Knowledge Representation

Knowledge Representation:

- Knowledge representation is a subject in **cognitive science** as well as in **artificial intelligence and knowledge modeling.**
- In cognitive science it is concerned with how people **store and process information.**
- In **artificial intelligence (AI) and knowledge modeling (KM)** it is a way to store **knowledge** so that **programs can process it and use it**, for example to support computer-aided design or to emulate human intelligence.
- AI researchers have borrowed representation theories from cognitive science.

Knowledge Representation Issues

- The aim is to show how **logic can be used to form representations** of the world and how a **process of inference** can be used to derive **new representations about the world** and how these can be used by an **intelligent agent to deduce what to do.**

We require:

- A ***formal language*** to represent knowledge in a computer tractable form.
- ***Reasoning*** - Processes to manipulate this knowledge to deduce non-obvious facts.

- **Knowledge:** Knowledge is **awareness or familiarity gained by experiences of facts, data, and situations.**

Following are the types of knowledge in artificial intelligence:

1. **Declarative Knowledge:**

- Declarative knowledge is to know about something.
- It includes concepts, facts, and objects.

2. **Procedural Knowledge**

- It is also known as imperative knowledge.
- Procedural knowledge is a type of knowledge which is responsible for knowing how to do something.
- It includes rules, strategies, procedures, agendas, etc.
- Procedural knowledge depends on the task on which it can be applied.

3. Meta-knowledge:

- Knowledge about the other types of knowledge is called Meta-knowledge.

4. Heuristic knowledge:

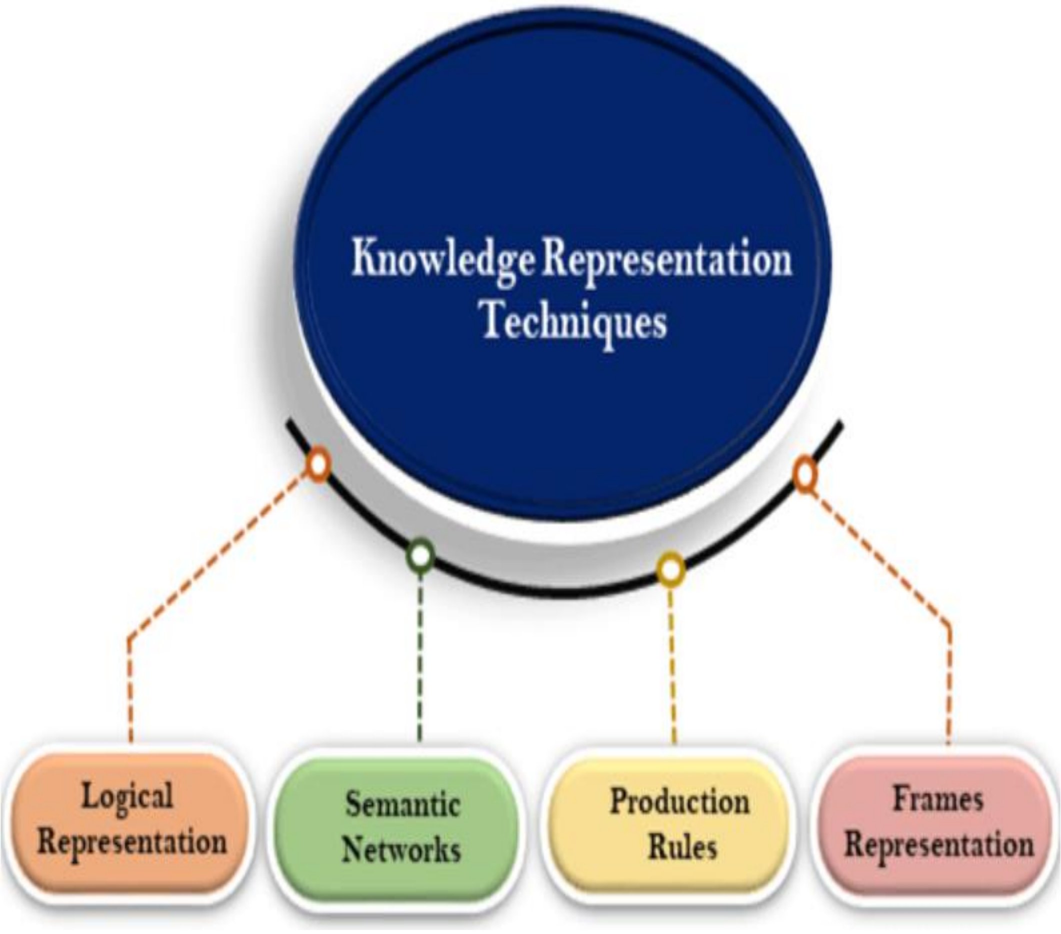
- Heuristic knowledge is representing knowledge of some experts in a filed or subject.

5. Structural knowledge:

- Structural knowledge is basic knowledge to problem-solving.
- It describes relationships between various concepts such as kind of, part of, and grouping of something.

- There are mainly four ways of knowledge representation which are given as follows

1. **Logical Representation**
2. **Semantic Network Representation**
3. **Frame Representation**
4. **Production Rules**



Why logic?

- The challenge is to design a language which allows one to represent all the necessary knowledge.
- **Logic** makes statements about the world which are true (or false) if the state of affairs it represents is the case (or not the case).
- Compared to natural languages (expressive but context sensitive) and programming languages (good for concrete data structures but not expressive) **logic combines the advantages of natural languages and formal languages.**

Logical Representation

- Logical representation is a language with **some concrete rules** which deals with **propositions** and has **no ambiguity in representation**.
- Logical representation means **drawing a conclusion based on various conditions.**
- This representation lays down some **important communication rules**.
- It consists of precisely defined **syntax and semantics** which supports the sound inference.
- Each sentence can be translated into **logics using syntax and semantics.**

Syntax:

- Syntaxes are the rules which decide how we can **construct legal sentences in the logic.**
- It determines which **symbol we can use in knowledge representation**.
- How to **write those symbols.**

Semantics:

- Semantics are the rules by which **we can interpret the sentence in the logic.**
- Semantic also involves **assigning a meaning to each sentence.**

Logical representation can be categorized into mainly two logics:

1. **Propositional Logics**
2. **Predicate logics**

Propositional logic (PL)

- Propositional logic (PL) is the simplest form of logic where all the statements are made by propositions.
- A proposition is a declarative statement which is either true or false.
- It is a technique of knowledge representation in logical and mathematical form.

Example:

a) It is Sunday.

b) The Sun rises from West (False proposition)

c) 3+3= 7(False proposition)

d) 5 is a prime number.

Following are some basic facts about propositional logic:

- Propositional logic is also called **Boolean logic** as it works **on 0 and 1.**
- In propositional logic, we use **symbolic variables** to represent the **logic**, such **A, B, C, P, Q, R, etc.**
- Propositions can be either **true or false**, but it cannot be both.
- Propositional logic consists of an **object, relations or function**, and **logical connectives**.
- Connectives can be said as **a logical operator** which connects two sentences.
- A proposition formula which is always true is called **tautology**, and it is also called a valid sentence.
- A proposition formula which is always false is called **Contradiction**.
- Statements which are questions, commands, or opinions are not propositions such as "**Where is Rohini**", "**How are you**", "**What is your name**", are not propositions.

Syntax of propositional logic:

- The syntax of propositional logic defines the **allowable sentences for the knowledge representation**. There are two types of Propositions:
- **Atomic Propositions**
- **Compound propositions**

- **Atomic Proposition:** Atomic propositions are the simple propositions. It consists of a single proposition symbol. These are the sentences which must be either true or false.

Example:

a) 2+2 is 4, it is an atomic proposition as it is a **true** fact.

b) "The Sun is cold" is also a proposition as it is a **false** fact.

- **Compound proposition:** Compound propositions are constructed by **combining simpler or atomic propositions,** using parenthesis and logical connectives.

Example:

a) "It is raining today, and street is wet."

b) "Ankit is a doctor, and his clinic is in Mumbai."

Logical Connectives:

- Logical connectives are used to connect **two simpler propositions** or **representing a sentence logically.** We can create compound propositions with the help of **logical connectives.**

There are mainly five connectives, which are given as follows:

- **Negation:** A sentence such as $\neg$ P is called negation of P. A literal can be either Positive literal or negative literal.

- **Conjunction:** A sentence which has ∧ connective such as, **P ∧ Q** is called a conjunction.
  Example: Rohan is intelligent and hardworking. It can be written as,
  P= Rohan is intelligent,
  Q= Rohan is hardworking. → **P∧ Q.**

- **Disjunction:** A sentence which has ∨ connective, such as **P ∨ Q**. is called disjunction, where P and Q are the propositions.
  Example: "Ritika is a doctor or Engineer",
  Here P= Ritika is Doctor. Q= Ritika is Doctor, so we can write it **as P ∨ Q.**

- **Implication:** A sentence such as P → Q, is called an implication. **Implications are also known as if-then rules. It can be represented as**
  If it is raining, then the street is wet.
  Let P= It is raining, and Q= Street is wet, so it is represented as **P → Q**

- **Biconditional:** A sentence such as **P⇔ Q is a Biconditional sentence, example If I am breathing, then I am alive**

  P= I am breathing, Q= I am alive, it can be represented as **P ⇔ Q.**

Precedence of connectives:

- Parenthesis, Negation, Conjunction(AND), Disjunction(OR), Implication, Biconditional

Rules of Inference

Inference:

- In artificial intelligence, we need intelligent computers which can create new logic from old logic or by evidence, **so generating the conclusions from evidence and facts is termed as Inference**.

Inference rules:

- Inference rules are the templates for generating valid arguments.
- **Inference rules are applied to derive proofs in artificial intelligence, and the proof is a sequence of the conclusion that leads to the desired goal.**
- In inference rules, the **implication** among all the **connectives plays an important role.** Following are some terminologies related to inference rules:

- **Implication:** It is one of the logical connectives which can be represented as $\mathbf{P \rightarrow Q}$. It is a Boolean expression.
- **Converse:** The converse of implication, which means the right-hand side proposition goes to the left-hand side and vice-versa. It can be written as $\mathbf{Q \rightarrow P}$.
- **Contrapositive:** The negation of converse is termed as contrapositive, and it can be represented as $\mathbf{\neg Q \rightarrow \neg P}$.
- **Inverse:** The negation of implication is called inverse. It can be represented as $\mathbf{\neg P \rightarrow \neg Q}$.

Types of Inference rules:

1. **Modus Ponens:**

- The Modus Ponens rule is one of the most important rules of inference, and it states that if **P and P → Q** is true, then we can infer that Q will be true. It can be represented as:

Notation for Modus ponens: $$\frac{P \rightarrow Q, \quad P}{\therefore Q}$$

Example:

Statement-1: "If I am sleepy then I go to bed" ==> P→ Q
Statement-2: "I am sleepy" ==> P
Conclusion: "I go to bed." ==> Q.
Hence, we can say that, if P→ Q is true and P is true then Q will be true.

2.**Modus Tollens:**

- The Modus Tollens rule state that if P$\rightarrow$ Q is true and **¬ Q is true, then ¬ P** will also true. It can be represented as:

Notation for Modus Tollens: $\dfrac{P \rightarrow Q, \ \sim Q}{\sim P}$

Example

Statement-1: "If I am sleepy then I go to bed" ==> P$\rightarrow$ Q
Statement-2: "I do not go to the bed."==> ~Q
Statement-3: Which infers that "**I am not sleepy**" => ~P

3. **Hypothetical Syllogism:**

- The Hypothetical Syllogism rule state that if P→R is true whenever P→Q is true, and Q→R is true. It can be represented as the following notation:
- **Example:**

Statement-1: If you have my home key then you can unlock my home. **P→Q**
Statement-2: If you can unlock my home then you can take my money. **Q→R**
Conclusion: If you have my home key then you can take my money. **P→R**

4. **Disjunctive Syllogism:**

- The Disjunctive syllogism rule state that if $P \vee Q$ is true, and $\neg P$ is true, then Q will be true. It can be represented as:

Notation of Disjunctive syllogism: $\dfrac{P \vee Q, \quad \neg P}{Q}$

- **Example:**

Statement-1: Today is Sunday or Monday. ==>$P \vee Q$
Statement-2: Today is not Sunday. ==> $\neg P$
Conclusion: Today is Monday. ==> Q

5. Addition:

- The Addition rule is one the common inference rule, and it states that If P is true, then P∨Q will be true.

Notation of Addition: $\dfrac{P}{P \lor Q}$

6. Simplification:

- The simplification rule state that if **P∧ Q** is true, then **Q or P** will also be true. It can be represented as:

Notation of Simplification rule: $\dfrac{P \land Q}{Q}$ Or $\dfrac{P \land Q}{P}$

7. Resolution:

- The Resolution rule state that if P∨Q and ¬ P∧R is true, then Q∨R will also be true. **It can be represented as**

Notation of Resolution $$\frac{P \lor Q, \quad \neg P \land R}{Q \lor R}$$

First-Order logic:

- First-order logic is another way of knowledge representation in artificial intelligence. It is an extension to propositional logic.
- First-order logic is also known as **Predicate logic or First-order predicate logic**.
- First-order logic is a powerful language that develops **information** about the **objects in a more easy way** and can also **express the relationship between those objects.**

- **First-order logic** (like natural language) does not only assume that the world contains facts like propositional logic but also assumes the following things in the world:
  - **Objects:** A, B, people, numbers, colors, wars, theories, squares, pits, wumpus, ...
  - **Relations: It can be unary relation such as:** red, round, is adjacent, **or n-any relation such as:** the sister of, brother of, has color, comes between
  - **Function:** Father of, best friend, third inning of, end of, ...

- As a natural language, first-order logic also has two main parts:
  - **Syntax**
  - **Semantics**

Syntax of First-Order logic:

- The syntax of FOL determines which **collection of symbols is a logical expression** in first-order logic. The basic syntactic elements of first-order logic are **symbols**. We **write statements in short-hand notation in FOL**.

Basic Elements of First-order logic:

- Following are the basic elements of FOL syntax:

Constant	1, 2, A, John, Mumbai, cat,....
Variables	x, y, z, a, b,....
Predicates	Brother, Father, >,....
Function	sqrt, LeftLegOf, ...
Connectives	∧, ∨, ¬, ⇒, ⇔
Equality	==
Quantifier	∀, ∃

Atomic sentences:

- Atomic sentences are the most basic sentences of first-order logic. These sentences are formed from a **predicate symbol** followed by a **parenthesis with a sequence of terms.**
- We can represent atomic sentences as **Predicate (term1, term2, ..., term n)**.
- **Example: Ravi and Ajay are brothers: => Brothers(Ravi, Ajay).**
  Chinky is a cat: => cat (Chinky).

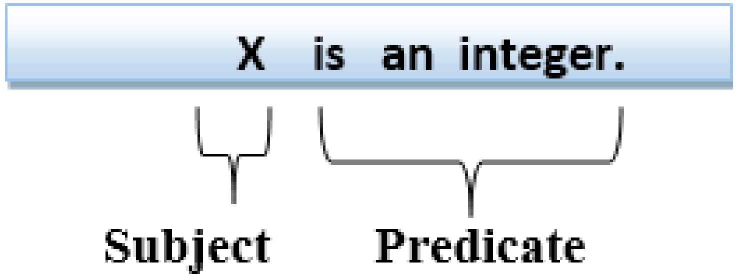
Complex Sentences:

- Complex sentences are made by combining atomic sentences using connectives.

First-order logic statements can be divided into two parts:

- **Subject:** Subject is the main part of the statement.
- **Predicate:** A **predicate can be defined as a relation,** which binds **two atoms together in a statement.**

- **Consider the statement: "x is an integer."**, it consists of two parts, the first part x is the **subject of the statement** and second part "is an integer," **is known as a predicate.**



- **All Boys like cricket**

predicate **like(boys,cricket)** **parameters**

- **Some boys like cricket**

like(boys,cricket)

Quantifiers in First-order logic:

- A **quantifier is a language element** which generates **quantification**, and **quantification specifies the quantity of specimen in the universe of discourse.**
- *****Note: Quantification in logic refers to the use of quantifiers to express statements about the quantity of objects that satisfy a certain property.*****
- These are the symbols that permit to determine or identify the range and scope of the variable in the logical expression. There are two types of quantifier:

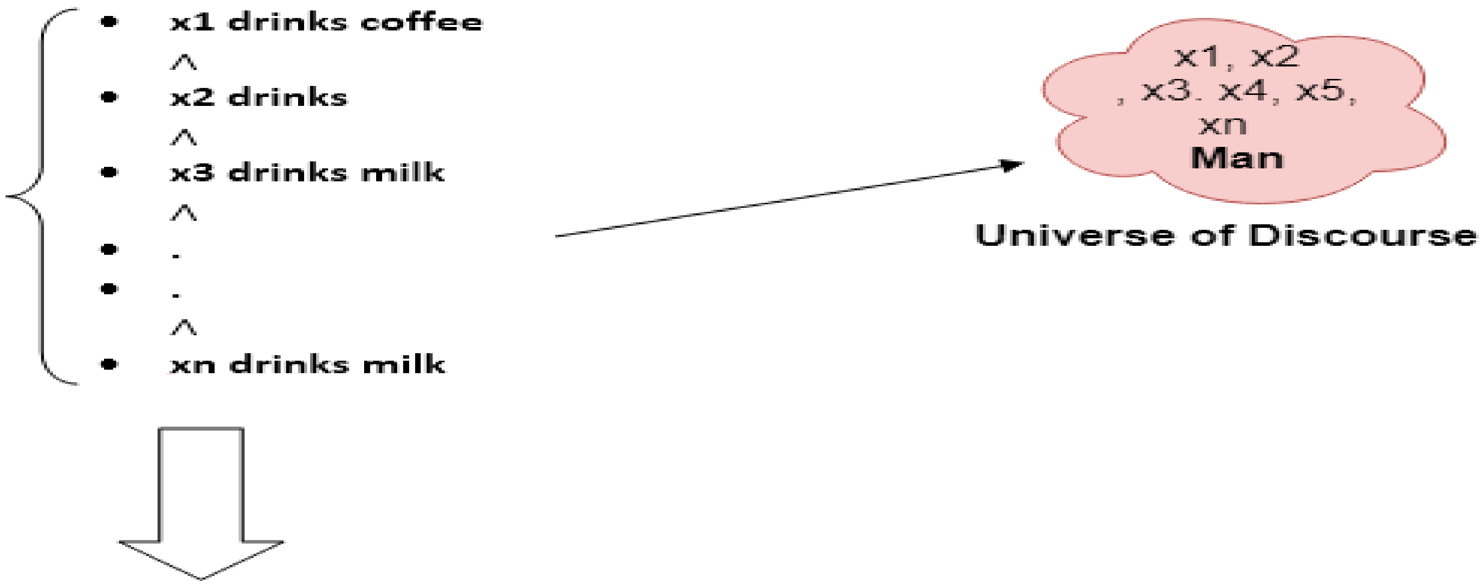
  - **Universal Quantifier, (for all, everyone, everything)**
  - **Existential quantifier, (for some, at least one).**

Universal Quantifier:

- Universal quantifier is a symbol of logical representation, which specifies that the statement within its range is true for everything or every instance of a particular thing.
- The Universal quantifier is represented by a symbol ∀, which resembles an inverted A.
- If x is a variable, then ∀x is read as:
- **For all x**
- **For each x**
- **For every x**

Example:

- **All man drink coffee.**



So in shorthand notation, we can write it as :

$\forall x \; man(x) \rightarrow drink\,(x, coffee).$

It will be read as: There are all x where x is a man who drink coffee.

Existential Quantifier:

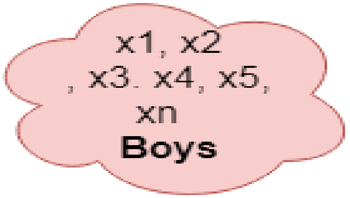
- Existential quantifiers are the type of quantifiers, which express that the statement within its scope is true for at least one instance of something.
- It is denoted by the logical operator ∃, which resembles as inverted E. When it is used with a predicate variable then it is called as an existential quantifier.

- If x is a variable, then existential quantifier will be ∃x or ∃(x). And it will be read as:
- **There exists a 'x.'**
- **For some 'x.'**
- **For at least one 'x.'**

Example:

Some boys are intelligent.

- **x1 is intelligent**
  V
- **x2 is intelligentV**
- **x3 is intelligent**
  V
- **.**
- **.**
  V
- **xn is intelligent**



Universe of Discourse

So in short-hand notation, we can write it as:

∃x: boys(x) ∧ intelligent(x)

It will be read as: There are some x where x is a boy who is intelligent.

Properties of Quantifiers:

- **In universal quantifier, ∀x∀y is similar to ∀y∀x.**
- **In Existential quantifier, ∃x∃y is similar to ∃y∃x.**
- **∃x∀y is not similar to ∀y∃x.**

Some Examples of FOL using quantifier:

1. All birds fly.
In this question the predicate is "**fly(bird)**."
And since there are all birds who fly so it will be represented as follows.
∀x bird(x) →fly(x).

2. Every man respects his parent.
In this question, the predicate is "**respect(x, y)," where x=man, and y= parent**.
Since there is every man so will use ∀, and it will be represented as follows:
∀x man(x) → respects (x, parent).

3. Some boys play cricket.
In this question, the predicate is "**play(x, y)**," where x= boys, and y= game. Since there are some boys so we will use **∃, and it will be represented as**:
∃x boys(x) → play(x, cricket).

- **4. Not all students like both Mathematics and Science.**
  In this question, the predicate is "**like(x, y),**" **where x= student, and y= subject**.
  Since there are not all students, so we will use **∀ with negation, so** following representation for this:
  $\neg\forall (x)\ [\ student(x) \rightarrow like(x, Mathematics) \wedge like(x, Science)]$.
- **5. Only one student failed in Mathematics.**
  In this question, the predicate is "**failed(x, y),**" **where x= student, and y= subject**.
  Since there is only one student who failed in Mathematics, so we will use following representation for this:

$$\exists x(F(x) \wedge \forall y(F(y) \rightarrow y=x))$$

- **$\exists\ x$: There exists a student x.**
- **$F(x)$: x failed in Mathematics.**
- **$\forall y\ (F(y) \rightarrow y=x)$: For any student y, if y failed in Mathematics, then y must be the same student as x.**

PL

- It uses prepositions in which complete sentence is denoted by symbol.
- Pl Cannot represent individual entities Eg: meera is short.
- It cannot express generalization, specialization or pattern.

  ex : triangles have 3 sides.

FOL

- Fol uses predicates which involve constants, variables, functions relations
- Fol can represent individual properties {short(meera)}
- It can express generalization, specialization or pattern.
- Ex: no_of_sides(triangle,3)

- We use the rules to derive the soundness of the argument in Example.
- Again let the following propositions stand for these statements about the world:
- **P: Rani writes books.**
- **Q: Rani helps other people to write books.**
- **R: Rani earns her living as an editor**

The following derivation determines Rani's occupation:

	Derivation	Rule
1	$P \vee Q$	Premise
2	$\neg P$	Premise
3	$Q \Rightarrow R$	Premise
4	Q	1, 2, DS
5	R	3, 4, MP

****Premises are the assumptions or facts that are assumed to be true within the context of the argument****

- Let the following propositions stand for these statements about the world:

- **P: Robbery was the reason for the murder.**
- **Q: Something was taken.**
- **R: Politics was the reason for the murder.**
- **S: A woman was the reason for the murder.**
- **T: The murderer left immediately.**
- **U: The murderer left tracks all over the room.**

- The following derivation determines the **reason for the murder**

FOPL Representation:

We can express the relationships between these propositions using implication statements:

1. $P \rightarrow Q$: If robbery was the reason for the murder, then something was taken.
   - FOPL: $\forall x\ (P(x) \rightarrow Q(x))$
2. $R \rightarrow T$: If politics was the reason for the murder, then the murderer left immediately.
   - FOPL: $\forall x\ (R(x) \rightarrow T(x))$
3. $S \rightarrow U$: If a woman was the reason for the murder, then the murderer left tracks all over the room.
   - FOPL: $\forall x\ (S(x) \rightarrow U(x))$

Given Observations (Assumptions):

- $\neg Q$: Something was not taken.
  - FOPL: $\neg Q(x)$
- $\neg T$: The murderer did not leave immediately.
  - FOPL: $\neg T(x)$
- U: The murderer left tracks all over the room.
  - FOPL: $U(x)$

Logical Reasoning:

Let's use the above observations to deduce the cause of the murder.

1. **Analyze $P \to Q$ and $\neg Q$:**
   - Premise: $P \to Q$ (If robbery was the reason, something was taken).
   - Observation: $\neg Q$ (Something was not taken).

   Modus Tollens:

   - Since Q is false ($\neg Q$), P must be false ($\neg P$).
   - **Conclusion:** Robbery was **not** the reason for the murder.
2. **Analyze $R \to T$ and $\neg T$:**
   - Premise: $R \to T$ (If politics was the reason, the murderer left immediately).
   - Observation: $\neg T$ (The murderer did not leave immediately).

   Modus Tollens:

   - Since T is false ($\neg T$), R must be false ($\neg R$).
   - **Conclusion:** Politics was **not** the reason for the murder.

3. **Analyze $S \to U$ and U:**
    - Premise: $S \to U$ (If a woman was the reason, the murderer left tracks all over the room).
    - Observation: U (The murderer left tracks all over the room).

    Modus Ponens:

    - Since U is true and $S \to U$ holds, S must be true.
    - **Conclusion:** A woman **was** the reason for the murder.

Final Conclusion:

Based on the logical derivation using the given premises and observations:

- **Robbery was not the reason** for the murder ($\neg P$).
- **Politics was not the reason** for the murder ($\neg R$).
- **A woman was the reason** for the murder (S).

Propositions:

- **A**: An electrical fault caused the fire.
- **B**: The fire started in the wiring.
- **C**: Arson was the cause of the fire.
- **D**: There were signs of accelerants at the scene.
- **E**: The fire department responded quickly.
- **F**: The fire was contained to a small area.

Logical Relationships Between Propositions:

1. $A \rightarrow B$: If an electrical fault caused the fire, then the fire started in the wiring.
   - **FOPL**: $\forall x \, (A(x) \rightarrow B(x))$
2. $C \rightarrow D$: If arson was the cause of the fire, then there were signs of accelerants at the scene.
   - **FOPL**: $\forall x \, (C(x) \rightarrow D(x))$
3. $E \rightarrow F$: If the fire department responded quickly, then the fire was contained to a small area.
   - **FOPL**: $\forall x \, (E(x) \rightarrow F(x))$

Assumptions (Observations) Given:

- $\neg B$: The fire did not start in the wiring.
  - FOPL: $\neg B(x)$
- D: There were signs of accelerants at the scene.
  - FOPL: $D(x)$
- F: The fire was contained to a small area.
  - FOPL: $F(x)$

Step-by-Step Logical Reasoning:

Step 1: Analyze $A \rightarrow B$ and $\neg B$

- Premise: $A \rightarrow B$
  - This means that if an electrical fault caused the fire, then the fire would have started in the wiring.
- Observation: $\neg B$ (The fire did not start in the wiring).
  - This observation contradicts the implication that an electrical fault was the cause.

Applying Modus Tollens:

- **Modus Tollens** states: If $A \rightarrow B$ and $\neg B$ is true, then $\neg A$ must be true.
- Since B is false ($\neg B$), A must also be false ($\neg A$).

Conclusion: An electrical fault was **not** the cause of the fire.

Step 2: Analyze $C \rightarrow D$ and D

- Premise: $C \rightarrow D$
  - This means that if arson was the cause of the fire, then there would be signs of accelerants at the scene.
- Observation: D (There were signs of accelerants at the scene).
  - This observation matches the consequence of arson being the cause.

Applying Modus Ponens:

- Modus Ponens states: If $C \rightarrow D$ and D is true, then C must be true.
- Since D is true and $C \rightarrow D$, we conclude C must be true.

Conclusion: Arson was the cause of the fire.

Step 3: Analyze $E \rightarrow F$ and F

- Premise: $E \rightarrow F$
  - This means that if the fire department responded quickly, then the fire would have been contained to a small area.
- Observation: F (The fire was contained to a small area).
  - This observation suggests that the fire department's quick response is a likely scenario.

Applying Modus Ponens:

- Since F is true and $E \rightarrow F$, E must also be true.

Conclusion: The fire department responded quickly.

Final Conclusions:

1. **The fire was not caused by an electrical fault:**
   - Based on the premise $A \to B$ and the observation $\neg B$, we conclude $\neg A$ (no electrical fault).
2. **The fire was caused by arson:**
   - The presence of accelerants at the scene (D) confirms C, meaning arson was the cause.
3. **The fire department responded quickly:**
   - The fire was contained to a small area (F), which, according to the logic, implies that the fire department responded quickly (E).

Summary:

- **An electrical fault did not cause the fire.**
- **Arson was the cause of the fire**, supported by the evidence of accelerants.
- **The fire department's quick response** helped contain the fire to a small area.

1. Lucy* is a professor
2. All professors are people.
3. John is the dean.
4. Deans are professors.
5. All professors consider the dean a friend or don’t know him.
6. Everyone is a friend of someone.
7. People only criticize people that are not their friends.
8. Lucy criticized John .

Knowledge base:

- is-prof(lucy)
- $\forall$ x (is-prof(x) $\rightarrow$ is-person(x))
- is-dean(John)
- $\forall$ x (is-dean(x) $\rightarrow$ is-prof(x))
- $\forall$ x ($\forall$ y (is-prof(x) $\wedge$ is-dean(y) $\rightarrow$ is-friend-of(y,x) $\vee$ $\neg$knows(x, y)))
- $\forall$ x ($\exists$ y (is-friend-of (y, x)))
- $\forall$ x ($\forall$ y (is-person(x) $\wedge$ is-person(y) $\wedge$ criticize (x,y) $\rightarrow$ $\neg$is-friend-of (y,x)))
- criticize(lucy, John)

Question: Is John no friend of Lucy?

Inference in First-Order Logic

- **Inference in First-Order Logic is used to deduce new facts or sentences from existing sentences.** Before understanding the FOL inference rule, let's understand some basic terminologies used in FOL.

Substitution:

- **Substitution is a fundamental operation performed on terms and formulas**. It occurs in all inference systems in first-order logic.
- **The substitution is complex in the presence of quantifiers in FOL**. If we write **F[a/x],** so it refers to **substitute a constant "a" in place of variable "x".**

Equality

- **Equality:**First-Order logic does not only use **predicate and terms** for making atomic sentences but also uses another way, which is **equality in FOL**. For this, we can use **equality symbols** which specify that the two terms refer to the same object.
- **Example: Brother (John) = Smith.**

- As in the above example, the object referred by the **Brother (John)** is similar to the object referred by **Smith**.
- The **equality symbol can also be used with negation to represent that two terms are not the same objects**.
- **Example: ¬(x=y) which is equivalent to x ≠y.**

FOL inference rules for quantifier:

- As propositional logic we also have inference rules in first-order logic, so following are some basic inference rules in FOL:
- **Universal Generalization**
- **Universal Instantiation**
- **Existential Instantiation**
- **Existential introduction**

Universal Generalization:

- **Universal generalization is a valid inference rule** which states that **if premise P(c) is true for any arbitrary element c in the universe of discourse, then we can have a conclusion as $\forall x\, P(x)$.**

It can be represented as: $\frac{P(c)}{\forall x\ P(x)}$.

Example: Let's represent, P(c): "**A byte contains 8 bits**", so for **∀ x P(x)** "**All bytes contain 8 bits.**", it will also be true.

Universal Instantiation:

- **Universal instantiation is also called as universal elimination or UI** is a valid inference rule. It can be applied multiple times to add new sentences.
- The **new KB is logically equivalent to the previous KB.**
- As per UI, **we can infer any sentence obtained by substituting a ground term for the variable**.

It can be represented as: $\frac{\forall x\ P(x)}{P(c)}$.

Example:1.

IF "Every person like ice-cream"=> ∀x P(x) so we can infer that "John likes ice-cream" => P(c)

Example: 2.

- Let's take a famous example,
- "All kings who are greedy are Evil." So let our knowledge base contains this detail as in the form of FOL:
- **∀x king(x) ∧ greedy (x) → Evil (x),**
- So from this information, we can infer any of the following statements using Universal Instantiation:
- **King(John) ∧ Greedy (John) → Evil (John),**
- **King(Richard) ∧ Greedy (Richard) → Evil (Richard),**
- **King(Father(John)) ∧ Greedy (Father(John)) → Evil (Father(John))**

Existential Instantiation:

- **Existential instantiation** is also called as **Existential Elimination**, It can be applied only once to replace the existential sentence.
- The **new KB is not logically equivalent** to **old KB**, but it will be satisfiable if old KB was satisfiable.

It can be represented as: $$\frac{\exists x\, P(x)}{P(c)}$$

Example:

- From the given sentence: **∃x Crown(x) ∧ OnHead(x, John),**
- So we can infer: **Crown(K) ∧ OnHead(K, John),** as long as K does not appear in the knowledge base.
- The above used **K is a constant symbol**, which is called **Skolem constant**.
- The Existential instantiation is a special case of **Skolemization process**.

Existential Introduction

- An existential introduction is also known as an **existential generalization**, which is a valid inference rule in first-order logic.
- This rule states that **if there is some element c** in the universe of discourse which has a **property P,** then we can infer that there **exists something in the universe which has the property P.**

It can be represented as: $$\frac{P(c)}{\exists x P(x)}$$

Example:

Let's say that,

- “Priyanka got good marks in English.”
- "Therefore, someone got good marks in English."

Well Formed Formulas

- In logic, a Well-Formed Formula (WFF) refers to a syntactically correct expression in propositional or first-order logic that adheres to the formal rules of the logical system.
- These rules ensure that the formula is meaningful and can be evaluated as either true or false.

Rules for constructing Wffs

- A predicate name followed by a list of variables such as ***P(x, y)*,** where ***P*** **is a predicate name, and *x* and *y* are variables**, is called an **atomic formula**.
- Examples of Well-Formed Formulas:
- In propositional logic: $P \wedge Q$, $\neg P \rightarrow Q$
- In first-order logic: $\forall x(P(x) \rightarrow Q(x))$, $\exists y(R(y) \wedge S(y))$

Wffs are constructed using the following rules:

1. ***True and False*** *are* ***wffs.***

2. *Each* ***propositional constant*** *(i.e. specific proposition), and each* ***propositional variable*** *(i.e. a variable representing propositions) are* ***wffs.***

3. *Each* ***atomic formula*** *(i.e. a specific predicate with variables) is a* ***wff.***

4. ***If A,B are wffs then*** **¬A, (A ∧B), (A ∨ B), (A →B) and (A ⇔ B).**

5. *If* ***x is a variable*** *(representing objects of the universe of discourse) and* ***A is a wff then so are*** **∀x A and ∃x A.**

Resolution

- Resolution is a **theorem proving technique** that proceeds by building **refutation proofs, i.e., proofs by contradictions**.
- **Resolution is used**, if there are **various statements are given**, and we need to **prove a conclusion of those statements**.
- Resolution is a single inference rule which can **efficiently operate on the conjunctive normal form or clausal form**.
- **Conjunctive Normal Form**: A sentence represented as a **conjunction of clauses** is said to be **conjunctive normal form** or **CNF.**
- **Note: An expression in CNF is a 'product of sums'.**

- Example:

a) **Ravi likes all kind of food.**
b) **Apple and chicken are food.**
c) **Anything anyone eats and is not killed is food.**
d) **Ajay eats peanuts and still alive.**

Prove: Ravi likes peanuts.

- ❖ **likes(Ravi, Peanuts)**

write in FOL form

- **likes(Ravi, Peanuts) => ¬ likes(Ravi, Peanuts)**

- **Steps to solve Resolution**

1. Negate the **statement to be proved**.
2. Convert given **facts into FOL**
3. Convert **FOL into CNF**
4. Draw **resolution graph**

Convert the facts into FOL

1. Ravi likes all kind of food.

   $\forall x$: food(x) -> likes(Ravi,x)

2. Apple and chicken are food

   i) food(apple)

   ii) food(chicken)

3. Anything anyone eats and is not killed is food.

   $\forall x \forall y$: eats(x,y) $\wedge$ ¬killed(x) -> food(y)

4. Ajay eats peanuts and still alive.

   eats(ajay,peanuts) $\wedge$ alive(ajay)

5. **$\forall x$: ¬killed(x) -> alive(x)**

   added predicates

6. **$\forall x$: alive(x) -> ¬killed(x)**

Convert FOL into CNF

1. Eliminate '->' & '<->'

   a->b : ¬a V b

   a<->b : a->b $\wedge$ b->a

2. Move ¬ inward
    - **$\neg(\forall x p) = \exists x \neg p$**
    - **$\neg(\exists x p) = \forall x \neg p$**
    - **$\neg(a \vee b) = \neg a \wedge \neg b$**
    - **$\neg(a \wedge b) = \neg a \vee \neg b$**
    - **$\neg(\neg a) = a$**
3. Rename Variable.
4. Replace Existential Quantifier by **skolem constant**

    $\exists x$ Rich(x) = Rich(G1)
5. Drop Universal Quantifier

1. $food(x) \to likes(Ravi,x)$

   ¬ food(x) V likes(Ravi,x) [a->b = ¬a V b]

2. $eats(x,y) \wedge \neg killed(x) \to food(y)$

   ¬ [eats(x,y) ∧ ¬killed(x)] V food(y)

   ¬ eats(x,y) V killed(x) V food(y)

3. **eats(ajay,peanuts) ∧ alive(ajay)** (Divide in resolution graph)
4. **¬ ¬killed(x) V alive(x) : killed(x) V alive(x)**
5. **¬ alive(x) -> ¬ ¬ killed(x) : ¬ alive(x) V killed(x)**

Resolution Graph

- ¬ likes(Ravi, Peanuts)

(I)

x/peanuts

¬ food(peanuts)

(II)

y/peanuts

¬ eats(x,peanuts) V killed(x)

x/Ajay (III)

Killed(Ajay)

(v)

¬ alive(ajay)

alive(ajay)

Uncertain Knowledge and Reasoning

- In real life, it is **not always possible to determine the state of the environment** as it might not be clear.
- Due to partially observable or non-deterministic environments, **agents may need to handle uncertainty and deal with:**
  - **Uncertain data:** Data that is missing, unreliable, inconsistent or noisy.
  - **Uncertain knowledge:** When the available knowledge has multiple causes leading to multiple effects or incomplete knowledge of causality in the domain.
  - **Uncertain knowledge representation:** The representations which provides a restricted model of the real system, or has limited expressiveness.
  - **Inference:** In case of incomplete or default reasoning methods, conclusions drawn might not be completely accurate.

Let’s understand this better with the help of an example.

- **IF** the patient has a history of smoking
- **AND** the patient has shortness of breath
- **AND** the X-ray shows a mass in the lungs
- **THEN** the diagnosis is lung cancer (0.85)

- In uncertain situations, the medical system (agent) does not guarantee a **definitive diagnosis** but operates based on the **available evidence**, **assumptions**, and **probabilities**.
- The confidence in the **diagnosis is 85%,** representing a **degree of belief** that this conclusion is correct given the circumstances, though further testing may be required.

Such uncertain situations can be dealt with using

- **Probability theory**
- **Truth Maintenance systems**
- **Fuzzy logic.**

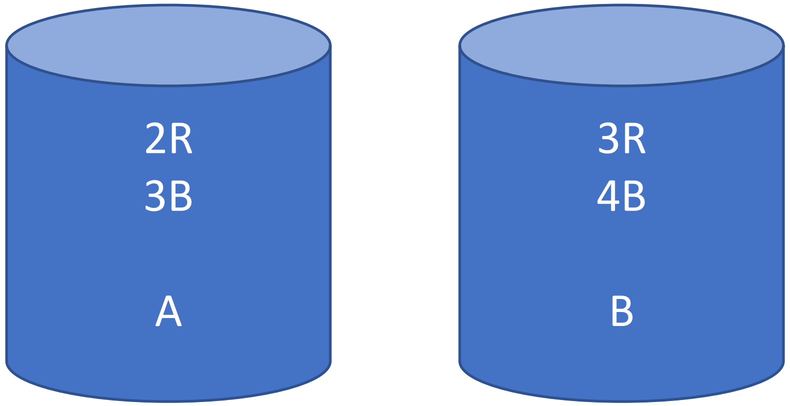
Probability

- Probability is the **degree of likeliness** that an event will occur.
- It provides a certain **degree of belief** in case of uncertain situations.
- It is defined over a set of **events U** and assigns value **P(e) i.e.** probability of occurrence of **event e** in the **range [0,1].**
- Here each sentence is labeled with a real number in the range of **0 to 1**, 0 means the sentence is false and 1 means it is true.

- **Conditional Probability or Posterior Probability** is the probability **of event A** given that **B has already occurred**.
- **P(A|B) = (P(B|A) * P(A)) / P(B)**
- For example, P(It will rain tomorrow| It is raining today) represents conditional probability of it raining tomorrow as it is raining today.
- **P(A|B) + P(NOT(A)|B) = 1**
- **Joint probability** is the probability of 2 independent events happening simultaneously like rolling two dice or tossing two coins together.

Bayes Theorem

- Bayes' theorem is also known as **Bayes' rule, Bayes' law**, or **Bayesian reasoning**, which determines the probability of an event with uncertain knowledge.
- In probability theory, it relates the **conditional probability** and **marginal probabilities** of two random events.
- It is a way to calculate the value of $P(B|A)$ with the knowledge of $P(A|B)$.
- Marginal probability refers to the probability of a single event occurring, regardless of the state of other events.
- Joint probability refers to the probability of multiple events occurring together.



1. What is the probability of red ball given that bag A is chosen

- P(R/A) = 2/5 { Bag A is Chosen}

2. What is the probability that red ball is drawn from bag A.

{ Here we need to select Red ball and Bag A – Two Conditions}

$$P(A \wedge R) = P(A).\ P(R/A)$$

3. What is the probability of getting red ball

$$P(R) = P(A \wedge R) + P(B \wedge R)$$

$P(A \wedge R)$: Probability of getting a red ball from bag A

$P(B \wedge R)$: Probability of getting a red ball from bag B

Bayes Theorem: { Reverse Probability}

- Given that red ball is drawn what is the probability that the ball is from bag A.

- $P(A/R) = P(A \wedge R) /((P(A \wedge R) + P(B \wedge R))$

$P(A \wedge R)$: from Bag A

$((P(A \wedge R) + P(B \wedge R))$:Total probability of Red Ball.

- P(A|B) is known as **posterior**, which we need to calculate, and it will be read as Probability of hypothesis A when we have occurred an evidence B.

- P(B|A) is called the **likelihood**, in which we consider that hypothesis is true, then we calculate the probability of evidence.

- P(A) is called the **prior probability**, probability of hypothesis before considering the evidence.

- P(B) is called **marginal probability**, pure probability of an evidence.

$$P(\text{cause}\,|\,\text{effect}) = \frac{P(\text{effect}\,|\,\text{cause})\, P(\text{cause})}{P(\text{effect})}$$

- **Question:** What is the probability that a person has lung cancer given that they are experiencing a persistent cough?

Given Data:

- A doctor is aware that lung cancer causes a patient to have a persistent cough, and it occurs 70% of the time.
- The known probability that a patient has lung cancer is 1 in 10,000.
- The known probability that a patient has a persistent cough is 5%.

Let:

- **a** be the proposition that the patient has a persistent cough.
- **b** be the proposition that the patient has lung cancer.

- Thus, we have the following information:
- **P(a|b)=0.7** (The probability of having a persistent cough given that the person has lung cancer).
- **P(b)=1/10000=0.0001**(The probability of having lung cancer).
- **P(a)=0.05** (The probability of having a persistent cough).
- Now, we can calculate the probability that a person has lung cancer given that they have a persistent cough using Bayes' Theorem:

$$P(b|a) = \frac{P(a|b) \cdot P(b)}{P(a)}$$

$$P(b|a) = \frac{0.7 \cdot 0.0001}{0.05} = \frac{0.00007}{0.05} = 0.0014$$

- So, the probability that a person has lung cancer given that they have a persistent cough is 0.0014, or 0.14%.

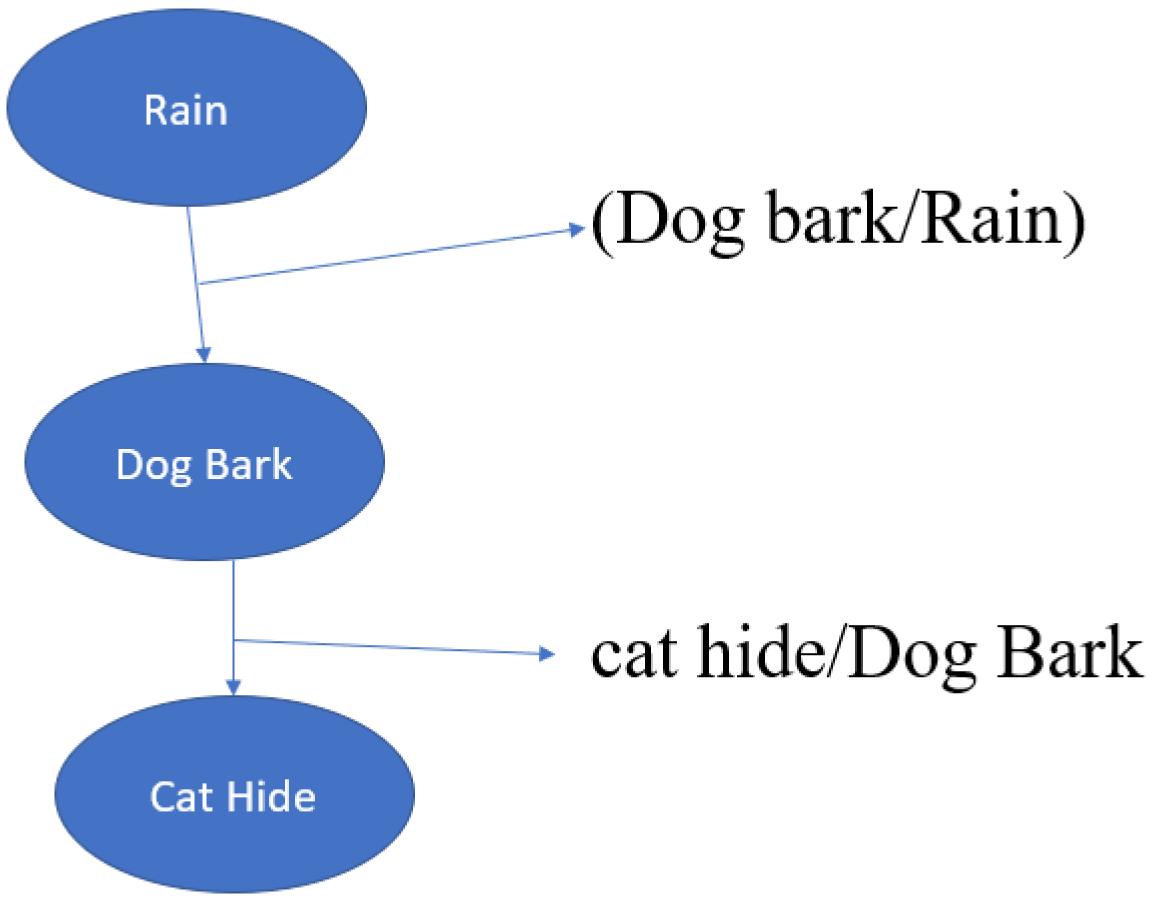
Bayesian Belief Network

- Directed Acyclic Graph
- Conditional Probability Table

Directed Acyclic Graph (DAG)

❖ Convenient for representing probabilistic relation between multiple events.

- Nodes
- Uncertainity
- Ditrected Edges



- Directed Arrow represents conditional probability

- Dog bark/Rain: Dog will bark if it rains

- Cat hide/dog bark: Cat will hide if dog barks

Conditional Probability Table:

- P(Dog/Rain): Probability of considered node with respect to its parent node.

	R	~R
B	9/48	18/48
~B	3/48	18/48

The table you've provided is a **joint probability distribution table** for two binary events, **R** and **B**, and their complements (~R and ~B). Here's an explanation of how to interpret this table:

Variables:

- **R**: Event R happens.
- **~R**: Event R does not happen.
- **B**: Event B happens.
- **~B**: Event B does not happen.

Probability Entries:

- The numbers in the table (fractions over 48) represent the **joint probabilities** of combinations of these events. For example:
- $P(B \wedge R) = 9/48$: The probability that both B and R happen is 9/48.
- $P(B \wedge \sim R) = 18/48$: The probability that B happens and R does not happen is 18/48.
- $P(\sim B \wedge R) = 3/48$: The probability that B does not happen and R happens is 3/48.
- $P(\sim B \wedge \sim R) = 18/48$: The probability that neither B nor R happens is 18/48.

Interpretation:

- This table summarizes the likelihood of the various combinations of events (R, B) and their complements occurring together.
- To calculate any specific probability, such as the marginal or conditional probabilities, we can sum up the relevant values:

1. **Marginal Probability of R:**

- $P(R) = P(B \wedge R) + P(\sim B \wedge R) = \frac{9}{48} + \frac{3}{48} = \frac{12}{48} = \frac{1}{4}$

2. **Marginal Probability of B:**

- $P(B) = P(B \wedge R) + P(B \wedge \sim R) = \frac{9}{48} + \frac{18}{48} = \frac{27}{48} = \frac{9}{16}$

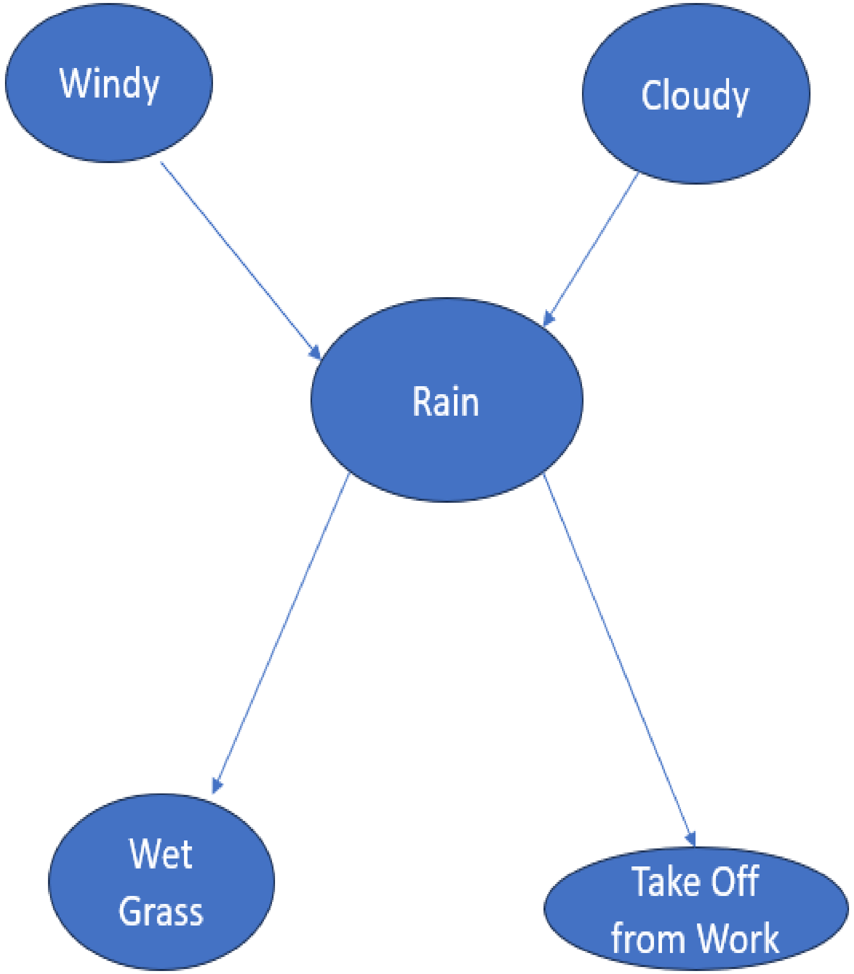
3. **Conditional Probability:**

You can also compute conditional probabilities, such as:

- $P(B|R) = \frac{P(B \wedge R)}{P(R)} = \frac{9/48}{12/48} = \frac{9}{12} = \frac{3}{4}$

Example

- P(windy)=0.001, P(~windy) = 1-0.001=0.999
- P(cloudy)=0.002, P(~cloudy)= 1-0.002=0.998



Windy	Cloudy	P(Rain)
T	T	0.95
T	F	0.95
F	T	0.29
F	F	0.001

Rain	P(Wet)
T	0.95
F	0.05

Rain	P(Off)
T	0.91
F	0.09

Q) Lets find the probability of having a wet grass

- P(wet) = P(Wet/rain) * P(rain) + P(wet/~rain) * P(~rain)

  = 0.95 * P(rain) + 0.05 * P(~rain)

P(rain) = [P(rain/windy, cloudy) * P(windy ∧ cloudy)] + [P(rain/~windy , Cloudy) * P(~windy ∧ cloud)] + [P(rain/windy,~cloudy) * P(windy ∧~cloudy)]+ [P(rain/~windy, ~cloudy) * P(~windy ∧~cloudy)]

= 0.95 * 0.001 * 0.002 + 0.29 * 0.999 * 0.002 + 0.95 * 0.001 * 0.998 + 0.001*0.999* 0.998

= 0.00252

- Like calculate P(~rain)

=P(~rain/windy,cloudy)* P(windy ∧ cloudy) + P(~rain/~windy,cloudy)* P(~windy ∧ cloudy) + P(~rain/windy,~cloudy)* P(windy ∧ ~cloudy) + P(~rain/~windy,~cloudy)* P(~windy ∧ ~cloudy)

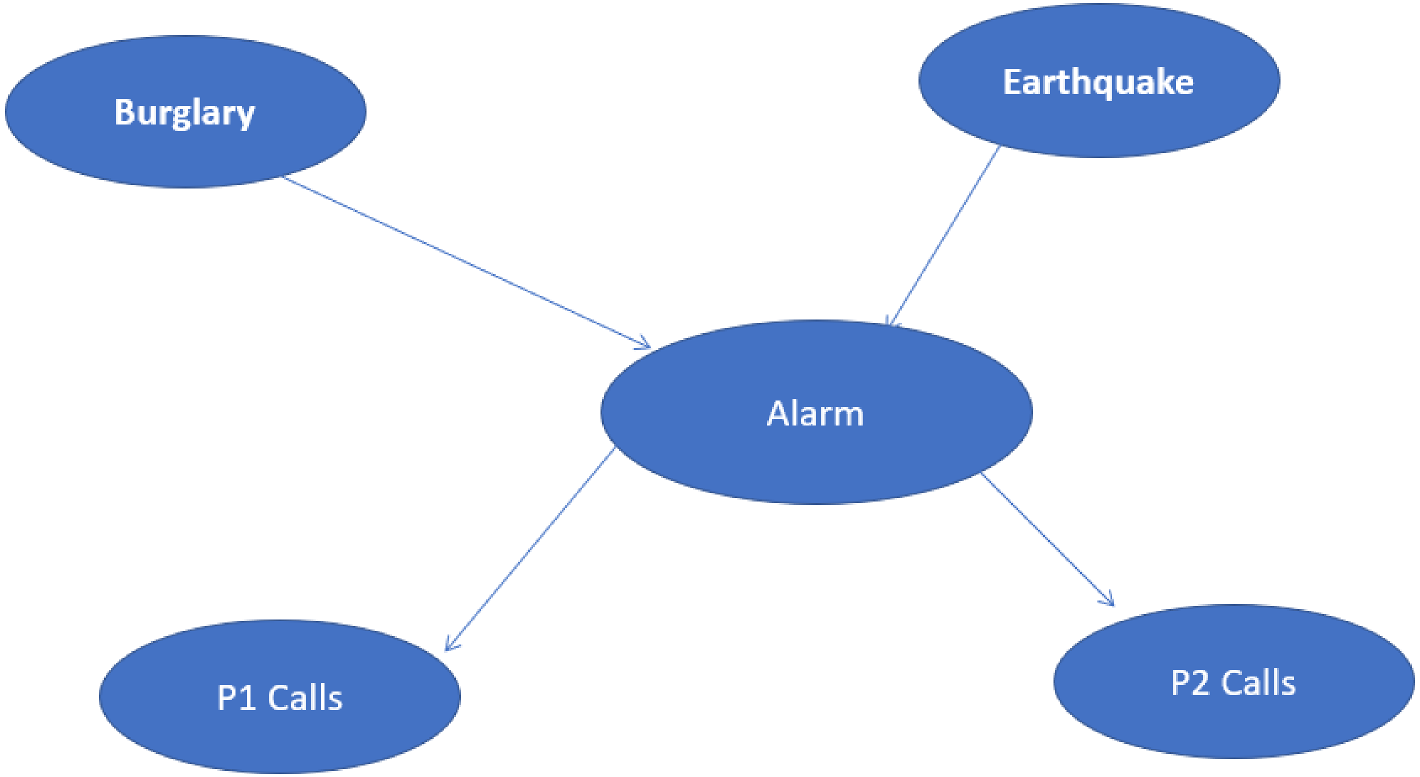
=[(1-0.95) * 0.001* 0.002] + [(1-0.29)* 0.999 * 0.002] + [(1-0.95)*0.001*0.998] + [(1-0.001)*0.999*0.998]

=0.99744

- **P(wet)=0.052**

Problem:

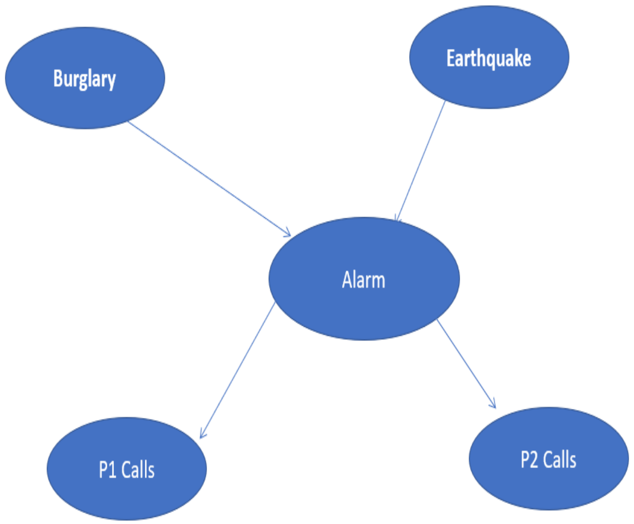
Calculate the probability that alarm has sounded, but there is neither a burglary, nor an earthquake occurred, and P1 and P2 both called the X.



- The Bayesian network for the above problem is given below. The network structure is showing that burglary and earthquake is the parent node of the alarm and directly affecting the probability of alarm's going off, but P1 and P2 calls depend on alarm probability.
- The network is representing that our assumptions do not directly perceive the burglary and also do not notice the minor earthquake, and they also not confer before calling.
- The conditional distributions for each node are given as conditional probabilities table or CPT.
- Each row in the CPT must be sum to 1 because all the entries in the table represent an exhaustive set of cases for the variable.

List of all events occurring in this network:

- Burglary (B)
- Earthquake(E)
- Alarm(A)
- P1 Calls(P1)
- P2 calls(P2)



- **P(B=T) = 0.001**
- **P(B=F) = 0.999**
- **P(E=T) = 0.002**
- **P(E=F) = 0.998**

B	E	P(A=T)	P(A=F)
T	T	0.95	0.05
T	F	0.94	0.06
F	T	0.29	0.71
F	F	0.001	0.999

Alarm(A)	P(P1=T)	P(P1=F)
T	0.90	0.10
F	0.05	0.95

Alarm(A)	P(P2=T)	P(P2=F)
T	0.70	0.30
F	0.01	0.99

- Calculate the probability that alarm has sounded, but there is neither a burglary, nor an earthquake occurred, and P1 and P2 both called the **X**.

- **P (P1,P2,A,~B,~E)**

  = P(P1/A). P(P2/A). P(A/~B,~E). P(~B). P(~E)

  = 0.90*0.70*0.001* 0.999* 0.998

  = 0.00062