Assignment -1

1) a) If $A=[a,b,c,d]$ and $B=[x,y,z]$. let R be the following relation from A to B $R=\{(a,x),(a,z),(d,y),(c,x),(b,z),(d,x)\}$

(i) Determine the matrix of the relation

To determine the matrix of the relation R, we need to create a matrix.

$A=[a,b,c,d]$

$B=[x,y,z]$

$R=\{(a,x),(a,z),(d,y),(c,x),(b,z),(d,x)\}$

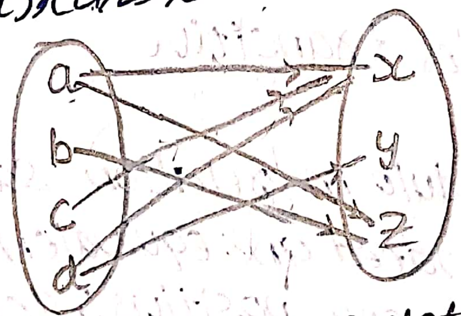
$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \end{array}$$

'1' indicates the corresponding pair is in the relation.

'0' not in relation.

(ii) Draw the arrow diagram of R

$R=\{(a,x),(a,z),(d,y),(c,x),(b,z),(d,x)\}$



iii) Find the Inverse relation R^{-1} of R

$R=\{(a,x),(a,z),(d,y),(c,x),(b,z),(d,x)\}$

To find Inverse we need to Reverse ordered pairs

$R^{-1}=\{(x,a),(z,a),(y,d),(x,c),(z,b),(x,d)\}$

This is Inverse relation

1 b) prove that for any integer m, the relation congruence modulo m is an equivalence relation on the integers

sol) equivalence relations on integers
∴ congruence modulo.

1) Reflexivity:
For any integer a, $a-a=0$ & 0 is divisible by m since m is a positive integer
∴ $a \equiv a \pmod m$, which satisfies reflexive

2) Symmetry:
If $a \equiv b \pmod m$ it means that $a-b$ is divisible by m
since m is a positive integer $b-a$ is also divisible by m
Hence, $b \equiv a \pmod m$, which satisfies symmetry

3) Transitive:
If $a \equiv b \pmod m$ and $b \equiv c \pmod m$ it means that $a-b$ & $b-c$ are both divisible by m.
$b-c$ are both divisible by m.
∴ $(a-b)+(b-c)=a-c$ is also divisible by m. which implies
$a \equiv c \pmod m$ which satisfies transitive

Since, the congruence modulo m relation satisfies all three properties it is indeed an equivalence relation on the integers for any positive integer m

congruence modulo
let $a, b, m \in Z$, $m \geq 2$ A is said to be congruent to B modulo m if and only if $(a-b)$ is divisible by m

2a) let R is a relation on set of real numbers and it is defined as $(a,b) \in R$ iff $x-y$ is an integer. then show that R is an equivalence relation

Sol) Given $R = \{(a,b) : (a-b) \in Z\}$
To prove that a relation R is an equivalence relation you need to show that it satisfies following properties
1. Reflexive 2. symmetric 3. transitive

1) Reflexive

$\forall a \in R$

$(a-b) \in R$

$a-a=0 \in Z$

$(a,a) \in R$

$\therefore$ R is reflexive

2) Symmetric

let $(a,b) \in R$

$(a-b) \in Z$

$(b-a) \in Z$, $\therefore (b,a) \in R$

$\therefore$ R is symmetric

3) Transitive

let $(a,b), (b,c) \in R$

$(a,b) \in Z$, $(b,c) \in Z$

$a-b+b-c \in Z$

$(a-c) \in Z$, $(a,c) \in R$

$\therefore$ R is Transitive

It satisfies all the 3 properties

Hence R is an equivalence relation

b) Suppose $(a,b) \in R$ iff the price of book a is greater than or equal to the price of book b & the no. of pages of book a greater than or equal to no. of pages in b. Show that R is partially ordered Relation

Sol) To show that the relation R is partially ordered relation we need to verify the properties - reflexive, Anti-symmetric transitive

1) Reflexive:
In the case, for any ~~eas~~ book a it is true that the price of book a is $\geq$ the price of it self $(a \geq a)$. The no. of pages of a book a is $\geq$ the no. of pages in it self $(a \geq a)$ $\therefore (a,a) \in R$

$\therefore$ R is reflexive

2) Anti-symmetric:
R is Anti-symmetric for a & b such that $(a,b) \in R$, $(b,a) \in R$ then $a=b$
we have a & b two books such that $(a,b) \in R$ & $(b,a) \in R$.
① The price of book a is $\geq$ to price of book b
② the no. of pages in book a is $\geq$ to no. of pages in book b
$\therefore a \geq b$ & $b \geq a$ we can conclude that $a=b$
$\therefore$ R is anti-symmetric

3) Transitive
Suppose we have 3 books a, b, c $(a,b) \in R$, $(b,c) \in R$
① The price of book a is $\geq$ to book b $(a \geq b)$
② The no. of pages in book a is $\geq$ book b $(a \geq b)$.
similarly.
① The price of book b is $\geq$ to book c $(b \geq c)$
② The no. of pages in book b is $\geq$ to book b $(b \geq c)$
now by property $a \geq c$ $\therefore (a,c) \in R$ $\therefore$ R is transitive
$\therefore$ It obeys 3 properties
$\therefore$ R is partially ordered Relation

3a) prove that $f(x)=x^3$ is a one-one function from $R \to R$ where R is set of real numbers. Also prove that $f^{-1} o\, g^{-1} = (gof)^{-1}$ for $f, g : Q \to Q$ such that $f(x)=4x$, $g(x)=x+5$

Sol) If f is a function. The function is said to be one-one if for all x & y whenever $f(x)=f(y)$ then $x=y$

given $f(x)=x^3$

let us consider x_1 & x_2

$$x_1^3 = x_2^3 \;;\; f(x_1)^3 = f(x_2)^3$$

$$x_1^3 - x_2^3 = 0$$

$$=(x_1-x_2)(x^2+xy+y^2)=0$$

$$x_1-x_2=0 \quad \boxed{x_1=x_2}$$

$\therefore$ $f(x)$ is one-one function

$f, g : Q \to Q$

$f(x)=4x \quad g(x)=x+5 \qquad f(x)=4x,\; g(x)=x+5$

$f(x)=4x = y \quad g(x)=x+5 = y \qquad f^{-1}(y)=y/4$ & $g^{-1}(y)=y-5$

$4x=y \quad x+5=y$

$x=y/4 \quad x=y-5$

$f^{-1}(y)=y/4 \quad g^{-1}(y)=y-5$

$gof(x)=g[f(x)] \qquad gof(x)=y$

$=g(4x) \qquad 4x+5=y$

$gof(x)=4x+5 \qquad 4x=y-5 \quad x=y-5/4$

$(gof)^{-1}(y) = \frac{y-5}{4} \Rightarrow (gof)^{-1}(x) = \frac{x-5}{4}$ — ①

Now, $f^{-1} o\, g^{-1} = f^{-1}[g^{-1}] = f^{-1}[x-5]$,

$f^{-1} o\, g^{-1} = \frac{x-5}{4}$ — ②

eq ① = eq ②

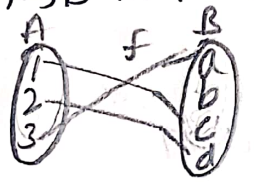
$f^{-1} o\, g^{-1} = (gof)^{-1}(x)$ Hence proved

3b) Define one-one & onto functions and explain the composition of functions with diagram. let f, g are two functions from $R \rightarrow R$ where R is set of real numbers Find $(fof)(x)$ if $f(x)=3x^2$ & $(fog)(x)$, $(gof)(x)$ if $f(x)=x^2-2$ $g(x)=x+4$.

Sol) one-one (Injective) function

A function f from A to B is said to be one-one function if all elements in set A having different images in set B.

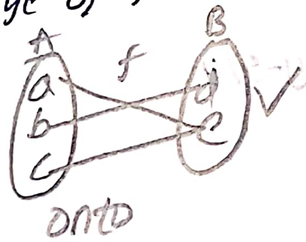
If function $f: A \rightarrow B$ is 1-1 iff $x_1, x_2 \in A$ $f(x_1)=f(x_2)$ $x_1=x_2$

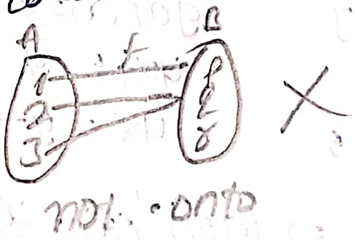


on-to (surjection) function

If function $f: A \rightarrow B$ is said to be onto function if each element of set B is mapping atleast one element of set A

A function $f: A \rightarrow B$ is said to be onto function if range of f is equal to co-domain of f

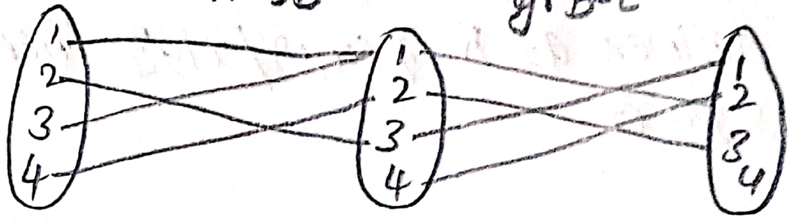


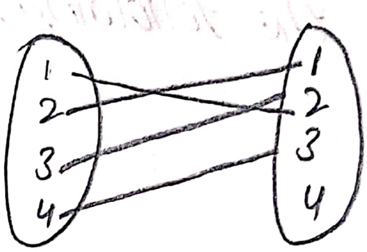


composition of functions:

let f from A to B ; g from B to C are two functions then the composition of f & g is denoted gof & it is defined as $gof: A \rightarrow C$.

$f: A \to B$ $\qquad$ $g: B - C$





$gof: A \to C$

given

$f(x) = 3x^2 - 2$ & $g(x) = x + 4$

1) $fof(x) = f[f(x)]$

$f[f(x)] = 3(3x^2)^2$

$= 3x(9x^4) = 27x^4$

2) $f(x) = x^2 - 2, \; g(x) = x + 4$

$(fog)(x) = f[g(x)]$

$= (x+4)^2 - 2 = x^2 + 16 + 8x - 2$

$= x^2 + 8x + 14$

$(gof)(x) = g[f(x)] = (x^2 - 2) + 4$

$= x^2 + 2$

$(fog)(x) \cdot (gof)(x) = (x^2 + 8x + 14)(x^2 + 2)$

$= x^4 + 2x^2 + 8x^3 + 16x + 14x^2 + 28$

$= x^4 + 16x^2 + 16x + 28$

$\therefore (fog)(x) \cdot (gof)(x) = x^4 + 16x^2 + 16x + 28$

4) Define poset. let R is a relation on set of integers (z) & defined as $R = \{(x,y) / x|y\}$ then prove that z is poset

Sol) POSET

If relation R satisfies the following 3 conditions

1) Reflexive
2) Anti-symmetric
3) Transitive

then R is called poset

Given $R = \{(x,y) / x|y\}$

1) Reflexive

let $\forall x \in A$ & $(x,x) \in R$

$x|x$ which is true

$\therefore$ R is reflexive

2) Anti symmetric

let $\forall x,y \in A$ & $(x,y) \in R$, $(y,x) \in R$

$x|y$ & $y|x$, $x=y$

$\therefore$ R is anti-symmetric

3) Transitive

$\forall x,y,z \in A$ $(x,y) \in R$, $(y,z) \in R$

$x|y$ & $y|z \Rightarrow x|z$

ex: (2,4) (4,8)

$4/2=2$, $8/4=2$ $2/8=2$

$\therefore$ R is transitive

3 conditions satisfie than

$\therefore$ R is POSET

5a) Draw Hasse diagram for POSET $(P(S), \subseteq)$ where $S = \{1,2,3,4\}$

Sol) $P(S) = \{\phi, \{1\}, \{2\}, \{3\}, \{4\}, \{1,2\}, \{1,3\},$
$\{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}, \{1,2,3\},$
$\{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\}$