* Save Space *

14/3/23 :-

2. Mathematical Logic and Induction

Proposition / Statement: A proposition is a declarative sentence either it is True or False but not both.

Ex:- $2+3=5$ — True (T)

$2<5$ — True (T)

$5>8$ — False (F).

- Propositions are denoted by p, q, r, s ...

Some sentences which are not propositions are

Ex:- Close the door;

where are you going?

Shut up

$x+y=3$

Compound proposition: when one or more propositions are connected through various connectivities is called compound proposition.

Ex:- $\underbrace{\text{Roses are Red}}_{p}$ $\underset{\text{connectivity.}}{\text{and}}$ $\underbrace{\text{Milk is white}}_{q}$ (T)

we have 5 connectivities

i) Negation - NOT - $\sim$

ii) Conjunction - AND - $\wedge$

iii) Disjunction - OR - $\vee$

4) Conditional - if ... then — $p \to q$

5) Biconditional - iff — $p \leftrightarrow q$

1. Negation :- if P is a proposition then 'P is not true' is also a proposition. this is represented by "$\sim p$".

Ex:- P : Sunday is a holiday

$\sim p$: Sunday is not a holiday.

Truth Table:

P	$\sim P$
T	F
F	T

2. conjunction: 2 propositions p, q can be combined by the word "and" two form a compound proposition is called conjunction. It is represented by "$P \wedge q$".

Truth Table:-

P	q	$P \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Note: Sometimes we use "but" instead of "AND"

3. disjunction: It is divided into 2 parts

i. Inclusive disjunction.

2 propositions p, q can be combined by the word "or" two form a compound

proposition is called disjunction represented by "PVq"

Truth Table:

P	q	PVq
T	T	T
T	F	T
F	T	T
F	F	F

ii) Exclusive disjunction: It is denoted by $P \underline{\vee} q$ or $P \oplus q$. If 2 statements are differenent then $P \underline{\vee} q$ is True otherwise False

Truth Table:

P	q	$P \underline{\vee} q$
T	T	F
T	F	T
F	T	T
F	F	F

Conditional Statement: If p, q are any 2 propositions then "P implies q" (if p then q) is also a proposition is called conditional statement and also called as implication Represented by "$P \rightarrow q$".

by

The conditional statement $p \to q$ is False only if p is True, q is False otherwise True.

Truth Table

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

NOTE: if p then q

p only if q

q whenever p

p implies q

p is sufficient condition for q

q is necessary condition for p

q follows from p

q provides that p

Biconditional Statements: if p, q are any 2 propositions then ". p if and if q" (p iff q) is also a proposition is called Biconditional Statement and it is represented by "$p \longleftrightarrow q$" (or) "$p \Longleftrightarrow q$"

"$p \Longleftrightarrow q$" is True if p, q have same behaviour otherwise it is false.

Truth Table:

p	q	$p \Longleftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Construct the Truth Table

p	q	$\sim p$	$\sim q$	$p \wedge q$	$p \veebar q$	$p \rightarrow q$	$p \leftrightarrow q$
T	T	F	F	T	F	T	T
T	F	F	T	F	T	F	F
F	T	T	F	F	T	T	F
F	F	T	T	F	F	T	T

15/3/23:-

Construct Truth Table for

1. $(p \to q) \wedge (q \to p)$ 2. $(p \vee q) \wedge r$

3. $p \vee (q \wedge r)$ 4. $p \wedge \sim p$ 5. $\sim p \vee q$

6. $\sim p \veebar \sim q$ 7. $p \to \sim p$ 8) $\sim p \to \sim q$

Sol:-

1. $(p \to q) \wedge (q \to p)$

p	q	$p \to q$	$q \to p$	$(p \to q) \wedge (q \to p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

2. $(p \vee q) \wedge r$ (No linear pairs try three set pairs)

p	q	$p \vee q$	r	$(p \vee q) \wedge r$
T	T	T	T	
T	F	T	F	
F	T	T	T	
F	F	F	F	

4. $p \wedge \sim p$

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

2.

p	q	r	$p \vee q$	$p \vee q \wedge r$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
F	T	T	T	T
T	F	F	T	F
F	F	T	F	F
F	T	F	T	F
F	F	F	F	F

3.

$q \wedge r$	$p \vee (q \wedge r)$
T	T
F	T
F	T
T	T
F	T
F	F
F	F
F	F

5. $\sim p \vee$

p
T
F
T
F

6. $\sim$

p
T
F

7.

5. $\sim p \vee q$

p	$\sim p$	q	$\sim p \vee q$
T	F	F	F
F	T	T	T
T	F	T	T
F	T	F	T

6. $\sim p \veebar \sim q$

p	q	$\sim p$	$\sim q$	$\sim p \veebar \sim q$
T	F	F	T	T
F	T	T	F	T
F	F	T	T	F
T	T	F	F	F

7. $p \rightarrow \sim p$

p	$\sim p$	$p \rightarrow \sim p$
T	F	F
F	T	T

8) $\sim p \rightarrow \sim q$

p	q	$\sim p$	$\sim q$	$\sim p \rightarrow \sim q$
T	F	F	T	T
F	T	T	F	F
T	T	F	F	T
F	F	T	T	T

* Tautology :- A compound proposition, which is True for all possible truth values of it's proposition's is called Tautology.

* Contradiction:- A compound proposition, which is false for all possible truth values of its proposition is called Contradiction.

* Contingency :- A compound proposition which is neither a tautology nor a contradictson is called contingency.

Examples:

p	~p	p∨~p	p∧~q	p→~p
T	F	T	F	F
F	T	T	F	T
		Tautology	contradiction	contingency

Logically Equivalent (≡): Compound propositions that have same truth values in all possible cases are logically Equivalent and it is denoted by

"$p \equiv q$" read as "p, q are logically equivalent".

* S.T

1. $(p \veebar q) \vee (p \Leftrightarrow q)$ is tautology

2. $(p \veebar q) \wedge (p \Leftrightarrow q)$ is contradiction

3. $(p \veebar q) \wedge (p \rightarrow q)$ is contingency

1

p	q	$p \veebar q$	$p \Leftrightarrow q$	$(p \veebar q) \vee (p \Leftrightarrow q)$
T	T	F	T	T
T	F	T	F	T
F	T	T	F	T
T	T	F	T	T

$\Downarrow$ tautology

② $(p \veebar q) \wedge (p \Leftrightarrow q)$	$(p \veebar q) \wedge (p \rightarrow q)$ ③	$p \rightarrow q$
F	F	T
F	F	F
F	T	T
F	F	T

$\Downarrow$ contradiction $\Downarrow$ contingency

$\therefore$ Hence proved.

* S.T.

$\sim(p \vee q)$ & $\sim p \wedge \sim q$ are logically equivalent

S.T.

p	q	$\sim p$	$\sim q$	$p \vee q$	$\sim(p \vee q)$	$\sim p \wedge \sim q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

$\therefore \sim(p \vee q) \equiv \sim p \wedge \sim q$

whether

2. $p \to q$ and $\sim q \to \sim p$ are logically equivalent

p	q	$\sim p$	$\sim q$	$p \to q$	$\sim q \to \sim p$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

$$\therefore \; p \to q \equiv \sim q \to \sim p.$$

* Construct Truth Tables for i) $[(p \to q) \wedge (q \to r)] \to (p \to r)$

ii) $(p \to q) \leftrightarrow (\sim p \vee q)$

iii) $(p \wedge q) \to r$

4. $q \wedge (\sim r \to p)$

5. $p \to (q \wedge r)$.

$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$p \rightarrow r$	$(p \rightarrow q) \wedge (q \rightarrow r)$	$(p \rightarrow q) \wedge (q \rightarrow r) \rightarrow p \rightarrow r$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	T
F	T	T	T	T	T	T	T
T	F	F	F	T	F	F	T
F	F	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	F	F	T	T	T	T	T

2. $(p \rightarrow q) \Longleftrightarrow (\sim p \vee q)$

p	q	$\sim p$	$p \rightarrow q$	$\sim p \vee q$	$(p \rightarrow q) \Longleftrightarrow (\sim p \vee q)$
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	T	T	True
F	F	T	T	T	True
					Tautology

3. $(p \wedge q) \rightarrow r$

p	q	r	$p \wedge q$	$(p \wedge q) \rightarrow (r)$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
F	T	T	F	T
T	F	F	F	T
F	F	T	F	T
F	T	F	F	T
F	F	F	F	T

4. $q \wedge (\sim r \rightarrow p)$

p	q	r	$\sim r$	$\sim r \rightarrow p$	$q \wedge (\sim r \rightarrow p)$
T	T	T	F	T	T
T	T	F	T	T	T
T	F	T	F	T	F
F	T	T	F	T	T
T	F	F	T	T	F
F	F	T	F	T	F
F	T	F	T	F	F
F	F	F	T	F	F

5. $p \rightarrow (q \wedge r)$

p	q	r	$q \wedge r$	$p \rightarrow (q \wedge r)$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
F	T	T	T	T
T	F	F	F	T
F	F	T	F	F
F	T	F	F	T
F	F	F	F	T

16/3/23 :-

Converse, Contra positive and Inverse :-

we can form some new conditional statements starting with a conditional statement "$p \to q$" in particular there are 3 related conditional statements

- → converse
- → contra +ve
- → Inverse

i) Converse: The proposition "$q \to p$" is called converse of "$p \to q$".

ii) "$\sim q \to \sim p$" is called contrave +ve of $p \to q$

iii) "$\sim p \to \sim q$" is called Inverse of $p \to q$

Ex: i) If it is raining then the grass is wet. write converse, contra +ve and Inverse of above proposition

i) Converse: if the grass is wet then it is raining.

ii) Contra +ve. ($\sim q \to \sim p$): If the grass is not wet, then it is not raining

iii) Inverse : $\sim p \rightarrow \sim q$: If it is not raining, then grass is not wet.

* " The Home team wins whenever it is raining " (q whenever p)

⇓

If $\underbrace{\text{it is raining}}_{p}$ then $\underbrace{\text{the home team wins}}_{q}$.

i) Converse :- ($q \rightarrow p$) If the home team wins then it is raining.

ii) Contra +ve. : ($\sim q \rightarrow \sim p$) If the home team doesn't (don't) win / looses then it is not raining.

iii) Inverse. : ($\sim p \rightarrow \sim q$) If it is not raining, then the home team don't win.

(Since "q whenever p" is one of the ways to express the conditional stmt "$p \rightarrow q$" the original stmt can be written as above)

* "If the figure is Square then it is Quadrilateral."

$\underbrace{\text{the figure is Square}}_{p}$ ... $\underbrace{\text{Quadrilateral}}_{q}$

i) **Converse** ($q \to p$): If the figure is Quadrilateral then it is Square

ii) **Contrapositive** ($\sim q \to \sim p$): If the figure is not Quadrilateral then it is not Square

iii) **Inverse** ($\sim p \to \sim q$): If the figure is not square, then it is not Quadrilateral.

Laws of Logic :-

i) **Double Negation** :- $\sim(\sim p) = p$

ii) **Identity law** :- $p \vee F_0 = p$, $p \wedge T_0 = p$

(F_0 → contradiction, T_0 → Tautology)

iii) **Inverse Law** :- a) $p \vee (\sim p) = T_0$

b) $p \wedge (\sim p) = F_0$

iv) **Idempotent law** :- a) $p \vee p = p$

b) $p \wedge p = p$

v) **Domination law** :- a) $p \vee T_0 = T_0$

b) $p \wedge F_0 = F_0$

vi) **Absorption law** :- a) $p \vee (p \wedge q) = p$

b) $p \wedge (p \vee q) = p$

vii) **Commutative law** :- a) $p \vee q = q \vee p$

b) $p \wedge q = q \wedge p$

viii) **Associative law** :- a) $p \vee (q \vee r) = (p \vee q) \vee r$

b) $p \wedge (q \wedge r) = (p \wedge q) \wedge r$

ix) **De-Morgan's law** :-

a) $\sim(p \vee q) = \sim p \wedge \sim q$

b) $\sim(p \wedge q) = \sim p \vee \sim q$

p	q	$p \vee q$	$\sim(p \vee q)$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

x) **Distributive law** :- a) $p \vee (q \wedge r)$

$= (p \vee q) \wedge (p \vee r)$

b) $p \wedge (q \vee r)$

$= (p \wedge q) \vee (p \wedge r)$

XI) Logically Equivalence involving Conditional stmts

a) $p \to q \equiv \sim p \vee q$ *

b) $p \to q \equiv \sim q \to \sim p$ *

c) $\sim(p \to q) \equiv p \wedge \sim q$

d) $p \vee q \equiv \sim p \to q$

e) $p \wedge q \equiv \sim(p \to \sim q)$

f) $(p \to q) \vee (p \to r) \equiv p \vee (q \to r)$

$\equiv p \to (q \vee r)$

g) $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$

h) $(p \to q) \vee (r \to q) \equiv (p \wedge r) \to q$

follow this $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

i) $(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$.

$(p \to q)$

* P.T the following are Tautology

i) $[p \wedge (p \to q)] \to q$

ii) $\sim(p \vee \sim q) \to \sim p$

iii) $p \to (p \vee q)$

* For any 3 propositions,

$(p \vee q) \to r \equiv (p \to r) \wedge (q \to r)$

$\Rightarrow (p \to q) \to r \iff [(p \wedge \sim r) \to \sim q]$

Sol^n

p	q	$p \to q$	$p \wedge (p \to q)$	$p \wedge (p \to q) \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T
				Tautology

p	q	r	$p \to q$	$(p \to q) \to r$	$\sim r$	$(p \wedge \sim r)$	$\sim q$	$(p \wedge \sim r) \to \sim q$ *	Result $\Leftrightarrow$
T	T	T	T	T	F	F	F	T	T
T	T	F	T	F	T	T	F	F	T
T	F	T	F	T	F	F	T	T	T
F	T	T	T	T	F	F	F	T	T
T	F	F	F	T	T	T	T	T	T
F	F	T	T	T	F	F	T	T	T
F	T	F	T	F	T	F	F	T	F
F	F	F	T	F	T	F	T	T	F

$p \quad q \quad r \quad p \to q \quad (p \to q) \to r$

$(p \to q) \to r \not\equiv (p \wedge \sim r) \to \sim q$

(not logically equivalent).

p	q	r	p∨q	$(p \vee q) \to r$	$p \to r$	$q \to r$	$(p \to r) \wedge (q \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	T	T
F	T	T	T	T	T	T	T
T	F	F	T	F	F	T	F
F	F	T	F	T	T	T	T
F	T	F	T	F	T	F	F
F	F	F	F	T	T	T	T

ii) $\sim(p \vee \sim q) \to \sim p$

p	q	$\sim p$	$\sim q$	$p \vee \sim q$	$\sim(p \vee \sim q)$	$\to \sim p$
1	1	0	0	1	0	
1	0	0	1	1	0	
0	1	1	0	0	1	
0	0	1	1	1	0	

23/3/2023 :

Q. Prove that the following compound propositions using laws of logic

i) $p \vee [p \wedge (p \vee q)] = p$

Sol. Take LHS,

$p \vee [p \wedge (p \vee q)] \Rightarrow p \vee p$ [$\because$ Absorption law $p \wedge (p \vee q) = p$]

$\Rightarrow p$ [$\because$ Idempotent law $p \vee p = p$]

ii) $[(p \vee q) \wedge (p \vee \sim q)] \vee q = p \vee q$

Sol. Take LHS,

$[(p \vee q) \wedge (p \vee \sim q)] \vee q = [p \vee (q \wedge \sim q)] \vee q$

$= [p \vee F_0] \vee q$

$= p \vee q$

(∵ distribution law
$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$]

Inverse law, $q \wedge \sim q = F_0$

By Identity law
$p \vee F_0 = p$]

iii) $(p \vee q) \wedge \sim[\sim p \vee q] = p \wedge \sim q$

Sol: Take,

LHS $(p \vee q) \wedge \sim[\sim p \vee q]$ (By demorgans law

$\Rightarrow \underbrace{(p \vee q)}_{p} \wedge [\underbrace{p}_{q} \wedge \underbrace{\sim q}_{r}]$

$\sim(p \vee q) = (\sim p \wedge \sim q)$

$\sim(\sim p \vee q) = \sim(\sim p) \wedge (\sim q) = p \wedge \sim q$

$\Rightarrow ((p \vee q) \wedge p) \wedge \sim q$

(By associative law
$p \wedge (q \wedge r) = (p \wedge q) \wedge r$]

(By absorption law
$p \wedge (p \vee q) = p$

$\Rightarrow$ $p \wedge \sim q$

iv) $\sim[\sim[(p \vee q) \wedge r] \vee \sim q] = q \wedge r$

$\sim[\sim(p \vee q) \wedge r] \vee \sim q]$

[By De-morgan's law
$\sim(p \wedge q) = \sim p \vee \sim q$]

$= \sim[\sim(\sim p \vee q) \vee \sim r] \vee \sim q]$

[∵ By de-morgon's law
$\sim[\sim(p \vee q) \vee \sim r]$

$= \sim[[(p \vee q) \wedge r] \vee \sim q]$

$= \sim(\sim(p \vee q)) \wedge \sim(\sim r))$
$\rightarrow (p \vee q) \wedge r$

$= \sim[p \wedge (q \wedge r) \vee \sim q]$

[∵ Associative law
$(p \vee q) \wedge r = p \wedge (q \wedge r)$]

demorgan law

Actual solution $\sim[\sim[((p \vee q) \wedge r) \wedge q)]$

$= [(p \vee q) \wedge r] \wedge q \Rightarrow$ commutative

$= q \wedge [(p \vee q) \wedge r] \Rightarrow$ Associative

$= [q \wedge (p \vee q)]] \wedge r \rightarrow$ absorbtion law

$= q \wedge r$

5) $(p \rightarrow q) \wedge [\sim q \wedge (r \vee \sim q)] = \sim(q \vee p)$

$\Rightarrow (p \rightarrow q) \wedge [\sim q \wedge (\sim q \vee r)]$ commutative law

$\Rightarrow (p \rightarrow q) \wedge \sim q$ $\Rightarrow$ Absorbtion law

$\Rightarrow (\sim p \wedge q) \wedge \sim q$ $\Rightarrow$ logically equivalence

$\Rightarrow \sim q \wedge (\sim p \vee q)$ $\Rightarrow$ Distributive law

$\Rightarrow (\sim q \wedge \sim p) \vee (\sim q \wedge q)$ $\rightarrow$ Identity law

$\Rightarrow \sim(q \vee p) \vee F_0$

$\Rightarrow \sim(q \vee p)$

Q. S.T $\{(p \vee q) \wedge \sim[\sim p \wedge (\sim q \vee \sim r)]\}$

$\vee$

$\{[(\sim p \wedge \sim q) \vee (\sim p \wedge \sim r)]$ in tautology

By demorgans law $\sim p \wedge \sim q = \sim(p \vee q)$

$\sim p \vee \sim q = \sim(p \wedge q)$

$\{(p \vee q) \wedge \sim[\sim p \wedge \sim(\sim q \wedge \sim r)]\}$

$\vee\ [\sim(\sim p \vee \sim q) \vee (\sim(\sim p \vee \sim q))]$

$\Rightarrow \{(p \vee q) \wedge \sim[\sim p \wedge \sim(q \wedge r)]\} \vee [\sim(p \vee q) \vee \sim(p \vee q)]$

$\Rightarrow \{(p \vee q) \wedge \sim[\sim(p \vee (q \wedge r))]$ (By distributive law

$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)]$

$\Rightarrow \{(p \vee q) \wedge \sim[\sim((p \vee q) \wedge (p \vee r))]$

(By demorgans law).

24/3/23 : Q. $p \to (q \to r) \equiv (p \wedge q) \to r$

Take LHS,

$p \to (q \to r)$ (conditional stmt $p \to q = \sim p \vee q$)

$\Rightarrow p \to (\sim q \vee r)$ $[\because p \to q = \sim p \vee q]$

$\Rightarrow \sim p \vee (\sim q \vee r)$ associative law $p \vee (q \vee r) = (p \vee q) \vee r$.

$\Rightarrow (\sim p \vee \sim q) \vee r$ demorgan's law $\sim p \vee \sim q = \sim (p \wedge q)$

$\Rightarrow \underbrace{\sim (p \wedge q)}_{p} \vee \underbrace{r}_{q}$ conditional statements $\sim p \vee q = p \to q$

$\Rightarrow (p \wedge q) \to r$

Rules of Inferences.

Name	Rule of inferences	Tautology
modus pones	$p \to q$ p $\therefore q$	$[(p \to q) \wedge p] \to q$
modus tollens	$p \to q$ $\sim q$ $\therefore \sim p$	$[(p \to q) \wedge \sim q] \to \sim p$
Transitive (or) Hypothetical Syllogism	$p \to q$ $q \to r$ $p \to r$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$

Disjunction syllogism	$p \vee q$ $\sim p$ $\therefore q$	$[(p \vee q) \wedge \sim p] \rightarrow q$ (or) $[(p \vee q) \wedge \sim q] \rightarrow p$
Addition	p / $\therefore p \vee q$ (or) q / $p \vee q$	$p \rightarrow (p \vee q)$ $q \rightarrow (p \vee q)$ (or)
Simplification	$p \wedge q$ / $\therefore p$ (or) $p \wedge q$ / $\therefore q$	$(p \wedge q) \rightarrow p$ $(p \wedge q) \rightarrow q$ (or)
Conjunction	p q $\therefore p \wedge q$	$[(p) \wedge (q)] \rightarrow [(p \wedge q)]$
Resolution	$p \vee q$ $\sim p \vee r$ $\therefore q \vee r$	$(p \vee q) \wedge (\sim p \vee r)$ $\rightarrow q \vee r$
Contra -tve	$p \rightarrow q$ $\therefore \sim q \rightarrow \sim p$	$(p \rightarrow q) \rightarrow (\sim q \rightarrow \sim p)$
law of implication	$p \vee q$ $\Downarrow$ $\therefore \sim p \rightarrow q$; $\sim p \vee q$ $\Downarrow$ $\therefore p \rightarrow q$	$(p \vee q) \rightarrow (\sim p \rightarrow q)$ (or) $(\sim p \vee q) \rightarrow (p \rightarrow q)$

Q. If I s...

If I'll p...

If I stu...

Let $p =$ ...

$r =$ ...

Step	
1.	$p \rightarrow q$
2	$q \rightarrow r$
3.	$p \rightarrow r$

Given a...

Q. $p \rightarrow q$

$q \rightarrow r$

$r \rightarrow s$

$\therefore p \rightarrow s$

Step	
1.	$p \rightarrow q$
2.	$q \rightarrow r$
3	$p \rightarrow r$
4.	...

Q. If I study, I'll pass in DM

If I'll pass in DM then my father will give gifts

If I study then my father will give gift

Let p = If I study ; q = I'll pass in DM

r = my father will give gifts

	Step	reason
1.	$p \to q$	Given premises
2	$q \to r$	Given premises
3.	$p \to r$	① & ② are transitive

Given argument is valid.

Q. $p \to q$

$q \to r$

$r \to s$

$\therefore p \to s$

	Step	reason
1.	$p \to q$	Given premises
2.	$q \to r$	Given premises
3	$p \to r$	① & ② are transitive
4.	$r \to s$	Given premises

$p \to s$ ③ & ④ are transitive

∴ Hence given argument is valid

Q. $r \to s$

$\sim s$

$\therefore \sim r$

Sol:

step	reason
① $r \to s$	Given premises
② $\sim s$	Given premises
③ $\sim r$	① & ② are modus Pollens

∴ Given arguments is valid

Q. $r \to s$

$p \to q$

$r \vee p$

$\therefore s \vee q$

Step	reason
① $r \to s$	Given premises
② $\sim r \vee s$	① is law of implication
③ $r \vee p$	Given premises
④ $\sim r \to p$	③ is law of implication
⑤ $p \vee s$	③ & ② is resolution
⑥ $p \to q$	Given premises
⑦ $\sim p \vee q$	⑥ is law of implication
⑧ $s \vee q$	⑤ & ⑦ is resolution

Q: $p \to r$

$\sim p \to q$

$q \to s$

$\sim r \to s$

Q. S.T $p \to s$ can be derived from the premises $\sim p \vee q$, $\sim q \vee r$, $r \to s$

$$\begin{array}{c} \sim p \vee q \\ \sim q \vee r \\ r \to s \\ \hline \therefore p \to s \end{array}$$

Step/ state	reason
① $\sim p \vee q$	Given premises
② $p \to q$	① is law of implication
③ $\sim q \vee r$	Given premises
④ $q \to r$	③ is law of implication
⑤ $r \to s$	Given premises
⑥ $p \to r$	② & ④ are transitive
⑦ $p \to s$	⑥ & ⑤ are transitive

$\therefore$ The given argument is valid.

28/3/23

1) If Sachin hits a century (p) he gets a free car (q)

2) Sachin hits a century

∴ Sachin gets a free car

P_1: $p \to q$

P_2: p

By using Modus Pones

$$\begin{array}{ll} P_1: & p \to q \\ P_2: & p \\ \hline \therefore & q \end{array}$$

$$[(p \to q) \wedge p] \to q$$

Step	Premises/reason
1. $p \to q$	Given premise
2. p	Given premise
3. $(p \to q) \wedge p \Rightarrow q$	from ① & ② Modus Pones

∴ Given Argument is valid.

2. P_1:

P_2:

P_3

1. p

2. q

3. $\sim$

4. $\sim p$

5. q

6.

7.

$q \to \sim p$

$\sim r \to p$ $\quad q \to r$

$\sim p \to r$

2. P1: If I study, then I'll not fail in exams
$\underbrace{\text{study}}_{p}$, $\underbrace{\text{not fail in exams}}_{\sim q}$

P2: If I do not watch tv in the evening then I'll study
$\underbrace{\text{do not watch tv in the evening}}_{\sim r}$, $\underbrace{\text{I'll study}}_{p}$

P3: I failed in the examination
$\underbrace{\text{failed in the examination}}_{q}$

I must have watched tv in the evening
$\underbrace{\text{watched tv}}_{r}$

Step	reason
1. $p \to \sim q$	given premises
2. $q \to \sim p$	contra +ve — 1
3. $\sim r \to p$	given
4. $\sim p \to r$	contra +ve
5. $q \to r$	from 2 & 4 Transitive
6. q	given
7. r	from 5 & 6, modus ponens

Given Arguments are True

* P1: If a baby is hungry, then the baby cries

P2: If the baby is not mad, then the baby does not cry.

P3: If a baby is mad, then the baby has red face

∴ If a baby is hungry, then the baby has red face.

Sol:

$p \to$ baby is hungry

$q \to$ baby cries

$r \to$ baby is mad

$s \to$ baby has red face

Given

P1 : $p \to q$

P2 : $\sim r \to \sim q$

P3 : $r \to s$

$\therefore p \to s$

Step		reason
1.	$p \to q$	Given premise
2.	$\sim r \to \sim q$	Given premise
3.	$q \to r$	Contra tve of 2
4.	$p \to r$	from 1 & 3, using Transitive
5.	$r \to s$	Given
6.	$p \to s$	from 4 & 5 using Transitive

* P1 :-

P2 :

P3 :

∴

Sol.

1.

2.

3

4

5

$\therefore$ Hence, Given Argument is True (valid).

P1 :- Ravi works hard,

P2 : If Ravi works hard, then he is a dull boy
p q

P3 : If Ravi is dull boy, then he will not get a job.
q $\sim r$

$\therefore$ He will not get a job
$\sim r$

Sol.

Step	reason
1. $p \to q$	Given
2. $q \to \sim r$	Given
3. $p \to \sim r$	from 1 & 2, Transitive
4. p	Given
5. $\sim r$	from 3 & 4, Modus pones

$\therefore$ Given argument is valid

* P1: I will become famous or I will not become a musician

p $\qquad$ $\sim q$

P2: I will become a musician

q

∴ Conclusion: I will become famous

p

Sol. Given

$$\begin{array}{c} p \vee q \\ \sim p \\ \hline q \end{array}$$

p : I will become famous

q : I will not become musician

$$p \to q = \sim p \vee q$$

	Step	Reason
1.	$p \vee q$	Given
2.	$\sim q$	Given
3.	$\therefore p$	from ① & ② using Disjunction

From law of implication

①
$$\begin{array}{c} p \wedge q \\ p \to (q \to r) \\ \hline r \end{array}$$

②
$$\begin{array}{c} p \to q \\ r \to s \\ p \vee r \\ \hline \therefore q \vee s \end{array}$$

not
ill become

$\sim q$

3. $p \to q$

$r \to s$

$\sim q \vee \sim s$

$\sim(p \wedge r)$

4. $p \to r$

$q \to r$

$(p \vee q) \to r$

5. $p \to q$

$r \to s$

$p \vee r$

$\therefore q \vee s$

---

1. Given,

$p \wedge q$

$p \to (q \to r)$

$\therefore r$

Step	reason
$p \wedge q$	given premises
$p \to (q \to r)$	given premises
$(p \wedge q) \to r$	2 is law of logic
r	3 & 1 Are Modus pones

$\therefore$ Given Arguments

2. $p \to q$

$r \to s$

$p \vee r$

$\therefore q \vee s$

Step	reason
1. $p \to q$	
2. $\sim p \vee q$	// sol^n back side
3. $p \vee r$	
4. $r \vee q$	
5. $\sim p \to r$	
6. $r \to s$	
7. $\sim r \vee s$	

2. $p \to q$

$r \to s$

$p \vee r$

$\therefore q \vee s$

Step	Reason
1. $p \to q$	Given premises
2. $\sim p \vee q$	1 is law of implication
3. $r \to s$	Given premises
4. $\sim r \vee s$	3 is law of implication
5. $p \vee r$	Given premises
6. $r \vee p$	5 is commutative
7. $p \vee s$	6 & 4 are Resolution
8. $q \vee s$	7 & 2 are resolution

3. $p \to q$

$r \to s$

$\sim q \vee \sim s$

$\therefore \sim(p \wedge r)$

Step	reason
1. $p \to q$	Given premises
2. $\sim p \vee q$	1 is law of implication
3. $r \to s$	Given premises
4. $\sim r \vee s$	3 is law of implication
5. $\sim q \vee \sim s$	Given premises
6. $\sim r \vee \sim q$	4 & 5 are Resolution
7. $\sim p \vee \sim r$	2 & 6 are resolution
8. $\sim(p \wedge r)$	De morgan's law

4. $p \to r$

$q \to r$

$\therefore (p \vee q) \to r$

$(r \vee \sim p) \wedge (r \vee \sim q)$

Step	reason
$p \to r$	
$q \to r$	
$(p \to r) \wedge (q \to r)$	
$(p \vee q) \to r$	

---

* Q. If you send me an email message, then I will finish writing the program

If you don't send me an email message, then I will go sleep early

If I go to sleep early then I will wakeup feeling refreshed.

$\therefore$ If I donot finish writing the program then I will wake up feeling refreshed.

Sol: Let p = Send me an email message

q = I will finish writing the program

r = I will go to sleep early

s = I will wakeup feeling refreshed

Given, $p \to q$
$\sim p \to r$
$r \to s$
$\therefore \sim q \to s$

Step	Reason
1. $p \to q$	given premises
2. $\sim p \vee q$	(1) is law of implication
3. $\sim p \to r$	Given premises
4. $p \vee r$	(3) is law of implication
5. $q \vee r$	(4) & (2) are Resolution
6. $r \to s$	given premises
7. $\sim r \vee s$	(6) is law of implication
8. $q \vee s$	(5) & (7) are Resolution
9. $\sim q \to s$	(8) is law of implication

Shortcut :-
$p \to q$
$\sim q \to \sim p$
$\sim p \to r$
$\sim q \to r$
$r \to s$
$\sim q \to s$

Q. If Ravi Studies then he will pass DM
If Ravi does not play cricket then he will study
Ravi failed in DM

$\therefore$ Ravi played cricket

Sol:- Given, p = Ravi studies,
q = he will pass DM
r = Ravi plays cricket

$p \to q$
$\sim r \to p$
$\sim q$

$\therefore r$

St
1. p
2. ~
3. ~
4. ~
5. r
6. r

Q. It is n
than
we
If w
If we

$\therefore$ w

Sol: Let ,

Step	Reason
1. $p \rightarrow q$	given premises
2. $\sim r \rightarrow p$	Given premises
3. $\sim q$	Given premises
4. $\sim p$	1 & 3 are modus tollens
5. $r \vee p$	2 is law of implication
6. r	5 & 4 are Disjunction syllogism

$\therefore$ Given Argument is valid

Q. It is not Sunny this afternoon and it is colder than yesterday.

we will go swimming only if it is sunny

If we do not go swimming, then we will take a hyd trip

If we take the hyd trip, then we will be home by sunset

$\therefore$ we will be home by sunset

Sol: Let, p = It is sunny this aftenoon

q = It is colder than yesterday

r = we will go swimming

s = we will take a hyd trip

t = we will be home by sunset

Given $\sim p \wedge q$
$r \rightarrow p$
$\sim r \rightarrow s$
$s \rightarrow t$
$\therefore t$

Step		reason
1.	$\sim r \rightarrow s$	Given premises
2.	$s \rightarrow t$	Given premises
3.	$\sim r \rightarrow t$	1 & 2 are transitive
4.	$r \rightarrow p$	Given premises
5.	$\sim r \vee p$	4 is law of implication
6.	$r \vee t$	3 is law of implication
7.	$p \vee t$	5 & 6 are resolution
8.	$\sim p \wedge q$	Given premises
9.	$\sim p$	8 is simplification
10.	t	7 & 9 are Disjunction Syllogism

$\therefore$ Hence Given argument is valid

First Order Logic (or) Predicate logic:-

In order to understand predicate logic, we must understand the following:

i) predicate.
ii) Quantifiers

predicates (open propositions):-

predicates are the statements involving variable which are neither True nor false until or unless the values of the variables are specified and it is denoted by $P(x)$.

Ex: 'x is an animal'

"$x<4$", "$y>5$", "$x+y=5$", "x is a rational no." etc.

→ All the above statements are neither True nor false, so they are not propositions.

Consider the Statement "x is an animal".

Here "x" is subject.

"is an animal" → predicate

∴ $A(x)$: x is an animal

"x is rational number"

∴ $R(x)$: x is rational number

"$y>5$"

∴ $G(y)$: y is greater than 5.

* Predicates can be Combined with logical connectivities like propositions.

Ex: "x is rational number or $x<5$" can be symobilized by $R(x) \vee L(x)$.

31/3/23 :-

1) 'x is a Rational Number or $x>3$'

Can be Symbolized by $R(x) \vee G(x)$

2) If x is a Rational Number then $x>5$

Can be Symbolized by $R(x) \rightarrow G(x)$

* **Quantifiers**:- Certain declarative Sentences involve words that indicates Quantity such as "all", "some", "one", "none" etc. These words helps to determine, for how many elements the predicate is true. Such words are called quantifiers.

Quantifiers are 2 types :-

1) Universal Quantifiers

2) Existential Quantifiers

1) Universal Quantifiers:- The Quantifier "all" is called Universal Quantifier and it is denoted by $\forall x, P(x)$ [$P(x)$ is true for all x]

Ex:- $P(x): x+1>x$

$P(1): 2>1\,(T)$, $P(2): 2+1>1\,(T)$, $P(3): 3+1>3\,(T)$

- - - - - $\forall x, P(x)$

$\therefore$ $P(x)$ is true for all 'x'; $\therefore$ $\boxed{\forall x, P(x)}$
integers.

ii) Existential Quantifier :- The Quantifier "Some" is called existential Quantifier and it can be denoted by "$\exists x, P(x)$".

($P(x)$ is true for some x).

Ex:- $G(x) : x < 5$

$G(1): 1<5\,(T)$, $G(2): 2<5\,(T)$, $G(3): 3<5\,(T)$

$G(4): 4<5\,(T)$, $G(5): 5<5\,(F)$, $G(6): 6<5\,(F)$ - - -

$\therefore$ $G(x)$ is true for some x;

It is denoted by $\boxed{\exists x, G(x)}$

Ex:- "all birds can fly"

$B(x)$: x is a bird

$F(x)$: x can fly

$\wedge / \vee$

* $\forall x, [B(x) \wedge F(x)]$

* $\forall x, [B(x) \rightarrow F(x)]$ (final)

* 2) Lions are dangerous

$L(x)$: x is a lion

$D(x)$: x is dangerous

$[S(x) \to (D(x) \wedge \sim A(x)$

$$\forall x, \ [L(x) \to D(x)]$$

3) There is a student who likes DM and does not likes AI.

$S(x)$: x is a student

$D(x)$: x likes DM

$A(x)$: x likes AI

$$\exists x, \ [S(x) \wedge D(x) \wedge \sim A(x)]$$

* Rules of Inferences for Quantified Propositions

	Name	Rule of Inference
1.	Universal Speification	$\dfrac{\forall x, P(x)}{\therefore P(c) \text{ for all } c}$
2.	Universal Generalization	$\dfrac{P(c) \text{ for all 'c'}}{\therefore \forall x, P(x)}$
3.	Existential Specification	$\dfrac{\exists x, P(x)}{\therefore P(c) \text{ for some } c.}$
4	Existential Generalization	$\dfrac{P(c) \text{ for some } c.}{\therefore \exists x, P(x)}$

$\sim A(x)$

Q. All men are mortal
Sachin is a man

∴ Sachin is mortal

$M(x)$:- x is a man
$T(x)$: x is mortal

Given,
$$\forall x, M(x) \to T(x)$$
$$M(S)$$
$$\therefore T(S)$$

	Step	reason
1.	$\forall x, M(x) \to T(x)$	Given
	$M(c) \to T(c)$	
2.	$M(S) \to T(S)$	① is Universal Specification $C =$ Sachin
3.	$M(S)$	Given
4.	$T(S)$	② & ③ are Modus ponens
	∴ Hence	Given argument is valid

* **Problem** :- check whether the following arguments is valid

1. All men are fallible
2. All Kings are men

∴ All Kings are fallible

Sol:

$M(x)$: x is men

$F(x)$: x is fallible

$K(x)$: x is King

Given,

$$\forall x, [M(x) \to F(x)]$$

$$\forall x, [K(x) \to M(x)]$$

$$\therefore \forall x, (K(x) \to F(x))$$

Step	reason
1. $\forall x, [M(x) \to F(x)]$	Given
2. $M(c) \to F(c)$	1 is Universal Specification
3. $\forall x, [K(x) \to M(x)]$	Given
4. $K(c) \to M(c)$	3 is Universal Specification
5. $K(c) \to F(c)$	4 & 2 are Transitive
6. $\forall x, [K(x) \to F(x)]$	5 is Universal Generalization

$\therefore$ Given argument is valid

2 Tigers are dangerous animal

There is / are tigers.

$\therefore$ There are dangerous animals

$T(x)$: x is a Tiger

$D(x)$: x is a dangerous animal

Given,

$$\forall x,\ T(x) \rightarrow D(x)$$

$$\underline{\exists x,\ T(x)}$$

$$\therefore\ \exists x,\ D(x)$$

Step | **reason**

1. $\forall x,\ T(x) \rightarrow D(x)$ — Given proposition
2. $T(c) \rightarrow D(c)$ — ① is Universal Specification
3. $\exists x,\ T(x)$ — Given
4. $T(c)$ — ③ is Universal Existential Specification
5. $D(c)$ — ② & ④ are Modus pones
6. $\exists x,\ D(x)$ — ⑤ is Existential Generalization

$\therefore$ Given argument is valid

4. All Integers are rational numbers

Some Integers are powers of 2

∴ Some rational numbers are powers of 2

Sol. Given, $I(x)$: x is a Integer

$R(x)$: x is a Rational no.

* $P(x)$: x is a power of 2

Given,

$$\forall x,\ I(x) \rightarrow R(x)$$

$$\exists x,\ I(x) \wedge P(x)$$

$$\therefore \exists x,\ R(x) \wedge P(x)$$

Step	reason
1. $\forall x,\ I(x) \rightarrow R(x)$	Given
2. $I(c) \rightarrow R(c)$	(1) is Universal Specification
3. $\exists x,\ I(x) \wedge P(x)$	Given
4. $I(c) \wedge P(c)$	(3) is Existential Specification
5. $I(c)$	~~(2) & (4)~~ (4) is simplification
6. $P(c)$	(4) is simplification

7. $R(c$

8. $P(c$

9. $\exists x,$

5. All

All

All

∴

Sol: Given

7. $R(c)$

8. $P(c) \wedge R(c)$

(2) & (5) are modus pones

(7) & (6) are conjunction

9. $\exists x, R(x) \xrightarrow{\wedge} P(x)$

Existential Generalization

$\therefore$ Given argument is valid.

5. All Squares are Rectangles

All Rectangles are parallelograms

All Parallelograms are Quadrilateral

$\therefore$ All Squares are Quadrilaterals

Sol: Given,

$S(x)$: x is a Square

$R(x)$: x is a Rectangle

$P(x)$: x is a parallelogram

$Q(x)$: x is a Quadrilateral

$\forall x, S(x) \rightarrow R(x)$

$\forall x, R(x) \rightarrow P(x)$

$\forall x, P(x) \rightarrow Q(x)$

$\forall x, S(x) \rightarrow Q(x)$

Step	reason
1. $\forall x, S(x) \to R(x)$	Given proposition
2. $S(c) \to R(c)$	Universal Specification
3. $\forall x, R(x) \to P(x)$	Given
4. $R(c) \to P(c)$	U.S
5. $\forall x, P(x) \to Q(x)$	Given
6. $P(c) \to Q(c)$	U.S
7. $S(c) \to P(c)$	Transitive from 2 & 4
8. $S(c) \to Q(c)$	Transitive from 7 & 6
9. $\forall x, S(x) \to Q(x)$	E.G

6. All Engineering students are good in Studies
William is Engineering Student

$\therefore$ William is good at Study

Sol $E(x)$: x is a Engineering student

$S(x)$: x is good at study

Given,

$$\forall x, E(x) \to S(x)$$
$$E(william)$$
$$\overline{\therefore S(william)}$$

Step	Reason
1. $\forall x, E(x) \to S(x)$	Given
2. $E(c) \to S(c)$	U.S
3. $E(william) \to S(william)$	c = william
4. $E(william)$	Given
5. $S(william)$	3 & 4 modus pones

$\therefore$ Hence Given Argument is valid

$\forall x, \sim L(x) \longrightarrow \sim N(x)$

7. All Humming birds are Richly coloured

$\Rightarrow$ No large birds live on honey

$\Rightarrow$ large birds doesnot live on honey

Birds that donot live on honey are dull in coloured

$\therefore$ Hummings birds are small.

Sol

$H(x)$: x is Humming bird

$R(x)$: x is Richly coloured

$N(x)$: x lives on honey

$L(x)$: x is alarge bird.

Given,

$$\left.\begin{array}{l} \forall x, H(x) \rightarrow R(x) \\ \forall x, L(x) \rightarrow \sim N(x) \\ \forall x, \sim N(x) \rightarrow \sim R(x) \end{array}\right\}$$

$\therefore \forall x, H(x) \rightarrow \sim L(x)$

Step	Reason
1. $\forall x, H(x) \rightarrow R(x)$	Given
2. $H(c) \rightarrow R(c)$	U.S True
3. $\forall x, L(x) \rightarrow \sim N(x)$	Given
4. $L(c) \rightarrow \sim N(c)$	U. (2)
5. $\forall x, \sim N(c) \rightarrow \sim R(c)$	$\frac{K(K+1)}{2}$
6. $\sim N(c) \rightarrow \sim R(c)$	

$p \vee q$ $\quad$ $P \to q$

$\sim p \vee r$ $\quad$ $q \vee r$ $\quad$ $\sim p \vee q$

7.	$L(c) \to \sim R(c)$	4 & 6 are Transitive
8.	$\sim H(c) \vee R(c)$	2 is implication
9.	$\sim L(c) \vee \sim R(c)$	7 is implication
10.	$R(c) \vee \sim H(c)$	8 is Commutative
11.	$\sim R(c) \vee \sim L(c)$	9 is commutative
12.	$\sim H(c) \vee \sim L(c)$	10, 11 are Resolution
13.	$H(c) \to \sim L(c)$	12 is implication
14.	$\forall x, H(x) \to \sim L(x)$	13 is U.G

$\therefore$ Hence, Given Argument is valid.

11/4/23 :- Mathematical Induction :-

Let $P(n)$ be a statement which may be either True or false for each positive number (Integer) 'n'. To prove $P(n)$ is true for all positive integer ($n \in N$). There are 3 steps to a proof using the principle of Mathematical Induction.