Definition of MVD

- A *multivalued dependency* (MVD) *X* ->->*Y* is an assertion that if two tuples of a relation agree on all the attributes of *X*, then their components in the set of attributes *Y* may be swapped, and the result will be two tuples that are also in the relation.

Definition of MVD

- A multi-valued dependency (MVD or MD) is an assertion that two sets of attributes are independent of each other.
- The *multi-valued dependency* $A_1 A_2 \ldots A_n \twoheadrightarrow B_1 B_2 \ldots B_m$ holds in a relation R if in every instance of R,

  for every pair of tuples t and u in R that agree on all the A's, we can find a tuple v in R that agrees
  1. with both t and u on A's,
  2. with t on the B's, and
  3. with u on all those attributes of R that are not A's or B's.

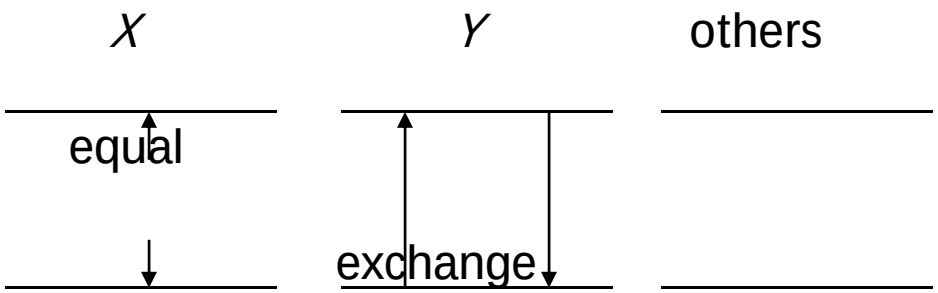
Number	DeptName	Textbook	Professor
4604	CS	FCDB	Ullman
4604	CS	SQL Made Easy	Ullman
4604	CS	FCDB	Widom
4604	CS	SQL Made Easy	Widom

Example

	Number	DeptName	Textbook	Professor
	4604	CS	FCDB	Ullman
t	4604	CS	SQL Made Easy	Ullman
u	4604	CS	FCDB	Widom
v	4604	CS	SQL Made Easy	Widom
	2604	CS	Data Structures	Ullman
	2604	CS	Data Structures	Widom

- Number DeptName $\twoheadrightarrow$ Textbook is an MD. For every pair of tuples t and u that agree on Number and DeptName, we can find a tuple v that agrees
  1. with both t and u on Number and DeptName,
  2. with t on Textbook, and with u on Professor.
- Number DeptName $\twoheadrightarrow$ Professor is an MD. For every pair of tuples t and u that agree on Number and DeptName, we can find a tuple v that agrees
  1. with both t and u on Number and DeptName,
  2. with t on Professor, and with u on Textbook.

Picture of MVD *X* ->->*Y*



Number	DeptName	Textbook	Professor
4604	CS	FCDB	Ullman
4604	CS	SQL Made Easy	Ullman
4604	CS	FCDB	Widom
4604	CS	SQL Made Easy	Widom
2604	CS	Data Structures	Ullman
2604	CS	Data Structures	Widom

- Does X -> Y imply X ->> Y ?

MVD Rules

- Every FD is an MVD
  - If *X* ->*Y*, then swapping *Y*'s between two tuples that agree on *X* doesn't change the tuples.
  - Therefore, the "new" tuples are surely in the relation, and we know *X* ->->*Y*.
- Definition of keys depend on FDs and not MDs

Rules for Manipulating MDs

- *Trivial dependencies rule:* If $A \twoheadrightarrow B$ is an MD, then $A \twoheadrightarrow AB$ is also an MD.

Splitting Doesn’t Hold

- Like FD’s, we cannot generally split the left side of an MVD.

- But unlike FD’s, we cannot split the right side either --- sometimes you have to leave several attributes on the right side.

Fourth Normal Form

- The redundancy that comes from MVD's is not removable by putting the database schema in BCNF.

- There is a stronger normal form, called 4NF, that (intuitively) treats MVD's as FD's when it comes to decomposition, but not when determining keys of the relation.

4NF Definition

- A relation *R* is in 4NF if whenever *X* ->->*Y* is a nontrivial MVD, then *X* is a superkey.
  - Nontrivial means that:
    1. *Y* is not a subset of *X*, and
    2. *X* and *Y* are not, together, all the attributes.
  - Note that the definition of "superkey" still depends on FD's only.

BCNF Versus 4NF

- Remember that every FD *X* ->*Y* is also an MVD, *X* ->->*Y*.
- Thus, if *R* is in 4NF, it is certainly in BCNF.
  - Because any BCNF violation is a 4NF violation.

- But *R* could be in BCNF and not 4NF, because MVD's are "invisible" to BCNF.