ 20 & 25 & 30 & 35 \\ 25 & 30 & 35 & 40\} \end{matrix}\right. \quad \text{Total} = 400$$

Mean of sample means $= \bar{x} = \dfrac{10+15+\dots+40}{N_1} = \dfrac{400}{16}$

$= 25$

(4) Variance of sampling distribution of means

$$\sigma^2 = \frac{\Sigma(x_i - \mu)^2}{N_1} = \frac{(10-25)^2 + (15-25)^2 + \dots + (40-25)^2}{16}$$

$$= \frac{1000}{16} = 62.5$$

Without Replacement

(5) The sample of size 2 without replacement $= {}^{N}C_{n} = {}^{4}C_{2}$
$= 6 = N_2$

$$\begin{matrix} \{(10,20) & (10,30) & (10,40) \\ (20,30) & (20,40) & \\ (30,40)\} & & \end{matrix} \qquad N_2 = 6$$

Sample means are { 15 20 25

25 30

35 } Total = 150

Mean of the sampling means $= \frac{150}{6} = 25$

(b) Variance of the sampling distribution of mean

$$\sigma^2 = \frac{\Sigma(x_j-\mu)^2}{N_2} = \frac{(15-25)^2+(20-25)^2+\dots+(35-25)^2}{6}$$

$$= \frac{250}{6} = 41.66$$

---

18)

(a) Given sample size $n = 100$

$SD = \sigma = 5$

$$\boxed{\text{Maximum Error} = Z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}}$$

$$\boxed{95\% \text{ of } Z_{\frac{\alpha}{2}} = 1.96}$$

$$\Rightarrow E = 1.96 \times \frac{5}{\sqrt{100}} = 1.96 \times \frac{5}{10} = 0.98$$

(b) Sample mean $\bar{x} = 21.6$

$n = 100,\ \sigma = 5.1$

$$\text{Confidence limits} = \left(\bar{x} - Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}},\ \bar{x} + Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}\right)$$

$$= \left(21.6 - (1.96)\frac{5}{10},\ 21.6 + (1.96)\frac{5}{10}\right)$$

$= (21.6 - 0.98 , 21.6 + 0.98)$

$= (20.08, 22.04)$

19)

(a) Point estimation :- A point estimate of a parameter θ is a single numerical value, which is computed from the given sample & serves as an approximation of the unknown exact value of the parameter. A point estimator is a statistic for estimating the population parameter θ & will be denoted by $\hat{\theta}$.

Interval estimation :- An interval estimate of a population parameter θ is an interval of the form $\hat{\theta}_L < \theta < \hat{\theta}_U$ where $\hat{\theta}_L$ and $\hat{\theta}_U$ depend on the value of the statistic $\hat{\theta}$ for a particular sample & also on the sampling distribution of $\hat{\theta}$.

(b) Sample size $n = 10$ (small sample)

$$\bar{x} = \text{Mean} = \frac{15+17+10+18+16+9+7+11+13+14}{10}$$

$$= \frac{130}{10} = 13$$

$$s^2 = \text{Variance} = \frac{\Sigma (x_i - \bar{x})^2}{n-1}$$

$$= \frac{(15-13)^2 + (17-13)^2 + \dots + (14-13)^2}{10-1}$$

$$= 13.33$$

$$s = \sqrt{13.33} = 3.6518$$

$$\alpha = 5\% \Rightarrow \alpha = 0.05 \Rightarrow \frac{\alpha}{2} = 0.025$$

$$\Rightarrow t_{\frac{\alpha}{2}} = t_{0.025} = 2.262 \qquad \nu = n-1 = 10-1 = 9 \text{ dof}$$

$$\text{Confidence Interval} = \left(\bar{x} - t_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}},\ \bar{x} + t_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}\right)$$

$$= \left(13 - (2.26)\left(\frac{3.6518}{\sqrt{10}}\right),\ 13 + (2.26)\left(\frac{3.6518}{\sqrt{10}}\right)\right)$$

$$= (10.3901,\ 15.6098)$$

14) Given $\bar{x} = 10.65$ min

$SD = S = 2.48$ min

$n = 20$ (small sample)

$\alpha = 5\% \Rightarrow \alpha = 0.05 \Rightarrow \frac{\alpha}{2} = 0.025$

$\therefore t_{\frac{\alpha}{2}} = t_{0.025} = 2.093$ at $\nu = n-1 = 19$ dof

Maximum Error $= E = t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$

$= 2.093 \times \frac{2.48}{\sqrt{20}} = 1.1606$

---

15) (a) $P(Z > k) = 0.2946$

We know that total probability $= 1$

$P(Z > k) = 1 - P(Z \leq k) = 0.2946$

$P(Z \leq k) = 1 - 0.2946 = 0.7054$

$A = P(Z \leq k) = 0.7054 > 0.5$

To find k remove 0.5 from Area (A)

$\Rightarrow 0.7054 - 0.5 = 0.2054$

$\boxed{\text{table}(k) = 0.2054}$

$\Rightarrow \boxed{k = 0.54}$

(b) $P(Z < k) = 0.0427$

$\Rightarrow 1 - P(Z \geq k) = 0.0427$

$\Rightarrow 1 - 0.0427 = P(Z \geq k)$

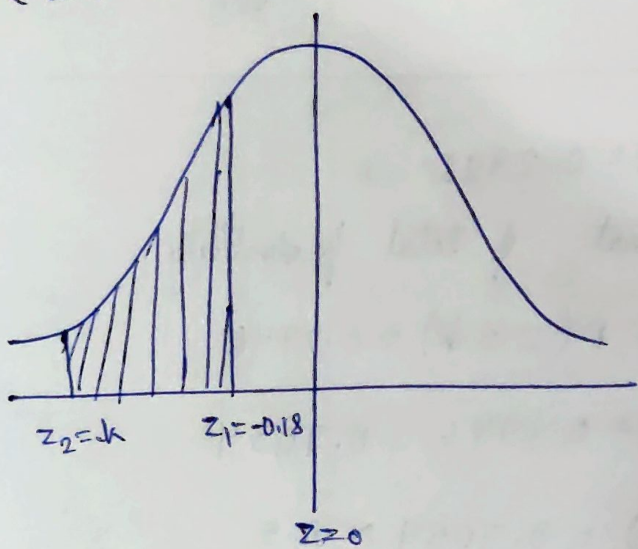
$\Rightarrow 0.9573 = P(Z \geq k) = A$

To find k subtract 0.5 from A

table $(k) = 0.9573 - 0.5 = 0.4573$

$\Rightarrow \boxed{k = 1.72}$

(c) $P(k < Z < -0.18) = 0.4197$



$\Rightarrow P(-0.18 < Z < 0) - P(k < Z < 0) = 0.4197$

$\Rightarrow 0.0714 - A_1 = 0.4197$

$\Rightarrow A_1 = 0.0714 - 0.4197$

$\Rightarrow A_1 = -0.3483$

$\Rightarrow A_1 = 0.3483$ ($\because$ Area is always +ve)

$P(k < Z < 0) = 0.3483$

$\Rightarrow P(Z > k) = 0.3483$

$\Rightarrow 1 - P(Z \le k) = 0.3483$

$\Rightarrow P(Z \le k) = 1 - 0.3483$

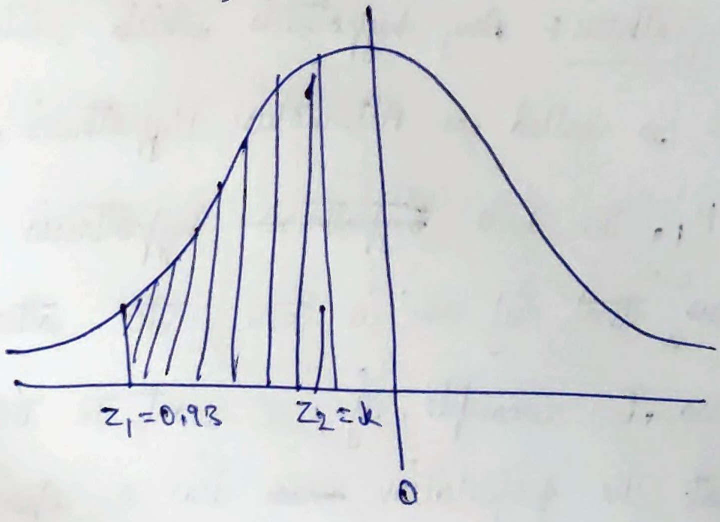
$\Rightarrow P(Z \le k) = 0.6517 \rightarrow (1) \quad > 0.5$

Subtract 0.5 from (1)

$\Rightarrow$ table $(k) = 0.6517 - 0.5 = 0.1517$

$\Rightarrow \boxed{k = 0.39}$

(d) $P(-0.93 < Z < k) = 0.7235$



$\Rightarrow P(-0.93 < Z < 0) - P(k < Z < 0) = 0.7235$

$\Rightarrow 0.3238 - A_1 = 0.7235$

$\Rightarrow A_1 = 0.3238 - 0.7235$

$\Rightarrow A_1 = -0.3997$

$\Rightarrow A_1 =$ table $(k) = 0.3997$ ($\because$ Area is always +ve)

$\Rightarrow \boxed{k = 1.28}$

21)

(a) Null Hypothesis :- For applying the Tests of significance we first set up a hypothesis — a definite statement about the population parameter. Such a hypothesis is usually a hypothesis of no-difference, is called Null Hypothesis

It is in the form $H_0 : \mu = \mu_0$

μ_0 is the value which is assumed ~~to~~ or claimed for the population characteristic. It is the reference point against which the Alternative Hypothesis is set up.

Alternative Hypothesis :- Any hypothesis which contradicts the Null Hypothesis is called an Alternative Hypothesis, usually denoted by H_1. The two ~~hypothesis~~ hypothesis H_0 & H_1 are ~~hypothesis~~ such that if one is true, the other is false & vice versa. For example, if we want to test the null hypothesis that the population ~~mean~~ has a specified mean μ_0 (say) i.e $H_0 : \mu = \mu_0$ Then the Alternative hypothesis would be

1. $H_1 : \mu \neq \mu_0$ (Two Tailed Test)
2. $H_1 : \mu > \mu_0$ (Right Tailed Test)
3. $H_1 : \mu < \mu_0$ (Left Tailed Test)

(b) Given size of the sample $n = 30$ (large sample)

Mean of the population $\mu = 800$

Mean of the sample $\bar{x} = 788$

SD of the population $\sigma = 40$

① <u>NH</u> : $H_0 : \mu = 800$

② <u>AH</u> : $H_1 : \mu \neq 800$ (Two tailed test)

③ <u>LOS</u> : $\alpha = 0.05$

④ <u>Critical Region</u> : In Two Tailed Test $\Rightarrow Z_{\frac{\alpha}{2}} = 1.96$

at $\alpha = 0.05$ LOS

★ Remember :- Table : Critical Values of Z

LOS (α)	1%	5%	10%
Critical values for TTT	$\lvert Z_\alpha \rvert = 2.58$	$\lvert Z_\alpha \rvert = 1.96$	$\lvert Z_\alpha \rvert = 1.645$
Critical values for RTT	$Z_\alpha = 2.33$	$Z_\alpha = 1.645$	$Z_\alpha = 1.28$
Critical values for LTT	$Z_\alpha = -2.33$	$Z_\alpha = -1.645$	$Z_\alpha = -1.28$

⑤ Test statistic : $\boxed{Z = \dfrac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}} = \dfrac{788 - 800}{\frac{40}{\sqrt{30}}} = -1.6431$

$|Z| = 1.64$

⑥ <u>Conclusion</u>:- $Z = 1.64 < Z_{\frac{\alpha}{2}} = 1.96$ at 5 % LOS

$\Rightarrow Z_{cal} < Z_{Tab}$

∴ Accept the NH at 5 % LOS

22) $n_1 = 71$, $n_2 = 75$ (large samples)

$\bar{x}_1 = 83.2$, $\bar{x}_2 = 90.8$

$\sigma_1 = 19.3$, $\sigma_2 = 21.4$

① <u>NH</u>: $H_0 : \mu_1 - \mu_2 = 0$

② <u>AH</u>:- $H_1 : \mu_1 - \mu_2 \neq 0$ (TTT)

③ <u>LOS</u>:- $\alpha = 0.05$

④ <u>CR</u>:- for TTT ; $Z_{\frac{\alpha}{2}} = 1.96$ at 5 % LOS

⑤ <u>Test statistic</u>:-

$$Z = \frac{(\bar{x}_1 - \bar{x}_2) - \delta}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$Z = \frac{(83.2 - 90.8) - 0}{\sqrt{\frac{(19.3)^2}{71} + \frac{(21.4)^2}{75}}}$$

$$Z = -2.255$$

$$|Z| = 2.255$$

⑥ <u>Conclusion</u>:- $Z = 2.255 > Z_{\frac{\alpha}{2}} = 1.96$ at 5 % LOS

$\Rightarrow Z_{cal} > Z_{tab}$

∴ Reject The NH at 5 % LOS

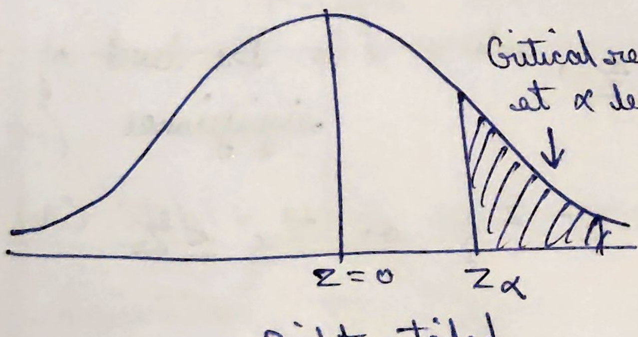
23)

(a) **One tailed test:-**

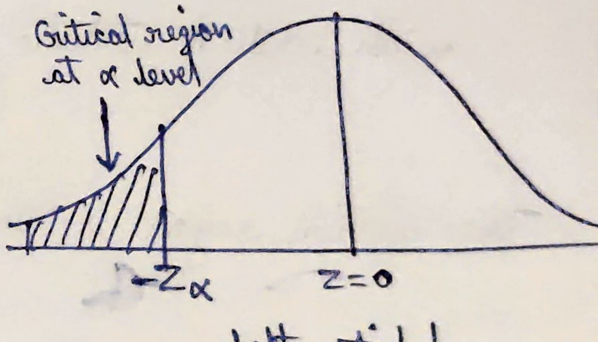
If the Alternative Hypothesis H_1 in a test of statistical hypothesis be one-tailed ~~the~~ (i.e either right-tailed or left-tailed but not both), then the test is called a one-tailed test. For example, to test whether the population mean $\mu = \mu_0$, we have $H_0 : \mu = \mu_0$ against the alternative hypothesis H_1 given by

1. $H_1 : \mu > \mu_0$ (Right tailed)
2. $H_1 : \mu < \mu_0$ (Left tailed)

and the corresponding test is a single-tailed or one-tailed or one sided.



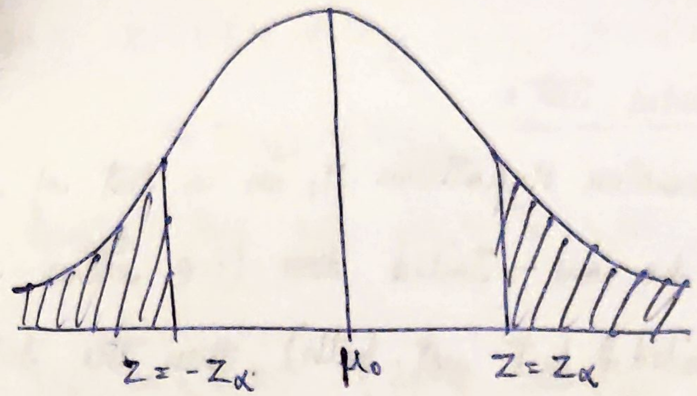
Right-tailed



Left-tailed

Two tailed test:-

Suppose we want to test the Null Hypothesis $H_0 : \mu = \mu_0$ against the Alternative Hypothesis $H_1 : \mu \neq \mu_0$



Since H_1 is two-tailed alternative hypothesis, the critical region under the curve is equally distributed on both the sides of the mean.

Thus, the critical area under the right-tail = The critical area under the left-tail.

= Half of the total area

= $\frac{1}{2}$ probability of rejection = $\frac{\alpha}{2}$

with critical statistic $Z_{\frac{\alpha}{2}}$, where α is the level of significance

The critical region is then $Z \leq -Z_{\frac{\alpha}{2}}$ or $Z_{\frac{\alpha}{2}} \leq Z$

(b) $n_1 = n_2 = 50$ (large samples)

$\bar{x}_1 = 86.7$, $\bar{x}_2 = 77.8$

$\sigma_1 = 6.28$, $\sigma_2 = 5.61$

① NH :- $H_0: \mu_1 - \mu_2 > 12$

② AH :- $H_1: \mu_1 - \mu_2 < 12$ (LTT)

③ LOS :- $\alpha = 0.05$

(4) Critical Region: In LTT, $Z_\alpha = -1.645$ at 5% LOS

(5) Test Statistic: $Z = \dfrac{(\bar{x}_1 - \bar{x}_2) - \delta}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$

$$= \frac{(86.7 - 77.8) - 12}{\sqrt{\dfrac{(6.28)^2}{50} + \dfrac{(5.61)^2}{50}}}$$

$$= -2.603$$

(6) Conclusion: $Z = -2.603 < Z_\alpha = -1.645$ at 5% LOS

$\Rightarrow Z_{cal} < Z_{tab}$

$\therefore$ Accept the NH at 5% LOS

---

24)

(a) Step 1: NH: Define or set up a NH (H_0) taking into consideration the nature of the problem & data involved

Step 2: AH: Set up the AH (H_1) so that we could decide whether we should use one-tailed or two-tailed test.

Step 3: LOS: Select the appropriate level of significance (α) depending on the reliability of the estimates & permissible risk, i.e, a suitable α is selected in advance if it is not given in the problem (Usually we choose 5% LOS)

Step 4: **Critical Region:** Depending on the LOS & AH, select the tabular value with suitable distribution

Step 5: **Test statistic:** The test statistic is $\boxed{t = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}}$

where s^2 is unbiased estimate of population variance with $v = (n-1)$ dof.

If the SD of the sample is given directly then the test statistic is $\boxed{t = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}}}$ with $v = (n-1)$ dof

Step 6: **Conclusion:-** We compare the computed value of the test statistic t with the critical value t_α at given LOS (α)

If $|t| < t_\alpha$, we accept the NH

If $|t| > t_\alpha$, we reject the NH

(b) Given no. of samples $(n) = 10$ (small sample)

$$\text{Mean}(\bar{x}) = \frac{63+63+66+67+68+69+70+70+71+71}{10}$$

$$\bar{x} = 67.8$$

Variance $(s^2) = \dfrac{\sum (x_i - \bar{x})^2}{n-1}$

$$= \frac{[(63-67.8)^2 + (63-67.8)^2 + \dots + (71-67.8)^2]}{10-1}$$

$$= 8.16$$

$$s = \sqrt{8.16} = 2.85$$

① NH: $H_0 : \mu = 66$

② AH: $H_1 : \mu \neq 66$ (TTT)

③ LOS: $\alpha = 0.05$

④ CR: In TTT $t_{\frac{\alpha}{2}} = t_{\frac{0.05}{2}} = t_{0.025} = 2.262$ at

$\nu = 10 - 1 = 9$ dof

⑤ Test statistic: $t = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \dfrac{67.8 - 66}{\frac{2.85}{\sqrt{10}}} = 1.996$

⑥ Conclusion: $t = 1.996 < t_{\frac{\alpha}{2}} = 2.262$ at 9 dof

$\Rightarrow t_{cal} < t_{tab}$

∴ Accept the NH at 5% LOS

25) Given $n=7$ (small sample)

$$\text{Mean}(\bar{x}) = \frac{3005+2935+2965+2995+3905+2935+2905}{7}$$

$$= \cancel{3235}\ \cancel{3377.857}\ 3092.142$$

$$\text{Variance}(s^2) = \frac{\sum(x_i-\bar{x})^2}{n-1}$$

$$\cancel{= \frac{\left[(3005-3235)^2+(2935-3235)^2+\dots+(2905-3235)^2\right]}{7-1}}$$

$$= \cancel{210200}\ 129723.8095$$

$$s = \cancel{\sqrt{210200} = 458.475}\ \sqrt{129723.8095} = 360.1719$$

① NH: $H_0: \mu = 3000$

② AH: $H_1: \mu \neq 3000$ (TTT)

③ LOS: $\alpha = 0.05$

④ CR: In TTT, $t_{\frac{0.05}{2}} = t_{0.025} = 2.447$ at $\nu = 7-1 = 6$ dof

⑤ Test statistic: $t = \dfrac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}} = \dfrac{\cancel{3235}\ 3092.142 - 3000}{\frac{\cancel{360.1719}\ 360.1719}{\sqrt{7}}} = \cancel{1.3561}\ 0.6768$

⑥ Conclusion: $t = \cancel{1.3561}\ 0.6768 < t_{\frac{\alpha}{2}} = 2.447$ at 6 dof

$\Rightarrow t_{cal} < t_{\frac{\alpha}{2}}$ (tab)

$\therefore$ Accept the NH at 5% LOS

26)

Diet A	13	14	10	11	2	16	10	8	
Diet B	7	10	12	8	10	11	9	10	11

Diet A = x $n_1 = 8$
Diet B = y $n_2 = 9$ } (small samples)

$$\bar{x} = \frac{\sum x_i}{n_1} = \frac{13+14+10+11+2+16+10+8}{8} = 10.5$$

$$\bar{y} = \frac{\sum y_i}{n_2} = \frac{7+10+12+8+10+11+9+10+11}{9} = 9.77$$

$$\boxed{s^2 = \frac{1}{n_1+n_2-2}\left[\sum(x_i-\bar{x})^2 + \sum(y_i-\bar{y})^2\right]}$$

$$= \frac{1}{8+9-2}[128 + 19.5561]$$

$$= 9.8370 \Rightarrow \boxed{s^2 = 9.8370}$$

① NH :- $H_0 : \mu_1 = \mu_2$

② AH :- $H_1 : \mu_1 \neq \mu_2$ (TTT)

③ LOS :- $\alpha = 0.05$

④ CR :- In TTT, $t_{\frac{\alpha}{2}} = t_{\frac{0.05}{2}} = t_{0.025} = 2.131$

at $\nu = n_1 + n_2 - 2 = 8+9-2 = 15$ dof

⑤ Test statistic :- $t = \dfrac{\bar{x}-\bar{y}}{\sqrt{s^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$

$$= \frac{10.5 - 9.77}{\sqrt{(9.8370)\left(\frac{1}{8}+\frac{1}{9}\right)}} = 0.47899$$

⑥ Conclusion: $t = 0.4789 < t_{\frac{\alpha}{2}} = 2.131$ at 15 dof

$\Rightarrow t_{cal} < t_{\frac{\alpha}{2}} (tab)$

$\therefore$ Accept the NH at 5% LOS

---

27)

Sample I	9	11	13	11	15	9	12	14
Sample II	10	12	10	14	9	8	10	

Sample - I

$n_1 = 8$

$\bar{x} = \frac{\sum x_i}{n_1} = 11.75$

$v_1 = n_1 - 1 = 7$

$S_1^2 = \frac{\sum (x_i - \bar{x})^2}{n_1 - 1}$

$= 4.785$

Sample - II

$n_2 = 7$

$\bar{y} = \frac{\sum y_i}{n_2} = 10.42$

$v_2 = n_2 - 1 = 6$

$S_2^2 = \frac{\sum (y_i - \bar{y})^2}{n_2 - 1}$

$= 3.95$

① NH: $H_0 : \sigma_1^2 = \sigma_2^2$

② AH: $H_1 : \sigma_2^2 \neq \sigma_2^2$

③ LOS: $\alpha = 0.05$

④ CR: $F_{0.05}(7,6) = 4.21$

⑤ Test statistic: $F = \frac{S_1^2}{S_2^2} \quad (\because S_1^2 > S_2^2)$

$= \frac{4.785}{3.95} = 1.211$

⑥ Conclusion:- $F = 1.211 < F_\alpha = 4.21$ at 5% LOS

$\Rightarrow F_{cal} < F_{tab}$

$\therefore$ Accept the NH at 5% LOS

---

29)

Digits	0	1	2	3	4	5	6	7	8	9
Frequency	18	19	23	21	16	25	22	20	21	15

① NH:- The digits are equally distributed

(OR)

$H_0 : O_i = E_i$

② AH:- $H_1 : O_i \neq E_i$

Expected frequency of each digit

$$E_i = \frac{18+19+23+21+16+25+22+20+21+15}{10} = \frac{200}{10}$$

$$= 20$$

③ LOS:- $\alpha = 0.05$

④ CR:- $\chi^2_{0.05} = 16.919$ at $v = 10 - 1 = 9$ dof

Observed Frequency (O_i)	Expected Frequency (E_i)	$O_i - E_i$	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
18	20	-2	4	0.2
19	20	-1	1	0.05
23	20	3	9	0.45
21	20	1	1	0.05
16	20	-4	16	0.8
25	20	5	25	1.25
22	20	2	4	0.2
20	20	0	0	0
21	20	1	1	0.05
15	20	-5	25	1.25
				4.3

(5) Test statistic :- $\chi^2 = \sum_{i=0}^{9} \left[\frac{(O_i - E_i)^2}{E_i} \right] = 4.3$

(6) Conclusion :- $\chi^2 = 4.3 < \chi^2_{0.05} = 16.919$ at 5% LOS

$\Rightarrow \chi^2_{cal} < \chi^2_{tab}$

$\therefore$ Accept the NH at 5% LOS

30)

		Performance in Training Program			
		Below Avg	Average	Above Avg	
Success in Job (Employers Rating)	Poor	23	60	29	112
	Average	28	79	60	167
	Good	9	49	63	121
		60	188	152	400

① NH: There is a relationship between an employee's performance in company's training program & his or her ultimate success in the job.

② AH: There is no relationship between an employee's performance in company's Training program & his or her ultimate success in the job.

③. LOS: $\alpha = 0.05$

Table of expected Frequencies

$\frac{112 \times 60}{400} = 16.8$	$\frac{112 \times 188}{400} = 52.64$	$\frac{112 \times 152}{400} = 42.56$
$\frac{167 \times 60}{400} = 25.05$	$\frac{167 \times 188}{400} = 78.49$	$\frac{167 \times 152}{400} = 63.46$
$\frac{121 \times 60}{400} = 18.15$	$\frac{121 \times 188}{400} = 56.87$	$\frac{121 \times 152}{400} = 45.98$

④ CR :- $\chi^2_{0.05} = 9.488$ at $\nu = \underbrace{(3-1)\times(3-1)}_{(rows-1)\times(cols-1)} = 4$ d.o.f

Observed Frequency (O_{ij})	Expected Frequency (E_{ij})	$O_{ij} - E_{ij}$	$(O_{ij} - E_{ij})^2$	$\frac{(O_{ij} - E_{ij})^2}{E_{ij}}$
23	16.8	6.2	38.44	2.288
60	52.64	7.36	54.1696	1.029
29	42.50	-13.56	183.8736	4.326
28	25.05	2.95	8.7025	0.347
79	78.49	0.51	0.2601	0.003
60	63.46	-3.46	11.9716	0.188
9	18.15	-9.15	83.7225	4.612
49	56.87	-7.87	61.9369	1.089
63	45.98	17.02	289.6804	6.300
				20.182

⑤ Test statistic :- $\chi^2 = \sum_{i=1}^{3}\sum_{j=1}^{3}\left[\frac{(O_{ij}-E_{ij})^2}{E_{ij}}\right] = 20.182$

⑥ $\chi^2 = 20.182 > \chi^2_{0.05} = 9.488$ at 5% LOS

$\Rightarrow \chi^2_{cal} > \chi^2_{tab}$

$\therefore$ Reject the NH at 5% LOS

31)

(a) Positive / direct Correlation :- If two variables tend to move together in same direction i.e, an increase in the value of one variable is accompanied by an increase in the value of the other variable, or a decrease in the value of one variable is accompanied by a decrease in the value of the other variable, then the correlation is called positive or direct correlation.

Negative / inverse correlation :- If two variables tend to move together in opposite directions so that an increase or decrease in the values of one variable is accompanied by a decrease or increase in the value of the other variable, then the correlation is called negative or inverse correlation.

(b)

x	65	66	67	67	69	68	70	72
y	67	68	65	68	72	72	69	71

Correlation coefficient $(r) = \dfrac{\Sigma UV}{\sqrt{\Sigma U^2 \Sigma V^2}}$

where $U = x - \bar{X} = x - 68$

$V = y - \bar{Y} = y - 69$

$\Rightarrow \bar{X} = \dfrac{544}{8} = 68$ | $\bar{Y} = \dfrac{552}{8} = 69$

x	y	$U = x - 68$	$V = y - 69$	U^2	V^2	UV
65	67	-3	-2	9	4	6
66	68	-2	-1	4	1	2
67	65	-1	-4	1	16	4
67	68	-1	-1	1	1	1
69	72	0	3	0	9	0
68	72	1	3	1	9	3
70	69	2	0	4	0	2
72	71	4	2	16	4	8
		0	0	36	44	24

$$\rightarrow r = \frac{\Sigma UV}{\sqrt{\Sigma U^2 \Sigma V^2}} = \frac{24}{\sqrt{36 \times 44}} = 0.6030$$

32)

Roll No	1	2	3	4	5	6	7	8	9	10
x	75	30	60	80	53	35	15	40	38	48
y	85	45	54	91	58	63	35	43	45	44

Correlation coefficient (r) $\Rightarrow \dfrac{\Sigma UV}{\sqrt{\Sigma U^2 \Sigma V^2}}$

where $U = x - \overline{X}$

$V = y - \overline{Y}$

$$\Rightarrow \overline{X} = \frac{75+30+60+80+53+35+15+40+38+48}{10} = 47.4$$

$\simeq 47$

$\Rightarrow \bar{Y} = \dfrac{85+45+54+91+58+63+35+43+45+44}{10}$

$= 56.3 \simeq 56$

x	y	$U = x - 47$	$V = y - 56$	U^2	V^2	UV
75	85	28	29	784	841	812
30	45	-17	-11	289	121	187
60	54	13	-2	169	4	-26
80	91	33	35	1089	1225	1155
53	58	6	2	36	4	12
35	63	-12	7	144	49	-84
15	35	-32	-21	1024	441	672
40	43	-7	-13	49	169	91
38	45	-9	-11	81	121	99
48	44	1	-12	1	144	-12
			3	3666	3119	2906

$\Rightarrow r = \dfrac{\Sigma UV}{\sqrt{\Sigma U^2 \Sigma V^2}} = \dfrac{2906}{\sqrt{3666 \times 3119}} = 0.8593$

33)

X	10	15	12	17	13	16	24	14	22	20
Y	30	42	45	46	33	34	40	35	39	38

Here $N = 10$

X	Rank of X	Y	Rank of Y	D = Rank of x − Rank of y	D^2
10	1	30	1	0	0
15	5	42	8	-3	9
12	2	45	9	-7	49
17	7	46	10	-3	9
13	3	33	2	1	1
16	6	34	3	3	9
24	10	40	7	3	9
14	4	35	4	0	0
22	9	39	6	3	9
20	8	38	5	3	9
					104

Rank Correlation coefficient $(\rho) = \cancel{1 - 6\Sigma D^2}$ $1 - \frac{6 \Sigma D^2}{N(N^2-1)}$

$$= 1 - \frac{6 \times 104}{10 \times (100-1)}$$

$$= 0.3696$$

34)

Aptitude Score	57	58	59	59	60	61	60	64
IQ Score	97	108	95	106	120	126	113	110

Here N = 8

Aptitude (X)	Rank of X	IQ (Y)	Rank of Y	D = Rank of X − Rank of Y	D^2
57	8	97	7	1	1
58	7	108	5	2	4
59	5.5	95	8	−2.5	6.25
59	5.5	106	6	−0.5	0.25
60	3.5	120	2	1.5	2.25
61	2	126	1	1	1
60	3.5	113	3	0.5	0.25
64	1	110	4	−3	9
					$\Sigma D^2 = 24$

In X-series, 60 occurs two times so rank of $60 = \frac{3+4}{2} = 3.5$

In X-series, 59 occurs two times so rank of $59 = \frac{5+6}{2} = 5.5$

To ΣD^2, we add $\frac{m(m^2-1)}{12}$ for each value repeated

So for 60, $m_1 = 2$ & for 59, $m_2 = 2$

So, for X-series $\Rightarrow$ $\frac{m_1(m_1^2-1)}{12} + \frac{m_2(m_2^2-1)}{12}$

$$= \frac{2(2^2-1)}{12} + \frac{2(2^2-1)}{12}$$

$$= \frac{1}{2} + \frac{1}{2} = 1$$

$\Rightarrow$ Rank Correlation Coefficient $(\rho) = 1 - \frac{6\left[\Sigma D^2 + \frac{m_1(m_1^2-1)}{12} + \frac{m_2(m_2^2-1)}{12}\right]}{N(N^2-1)}$

$$= 1 - \frac{6[24+1]}{8(8^2-1)} = 0.7023$$

35)

x	27	45	41	19	35	39	19	49	15	31
y	57	64	80	46	62	72	52	77	57	68

The required regression lines are :-

$$x - \bar{x} = r \frac{\sigma_x}{\sigma_y} (y - \bar{y}) \rightarrow 1$$

$$y - \bar{y} = r \frac{\sigma_y}{\sigma_x} (x - \bar{x}) \rightarrow 2$$

$$r = \frac{Cov(x,y)}{\sigma_x \cdot \sigma_y} = \frac{\frac{\Sigma xy}{n} - \bar{x}\,\bar{y}}{\sqrt{\left(\frac{\Sigma x^2}{n} - (\bar{x})^2\right)\left(\frac{\Sigma y^2}{n} - (\bar{y})^2\right)}}$$

$\Rightarrow \bar{x} = \frac{320}{10} = 32 \quad \Rightarrow \bar{x}^2 = 1024$

$\Rightarrow \bar{y} = \frac{635}{10} = 63.5 \quad \Rightarrow \bar{y}^2 = 4032.25$

x	y	x^2	y^2	xy
27	57	729	3249	1539
45	64	2025	4096	2880
41	80	1681	6400	3280
19	46	361	2116	874
35	62	1225	3844	2170
39	72	1521	5184	2808
19	52	361	2704	988
49	77	2401	5929	3773
15	57	225	3249	855
31	68	961	4624	2108
		11490	41395	21275

$$\Rightarrow r = \frac{\frac{21275}{10} - 32 \times 63.5}{\sqrt{\left(\frac{11490}{10} - 1024\right)\left(\frac{41395}{10} - 4032.25\right)}}$$

$$= 0.8248$$

$$\Rightarrow \sigma_x = \sqrt{\frac{\Sigma x^2}{n} - (\bar{x})^2} = \sqrt{\frac{11490}{10} - 1024} = 11.1803$$

$$\Rightarrow \sigma_y = \sqrt{\frac{\Sigma y^2}{n} - (\bar{y})^2} = \sqrt{\frac{41395}{10} - 4032.25} = 10.3561$$

Substitute the above values in 1 & 2

$$1 \Rightarrow (x-32) = (0.8248)\left(\frac{11.1803}{10.3561}\right)(y-63.5) \rightarrow 3$$

$$2 \Rightarrow (y-63.5) = (0.8248)\left(\frac{10.3561}{11.1803}\right)(x-32) \rightarrow 4$$

Simplifying 3 we get

$$3 \Rightarrow (x-32) = (0.8248)\left(\frac{11.1803}{10.3561}\right)(y-63.5)$$

$$\Rightarrow x - 32 = (0.8904)(y - 63.5)$$

$$\Rightarrow x - 32 = 0.8904y - 56.5404$$

$$\Rightarrow \boxed{x = 0.8904y - 24.5405} \rightarrow 5$$

Simplifying 4 we get

④ $\Rightarrow (y-63.5) = (0.8248)\left(\frac{10.3561}{11.1803}\right)(x-32)$

$\Rightarrow y-63.5 = (0.7689)(x-32)$

$\Rightarrow y-63.5 = 0.7689x - 24.6063$

$\Rightarrow \boxed{y = 0.7689x + 38.8936} \rightarrow ⑥$

For $x = 35$ minutes we need to find y

Substitute $x = 35$ in ⑥

$\Rightarrow y = (0.7689)(35) + 38.8936$

$\Rightarrow \boxed{y = 65.8051}$

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36)

Experience (X)	16	12	18	4	3	10	5	12
Performance Ratings (Y)	88	87	89	68	78	80	75	83

The required regression lines are:-

$$x - \bar{x} = r\frac{\sigma_x}{\sigma_y}(y-\bar{y}) \rightarrow ①$$

$$y - \bar{y} = r\frac{\sigma_y}{\sigma_x}(x-\bar{x}) \rightarrow ②$$

$$r = \frac{\text{Cov}(x,y)}{\sigma_x \cdot \sigma_y} = \frac{\frac{\Sigma xy}{n} - \bar{x}\,\bar{y}}{\sqrt{\left(\frac{\Sigma x^2}{n} - (\bar{x})^2\right)\left(\frac{\Sigma y^2}{n} - (\bar{y})^2\right)}}$$

x	y	x^2	y^2	xy
16	88	256	7744	1408
12	87	144	7569	1044
18	89	324	7921	1602
4	68	16	4624	272
3	78	9	6084	234
10	80	100	6400	800
5	75	25	5625	375
12	83	144	6889	996
		1018	52856	6731

$\bar{x} = \frac{80}{8} = 10$

$\Rightarrow (\bar{x})^2 = 100$

$\bar{y} = \frac{648}{8} = 81$

$\Rightarrow (\bar{y})^2 = 6561$

$$\Rightarrow r = \frac{\frac{6731}{8} - 10 \times 81}{\sqrt{\left(\frac{1018}{8} - 100\right)\left(\frac{52856}{8} - 6561\right)}}$$

$$= 0.8861$$

$$\sigma_x = \sqrt{\frac{\Sigma x^2}{n} - (\bar{x})^2} = \sqrt{\frac{1018}{8} - 100} = 5.2201$$

$$\sigma_y = \sqrt{\frac{\Sigma y^2}{n} - (\bar{y})^2} = \sqrt{\frac{52856}{8} - 6561} = 6.7823$$

Substitute the above values in ① & ②

$$① \Rightarrow (x - 10) = (0.8861)\left(\frac{5.2201}{6.7823}\right)(y - 81) \rightarrow ③$$

$$② \Rightarrow (y - 81) = (0.8861)\left(\frac{6.7823}{5.2201}\right)(x - 10) \rightarrow ④$$

Simplifying ③ we get

③ $\Rightarrow (x-10) = (0.6820)(y-81)$

$\Rightarrow x - 10 = 0.6820y - 55.242$

$\Rightarrow \boxed{x = 0.6820y - 45.242} \rightarrow ⑤$

Simplifying ④ we get

④ $\Rightarrow (y-81) = (1.1512)(x-10)$

$\Rightarrow y - 81 = 1.1512x - 11.512$

$\Rightarrow \boxed{y = 1.1512x + 69.488} \rightarrow ⑥$

For $x = 7$ years we need to find y

Sub $x = 7$ in ⑥

$\Rightarrow y = (1.1512)(7) + 69.488$

$\Rightarrow \boxed{y = 77.5464}$

37)

x	1	2	3	4	5	6
y	14	33	40	63	76	85

The required regression lines are

$$x - \bar{x} = r\frac{\sigma_x}{\sigma_y}(y - \bar{y}) \rightarrow ①$$

$$y - \bar{y} = r\frac{\sigma_y}{\sigma_x}(x - \bar{x}) \rightarrow ②$$

$$r = \frac{Cov(x,y)}{\sigma_x \cdot \sigma_y} = \frac{\frac{\Sigma xy}{n} - \bar{x}\,\bar{y}}{\sqrt{\left(\frac{\Sigma x^2}{n} - (\bar{x})^2\right)\left(\frac{\Sigma y^2}{n} - (\bar{y})^2\right)}}$$

$\Rightarrow \bar{x} = \frac{21}{6} = 3.5$ $\Rightarrow \bar{x}^2 = 12.25$

$\Rightarrow \bar{y} = \frac{311}{6} = 51.83$ $\Rightarrow \bar{y}^2 = 2686.3489$

x	y	x^2	y^2	xy
1	14	1	196	14
2	33	4	1089	66
3	40	9	1600	120
4	63	16	3969	252
5	76	25	5776	380
6	85	36	7225	510
		91	19855	1342

$$\Rightarrow r = \frac{\frac{1342}{6} - 3.5 \times 51.83}{\sqrt{\left(\frac{91}{6} - 12.25\right)\left(\frac{19855}{6} - 2686.3489\right)}}$$

$$= 0.9915$$

~~Substitute the above val~~

$$\sigma_x = \sqrt{\frac{91}{6} - 12.25} = 1.7078$$

$$\sigma_y = \sqrt{\frac{19855}{6} - 2686.3489} = 24.9563$$

Substitute the above values in ① & ②

$$\Rightarrow ① \Rightarrow (x - 3.5) = (0.9915)\left(\frac{1.7078}{24.9563}\right)(y - 51.83) \rightarrow ③$$

$$② \Rightarrow (y - 51.83) = (0.9915)\left(\frac{24.9563}{1.7078}\right)(x - 3.5) \longrightarrow ④$$

Simplifying ③ we get

③ $\Rightarrow (x-3.5) = (0.0678)(y-51.83)$

$\Rightarrow x - 35 = 0.0678y - 3.5140$

$\Rightarrow \boxed{x = 0.0678y - 7.0140} \rightarrow ⑤$

Simplifying ④ we get

④ $\Rightarrow (y-51.83) = (14.4889)(x-3.5)$

$\Rightarrow y - 51.83 = 14.4889x - 50.7111$

$\Rightarrow \boxed{y = 14.4889x + 1.1189} \rightarrow ⑥$

For $x = 3.5$ we need to find y

Substitute $x = 3.5$ in ⑥

$\Rightarrow y = (14.4889)(3.5) + 1.1189$

$\Rightarrow \boxed{y = 51.83005}$

38)

x	1	2	3	4	5	6	7	8
y	1.0	1.2	1.8	2.5	3.6	4.7	6.6	9.1

Let the line of best fit is of the form

$y = a + bx \rightarrow \text{I}$

Its Normal Equations are

$$\sum y = na + b\sum x \rightarrow (1)$$

$$\sum xy = a\sum x + b\sum x^2 \rightarrow (2)$$

Calculation Table

x	y	x^2	xy
1	1.0	1	1
2	1.2	4	2.4
3	1.8	9	5.4
4	2.5	16	10
5	3.6	25	18
6	4.7	36	28.2
7	6.6	49	46.2
8	9.1	64	72.8
36	30.5	204	184

$\boxed{n=8}$

Substitute all the summation values in (1) & (2)

$(1) \Rightarrow 30.5 = 8a + 36b \rightarrow (3)$

$(2) \Rightarrow 184 = 36a + 204b \rightarrow (4)$

Solving (3) & (4) we get

$\boxed{a=-1.1964}$, $\boxed{b = 1.1130}$

Substituting values of a & b in (I), we get the line of best fit as

$$\boxed{y = -1.1964 + 1.1130\,x}$$

39)

Overtime (x)	1	1	2	2	3	3	4	5	6	7
Additional Units (y)	2	7	7	10	8	12	10	14	11	14

Let the parabola of best fit is of the form

$y = a + bx + cx^2 \longrightarrow$ I $\boxed{n=10}$

Its Normal Equations are

$\sum y = na + b\sum x + c\sum x^2 \longrightarrow$ ①

$\sum xy = a\sum x + b\sum x^2 + c\sum x^3 \longrightarrow$ ②

$\sum x^2 y = a\sum x^2 + b\sum x^3 + c\sum x^4 \longrightarrow$ ③

Calculation Table

x	y	x^2	x^3	x^4	xy	x^2y
1	2	1	1	1	2	2
1	7	1	1	1	7	7
2	7	4	8	16	14	28
2	10	4	8	16	20	40
3	8	9	27	81	24	72
3	12	9	27	81	36	108
4	10	16	64	256	40	160
5	14	25	125	625	70	350
6	11	36	216	1296	66	396
7	14	49	343	2401	98	686
34	95	154	820	4774	377	1849

Substitute the above values in 1, 2, 3

1 $\Rightarrow$ $95 = 10a + 34b + 154c \rightarrow$ 4

2 $\Rightarrow$ $377 = 34a + 154b + 820c \rightarrow$ 5

3 $\Rightarrow$ $1849 = 154a + 820b + 4774c \rightarrow$ 6

Solving 4, 5, 6 we get

$\boxed{a = 1.8022}$

$\boxed{b = 3.4822}$

$\boxed{c = -0.2689}$

Substituting the values of a, b, c in I we get parabola of best fit

$$\boxed{y = 1.8022 + 3.4822x - 0.2689x^2}$$

40)

x	1	5	7	8	12
y	10	15	12	15	21

Let the exponential curve of best ~~form~~ fit is of the form $y = ae^{bx} \rightarrow$ I

Take $\log_e$ on both sides

$\log_e y = \log_e a + \log_e e^{bx} \rightarrow$ 1

$Y = A + bx \rightarrow$ 2

where $Y = \log_e y$

$A = \log_e a \Rightarrow a = e^A$

Its Normal Equations are

$$\Sigma Y = nA + b\Sigma x \rightarrow (3)$$

$$\Sigma xY = A\Sigma x + b\Sigma x^2 \rightarrow (4)$$

Calculation Table

x	y	$Y = \log_e y$	x^2	xY
1	10	2.3025	1	2.3025
5	15	2.7080	25	13.54
7	12	2.4849	49	17.3943
8	15	2.7080	64	21.664
12	21	3.0445	144	36.534
33	73	13.2479	283	91.4348

$\boxed{n = 5}$

Substituting the above values in (3) & (4) :

(3) $\Rightarrow 13.2479 = 5A + 33b \rightarrow (5)$

(4) $\Rightarrow 91.4348 = 33A + 283b \rightarrow (6)$

Solving (5) & (6)

$\Rightarrow \boxed{A = 2.2448}$

$\boxed{b = 0.06132}$

Now $a = e^A$

$\Rightarrow \boxed{a = 9.4385}$

Substituting values of a & b in (I) we get the exponential curve of best fit as

$$\boxed{y = (9.4385)\, e^{0.06132x}}$$

28)

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
No. of accidents	14	18	12	11	15	14	84

① NH :- The accidents are uniformly distributed over the week

(OR)

$$H_0 : O_i = E_i$$

② AH :- $H_1 : O_i \neq E_i$

Expected Frequency of each day

$$E_i = \frac{14+18+12+11+15+14+84}{7} = 24$$

③ LOS :- $\alpha = 0.05$

④ CR :- $\chi^2_{0.05} = 12.592$ at $\nu = 7-1 = 6$ dof

Observed Frequency (O_i)	Expected Frequency (E_i)	$O_i - E_i$	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
14	24	-10	100	4.1666
18	24	-6	36	1.5
12	24	-12	144	6
11	24	-13	169	7.0416
15	24	-9	81	3.375
14	24	-10	100	4.1666
84	24	60	3600	150
				176.2498

⑤ Test statistic :- $\chi^2 = \sum_{i=1}^{7} \left[\frac{(O_i - E_i)^2}{E_i} \right] = 176.2498$

⑥ Conclusion :- $\chi^2 = 176.2498 > \chi^2_{0.05} = 12.592$ at 5% LOS

$\Rightarrow \chi^2_{cal} > \chi^2_{Tab}$

$\therefore$ Reject the NH at 5% LOS

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4) (a) Pure birth & death process :-

→ The term birth refers to the arrival of a new calling unit in the system & the term death refers to the departure of a served unit.

→ The model in which only arrivals are counted & no departure takes place is called pure birth process (or) model.

→ In a queuing model, counting between arrival & departure is called pure birth & death model.

→ When the arrivals are completely random then the probability of distribution of no. of arrivals in a fixed time interval follows a poisson distribution.

(b) Given $\mu = \frac{1}{30}$ per minute, $\lambda = \frac{10}{8} = \frac{10}{8 \times 60} = \frac{1}{48}$ per min

Expected no. of jobs $= L_s = \frac{\frac{\lambda}{\mu}}{1 - \frac{\lambda}{\mu}} = \frac{\lambda}{\mu - \lambda}$

$$= \frac{\frac{1}{48}}{\frac{1}{30} - \frac{1}{48}} = \frac{30}{18} = \frac{5}{3} = 1\frac{2}{3} \text{ jobs}$$

The fraction of time, the repairman is busy $= \rho = \frac{\lambda}{\mu}$

$$= \frac{30}{48}$$

The no. of hours for which the repairman is busy in an 8-hour day $= 8\left(\frac{\lambda}{\mu}\right) = 8 \times \frac{30}{48} = 5$ hours

$\therefore$ No. of hours the repairman is idle $= 8 - 5 = 3$ hours

42)

(a) The Queue Discipline :-

This concerns the way in which customers in the queue are served. The usual type that we come across methodically is "First Come First Serve". This is the general situation in queue to get tickets at cinema theatre, ticket counter in a railway station, Barber shop, Doctor's clinic, established car garages, etc. The queue discipline may be "Last In First out" as in the case of units stacked in a godown. The first to enter will be away from the main door & the last to enter will be nearer to the main door. When the items are to be taken out, the units nearer to the main door will be out first & those away will be out later.

Sometimes, the queue discipline may be random just like in the case of choosing a volunteer in a magic show. Here, the queue discipline may be To handle "service at random".

Sometimes, the service may be on priority basis just like in the case of treatment recieved to patients by a doctor based on their emergency. Here, the queue discipline is termed as "service with priority" or "priority queue".

(b) Given $\lambda = 40$ per hour

$\mu = 50$ per hour

Average waiting time of a student before recieving service $= W_q = \frac{\lambda}{\mu(\lambda-\mu)} = \frac{40}{50(50-40)}$

$= 0.08$ hour

$= 0.08 \times 60$

$= 4.8$ minutes

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43) Given $\lambda = 10$ per hour

$\mu = 12$ per hour

(i) Probability that the counter is free $= P_0 = 1 - \frac{\lambda}{\mu}$

$= 1 - \frac{10}{12}$

$= \frac{1}{6} = 0.1666$

(ii) Average number of customers in the queue

$$= L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{100}{12(12-10)} = 4.16$$

$\simeq 4$ customers

---

44) Given $\lambda = 30/\text{day} = \frac{30}{24} = 1.25$ per hour

$$\mu = \frac{1}{36} \times 60 = 1.66 / \text{hour.}$$

(a) Mean queue size $= L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{(1.25)^2}{1.66(1.66-1.25)}$

$= 2.295$

(b) Probability that there are more than 10 trains in the system $= \rho^{10+1} = \left(\frac{\lambda}{\mu}\right)^{10+1} = \left(\frac{1.25}{1.66}\right)^{11} = 0.0441$

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45) Given $\lambda = 5/\text{hour}$

$$\mu = \frac{1}{10} \times 60 = 6/\text{hour}$$

(a) Average number of customers in the shop $= L_s$

$$= \frac{\lambda}{\mu-\lambda}$$

$$= \frac{5}{6-5}$$

$= 5$ customers

(b) Average waiting time of a customer $= W_q$

$$= \frac{\lambda}{\mu(\mu-\lambda)}$$

$$= \frac{5}{6(6-5)}$$

$$= 0.833 \text{ hours}$$

(c) The percent of time a person on arrival can walk right without having to wait $= P_0 = \left(1-\frac{\lambda}{\mu}\right) \times 100$

$$= \left(1-\frac{5}{6}\right) \times 100$$

$$= \frac{1}{6} \times 100 = 16.66\%$$

(d) Probability of a customer waiting for more than 5 minutes $= \left(\frac{\lambda}{\mu}\right) e^{(\lambda-\mu)W}$

$$\left[\text{Here } W = 5 \text{ min} = \frac{5}{60} = \frac{1}{12} \text{ hour}\right]$$

$$= \left(\frac{5}{6}\right) e^{(5-6)\left(\frac{1}{12}\right)}$$

$$= 0.7667.$$

46) Given $\lambda = 3$ units / hours

$$\mu = \frac{1}{0.25} = \frac{100}{25} = 4 \text{ units / hour}$$

Traffic intensity $= \rho = \frac{\lambda}{\mu} = \frac{3}{4}$

No. of units allowed in the system $= N = 2$

Let P_n be the probability for n units to be in the system

$$\Rightarrow P_0 = \frac{1-\rho}{1-\rho^{N+1}} = \frac{1-\frac{3}{4}}{1-\left(\frac{3}{4}\right)^3} = \frac{\frac{1}{4}}{1-\frac{27}{64}} = \frac{16}{37}$$

$$\boxed{P_N = \rho^N P_0}$$

$= 0.4324$

$$\Rightarrow P_1 = \rho . P_0 = \frac{3}{4} \times \frac{16}{37} = \frac{12}{37} = 0.3243$$

$$\Rightarrow P_2 = \rho^2 . P_0 = \frac{9}{16} \times \frac{16}{37} = \frac{9}{37} = 0.2432$$

Note :- $P_0 + P_1 + P_2 = 1$ (Here)

The expected number in the system

$$= E(n) = \sum_{n=0}^{2} n P_n = 0 P_0 + 1 . P_1 + 2 P_2$$

$$= \frac{12}{37} + \frac{18}{37}$$

$$= \frac{30}{37}$$

There will be $\frac{30}{37}$ units on average in the system

47) Given $\lambda = 6$ per hour

$\mu = 12$ per hour

Traffic intensity $= \rho = \frac{\lambda}{\mu} = \frac{1}{2}$

While one train is served only 2 can wait

Hence, the maximum number of Trains allowed in the system $= N = 1 + 2 = 3$

Probability that there is no Train in the system $= P_0 = \frac{1-\rho}{1-\rho^{N+1}} = \frac{1-\frac{1}{2}}{1-\left(\frac{1}{2}\right)^4} = \frac{8}{15} = 0.5333$

We know that $\boxed{P_n = \rho^n P_0}$

$\Rightarrow P_1 = \rho P_0 = \frac{1}{2} \times \frac{8}{15} = \frac{4}{15} = 0.2666$

$\Rightarrow P_2 = \rho^2 P_0 = \left(\frac{1}{2}\right)^2 \times \frac{8}{15} = \frac{2}{15} = 0.1333$

$\Rightarrow P_3 = \rho^3 P_0 = \left(\frac{1}{2}\right)^3 \times \frac{8}{15} = \frac{1}{15} = 0.0666$

Note: $P_0 + P_1 + P_2 + P_3 = 1$ (Here)

Average no. of Trains in the system $= E(n) = \sum_{n=1}^{3} n P_n$

$= 1P_1 + 2P_2 + 3P_3$

$= \frac{4}{15} + 2 \times \frac{2}{15} + 3 \times \frac{1}{15} = \frac{11}{15}$

$\Rightarrow$ On average there are $\frac{11}{15}$ trains in the system.

It is given that each train on average takes

$\frac{1}{12}$ hours $\times 60 = 5$ minutes

$\because$ A new arrival expects $\frac{11}{15}$ trains in the system,

the expected waiting time of a new train

entering into the system $= \frac{11}{15} \times 5 = \frac{11}{3}$ minutes

$= 3.666$ minutes

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48) Given $\lambda = \frac{1}{20}$ per minute

$\mu = \frac{1}{36}$ per minute

Traffic intensity $= \rho = \frac{\lambda}{\mu} = \frac{\frac{1}{20}}{\frac{1}{36}} = \frac{36}{20} = 1.8$

$N = 4$ trains

(a) Probability that the yard is empty is given by

$$P_0 = \frac{1-\rho}{1-\rho^{N+1}} = \frac{1-(1.8)}{1-(1.8)^{4+1}} = 0.0447 = \frac{625}{13981}$$

(b) Average no. of trains in the system is given

by $L_s = \sum_{n=0}^{4} n P_n$

We know that $\boxed{P_n = \rho^n P_0}$

$\Rightarrow P_1 = 1.8 \times \frac{625}{13981} = \frac{1125}{13981}$

$\Rightarrow P_2 = 1.8^2 \times \frac{625}{13981} = \frac{2025}{13981}$

$\Rightarrow P_3 = 1.8^3 \times \frac{625}{13981} = \frac{3645}{13981}$

$\Rightarrow P_4 = 1.8^4 \times \frac{625}{13981} = \frac{6561}{13981}$

$\Rightarrow L_s = \sum_{n=0}^{4} n P_n$

$= 0P_0 + 1P_1 + 2P_2 + 3P_3 + 4P_4$

$= 0 + \frac{1125}{13981} + \frac{2 \times 2025}{13981} + \frac{3 \times 3645}{13981}$

$+ \frac{4 \times 6561}{13981}$

$= 3.029 \simeq 3$ trains

---

49) Given $\lambda = 3$ customers per hour

$\mu = \frac{1}{15}$ per minute $= \frac{1}{15} \times 60 = 4$ customers per hour

$N =$ Capacity of the system

$=$ Chairs to accomodate waiting people + one chair in service $= 6 + 1 = 7$

Traffic Intensity $= \rho = \frac{\lambda}{\mu} = \frac{3}{4}$

(a) Probability that a customer can get directly into the barber chair upon arrival can be given by

$$P_0 = \frac{1-\rho}{1-\rho^{N+1}} = \frac{1-\frac{3}{4}}{1-\left(\frac{3}{4}\right)^{7+1}} = 0.2778$$

(b) Expected no. of customers waiting for haircut is given by $L_q = L_s - \frac{\lambda}{\mu}$

$$= P_0 \sum_{n=0}^{N} n \rho^n - \frac{\lambda}{\mu}$$

$$= (0.2778) \times \sum_{n=0}^{7} n \left(\frac{3}{4}\right)^n - \frac{3}{4}$$

$$= 1.3598$$

(c) Effective arrival rate $=$ (service rate) $(1-P_0)$

$$= \mu(1-P_0)$$

$$= 4(1-0.2778)$$

$$= 2.89 \simeq 3 \text{ per hour}$$

(d) The time a customer can expect to spend in the barbershop = $W_s = \frac{L_s}{\lambda} = \frac{P_0 \sum_{n=0}^{7} n \rho^n}{\lambda}$

$$= \frac{(0.2778) \times \sum_{n=0}^{7} n \times \left(\frac{3}{4}\right)^n}{3}$$

$$= 0.7032 \text{ hours.}$$

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50) Given $\lambda = \frac{1}{15}$ per minute

$\mu = \frac{1}{33}$ per minute

$N = 4$

Traffic intensity $= \rho = \frac{\lambda}{\mu} = \frac{\frac{1}{15}}{\frac{1}{33}} = \frac{11}{5} = 2.2$

(a) Probability that the yard is empty is given by

$$P_0 = \frac{1-\rho}{1-\rho^{N+1}} = \frac{1-2.2}{1-(2.2)^{4+1}} = 0.0237$$

(b) The average number of trains in the system is given by $L_s = P_0 \sum_{n=0}^{4} n \rho^n$

$$= (0.0237) \times \sum_{n=0}^{4} n \times (2.2)^n$$

$$= 3.2593$$

2) (a) Discrete Random Variable :- A random variable X which can take only a finite number of discrete values in an interval of domain is called discrete random variable. In other words, if the random variable takes the values only on the set {0, 1, 2, ..., n} is called a Discrete Random Variable.

Continuous Random Variable :- A random variable X which can take values continuously, i.e, which takes all possible values in a given interval is called a Continuous Random Variable.