(iii) P(Atmost two rivets will be defective)

i.e $P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$

$= 0.59 + 0.32 + {}^5C_2 (0.1)^2 (0.9)^3$

$= 0.59 + 0.32 + 0.07$

$= 0.99$

Binomial Frequency Distribution:

If n independent trials constitute one experiment. This experiment repeated N times then the binomial frequency distribution is $N(p+q)^n$.

① Four coins are tossed 160 times. The no. of times x heads occur is given below.

x	0	1	2	3	4
No. of times $f(x)$	8	34	69	43	6

Fit a Binomial Distribution to this data on the hypothesis that coins are unbiased.

soln: The coin is unbiased

$p = \frac{1}{2}$, $q = \frac{1}{2}$, $n = 4$ ($\because$ four coins are tossed.)

No. of times repeated $N = 160$

By Binomial Distribution $P(x=r) = {}^{n}C_{r}\, p^{r} q^{n-r}$

so $P(x=0) = {}^{4}C_{0}\, p^{0} q^{4} = 1 \cdot \left(\frac{1}{2}\right)^{0}\left(\frac{1}{2}\right)^{4} = \frac{1}{16}$

$p(x=1) = {}^{4}C_{1} \cdot \left(\frac{1}{2}\right)^{1}\left(\frac{1}{2}\right)^{3} = 4 \cdot \frac{1}{2} \cdot \frac{1}{8} = \frac{1}{4}$

$P(x=2) = {}^{4}C_{2}\left(\frac{1}{2}\right)^{2}\left(\frac{1}{2}\right)^{2} = 6 \cdot \frac{1}{4} \cdot \frac{1}{4} = 3/8$

$P(x=3) = {}^{4}C_{3}\left(\frac{1}{2}\right)^{3}\left(\frac{1}{2}\right) = 4 \cdot \frac{1}{8} \cdot \frac{1}{2} = 1/4$

No. of heads (x)	No. of times $f(x)$	Probability $P(x=x) = {}^{n}C_{x}\, p^{x} q^{n-x}$	expected or Theoritical frequency $f(a) = N \cdot P(x=x)$
0	08	$P(0) = 1/16$	$160 \times \frac{1}{16} = 10$
1	34	$P(1) = 1/4$	$160 \times \frac{1}{4} = 40$
2	69	$P(2) = 3/8$	$160 \times \frac{3}{8} = 60$
3	43	$P(3) = 1/4$	$160 \times \frac{1}{4} = 40$
4	06	$P(4) = 1/16$	$160 \times \frac{1}{16} = 10$
	160		160 expected

(2) Fit a binomial Distribution for the following data. Write down x values & corresponding frequencies. Here x values are 0, 1, 2, 3, 4, 5, 6 corresponding frequencies 13, 25, 52, 58, 32, 16, 4.

Soln: Let make a table from the given data

x	0	1	2	3	4	5	6
f(x)	13	25	52	58	32	16	4

Here W.K.T $N = \sum f_i = 13+25+52+58+32+16+4$

$$\boxed{N = 200}$$

To find p and q

W.K.T mean $= np = \dfrac{\sum x_i f_i}{N} =$

$$= \frac{0\times13+1\times25+2\times52+3\times58+4\times32+5\times16+6\times4}{200}$$

$$= \frac{535}{200} = 2.67$$

Here $n = 6$

$\therefore\ 6p = 2.67$

$$p = \frac{2.67}{6} = 0.38$$

$$\Rightarrow \boxed{p = 0.38 \simeq 0.4}$$

w.k.T $\quad p+q=1 \Rightarrow q = 1-0.38 \simeq 1-0.4$

$$\boxed{q = 0.6}$$

By Binomial Distribution.

$P(x=0) = {}^6C_0 \cdot (0.4)^0 (0.6)^6 = (0.6)^6 = 0.0466$

$P(x=1) = {}^6C_1 (0.4)^1 (0.6)^5 = 6(0.4)(0.6)^5 = 0.186$

$P(x=2) = {}^6C_2 (0.4)^2 (0.6)^4 = \frac{\cancel{6}^{3}\times 5}{\cancel{2}}(0.4)^2(0.6)^4 = 0.311$

$P(x=3) = {}^6C_3 (0.4)^3 (0.6)^3 = \frac{6\cdot 5\cdot 4}{1\cdot 2\cdot 3}(0.4)^3(0.6)^3 = 0.276$

$P(x=4) = {}^6C_4 (0.4)^4 (0.6)^2 = \frac{\cancel{6}^{3}\times 5\times \cancel{4!}}{\cancel{4!}\times \cancel{2!}}(0.4)^4(0.6)^2 = 0.138$

$P(x=5) = {}^6C_5 (0.4)^5 (0.6) = 6(0.4)^5(0.6)^1 = 0.036$

$P(x=6) = {}^6C_6 (0.4)^6 (0.6)^0 = (0.4)^6 = 0.004$

Then Expected frequencies are

$f(0) = N.P(x=0) = 200 \times 0.0466 = 9.32$

$f(1) = N.P(x=1) = 200 \times 0.186 = 37.2$

$f(2) = N.P(x=2) = 200 \times 0.311 = 62.2$

$f(3) = N.P(x=3) = 200 \times 0.276 = 55.2$

$f(4) = N.P(x=4) = 200 \times 0.138 = 27.6$

$f(5) = N.P(x=5) = 200 \times 0.036 = 7.2$

$f(6) = N.P(x=6) = 200 \times 0.004 = 0.8$

199.52

$\simeq 200$

③ Fit a Binomial Distribution for the following distribution.

x	0	1	2	3	4	5
$f(x)$	2	14	20	34	22	8

Soln: $N = \sum f_i = 100$

To find p: $np = \text{mean} = \dfrac{\sum x_i f_i}{N}$

$$np = \frac{(0\times2)+(1\times14)+(2\times20)+(3\times34)+(4\times22)+(5\times8)}{100}$$

$$np = 2.84 \Rightarrow p = \frac{2.84}{5} = 0.568$$

$$\boxed{p = 0.568}$$

$\therefore p+q = 1 \Rightarrow q = 1-p = 1-0.568$

$$\boxed{q = 0.432}$$

By Binomial Distribution

$P(X=0) = 5C_0 (0.568)^0 (0.432)^5 \cong 0.059$

$P(X=1) = 5C_1 (0.568)^1 (0.432)^4 \simeq 0.2248$

$P(X=2) = 5C_2 (0.568)^2 (0.432)^3 \simeq 0.34198$

$P(X=3) = 5C_3 (0.568)^3 (0.432)^2 \simeq 0.26014$

$P(X=4) = 5C_4 (0.568)^4 (0.432)^1 \simeq 0.0989$

$P(X=5) = 5C_5 (0.568)^5 (0.432)^0 \simeq 0.0150$

Then Expected frequencies are

$f(0) = N\cdot P(0) = 100 \times 0.059 = 5.9 \simeq 6$

$f(1) = N\cdot P(1) = 100 \times 0.2248 = 22.48 \simeq 22$

$f(2) = N\cdot P(2) = 100 \times 0.34198 = 34.198 \simeq 34$

$f(3) = N\cdot P(3) = 100 \times 0.26014 = 26.014 \simeq 26$

$f(4) = N\cdot P(4) = 100 \times 0.0989 = 9.89 \simeq 9.8$

$f(5) = N\cdot P(5) = 100 \times 0.0150 = 1.50 \simeq 1.5$

$\underline{100.7}$

(4) The mean of binomial Distribution is 3 and variance is 9/4. Find (i) the value of n (ii) $P(X \geq 7)$ (iii) $P(1 \leq x \leq 6)$

Soln: Given mean $(np) = 3$

Variance $npq = 9/4$

$(np)q = \frac{9}{4}$

$3q = \frac{9}{4} \Rightarrow q = \frac{3}{4}$,

so $p = 1 - q = 1 - \frac{3}{4} = 1/4$

$\therefore \boxed{p = 1/4 \quad q = \frac{3}{4}}$

(i) value of 'n'

mean $np = 3$.

$n \cdot \frac{1}{4} = 3 \Rightarrow \boxed{n = 12}$

(ii) $P(x \geq 7) = 1 - P(x < 7)$

$= 1 - [P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4) + P(x=5) + P(x=6)]$

$= 1 - \{ {}^{12}C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{12} + {}^{12}C_1 \left(\frac{1}{4}\right)^1 \left(\frac{3}{4}\right)^{11} + {}^{12}C_2 \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)$

$+ {}^{12}C_3 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^9 + {}^{12}C_4 \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right)^8 + {}^{12}C_5 \left(\frac{1}{4}\right)^5 \left(\frac{3}{4}\right)^7$

$+ {}^{12}C_6 \left(\frac{1}{4}\right)^6 \left(\frac{3}{4}\right)^6 \} = 0.014$

$P(1 \leq x \leq 6) = P(x=1) + P(x=2) + P(x=3) + P(x=4) + P(x=5) + P(x=6)$

$= {}^{12}C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{12} + {}^{12}C_1 \left(\frac{1}{4}\right)^1 \left(\frac{3}{4}\right)^{11} + {}^{12}C_2 \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^{10}$

$+ {}^{12}C_3 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^9 + {}^{12}C_4 \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right)^8 + {}^{12}C_5 \left(\frac{1}{4}\right)^5 \left(\frac{3}{4}\right)^7 + {}^{12}C_6 \left(\frac{1}{4}\right)^6 \left(\frac{3}{4}\right)$

$= 0.952$

Poisson's Distribution:

It is suitable for rare events for which the probability of occurence of p is very small and the no. of trials 'n' is very large, where np is finite. Then poisson's distribution is

$$P(x, \lambda) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}, \quad x = 0, 1, 2, \ldots \infty$$

or

$$\frac{e^{-\mu} \cdot \mu^{x}}{x!} \qquad \begin{pmatrix} n \to \infty \\ np = \lambda \\ \text{or} \\ np = \mu \end{pmatrix}$$

(i) Mean of P.D (μ or λ):

W.K.T mean of D.R.V $= \sum x_i P_i$

$$= \sum x_i P(x, \lambda)$$

i.e $$E(x) = \sum_{x=0}^{\infty} x_i \cdot \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$$

$$= e^{-\lambda} \cdot \sum_{x=0}^{\infty} \frac{\cancel{x} \cdot \lambda^{x}}{\cancel{x}(x-1)!}$$

$$= e^{-\lambda}\left[\frac{\lambda}{1} + \frac{\lambda^2}{1!} + \frac{\lambda^3}{2!} + \ldots\right]$$

$$= e^{-\lambda} \cdot \lambda\left[1 + \lambda + \frac{\lambda^2}{2!} + \ldots\right]$$

$$= e^{-\lambda} \cdot \lambda e^{\lambda} \qquad \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots = e^{x}\right)$$

$$= \lambda$$ $\therefore$ Mean of P.D is λ or μ i.e $\boxed{E(x) = \lambda}$

(ii) Variance (σ^2)

$$\sigma^2 = E(x^2) - [E(x)]^2$$

$$= \sum_{x=0}^{\infty} x^2 P(x,\lambda) - (\lambda)^2$$

$$= \left[\sum_{x=0}^{\infty} (x^2+x-x) \cdot \frac{e^{-\lambda} \cdot \lambda^x}{x!}\right] - \lambda^2$$

$$= \sum_{x=0}^{\infty} \left(\frac{x(x-1)+x}{x!}\right) e^{-\lambda} \cdot \lambda^x - \lambda^2$$

$$= \sum_{x=0}^{\infty} \frac{x(x-1)}{x!} e^{-\lambda} \cdot \lambda^x + \sum_{x=0}^{\infty} \frac{x}{x!} e^{-\lambda} \cdot \lambda^x - \lambda^2$$

$$= \sum_{x=0}^{\infty} \frac{\cancel{x(x-1)}}{\cancel{x(x-1)}(x-2)!} e^{-\lambda} \cdot \lambda^x + \sum_{x=0}^{\infty} \frac{\cancel{x} \cdot e^{-\lambda} \cdot \lambda^x}{\cancel{x}(x-1)!} - \lambda^2$$

$$= e^{-\lambda} \cdot \sum_{x=0}^{\infty} \frac{\lambda^x}{(x-2)!} + \lambda - \lambda^2$$

$$= e^{-\lambda}\left[\frac{\lambda^2}{1} + \frac{\lambda^3}{1!} + \frac{\lambda^4}{2!} + \frac{\lambda^5}{3!} + \cdots\right] + \lambda - \lambda^2$$

$$= e^{-\lambda} \cdot \lambda^2\left[1 + \lambda + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \cdots\right] + \lambda - \lambda^2$$

$$= \cancel{e^{-\lambda}} \cdot \lambda^2 \cdot \cancel{e^{\lambda}} + \lambda - \lambda^2$$

$$= \cancel{\lambda^2} + \lambda - \cancel{\lambda^2}$$

$$\boxed{\sigma^2 = \lambda}$$

and S.D $\sigma = \sqrt{\lambda}$

Problems

(1) If a R.V has a P.D such that $P(1) = P(2)$.
Find (i) the mean of the distribution
(ii) $P(4)$ (iii) $P(X \geq 4)$ (iv) $P(1 < x < 4)$

soln: Given that $P(1) = P(2)$

(i) w.k.t $P(X) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$

Given $P(1) = P(2)$

$\Rightarrow \frac{\cancel{e^{-\lambda}} \cdot \cancel{\lambda}}{1!} = \frac{\cancel{e^{-\lambda}} \cdot \lambda^{\cancel{2}}}{2!}$

$\Rightarrow \boxed{\lambda = 2}$

or $\lambda^2 - 2\lambda = 0$
$\lambda(\lambda - 2) = 0$
$\lambda = 0, \lambda = 2$
But λ is always +ve $\therefore \lambda = 2$

$\therefore \boxed{\text{Mean} = \lambda = 2}$

(ii) $P(4) = \frac{e^{-\lambda} \cdot \lambda^{4}}{4!} = \frac{e^{-2} \cdot (2)^{4}}{4 \times 3 \times 2 \times 1} = \frac{e^{-2} \cdot \cancel{16}^{2}}{\cancel{4} \times 3 \times \cancel{2} \times 1}$

$= \frac{2}{3} e^{-2} = 0.09$

iii) $P(X \geq 4) = 1 - P(X < 4)$

$= 1 - [P(0) + P(1) + P(2) + P(3)]$

$= 1 - \left[\frac{e^{-\lambda} \cdot \lambda^{0}}{0!} + \frac{e^{-\lambda} \cdot \lambda^{1}}{1!} + \frac{e^{-\lambda} \cdot \lambda^{2}}{2!} + \frac{e^{-\lambda} \cdot \lambda^{3}}{3!} \right]$

$= 1 - e^{-\lambda}\left[1 + \lambda + \frac{\lambda^2}{2} + \frac{\lambda^3}{6} \right]$

$= 1 - e^{-2}\left[1 + 2 + \frac{(2)^2}{2} + \frac{(2)^3}{6}\right]$

$= 1 - e^{-2}\left[1 + 2 + 2 + \frac{4}{3}\right]$

$= 1 - e^{-2}\left[5 + \frac{4}{3}\right] = 1 - e^{-2}\left[\frac{15+4}{3}\right]$

$= 1 - e^{-2}\left[\frac{19}{3}\right] = 0.15$

(iv) $P(1 < x < 4) = P(2) + P(3)$

$= \frac{e^{-\lambda}\cdot\lambda^2}{2!} + \frac{e^{-\lambda}\cdot\lambda^3}{3!}$

$= e^{-\lambda}\left[\frac{\lambda^2}{2} + \frac{\lambda^3}{6}\right] = e^{-2}\left[\frac{\cancel{4}^{2}}{\cancel{2}} + \frac{\cancel{8}^{4}}{\cancel{6}_{3}}\right]$

$= e^{-2}\left[2 + \frac{4}{3}\right] = e^{-2}\left[\frac{10}{3}\right] = 0.45$

(2) If a P.D $\ni P(X=1)\cdot\frac{3}{2} = P(X=3)$ then Find (i) $P(X \geq 1)$ (ii) $P(X \leq 3)$ (iii) $P(2 \leq x \leq 5)$

Solution: Given $P(X=1)\cdot\frac{3}{2} = P(X=3)$

$\frac{e^{-\lambda}\cdot\lambda^1}{1!}\cdot\frac{3}{2} = \frac{e^{-\lambda}\cdot\lambda^3}{3!}$

$\frac{3\lambda}{\cancel{2}} = \frac{\lambda^3}{\cancel{6}_{3}} \Rightarrow 9\lambda = \lambda^3$

$\lambda^3 - 9\lambda = 0 \Rightarrow \lambda(\lambda^2 - 9) = 0 \Rightarrow$

$\lambda = 0, \lambda = 3, \lambda = -3$

In a Poisson Distribution λ is always +ve

$\therefore \boxed{\lambda = 3}$

(i) $P(X \geq 1) = 1 - P(X < 1)$

$= 1 - P(X=0)$

$= 1 - \frac{e^{-\lambda} \cdot \lambda^0}{0!} = 1 - \frac{e^{-3} \cdot (3)^0}{0!}$

$= 1 - e^{-3} = 0.950$

(ii) $P(X \leq 3) = P(0) + P(1) + P(2) + P(3)$

$= \frac{e^{-3} \cdot (3)^0}{0!} + \frac{e^{-3} (3)^1}{1!} + \frac{e^{-3} \cdot (3)^2}{2!} + \frac{e^{-3} (3)^3}{3!}$

$= e^{-3} \left[1 + 3 + \frac{9}{2} + \frac{27}{6} \right]$

$= 0.62$

iii) $P(2 \leq x \leq 5) = P(2) + P(3) + P(4) + P(5)$

$= \frac{e^{-3} \cdot 3^2}{2!} + \frac{e^{-3} \cdot 3^3}{3!} + \frac{e^{-3} \cdot 3^4}{4!} + \frac{e^{-3} \cdot 3^5}{5!}$

$= e^{-3} \left[\frac{9}{2} + \frac{27}{6} + \frac{81}{24} + \frac{243}{120} \right]$

$= 0.7$

Mode of the P.D:

Mode of the P.D lies b/w $\lambda - 1$ & λ

$\Rightarrow$ λ is not an integer, the mode of P.D is integral part of λ

Recurrence relation:

$$\boxed{\frac{P(x+1)}{P(x)} = \frac{\lambda}{x+1}} \quad \text{or} \quad \boxed{P(x+1) = \frac{\lambda}{x+1} P(x)}$$

Where $p(x+1) = \dfrac{e^{-\lambda} \cdot \lambda^{x+1}}{(x+1)!}$; $P(x) = \dfrac{e^{-\lambda} \cdot \lambda^{x}}{x!}$

Proof: $\dfrac{P(x+1)}{P(x)} = \dfrac{e^{-\lambda} \cdot \lambda^{x+1}}{(x+1)!} \cdot \dfrac{x!}{e^{-\lambda} \cdot \lambda^{x}} = \dfrac{\lambda}{(x+1)}$

3) If x is a P.D such that $P(X=0) = P(X=1)$ find (i) $P(X=0)$ (ii) Using recurrence formula (iii) find the probability at $x = 1, 2, 3, 4, 5, 6$

soln: Given $P(x=0) = P(x=1)$

$$\frac{e^{-\lambda} \cdot \lambda^0}{0!} = \frac{e^{-\lambda} \cdot \lambda^1}{1!}$$

$$\Rightarrow \boxed{\lambda = 1}$$

(i) $P(X=0) = \dfrac{e^{-1}(1)^0}{0!} = e^{-1} = 0.36$

(ii) W.K.T Recurrence formula is

$$P(x+1) = \frac{\lambda}{x+1} P(x)$$

let $x=0$: $P(0+1) = \dfrac{1}{0+1} P(0)$

$$P(1) = P(0)$$

$$= \frac{e^{-1}(1)^0}{0!} = 0.36$$

$$\boxed{P(1) = 0.36}$$

let $x=1$, $P(1+1) = \dfrac{1}{1+1} P(1)$

$$P(2) = \frac{1}{2} \cdot 0.36 = 0.18$$

$$\boxed{P(2) = 0.18}$$

at $x=2$, $P(2+1) = \dfrac{1}{2+1} P(2)$

$$= \frac{1}{3}(0.18)$$

$$\boxed{P(3) = 0.06}$$

at $x=3$, $P(3+1) = \dfrac{1}{3+1} P(3)$

$$= \frac{1}{4}(0.6)$$

$$\boxed{P(4) = 0.015}$$

at $x=4$, $P(4+1) = \frac{1}{4+1} P(4)$

$= \frac{1}{5}(0.015)$

$$\boxed{P(5) = 0.003}$$

at $x=5$, $P(5+1) = \frac{1}{5+1} P(5)$

$= \frac{1}{6}(0.003)$

$$\boxed{P(6) = 0.0005}$$

(4) The number of breakdowns of a computer is a random variable having poisson distribution with mean of 1.8 per month. Find the probability that the computer will function for a month

(a) without any breakdowns

(b) with only one breakdown

(c) with atleast 2 breakdown

Soln: Given $\lambda = 1.8$

Let take X is no. of breakdowns

(a) $P(X=0) = \frac{e^{-\lambda} \cdot \lambda^x}{x!} = \frac{e^{-1.8}(1.8)^0}{0!} = e^{-1.8} = 0.165$

(b) $P(X=1) = \frac{e^{-\lambda} \cdot \lambda^x}{x!} = \frac{e^{-1.8}(1.8)^1}{1!} = e^{-1.8}(1.8)$

$= 0.29$

(c) $P(X = \text{atleast 2 breakdowns})$

$P(X \geq 2) = 1 - P(X < 2)$

$= 1 - [P(X=0) + P(X=1)]$

$= 1 - \left[\frac{e^{-1.8}(1.8)^0}{0!} + \frac{e^{-1.8}(1.8)^1}{1!}\right]$

$= 0.54$

5) Fit a P.D for following data and calculate the expected frequencies

x	0	1	2	3	4
$f(x)$	109	65	22	3	1

soln: W.K.T mean $= \lambda = \frac{\sum f_i x_i}{N}$ (where $N = \sum f_i$

$\sum f_i x_i = 0(109) + 1(65) + 2(22) + 3(3) + 4(1)$

$= 122$

$N = \sum f_i = 109 + 65 + 22 + 3 + 1 = 200$

Mean $(\lambda) = \frac{122}{200} = 0.61$

X	Given frequencies $f(x)$	Probability Distribution by P.D $P(X=x)=\frac{e^{-\lambda}\lambda^x}{x!}$	Theoritical expectation of frequencies $N \cdot P(X=x)$
0	109	$P(X=0)=\frac{e^{-0.61}(0.61)^0}{0!} = 0.543$	$200 \times 0.543 = 108.6$
1	65	$P(X=1)=\frac{e^{-0.61}(0.61)^1}{1!} = 0.331$	$200 \times 0.331 = 66.2$
2	22	$P(X=2)=\frac{e^{-0.61}(0.61)^2}{2!} = 0.1010$	$200 \times 0.1010 = 20.2$
3	3	$P(X=3)=\frac{e^{-0.61}(0.61)^3}{3!} = 0.0205$	$200 \times 0.0205 = 4.1$
4	1	$P(X=4)=\frac{e^{-0.61}(0.61)^4}{4!} = 0.00313$	$200 \times 0.00313 = 0.62$

Expected frequency is $108.6 + 66.2 + 20.2 + 4.1 + 0.62 = 199.72$

$\simeq 200$

(6) Fit a P.D for following data and calculate the expected frequencies

$X = x$	0	1	2	3	4	5
$f(X=x)$	142	156	69	27	5	1

soln: We know that Mean $\lambda = \frac{\Sigma f_i x_i}{N}$

$$\Sigma f_i x_i = 0(142) + 1(156) + 2(69) + 3(27) + 4(5) + 5(1)$$

$$= 400$$

$$N = \Sigma f_i = 142 + 156 + 69 + 27 + 5 + 1 = 400$$

$$\therefore \lambda = \frac{\Sigma f_i x_i}{N} = \frac{400}{400} = 1$$

$X = x$	Given frequencies	$P(X=x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}$	$N \cdot P(X=x)$ Theoritical
0	142	$P(x=0) = \frac{e^{-1}(1)^0}{0!} = 0.3678$	$400 \times 0.3678 = 147$
1	156	$P(x=1) = \frac{e^{-1}(1)^1}{1!} = 0.3678$	$400 \times 0.3678 = 147$
2	69	$P(x=2) = \frac{e^{-1}(1)^2}{3!} = \cancel{0.0613}$ 0.1839	73.5
3	27	$P(x=3) = \frac{e^{-1}(1)^3}{3!} = 0.0613$	24.5
4	5	$P(x=4) = \frac{e^{-1}(1)^4}{4!} = 0.0153$	6
5	1	$P(x=5) = \frac{e^{-1}(1)^5}{5!} = 3.06 \times 10^{-3}$	$400 \times 3.06 \times 10^{-3} = 1$

Expected frequencies $N \cdot P(X=x)$

$$= 147 + 147 + 73.5 + 24.5 + 6 + 1 = 399 \simeq 400$$