UNIT - I

1. State Rolle's, Lagrange's and Cauchy's mean value theorems.
2. State Geometrical interpretation of Rolle's Theorem and Lagrange's theorem.
3. Verify Rolle's theorem for $x(x+3)e^{-x/2}$ in [-3, 0]
4. Using mean value theorem S.T. $g(x)=8x^3-6x^2-2x+1$ has a zero between 0 and 1.
5. Prove that $\frac{\pi}{4}+\frac{3}{25}<\tan^{-1}\frac{4}{3}<\frac{\pi}{4}+\frac{1}{6}$ using Lagrange's Mean Value Theorem.
6. Prove that using mean value theorem $x>\log(1+x)>\frac{x}{1+x},\ if\ x>0$.
7. Show That $\log(1+e^x)=\log 2+\frac{x}{2}+\frac{x^2}{8}-\frac{x^4}{192}+-------$.
8. Verify Cauchy's Mean value theorem for $f(x)=\sqrt{x},\ g(x)=\frac{1}{\sqrt{x}}$ in [a, b].

UNIT - II

9. If $u=x+y+z, uv=y+z, uvw=z$ *then* show that $JJ'=1$.
10. If $u=x^2-y^2, v=2xy\ where\ x=r\cos\theta, y=r\sin\theta$ then find $J\left(\frac{u,v}{r,\theta}\right)$.
11. If $x=r\cos\theta\ \ and\ \ y=r\sin\theta\ \ then$ show that $JJ^1=1$.
12. Check whether the following functions are functionally dependent or not if so find the relation between them where $u=\frac{x^2-y^2}{x^2+y^2}, v=\frac{2xy}{x^2+y^2}$
13. Divide 24 into three parts such that the continued product of the first, square of the second and cube of the third is maximum.
14. A rectangular box open at the top is to have volume of 32 cubic feet, Find the dimensions of the box requiring least material for its construction.
15. Find the extreme values of $f(x,y)=\sin x+\sin y+\sin(x+y)$.
16. Find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid $4x^2+4y^2+9z^2=36$.

UNIT - III

17. Evaluate $\iint_R xy\,dx\,dy$ where R is the region bounded by x-axis, $x = 2a$ and the curve $x^2 = 4ay$

18. Evaluate $\iint_R (x+y)\,dy\,dx$ where the region is bounded by xy=6 and x+y=7

19. Evaluate $\iint_R dx\,dy$ over the area bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

20. Change the order of integration and Evaluate $\int_0^1\int_y^{2-y} xy\,dx\,dy$

21. Change the order of integration and Evaluate $\int_0^1\int_{x^2}^{2-x} xy\,dy\,dx$

22. Change the order of integration and Evaluate $\int_0^a\int_y^a \frac{x}{x^2+y^2}\,dx\,dy$

23. Evaluate $\int_1^3\int_{-x+2}^{x} (2x+1)\,dy\,dx$ by changing order of integration.

24. Evaluate $\int_0^{4a}\int_{\frac{y^2}{4a}}^{y} \frac{x^2-y^2}{x^2+y^2}\,dx\,dy$ by changing into polar coordinates.

25. Evaluate $\iint_R dx\,dy$ where R is the parallelogram in the xy-plane with vertices (1,0), (3,1), (2,2) and (0,1) by using transformation u = x + y and v = x – 2y.

26. Find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid $4x^2+4y^2+9z^2=36$.

27. Evaluate (a) $\int_{-1}^{1}\int_0^z\int_{x-z}^{x+z}(x+y+z)\,dx\,dy\,dz$ (b) $\int_0^1\int_1^2\int_2^3(x+y+z)\,dx\,dy\,dz$

28. Find the volume of the surface in first octant and bounded by $2x+3y+4z=12$

29. Evaluate $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} \frac{dx\,dy\,dz}{\sqrt{1-x^2-y^2-z^2}}$ by changing to spherical coordinates.

30. Evaluate $\iiint (x^2+y^2+z^2)\,dx\,dy\,dz$ taken over the volume enclosed by the sphere $x^2+y^2+z^2=1$

UNIT–IV

31. Find directional derivative of the function $f = 4xy^2 + 2x^2yz$ at the point P (1, 2, 3) in the direction of PQ where Q (5, 0, 4).
32. Prove that $r^n\overline{r}$ is Solenoidal if n=-3.
33. Prove that $\nabla(r^n) = nr^{n-2}\overline{r}$.
34. Prove that $div(gradr^m) = m(m+1)r^{m-2}$ (or) $\nabla^2(r^m) = m(m+1)r^{m-2}$
35. Prove that $r^n\overline{r}$ is irrotational.
36. Find constants a, b, c so that the vector $\overline{F} = (x+2y+az)\ i + (b\ x - 3y - z)\ j + (4x+cy+2z)\ k$ is irrotational.
37. If $\overline{f} = (x+3y+z)i + (2py-z)j - (xy+3z)k$ is solenoidal, then find p.
38. Find directional derivative of $xyz^2 + xz$ at $(1,1,1)$ in the direction of normal to the surface $3xy^2 + y = z$ at $(0,1,1)$.
39. If the temperature at any point in space is given by $t = xy + yz + zx$. Find rate of change and determine the maximum rate of change.
40. Find the angle of intersection of spheres $x^2 + y^2 + z^2 = 29$ and $x^2 + y^2 + z^2 + 4x - 6y - 8z = 47$ at the point (4,-3,2).
41. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2, -1, 2).
42. Prove that $\overline{F} = (2xyz^2)\overline{i} + (x^2z^2 + z\cos yz)\overline{j} + (2x^2yz + y\cos yz)\overline{k}$ is conservative. Find its scalar potential.

UNIT–V

43. If $\overline{f} = (x^2 + y^2)i - 2xyj$, evaluate $\int_c \overline{f}.\overline{dr}$ where C is the rectangle in xy-plane bounded by x=0, x=a, y=0, y=b.
44. Find the work done by the force $\overline{F} = (x^2 - y^2 + x)\overline{i} + (2xy + y)\overline{j}$ which moves a particle in xy-plane from (0, 0) to (1,1) along the curve $y^2 = x$.
45. Find the work done by the force $\overline{F} = (x^2 - yz)\overline{i} + (y^2 - zx)\overline{j} + (z^2 - xy)\overline{k}$ in taking a particle from (1, 1, 1) to (3, -5, 7).
46. Evaluate $\iint_s \overline{F}.\hat{n}ds$ where $\overline{F} = z\ \overline{i} + x\overline{j} - 3y^2z\ \overline{k}$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between z=0 and z=5.
47. Verify Green's theorem for $\int_c (xy + y^2)dx + x^2dy$ Where C is bounded by $y = x$ and $y = x^2$.
48. Verify Green's theorem for $\int_c (2xy - x^2)dx + (x^2 + y^2)dy$ Where C is bounded by $x = y^2$ and $y = x^2$.
49. Verify Gauss Divergence theorem for the function $\overline{F} = 4x\overline{i} - 2y^2\overline{j} - z^2\overline{k}$ taken over the surface bonded by the cylinder $x^2 + y^2 = 4$, $z = 0$ and $z = 3$.
50. Verify Gauss theorem for $\overline{f} = 4xzi - y^2j + yzk$ taken over the surface of the cube bounded by x=0, x=a, y=0, y=a, z=0, z=a.
51. Evaluate $\int curl\overline{f} \cdot \hat{n}ds$ where $\overline{f} = (x^2 + y - 4)\overline{i} + 3xy\overline{j} + (2xz + z^2)\overline{k}$, where S is the surface of the semi sphere $x^2 + y^2 + z^2 = 16$ above the xy-plane.
52. Verify Stoke's theorem for $\overline{F} = (2x - y)\overline{i} - yz^2\overline{j} - y^2z\overline{k}$ over upper half of the surface of the sphere $x^2 + y^2 + z^2 = 1$ bounded by the projection in xy-plane.