ENGINEERING DRAWING

[As per Latest Syllabus of JNTU-Hyderabad]

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Both the authors have also written the textbook, *Engineering Graphics*, published by McGraw Hill Education (India).

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PREFACE

Engineering Drawing is a core subject taught at the first-year level in all disciplines of engineering, both at the degree as well as the diploma level. It is also a pre-requisite to all engineering professionals since it acts as an 'international language of engineers'. It is a viable method of communicating technical ideas in a recorded form. When exact visual understanding is necessary, engineering graphics is the accurate technique that can be used. It develops the ability to visualise any object with all the physical and dimensional configurations.

Making of the Book

Engineering Drawing book is mainly intended to meet the requirements of the first year BE/B.Tech. students of all the technical universities and institutes and other basic courses of professional technical bodies. As it is essentially intended to be a classroom textbook, it contains a large number of solved problems covering every phase of the subject in a simple and understandable form. Problems have been classified from simple to typical ones and step-by-step procedures are given for solving them. The presentation of the subject matter and illustrations is simplified so as to enable the readers understand the basic concepts of the subject easily. It can also be used as a reference book for engineers working in the design office as well as on the shop floor.

This book has been written considering the newly revised syllabus of various universities, the new pattern of university examinations, previous exam question papers and will fulfill the requirement of engineering drawing for the future learning of drawing-oriented subjects like machine drawing, building drawing, etc. One of the first things that attracts everyone's attention is the excellent presentation in a clear, logical and concise manner. The work is an extract of the knowledge gained by the experience of successful classroom teaching of this subject with utmost devotion. Anyone who goes through the book cannot miss the enormous work that has gone into preparing the text in the present form.

This book is designed to be a comprehensive guide to cover the basic principles and also includes every significant feature of graphics software to make use of computers as drawing instruments. The miscellaneous problems on almost every topic will develop professional-level drawing skills. Although the basics are fully covered, many advanced features are also included. Therefore, the beginners should not feel concerned if some of the material seems too advanced. It will be useful in their professional career.

Salient Features of the Book

The drawings have been prepared to the scale with the help of advanced software packages, maintaining the required recommendations of ISO and the latest BIS standards. Simple language, systematic introduction of concepts, variety of solved and exercise questions and easy-to-grasp formatting are some of the major features of the text. The salient features include the following:

- Emphasis on basic concepts with simplified presentation of the subject matter and illustrations
- Use of latest BIS and ISO standards
- Classification of problems from simple to typical ones
- Large number of solved problems from university question papers provided with step-by-step procedures
- Questions for viva-voce and Multiple-choice questions on each chapter
- Excellent illustrations (2D and 3D) for effective visualization of the objects
- Enhanced pedagogy includes
  - *600 Solved problems*
  - *800 Practice questions*
  - *200 Questions for viva-voce*
  - *240 Multiple-choice questions*

Chapter Organisation

This book is divided into 18 chapters. The overall organization of the book goes from simple to complex, and each chapter has exhaustive pedagogy to support the text.

Chapter 1 provides the list of essential drawing equipments and instruments required in engineering graphics and their uses. **Chapter 2** highlights the latest recommendations of The Bureau of Indian Standards in its bulletin 'SP 46: 2003 Engineering Drawing Practice for Schools and Colleges'. **Chapter 3** reviews the elementary geometrical constructions. **Chapter 4** describes the different types of engineering scales and their typical applications. **Chapters 5 and 6** deal with the construction of curves used in engineering practice.

Chapter 7 begins with the fundamentals of orthographic projections. **Chapters 8 through 12** present the orthographic projections of points, straight lines, planes, solids and sections of solids. **Chapter 13** describes the development of surfaces as applied to sheet metal work. **Chapter 14** deals with the curves of intersection of interpenetrating solids. It is recommended that beginners read chapters 8 through 14 in the same chronological order as given in this book. **Chapter 15 through Chapter 17** describe the principal methods of construction of pictorial views.

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Any further suggestion or criticism from the readers for improvement of the text will be highly appreciated and shall be incorporated in the next edition. For any assistance or clarification, readers may contact the authors at *bas_agr@yahoo.co.in* and/or *cma2004@rediffmail.com*.

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Publisher's Note

We look forward to receiving valuable views, comments and suggestions for improvements from teachers and students, all of which can be sent to *tmh.corefeedback@gmail.com*, mentioning the title and author's name on the subject line.

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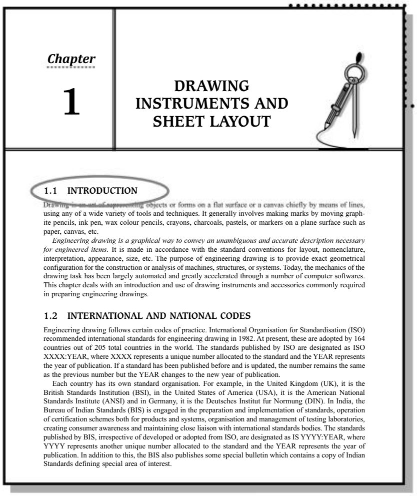
VISUAL WALKTHROUGH



Chapter

1

DRAWING INSTRUMENTS AND SHEET LAYOUT

1.1 INTRODUCTION

Drawing is an art of representing objects or forms on a flat surface or a canvas chiefly by means of lines, using any of a wide variety of tools and techniques. It generally involves making marks by moving graphite pencils, ink pen, wax colour pencils, crayons, charcoals, pastels, or markers on a plane surface such as paper, canvas, etc.

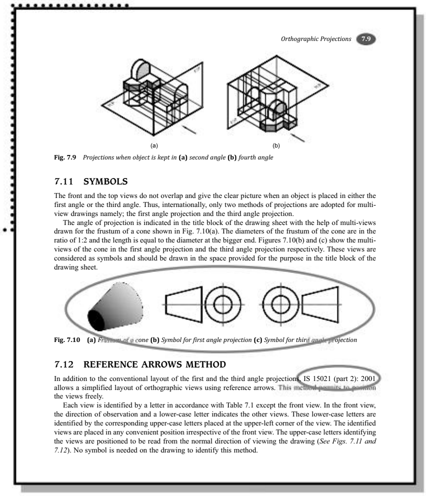
Engineering drawing is a graphical way to convey an unambiguous and accurate description necessary for engineered items. It is made in accordance with the standard conventions for layout, nomenclature, interpretation, appearance, size, etc. The purpose of engineering drawing is to provide exact geometrical configuration for the construction or analysis of machines, structures, or systems. Today, the mechanics of the drawing task has been largely automated and greatly accelerated through a number of computer softwares. This chapter deals with an introduction and use of drawing instruments and accessories commonly required in preparing engineering drawings.

1.2 INTERNATIONAL AND NATIONAL CODES

Engineering drawing follows certain codes of practice. International Organisation for Standardisation (ISO) recommended international standards for engineering drawing in 1982. At present, these are adopted by 164 countries out of 205 total countries in the world. The standards published by ISO are designated as ISO XXXX:YEAR, where XXXX represents a unique number allocated to the standard and the YEAR represents the year of publication. If a standard has been published before and is updated, the number remains the same as the previous number but the YEAR changes to the new year of publication.

Each country has its own standard organisation. For example, in the United Kingdom (UK), it is the British Standards Institution (BSI), in the United States of America (USA), it is the American National Standards Institute (ANSI) and in Germany, it is the Deutsches Institut fur Normung (DIN). In India, the Bureau of Indian Standards (BIS) is engaged in the preparation and implementation of standards, operation of certification schemes both for products and systems, organisation and management of testing laboratories, creating consumer awareness and maintaining close liaison with international standards bodies. The standards published by BIS, irrespective of developed or adopted from ISO, are designated as IS YYYY:YEAR, where YYYY represents another unique number allocated to the standard and the YEAR represents the year of publication. In addition to this, the BIS also publishes some special bulletin which contains a copy of Indian Standards defining special area of interest.

Introduction *provides a quick look into the concepts the reader is going to learn.*



Orthographic Projections 7.9

(a) (b)

Fig. 7.9 *Projections when object is kept in* **(a)** *second angle* **(b)** *fourth angle*

7.11 SYMBOLS

The front and the top views do not overlap and give the clear picture when an object is placed in either the first angle or the third angle. Thus, internationally, only two methods of projections are adopted for multiview drawings namely; the first angle projection and the third angle projection.

The angle of projection is indicated in the title block of the drawing sheet with the help of multi-views drawn for the frustum of a cone shown in Fig. 7.10(a). The diameters of the frustum of the cone are in the ratio of 1:2 and the length is equal to the diameter at the bigger end. Figures 7.10(b) and (c) show the multiviews of the cone in the first angle projection and the third angle projection respectively. These views are considered as symbols and should be drawn in the space provided for the purpose in the title block of the drawing sheet.

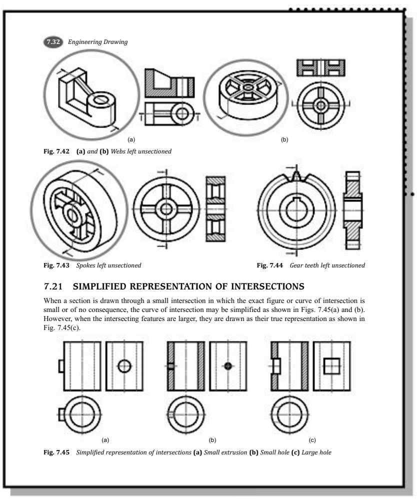
Fig. 7.10 **(a)** *Frustum of a cone* **(b)** *Symbol for first angle projection* **(c)** *Symbol for third angle projection*

7.12 REFERENCE ARROWS METHOD

In addition to the conventional layout of the first and the third angle projections, IS 15021 (part 2): 2001 allows a simplified layout of orthographic views using reference arrows. This method permits to position the views freely.

Each view is identified by a letter in accordance with Table 7.1 except the front view. In the front view, the direction of observation and a lower-case letter indicates the other views. These lower-case letters are identified by the corresponding upper-case letters placed at the upper-left corner of the view. The identified views are placed in any convenient position irrespective of the front view. The upper-case letters identifying the views are positioned to be read from the normal direction of viewing the drawing (*See Figs. 7.11 and 7.12*). No symbol is needed on the drawing to identify this method.

Diagrams *have been prepared with the help of advanced software packages maintaining the recommendations of latest BIS code.*



7.32 Engineering Drawing

(a) (b)

Fig. 7.42 **(a)** *and* **(b)** *Webs left unsectioned*

Fig. 7.43 *Spokes left unsectioned*

Fig. 7.44 *Gear teeth left unsectioned*

7.21 SIMPLIFIED REPRESENTATION OF INTERSECTIONS

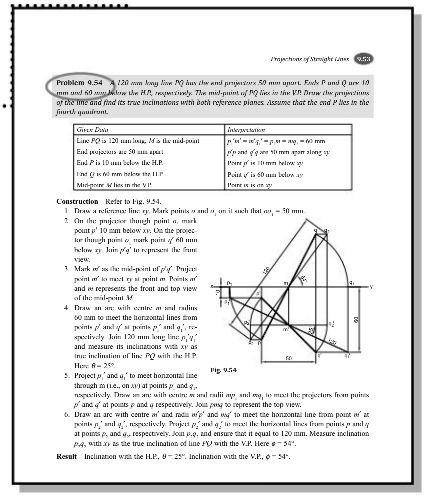
When a section is drawn through a small intersection in which the exact figure or curve of intersection is small or of no consequence, the curve of intersection may be simplified as shown in Figs. 7.45(a) and (b). However, when the intersecting features are larger, they are drawn as their true representation as shown in Fig. 7.45(c).

(a) (b) (c)

Fig. 7.45 *Simplified representation of intersections* **(a)** *Small extrusion* **(b)** *Small hole* **(c)** *Large hole*

3D Illustrations *assist in visualisation of the object in a lucid manner.*

Problems *are simplified to enable the readers understand the basic concepts in a clear, logical and concise manner easily.*



Projections of Straight Lines 9.53

Problem 9.54 *A 120 mm long line PQ has the end projectors 50 mm apart. Ends P and Q are 10 mm and 60 mm below the H.P., respectively. The mid-point of PQ lies in the V.P. Draw the projections of the line and find its true inclinations with both reference planes. Assume that the end P lies in the fourth quadrant.*

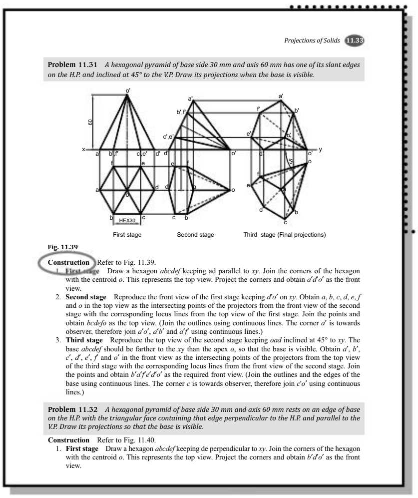
Given Data	*Interpretation*
Line PQ is 120 mm long, M is the mid-point	$p_1'm' = m'q_1' = p_2m = mq_2 = 60$ mm
End projectors are 50 mm apart	$p'p$ and $q'q$ are 50 mm apart along xy
End P is 10 mm below the H.P.	Point p' is 10 mm below xy
End Q is 60 mm below the H.P.	Point q' is 60 mm below xy
Mid-point M lies in the V.P.	Point m is on xy

Construction Refer to Fig. 9.54.

1. Draw a reference line xy. Mark points o and o_1 on it such that $oo_1 = 50$ mm.
2. On the projector though point o, mark point p' 10 mm below xy. On the projector though point o_1 mark point q' 60 mm below xy. Join $p'q'$ to represent the front view.
3. Mark m' as the mid-point of $p'q'$. Project point m' to meet xy at point m. Points m' and m represents the front and top view of the mid-point M.
4. Draw an arc with centre m and radius 60 mm to meet the horizontal lines from points p' and q' at points p_1' and q_1', respectively. Join 120 mm long line $p_1'q_1'$ and measure its inclinations with xy as true inclination of line PQ with the H.P. Here $\theta = 25°$.
5. Project p_1' and q_1' to meet horizontal line through m (i.e., on xy) at points p_1 and q_1, respectively. Draw an arc with centre m and radii mp_1 and mq_1 to meet the projectors from points p' and q' at points p and q respectively. Join pmq to represent the top view.
6. Draw an arc with centre m' and radii $m'p'$ and mq' to meet the horizontal line from point m' at points p_2' and q_2', respectively. Project p_2' and q_2' to meet the horizontal lines from points p and q at points p_2 and q_2, respectively. Join p_2q_2 and ensure that it equal to 120 mm. Measure inclination p_2q_2 with xy as the true inclination of line PQ with the V.P. Here $\phi = 54°$.

Fig. 9.54

Result Inclination with the H.P., $\theta = 25°$. Inclination with the V.P., $\phi = 54°$.



Projections of Solids 11.33

Problem 11.31 *A hexagonal pyramid of base side 30 mm and axis 60 mm has one of its slant edges on the H.P. and inclined at 45° to the V.P. Draw its projections when the base is visible.*

First stage Second stage Third stage (Final projections)

Fig. 11.39

Construction Refer to Fig. 11.39.

1. **First stage** Draw a hexagon $abcdef$ keeping ad parallel to xy. Join the corners of the hexagon with the centroid o. This represents the top view. Project the corners and obtain $a'd'o'$ as the front view.
2. **Second stage** Reproduce the front view of the first stage keeping $d'o'$ on xy. Obtain a, b, c, d, e, f and o in the top view as the intersecting points of the projectors from the front view of the second stage with the corresponding locus lines from the top view of the first stage. Join the points and obtain $bcdefo$ as the top view. (Join the outlines using continuous lines. The corner a' is towards observer, therefore join $a'o'$, $a'b'$ and $a'f'$ using continuous lines.)
3. **Third stage** Reproduce the top view of the second stage keeping oad inclined at 45° to xy. The base $abcdef$ should be farther to the xy than the apex o, so that the base is visible. Obtain a', b', c', d', e', f' and o' in the front view as the intersecting points of the projectors from the top view of the third stage with the corresponding locus lines from the front view of the second stage. Join the points and obtain $b'a'f'e'd'o'$ as the required front view. (Join the outlines and the edges of the base using continuous lines. The corner c is towards observer, therefore join $c'o'$ using continuous lines.)

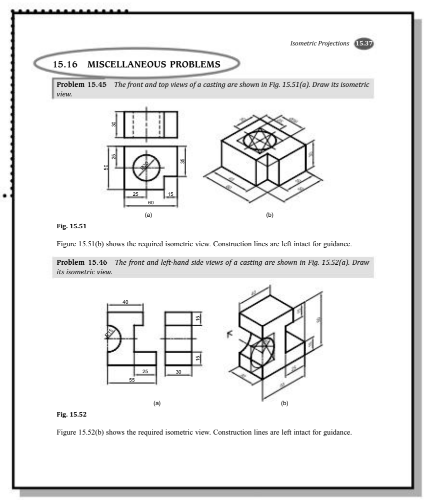
Problem 11.32 *A hexagonal pyramid of base side 30 mm and axis 60 mm rests on an edge of base on the H.P. with the triangular face containing that edge perpendicular to the H.P. and parallel to the V.P. Draw its projections so that the base is visible.*

Construction Refer to Fig. 11.40.

1. **First stage** Draw a hexagon $abcdef$ keeping de perpendicular to xy. Join the corners of the hexagon with the centroid o. This represents the top view. Project the corners and obtain $b'd'o'$ as the front view.

Step-by-step Construction *procedure is given to understand the solved problems.*

Miscellaneous Problems with Solutions *of some typical problems are given to develop professional-level drawing skills.*



Isometric Projections 15.37

15.16 MISCELLANEOUS PROBLEMS

Problem 15.45 *The front and top views of a casting are shown in Fig. 15.51(a). Draw its isometric view.*

(a) (b)

Fig. 15.51

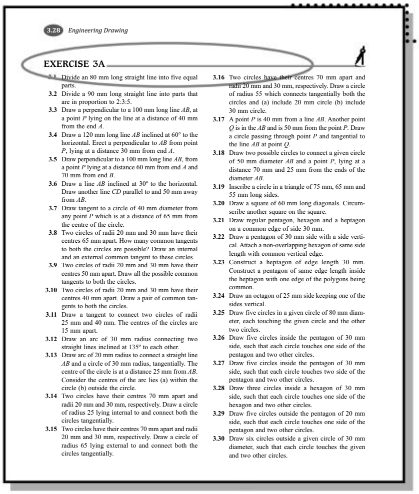
Figure 15.51(b) shows the required isometric view. Construction lines are left intact for guidance.

Problem 15.46 *The front and left-hand side views of a casting are shown in Fig. 15.52(a). Draw its isometric view.*

(a) (b)

Fig. 15.52

Figure 15.52(b) shows the required isometric view. Construction lines are left intact for guidance.

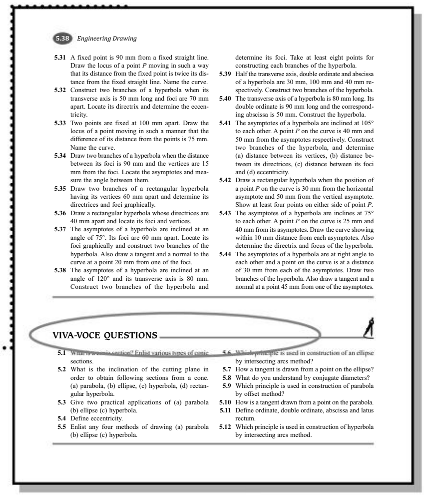


3.28 *Engineering Drawing*

EXERCISE 3A

3.1 Divide an 80 mm long straight line into five equal parts.
3.2 Divide a 90 mm long straight line into parts that are in proportion to 2:3:5.
3.3 Draw a perpendicular to a 100 mm long line *AB*, at a point *P* lying on the line at a distance of 40 mm from the end *A*.
3.4 Draw a 120 mm long line *AB* inclined at 60° to the horizontal. Erect a perpendicular to *AB* from point *P*, lying at a distance 30 mm from end *A*.
3.5 Draw perpendicular to a 100 mm long line *AB*, from a point *P* lying at a distance 60 mm from end *A* and 70 mm from end *B*.
3.6 Draw a line *AB* inclined at 30° to the horizontal. Draw another line *CD* parallel to and 50 mm away from *AB*.
3.7 Draw tangent to a circle of 40 mm diameter from any point *P* which is at a distance of 65 mm from the centre of the circle.
3.8 Two circles of radii 20 mm and 30 mm have their centres 65 mm apart. How many common tangents to both the circles are possible? Draw an internal and an external common tangent to these circles.
3.9 Two circles of radii 20 mm and 30 mm have their centres 50 mm apart. Draw all the possible common tangents to both the circles.
3.10 Two circles of radii 20 mm and 30 mm have their centres 40 mm apart. Draw a pair of common tangents to both the circles.
3.11 Draw a tangent to connect two circles of radii 25 mm and 40 mm. The centres of the circles are 15 mm apart.
3.12 Draw an arc of 30 mm radius connecting two straight lines inclined at 135° to each other.
3.13 Draw arc of 20 mm radius to connect a straight line *AB* and a circle of 30 mm radius, tangentially. The centre of the circle is at a distance 25 mm from *AB*. Consider the centres of the arc lies (a) within the circle (b) outside the circle.
3.14 Two circles have their centres 70 mm apart and radii 20 mm and 30 mm, respectively. Draw a circle of radius 25 lying internal to and connect both the circles tangentially.
3.15 Two circles have their centres 70 mm apart and radii 20 mm and 30 mm, respectively. Draw a circle of radius 65 lying external to and connect both the circles tangentially.
3.16 Two circles have their centres 70 mm apart and radii 20 mm and 30 mm, respectively. Draw a circle of radius 55 which connects tangentially both the circles and (a) include 20 mm circle (b) include 30 mm circle.
3.17 A point *P* is 40 mm from a line *AB*. Another point *Q* is in the *AB* and is 50 mm from the point *P*. Draw a circle passing through point *P* and tangential to the line *AB* at point *Q*.
3.18 Draw two possible circles to connect a given circle of 50 mm diameter *AB* and a point *P*, lying at a distance 70 mm and 25 mm from the ends of the diameter *AB*.
3.19 Inscribe a circle in a triangle of 75 mm, 65 mm and 55 mm long sides.
3.20 Draw a square of 60 mm long diagonals. Circumscribe another square on the square.
3.21 Draw regular pentagon, hexagon and a heptagon on a common edge of side 30 mm.
3.22 Draw a pentagon of 30 mm side with a side vertical. Attach a non-overlapping hexagon of same side length with common vertical edge.
3.23 Construct a heptagon of edge length 30 mm. Construct a pentagon of same edge length inside the heptagon with one edge of the polygons being common.
3.24 Draw an octagon of 25 mm side keeping one of the sides vertical.
3.25 Draw five circles in a given circle of 80 mm diameter, each touching the given circle and the other two circles.
3.26 Draw five circles inside the pentagon of 30 mm side, such that each circle touches one side of the pentagon and two other circles.
3.27 Draw five circles inside the pentagon of 30 mm side, such that each circle touches two side of the pentagon and two other circles.
3.28 Draw three circles inside a hexagon of 30 mm side, such that each circle touches one side of the hexagon and two other circles.
3.29 Draw five circles outside the pentagon of 20 mm side, such that each circle touches one side of the pentagon and two other circles.
3.30 Draw six circles outside a given circle of 30 mm diameter, such that each circle touches the given and two other circles.

Exercise *covers a large number of unsolved problems for practice.*



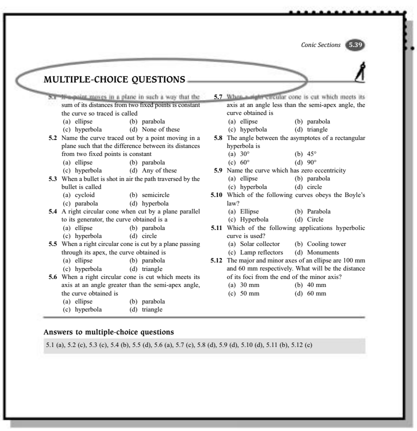
5.38 *Engineering Drawing*

5.31 A fixed point is 90 mm from a fixed straight line. Draw the locus of a point *P* moving in such a way that its distance from the fixed point is twice its distance from the fixed straight line. Name the curve.
5.32 Construct two branches of a hyperbola when its transverse axis is 50 mm long and foci are 70 mm apart. Locate its directrix and determine the eccentricity.
5.33 Two points are fixed at 100 mm apart. Draw the locus of a point moving in such a manner that the difference of its distance from the points is 75 mm. Name the curve.
5.34 Draw two branches of a hyperbola when the distance between its foci is 90 mm and the vertices are 15 mm from the foci. Locate the asymptotes and measure the angle between them.
5.35 Draw two branches of a rectangular hyperbola having its vertices 60 mm apart and determine its directrices and foci graphically.
5.36 Draw a rectangular hyperbola whose directrices are 40 mm apart and locate its foci and vertices.
5.37 The asymptotes of a hyperbola are inclined at an angle of 75°. Its foci are 60 mm apart. Locate its foci graphically and construct two branches of the hyperbola. Also draw a tangent and a normal to the curve at a point 20 mm from one of the foci.
5.38 The asymptotes of a hyperbola are inclined at an angle of 120° and its transverse axis is 80 mm. Construct two branches of the hyperbola and determine its foci. Take at least eight points for constructing each branches of the hyperbola.
5.39 Half the transverse axis, double ordinate and abscissa of a hyperbola are 30 mm, 100 mm and 40 mm respectively. Construct two branches of the hyperbola.
5.40 The transverse axis of a hyperbola is 80 mm long. Its double ordinate is 90 mm long and the corresponding abscissa is 50 mm. Construct the hyperbola.
5.41 The asymptotes of a hyperbola are inclined at 105° to each other. A point *P* on the curve is 40 mm and 50 mm from the asymptotes respectively. Construct two branches of the hyperbola, and determine (a) distance between its vertices, (b) distance between its directrices, (c) distance between its foci and (d) eccentricity.
5.42 Draw a rectangular hyperbola when the position of a point *P* on the curve is 30 mm from the horizontal asymptote and 50 mm from the vertical asymptote. Show at least four points on either side of point *P*.
5.43 The asymptotes of a hyperbola are inclines at 75° to each other. A point *P* on the curve is 25 mm and 40 mm from its asymptotes. Draw the curve showing within 10 mm distance from each asymptotes. Also determine the directrix and focus of the hyperbola.
5.44 The asymptotes of a hyperbola are at right angle to each other and a point on the curve is at a distance of 30 mm from each of the asymptotes. Draw two branches of the hyperbola. Also draw a tangent and a normal at a point 45 mm from one of the asymptotes.

VIVA-VOCE QUESTIONS

5.1 What is a conic section? Enlist various types of conic sections.
5.2 What is the inclination of the cutting plane in order to obtain following sections from a cone. (a) parabola, (b) ellipse, (c) hyperbola, (d) rectangular hyperbola.
5.3 Give two practical applications of (a) parabola (b) ellipse (c) hyperbola.
5.4 Define eccentricity.
5.5 Enlist any four methods of drawing (a) parabola (b) ellipse (c) hyperbola.
5.6 Which principle is used in construction of an ellipse by intersecting arcs method?
5.7 How a tangent is drawn from a point on the ellipse?
5.8 What do you understand by conjugate diameters?
5.9 Which principle is used in construction of parabola by offset method?
5.10 How is a tangent drawn from a point on the parabola.
5.11 Define ordinate, double ordinate, abscissa and latus rectum.
5.12 Which principle is used in construction of hyperbola by intersecting arcs method.

Viva-voce Questions *are added at the end of each chapter to prepare students for viva-voce held during practical examinations.*



Conic Sections 5.39

MULTIPLE-CHOICE QUESTIONS

5.1 If a point moves in a plane in such a way that the sum of its distances from two fixed points is constant the curve so traced is called
(a) ellipse (b) parabola
(c) hyperbola (d) None of these
5.2 Name the curve traced out by a point moving in a plane such that the difference between its distances from two fixed points is constant
(a) ellipse (b) parabola
(c) hyperbola (d) Any of these
5.3 When a bullet is shot in air the path traversed by the bullet is called
(a) cycloid (b) semicircle
(c) parabola (d) hyperbola
5.4 A right circular cone when cut by a plane parallel to its generator, the curve obtained is a
(a) ellipse (b) parabola
(c) hyperbola (d) circle
5.5 When a right circular cone is cut by a plane passing through its apex, the curve obtained is
(a) ellipse (b) parabola
(c) hyperbola (d) triangle
5.6 When a right circular cone is cut which meets its axis at an angle greater than the semi-apex angle, the curve obtained is
(a) ellipse (b) parabola
(c) hyperbola (d) triangle
5.7 When a right circular cone is cut which meets its axis at an angle less than the semi-apex angle, the curve obtained is
(a) ellipse (b) parabola
(c) hyperbola (d) triangle
5.8 The angle between the asymptotes of a rectangular hyperbola is
(a) 30° (b) 45°
(c) 60° (d) 90°
5.9 Name the curve which has zero eccentricity
(a) ellipse (b) parabola
(c) hyperbola (d) circle
5.10 Which of the following curves obeys the Boyle's law?
(a) Ellipse (b) Parabola
(c) Hyperbola (d) Circle
5.11 Which of the following applications hyperbolic curve is used?
(a) Solar collector (b) Cooling tower
(c) Lamp reflectors (d) Monuments
5.12 The major and minor axes of an ellipse are 100 mm and 60 mm respectively. What will be the distance of its foci from the end of the minor axis?
(a) 30 mm (b) 40 mm
(c) 50 mm (d) 60 mm

Answers to multiple-choice questions

5.1 (a), 5.2 (c), 5.3 (c), 5.4 (b), 5.5 (d), 5.6 (a), 5.7 (c), 5.8 (d), 5.9 (d), 5.10 (d), 5.11 (b), 5.12 (c)

Multiple-Choice Questions *are added at the end of each chapter for the purpose of competitive examinations.*

ABBREVIATIONS, NOTATIONS AND SYMBOLS

2D Two-dimensional
3D Three-dimensional
A.G.P. Auxiliary Ground Plane
A.I.P. Auxiliary Inclined Plane
AV Axis of Vision
A.V.P. Auxiliary Vertical Plane
B.I.S. Bureau of Indian Standards
CL Centre Line
CP Central Plane
F.V. Front View
GL Ground Line
GP Ground Plane
HL Horizon Line
H.P. Horizontal Plane
HP Horizon Plane
H.T. Horizontal Trace
IS Indian Standards
ISO International Standards Organization
LC Least Count
L_S Length of Scale
M.S.D. Main Scale Division
P.P. Profile Plane
PP Picture Plane
R.F. Representative Fraction
SP Station point
S.V. Side View
T.S. True Shape
TL True Length
T.V. Top View
V.P. Vertical Plane
V.S.D. Vernier Scale Division
V.T. Vertical Trace

Abbreviations and Symbols used in dimensioning
ϕ Diameter of circle
R Radius of circle
$S\phi$ Diameter of sphere
SR Radius of sphere
□ Side of square
HEX Side of regular hexagon

Abbreviations for units of length
km kilometre
Hm hectometre
Dm or dam decametre
m metre
dm decimetre
cm centimetre
mm millimetre
mi mile
fur furlong
ch chain
yd yard
ft foot
in inch

Symbols
α Apparent inclination of line or element with the H.P.
β Apparent inclination of line or element with the V.P.
θ True inclination of line or element of plane/solid with the H.P.
ϕ True inclination of line or element of plane/solid with the V.P.
e Eccentricity of conic sections

Notations

a, b, c Top views of points A, B, C

a', b', c' Front views of points A, B, C

a'', b'', c'' Side views of points A, B, C

h Horizontal trace

h' Front view of horizontal trace

o Origin or centre point

v Top view of vertical trace

v' Vertical trace

xy, x_1y_1, x_2y_2 Reference lines

Roadmap to the Syllabus

R13 B.Tech. First Year
Jawaharlal Nehru Technological University Hyderabad
Engineering Drawing

Unit – I

Introduction to Engineering Drawing: Principles of engineering drawing/graphics, Various drawing instruments, Conventions in drawing, Lettering practice, BlS Conventions.

GO TO
Chapter 1 – Drawing Instruments and Sheet Layout
Chapter 2 – Lines, Lettering and Dimensioning

Curves – Construction of Curves used in Engineering Practice: (a) Conic Sections including the rectangular hyperbola- general method only. (b) Cycloid, epicycloid and hypocycloid (c) involute.

GO TO
Chapter 5 – Conic Sections
Chapter 6 – Engineering Curves

Scales: Construction of different types of scales. Plain, diagonal and vernier scales.

GO TO
Chapter 4 – Scales

Unit – II

Orthographic Projections in First Angle Projection: Principles of orthographic projections, conventions, first and third Angle Projections.

GO TO
Chapter 7 – Orthographic Projections

Projections of Points: Including points in all four quadrants.

GO TO
Chapter 8 – Projections of Points

Projections of Lines: Parallel, perpendicular, inclined to one plane and inclined to both planes. True length and true angle of a line. Traces of a line.

GO TO
Chapter 9 – Projections of Straight Lines

Projections of Planes: Plane parallel, perpendicular and inclined to one reference plane. Plane inclined to both the reference planes.

GO TO
Chapter 10 – Projections of Planes

Unit – III

Projections of Solids: Projections of regular solids, cube, prisms, pyramids, tetrahedran, cylinder and cone, axis inclined to both planes.

GO TO
Chapter 11 – Projections of Solids

Sections and Sectional Views: Right regular solids- prism, cylinder, pyramid, cone. Use of auxiliary views.

GO TO
Chapter 12 – Sections of Solids

Unit – IV

Development of Surfaces: Development of surfaces of right regular solids, prisms, cylinder, pyramid, cone and their parts, frustum of solids.

GO TO
Chapter 13 – Development of Surfaces

Intersection of Solids: Intersection of cylinder vs cylinder, cylinder vs prism. cylinder vs cone.

GO TO
Chapter 14 – Intersection of Surfaces

Unit – V

Isometric Projections: Principles of isometric projection, isometric scale, isometric views, conventions. lsometric views of plane figures, simple and compound solids. lsometric projection of objects having non-isometric lines. lsometric projection of parts with spherical parts.

GO TO
Chapter 15 – Isometric Projections

Transformation of Projections: Conversion of isometric views to orthographic views- simple objects.

GO TO
Chapter 7.15 – Conversion of Pictorial View into Orthographic View

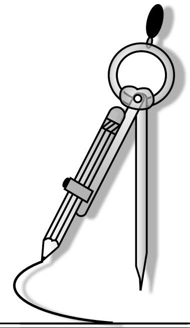
Perspective Projections: Perspective view of points, lines, plane figures. Vanishing point methods (general method only).

GO TO
Chapter 17 – Perspective Projections

Chapter

1

DRAWING INSTRUMENTS AND SHEET LAYOUT



1.1 INTRODUCTION

Drawing is an art of representing objects or forms on a flat surface or a canvas chiefly by means of lines, using any of a wide variety of tools and techniques. It generally involves making marks by moving graphite pencils, ink pen, wax colour pencils, crayons, charcoals, pastels, or markers on a plane surface such as paper, canvas, etc.

Engineering drawing is a graphical way to convey an unambiguous and accurate description necessary for engineered items. It is made in accordance with the standard conventions for layout, nomenclature, interpretation, appearance, size, etc. The purpose of engineering drawing is to provide exact geometrical configuration for the construction or analysis of machines, structures, or systems. Today, the mechanics of the drawing task has been largely automated and greatly accelerated through a number of computer softwares. This chapter deals with an introduction and use of drawing instruments and accessories commonly required in preparing engineering drawings.

1.2 INTERNATIONAL AND NATIONAL CODES

Engineering drawing follows certain codes of practice. International Organisation for Standardisation (ISO) recommended international standards for engineering drawing in 1982. At present, these are adopted by 164 countries out of 205 total countries in the world. The standards published by ISO are designated as ISO XXXX:YEAR, where XXXX represents a unique number allocated to the standard and the YEAR represents the year of publication. If a standard has been published before and is updated, the number remains the same as the previous number but the YEAR changes to the new year of publication.

Each country has its own standard organisation. For example, in the United Kingdom (UK), it is the British Standards Institution (BSI), in the United States of America (USA), it is the American National Standards Institute (ANSI) and in Germany, it is the Deutsches Institut fur Normung (DIN). In India, the Bureau of Indian Standards (BIS) is engaged in the preparation and implementation of standards, operation of certification schemes both for products and systems, organisation and management of testing laboratories, creating consumer awareness and maintaining close liaison with international standards bodies. The standards published by BIS, irrespective of developed or adopted from ISO, are designated as IS YYYY:YEAR, where YYYY represents another unique number allocated to the standard and the YEAR represents the year of publication. In addition to this, the BIS also publishes some special bulletin which contains a copy of Indian Standards defining special area of interest.

SP 46:2003 Engineering Drawing Practices for School and Colleges is such a special bulletin of Bureau of Indian Standards, which provides standard codes to be used for engineering drawing practice by all the students and practicing engineers.

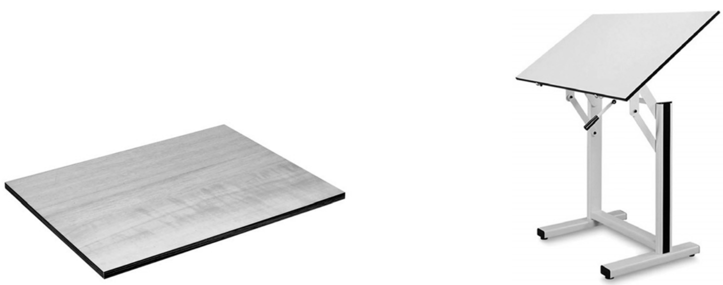
1.3 DRAWING INSTRUMENTS

To be proficient in engineering drawing, it is essential to be familiar with the drawing instruments and the techniques of using them. Great care must be taken for the proper choice of drawing instruments to get the desired accuracy with ease. Following is a list of common drawing instruments and accessories:

1. Drawing board
2. Mini drafter
3. Drawing sheet
4. Drawing Pencil
5. Compass (pivot joint type and spring bow type)
6. Divider (pivot joint type and spring bow type)
7. Protractor
8. Ruler (scale)
9. French curves
10. Set squares
11. Eraser or rubber
12. Sheet fasteners
13. Template
14. Pencil cutter
15. Sand paper pad
16. Brush or towel cloth

1.4 DRAWING BOARD

Figure 1.1(a) shows a conventional drawing board. *It is made of softwood which provides a flat surface.* Engineers and draftsmen use the drawing board for making and modifying drawings on paper with pencil or ink. The working surface must be smooth and free from cracks, bumps and holes so that pencils can easily draw lines. Usually, the surface is covered with a thin vinyl sheet to prevent damage while using the compasses and the dividers. The standard sizes of drawing boards prepared according to recommendations of Bureau of Indian Standards are given in Table 1.1. Drawing boards designated by D00 and D0 are used for drawing offices whereas D1 and D2 are used by engineering students.



(a) (b)

Fig. 1.1 *Drawing board* **(a)** *Conventional* **(b)** *Modern*

Table 1.1 *Designation and size of Drawing Board (All dimensions in are millimetres)*

Designation	*Length × Width*	*Tolerance on Length/Width*	*Thickness*	*Tolerance on Thickness*	*Recommended for use with sheet sizes*
D00	1525 × 1220	±5	22	±1	A0
D0	1270 × 920	±5	22	±1	A0, A1
D1	920 × 650	±5	22	±1	A1, A2
D2	650 × 470	±5	22	±1	A2, A3
D3	500 × 350	±5	22	±1	A3, A4

Ancient drawing boards were provided with ebony strips on their left edge to guide the T-squares. As the T-squares are outdated, the modern boards do not require such strips. Presently drawing boards are supported on steel frame as shown in Fig. 1.1(b). The steel frame provides mechanical linkages which help in controlling the height and the inclination of the drawing board to suit comfortable working in standing position. A tall drawing stool prepared according to IS 4209:1989 is generally used for sitting purpose.

1.5 MINI DRAFTER

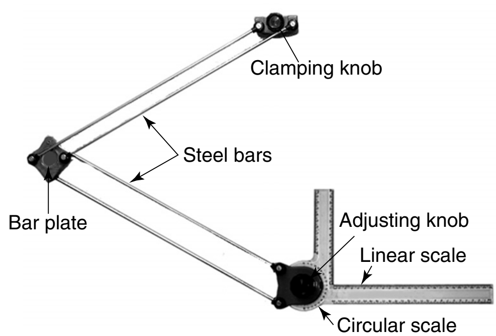
A mini drafter is used to draw horizontal, vertical or inclined parallel lines of desired lengths anywhere on the drawing sheet with considerable ease. It is basically an arm-type drafting machine which combines the functions of a T-square, set-square, protractor and scale. The size of the mini drafter is specified by the length of arms, usually 43 cm long. Figure 1.2(a) shows a mini drafter used by the students of school and colleges. It consists of linear and circular scales, an adjusting knob, steel bars, a bar plate and a clamping knob. The linear scale is in the form of a pair of blades fixed at right angles and graduated in millimetres. In normal position, one of the blades of the scale is horizontal and the other is vertical. This can be set and clamped at any angle with the help of the circular scale and the adjusting knob.

The bigger version of the mini drafter is called a *drafting machine*. It is permanently fixed on a large drawing board and is used in industries.

1.5.1 Clamping the Mini Drafter

The following procedure should be adopted for clamping the mini-drafter on a drawing board:

1. Set the circular scale of the drafter at zero degree and tighten the adjusting knob.
2. Take the clamping knob towards the top edge of the board and align the linear scales with the vertical and the horizontal boundary lines (or edges in case boundary lines are not drawn) of the drawing sheet.
3. Firmly grip the scales and tighten the clamping knob of the mini drafter, as shown in Fig. 1.2(b). The mini-drafter mechanism will keep the scale always parallel to the original position wherever it may slide over the board.



(a)

(b)

Fig. 1.2 **(a)** *Mini-drafter* **(b)** *Clamping mini-drafter*

1.5.2 Rolling Ruler

Figure 1.3 shows a rolling ruler. It consists of a rolling cylinder which enables it to roll over the drawing sheet. This facilitates to draw parallel straight lines. Sometimes a small protractor is also attached in the middle portion of it. The rolling ruler must be handled and rolled carefully otherwise it may lead to error. However, a mini drafter should be preferred as it serves the desired purpose with ease and accuracy.

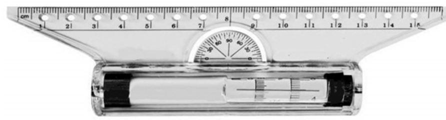


Fig. 1.3 *Rolling ruler*

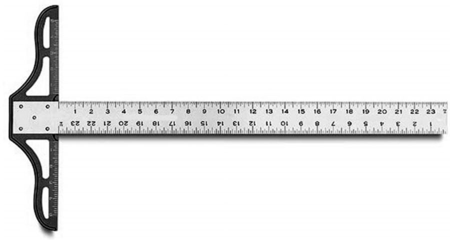


Fig. 1.4 *T-square*

1.5.3 T-Square

Figure 1.4 shows a T-square. Its name comes from the general shape of the instrument where the head is supported on the edge of the drawing board and the long transparent plastic scale slides on the drawing sheet. This scale is used to draw parallel horizontal lines. The T-square serves as a guide for the set-squares for drawing parallel lines at commonly used angles 30º, 45º and 60º. Some T-squares are designed with adjustable heads to allow angular adjustments of the blade. A mini-drafter is used as an alternative of a T-square.

1.6 DRAWING SHEET

A drawing sheet comprises a thick paper onto which the drawing is made. It is available in standard thickness and size. The thickness is specified by weight in grams per square metre (gsm). A sheet with 150 to 250 gsm is suitable for drawing. Size of the sheet depends upon the size of drawing. The Bureau of Indian Standards in its bulletin is 10711:2001 recommends "ISO-A series" of paper size for the drawing sheet as given in Table 1.2. This series of paper always has a length-to-width ratio of $\sqrt{2}:1$ rounded off to the nearest millimetre. Paper size A0 has an area of 1 square metre. Successive sizes in the series are designated as A1, A2, A3, etc., which are obtained by halving the preceding size. Figure 1.5 shows the relationship among various sizes.

Table 1.2 *Paper sizes for ISO-A series*

Series	*Paper size (mm × mm)*
A0	841 × 1189
A1	594 × 841
A2	420 × 594
A3	297 × 420
A4	210 × 297
A5	148 × 210
A6	105 × 148
A7	74 × 105
A8	52 × 74

In addition to the ISO-A series, there is a less common special elongated size and exceptional elongated size of drawing sheets. These sizes are obtained by extending the shorter sides of the ISO-A series to lengths that are multiple of the shorter side of the chosen basic format. Such elongated size sheets are used when greater length is required.

Coloured sheet with thickness equivalent to drawing sheet is commonly called card sheet. It is spread on the drawing board before fixing the drawing sheet. The card sheet helps to prevent the drawing sheet from getting impressions of the flaws, holes or knots that may be present on the surface of the drawing board.

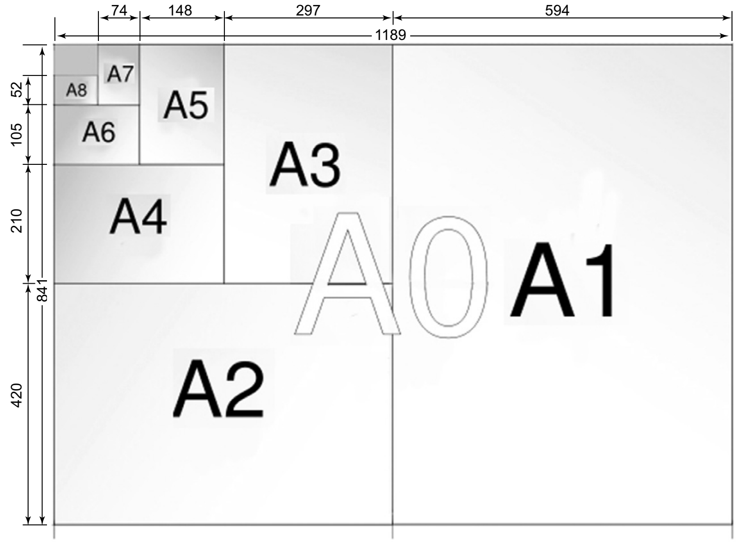


Fig. 1.5 *Drawing sheet sizes of ISO-A series (in mm)*

1.6.1 Selection of Sheets

The drawing sheet should be white enough to produce better impression of the drawing than any dull white or light yellowish type. The card sheet of black colour is preferred as it gives less strain on eyes while working. Students generally use A1 size card sheet and A2 size drawing sheet.

1.6.2 Fixing the Sheets

First, spread the card sheet on the drawing board and fix all its four corners with the help of drawing clips. The edges of the card sheet should be parallel to the edges of the drawing board. Now spread the drawing sheet over the card sheet aligning the edges and fix with adhesive tape. It is suggested to place the drawing sheet slightly towards the lower right corner of the card sheet to enable the drafter easily move over the whole working area of the drawing sheet.

1.6.3 Keeping the Drawing Sheet

Valuable drawings need satisfactory handling and storage facilities in order to preserve them in good condition. Figure 1.6 shows *a sheet holder which can be used to store sheets* (both card and drawing sheets) *and carry during travel*. It is a cylinder shaped box made up of plastic. The inner diameter is 8 cm and the length can be varied from 70 cm to 130 cm.

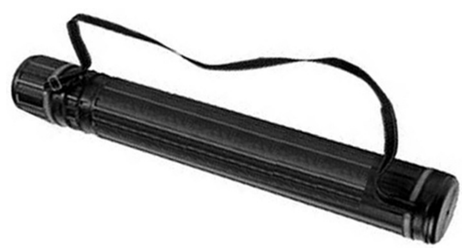


Fig. 1.6 *Sheet holder*

1.7 DRAWING PENCIL

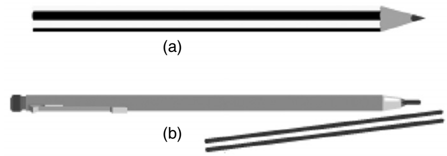


Fig. 1.7 *Pencil* **(a)** *Wooden* **(b)** *Mechanical*

In engineering drawing, *a pencil is used to create marks on the drawing sheet* via physical abrasion. It contains a graphite lead with either wooden or mechanical type protective casing. A good quality pencil draws line of uniform shade and thickness. A wooden pencil with hexagonal cross-sectional shape is shown in Fig. 1.7(a). A mechanical pencil shown in Fig. 1.7(b) is basically a lead holder that requires a piece of lead to be manually inserted. It contains a mechanical system, either propeller or clutch type, to push lead through a hole at the end. Such a pencil is easier to use and always guarantees a sharp point.

1.7.1 Grading of Pencils

Pencils are graded according to the proportion of graphite to clay mixture in the pencil lead. A set of pencils ranges from hardest to softest as follows:

9H	8H	7H	6H	5H	4H	3H	2H	H	F	HB	B	2B	3B	4B	5B	6B	7B

← Hard ← Medium → Soft →

A pencil that is considered the medium grade is designated by the letter HB. The grade becomes harder shown by the value of the figure preceding the letter H, viz. 2H, 3H, 4H, etc. Similarly, the grade becomes softer shown by the figure preceding the letter B, viz. 2B, 3B, 4B, etc. A hard pencil produces thin, grey line while a soft pencil produces thick line.

1.7.2 Selection of Pencil

Engineers prefer harder pencils which allow for a greater control in the shape of the lead and line intensity. Usually, three line qualities are needed in engineering drawing. Thick black lines are used to represent visible and outlines, thin grey lines are used for construction work and medium thick lines are used for dimensioning. Pencil manufacturers have not established uniformity in grades. A pencil with grade H may vary in hardness from one brand to other brand. Moreover, amount of pressure exerted on the pencil also varies with user. Hence, with experience and preference one should select the trade name and grade of pencil that suits the needs.

Humidity affects the graphite core of lead pencils. On dry days, the pencil leaves more dust or residue than on days of high humidity. On damp days, pencil lines appear more black or dense. When continuing the drawing on a day of high humidity, use a pencil with one grade harder to produce drawing quality similar to that on a dry day. Different line qualities may also be obtained by varying the amount of pressure exerted on the pencil, but this should not be attempted without experience.

1.7.3 Working End of Pencil

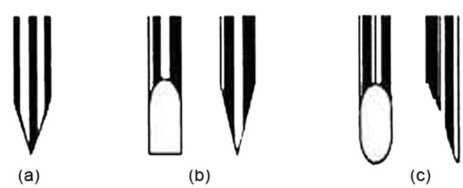


Fig. 1.8 *Working ends of pencil* **(a)** *Conical* **(b)** *Chiseled* **(c)** *Beveled*

The working end of a pencil may have a number of different shapes, namely; conical, chiselled or bevelled as shown in Fig. 1.8. These ends are carefully prepared by blade-type pencil cutters and sand papers. Conical point is used for general purpose including writing, dimensioning and making arrowheads. Chisel edge is suitable for drawing straight lines while bevelled is preferred for drawing circles and arcs.

Always maintain uniform sharpness of the pencil lead to produce a uniform thick line. A pencil with too sharp point breaks easily and a too dull point produces fuzzy lines. Hold the pencil comfortably and naturally. Keep the pencil aligned with the drafting instrument and tilt at an angle of approximately 45° in the direction of pulling. Lines may increase or decrease in thickness when direction of the stroke is changed. Maintain even pressure to produce the line of uniform thickness.

1.8 COMPASS

A compass is used to draw circles, arcs and curves. In engineering drawing, use of a pivot joint compass and a spring bow compass are recommended.

1.8.1 Pivot Joint Compass

A pivot joint compass shown in Fig. 1.9(a) is used to draw circles, arcs and circular curves of diameter greater than 20 mm. It consists of two legs pivoted together at its upper end which provides enough friction to hold the legs of the compass in a set position. One of the legs is equipped with pointed needle at the lower end while the other leg is equipped with a setscrew for mounting either a pen or a pencil attachment on the compass. Both legs of the compass are provided with a knee joint so that when bigger circles are drawn, the needle point and the pencil lead point may be kept perpendicular to the drawing sheet (see Fig. 1.9(b)). An extension bar can be inserted in the leg equipped with marking leg to increase the radius of the circle (see Fig. 1.9(c)).

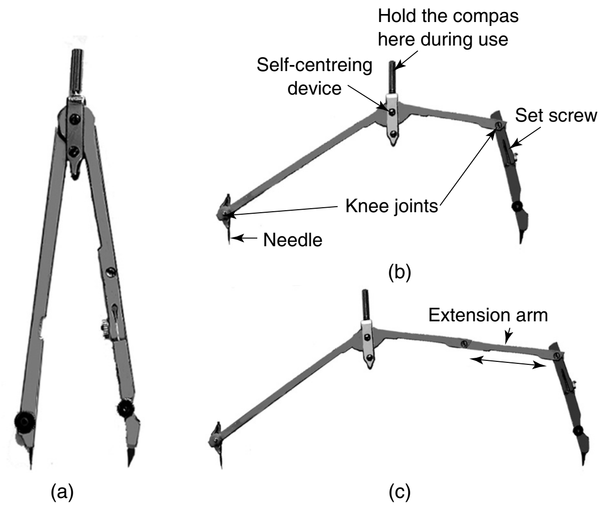


Fig. 1.9 **(a)** *Pivot joint compass* **(b)** *Legs bended at knee joints* **(c)***Extension arm inserted to draw large circles*

1.8.2 Spring Bow Compass

A spring bow compass shown in Fig. 1.10 is used to draw circles, arcs and circular curves of diameter less than 50 mm. They are usually of the centre adjustment type in which a knurled nut is placed at the centre to adjust the distance between the legs. Side adjustment type bow-compasses are also available in which a knurled nut is placed at the side to adjust the distance between the legs. Both legs of the compass are provided with a knee joint to keep the needle point and the pencil lead point perpendicular to the drawing sheet.

Fig. 1.10 *Spring bow compass*

1.8.3 Working with Compass

To draw a circle with a compass, adjust the opening of the legs of the compass to the required radius. Hold the compass and place the needle point lightly on the centre. Slightly press the needle point into the drawing sheet and rotate the

Fig. 1.11 **(a)** *Setting length of needle point* **(b)** *Preparing pencil lead*

marking leg around it. Always rotate the compass clockwise. While drawing circles and arcs with compass, consider the following:

1. As the needle is required to be inserted slightly inside the paper, it is kept slightly longer than the lead point (see Fig. 1.11(a)).
2. The lead of the compass should be sharpened with a single elliptical face (see Fig. 1.11(b)).
3. An even pressure should be applied during rotating the compass to have uniform thickness of the line.
4. It is important that the circles and arcs produced with the compass are of the same quality as corresponding pencil lines. Since much pressure on the compass cannot be exerted as with pencils, a lead of the compass that is one grade softer than the lead of the pencil should be used for corresponding line work.
5. When many circles are drawn using the same centre, the needle of the compass may tend to bore an oversized hole in the drawing sheet. To prevent these holes, a device called a horn centre or centre disk may be is placed over the centre point.

Fig. 1.12 *Pivot joint divider*

1.9 DIVIDER

A divider is used to divide lines or curves into a number of equal parts (using trail method), to transfer measurement from one part of the drawing to another part and to step-off a series of equal distances on the drawing. In engineering drawing, use of a pivot joint divider and a spring bow divider are recommended.

1.9.1 Pivot Joint Divider

A pivot joint divider shown in Fig. 1.12 consists of two legs pivoted together at its upper end which provides enough friction to hold the legs of the divider in a set position. At the lower end both legs are equipped with pointed needle, but it does not have knee joint. In most of the instrument boxes, a needle attachment is provided which has to be mounted on the setscrew of the compass for converting it into the divider.

Fig. 1.13 *Spring bow divider*

1.9.2 Spring Bow Divider

A spring bow divider shown in Fig. 1.13 is used for marking minute divisions and large number of short distances. They are usually of centre adjustment type in which a knurled nut is placed at the centre to adjust the distance between the legs.

1.9.3 Working with Divider

To divide either a line or a curve into a given number of equal parts by trial, open the dividers to a rough approximation of the first division and step-off the distance lightly, holding the dividers by the handle and pivoting the instrument on alternate sides of the line at each step. If the dividers fall short of the end of the line, hold the back leg in place and advance the forward leg, by guess, one division of the remaining distance. Repeat the procedure until the last step falls at the end of the line. During this process, do not punch holes in the paper, but just barely mark the surface for future reference.

To transfer measurements from one part of the drawing to another part, set the dividers to the correct distance then transfer the measurements to the drawing by pricking the drawing surface very lightly with the points of the dividers.

To measure off a series of equal distances on the line, set the dividers to the given distance. Then step-off this distance as many times as desired by swinging the dividers from one leg to the other along the line, first swinging clockwise 180 degrees, then anticlockwise 180 degrees, and so on.

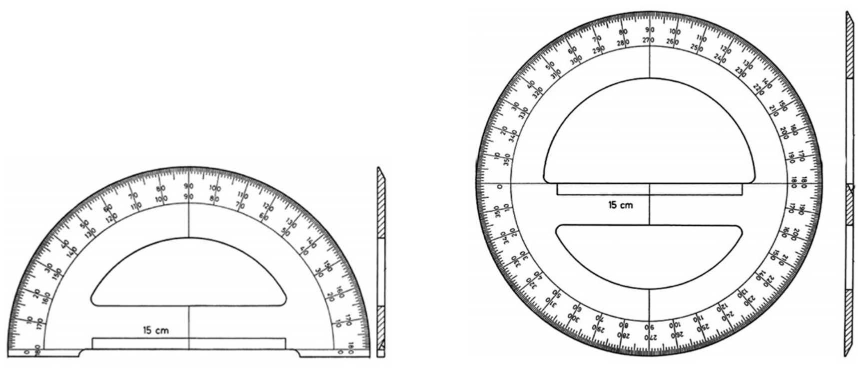
1.9.4 Equal Space Divider

An equal space divider shown in Fig. 1.14 has usually 11 legs. They use multiple metal strips to form ten equal spaces that may be expanded or compressed to set at desired lengths. They can also be used to divide a line into equal number of parts with ease. However, their use in engineering drawing is limited as it may lead to slight error in the division.

Fig. 1.14 *Equal space divider*

1.10 PROTRACTOR

A protractor is used to draw and measure angles, and to divide circles or sectors into desired number of equal parts. They are available in semi-circular and circular shapes. A semi-circular protractor used by engineers, as shown in Fig. 1.15(a), is generally labelled from 0º to 180º in both directions and graduated in increments of 1/2º. The line joining 0º-180º is called the baseline of the protractor and centre of the baseline is called *origin of the protractor*. Circular protractors as shown in Fig. 1.15(b) may be labelled from 0º to 360º (both clockwise and counter clockwise), or they may be labelled from 0º to 90º in four quadrants.



(a) (b)

Fig. 1.15 *Protractor* **(a)** *Semi-circular* **(b)** *Circular*

1.10.1 Use of the Protractor

To measure the given angle, first align the origin of the protractor over the vertex of the angle to be measured. Then align one of the edges of the angle along the baseline of the protractor. The other edge of the angle is read from one of the two scales of the protractor, whichever is more convenient.

Similarly to draw an angle, first align the origin of the protractor over the origin of the given line and align the line along the baseline of the protractor. Mark the point at required angle and join it with the origin of the line.

1.11 RULER (SCALE)

A ruler is used to measure distances and to draw straight lines in centimetres and millimetres. A flat ruler with bevel edges is shown in Fig. 1.16(a). It is available in 15 cm or 30 cm length. One edge is calibrated in millimetres while the other is in half millimetres. Another variant of metric ruler has triangular cross section as shown in Fig. 1.16(b). It has three sides providing six scales, with each side showing two scales. They are easy to pick up. A clip is attached to identify the scale in use.

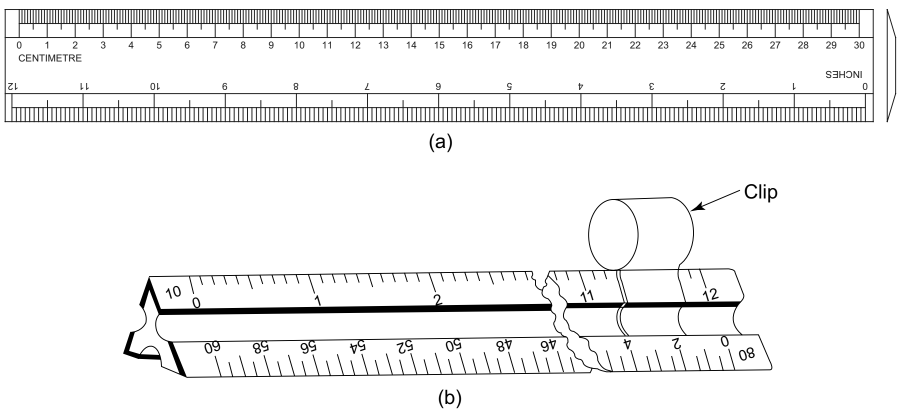


Fig. 1.16 *Ruler* **(a)** *Flat with bevel edge* **(b)** *Triangular*

1.11.1 Working with a Ruler

Keep the edge of the ruler on the line on which the measurement is to be marked, looking from exactly above the required division. Mark the desired dimension with a fine pencil point.

1.11.2 Engineer's Scale

The word 'scale' usually employs for an instrument used for drawing or measuring the length of a straight line. *It is also used to represent the proportion in which the drawing is made with respect to the object.* It is used to make full size, reduced size or enlarged size drawing conveniently depending upon the size of the object and that of the drawing sheet. Usually, the engineers scale is made up of cardboard and as recommended by Bureau of Indian Standards are available in a set of eight scales. These are designated from M1 to M8 as shown in Table 1.3.

Table 1.3 *Designation and description of Engineer's scale*

Designation	*Description*	*Scales*	*Designation*	*Description*	*Scales*
M1	Full size 50 cm to a metre	1:1 1:2	M5	5 mm to a metre 2 mm to a metre	1:200 1:500
M2	40 cm to a metre 20 cm to a metre	1:2.5 1:5	M6	3.3 mm to a metre 1.66 mm to a metre	1:300 1:600
M3	10 cm to a metre 5 cm to a metre	1:10 1:20	M7	2.5 mm to a metre 1.25 mm to a metre	1:400 1:800
M4	2 cm to a metre 1 cm to a metre	1:50 1:100	M8	1 mm to a metre 0.5 mm to a metre	1:1000 1:2000

1.12 FRENCH CURVES

French curves are used to draw smooth curves of almost any desired curvature in mechanical drawings. They are made of transparent plastic having an edge composed of several different curves. They are available in a variety of shapes and sizes. A typical set of three French curves available in Indian market are illustrated in Fig. 1.17.

Fig. 1.17 *French curves*

1.12.1 Use of the French Curves

French curves are used to draw a perfectly smooth curve through predetermined points in short steps. First plot the points by a light pencil to connect the points freehand resulting into a smooth flowing curve. Select a suitable French curve and match a part of its edge with the freehand curve already drawn. Move the dark pencil along the part of the curve matching with the French curve edge. Now move the French curve forward to match the next segment of the freehand curve and darken it in the same manner. This will result into a series of plotted points.

Figure 1.18 shows how a smooth line is drawn through a series of plotted points. The French curve in view A matches points 1, 2, 3, and 4. Draw a line from 1 to 3 only (not to 4). At B, the curve matches points 3 to beyond 6. Draw a line from 3 to 6. At C, it matches a point short of 6 to beyond 7. Draw a line from 6 to 7. At D, it matches a point short of 7 to beyond 9. Draw a line from 7 to 9. At E, it matches a point short of 9 to beyond 11. Draw a line from 9 to 11. While plotting the desired curve, consider the following:

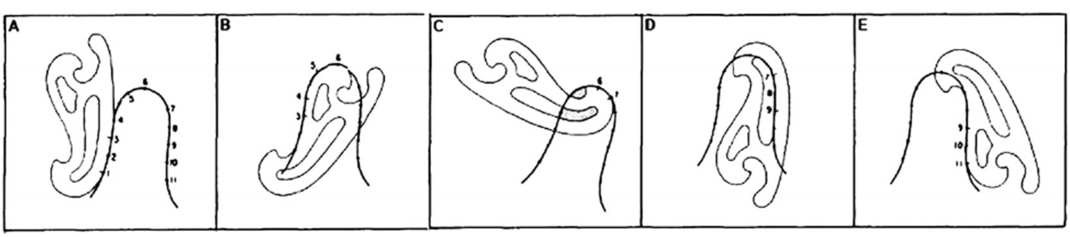


Fig. 1.18 *Use of French curve in drawing predetermined curve*

1. French curve should be so placed that it intersects at least two points of the curve. If the curve is sharp enough, you may consider some more points on the curve.
2. Avoid abrupt changes in curvature by placing the short radius of the French curve toward the short radius portion of the curve.
3. Avoid working on the underside of the French curve. You may need to change your position around the drawing board, when necessary, so that you can work on the side of the French curve that is away from you.

1.12.2 Flexible Cord

Recently a flexible cord shown in Fig. 1.19 is also used in place of French curves for drawing smooth curves with relatively great ease. It consists of a lead bar embedded in rubber covering. The flexibility of the material allows it to bend to any contour.

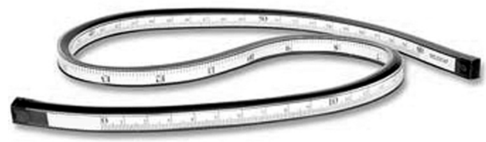


Fig. 1.19 *Flexible cord*

1.13 SET SQUARES

A pair of right angled triangle is called *set squares*. A set square has either 45º-45º angle or 30º–60º angle. The 45º set square shown in Fig. 1.20(a) is a right-angled triangle in which acute angles measure 45º. The 30º–60º set-square shown in Fig. 1.20(b) is a right-angled triangle in which acute angles measure 30º and 60º. Set squares are usually made of transparent plastic to see the work underneath. *They are used to draw lines inclined at 30º, 45º and 60º with the horizontal. By using two set squares, lines inclined at 15º and 75º can also be drawn.*

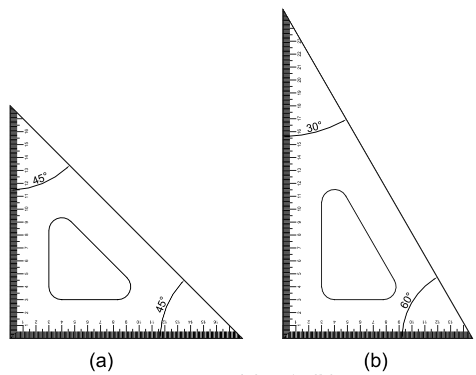


Fig. 1.20 *Seta square* **(a)** *45°* **(b)** *30°–60°*

1.14 ERASER

An eraser, also known as a rubber, is used for removing pencil markings. An eraser that is popular is the *art gum eraser*, made of soft pliable gum. It is especially suited to removing large areas, and does not damage the paper. It leaves much residue which should be whisked away with a draftsman's brush or cloth. A *kneaded eraser* is usually made of a grey or light blue material that resembles putty or gum. It erases by absorbing graphite particles and can be used for precision erasing. Generally, it leaves no residue. If this eraser becomes overly warm, the substance may break down leaving a stain on the drawing surface. A *soft vinyl eraser* has a plastic-like texture and is commonly white in colour. When large areas or dark marks are erased, the eraser causes smearing. Therefore, it is generally used to erase light marks and for precision erasing. Engineers favour this type of eraser for work on technical drawings due to their gentleness on paper. A harder eraser is designed for erasing lines in ink.

Figure 1.21 shows an *electric eraser* with refill box. It has a knob in a short thin rod attached to a motor. The eraser knob turns at a uniform speed achieving a smooth erasure with a minimum of paper trauma. Electric erasers work quickly and completely. Holding the electric eraser steady in one spot may easily wear a hole or damage the surface of the material being erased.

Fig. 1.21 *Electric eraser with refill*

When there are many lines close together and only one of them is to be removed or changed, the desired lines may be protected by an *erasing shield* shown in Fig. 1.22.

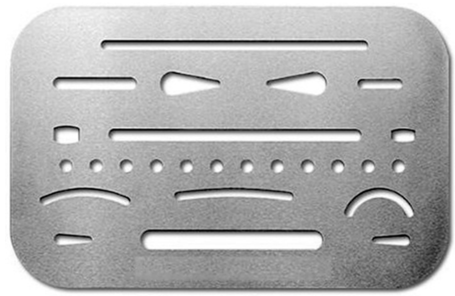


Fig. 1.22 *Eraser shield*

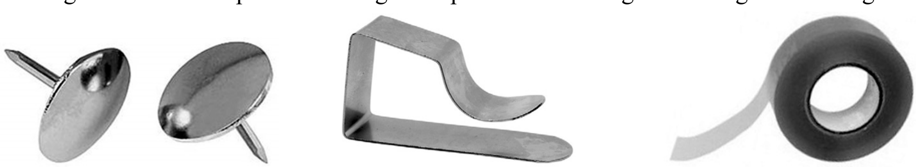
1.15 SHEET FASTENERS

Sheet fasteners are used to fix the card sheet and the drawing sheets on the drawing board. Figures 1.23(a-c) show drawing pins, drawing clips and adhesive tapes that can be used as sheet fasteners.

1. Drawing pins or thumb tacks They are easy to use and remove. They offer a firm grip on the drawing sheet. Their use should be avoided as it damages the drawing board surface. Moreover the heads of the pin may obstruct the free movement of the mini-drafter.

2. Drawing clips They are made of steel or plastics and have a spring action. They can be used for fixing the card sheet and drawing sheets at all the four corners when their sizes are compatible with the drawing board. In case the drawing sheet is much smaller than the drawing board, it is not possible to use such clips.

3. Adhesive tape Generally, adhesive tapes having width of 10 mm to 15 mm are used to fix the drawing sheet on the card sheet. A length of around 40 mm is cut and fixed across the corners of the drawing sheet. Lighter coating of adhesive helps in removing the tape without tearing or marring the drawing sheet.

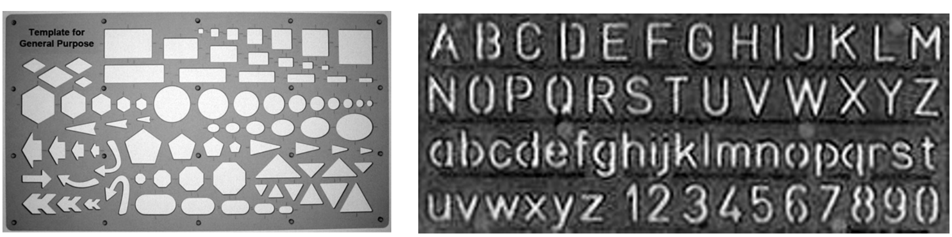


(a) (b) (c)

Fig. 1.23 *Sheet fasteners* **(a)** *Drawing pins* **(b)** *Drawing clip* **(c)** *Adhesive tape*

1.16 TEMPLATES

Drawing templates or stencils are time saving devices that are used for drawing various shapes and standard symbols. They are especially useful when shapes and symbols repeatedly appear on the drawing. They are available in a wide variety of shapes including lettering, circles, ellipses, isometric circles, polygons and arrowheads. Figures 1.24(a) and (b) show a few of them that can be used to draw circles, polygons, arrowheads, isometric circles and lettering. A template should be hold firmly while using to keep it from slipping out of position. They should be used only when accuracy can be sacrificed for speed.



(a) (b)

Fig. 1.24 *Drawing templates* **(a)** *General purpose* **(b)** *Lettering*

1.17 PENCIL CUTTERS

A pencil cutter or sharpener is used to prepare the working end of a pencil. A pencil with softer lead requires sharpening more often than with hard lead. Figure 1.25(a) shows a conventional sharpener which can produce only short length conical point. Figures 1.25(b) and (c) show blade-type cutters and mechanical sharpener. They are suitable for removing the wood from pencil. Thereafter, the desired working end can be prepared by rubbing over the sandpaper pad.

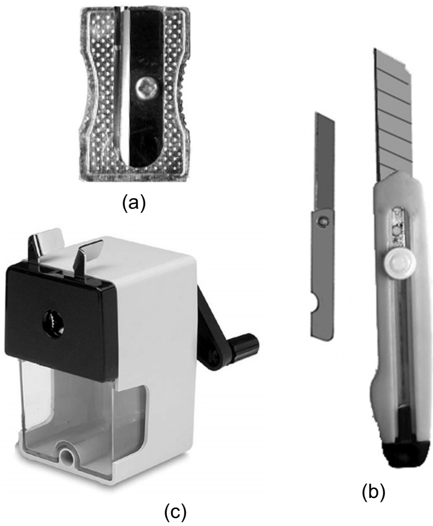


Fig. 1.25 *Sharpeners* **(a)** *Conventional* **(b)** *Blade cutter* **(c)** *Mechanical*

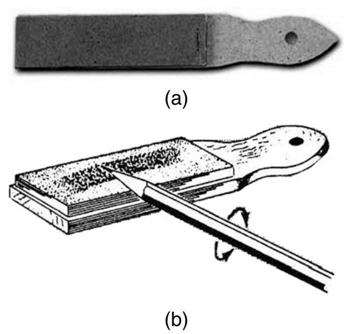


Fig. 1.26 **(a)** *Sandpaper Pad* (b) *Conical point is produced*

Fig. 1.27 *Brush*

1.18 SAND PAPER PAD

A sand paper pad or block is used to sharpen the pencil lead. It should be kept within reach of the user as it is frequently required to prepare the pencil edge. Figure 1.26(a) shows a sand paper pad. Figure 1.26(b) shows a conical or needlepoint being produced by rubbing pencil lead on the sandpaper pad.

1.19 BRUSH OR TOWEL

Figure 1.27 shows a brush. *The brush, duster or towel cloth is used to keep the drawing surface clean by removing crump (formed after the use of eraser), graphite particles or accumulated dirt before they spread over the drawing sheet.* Crump and graphite particles should not be brushed off with hands as they may spoil the drawing instead of cleaning it.

1.20 GENERAL PREPARATION FOR DRAWING

Arrange the drawing board and stool so that work could be done comfortably without fatigue or eye strain. The working area should be well lighted. Natural light is the best, if available and ample. The drawing board should be arranged such that the light may come from the front-left (from the front right in case of a left-handed person). This minimises shadows cast by drawing instruments and hands. Every possible care must be taken to eliminate eye strain.

Clean all the drawing instruments and accessories so that their surface may not spoil the sheet. Arrange them in a systematic manner, which is essential for saving time. Place the drawing instruments and reference publications on a small worktable adjacent to the drawing board. Clamp the mini-drafter on the drawing board and fasten the drawing sheet such that the mini-drafter can slide over the entire working area of the sheet. Switch to a harder pencil lead to draw fine or precise details.

1.21 PLANNING AND LAYOUT OF SHEET

A proper planning and layout of drawing sheet facilitates the easy reading of drawings and interchange of information. A standard arrangement should ensure that all necessary information for understanding the content of drawing is included and sufficient extra margin is left to facilitate easy filing and binding wherever necessary. The Bureau of Indian Standards in its bulletin IS 10711:2001 specifies the size and layout of the standard drawing sheets. It is recommended that standard formats should be followed to improve readability, handling, filing and reproduction.

Individual companies may use a slightly different layout for the sake of their own convenience but all necessary information is located at approximately the same place on most engineering drawings. Companies generally use pre-printed title block, borders and frames on drawing sheet to reduce drafting time and cost.

1.22 FRAMES AND BORDERS

The drawing sheets of sizes greater than that of the ISO-A series sizes are called *untrimmed sheet*. The sheet cut from the untrimmed sheet are called *trimmed sheet*. The frame limits the drawing space. It is recommended that the frame must be provided on the drawing sheets of all sizes and should be executed with continuous lines of 0.7 mm width. Table 1.4 provide the sizes of the untrimmed sheet, trimmed sheet and drawing space. Figure 1.28 shows relation between untrimmed sheet and trimmed sheet of A3 size along with other information.

Table 1.4 *Preferred sizes of untrimmed sheet, trimmed sheets and drawing space*

Designation	*Untrimmed sheet (in mm)*	*Trimmed sheet (in mm)*	*Drawing space (in mm)*	*Number of grid reference fields*
A0	880 × 1230	841 × 1189	821 × 1159	16 × 24
A1	625 × 880	594 × 841	574 × 811	12 × 16
A2	450 × 625	420 × 594	400 × 564	8 × 12
A3	330 × 450	297 × 420	277 × 390	6 × 8
A4	240 × 330	210 × 297	180 × 277	4 × 6

The space between the edges of the trimmed sheet and the frame is called a *border*. The width of the border is 20 mm on the left edge and 10 mm on the other edges. The larger border on the left edge helps in filing without damaging the drawing space. The border contains the following items:

1. Trimming marks *Trimming marks are used as an aid to trimming the untrimmed sheet.* The marks are provided in the borders at the four corners of the sheet. The marks are either in the form of two overlapping

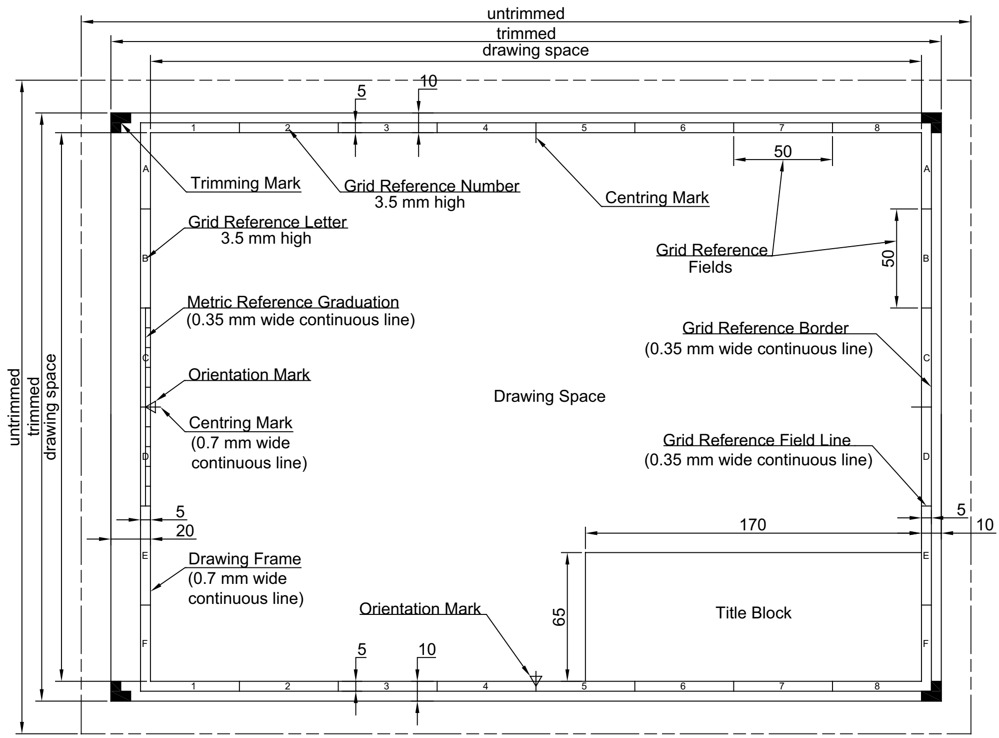


Fig. 1.28 *Frames and borders*

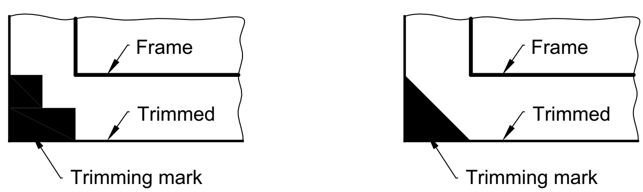


Fig. 1.29 *Trimming marks*

filled rectangles having 10 mm × 5 mm size or right angled isosceles triangles having 10 mm long sides as shown in Fig. 1.29. The sheets are trimmed to the outer edges of these marks and therefore remain on the sheet after it has been trimmed.

2. Grid references *A grid reference system, also called alpha-numeric referencing, provides an easy reference method to locate a specific area on the drawing for additions, modifications, revisions, etc.* For execution, a grid reference border is drawn all around the outside of the frame at a distance of 5 mm. Starting from the centre of each sides, short lines are drawn at every 50 mm to form a reference field of size 5 mm × 50 mm. The corner reference fields may be longer or shorter than 50 mm to account for the remainders resulting from the divisions. Table 1.4 provides the number of reference fields on short × long sides of the standard size drawing sheets.

The letters and numerals of nearly 3.5 mm height are written in vertical characters within the grid reference field. Usually, letters are placed in chronological order from the top to the downwards on both left and right side reference fields (except for the A4 size sheet where they are placed in the right side area only). Letters I and O are not used. The numbers are placed in chronological order from the left to the right side on both top and bottom reference fields (except for the A4 size sheet where they are placed in the top side area only). Figures 1.30(a) and (b) illustrates the grid reference system for A1 and A2 size drawing sheets.

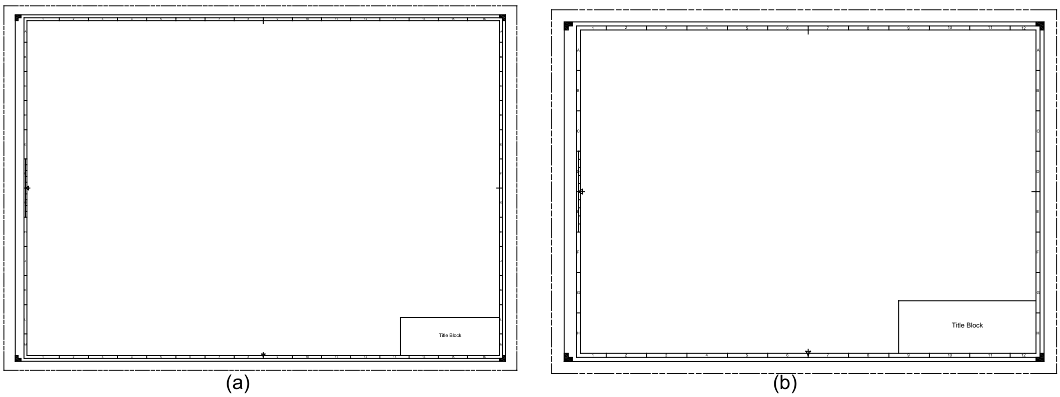


Fig. 1.30 *Planning and layout of drawing sheet for* **(a)** *A1 size* **(b)** *A2 size*

3. Centring mark *Four centring marks are provided to facilitate positioning of the drawing sheet for photocopying, scanning, etc.* The marks are placed at the centre of each of the four sides. The centring marks are 10 mm long starting from the grid reference border enters in the area of drawing space as shown in Fig. 1.28. The marks should be executed with a continuous line of 0.7 mm width.

4. Orientation mark *Two orientation marks are provided to indicate the orientation of the drawing sheet on the drawing board*. The marks are in the form of arrows as shown in Fig. 1.28. The marks are placed across the frame, one at a shorter side and one at a longer side, coinciding with the centring marks on those sides. One of the orientation marks always points to the draftsman.

5. Metric reference graduation *A metric reference graduation shows usefulness in knowing the scale factor of the drawing which has been scanned or photocopied in a scale different than that of the original.* The metric reference graduation starts from the left side frame and extends into the border for nearly 3 mm width. The graduation is 100 mm long, divided into 10 mm intervals and disposed symmetrically about a centring mark as shown in Fig. 1.28.

1.23 TITLE BLOCK

A title block provides information for identification, administration and interpretation of the whole drawing. It is placed in the bottom right-hand corner of the drawing frame, where it is readily seen when the prints are folded in the prescribed manner. The size of the title block recommended is 170 mm × 65 mm for all sizes of drawing sheets. Figures 1.31 and 1.32 show the sample title block used by draftsmen in industries and engineering students in colleges respectively.

A title block should contain the following information:

1. Name of the legal owner of the drawing (company, firm, organisation or enterprises)
2. Title of the drawing

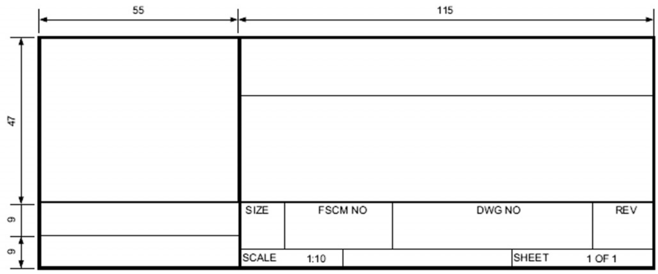


Fig. 1.31 *Title block used by draughtsmen in industries*

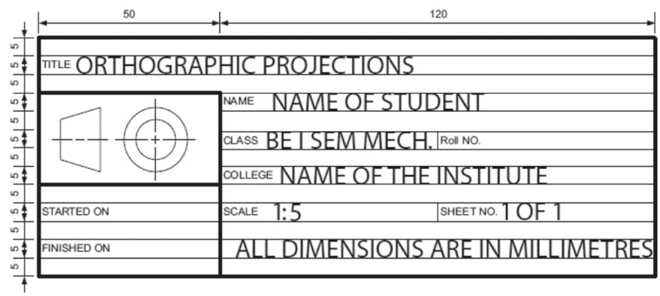


Fig. 1.32 *Title block used by students in engineering colleges*

3. Drawing sheet number
4. The scale
5. Symbol indicating the angle of projection used
6. The signature or initials of the staff designing, drawing, checking, approving officer and issuing officer, along with dates.
7. Other information, as required

Titles of drawings should be as concise as possible, consistent with adequate description. Multiple sheet drawings with the same identification number should be indicated as '*N* of *P*', where *N* is the sheet number and *P* is the total number of sheet.

The scale is the ratio of the linear dimension of an element of an object as represented in the drawing to the real linear dimension of the same element of the object itself. All drawings should be drawn to the scale for which, the selected scale should be large enough to permit easy and clear interpretation of the information depicted. The scale should be noted in the title block. When more than one scale is used, they should be shown close to the views to which they refer, and the title block should read 'scales as shown'. If a drawing uses predominantly one scale, it should be noted in the title block together with the wording 'or as shown'.

All orthographic drawings are made either according to first angle or third angle projection. These are depicted in the title block by their corresponding symbols. Title block should also contain statement "All dimensions are in millimetres unless otherwise specified". This means that all the features or dimension on drawing have a relationship or specifications given in the title block unless a specific note or dimensional tolerances is provided at a particular location in the drawing.

1.24 SPACE FOR TEXT

The space for text on a drawing sheet should provide all information necessary for the understanding the contents of the drawing. The space for text should be provided at the right-hand frame of the drawing space as shown in Fig. 1.33(a). The width of the space shall be equal to that of the title block, i.e., maximum 170 mm or at least 100 mm. If a figure takes up the whole width of the drawing sheet then the space for the text shall be provided at the bottom edge of the drawing sheet as shown in Fig. 1.33(b). The height of the space for text shall be chosen as required.

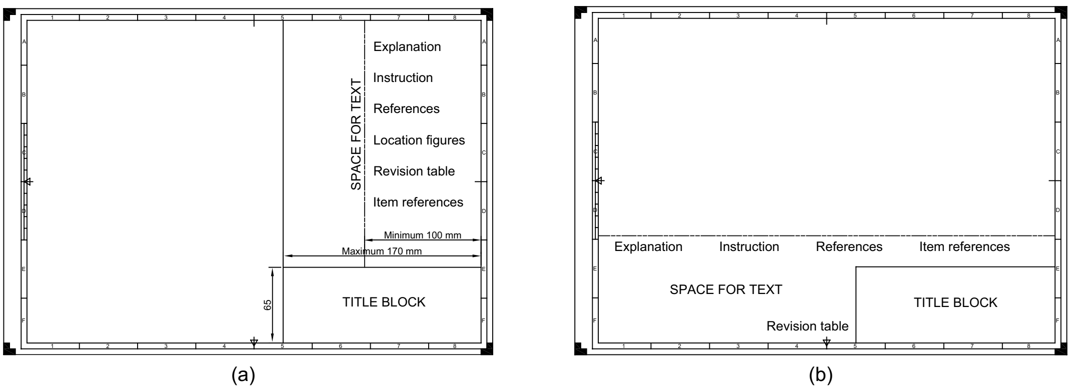


Fig. 1.33 *Space for text* **(a)** *Right-hand edge* **(b)** *Bottom edge*

The space for text should provide the following information:

1. Explanations Here the explanation of special symbols, designation, abbreviations and units of dimensions should be given which are needed to read the drawing.

2. Instructions Here the instruction related to material, realisation, surface treatment, assembly placing, number of units and combined dimensions should be given.

3. References Here the reference should be made to supplementary drawings and other documents.

4. Location figures Location figures are used in architectural and building drawings. A location figure may comprise the following:

(a) Schematic site plan with area, arrow indicating the north, building, part of building, etc.
(b) Schematic plan of building with area, part, etc.
(c) Schematic section through building with floor plan direction of view, etc.

5. Revision table Revision tables are used to record all document modifications, alterations or revisions which are made time to time to the drawing. In addition, any other factor which might influence the validity of the drawing shall be located in the revision table. The method of recording may vary in detail, but commonly the necessary information is entered in a table made of thin or thick continuous lines. The word 'ditto' or its equivalent abbreviations should not be used.

To facilitate extension of revision panel, entries for revision should begin from bottom upwards if the revision panel is a part of the title block as shown in Fig. 1.34 and from top downwards when revision panel is at the top right hand corner on drawing. It may contain the following information:

5 mm

DESIGNATION	DETAILS OF REVISION	DATE	SIGNATURE

Fig. 1.34 *Revision table*

(a) Designation The identification of a change on a drawing may be a symbol, number or letter enclosed within a circle, square or triangle. The designation column should show the reference to this identification mark or appropriate grid reference.

(b) Detail of revision The detail of revision column should show brief record of the changes in the drawing.

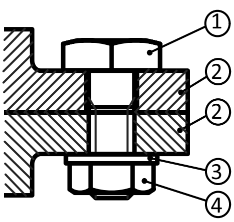
(c) Date The date column should show dated initials of the person who carried out the revision.

(d) Signature The signature column should show dated initials of the approving authority.

(e) Other applicable information To accompany other information necessary for clarity regarding revision in the drawing, more columns can be added.

1.25 ITEM REFERENCES ON DRAWING AND ITEM LISTS

If the drawing contains a number of items, or if it is an assembly drawing, a tabulated list of items is attached to the bottom right of the drawing frame, just above the title block. The list may be in conjunction with the title block. The item list included in the drawing should have its sequence from bottom to top, with headings of the column immediately underneath as shown in Fig. 1.35. It should be such as to be read in the viewing direction of the drawing. It is recommended that the item list be arranged in columns to allow information to be entered under the following headings:



5 mm.

4.	1	HEXAGONAL NUT		
3.	1	WASHER		
2.	1	SQUARE HEADED BOLT		
1.	2	PLATES		M.S.
S. NO.	QUANTITY	DESCRIPTION	REFERENCE	MATERIAL

Fig. 1.35 *Item list (bill of materials)*

1. Quantity The quantity column should show the total number of that particular item necessary for one complete assembly.

2. Description The description column should show the designation of the item. If the item concerns a standard part such as bolt, nut, stud, etc., its standard designation may be used.

3. Item reference The reference column should show the reference to the relevant item reference number. It is generally composed of Arabic numerals. To distinguish them from other indications the numerals used have either (a) height twice as used for dimensioning and similar indications, and/or (b) encircling. The item reference must assign in a sequential order to each component part shown in assembly and each detailed item on the drawing. The identical parts in an assembly should have the same item reference.

4. Material The material column should show the type and quality of the material to be used. If this is a standard material, its standard designation should be mentioned.

5. Other applicable information To accompany other information necessary for finish products such as stock number, unit mass, state of delivery, etc., more columns can be added.

1.26 FOLDING OF DRAWING SHEETS

The drawing sheet after completion of the drawing should be folded properly according to IS 11664:1986 recommended by Bureau of Indian Standards. Figure 1.36(a) shows the method of folding the drawing sheet

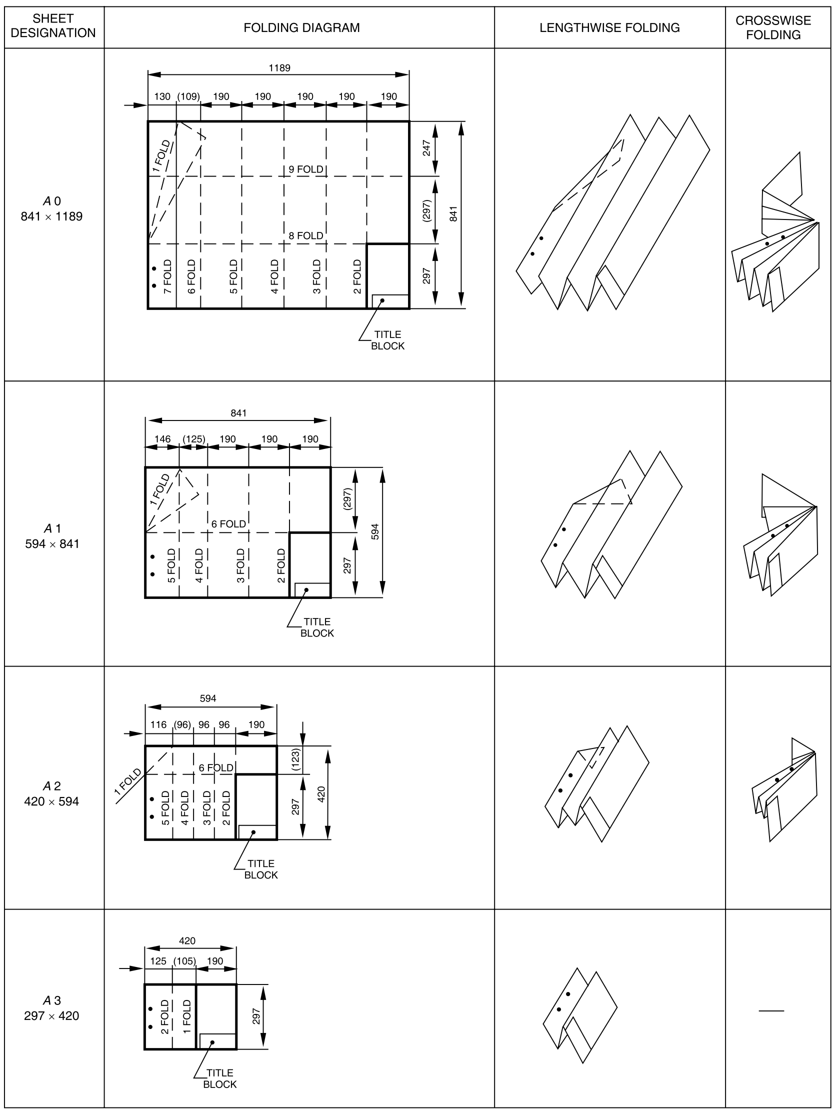
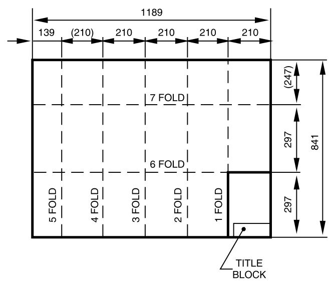
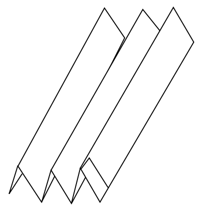
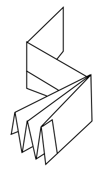


Fig. 1.36(a) *Folding of drawing sheets for filing or binding, all dimensions are in millimetres*

intended for filing or binding while Fig. 1.36(b) shows the method of folding the drawing sheet intended to keep individually in filing cabinet. It can be seen that the title block of all the folded prints appears in topmost position. Depending upon the folding method adopted, suitable folding marks are to be introduced in the tracing sheets as a guide.

<table>
<tr><th>SHEET DESIGNATION</th><th>FOLDING DIAGRAM</th><th>LENGTHWISE FOLDING</th><th>CROSSWISE FOLDING</th></tr>
<tr><td>A 0
841 × 1189</td><td>
</td><td></td><td></td></tr>
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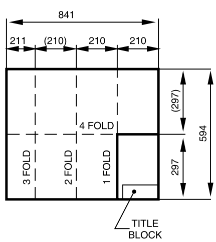
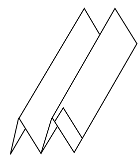
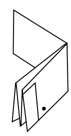
594 × 841</td><td>
</td><td></td><td></td></tr>
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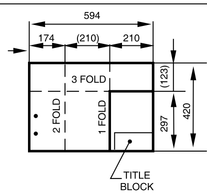
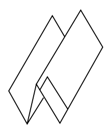
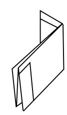
420 × 594</td><td>
</td><td></td><td></td></tr>
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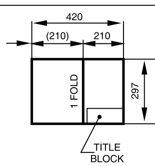
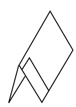
297 × 420</td><td>
</td><td></td><td>—</td></tr>
</table>

Fig. 1.36(b) *Folding of drawing sheets for storing in filing cabinet, all dimensions are in millimetres*

1.27 CONCLUSION

One should practice handling and using drawing instruments before working with complex drawing problems. Developing correct drawing habits will enable to make continuous improvement in the quality of drawings. Each drawing will offer an opportunity for practice. Later on, good form in the use of instruments will become a natural habit.

EXERCISE 1A

1.1 Use a mini-drafter to draw Figs. E1.1 to E1.3 in a square of 100 mm side. Take the distance between consecutive parallel lines as 10 mm.

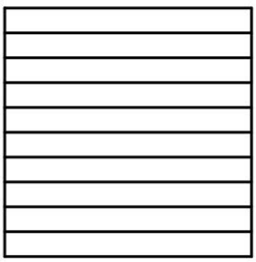


Fig. E1.1

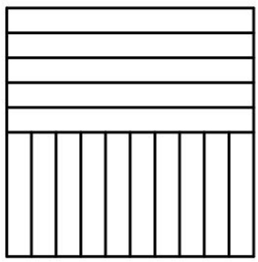


Fig. E1.2

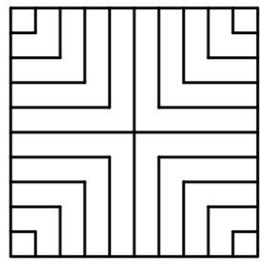


Fig. E1.3

1.2 Use a mini-drafter to draw Figs. E1.4 to E1.6 in a square of 90 mm side. Take the distance between consecutive parallel lines as 10 mm.

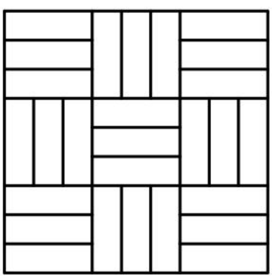


Fig. E1.4

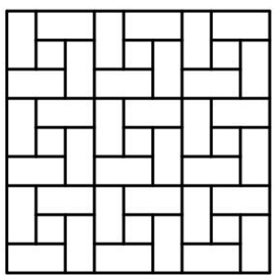


Fig. E1.5

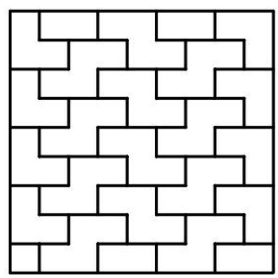


Fig. E1.6

1.3 Use a mini-drafter to draw Fig. E1.7 to E1.9 in a square of 100 mm side. In Figs. E1.7 and Fig. E1.8, take the distance between consecutive parallel lines as 10 mm.

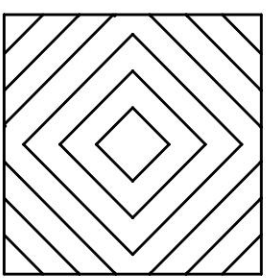


Fig. E1.7

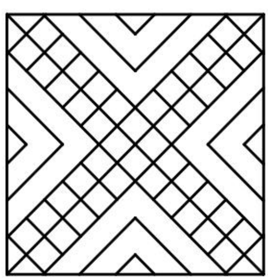


Fig. E1.8

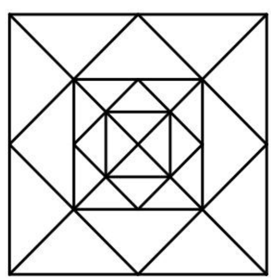


Fig. E1.9

1.4 Use necessary drawing instruments to draw Figs. E1.10 to E1.12 in a square of 100 mm.

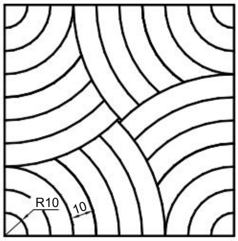


Fig. E1.10

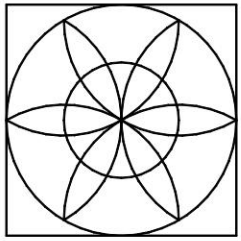


Fig. E1.11

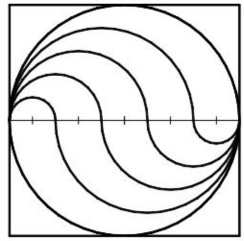


Fig. E1.12

1.5 Use necessary drawing instruments to draw Figs. E1.13 to E1.15 in a square of 100 mm.

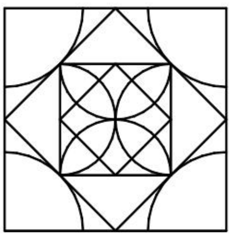


Fig. E1.13

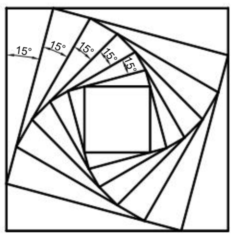


Fig. E1.14

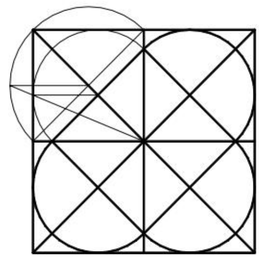


Fig. E1.15

1.6 Use necessary drawing instruments to draw Figs. E1.16 to E1.18 in a circle of diameter 100 mm.

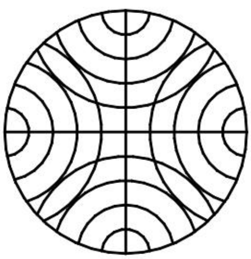


Fig. E1.16

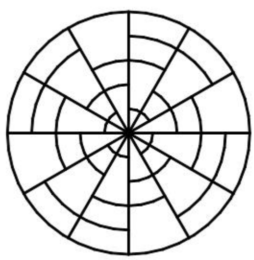


Fig. E1.17

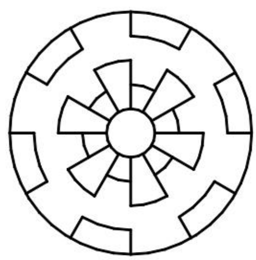


Fig. E1.18

1.7 Use necessary drawing instruments to draw Figs. E1.19 to E1.21 in a circle of diameter 100 mm.

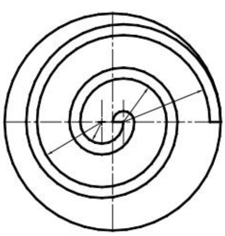


Fig. E1.19

Fig. E1.20

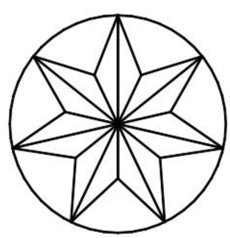


Fig. E1.21

VIVA-VOCE QUESTIONS

1.1 What do you mean by the International Organisation for Standardisation and Bureau of Indian Standards?

1.2 What is the standard designation of the drawing boards recommended by Bureau of Indian Standards? Which drawing board is recommended for the A2 size drawing sheet?

1.3 How pencils are graded? Which grade of pencil is suitable for lettering, drawing outlines and visible edges, and construction lines?

1.4 What is the use of Engineer's scales? List the types of engineer's scale as recommended by Bureau of Indian Standards.

1.5 What is the use of French curves? Explain its working in brief.

1.6 What is the use of a pair of set-squares? How parallel lines are drawn with the help of a set-square?

1.7 How a pair of set-squares can be used to draw an angle of 15° and 75°?

1.8 Name different types of fasteners that can be used to fix a drawing sheet on the drawing boards. Give their relative merits and demerits.

1.9 What do you mean by drawing frames and borders? Enlist different items contained by them.

1.10 Explain the purpose of the items contained by frames and borders.

1.11 What information should be contained in the title block of a drawing sheet?

1.12 Write short notes on (a) grid reference and (b) metric reference graduation.

1.13 What information may be included in the space for text?

1.14 What is a revision table? What information should be contained by it?

1.15 When an item references is required? How is the item list prepared?

MULTIPLE-CHOICE QUESTIONS

1.1 Which of the following is bulletin is the recent publication of Bureau of Indian Standards, contains codes for practice in engineering drawing?
(a) IS 696 (b) SP 46
(c) BS 8888 (d) ASME Y14.100

1.2 A device which combines the functions of a T-square, set square, protractor and scale is called
(a) fasteners (b) mini drafter
(c) templates (d) combination set

1.3 A drafter helps in drawing
(a) parallel and perpendicular lines
(b) concentric circles
(c) smooth curves
(d) All the these

1.4 Paper size 'A0' has an area of
(a) 1 m^2 (b) 0.75 m^2
(c) 0.5 m^2 (d) 0.25 m^2

1.5 In the engineering system of paper sizes, which of the following is 'A2' size?
(a) 841 mm × 1189 mm
(b) 594 mm × 841 mm
(c) 420 mm × 594 mm
(d) 210 mm × 297 mm

1.6 Which of the following pencil leads is hardest?
(a) HB (b) H
(c) B (d) F

1.7 In a compass, lengthening bar is used to
(a) draw circles of large diameters
(b) draw circle of uniform thickness
(c) increase the overall height
(d) grip firmly while drawing circles

1.8 Usually, a bow compass is used to draw circles and arcs of diameter
(a) greater than 50 mm
(b) less than 50 mm
(c) in steps of 10 mm
(d) in steps of 5 mm

1.9 To draw smooth curves of any nature, the drafting instrument used is a/an
(a) mini drafter
(b) French curve
(c) template
(d) eraser shield

1.10 Set-squares can draw lines with precision, inclined with the horizontal at an angle in multiples of
(a) 5 degrees (b) 10 degrees
(c) 15 degrees (d) 20 degrees

1.11 Parallel lines can be drawn with the help of
(a) mini drafter
(b) T-square
(c) pair of set-squares
(d) All of these

1.12 Grid References on a drawing sheet provide the following information
(a) Location of details, additions, modifications, revisions, etc. of drawing
(b) To facilitate the positioning of the drawing when reproduced
(d) To facilitate brief record and initials of the person responsible
(c) To facilitate trimming

1.13 A title block does not provide which of the following information?
(a) Name of the legal owner
(b) Drawing sheet number
(c) Angle of projection used
(d) Explanation of symbols used

1.14 Which of the following statements is preferred containing the statement "All dimensions are in millimetres unless otherwise specified"?
(a) Frames and borders
(b) Title block
(c) Item list
(d) Revision table

1.15 An item list on a drawing sheet provides which of the following information?
(a) Name of the company, title of the drawing, scale and angle of projection
(b) Item, description, quantity and material
(c) Explanation of special symbols, abbreviations and units of dimensions
(d) All the above

1.16 Number of orientation mark generally contained by a drawing sheet is
(a) One (b) Two
(c) Three (d) Four

1.17 What should appear on the top position after folding the drawing sheet?
(a) Revision table (b) Item references
(c) Title block (d) Any of these

Answers to multiple-choice questions

1.1 (b), 1.2 (b), 1.3 (a), 1.4 (a), 1.5 (c), 1.6 (b), 1.7 (a), 1.8 (b), 1.9 (b), 1.10 (c), 1.11 (d), 1.12 (a), 1.13 (d), 1.14 (b), 1.15 (b), 1.16 (b), 1.17 (c)

Chapter

2 LINES, LETTERING AND DIMENSIONING

2.1 INTRODUCTION

Engineering drawing is supposed to give complete information about the shape and size of the objects like machine parts, buildings, etc. The shape of the object is conveyed through the appearance of the drawing while the size description is expressed in the form of figured dimensions and notes. The Bureau of Indian Standards has recommended various types of lines, letters and dimensions to be used. This chapter introduces the standard practice for configuration of lines to specify shape, letters for writing note, dimensions to convey the size and the correct way of their implementation.

2.2 LINES

Engineering drawings are prepared with the help of symbolic lines. Table 2.1 shows the basic types of lines recommended by Bureau of Indian Standard in its bulletin IS 10714 (Part 20):2001. Table 2.2 shows the length of the line elements for configuration of lines depicted in the Table 2.1, where *d* is the width of the line.

The selection of width of the line depends on the type and size of drawing. The width of the line should be opted from one of the following:

0.13 mm; 0.18 mm; 0.25 mm; 0.35 mm; 0.5 mm; 0.7 mm; 1.0 mm; 1.4 mm; 2.0 mm.

The series is based on a common ratio $1{:}\sqrt{2}$. Usually, lines of three different widths specified as narrow, wide and extra wide lines are used. The narrow, wide and extra wide lines have a width ratio of 1:2:4. The line having width in between wide and narrow lines are used for lettering of graphical symbols and special field applications. In a drawing, all narrow lines should be of same uniform width and similarly all wide lines should have same uniform width. The lines should be sharp and dense to obtain good reproduction.

2.2.1 Application of Popular Types of Lines

In a drawing sheet of *A*1 and *A*2 sizes, the wide lines should have a width of 0.7 mm and the narrow lines have a width of 0.35 mm. The directly visible edges of the object depicted in a drawing are called outlines. Continuous wide line is used to draw outlines as shown in Fig. 2.1(a). Dashed narrow line is used to depict the edges that are not directly visible. The dashed narrow line has dashes of about 2 mm long with spacing between the two dashes of about 1 mm.

Long-dashed dotted narrow line is used to indicate axis of the object, and centres of circles and arcs. The line has alternate long-dashes and dots. The long-dashes are about 10 mm long and the spacing between the long-dash and the dot is about 1 mm. It is preferred to extend centre lines for a short distance beyond the outlines.

Continuous narrow line is used for configuring dimension lines, leader lines and construction lines. The line is also used for indicating surfaces in section views resulting from cutting as shown in Fig. 2.1(b), generally called *hatching*. The hatching has a distance of 1 mm to 2 mm between the two parallel lines. Table 2.3 show the overall applications of popular lines in the mechanical engineering drawing.

Table 2.1 *Basic types of lines*

S. No.	*Representation*	*Description*
01		Continuous line
02		Dashed line
03		Dashed space line
04		Long dashed dotted line
05		Long dashed double-dotted line
06		Long dashed triplicate-dotted line
07		Dotted line
08		Long dashed short dashed line
09		Long dashed double-short dashed line
10		Dashed dotted line
11		Double-dashed dotted line
12		Dashed double-dotted line
13		Double-dashed double-dotted line
14		Dashed triplicate-dotted line
15		Double-dashed triplicate-dotted line

Table 2.2 *Configuration of basic lines (where d is the width of line)*

Line Element	*Line type no.*	*Description*
Dots	04 to 07 and 10 to15	≤ 0.5 d
Gaps	02 and 04 to 15	3 d
Short dashes	08 and 09	6 d
Dashes	02, 03 and 10 to 15	12 d
Long dashes	04 to 06, 08 and 09	24 d
spaces	03	18 d

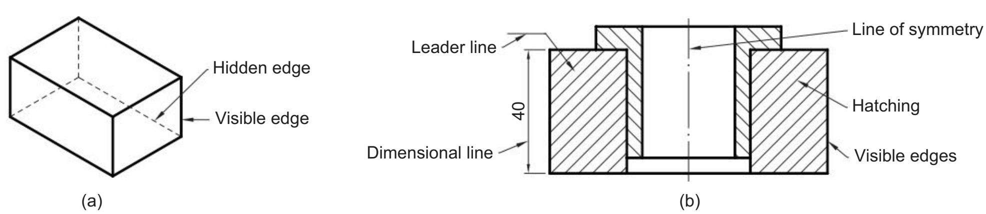


Fig. 2.1 *Applications of popular type of lines for depicting* **(a)** *visible and hidden edges* **(b)** *hatching, line of symmetry, dimensional line and leader line*

Table 2.3 *Applications of various types of lines in engineering drawing*

S. No.	*Line description and representation*		*Applications*
1.	Continuous narrow lines	(a)	Imaginary lines of intersection
		(b)	Dimension line
		(c)	Extension line
		(d)	Leader line with reference line
		(e)	Hatching
		(f)	Outlines of revolved section
		(g)	Short centre lines
		(h)	Root of screw threads
		(i)	Dimension lines termination, arrowheads
		(j)	Diagonals for indication of flat surfaces
		(k)	Bending lines on blanks and processing parts
		(l)	Framing of details
		(m)	Indication of repetitive details
		(n)	Interpretation lines of tapered features
		(o)	Location of laminations
		(p)	Projection lines
		(q)	Grid lines
2.	Continuous narrow freehand line		[a] Preferably manually represented termination of partial or interrupted views, cuts and sections, if the limit is not a line of symmetry or a centre line
3.	Continuous narrow line with zigzags		[a] Preferably mechanically represented termination of partial or interrupted views, cuts and sections. If the limit is not a line of symmetry or a centre line
4.	Continuous wide line	(a)	Visible edges
		(b)	Visible outlines
		(c)	Crests of screw threads
		(d)	Limit of length of full depth thread
		(e)	Main representation in diagrams, maps, flow charts
		(f)	System lines (structural metal engineering)
		(g)	Parting lines of moulds in views
		(h)	Lines of cuts and section arrows
5.	Dashed narrow line	(a)	Hidden edges
		(b)	Hidden outlines
6.	Dashed wide line	(a)	Indication of permissible areas of surface treatment

7.	Long-dashed dotted narrow line	(a) Centre lines (b) Line of symmetry (c) Pitch circle of gears (d) Pitch circle of holes
8.	Long-dashed dotted wide line	(a) Indication of (limited) required areas of surface treatment, eg. heat treatment (b) Indication of cutting planes
9.	Long-dashed double-dotted narrow line	(a) Outlines of adjacent parts (b) Extreme positions of movable parts (c) Centroidal lines (d) Initial outlines prior to forming (e) Parts situated in front of a cutting plane (f) Outlines of alternative executions (g) Outlines of the finished part within blanks (h) Framing of particular fields/areas (i) Projected tolerance zone

[a] It is recommended to use only one type of line on one drawing.

2.2.2 Rules for Drafting Lines

The following rules should be observed while drafting lines:

1. Lines should be drawn in black or white depending on the colour of the background.
2. The minimum space between parallel lines should preferably be greater than 0.7 mm.
3. The dotted line should preferably meet at a dot as shown in Fig. 2.2(a). Other types of lines should preferably meet at a dash as shown in Figs. 2.2(b) and (c).

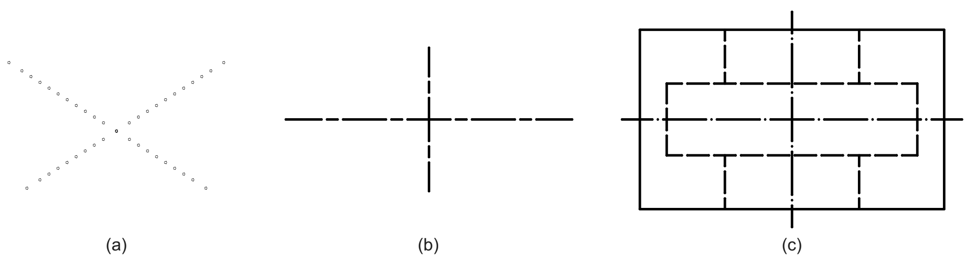


Fig. 2.2 **(a)** *Dotted line meet at dot* **(b)** *and* **(c)** *Dashed lines meet at a dash long*

4. In case of two or more lines of different type which may overlap or coincide, the drawing priority may be given in the following order:
   (a) Visible outlines and edges
   (b) Hidden outlines and edges
   (c) Cutting planes
   (d) Centre lines and lines of symmetry

(e) Centroidal lines
(f) Projection lines

For example, if a visible line coincides with a hidden line, then only visible line is to be drawn ignoring the hidden line. Similarly, if a hidden line coincides with a projection line, then only hidden line is to be drawn ignoring the projection line.

2.3 LETTERING

Writing of titles, dimension value, notes and other particulars on a drawing is called lettering. It is used to provide detailed specifications of an object. There could be various styles of writing Latin alphabets. With the goals of legibility and uniformity, styles are standardised. The Bureau of Indian Standards recommends Latin alphabets and numerals for technical drawing in its bulletin IS 9609 (Part 1):2006.

Single-stroke vertical capital letters and numerals shown in Fig. 2.3(a) are the simplest form of letters generally used in practice. The figure also shows single-stroke vertical lower-case letters. The term ‘single-stroke’ do not mean that the entire letter should be made in one stroke without lifting the pencil. It actually means that the width of the line of the letter should be such as is obtained in one stroke of pencil. Figure 2.3(b) shows single stroke inclined capital letters, lower-case letters, mathematical operators and numerals. The letters have an inclination towards right of about 75° with the horizontal and are used in special conditions.

(a) (b)

Fig. 2.3 *Lettering A* **(a)** *Single-stroke vertical letters* **(b)** *Single-stroke inclined letters*

The nominal size of letters and numerals are designated by their heights (h) of the outline contour of the upper-case letters. The selection of heights of the letter depends on the type and size of drawing and should be opted from 1.8 mm, 2.5 mm, 3.5 mm, 5 mm, 7 mm, 10 mm, 14 mm and 20 mm. Table 2.4 shows the recommended spacing between characters, words and lines. Figure 2.4(a) shows style of lettering for

Table 2.4 *Dimensioning of lettering Type A*

Characteristic	*Ratio*	*Dimensions (mm)*							
Height of capital lettering, h	$\left(\frac{14}{14}\right)$h	1.8	2.5	3.5	5	7	10	14	20
Height of lower-case lettering, c_1	$\left(\frac{10}{14}\right)$h	1.3	1.8	2.5	3.5	5	7	10	14
Tails of lower case letters, c_2	$\left(\frac{4}{14}\right)$h	0.52	0.72	1	1.4	2	2.8	4	5.6
Stem of lower case letters, c_3	$\left(\frac{4}{14}\right)$h	0.52	0.72	1	1.4	2	2.8	4	5.6
Area of diacritical marks (upper case letters), f	$\left(\frac{5}{14}\right)$h	0.65	0.9	1.25	1.75	2.5	3.5	5	7
Spacing between characters, a	$\left(\frac{2}{14}\right)$h	0.26	0.36	0.5	0.7	1	1.4	2	2.8
Minimum spacing of baselines[1)], b_1	$\left(\frac{25}{14}\right)$h	3.25	4.5	6.25	8.75	12.5	17.5	25	35
Minimum spacing of baselines[2)], b_2	$\left(\frac{21}{14}\right)$h	2.73	3.78	5.25	7.35	10.5	14.7	21	29.4
Minimum spacing of baselines[3)], b_3	$\left(\frac{17}{14}\right)$h	2.21	3.06	4.25	5.95	8.5	11.9	17	23.8
Minimum spacing between words, e	$\left(\frac{6}{14}\right)$h	0.78	1.08	1.5	2.1	3	4.2	6	8.4
Line width, d	$\left(\frac{1}{14}\right)$h	0.13	0.18	0.25	0.35	0.5	0.7	1	1.4

[1)]Lettering style: Upper-case and lower-case letters with diacritical marks (see Fig. 2.4(a))
[2)]Lettering style: Upper-case and lower-case letters without diacritical marks (see Fig. 2.4(b))
[3)]Lettering style: Upper-case letters only (see Fig. 2.4(c))

upper-case and lower-case letters with diacritical marks, Fig. 2.4(b) shows style of lettering for upper-case and lower-case letters without diacritical marks and Fig. 2.4(c) shows lettering style for upper-case letters only. Lettering should be done on the drawing in such a manner that it may be read when the drawing is viewed from the bottom edge, except when it is used for dimensioning purpose.

2.3.1 Rules for Lettering

A freehand lettering practice allows to represent ideas in a short time in an efficient manner. A very hard pencil will produce lettering that cannot reproduced easily while a soft pencil will produce lettering that may smear easily on a drawing. An HB or H pencil with conical-shaped point works best for most lettering. The ability to letter well can be acquired only by continued and careful practice. The following rules should be observed while lettering:

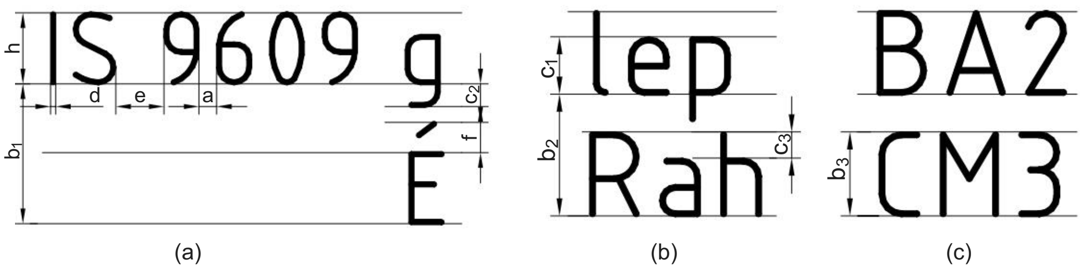


Fig. 2.4 *Lettering* **(a)** *With diacritical marks* **(b)** *and* **(c)** *Without diacritical marks*

1. Light guidelines drawn using sharp pencil ensures consistency in the size of the letter characters. If the lettering consists of capitals, draw only the cap line and base line. If the lowercase letters are included as well, draw the waist line and the drop line.
2. The thickness of the line of the letter should be such as is obtained in one stoke of pencil. Exert a firm uniform pressure, but not so heavy as to cut grooves in the sheet.
3. The width-to-height ratio should be around 1:2 for all capital alphabets (except I and W) and 1:3 for all numerals (except 1).
4. Letters should be written in capitals. Lower-case alphabet should be used only when they are accepted in international usage for abbreviations.
5. The horizontal lines of the letter should be drawn from left to right and vertical or inclined lines from top to bottom. Alphabet and numerals should neither touch each other nor the lines.
6. Letters and numerals should neither touch each other nor the lines.
7. Letters should be so spaced that the area between letters appears equal. It is not necessary to keep clearances between adjacent letters equal. For example, letters LA, TV or Tr.
8. Words should be spaced one letter apart.
9. Letters should be so written that they appear upright from the bottom edge, except when they are used for dimensioning. For dimensioning, they may appear upright from the bottom edge for the right hand side or the corner in between.

2.4 DIMENSIONING

Dimensions are indicated on the drawing to define the size characteristics such as length, breadth, height, diameter, radius, angle and location of hole, slot, etc. They should be mentioned directly on the drawing to describe a component clearly and completely in its finished form. The Bureau of Indian standards in its bulletin IS 11669:1986 (reaffirmed 1999) recommends general principle of dimensioning in technical drawing.

2.4.1 Dimensioning Terminology

Figure 2.5 shows the methodology of dimensioning a drawing. The terminologies related to the dimensioning are as follows:

1. Dimension value It is a numerical value that is being assigned to the size, shape or location of the feature being dimensioned. They are expressed in a specific unit (preferably millimetres) on drawings with relevant information.

2. Dimension lines These are thin continuous lines that show the extent and direction of the dimension. They should be placed 8 to 10 mm away from the outlines and should be placed uniformly 6 to 8 mm from each other. The dimension values are placed preferably near the middle of the dimension lines.

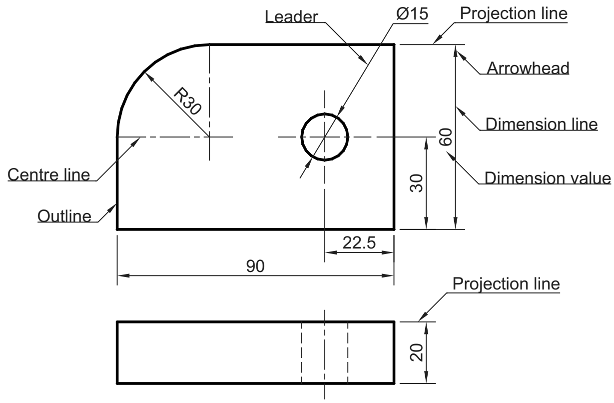


Fig. 2.5 *Dimensioning terminology*

3. Projection lines These are the thin continuous lines stretched out from the outlines for dimensioning and extended 2 to 3 mm beyond the dimension lines. They should be drawn in a direction perpendicular to the feature to be dimensioned. Projection lines and dimension lines should not cross other lines, unless this is unavoidable. Under special circumstances, projection lines may be drawn obliquely, but parallel with each other as shown in Fig. 2.6.

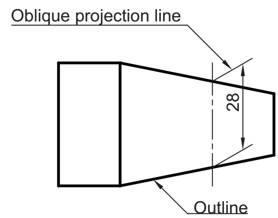


Fig. 2.6 *Projection lines drawn obliquely*

4. Leaders or pointer lines These are the lines referring to a feature and notes. It is executed using the thin continuous lines and terminated by arrow heads or dots as shown in Fig. 2.7(a). Notes and figures are written above the extended dimension lines.

Leaders should not to be inclined at an angle less than 30° as shown in Fig. 2.7(b) or parallel to adjacent dimensions or projection lines where confusion might arise. Leaders are never drawn vertical, horizontal, curved or free hand. Usually they are drawn at any convenient angle 30°, 45° and 60. Use of long leaders should be avoided.

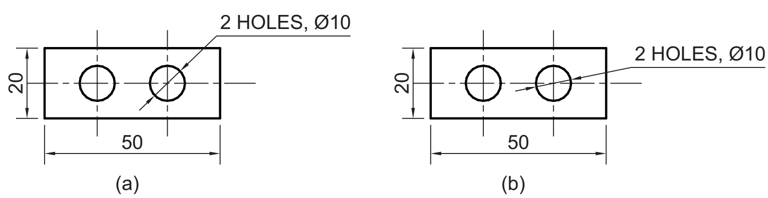


Fig. 2.7 *Leader line* **(a)** *Correct method* **(b)** *Inclination less than 30° is not permitted*

5. Arrowheads Usually, arrowheads are used for the termination of dimension lines. They may be open at a convenient angle of 30° to 90°, closed blank or closed filled as shown in Fig. 2.8(a). The closed filled arrowheads have length about three times the depth/width as shown in Fig. 2.8(b), and are preferred in the engineering drawings. Usually, length of closed filled arrowheads is 3 mm for small drawings and 4 to 5 mm for large drawings. Oblique stroke shown in Fig. 2.8(c) and point shown in Fig. 2.8(d) may be used in lieu of arrowheads when the space to accommodate the arrowheads termination is insufficient.

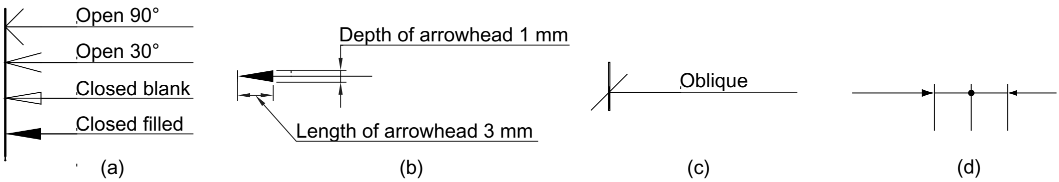


Fig. 2.8 *Arrowheads* **(a)** *Various types* **(b)** *Closed filled type* **(c)** *Oblique stroke* **(d)** *Point*

2.5 PLACEMENT OF DIMENSIONS

Dimensions should be placed on the view which shows the relevant features more clearly. The two recommended systems of placing the dimensions are as follows:

2.5.1 Aligned System

1. Linear dimensioning All dimension values are placed above the dimension lines as shown in Fig. 2.9(a). The values can be read from the bottom or the right-hand edges of the drawing sheet. Figure 2.9(b) shows the recommended direction that has to be used for writing the dimension values for the inclined dimension lines. As far as possible, dimension lines should not be placed in 30º zone, shown by hatching in Fig. 2.9(b).

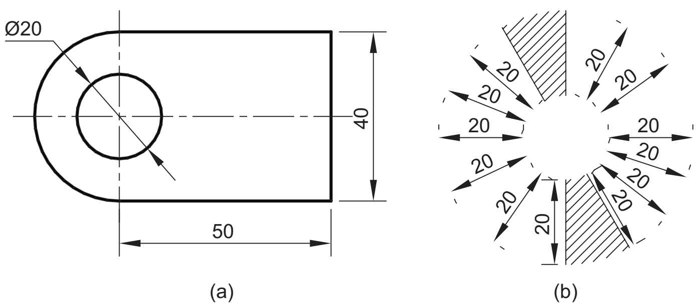


Fig. 2.9 *Aligned system for linear dimensioning* **(a)** *An example* **(b)** *Possible cases*

2. Angular dimensioning Angular dimensions and their deviations are dimensioned in the same manner as that of linear dimensions. Figure 2.10(a) shows the recommended direction that has to be used for writing the dimension values. In certain cases, dimension values may be written horizontally as shown in Fig. 2.10(b), if this improves clarity.

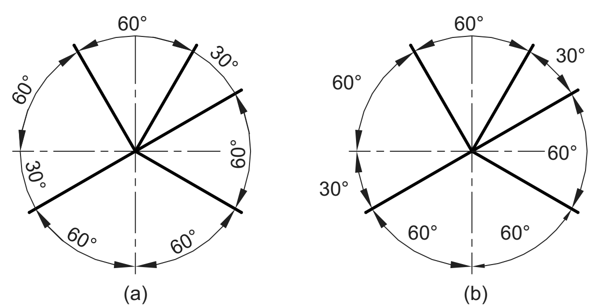


Fig. 2.10 *Aligned system for angular dimensioning* **(a)** *General purpose* **(b)** *Special purpose*

2.5.2 Unidirectional System

1. Linear dimensioning All dimension values are placed upright as shown in Fig. 2.11(a), so that they may be read from the bottom edge

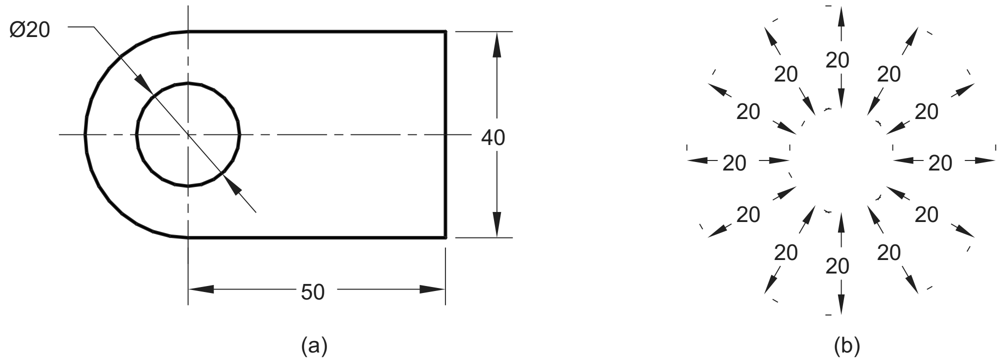


Fig. 2.11 *Unidirectional system for linear dimensioning* **(a)** *An example* **(b)** *Possible cases*

of the drawing sheet. For inserting a dimension value, the dimension line is broken at the middle. Figure 2.11(b) conveys that there is no restriction for writing the dimension values for the inclined dimension lines. This system is advantageous on large drawings where it is inconvenient to read dimensions from the right-hand side.

2. Angular dimensioning Angular dimensions and their deviations are dimensioned in the same manner as that of linear dimensions. Figure 2.12 suggests the correct orientation and method for writing dimension values.

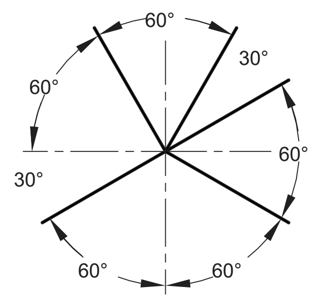


Fig. 2.12 *Unidirectional system for angular dimensioning*

2.6 ARRANGEMENT OF DIMENSIONS

When several dimensions are to be placed on the drawing, they need to be arranged such that it provides a unique interpretation. The classification of dimensions on the basis of arrangement is as follows:

2.6.1 Continuous or Chain Dimensioning

In chain dimensioning, the dimensions are aligned such that an arrowhead of one dimension touches tip to tip the arrowhead of the adjacent dimension as shown in Figs. 2.13(a) and (b). The overall dimension is placed outside the other smaller dimensions. The arrangement should be used only where the possible accumulation of tolerances does not endanger the functional requirement of the part (A tolerance is an indication of the accuracy the product has to be made).

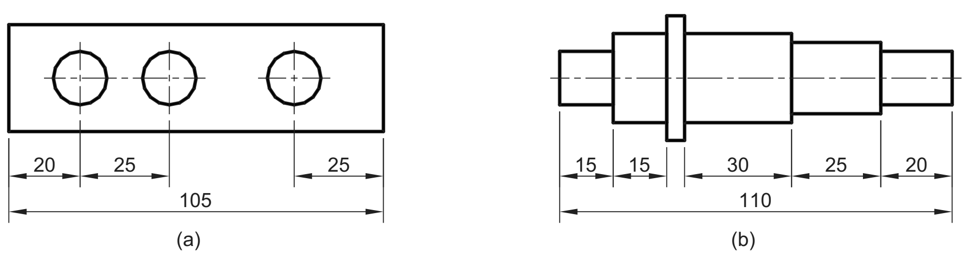


Fig. 2.13 **(a)** *and* **(b)** *Continuous or chain dimensioning*

2.6.2 Dimensioning from a Common Feature

Here a number of dimensions are measured in the same direction from a common feature. Obviously, all these dimensions share a common extension line. The arrangement should be used when dimensions have to be established from a particular datum surface.

1. Progressive or parallel dimensioning In parallel dimensioning, the dimension lines are spaced out parallel one to another. Smaller dimension is placed nearer the outline. The next smaller dimension is placed next and so on, as shown in Figs. 2.14(a) and (b). Dimensions should be staggered when number of parallel dimensions is more.

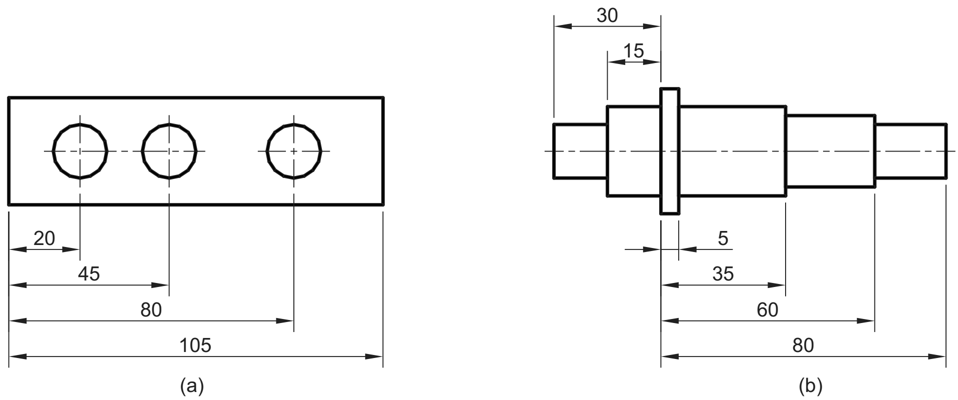


Fig. 2.14 **(a)** *and* **(b)** *Progressive or parallel dimensioning*

2. Superimposed running dimensioning It is a simplified parallel dimensioning and is used where there are space limitations and where no legibility problems would occur. All the dimensions begin from a common origin (indicated by a small circle of approximately 3 mm diameter) and terminate with arrowheads where the individual dimension ends. The dimension values are rotated through 90° and placed in-line with the projection line as shown in Figs. 2.15(a) and (b) or above the dimension line near the arrowhead.

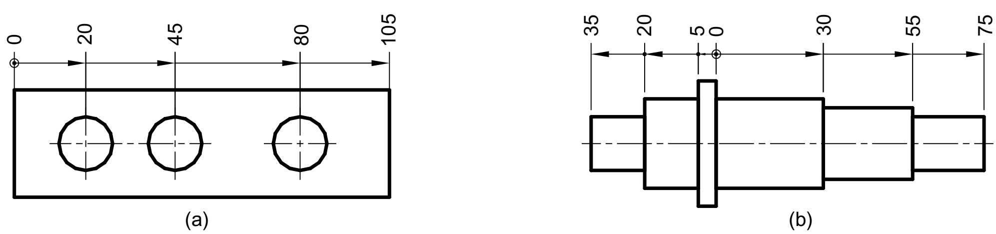


Fig. 2.15 **(a)** *and* **(b)** *Superimposed running dimensioning*

2.6.3 Combined dimensioning

This combined dimensioning results from simultaneous use of chain dimensioning, parallel dimensioning and superimposed running dimensioning in a single drawing as shown in Figs. 2.16(a) and (b).

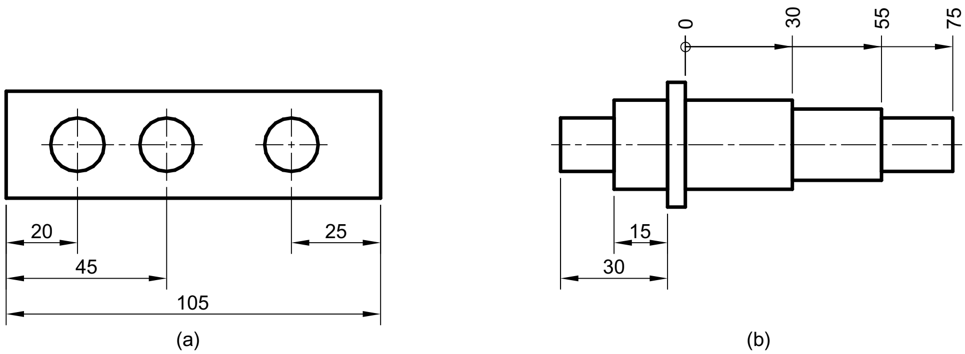
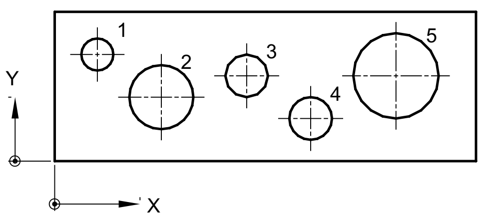


Fig. 2.16 **(a)** *and* **(b)** *Combined dimensioning*

2.6.4 Coordinate dimensioning

Dimensioning by using a coordinate table make the drawing easier to read, especially when the part have too many dimensions when drawn to other style. Figure 2.17 shows a plate with many holes where coordinate dimensioning should be used, as other style of dimensioning will give an overly cluttered look. Origin identification (X and Y) should be given for the part and all the features (in this case, case holes) should be numerically labelled. A coordinate table containing the dimensional details should be place near to the title block.



	X	Y	∅
1	20	50	15
2	50	30	30
3	90	40	20
4	120	20	20
5	160	40	40

Fig. 2.17 *Coordinates dimensioning*

2.7 SYMBOLS AND NOTES FOR DIMENSIONING

Symbols are incorporated to indicate specific geometry wherever necessary. Notes are provided to give specification of a particular feature or to give specific information necessary during the manufacturing of the object. The section describes use of symbols and notes for dimensioning under specific situation that are recurrent in engineering drawing.

1. Circle Circle is drawn to represent the circular feature such as cylinders, holes or a series of holes in a component. The circles are generally specified by its diameter. The dimension value of diameter should be preceded by a symbol '∅'. When leader is used to specify diameter it should be a radial line and when dimension line is used to specify diameter it should be placed either horizontal or vertical. Alternative methods of dimensioning diameters of the circles are shown in Fig. 2.18. The size of the circle and the space available on the drawing

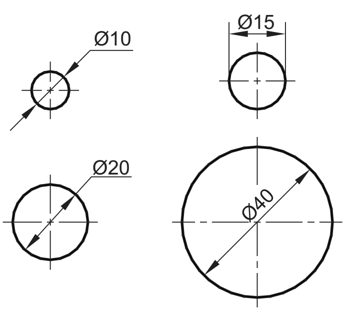


Fig. 2.18 *Methods of diameter dimensioning*

generally dictates the method to be chosen. Figure 2.19 shows an example of dimensioning a cylinder with a hole. The dimension for diameter should be placed on the most appropriate view to ensure clarity.

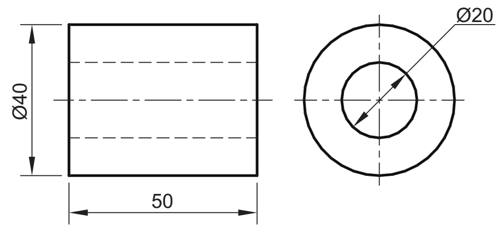


Fig. 2.19 *Dimensioning cylinder with a hole*

2. Radius Radius is used to represent a curved surface that can be defined by arcs such as fillets and rounds†. The dimension value of radius should be preceded by a letter '*R*'. Leader which is basically a radial line with one arrowhead should be used to specify radius. The arrowhead must touch the arc contour either on the inside or outside. Alternative methods of radius dimensioning are shown in Figs. 2.20(a) and (b). As far as possible, the centre of the arc should be denoted either by a dot or small cross, and the leader should pass through the centre.

In case there is not enough space available, the radius does not need to have their centre located. In case the size of the radius can be derived from other dimensioning, the radius should be indicated with a leader and symbol R without an indication of the value.

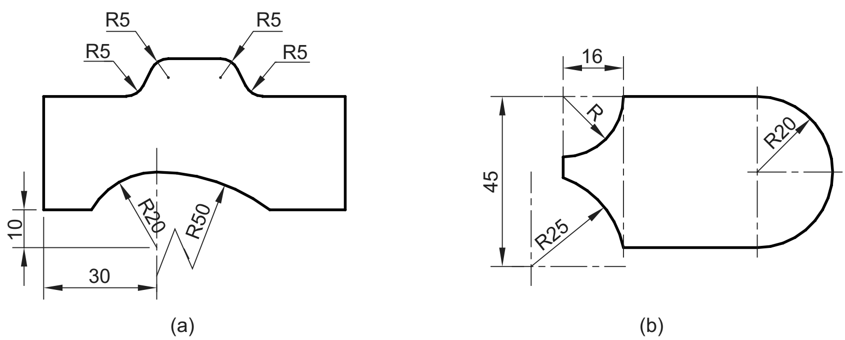


Fig. 2.20 **(a)** *and* **(b)** *Methods of radius dimensioning*

3. Angle Angular dimensions are given when the outline of a surface is at an angle to the horizontal, vertical or any other radial outlines. For dimensioning an angle, a curved dimension line is drawn which is basically a circular arc having its centre at the vertex. Radius of the arc depends upon the space required for the dimension vales. Lesser the angle more is the radius required for the dimensioning arc. The dimension value is placed over the dimension line and expressed in degree, minutes and seconds such as 67° or 65°32′2″. Alternative methods of angle dimensioning are shown in Fig. 2.21. The angle dimensioning can also be used to dimension an arc as shown in Fig. 2.22

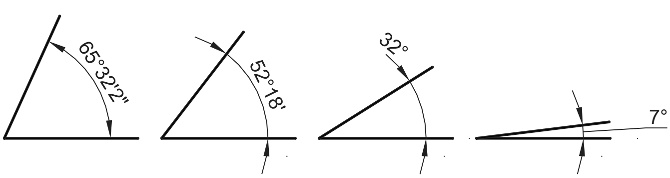


Fig. 2.21 *Methods of angle dimensioning*

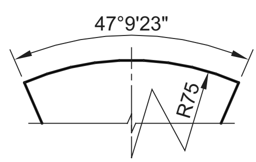


Fig. 2.22 *Dimensioning curve using angle dimensioning*

†rounding off of an interior corner is called *fillet* whereas rounding of an exterior corner is called *round*

4. Chord and arc length Chord and arc length specified to give information of the rounded features. The method is relatively less common as comparison to radii or angles. For dimensioning the chord length, a straight dimension line is stretched out from the ends points of the curved outline using projection lines, as shown in Fig 2.23(a). The dimension value is placed over the dimension line.

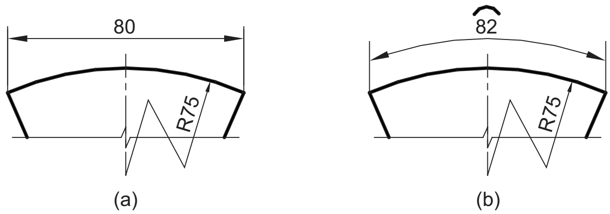
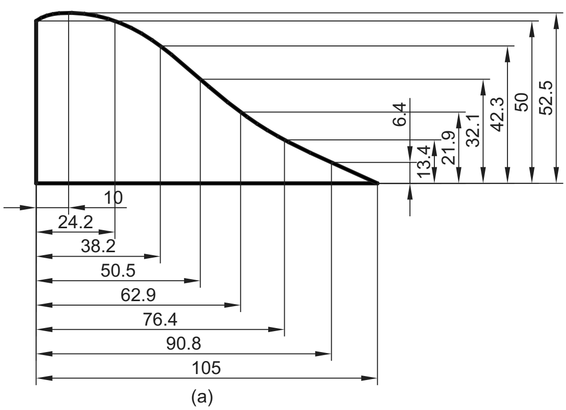
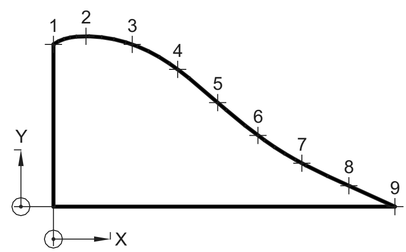


Fig. 2.23 *Dimensioning curve using* **(a)** *chord length* **(b)** *arc length*

The arc length is specified when it is required to measure along the actual part surface. For dimensioning the arc length, a curved dimension line which has radius that is offset from the curved outline is stretched out using straight projection lines, as shown in Fig. 2.23(b). The dimension value of arc length is placed over the dimension line and a curved symbol is placed above it.

5. Curved surface Sometimes contour of a curved surface cannot be defined by arcs. Such curved surfaces are dimensioned by locating points along the contour using parallel dimensioning as shown in Fig. 2.24(a). The points along the contour can also be locating using coordinate dimensioning as shown in Fig. 2.24(b).





Point	1	2	3	4	5	6	7	8	9
X	0	10.0	24.2	38.2	50.5	62.9	76.4	90.8	105
Y	50	52.5	50	42.3	32.1	21.9	13.4	6.4	0

(b)

Fig. 2.24 *Curved surface dimensioning using* **(a)** *parallel dimensioning* **(b)** *coordinate dimensioning*

6. Sphere Figure 2.25 shows method used for dimensioning a spherical part. The dimension value of spherical diameter should be preceded by a symbol 'S∅'. When leader is used to specify spherical diameter

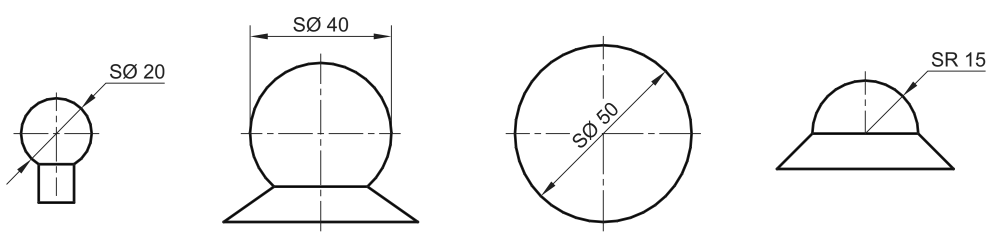


Fig. 2.25 *Dimensioning spheres*

it should be a radial line and when dimension line is used to specify spherical diameter it should be placed either horizontal or vertical

The dimension value of spherical radius should be preceded by a letter 'SR'. Leader which is basically a radial line with one arrowhead should be used to specify spherical radius. The arrowhead must touch the arc contour either on the inside or outside.

7. Square and hexagonal Cross section Usually, square is machined at the end of the shaft so that it can be turned by means of spanner. Figure 2.26(a) shows method used for dimensioning an object that has square cross section. The dimension value of square side should be preceded by a symbol '□'. Two continuous narrow diagonal lines are added to indicate the visible flat surface.

Figure 2.26(b) shows method used for dimensioning an object that has hexagonal cross section. The dimension value of hexagonal side should be preceded by a word 'HEX'. In the figure, although there appears that the distance between flat faces are given a dimensional value but in actual the dimension value represents the length of the side of the hexagon. The continuous narrow diagonal lines are added to indicate two visible flat surfaces.

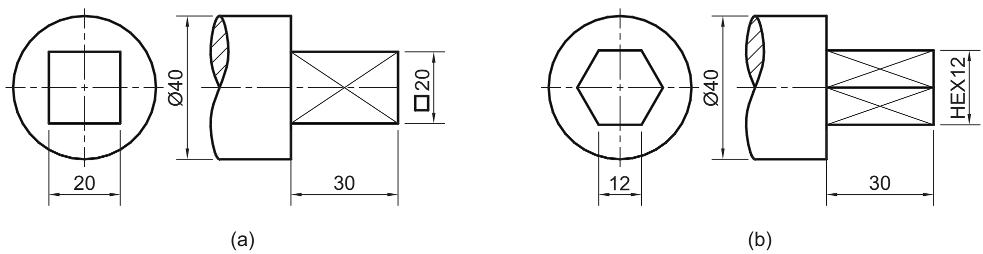


Fig. 2.26 *Dimensioning* **(a)** *Square cross section* **(b)** *Hexagonal cross section*

8. Chamfers A chamfer is obtained by cutting off the inside or outside edges of a cylindrical part. The chamfer of 45° to the surface is commonly applied and can be dimensioned with a note as shown in Figs. 2.27(a) and (b). The chamfer of angle other than 45° should be dimensioned by mentioning the angle and a side length as shown in Figs. 2.27(c) and (d). Figure 2.27(e) shows method used for dimensioning internal chamfer at 45° and 30°.

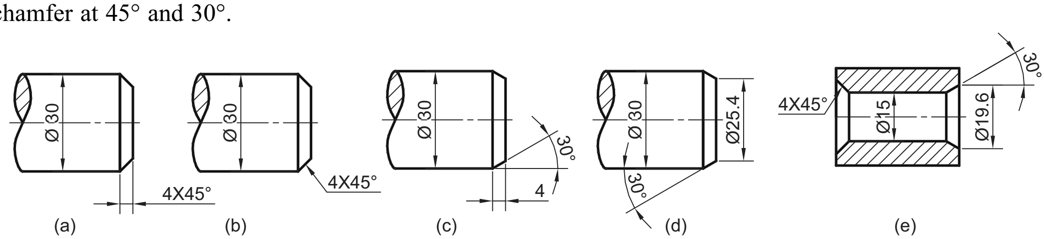


Fig. 2.27 *Dimensioning chamfers* **(a)** *and* **(b)** *External at 45°* **(c)** *and* **(d)** *External angle other than 45°* **(e)** *Internal*

9. Slope or flat taper Slope or flat taper is the inclination of a line representing the inclined surface of the wedge. The rate of slope is defined as the difference in the height of the two sections of the wedge to their baseline distance. If H and h are the heights at the two sections and L is the distance between them, then the rate of slope is given by

$$S = \frac{H - h}{L} = \tan(\alpha),$$

where α is the slope angle. The slopes are used in the locking devices such as taper keys and adjusting shims. Figure 2.28(a) and (b) shows conventional methods used for dimensioning an object that has slope. The slope can also be indicated using graphical symbol preceding the value for rate of slope, as shown in Fig. 2.28(c). The orientation of the graphical symbol should coincide with that of the wedge.

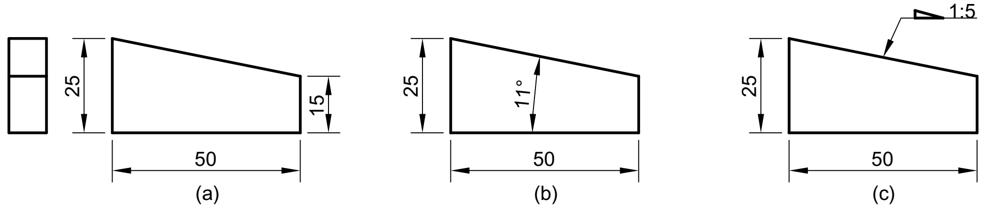


Fig. 2.28 **(a)** *to* **(c)** *Dimensioning tapered surfaces*

10. Conical taper The conical taper is used on the shanks of small tools such as drills, reamers, counterbores, etc. to hold them accurately in the machine spindle. The *rate of taper* is defined as the difference in the diameters of the two sections of a cone to the distance between them. If D and d are the diameters at the two sections and L is the distance between them, see Fig. 2.29(b), then the rate of taper is given by

$$C = \frac{D - d}{L} = 2\tan\left(\frac{\alpha}{2}\right),$$

where α is the cone angle. The taper can be dimensioned by any one of the following methods:

(a) Using standard taper, like Morse taper as shown in Fig. 2.29(a) or
(b) Using conventional dimensioning system, as shown in Fig. 2.29(b) and (c) or
(c) Using graphical symbol preceding the value for rate of taper, as shown in Fig. 2.29(d). The orientation of the graphical symbol should coincide with that of the cone.

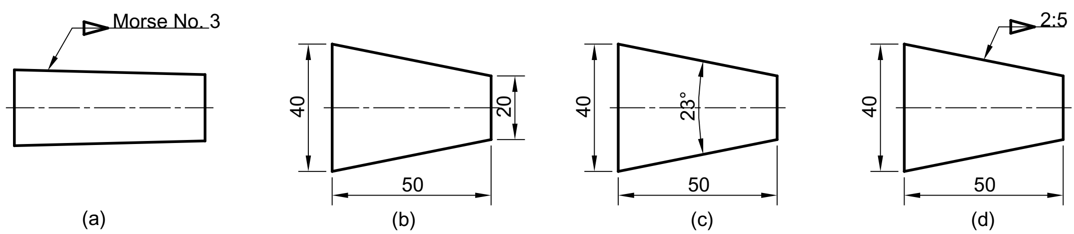


Fig. 2.29 *Rate of conical taper is given by* **(a)** *standards* **(b)** *linear dimensions* **(c)** *angular dimension* **(d)** *symbol*

11. Countersink A countersink is a conical hole cut into a manufactured object. The hole is used to accommodate the head of a countersunk screw or bolt, so that the head lies below the surface of the object as shown in Fig. 2.30(a). The countersink holes have usually an included angle of 90°. Figure 2.30(b) and (c) show conventional methods to dimension the countersink in the front view. Figure 2.30(d) show methods to dimension the countersink in the top view.

12. Counterbore A counterbore is a cylindrical flat-bottomed hole, which enlarges another hole. The hole is used to accommodate the head of a fastener (i.e., hexagonal head or socket head capscrew), so that the head lies below the surface of the object as shown in Fig. 2.31(a). Thus, a counterbore has a perpendicular surface for fitting a fastener head. Figure 2.31(b) and (c) show conventional methods to dimension the counterbore in the front view. Figure 2.31(d) show methods to dimension the counterbore in the top view.

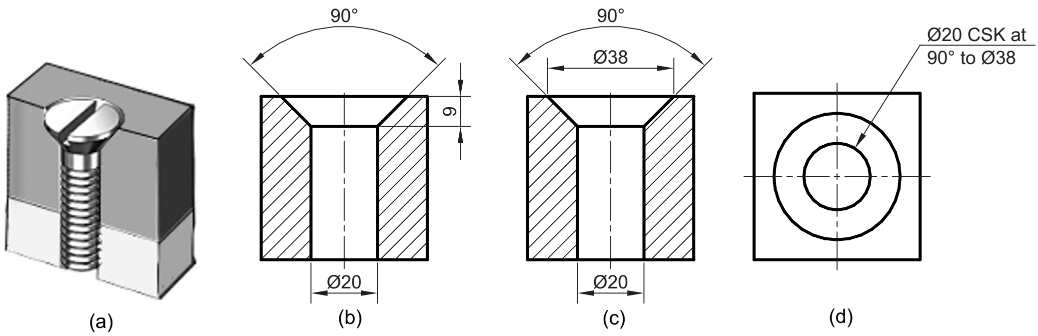


Fig. 2.30 *Countersink* **(a)** *With countersunk screw* **(b)** *Dimensioning by giving depth and included angle* **(c)** *Dimensioning by giving diameter and included angle* **(d)** *Dimensioning in the top view*

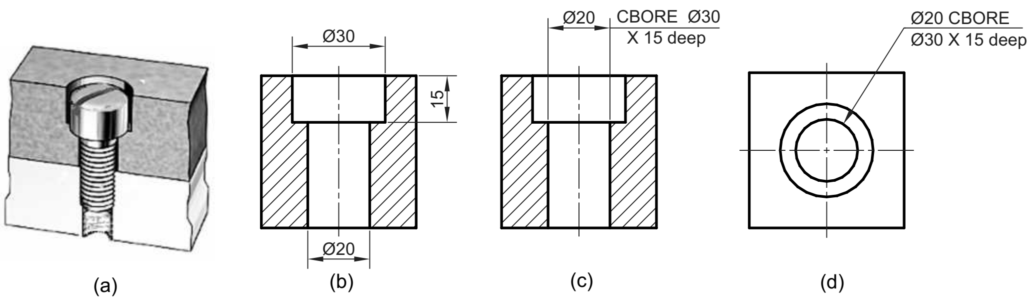


Fig. 2.31 *Counterbore* **(a)** *With pan screw* **(b)** *Dimensioning by giving diameter and depth* **(c)** *Dimensioning by using abbreviation* **(d)** *Dimensioning in the top view using abbreviation*

13. Spotface A spotface is an area where the surface is machined just enough to provide a level seating surface for a bolt heat, nut or washer. It resembles to a counterbore, except that a spotface is generally 2 mm or less in depth. Figure 2.32(a) shows a conventional method to dimension the spotface in the front view. The depth of a spotface is normally not shown. Figure 2.32(b) shows the method to dimension the spotface in the top view.

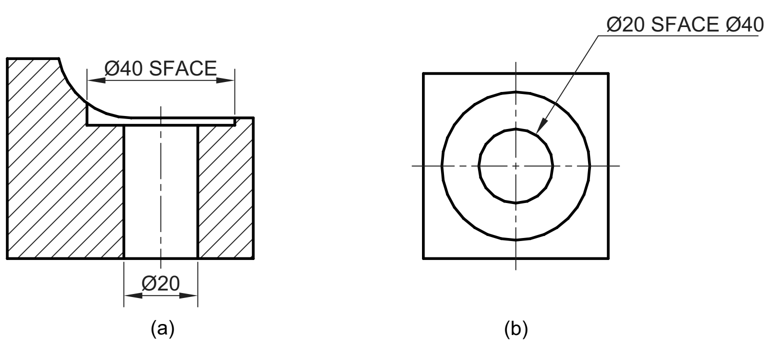


Fig. 2.32 *Spotface dimensioning in the* **(a)** *sectional front view* **(b)** *top view*

14. Equidistant and repeated features It is common to use same size of hole or other feature more than once in the design of a part. The features equidistant and repeated in linear space can be dimensioned by giving one typical dimension, noting the number of times a dimension is repeated and total length as reference (see Figs. 2.33(a) and (b)).

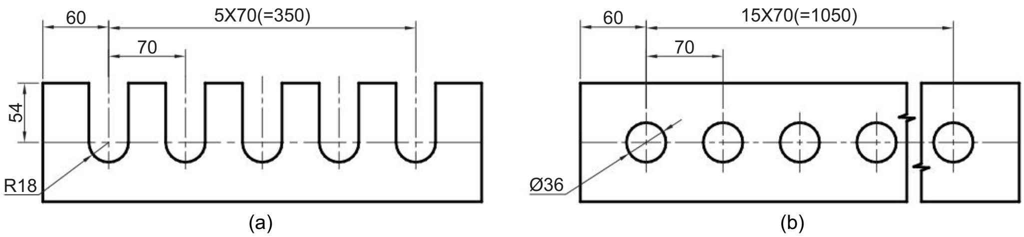


Fig. 2.33 *Dimensioning features repeated in linear space* **(a)** *When entire part drawn* **(b)** *With break lines*

The features equidistant and repeated in angular space can be dimensioned by giving one typical dimension and noting the number of times a dimension is repeated and total angle as reference (see Fig. 2.34(a)). The angles of the spacing may be omitted if their number is evident without any confusion as shown in Fig. 2.34(b).

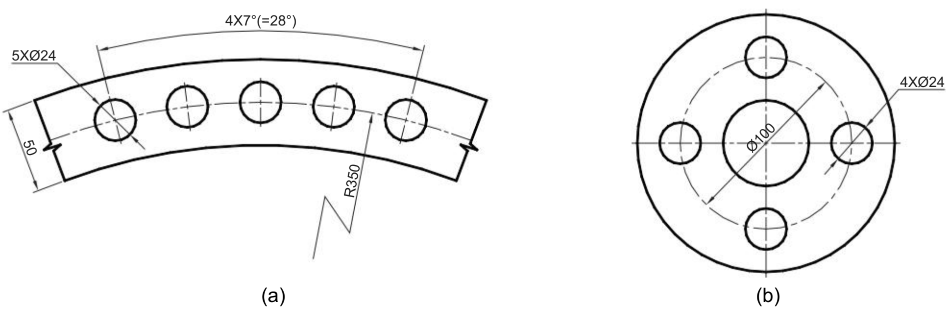


Fig. 2.34 *Dimensioning holes repeated in angular space for* **(a)** *specified angle* **(b)** *full peripheral*

15. Partial drawn views It is common to draw partial view of parts that are too long for the drawing sheet. In this case a symmetry symbol can be placed as shown in Fig. 2.35(a) and (b). The symmetry symbol is two short thick parallel lines places near the centreline ends outside the view and drawn at 90° to the centreline. The symmetry symbol should not be used if the entire part is drawn.

16. Dimension not to scale All drawings should be made to a scale. However, it is often simpler to revise a dimension rather than redraw the object with revised dimensions. In Fig. 2.36, a rectangle of 80 mm × 40

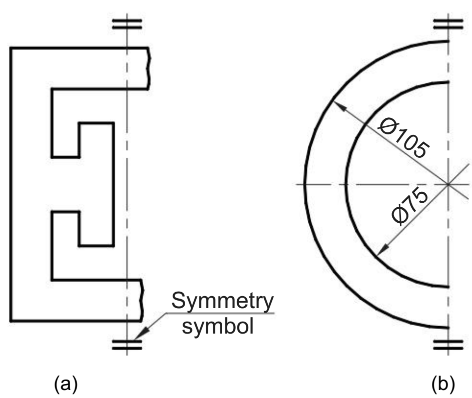


Fig. 2.35 **(a)** *and* **(b)** *Partial drawn view dimensioning*

mm has been revised by new values 100 mm × 40 mm without redrawing. When such a situation occurs, then the dimension which is not to scale should be underscored with a heavy straight line or marked NTS.

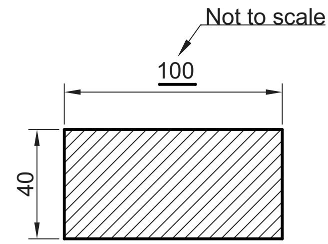


Fig. 2.36 *Not to scale dimensioning*

2.8 RULES OF DIMENSIONING

Following rules should be observed while dimensioning:

1. Dimensions should be clear and permit only one interpretation. Numerals and letters should be large enough (about 3 to 5 mm high for whole numbers and 6 mm high for fractions) to ensure easy reading.
2. In general, a circle is dimensioned by its diameter and an arc by its radius. The centre lines should not extend from view to view.
3. Dimensions should be quoted in millimetres to the minimum number of significant figures. For example, 12 and not 12.0.
4. The decimal point in a dimension should be bold and should be in line with the bottom line of the figure. In case the dimension is less than unity, a zero should precede the decimal point such as 0.35.
5. Functional dimensions† should be shown directly on the drawing wherever possible, while non-functional dimensions† should be placed in a way that is most convenient for production and inspection.
6. Projection lines should be drawn perpendicular to the feature being dimensioned. However, they may be drawn obliquely and parallel to each other as shown in Fig. 2.6, if necessary.
7. Dimension values should be placed preferably near the middle. If unavoidable due to lack of space, they may be placed above the extended portion of the dimension line beyond the arrowheads, preferably on right hand side (see Figs. 2.37(a) and (b)).

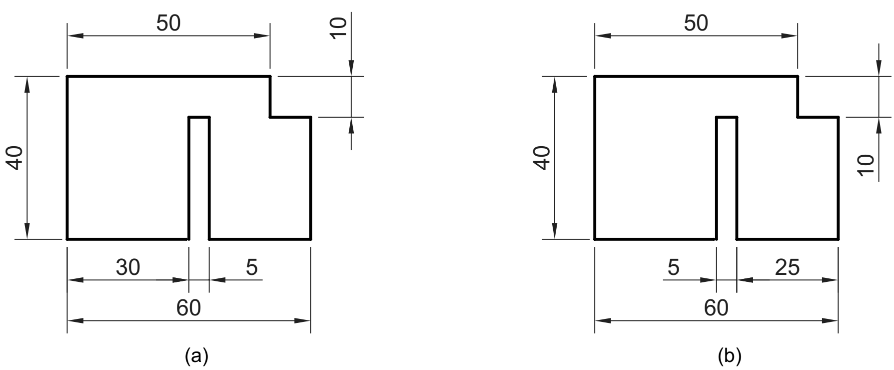
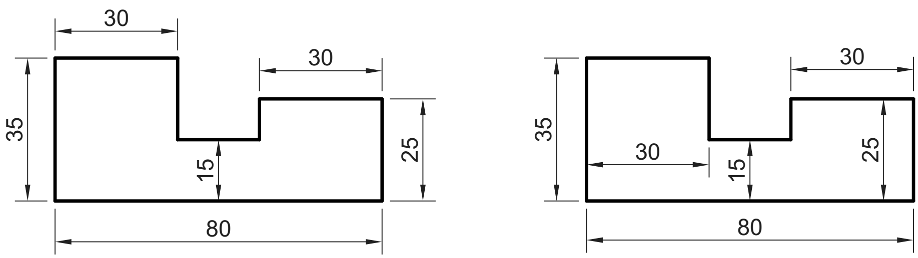


Fig. 2.37 *Placing dimension 5 and 10* **(a)** *Correct method* **(b)** *Towards left not permitted*

8. As far as possible, dimensions should be placed outside the views. In case it is not possible, they may be placed within the view, as shown in Fig. 2.38. However, dimensions should not be placed within a view unless drawing becomes clear by doing so.

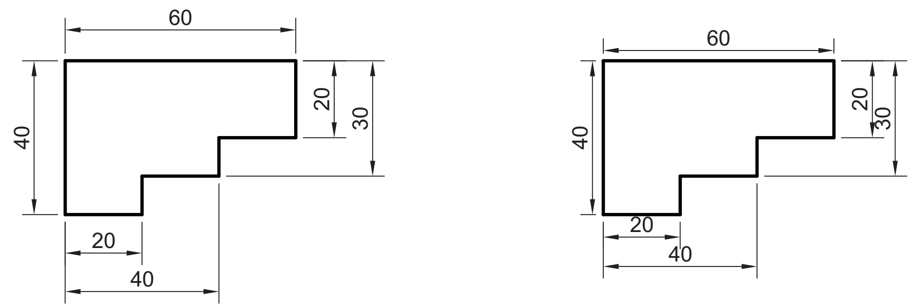
†A dimension that is essential to the function of the part is called *functional dimension* such as a screw thread size. A dimension that is not essential to the function of the part is called *non-functional dimension*, such as the depth of the tap hole for an internal screw thread.



(a) (b)

Fig. 2.38 *Placing dimension* **(a)** *Correct method* **(b)** *Inside a view not permitted*

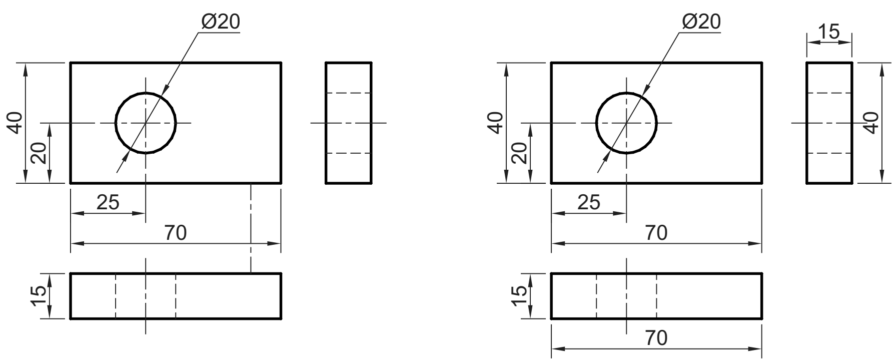
9. Line of the drawing should never be used as a dimension line or coincide with a dimension line. Dimension lines should be spaced uniformly throughout the drawing. They should be 8 mm to 10 mm from the object outline and 6 mm to 10 mm from each other (see Fig. 2.39).



(a) (b)

Fig. 2.39 *Placing dimension* **(a)** *Correct method* **(b)** *Too close not permitted*

10. The dimensions should be staggered when number of parallel dimensions is more.
11. Dimensions shall be placed on the view that most clearly shows the corresponding features.
12. Dimensions indicated in one view need not be repeated in another view, except for purpose of identification, clarity or both (see Fig. 2.40).



(a) (b)

Fig. 2.40 *Placing dimension* **(a)** *Correct method* **(b)** *Repetition not permitted*

13. Dimensions should be attached to the view where the contour shape is best shown.
14. Dimensions should be marked with reference to the visible outlines, rather than from the hidden lines. Dimensions should be marked from a base line or centre line of a hole or cylindrical parts or finished surfaces, etc, which may be readily established, based on design requirements and the relationship to other parts as shown in Fig. 2.41(a) and not as shown in Fig. 2.41(b).

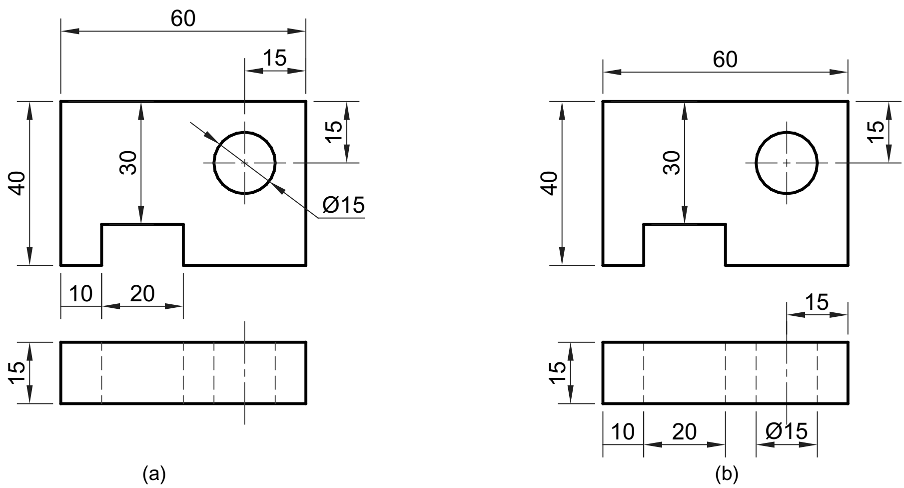


Fig. 2.41 *Placing dimension* **(a)** *Correct method* **(b)** *From hidden lines are not permitted*

15. Dimensioning to a centre line should be avoided, except when the centre line passes through the centre of a hole, or a cylindrical part (see Fig. 2.42).

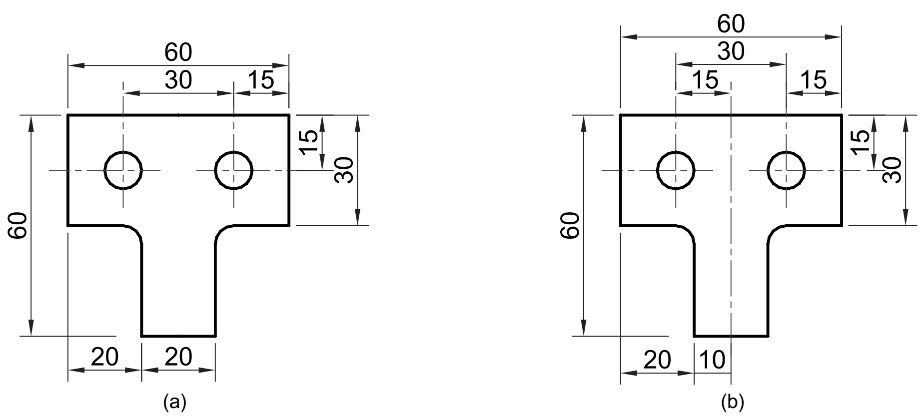


Fig. 2.42 *Placing dimension* **(a)** *Correct method* **(b)** *From centre line not permitted*

16. An axis or a contour line should never be used as a dimension line but may be used as a projection lines (see Fig. 2.43).

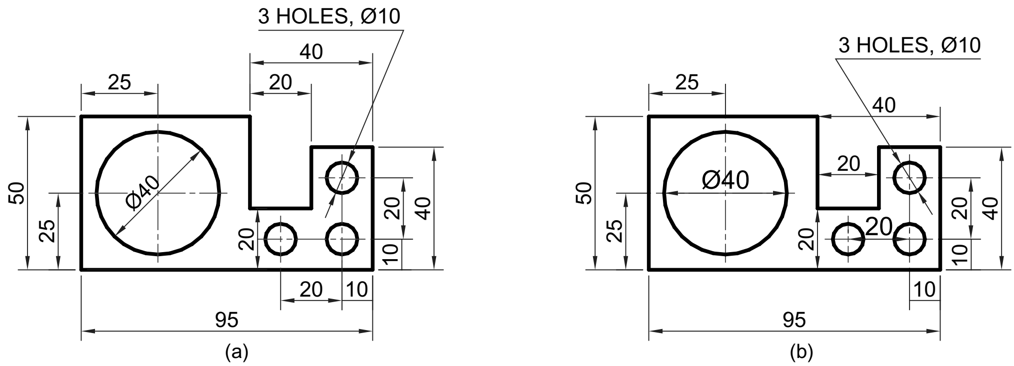


Fig. 2.43 *Placing dimension* **(a)** *Correct method* **(b)** *With contour lines not permitted*

17. As far as possible, the intersection of dimension lines should be avoided. However, if the intersection of two dimension lines is unavoidable, the lines should not be broken. It may be noted that in case of unidirectional dimensioning, the dimension lines are broken for inserting the dimension value.
18. When several dimensions are placed on the same side of the drawing, position the shortest dimension nearest to the component. This will avoid intersection of dimension lines with projection lines. However, if their intersection is unavoidable, neither line should be shown with break (see Fig. 2.44).
19. Overall dimensions should be placed outside the intermediate dimensions. If an overall dimension is shown, one of the intermediate dimensions is redundant and should not be dimensioned (see Fig. 2.45).

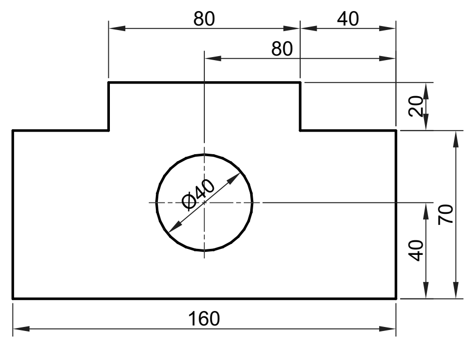


Fig. 2.44 *Intersection of projection and dimension lines is unavoidable*

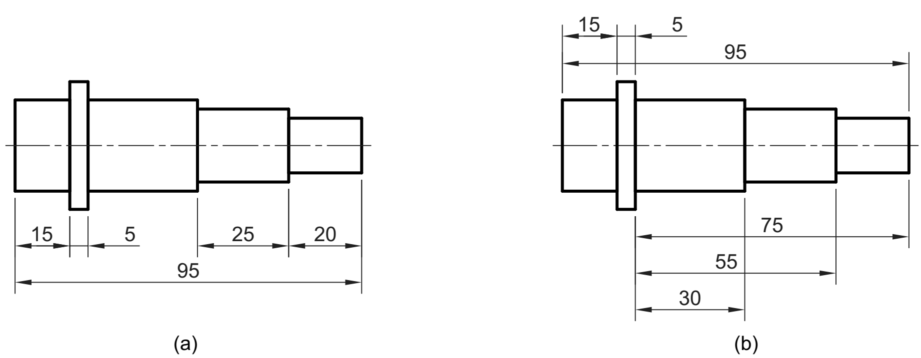


Fig. 2.45 *Dimension* **(a)** *Correct method* **(b)** *Overall length inside the intermediate one, not permitted*

20. If the space for arrowhead termination is sufficient, it should be shown within the limit of dimension lines. If the space is limited, the arrowhead termination may be shown outside the intended limits of the dimension lines that are extended for that purpose. However, where space is too small for an arrowhead, it may be replaced by oblique stroke or a dot (see Fig. 2.46).

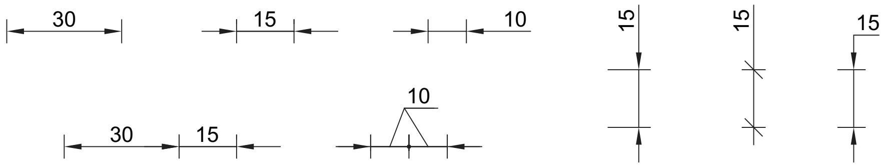


Fig. 2.46 *Methods for arrowhead termination*

21. As far as possible, all dimensions in one particular drawing should be expressed in one unit only. The recommended unit is being millimetre. There is no necessity to add the symbol for the unit, for example, a dimension 35 means 35 mm, even though the symbol for the unit 'mm' is omitted.
22. If in a particular drawing it is not possible to express dimensions in millimetres but a different unit such as metre, kilometre, etc., is used for all values, then also only the dimension value and at a prominent place a footnote is added such as, "ALL DIMENSIONS IN METRES" or "ALL DIMENSIONS IN KILOMETRES", etc.
23. No more dimensions should be given than are necessary to describe the finished product.
24. A dimensional line should be shown unbroken where the feature to which it refers is shown broken (see Fig. 2.47(a) and (b)).

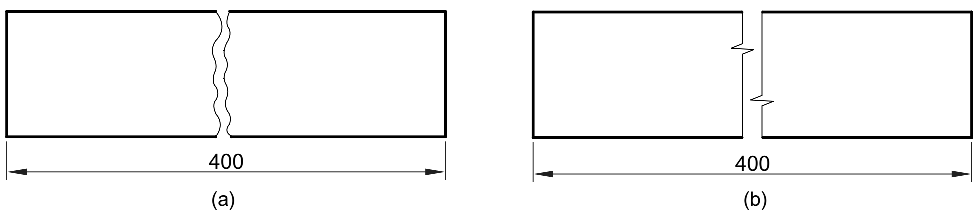


Fig. 2.47 **(a)** *and* **(b)** *Feature is broken but dimension line is unbroken*

25. When a dimension line cannot be completely drawn to its normal termination point, the free end should be terminated in a double arrowhead as shown in Fig. 2.48.

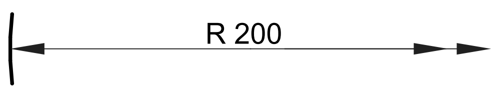


Fig. 2.48 *Free end is terminated with double arrowheads*

EXERCISE 2A

2.1 Taking the gridlines 10 mm apart, redraw Figs. E2.1 to E2.3 and dimension them.

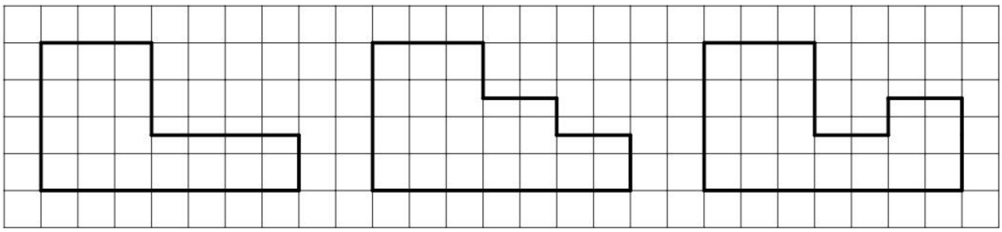


Fig. E2.1 **Fig. E2.2** **Fig. E2.3**

2.2 Taking the gridlines 10 mm apart, redraw Figs. E2.4 to E2.6 and dimension them.

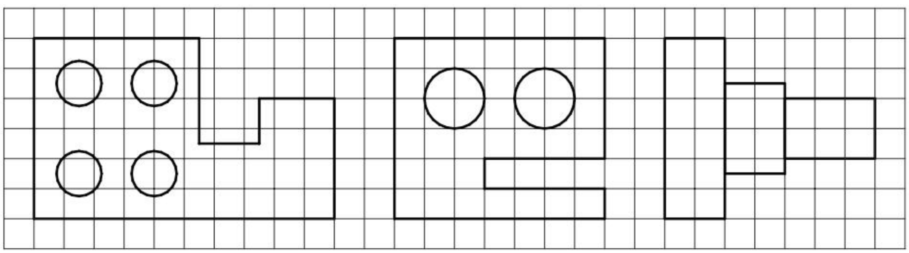


Fig. E2.4 **Fig. E2.5** **Fig. E2.6**

2.3 Taking the gridlines 10 mm apart, redraw Figs. E2.7 to E2.9 and dimension them.

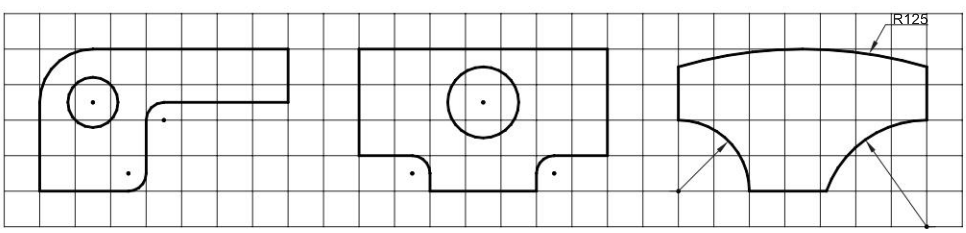


Fig. E2.7 **Fig. E2.8** **Fig. E2.9**

2.4 Taking the gridlines 10 mm apart, redraw Figs. E2.10 to E2.12 and dimension them.

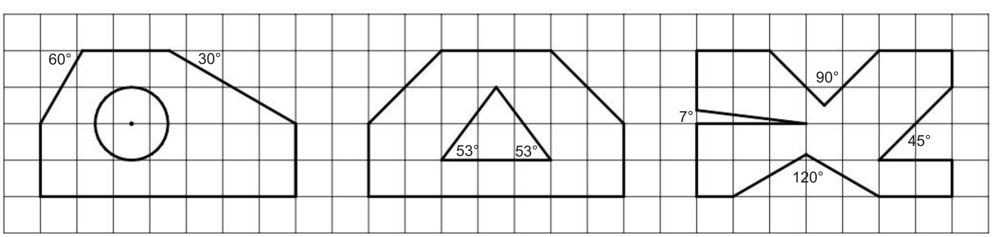


Fig. E2.10 **Fig. E2.11** **Fig. E2.12**

2.5 Taking the gridlines 10 mm apart, redraw Figs. E2.13 to E2.15 and dimension them.

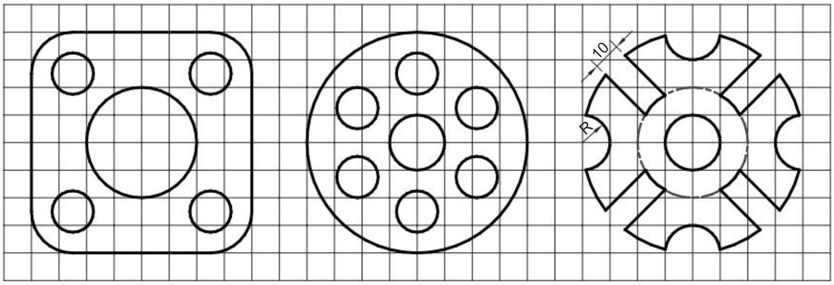


Fig. E2.13 **Fig. E2.14** **Fig. E2.15**

VIVA-VOCE QUESTIONS

2.1 Draw basic types of lines recommended by Bureau of Indian Standards.

2.2 Configure lines of length 100 mm to represent (a) continuous narrow lines (b) continuous wide line (c) dashed narrow line (d) long-dashed dotted narrow lines.

2.3 Configure lines of length 100 mm used for (a) visible outlines (b) hidden edges (c) lines of symmetry (d) projection lines.

2.4 What is lettering? Which standard is recommended by Bureau of Indian Standards for lettering Latin alphabet in technical drawings?

2.5 Write the alphabet *A* to *Z* and the numerals 0 to 9 taking 14 mm height using single stroke vertical capital letters.

2.6 Write the phrase "the quick brown fox jumps over the lazy dog" taking 10 mm height using single stroke vertical capital letters.

2.7 What is dimensioning? What are the different terms and notations used for dimensioning?

2.8 Differentiate between aligned and unidirectional systems of dimensioning.

2.9 Differentiate between chain dimensioning and parallel dimensioning.

2.10 Differentiate between superimposed running dimensioning and coordinate dimensioning.

MULTIPLE-CHOICE QUESTIONS

2.1 Hidden lines are drawn as
- (a) dashed narrow line
- (b) dashed wide line
- (c) long-dashed dotted narrow line
- (d) long-dashed dotted wide line

2.2 Centre lines are drawn as
- (a) continuous narrow lines
- (b) dashed narrow line
- (c) long-dashed dotted narrow line
- (d) long-dashed double dotted narrow line

2.3 Long-dashed dotted narrow line is used to represent
- (a) line of symmetry
- (b) centre lines
- (c) pitch circle of gears and holes
- (d) All of these

2.4 The length-to-height ratio of a closed filled arrow head is
(a) 1:3 (b) 3:1 (c) 1:2 (d) 2:1

2.5 The inclined letters should have inclination of
(a) 75° towards right (b) 75° towards left
(c) 60° towards right (c) 60° towards left

2.6 The line connecting a view to a note is called a
(a) dimension line (b) projection line
(c) leader (d) arrowheads

2.7 Which of the following is not a specified method for arranging dimensions?
(a) parallel dimensioning
(b) perpendicular dimensioning
(c) continuous dimensioning
(d) dimensioning by coordinates

2.8 When dimensions are specified from a common origin and paced parallel to one another, it is called
(a) chain dimensioning
(b) parallel dimensioning
(c) superimposed running dimensioning
(d) coordinate dimensioning

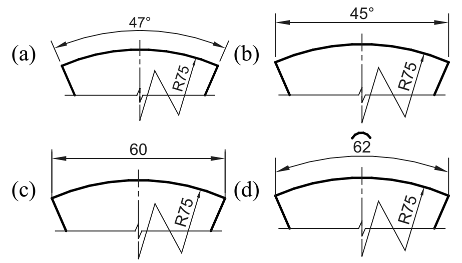
2.9 The two recommended systems of placing the dimensions are
(a) unidirectional and aligned systems
(b) vertical and inclined systems
(c) unidirectional and inclined systems
(d) vertical and aligned systems

2.10 The dimension value for diameter of a circle should be
(a) preceded by the symbol '∅'
(b) succeeded by the symbol '∅'
(c) preceded by the letter '*D*'
(d) succeeded by the letter '*D*'

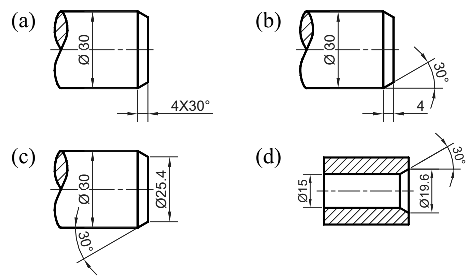
2.11 The dimension value for radius of an arc should be preceded by
(a) *R* (b) *CR* (c) *SR* (d) *RAD*

2.12 Rounded interior corner is called a
(a) round (b) chamfer
(c) fillet (d) countersink

2.13 Which of the following is an **incorrect** method of dimensioning?



2.14 Which of the following is an **incorrect** method of dimensioning chamfer?



2.15 The recommended method of dimensioning a sphere with 80 mm diameter is
(a) 80∅S (b) ∅80S (c) S80∅ (d) S∅80

2.16 In Fig. M2.1, the continuous narrow diagonal lines indicate
(a) circular cross section
(b) square cross section
(c) hexagonal cross section
(d) flat surface

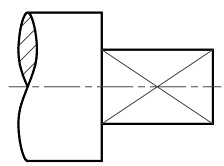


Fig. M2.1

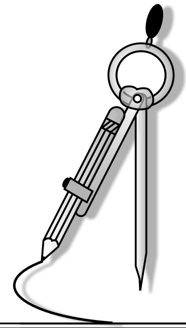
Answers to multiple-choice questions

2.1 (a), 2.2 (c), 2.3 (d), 2.4 (b), 2.5 (a), 2.6 (c), 2.7 (b), 2.8 (b), 2.9 (a), 2.10 (a), 2.11 (a), 2.12 (c), 2.13 (b), 2.14 (a), 2.15 (d), 2.16 (d)

Chapter

3

GEOMETRICAL CONSTRUCTIONS



3.1 INTRODUCTION

Engineers should be familiar with the principles of plane and solid geometry. A thorough knowledge of these principles is a prerequisite to solve engineering drawing problems. Plane figures such as circle, triangle, and different polygons frequently constitute an important part of various objects for preparing engineering drawings. This chapter presents some of the important methods of geometrical constructions based on the principles of plane geometry studied earlier.

3.2 BISECT A LINE AND AN ARC

An arc is defined as a segment of the circumference of a circle. Bisecting a line or an arc means to divide them into two equal halves. Geometrical method to bisect a line and an arc is illustrated in the following problem.

Problem 3.1 *Bisect (a) an 80 mm long line and (b) a circular arc AB.*

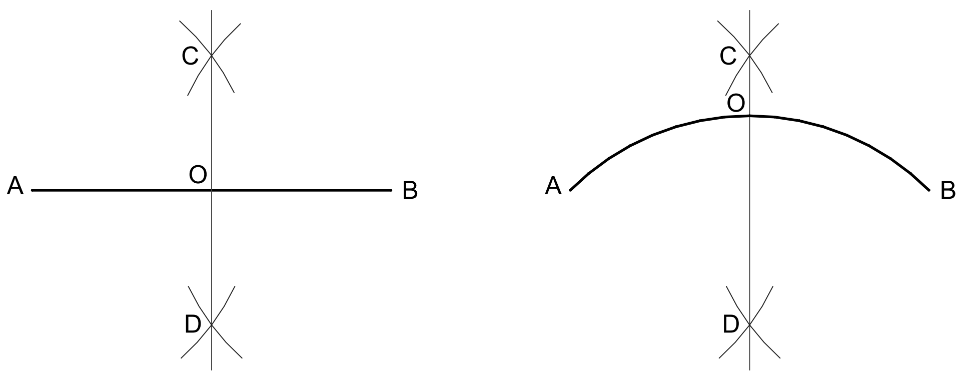


Fig. 3.1 *Bisector to* **(a)** *a line AB* **(b)** *a circular arc AB*

Construction Refer to Figs. 3.1(a) and (b).

1. Draw (a) an 80 mm long line *AB* or (b) a given arc *AB*.

2. Set the compass to a radius greater than half of the length *AB*. With centre *A*, draw arcs on both sides of *AB*.
3. With the same radius but centre *B*, draw arcs on both sides of *AB* to intersect previous drawn arcs at points *C* and *D*.
4. Join *C* to *D*. This is the perpendicular bisector which bisects the line/arc *AB*.

3.3 PERPENDICULAR TO A LINE

A perpendicular on a line subtends an angle of 90° with it. It may be required to draw perpendicular from a point lying either on the line or from a point outside it. It is recommended to draw perpendiculars with the help of drafter, as far as possible.

3.3.1 Perpendicular from a Point on the Line

To draw a perpendicular from a point lying on the line, set horizontal blade of the drafter along the given line *AB* and place it such that vertical blade gets in contact with the given point P as shown in Fig. 3.2(a). Draw a line *PD* as guided by the vertical blade, which is perpendicular to the line *AB*. A set-square or a protractor can also serve the purpose with ease. Geometrical constructions to draw a perpendicular line with the help of the compass are illustrated in the following problem.

Problem 3.2 *Draw a perpendicular line to an 80 mm long straight line AB, at a point P lying on the line at a distance of 30 mm from the end A.*

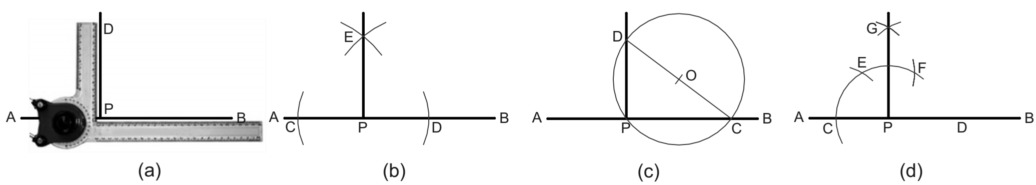


Fig. 3.2 *Perpendicular line from a point within it* **(a)** *Using drafter* **(b)** *to* **(d)** *Using compass*

Construction Draw an 80 mm long line *AB* and mark a point *P* on the line 30 mm away from *A*.

Method 1 Refer to Fig. 3.2(b).

1. Set the compass to any convenient radius and with centre *P* draw an arc to meet the line *AB* at points *C* and *D*.
2. Reset the compass to a radius greater than half of *CD*. With centres *C* and *D* respectively draw arcs to intersect at point *E*.
3. Join *P* to *E*. Line *PE* is perpendicular to *AB*.

Method 2 Refer to Fig. 3.2(c).

1. Mark any convenient point *O* outside the line. Draw a circle with centre *O* and radius *OP* to meet the line *AB* at point *C*.
2. Join *C* to *O* and produce it to meet the circle at point *D*.
3. Join *P* to *D*. Line *PD* is perpendicular to *AB*.

Method 3 Refer to Fig. 3.2(d).

1. Set the compass to any convenient radius and centre *P*, draw an arc *CF* to meet the line *AB* at point *C*.
2. With the same radius *PC*, strike two divisions *CE* and *EF* respectively on the arc *CD*.
3. Reset the compass to a radius greater than half of the length *EF*. With centres *E* and *F* respectively, draw arcs to intersect at point *G*.
4. Join *G* to *P*. Line *GP* is perpendicular to *AB*.

3.2.2 Perpendicular from a Point outside the Line

To draw a perpendicular from a point lying outside the line, set the horizontal blade of the drafter along the given line *AB* and place it such that vertical blade gets in contact with the given point *P* as shown in Fig. 3.3(a). Draw a line *PC* as guided by the vertical blade, which is perpendicular to *AB*. A set square can also be used for the purpose. Geometrical constructions to draw a perpendicular line with the help of the compass are illustrated in the following problem.

Problem 3.3 *Draw a perpendicular to an 80 mm long line AB, from a point P lying at a distance 50 mm from end A and 60 mm from end B.*

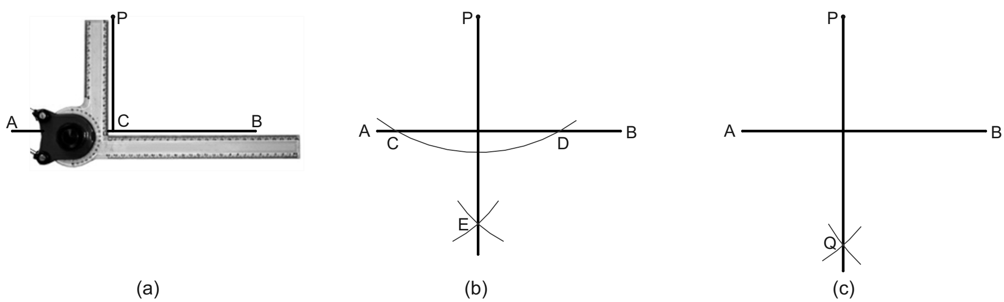


Fig. 3.3 *Perpendicular line from a point outside it* **(a)** *Using drafter* **(b)** *and* **(c)** *Using compass*

Construction

1. Draw an 80 mm long line *AB*.
2. With centre *A* and radius 50 mm draw an arc.
3. With centre *B* and radius 60 mm draw another arc to intersect previous drawn arc at point *P*.

Method 1 Refer to Fig. 3.3(b).

1. Set the compass to any convenient radius and with centre *P*, draw an arc *CD* to meet the line *AB* at points *C* and *D*.
2. Reset the compass to a radius greater than half of *CD* and centres *C* and *D* respectively, draw arcs to intersect at point *E*.
3. Join *P* to *E*. Line *PE* is perpendicular to *AB*.

Method 2 Refer to Fig. 3.3(c).

1. Draw an arc with centre *A* and radius *AP*.

2. Draw another arc with centre *B* and radius *BP* to intersect the previous drawn arc at point *Q*. It may be noted that the points *P* and *Q* lies on the opposite sides of the line *AB*.
3. Join *P* to *Q*. Line *PQ* is perpendicular to *AB*.

3.4 PARALLEL LINES

The lines lying in a plane which never intersect each other are called *parallel lines*. As far as possible, the parallel lines should be drawn with the help of drafter. To draw a line parallel to the given line *AB*, set the horizontal blade of the drafter along the given line *AB*. Now with the same horizontal blade draw lines *CD*, *EF*, etc., as shown in Fig. 3.4(a). The lines *CD* and *EF* are parallel to *AB*. A pair of set squares can also be used to draw parallel lines. Geometrical constructions to draw parallel lines with the help of a compass are illustrated in the following problem.

Problem 3.4 *Draw a line parallel to a given straight line AB through a point 50 mm away from it.*

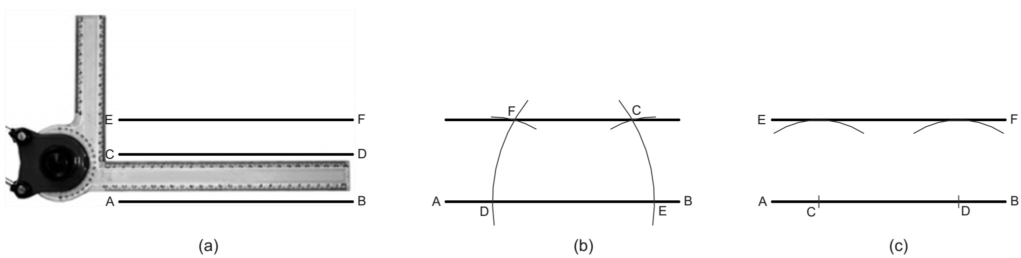


Fig. 3.4 *Parallel line* **(a)** *Using drafter* **(b)** *and* **(c)** *Using compass*

Construction Draw the given line *AB* and mark point *C* through which parallel line has to be drawn.

Method 1 Refer to Fig. 3.4(b).

1. Mark any point *D* on the line *AB*. With centre *D* and radius *DC*, draw arc *CE* to meet *AB* at point *E*.
2. With centre *E* and the same radius *DC*, draw an arc *DF*.
3. With centre *D* and radius of chord length *CE*, draw an arc to intersect arc *DF* at point *F*.

Join *C* to *F*. Line *CF* is parallel to the line *AB*.

Method 2 Refer to Fig. 3.4(c).

1. Mark any two points *C* and *D* on line *AB*. Draw two arcs each of radius 50 mm (distance of parallel line from *AB*) with centres *C* and *D*, respectively
2. Draw a line *EF* touching both of these arcs. Line *EF* is parallel to the line *AB*.

3.5 DIVIDE A LINE

3.5.1 Equal Parts

A line can be divided into any number of equal parts by trial and error with the help of a divider. Geometric method to divide a line into desired number of equal parts is illustrated in the following problem.

Problem 3.5 *Divide an 80 mm long straight line into seven equal parts.*

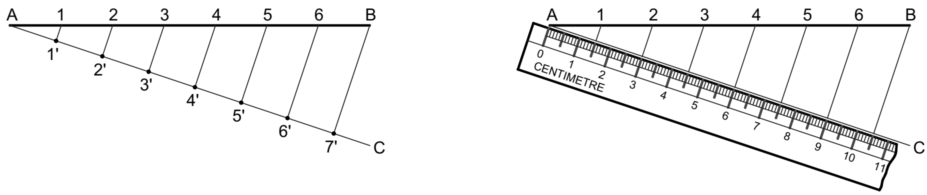


Fig. 3.5 *Divide a line into equal parts* **(a)** *Using divider and drafter* **(b)** *Using scale and drafter*

Construction Refer to Fig. 3.5(a).

1. Draw an 80 mm long straight line *AB*.
2. Draw another line *AC* at any convenient acute angle with *AB*.
3. Set the divider to a convenient length and mark off seven equal spaces on *AC*. Let these points be 1′, 2′, 3′, 4′, 5′, 6′, and 7′.
4. Join 7′ to *B*.
5. Set the drafter along 7′*B* and draw parallel lines through points 1′, 2′, 3′, 4′, 5′ and 6′ to meet *AB* at points 1, 2, 3, 4, 5 and 6 respectively. These points divide *AB* in seven equal parts.
   A scale can also be used to lay off equal intercepts on line *AC* as shown in Figure 3.5(b).

3.5.2 Given Proportion

Geometrical method of dividing a line into given proportion is illustrated in the following problem.

Problem 3.6 *Divide an 80 mm long straight line in proportion of 2:3:4.*

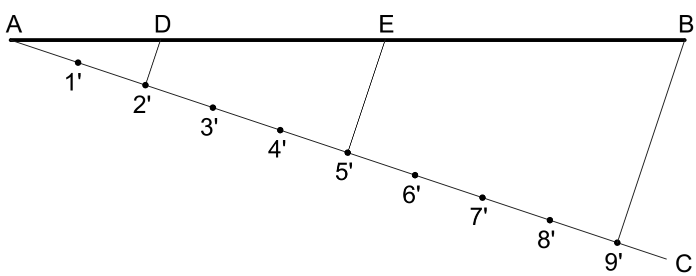


Fig. 3.6 *Division of line AB in proportion of 2:3:4*

Construction Refer to Fig. 3.6.

1. Draw an 80 mm long straight line *AB*.
2. Draw a line *AC* at any convenient acute angle with *AB*.
3. Set the divider to a convenient length and mark off (2 + 3 + 4 = 9) nine equal spaces on *AC*. Let the points be 1′, 2′, 3′, 4′, 5′, 6′, 7′, 8′, and 9′.

4. Join 9′ to *B*.
5. Draw lines through points 2′ and 5′ parallel to 9′*B* to meet *AB* at points *D* and *E*. These points divide *AB* in proportion of 2:3:4.

3.6 ANGLE BISECTOR

An angle bisector divides the given angle in two equal halves. If the measurement of the angle is known, the bisector can be drawn by mathematically dividing the angle by two and laying off the result with the help of a protractor. Geometrical method for bisecting an angle with the help of a compass is illustrated in the following problem.

Problem 3.7 *Draw an angle of 75° and bisect it with the help of a compass.*

Construction Refer to Fig. 3.7.

1. Draw an angle *AOB* which is equal to 75°.
2. Set compass to any convenient radius and with centre *O*, draw an arc *CD* to meet lines *OA* and *OB* at points *C* and *D* respectively.
3. Reset the compass to any radius greater than half of chord length *CD*. With centres *C* and *D* respectively, draw arcs to intersect each other at point *P*.
4. Join *O* to *P*. Line *OP* is the bisector of the angle *AOB*.

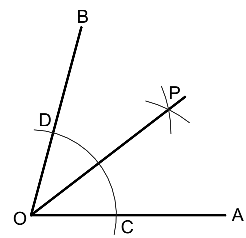


Fig. 3.7 *Bisecting the angle AOB*

3.7 CENTRE OF AN ARC OR CIRCLE

Problem 3.8 *Locate the centre of the arc shown in Fig. 3.8.*

Construction Refer to Fig. 3.8.

1. Draw two chords *CD* and *EF* of any convenient lengths in the arc.
2. Draw *PQ* as the bisector of chord *CD* (Problem 3.1). Similarly, draw *RS* as the bisector of chord *EF*.
3. Let the lines *PQ* and *RS*, produce if required, to intersect each other at point *O*. The point *O* is the required centre of the arc.

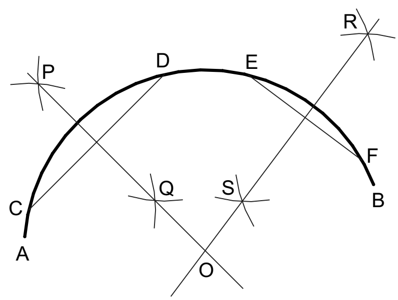


Fig. 3.8 *Locate centre O for the arc AB*

This method can also be used to determine the centre of a circle.

3.8 CIRCLE THROUGH THREE POINTS

Problem 3.9 *Draw a circle passing through three points A, B and C not lying in a straight line.*

Construction Refer to Fig. 3.9.

1. Mark points A, B and C. Join AB and BC.
2. Draw PQ as the perpendicular bisector of line AB (Problem 3.1). Similarly, draw RS as the perpendicular bisector of line BC.
3. Let the lines PQ and RS, produced if required, to intersect each other at point O. This point O is the required centre of the circle.
4. Draw a circle with centre O and radius OA (= OB = OC). This circle passes through points A, B and C.

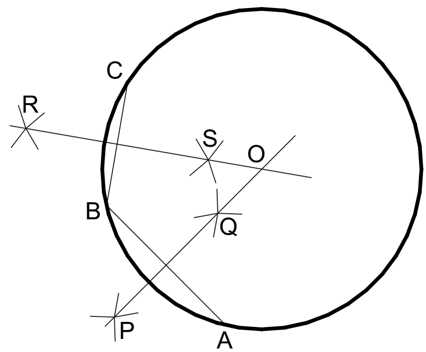


Fig. 3.9 *Circle passes through three points A, B and C*

3.9 DIVIDE A CIRCLE

A protractor may be used to divide a circle into any number of parts. In case it is required to divide a circle into 12 equal parts, then the mini-drafter and compass serves the purpose with ease. The following problem illustrates the use of the mini-drafter and the compass to divide a circle in 12 equal parts.

Problem 3.10 *Divide a 50 mm diameter circle into 12 equal segments.*

Construction Refer to Fig. 3.10.

1. Draw a circle with centre O and 50 mm diameter.
2. Using drafter, draw diameters AG and DJ, perpendicular of each other.
3. Draw arcs of radius equal to the radius of the circle (= 25 mm) and centre A to meet the circumference of the circles at points C and K.
4. Similarly, draw arcs of the same radius (= 25 mm) and centres D, G and J respectively, to meet the circumference of the circle at points B, F, E, I, H and L. The points divide the circumference of the circle into 12 equal segments.

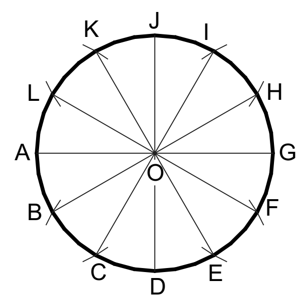


Fig. 3.10 *Divide a circle into 12 segments*

3.10 TANGENT TO A CIRCLE

A tangent to a circle is a line that intersects the circle at exactly one point. This intersection point is called the point of tangency.

3.10.1 Tangent from a Point on the Circle

It is well known that if a line is perpendicular to the radius of a circle at its outer endpoint, then that line is tangent to the circle.

Problem 3.11 *Draw a tangent to a circle of 50 mm diameter through point P. The point P lies on the circumference of the circle.*

Construction Refer to Fig. 3.11.

1. Draw a circle with centre *O* and 50 mm diameter. Mark a point *P* on the circumference of the circle as the point of tangency.
2. Join *OP*.
3. Through point *P*, draw a line *TPT′* perpendicular to *OP*. This is the required tangent.

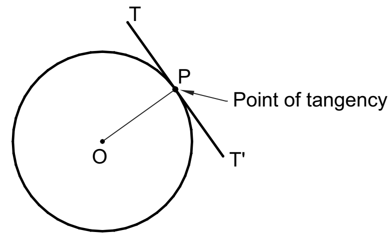


Fig. 3.11 *Tangent TT¢ from point P*

3.10.2 Tangent from a Point outside the Circle

Problem 3.12 *Draw a tangent to a circle of 50 mm diameter through point P. The point P is at a distance of 90 mm from the centre of the circle.*

Construction Refer to Fig. 3.12.

1. Draw a circle with centre *O* and 50 mm diameter. Mark a point *P* at a distance 90 mm from the centre *O*.
2. Join *OP*. Mark *A* as the midpoint of the line *OP* (Problem 3.1).
3. Draw an arc with centre *A* and radius *OA*, to meet the circle at points *T* and *T′*.
4. Join *PT* and *PT′* and produce. The lines *PT* and *PT′* represents two possible tangents.

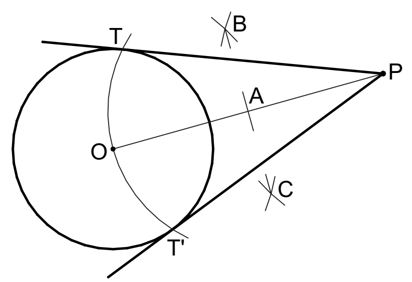


Fig. 3.12 *Tangent PT and PT′ from point P*

3.10.3 Tangent to an Arc from a Point on it

Problem 3.13 *Draw a tangent to the arc passing through point P. The point P lies on the circumference of the arc.*

Construction Draw the given arc *AB* and mark point *P* on it.

Method 1 Refer to Fig. 3.13(a).

1. Draw two chords *CD* and *EF*. Bisect the chords and locate the centre *O* for the arc (Problem 3.8).
2. Join *OP*. Draw a line *TPT′* perpendicular to *OP*. This is the required tangent.

Method 2 Refer to Fig. 3.13(b).

1. Draw an arc with centre *P* and any convenient radius, to meet the arc *AB* at points *C* and *D*.
2. Draw line *EF* as the bisector to the arc *CD* (Problem 3.1).
3. Through point *P*, draw a line *TPT′* perpendicular to *EF*. This is the required tangent.

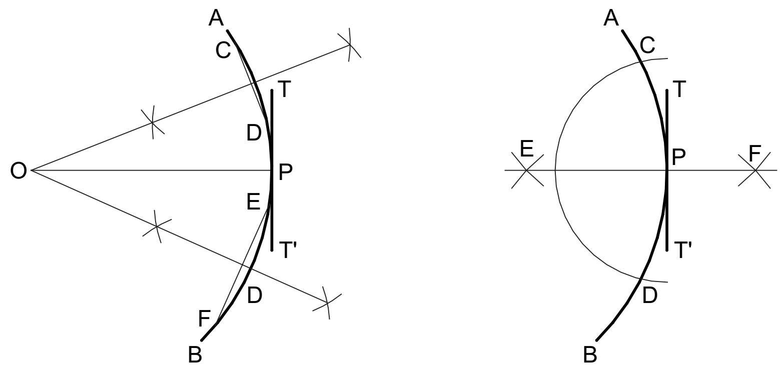


Fig. 3.13 *Tangent TT′ to the arc*

3.11 TANGENT TO TWO CIRCLES

When the centres of two circles are at a distance greater than the sum of their radius then there will be four tangents common to the circles, two exterior and two interior. If the circles touch each other at a point such that the distance between the centres is equal to the sum of their radius, then there will be three tangents common to the circle, two exterior and one through the point of contact. If the circles intersect each other then there will be two exterior tangents only. If the circles touch at a point such that one of them lie within the other then there will be only tangents at the point of contact. The construction of exterior and interior tangents common to two circles is illustrated in the following problem.

Problem 3.14 *Draw exterior and interior tangents connecting two circles of radii 25 mm and 40 mm having their centres 100 mm apart.*

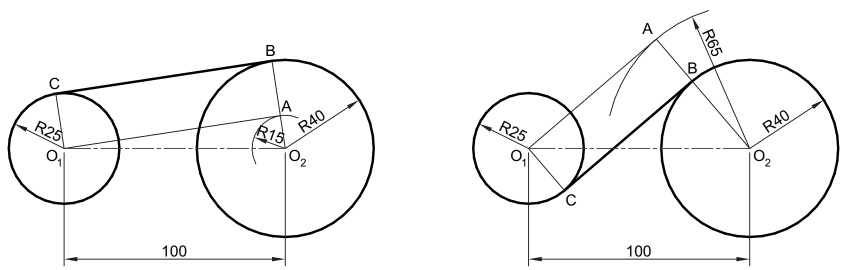


Fig. 3.14 *Tangent BC common to two circles* **(a)** *Exterior* **(b)** *Interior*

Construction Refer to Fig. 3.14.

1. Draw a 100 mm long line O_1O_2.
2. Draw a circle with centre O_1 and radius R_1 (= 25 mm).
3. Draw another circle with centre O_2 and radius R_2 (= 40 mm).

3.11.1 Exterior Tangent

1. Draw an arc with centre O_2 and radius $R_2 - R_1$ (= 15 mm).
2. Through point O_1, draw a tangent to this arc to touch at point A (Problem 3.12).
3. Join O_2A and produce it to meet the circle at point B.
4. Through B, draw a line BC parallel to O_1A, to touch the smaller circle at point C. Line BC is the required tangent exterior to the circles.

3.11.2 Interior Tangent

1. Draw an arc with centre O_2 and radius $R_2 + R_1$ (= 65 mm).
2. Through point O_1, draw a tangent to this arc to touch at point A (Problem 3.12).
3. Join O_2 to A which intersect the bigger circle at point B.
4. Through B, draw a line BC parallel to O_1A, to touch the smaller circle at point C. Line BC is the required tangent interior to the circles.

3.12 ARC TO CONNECT LINES AND CIRCLES TANGENTIALLY

An arc of given radius can be used to connect two non-parallel straight lines, a line and a circle or two circles, tangentially.

3.12.1 Arc to Connect Two Straight Lines

Problem 3.15 *Draw an arc of 20 mm radius connecting two straight lines inclined at 75° to each other.*

Construction Refer to Fig. 3.15.

1. Draw an angle CAB of 75°.
2. Draw a line PQ parallel to AB at a distance 20 mm.
3. Draw another line RS parallel to AC at a distance 20 mm to intersect line PQ at point O.
4. With centre O and radius 20 mm, draw an arc EF. The arc EF is tangential to both lines AB and AC.

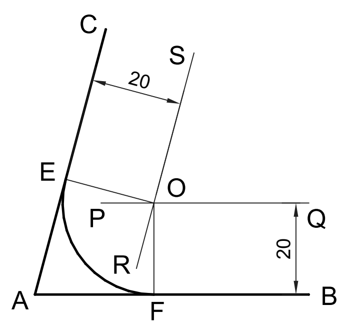


Fig. 3.15 *Arc EF to connect lines AB and AC tangentially*

3.12.2 Arc to Connect a Line and an Arc/Circle

Problem 3.16 *Draw an arc of 20 mm radius to connect the straight line AB and the arc CD of 60 mm radius. The centre of the arc CD is at a distance 30 mm from the line AB.*

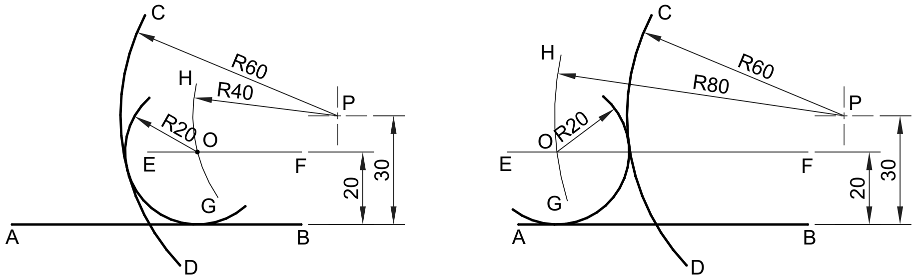


Fig. 3.16 *Arc with 20 mm radius is tangent to the line AB and arc CD, centres of both arcs lie on the* **(a)** *same side* **(b)** *opposite side*

Construction

1. Draw a line *AB*. Mark a point *P* at a distance 30 mm from the line *AB*.
2. Draw an arc *CD* with centre *P* and radius 60 mm.

Centre of Two Arcs Lie on the Same Side (Fig. 3.16(a))

1. Draw a line *EF* at a distance 20 mm from *AB* and parallel to it.
2. Draw an arc *GH* with centre *P* and radius 40 mm (60 – 20 = 40) to intersect the line *EF* at point *O*.
3. Draw an arc with centre *O* and radius 20 mm. This arc is tangential to both, the line *AB* and the arc *CD*.

Centre of Two Arcs Lie on Opposite Side (Fig. 3.16(b))

1. Draw a line *EF* at a distance 20 mm from *AB* and parallel to it.
2. Draw an arc *GH* with centre *P* and radius 80 mm (60 + 20 = 80) to intersect the line *EF* at point *O*.
3. Draw an arc with centre *O* and radius 20 mm. This arc is tangential to both, the line *AB* and the arc *CD*.

3.12.3 Arc to Connect Two Circles

Problem 3.17 *Two circles of 25 mm and 35 mm radii have their centres 100 mm apart. Draw an arc to connect both these circles tangentially, the radius of which is*

(a) 50 mm and is internal to both the circles,
(b) 100 mm and is external to both the circles and
(c) 100 mm and internal to the circle of 25 mm radius and external to the circle of 35 mm radius.

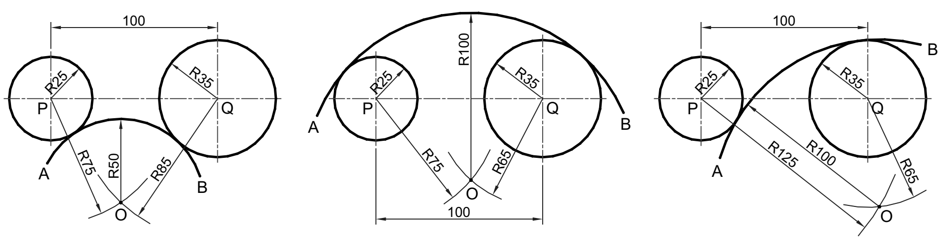


Fig. 3.17 *Arc AB connect two circles, the arc is* **(a)** *internal to both circles* **(b)** *external to both circles* **(c)** *internal to smaller circle and external to larger circle*

Construction

1. Mark points P and Q, 100 mm apart.
2. Draw a circle with centre P and radius 25 mm.
3. Draw another circle with centre Q and radius 35 mm.

(a) Internal arc Refer to Fig. 3.17(a).

1. Draw an arc with centre P and radius 75 mm (25 + 50 = 75).
2. Draw another arc with centre Q and radius 85 mm (35 + 50 = 85) to intersect the previous arc at point O.
3. Draw an arc AB with centre O and radius 50 mm. This arc is internal to both the circles and touches them tangentially.

(b) External arc Refer to Fig. 3.17(b).

1. Draw an arc with centre P and radius 75 mm (100 – 25 = 75).
2. Draw another arc with centre Q and radius 65 mm (100 – 35 = 65) to intersect the previous arc at point O.
3. Draw an arc AB with centre O and radius 100 mm. This arc is external to both the circles and touches them tangentially.

(c) Internal to smaller circle and external to the larger circle Refer to Fig. 3.17(c).

1. Draw an arc with centre P and radius 125 mm (100 + 25 = 125).
2. Draw another arc with centre Q and radius 65 mm (100 – 35 = 65) to intersect the previous arc at point O.
3. Draw an arc AB with centre O and radius 100 mm. This arc is internal to the circle with centre P and external to the circle with centre Q and touches them tangentially.

3.13 ARC TO CONNECT LINE AND POINT

Problem 3.18 *Draw an arc to connect a straight line AB tangentially at point Q and a point P lying at a distance 65 mm from the line AB.*

Construction Refer to Fig. 3.18.

1. Draw a line AB and mark point Q on it. Also, mark point P outside AB.

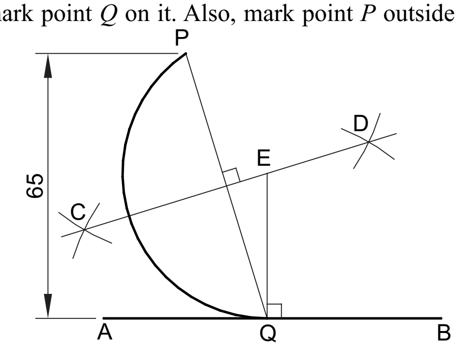


Fig. 3.18 *Arc to connect line AB tangentially at point Q and a point P*

2. Join PQ. Draw CD as the bisector of line PQ (Problem 3.1).
3. Draw another line QE perpendicular to AB to intersect the line CD at point E.
4. Draw an arc PQ with centre E and radius EP (= EQ). This arc meets the line AB tangentially at point Q and also passes through point P.

3.14 CIRCLE TO CONNECT ANOTHER CIRCLE AND A POINT

Problem 3.19 *Draw a circle of 60 mm diameter and mark a point P 70 mm away from its centre. Draw another circle which passes through the point P and is tangential to that circle at point Q.*

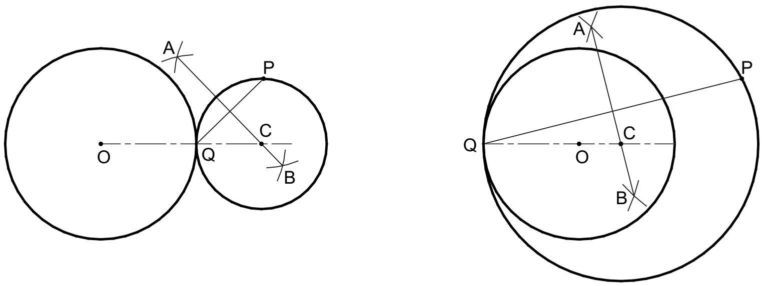


Fig. 3.19 (a) *and* **(b)** *Circle passing through a given point P and tangential to a given circle at point Q*

Construction Refer to Figs. 3.19(a) and (b).

1. Draw a circle of diameter 60 mm with centre O. Mark a point Q on the circle.
2. Mark point P at a distance 70 mm from O.
3. Join PQ. Draw line AB as the bisector of line PQ.
4. Let the line AB, produced if required, intersect the line OQ produced at point C. Draw a circle with centre C and radius CQ (= CP). This is the required circle.

3.15 POLYGONS

Polygons are defined as a closed curve consisting of a set of line segments connected such that no two segments cross. The straight line segments that make up the polygon are called its *sides* or *edges* and the points where the sides meet are the *polygon's vertices*. The simplest polygons are triangles (three sides), quadrilaterals (four sides) and pentagons (five sides). A polygon is convex if any two points inside the polygon can be connected by a line segment that does not intersect any side. If a side is intersected, the polygon is knows as a *concave*.

A polygon with all sides equal is equilateral. One with all interior angles equal is equiangular. Any polygon that is both equilateral and equiangular is a regular polygon (e.g., equilateral triangle, square). Construction of regular polygons is required in making drawing of engineering parts very frequently, e.g., Nut-bolt assembly, spanner, gears, etc.

3.15.1 Construction of Regular Polygons (General Methods)

Regular polygons can be drawn with the help of protractor taking internal angle of the polygon equal to $\left[\frac{(n-2)}{n}\times 180°\right]$ or external angle equal to $\left[\frac{2}{n}\times 180°\right]$, where n is the number of sides of the polygon. General method for construction of a regular polygon is illustrated in the following problem.

Problem 3.20 *Draw a regular pentagon and a regular heptagon of 40 mm sides, using general method.*

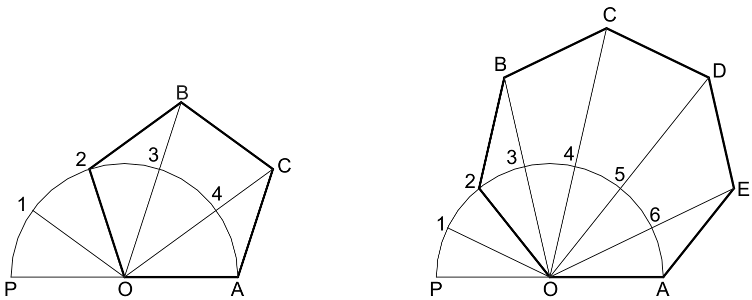


Fig. 3.20 *General method 1 for* **(a)** *pentagon* **(b)** *heptagon*

Construction

Method 1 Refer to Fig. 3.20(a) for pentagon and Fig. 3.20(b) for heptagon.

1. Draw a 40 mm long line OA.
2. Draw a semicircle AP with centre O and radius OA.
3. Divide the semi-circular AP into n equal parts, where n is the number of sides of the polygon ($n = 5$ for pentagon and $n = 7$ for heptagon). Starting from point P, mark these divisions as 1, 2, 3, 4, etc.
4. Join $O2$, $O3$, $O4$, etc.
5. Draw an arc with centre 2 and radius OA to cut the line $O3$ produced at B.
6. Draw another arc with centre B and the same radius OA to cut line $O4$ produced at C. For heptagon, proceed to draw arcs with centres C and D, radius OA to meet lines $O5$ and $O6$ produced at points D and E, respectively.
7. Join the points O, 2, B, C, …, A and obtain the required polygon.

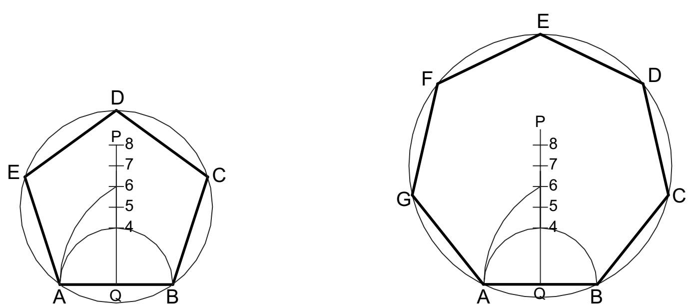


Fig. 3.21 *General method 2 for* **(a)** *pentagon* **(b)** *heptagon*

Method 2 Refer to Fig. 3.21(a) for pentagon and Fig. 3.21(b) for heptagon.

1. Draw a 40 mm long line *AB*.
2. Draw *PQ* as perpendicular bisector to *AB* (Problem 3.1).
3. Describe a semicircle with diameter *AB*, to intersect *PQ* at 4.
4. With either *A* or *B* as the centre, draw an arc of radius *AB* to meet *PQ* at 6.
5. Bisect the line 4-6 and obtain point 5 on *PQ*.
6. Mark points 7 and 8 on *PQ* such that the lengths 4–5, 5–6, 6–7, 7–8 are equal.
7. Draw a circle with centre 5 for pentagon and 7 for heptagon, to pass through points *A* and *B*.
8. Taking *AB* as radius cut the circle at *C*, *D*, *E*, etc., and join them to get the required polygon.

3.15.2 Construction of a Regular Hexagon

Methods explained in problem 3.20 can be used to draw a regular hexagon. However much convenient method for construction of hexagon is illustrated in following problem.

Problem 3.21 *Draw a regular hexagon of 40 mm sides, keeping a side (a) vertical (b) horizontal.*

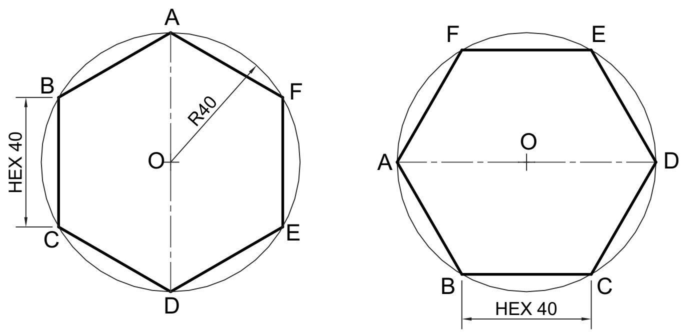


Fig. 3.22 *Hexagon with a side* **(a)** *Vertical* **(b)** *Horizontal*

Construction Refer to Figs. 3.22(a) and (b).

1. Draw a circle with centre *O* and radius 40 mm.
2. Mark a diameter *AD* in vertical position for case (a) and in horizontal position for case (b).
3. With radius *OA* and centres *A* and *D*, draw arcs to cut the circle at points *B*, *F*, *C* and *E*.
4. Join *ABCDEF* to get the required hexagon.

3.16 CONSTRUCTION OF A TRIANGLE

3.16.1 Triangle with Given Length of Sides

The following problem illustrates the method of drawing a triangle when all sides are given.

Problem 3.22 *Draw a triangle of sides 80 mm, 60 mm and 50 mm.*

Construction Refer to Fig. 3.23.

1. Draw an 80 mm long line *AB*.
2. Draw an arc of radius 60 mm with centre *A*.
3. Draw another arc of radius 50 mm with centre *B* to intersect the previous arc at point *C*.
4. Join *ABC* to get the required triangle.

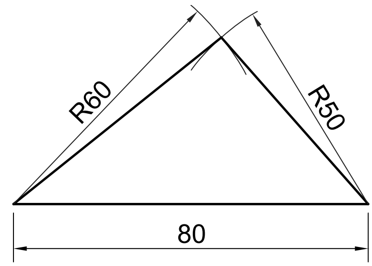


Fig. 3.23 *Triangle*

3.16.2 Equilateral Triangle

All the sides of an equilateral triangle are of equal lengths and each angle measures 60°. An equilateral triangle can be constructed by using a 30°–60° set square as shown in Fig. 3.24(a). The 60° angle could also be drawn with the help of a protractor.

If the side of the triangle is given, it can be constructed with the help of a compass by the method illustrated in problem 3.22 and Fig. 3.24(b). If the length of the altitude is given, it can be constructed as illustrated in the following problem.

Problem 3.23 *Draw an equilateral triangle of 50 mm long altitude.*

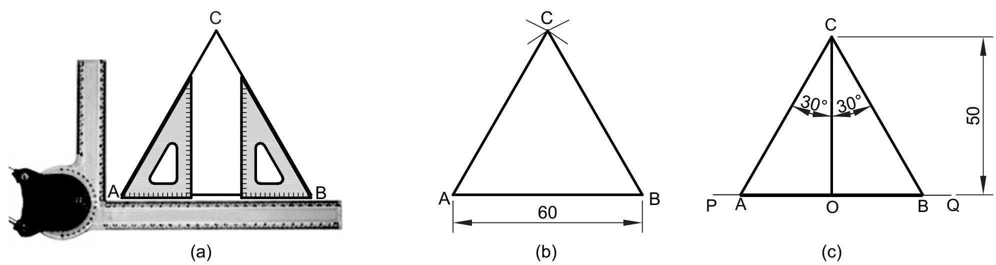


Fig. 3.24 *Equilateral triangle* **(a)** *Using set square* **(b)** *When side is given* **(c)** *When altitude is given*

Construction Refer to Fig. 3.24 (c).

1. Draw a line *PQ*. Mark a point *O* on it.
2. Draw a 50 mm long line *OC*, perpendicular to the line *PQ* (Problem 3.2).
3. Draw lines *CA* and *CB* inclined at 30° to the line *OC*, with the help of either 30–60° set square or a protractor or a compass.
4. Let *CA* and *CB* meet *PQ* at points *A* and *B*. *ABC* represents the required equilateral triangle.

3.17 RECTANGLE AND SQUARE

3.17.1 Rectangle and Square of Given Length of Sides

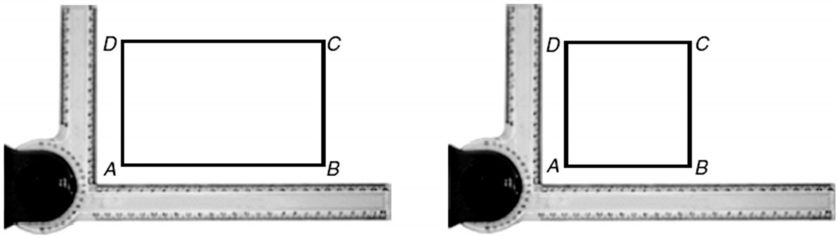


Fig. 3.25 **(a)** *Rectangle* **(b)** *Square*

A mini-drafter can be used to draw the rectangle of given sides. First draw a line *AB* of given length. Set the drafter along the line *AB* and erect perpendiculars *AD* and *BC*, equal to the given width as shown in Fig. 3.25(a). Join points *C* and *D* to obtain the required rectangle. It becomes a square if all the sides are equal as shown in Fig. 3.25(b).

3.17.2 Square with Given Length of Diagonals

The diagonals of a square are equal in length and perpendicular bisector of each other. The angle between a side of the square and the diagonal is 45°. A mini-drafter can be used to draw the square when length of diagonals is known. Using drafter, draw lines *AC* and *BD* perpendicular to each other. Set the drafter at 45° to AC or *BD*. Now draw 45° lines *AB* and *AD* as shown in Fig. 3.26(a). Proceed to draw lines *BC* and *CD*. A 45° set square or a protractor may also be used to draw the square. It can also be constructed with the help of compass as illustrated in the following problem.

Problem 3.24 *Draw a square of 70 mm long diagonals.*

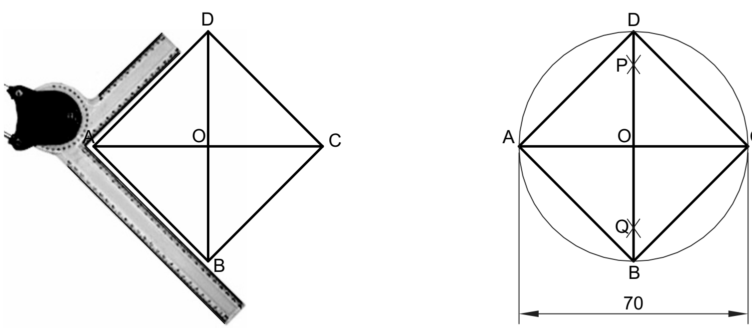


Fig. 3.26 *Square of given diagonal lengths* **(a)** *Using drafter* **(b)** *Using compass*

Construction Refer to Fig. 3.26(b).

1. Draw a 70 mm long line *AC*.

2. Draw line *PQ* as perpendicular bisector of *AC* to meet at *O* (Problem 3.1).
3. Draw a circle with centre *O* and radius *OA* to meet the line *PQ* produced at points *B* and *D*.
4. Join *ABCD*. This is the required square.

3.18 CONSTRUCTION OF A REGULAR PENTAGON

Special methods for the construction of regular pentagon are illustrated in the following problem.

Problem 3.25 *Draw a regular pentagon of 40 mm side.*

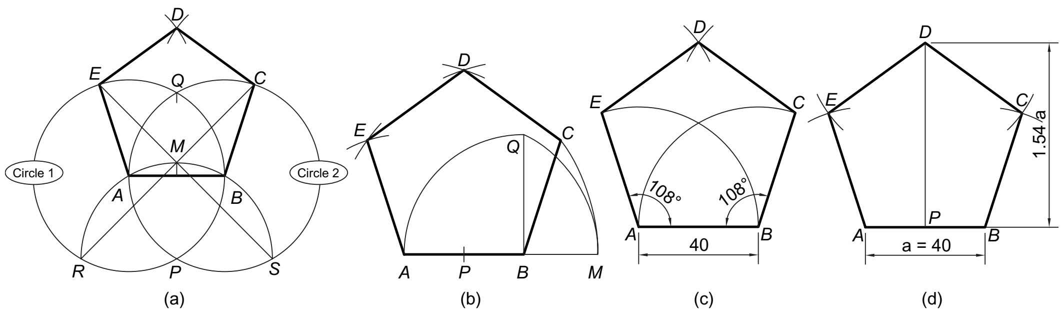


Fig. 3.27 *Pentagon* **(a)** *Method 1* **(b)** *Method 2* **(c)** *Method 3* **(d)** *Method 4*

Construction

Method 1 Refer to Fig. 3.27(a).

1. Draw a 40 mm long line *AB*.
2. Set compass to radius *AB* and draw circle 1 and circle 2 with centres *A* and *B* respectively. Let circles 1 and 2 intersect each other at points *P* and *Q*.
3. With centre *P* and same radius *AB*, draw an arc to intersect circle 1 and circle 2 at points *R* and *S* respectively.
4. Join *PQ* to meet arc *RS* at point *M*.
5. Join *RM* and produce it to meet circle 2 at point *C*.
6. Join *SM* and produce it to meet circle 1 at point *E*.
7. Draw arcs with radius *AB* and centres *C* and *E* respectively, to intersect each other at point *D*.
8. Join *ABCDE* to obtain the required pentagon.

Method 2 Refer to Fig. 3.27(b).

1. Draw a 40 mm long line *AB* and mark its mid-point *P* (Problem 3.1).
2. Erect line *BQ* perpendicular and equal to *AB*.
3. Draw an arc with centre *P* and radius *PQ* to meet *AB* produced at point *M*. Line *AM* is the diagonal of the pentagon.
4. Draw an arc with centre *A* and radius AM to intersect the arc with centre *B* and radius *AB* at point *C*.
5. Draw another arc with centre *A* and radius *AB* to intersect the arc with centre *B* and radius AM at point *E*.

6. Draw arcs of radius AM with centres *A* and *B* respectively, to intersect each other at point *D*.
7. Join *ABCDE* to obtain the required pentagon.

Method 3 Refer to Fig. 3.27(c).

1. Draw a 40 mm long line *AB*.
2. Draw 40 mm long lines *AE* and *BC*, both inclined at 108° with line *AB*.
3. With centres *C* and *E* and radius *AB*, draw arcs to intersect each other at point *D*.
4. Join *ABCDE* to obtain the required pentagon.

Method 4 Refer to Fig. 3.27(d).

1. Draw a 40 mm long line *AB*.
2. Draw a line *PD* as the perpendicular bisector of line *AB* (Problem 3.1).
3. Set the length of *PD* equal to 1.54 times of the length of the side *AB*.
4. Draw arcs with radius *AB* and centres *B* and *D* respectively, to intersect each other at point *C*.
5. Draw arcs with radius *AB* and centres *A* and *D* respectively, to intersect each other at point *E*.
6. Join *ABCDE* to obtain the required pentagon.

3.19 CONSTRUCTION OF A REGULAR HEXAGON

A special method for the construction of a regular hexagon is illustrated in Problem 3.21. Another method for the construction of a regular hexagon is illustrated below.

Problem 3.26 *Draw a regular hexagon of 40 mm side.*

Construction Refer to Fig. 3.28.

1. Draw a 40 mm long line *AB*.
2. Draw lines *A*2 and *A*4 inclined at 60° and 120° respectively with *AB*. Also, draw lines *B*1 and *B*3 inclined at 60° and 120° respectively with *AB*.
3. Draw arcs with radius *AB* and centres *A* and *B* to intersect *A*4 and *B*1 at *F* and *C* respectively. Also, draw arcs with radius *AB* and centres *C* and *F* to intersect *A*2 and *B*3 at *D* and *E* respectively.
4. Join *ABCDEF*, to obtain the required hexagon.

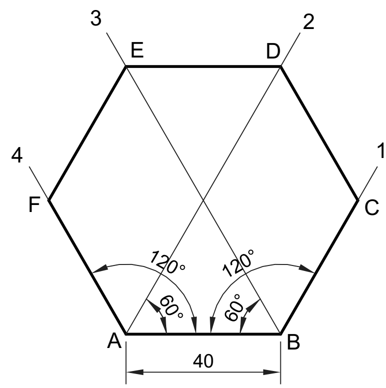


Fig. 3.28 *Regular hexagon*

3.20 INSCRIBE POLYGON IN A CIRCLE

Inscribing a regular polygon in a circle is to draw the maximum size of polygon in the circle. It is possible only if all the vertices of the polygon touch the circumference of the circle. The simplest way to inscribe a polygon of *n*-sides is to divide the circle into *n*-equal parts (with the help of a protractor) using the radial lines from the centre as shown in Figs. 3.28(a) to (f). Join the points of the circumference of the circle and obtain the required polygon. The set-squares can be used to make 120°, 90°, 60° and 45° angles and therefore can be used to draw an equilateral triangle, square, regular hexagon and regular octagon.

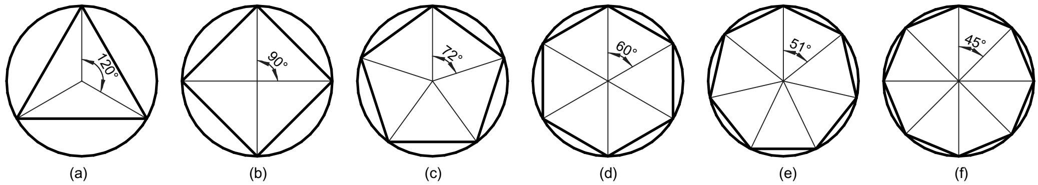


Fig. 3.29 *Inscribe a* **(a)** *triangle* **(b)** *square* **(c)** *pentagon* **(d)** *hexagon* **(e)** *heptagon* **(f)** *octagon*

3.20.1 General Method to Inscribe a Polygon

General method to inscribe a regular polygon in a circle with the help of a compass is illustrated in the following problem.

Problem 3.27 *In a circle of 70 mm diameter, inscribe (a) a regular pentagon and (b) a regular heptagon.*

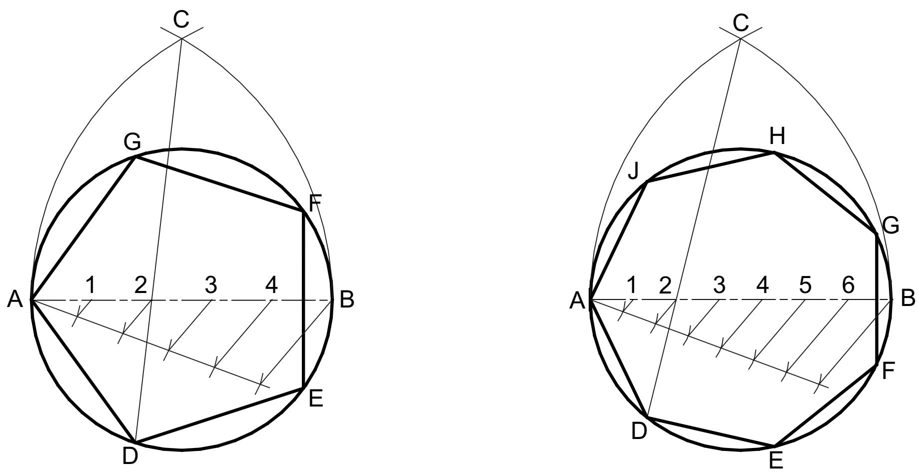


Fig. 3.30 *Inscribe a polygon in a circle* **(a)** *pentagon* **(b)** *heptagon*

Construction Refer to Figs. 3.30(a) and (b).

1. Draw a circle of 70 mm diameter *AB*.
2. Divide *AB* into *n* equal parts, where *n* is number of sides of the polygon ($n = 5$ for pentagon and $n = 7$ for heptagon). Call the second point from the left as 2.
3. Draw two arcs of radius *AB* with centres *A* and *B* respectively to intersect each other at *C*.
4. Join *C*2 and extend it to meet the circle at *D*. Chord *AD* is the side of the required polygon.
5. Taking *AD* as the radius, cut the circle progressively to get points *E*, *F*, *G*, etc. Now join them to obtain the required polygon.

3.20.2 Special Method to Inscribe a Pentagon

Problem 3.28 *Inscribe a pentagon in a circle of 70 mm diameter.*

Construction Refer to Fig. 3.31.

1. Draw a circle of 70 mm diameter with centre *O*.
2. Draw diameters *AB* and *CD* to the circle, perpendicular to each other.
3. Mark *M* as the mid-point of *OB* (Problem 3.1).
4. With centre *M* and radius *MC*, draw an arc to intersect *OA* at *N*.
5. Join CN. The line CN is equal to the side of the regular pentagon.
6. Taking CN as the radius, cut the circle progressively to get points 1, 2, 3 and 4.
7. Join C1234 to obtain the required pentagon.

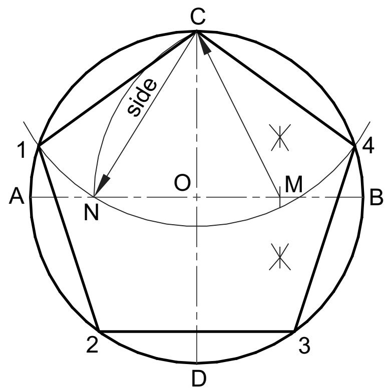


Fig. 3.31 *Inscribe a pentagon in a circle*

3.20.3 Special Method to Inscribe a Hexagon

Special method to inscribe a hexagon in a circle is illustrated in Problem 3.21.

3.20.4 Special Method to Inscribe a Heptagon

Problem 3.29 *Inscribe a regular heptagon in a circle of 80 mm diameter.*

Construction Refer to Fig. 3.32.

1. Draw a circle of 80 mm diameter with centre *O*.
2. Draw *OA* as the radius of the circle.
3. Draw an arc with centre *A* and radius *AO* to meet the circle at points *B* and *C*.
4. Join *BC* to intersect at *D*. The line *BD* is the length of the side of the regular heptagon.
5. Taking *BD* as the radius, cut the circle progressively to get points 1, 2, 3, 4, 5 and 6.
6. Join B123456 to obtain the required heptagon.

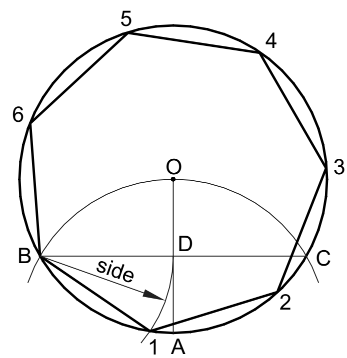


Fig. 3.32 *Inscribe a heptagon in a circle*

3.21 INSCRIBE AN OCTAGON IN A SQUARE

Problem 3.30 *Inscribe a regular octagon in a square of side 80 mm.*

Construction Refer to Fig. 3.33.

1. Draw a square *ABCD* of sides 80 mm long.
2. Join diagonals *AC* and *BD* to intersect each other at centre *O*.
3. Draw an arc with centre *A* and radius *AO* to meet sides of the square at 1 and 4.
4. Similarly, draw arcs of radius *AO* with centres *B*, *C* and *D* respectively, to meet the sides of the square at points 3, 6, 5, 8, 7 and 2.
5. Join 12345678 to obtain the required octagon.

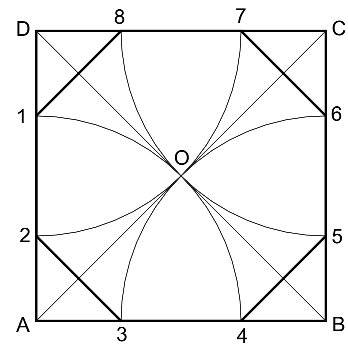


Fig. 3.33 *Inscribe an octagon in a circle*

3.22 CIRCUMSCRIBE POLYGON TO A CIRCLE

Circumscribing a regular polygon on a circle is to draw the minimum size of the polygon bounding the circle. This is only possible when all the sides of the polygon meet the circle tangentially. First divide the circumference of the circle into n number of equal parts using radial lines as shown in Fig. 3.34(a) to (f). A protractor may be used for the purpose. Draw tangents to the circle where these radial lines meet the circle. These tangents make the required polygon.

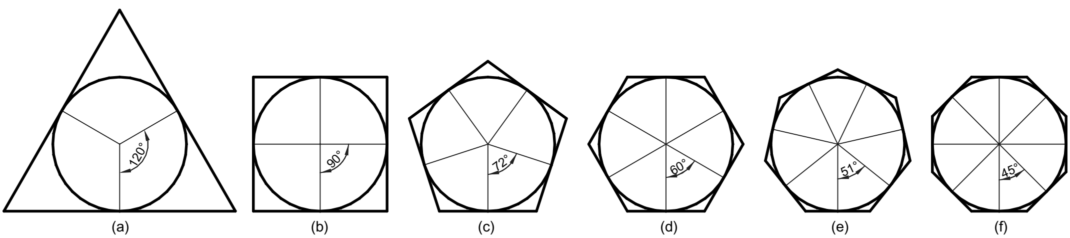


Fig. 3.34 *Circumscribe a* **(a)** *triangle* **(b)** *square* **(c)** *pentagon* **(d)** *hexagon* **(e)** *heptagon* **(f)** *octagon*

3.23 INSCRIBE A CIRCLE IN A POLYGON

Inscribing a circle means to draw the biggest circle inside the polygon.

3.23.1 Inscribe a Circle in a Triangle

Problem 3.31 *Draw a triangle of sides 80 mm, 60 mm and 50 mm and inscribe a circle in it.*

Construction Refer to Fig. 3.35.

1. Draw a triangle *ABC* of given sides (Problem 3.22).
2. Draw the bisectors of any two angles. Lines *AM* and *BN* are the bisector of angles *BAC* and *ABC* which meet at *O* (Problem 3.7).

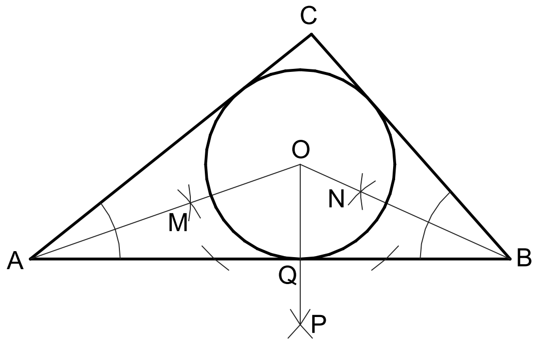


Fig. 3.35 *Inscribe a circle in a triangle*

3. Draw *OQ* perpendicular to *AB* (Problem 3.3).
4. With centre *O* and radius *OQ* draw the required circle.

3.23.2 Inscribe a Circle in the Quadrant

Problem 3.32 *Inscribe a circle in the quadrant ABCD. Take AB = AD = 60 mm, BC = CD = 35 mm and AC = 70 mm.*

Construction Refer to Fig. 3.36.

1. Draw a 70 mm long line *AC*.
2. Draw an arc with centre *A* and radius *AB* to intersect the arc with centre *C* and radius *BC* at points *B* and *D*. Join *ABCD* to obtain the quadrant.
3. Draw *BM* as the bisector of angle *ABC*. Produce line *BM* to meet *AC* at *P* (Problem 3.7).
4. Draw a perpendicular *PN* on one of the sides of the triangle say *BC* to meet at *Q* (Problem 3.3).
5. Draw a circle with centre *P* and radius *PQ*. This circle touches all the sides of the quadrant.

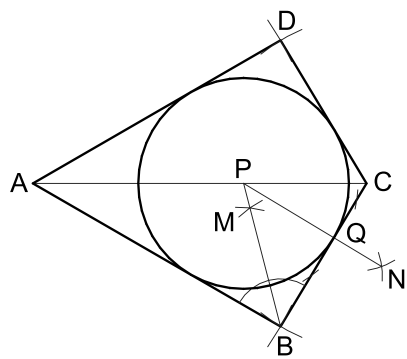


Fig. 3.36 *Inscribe a circle in a quadrant*

3.23.3 Inscribe a Circle in a Regular Polygon

Problem 3.33 *Inscribe a circle in a regular pentagon of 35 mm side.*

Construction Refer to Fig. 3.37.

1. Draw a regular pentagon *ABCDE* of 35 mm side (Problem 3.25).
2. Draw the bisectors of any two angles. Lines *AM* and *BN* are the bisector of angles *BAE* and *ABC* which meet at *O* (Problem 3.7).
3. Draw a perpendicular from point *O* on one of the sides of the triangle. Here *OP* is perpendicular to *AB* which meets at *Q* (Problem 3.3).
4. Draw a circle with centre *O* and radius *OQ*. This circle touches all the sides of the pentagon.

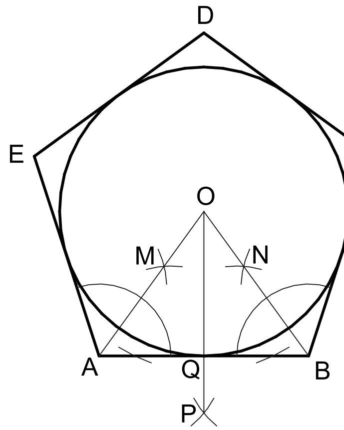


Fig. 3.37 *Inscribe a circle in a regular pentagon*

3.23.4 Inscribe a Number of Equal Circles Inside a Given Circle

Problem 3.34 *In a given circle of 70 mm diameter, draw four equal circles such that each touches the given circle and the other two circles.*

Construction Refer to Fig. 3.38.

1. Draw a circle of 70 mm diameter.
2. Divide the circle into 8 equal parts and draw radial lines. The centres of the circle will lie on the alternate radial lines.
3. Draw tangent *AB* to the circle at from the end of the radial line *O*1.
4. Extend the radial line *O*2 and *O*8 to meet this tangent at *B* and *A*, respectively.
5. Inscribe a circle with centre *P* in triangle *ABO* (Problem 3.31).
6. Proceed to draw other circles within the given circle.

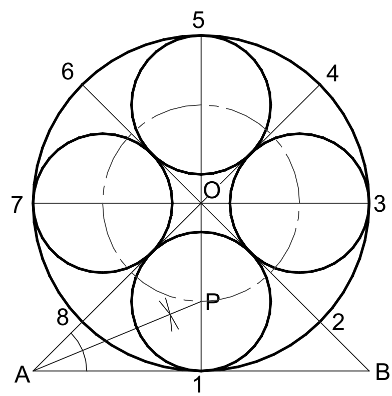


Fig. 3.38 *Inscribe four circles in a given circle*

3.23.5 Inscribe *n* Equal Circles in *n* Sided Regular Polygon

Problem 3.35 *Inscribe inside the polygon same number of circles as number of sides of the polygon, such that each circle touches one side of the polygon and two other circles. Consider polygon as (a) a pentagon of 40 mm side, (b) a hexagon of 40 mm side.*

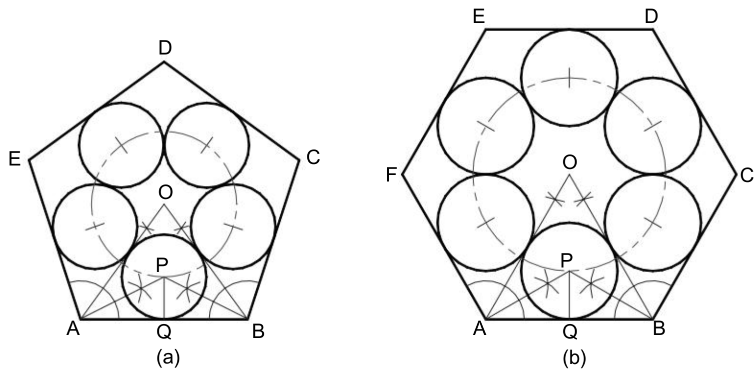


Fig. 3.39 *Inscribe circles in a* **(a)** *pentagon* **(b)** *hexagon*

Construction Refer to Figs. 3.39(a) and (b).

1. Draw a regular polygon (pentagon or hexagon) of 40 mm sides (Problem 3.20).
2. Draw the *bisectors of any two angles* of the polygon to meet at point *O*. In the figure, lines *AO* and *BO* are the angle bisectors (Problem 3.7).
3. Inscribe a circle with centre *P* in the triangle *ABO* (Problem 3.31).
4. Proceed to draw other circles within the polygon.

Problem 3.36 *Inscribe inside the polygon same number of circles as number of sides of the polygon, such that each circle touches two sides of the polygon and two other circles. Consider polygon as (a) a pentagon of 30 mm side, (b) a hexagon of 30 mm side.*

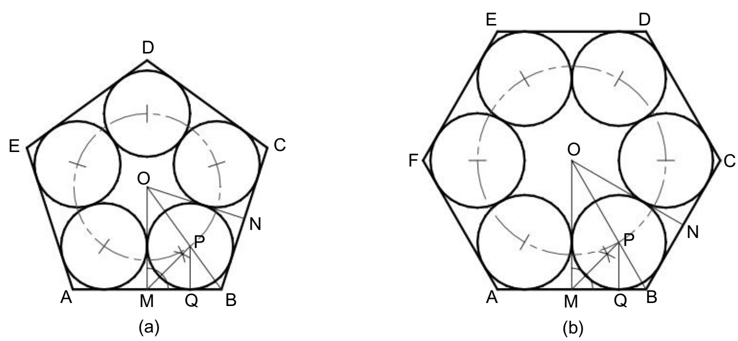


Fig. 3.40 *Inscribe circles in a* **(a)** *pentagon* **(b)** *hexagon*

Construction Refer to Figs. 3.40(a) and (b).

1. Draw a regular polygon (pentagon or hexagon) of 30 mm sides (Problem 3.20).
2. Draw *perpendicular bisectors of any two sides* of the polygon to meet at point *O*. In the figure, lines *MO* and *NO* are the perpendicular bisectors of sides *AB* and *BC* (Problem 3.1).
3. Inscribe a circle with centre *P* in the quadrant *OMBN* (Problem 3.32).
4. Proceed to draw other circles within the polygon.

3.23.6 Inscribe Three Equal Circles in a Hexagon

Problem 3.37 *Draw three circles inside a hexagon of 40 mm side, such that each circle touches one side of the hexagon and two other circles.*

Construction Refer to Fig. 3.41.

1. Draw a hexagon of 40 mm side (Problem 3.21).
2. Draw *perpendicular bisectors OM and ON of any two alternate sides*, say *AF* and *BC*.
3. Produce side *AB* to meet line *OM* and *ON* at points *M* and *N* respectively.
4. Inscribe a circle with centre *P* in the triangle *MNO* (Problem 3.31).
5. Proceed to draw other circles within the hexagon.

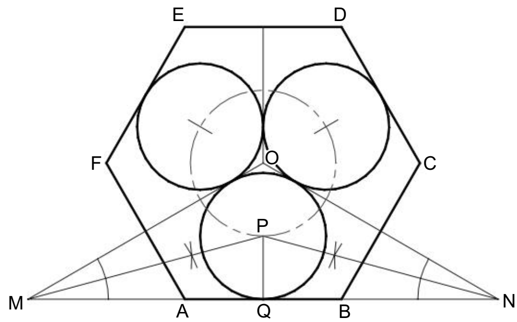


Fig. 3.41 *Inscribe three circles in a regular hexagon*

3.23.7 Same Number of Equal Circles Outside as the Number of Sides in a Regular Polygon

Problem 3.38 *Draw outside a regular polygon, the same number of equal circles as number of sides of the polygon, such that each circle touches one side of the polygon and two other circles. Consider polygon as (a) a pentagon of 15 mm side, (b) a hexagon of 15 mm side.*

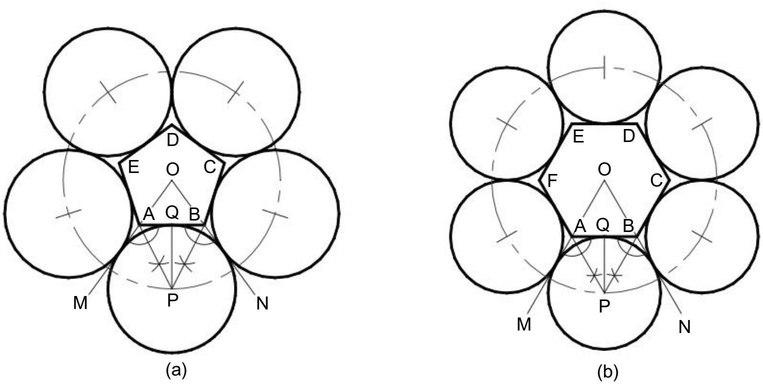


Fig. 3.42 *Number of circles outside a* **(a)** *pentagon* **(b)** *hexagon*

Construction Refer to Fig. 3.42.

1. Draw a regular polygon (pentagon or hexagon) of 15 mm sides (Problem 3.20).
2. Join the centre of the polygon with any two corners of the polygon and extend to obtain lines *OM* and *ON*.

3. Draw the bisectors *AP* and *BP* of angles *BAM* and *ABN* to intersect each other at point *P* (Problem 3.7).
4. From point *P* draw *PQ* perpendicular to *AB* (Problem 3.3).
5. Draw a circle with centre *P* and radius *PQ*.
6. Proceed to draw other circles outside the polygon.

3.24 ENGINEERING APPLICATIONS

The geometrical constructions are used to draw multi-views of objects in orthographic projections. The following problem illustrates one of such applications.

Problem 3.39 *Redraw the C-clamp shown in Fig. 3.43(a).*

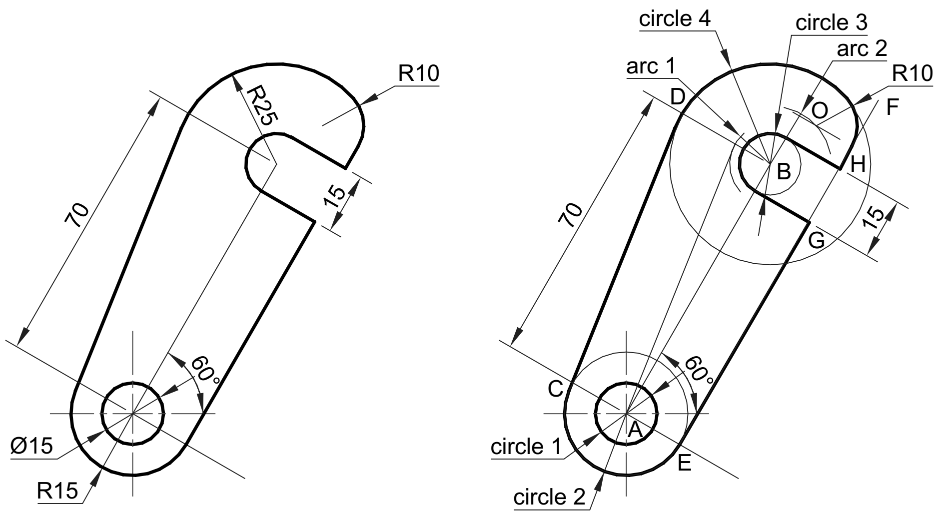


Fig. 3.43 *View of a C-clamp*

Construction Refer to Fig. 3.43(b).

1. Draw a 70 mm long line *AB*, inclined at 60° to the horizontal.
2. Taking *A* as centre draw circle 1 of 15 mm diameter and circle 2 of 30 mm diameter.
3. Taking *B* as centre draw circle 3 of 15 mm diameter and circle 4 of 50 mm diameter.
4. Draw a tangent line *CD* to the circle 2 and circle 4 (Problem 3.14).
5. Draw line *EF* which is tangent to the circle 2 and parallel to *AB*.
6. Draw two lines which are perpendicular to *AB* and tangent to circle 3 to meet *EF* at points *G* and *H* (Problem 3.11).
7. Locate centre *O* to draw the arc of radius 10 mm (Problem 3.16).

EXERCISE 3A

3.1 Divide an 80 mm long straight line into five equal parts.

3.2 Divide a 90 mm long straight line into parts that are in proportion to 2:3:5.

3.3 Draw a perpendicular to a 100 mm long line *AB*, at a point *P* lying on the line at a distance of 40 mm from the end *A*.

3.4 Draw a 120 mm long line *AB* inclined at 60° to the horizontal. Erect a perpendicular to *AB* from point *P*, lying at a distance 30 mm from end *A*.

3.5 Draw perpendicular to a 100 mm long line *AB*, from a point *P* lying at a distance 60 mm from end *A* and 70 mm from end *B*.

3.6 Draw a line *AB* inclined at 30º to the horizontal. Draw another line *CD* parallel to and 50 mm away from *AB*.

3.7 Draw tangent to a circle of 40 mm diameter from any point *P* which is at a distance of 65 mm from the centre of the circle.

3.8 Two circles of radii 20 mm and 30 mm have their centres 65 mm apart. How many common tangents to both the circles are possible? Draw an internal and an external common tangent to these circles.

3.9 Two circles of radii 20 mm and 30 mm have their centres 50 mm apart. Draw all the possible common tangents to both the circles.

3.10 Two circles of radii 20 mm and 30 mm have their centres 40 mm apart. Draw a pair of common tangents to both the circles.

3.11 Draw a tangent to connect two circles of radii 25 mm and 40 mm. The centres of the circles are 15 mm apart.

3.12 Draw an arc of 30 mm radius connecting two straight lines inclined at 135º to each other.

3.13 Draw arc of 20 mm radius to connect a straight line *AB* and a circle of 30 mm radius, tangentially. The centre of the circle is at a distance 25 mm from *AB*. Consider the centres of the arc lies (a) within the circle (b) outside the circle.

3.14 Two circles have their centres 70 mm apart and radii 20 mm and 30 mm, respectively. Draw a circle of radius 25 lying internal to and connect both the circles tangentially.

3.15 Two circles have their centres 70 mm apart and radii 20 mm and 30 mm, respectively. Draw a circle of radius 65 lying external to and connect both the circles tangentially.

3.16 Two circles have their centres 70 mm apart and radii 20 mm and 30 mm, respectively. Draw a circle of radius 55 which connects tangentially both the circles and (a) include 20 mm circle (b) include 30 mm circle.

3.17 A point *P* is 40 mm from a line *AB*. Another point *Q* is in the *AB* and is 50 mm from the point *P*. Draw a circle passing through point *P* and tangential to the line *AB* at point *Q*.

3.18 Draw two possible circles to connect a given circle of 50 mm diameter *AB* and a point *P*, lying at a distance 70 mm and 25 mm from the ends of the diameter *AB*.

3.19 Inscribe a circle in a triangle of 75 mm, 65 mm and 55 mm long sides.

3.20 Draw a square of 60 mm long diagonals. Circumscribe another square on the square.

3.21 Draw regular pentagon, hexagon and a heptagon on a common edge of side 30 mm.

3.22 Draw a pentagon of 30 mm side with a side vertical. Attach a non-overlapping hexagon of same side length with common vertical edge.

3.23 Construct a heptagon of edge length 30 mm. Construct a pentagon of same edge length inside the heptagon with one edge of the polygons being common.

3.24 Draw an octagon of 25 mm side keeping one of the sides vertical.

3.25 Draw five circles in a given circle of 80 mm diameter, each touching the given circle and the other two circles.

3.26 Draw five circles inside the pentagon of 30 mm side, such that each circle touches one side of the pentagon and two other circles.

3.27 Draw five circles inside the pentagon of 30 mm side, such that each circle touches two side of the pentagon and two other circles.

3.28 Draw three circles inside a hexagon of 30 mm side, such that each circle touches one side of the hexagon and two other circles.

3.29 Draw five circles outside the pentagon of 20 mm side, such that each circle touches one side of the pentagon and two other circles.

3.30 Draw six circles outside a given circle of 30 mm diameter, such that each circle touches the given and two other circles.

3.31 Use necessary drawing instruments to draw Figs. E3.1 to E3.3 in a circle of diameter 100 mm.

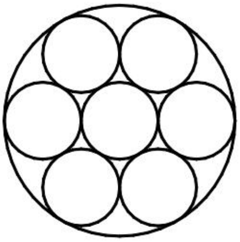


Fig. E3.1

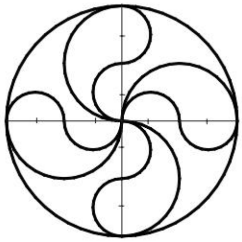


Fig. E3.2

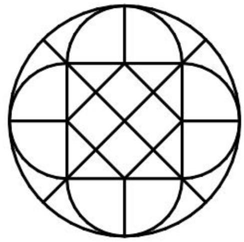


Fig. E3.3

3.32 Use necessary drawing instruments to reproduce Figs. E3.4 to E3.12.

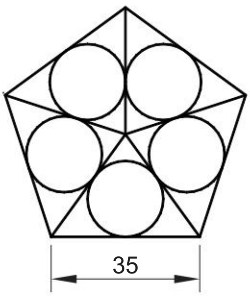


Fig. E3.4

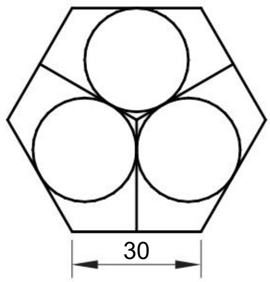


Fig. E3.5

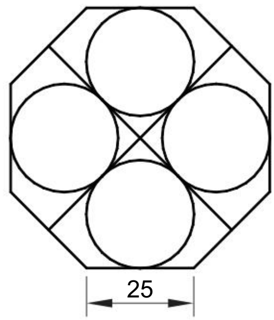


Fig. E3.6

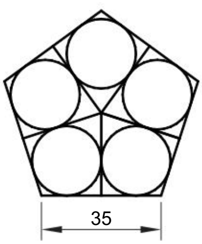


Fig. E3.7

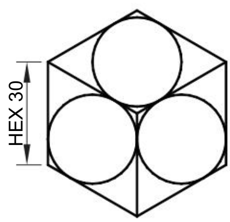


Fig. E3.8

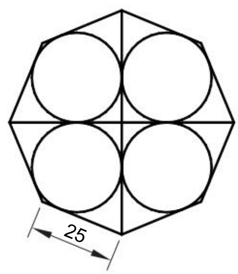


Fig. E3.9

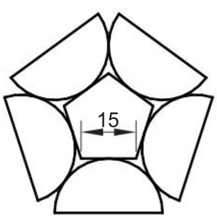


Fig. E3.10

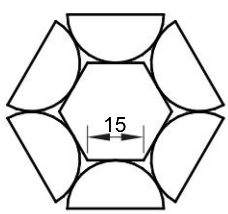


Fig. E3.11

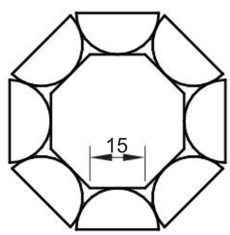


Fig. E3.12

3.33 Use necessary drawing instruments to reproduce Figs. E3.13 to E3.18.

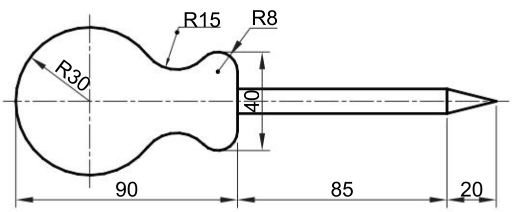


Fig. E3.13 *Screw driver*

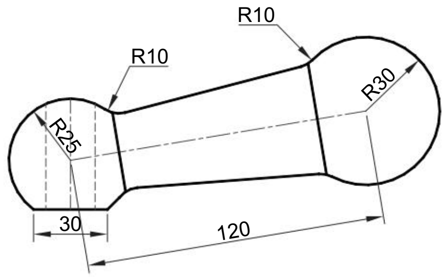


Fig. E3.14 *Knob*

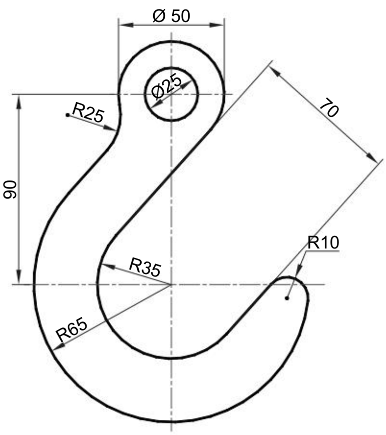


Fig. E3.15 *Hook*

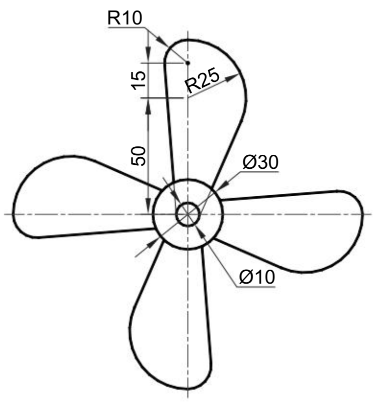


Fig. E3.16 *Fan*

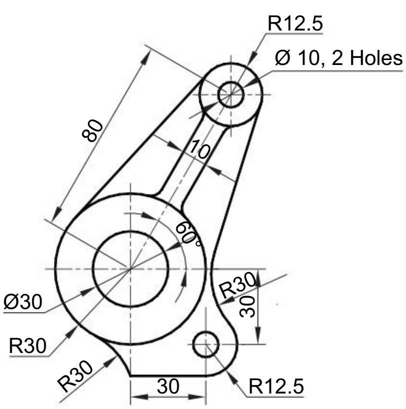


Fig. E3.17 *Lever*

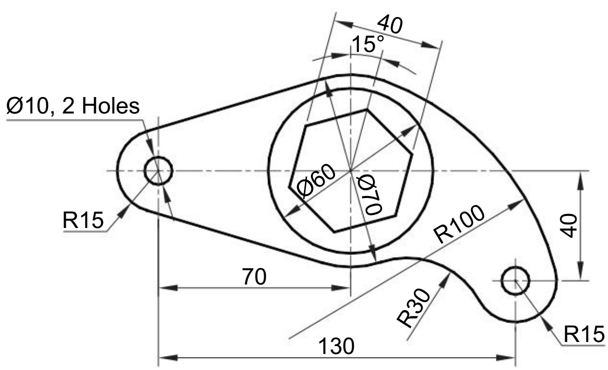


Fig. E3.18 *Rocker arm*

VIVA-VOCE QUESTIONS

3.1 What do you understand by a perpendicular bisector?
3.2 How can a drafter be used for drawing perpendicular lines and parallel lines?
3.3 How can you divide a line into a number of equal parts?
3.3 State possible number of tangents to two circles.
3.5 Explain possible ways by which an arc can connect two circles.
3.6 Explain the method of inscribing hexagon in a circle.
3.7 Distinguish between inscribed figures and circumscribed figures.

MULTIPLE-CHOICE QUESTIONS

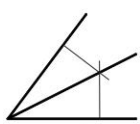
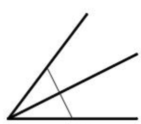
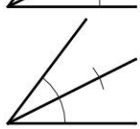
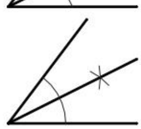
3.1 When two graphic entities are at a constant distance apart along the length, it is commonly referred as
(a) concentricity (b) parallelism
(c) perpendicularity (d) chordality

3.2 There are two fixed points *A* and *B*. Another point *P* moves in such a way that *PA* and *PB* are always at right angles. The locus of *P* is
(a) a circle (b) an ellipse
(c) a parabola (d) a hyperbola

3.3 If a line intersects a circle at two points and does not pass through the centre, the line segment inside the circle is referred as
(a) radial line (b) chord
(c) quadrant (d) sequent

3.4 A point *P* moves such that it is always at the same distance from each of the two straight lines which intersect at an angle of 60°. The locus of the point *P* is
(a) a bisector of the angle
(b) an arc connecting lines tangentially
(c) a line which forms a triangle
(d) None of these

3.5 Which diagram below shows a correct geometrical construction to bisect an angle?

(a)

(b)

(c)

(d)


3.6 The number of tangents that can be drawn to a circle from a point outside is
(a) 1 (b) 2 (c) 3 (d) 4

3.7 The number of common tangents that can be drawn to two circles which touch each other externally?
(a) 1 (b) 2 (c) 3 (d) 4

3.8 An external tangent connecting two circles of equal radii is
(a) parallel to the line joining the centres
(b) perpendicular to the line joining the centres
(c) inclined to the line joining the centres
(d) Any of these

3.9 Maximum possible exterior angle in a regular polygon is
(a) 60° (b) 90° (c) 120° (d) 135°

3.10 The point where the medians of a triangle are concurrent is called the
(a) incentre (b) orthocentre
(c) circumcentre (d) centroid

3.11 The circumcentre of an obtuse angled triangle is located
(a) on the triangle (b) outside the triangle
(c) inside the triangle (d) the location varies

3.12 A polygon having the sum of the measures of the interior angles equal the sum of the measures of the exterior angles is
(a) triangle (b) quadrilateral
(c) hexagon (d) octagon

3.13 The included angle of a pentagon is
(a) 68° (b) 72° (c) 108° (d) 112°

3.14 Number of diagonal that a hexagon can have
(a) 3 (b) 6 (c) 9 (d) 12

3.15 A ten-sided polygon is referred as
(a) hexagon (b) octagon
(c) decagon (d) dodecagon

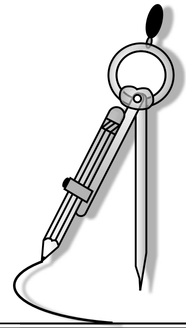
Answers to multiple-choice questions

3.1 (b), 3.2 (a), 3.3 (b), 3.4 (a), 3.5 (d), 3.6 (b), 3.7 (c), 3.8 (a), 3.9 (c), 3.10 (d), 3.11 (b), 3.12 (b), 3.13 (c), 3.14 (c), 3.15 (c)

Chapter

4

SCALES



4.1 INTRODUCTION

The word ‘scale’ usually employs for an instrument used for drawing or measuring the length of a straight line. *It is also used to represent the proportion in which a drawing is made with respect to the size of the object.*

It is always convenient to make the drawing of an object to its actual size provided its size permits, e.g., a 50 mm diameter plain disc should be represented by a circle of 50 mm diameter on the drawing. When a drawing is prepared to the actual size of the object, the scale is said to be *full size scale* and the drawing is said to be *full size drawing*.

However, it is not possible to make the drawings of machines, buildings, town plans, etc., to their actual size. When the object is of large size, the actual dimensions of the object have to be reduced on some regular proportion to make its drawing, e.g., a rectangular plot of size 25 m × 10 m can be represented by a rectangle of 250 mm × 100 mm. The scale selected in the present case is 1 mm = 0.10 m. In other words, 1 mm on the drawing represents 0.10 m length of the object. When a drawing is prepared smaller than the actual size of the object, the scale is said to be *reduction scale* and the drawing is said to be *reduce sized drawing*.

Similarly very small object such as the gear mechanism of a wrist watch, component of an electronic instrument, etc., is enlarged on some regular proportion to make its drawing. When the drawing is prepared larger than the actual size, the scale is said to be an *enlargement scale* and the drawing is said to be *enlarge sized drawing*.

4.2 REPRESENTATION OF SCALE

A scale can be expressed in one of the following ways:

1. **Engineering scale** Engineering scale is represented by indicating the relation between the dimension on the drawing and the corresponding actual dimension of the object. It is expressed as
   1 mm = 1 mm for full size drawing
   1 mm = 5 m, 1 mm = 8 km, etc., for reduce size drawing
   1 mm = 0.2 mm, 1 mm = 5 μm, etc., for enlarge size drawing
   They are usually written on the drawings in numerical forms.
2. **Graphical scale** Graphical scale is expressed by its representative fraction and is captioned on the drawing itself. As the drawing becomes old, the drawing sheet may shrink and the engineering scale would provide inaccurate results. However, the scale made on the drawing sheet along with drawing of object will shrink in the same proportion. This will always provide an accurate result. It is the basic advantage gained by graphical representation of a scale.

4.3 UNITS OF MEASUREMENTS

Table 4.1 provides the relationship of various units used for linear measurement.

Table 4.1 *Units of measurements*

Metric system for linear measurement	*British system for linear measurement*
1 kilometre (km) = 10 hectometres	1 league = 3 miles
1 hectometre (Hm) = 10 decametres	1 mile (mi) = 8 furlongs
1 decametre (Dm or dam) = 10 metres	1 furlong (fur) = 10 chains
1 metre (m) = 10 decimetres	1 chain (ch) = 22 yards
1 decimetre (dm) = 10 centimetres	1 yard (yd) = 3 feet
1 centimetre (cm) = 10 millimetres (mm)	1 foot (ft) = 12 inches
	1 inch (in) = 8 eighth

The following linear and area conversions is also useful in construction of scales.

Linear conversion 1 mile = 1.609 km
1 inches = 25.4 mm

Area conversion 1 are (a) = 100 m^2
1 hectare (ha) = 100 ares = 10000 m^2
1 square mile = 640 acres
1 acre (ac) = 10 square chain = 4840 square yards

4.4 TYPES OF SCALES

Scales are classified as the following:

1. Plain scale
2. Diagonal scale
3. Comparative scale (plain and diagonal type)
4. vernier scale
5. Scale of chords

4.5 REPRESENTATIVE FRACTION (RF)

Representative fraction is defined as the ratio of the linear dimensions of an element of the object in the drawing to its actual linear dimension of the same element of the object itself.

$$\text{R.F.} = \frac{\text{Length of an element in the drawing}}{\text{Actual length of the same element}}$$

In the above formula, both the numerator and the denominator are converted into same units. The value of RF is always kept in fraction, i.e., expressed in the form of $x{:}y$ in which both x and y are lengths brought into the same units. Thus, RF has *no decimal and no unit*. It may be noted that if (i) x is smaller than y it represents a reduction scale, (ii) x is greater than y it represents an enlargement scale, (iii) both x and y are equal it is a full size scale. It is further emphasized that RF is the ratio of lengths and not the ratio of areas or volumes.

In case, the quantities for the numerator or the denominator are given in units of areas such as km^2, m^2, mm^2, etc., then take the square root to convert them in lengths units. Similarly, if the quantities in the numerator and the denominator are given in unit of volume such as km^3, m^3, mm^3, etc., then take the cube root to convert them in length units. The following problems illustrate the method to determine RF of a scale.

Problem 4.1 *If a 1 centimetre long line on a map represents a real length of 4 metres, calculate the RF.*

$$\text{R.F.} = \frac{1\text{ cm}}{4\text{ m}} = \frac{1\text{ cm}}{400\text{ cm}} = \frac{1}{400}$$

Problem 4.2 *The distance between two stations by road is 200 kilometres. It is represented on a certain map by a 5 centimetres long line. Find the RF.*

$$\text{R.F.} = \frac{5\text{ cm}}{200\text{ km}} = \frac{5\text{ cm}}{200\times 10^5\text{ cm}} = \frac{1}{4\times 10^6}$$

Problem 4.3 *A circle of 15 cm diameter on a drawing represents a disc of 6 mm diameter. Find the RF.*

$$\text{R.F.} = \frac{15\text{ cm}}{6\text{ mm}} = \frac{150\text{ mm}}{6\text{ mm}} = \frac{25}{1}$$

Problem 4.4 *An area of 400 square centimetres on a map represents an area of 25 square kilometres on a field. Find the RF.*

$$\text{R.F.} = \frac{\sqrt{400}\text{ cm}}{\sqrt{25}\text{ km}} = \frac{20\text{ cm}}{5\text{ km}} = \frac{20\text{ cm}}{5\times 10^5\text{ cm}} = \frac{1}{25000}$$

Problem 4.5 *A rectangular field of 0.54 hectares is represented on a map by a rectangle of 3 centimetres × 2 centimetres. Find the RF.*

$$\text{R.F.} = \frac{\sqrt{3\times 2}\text{ cm}}{\sqrt{0.54\times 10^4}\text{ m}} = \sqrt{\frac{6}{0.54\times 10^4}} \times \frac{1\text{ cm}}{100\text{ cm}} = \frac{1}{3000}$$

Problem 4.6 *A room of 1728 cubic metres volume is represented in a model by a cube of 4 centimetres side. Find the RF.*

$$\text{R.F.} = \frac{4\text{ cm}}{\sqrt[3]{1728}\text{ m}} = \frac{4\text{ cm}}{12\text{ m}} = \frac{4\text{ cm}}{12\times 100\text{ cm}} = \frac{1}{300}$$

The RF of the scale should be written below the individual drawing. However, if the scale is common for all drawings on a particular drawing sheet, it should be mentioned in the title block. The standard values of RF as recommended by the Bureau of Indian Standards in its bulletin IS 10713:1983 (reaffirmed 1999) are shown in Table 4.2.

Table 4.2 *Scale for use in engineering drawing*

Category	*Recommended scale*									
Enlargement scale	50:1	20:1	10:1	5:1	2:1					
Full Scale	1:1									
Reduction scale	1:2	1:5	1:10	1:20	1:50	1:100	1:200	1:500	1:1000	1:2000	1:5000

The scale to be chosen for the drawing depends upon the functionality of the drawing, complexity and the size of the object. Intermediate scales can be used in exceptional cases where recommended scales cannot be applied for functional reasons. In all cases, the selected scale should be large enough to permit easy and clear interpretation of the desired details.

4.6 REQUIREMENTS OF A SCALE

The data required for the construction of a plain, a diagonal or a vernier scale are as follows:

1. RF of the scale.
2. The maximum length which the scale can measure.
3. Least count of the scale, i.e., minimum length which the scale can measure.

The following steps are required in construction of all types of scales except the scale of chords.

1. Determine the value of R.F.
2. Calculate the length of scale as L_s = R.F. × Maximum length. It is usually expressed in cm.
3. Actual drawing of the scale.

4.7 PLAIN SCALE

A plain scale is used to measure up to two consecutive units, i.e., a unit and its immediate sub division. For example, (a) metre and decimetre, (b) kilometre and hectometre, (c) feet and inches, etc.

4.7.1 Construction of Plain Scale

The following problems illustrate the method of construction of plain scales.

Problem 4.7 *Construct a scale of 1:40 to read metres and decimetres and long enough to measure up to 6 metres. Mark a distance of 4.7 m on it.*

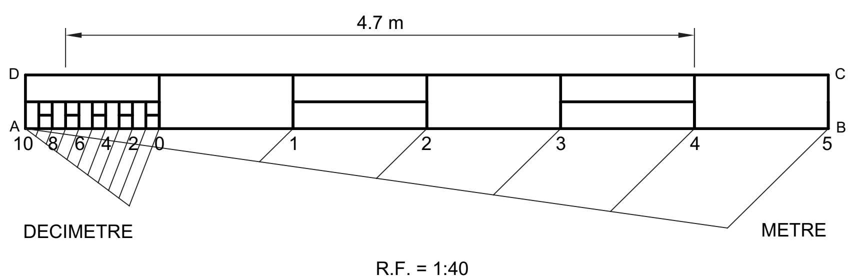


Fig. 4.1

Construction Refer to Fig. 4.1.

1. Given (a) R.F. = 1/40, (b) maximum length = 6 m and (c) least count = 1 dm.
2. Length of scale, L_s = R.F. × Maximum length = $\frac{1}{40} \times 6 \times 100 \text{ cm} = 15 \text{ cm}$
3. Draw a rectangle having length AB = 15 cm and arbitrary width, say, AD = 10 mm.
4. Here the length of scale represents 6 m. Divide it into 6 equal parts#. Each part represents 1 metre. Mark the main units.
5. Divide the first division $A0$ of the scale into 10 subdivisions#. Each subdivision represents 1 decimetre. Mark subunits on the scale.
6. Write "R.F. = 1:40" below the scale.
7. Mark a length of 4.7 m on the scale, i.e., 4 metres on the right side of the zero mark and 7 decimetres on the left side of zero mark.

Problem 4.8 *If 1 centimetre long line on a map represents a real length of 4 metres. Calculate the R.F. and draw a plain scale long enough to measure up to 50 metres. Show a distance of 44 m on it.*

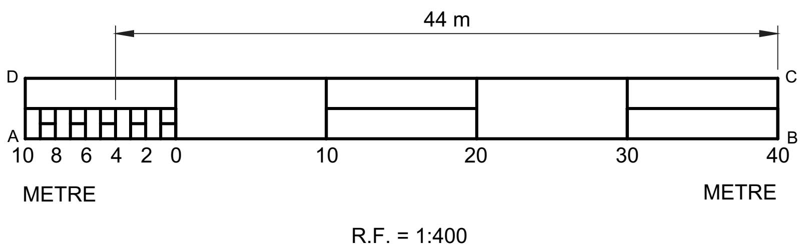


Fig. 4.2

Construction Refer to Fig. 4.2.

1. R.F. = $\frac{1 \text{ cm}}{4 \text{ m}} = \frac{1 \text{ cm}}{4 \times 100 \text{ cm}} = \frac{1}{400}$
2. Since scale has to show a distance of 44 m, assume the least count as 1 m.
3. Length of scale, L_s = R.F. × max. length = $\frac{1}{400} \times 50 \times 100 = 12.5 \text{ cm}$
4. Draw a rectangle having a 12.5 cm length and 10 mm width.
5. Divide the length of scale into five equal parts, each part representing 10 metres.
6. Divide first division of the scale into 10 equal parts, each representing 1 metre.
7. Mark units and subunits on the scale and write the value of R.F.
8. Mark a length 44 m on the scale, i.e., 40 m on the right side of the zero mark and 4 m on the left side of zero mark.

Readers are advised to refer to Chapter 3 for the details of methods of dividing a straight line into equal number of parts.

Problem 4.9 *A real length of 1 decametre is represented by a line of 5 cm in a drawing. Find the R.F. and construct a plain scale to measure up to 2.5 decametres. Mark a distance of 19 m on it.*

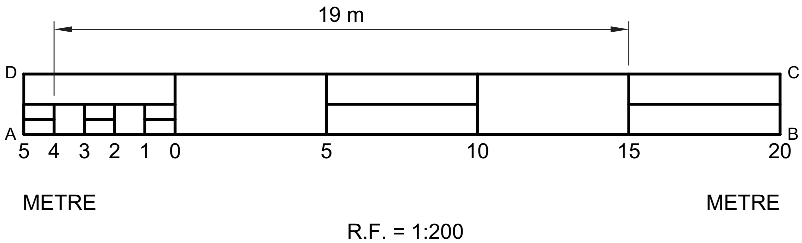


Fig. 4.3

Construction Refer to Fig. 4.3.

1. R.F. = $\dfrac{5\text{ cm}}{1\text{ Dm}} = \dfrac{5\text{ cm}}{1000\text{ cm}} = \dfrac{1}{200}$
2. Length of scale, L_s = R.F. × Maximum length = $\dfrac{1}{200} \times 2.5 \times 1000 = 12.5$ cm
3. Draw a rectangle having a 12.5 cm length and 10 mm width.
4. Divide the length of scale into five equal parts, each part representing 5 m.
5. Divide first division of the scale into five equal parts, each representing 1 m.
6. Mark units and subunits on the scale and write the value of R.F.
7. Mark a length 19 m on the scale, i.e., 15 m on the right side of the zero mark and 4 m on the left side of the zero mark.

Problem 4.10 *Construct a scale of 1:5 to show decimetres and centimetres and long enough to measure up to 1 m. Show a distance of 6.3 dm on it.*

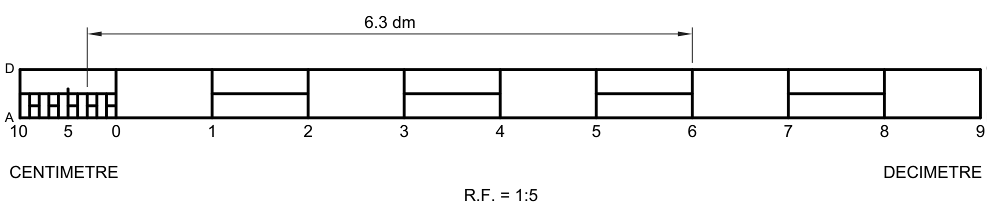


Fig. 4.4

Construction Refer to Fig. 4.4.

1. R.F. = 1:5
2. Length of scale, L_s = R.F. × Maximum length = $\dfrac{1}{5} \times 1 \times 100 = 20$ cm
3. Draw a rectangle having a 20 cm length and 10 mm width.
4. Divide the length of scale in 10 equal parts, each part representing 1 decimetre.
5. Divide first division of the scale in 10 equal parts, each representing 1 cm.

6. Mark units and subunits on the scale and write the value of R.F.
7. Mark a length 6.3 dm on the scale, i.e., 6 decimetres on the right side of the zero mark and 3 centimetres on the left side of the zero mark.

Problem 4.11 *A line of 1 centimetre represents an actual length of 4 dm. Draw a plain scale and mark a distance of 6.7 m on it.*

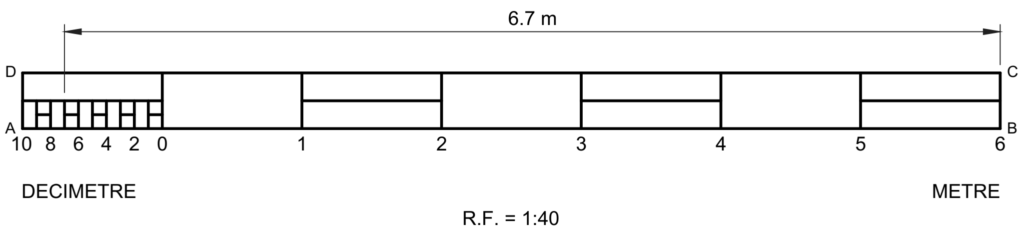


Fig. 4.5

Construction: Fig. 4.5

1. R.F. = $\dfrac{1\text{ cm}}{4\text{ dm}} = \dfrac{1\text{ cm}}{4 \times 10\text{ cm}} = \dfrac{1}{40}$
2. Here maximum length is not given. Since it is required to show a distance of 6.7 m, the maximum length should be an integer greater than 6.7 m. Therefore, assume the maximum length at least 7 m.
3. Length of scale, L_s = R.F. × Maximum length = $\dfrac{1}{40} \times 7 \times 100 = 17.5\text{ cm}$
4. Draw a rectangle having a 17.5 cm length and 10 mm width.
5. Divide the length of scale into seven equal parts, each part representing 1 m.
6. Divide first division of the scale into 10 equal parts, each representing 1 dm.
7. Mark units and subunits on the scale and write the value of R.F.
8. Mark a length 6.7 m on the scale, i.e., 6 metres on the right side of the zero mark and 7 dm on the left side of the zero mark.

Problem 4.12 *An area of 49 square centimetres on a map represents an area of 16 sq. m on a field. Draw a scale long enough to measure 8 m. Mark a distance of 6 m 9 dm on the scale.*

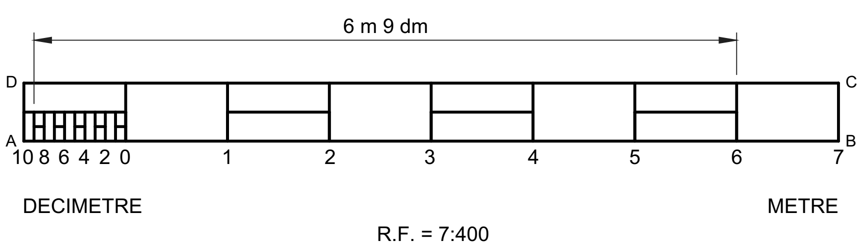


Fig. 4.6

Construction Refer to Fig. 4.6.

1. We know that R.F. is the ratio of lengths, therefore,

$$\text{R.F.} = \sqrt{\frac{49\ \text{cm}^2}{16\ \text{m}^2}} = \frac{7\ \text{cm}}{4\ \text{m}} = \frac{7\ \text{cm}}{4 \times 100\ \text{cm}} = \frac{7}{400}$$

2. Length of scale, L_s = R.F. × Maximum length = $\dfrac{7}{400} \times 8 \times 100 = 14\ \text{cm}$
3. Draw a rectangle having a 14 cm length and 10 mm width.
4. Divide the length of scale into eight equal parts, each part representing 1 m.
5. Divide the first division of the scale in 10 equal parts, each representing 1 dm.
6. Mark units and subunits on the scale and write the value of R.F.
7. Mark a length 6 m 9 dm on the scale as shown, i.e., 6 m on the right side of the zero mark and 9 dm on the left side of the zero mark.

Problem 4.13 *A cube of 5 cm side represents a tank of 1000 cubic metres volume. Find the R.F. and construct a scale to measure up to 35 m. Mark a distance of 27 m on it.*

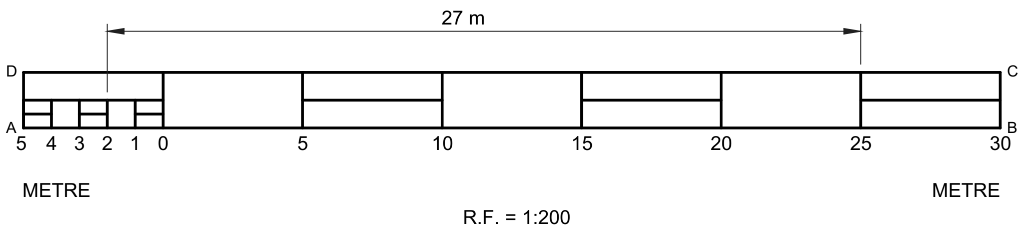


Fig. 4.7

Construction Refer to Fig. 4.7.

1. We know that R.F. is the ratio of lengths, therefore R.F. = $\dfrac{5\ \text{cm}}{\sqrt[3]{1000}\ \text{m}} = \dfrac{5\ \text{cm}}{10 \times 100\ \text{cm}} = \dfrac{1}{200}$
2. Length of scale, L_s = R.F. × Maximum length = $\dfrac{1}{200} \times 35 \times 100\ \text{cm} = 17.5\ \text{cm}$
3. Draw a rectangle having a 17.5 cm length and 10 mm width.
4. Divide the length of scale into seven equal parts, each part representing 5 m.
5. Divide first division of the scale into five equal parts, each representing 1 m.
6. Mark units and subunits on the scale and write the value of R.F.
7. Mark a distance of 27 metres on the scale, i.e., 25 m on the right side of the zero mark and 2 m on the left side of the zero mark.

Problem 4.14 *Construct a scale of 1:14 to read feet and inches and long enough to measure 7 feet. Show a distance of 5 feet 10 inches on it.*

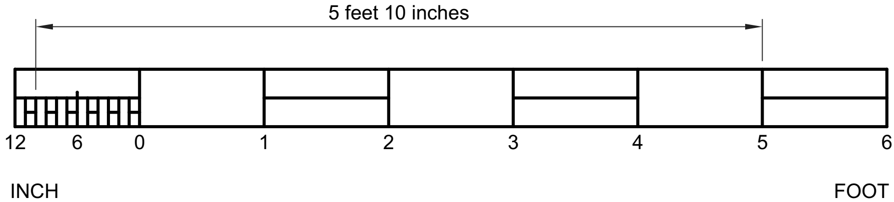


Fig. 4.8

Construction Refer to Fig. 4.8.

1. R.F. $= \dfrac{1}{14}$
2. Length of scale, L_s = R.F. × Maximum length $= \dfrac{1}{14} \times 7 \times 12 = 6$ inches $= 15.24$ cm
3. Draw a rectangle of length 6 inches, i.e., 15.24 cm and width 10 mm.
4. Divide the length of scale into seven equal parts, each part representing 1 foot.
5. Divide the first division of scale into 12 equal parts, each representing 1 inch.
6. Mark units and subunits on the scale and write the value of R.F.
7. Mark a distance of 5 feet 10 inches on the scale, i.e., 5 feet on the right side of the zero mark and 10 inches on the left side of the zero mark.

Problem 4.15 *Construct a scale of 1:54 to show yards and feet and long enough to measure 9 yards.*

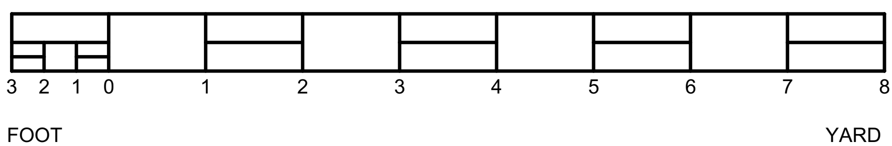


Fig. 4.9

Construction Refer to Fig. 4.9.

1. R.F. $= \dfrac{1}{54}$
2. Length of scale, L_s = R.F. × Maximum length $= \dfrac{1}{54} \times 9 \times 3 \times 12 = 6$ inches $= 15.24$ cm
3. Draw a rectangle of length 6 inches, i.e., 15.24 cm and width 10 mm.
4. Divide the length of scale in 9 equal parts, each part representing 1 yard.
5. Divide the first division of scale in three equal parts, each representing 1 foot.
6. Mark units and subunits on the scale and write the value of R.F.

4.8 DIAGONAL SCALE

A diagonal scale is used to represent three consecutive units, i.e., a unit and its immediate two sub divisions. For example, (a) metre, decimetre and centimetre (b) kilometre, hectometre and decametre, (c) yards, feet and inches, etc. A diagonal scale can measure more accurately than a plain scale.

4.8.1 Principle of Diagonal Scale

The third unit in a diagonal scale is obtained by diagonal principle which is described as follows (Fig. 4.10):

1. Draw a line *AB* of unit length.
2. Draw *BC* perpendicular to line *AB* taking any convenient length, say 50 mm. Join *AC*.
3. Divide *BC* into 10 equal parts and name the points as 1, 2, 3, etc.
4. Draw lines parallel to *AB* through points 1, 2, 3, etc. to meet *AC* at points 1′, 2′, 3′, etc.
5. The triangles *C*11′, *C*22′, *C*33′, *C*44′, etc., are similar to triangle CBA. Therefore, their sides are in proportional.

   i.e., $$11' = \frac{1}{10} \text{ of } AB, \quad 22' = \frac{2}{10} \text{ of } AB, \quad 33' = \frac{3}{10} \text{ of } AB, \text{ etc.}$$

   where *AB* is equal to one major unit. The lines 11′, 22′, 33′, etc., represent lengths in multiples of one-tenth of a unit.

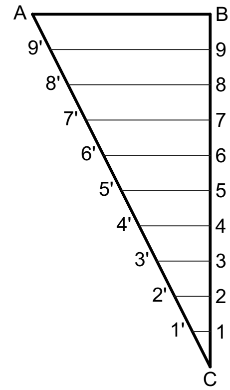


Fig. 4.10

4.8.2 Construction of Diagonal Scale

The following problems illustrate the construction of the diagonal scale. Let us first consider Problem 4.7, to read three units, i.e., metres, decimetres and centimetres.

Problem 4.16 *Construct a scale of 1:40 to read metres, decimetres and centimetres and long enough to measure up to 6 m. Mark a distance of 4.76 m on it.*

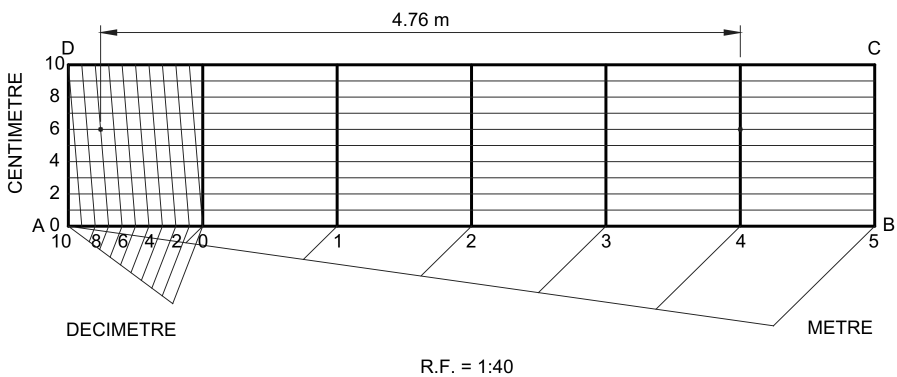


Fig. 4.11

Construction Refer to Fig. 4.11.

1. Given (a) R.F. = 1/40, (b) maximum length = 6 m and (c) least count = 1 cm.
2. Length of scale, L_s = R.F. × Maximum length $= \dfrac{1}{40} \times 6 \times 100\text{ cm} = 15\text{ cm}$
3. Draw a rectangle of length *AB* = 15 cm and width *AD* either 30 or 40 mm.
4. Here line *AB* represents 6 m. Divide *AB* into six equal parts so that each part represents 1 metre. Erect perpendicular lines through them to meet line *CD*. Mark the main units.
5. Divide the first division *A*0 into 10 equal subdivisions. Each subdivision represents 1 decimetre. Mark second unit on the scale. Also erect diagonal lines through them as shown.
6. Divide *AD* into 10 equal parts. Draw horizontal lines through each of them to meet *BC*. Mark third unit of the scale as shown.
7. Write the value of R.F. below the scale.
8. Mark a length of 4.76 m on the scale, i.e., 4 m on the right side of the zero mark, 7 dm on the left side of zero mark and move up along the diagonal line to six divisions.

Problem 4.17 *If 1 cm long line on a map represents a real length of 4 m. Calculate the R.F. and draw a diagonal scale long enough to measure up to 50 metres. Show a distance of 44.5 m on it.*

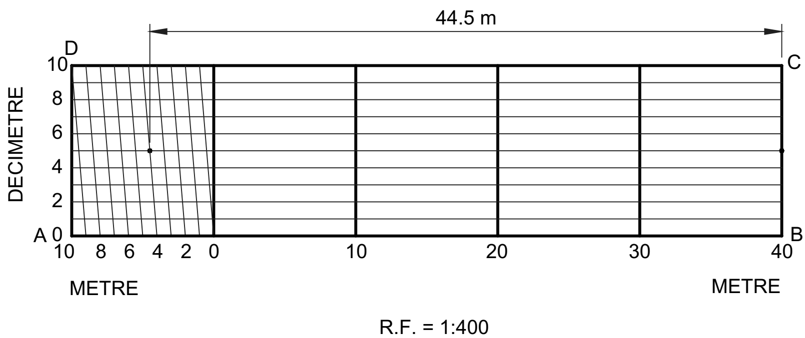


Fig. 4.12

Construction Refer to Fig. 4.12.

1. R.F. = $\dfrac{1\text{ cm}}{4\text{ m}} = \dfrac{1\text{ cm}}{4 \times 100\text{ cm}} = \dfrac{1}{400}$
2. Since scale has to show a distance of 44.5 m, assume the least count as 0.1 m.
3. Length of scale, L_s = R.F. × Maximum length = $\dfrac{1}{400} \times 50 \times 100 = 12.5\text{ cm}$
4. Draw a rectangle *ABCD* of length *AB* = 12.5 cm and width *AD* either 30 or 40 mm.
5. Here line *AB* represents 50 m. Divide *AB* into five equal parts so that each part represents 10 metres. Erect perpendicular lines through them to meet line *CD*. Mark the main units.
6. Divide the first division *A*0 into 10 equal subdivisions. Each subdivision represents 1 m. Mark second unit on the scale. Also erect diagonal lines through them as shown.
7. Divide *AD* into 10 equal parts. Draw horizontal lines through each of them to meet *BC*. Mark third unit of the scale as shown.

8. Write the value of R.F. below the scale.
9. Mark a length of 44.5 m on the scale, i.e., 40 metres on the right side of the zero mark, 4 m on the left side of zero mark and move up along the diagonal line to five divisions.

Problem 4.18 *The distance between two stations by road is 200 km and it is represented on a certain map by a 5 cm long line. Find the R.F. and construct a diagonal scale showing single kilometre and long enough to measure up to 600 km. Show a distance of 467 km on this scale.*

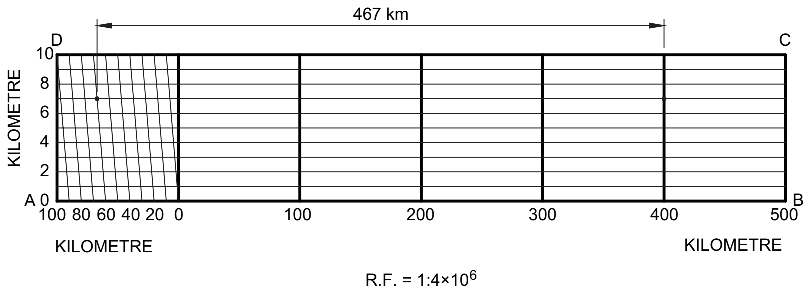


Fig. 4.13

Construction Refer to Fig. 4.13.

1. R.F. = $\dfrac{5\text{ cm}}{200\text{ km}} = \dfrac{5\text{ cm}}{200\times 10^5\text{ cm}} = \dfrac{1}{4\times 10^6}$
2. Length of the scale, L_s = R.F. × Maximum length = $\dfrac{1}{4\times 10^6}\times 600\times 10^5\text{ cm} = 15\text{ cm}$
3. Draw a rectangle *ABCD* of length *AB* = 15 cm and width *AD* either 30 or 40 mm.
4. Divide *AB* into six equal parts so that each part represents 100 km. Erect perpendicular lines through them to meet the line *CD*. Mark the main unit.
5. Divide first division *A*0 into 10 equal subdivisions, each representing 10 km. Mark second unit on the scale. Also erect diagonal lines through them as shown.
6. Divide *AD* into 10 equal parts and draw horizontal lines through each of them to meet line *BC*. Mark third unit of the scale along it as shown.
7. Write the value of R.F. below the scale.
8. Mark a length 467 km on the scale, i.e., 400 km on the right side of the zero mark, 60 km on the left side of zero mark and move up along the diagonal line by seven divisions.

Problem 4.19 *The distance between two points on a map is 15 cm. The real distance between them is 20 km. Draw a diagonal scale to measure up to 25 km and show a distance of 18.6 km on it.*

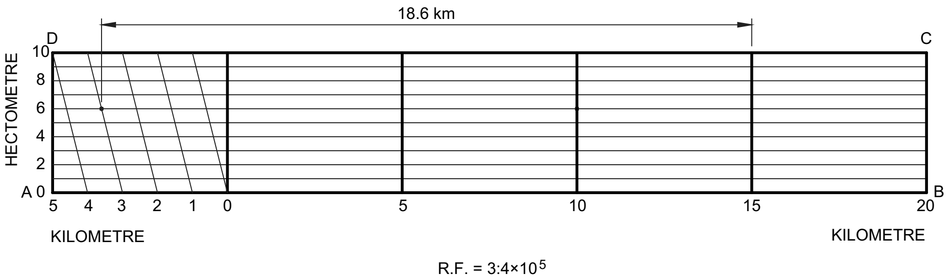


Fig. 4.14

Construction Refer to Fig. 4.14.

1. R.F. = $\dfrac{15\text{ cm}}{20\text{ km}} = \dfrac{15\text{ cm}}{20\times10^5\text{ cm}} = \dfrac{3}{4\times10^5}$
2. Since scale has to show a distance of 13.6 km, assume the least count as 0.1 km.
3. Length of scale, L_s = R.F. × Maximum length = $\dfrac{3}{4\times10^5}\times 25\times10^5\text{ cm} = 18.75\text{ cm}$
4. Draw a rectangle *ABCD* of length 18.75 cm and width either 30 or 40 mm.
5. Divide AB into five equal parts so that each part may represent 5 km. Erect perpendicular lines through them to meet the line *CD*.
6. Divide first division *A*0 into five equal subdivisions, each representing 1 km. Erect diagonal lines through them as shown.
7. Divide *AD* into 10 equal parts and draw horizontal lines through each of them to meet at *BC*.
8. Write main unit, second unit, third unit and the value of R.F.
9. Mark the distance 18.6 km on the scale, i.e., 15 km on the right side of the zero mark, 3 km on the left side of zero mark and move up along the diagonal line by six divisions.

Problem 4.20 *A length of 1 mm is enlarged 20 times in a drawing. What is the R.F. of this scale? Draw a diagonal scale of 0.01 mm least count and mark a distance of 7.83 mm on it.*

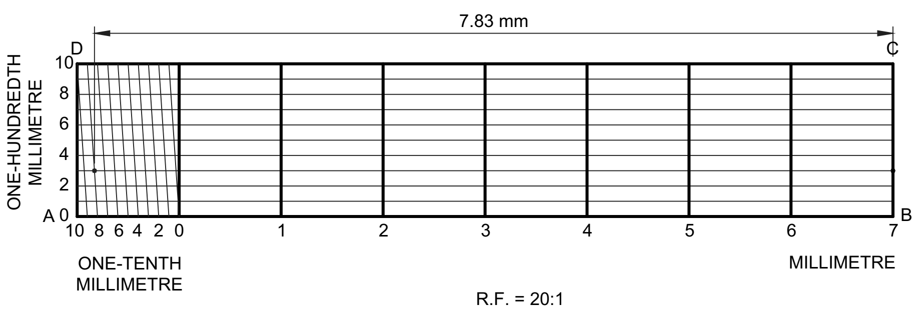


Fig. 4.15

Construction Refer to Fig. 4.15.

1. R.F. = 20:1; least count = 0.01 mm.
2. Since scale has to show a distance of 7.83 mm, the maximum length should be at least 8 mm.
3. Length of scale, L_s = R.F. × Maximum length = $20 \times 8 \text{ mm} = 160 \text{ mm} = 16 \text{ cm}$
4. Draw a rectangle *ABCD* of length 16 cm and width either 30 or 40 mm.
5. Divide *AB* into 8 equal parts so that each part may represent 1 mm. Erect perpendicular lines through them to meet the line *CD*.
6. Divide first division *A*0 into 10 equal subdivisions, each representing 0.1 mm. Erect diagonal lines through them as shown.
7. Divide *AD* into 10 equal parts and draw horizontal lines through each of them to meet at *BC*.
8. Write main unit, second unit, third unit and the value of R.F.
9. Mark the distance 7.83 mm on the scale, i.e., 7 mm on the right side of the zero mark, 0.8 mm on the left side of zero mark and move up along the diagonal line by three divisions.

Problem 4.21 *The distance between two stations is 100 km and on a road map it is shown by 30 cm. Draw a diagonal scale and mark 46.8 km and 32.4 km on it.*

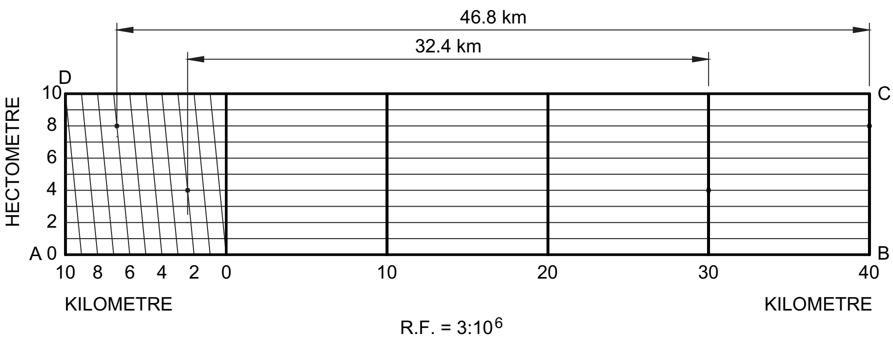


Fig. 4.16

Construction Refer to Fig. 4.16.

1. $\text{R.F.} = \dfrac{30 \text{ cm}}{100 \text{ km}} = \dfrac{30 \text{ cm}}{100 \times 10^5 \text{ cm}} = \dfrac{3}{10^6}$
2. Since scale has to show a distance of 46.8 km, the maximum length should be at least 50 km and the least count 0.1 km.
3. Length of scale, L_s = R.F. × Maximum length = $\dfrac{3}{10^6} \times 50 \times 10^5 \text{ cm} = 15 \text{ cm}$
4. Draw a rectangle *ABCD* of length 15 cm and width either 30 or 40 mm.
5. Divide *AB* into five equal parts, each representing 10 km. Erect perpendicular lines through them to meet the line *CD*.
6. Divide first division *A*0 into 10 equal subdivisions, each representing 1 km. Erect diagonal lines through them as shown.
7. Divide *AD* into 10 equal parts and draw horizontal lines through each of them to meet at *BC*.

8. Write main unit, second unit, third unit and the value of R.F.
9. Mark 46.8 km and 32.4 km as shown.

Problem 4.22 *Construct a scale to measure kilometre, 1/8th of a kilometre and 1/40th of a kilometre, in which 1 km is represented by 3 cm. Mark on this scale a distance of 4.825 km.*

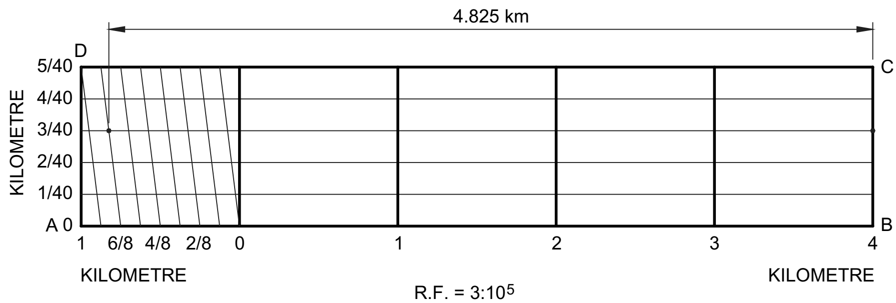


Fig. 4.17

Construction Refer to Fig. 4.17.

1. R.F. $= \dfrac{3\text{ cm}}{1\text{ km}} = \dfrac{3\text{ cm}}{1\times10^5\text{ cm}} = \dfrac{3}{10^5}$
2. Since scale has to show a distance of 4.825 km, the maximum length should be at least 5 km.
3. Length of scale, L_s = R.F. × Maximum length = $\dfrac{3}{10^5}\times 5\times 10^5\text{ cm} = 15\text{ cm}$
4. Draw a rectangle *ABCD* of length 15 cm and width either 30 or 40 mm.
5. Divide *AB* into five equal parts, each representing 1 km. Erect perpendicular lines through them to meet the line *CD*.
6. Divide 0A into eight equal subdivisions, each representing 1/8 km (i.e., 0.125 km). Erect diagonal lines through them as shown.
7. Divide *AD* into five equal parts, each representing 1/40 km (i.e., 0.025 km). Draw horizontal lines through each of them to meet *BC*.
8. Write main unit, second unit, third unit and the value of R.F.
9. The breakup of 4.825 km is $4\text{ km} + 6\times\dfrac{1}{8}\text{km} + 3\times\dfrac{1}{40}\text{km}$. Therefore, take 4 km on the right side of the zero mark, six subdivisions on the left side of zero mark and move up along the diagonal line by three divisions.

Problem 4.23 *An area of 400 square centimetres on a map represents an area of 25 square kilometres on a field. Construct a scale to measure up to 5 km and capable to show a distance of 3.56 km. Indicate this distance on the scale.*

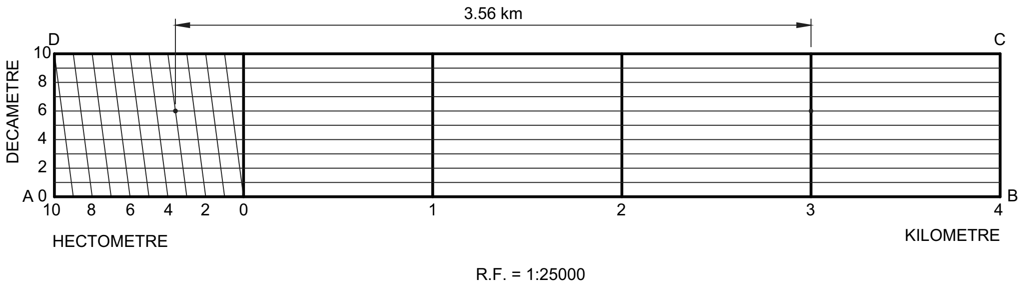


Fig. 4.18

Construction Refer to Fig. 4.18.

1. We know that R.F. is the ratio of lengths, therefore,

$$\text{R.F.} = \frac{\sqrt{400}\ \text{cm}}{\sqrt{25}\ \text{km}} = \frac{20\ \text{cm}}{5 \times 10^5\ \text{cm}} = \frac{1}{25000}$$

2. Since scale has to show a distance of 3.56 km, assume the least count as 0.01 km.
3. Length of scale, L_s = R.F. × Maximum length = $\frac{1}{25000} \times 5 \times 10^5\ \text{cm} = 20\ \text{cm}$
4. Draw a rectangle *ABCD* of length 20 cm and width either 30 or 40 mm.
5. Divide *AB* into five equal parts, each representing 1 km. Erect perpendicular lines through them to meet the line CD.
6. Divide first division *A*0 into 10 equal subdivisions, each representing 1 hm. Erect diagonal lines through them as shown.
7. Divide *AD* into 10 equal parts and draw horizontal lines through each of them to meet at *BC*.
8. Write main unit, second unit, third unit and the value of R.F.
9. Mark a distance of 3.56 km as shown.

Problem 4.24 *A rectangular field of 0.54 hectares is represented on a map by a rectangle of 3 cm × 2 cm. Calculate the R.F. Draw a diagonal scale to read up to a single metre and long enough to measure up to 500 metres. Show a distance of 438 m on it.*

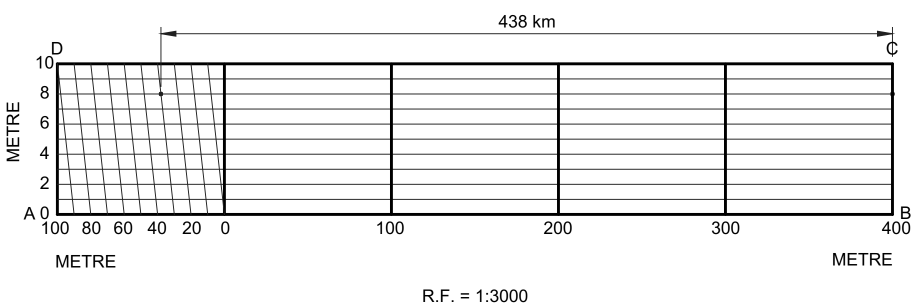


Fig. 4.19

Construction Refer to Fig. 4.19.

1. We know that R.F. is the ratio of lengths, therefore,

$$\text{R.F.} = \frac{\sqrt{3\times 2}\ \text{cm}}{\sqrt{0.54\times 10^4}\ \text{m}} = \sqrt{\frac{6}{0.54\times 10^4}} \times \frac{1\ \text{cm}}{100\ \text{cm}} = \frac{1}{3000}$$

2. Length of scale, L_s = R.F. × Maximum length = $\frac{1}{3000}\times 500\times 100\ \text{cm} = 16.7\ \text{cm}$
3. Draw a rectangle *ABCD* of length 16.7 cm and width either 30 or 40 mm.
4. Divide *AB* into five equal parts, each representing 100 m. Erect perpendicular lines through them to meet the line *CD*.
5. Divide first division *A*0 into 10 equal subdivisions, each representing 10 m. Erect diagonal lines through them as shown.
6. Divide *AD* into 10 equal parts and draw horizontal lines through each of them to meet at *BC*.
7. Write main unit, second unit, third unit and the value of R.F.
8. Mark a distance of 438 m as shown.

Problem 4.25 *A room of 1728 cubic metres volume is shown by a cube of 4 cm side. Find the R.F. and construct a scale to measure up to 50 m. Indicate a distance of 37.6 m on the scale.*

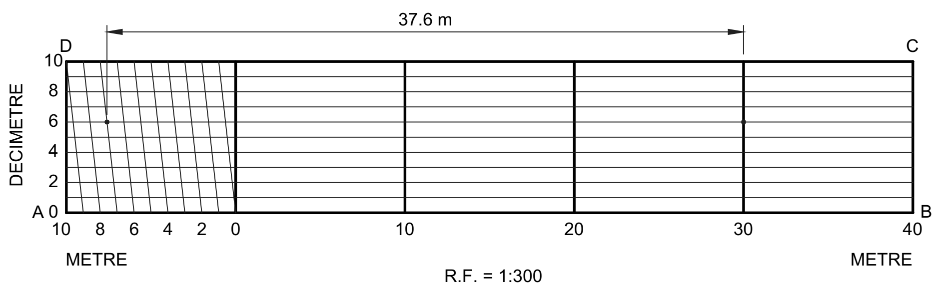


Fig. 4.20

Construction Refer to Fig. 4.20.

1. We know that R.F. is the ratio of lengths, therefore, $\text{R.F.} = \frac{4\ \text{cm}}{\sqrt[3]{1728}\ \text{m}} = \frac{4\ \text{cm}}{12\times 100\ \text{cm}} = \frac{1}{300}$
2. Length of scale, L_s = R.F. × Maximum length = $\frac{1}{300}\times 50\times 100\ \text{cm} = 16.67\ \text{cm}$
3. Draw a rectangle *ABCD* of length 16.67 cm and width either 30 or 40 mm.
4. Divide *AB* into five equal parts, each representing 10 m. Erect perpendicular lines through them to meet the line *CD*.
5. Divide first division *A*0 into 10 equal subdivisions, each representing 1 m. Erect diagonal lines through them as shown.
6. Divide *AD* into 10 equal parts and draw horizontal lines through each of them to meet at *BC*.
7. Write main unit, second unit, third unit and the value of R.F.
8. Mark a distance of 37.6 m as shown.

Problem 4.26 *Construct a diagonal scale of 1:63360 to read miles, furlongs and chains and long enough to measure up to 6 miles. Show a distance of 4 miles 3 furlongs 2 chains on it.*

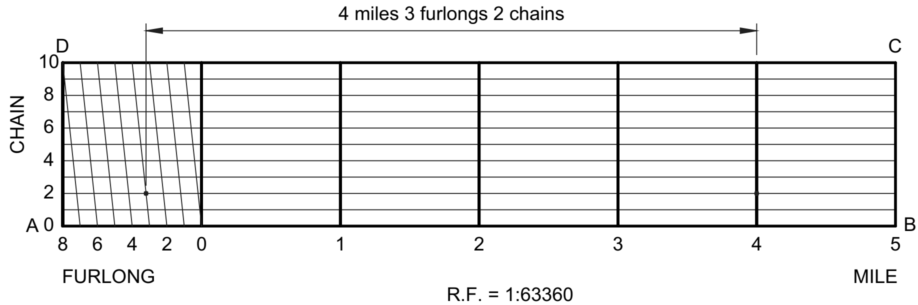


Fig. 4.21

Construction Refer to Fig. 4.21.

1. Given (a) R.F. = 1/63360, (b) maximum length = 6 miles and (c) least count = 1 chain
2. Length of scale, L_s = R.F. × Maximum length

$$= \frac{1}{63360} \times 6 \times 8 \times 10 \times 22 \times 3 \times 12 \text{ inches} = 6 \text{ inches} = 15.24 \text{ cm}$$

3. Draw a rectangle of length *AB* = 15.24 cm and width *AD* either 40 or 50 mm.
4. Divide *AB* into six equal parts, each part may represent 1 mile. Erect perpendicular lines through them to meet the line *CD*.
5. Divide first division *A*0 into eight equal subdivisions, each representing 1 furlong. Erect diagonal lines through them as shown.
6. Divide *AD* into 10 equal parts to represent 1 chain. Draw horizontal lines through each of them to meet at *BC*.
7. Write main unit, second unit, third unit and the value of R.F.
8. Mark a distance of 4 miles 3 furlongs 2 chains as shown.

Problem 4.27 *Construct a diagonal scale showing yards, feet and inches in which 2 inches long line represents 1.25 yards and is long enough to measure up to 5 yards. Find R.F. and mark a distance of 4 yards 2 feet 8 inches.*

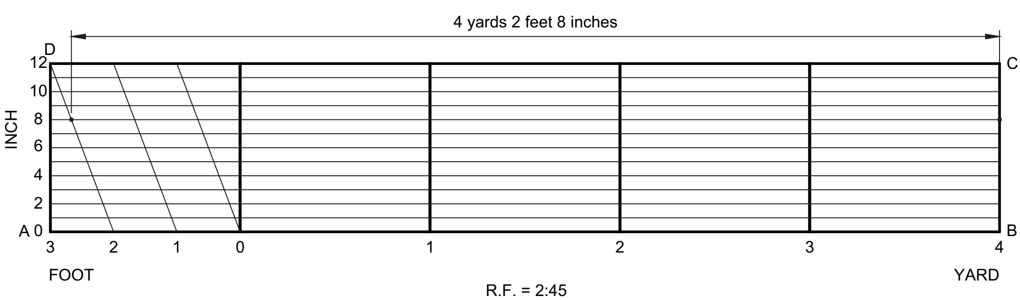


Fig. 4.22

Construction Refer to Fig. 4.22.

1. $\text{R.F.} = \dfrac{2 \text{ inches}}{1.25 \text{ yards}} = \dfrac{2 \text{ inches}}{1.25 \times 3 \times 12 \text{ inches}} = \dfrac{2}{45}$
2. Length of scale, L_s = R.F. × Maximum length $= \dfrac{2}{45} \times 5 \times 3 \times 12 \text{ inches} = 8 \text{ inches} = 20.32 \text{ cm}$
3. Draw a rectangle of length *AB* = 20.32 cm and width *AD* either 36 or 48 mm.
4. Divide *AB* into five equal parts to represent 1 yard each. Erect perpendicular lines through them to meet line *CD*.
5. Divide first division *A*0 into three equal subdivisions, each representing 1 foot. Erect diagonal lines through them as shown.
6. Divide *AD* into 12 equal parts to represent 1 inch. Draw horizontal lines through each of them to meet *BC*.
7. Write main unit, second unit, third unit and the value of R.F.
8. Mark a distance of 4 yards 2 feet 8 inches as shown.

4.9 COMPARATIVE SCALE

Comparative scale is a pair of scales having a common representative fraction but graduated to read different units. A map drawn in miles and furlongs can be measured directly in kilometres and hectometres with the help of a comparative scale. Comparative scales may be either plain or diagonal scales depending upon the requirement.

4.9.1 Construction of Comparative Scale

The following problems illustrate the construction of the comparative scales. Let us first consider Problem 4.7, to compare the metric scale with an equivalent British scale.

Problem 4.28 *Construct a scale of 1:40 to read metres and decimetres and long enough to measure up to 6 m. Also draw a comparative scale to show yards and feet and measure up to 7 yards. On this scale, determine the distance in British system which is equivalent to 5.2 m. Take 1 yard = 91.44 cm.*

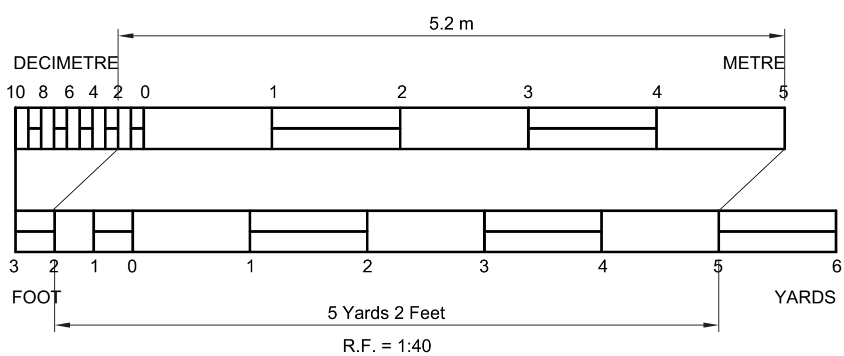


Fig. 4.23

Construction Refer to Fig. 4.23.

1. Given R.F. = 1/40.
2. For metric scale, maximum length = 6 metres and least count = 1 dm.

   Length of scale, L_s (metric system) = R.F. × Maximum length = $\frac{1}{40} \times 6 \times 100\text{ cm} = 15\text{ cm}$
3. For British scale, maximum length = 7 yards and least count = 1 foot.

   Length of scale, L_s (British system) = R.F. × Maximum length = $\frac{1}{40} \times 7 \times 91.44 = 16\text{ cm}$
4. Draw a 15 cm long plain scale to represent 6 m. Make its divisions and subdivisions so that its least count is 1 decimetre.
5. Draw another 16 cm long plain scale to represent 7 yards. Make its divisions and subdivisions so that its least count is 1 foot.
6. Write the main unit and the second unit for both the scales. Also write the value of R.F. below the scales.
7. Mark a distance of 5.2 m on the metric scale and show an equivalent length on the British scale as 5 yards 2 feet.

Problem 4.29 *On a railway map, an actual distance of 36 miles between two stations is represented by a line 10 cm long. Draw a plain scale to show mile and long enough to read up to 60 miles. Also draw a comparative scale attached to it to show kilometre and read up to 90 kilometres. On the scale show the distance in kilometres equivalent to 46 miles. Take 1 mile = 1609 metres.*

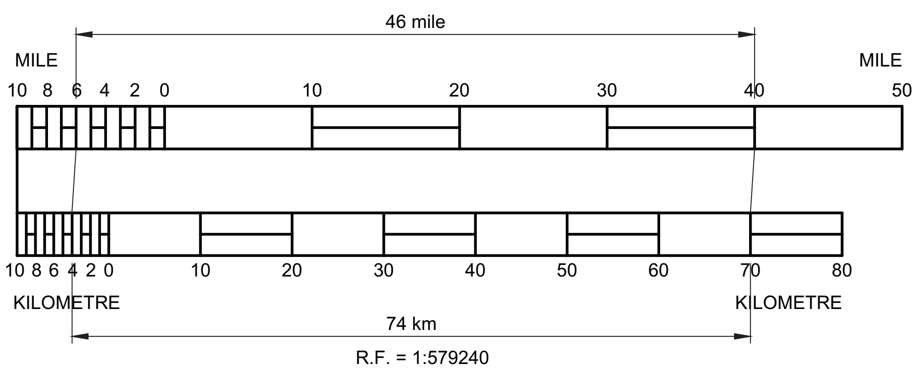


Fig. 4.24

Construction Refer to Fig. 4.24.

1. $\text{R.F.} = \frac{10\text{ cm}}{36\text{ miles}} = \frac{10\text{ cm}}{36 \times 1609 \times 100\text{ cm}} = \frac{1}{579240}$
2. For British scale, maximum length = 60 miles and least count = 1 mile.

   L_s (British system) = R.F. × Maximum length = $\frac{1}{579240} \times 60 \times 1609 \times 10^2 = 16.67\text{ cm}$

3. For metric scale, maximum length = 90 kilometres and least count = 1 kilometre.

   L_s (metric system) = R.F. × Maximum length = $\dfrac{1}{579240} \times 90 \times 10^5 = 15.54$ cm
4. Draw a 16.67 cm long plain scale to represent 60 miles. Make its divisions and subdivisions so that its least count is 1 mile.
5. Draw another 15.54 cm long plain scale to represent 90 km. Make its divisions and subdivisions so that its least count is 1 km.
6. Write the main unit and the second unit for both the scales. Also write the value of R.F.
7. Mark a distance of 46 miles on the British scale and show an equivalent length on the metric scale as 74 km.

Problem 4.30 *The distance between two cities is 150 km. A passenger train covers this distance in 5 hr. Construct a scale to measure the distance covered by the train in a single minute and up to 1 hour. The scale is drawn to 1:2 × 10^5. Show the distance travelled in 38 minutes.*

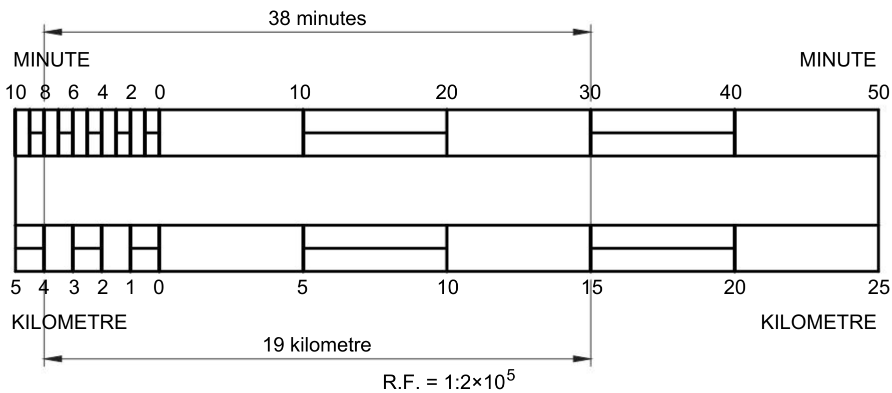


Fig. 4.25

Construction Refer to Fig. 4.25.

1. The distance covered in 1 hour = 150/5 = 30 km.
2. Length of scales, L_s = R.F. × Maximum length

   L_s (time scale) $= \dfrac{1}{2 \times 10^5} \times 1 \text{ hour} = \dfrac{1}{2 \times 10^5} \times 30 \text{ km} = \dfrac{1}{2 \times 10^5} \times 30 \times 10^5 \text{ cm} = 15 \text{ cm}$

   L_s (metric scale) $= \dfrac{1}{2 \times 10^5} \times 30 \times 10^5 \text{ cm} = 15 \text{ cm}$
3. Draw a 15 cm long plain scale to represent 60 minutes. Make its divisions and subdivisions so that its least count is one minute.
4. Draw another 15 cm long plain scale to represent 30 km. Make its divisions and subdivisions so that its least count is 1 km.
5. Write the main unit and the second unit for both the scales. Also write the value of R.F.
6. Mark a distance of 38 minute on the minute-scale and show an equivalent length on the metric scale as 19 km.

Problem 4.31 *A motorcycle is running at a speed of 40 km/hr. Construct a plain scale to read up to a kilometre and a minute. The scale should measure up to 50 km. The R.F. of the scale is 1:250000. On the scale show the distance covered in 39 min.*

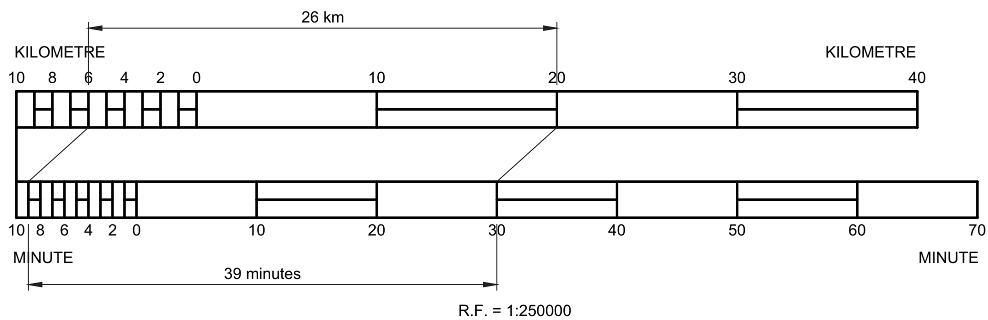


Fig. 4.26

Construction Refer to Fig. 4.26.

1. R.F. $= \dfrac{1}{250000}$; maximum length of metric scale = 50 km; least count of metric scale = 1 km.
   Assume maximum length of time scale = 80 minute; least count of metric scale = 1 minute.
2. Length of scales, L_s = R.F. × Maximum length

   $$L_s \text{ (metric scale)} = \frac{1}{250000} \times 50 \times 10^5 \text{ cm} = 20 \text{ cm}$$

   $$L_s \text{ (time scale)} = \frac{1}{250000} \times 80 \text{ minutes} = \frac{80}{250000} \times \frac{40 \times 10^5}{60} \text{ cm} = 21.33 \text{ cm}$$

3. Draw a 20 cm long plain scale to represent 50 km. Make its divisions and subdivisions so that its least count is 1 km.
4. Draw another 21.33 cm long plain scale to represent 80 minutes. Make its divisions and subdivisions so that its least count is 1 minute.
5. Write the main unit and the second unit for both the scales. Also write the value of R.F.
6. Mark a distance of 39 minute on the time scale and show an equivalent length on the metric scale as 26 km.

Problem 4.32 *If 1 centimetre long line on a map represents a real length of 4 m. Calculate the R.F. and draw a diagonal scale long enough to measure up to 50 m. Also draw a comparative scale to show chains, yards and feet and read up to 2.5 chains. On the scale show the distance in British system equivalent to 44.5 m. Take 1 chain = 20.1168 m.*

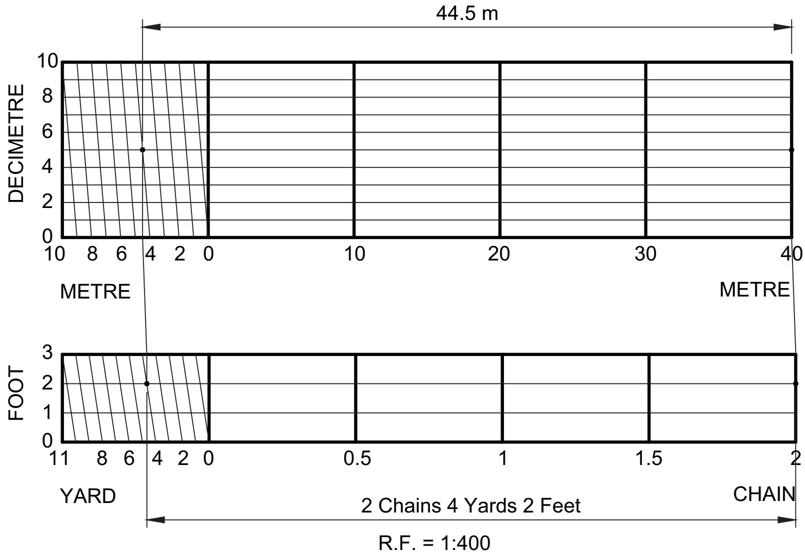


Fig. 4.27

Construction Refer to Fig. 4.27.

1. R.F. = $\dfrac{1\text{ cm}}{4\text{ m}} = \dfrac{1\text{ cm}}{4 \times 100\text{ cm}} = \dfrac{1}{400}$
2. For metric scale, maximum length = 50 metres and least count = 1 dm.

   Length of scale, L_s (metric) = R.F. × Maximum length = $\dfrac{1}{400} \times 50 \times 100 = 12.5\text{ cm}$
3. For British scale, maximum length = 2.5 chains and least count = 1 foot.

   Length of scale, L_s (British) = R.F. × Maximum length = $\dfrac{1}{400} \times 2.5 \times 20.1168 \times 10^2\text{ cm} = 12.57\text{ cm}$
4. Draw a metric diagonal scale 12.5 cm long to represent 50 m. Make its divisions and subdivisions such that least count of the scale is 1 dm.
5. Draw another British diagonal scale 12.57 cm long to represent 2.5 chains. Make its divisions and subdivisions such that least count of the scale is 1 foot.
6. Write main unit, second unit, third unit and R.F. on both the scales.
7. Mark a distance of 44.5 m on the metric scale and show an equivalent length on the British scale as 2 chains 4 yards 2 feet.

Problem 4.33 *A city bus is running at a speed of 50 km/hr. Construct a diagonal scale to show 1 km by 3 cm and to measure up to 6 km. Also mark on the scale the time taken by the bus to cover a distance of 4.56 km.*

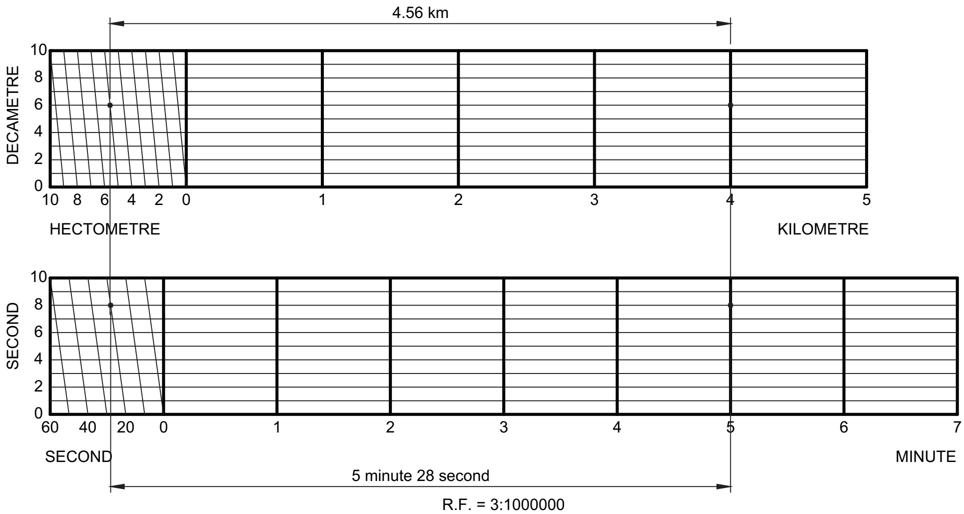


Fig. 4.28

Construction Refer to Fig. 4.28.

1. R.F. $= \dfrac{3\text{ cm}}{1\text{ km}} = \dfrac{3}{100000}$
2. Length of scales,

   L_s (metric) = R.F. × Maximum length = $\dfrac{3}{100000} \times 6 \times 10^5 \text{ cm} = 18 \text{ cm}$

   L_s (time) = R.F. × Maximum length = $\dfrac{3}{100000} \times 8 \text{ minutes} = \dfrac{3}{100000} \times 8 \times \dfrac{50 \times 10^5}{60} \text{ cm} = 20 \text{ cm}$
3. Draw an 18 cm long diagonal scale to represent 6 km. Make its divisions and subdivisions such that least count of the scale is 1 decametre.
4. Draw another 20 cm long diagonal scale to represent eight minutes. Make its divisions and subdivisions such that its least count of the scale is one second.
5. Write the main unit, the second unit and third unit for both the scales. Also write the value of R.F.
6. Mark a distance of 4.56 km on the metric scale and show an equivalent length on the time scale as five minutes and 28 seconds.

4.10 VERNIER SCALE

A vernier scale is used to measure three consecutive units of a metric scale. Thus, the measuring accuracy of the vernier scale is equivalent to that of a diagonal scale. The vernier scale consists of two parts: (a) a main scale and (b) a vernier. The vernier is an auxiliary scale that is attached at one end of the main scale. As it is difficult to subdivide the minor divisions of the main scale in an ordinary way, it is done with the help of the vernier. Most often, the vernier scale is used on length measuring devices such as vernier callipers and micrometres. Vernier scales are of two types namely direct vernier and retrograde vernier. Both are equally sensitive and easy to read.

1. Direct vernier or forward vernier The direct vernier has spaces shorter than those of the main scale. Figure 4.29(a) shows a direct vernier scale. Here the space of nine divisions of the main scale is equally distributed in 10 divisions on the vernier. As the main scale divisions are 1 unit apart, the vernier scale divisions will be 0.9 unit apart. In other words, each division on the vernier scale is 0.1 division smaller than one division on the main scale. This 0.1 division difference is utilised to read the third unit.

Thus, a direct vernier is one in which the space of (n–1) main scale divisions are divided into n vernier divisions. The vernier scale ratio is [(n):(n–1)].

2. Retrograde vernier or backward vernier The retrograde vernier has spaces longer than those of the main scale. Figure 4.29(b) shows a retrograde vernier scale. Here the space of 11 divisions of the main scale is equally distributed in 10 divisions on the vernier. As the main scale divisions are one unit apart, the vernier scale divisions will be 1.1 units apart. In other words, each division on the vernier scale is 0.1 division larger than one division on the main scale. This 0.1 division difference is utilised to read the third unit.

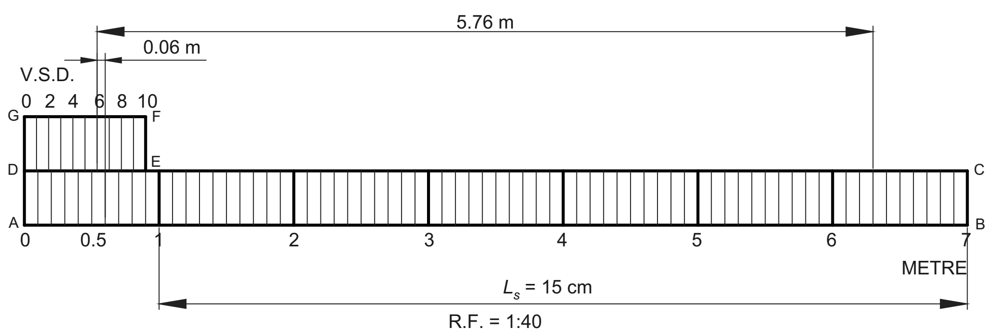
Thus, a retrograde vernier is one in which the space of (n + 1) main scale divisions are divided into n vernier divisions. The vernier scale ratio is [(n):(n + 1)].

The vernier enables an unambiguous interpolation between the smallest divisions on the main scale. The smallest distance measured is popularly called the *least count*. It is the difference between one main scale division (m.s.d.) and one vernier scale division (v.s.d.). In other words, the least count = |1 m.s.d. – 1 v.s.d.|

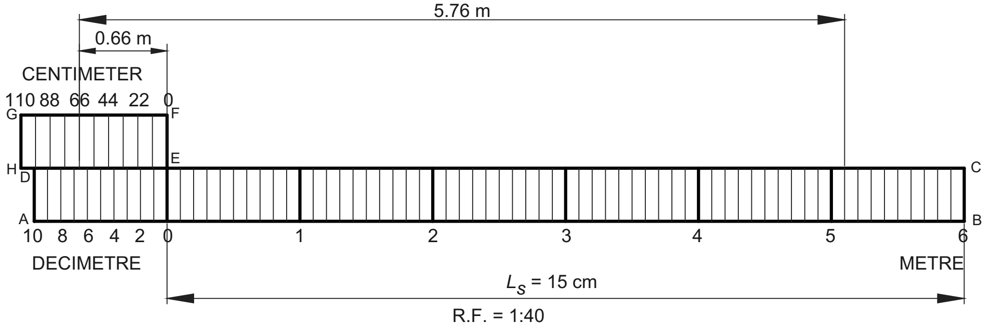
4.10.1 Construction of Vernier Scale

The following problems illustrate the method of construction of direct and retrograde-type vernier scales. Let us first consider Problem 4.16 in which a vernier scale is to be drawn to read three units, i.e., metres, decimetres and centimetres.

Problem 4.34 *Construct a vernier scale of 1:40 to read metres, decimetres and centimetres and long enough to measure up to 6 m. Mark a distance of 5.76 m on it.*



(a)



(b)

Fig. 4.29 **(a)** *Direct or forward vernier* **(b)** *Retrograde or backward vernier*

Construction

1. Given (a) R.F. = 1/40, (b) maximum length = 6 m and (c) least count = 1 cm.
2. Length of scale, L_s = R.F. × Maximum length = $\frac{1}{40} \times 6 \times 100 \text{ cm} = 15 \text{ cm}$
3. Use the following steps to draw the main scale:
   (a) Draw a rectangle *ABCD* of length 15 cm and width 10 mm.
   (b) Here the length of scale represents 6 m, therefore, divide it into six equal parts. Thus each main scale division represents 1 m.
   (c) Extend the length of the scale by one main scale division (i.e., 1 m) for placing vernier over it.
   (d) Further divide all the main scale divisions into 10 equal parts. Thus each subdivision represents 1 dm.
   (e) Mark the units for the main scale divisions as shown.

Method 1: Direct or forward vernier [Fig. 4.29(a)]

4. Draw direct vernier using the following steps:
   (f) Draw a 10 mm wide rectangle *DEFG* just above the left end of the main scale by taking *DE* = 9 main scale subdivisions. Here *DE* represents 9 dm.
   (g) Divide line *DE* on the vernier into 10 equal parts. Thus each small division of vernier represents 0.9 dm.
   (h) The difference between one main scale subdivision and one vernier division is 1 centimetre (1 dm – 0.9 dm = 0.1 dm = 1 cm). The distance between the consecutive divisions of the main scale and vernier increases in steps of 1 cm.
   (i) Mark the vernier scale divisions as shown.
5. The difference between sixth division of the vernier to the sixth division of the main scale is 0.06 m. Adding 5.7 m to this will give a length of 5.76 m. Therefore, mark a starting point at 6th division on the V.S.D. and end point at 6.3 m on the main scale.

Method 2: Retrograde or backward vernier [Fig. 4.29(b)]

6. Draw retrogate vernier using the following steps:
   (a) Draw a 10 mm wide rectangle *EFGH* above the left end of the main scale as shown by taking *EH* = 11 divisions of the main scale. Here *EH* represents 11 dm.

(b) Divide line *EH* on the vertical into 10 equal parts. Thus each small division of vernier represents 1.1 dm or 11 cm.

(c) The difference between one vernier division and one main scale division is 1 centimetre (1.1 dm – 1 dm = 0.1 dm = 1 cm).

(d) Mark the vernier scale units as shown.

7. Mark a starting point at 66 centimetres on the vernier and end point at 5.1 m on the main scale. This will give a length of 5.76 m.

Note

1. Both the direct and the retrograde vernier scales require a main scale for placing the vernier over it.
2. To show a distance of 5.76 m on either of the vernier scales, the 6 m long main scale is insufficient. There is a need of 7 m long main scale of which the first division will be used for placing the vernier.
3. Reading of the retrograde vernier is much simpler than the direct vernier. Therefore, the retrograde vernier is generally preferred in the drawings.

Problem 4.35 *If 1 cm long line on a map represents a real length of 4 m. Calculate the R.F. and draw a vernier scale long enough to measure up to 50 m. Show a distance of 44.5 m on it.*

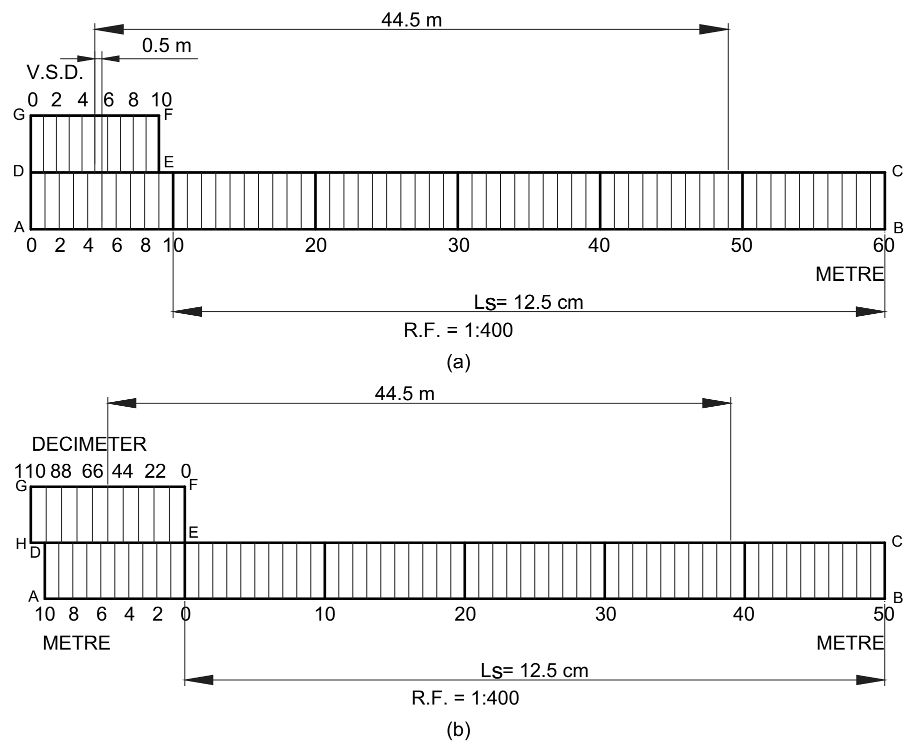


Fig. 4.30 **(a)** *Direct or forward vernier* **(b)** *Retrograde or backward vernier*

Construction Any one of the vernier scale (direct or retrograde) may be used.

1. R.F. = $\dfrac{1\,\text{cm}}{4\,\text{m}} = \dfrac{1\,\text{cm}}{4 \times 100\,\text{cm}} = \dfrac{1}{400}$
2. Length of scale, L_s = R.F. × Maximum length = $\dfrac{1}{400} \times 50 \times 100 = 12.5\,\text{cm}$
3. Use the following steps to draw the main scale:
   (a) Draw a rectangle *ABCD* of length 12.5 cm and width 10 mm.
   (b) Here the length of scale represents 50 m. Divide the length of rectangle into five equal parts. Each part represents 10 metre length.
   (c) Extend the length of the scale equivalent to one main unit (i.e., 10 metres) for placing vernier over it.
   (d) Divide all major unit lengths into 10 equal divisions so that each subdivision represents 1 m length.
   (e) Mark the units for the main scale divisions as shown.

Method 1: Direct vernier or forward vernier [Fig. 4.30(a)]

4. Draw vernier using the steps as follows:
   (a) Draw another rectangle *DEFG* over the first division of the main scale. Take length equivalent to 9 divisions of the main scale, i.e., 9 dm and width 10 mm.
   (b) Divide the lengths of vernier into 10 equal divisions. Each division of vernier represents 0.9 dm length.
   (c) The difference between one main scale division and one vernier division is 1 dm (1 m – 0.9 m = 0.1 m). The distance between the consecutive divisions of the main scale and vernier increases in steps of 1 dm.
   (d) Mark the vernier scale units as shown.
5. The difference between fifth division of the vernier to the fifth division of the main scale is 0.5 m. Adding 44 m to this will give a length of 44.5 m. Therefore, mark a starting point at fifth division on the V.S.D. and end at 49 m on the main scale.

Method 2: Retrograde vernier or backward vernier [Fig. 4.30(b)]

6. Draw vernier using the steps as follows:
   (a) Draw another rectangle *EFGH* over the first division of the main scale. Take length equivalent to 11 divisions of the main scale, i.e., 11 dm and width 10 mm.
   (b) Divide the lengths of vernier into 10 equal divisions. Each division of vernier represents 1.1 dm length.
   (c) The difference between one vernier division and one main scale division is 1 cm (1.1 dm – 1 dm = 1 cm).
   (d) Mark the vernier scale units as shown.
7. Mark a starting point at 55 decimetres on the vernier and end at 39 m on the main scale. This will give a length of 44.5 m.

Problem 4.36 *A real length of 10 m is represented by a line of 5 cm on a drawing. Find the R.F. and construct a vernier scale such that the least count is 2 dm and can measure up to 25 m. Mark a distance of 19.4 m on it.*

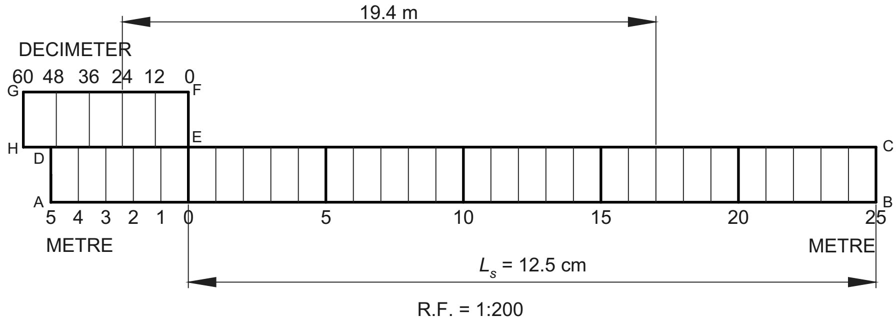


Fig. 4.31

Construction Refer to Fig. 4.31.

1. R.F. = $\dfrac{5\text{ cm}}{10\text{ m}} = \dfrac{5\text{ cm}}{1000\text{ cm}} = \dfrac{1}{200}$
2. Length of scale, L_s = R.F. × Maximum length = $\dfrac{1}{200} \times 25 \times 100 = 12.5\text{ cm}$
3. Use the following steps to draw the main scale:
   (a) Draw a rectangle *ABCD* of length 12.5 cm and width 10 mm.
   (b) Here the length of scale represents 25 m. Divide the length of rectangle into five equal parts. Each part represents 5 m length.
   (c) Extend the length of the scale equivalent to one main unit (i.e., 5 m) for placing vernier over it.
   (d) Divide all major unit lengths into five equal divisions so that each subdivision represents 1 m length.
   (e) Mark the units for the main scale divisions as shown.
4. Draw vernier using the steps as follows:
   (a) Draw another rectangle *EFGH* over the first division of the main scale. Take length equivalent to 6 divisions of the main scale, i.e., 6 m and width 10 mm.
   (b) Divide the lengths of vernier into five equal divisions. Each division of vernier represents 1.2 m length.
   (c) The difference between one vernier division and one main scale division is 2 dm (1.2 m – 1 m = 2 dm).
   (d) Mark the vernier scale units as shown.
5. Mark a starting point at 24 dm on the vernier and end at 17 m on the main scale. This will give a length of 19.4 m.

Problem 4.37 *On a map a rectangle of 125 cm × 200 cm represents an area of 6250 square kilometres. Draw a backward vernier scale to show decametre and long enough to measure up to 7 km. Show a distance of 6.43 km on it.*

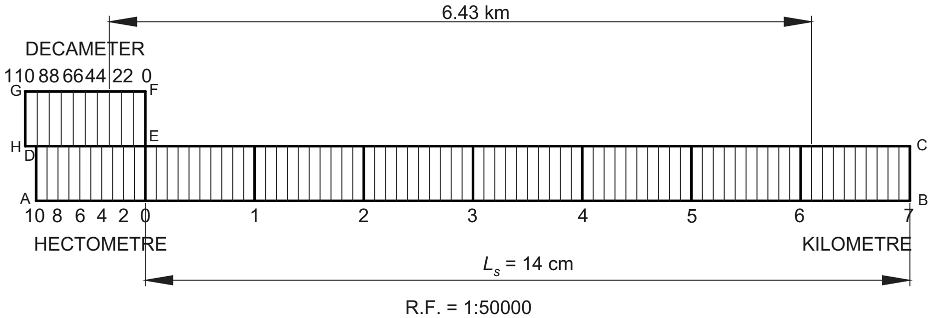


Fig. 4.32 *Retrograde or backward vernier*

Construction Refer to Fig. 4.32.

1. We know that R.F. is the ratio of lengths, therefore,

$$\text{R.F.} = \frac{\sqrt{125 \times 200}\ \text{cm}}{\sqrt{6250}\ \text{km}} = \sqrt{\frac{25000}{6250}} \times \frac{1\ \text{cm}}{10^5\ \text{cm}} = \frac{1}{50000}$$

2. Length of scale, L_s = R.F. × Maximum length = $\dfrac{1}{50000} \times 7 \times 10^5\ \text{cm} = 14\ \text{cm}$
3. Use the following steps to draw the main scale:
   (a) Draw a rectangle *ABCD* of length 14 cm and width 10 mm.
   (b) Here the length of scale represents 7 km. Divide the length of rectangle into seven equal parts. Each part represents 1 km length.
   (c) Extend the length of the scale equivalent to one main unit (i.e., 1 km) for placing vernier over it.
   (d) Divide all major unit lengths into 10 equal divisions so that each subdivision represents 1 hm length.
   (e) Mark the units for the main scale divisions as shown.
4. Draw vernier using the steps as follows:
   (a) Draw another rectangle *EFGH* over the first division of the main scale. Take length equivalent to 11 divisions of the main scale, i.e., 11 hm and width 10 mm.
   (b) Divide the lengths of vernier into 10 equal divisions. Each division of vernier represents 1.1 hm length.
   (c) The difference between one vernier division and one main scale division is 1 cm (1.1 Hm – 1 Hm = 1 Dm).
   (d) Mark the vernier scale units as shown.
5. Mark a starting point at 33 Dm on the vernier and end at 6.1 km on the main scale. This will give a length of 6.43 km.

Problem 4.38 *Construct a full size retrograde vernier scale of inches and show on it lengths 4.67 inches.*

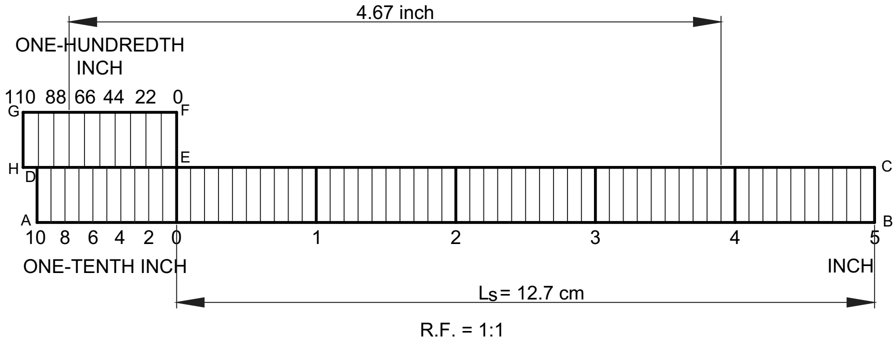


Fig. 4.33 *Use of retrograde vernier*

Construction Refer to Fig. 4.33.

1. Given (a) R.F. = 1 (full size scale),
2. Since scale has to show a distance of 4.67 inches, it is essential to have division of main scale and vernier in decimal system. The maximum length should be at least 5 in. and the least count 0.01 in.
3. Length of scale, L_s = R.F. × Maximum length = $5 \text{ inches} = 12.7 \text{ cm}$
4. Use the following steps to draw the main scale:
   (a) Draw a rectangle *ABCD* of length 12.7 cm and width 10 mm.
   (b) Here the length of scale represents 5 in. Divide the length of rectangle into 5 equal parts. Each part represents 1 inch length.
   (c) Extend the length of the scale equivalent to one main unit (i.e., 1 inch) for placing vernier over it.
   (d) Divide all major unit lengths into 10 equal divisions so that each subdivision represents 0.1 in. length.
   (e) Mark the units for the main scale divisions as shown.
5. Draw vernier using the steps as follows:
   (a) Draw another rectangle *EFGH* over the first division of the main scale. Take length equivalent to 11 divisions of the main scale, i.e., 1.1 inches and width 10 mm.
   (b) Divide the lengths of vernier into 10 equal divisions. Each division of vernier represents 0.11 in. length.
   (c) The difference between one vernier division and one main scale division is 1 centimetre (0.11 in. – 0.1 in. = 0.01 in.).
   (d) Mark the vernier scale units as shown.
6. Mark a starting point at 77 in. on the vernier and end at 3.9 in. on the main scale. This will give a length of 4.67 km.

4.11 SCALE OF CHORDS

In absence of a protractor, a scale of chords may be used to measure the angle or to set the required angle. The construction is based on the lengths of chords of angles measured on the same arc.

4.11.1 Construction of Scale of Chords

Figure 4.34 shows the scale of chords. The steps to construct the scale of chords are as follows:

1. Draw a line *AB* of any convenient length.
2. At point *B*, erect a line *BF* perpendicular to *AB*.
3. With centre *B* draw an arc *AC* cutting *BF* at point *C*. This arc subtends 90° angle at the centre *B*.
4. Divide this arc *AC* into nine equal parts, each representing 10º division at *B*.
5. With centre *A* turn down the divisions to line *AB* produced. Thus, the distance *AE* on the scale represents the length of chord *A*-30, which subtends an angle 30° at the point *B*.
6. Similarly, mark 5° divisions on the line *AB*.
7. Complete the scale by drawing a rectangle below *AD* as shown. It may be noted that the divisions obtained are unequal, decreasing gradually from *A* to *D*. Scale *AD* is the required scale of chords.

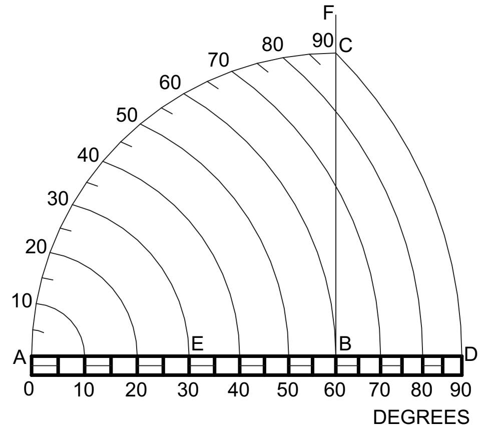


Fig. 4.34 *Scale of chords showing 5º divisions*

4.11.2 Application of Scale of Chords

Scale of chords are used to set off the given angle and measure the given angle.

Set-off the Given Angle

Consider the following problems.

Problem 4.39 *Construct a scale of chords showing 5° divisions and with its aid set-off angles 40°, 55° and 130°.*

Construction Refer to Fig. 4.35.

1. Draw scale of chords as shown in Fig. 4.34.
2. Draw a line *PQ*.
3. Draw an arc *RS* with *P* as the centre and *AB* as the radius.
4. Draw an arc with *Q* as centre and radius equal to 0°–40° chord length, to intersect arc *RS* at point *T*. Join *PT* and produce. The ∠*TPQ* represents 40°.
5. Similarly, draw an arc with *Q* as centre and radius equal to 0°–55° chord length, to intersect arc *RS* at point *U*. Join *PU* and produce. The ∠*UPQ* represents 55°.
6. Draw an arc with *Q* as centre and radius equal to 0°–90° chord length, to intersect arc *RS* at point *V*. Draw another arc with *V* as centre and radius equal to 0°–40° chord length, to intersect arc *VS* at point *W*. Join *PW* and produce. The ∠*WPQ* represents 130° (90° + 40° = 130°).

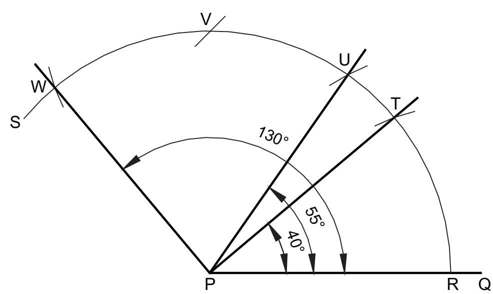


Fig. 4.35

Problem 4.40 *Construct a scale of chords showing 6° divisions and with its aid set-off an angle of 54°.*

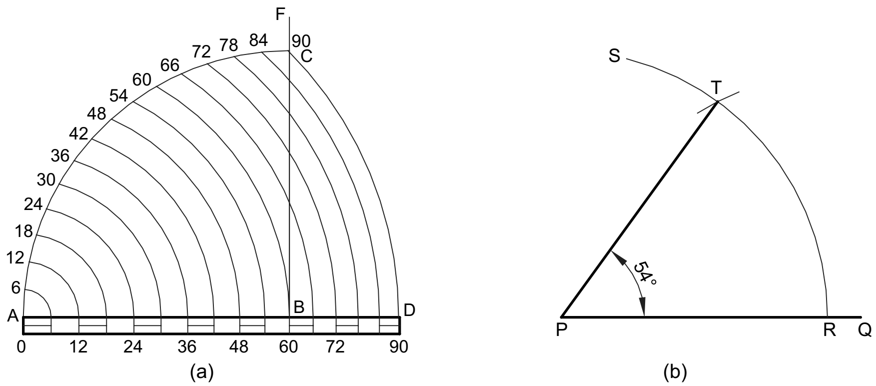


Fig. 4.36 **(a)** *Scale of chords showing 6° divisions* **(b)** *Mark an angle of 54°*

Construction Refer to Figs. 4.36(a) and (b).

1. Draw a line *AB* of any convenient length.
2. At point *B*, erect a line *BF* perpendicular to *AB*.
3. With centre *B* draw an arc *AC* cutting *BF* at point *C*. This arc subtends 90° angle at the centre *B*.
4. Divide this arc *AC* into 15 equal parts, each representing 6° division at *B*.
5. With centre *A* turn down the divisions to line *AB* produced.
6. Complete the scale by drawing a rectangle below *AD* to represent the required scale of chords.
7. Draw a line *PQ*. Draw an arc *RS* with *P* as the centre and *AB* as the radius.
8. Draw an arc with *Q* as centre and radius equal to 0°–54° chord length, to intersect arc *RS* at point *T*. Join *PT* and produce. The ∠*TPQ* represents 54°.

Measure the Given Angle

Consider the following problem.

Problem 4.41 *Construct a scale of chords showing 5° divisions and with its aid measure angle PQR shown in Fig. 4.37.*

Construction Refer to Fig. 4.37.

1. Draw scale of chord as shown in Fig. 4.34.
2. Draw an arc with *Q* as the centre and radius equal to *AB* the scale of chords, to meet line *PQ* at *T* and line *RQ* at *S*.
3. Transfer the chord length *ST* on the scale of chords and read the angle. Here it is 55°.

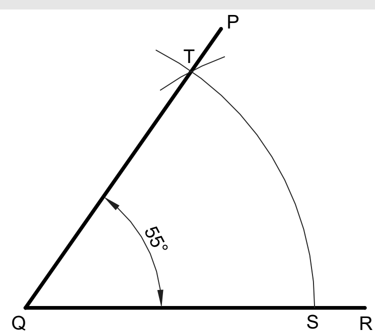


Fig. 4.37

EXERCISE 4A

Plain scale

4.1 Construct a scale of 1:500 to read a single meter and long enough to measure up to 70 m. Mark on it a distance of 46 m.

4.2 A length of 2 dm is represented by 5 cm. Find the R.F. and construct a plain scale to measure up to 8 dm and mark a distance of 68 m on it.

4.3 Construct a scale of R.F. 1:125 to read a single metre and long enough to measure lengths up to 25 m. Show the length of 16 m on this scale.

4.4 A 3.2 cm long line represents a distance of 4 m. Extend the line to measure up to 20 m and show on it units of metre and 5 m. Show a length of 18 m on this scale.

4.5 A line of 1 cm represents a length of 1.25 decimetres. Draw a plain scale and mark a distance of 2.3 m on it.

4.6 An area of 81 sq. cm on a map represents an area of 36 sq. m on a field. Draw a scale long enough to measure 9 m. Mark a distance of 7 m 4 dm on the scale.

4.7 A rectangular plot of 100 sq. km is represented on a certain map by a rectangular area of 4 sq. cm. Draw a scale to show 50 km and mark a distance of 41 km on it.

4.8 A rectangular plot of land of area 16 sq. m is represented on a map by a similar rectangle of 1 square centimetre. Calculate the R.F. of the scale of the map. Construct a plain scale to read metres and long enough to measure up to 60 metres. Indicate a distance of 45 m on the scale.

4.9 A cube of 5 cm side represents a tank of 8000 cubic metres volume. Find the R.F. and construct a scale to measure up to 60 metres and mark on it a distance of 47 m. Indicate R.F. of the scale.

4.10 A container of 1000 cubic metres volume is represented by a block of 125 cubic cm volume. Fine R.F. and construct a scale to measure up to 30 m. Measure a distance of 19 m on this scale.

4.11 Construct a scale of 1:5 to read feet and inches and long enough to measure 2 feet 6 inches. Show a distance of 1 foot 3 inches on it.

Diagonal scale

4.12 The R.F. of the scale is 3:100. Construct a scale to measure a maximum distance of 5 m and show a distance 3.49 m on it.

4.13 Construct a diagonal scale of 2:75 to read meters, decimetres and centimetres and long enough to measure up to 6 m. Mark on it a distance of 2.47 m.

4.14 Construct a diagonal scale showing hectometres, decametres and metres, in which 1 cm long line represents 50 metres and long enough to measure up to 1 km. Find R.F. and mark a distance 5 Hm 3 Dm 7 m on it.

4.15 The distance between two stations by rail is 50 km and it is represented on a certain map by a 1 cm long line. Find the R.F. and construct a diagonal scale showing single km and long enough to measure up to 700 km. Indicate a distance of 538 km on this scale.

4.16 The distance between two points on a map is 5 cm. The real distance between them is 20 metres. Draw a diagonal scale to measure up to 60 m and show a distance of 43.6 m on it.

4.17 The distance between two stations is 10 km and on a road map it is shown by 3 cm. Draw a diagonal scale and indicate a distance of 47.9 km and 32.3 km on it.

4.18 Construct a scale to measure kilometre, 1/8th of a kilometre and 1/40th of a kilometre, in which 1 km is represented by $3\frac{1}{2}$ centimetre. The scale should be long enough to measure up to 5 kilometres. Show on this a distance of $3\frac{23}{40}$ km.

4.19 An area of 63 sq. cm on a map represents an area of 1.75 sq. km on a field. Construct a scale to measure up to 2.5 km and capable to show hundredth of a kilometre. Indicate 1.87 km on the scale.

4.20 A rectangular field of 25000 sq. m is represented on a map by a rectangle of 5 cm × 4 cm sides. Calculate then R.F. Draw a diagonal scale to read single meter and long enough to measure up to 500 m. Mark a length of 362 m on the scale.

4.21 A cube of 5 cm side shows a tank of 8000 cubic metres volume. Find the R.F. and construct a scale to measure up to 70 m and mark a distance of 53 m on it.

4.22 Construct a diagonal scale of R.F 1:4000 and having a least count of 1 m. Using this scale, construct a right-angled triangle of sides 301 m and 402 m and find the length of hypotenuse.

Comparative scale

4.23 Construct a scale of 1:60 to show metres and decimetres and long enough to measure up to 7 metres. Also draw a comparative scale attached to it to show yards and feet and read up to 8 yards. On the scale show the distance in British system equivalent to 6.7 m. Take 1 yard = 91.44 cm.

4.24 What is the R.F. of a scale which measures 1/8th inches to a mile? Draw plain comparative scales of two units to measure up to 60 miles and 100 km. Take 1 mile = 1.609 km.

4.25 Construct a plain scale to compute time in minutes and distance covered by a train in kilometres, when the train passes between two stations 240 kilometres apart in four hours. The scale should have R.F. 1/400000. Show the distance covered in 45 min. on the scale.

4.26 A 4 cm long line on a map represents an actual length of 200 miles. Construct a comparative diagonal scale to read up to a single mile and a single kilometre. Mark a distance of 653 miles and find the corresponding length in kilometres. Take 1 mile = 1.6 km.

4.27 The distance between Bhopal and Indore by train is 140 miles. It is represented by a 7 cm long line on a map. Draw a comparative diagonal scale to represent 350 miles. Mark a length of 237 miles and with the help of the scale find the corresponding distance in kilometres. Take 1 mile = 1.6 km.

4.28 The distance between two stations A and B is 144 km and is covered by train in four hours. Draw a plain scale to measure the time up to single minute. R.F. of the scale is 1:240000. Calculate the distance covered by the train in 45 minutes and show minutes on the scale.

4.29 An aeroplane is flying at a speed of 360 km/hr. Draw a diagonal scale to represent 6 kilometres by 1 centimetre and to show distance up to 60 km. Find the R.F. of the scale and find from the scale the distance covered by the aeroplane in (a) 3 minutes 22 seconds and (b) 7 minutes 49 seconds.

4.30 The distance between Hyderabad and Bangalore is 600 km. An express train takes 12 hours to cover this distance. Construct a comparative plain scale to measure the distance covered up to 1 hour. With the help of the scale find the following: (a) the distance covered in 36 minutes and (b) the time taken to travel 354 km. Take R.F. as 1:312500.

4.31 A train is running at a speed of 40 km/hr. Construct a comparative scale to read a kilometre and a minute and long enough to read up to 50 kilometres. The R.F. of the scale is 1:250000. On the scale show the distances travelled by the train in 42 minutes.

Vernier scale

4.32 Construct a retrograde vernier scale to be used with a map, the scale of which is 1 cm = 40 m. The scale should read in metres and maximum up to 500 m. Mark a distance of 456 m on it.

4.33 The actual length of 300 metres is represented by a line of 10 cm on a drawing. Draw a vernier scale to read up to 500 m. Mark on it a length of 367 m.

4.34 Construct a vernier scale to the read meters, decimetres, centimetres and long enough to measure up to 6 m, when 1 metre is represented by 25 mm. Find R.F. and show a distance of 4.36 m on it.

4.35 Construct a vernier scale to read distance correct to decametre on a map in which the actual distances are reduced in the ratio of 1:40000. The scale should be long enough to measure up to 6 kilometres. Mark on the scale a length of 5.64 km.

4.36 The area of a field is 50000 sq. m. The length and breadth of the field on the map is 10 centimetres and 8 cm respectively. Construct a backward vernier scale which can read up to single metre. Mark the length of 235 m on the scale. What is the R.F. of the scale?

Scale of chords

4.37 Construct a scale of chords showing 10° divisions and with its aid set-off angles of 40° and 150°.

4.38 With the help of the scale of chords and a least count of 15°, construct the following angles: (a) 75°, (b) 105° and (c) 225°.

4.39 The least count of a scale of chords is 18°. With the help of this scale erect the angles of 72° and 108°.

4.40 Using 6° least count in a scale of chords construct the following angles: (a) 42°, (b) 108° and (c) 210°.

VIVA-VOCE QUESTIONS

4.1 Distinguish among full size, reduced size and enlarged size drawing.
4.2 Explain reducing scale and give two practical applications.
4.3 Explain enlargement scale and give two practical applications.
4.4 State the advantages of graphical scale over an engineering scale?
4.5 What is representative fraction?
4.6 Enlist types of scales used in engineering practices.
4.7 Explain the principle of diagonal scale?
4.8 What are the advantages of a diagonal scale over a plain scale?
4.9 What is a comparative scale?
4.10 What is the difference between a direct and a retrograde vernier?
4.11 What is the difference between a diagonal and a vernier scale?
4.12 What are the applications of the scale of chords?

MULTIPLE-CHOICE QUESTIONS

4.1 For drawing the components of a wrist watch, the scale used is
(a) reduction scale (b) full size scale
(c) enlargement scale (d) Any of these

4.2 Drawing of a building can be made on *A*0 size drawing sheet using
(a) reduction scale (b) full size scale
(c) enlargement scale (d) None of these

4.3 The R.F. is always
(a) less than 1 (b) equal to 1
(c) greater than 1 (d) Any of these

4.4 The unit of R.F. is
(a) cubic centimetre (b) square centimetre
(c) centimetre (d) None of these

4.5 The full form of R.F. is
(a) reduction fraction (b) representative fraction
(c) reduction factor (d) representative factor

4.6 The R.F. of the scale on a mini-drafter is
(a) 0 (b) 1
(c) 10 (d) none of these

4.7 A map of 10 cm $\times$ 8 cm represents an area of 50000 sq. m of a field. The R.F. of the scale is
(a) 1/25 (b) 1/625
(c) 1/2500 (d) 1/6250000

4.8 An area of 36 sq. km is represented by 144 square centimetres on a map. The R.F. is
(a) 1/4 (b) 1/2
(c) 1/5000 (d) 1/50000

4.9 The length of scale with R.F. 1/40 to measure up to 6 m will be
(a) 10 cm (b) 12 cm
(c) 15 cm (d) 20 cm

4.10 When measurements are required in three consecutive units, the appropriate scale is
(a) plain scale (b) diagonal scale
(c) isometric scale (d) scales of chords

4.11 The diagonal scale is most suitable to take a measurement of
(a) diameter of a circle
(b) diagonal of a square
(c) side of a pentagon
(d) All of these

4.12 The scale used for measuring in two systems of units is
(a) plain scale (b) diagonal scale
(c) comparative scale (d) vernier scale

4.13 The diagonal of a square can be measured by
(a) plain scale (b) diagonal scale
(c) vernier scale (d) All of these

4.14 Comparative scale is a pair of scale having a common
(a) units (b) R.F.
(c) length of scale (d) least count

4.15 An angle can be set off and measured with the help of
(a) plane scale (b) diagonal scale
(c) comparative scale (d) scale of chords

Answers to multiple-choice questions

4.1 (c), 4.2 (a), 4.3 (d), 4.4 (d), 4.5 (b), 4.6 (b), 4.7 (c), 4.8 (d), 4.9 (c), 4.10 (b), 4.11 (d), 4.12 (c), 4.13 (d), 4.14 (b), 4.15 (d)

Chapter

5 CONIC SECTIONS

5.1 INTRODUCTION

Historically, the term straight line and curved lines were used to distinguish the line without curvature and with curvature. Today, line is considered to have null curvature whereas the curve is considered to have certain curvature. The curves may be a two-dimensional called plane curve or a three-dimensional called space curve. In engineering practice, a number of objects contain *plane algebraic curve of two-degree called conic sections*. This chapter deals with a few common methods of construction of the conic sections and the field of their application.

5.2 CONE

A cone is formed if a right-angled triangle with an apex angle α is rotated about its altitude as the axis, shown in Fig. 5.1(a). The height and the radius of the base of the cone are respectively equal to the altitude and the base of the triangle. The apex angle of the cone is 2α, shown in Fig. 5.1(b). Any imaginary line joining the apex to the circumference of the base circle is called a generator. A cone has an infinite number of generators.

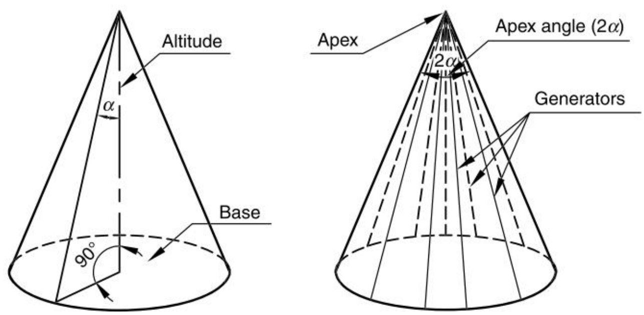


Fig. 5.1 *Cone and nomenclature*

5.3 CONIC SECTIONS

A conic section is a curve obtained by cutting a right-circular cone with the help of a plane in different positions relative to the axis. Traditionally, there are three types of conic section, namely; the ellipse, the parabola, and the hyperbola. The circle is a special case of the ellipse is sometimes called the fourth type of conic section. The isosceles triangle can also obtained by cutting the cone with a section plane, but it is not considered as the conic section. Various types of conic sections and their applications are as follows:

1. Circle When the cutting plane is perpendicular to the axis of the cone, i.e., $\theta = 90^{o}$, the curve of intersection obtained is a circle (*see Fig. 5.2*). Often the circle is known as the fourth type of conic section.

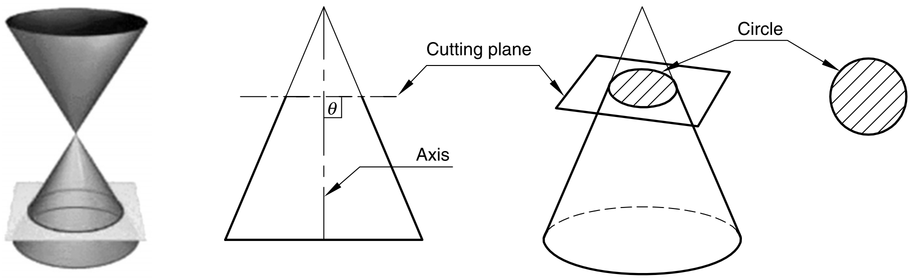


Fig. 5.2 *Formation of a circle by cutting a cone*

Applications Circles find their application in a vast number of objects such as diaphragms, discs, rings, plates, etc. A circle revolving around its axis forms a surface called a *sphere*.

2. Isosceles triangle

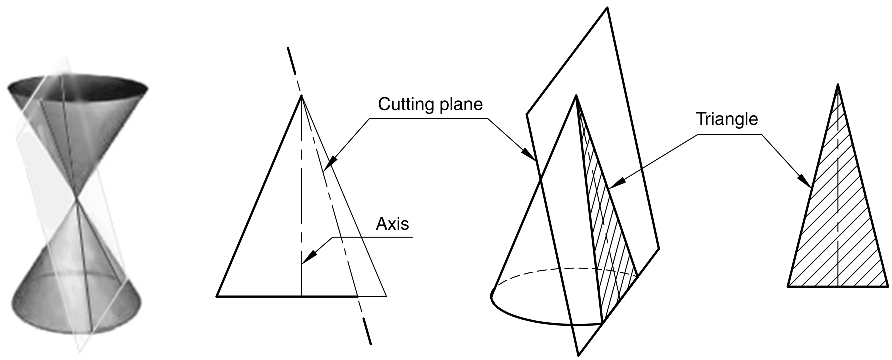


Fig. 5.3 *Formation of an isosceles triangle by cutting a cone*

When the cutting plane is passing through the apex and cuts the base of the cone, the curve of intersection is an isosceles triangle (*see Fig. 5.3*). The inclination of the cutting plane is obviously less than half of the apex angle, i.e., $\theta < \alpha$. Although, the isosceles triangle can also obtained by cutting the cone with a section plane, but it is not considered as the conic section.

3. Ellipse When the cutting plane is inclined to the axis and cut all the generators of the cone, the section is an ellipse. The inclination of the cutting plane for an ellipse should be greater than half of the apex angle, i.e., $\theta > \alpha$ (*see Fig. 5.4*).

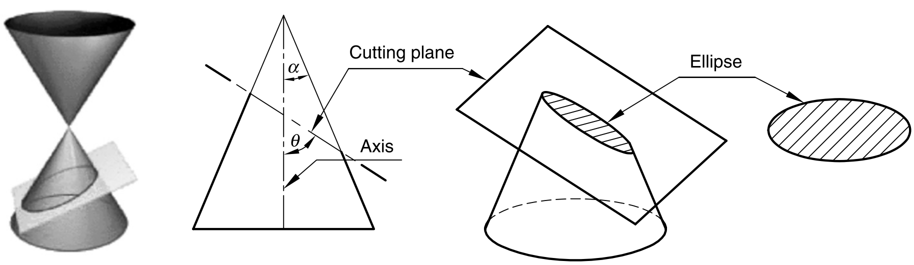


Fig. 5.4 *Formation of an ellipse by cutting a cone*

Applications Elliptical curves find their use in concrete arches shown in Fig. 5.5(a), stone bridges, dams, monuments (memorial structure), man-holes, glands, stuffing boxes, etc. A planet travels around the sun in an elliptical orbit with the sun at one of its foci, shown in Fig. 5.5(b). The orbits of the moon and artificial satellites of the earth are also elliptical. On a far smaller scale, the electrons of an atom move in an approximately elliptical orbit with the nucleus at one focus, shown in Fig. 5.5(c).

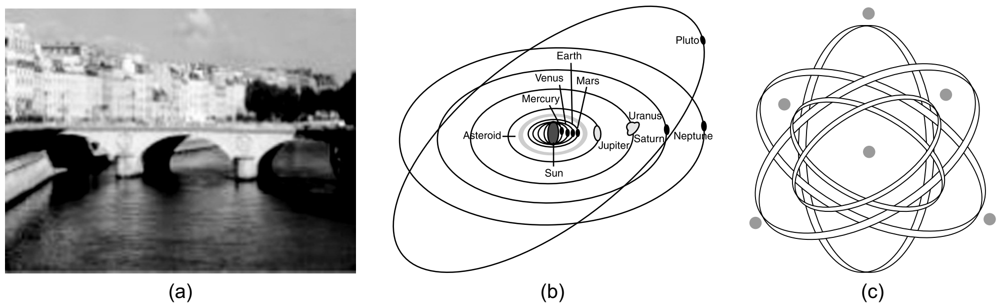


Fig. 5.5 **(a)** *Concrete arches* **(b)** *Orbit of planets* **(c)** *Movement of electrons*

The ellipse has an important property that is used in the reflection of light and sound waves. Any light or signal that starts at one focus will be reflected to the other focus, shown in Fig. 5.6(a). This principle is used in lithotripsy, a medical procedure for treating kidney stones. The principle is also used in the construction of "whispering galleries" such as in St. Paul's Cathedral in London, in which a person whispers near one focus is heard at the other focus although he cannot be heard at many places in between, as shown in Fig. 5.6(b).

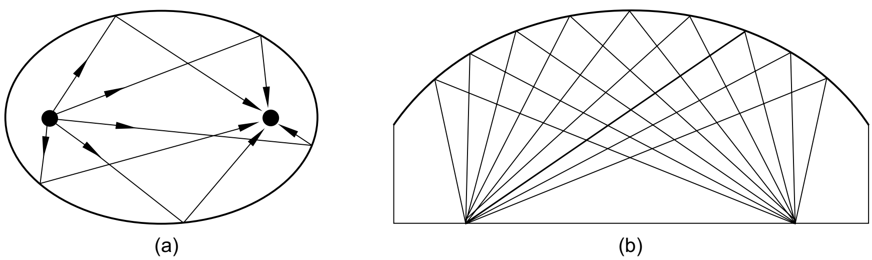


Fig. 5.6 *Principle of reflection of light and sound waves in* **(a)** *lithotripsy* **(b)** *whispering galleries*

4. Parabola When the cutting plane is inclined to the axis and is parallel to one of the generators of the cone, the section is a parabola. The inclination of the cutting plane is equal to half of the apex angle, i.e., $\theta = \alpha$ (*see Fig. 5.7*).

Applications One of nature's best known approximations to parabolas is the path taken by a body projected upward and obliquely to the pull of gravity, as in the parabolic trajectory of a golf ball (*see Figs. 5.8(a) and (b)*). The friction of air and the pull of gravity will change slightly the projectile's path from that of a true parabola, but in many cases the error is insignificant. Many bridge designs such as Golden Gate Bridge, Howrah Bridge, etc., use as parabolic supports (*see Fig. 5.8(c)*).

If light is placed at the focus of a parabolic mirror, it will be reflected in rays parallel to the axis as shown in Fig. 5.9(a). This principle is used for getting straight beam of light in headlamps, torches, etc. The opposite principle is used in the giant mirrors, reflecting telescopes and antennas to collect light and radio waves from outer space and focus them at the focal point, as shown in Fig. 5.9(b). A solar furnace and

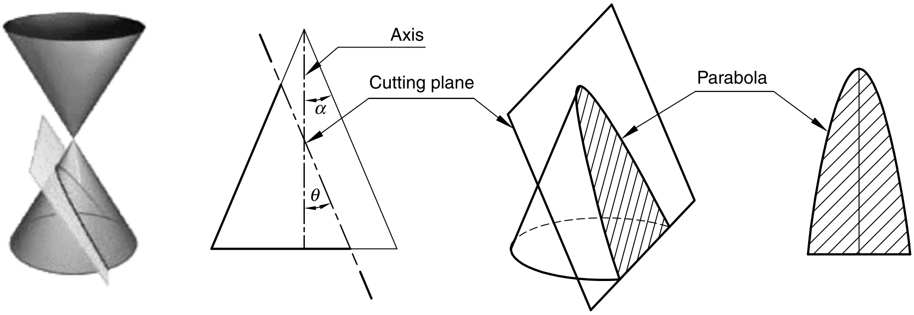


Fig. 5.7 *Formation of a parabola by cutting a cone*

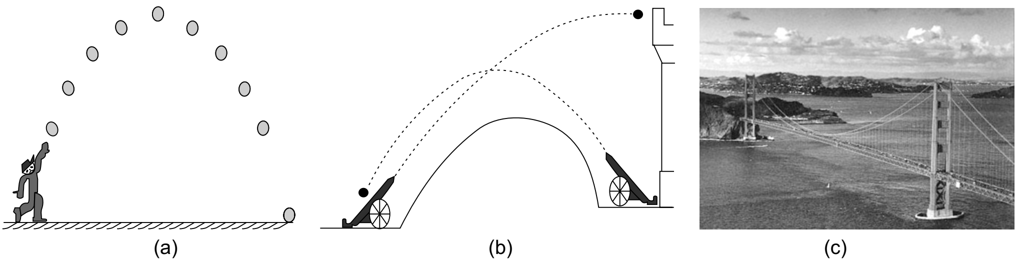


Fig. 5.8 **(a)** *and* **(b)** *Parabolic trajectory* **(c)** *Bridge*

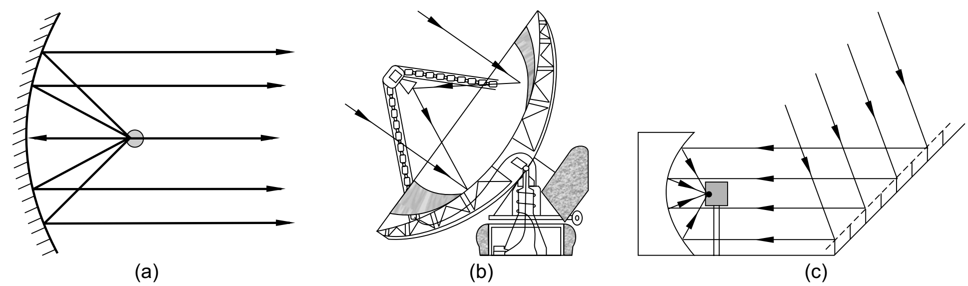


Fig. 5.9 **(a)** *Parabolic mirror* **(b)** *Antenna* **(c)** *Solar furnace*

solar cooker produces heat by focusing sunlight by means of a parabolic mirror arrangement, as shown in Fig. 5.9(c).

5. Hyperbola When the cutting plane cuts both the parts of the double cone, the section is a hyperbola. The cutting plane for the hyperbola should not pass through the apex and its inclination should be less than half of the apex angle, i.e., $\theta < \alpha$ (*see Fig. 5.10*).

Rectangular hyperbola When the cutting plane is parallel to the axis of the cone at a distance, the section is a rectangular hyperbola, i.e., $\theta = 0°$ (*see Fig. 5.11*).

Applications The hyperbolic curve graphically represents the Boyle's law, i.e., PV = constant. A comet that does not return to the sun follows a hyperbolic path. A household lamp shown in Fig. 5.12(a) casts hyperbolic shadows on a wall. A hyperbola revolving around its axis forms a surface called a *hyperboloid*.

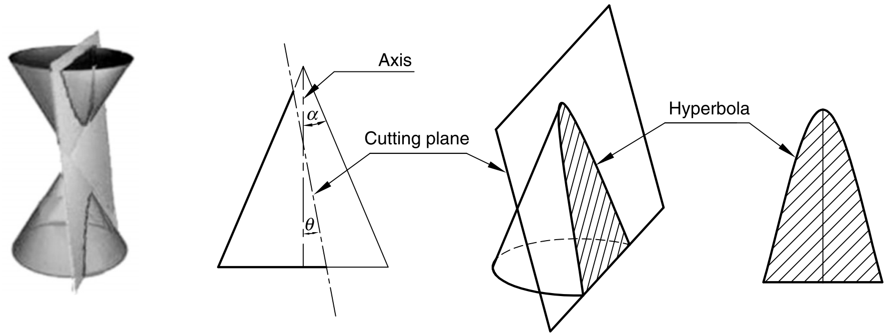


Fig. 5.10 *Formation of a hyperbola by cutting a cone*

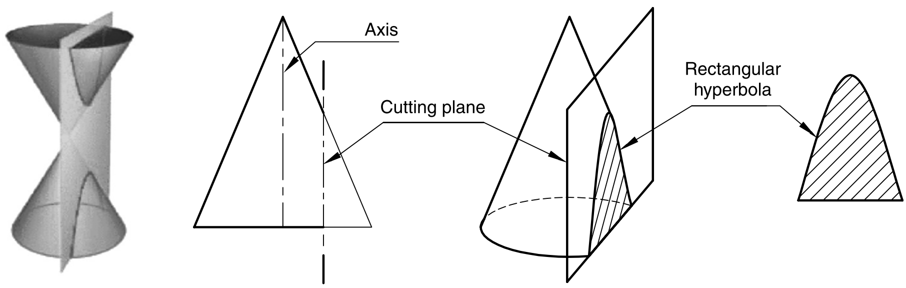


Fig. 5.11 *Formation of a rectangular hyperbola by cutting a cone*

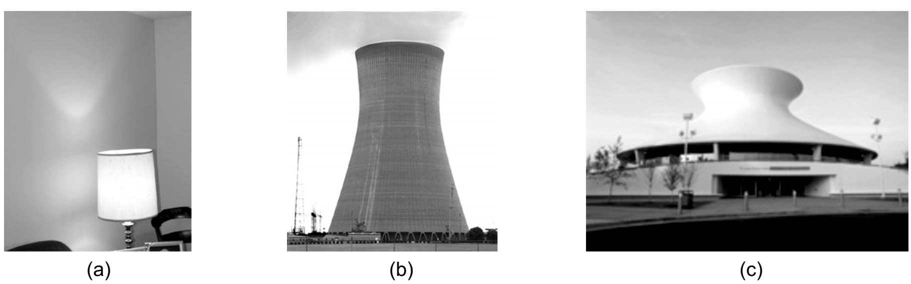


Fig. 5.12 **(a)** *Lamp shadows* **(b)** *Cooling tower* **(c)** *St. Louis Science Centre*

The hyperboloid is useful in design of water channels, cooling towers, etc. Figure 5.12(b) shows use of cooling tower for nuclear plant. The architecture of the James S. McDonnell Planetarium of the St. Louis Science Centre, shown in Fig. 5.12(c) is also hyperboloid.

Two hyperboloids of revolution shown in Fig. 5.13(a) can provide gear transmission between two skew axes. The cogs of each gear are a set of generating straight lines. Reflecting telescopes shown in Fig. 5.13(b) use the hyperbolic mirrors. If the centre of each of two sets of concentric circles shown in Fig. 5.13(c) is the source of a radio signal, the synchronised signals would intersect one another in associated hyperbolas. This principle forms the basis of a hyperbolic radio navigation system known as Long Range Navigation. Figure 5.13(d) shows a wooden vase having hyperboloid shape.

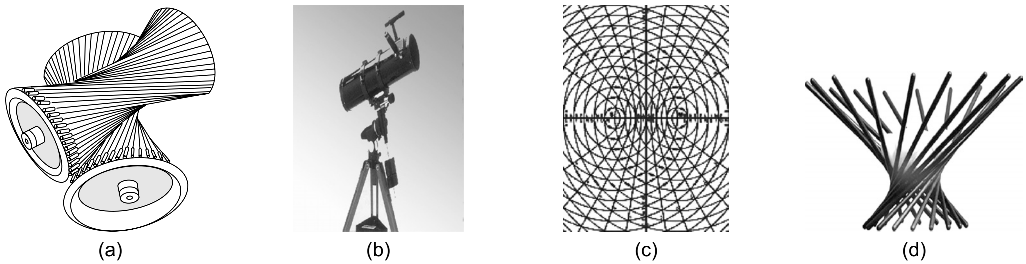


Fig. 5.13 **(a)** *Gear transmission* **(b)** *Reflecting telescopes* **(c)** *Radio signal* **(d)** *Wooden vase*

5.3.1 Comparison of Shapes

Since circles, parabolas and rectangular hyperbolas are formed by cutting the cone at specific angles, they have unique shapes. All circles are identical in shape, all parabolas are identical in shape and all rectangular hyperbolas are identical in shape; only their size and orientation differ. However, for ellipses and hyperbolas (other than rectangular hyperbola) there is a wide range of angles between the plane and the axis of the cone, so they have a wide range of shapes. Ellipses can vary in shape from very nearly circular, to very nearly parabolic. Hyperbola may vary in shape from very nearly parabolic to rectangular hyperbola. The arms of a parabola eventually become parallel to each other, while the arms of a hyperbola always make an angle relative to each other.

5.4 CONSTRUCTION OF ELLIPSE

An ellipse can be construction by the following methods:

1. Eccentricity method (general method)
2. Intersecting arcs method or arcs of circles method
3. Concentric circles method
4. Oblong method
   (a) Rectangle method
   (b) Parallelogram method

5.4.1 Eccentricity Method

An ellipse is defined as the locus of a point P moving in a plane in such a way that the ratio of its distance from a fixed point F_1 to the fixed straight line DD' is a constant and is always less than unity (*see Fig. 5.14*).

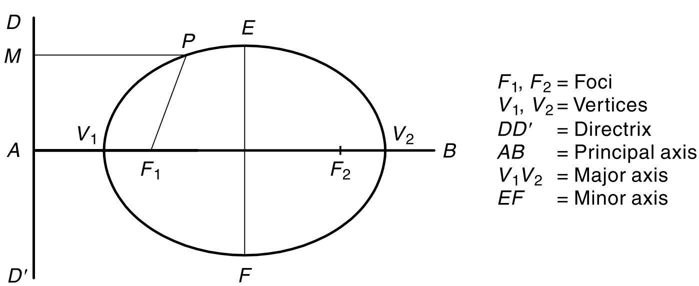


Fig. 5.14 *Ellipse and its terminology*

$$\text{Eccentricity, } e = \frac{\text{Distance of the point from the focus}}{\text{Distance of the point from the directrix}} = \frac{PF_1}{PM} < 1$$

The eccentricity method for the construction of the ellipse is based on this definition.

Problem 5.1 *Draw an ellipse when the distance of its focus from its directrix is 50 mm and eccentricity is 2/3. Also, draw a tangent and a normal to the ellipse at a point 70 mm away from the directrix.*

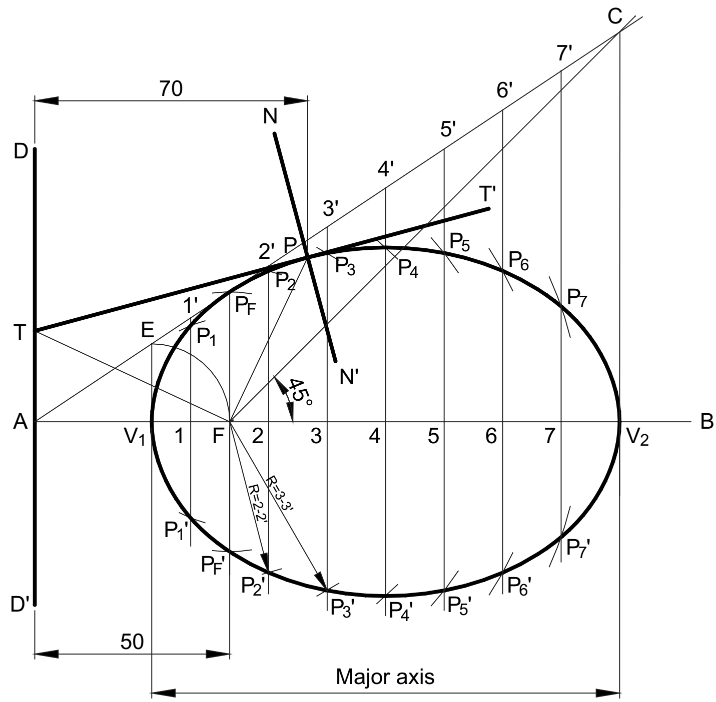


Fig. 5.15 *Eccentricity method for ellipse*

Construction Refer to Fig. 5.15.

1. Draw a vertical line DD' to represent directrix.
2. Draw the principal axis AB perpendicular to DD'.
3. Mark focus F on the axis AB such that $AF = 50$ mm.
4. Divide AF into five equal divisions. Mark the vertex V_1 on the third division point from A. Therefore, $\frac{V_1F}{V_1A} = \frac{2}{3}$ and thereby the vertex V_1 represents a locus point of the ellipse.
5. Draw a vertical line V_1E equal to V_1F. Join A to E and produce it to some distance. Therefore, in the triangle AV_1E, $\frac{V_1E}{V_1A} = \frac{V_1F}{V_1A} = \frac{2}{3}$.

6. Draw a line from the focus F, inclined at 45° to the axis AB to intersect the line AE produced at point C. Draw a perpendicular line from point C to meet the axis AB at point V_2. The line V_1V_2 represents the major axis of the ellipse.
7. Mark a point 1 anywhere on the major axis V_1V_2. Draw a line through point 1, perpendicular to the axis AB to meet line AE produced at point 1′. Therefore, $\frac{11'}{1A} = \frac{V_1E}{V_1A} = \frac{2}{3}$
8. With centre F and radius equal to 1-1′, draw arcs to intersect the perpendicular line 1-1′ at points P_1 and P_1'. These are the loci points of the ellipse because $\frac{FP_1}{1A} = \frac{FP_1'}{1A} = \frac{11'}{1A} = \frac{2}{3}$
9. Similarly, mark some more points, say 2, 3, 4, etc., on the major axis V_1V_2 which need not be equidistant and repeat steps 7 and 8. This will give some more loci points of the ellipse like; P_2 and P_2', P_3 and P_3', P_4 and P_4', etc.
10. Join all the loci points of the ellipse and obtain the required ellipse.

Tangent and normal to an ellipse

1. Mark a point P on the ellipse at a distance of 70 mm from the directrix and Join PF.
2. Draw a line FT perpendicular to the line PF to meet the directrix DD' at point T.
3. Join TP and produce to some point T'. The line TT' is the required tangent.
4. Through point P, draw a line NN' perpendicular to TT'. The line NN' is the required normal.

5.4.2 Intersecting Arcs Method or Arcs of Circles Method

Consider two fixed points F_1 and F_2 on the board as shown in Fig. 5.16. Take a piece of thread with length AB and tie its end at F_1 and F_2. Stretch the thread tight with the help of pencil and move the pencil point. It can be observed that the curve traced is an ellipse.

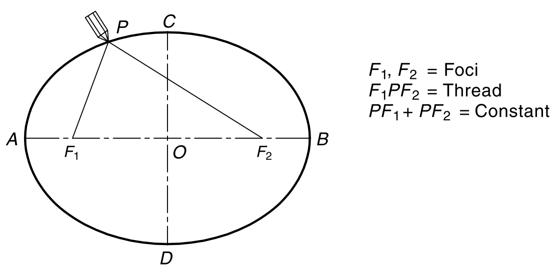


Fig. 5.16 *Principle of Intersecting Arcs method*

Thus, *the ellipse is a curve traced by a point P moving such that the sum of its distance from the two fixed points, F_1 and F_2 is constant, and equal to the major axis.* Intersecting arcs method for constructing the ellipse is based on this principle. The method is also known as *arcs of circles* method. Consider the following problem for the construction of the ellipse using intersecting arcs method.

Problem 5.2 *A point moves in a plane in such a way that the sum of its distances from two fixed points 100 mm apart is 130 mm. Name and draw the locus of this point.*

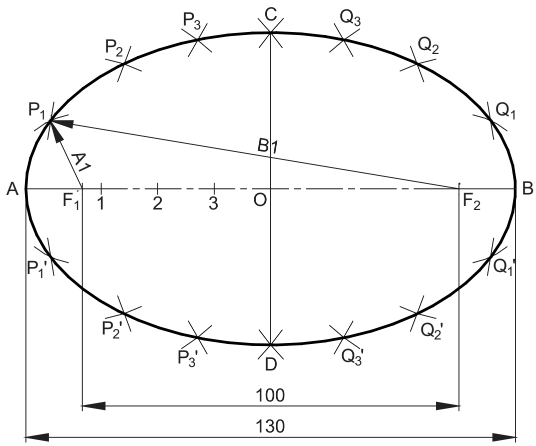


Fig. 5.17 *Intersecting arcs method for ellipse*

Construction Refer to Fig. 5.17.

1. Draw the major axis AB = 130 mm and mark its mid-point O.
2. Mark points F_1 and F_2 as foci on line AB, 100 mm apart such that $OF_1 = OF_2$.
3. Mark points 1, 2, 3, etc., on OF_1 at any convenient distances, which need not be equal.
4. With foci F_1 and F_2 as the centres and radius $A1$, draw arcs on both sides of AB.
5. With foci F_1 and F_2 as the centres and radius $B1$, draw arcs on both sides of AB to intersect the previous arcs at four points P_1, P_1', Q_1 and Q_1'.
6. Repeat step 4 and step 5 with radii $A2$ and $B2$, $A3$ and $B3$, etc., and obtain points P_2, P_2', Q_2, Q_2', P_3, P_3', Q_3, and Q_3'.
7. Draw a smooth curve passing through all the points. The curve is the required ellipse.

Corollary 1 If the distances between foci (F_1F_2) and the length of major axis (AB) are known, draw an arc with centre F_1 or F_2 and radius AO to intersect the perpendicular bisector through mid-point O at points C and D. The line CD is the minor axis.

Corollary 2 If the lengths of major axis (AB) and minor axis (CD) are known, draw an arc with centre C or D and radius AO to meet the major axis (AB) at points F_1 and F_2. The points F_1 and F_2 are the foci of the ellipse.

Corollary 3 If the distances between the foci (F_1F_2) and length of minor axis (CD) are known, draw an arc with centre O and radius CF_1 (= CF_2) to meet the line joining the foci at points A and B. The line AB is the major axis.

5.4.3 Concentric Circles Method

This is a special method for construction of ellipse.

Problem 5.3 *The major axis of an ellipse is 110 mm and minor axis is 70 mm long. Draw an ellipse by concentric circle method.*

Construction Refer to Fig. 5.18.

1. Draw major axis AB = 110 mm and minor axis CD = 70 mm. Lines AB and CD are perpendicular bisectors of each other meeting at point O.
2. Draw two concentric circles with centre O of diameters AB and CD.
3. Divide both the circles into twelve equal parts. Mark points 1′, 2′, 3′, …etc., on the circumference of the circle with diameter AB and points 1, 2, 3,…etc., on circumference of the circle with diameter CD.
4. Draw vertical line through point 1′ to meet the horizontal line drawn through point 1 at point P_1. The point P_1 represents a point on the ellipse.
5. Similarly, draw vertical lines from other points 2′, 3′,… , etc., to meet the corresponding horizontal lines drawn from points 2, 3, …, etc., at points P_2, P_3, … etc. The point P_2, P_3, …, etc., lie on the ellipse.
6. Draw a smooth curve passing through all these points to get the required ellipse.

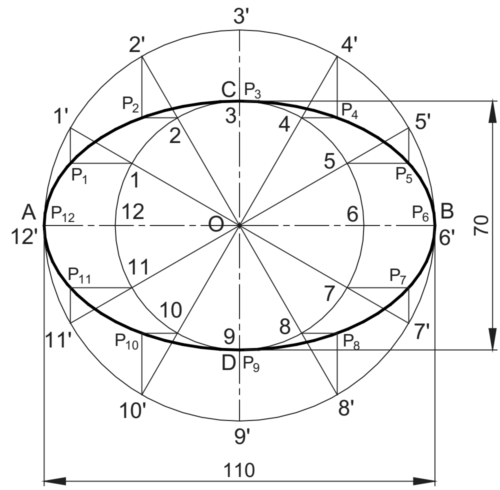


Fig. 5.18 *Concentric circles method for ellipse*

5.4.4 Rectangle Method

It is basically a method of inscribing an ellipse in a rectangle.

Problem 5.4 *Inscribe the largest possible ellipse in a rectangle of sides 160 mm and 100 mm.*

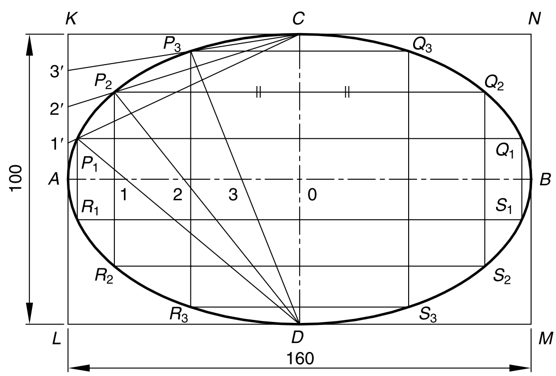


Fig. 5.19 *Rectangle method for ellipse*

Construction Refer to Fig. 5.19.

1. Draw a rectangle $KLMN$ with sides KL = 100 mm and LM = 160 mm.
2. Mark A, B, C and D as mid-points of sides KL, MN, NK and LM respectively.

3. Mark O as the intersecting point of line AB and CD.
4. Divide lines AO and AK into same number of equal parts, say 4. Mark 1, 2, 3 on AO and 1′, 2′, 3′ on AK.
5. Join C with points 1′, 2′, 3′.
6. Draw lines from point D, to pass through points 1, 2 and 3, and intersect lines $C1'$, $C2'$ and $C3'$ at points P_1, P_2 and P_3 respectively.
7. Draw the curve passing through A, P_1, P_2, P_3 and C. This is one quarter of the ellipse.
8. As the curve is symmetric about axes, obtain points Q_1, Q_2, Q_3 of the curve in the quadrant CNBO by drawing horizontal lines from points P_1, P_2, P_3 and make their lengths equal on both sides of OC.
9. Similarly, obtain points R_1, R_2, R_3, S_1, S_2, S_3 of the curve in ABML by drawing vertical lines from points P_1, P_2, P_3, Q_1, Q_2, Q_3 and make their lengths equal on both sides of AB.
10. Join the points obtained in steps 8 and 9 by a smooth curve to get the required ellipse.

5.4.5 Parallelogram Method

Two diameters of a conic section are said to be conjugate if each chord parallel to one diameter is bisected by the other diameter. For example, two diameters of a circle are conjugate if and only if they are perpendicular.

For an ellipse, *conjugate diameters are the lines passing through the centre of ellipse and parallel to the tangents on the curve at the point of intersection of the other diameter with the ellipse.* In Fig. 5.20, line AB is parallel to the tangent KN passing point C and tangent LN passing through point D. Similarly, line CD is parallel to the tangent KL passing point A and tangent MN passing through point B. Also, both AB and CD pass through the centre O. Therefore, AB and CD is a pair of conjugate diameters.

An ellipse can have an unlimited pair of conjugate diameters. The tangents at the end of each pair of conjugate diameters of an ellipse form a parallelogram of constant area. It is possible to construct an ellipse when a pair of conjugate diameters or bounding parallelogram is known.

Problem 5.5 *The conjugate diameters of an ellipse are 120 mm and 80 mm. The included angle between them is 75°. Draw an ellipse by parallelogram method.*

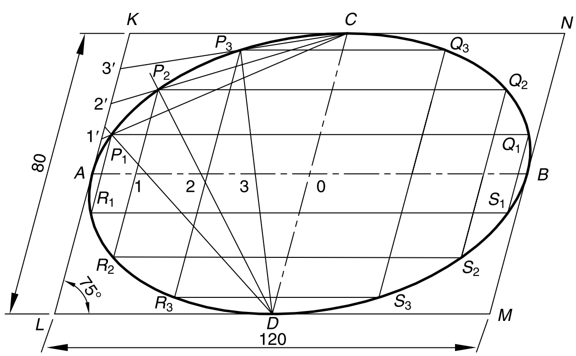


Fig. 5.20 *Parallelogram method for ellipse*

Construction Refer to Fig. 5.20.

1. Draw a parallelogram *KLMN* with sides *KL* = 80 mm, *LM* = 120 mm and $\angle KLM = 75^\circ$.
2. Mark *A*, *B*, *C* and *D* as mid-points of sides *KL*, *MN*, *NK* and *LM* respectively.
3. Join *AB* and *CD* as the conjugate diameters of the ellipse. The intersecting point *O* of the conjugate diameters is the centre of the ellipse.
4. Divide lines *OA* and *KA* into same number of equal parts, say 4. Mark 1, 2, 3 on *OA* and 1′, 2′, 3′ on *KA*.
5. Join point *C* with 1′, 2′, 3′.
6. Draw lines from point *D*, to pass through points 1, 2 and 3 and intersect lines *C*1′, *C*2′ *C*3′ at points P_1, P_2, P_3 respectively.
7. Draw smooth curve through points *A*, P_1, P_2, P_3 and *C*. This is one quarter of the ellipse.
8. Obtain Q_1, Q_2, Q_3 of the curve in quadrant *AOCK* by drawing lines parallel to *AB* from points P_1, P_2, P_3 and make their lengths equal on both sides of *CD*.
9. Similarly, obtain points R_1, R_2, R_3, S_1, S_2, S_3 of the curve in *ABML* by drawing lines parallel to *KL* from points P_1, P_2, P_3, Q_1, Q_2, Q_3 and make their lengths equal on both sides of *AB*.
10. Join the points obtained in steps 8 and 9 by a smooth curve to get the required ellipse.

5.5 LOCATE CENTRE, MAJOR AXIS AND MINOR AXIS

Problem 5.6 *Determine the centre, major axis and minor axis of the given ellipse.*

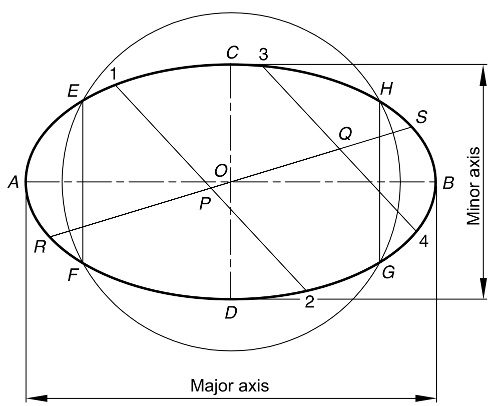


Fig. 5.21 *Locate the centre, the major axis and the minor axis*

Construction Refer to Fig. 5.21.

1. Draw two parallel chords 1-2 and 3-4 across the ellipse, separated by some distance.
2. Mark *P* and *Q* as mid-points of the chords 1-2 and 3-4 respectively.
3. Join *PQ* and produce it on both sides to meet the ellipse at points *R* and *S*.
4. Mark *O* as the mid-point of *RS*. The point *O* represents the centre.
5. With centre *O* and radius of any convenient length, draw a circle to cut the ellipse at points *E*, *F*, *G* and *H*.

6. Complete the rectangle *EFGH*.
7. Through point *O*, draw a line *AB* parallel to *EH*, and line *CD* parallel to *EF*. The lines *AB* and *CD* represent the major and minor axes, respectively.

5.6 TANGENTS AND NORMAL TO THE ELLIPSE

5.6.1 From a Point on the Ellipse

Problem 5.7 *Draw a tangent and a normal to the ellipse through a point P, lying on it.*

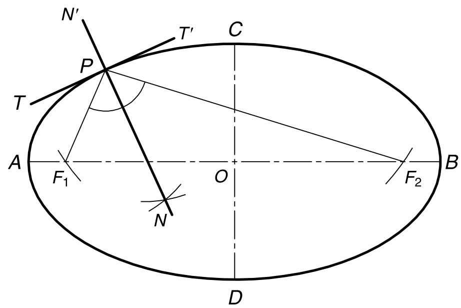


Fig. 5.22 *Tangent and normal through a point, lying on the ellipse*

Construction Refer to Fig. 5.22.

1. Draw the ellipse and locate point P through which tangent is to be drawn.
2. Draw the centre *O*, major axis *AB* and the minor axis *CD* of the ellipse. (Problem 5.6)
3. Draw an arc with centre *C* and radius *OA* to meet the major axis *AB* at points F_1 and F_2. The points F_1 and F_2 represent the foci.
4. Join the point *P* with foci F_1 and F_2.
5. Draw *NN′* as the bisector of the angle F_1PF_2. The line *NN′* represents the normal.
6. Through point *P*, draw a line *TT′* perpendicular to *NN′*. The line *TT′* represents the tangent.

5.6.2 From a Point Outside the Ellipse

Problem 5.8 *Draw a pair of tangents to the ellipse through a point P, lying outside.*

Construction Refer to Fig. 5.23.

1. Draw the ellipse and locate point P through which tangent is to be drawn.
2. Draw the centre *O*, major axis *AB* and the minor axis *CD* of the ellipse. (Problem 5.6)
3. Draw an arc with centre *C* and radius *OA* to meet the major axis *AB* at points F_1 and F_2. The points F_1 and F_2 represent the foci.
4. Draw an arc with centre *P* and radius PF_2 to intersect another arc with centre F_1 and radius *AB* at points *G* and *H*.

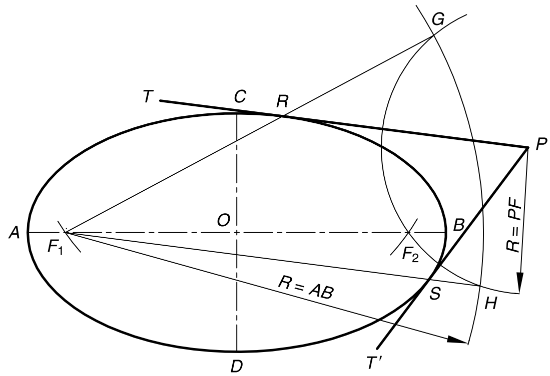


Fig. 5.23 *Pair of tangents through a point, lying outside the ellipse*

5. Join F_1G and F_1H. Let they intersect the ellipse at points R and S, respectively.
6. Join PR and PS and produce them to points T and T' respectively.
7. Lines PT and PT' are the tangents to the ellipse.

5.7 CONSTRUCTION OF PARABOLA

A parabola can be constructed by the following methods:

1. Eccentricity method
2. Offset method
3. Tangent method
4. Oblong method
   (a) Rectangle method
   (b) Parallelogram method

5.7.1 Eccentricity Method

A parabola is defined as the locus of a point P moving in a plane in such a way that the ratio of its distance from a fixed point F to the fixed straight line DD′ is a constant and is always equal to unity (*see Fig. 5.24*).

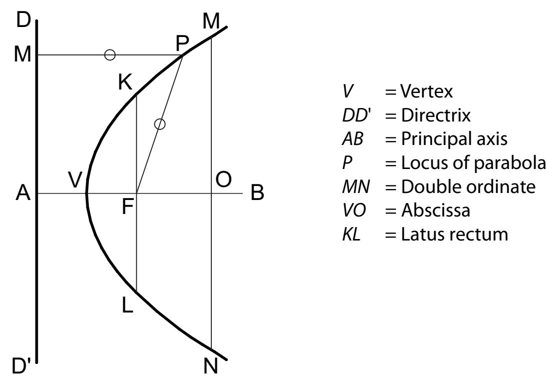


Fig. 5.24 *Parabola and its terminology*

$$\text{Eccentricity, } e = \frac{\text{Distance of the point from the focus}}{\text{Distance of the point from the directrix}} = \frac{PF}{PM} = 1$$

The eccentricity method for construction of the ellipse is based on this definition.

Any chord perpendicular to the axis of the parabola is called the *double ordinate*. Ordinate is half of the double ordinate. The distance between the vertex and the double ordinate is called *abscissa*. The parabola has unlimited number of double ordinate and for each double ordinate there is an abscissa. The chord through focus and perpendicular to the axis of the parabola is called the *latus rectum*. In Fig. 5.24, MN is the double ordinate, OM is the ordinate, VO is the abscissa and KL is the latus rectum.

Problem 5.9 *Draw a parabola when the distance between its focus and directrix is 50 mm. Also, draw a tangent and a normal at a point 70 mm from the directrix.*

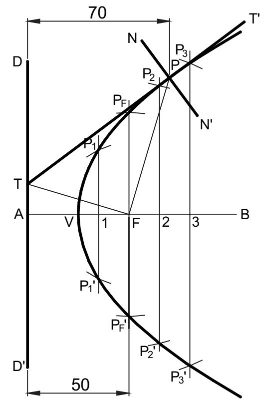


Fig. 5.25 *Eccentricity method for parabola*

Construction Refer to Fig. 5.25.

1. Draw a vertical line DD' to represent the directrix.
2. Draw the principal axis AB perpendicular to the directrix DD'.
3. Mark focus F on the axis AB such that $AF = 50$ mm.
4. Mark vertex V at the mid of AF. This vertex V is the locus point of the parabola because $e = \dfrac{VF}{AV} = 1$.
5. Mark a point 1 anywhere on the axis AB and draw a perpendicular line through it.
6. With centre F and radius equal to A1, draw arcs to intersect the perpendicular line through point 1 at points P_1 and P_1'. These are the loci points of the parabola because $\dfrac{FP_1}{A1} = \dfrac{FP_1'}{A1} = 1$.
7. Similarly, mark some more points, say 2, 3, etc., on the axis AB which need not be at equidistant and repeat Step 6. This will give some more loci points of the parabola like; P_2 and P_2', P_3 and P_3', etc.
8. Join all the loci points P_2', P_1', V, P_1, P_2, etc., by a smooth curve and obtain the required parabola.

Tangent and normal to a parabola

1. Mark a point P on the parabola at a distance of 70 mm from the directrix and join PF.
2. Draw a line FT perpendicular to the line PF to meet the directrix DD' at point T.
3. Join T to P and produce to some point T'. The line TT' is the required tangent.
4. Through point P, draw a line NN' perpendicular to TT'. The line NN' is the required normal.

5.7.2 Offset Method

The equation for the parabola is $x^2 = 4ay$. Thus, x^2 is directly proportional to y. Therefore, a curve passing through coordinates (0,0), (1,1), (2,4), (3,9), (4,16), etc., will generate a parabolic curve. Offset method is based on this principle.

Problem 5.10 *A parabolic arch has a span[1#] of 160 mm and a maximum rise of 100 mm. Draw a curve using offset method.*

Construction Refer to Fig. 5.26.

1. Draw a rectangle $ABCD$, with span $AB = 160$ mm and rise $AD = 100$ mm.
2. Mark E and F as the mid-points of sides AB and CD respectively. Join EF.
3. Divide FD into number of equal parts, say 4. Also, divide line DA into square of the number of parts made for line FD, i.e., $4^2 = 16$. Number the points as shown.

[1#]Span is defined as the distance between two extremities, such as the ends of a bridge or arch. It is also called range.

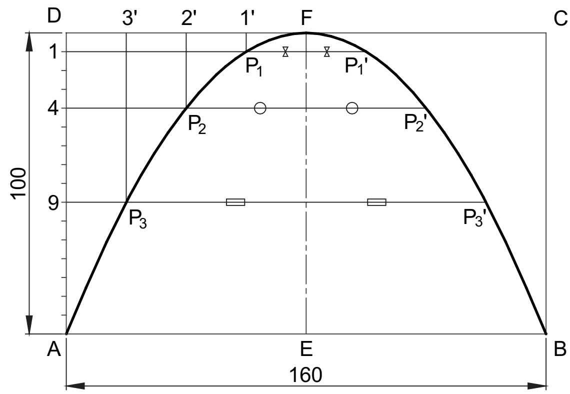


Fig. 5.26 *Offset method for parabola*

4. Draw horizontal lines from points 1, 4, 9 of side *AD* to meet vertical lines from points 1′, 2′, 3′ of side *FD* at points P_1, P_2, P_3, respectively.
5. As the curve is symmetric about axis, obtain points P_1', P_2', P_3' of the curve by drawing horizontal lines through points P_1, P_2, P_3 and making them equal on both side of axis *EF*.
6. Draw a smooth curve to pass through *A*, P_3, P_2, P_1, *F*, P_1', P_2', P_3' and *B*. This curve is the required parabola.

5.7.3 Tangent Method

This is a special method for construction of a parabola.

Problem 5.11 *Draw a parabola of base 150 mm and axis 80 mm by tangent method.*

Construction Refer to Fig. 5.27.

1. Draw the base *AB* = 150 mm and mark the mid-point *C*.
2. Draw the axis *CD* = 80 mm perpendicular to *AB*.
3. Produce *CD* to point *O* taking *DO* = *CD*. Join *OA* and *OB*.
4. Divide lines *OA* and *OB* into same number of equal parts, say 8. Mark divisions of side *AO* as 1, 2, 3, etc., and side *OB* as 1′, 2′, 3′, etc.
5. Join 11′, 22′, 33′, etc. These lines are tangents to the parabola.
6. Draw a smooth curve starting from point A, tangent to lines 11′, 22′, 33′, etc., and ending at point *B*. The curve is the required parabola.

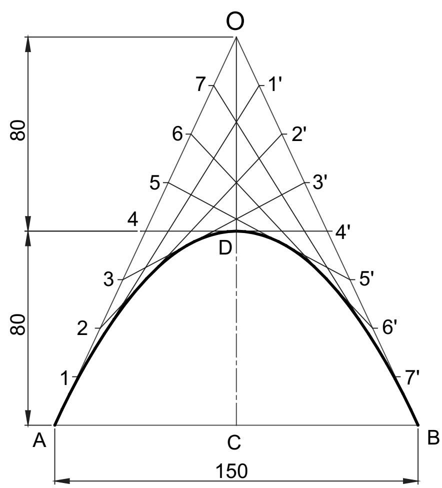


Fig. 5.27 *Tangent method for parabola*

5.7.4 Rectangle Method

Rectangle method is basically a method of inscribing parabola in a rectangle.

Problem 5.12 *Draw a parabola of base 120 mm and axis 80 mm by oblong method.*

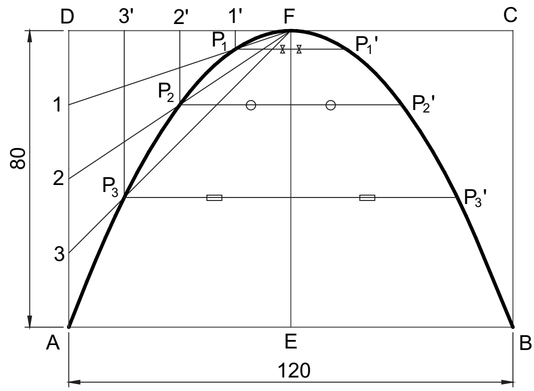


Fig. 5.28 *Rectangle method for parabola*

Construction Refer to Fig. 5.28.

1. Draw a rectangle *ABCD* taking *AB* = 120 mm and *AD* = 80 mm.
2. Mark *E* and *F* as the mid-points of *AB* and *CD* respectively. Join *EF* to represent the axis.
3. Divide *FD* and *DA*, into equal number of parts, say 4. Mark division of side *DA* as 1, 2, 3 and divisions of *FD* as 1′, 2′, 3′. Now join *F* with points 1, 2, 3.
4. Through 1′, 2′, 3′ draw lines parallel to the axis *EF* to meet *F*1, *F*2, *F*3 at P_1, P_2, P_3 respectively.
5. As the curve is symmetric about axis, obtain points P_1', P_2', P_3' of the curve by drawing horizontal lines through points P_1, P_2, P_3 and making them equal on both side of axis *EF*.
6. Draw a smooth curve passing through *A*, P_3, P_2, P_1, *F*, P_1', P_2', P_3' and *B* to get the required parabola.

5.7.5 Parallelogram Method

Parallelogram method is basically a method of inscribing parabola in a parallelogram.

Problem 5.13 *Inscribe a parabola in a parallelogram of sides 110 mm and 80 mm, the included angle being 60°. Consider the longer side of the parallelogram as the base of the parabola.*

Construction Refer to Fig. 5.29.

1. Draw a parallelogram *ABCD* taking *AB* = 120 mm and *AD* = 80 mm and ∠*DAB* = 60°.
2. Mark *E* and *F* as the mid-points of *AB* and *CD* respectively. Join *EF* to represent the axis.
3. Divide lines *FD* and *DA*, into equal number of parts, say 4. Mark division of side *DA* as 1, 2, 3 and divisions of *FD* as 1′, 2′, 3′. Join *F* with points 1, 2, 3.
4. Through 1′, 2′, 3′ draw lines parallel to the axis *EF* to meet lines *F*1, *F*2, *F*3 at points P_1, P_2, P_3 respectively.

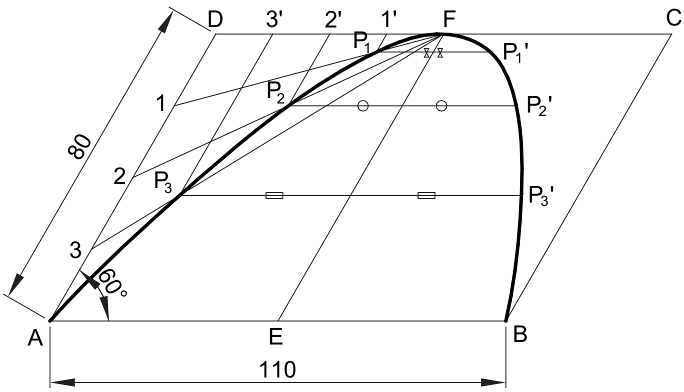


Fig. 5.29 *Parallelogram method for parabola*

5. As the curve is symmetric about axis, obtain points P_1', P_2', P_3' of the curve by drawing horizontal lines through points P_1, P_2, P_3 and making them equal on both side of axis *EF*.
6. Draw a smooth curve passing through A, P_3, P_2, P_1, F, P_1', P_2', P_3' and B. This curve is the required parabola.

5.8 LOCATE AXIS, FOCUS AND DIRECTRIX

Problem 5.14 *Determine the axis, focus and directrix of a parabola.*

Construction Refer to Fig. 5.30.

1. Draw two parallel chords *KL* and *MN* across the parabola, separated by some distance.
2. Mark *T* and *U* as the mid-points of the chords *KL* and *MN* respectively.
3. Join *T* to *U* and produce it to some length.
4. Draw a line perpendicular to *TU* at any convenient distance to meet the parabola at points *P* and *Q*. The line *PQ* is a double ordinate to the parabola.
5. Draw *AB* as the perpendicular bisector of *PQ*. The line *AB* is the required axis of the parabola. The axis *AB* meets the parabola at point *V*, called vertex.
6. Let *PQ* meet the axis *AB* at point *R*. Mark a point *S* on the axis *AB* such that $RV = VS$.
7. Join *SP*. This is tangent to the parabola at point *P*.
8. Draw a line *EG* as the perpendicular bisector of line *SP*. Let *EG* meet the axis *AB* at point *F*. The point *F* is the required focus of the parabola.

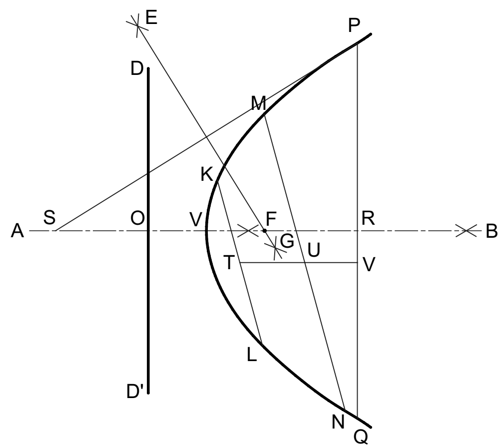


Fig. 5.30 *Locate the axis, focus and directrix*

9. Mark a point O on the axis AB such that $VF = OV$.
10. Draw a line DD' through O, perpendicular to the axis AB. The line DD' is the required directrix of the parabola.

5.9 TANGENT AND NORMAL TO THE PARABOLA

5.9.1 From a Point on the Parabola

Problem 5.15 *Draw a tangent and a normal to the given parabola at a point P.*

Construction Refer to Fig. 5.31.

1. Determine axis AB of the parabola and mark the vertex V (Problem 5.14).
2. Mark a point P on the parabola from which tangent is to be drawn.
3. From point P, draw a line PM perpendicular to AB.
4. Mark a point T on the axis AB such that $TV = VM$.
5. Join TP and extend it to a point T'. The line TT' is the tangent to the parabola.
6. Draw line NN' as the perpendicular bisector of TT'. The line NN' is the required normal.

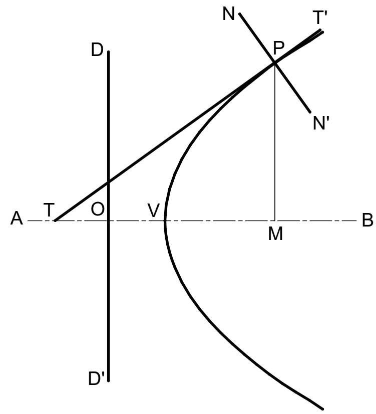


Fig. 5.31 *Tangent and normal through a point, lying on the parabola*

5.9.2 From a Point Outside the Parabola

Problem 5.16 *Draw a pair of tangents to the given parabola from a point P lying outside it.*

Construction Refer to Fig. 5.32.

1. Determine axis AB, focus F and the directrix DD' of the parabola (Problem 5.14).
2. Locate point P outside the parabola through which tangents are to be drawn.
3. Draw a circle with centre P and radius PF to meet the directrix DD' at points Q and R.
4. Draw horizontal lines from Q and R to meet the parabola at points S and T respectively.
5. Join PS and PT and extend them. These are the required tangents.

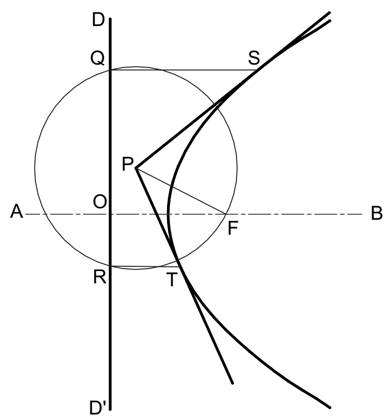


Fig. 5.32 *Pair of tangents through a point, lying outside the parabola*

5.10 CONSTRUCTION OF HYPERBOLA

A hyperbola can be constructed by the following methods:

1. Eccentricity method
2. Intersecting arcs method
3. Oblong method
4. Intercept method
5. Asymptotes method
   (a) Orthogonal asymptotes method
   (b) Oblique asymptotes method

5.10.1 Eccentricity Method

A hyperbola is defined as the locus of a point *P* moving in a plane in such a way that the ratio of its distance from a fixed point *F* to the fixed straight line *DD′* is a constant and is always greater than unity (see Fig. 5.33).

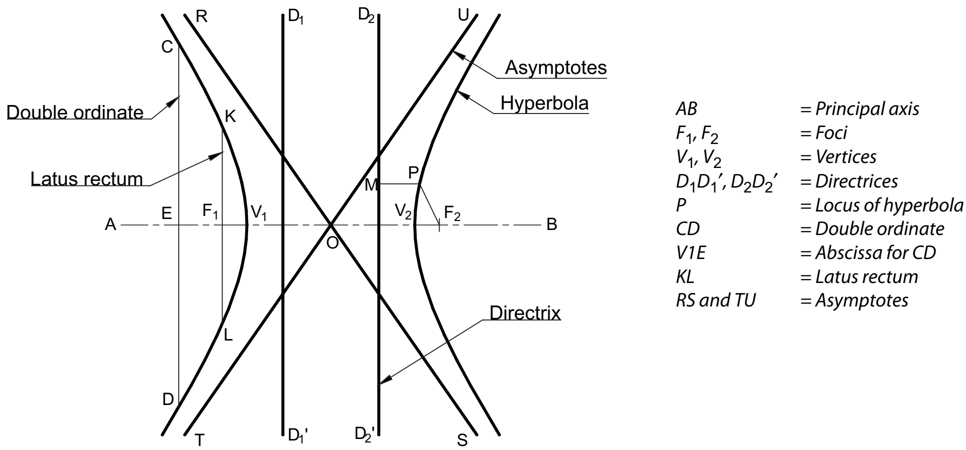


Fig. 5.33 *Hyperbola and its terminology*

$$\text{Eccentricity, } e = \frac{\text{Distance of the point from the focus}}{\text{Distance of the point from the directrix}} = \frac{PF_2}{PM} > 1$$

The eccentricity method for the construction of the hyperbola is based on this definition.

A hyperbola has two vertices V_1 and V_2. The line joining the two vertices is called the *major axis or the transverse axis*. Any chord perpendicular to the axis of the hyperbola is called the *double ordinate*. The distance between the vertex and the double ordinate is called the *abscissa*. Similar to the parabola, the hyperbola also has unlimited number of double ordinates and for each double ordinate there is an abscissa. The chord through focus and perpendicular to the axis of the hyperbola is called the *latus rectum*. In Fig. 5.33, *CD* is the double ordinate, *CE* is the ordinate, V_1E is the abscissa and *KL* is the latus rectum.

Asymptotes are the straight line that pass through the centre of the transverse axis and tangential to the hyperbola at infinity. They approach nearer and nearer to the hyperbola while moving away from the centre and assumed to touch the hyperbola at infinity. In Fig. 5.33, *RS* and *TU* are the asymptotes. When the asymptotes to the hyperbola intersect at right angles, the curve is known as rectangular hyperbola and its eccentricity is $\sqrt{2}$.

Problem 5.17 *Draw a hyperbola when the distance of its focus from its directrix is 50 mm and eccentricity is 3/2. Also, draw a tangent and a normal to the hyperbola at a point 25 mm from the directrix.*

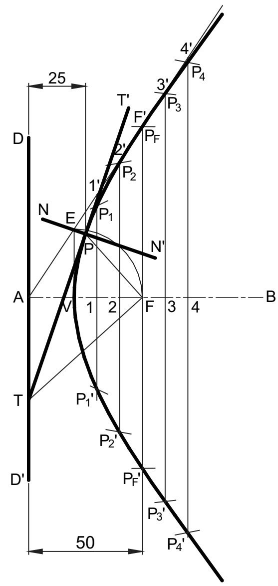


Fig. 5.34 *Eccentricity method for hyperbola*

Construction Refer to Fig. 5.34.

1. Draw a directrix DD'.
2. Draw the principal axis AB perpendicular to the directrix DD'.
3. Mark focus F on the axis AB such that $AF = 50$ mm.
4. Divide AF into five equal divisions. Mark the vertex V on the second division-point from A. Therefore, $\dfrac{VF}{VA} = \dfrac{3}{2}$ and thereby the vertex V represents a locus point of the hyperbola.
5. Draw a vertical line VE equal to VF. Join A to E and produce it to some distance. Therefore, in the triangle AVE, $\dfrac{VE}{VA} = \dfrac{VF}{VA} = \dfrac{3}{2}$.
6. Mark a point 1 anywhere on the axis AB. Draw a line through point 1, perpendicular to the axis AB to meet line AE produced at point $1'$. Therefore, $\dfrac{11'}{1A} = \dfrac{VE}{VA} = \dfrac{3}{2}$
7. With centre F and radius equal to $1\text{-}1'$, draw arcs to intersect the perpendicular line $1\text{-}1'$ at points P_1 and P_1'. These are the loci points of the hyperbola because $\dfrac{FP_1}{1A} = \dfrac{FP_1'}{1A} = \dfrac{11'}{1A} = \dfrac{3}{2}$
8. Similarly, mark some more points, say 2, 3, 4, etc., on the axis AB which need not be equal and repeat Steps 6 and 7. This will give some more loci points of the ellipse like; P_2 and P_2', P_3 and P_3', P_4 and P_4', etc.
9. Join points P_4', P_3', P_2', P_1', V, P_1, P_2, P_3, P_4 by a smooth curve. This is the required hyperbola.

Tangent and normal to a hyperbola

1. Mark a point P on the hyperbola at a given distance, 25 mm from the directrix. Join PF.
2. Draw a line FT perpendicular to PF to meet the directrix at point T.
3. Join TP and extend it to some point T'. The line TT' is the required tangent.
4. Through point P, draw a line NN' perpendicular to TT'. The line NN' is the required normal.

5.10.2 Intersecting Arcs Method

The hyperbola is a curve traced by a point P moving such that the difference of its distance from the two fixed points, F_1 and F_2 is constant, and equal to the distance between the vertices of the two branches of the hyperbola.

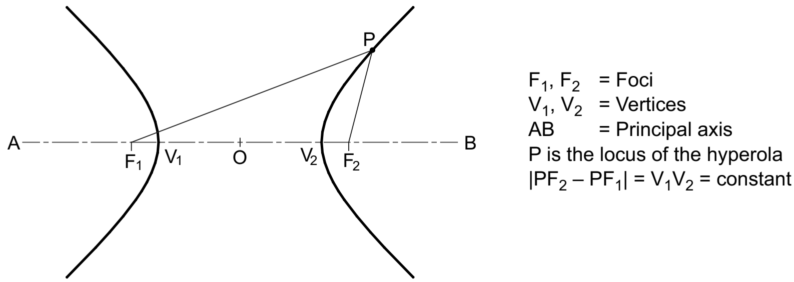


Fig. 5.35 *Principle of intersecting arcs method*

Consider foci F_1 and F_2, and vertices V_1 and V_2 as shown in Fig. 5.35. Then for hyperbola, the locus of any point P should satisfy $|PF_2 - PF_1| = V_1V_2$. The distance between the vertices V_1V_2 is also known as *transverse axis* or *major axis*. Intersecting arc method is based on this definition.

Problem 5.18 *Two fixed points are 80 mm apart. Draw the locus of a point P which moves in such a manner that the difference of its distance from the fixed points is always the same and equal to 60 mm. Name the curve.*

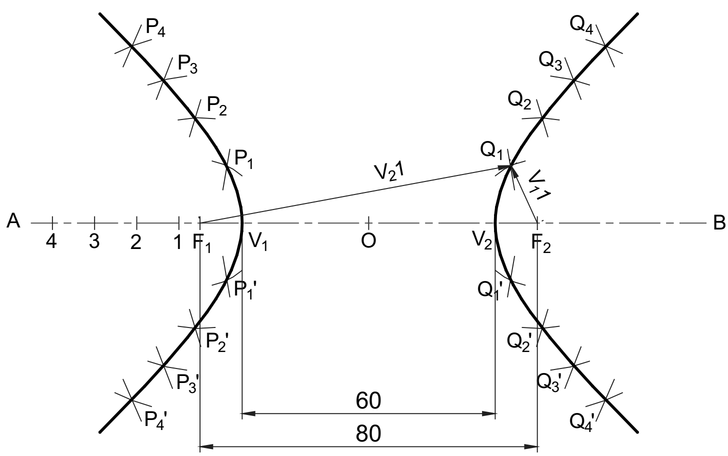


Fig. 5.36 *Intersecting arcs method for hyperbola*

Construction Refer to Fig. 5.36.

1. Draw the principal axis AB and mark a point O on it.
2. Mark F_1, F_2 foci on AB such that $F_1F_2 = 80$ mm. Also mark V_1, V_2 as vertices on AB such that $V_1V_2 = 60$ mm. The points are symmetry about point O.
3. Mark points 1, 2, 3, etc., on AF_1 at any convenient distances, which need not be equal.
4. With foci F_1 and F_2 as the centres and radius V_11, draw arcs on both sides of AB.
5. With foci F_1 and F_2 as the centres and radius V_21, draw arcs on both sides of AB to intersect the previous arcs at four points P_1, P_1', Q_1 and Q_1'.

6. Repeat Steps 4 and 5 with radii $V_1 2$ and $V_2 2$, $V_1 3$ and $V_2 3$, etc., and obtain points P_2, P_2', Q_2, Q_2', P_3, P_3', Q_3, Q_3', etc.
7. Draw a pair of smooth curves to pass through all the points. This gives required two branches of the hyperbola.

5.10.3 Oblong Method

The oblong method is used to draw hyperbola when abscissa, double ordinate and transverse axis are known.

Problem 5.19 *Draw a hyperbola when half the transverse axis, double ordinate and abscissa are 50 mm, 120 mm and 40 mm long respectively.*

Construction Refer to Fig. 5.37.

1. Draw a rectangle *KLMN* such that *KL* = 120 mm (double ordinate) and *LM* = 40 mm.
2. Mark *C* and *V* as the mid-points of *KL* and *NM* respectively.
3. Join *CV* and produce to point *O* such that *VO* = 50 mm (half the transverse axis).
4. Divide *KN* and *KC* into equal number of parts, say 4. Name the points of line *KN* as 1, 2, 3 and line *KC* as 1′, 2′, 3′.
5. Join point *V* with points 1, 2, 3 and join point *O* with 1′, 2′, 3′.
6. Mark points P_1, P_2, P_3 as the intersection of *V*1 and *O*1′, *V*2 and *O*2′, *V*3 and *O*3′, respectively.
7. Draw perpendicular lines from P_1, P_2, P_3 on the axis *AB* and obtain points P_1', P_2', P_3' such that they are symmetry about the axis *AB*.
8. Join *K*, P_1, P_2, P_3, *V*, P_1', P_2', P_3', *L* with a smooth curve. This is the required hyperbola.

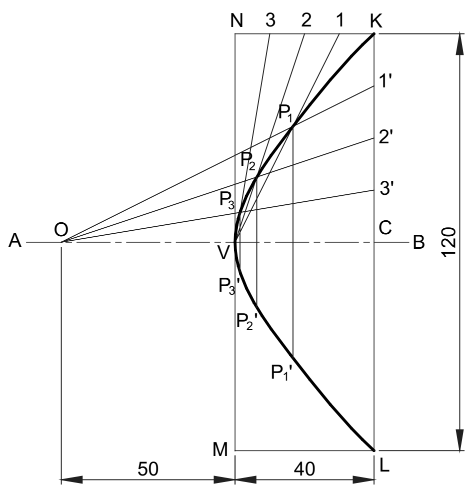


Fig. 5.37 *Oblong method for hyperbola*

5.10.4 Intercept Method

In a rectangular hyperbola if the chord of the hyperbola is extended to intersect the axes, the intercept between the curves and the axes are equal. This principle of intercept for the construction of a rectangular hyperbola is illustrated in the following problem.

Problem 5.20 *Draw a rectangular hyperbola when the position of a point P on the curve is at a distance of 30 mm and 50 mm from two asymptotes.*

Construction Refer to Fig. 5.38.

1. Draw asymptotes *OX* and *OY* at right angle to each other.
2. Mark a point *P* such that its distance from *OX* is 30 mm and from *OY* is 50 mm.

3. Draw a line *AB* at any convenient inclination to pass through point *P*. The points *A* and *B* should lie on the asymptotes. Mark point *Q* on line *AB* such that *PA* = *BQ*.
4. Similarly, draw another line *CD* at any convenient inclination to pass through point *P*. The points *C* and *D* should lie on the asymptotes. Mark point *R* on line *CD* such that *PD* = *CR*.
5. Draw a line *EF* at any convenient inclination to pass through point *Q*. The points *E* and *F* should lie on the asymptotes. Mark point *S* on line *EF* such that *QF* = *ES*.
6. Similarly, draw another line *GH* at any convenient inclination to pass through point *Q*. The points *G* and *H* should lie on the asymptotes. Mark point *T* on line *GH* such that *QH* = *GT*.
7. Proceed to mark sufficient number of points of the hyperbola. Draw a smooth curve passing through the points *P*, *Q*, *R*, *S*, *T*, etc. This is the required rectangular hyperbola.

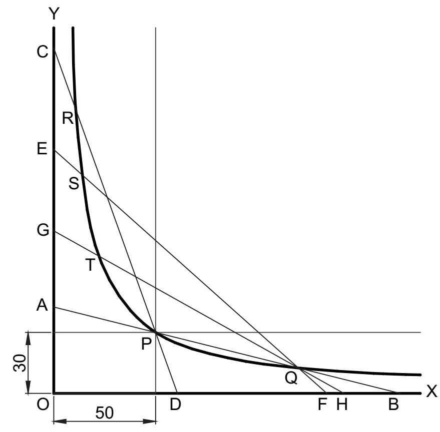


Fig. 5.38 *Intercept method for hyperbola*

5.10.5 Orthogonal Asymptotes Method

Orthogonal asymptotes method is used to draw rectangular hyperbola. The asymptotes are perpendicular to each other.

Problem 5.21 *A point P of the hyperbola is situated at a distance of 35 mm and 50 mm from the pair of asymptotes. The asymptotes are perpendicular to each other. Draw a hyperbola using orthogonal asymptotes method.*

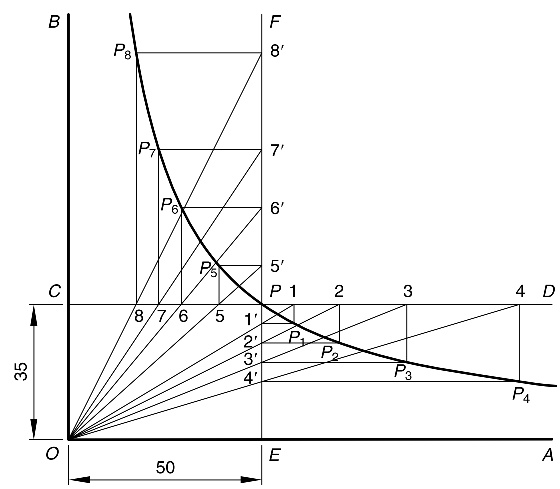


Fig. 5.39 *Orthogonal asymptotes method for hyperbola*

Construction Refer to Fig. 5.39.

1. Draw asymptotes *OA* and *OB* perpendicular to each other.
2. Mark a point *P* such that its distance from *OA* is 35 mm and from *OB* is 50 mm.
3. Draw lines *CD* and *EF* parallel to *OA* and *OB* respectively, and pass through point *P*,
4. Mark points 1, 2, 3, etc., on line *PD* at any convenient distance, which need not be equal.
5. Join *O*1, *O*2, *O*3, etc., to intersect the line *EP* at points 1′, 2′, 3′, etc.
6. Draw lines from points 1, 2, 3, etc., parallel to *OB* to intersect lines drawn from points 1′, 2′, 3′, parallel to *OA* at points P_1, P_2, P_3, etc.
7. Mark points 5, 6, 7, etc., on line *CP* at any convenient distance, which need not be equal.
8. Join *O*5, *O*6, *O*7, etc., and produce to intersect the line *PF* at points 5′, 6′, 7′, etc.
9. Draw lines from points 5, 6, 7, etc., parallel to *OB* to intersect lines drawn from points 5′, 6′, 7′, parallel to *OA* at points P_5, P_6, P_7, etc.
10. Draw a smooth curve passing through P_1, P_2, P_3, P_5, P_6, P_7, etc., to get the required rectangular hyperbola.

5.10.6 Oblique Asymptotes Method

The oblique asymptotes method is used to draw hyperbola other than rectangular hyperbola. The asymptotes are inclined at an angle other than 90º.

Problem 5.22 *Draw a hyperbola when its asymptotes are inclined at 60º to each other and passes through a point P. The point P is at a distance of 40 mm and 50 mm from the asymptotes.*

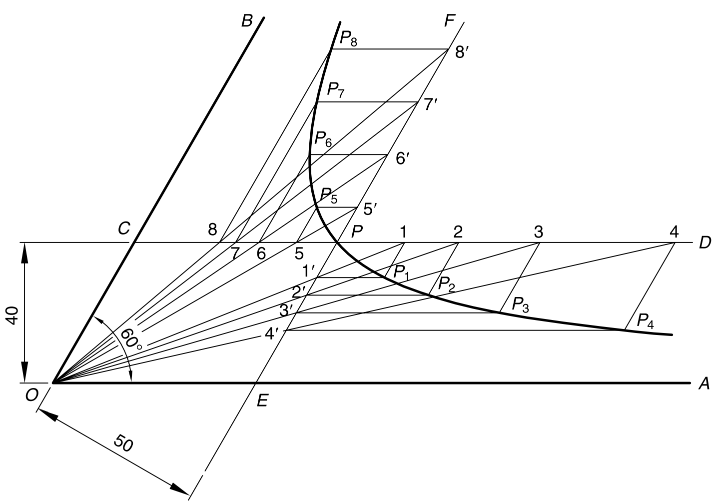


Fig. 5.40 *Oblique asymptotes method for hyperbola*

Construction Refer to Fig. 5.40.

1. Draw asymptotes *OA* and *OB* inclined at an angle of 60º.
2. Mark a point *P* such that its distance from *OA* is 40 mm and from *OB* is 50 mm.

3. Draw lines CD and EF parallel to OA and OB respectively, and pass through point P,
4. Mark points 1, 2, 3, etc., on line PD at any convenient distance, which need not be equal.
5. Join $O1$, $O2$, $O3$, etc., to intersect the line EP at points $1'$, $2'$, $3'$, etc.
6. Draw lines from points 1, 2, 3, etc., parallel to OB to intersect lines drawn from points $1'$, $2'$, $3'$, parallel to OA at points P_1, P_2, P_3, etc.
7. Mark points 5, 6, 7, etc., on line CP at any convenient distance, which need not be equal.
8. Join $O5$, $O6$, $O7$, etc., and produce to intersect the line PF at points $5'$, $6'$, $7'$, etc.
9. Draw lines from points 5, 6, 7, etc., parallel to OB to intersect lines drawn from points $5'$, $6'$, $7'$, parallel to OA at points P_5, P_6, P_7, etc.
10. Draw a smooth curve passing through P_1, P_2, P_3, P_5, P_6, P_7, etc., to get the required rectangular hyperbola.

5.11 LOCATE ASYMPTOTES AND DIRECTRIX

Problem 5.23 *Draw two branches of hyperbola having foci 70 mm apart and vertices 40 mm apart. Locate the asymptotes and measure the included angle. Also locate the directrix of the hyperbola.*

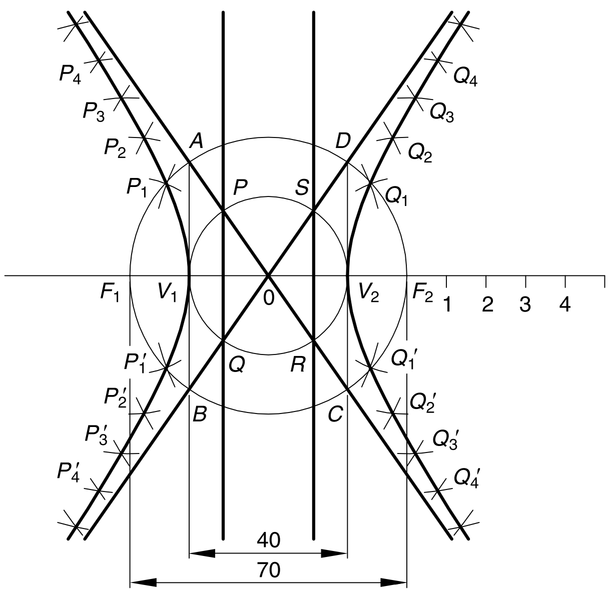


Fig. 5.41 *Locate asymptotes and directrix of the hyperbola*

Construction Refer to Fig. 5.41.

1. Draw two branches of the hyperbola using intersecting arcs method. (Problem 5.18)
2. Draw a circle with centre O and diameter F_1F_2.

3. At V_1 and V_2 draw two lines perpendicular to the axis to intersect the circle at points A, B, C and D.
4. Join AC and BD and produce on both sides. These are the required asymptotes. Measure the included angle as 70° or 110°.
5. With centre O and diameter V_1V_2 draw a circle to intersect asymptotes at points P, Q, R and S.
6. Join PQ and RS and produce on both sides. These are the required directrices.

5.12 TANGENT AND NORMAL TO THE HYPERBOLA

5.12.1 From a Point on the Parabola

Problem 5.24 *Draw a tangent and a normal to the given hyperbola at a point P, when distance between foci is known.*

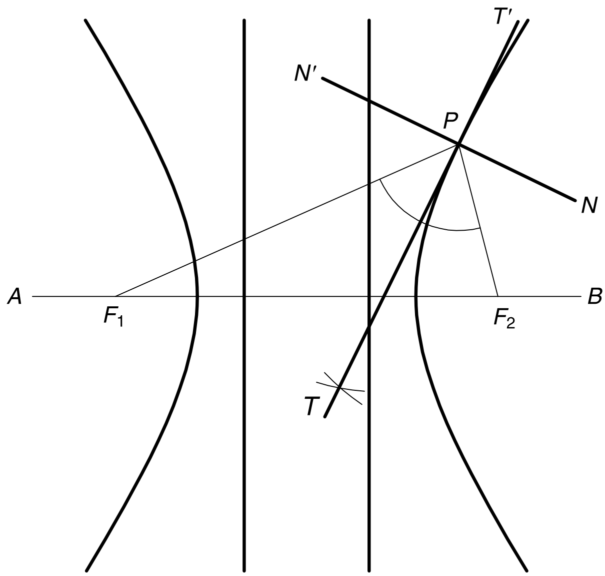


Fig. 5.42 *Tangent and normal through a point, lying on the hyperbola*

Construction Refer to Fig. 5.42.

1. Let AB is the axis and F_1 and F_2 are the foci of the given hyperbola.
2. Locate a point P on the hyperbola. Join PF_1 and PF_2.
3. Draw TT' as the bisector of $\angle F_1PF_2$. This is the required tangent.
4. Draw a line NN' to pass through point P and perpendicular to TT'. This is the required normal.

5.13 MISCELLANEOUS PROBLEMS

Problem 5.25 *The major and minor axes of an ellipse are 140 mm and 90 mm respectively. Find the foci and draw the ellipse using arcs of circle method. Draw a tangent and a normal to the ellipse at a point 40 mm above the major axis.*

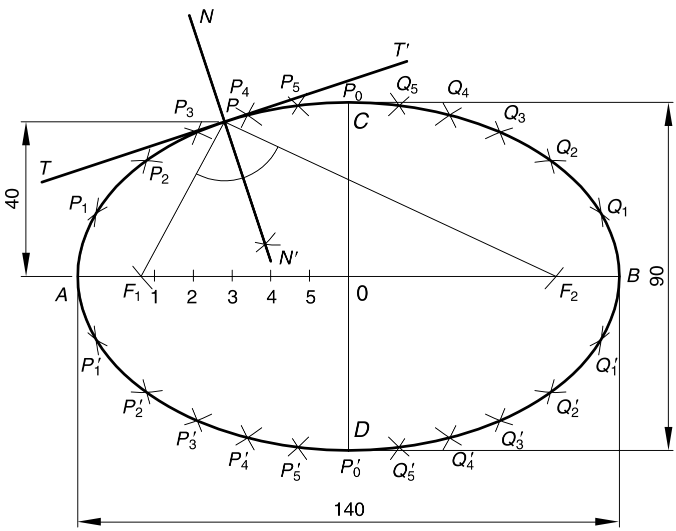


Fig. 5.43

Construction Refer to Fig. 5.43.

1. Draw the major axis AB = 140 mm and minor axis CD = 90 mm bisecting each other at O.
2. Draw an arc with centre C and radius OA to meet line AB at points F_1 and F_2 called foci.
3. Draw the ellipse using 'arcs of circle' method (Problem 5.2).
4. Draw a line at a distance 40 mm and parallel to AB to meet the ellipse at point P.
5. Draw tangent and normal to the ellipse from the point P (Problem 5.7).

Problem 5.26 *Construct an ellipse having a major axis 100 mm and minor axis 65 mm. Locate its foci, directrices and find the eccentricity.*

Construction Refer to Fig. 5.44.

1. Draw the ellipse using 'concentric circles' method (Problem 5.3).
2. Draw an arc with centre C and radius OA to cut the major axis AB at points F_1 and F_2. Points F_1 and F_2 are the foci of the ellipse.
3. From focus F_1, draw a line F_1P perpendicular to the axis AB to meet the ellipse at a point P.
4. Draw a tangent TT' to the ellipse at point P (Problem 5.7).
5. Let the tangent TT' meet the axis AB produced at point Q. Through point Q, draw a line D_1D_1' perpendicular to the axis AB. The line D_1D_1' represents the directrix.
6. Draw a line D_2D_2' parallel to D_1D_1' at distance equal to OQ on the other side of ellipse. D_2D_2' is the second directrix.

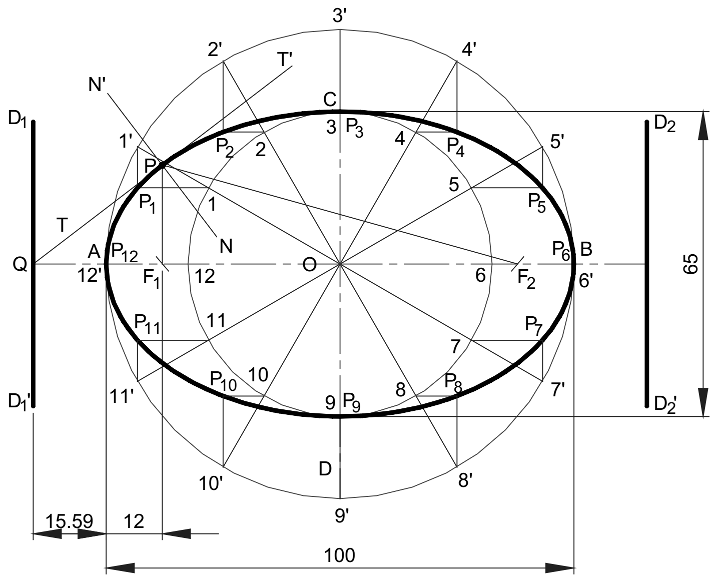


Fig. 5.44

7. Determine the ratio of AF_1 and AQ as eccentricity. Here, $e = \dfrac{AF_1}{AQ} = \dfrac{12}{15.6} = 0.77$.

Problem 5.27 *The conjugate diameters of an ellipse are 130 mm and 90 mm. The included angle between them is 60°. Draw the ellipse and determine its major and minor axes.*

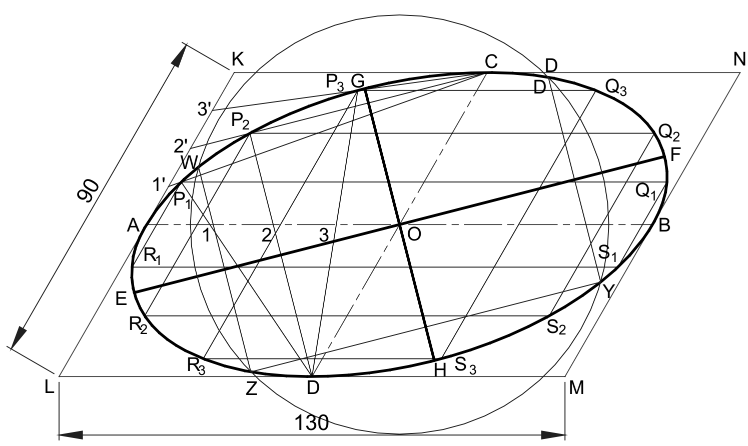


Fig. 5.45

Construction Refer to Fig. 5.45.

1. Draw conjugate diameters AB = 130 mm and CD = 90 mm inclined at an angle of 60° and intersect each other at a mid-point O.
2. Draw parallelogram $KLMN$ such that sides KL and MN are parallel to line CD and sides LM and KN are parallel to line AB.
3. Inscribe an ellipse in the parallelogram $KLMN$ (Problem 5.5).
4. Locate the major axis EF and the minor axis GH (Problem 5.6).

Problem 5.28 *The major axis of an ellipse is 110 mm long and the foci are at a distance of 15 mm from the ends. Draw the ellipse, one-half of it by concentric circles method and the other half by rectangle method. Find the directrix and eccentricity of this ellipse.*

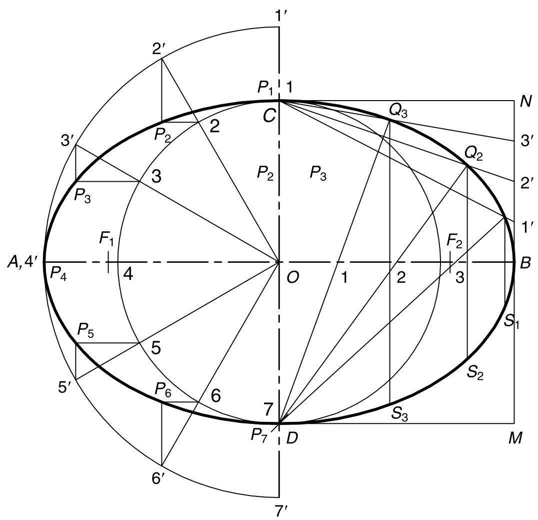


Fig. 5.46

Construction Refer to Fig. 5.46.

1. Draw the major axis AB = 110 mm and mark foci F_1 and F_2 80 mm apart. Let O is the mid-point of both AB and F_1F_2.
2. Draw an arc with centre F_1 and radius OA to meet a perpendicular line through point O at points C and D. Line CD is the minor axis.
3. Draw left half of the ellipse using concentric circles method (Problem 5.3).
4. Draw a rectangle $CDMN$ and inscribe other half of the ellipse using rectangle method (Problem 5.4).
5. Determine eccentricity, $e = \dfrac{\text{Distance between foci}}{\text{Length of major axis}} = \dfrac{F_1F_2}{AB} = \dfrac{110 - 2 \times 15}{110} = \dfrac{8}{11}$

Problem 5.29 *Draw an ellipse passing through end points of a triangle ABC having sides 100 mm, 75 mm and 50 mm long. Also, draw a curve parallel to and 25 mm away from the ellipse.*

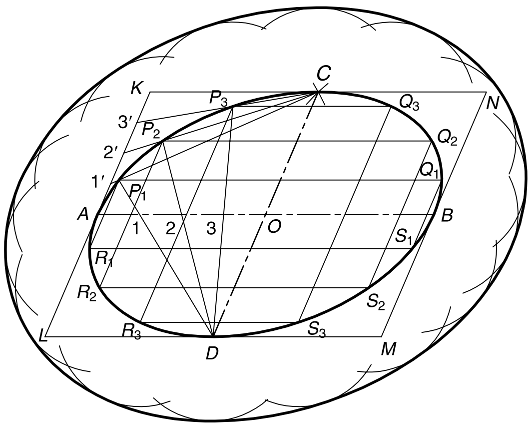


Fig. 5.47

Construction Refer to Fig. 5.47.

1. Draw a triangle *ABC* taking *AB* = 100 mm, *AC* = 75 mm and *BC* = 50 mm.
2. Mark *O* as the mid-point of *AB*. Join *CO* and extend it to *D* such that *OC* = *OD*.
3. Draw a parallelogram *KLMN* and inscribe the ellipse (Problem 5.5).
4. Draw a curve parallel to and 25 mm away to the ellipse (Problem 5.9).
   It may be noted that the curve drawn parallel to the ellipse is not an ellipse.

Problem 5.30 *Draw an ellipse of major axis 90 mm to pass through a point, 75 mm and 28 mm from the ends of the major axis.*

Construction Refer to Fig. 5.48.

1. Draw the major axis *AB* = 90 mm. Mark *O* as the mid-point of *AB*.
2. Draw an arc with centre *A* and radius 75 mm to intersect another arc with centre *B* and radius 28 mm at point *P*. The ellipse should pass through point *P*.
3. Draw a circle with centre *O* and diameter 90 mm.
4. Draw a line from *P* perpendicular to the *AB*, to meet the circle at *Q*. Join *OQ*.
5. Draw a line from point *P* parallel to *AB*, to meet the line *OQ* at point *R*.
6. Draw a circle with centre *O* and radius *OR*, to intersect a line through point *O* and perpendicular to *AB* at points *C* and *D*. The line *CD* is the minor axis.
7. Draw the ellipse using concentric circles method with major axis *AB* and minor axis *CD* (Problem 5.3).

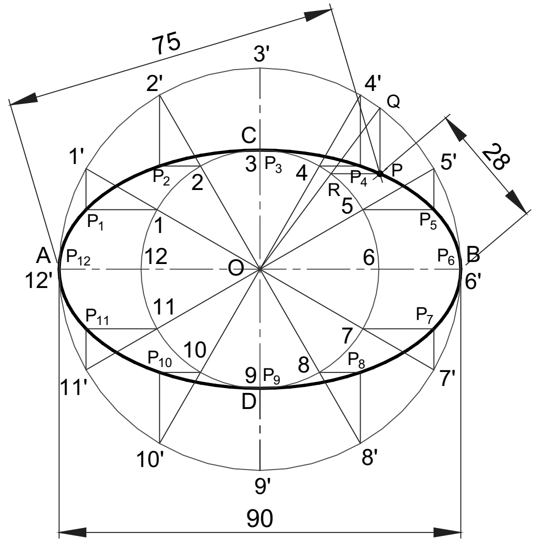


Fig. 5.48

Problem 5.31 *Inscribe a parabola in a rectangle of sides 100 mm and 80 mm with the longer side as the base. Locate the focus and the directrix of the parabola.*

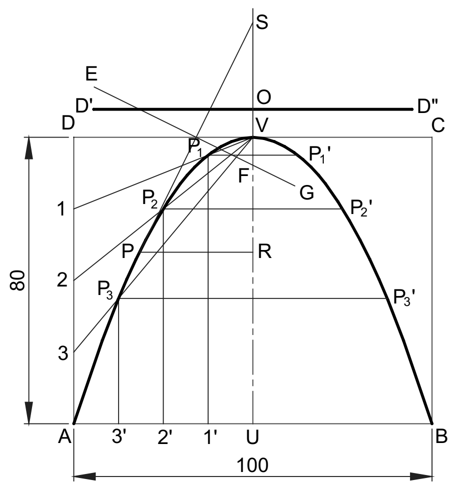


Fig. 5.49

Construction Refer to Fig. 5.49.

1. Draw a rectangle of sides 100 mm and 80 mm. Inscribe a parabola (Problem 5.12).
2. Locate the focus and the directrix of the parabola (Problem 5.14).

Problem 5.32 *A shot is discharged from the ground at an inclination of 45° to the ground which is horizontal. The shot returns to the ground at a point 120 m away from the point of discharge. Draw the path traced by the shot. Find the direction of the shot after it has travelled a horizontal distance of 100 m.*

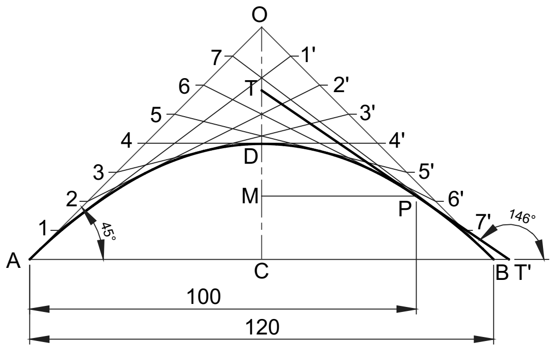


Fig. 5.50

Construction Refer to Fig. 5.50.

1. Draw an isosceles triangle *ABO* taking *AB* = 120 mm and ∠*OAB* = ∠*OBA* = 45°.
2. Mark *C* as the mid-point of *AB*. Join *OC*.
3. Draw the parabola using tangent method with base *AB* and half of *OC* as altitude (Problem 5.11).
4. Draw a vertical line at a horizontal distance of 100 m from *A* to meet the parabola at *P*.
5. Draw tangent *TT′* from point *P* of the parabola (Problem 5.15).
6. Measure inclination of the tangent *TT′* with line *AB* as the direction of the shot at *P*. Here it is 146°.

Problem 5.33 *Two points A and B are 110 mm apart. Point C is 90 mm and 60 mm from A and B respectively. Draw a parabola passing through points A, B and C.*

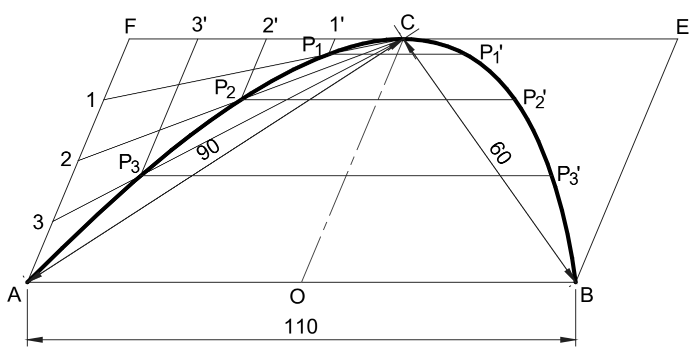


Fig. 5.51

Construction Refer to Fig. 5.51.

1. Draw a triangle *ABC* with *AB* = 110 mm, *AC* = 90 mm and *BC* = 60 mm.

2. Mark O as its mid-point of AB. Join OC.
3. Draw a parallelogram ABEF and inscribe a parabola using oblong method (Problem 5.13).

Problem 5.34 *A stone is thrown from a 4 m high building and at its highest flight, the stone just crosses the top of a 10 m high tree from the ground. Trace the path of the projectile, if the horizontal distance between the building and the tree is 5 m. Find the distance of the point from the building where the stone falls on the ground.*

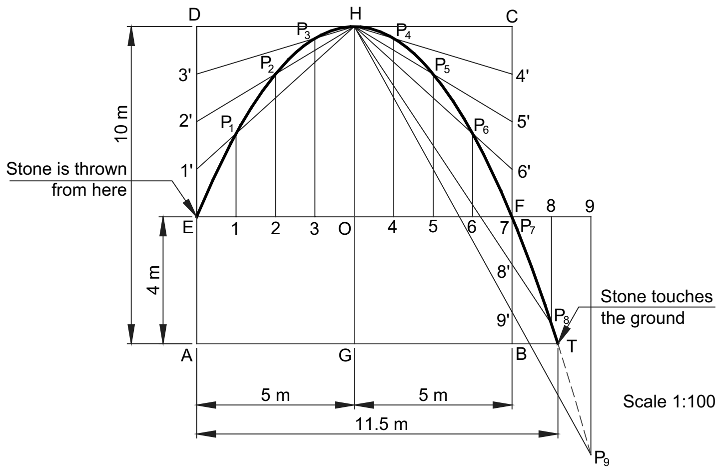


Fig. 5.52

Construction Refer to Fig. 5.52.

1. Take a scale 1 cm = 1 m.
2. Draw a rectangle $ABCD$ of 10 m × 10 m. The horizontal line AB represents the ground level.
3. Mark a point E, 4 m above point A. The line AE represents the building and point E represents the location from where the ball is thrown.
4. Mark G and H as the mid-points of AB and CD respectively. Join GH to represent the palm tree.
5. Draw a parabola in rectangle $EFCD$ (Problem 3.12). It may be noted that here sides ED, EO, FO and FC are divided into four equal parts.
6. Mark points 8, 9 on line EF produced such that lengths F-8 and 8-9 are equal to E-1. Similarly, mark points 8′ and 9′ on BF such that 7′-8′ and 8′-9′ are equal to C-4′.
7. Join H8′ and H9′ produce to intersect the vertical lines from points 8 and 9 at points at P_8 and P_9, respectively.
8. Extend the parabola to pass through points P_8 and P_9.
9. Locate point T of the parabola on line AB produced. This is the point where ball will touch the ground. Measure AT as 11.5 m.

Problem 5.35 *The directrices of a hyperbola are 50 mm apart and the vertices are 70 mm apart. Locate the asymptotes and foci graphically and construct two branches of the hyperbola.*

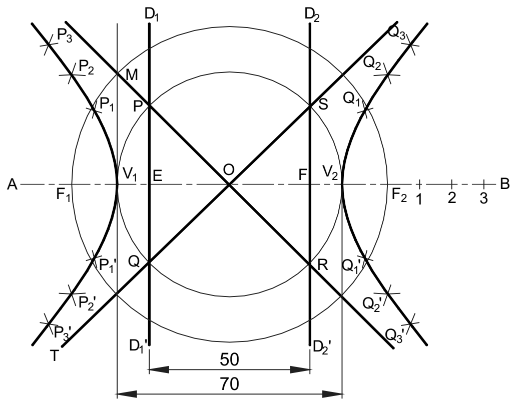


Fig. 5.53

Construction Refer to Fig. 5.53.

1. Draw principal axis *AB* and mark a point *O*.
2. Mark vertices V_1 and V_2 on *AB*, 70 mm apart and symmetric about *O*.
3. Mark points *E* and *F* on *AB*, 50 mm apart and symmetric about *O*.
4. Draw vertical lines D_1D_1' and D_2D_2' from points *E* and *F*, perpendicular to *AB* as the directrices.
5. With centre *O* and diameter V_1V_2, draw a circle to intersect the directrices D_1D_1' and D_2D_2' at points *P*, *Q*, *R* and *S*.
6. Join *PR* and *QS* and produce on both sides. These are the required asymptotes.
7. Draw vertical lines from points V_1 to meet the asymptotes at points *M*.
8. With centre *O* and radius *OM* draw a circle to meet *AB* at points F_1 and F_2. These are the required foci.
9. Draw two branches of the hyperbola using intersecting arcs method (Problem 5.18).

Problem 5.36 *The foci of a rectangular hyperbola are 70 mm apart. Draw two branches of the hyperbola and locate vertices and directrices graphically.*

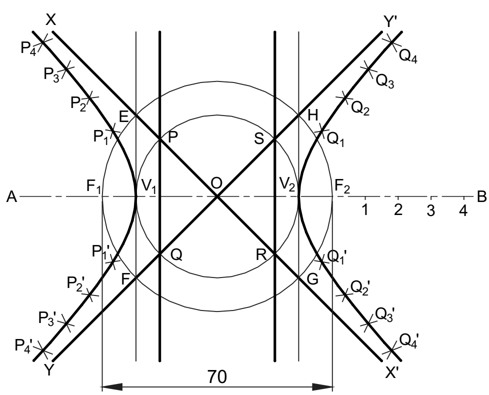


Fig. 5.54

Construction Refer to Fig. 5.54.

1. Draw the principal axis AB and mark O on it.
2. Draw two asymptotes XX' and YY' passing through O and inclined at 45° with AB.
3. Mark F_1 and F_2 on AB taking $OF_1 = OF_2 = 35$ mm.
4. With centre O and diameter F_1F_2, draw a circle to cut the asymptotes at points E, F, G and H.
5. Join EF and GH to cut the principal axis at points V_1 and V_2. These are the required vertices.
6. With centre O and diameter V_1V_2 draw a circle to cut the asymptotes at point P, Q, R and S.
7. Join PQ and RS and produce on both sides. These are the required directrices.
8. Draw the hyperbola using intersecting arcs method (Problem 5.18).

Problem 5.37 *A pair of asymptotes of the hyperbola is inclined at an angle of 120°. A point P of the hyperbola lies at a distance of 20 mm and 30 mm from the asymptotes. Draw two branches of the hyperbola and determine distance between the vertices.*

Construction Refer to Fig. 5.55.

1. Draw asymptotes XX' and YY' inclined at 120°. Locate point P is at a distance of 40 mm and 50 mm from the asymptotes. Draw one branch of the hyperbola using oblique asymptotes method (Problem 5.22).
2. Draw other branch of the hyperbola making it symmetrical about line MN.

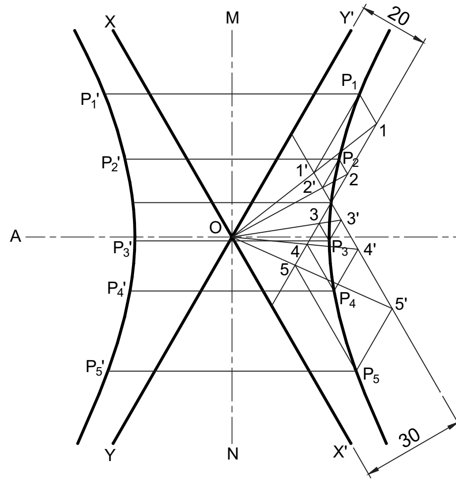


Fig. 5.55

EXERCISE 5A

Ellipse

5.1 Draw the ellipse when the distance of its focus from its directrix is equal to 60 mm and eccentricity is 3/5. Also draw a tangent and a normal to the ellipse at a point 100 mm away from the directrix.

5.2 Draw an ellipse when the distance of its vertex from its directrix is 24 mm and distance of its focus from directrix is 42 mm.

5.3 A fixed point is 90 mm from a fixed straight line. Draw the locus of a point P moving in such a way that its distance from the fixed straight line is twice its distance from the fixed point. Name the curve. Draw a tangent and a normal at a point 40 mm away from the fixed point.

5.4 Two fixed point A and B are 80 mm apart. Trace the complete path of a point P moving in such a way that the sum of its distance from A and B is always the same and equal to 110 mm. Name the curve and the construction method employed.

5.5 The distance between the directrices of an ellipse are 170 mm and the distance between its foci are 70 mm. Determine its major and minor axes and construct the ellipse using intersecting arcs method.

5.6 Construct an ellipse having major and minor axes 140 mm and 80 mm respectively. Draw a pair of tangent from a point P situated outside at a distance of 90 mm from both the axes.

5.7 An ellipse has the major axis and the minor axis in the ratio of 3:2. Draw the ellipse when the major axis is 135 mm.

5.8 The major axis of an ellipse is 120 mm long and the foci are at a distance of 20 mm from its ends. Draw the ellipse using one-half of it by concentric circles method and the other half by rectangle method.

5.9 The directrices of an ellipse are 150 mm apart and its vertices are 100 mm apart. Construct the ellipse, one half of it by 'intersecting circles' method and the other half by 'concentric circles' method.

5.10 Inscribe an ellipse in a rectangle of sides 150 mm and 110 mm. Draw a curve parallel to the ellipse and 25 mm away from it.

5.11 Construct an ellipse in a parallelogram of sides 150 mm and 90 mm with an included angle of 60°. Draw tangent and normal to it at a point 60 mm from centre of the ellipse.

5.12 The conjugate diameters of an ellipse are 120 mm and 80 mm long and they intersect at an angle of 120°. Construct an ellipse and determine its major and minor axes graphically.

5.13 Two points *A* and *B* are 120 mm apart. Third point *C* is 90 mm from *A* and 60 mm from *B*. Draw an ellipse passing through points *A*, *B* and *C*.

5.14 The directrices of an ellipse are 150 mm apart and the major axis is 110 mm long. Draw the ellipse and determine the eccentricity, length of the minor axis and distance between the foci.

5.15 In a triangle *ABC*, *AB*, *AC*, and *BC* are 110 mm, 60 mm and 70 mm respectively. Draw an ellipse such that *A* and *B* are foci, and *C* is a point on the curve. Find the directrix and eccentricity of this ellipse.

5.16 Draw an ellipse having 120 mm long major axis and pass through a point lying at a distance of 100 mm and 35 mm from the ends of the major axis. Find the directrix and eccentricity of this ellipse.

5.17 Draw an ellipse having 60 mm long minor axis passing through a point *P* lying at a distance of 65 mm and 35 mm from the ends of the minor axis. Also draw a tangent and a normal to the ellipse at the point *P*.

Parabola

5.18 Construct a parabola whose focus is at a distance of 40 mm from the directrix. Draw a tangent and a normal to the parabola at a point 50 mm away from the principal axis. Determine the double ordinate through a point 90 mm from the directrix.

5.19 Construct a parabola using 'eccentric method' whose vertex is at a distance of 30 mm from the focus. Draw a pair of tangents from a point *P*, outside the curve, 20 mm from the vertex and 40 mm from the focus.

5.20 Draw the locus of a point which moves in such a manner that its distance from a fixed point is equal to its distance from a fixed straight line. Consider the distance between the fixed point and the fixed line as 60 mm. name the curve.

5.21 Construct a parabola using 'Offset method' when its double ordinate is 150 mm and abscissa is 75 mm. Locate the focus and directrix to the parabola.

5.22 A fountain jet discharges water from ground level at an inclination of 60° to the ground. The jet travels a horizontal distance of 14 m from the point of discharge and falls on the ground. Trace the path of the jet and name the curve.

5.23 Draw a rectangle having its sides 150 mm and 90 mm long. Inscribe two parabolas in it with their axis bisecting each other.

5.24 Inscribe a parabola in the parallelogram of sides 110 mm and 70 mm long with longer side of it as the normal base. Consider one of the included angles between the sides as 60°.

5.25 In a triangle *ABC*; *AB*, *BC* and *CA* are 120 mm, 40 mm and 80 mm long respectively. Draw a parabola passing through the points *A*, *B* and *C*. also determine the axis of the parabola.

5.26 A ball is thrown from the ground level which reaches a height of 16 m and a horizontal distance of 28 m before coming to the ground. Trace the path of the ball and determine direction of the ball when it was at a height of 10 m from the ground.

5.27 When cricket a ball was thrown, it reached a maximum height of 7 m and fell on the ground at a distance of 16 m from the point of projection. Draw the path of the ball, calculate the angle of projection and name the curve.

Hyperbola

5.28 The focus of a hyperbola is 60 mm from its directrix. Draw the curve when eccentricity is 5/3. Draw a tangent and a normal to the curve at a point 45 mm from the directrix.

5.29 Draw the hyperbola when the focus and the vertex are 25 mm apart. Consider eccentricity as 3/2. Draw a tangent and a normal to the curve at a point 35 mm from the focus.

5.30 Draw the hyperbola when its vertex and its focus are at a distance of 15 mm and 40 mm respectively from the directrix. Plot at least six points.

5.31 A fixed point is 90 mm from a fixed straight line. Draw the locus of a point *P* moving in such a way that its distance from the fixed point is twice its distance from the fixed straight line. Name the curve.

5.32 Construct two branches of a hyperbola when its transverse axis is 50 mm long and foci are 70 mm apart. Locate its directrix and determine the eccentricity.

5.33 Two points are fixed at 100 mm apart. Draw the locus of a point moving in such a manner that the difference of its distance from the points is 75 mm. Name the curve.

5.34 Draw two branches of a hyperbola when the distance between its foci is 90 mm and the vertices are 15 mm from the foci. Locate the asymptotes and measure the angle between them.

5.35 Draw two branches of a rectangular hyperbola having its vertices 60 mm apart and determine its directrices and foci graphically.

5.36 Draw a rectangular hyperbola whose directrices are 40 mm apart and locate its foci and vertices.

5.37 The asymptotes of a hyperbola are inclined at an angle of 75°. Its foci are 60 mm apart. Locate its foci graphically and construct two branches of the hyperbola. Also draw a tangent and a normal to the curve at a point 20 mm from one of the foci.

5.38 The asymptotes of a hyperbola are inclined at an angle of 120° and its transverse axis is 80 mm. Construct two branches of the hyperbola and determine its foci. Take at least eight points for constructing each branches of the hyperbola.

5.39 Half the transverse axis, double ordinate and abscissa of a hyperbola are 30 mm, 100 mm and 40 mm respectively. Construct two branches of the hyperbola.

5.40 The transverse axis of a hyperbola is 80 mm long. Its double ordinate is 90 mm long and the corresponding abscissa is 50 mm. Construct the hyperbola.

5.41 The asymptotes of a hyperbola are inclined at 105° to each other. A point *P* on the curve is 40 mm and 50 mm from the asymptotes respectively. Construct two branches of the hyperbola, and determine (a) distance between its vertices, (b) distance between its directrices, (c) distance between its foci and (d) eccentricity.

5.42 Draw a rectangular hyperbola when the position of a point *P* on the curve is 30 mm from the horizontal asymptote and 50 mm from the vertical asymptote. Show at least four points on either side of point *P*.

5.43 The asymptotes of a hyperbola are inclines at 75° to each other. A point *P* on the curve is 25 mm and 40 mm from its asymptotes. Draw the curve showing within 10 mm distance from each asymptotes. Also determine the directrix and focus of the hyperbola.

5.44 The asymptotes of a hyperbola are at right angle to each other and a point on the curve is at a distance of 30 mm from each of the asymptotes. Draw two branches of the hyperbola. Also draw a tangent and a normal at a point 45 mm from one of the asymptotes.

VIVA-VOCE QUESTIONS

5.1 What is a conic section? Enlist various types of conic sections.

5.2 What is the inclination of the cutting plane in order to obtain following sections from a cone. (a) parabola, (b) ellipse, (c) hyperbola, (d) rectangular hyperbola.

5.3 Give two practical applications of (a) parabola (b) ellipse (c) hyperbola.

5.4 Define eccentricity.

5.5 Enlist any four methods of drawing (a) parabola (b) ellipse (c) hyperbola.

5.6 Which principle is used in construction of an ellipse by intersecting arcs method?

5.7 How a tangent is drawn from a point on the ellipse?

5.8 What do you understand by conjugate diameters?

5.9 Which principle is used in construction of parabola by offset method?

5.10 How is a tangent drawn from a point on the parabola.

5.11 Define ordinate, double ordinate, abscissa and latus rectum.

5.12 Which principle is used in construction of hyperbola by intersecting arcs method.

MULTIPLE-CHOICE QUESTIONS

5.1 If a point moves in a plane in such a way that the sum of its distances from two fixed points is constant the curve so traced is called

(a) ellipse (b) parabola
(c) hyperbola (d) None of these

5.2 Name the curve traced out by a point moving in a plane such that the difference between its distances from two fixed points is constant

(a) ellipse (b) parabola
(c) hyperbola (d) Any of these

5.3 When a bullet is shot in air the path traversed by the bullet is called

(a) cycloid (b) semicircle
(c) parabola (d) hyperbola

5.4 A right circular cone when cut by a plane parallel to its generator, the curve obtained is a

(a) ellipse (b) parabola
(c) hyperbola (d) circle

5.5 When a right circular cone is cut by a plane passing through its apex, the curve obtained is

(a) ellipse (b) parabola
(c) hyperbola (d) triangle

5.6 When a right circular cone is cut which meets its axis at an angle greater than the semi-apex angle, the curve obtained is

(a) ellipse (b) parabola
(c) hyperbola (d) triangle

5.7 When a right circular cone is cut which meets its axis at an angle less than the semi-apex angle, the curve obtained is

(a) ellipse (b) parabola
(c) hyperbola (d) triangle

5.8 The angle between the asymptotes of a rectangular hyperbola is

(a) 30° (b) 45°
(c) 60° (d) 90°

5.9 Name the curve which has zero eccentricity

(a) ellipse (b) parabola
(c) hyperbola (d) circle

5.10 Which of the following curves obeys the Boyle's law?

(a) Ellipse (b) Parabola
(c) Hyperbola (d) Circle

5.11 Which of the following applications hyperbolic curve is used?

(a) Solar collector (b) Cooling tower
(c) Lamp reflectors (d) Monuments

5.12 The major and minor axes of an ellipse are 100 mm and 60 mm respectively. What will be the distance of its foci from the end of the minor axis?

(a) 30 mm (b) 40 mm
(c) 50 mm (d) 60 mm

Answers to multiple-choice questions

5.1 (a), 5.2 (c), 5.3 (c), 5.4 (b), 5.5 (d), 5.6 (a), 5.7 (c), 5.8 (d), 5.9 (d), 5.10 (d), 5.11 (b), 5.12 (c)

Chapter

6 ENGINEERING CURVES AND LOCI OF POINTS

6.1 INTRODUCTION

The curve is considered to have certain curvature. The plane algebraic curve with two-degree such as circles and conic sections has been discussed in the previous chapter. The plane transcendental curve with variable degree such as roulettes, sprial and helix are also of engineering importance. This chapter deals with some common methods of construction of plane transcendental curves used in engineering practices.

6.2 ROULETTES

Curves generated by the rolling contact of one curve or line on another curve or line are called roulettes. There are infinite varieties of roulettes. The most common types of roulettes used in engineering applications are cycloids, epicycloids, hypocycloids, trochoids, and involutes.

6.3 CYCLOIDAL CURVES

Cycloidal curves are generated by a point lying on the circumference of a circle, when it rolls along a fixed straight or curved path without slipping. The circle which rolls is called the *rolling circle* or *generating circle* and the fixed straight line or the circle on which it rolls is called the *directing line* or the *directing circle*. Cycloidal curves are commonly used in kinematics (the study of motion) and in mechanisms that work with rolling contact.

6.3.1 Cycloid

A cycloid is a curve traced by a point on the circumference of a circle which rolls along a fixed straight line without slipping. Consider Fig. 6.2 where a circle with centre C rolls along a straight line PQ. The path traced by the point P lying on the circumference of the circle is called cycloid.

Cycloid was named by Galileo in 1599. The cycloid has been historically used in teeth profiles of gears. The cycloid is the solution to the brachistochrone problem (i.e., it is the curve of fastest descent under gravity) and the related tautochrone problem (i.e., the period of an object in descent without friction inside this curve does not depend on the object's starting position).

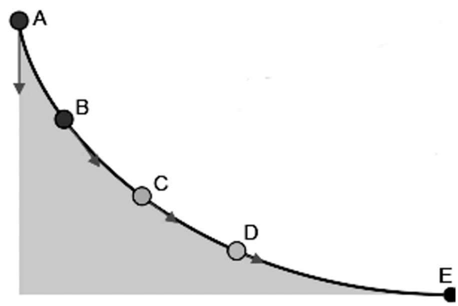


Fig. 6.1 *Tautochrone problem (objects at A, B, C and D will take same time to reach point E)*

Problem 6.1 *Draw a cycloid of a circle of diameter 50 mm for one revolution. Also, draw a tangent and a normal to the curve at a point 35 mm above the base line.*

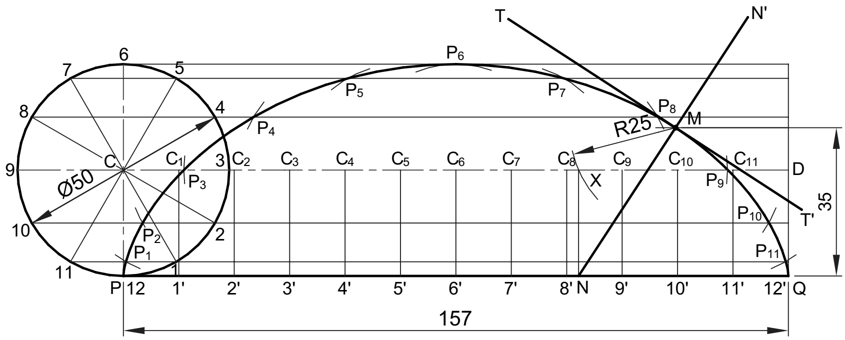


Fig. 6.2 *Cycloid*

Construction Refer to Fig. 6.2.

1. Draw a circle of diameter 50 mm with centre C.
2. Draw the directing line PQ = π D = 157 mm long, horizontal and tangential to the circle.
3. Divide the circle into 12 equal parts and mark the divisions as 1, 2, 3, etc. Draw lines through points 1, 2, 3, etc., parallel to PQ.
4. Divide PQ into 12 equal parts and mark the divisions as 1′, 2′, 3′, etc.
5. Erect vertical lines from points 1′, 2′, 3′, etc., to meet the centre line CD at C_1, C_2, C_3, etc. When the circle rolls through $1/12^{th}$ rotation, point 1 of the circle will coincide with 1′ and centre C will move to C_1. The point P will move to the new position P_1 lying on the horizontal line through point 1 at a distance of 25 mm from C_1.
6. Draw an arc with centre C_1 and radius 25 mm to intersect the horizontal line through point 1 at point P_1.
7. Similarly, draw arcs with centres C_2, C_3, C_4, etc., and radius 25 mm, to intersect the horizontal locus lines through points 2, 3, 4, etc., at points P_2, P_3, P_4, etc., respectively.
8. Draw a smooth curve passing through P_1, P_2, P_3, P_4, etc., to get the required cycloid.

Tangent and normal to the cycloid

1. Mark a point M on the cycloid 35 mm above PQ.
2. Draw an arc with centre M and radius 25 mm, to intersect the centre line at X.
3. Draw a vertical line from X to meet PQ at N.
4. Join NM and produce to N′. This line NN′ is the required normal.
5. Through point M draw a line TT′ perpendicular to NN′. This line TT′ is the required tangent.

6.3.2 Epicycloid

An epicycloid is a curve traced by a point on the circumference of a circle which rolls along another circle outside it, without slipping. Consider Fig. 6.4 where a circle with centre C rolls along the arc of circle with centre O and outside it. The path traced by a point P lying on the circumference of the rolling circle is called epicycloid. Epicycloids are used in rotary pumps, blowers and superchargers.

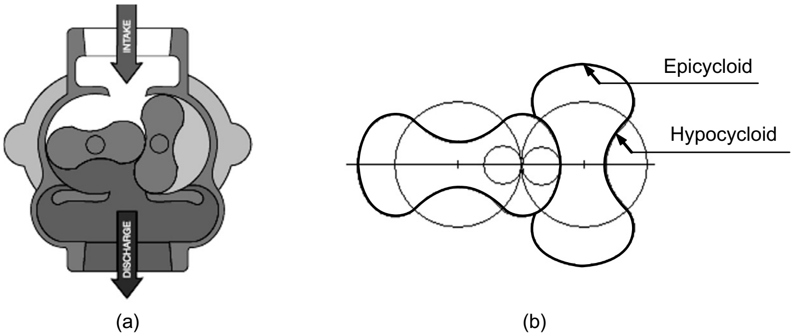


Fig. 6.3 *Rotary pump* **(a)** *Cross-sectional view* **(b)** *Lobe profile*

Problem 6.2 *Draw an epicycloid of a circle of diameter 50 mm, which rolls outside a circle of diameter 180 mm for one revolution. Also, draw a tangent and a normal to the epicycloid at a point 135 mm from the centre of the directing circle.*

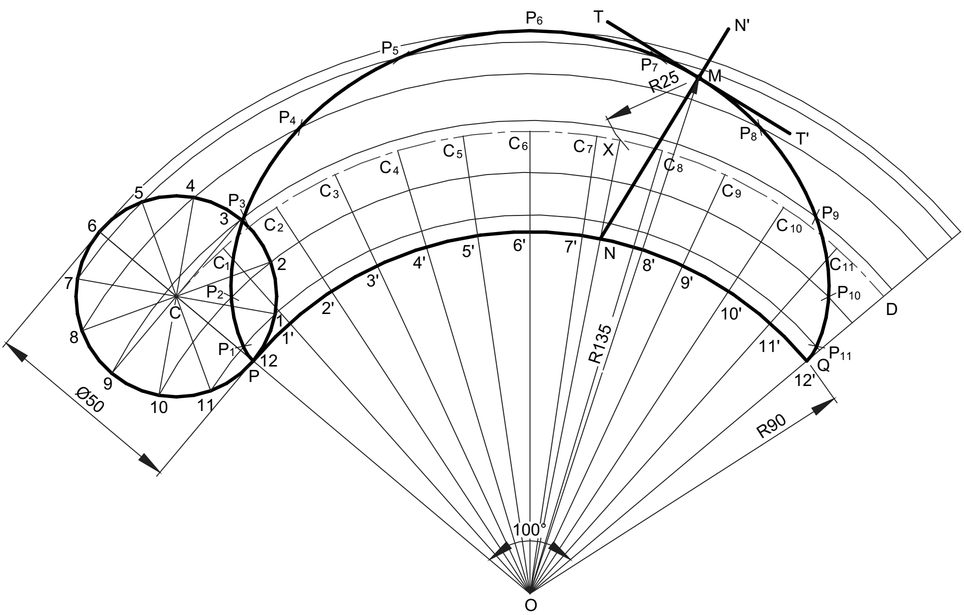


Fig. 6.4 *Epicycloid*

Construction Refer to Fig. 6.4.

A generating circle will cover an arc length $PQ = 2\pi r$ in one revolution. The angle subtended by the arc PQ at centre O is given by $\theta = \dfrac{d}{D} \times 360°$, where d and D are the diameters of the generating and directing circles respectively. Here d = 50 mm and D = 180 mm, therefore, $\theta = \dfrac{50}{180} \times 360° = 100°$.

1. Draw an arc PQ with centre O and radius 90 mm, to subtend angle of 100°. This represents the directing path.
2. Join OP and extend it to point C such that CP = 25 mm.
3. Draw a generating circle with centre C and radius CP. Divide it into 12 equal parts and mark its divisions as 1, 2, 3, 4, etc.
4. Also divide the arc PQ into 12 equal parts and mark the divisions as 1′, 2′, 3′, 4′, etc.
5. Draw arcs, with centre O and radii equal to $O1$, $O2$, $O3$, $O4$, etc., to meet line OQ produced.
6. Draw an arc with centre O and radius OC to meet OQ at D. The arc CD is known as the centre arc.
7. Join $O1'$, $O2'$, $O3'$, $O4'$, etc., and extend each of them to meet CD at C_1, C_2, C_3, C_4, etc., respectively. When the circle rolls through 1/12th rotation, point 1 of the circle will coincide with 1′ and centre C will move to C_1. The point P will move to the new position P_1 lying on the arc through point 1 at a distance of 25 mm from C_1.
8. Draw an arc with centre C_1 and radius 25 mm to intersect the arc through point 1 at P_1.
9. Similarly, draw arcs with centres C_2, C_3, C_4, etc. and radius 25 mm, to intersect arcs through points 2, 3, 4, etc., at P_2, P_3, P_4, etc., respectively.
10. Draw a smooth curve to pass through P_1, P_2, P_3, P_4, etc., and obtain the required epicycloid.

Tangent and normal to epicycloid

1. Mark a point M on the epicycloid at a radial distance of 135 mm from the centre O.
2. Draw an arc with centre M and radius 25 mm, to intersect the centre arc CD at X.
3. Join OX which intersects the arc PQ at N.
4. Join NM and produce it to N'. The line NN' is the required normal.
5. Through point M, draw a line TT' perpendicular to NN'. The line TT' is the required tangent.

6.3.3 Hypocycloid

A hypocycloid is a curve traced by a point on the circumference of a circle which rolls along another circle and inside it, without slipping. Consider Fig. 6.5 where a circle with centre C rolls along the arc of circle with centre O and inside it. The path traced by a point P lying on the circumference of the rolling circle is called hypocycloid. Hypocycloids are also used in rotary pumps, blowers, and superchargers.

Problem 6.3 *Draw a hypocycloid of a circle of diameter 50 mm, which rolls inside a circle of diameter 180 mm for one revolution. Also, draw a tangent and a normal to the hypocycloid at a point 50 mm from the centre of the directing circle.*

Construction Refer to Fig. 6.5.

A generating circle will cover an arc length $PQ = 2\pi r$ in one revolution. The angle subtended by the arc PQ at centre O is given by $\theta = \frac{d}{D} \times 360°$, where d and D are the diameters of the generating and directing circles respectively. Here d = 50 mm and D = 180 mm, therefore, $\theta = \frac{50}{180} \times 360° = 100°$.

1. Draw an arc PQ with centre O and radius 90 mm, to subtend angle of 100°. This represents the directing path.

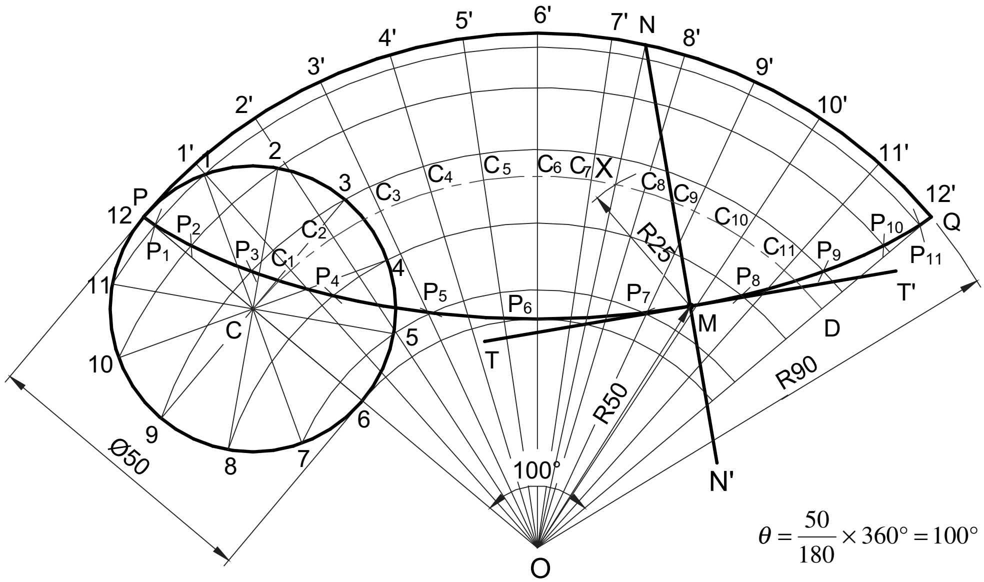


Fig. 6.5 *Hypocycloid*

2. Join OP and mark point C on it such that $CP = 25$ mm.
3. Draw the generating circle with centre C and radius CP. Divide the circle into 12 equal parts and mark the divisions as 1, 2, 3, 4, etc.
4. Also, divide the directing arc PQ into 12 equal parts and mark the divisions as 1′, 2′, 3′, 4′, etc.
5. Draw arcs, with centre O and radii equal to $O1$, $O2$, $O3$, etc., to meet line OQ.
6. Draw an arc with centre O and radius OC to meet OQ at point D. The arc CD is known as the centre arc.
7. Join $O1'$, $O2'$, $O3'$, $O4'$, etc., to meet the arc CD at C_1, C_2, C_3, C_4, etc., respectively.
   When the circle rolls through 1/12th rotation, point 1 of the circle will coincide with 1′ and centre C will move to C_1. The point P will move to the new position P_1 lying on the arc through point 1 at a distance of 25 mm from C_1.
8. Draw an arc with centre C_1 and radius 25 mm to intersect the arc through point 1 at point P_1.
9. Similarly, draw arcs with centres C_2, C_3, C_4, etc., and radius 25 mm, to intersect arcs through points 2, 3, 4, etc., at P_2, P_3, P_4, etc., respectively.
10. Draw a smooth curve to pass through P_1, P_2, P_3, P_4, etc., and obtain the required hypocycloid.

Tangent and normal to hypocycloid

1. Mark point M on the hypocycloid at a radial distance of 50 mm from the centre O.
2. Draw an arc with centre M and radius 25 mm to intersect the centre arc CD at X.
3. Join OX and produce to meet arc PQ at N.
4. Join NM and produce it to N'. The line NN' is the required normal.
5. Through point M, draw a line TT' perpendicular to NN'. The line TT' is the required tangent.

6.4 TROCHOID, EPITROCHOID AND HYPOTROCHOID

6.4.1 Trochoid

A trochoid is a curve traced by a point situated either inside or outside the circle, which rolls along a fixed straight line without slipping. If the tracing point is situated *inside* the rolling circle, the curve is called an *inferior trochoid*. If the tracing point is situated *outside* the rolling circle, the curve is called a *superior trochoid*. A trochoidal curve approximate wave profile and used in naval architecture.

Let a circle with centre *C* rolls along a straight line *AB*. In Fig. 6.6, a point *P* is situated inside the rolling circle and the path traced by it is called inferior trochoid. In Fig. 6.7, the point P is situated outside the rolling circle and the path traced by it is called superior trochoid.

Problem 6.4 *A circle of diameter 50 mm rolls along a straight line for one revolution. Draw the locus of a point, lying at a distance of 16 mm from the centre of the circle.*

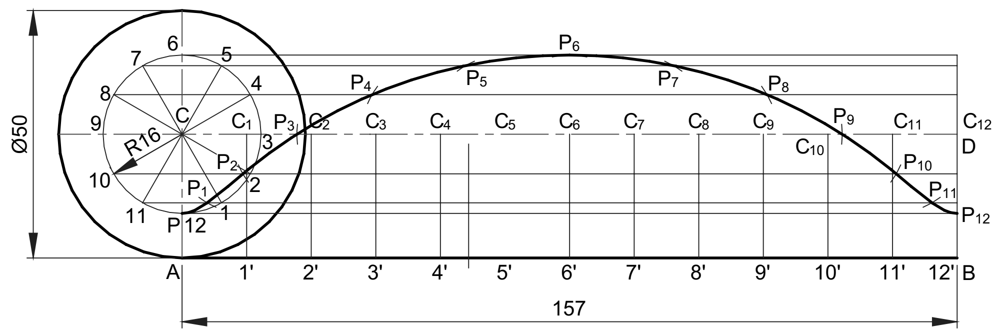


Fig. 6.6 *Inferior trochoid*

Construction Refer to Fig. 6.6.

1. Draw a rolling circle of diameter 50 mm with centre *C*. Mark a tracing point *P* at a distance 16 mm from the centre *C*.
2. Draw the directing line $AB = \pi D = 157$ mm long, horizontal and tangential to the circle.
3. Divide *AB* into 12 equal parts and mark the divisions as 1′, 2′, 3′, etc., on it.
4. Draw another circle with centre *C* and radius *CP*. Divide the circle into 12 equal parts and mark the divisions as 1, 2, 3, etc. Draw lines through points 1, 2, 3, etc., parallel to *AB*.
5. Erect vertical lines from 1′, 2′, 3′, etc., to meet the centre line *CD* at C_1, C_2, C_3, etc.

When the circle rolls through 1/12th rotation, the centre *C* will move to C_1. The point *P* will move up to the new position P_1 lying on the horizontal line through point 1 at a distance of 16 mm from C_1.

6. Draw an arc with centre C_1 and radius 16 mm to intersect the horizontal line through point 1 at P_1.
7. Similarly, draw arcs with centres C_2, C_3, C_4, etc., and radius 16 mm, to intersect the horizontal locus lines through points 2, 3, 4, etc., at P_2, P_3, P_4, etc., respectively.
8. Draw a smooth curve to pass through P_1, P_2, P_3, P_4, etc., and obtain the required inferior trochoid.

Problem 6.5 *A circle of diameter 50 mm rolls along a straight line for one revolution. Draw the locus of a point, lying at a distance of 36 mm from the centre of the circle.*

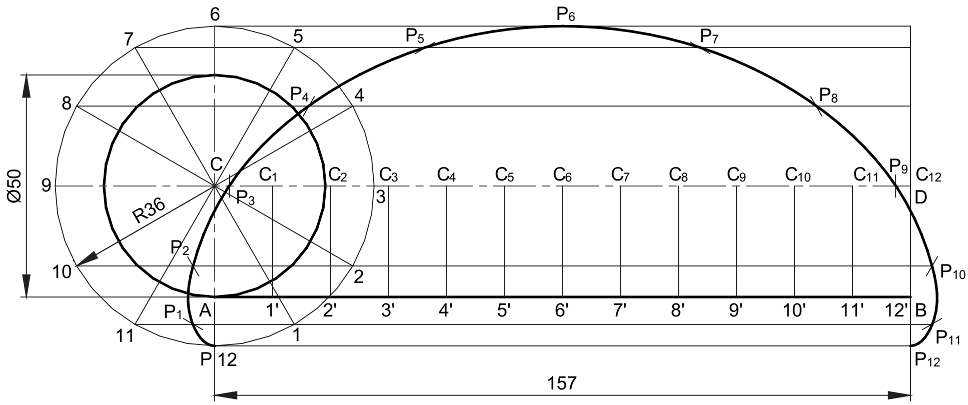


Fig. 6.7 *Superior trochoid*

Construction Refer to Fig. 6.7.

1. Draw a rolling circle of diameter 50 mm with centre C. Mark a tracing point P at a distance 36 mm from the centre C.
2. Draw the directing line $AB = \pi D = 157$ mm long, horizontal and tangential to the circle.
3. Divide AB into 12 equal parts and mark the divisions as $1'$, $2'$, $3'$, etc., on it.
4. Draw another circle with centre C and radius CP. Divide the circle into 12 equal parts and mark the divisions as 1, 2, 3, etc., as shown. Draw lines through 1, 2, 3, etc., parallel to PQ.
5. Erect vertical lines from points $1'$, $2'$, $3'$, etc., to meet the centre line CD at C_1, C_2, C_3, etc.

When the circle rolls through 1/12th rotation, the centre C will move to C_1. The point P will move up to the new position P_1 lying on the horizontal line through point 1 at a distance of 36 mm from C_1.

6. Draw an arc with centre C_1 and radius 36 mm to intersect the horizontal line through point 1 at point P_1.
7. Similarly, draw arcs with centres C_2, C_3, C_4, etc., and radius 36 mm, to intersect the horizontal locus lines through points 2, 3, 4, etc., at points P_2, P_3, P_4, etc., respectively.
8. Draw a smooth curve to pass through P_1, P_2, P_3, P_4, etc., and obtain the required superior trochoid.

6.4.2 Epitrochoid

An epitrochoid is a curve traced by a point situated either inside or outside the circle, which rolls along another circle outside it, without slipping. If the tracing point is situated *inside* the rolling circle, the curve is called an *inferior epitrochoid*. If the tracing point is situated *outside* the rolling circle, the curve is called a *superior epitrochoid.*

Let a circle with centre C rolls outside a circular arc AB. In Fig. 6.8, a point P is situated inside the rolling circle and the path traced by it is called inferior epitrochoid. In Fig. 6.9, the point P is situated outside the rolling circle and the path traced by it is called superior epitrochoid.

Problem 6.6 *A circle of diameter 50 mm rolls outside a circular arc of radius 90 mm for one revolution. Draw the locus of a point, lying at a distance of 16 mm from centre of the circle.*

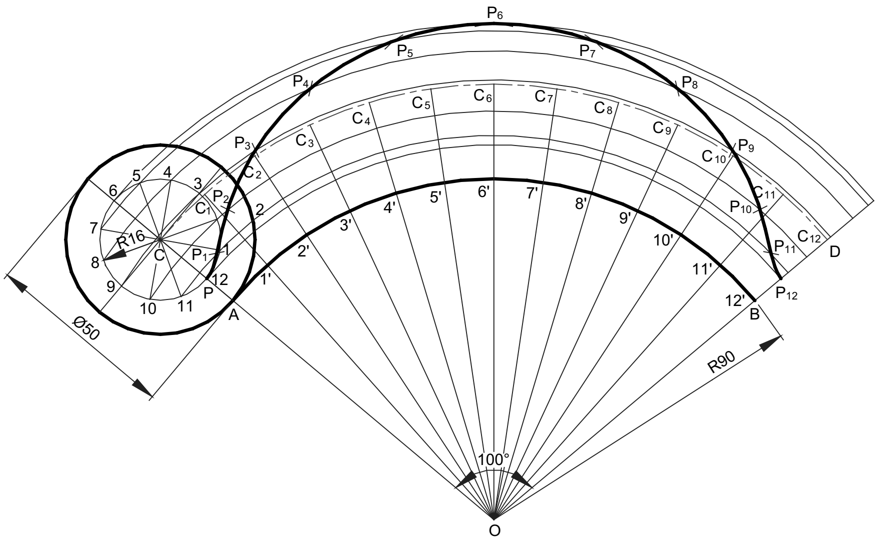


Fig. 6.8 *Inferior epitrochoid*

Construction Refer to Fig. 6.8.

1. Determine angle subtended by the arc *AB* at centre *O* for one revolution of the circle as $\theta = \dfrac{d}{D} \times 360°$ $= \dfrac{50}{180} \times 360° = 100°$.
2. Draw an arc *AB* with centre *O* and radius 90 mm, to subtend angle of 100°. This represents the directing path.
3. Divide arc *AB* into 12 equal parts and mark the divisions as 1′, 2′, 3′, 4′, etc.
4. Join *OA* and extend it to point *C*, such that *AC* = 25 mm. Draw a rolling circle with centre *C* and radius *AC*.
5. Mark a tracing point *P* at a distance 16 mm from the centre *C*. Draw a circle with centre *C* and radius *CP*. Divide the circle into 12 equal parts and mark the divisions as 1, 2, 3, etc.
6. Draw arcs, with centre *O* and radii equal to *O*1, *O*2, *O*3, *O*4, etc., to meet line *OB* produced.
7. Draw an arc with centre *O* and radius *OC* to meet *OB* at point *D*. The arc *CD* is known as the centre arc.
8. Extend lines *O*1′, *O*2′, *O*3′, *O*4′, etc., to meet the centre arc *CD* at points C_1, C_2, C_3, C_4, etc., respectively.

When the circle rolls through 1/12th rotation, the centre *C* will move to C_1. The point *P* will move to the new position P_1 lying on the arc through point 1 at a distance of 16 mm from C_1.

9. Draw an arc with centre C_1 and radius 16 mm to intersect the arc through point 1 at point P_1.
10. Similarly, draw arcs with centres C_2, C_3, C_4, etc. and radius 16 mm, to intersect arcs through points 2, 3, 4, etc., at points P_2, P_3, P_4, etc., respectively.
11. Draw a smooth curve to pass through P_1, P_2, P_3, P_4, etc., and obtain the required inferior epitrochoid.

Problem 6.7 *A circle of diameter 50 mm rolls outside a circular arc of radius 90 mm for one revolution. Draw the locus of a point, lying at a distance of 36 mm from the centre of the circle.*

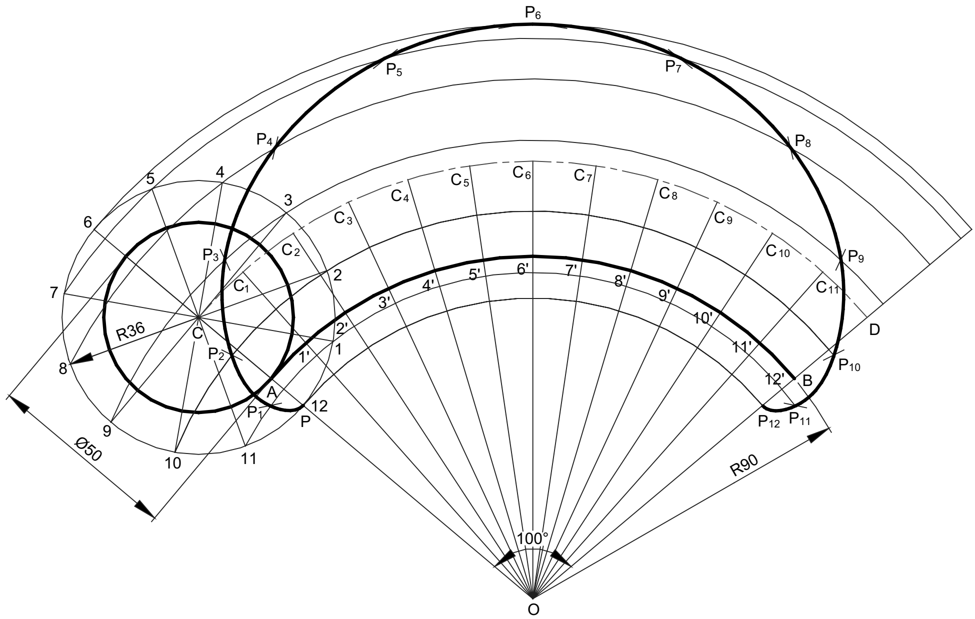


Fig. 6.9 *Superior epitrochoid*

Construction Refer to Fig. 6.9.

1. Determine angle subtended by the arc *AB* at centre *O* for one revolution of the circle as $\theta = \dfrac{d}{D} \times 360°$ $= \dfrac{50}{180} \times 360° = 100°$.
2. Draw an arc *AB* with centre *O* and radius 90 mm, to subtend angle of 100°. This represents the directing path.
3. Divide the directing arc AB into 12 equal parts and mark the divisions as 1′, 2′, 3′, 4′, etc.
4. Join *OA* and extend it to point *C*, such that *AC* = 25 mm. Draw a rolling circle with centre *C* and radius *AC*.
5. Mark a tracing point *P* at a distance 36 mm from the centre *C*. Draw a circle with centre *C* and radius *CP*. Divide the circle into 12 equal parts and mark the divisions as 1, 2, 3, etc.
6. Draw arcs, with centre *O* and radii equal to *O*1, *O*2, *O*3, *O*4, etc., to meet line *OB* produced.
7. Draw an arc with centre *O* and radius *OC* to meet *OB* at point *D*. The arc *CD* is known as the centre arc.
8. Extend lines *O*1′, *O*2′, *O*3′, *O*4′, etc. to meet the centre arc *CD* at points C_1, C_2, C_3, C_4, etc. respectively.

When the circle rolls through 1/12th rotation, the centre *C* will move to C_1. The point *P* will move to the new position P_1 lying on the arc through point 1 at a distance of 36 mm from C_1.

9. Draw an arc with centre C_1 and radius 36 mm to intersect the arc through point 1 at point P_1.
10. Similarly, draw arcs with centres C_2, C_3, C_4, etc., and radius 36 mm, to intersect arcs through points 2, 3, 4, etc., at points P_2, P_3, P_4, etc., respectively.
11. Draw a smooth curve to pass through P_1, P_2, P_3, P_4, etc., and obtain the required inferior epitrochoid.

6.4.3 Hypotrochoid

A hypotrochoid is a curve traced by a point situated either inside or outside the circle, which rolls along another circle inside it, without slipping. If the tracing point is situated *inside* the rolling circle, the curve is called an *inferior hypotrochoid*. If the tracing point is situated *outside* the rolling circle, the curve is called a *superior hypotrochoid*.

Let a circle with centre C rolls inside a circular arc *AB*. In Fig. 6.10, a point *P* is situated inside the rolling circle and the path traced by it is called *inferior hypotrochoid*. In Fig. 6.11, the point *P* is situated outside the rolling circle and the path traced by it is called superior hypotrochoid.

Problem 6.8 *A circle of diameter 50 mm rolls inside a circular arc of radius 90 mm for one revolution. Draw the locus of a point, lying at a distance of 16 mm from the centre of the circle.*

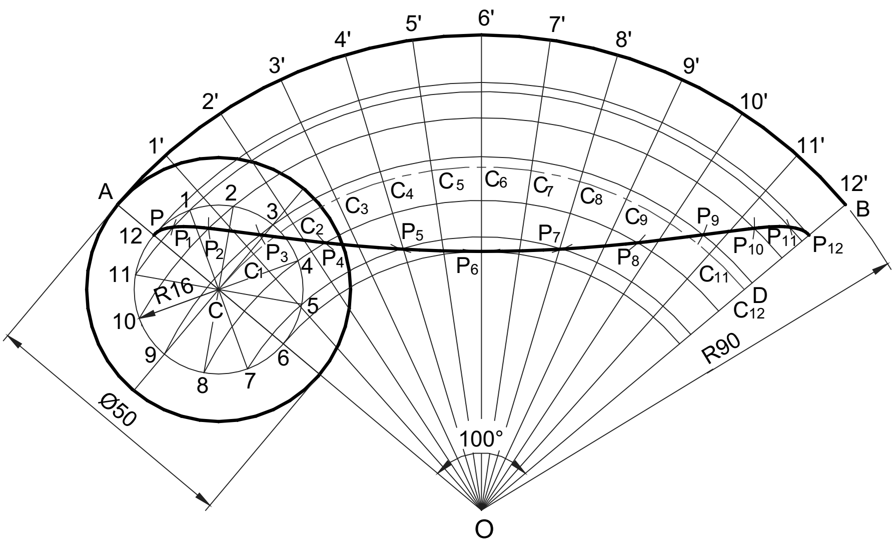


Fig. 6.10 *Inferior hypotrochoid*

Construction Refer to Fig. 6.10.

1. Determine angle subtended by the arc AB at centre O for one revolution of the circle as $\theta = \dfrac{d}{D} \times 360°$ $= \dfrac{50}{180} \times 360° = 100°$.

2. Draw an arc AB with centre O and radius 90 mm, to subtend angle of 100°. This represents the directing path.
3. Divide the directing arc AB into 12 equal parts and mark the divisions as 1′, 2′, 3′, 4′, etc.
4. Join OA and mark point C on it such that AC = 25 mm. Draw a rolling circle with centre C and radius AC.
5. Mark a tracing point P at a distance 16 mm from the centre C. Draw a circle with centre C and radius CP. Divide the circle into 12 equal parts and mark the divisions as 1, 2, 3, etc.
6. Draw arcs, with centre O and radii equal to $O1$, $O2$, $O3$, $O4$, etc., to meet line OB.
7. Draw an arc with centre O and radius OC to meet OB at point D. The arc CD is known as the centre arc.
8. Extend lines $O1'$, $O2'$, $O3'$, $O4'$, etc., to meet the centre arc CD at points C_1, C_2, C_3, C_4, etc., respectively.

When the circle rolls through 1/12th rotation, the centre C will move to C_1. The point P will move to the new position P_1 lying on the arc through point 1 at a distance of 16 mm from C_1.

9. Draw an arc with centre C_1 and radius 16 mm to intersect the arc through point 1 at point P_1.
10. Similarly, draw arcs with centres C_2, C_3, C_4, etc., and radius 16 mm, to intersect arcs through points 2, 3, 4, etc., at points P_2, P_3, P_4, etc., respectively.
11. Draw a smooth curve to pass through P_1, P_2, P_3, P_4, etc., and obtain the required inferior hypotrochoid.

Problem 6.9 *A circle of diameter 50 mm rolls inside a circular arc of radius 90 mm for one revolution. Draw the locus of a point, lying at a distance of 36 mm from the centre of the circle.*

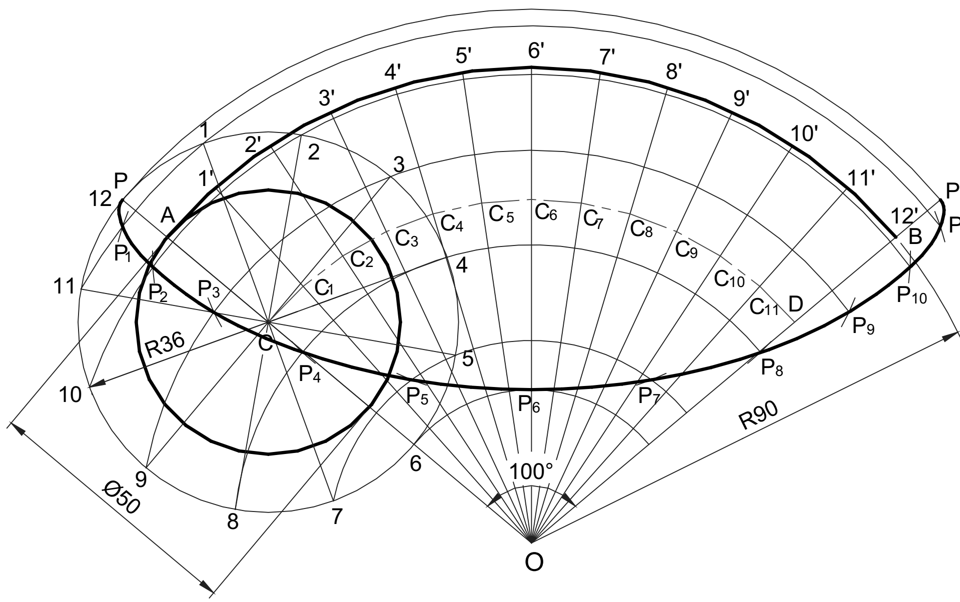


Fig. 6.11 *Superior hypotrochoid*

Construction Refer to Fig. 6.11.

1. Determine angle subtended by the arc AB at centre O for one revolution of the circle as $\theta = \frac{d}{D} \times 360°$ $= \frac{50}{180} \times 360° = 100°$.
2. Draw an arc AB with centre O and radius 90 mm, to subtend angle of 100°. This represents the directing path.
3. Divide the arc AB into 12 equal parts and mark the divisions as 1′, 2′, 3′, 4′, etc.
4. Join OA and mark point C on it such that AC = 25 mm. Draw a rolling circle with centre C and radius AC.
5. Mark a tracing point P at a distance 36 mm from the centre C. Draw a circle with centre C and radius CP. Divide the circle into 12 equal parts and mark the divisions as 1, 2, 3, etc.
6. Draw arcs, with centre O and radii equal to $O1$, $O2$, $O3$, $O4$, etc., to meet line OB.
7. Draw an arc with centre O and radius OC to meet OB at point D. The arc CD is known as the centre arc.
8. Extend lines $O1'$, $O2'$, $O3'$, $O4'$, etc., to meet the centre arc CD at points C_1, C_2, C_3, C_4, etc., respectively.

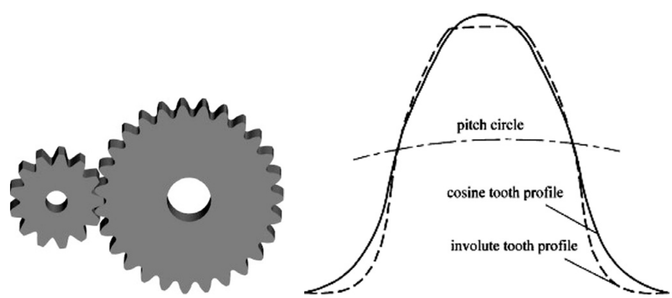
When the circle rolls through 1/12th rotation, the centre C will move to C_1. The point P will move to the new position P_1 lying on the arc through point 1 at a distance of 36 mm from C_1.

9. Draw an arc with centre C_1 and radius 36 mm to intersect the arc through point 1 at point P_1.
10. Similarly, draw arcs with centres C_2, C_3, C_4, etc., and radius 36 mm, to intersect arcs through points 2, 3, 4, etc., at points P_2, P_3, P_4, etc., respectively.
11. Draw a smooth curve to pass through P_1, P_2, P_3, P_4, etc., and obtain the required superior hypotrochoid.

6.5 INVOLUTE

An involute is a curve traced out by an end of a thread, when it is unwound from a circle or a polygon, the thread being kept tight. An involute of a circle is used as teeth profile of a large gear wheel and a gear reducer. In Fig. 6.13, the end of the thread is at point P of the circle. When the thread is unwound keeping it always tight, it is always tangential to the circle and the point P will generated an involute. An involute of a circle is used Payton's water meter and teeth profile of a gear.

(a)



(b)

Fig. 6.12 **(a)** *Payton's water meter* **(b)** *Gears with involute tooth profile*

Problem 6.10 *Draw an involute of a circle of diameter 50 mm. Also draw normal and tangent at a point 100 mm from the centre of the circle.*

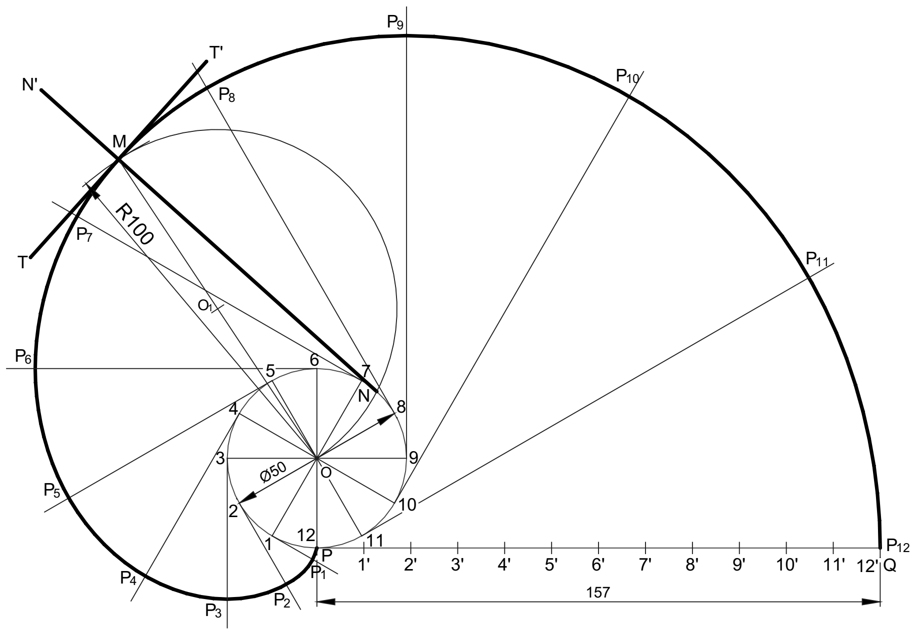


Fig. 6.13 *Involute of a circle*

Construction Refer to Fig. 6.13.

1. Draw a circle of diameter 50 mm and divide it into 12 equal parts.
   Let the end of thread before unwound is at point *P* of the circle. Consider the thread is unwound in the clockwise direction for one revolution. Then the length of the thread is πD and is tangent to the circle at point *P*.
2. Draw a line $PQ = \pi D = 157$ mm and divide it into 12 equal parts. Mark its divisions as 1′, 2′ 3′, 4′, etc.
3. Draw tangents to the circle at points 1, 2, 3, 4, etc., such that they represent the thread position during unwound.
4. The length of thread when unwound for 1/12th revolution is equal to $P1'$. Therefore, draw an arc with centre 1 and radius $P1'$ to intersect the tangent line through point 1 at point P_1.
5. The length of thread when unwound for 2/12th revolution is equal to $P2'$. Therefore, draw an arc with centre 2 and radius $P2'$ to intersect the tangent line through point 2 at point P_2.
6. Similarly, draw arcs with centres 3, 4, 5, etc., and radii $P3'$, $P4'$, $P5'$, etc., respectively, to intersect the tangent line through points 3, 4, 5, etc., at points P_3, P_4, P_5, etc., respectively.
7. Draw a smooth curve to pass through P_1, P_2, P_3, P_4, etc., and obtain the required involute.

Tangent and normal to involute

1. Mark a point M on the involute at a radial distance of 100 mm from the centre O.
2. Join OM and mark O_1 as its mid-point.
3. Draw a semi-circle in clockwise direction with centre O_1 and diameter OM to intersect the base circle at N.
4. Join NM and produce it to N'. The line NN' is the required normal.
5. Through point M, draw a line TT' perpendicular to NN'. The line TT' is the required tangent.

Problem 6.11 *Draw an involute of a hexagon of side 25 mm.*

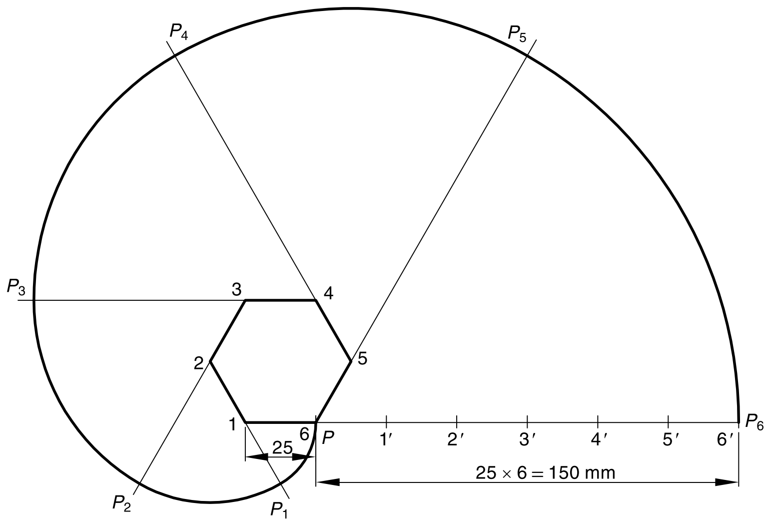


Fig. 6.14 *Involute of a hexagon*

Construction Refer to Fig. 6.14.
Draw a hexagon of side 25 mm. Mark a point P at corner 6. The end of thread before unwound is at point P. Consider the thread is unwound in the clockwise direction for one revolution. The length of the thread is $6 \times 25 = 150$ mm.

1. Produce sides 2-1, 3-2, 4-3, 5-4, 6-5 and 1-6 as shown.
2. Draw arcs with centres 1, 2, 3, 4, 5, 6 and radii in multiple of side length (i.e., 25 mm, 50 mm, 75 mm, 100 mm, 125 mm and 150 mm respectively) to meet sides 2-1, 3-2, 4-3, 5-4, 6-5 and 1-6 at P_1, P_2, P_3, P_4, P_5 and P_6 respectively.
3. Draw a smooth curve to pass through P_1, P_2, P_3, P_4, P_5, P_6 and obtain the required involute.

6.6 SPIRAL

If a line rotates in a plane about one of its ends and at the same time, if a point moves along the line continuously in one direction, the curve traced out by the moving point is called a spiral. The following terms are used in connection to the spiral.

1. Pole It is the fixed end of a line about which the line rotates.

2. Radius vector It is the line joining any point of the curve with the pole.

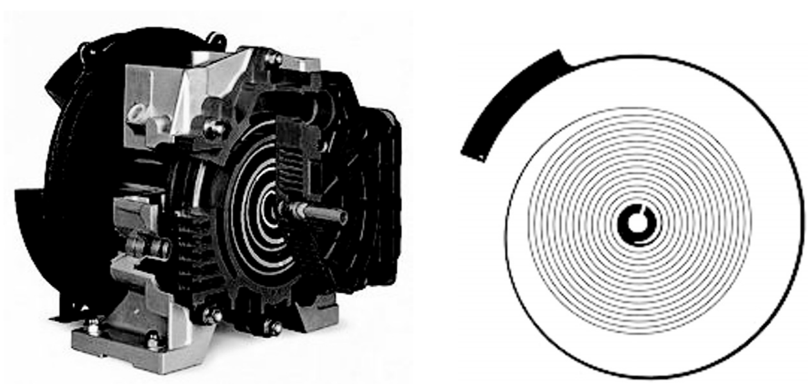
3. Vectorial angle It is the angle between the initial position and the instantaneous position of the line.

4. Convolution Rotation of the moving line through 360º is called one convolution. A spiral can make any number of convolutions before it reaches the final destination.
Two types of spirals, namely Archimedean and logarithmic spirals are commonly used in engineering practice.

6.6.1 Archimedean Spiral

The Archimedean spiral is named after the Greek mathematician Archimedes. *It is defined as a curve traced out by a point moving uniformly along a straight line towards or away from the pole, while the line revolves about its one of the ends with uniform angular velocity.* Consider Fig. 6.16 where point P is moving about a fixed point O such that for every increase in its vectorial angle of 30º, point P moves away through a distance of P1. Thus, point P traces an Archimedean spiral.

An Archimedean spiral is used in the mechanism of scroll compressors, grooves of early gramophone records, coils of watch balance springs, and screw mechanisms capable of raising water for irrigation. Archimedean spirals are used in digital light processing (DLP) projection systems to minimize the rainbow effect. They are used in food microbiology to quantify bacterial concentration through a spiral platter.



(a) (b)

Fig. 6.15 **(a)** *Scroll compressor* **(b)** *Watch spring*

Problem 6.12 *Draw an Archimedean spiral of 1½ convolutions for the shortest and the greatest radii as 20 mm and 90 mm, respectively. Also, draw a tangent and a normal to the curve at a point 50 mm from the pole.*

Construction Refer to Fig. 6.16.

1. Draw a circle of radius 90 mm and divide into 12 equal parts.

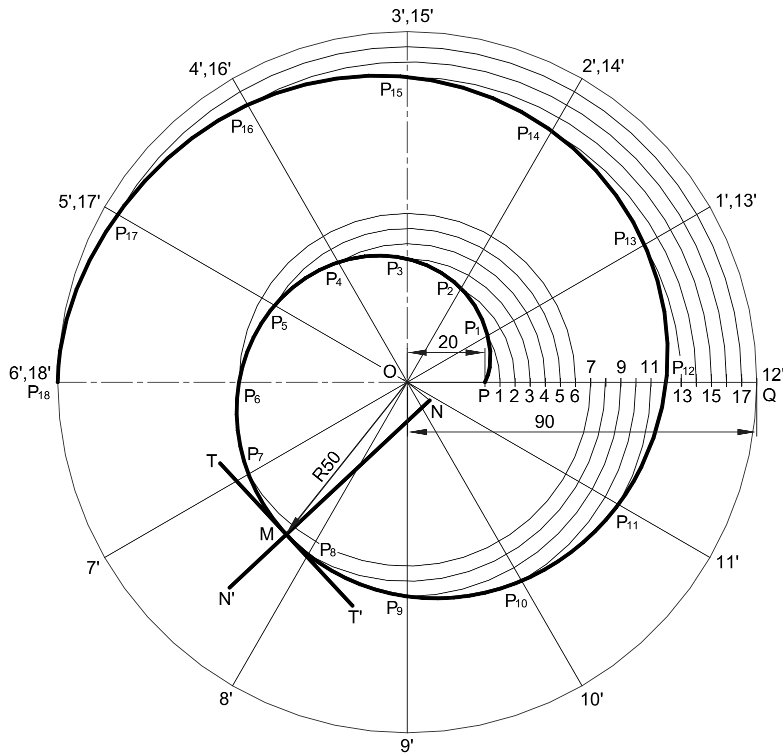


Fig. 6.16 *Archimedean spiral*

2. The angular movement for 1½ convolutions is 540°. Therefore, mark the divisions of the circle as 1′, 2′, 3′, 4′, …, 12′, …, 18′ (@12 marks per convolution).
3. On the radial line *OQ*, mark point *P* at a distance 20 mm from the centre *O*. Divide *PQ* into 18 parts (@12 parts per convolution). Mark the divisions as 1, 2, 3, 4, etc.
4. Draw an arc with the centre *O* and radius *O*1 to meet the radial line *O*1′ at point P_1.
5. Similarly, draw arcs with the centre *O* and radii *O*2, *O*3, *O*4, etc. to meet the radial lines *O*2′, *O*3′, *O*4′, etc., respectively, at points P_2, P_3, P_4, etc.
6. Draw a smooth curve to pass through P_1, P_2, P_3 etc., and obtain the required Archimedean spiral.

Tangent and normal to Archimedean Spiral

1. Mark a point *M* on the spiral at a radial distance of 50 mm from the centre *O* and join *OM*.
2. Draw $\text{ON} = \dfrac{90-20}{1.5\times 2\pi} = 7.43\text{ mm}$ perpendicular to the line *OM*.
3. Join *NM* and produce it to *N*′. The line *NN*′ is the required normal.
4. Through point *M* draw a line *TT*′ perpendicular to *NN*′. The line *TT*′ is the required tangent.

6.6.2 Logarithmic Spiral

A logarithmic spiral is a curve traced by a point moving along a rotating line such that for equal angular displacement of the line, the ratio of the lengths of consecutive radius vectors is constant. Thus, in logarithmic spiral, vectorial angles increase in arithmetic progression and the corresponding radius vectors are in geometrical progression. Consider Fig. 6.18(b) where $\frac{OP_1}{OP} = \frac{OP_2}{OP_1} = \frac{OP_3}{OP_2} = \text{constant}$ for uniform increase in the vectorial angle by 30°. So the curve is a logarithmic spiral.

Logarithmic spirals are used in volute casing of pumps. They are approximated by the shape of galaxies, nautilus shells, tornadoes, cyclones and hurricanes. The human ear is a logarithmic spiral too.

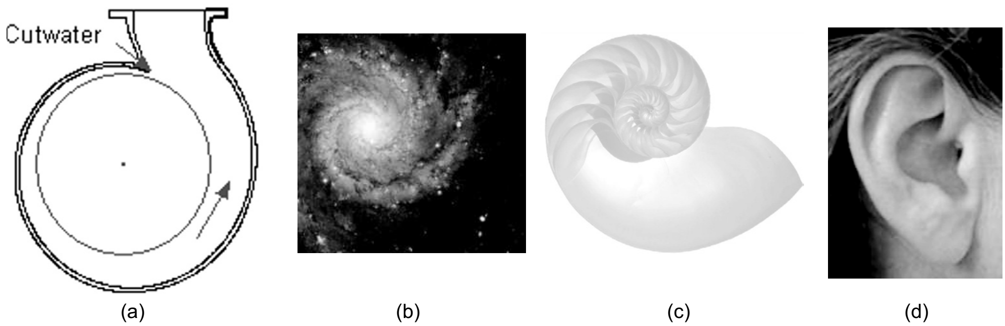


Fig. 6.17 **(a)** *Volute casing* **(b)** *Galaxies* **(c)** *Nautilus shells* **(d)** *Human ear*

Problem 6.13 *Draw a logarithmic spiral of one convolution, when the shortest distance is 16 mm and ratio of the length of radius vectors enclosing an angle of 30° is 9:8. Also, draw a tangent and a normal to the curve at a point 50 mm from the pole.*

Construction Refer to Fig. 6.18.

Determine the length of the radius vector graphically (Fig. 6.18a).

1. Draw an angle *CAB* = 30º.
2. Mark point *D* on *AB* such that *AD* = 16 mm (shortest distance).
3. Mark point *E* on *AC* such that $AE = \frac{9}{8} \times 16 \text{ mm} = 18 \text{ mm}$.
4. Join *DE*. Draw an arc with centre *A* and radius *AE* to meet *AB* at point 1.
5. Draw a line from point 1, parallel to *DE* to meet *AC* at point 1′. Draw an arc with centre *A* and radius *A*1′ to meet *AB* at point 2.
6. Draw a line from point 2, parallel to *DE* to meet *AC* at point 2′. Draw an arc with centre *A* and radius *A*2′ to meet *AB* at point 3.
7. Proceed in the manner explained in step 6 and obtain points 4, 5, 6, 7, 8, 9, 10, 11 and 12 on the line *AB*.

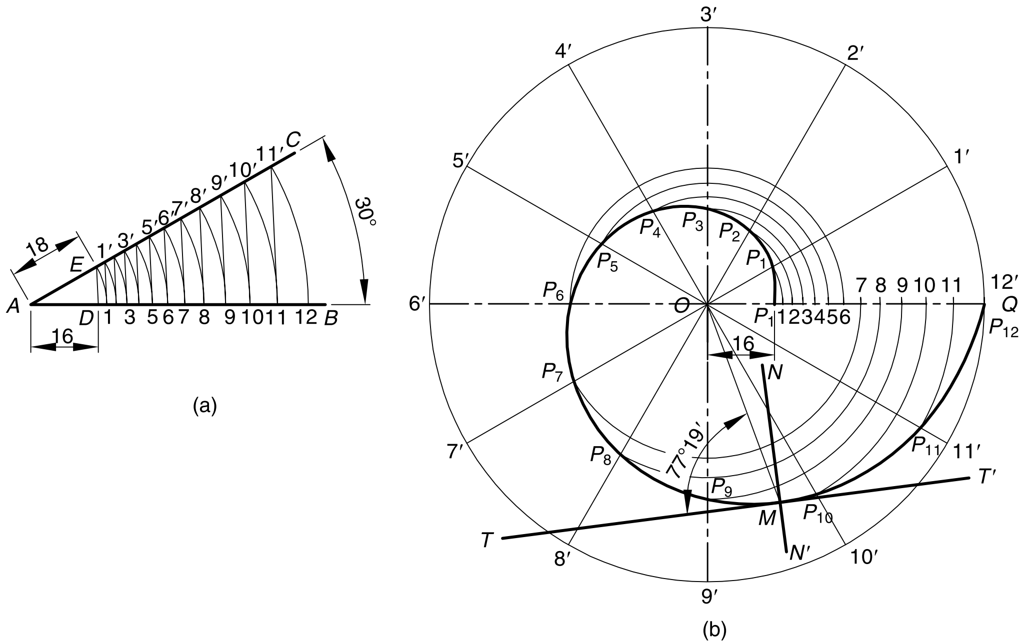


Fig. 6.18 **(a)** *Length of radius vector* **(b)** *Logarithmic spiral*

Draw logarithmic spiral (Fig. 6.18b).

8. Draw a circle with centre O and diameter equal to length A-12 of Fig. 6.18(a). Divide the circle into 12 equal parts and mark the divisions as 1′, 2′, 3′, 4′, etc., as shown.
9. Mark points P, 1, 2, 3, …, 12 on OQ such that $OP = AD$, $O1 = A1$, $O2 = A2$, $O3 = A3$, etc. Thus, OQ is the copy of the line AB.
10. Draw an arc with centre O and radius $O1$ to meet the radial line $O1'$ at point P_1.
11. Similarly, draw arcs with centre O and radii $O2$, $O3$, $O4$, etc., to meet radial lines $O2'$, $O3'$, $O4'$, etc., respectively at points P_2, P_3, P_4, etc.
12. Draw a smooth curve to pass through P_1, P_2, P_3 etc., and obtain the required logarithmic spiral.

Tangent and normal to the logarithmic spiral

1. Mark a point M on the spiral at a radial distance of 50 mm from the centre O.
2. Determine $\alpha = \tan^{-1}\dfrac{\theta}{\ln r} = \tan^{-1}\dfrac{\pi/6}{\ln 9/8} = 77^\circ 19'$.
3. Mark a point T such that angle $OMT = \alpha = 77^\circ 19'$. Produce TM to obtain T'. The line TT' is the required tangent.
4. Through point M, draw a line NN' perpendicular to TT'. The line NN' is the required normal.

6.7 HELIX

A helix is a curve traced by a point moving around the surface of a cylinder or a cone with a uniform speed along with a uniform axial movement. In other words, the helix is generated by a point lying on the surface of a cylinder or a cone which is moving along the direction of the axis with a uniform speed and also moving around the surface with another constant speed. The helix is used in screw threads, springs and pumps. Helices are important in biology, as the DNA molecule is formed as two intertwined helices, and many proteins have helical substructures, known as alpha helices. Handrails of spiral staircases and a road going round the hill to reach the top also generate the helix.

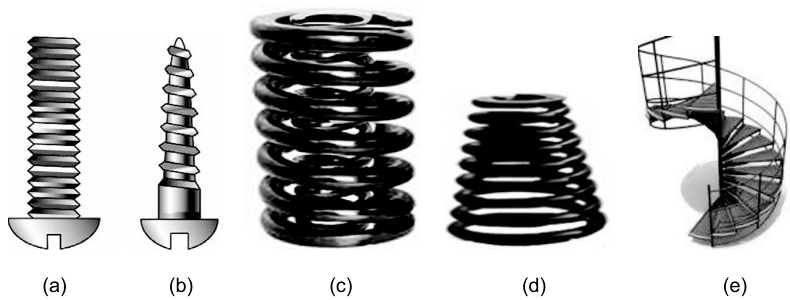


Fig. 6.19 **(a)** *and* **(b)** *Helix on a screw* **(c)** *and* **(d)** *Helical spring* **(e)** *Handrails of spiral staircases*

6.7.1 Cylindrical Helix

A cylindrical helix is a curve traced by a point moving around the surface of a cylinder with a uniform speed along with a uniform axial movement. In Fig. 6.20, a point P is moving upwards around the surface of the cylinder. Thus, the point P traces a cylindrical helix.

Problem 6.14 *Draw a helix of one convolution upon a cylinder of base diameter 50 mm and pitch 60 mm.*

Construction Refer to Fig. 6.20.

1. Draw a circle of diameter 50 mm and a rectangle of base 50 mm and height 60 mm. This represents the top and the front views of the cylinder.
2. Divide the circle into 12 equal parts and label the divisions as 1, 2, 3, etc.
3. Divide the 60 mm height into 12 equal parts and label the divisions as 1′, 2′, 3′, etc.

Let the helix start at point *P* and move upwards in anticlockwise direction.

4. Draw a horizontal line from point 2′ to intersect the vertical line from point 2 at point P_2.
5. Similarly, draw horizontal lines from point 3′, 4′, 5′, etc., to intersect the vertical lines from points 3, 4, 5, etc., at points P_3, P_4, P_5, etc.
6. Draw a smooth curve to pass through 1′, P_2, P_3, P_4, etc. and obtain the required helix.

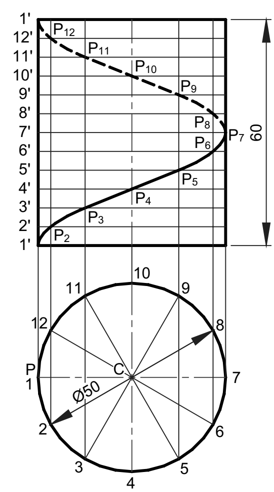


Fig. 6.20 *Cylindrical helix*

6.7.2 Conical Helix

A helix is a curve traced by a point moving around the surface of a cone with a uniform speed along with a uniform axial movement. In Fig. 6.21, a point P is moving upwards around the surface of the cone. Thus, the point P traces a conical helix.

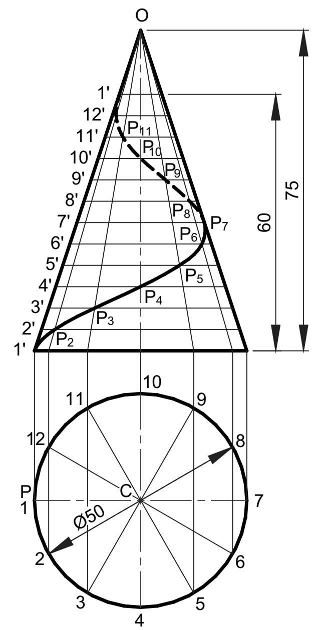


Fig. 6.21 *Conical helix*

Problem 6.15 *Draw a helix of one convolution upon a cone of base diameter 50 mm, axis 75 mm long and pitch 60 mm.*

Construction Refer to Fig. 6.21.

1. Draw a circle of diameter 50 mm and a triangle of base 50 mm and height 75 mm. This represents the top and the front views of a cone.
2. Divide the circle into 12 equal parts and label the divisions as 1, 2, 3, etc.
3. Project points 1, 2, 3, etc., to the base of the cone and then join them to the apex. These lines are called generators of the cone.
4. Divide the 60 mm height of the cone into 12 equal parts and label as 1′, 2′, 3′, etc.

Let the helix start at point P and move upwards in anticlockwise direction.

5. Draw a horizontal line from point 2′ to intersect the generator through point 2 at P_2.
6. Similarly, draw horizontal lines from points 3′, 4′, 5′, etc., to intersect the generators through points 3, 4, 5, etc., at P_3, P_4, P_5, etc.
7. Draw a smooth curve to pass through 1′, P_2, P_3, P_4, etc., and obtain the required helix.

6.8 MISCELLANEOUS PROBLEMS

Problem 6.16 *A circle of diameter 50 mm rolls on a horizontal line for a half revolution and then on a vertical line upwards for another half revolution. Draw the curve traced out by a point lying on the circumference of the circle.*

Construction Refer to Fig. 6.22.

1. Draw a circle of diameter 50 mm with centre C and a tangent $PQ = \dfrac{\pi D}{2} = 78.5$ mm long.
2. Divide the circle into 12 equal parts and mark the divisions as 1, 2, 3, etc. Draw lines parallel to PQ from points 1, 2, 3, etc.
3. Divide PQ into six equal parts and mark 1′, 2′, 3′, 4′, 5′, 6′ on it. Draw perpendicular lines through these points to meet the centre line CR at points C_1, C_2, C_3, etc.
4. Draw arcs with centre C_1, C_2, C_3, C_4, C_5, C_6 of radius 25 mm, to meet the horizontal lines from points 1, 2, 3, 4, 5, 6 at points P_1, P_2, P_3, P_4, P_5, P_6 respectively.
5. Draw a smooth curve to pass through P_1, P_2, P_3, P_4, P_5, P_6.

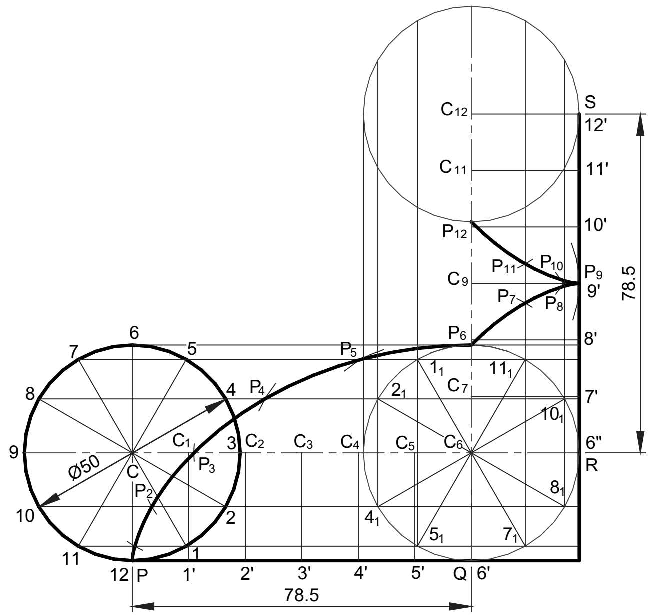


Fig. 6.22

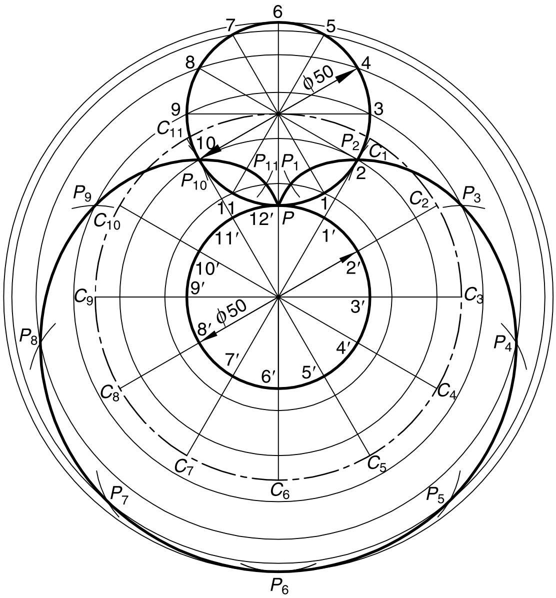
It may be noted that when the circle coincides with point Q, the centre is be at point C_6. In this position, the circle also touches the vertical wall at point R. Now the circle will roll along the line RS for another half revolution, i.e., $\dfrac{\pi D}{2} = 78.5$ mm.

6. Draw a 78.54 mm long line RS as the new directing line.
7. Draw lines parallel to RS from points 7_1, 8_1, 9_1, 10_1, 11_1, 12_1.
8. Divide RS into 6 equal parts and mark 7′, 8′, 9′, 10′, 11′, 12′ on it. Draw horizontal lines from these points to meet the vertical centre line at points C_7, C_8, C_9, C_{10}, C_{11}, C_{12}.
9. Draw arcs with centres C_7, C_8, C_9, C_{10}, C_{11}, C_{12} of radius 25 mm, to meet the horizontal lines from points 7_1, 8_1, 9_1, 10_1, 11_1, 12_1 at points P_7, P_8, P_9, P_{10}, P_{11}, P_{12} respectively.
10. Draw a smooth curve to pass through P_6, P_7, P_8, P_9, P_{10}, P_{11} and P_{12}.

Problem 6.17 *Draw an epicycloid, when the diameters of the rolling and the directing circles are 50 mm. Suggest an alternative name for this curve.*

When the diameters of the rolling and the directing circles are equal, the angle subtended by the rolling circle at centre of the directing circle is 360º and the epicycloid becomes a **cardioid**.

Follow the steps of constructions of Problem 6.2, where $r = R = 25$ mm, $\theta = 360°$ and obtain the epicycloid as shown in Fig. 6.23.



The alternative name for the curve is cardioid.

Fig. 6.23

Problem 6.18 *Construct a hypocycloid taking the diameter of the generating circle and radius of directing circle as 60 mm.*

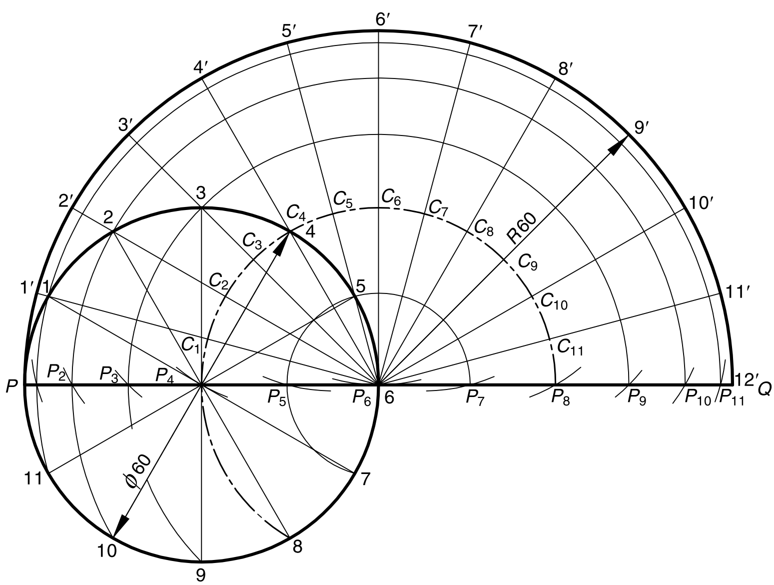


Fig. 6.24

Construction Refer to Fig. 6.24.

When the diameter of the generating (rolling) circle is half the diameter of the directing circle (i.e., $r = R/2$), the hypocycloid becomes a straight line.

For such a hypocycloid, $\theta = \dfrac{r}{R} \times 360° = \dfrac{30}{60} \times 360° = 180°$.

Follow the steps of constructions of Problem 6.3, where r = 30 mm, R = 60 mm, $\theta = 180^{\circ}$ and obtain the hypocycloid as a straight line as shown in Fig. 6.24.

Problem 6.19 *A disc is in the form of a square of side 30 mm surmounted by a semi-circle on one of the sides of the square and a half hexagon on the opposite side. Draw the path of the end of a string which is unwound from the circumference of the disc.*

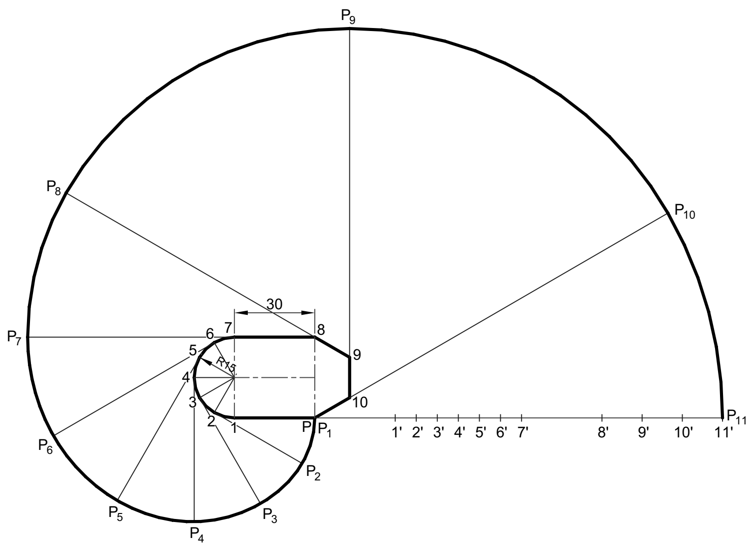


Fig. 6.25

Construction Refer to Fig. 6.25.

1. Draw the given disc and determine the length of its perimeter.
2. Draw the involute as shown. Refer to Problems 6.10 and 6.11 for details of construction.

Problem 6.20 *Draw a path traced out by an end of a piece of thread when unwound to a length of 150 mm from a circle of diameter 40 mm, the thread being kept tight during unwound. Name the curve traced.*

Construction Refer to Fig. 6.26.

The length of thread is 150 mm. Therefore, the angle through which the thread should unwound is given by $\theta = \dfrac{\text{Length of thread}}{\pi D} \times 360° = \dfrac{150}{\pi \times 40} \times 360° = 429.7°$. Thus, thread should be unwound by 360° + 69.7° (i.e., one revolution + 69.7°)

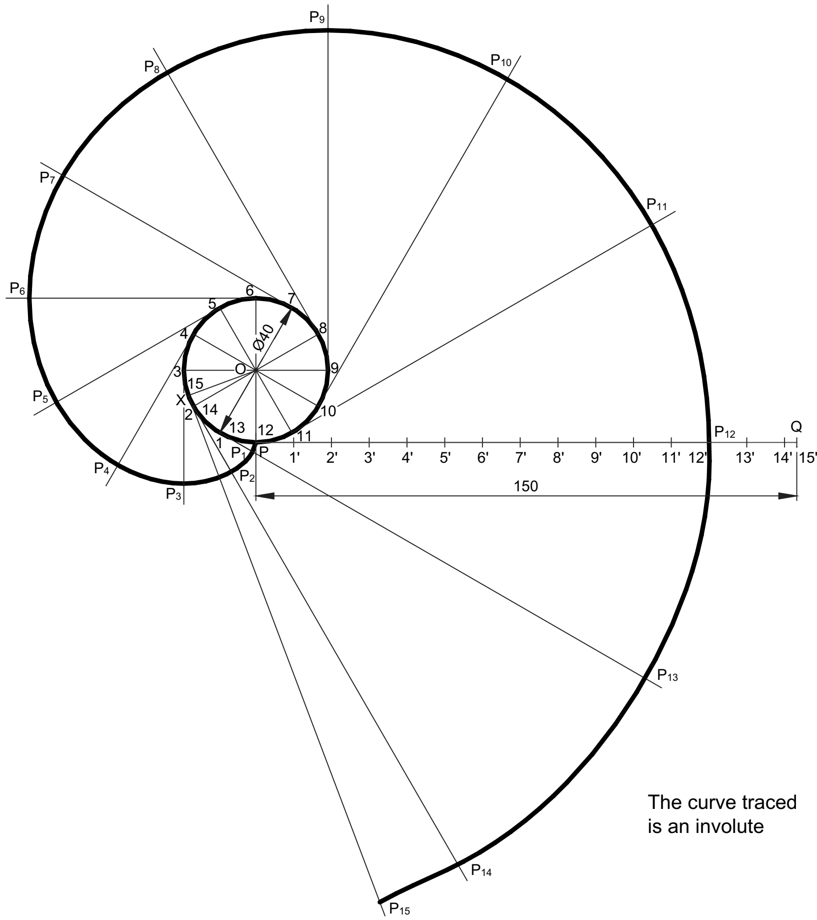


Fig. 6.26

1. Draw a circle of diameter 40 mm and a tangential line PQ = 150 mm.
2. Mark points 1, 2, 3, 4, etc., on line PQ starting from point P such that each of lengths P-1, 1-2, 2-3, 3-4, etc., are equal to $\frac{\pi \times D}{12} = 10.5\,\text{mm}$. The last division 14-15 will be of shorter length (150 -10.5 × 14 = 3 mm).
3. Draw tangents to the circle at points 1, 2, 3, 4, etc., in the direction of unwound.
4. Draw arcs with centres 1, 2, 3, 4, etc. and radii P1′, P2′, P3′, P4′, etc., respectively, to intersect the tangent lines through points 1, 2, 3, 4, etc., at points P_1, P_2, P_3, P_4 etc., respectively. (Refer to Problem 6.10).
5. Proceed to mark points P_{13} and P_{14}. Point P_{15} will lie on a tangent line through a point X, where the angle POX = 69.7°.
6. Draw a smooth curve to to pass through P_1, P_2, P_3, P_4, etc. The curve obtained is an involute.

Problem 6.21 *PQ is a rope 1.4 m long, tied to a peg at Q, as shown in Fig. 6.24(a). Keeping it always tight, the rope is wound around the pole O. Draw the curve traced out by the end P. Consider a scale of 1:10.*

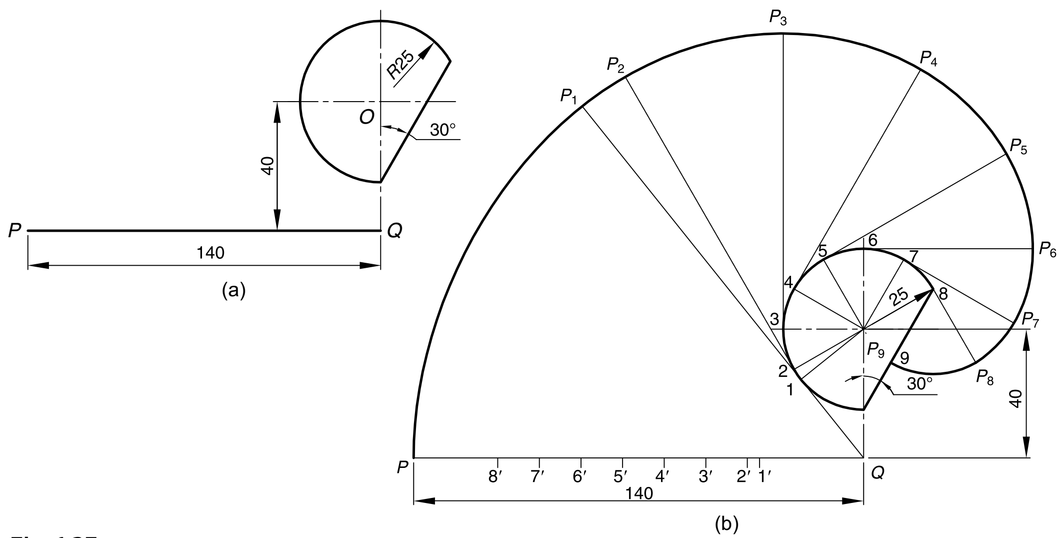


Fig. 6.27

Construction Refer to Fig. 6.27(b).

1. Draw Fig. 6.27(a). From point Q, draw a 140 mm long tangent QP_1 to touch the sector at a point 1. Measure Q-1.
2. Divide the semicircle 2-8 in 6 equal parts and label as 2, 3, 4, 5, 6, 7 and 8.
3. Mark points 1′, 2′, 3′, 4′, etc., on line PQ such that Q-1′ = tangent Q-1, 1′-2′ = arc 1-2, 2′-3′ = arc 2-3, 3′-4′ = arc 3-4, 4′-5′ = arc 4-5, 5′-6′ = arc 5-6, 6′-7′ = arc 6-7 and 7′-8′ = arc 7-8.
4. Draw tangents at 2, 3, 4, 5, 6, 7 and 8 of lengths P-2′, P-3′, P-4′, P-5′, P-6′, P-7′ and P-8′ respectively and locate points P_1, P_2, P_3, P_4, P_5, P_6, P_7 and P_8.
5. Draw a smooth curve to pass through P, P_1, P_2, P_3, P_4, P_5, P_6, P_7 and P_8. Draw an arc with centre 8 and radius 8-P_8 up to point 9. The curve is the required involute.

Problem 6.22 *Draw a triangle AOB such that corner O is at a distance of 30 mm and 42 mm from the other two corners and subtends an included angle of 60°. Draw an Archimedean spiral for one revolution starting from pole O and to pass through points A and B.*

Construction Refer to Fig. 6.28.

1. Draw a triangle AOB taking OA = 30 mm, angle AOB = 60° and OB = 42 mm. For the angular movement of 60°, the distance covered is $OB - OA$ = 12 mm. In other words, for every 30° angular movement the distance covered is 6 mm. In reaching from pole O to point A, the distance covered is 30 mm. The angular movement is $\frac{30}{6} \times 30° = 150°$. Therefore, movement is started from the pole 150° before reaching arm OA. The distance travelled for one convolution is 12×6 = 72 mm.

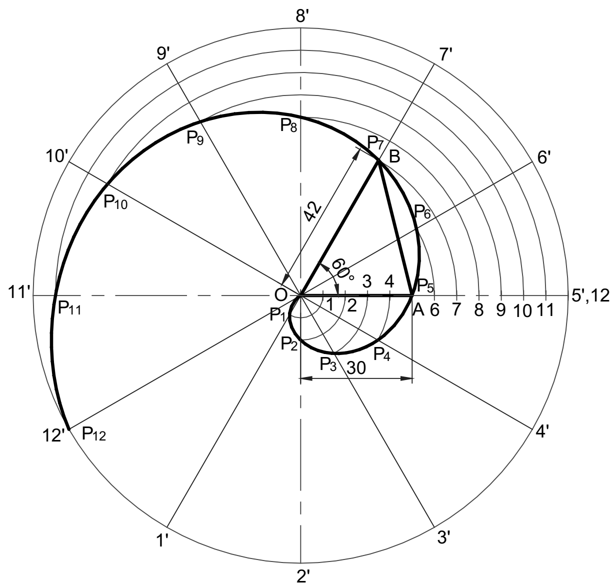


Fig. 6.28

2. Draw a circle of radius 72 mm with centre O. Mark points 1, 2, 3, 4,...,12 on OA produced such that they are 6 mm apart.
3. Divide the circle into 12 equal parts and name the divisions as 1′, 2′, 3′, etc.
4. Draw an arc with the centre O and radius $O1$ to meet the radial line $O1'$ at point P_1.
5. Similarly, draw arcs with the centre O and radii $O2$, $O3$, $O4$, etc., to meet the radial lines $O2'$, $O3'$, $O4'$, etc., respectively, at points P_2, P_3, P_4, etc.
6. Draw a smooth curve to pass through P_1, P_2, P_3, P_4, etc., and obtain the required spiral.

Problem 6.23 *A 150 mm long link swings on a pivot O from its vertical position of rest to the right, through an angle of 40°. After it swings to the left through an angle of 80°, it returns to its initial centre position. During this period, a point P moving at a uniform speed along the centre line of the link from a point at a distance of 22 from O, reaches the end of the link. Draw the locus of the point P.*

Construction Refer to Fig. 6.29.

1. Draw a vertical line OA = 150 mm.
2. Draw lines OB and OC making angles of 40° on the opposite side of OA.
3. Mark a point P on the line OA, at a distance 22 mm from point O.
4. Divide the angles AOB and AOC into 4 equal parts. Mark the divisions as 1, 2, 3, etc.
5. As P travels through an angle of 160º (40° + 80° + 40° = 160°), divide PA into 16 equal parts.
6. Draw an arc with centre O and radius $O1'$ to meet line $O1$ at point P_1.
7. Similarly, draw arcs with centre O and radius $O2'$, $O3'$, $O4'$, etc., to meet lines $O2$, $O3$, $O4$, etc., at points P_2, P_3, P_4,…on respectively.
8. Draw a smooth curve to pass through P_1, P_2, P_3, P_4, etc., and obtain the required locus of P.

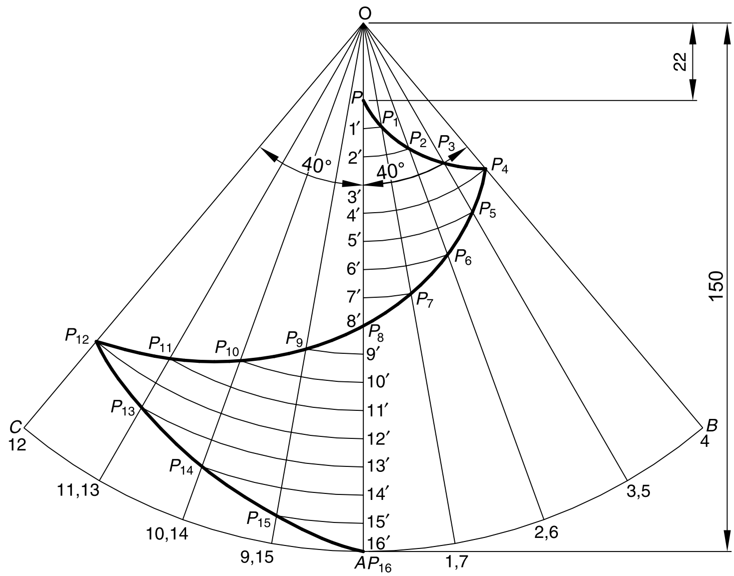


Fig. 6.29

Problem 6.24 *A circular disc of diameter 120 mm rotates about its centre with a uniform angular velocity. A point P which is at an end A of the diameter AB moves with the uniform linear velocity and reaches the end B when the disk completes one rotation. Trace the locus of the point P.*

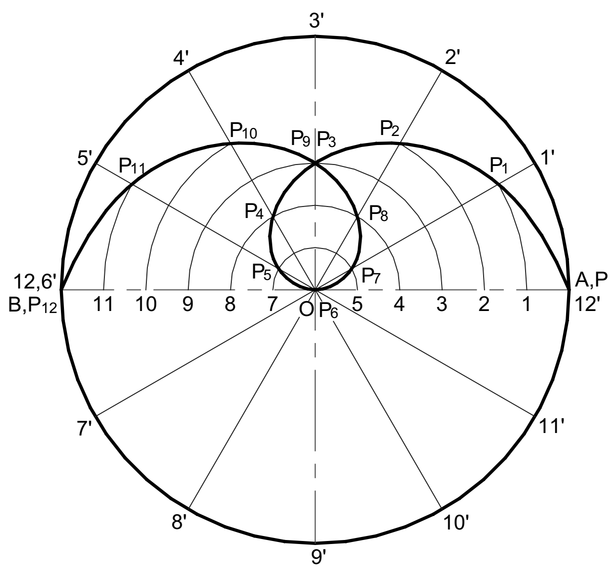


Fig. 6.30

Construction Refer to Fig. 6.30.

1. Draw a circle with centre O and diameter AB = 120 mm.
2. Divide the circle into 12 equal parts and mark the divisions as 1′, 2′, 3′, etc.
3. Divide the diameter AB into 12 equal parts and mark the divisions as 1, 2, 3, etc.
4. Draw arcs with centre O and radii $O1$, $O2$, $O3$, $O4$, $O5$, $O6$ to meet radial lines $O1'$, $O2'$, $O3'$, $O4'$, $O5'$, $O6'$ at points P_1, P_2, P_3, P_4, P_5, P_6.
5. Also draw arcs with the centre O and radii $O7$, $O8$, $O9$, $O10$, $O11$, $O12$ to meet radial lines $O1'$, $O2'$, $O3'$, $O4'$, $O5'$, $O6'$ at points P_7, P_8, P_9, P_{10}, P_{11}, P_{12}.
6. Draw a smooth curve passing through all the points P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}, P_{11}, P_{12}. The curve is the required locus of point P. It is an Archemedian spiral.

EXERCISE 6A

Cycloid

6.1 A circle of diameter 50 mm rolls for a distance of 180 mm along a straight line. Draw the locus of a point lying on the circumference of the circle. Name the curve traced.

6.2 A circular wheel of diameter 40 mm rolls without slipping along a straight line. Draw the curve traced by a point P, lying on the rim for 1.25 revolutions of the wheel. Name the curve traced. Also, draw a tangent and a normal at a point P, when the wheel has travelled 100 mm from its starting position.

6.3 A circle of diameter 60 mm rolls over a straight line. Draw the locus of a point lying on the circumference of the circle, initially at the topmost position and occupies the bottom most position after the revolution. Name the curve traced. Also, draw a tangent and a normal to the curve at a point 45 mm above the base line.

6.4 A circle of diameter 50 mm rolls on a straight line without slipping. In the initial position the diameter AB of the circle is parallel to the line on which it rolls. Draw the loci of the points A and B for one revolution of the circle.

6.5 ABC is an equilateral triangle of side 50 mm. Trace the loci of vertices A, B and C when the circle circumscribing triangle ABC rolls without slipping along a fixed straight line for one revolution.

6.6 A circle of diameter 50 mm rolls on a horizontal line for a half revolution and then on a vertical line downwards for another half revolution. Draw the curve traced out by a point lying on the circumference of the circle, taking the bottom-most point of the circle as the initial position.

6.7 A circle of diameter 50 mm rolls on a horizontal line for half revolution and then on an inclined line 60º with horizontal for another half. Draw the curve traced out by a point P, lying on the circumference of the circle, taking the topmost point of the rolling circle as the initial position of point P.

Epicycloid

6.8 Draw an epicycloid generated by a point on the circumference of a circle of diameter 50 mm with rolls outside another circle of diameter 150 mm, for one revolution. Also, draw a tangent and a normal to the curve, at a point 115 mm from the centre of the directing circle.

6.9 A 40 mm diameter circle rolls outside an arc of radius 70 mm for a circular distance of 120 mm. Trace the path of a point lying on the circumference of the rolling circle, which is in contact with the arc in its initial position. Name the curve.

6.10 The angle subtended by a directing circle when a circle of diameter 40 mm rolls outside it for one complete revolution is 108°. Determine the diameter of the directing circle. Trace the path of a point lying on the circumference of the rolling circle.

6.11 A circle of diameter 60 mm rolls over a circular arc of diameter 200 mm for one-half revolution and has an external contact. Draw the locus of a point lying on the circumference of the circle and initially farthest away from the arc. Name the curve traced. Also, draw a tangent and a normal to the curve at a point 45 mm above the base line.

6.12 Draw the locus of a point lying on the circumference of a wheel of diameter 60 cm for one revolution, when it passes over a segmental arched culvert of radius 1 m. Take a scale of 1:10.

6.13 A rolling circle of diameter AB = 40 mm, rolls on a fixed disc of diameter 60 mm with external contact. Draw the loci of path traced by the points A and B of the rolling circle for one revolution when one of the end points of diameter AB is in contact to the disc at the starting position.

6.14 Construct a cardioid taking the diameters of both the rolling and directing circles as 60 mm.

Hypocycloid

6.15 Draw a hypocycloid of a circle of diameter 50 mm, which rolls inside a circular arc of radius 85 mm for one revolution. Also, draw a tangent and a normal to the epicycloid at a point 50 mm from the centre of the directing circle.

6.16 A circus man rides on a motor cycle inside a globe of diameter 3.5 m. The motor cycle wheel is 1 m in diameter. Draw the locus of a point lying on the circumference of the wheel of motor cycle for one turn. Take a scale of 1:20.

6.17 Draw the locus of a point lying on the circumference of a circle of diameter 70 mm, which rolls inside a circle of diameter 140 mm for one rotation.

6.18 A hypocycloid is in the form of a 120 mm long straight line. Determine the diameters of the rolling and the directing circles. Also, draw the curve.

Trochoid, epitrochoid and hypotrochoid

6.19 A circle of diameter 50 mm rolls along a straight line for one revolution. Draw the locus of a point, lying at a distance of 18 mm from the centre of the circle.

6.20 A circle of diameter 50 mm rolls outside a circle of diameter 180 mm for one revolution. Draw the locus of a point, lying at a distance of 18 mm from the centre of the rolling circle.

6.21 A circle of diameter 50 mm rolls inside a circle of diameter 180 mm for one revolution. Draw the locus of a point, lying at a distance of 32 mm from the centre of the rolling circle.

6.22 A 50 mm diameter circle rolls for a distance of 180 mm along a straight line. Draw the locus of a point lying outside the rolling circle at a distance of 8 mm from the rim. Name the curve traced.

Involute

6.23 Draw an involute of a square of side 25 mm.

6.24 Draw an involute of a pentagon of side 25 mm.

6.25 An elastic string of length 150 mm has its one end attached to the circumference of a circular disc of diameter 40 mm. Draw the curve traced out by the other end of the string when it is completely wound around the disc, keeping the string always tight.

6.26 Draw the path that would traced by an end of the string, when is unwound from the circumference of the disc which is in the form of a square of side 30 mm surmounted by semicircles on the opposite sides.

Spiral

6.27 Draw an Archimedean spiral for one convolution with a shortest distance of 15 mm and radial increment of 5 mm for each 30º.

6.28 Draw a triangle AOB such that corner O is at a distance of 12 mm and 36 mm from the other two corners and subtends an included angle of 120°. Draw an Archimedean spiral for one revolution starting from pole O and to pass through points A and B.

6.29 An 80 mm long link OA, rotates at a uniform speed about the pivot O. A point P lying on the link at a distance 20 mm from O moves at a uniform speed and reaches the end A, while the link has rotated through 3/5 of a revolution. Draw the locus of the point P.

6.30 A circular disc of diameter 140 mm rotates about its centre with a uniform angular velocity. A point P which is at an end A of the diameter AB moves with the uniform linear velocity and reaches the end B when the disk completes one rotation. Trace the locus of the point P.

6.31 Draw a logarithmic spiral of one convolution taking smaller radius as 10 mm and the ratio of the length of adjacent radii enclosing 30° as 10:9. Draw a tangent and a normal to the curve at a point 50 mm from the pole.

6.32 Draw a logarithmic spiral for 1.25 convolutions such that the angle between the consecutive radii is 30°, the ratio of succeeding radii is 7:6 and the greatest radius is 120 mm. Also draw a tangent and a normal at a point 70 mm from the pole.

6.33 A circular disc of diameter AB = 100 mm rotates with a uniform angular velocity. The point P starting from A moves with uniform linear velocity and reaches the point B when the disk completes one revolution. Trace the locus of the point P moving from A to B.

6.9 LOCI OF POINTS

6.9.1 Four Bar Chain

A four-bar chain shown in Fig. 6.31 is the simplest movable closed chain linkage. It consists of four links connected in a loop by four joints. The four links may be of different lengths. The joints are configured so the links move in a plane. According to Grashof's law for a four bar mechanism, the sum of the shortest and longest link should not be greater than the sum of the remaining two link lengths if there is to be continuous relative motion between the two links. A very important consideration in designing a mechanism is to ensure that the input crank makes a complete revolution relative to the other links. This mechanism is used to transform a rotary motion into an oscillating motion.

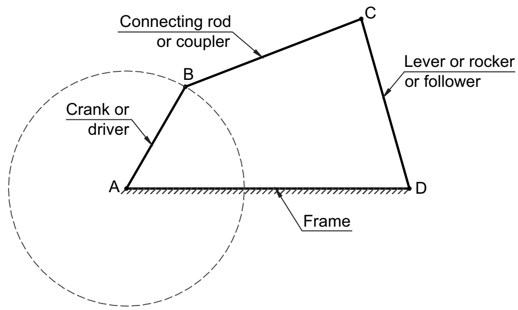


Fig. 6.31 *Four bar chain*

Problem 6.25 *In a link mechanism, two cranks AB and CD each of length 40 mm rotate in opposite directions. Draw the locus of a point P which is situated on the connecting rod BC at a distance of 25 mm from C. Take length of the connecting rod BC and the fixed link AD as 130 mm.*

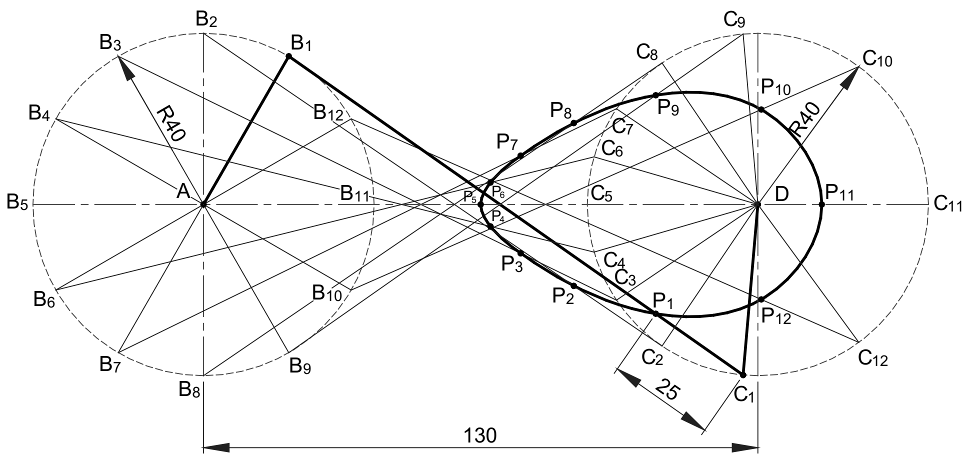


Fig. 6.32

Construction Refer to Fig. 6.32.

1. Draw a 130 mm long line *AD*. With centres *A* and *D*, draw circles each of radius 40 mm to represent the loci of points *B* and *C* respectively.
2. Divide the circle with centre *A* into 12 equal parts and mark the divisions as B_1, B_2, B_3, etc.
3. Consider the point *B* is situated initially at point B_1. Draw an arc with centre B_1 and radius 130 mm to intersect the circle with centre *D* at a point C_1.

4. Join B_1C_1 and mark a point P_1 on line B_1C_1 such that $C_1P_1 = 25$ mm.
5. Similarly, draw arcs of radius 130 mm from centres B_2, B_3, B_4, etc., to intersect the circle with centre D at points C_2, C_3, C_4, etc. Join B_2C_2, B_3C_3, B_4C_4, etc., and locate point P_2, P_3, P_4, etc., such that C_2P_2, C_3P_3, C_4P_4, etc., are 25 mm long.
6. Join P_1, P_2, P_3, P_4, etc., by a smooth curve to represent the locus of the point P.

Problem 6.26 *Two cranks AB and CD each of length 40 mm rotate in opposite directions. Draw the locus of a point P situated at a distance of 50 mm on the extension of the link BC. Take length of the connecting rod BC and the fixed link AD as 110 mm.*

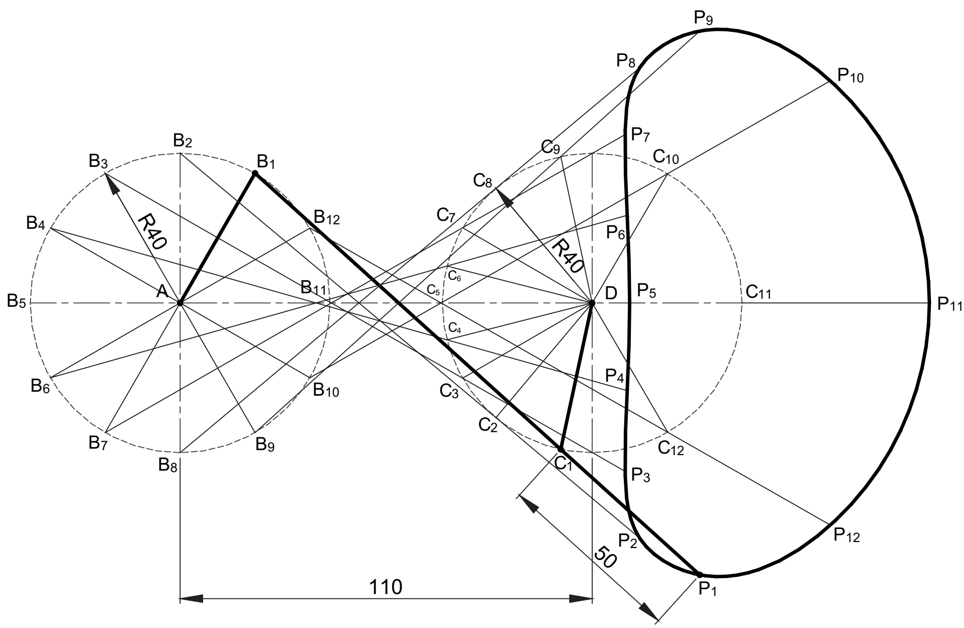


Fig. 6.33

Construction Refer to Fig. 6.33.

1. Draw a 110 mm long line AD. With centres A and D, draw circles each of radius 40 mm to represent the loci of points B and C respectively.
2. Divide the circle with centre A into 12 equal parts and mark the divisions as B_1, B_2, B_3, etc.
3. Consider the point B is situated initially at point B_1. Draw an arc with centre B_1 and radius 110 mm to intersect the circle with centre D at a point C_1.
4. Join B_1C_1 and produce to mark a point P_1 such that $C_1P_1 = 50$ mm.
5. Similarly, draw arcs of 110 mm radius from centres B_2, B_3, B_4, etc., to intersect the circle with centre D at points C_2, C_3, C_4, etc. Join B_2C_2, B_3C_3, B_4C_4, etc., and locate point P_2, P_3, P_4, etc., such that C_2P_2, C_3P_3, C_4P_4, etc., are 50 mm long.
6. Join P_1, P_2, P_3, P_4, etc., by a smooth curve to represent the locus of the point P.

Problem 6.27 *A four bar mechanism has a 50 mm long driving link AB, 120 long fixed link AD, and 90 mm long other links BC and CD. Draw the locus of the point P, lying on the connecting rod at a distance 30 mm from B for one revolution of the driving link.*

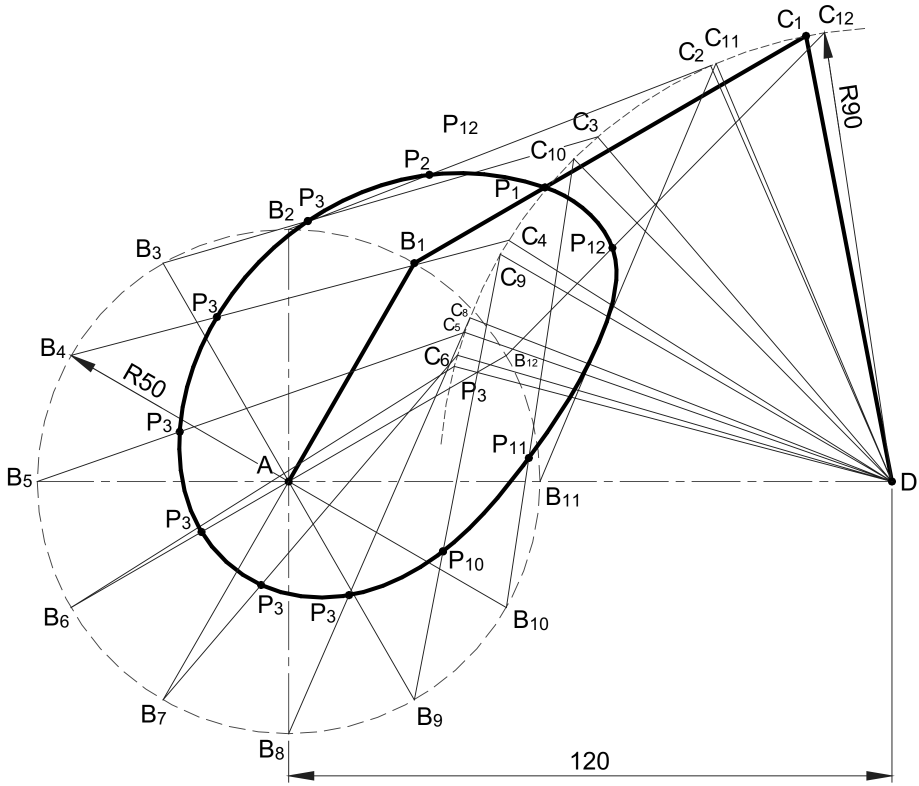


Fig. 6.34

Construction Refer to Fig. 6.34.

1. Draw a 120 mm long line *AD*.
2. Draw a circle with centre *A* and radius 50 mm to represent the locus of point *B*. Divide the circle into 12 equal parts and mark the divisions as B_1, B_2, B_3, etc.
3. Draw an arc with centre *D* and radius 90 mm to represent the locus of point *C*.
4. Consider the point B is situated initially at point B_1. Draw an arc with centre B_1 and radius 90 mm to intersect the arc at a point C_1.
5. Join B_1C_1 and mark a point P_1 on line B_1C_1 such that $B_1P_1 = 30$ mm.
6. Similarly, draw arcs of radius 90 mm from centres B_2, B_3, B_4, etc., to intersect the arc with centre *D* at points C_2, C_3, C_4, etc. Join B_2C_2, B_3C_3, B_4C_4, etc. and locate point P_2, P_3, P_4, etc., such that B_2P_2, B_3P_3, B_4P_4, etc., are 30 mm long.
7. Join P_1, P_2, P_3, P_4, etc., by a smooth curve to represent the locus of the point *P*.

Problem 6.28 *Two equal cranks connected through a link AB rotate in opposite directions. Draw the locus of the end point P of another link PQ connected at the mid-point of the link AB. $O_1A = O_2B = 40$ mm, $O_1O_2 = AB = 140$ mm, $PQ = 50$ mm and $AQP = 90°$.*

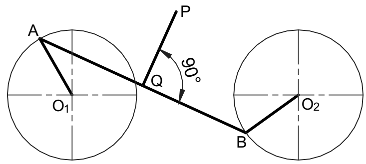


Fig. 6.35(a)

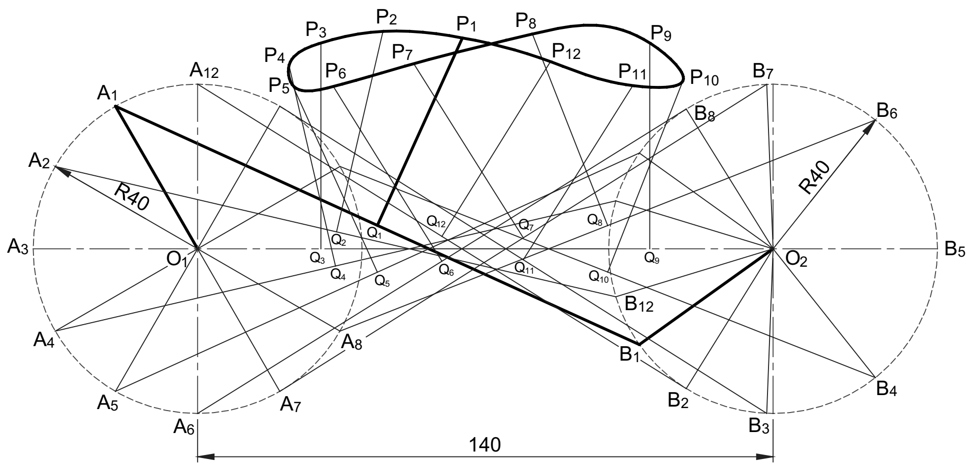


Fig. 6.35(b)

Construction Refer to Fig. 6.35(b).

1. Draw a 140 mm long line O_1O_2. With centres O_1 and O_2, draw circles each of radius 40 mm to represent the loci of points A and B respectively.
2. Divide the circle with centre O_1 into 12 equal parts and mark the divisions as A_1, A_2, A_3, etc.
3. Consider the point A is situated initially at point A_1. Draw an arc with centre A_1 and radius 140 mm to intersect the circle with centre O_2 at a point B_1.
4. Join A_1B_1 and mark its mid-point Q_1. Draw a 50 mm long line Q_1P_1 perpendicular to the line A_1B_1.
5. Similarly, draw arcs of radius 140 mm from centres A_2, A_3, A_4, etc., to intersect the circle with centre O_2 at points B_2, B_3, B_4, etc. Join A_2B_2, A_3B_3, A_4B_4, etc., and locate their mid-point Q_2, Q_3, Q_4, etc. Draw 50 mm long lines Q_2P_2, Q_3P_3, Q_4P_4, etc., perpendicular to the lines A_2B_2, A_3B_3, A_4B_4, etc.
6. Join P_1, P_2, P_3, P_4, etc., by a smooth curve to represent the locus of the point P.

6.9.2 Single Slider Crank Chain

A single slider crank chain is a modification of the basic four bar chain. It consists of one sliding pair and three turning pairs. This type of mechanism converts rotary motion into reciprocating motion and vice versa. Figure 6.36 shows a single slider crank mechanism. As the crank rotates, the cross-head reciprocates in the guides and thus the piston reciprocates in the cylinder. It is usually found in reciprocating steam engine mechanism.

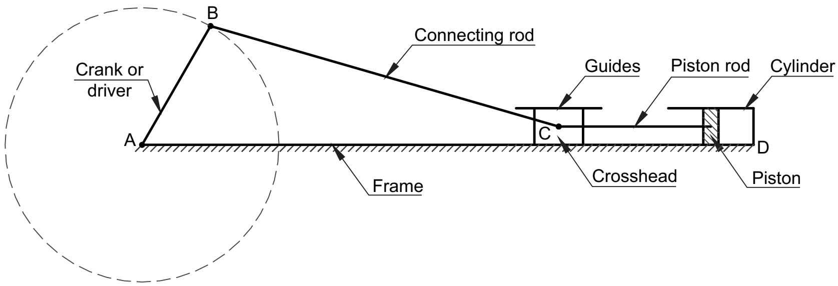


Fig. 6.36

Problem 6.29 *The slider end in an offset slider crank mechanism moves in guides along a line PQ situated 25 mm below the axis of the crank shaft. Draw the locus of a point C 50 mm from A along AB. Take crank length OA = 40 mm and slider length AB = 120 mm.*

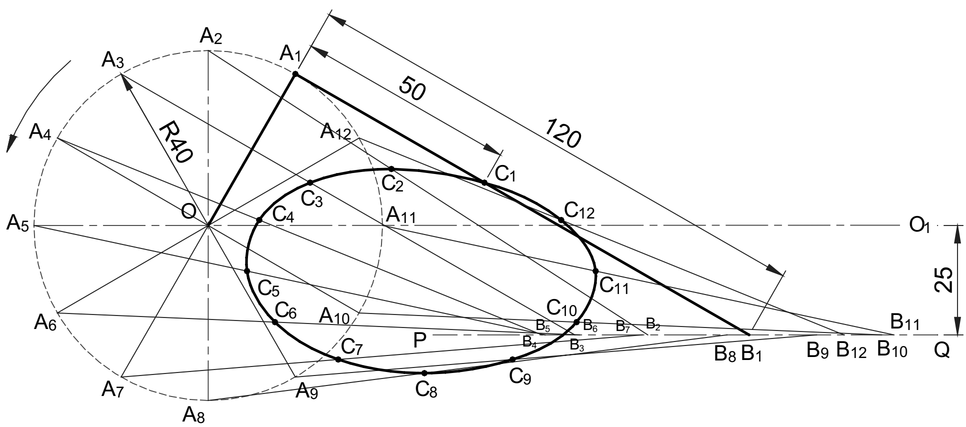


Fig. 6.37

Construction Refer to Fig. 6.37.

1. Draw parallel lines OO_1 and PQ, 25 mm apart.
2. Draw a circle with centre O and radius 40 mm as the locus of point A. Divide this circle into 12 equal parts and mark the divisions as A_1, A_2, A_3, etc.

3. Consider the point A is situated initially at point A_1. Draw an arc with centre A_1 and radius 120 mm to intersect the line PQ at a point B_1.
4. Join A_1B_1 and mark point C_1 such that A_1C_1 = 50 mm.
5. Similarly, draw arcs of radius 120 mm from centres A_2, A_3, A_4, etc., to intersect the line PQ at points B_2, B_3, B_4, etc. Join A_2B_2, A_3B_3, A_4B_4, etc., and locate points C_2, C_3, C_4, etc., at a distance 50 mm from points A_2, A_3, A_4, etc.
6. Join C_1, C_2, C_3, C_4, etc., by a smooth curve to represent the locus of the point C.

Problem 6.30 *A crank OA rotates about O and carries with it a rod PQ 120 mm pin-jointed to the crank at A. the rod is constrained to pass through a fixed point C. Determine the loci of the ends P and Q of the rod.*

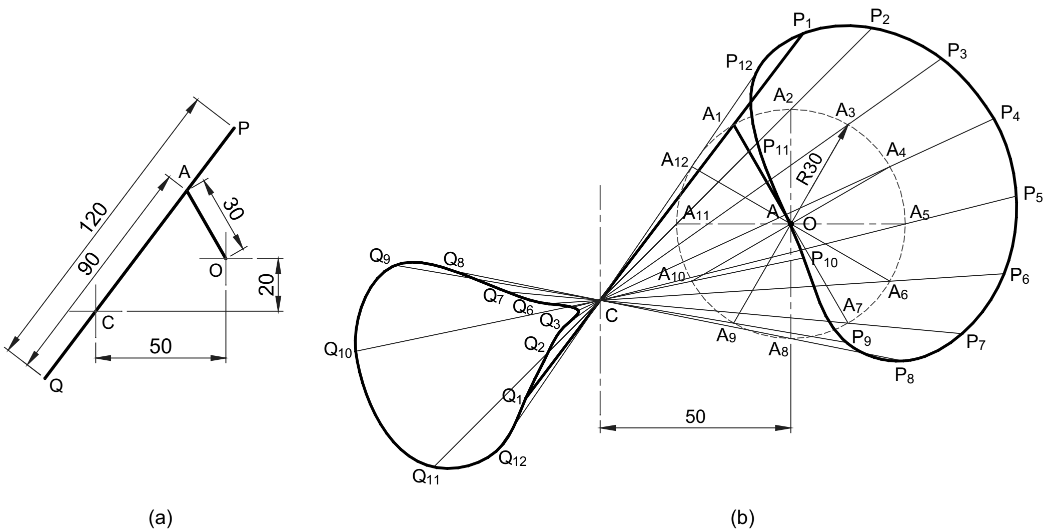


Fig. 6.38

Construction Refer to Fig. 6.38(b).

1. Locate points O and C as given in Fig. 6.38(a).
2. Draw a circle with centre O and radius 30 mm as the locus of point A. Divide this circle into 12 equal parts and mark the divisions as A_1, A_2, A_3, etc.
3. Consider the point A is situated initially at point A_1. Join A_1C and produce on both sides.
4. Locate points P_1 and Q_1 on A_1C produced such that A_1Q_1 = 90 mm, P_1Q_1 = 120 mm and A_1P_1 = 30 mm as shown.
5. Similarly, join A_2C, A_3C, A_4C, etc. Locate points Q_2, Q_3, Q_4, etc., on A_2C, A_3C, A_4C, etc., produced such that the distance of points Q_2, Q_3, Q_4, etc., from points A_2, A_3, A_4, etc., is 90 mm. Locate points P_2, P_3, P_4, etc., on CA_2, CA_3, CA_4, etc., produced such that the distance of points P_2, P_3, P_4, etc., from points A_2, A_3, A_4, etc., is 30 mm.
6. Join P_1, P_2, P_3,…etc., and join Q_1, Q_2, Q_3,… etc., by smooth curves to represent the loci of points P and Q respectively.

Problem 6.31 *A 30 mm long crank OA rotates about O. A rod AP pin-jointed to the crank at A, is constrained to always touch the circle of radius 20 mm. Plot the locus of the end P for a complete revolution of the crank. Take OC = 65 mm and AP = 75 mm.*

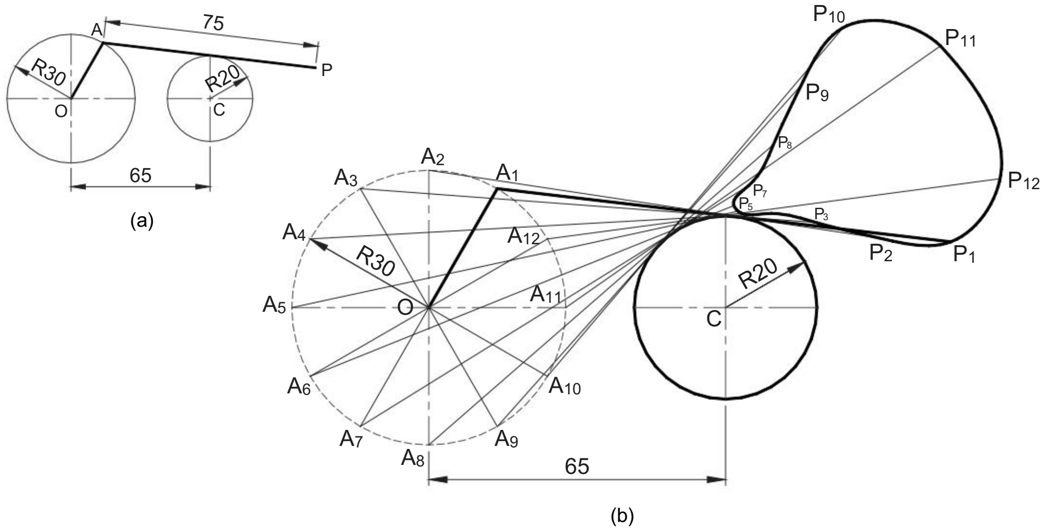


Fig. 6.39

Construction Refer to Fig. 6.38(b).

1. Draw a 65 mm long line *OC*.
2. Draw circles with centres *O* and *C* of radius 30 mm and 20 mm, respectively.
3. Divide the circle with centre *O* into 12 equal parts and mark the divisions as A_1, A_2, A_3, etc.
4. Draw 75 mm long line A_1P_1, A_2P_2, A_3P_3, etc., tangential to the circle with 20 mm radius.
5. Join P_1, P_2, P_3, P_4, etc., by a smooth curve to represent the locus of the point *P*.

EXERCISE 6B

6.1 Two equal cranks *OP* and *QR* in a link mechanism rotate in opposite directions. Draw the locus of point *S* which is situated on the connecting rod *PR* at a distance of 20 mm from *P*. Take length of the connecting rod *PR* and the fixed link *OQ* as 100 mm and crank length 30 mm.

6.2 Two cranks *OP* and *QR* each of length 30 mm rotate in opposite directions. Draw the locus of a point *S* situated at a distance of 40 mm on the extension of the link *PR*. Take *OQ* = *PR* = 110 mm.

6.3 Draw the locus of a point *P* lying on the midpoint of connecting rod in a four bar mechanism using the following dimensions: Driving link *AB* = 40 mm; Fixed link *AD* = 100 mm and other two links *BC* = *CD* = 70 mm.

6.4 Two equal cranks connected through a link *AB* rotate in opposite directions. Draw the locus of the end point *P* of another link *PQ* connected at 60 mm from A. Take $OA = O_1B = 40$ mm, $OO_1 = AB = 140$ mm, $PQ = 50$ mm and $AQP = 90º$.

6.5 Two cranks *AB* and *CD* each of lengths 45 mm rotate in opposite directions. Draw the locus of a point *P* situated at a distance of 60 mm on the extension of the link *BD*. Distance between the centres *AC* and length of link *BD* may be taken as 120 mm.

6.6 A 40 mm long crank *OA* rotates about *O* and carries with it a 150 mm long rod *PQ* pin-jointed to the crank at *A* as shown in Fig. E6.1. The rod is constrained to pass through a fixed point *C*. Determine the loci of the ends *P* and *Q* of the rod.

6.7 A 40 mm long crank *OA* rotates about a fixed point *O*. *A* 125 mm long rod *AP*, pin-jointed to the crank at *A*, is constrained to always touch a circle of diameter 40 mm, the centre of which is 75 mm from *O* as shown in Fig. E6.2. Plot the locus of *P* for one revolution of the crank.

6.8 In an offset slider crank mechanism, the slider end moves in guides along a line *PQ* situated 20 mm below the axis of the crank shaft. Draw the locus of the midpoint *C* of the slider AB if crank length *OA* = 35 mm and slider length *AB* = 110 mm.

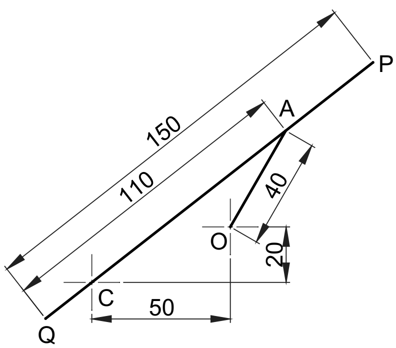


Fig. E6.1

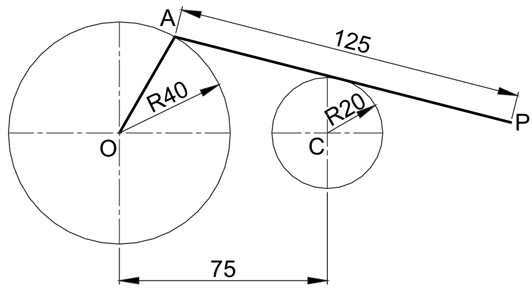


Fig. E6.2

VIVA-VOCE QUESTIONS

6.1 Define roulettes and cycloidal curves.
6.2 What is a cycloid? Give its practical applications.
6.3 What is an epicycloid? Give its practical applications.
6.4 What is a hypocycloid? Give its practical applications.
6.5 Differentiate between cycloid, epicycloid and hypocycloid.
6.6 What is a trochoid? Differentiate between inferior and superior trochoids.
6.7 What is an epitrochoid? Differentiate between inferior and superior epitrochoids.
6.8 What is a hypotrochoid? Differentiate between inferior and superior hypotrochoids.
6.9 Differentiate between trochoid, epitrochoid and hypotrochoids.
6.10 What is an involute? Give its practical applications.
6.11 Differentiate between Archimedean and logarithmic spirals.
6.12 What is a helix? Give its practical applications.
6.13 Differentiate between cylindrical and conical helixes.
6.14 What is the nature of epicycloid when radius of the directing circle is (a) equal to the diameter of the rolling circle, (b) twice the diameter of the rolling circle?
6.15 What is the nature of hypocycloid when radius of the directing circle is (a) equal to the diameter of the rolling circle, (b) twice the diameter of the rolling circle?
6.16 Explain the working of a four bar mechanism with the help of a neat sketch.
6.17 Explain the working of a slider crank mechanism with the help of a neat sketch.

MULTIPLE-CHOICE QUESTIONS

6.1 The curves generated by the rolling contact of one curve or line on another curve or line are called
(a) conic sections (b) roulettes
(c) spiral (d) helix

6.2 A curve generated by a point lying on the circumference of a circle which rolls along a fixed straight or curved path without slipping is called
(a) cycloidal curve (b) trochoidal curve
(c) involute (d) spiral

6.3 The curve traced out by a point on the circumference of a circle which rolls on another circle with external contact is
(a) epicycloids
(b) hypocycloid
(c) superior epitrochoid
(d) superior hypotrochoid

6.4 A curve traced out by a point on the circumference of a circle which rolls outside another circle of same diameter is
(a) cycloid (b) cardioid
(c) trochoid (d) None of these

6.5 The curve traced out by a point on the circumference of a circle which rolls on another circle with internal contact is
(a) epicycloids
(b) hypocycloid
(c) superior epitrochoid
(d) superior hypotrochoid

6.6 The hypocycloid having diameter of the rolling circle one-half that of the generating circle resembles
(a) straight line (b) circle
(c) ellipse (d) None of these

6.7 A curve traced by a point situated either inside or outside the circle which rolls along a fixed straight line is
(a) cycloid (b) cardioid
(c) trochoid (d) None of these

6.8 An involute of a circle is popularly used in
(a) projectile trajectory
(b) support of bridges
(c) teeth profile of gears
(d) All of these

6.9 The curve traced by a point on a straight line which rolls on a circle, without slipping is called
(a) cycloid (b) epicycloids
(c) hypocycloid (d) involute

6.10 Involute curve is used in
(a) chains (b) gears
(c) cams (d) pulleys

6.11 A curve traced out by a point moving uniformly along a straight line towards the pole, while the line revolves about its one of the ends with uniform angular velocity is
(a) cycloid
(b) involute
(c) Archimedean spiral
(d) logarithmic spiral

6.12 A curve traced by a point moving along a rotating line such that for equal angular displacement of the line, the ratio of the lengths of consecutive radius vectors is constant. The curve is called
(a) cycloid
(b) involute
(c) Archimedean spiral
(d) logarithmic spiral

6.13 A curve traced by a point moving around the surface of a cylinder or a cone with a uniform speed alongwith a uniform axial movement. The curve is called
(a) cycloid (b) involute
(c) spiral (d) helix

6.14 In a four bar mechanism the arm which rotates is called
(a) frame (b) follower
(c) crank (d) coupler

6.15 Single slider crank chain have
(a) three sliding pair and one turning pair
(b) two sliding pair and two turning pair
(c) one sliding pair and three turning pair
(d) four turning pairs

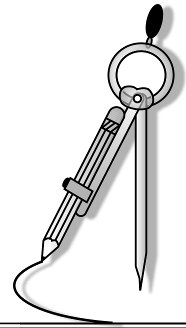
Answers to multiple-choice questions

6.1 (b), 6.2 (a), 6.3 (a), 6.4 (b), 6.5 (b), 6.6 (a), 6.7 (c), 6.8 (c), 6.9 (d), 6.10 (b), 6.11 (c), 6.12 (d), 6.13 (d), 6.14 (c), 6.15 (c)

Chapter

7

ORTHOGRAPHIC PROJECTIONS



7.1 PROJECTION

A projection is defined as an image or a drawing of an object made on a plane. All drawings used in the field of engineering are based on the principles of projection. This is the reason engineering drawings are capable of precisely conveying the external as well as internal features of objects in terms of their shape and size. Projections can be classified on the basis of the lines of sight and the positions of planes on which the drawing is made. The lines of sight are popularly called *projectors* and the planes on which the drawings are made are called *planes of projection*. Figure 7.1 shows the detailed classification of projections.

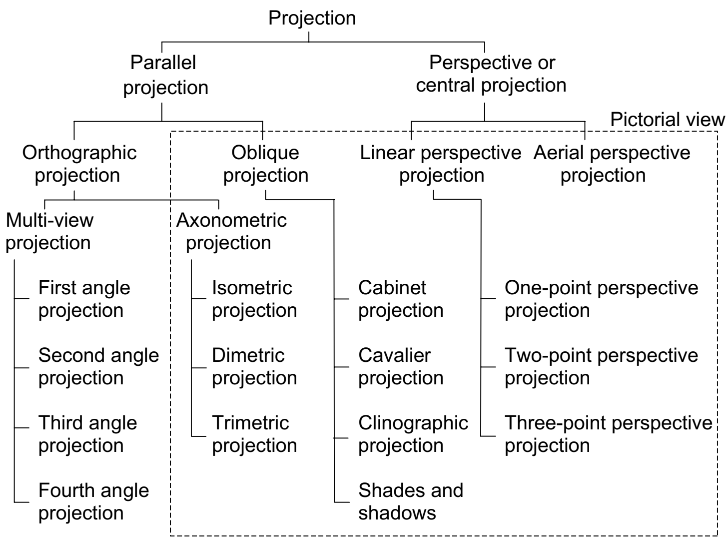


Fig. 7.1 *Classification of projections*

7.2 PICTORIAL VIEW AND MULTI-VIEW

A pictorial view is a means of representing a three-dimensional object so as to reveal all three directions (axes) of space in one picture. It includes perspective, oblique and axonometric projections. A perspective projection provides a realistic view similar to that seen by the eye or captured by a camera. The portion of an object nearer the observer appears larger than those at a distance. Figure 7.2(a) is an example of a perspective projection. An axonometric projection is an approximate perspective projection which provides great liberties for economy of effort and best effect. There is no difference in the size of the nearer or the farther portion of the object. Figure 7.2(b) is an example of an axonometric projection. An oblique projection provides the front face of an object in its true shape and size. This facilitates to draw front face containing curves and circular features with ease. However the other faces of the object are foreshortened and distorted. Figure 7.2(c) is an example of an oblique projection.

Thus, a pictorial drawing gives an idea of shape and outside appearance at a glance. However, this method cannot be universally adopted, mainly because all objects except the simplest cannot be drawn easily and rapidly. Also hidden parts and constructional details are not clearly shown.

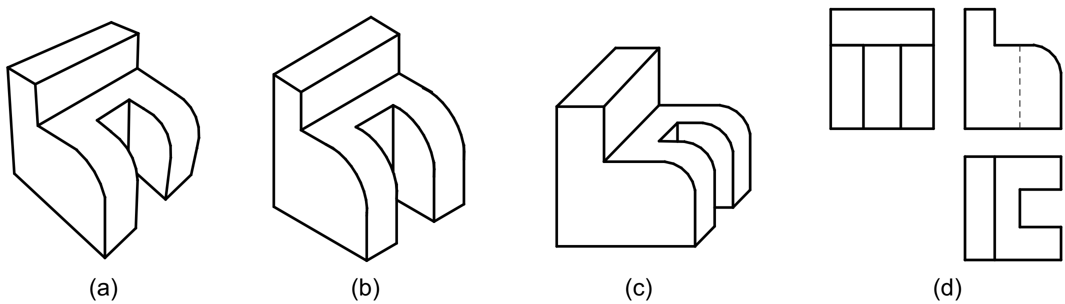


Fig. 7.2 *Projections* **(a)** *Perspective* **(b)** *Axonometric* **(c)** *Oblique* **(d)** *Multi-view*

A multi-view drawing is a means of representing a three-dimensional object in two dimensions. For complete description of an object, two to six views are drawn on a set of mutually perpendicular planes. Here each view indicates the true shape and size which facilitates ease of drawing and its interpretation. Figure 7.1(d) is an example of multi-view drawing/projection where three views are drawn to describe an object.

Multi-view drawings are universally adopted. The only disadvantage of this method is that it requires a thorough understanding of the principles of projections and a great practice to read multi-views to interpret the actual shape. This chapter deals with a detail study of orthographic projections limited to multi-view drawings.

7.3 ORTHOGRAPHIC PROJECTION

The term orthographic is derived from the word '*orthos*' which means perpendicular. An observer is considered to look at the given object from an infinite distance such that the rays of sight from the eyes are parallel to each other and perpendicular to the plane of projection, as shown in Fig. 7.3(a) and (b). These rays of sight are called projectors. Thus, *the orthographic projection is a parallel projection in which the projectors are parallel to each other and perpendicular to the plane of projection.*

Orthographic projection technique can produce either single pictorial drawing that shows all the three dimensions in one view, or multi-views that show only two dimensions of the object in each view.

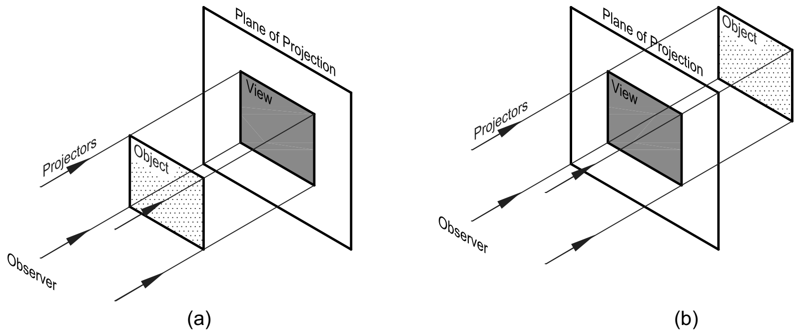


Fig. 7.3 *Object* **(a)** *In front of the plane of projection* **(b)** *Behind the plane of projection*

7.4 MULTI-VIEW DRAWING

Multi-view drawing requires two or more orthographic projections to define the exact shape of a three-dimensional object. Each orthographic view is a two-dimensional drawing showing only two out of three dimensions of the object. Consequently, no single view contains sufficient information to completely define the shape of the object. All orthographic views must be correlated together to interpret the object. The arrangement and relationship among the views are therefore inter-related. The standards and conventions of multi-view drawings have been developed over many years. Now let us get familiar with the concepts and principles of orthographic projections.

Orthographic projections are mainly obtained on two principal planes (*also known as reference planes*) namely vertical plane and horizontal plane as shown in Fig 7.4(a). These principal planes are perpendicular to each other and they divide the space into four segments or quadrants. They are popularly called *angles*.

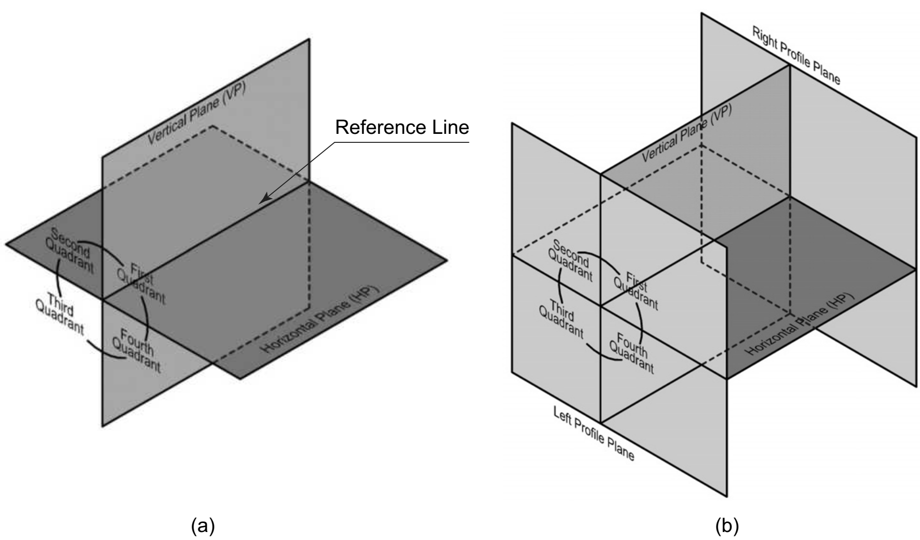


Fig. 7.4 **(a)** *Principal planes* **(b)** *Vertical, horizontal and profile planes*

Thus, we get four angles called first, second, third and fourth angle. The space which lies above H.P. and in front of V.P. is called the first angle. The space which lies above H.P. and behind V.P. is called the second angle. The space which lies below H.P. and behind V.P. is called the third angle. The space which lies below H.P. and in front of V.P. is called the fourth angle. The object is considered to lie in one of these angles for drawing the projections. Depending on the position of the object, the orthographic projection can be classified as follows:

1. First angle projection The object lies in the first angle, i.e., above H.P. and in front of V.P.

2. Second angle projection The object lies in the second angle, i.e., above H.P. and behind V.P.

3. Third angle projection The object lies in the third angle, i.e., below H.P. and behind V.P.

4. Fourth angle projection The object lies in the fourth angle, i.e., below H.P. and in front of V.P.

7.5 TERMINOLOGY

Multi-view drawings are made on three mutually perpendicular planes namely; vertical, horizontal and profile plane, as shown in Fig. 7.4(b). These planes are popularly called *reference planes* and sometimes *principal planes*. The following terms are frequently used in multi-view drawings:

1. Vertical plane Vertical plane, also known as *front reference plane,* is assumed to be placed vertically and is denoted by V.P.

2. Horizontal plane Horizontal plane, also known as *horizontal reference plane*, is assumed to be placed horizontally and is denoted by H.P. It is perpendicular to V.P.

3. Profile plane A plane perpendicular to both the above planes is known as a profile plane. The plane on the right end of the planes is known is right profile plane while the plane on the left end is known as left profile plane.

4. Reference plane All the above mentioned mutually perpendicular planes are called reference planes.

5. Principal plane It is an alternative name of the reference plane.

6. Reference line The line of intersection between the principal planes is known as a reference line. It is also popularly called *xy* line.

7. Front view The view of an object by observing it from the front and drawn on the V.P. is called front view (F.V.) or elevation.

8. Top view The view of an object by observing it from the top and drawn on the H.P. is called top view (T.V.) or plan.

9. Side view The view of an object by observing it from the left-hand side or right-hand side and drawn on a profile plane is called side view or end view.

7.6 FIRST ANGLE PROJECTION

In first angle projection, the object is considered to lie in the first angle, i.e., in front of the V.P. and above the H.P. The observer who is theoretically at infinite distance looks at the object from the front. The rays of sight are parallel to each other and perpendicular to the V.P. (*See Fig. 7.5(a)*). The view obtained on the V.P. is similar to the front face of the object and is known as front view. It may be noted that the front view shows only the length and height of the object. It does not indicate the width.

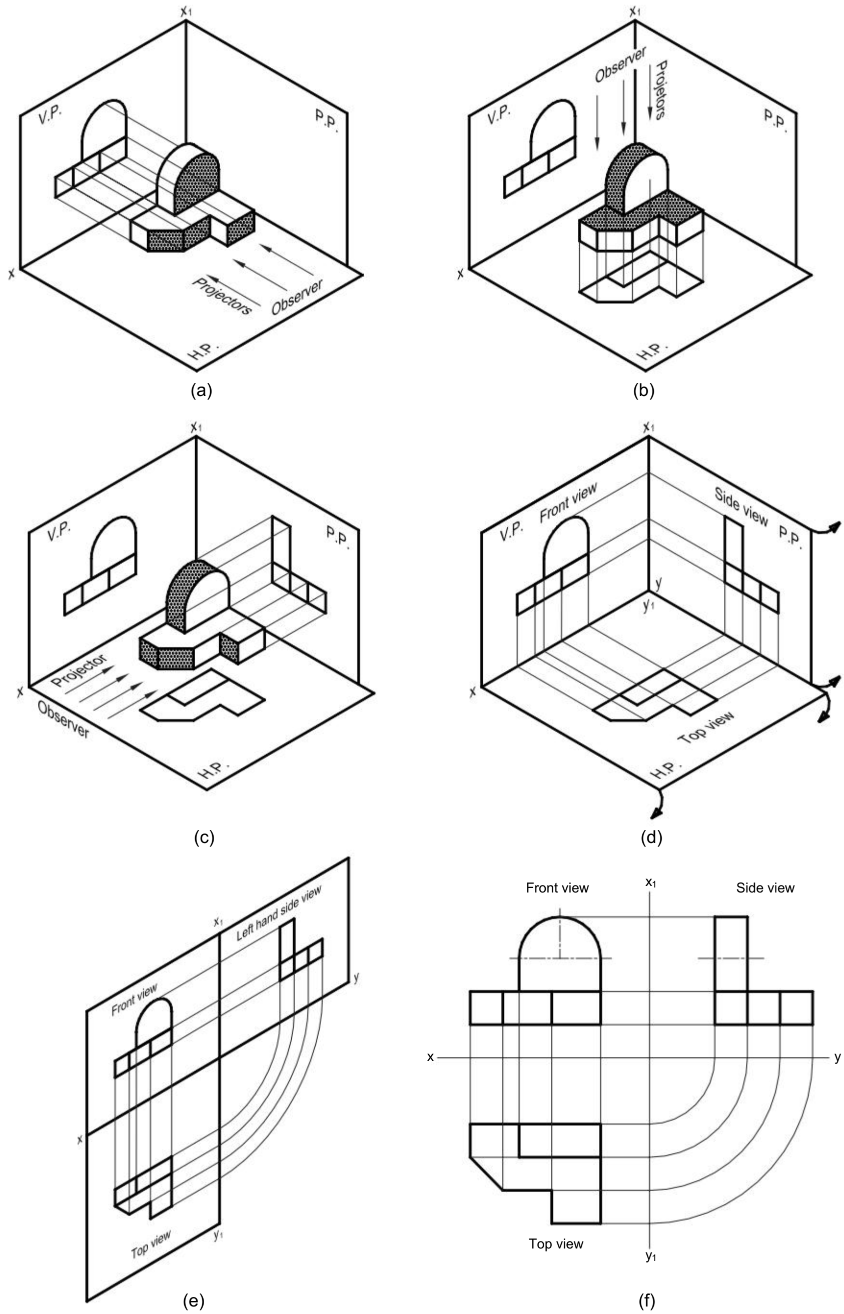


Fig. 7.5 **(a)** *Front view on V.P.* **(b)** *Top view on H.P.* **(c)** *Left-hand side view on P.P.* **(d)** *Co-relation among different views* **(e)** *Unfold the planes* **(f)** *Final representation of front, top and side views*

Again the observer looks at the object from the top such that the rays of sight are parallel to each other and perpendicular to the H.P. (*See Fig. 7.5(b)*). The view obtained on the H.P. is similar to the top face of the object and is known as top view. It may be noted that the top view shows only the length and width of the object. It does not indicate the height.

Now the observer looks at the object from the left hand side such that rays of the sight are parallel to each other and perpendicular to the profile plane (P.P.) (*See Fig. 7.5(c)*). The view obtained on the P.P. is similar to the side face of the object and is known as side view. It may be noted that the side view shows only the width and height of the object. It does not indicate the length.

The co-relation among the front, top and side views are shown in Fig. 7.5(d). The three views namely front, top and side views are obtained on three different planes which are perpendicular to each other. All of these views are to be drafted on a drawing sheet which is a single plane. Therefore, it is customary to keep V.P. fixed and rotate the H.P. about the reference line *xy* through 90º to make it co-planer with V.P. Similarly the profile plane (P.P.) is rotated about x_1y_1 line to make it co-planer with V.P. The direction of such rotation is indicated by arrows in Fig. 7.5(d). Thus top and side views become co-planer with the front view as shown in Fig. 7.5(e). Finally the relative position of the different views which appears on the drawing sheet is shown in Fig. 7.5(f).

Figure 7.6 shows another example of first angle projection. An attempt has been made to draw the front, top, left-hand side, right-hand side, bottom, and rear views.

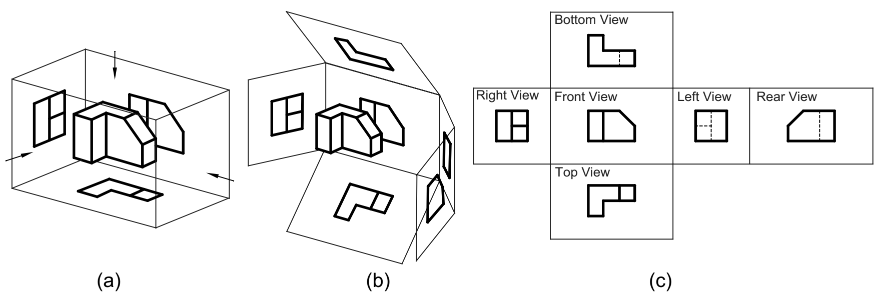


Fig. 7.6 **(a)** *Object kept in six-sided box* **(b)** *Unfold the box* **(c)** *Multi-view drawing in first angle*

7.7 FEATURES OF FIRST ANGLE PROJECTION

From the above discussions, the main features of the first angle projection can be summarised as follows:

1. The object lies in the first angle, i.e., in front of the V.P. and above the H.P.
2. The object lies between the observer and the plane of projection.
3. Top view is drawn below the front view.
4. Left-hand side view is drawn to the right side of the front view.
5. Right-hand side view is drawn to the left side of the front view.

7.8 THIRD ANGLE PROJECTION

In third angle projection, the object is considered to lie in the third angle, i.e. behind the V.P. and below the H.P. The observer who is theoretically at infinite distance looks at the object from the front. The rays of sight are parallel to each other and perpendicular to the V.P. (*See Fig. 7.7(a)*). The view obtained on the V.P. is similar to the front face of the object and is known as front view. It may be noted that the front view shows only the length and height of the object. It does not indicate the width.

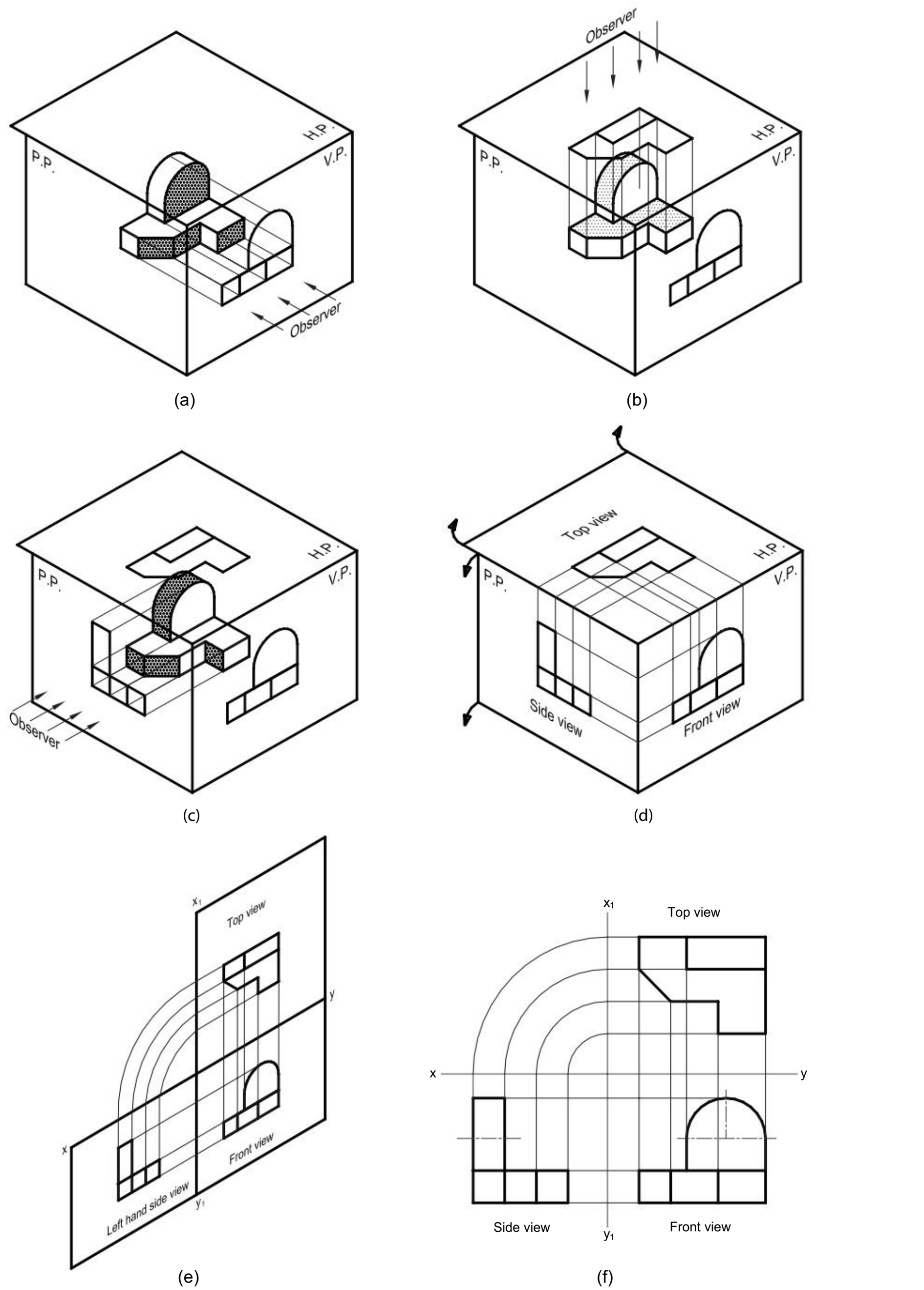


Fig. 7.7 **(a)** *Front view on V.P.* **(b)** *Top view on H.P.* **(c)** *Left-hand side view on P.P.* **(d)** *Co-relation among different views* **(e)** *Unfold the planes* **(f)** *Final representation of front, top and side views*

Again the observer looks at the object from the top such that the rays of sight are parallel to each other and perpendicular to the H.P. (*See Fig. 7.7(b)*). The view obtained on the H.P. is similar to the top face of the object and is known as top view. It may be noted that the top view shows only the length and width of the object. It does not indicate the height.

Now consider that the observer looks at the object from the left hand side such that rays of the sight are parallel to each other and perpendicular to the profile plane (P.P.) (*See Fig. 7.7(c)*). The view obtained on the P.P. is similar to the side face of the object and is known as side view. It may be noted that the side view shows only the width and height of the object. It does not indicate the length.

The co-relation among the front, top and side views are shown in Fig. 7.7(d). The three views namely front, top and side views are obtained on three different planes which are perpendicular to each other. All of these views are to be drafted on a drawing sheet which is a single plane. Therefore, it is customary to keep V.P. fixed and rotate the H.P. about the reference line *xy* through 90º to make it co-planer with V.P. Similarly the profile plane (P.P.) is rotated about x_1y_1 line to make it co-planer with V.P. The direction of such rotation is indicated by arrows in Fig. 7.7(d). Thus top and side views become co-planer with the front view as shown in Fig. 7.7(e). Finally the relative position of the different views which appears on the drawing sheet is shown in Fig. 7.7(f).

Figure 7.8 shows another example of third angle projection. An attempt has been made to draw the front, top, left-hand side, right-hand side, bottom and rear views.

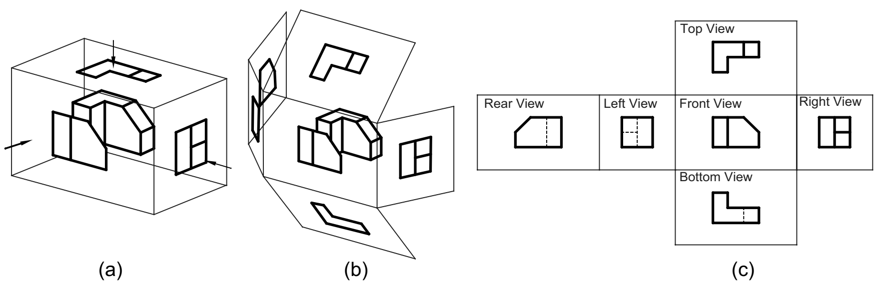


Fig. 7.8 **(a)** *Object kept in six-sided box* **(b)** *Unfold the box* **(c)** *Multi-view drawing in third angle*

7.9 FEATURES OF THIRD ANGLE PROJECTION

From the above discussions, the main features of the third angle projection can be summarised as follows:

1. The object lies in the third angle, i.e., behind the V.P. and below the H.P.
2. The plane of projection lies between the object and the observer.
3. Top view is drawn above the front view.
4. Left-hand side view is drawn to the left side of the front view.
5. Right-hand side view is drawn to the right side of the front view.

7.10 SECOND AND FOURTH ANGLE PROJECTIONS

In second angle projection, the object is considered to lie in the second angle, i.e., behind the V.P. and above the H.P. as shown in Fig. 7.9(a). In fourth angle projection, the object is considered to lie in the fourth angle, i.e., in front of V.P. and below the H.P. as shown in Fig. 7.9(b). In both of these cases, there is a possibility of overlapping the front and top views after rotation of the H.P. about the reference line *xy* and making co-planer with V.P. Thus these methods of projection are not useful in practice.

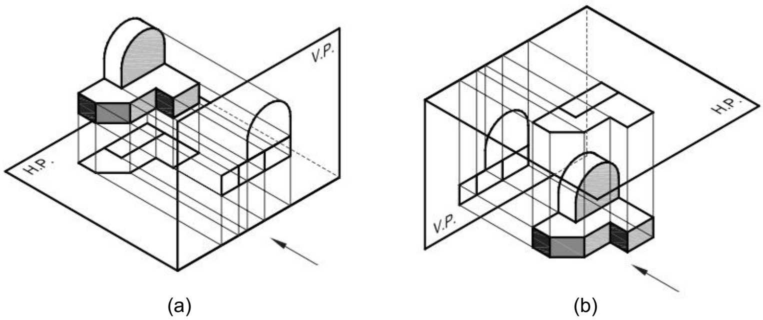


Fig. 7.9 *Projections when object is kept in* **(a)** *second angle* **(b)** *fourth angle*

7.11 SYMBOLS

The front and the top views do not overlap and give the clear picture when an object is placed in either the first angle or the third angle. Thus, internationally, only two methods of projections are adopted for multi-view drawings namely; the first angle projection and the third angle projection.

The angle of projection is indicated in the title block of the drawing sheet with the help of multi-views drawn for the frustum of a cone shown in Fig. 7.10(a). The diameters of the frustum of the cone are in the ratio of 1:2 and the length is equal to the diameter at the bigger end. Figures 7.10(b) and (c) show the multi-views of the cone in the first angle projection and the third angle projection respectively. These views are considered as symbols and should be drawn in the space provided for the purpose in the title block of the drawing sheet.

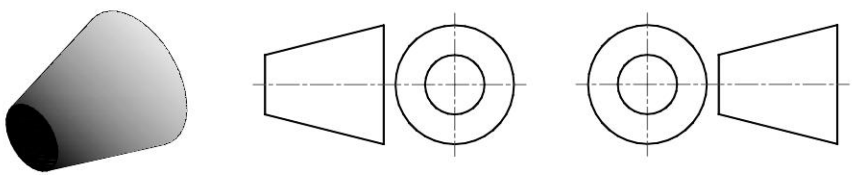


Fig. 7.10 **(a)** *Frustum of a cone* **(b)** *Symbol for first angle projection* **(c)** *Symbol for third angle projection*

7.12 REFERENCE ARROWS METHOD

In addition to the conventional layout of the first and the third angle projections, IS 15021 (part 2): 2001 allows a simplified layout of orthographic views using reference arrows. This method permits to position the views freely.

Each view is identified by a letter in accordance with Table 7.1 except the front view. In the front view, the direction of observation and a lower-case letter indicates the other views. These lower-case letters are identified by the corresponding upper-case letters placed at the upper-left corner of the view. The identified views are placed in any convenient position irrespective of the front view. The upper-case letters identifying the views are positioned to be read from the normal direction of viewing the drawing (*See Figs. 7.11 and 7.12*). No symbol is needed on the drawing to identify this method.

Table 7.1

Direction of observation		*Designation (Identification) of views*
View in direction	*view from*	
a	Front	A
b	Above	B
c	Left	C
d	Right	D
e	Below	E
f	Rear	F

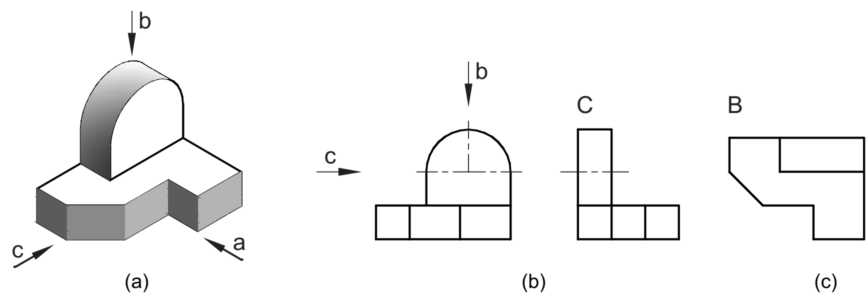


Fig. 7.11 **(a)** *Three-dimensional object* **(b)** *Orthographic views*

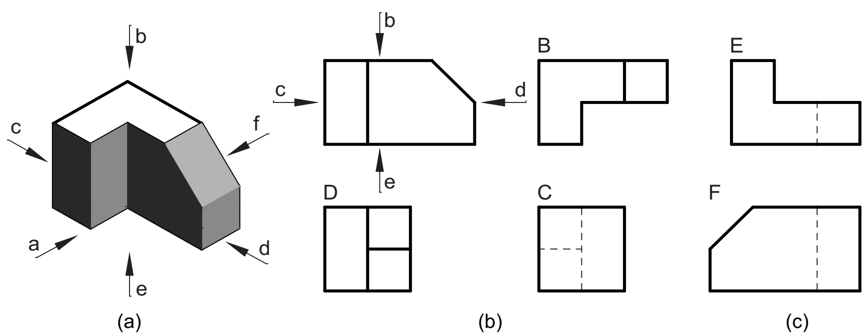


Fig. 7.12 **(a)** *Three-dimensional object* **(b)** *Orthographic views*

7.13 ASSUMPTIONS

1. The direction for the principal view is generally indicated on a pictorial view by an arrow. If it is not given, the view of the object showing the important features, which may be chosen from the point of design, assembly, sales, service or maintenance is considered as the front view. It is the most informative view.
2. Hidden part of a symmetrical object is to be treated similar to the visible part.
3. The holes, grooves, etc., are assumed to be drilled or cut through, unless otherwise specified.
4. The dimensions of radii for small curves of fillet may be suitably assumed, if not specified.

7.14 GENERAL PREPARATION FOR MULTI-VIEW DRAWINGS

1. Observe the shape and dimensions of the given object carefully and determine the overall dimension for each view. Select a suitable scale so as to accommodate all the views on the drawing sheet.
2. Decide the direction of side view and fix up the relative positions of the front, top and side views according to the method of projection used. In first angle projection, top view must be placed below the front view, left-hand side view must lie on the right side of the front view and right-hand side view must lie on the left side of the front view.
3. There should be sufficient space between the views (front, top and side views) to facilitate easy placement of dimensions and also to avoid crowding.
4. Preferably start drawing the views in which the circular parts of the object are seen as circles or part of it. It becomes simpler to project the points of the circle in other views.
5. The front and top views always lie between the same vertical projectors.
6. The front and side views always lie between the same horizontal projectors.
7. The surface parallel to the reference plane will be seen as true shape of the surface.
8. The surface perpendicular to the reference plane will be seen as a straight line.
9. The invisible edges of the object are represented by dotted lines.
10. All lines of symmetry and centre lines should be represented by long dashed dotted narrow lines.
11. In case two or more lines of different types overlap or coincide, the priority may be given according to their importance. For example, if a visible line coincides with a hidden line, then only visible line is to be drawn ignoring the hidden line. Similarly, if a hidden line coincides with a centre line, then only hidden line is to be drawn ignoring the centre line.
12. Appropriate symbol indicating the type of projection should be placed in the title block of the drawing sheet.
13. All the views should be dimensioned as discussed in Chapter 2.
14. All views should be properly labelled.

7.15 CONVERSION OF PICTORIAL VIEW INTO ORTHOGRAPHIC VIEWS

The majority of objects require three views to completely describe their geometrical and dimensional features. The front and the top views have common length dimension; the front and the side views have common height dimension; the top and the side views have common width dimension. The views should be aligned properly so as to share the common dimension. The distance between the views can vary according to the space available on the drawing sheet and number of dimensions to be shown. The following problems describe the basics for drawing three views of simple objects using first angle projection method.

7.15.1 Blocks

Problem 7.1 *Pictorial view of an object is shown in Fig. 7.13(a). Using first angle projection, draw its (a) front view from the X-direction, (b) top view and (c) left-hand side view.*

Visualisation Assume that the object is kept on the H.P. with the face *ABCD* parallel to the V.P. Since *ABCD* is parallel to V.P. its front view is $a'b'c'd'$. The front view of face EFGH overlaps with $a'b'c'd$. All other planes of the object are perpendicular to the V.P. Their front views are straight lines coinciding with $a'b'c'd'$. Thus $a'b'c'd'$ is the front view of the given object. Similarly *abfe* and $e''h''d''a''$ are the top and side views, respectively.

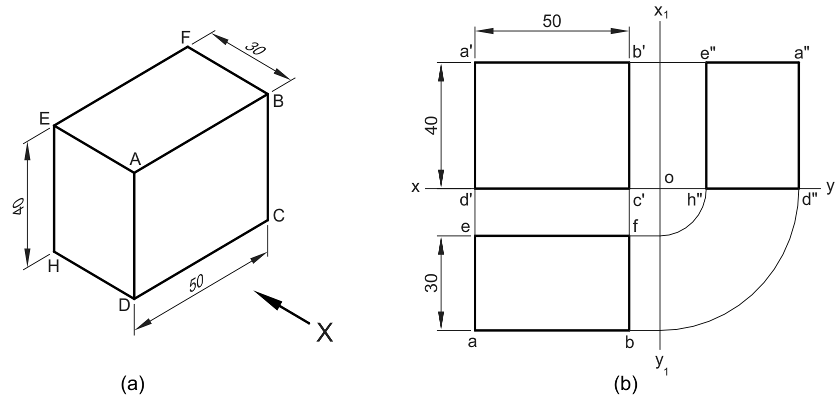


Fig. 7.13 **(a)** *Pictorial view* **(b)** *Orthographic views*

Construction Refer to Fig. 7.13(b).

1. Draw a reference line xy and another line x_1y_1 perpendicular to it.
2. Draw a rectangle $a'b'c'd'$ to represent the front view.
3. Project all the corners from the front view perpendicular to xy and draw $abfe$ as the top view.
4. Project all the corners from the top view up to x_1y_1. Transfer these points on line xy with the help of the compass taking o as the centre. Now project points from xy vertically upwards.
5. Draw horizontal projectors from the corners of the front view to intersect the vertical lines drawn in the previous step and obtain $e''h''d''a''$ as the side view.

Problem 7.2 *Pictorial view of an object is shown in Fig. 7.14(a). Using first angle projection, draw its (a) front view from the X-direction, (b) top view and (c) left-hand side view.*

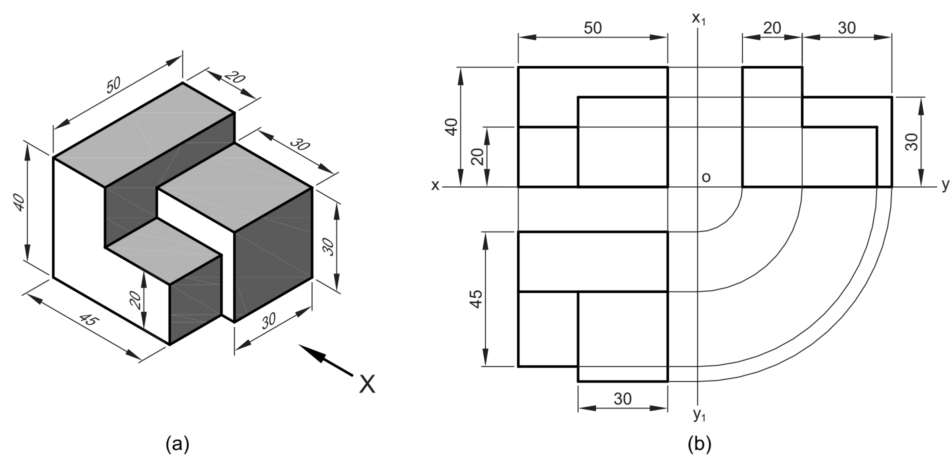


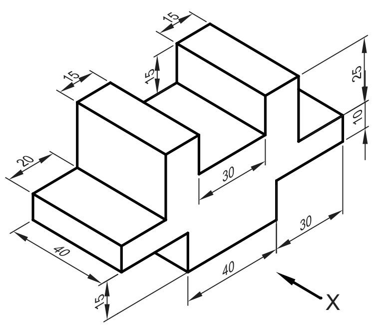
Fig. 7.14 **(a)** *Pictorial view of simple block* **(b)** *Orthographic views*

Construction Refer to Fig. 7.14(b).

Assume that the object is kept on the H.P.

1. The faces in dark grey shade are parallel to the V.P. and will appear as true shape and size in the front view. Draw the front view as shown and project all the points perpendicular to xy.
2. The faces in light grey shade are parallel to the H.P. and will appear as true shape and size in the top view. Draw the top view as shown and project all the points up to x_1y_1. Transfer these points on line oy with the help of the compass taking o as centre. Project these points vertically upwards.
3. The faces without shade are parallel to the P.P. and will appear as true shape and size in the left-hand side view. Draw the side view on the intersection of projectors from the front view and vertical lines drawn in the previous step.

Problem 7.3 *Pictorial view of an object is shown in Fig. 7.15(a). Using first angle projection, draw its (a) front view from the X-direction, (b) top view and (c) left-hand side view.*



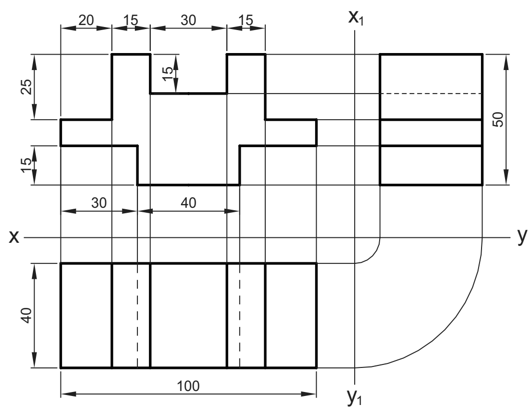


Fig. 7.15 **(a)** *Pictorial view* **(b)** *Orthographic views*

Looking at Fig. 7.15(a), it can be observed that the basic feature of the object lies in the plane parallel to V.P. Therefore, it is convenient to first draw the front view which is the true replica of this feature. Now project all the points from the front view and obtain the top and side views as shown in Fig. 7.15(b).

Problem 7.4 *Pictorial view of an object is shown in Fig. 7.16(a). Using first angle projection, draw its (a) front view from the X-direction, (b) top view and (c) left-hand side view.*

Looking at Fig. 7.16(a) it can be observed that the basic feature of the object lies in the plane parallel to H.P. Therefore, it is convenient to first draw the top view which is the true replica of this feature. Now project all the points from the top view and obtain the front and side views as shown in Fig. 7.16(b).

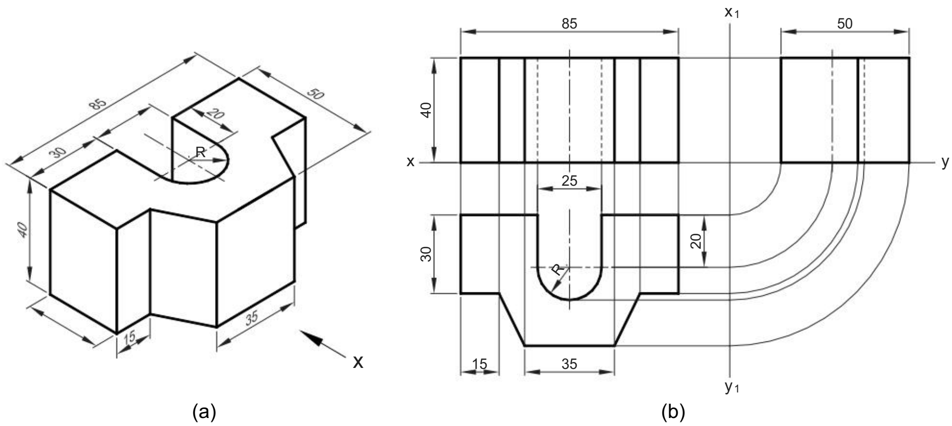


Fig. 7.16 **(a)** *Pictorial view* **(b)** *Orthographic views*

7.15.2 Angle Plates

Problem 7.5 *Pictorial view of an object is shown in Fig. 7.17(a). Using first angle projection, draw its (a) front view, (b) top view and (c) right-hand side view.*

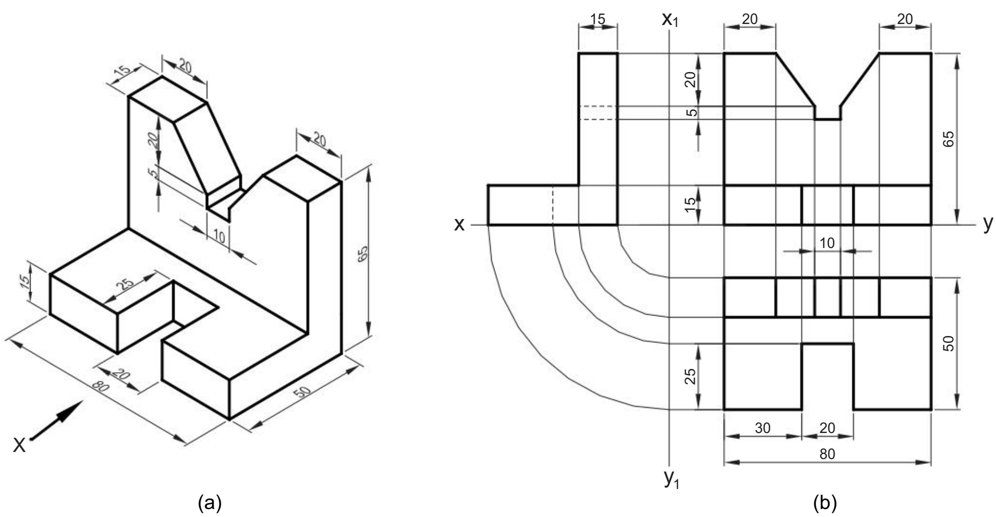


Fig. 7.17 **(a)** *Pictorial view* **(b)** *Orthographic views*

The object given in Fig. 7.17(a) may be analysed as a combination of a vertical plate and a horizontal plate shown in Fig. 7.18(a) and Fig. 7.19(a), respectively. The orthographic projections of the vertical plate can be drawn as Fig. 7.18(b) and those of the horizontal plate as Fig. 7.19(b). The final orthographic views of the object are illustrated in Fig. 7.17(b) which is a combination of Fig. 7.18(b) and Fig. 7.19(b).

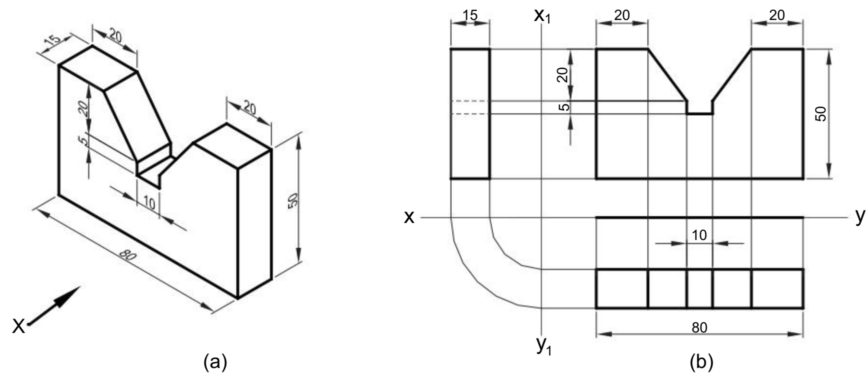


Fig. 7.18 **(a)** *Vertical plate* **(b)** *Orthographic views*

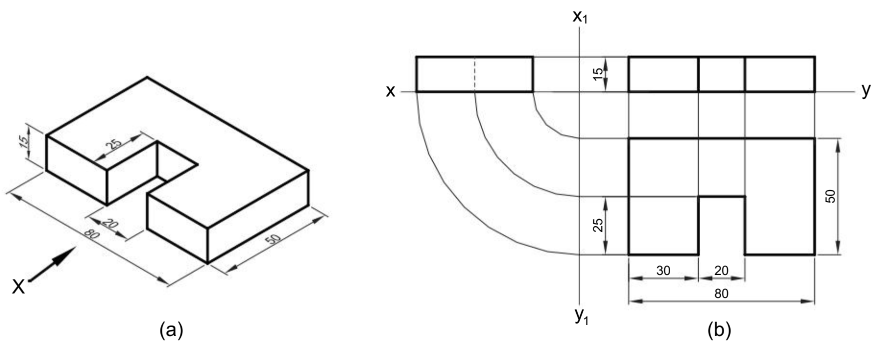


Fig. 7.19 **(a)** *Horizontal plate* **(b)** *Orthographic views*

7.15.3 Combination of Blocks, Plates, Ribs and Webs

Problem 7.6 *Pictorial view of an object is shown in Fig. 7.20(a). Using first angle projection, draw its (a) front view, (b) top view and (c) right-hand side view.*

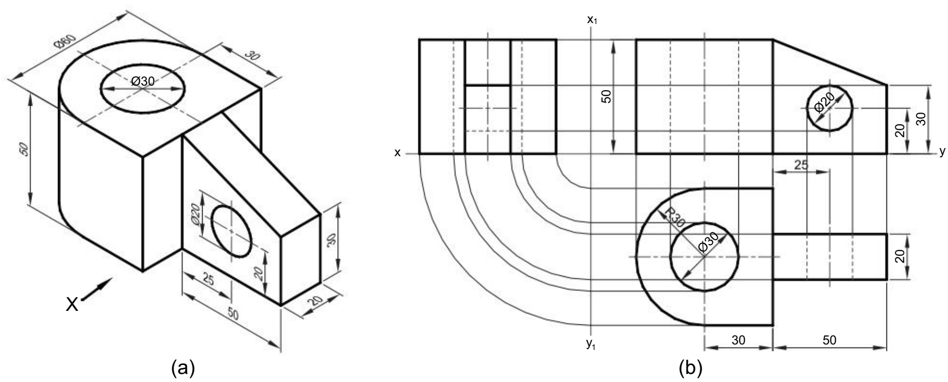


Fig. 7.20 **(a)** *Pictorial view* **(b)** *Orthographic views*

The object given in Fig. 7.20(a) may be analysed as a combination of a block and a rib shown in Fig. 7.21(a) and Fig. 7.22(a) respectively. The orthographic projections of the block can be drawn as Fig. 7.21(b) and those of the rib as Fig. 7.22(b). The final orthographic views of the object are illustrated in Fig. 7.20(b) which is a combination of Fig. 7.21(b) and Fig. 7.22(b).

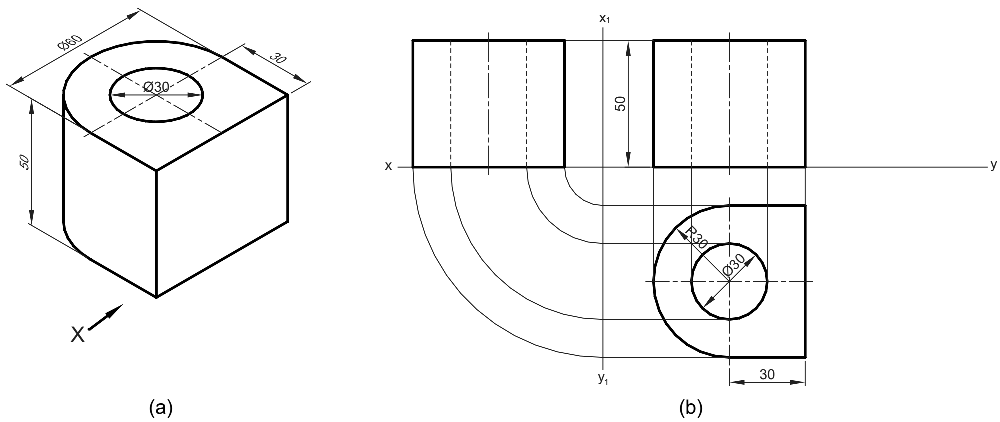


Fig. 7.21 **(a)** *Block* **(b)** *Orthographic views*

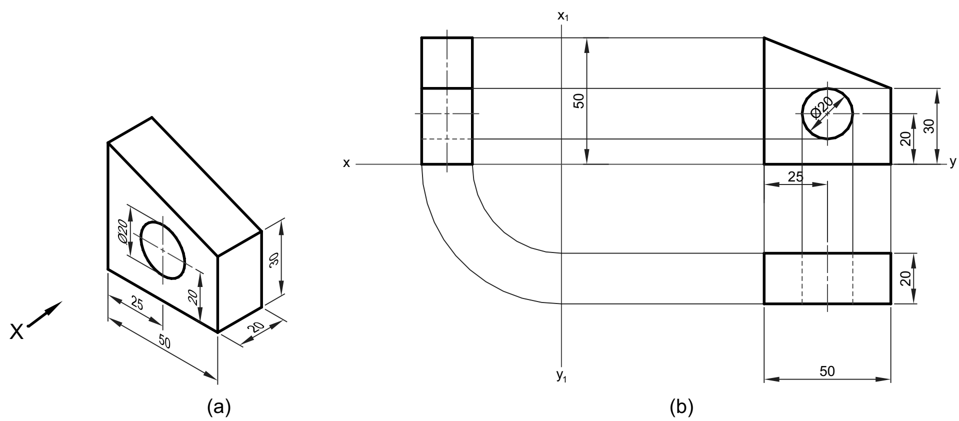


Fig. 7.22 **(a)** *Rib* **(b)** *Orthographic views*

Problem 7.7 *Pictorial view of an object is shown in Fig. 7.23(a). Using first angle projection, draw its (a) front view, (b) top view and (c) right-hand side view.*

The object given in Fig. 7.23(a) may be analysed as a combination of a horizontal plate, a block and a rib. First obtain the front view. Project the corners from the front view on xy to obtain the top view. Proceed to project corners from the front and top views to intersect each other and obtain the right-hand side view as shown in Fig. 7.23(b).

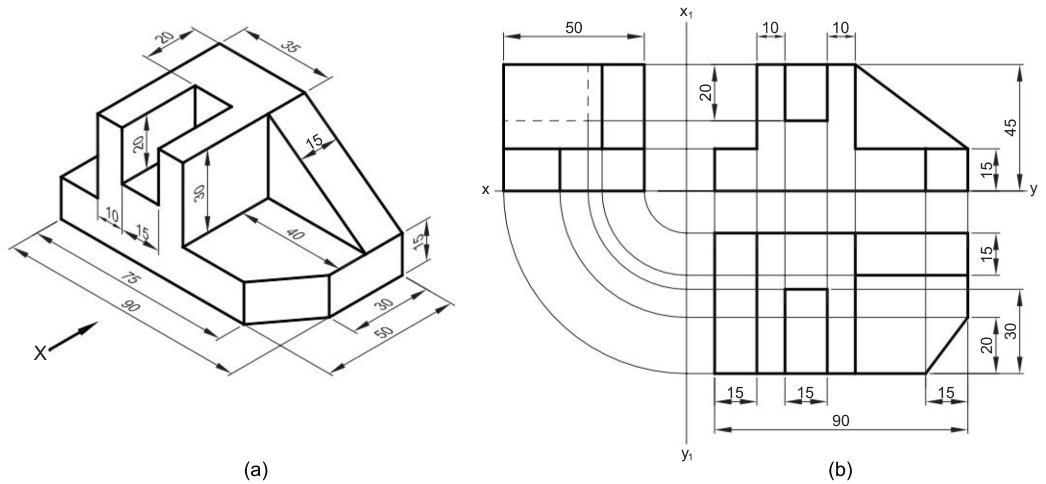


Fig. 7.23 **(a)** *Pictorial view* **(b)** *Orthographic views*

Problem 7.8 *Pictorial view of an object is shown in Fig. 7.24(a). Using first angle projection, draw its (a) front view, (b) top view and (c) side view.*

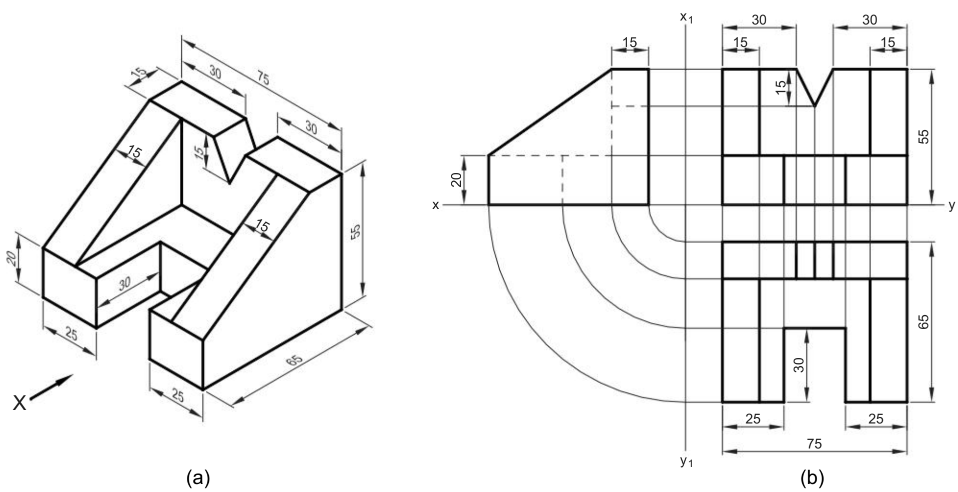


Fig. 7.24 **(a)** *Pictorial view* **(b)** *Orthographic views*

The object given in Fig. 7.24(a) may be analysed as a combination of a horizontal plate, a vertical plate and two ribs. The final orthographic views of the object can be drawn as illustrated in Fig. 7.24(b).

7.15.4 Object Containing Inclined Surfaces

Problem 7.9 *Pictorial view of an object is shown in Fig. 7.25(a). Using first angle projection, draw its (a) front view, (b) top view and (c) side view.*

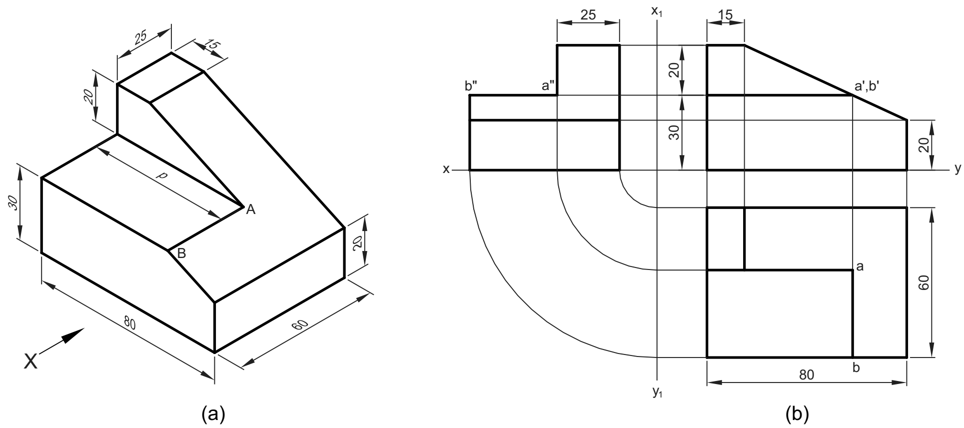


Fig. 7.25 **(a)** *Pictorial view* **(b)** *Orthographic views*

It can be observed that the inclined surface of the given object is perpendicular to the V.P. Therefore, its front view shall be a straight line whereas its top and side views shall appear as rectangles. At the first sight, it one may get an impression that the dimension value p is missing. This is obtained by drawing a horizontal line up to $a'b'$ in the front view. This is further projected to get line ab in the top view.

Problem 7.10 *Pictorial view of an object is shown in Fig. 7.26(a). Using first angle projection, draw its (a) front view, (b) top view and (c) side view.*

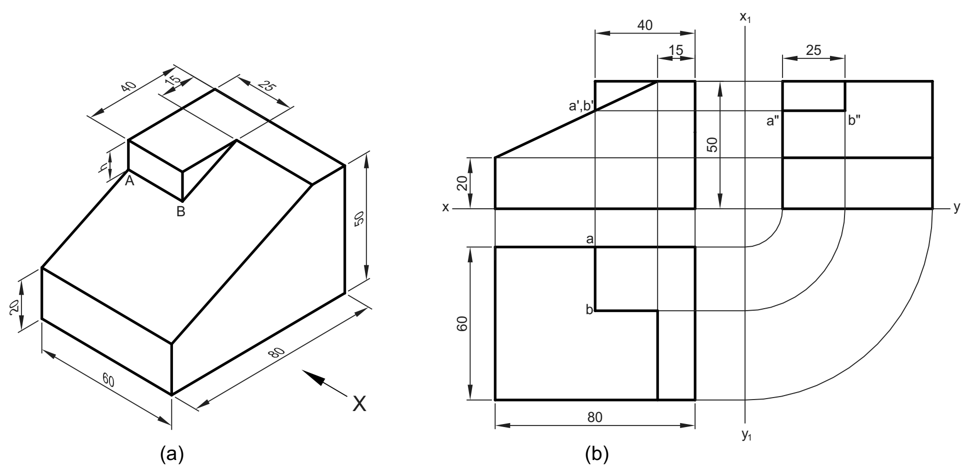


Fig. 7.26 **(a)** *Pictorial view* **(b)** *Orthographic views*

It can be observed that the inclined surface of the given object is perpendicular to the V.P. Therefore its front view shall be a straight line whereas its top and side views shall appear as rectangles. At the first sight, it one may get an impression that the dimension value h is missing. This is obtained by drawing a vertical line up to $a'b'$ in the front view. This is further projected to get line *ab* in the top view and *a*"*b*" in the side view.

Problem 7.11 *Pictorial view of an object is shown in Fig. 7.27(a). Using first angle projection, draw its (a) front view, (b) top view and (c) side view.*

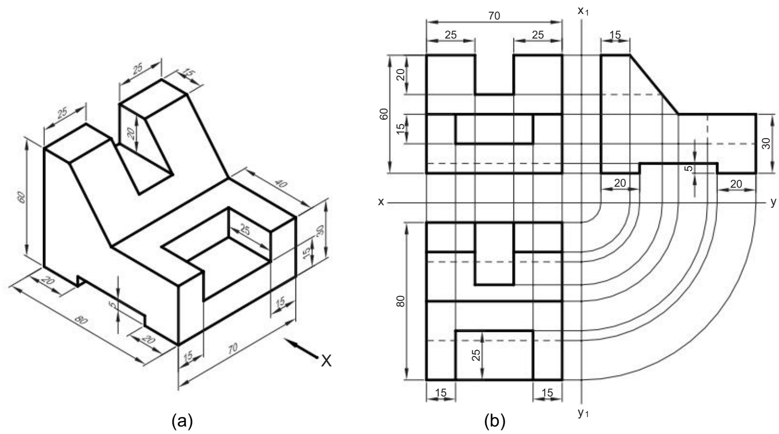


Fig. 7.27 **(a)** *Pictorial view* **(b)** *Orthographic views*

Problem 7.12 *Pictorial view of an object is shown in Fig. 7.28(a). Using first angle projection, draw its (i) front view, (ii) top view and (iii) side view.*

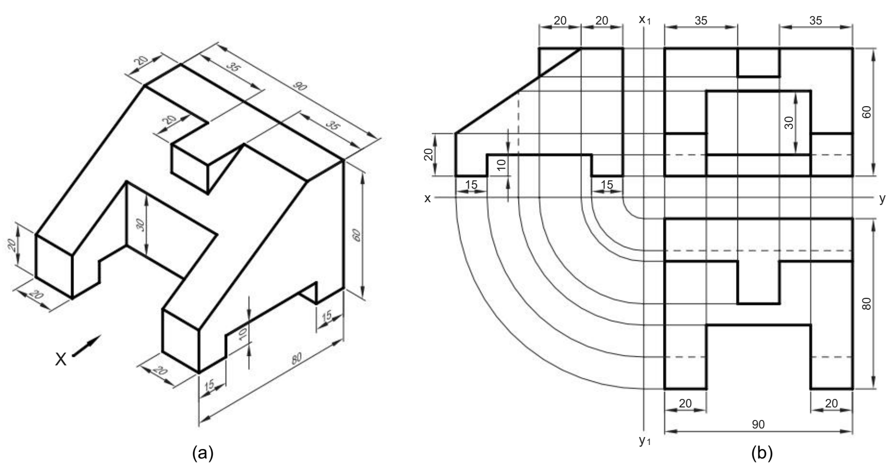


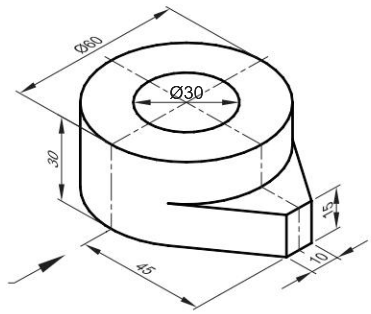
Fig. 7.28 **(a)** *Pictorial view* **(b)** *Orthographic views*

7.15.5 Objects with Fillets and Rounds

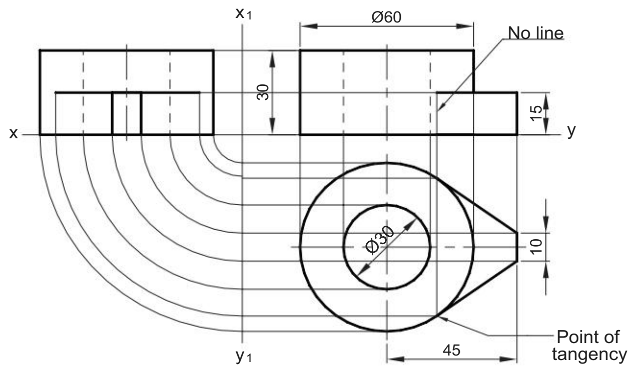
A fillet is a rounded interior corner whereas a round is a rounded exterior corner. They eliminate sharp corners on objects and are normally found on cast, forged, or plastic parts. In engineering drawings, it is necessary to represent fillets and rounded corners for a clearer representation of an object.

Filleted surface which is tangent to cylinder is represented by a small curvature called runout. The runout starts from the point of tangency, using a radius equal to that of the fillet and a curvature length of approximately one-eighth the circumference the fillet circle. Problems 7.13 and 7.14 describe the representation of objects without and with runout respectively.

Problem 7.13 *Pictorial view of an object is shown in Fig. 7.29(a). Using first angle projection, draw its (a) front view, (b) top view and (c) side view.*



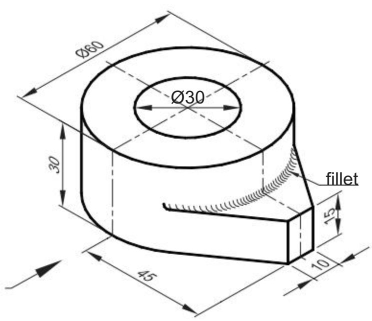
(a)



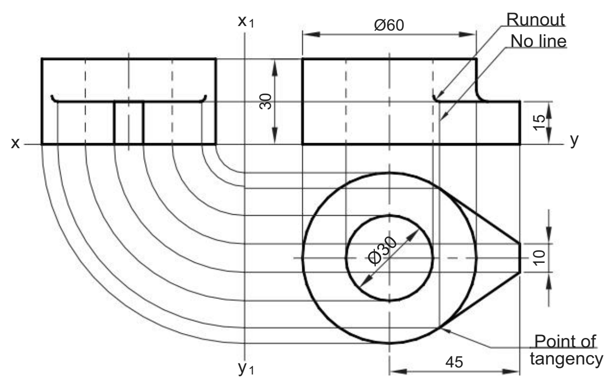
(b)

Fig. 7.29 **(a)** *Pictorial view* **(b)** *Orthographic views*

Problem 7.14 *Pictorial view of an object is shown in Fig. 7.30(a). Using first angle projection, draw its (a) front view, (b) top view and (c) side view.*



(a)



(b)

Fig. 7.30 **(a)** *Pictorial view* **(b)** *Orthographic views*

Problem 7.15 *Pictorial view of an object is shown in Fig. 7.31(a). Using first angle projection, draw its (a) front view, (b) top view and (c) side view.*

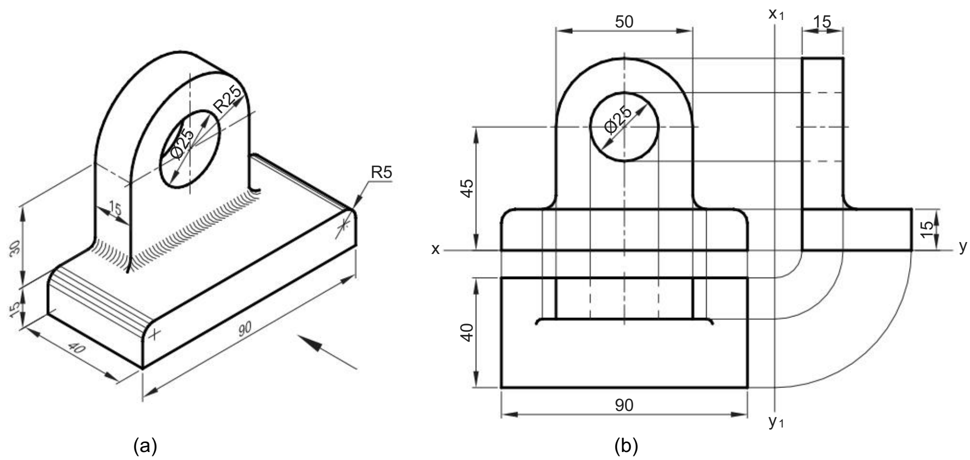


Fig. 7.31 **(a)** *Pictorial view* **(b)** *Orthographic views*

Problem 7.16 *Pictorial view of an object is shown in Fig. 7.32(a). Using first angle projection, draw its (a) front view, (b) top view and (c) side view.*

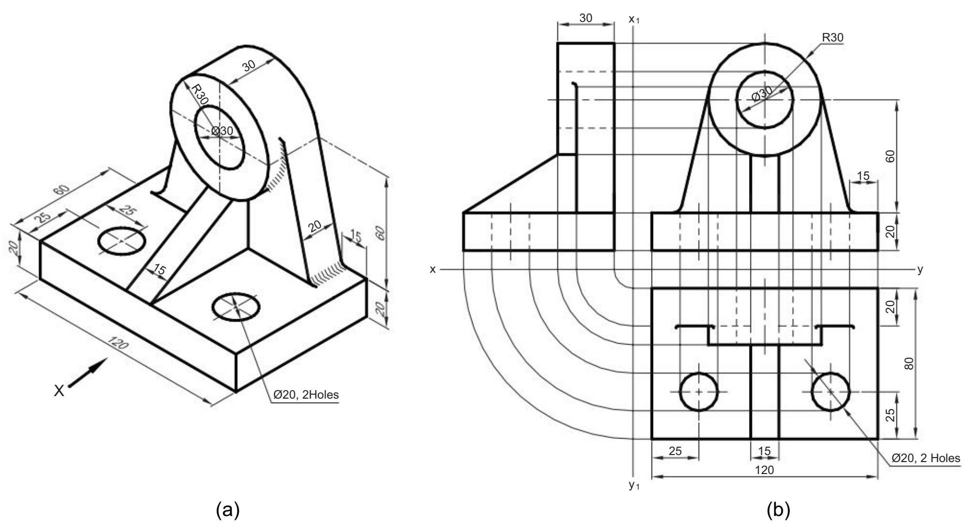


Fig. 7.32 **(a)** *Pictorial view* **(b)** *Orthographic views*

7.16 MISCELLANEOUS PROBLEMS

Problem 7.17 *Pictorial view of an object is shown in Fig. 7.33(a). Using first angle projection, draw its (a) front view, (b) top view and (c) left-hand side view.*

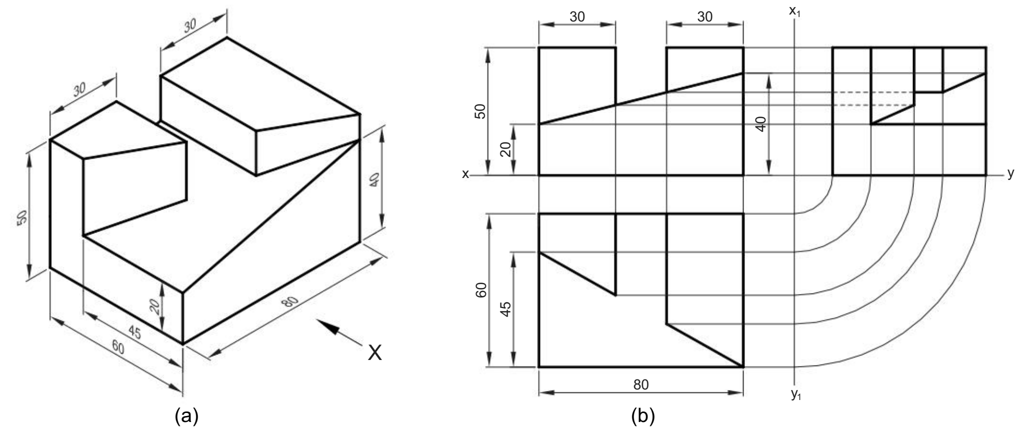


Fig. 7.33 **(a)** *Pictorial view* **(b)** *Orthographic views*

Problem 7.18 *Pictorial view of an object is shown in Fig. 7.34(a). Using first angle projection, draw its (a) front view, (b) top view and (c) side view.*

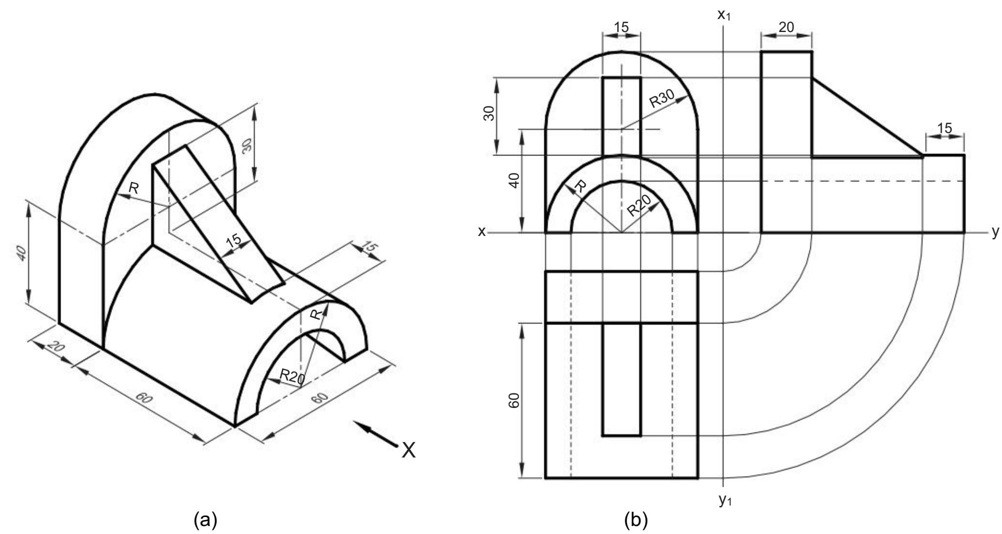
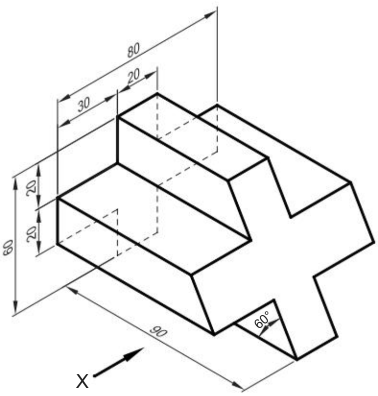


Fig. 7.34 **(a)** *Pictorial view* **(b)** *Orthographic views*

Problem 7.19 *Pictorial view of an object is shown in Fig. 7.35(a). Using first angle projection, draw its (a) front view, (b) top view and (c) side view.*



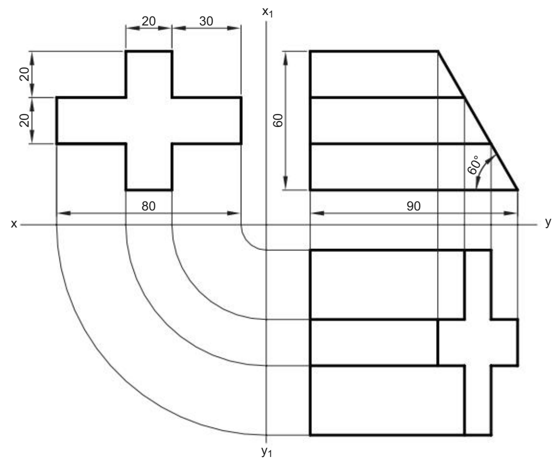


Fig. 7.35 **(a)** *Pictorial view* **(b)** *Orthographic views*

EXERCISE 7A

7.1 Draw three views of the objects shown in Figs. E7.1 to E7.6 using first angle projection.

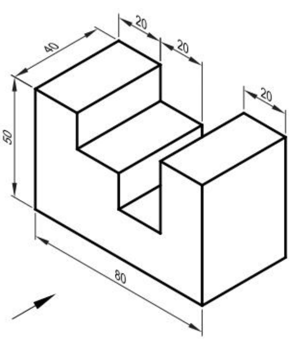


Fig. E7.1

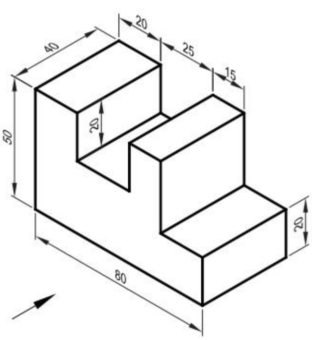


Fig. E7.2

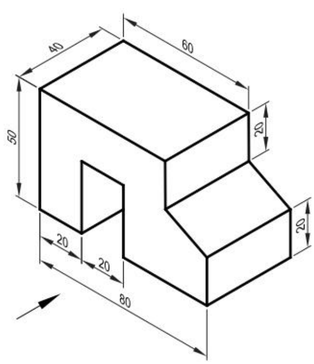


Fig. E7.3

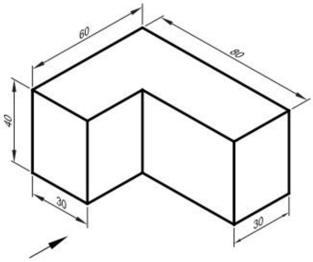


Fig. E7.4

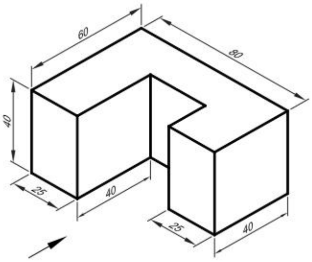


Fig. E7.5

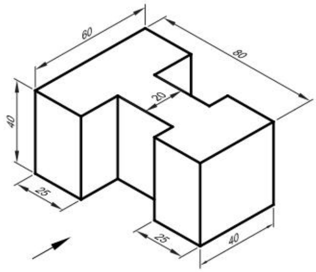


Fig. E7.6

7.2 Draw three views of the objects shown in Figs. E7.7 to E7.12 using first angle projection.

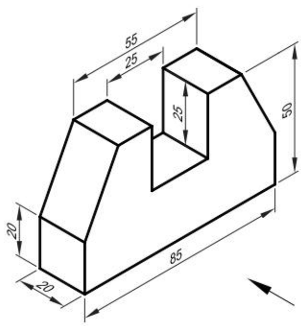


Fig. E7.7

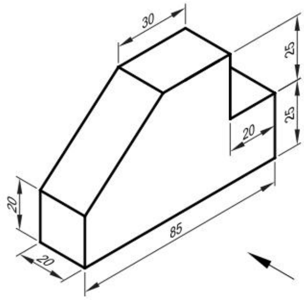


Fig. E7.8

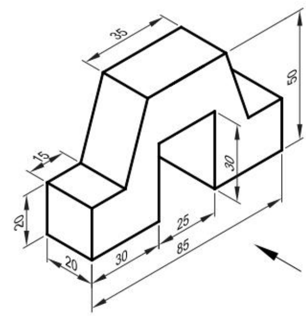


Fig. E7.9

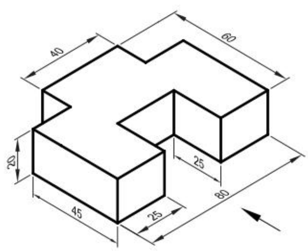


Fig. E7.10

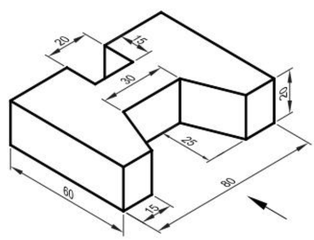


Fig. E7.11

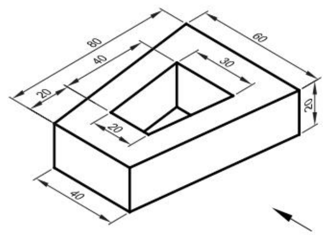


Fig. E7.12

7.3 Draw three views of the objects shown in Figs. E7.13 to E7.18 using first angle projection.

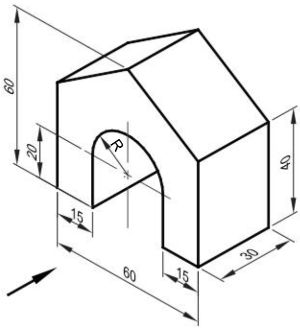


Fig. E7.13

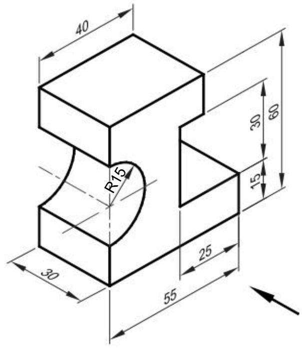


Fig. E7.14

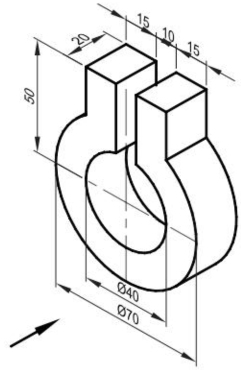


Fig. E7.15

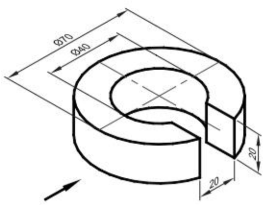


Fig. E7.16

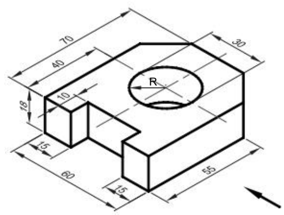


Fig. E7.17

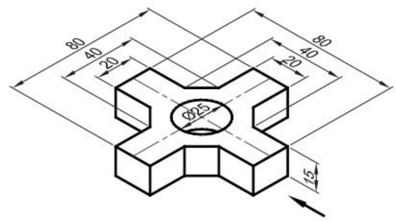


Fig. E7.18

7.4 Draw three views of the objects shown in Figs. E7.19 to E7.24 using first angle projection.

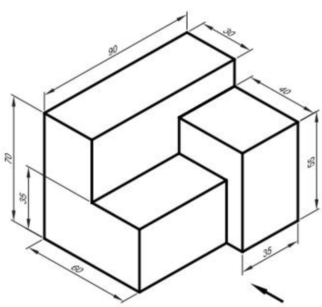


Fig. E7.19

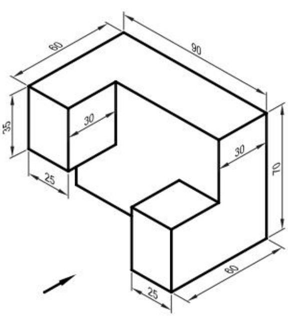


Fig. E7.20

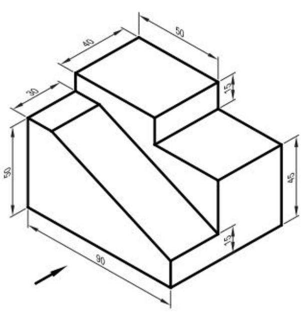


Fig. E7.21

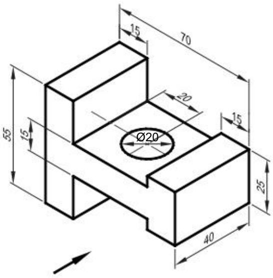


Fig. E7.22

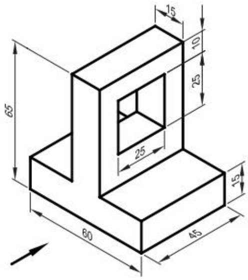


Fig. E7.23

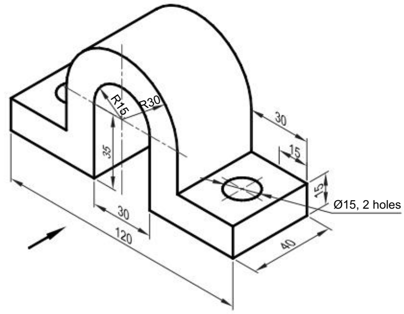


Fig. E7.24

7.5 Draw three views of the objects shown in Figs. E7.25 to E7.30 using first angle projection.

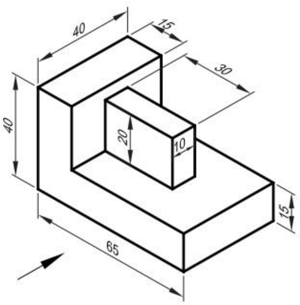


Fig. E7.25

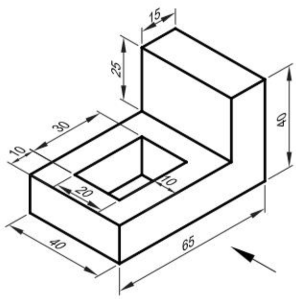


Fig. E7.26

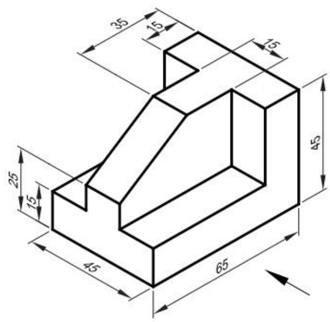


Fig. E7.27

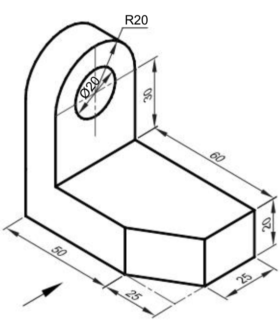


Fig. E7.28

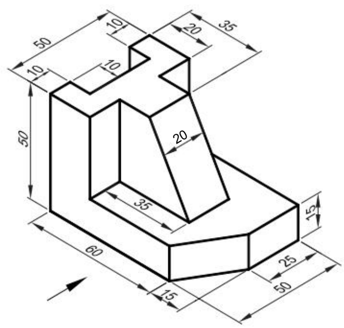


Fig. E7.29

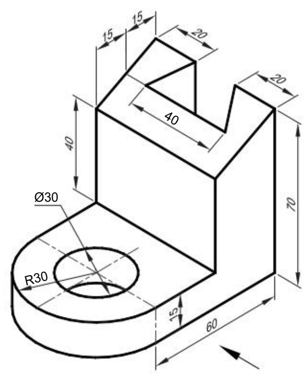


Fig. E7.30

7.6 Draw three views of the objects shown in Figs. E7.31 to E7.36 using first angle projection.

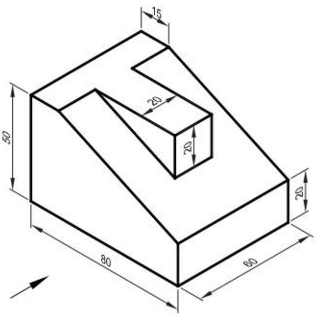


Fig. E7.31

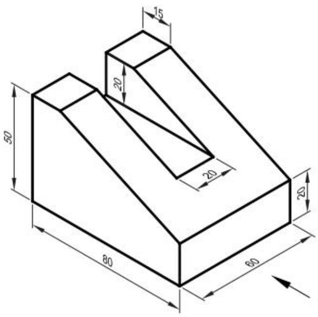


Fig. E7.32

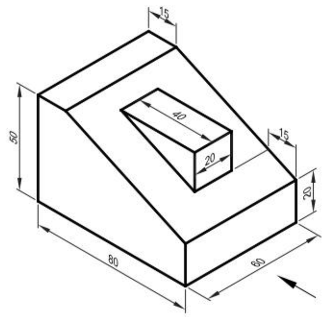


Fig. E7.33

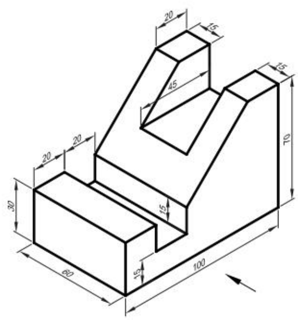


Fig. E7.34

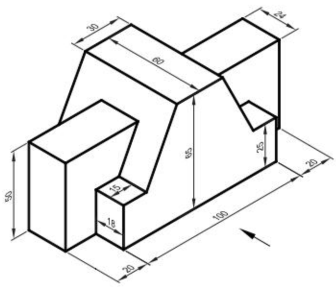


Fig. E7.35

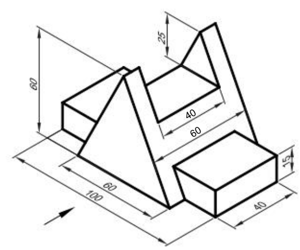


Fig. E7.36

7.7 Draw three views of the objects shown in Figs. E7.37 to E7.42 using first angle projection.

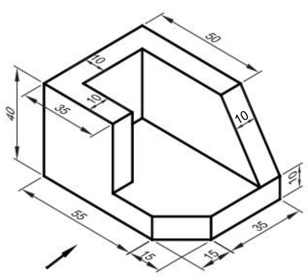


Fig. E7.37

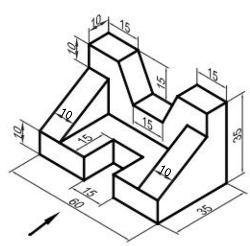


Fig. E7.38

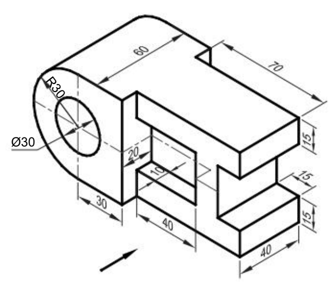


Fig. E7.39

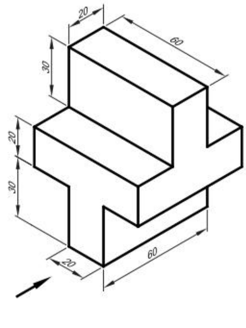


Fig. E7.40

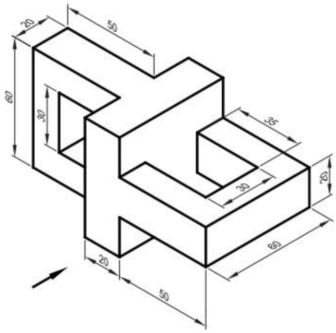


Fig. E7.41

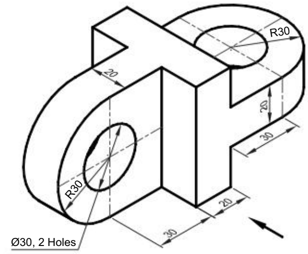


Fig. E7.42

7.8 Draw three views of the objects shown in Figs. E7.43 to E7.51 using first angle projection.

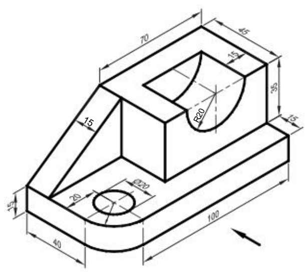


Fig. E7.43

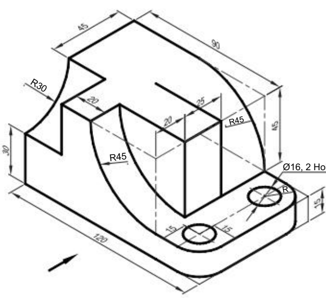


Fig. E7.44

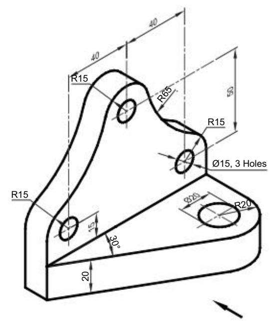


Fig. E7.45

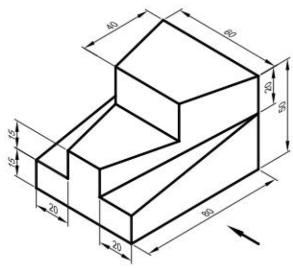


Fig. E7.46

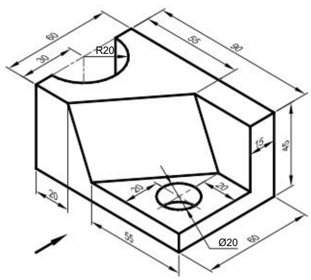


Fig. E7.47

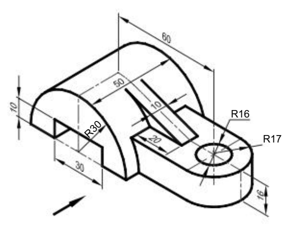


Fig. E7.48

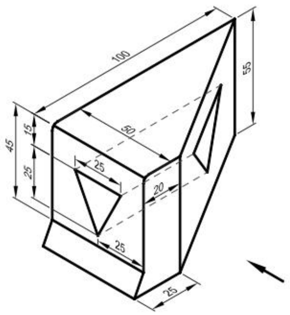


Fig. E7.49

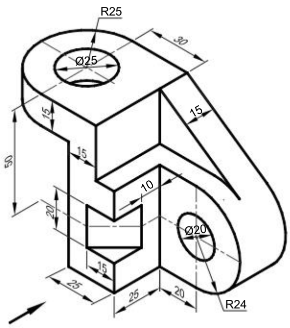


Fig. E7.50

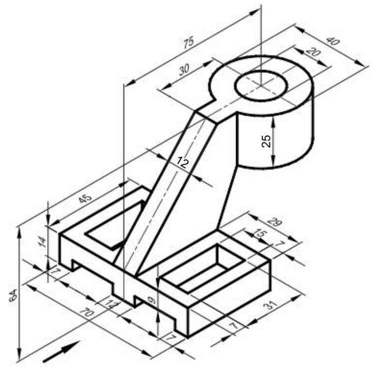


Fig. E7.51

7.17 SECTIONAL VIEWS

The multi-view drawing effectively communicates the information about the shape and the size of an object in which the internal details are represented by hidden lines. However, in engineering practices, there are many objects which are complicated in shapes. As a result, their multi-view projections may contain too many hidden lines. In such cases, it becomes very difficult to interpret or read the drawings.

Section views use a technique in which the object is assumed to be cut by an imaginary plane called a section plane or a cutting plane as shown in Fig. 7.36. The section plane is generally taken parallel to the plane of projection. The portion of the object which falls between the section plane and the observer is considered to be removed and a corresponding new view is drawn. This view is called the *sectional view*. It helps to interpret the internal details more clearly. The part of the object which is cut by the section plane is marked with equidistant parallel lines called *section lines* or *hatching*. The section lines are usually made inclined at 45º to the reference line. No hidden lines are drawn on the sectional views unless it is very essential for dimensioning or for clearly describing the shape. Sectional views are necessary for a clear understanding of complicated parts.

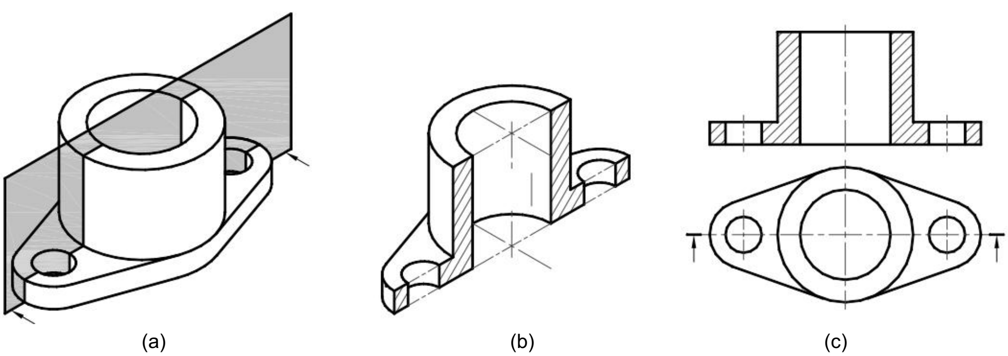


Fig. 7.36 *Object* **(a)** *Pictorial view* **(b)** *Cut by a section plane* **(c)** *Sectional views*

Creating section views requires visualisation skills and adherence to strict standards and conventional practices studied earlier in this chapter.

7.18 REPRESENTATION OF A CUTTING PLANE

The edge view or the trace of the cutting plane shows the exact location through which the cutting plane passes. It is represented by a long dashed dotted thin line capped with thick ends, which will always take precedence over lighter lines such as centre lines and hidden lines. Arrowheads are made near these ends to show the direction of sight of the sectional view. If the cutting plane passes through the centre line of the object such that it divides it into two equal halves, the line representing the cutting plane need not be shown on the drawing.

7.19 SECTION LINES OR HATCHING

Section lines are used to represent the areas common to the object and the section plane. These are thin continuous lines used in the sectional view to show the area where the cutting plane has actually cut the material. *Section lines are evenly spaced at 45º* as shown in Fig. 7.37(a). When the shape of the object is odd and section lines drawn at 45º becomes parallel or perpendicular to the outline of the object, they should be drawn at 45° to the outlines as shown in Fig. 7.37(b). Angles of 30º and 60º are also common, as shown in Figs. 7.37(c) and (d). Section lines at angles more than 75º or less than 15º from horizontal should be avoided.

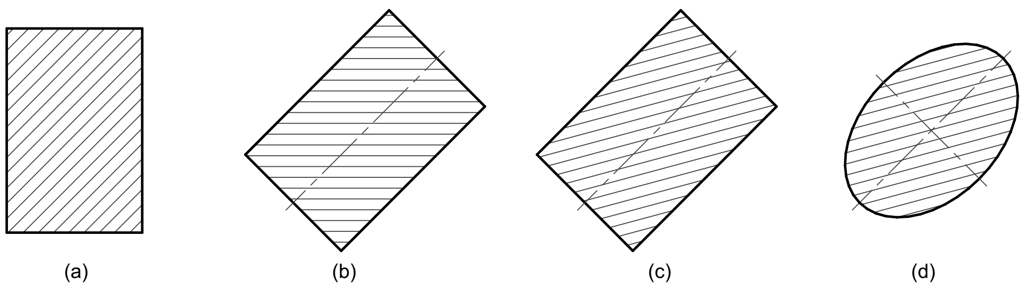


Fig. 7.37 *Section lines inclined at* **(a)** *45°* **(b)** *45° to outlines* **(c)** *and* **(d)** *30° to horizontal*

All section line of each part should be drawn in the same direction. Section lines for adjacent parts should be drawn in different direction or at a different angle, as shown in Fig. 7.38(a). The distance between section lines depends on the size of the part. For average drawing section lines are spaced uniformly 2 mm apart whereas large drawing may have section lines spaced uniformly up to 5 mm, as shown in Fig. 7.38(b).

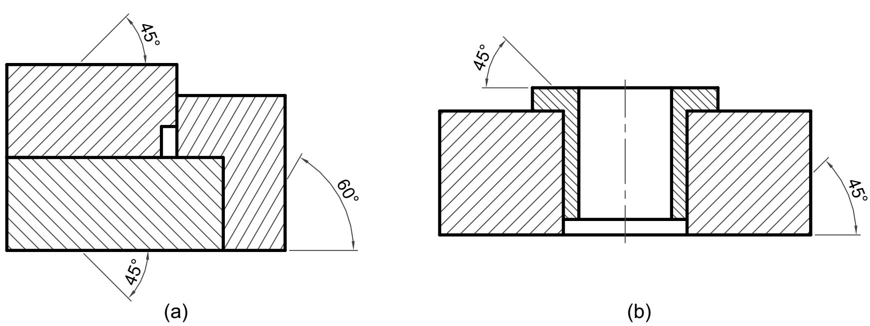


Fig. 7.38 *Section lines for* **(a)** *adjacent parts* **(b)** *spacing made according to their size*

For a very large drawing requiring section lines it is usual practice to draw outline section as shown in Fig. 7.39(a) to represent the cut surface. Many products are made from thin materials such as sheet-metal fabrications, gaskets, seals and packing. For such products, it is virtually impossible to draw sectional lines to represent the cut surface and is therefore preferred to make entirely black as shown in Fig. 7.39(b).

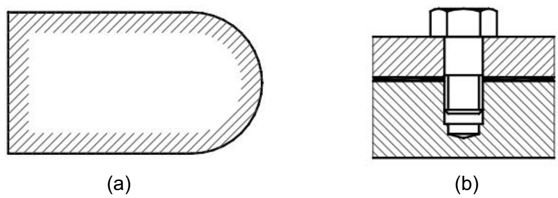


Fig. 7.39 *Section* **(a)** *in very large drawing* **(b)** *very thin material drawing such as gasket*

7.20 FEATURES LEFT UNCUT

Objects like shaft, rod, key, cotter, nut, bolt, rivet, rib, etc., are conventionally not cut as shown in Figs. 7.40 and 7.41, though the section plane may pass through them. This is merely to maintain clarity in the sectional view, if the cutting plane passes longitudinally (parallel) through them. However, if the cutting plane passes through them perpendicular to their axes, then section lines are shown. Other examples of objects left uncut in sectional views include screws, washers, bearings, webs, spokes, gear teeth, pins and springs. Some of these are shown in Figs. 7.42–7.44.

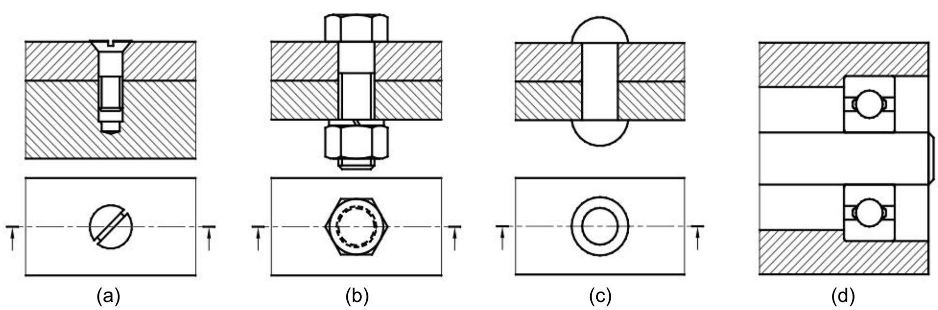


Fig. 7.40 *Parts left unsectioned* **(a)** *Screws* **(b)** *Nuts, bolts and washers* **(c)** *Rivets* **(d)** *Shaft and bearings*

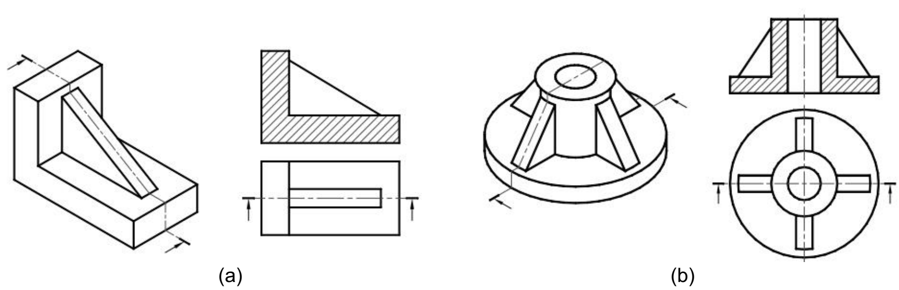
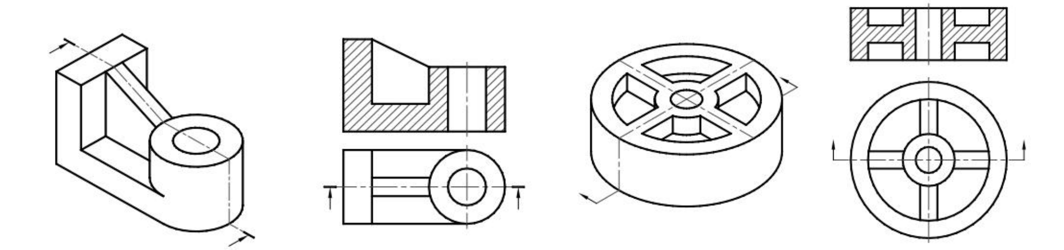


Fig. 7.41 **(a)** *and* **(b)** *Ribs left unsectioned*



(a) (b)

Fig. 7.42 **(a)** *and* **(b)** *Webs left unsectioned*

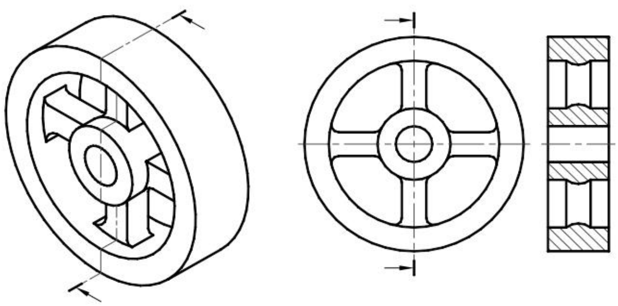


Fig. 7.43 *Spokes left unsectioned*

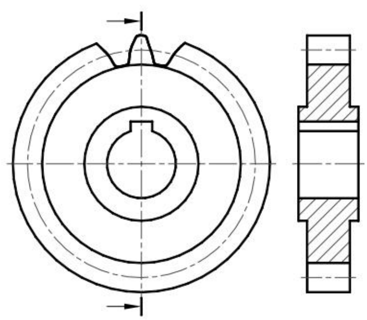
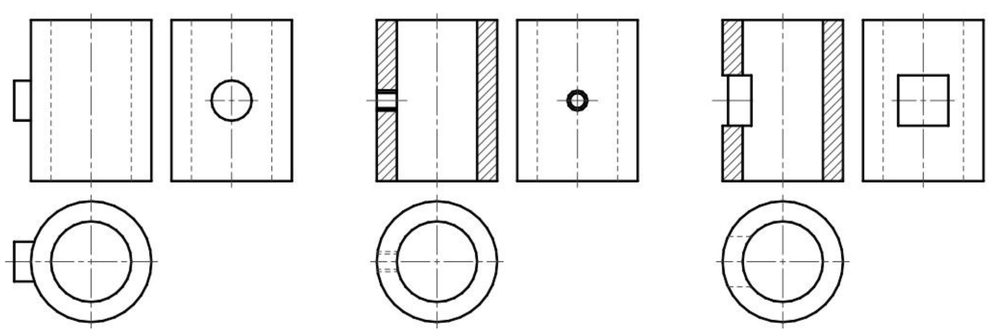


Fig. 7.44 *Gear teeth left unsectioned*

7.21 SIMPLIFIED REPRESENTATION OF INTERSECTIONS

When a section is drawn through a small intersection in which the exact figure or curve of intersection is small or of no consequence, the curve of intersection may be simplified as shown in Figs. 7.45(a) and (b). However, when the intersecting features are larger, they are drawn as their true representation as shown in Fig. 7.45(c).



(a) (b) (c)

Fig. 7.45 *Simplified representation of intersections* **(a)** *Small extrusion* **(b)** *Small hole* **(c)** *Large hole*

7.22 SECTION LINE CONVENTIONS

The commonly accepted symbols to indicate the type of material in the sectioned part as recommended by Bureau of Indian Standards in its bulletin IS 11663:1986 are shown in Fig. 7.46. These symbols indicate only the general type of materials. The same hatching pattern has been used for all the materials such as steel, cast iron and their various compositions and alloys. Therefore, it is recommended that different materials should be indicated by notes on the drawing or in the item references list on the drawing.

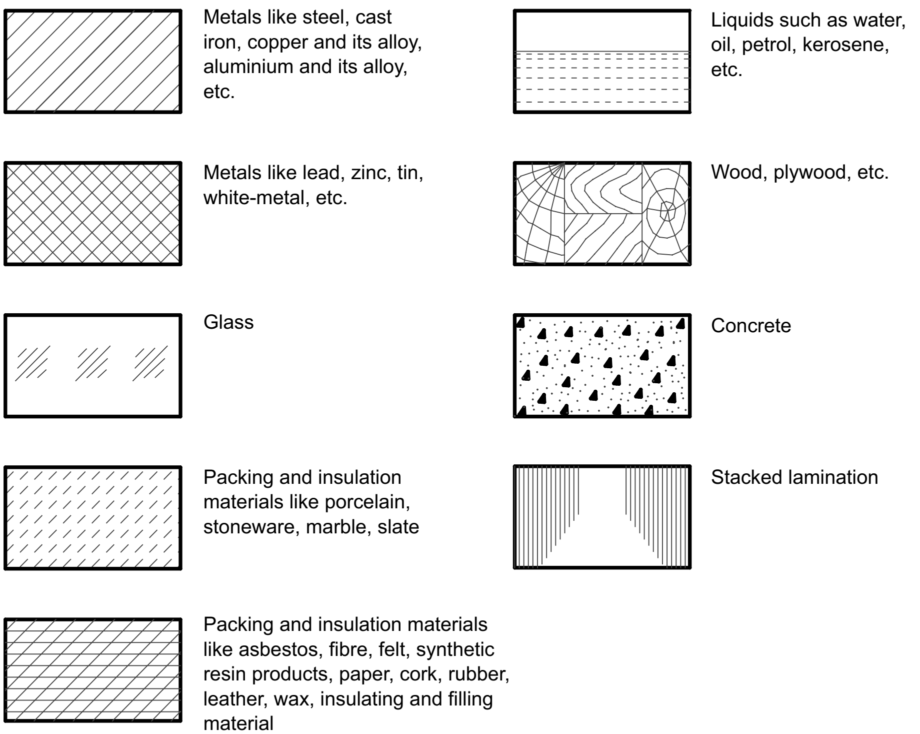


Fig. 7.46 *Conventional representation of common materials*

7.23 TYPES OF SECTIONAL VIEWS

There are several types of section views, namely; full section, half section, offset section, aligned section, broken section, revolved section and removed section. Phantom section is also used. The type of section view used for a particular purpose depends upon the one which most clearly and concisely represents the features of interest. The details of various types of section views are described as follows:

7.23.1 Full Section

The full section is drawn when the cutting plane passes entirely through the object, generally through its middle. The half of the object closer to the observer is removed. Figures 7.47(a) and (b) show the pictorial

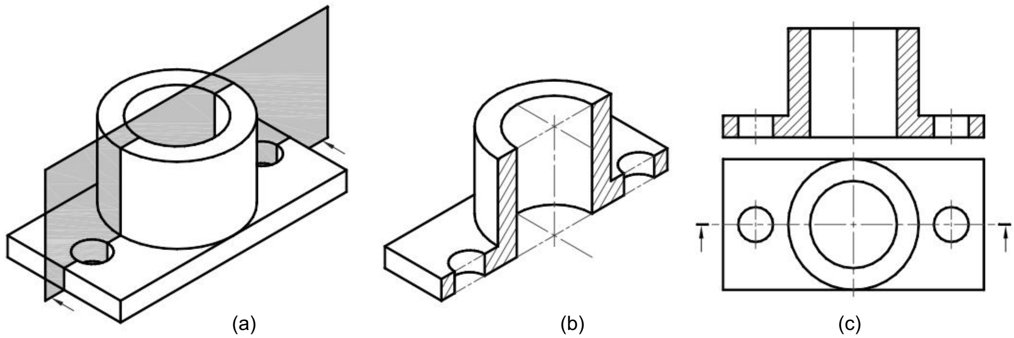


Fig. 7.47 *Object* **(a)** *Pictorial view* **(b)** *Cut by full-sectional plane* **(c)** *Full-sectional front and top views*

view of an object in which the cutting plane is made to pass through the centreline, dividing the object in two halves.

Figure 7.47(c) shows the full sectional front view and the top view. Only the areas of the object actually cut by the section plane have section lines. The top view is unaffected by the section. It is drawn for the complete object, as so because no part of the object is actually cut and removed. Top view shows the cutting plane line to indicate the sectional plane. The arrowheads attached to the cutting plane line indicate the viewing direction. The cutting plane line may be omitted in case it passes though the centre line of the object, dividing it into two equal halves.

7.23.2 Half Section

It is realised that the internal details of an object are clearly seen in the sectional views at the cost of the external features. Both the external as well as the internal features can simultaneously be retained in half-sectional views. *A half section is generally used in symmetrical objects where the cutting plane cuts away only one quarter of the object.* Figures 7.48(a) and (b) show the pictorial view of an object in which the cutting plane removes one-quarter of the object.

The name half comes from the idea that only half of the sectional view is sectioned. Figure 7.48(c) shows the front view left half in section and the top view. The front view has one-half view in section and the other half view without section. It is the conventional practice, not to draw hidden lines for the half without section. The top view shows the complete object, since no part is actually removed. Direction of view is

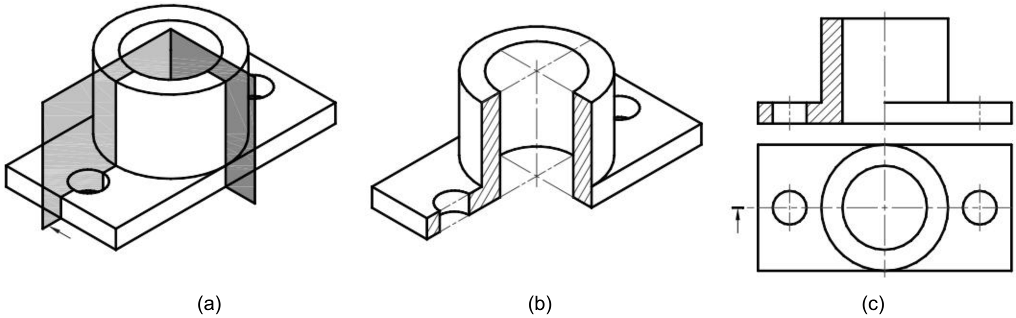


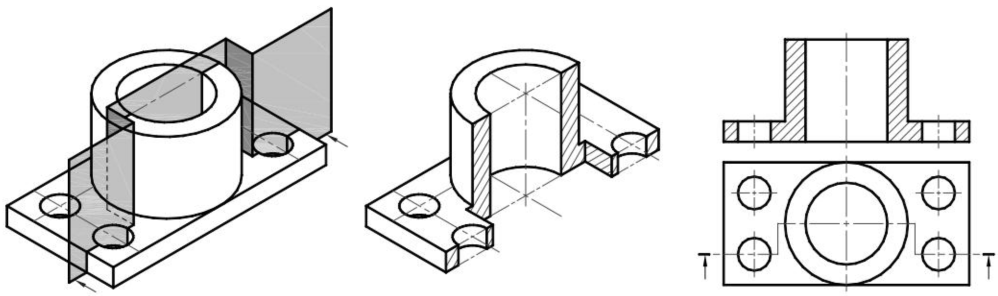
Fig. 7.48 *Object* **(a)** *Pictorial view* **(b)** *Cut by half-sectional plane* **(c)** *Half-sectional front and top views*

shown by one arrow only. The half-sectional view finds its greatest usefulness in the assembly drawing where it is often necessary to show both internal and external features in a single view.

7.23.3 Offset Section

Objects which do not have their internal features along the line of symmetry, it is often convenient to use a zigzag type of section plane to get what is known as offset section. In this type of section, the cutting plane changes direction backward and forward (zigzag) to include the features that are not in straight line and are important to show. The offset in the section plane are all at 90º. Figures 7.49(a) and (b) show the pictorial view of an object cut by the offset section plane. The cutting plane is positioned so that the holes on both sides are sectioned.

Figure 7.49(c) shows the sectional front view and the top view. The bend of the section plane is not visible in the front view. The front view appears as full section would appear. The top view shows the offset cutting plane line with two 90º bends for each offset. The offset section is made to make certain unique features visible.

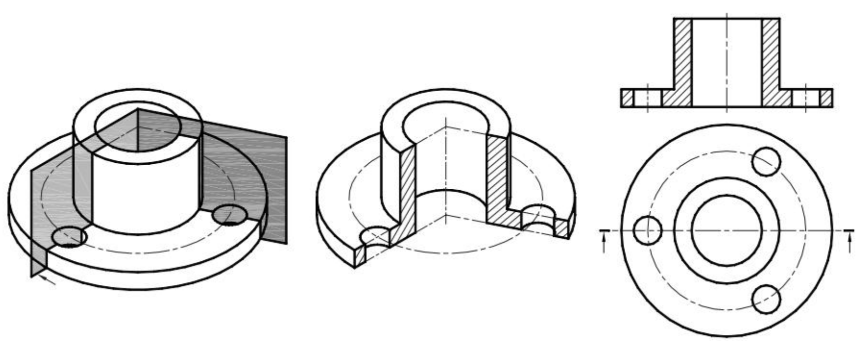


(a) (b) (c)

Fig. 7.49 *Object* **(a)** *Pictorial view* **(b)** *Cut by offset sectional plane* **(c)** *Sectional front and top views*

7.23.4 Aligned Section

A normal multi-view drawing is difficult to visualise and read especially when it contains number of ribs, arms and/or holes that are not aligned with horizontal or vertical centre line. Figures 7.50(a) and (b) show



(a) (b) (c)

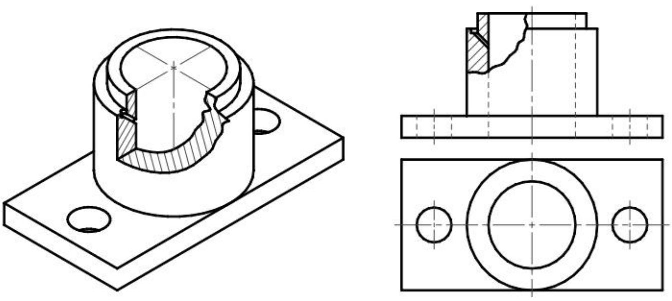
Fig. 7.50 *Object* **(a)** *Pictorial view* **(b)** *Cut by aligned sectional plane* **(c)** *Sectional front and top views* the pictorial view of an object containing holes that are equally spaced but not aligned with the centre line of the object. *In aligned section, the cutting plane changes its direction to include angular array elements in the sectional view. Normally, the change in direction is less than 90°.*

Figure 7.50(c) shows the aligned sectional front view and the top view. The front view is drawn as if the hole is rotated and aligned with the horizontal section line. The aligned cutting plane line is not drawn at an angle to include the holes but is drawn straight similar to full section cutting plane line. Thus, the aligned section does not show the section in true projection, yet are used to make the clearest possible representation of the unique features.

7.23.5 Broken-out Section

The broken-out section is also referred as local section or partial section. *The broken-out section is needed when a small part is required to be shown in section locally by removing the outside surface.* Figure 7.51(a) shows the pictorial view an object with oil hole. Figure 7.51(b) shows the broken-out sectional front view and the top view. The internal detail of the oil hole is better illustrated through the broken-out section in the front view and do not require a full or half sectional views. The broken out section has an irregular freehand line to define the boundary of the section, separating the sectioned area from the remaining view. It is not required to add the cutting plane line in this type of view.

Figure 7.52 shows a broken-out section where a keyway of a shaft is drawn in section. The broken-out sectional view finds its greatest usefulness to show internal features where sections are not drawn.



(a) (b)

Fig. 7.51 *Object* **(a)** *Broken-out pictorial view* **(b)** *Sectional front and top views*

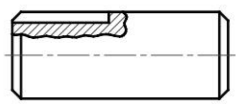


Fig. 7.52 *Sectional view*

7.23.6 Revolved Section

The revolved section is sometime referred as rotated section. *It is obtained by cutting an object by a section plane which is at right angles to the axis of the object. It is then revolved through right angles so as to merge with the plane of projection.* Thus, it eliminates the need to draw extra views to define the cross section of ribs, webs, bars, arms, spokes or other similar features of the object. It is in fact a drawing within a drawing, and it clearly describes the object's shape at a certain cross section.

Figures 7.53(a)(b), and (c) shows three separate examples the revolved section where single view is sufficient to define the shape and size of the object. The revolved section is not popular as they lead to confusion in interpreting the drawing. If possible, use of the removed section should always be preferred instead of revolved section.

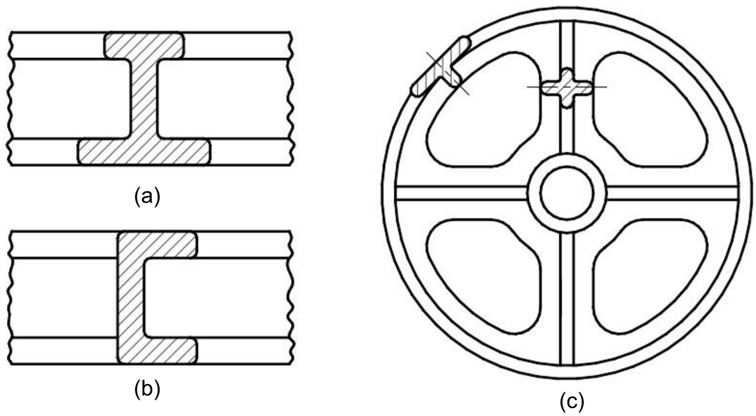


Fig. 7.53 *Revolved section* **(a)** *Shaft of I- section* **(b)** *Shaft with U section* **(c)** *Flywheel*

7.23.7 Removed Section

The removed section is somewhat similar to the revolved section because the section plane cuts the object exactly in the same way. However, the section in this case is removed elsewhere instead of superimposing on the plane of projection. The removed section is used to define the changing contour of a shaft with a variable features along the length, profile of a turbine blade, etc. They are also used to define the cross section of ribs, webs, bars, arms, spokes or other similar features of the object.

Figures 7.54 (a) and (b) show a shaft where multiple removed sections were used to indicate the profile change. Figure 7.54(c) shows a flywheel having different cross section of the rim and the spoke. It may be noted that a removed section must identify the cutting plane line for each section. The sectional view does not indicate features other than the actual section.

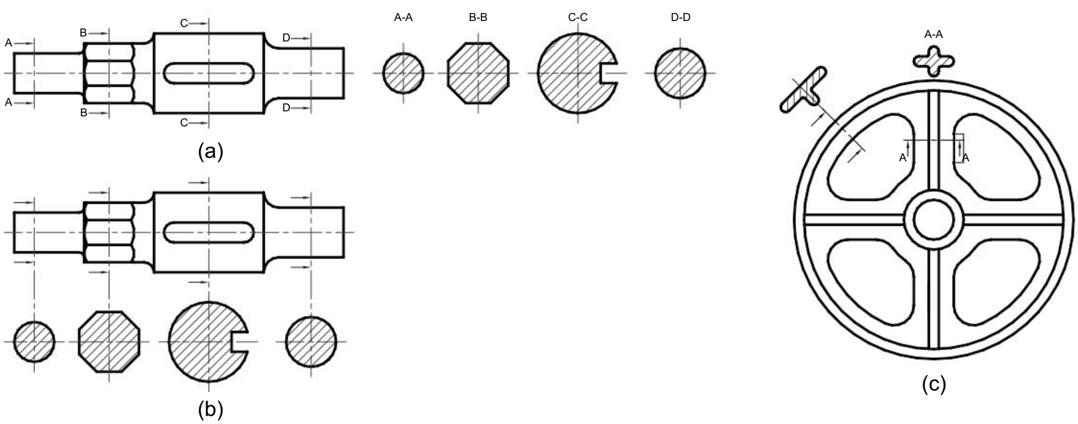


Fig. 7.54 *Removed section for a* **(a)** *and* **(b)** *shaft with a variable features* **(c)** *flywheel*

7.23.8 Phantom Section

A phantom section is a regular exterior view on which the interior construction is superimposed by cross-hatching an imaginary cut surface with dashed section lines. This type of section is used only when a regular or broken section may remove some important detail, or in some instances to show an accompanying part in relative position. Figure 7.55 shows an example of phantom section. The hidden lines are shown by long-dashed dotted narrow line whereas hatching is done with dashed narrow lines. This method of section may lead confusion and should be avoided.

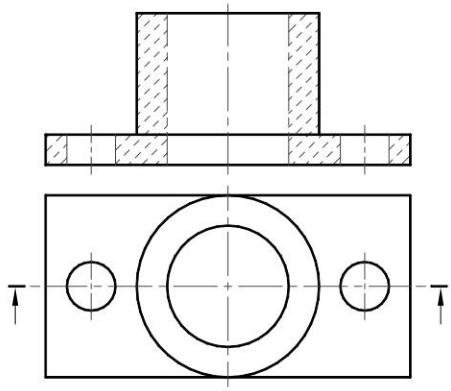


Fig. 7.55 *Phantom section*

7.24 CONVENTIONAL BREAKS

Conventional breaks permits to shorten the lengths of long objects without a reducing scale. Figure 7.56 shows typical conventional breaks. These breaks are usually drawn freehand. The section lines are drawn in the same direction in both the broken parts.

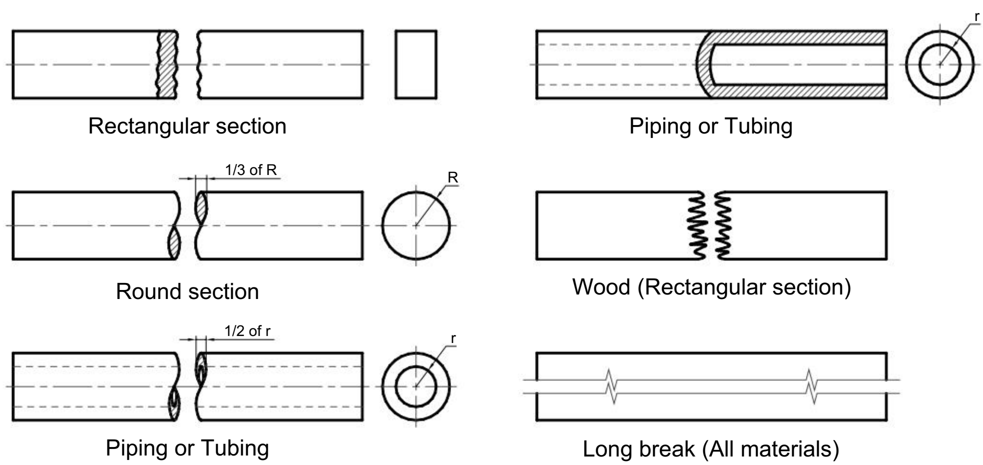


Fig. 7.56 *Conventional breaks*

7.25 PROBLEMS ON SECTIONAL VIEWS

Problem 7.20 *Pictorial view of an object is shown in Fig. 7.57(a). Using first angle projection, draw its (a) sectional front view (b) top view and (c) side view.*

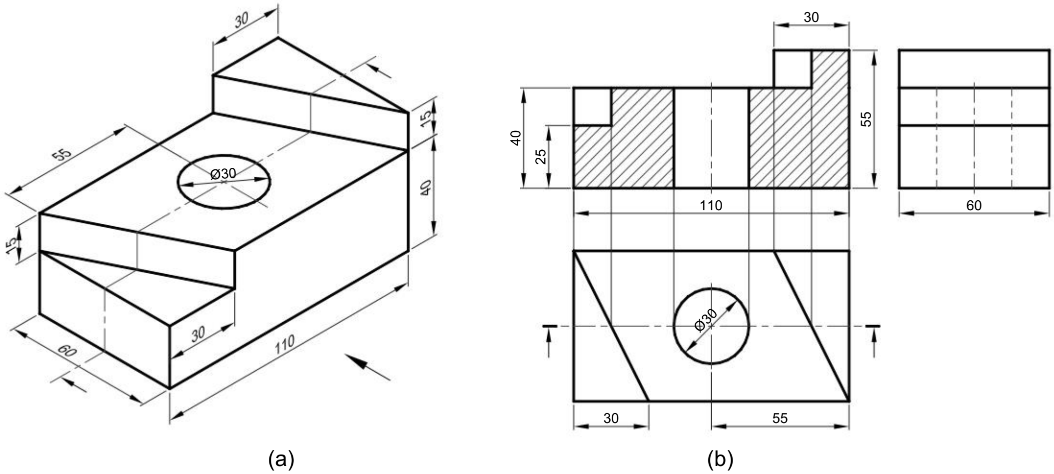


Fig. 7.57 **(a)** *Pictorial view* **(b)** *Orthographic views*

Problem 7.21 *Pictorial view of an object is shown in Fig. 7.58(a). Using first angle projection, draw its (a) sectional front view (b) sectional top view and (c) side view.*

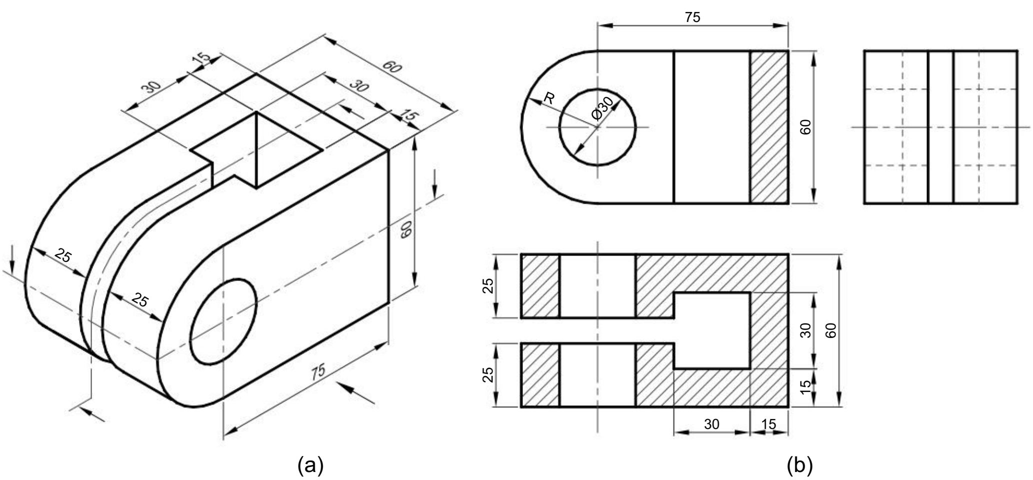


Fig. 7.58 **(a)** *Pictorial view* **(b)** *Orthographic views*

Problem 7.22 *Pictorial view of an object is shown in Fig. 7.59(a). Using first angle projection, draw its (a) front view and (b) sectional side view.*

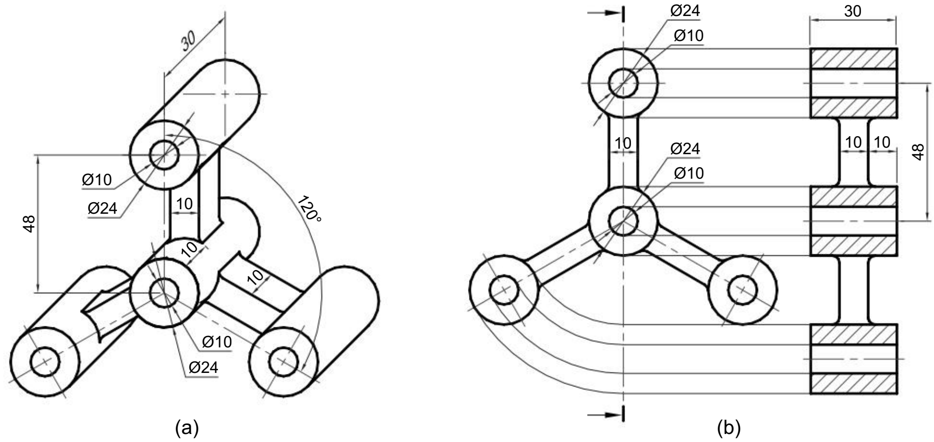


Fig. 7.59 **(a)** *Pictorial view* **(b)** *Orthographic views*

Problem 7.23 *Orthographic view of an object is shown in Fig. 7.60(a). Reproduce the front view and draw its sectional side view.*

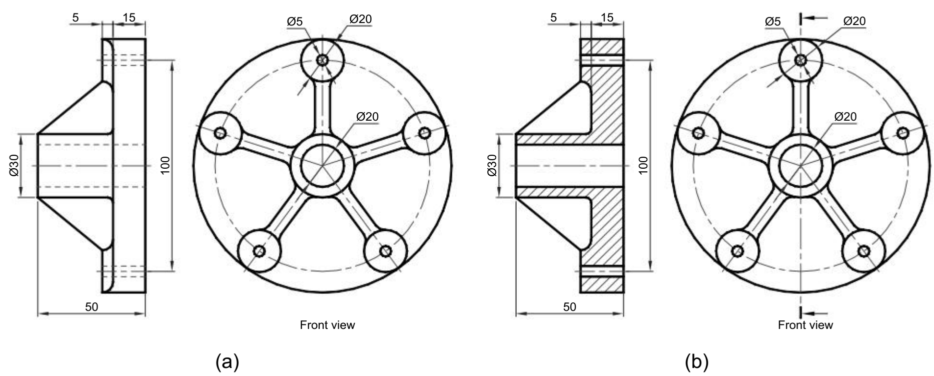


Fig. 7.60 **(a)** *Orthographic views* **(b)** *Front view and sectional side view*

EXERCISES 7B

7.1 Pictorial view of an object is shown in Fig. E7.52. Using first angle projection, draw its (a) sectional *A-A* front view, (b) sectional *B-B* top view and (c) side view.

7.2 Pictorial view of an object is shown in Fig. E7.53. Using first angle projection, draw its (a) sectional *A-A* front view, (b) sectional *B-B* top view and (c) side view.

7.3 Pictorial view of an object is shown in Fig. E7.54. Using first angle projection, draw its (a) sectional *A-A* front view, (b) top view and (c) side view.

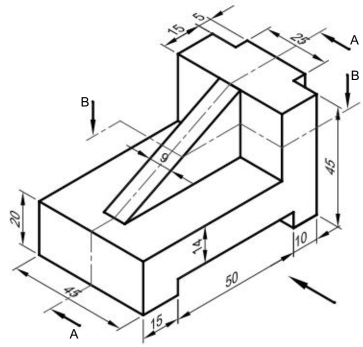


Fig. E7.52

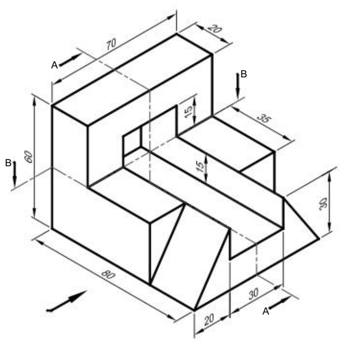


Fig. E7.53

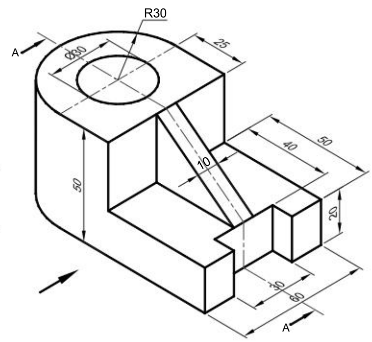


Fig. E7.54

7.4 Pictorial view of an object is shown in Fig. E7.55. Using first angle projection, draw its (a) full-sectional front view, (b) top view and (c) side view.

7.5 Pictorial view of an object is shown in Fig. E7.56. Using first angle projection, draw its (a) full-sectional front view, (b) top view and (c) side view.

7.6 Pictorial view of an object is shown in Fig. E7.57. Using first angle projection, draw its (a) full-sectional front view, (b) full sectional top view and (c) side view.

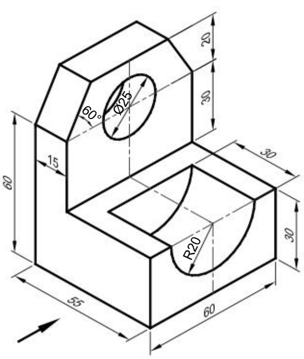


Fig. E7.55

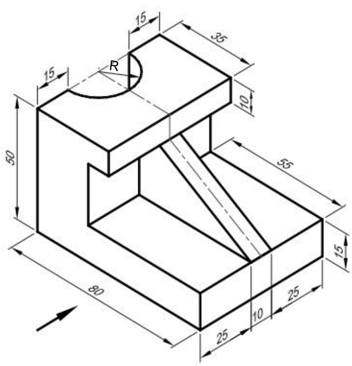


Fig. E7.56

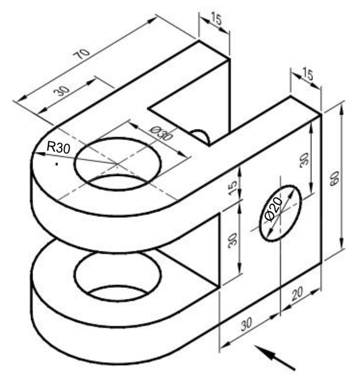


Fig. E7.57

7.7 Pictorial view of an object is shown in Fig. E7.58. Using first angle projection, draw its (a) full-sectional front view, (b) top view and (c) sectional A-A side view.

7.8 Pictorial view of an object is shown in Fig. E7.59. Using first angle projection, draw its (a) full sectional front view, (b) top view and (c) side view.

7.9 Pictorial view of an object is shown in Fig. E7.60. Using first angle projection, draw its (a) sectional front view, (b) sectional top view and (c) sectional side view.

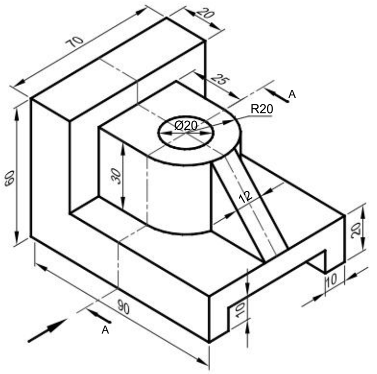


Fig. E7.58

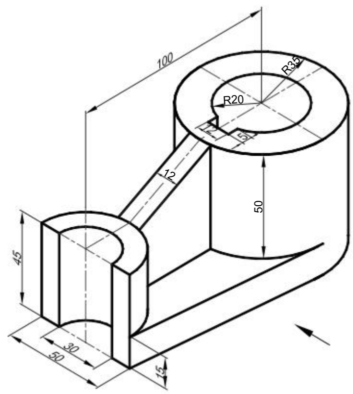


Fig. E7.59

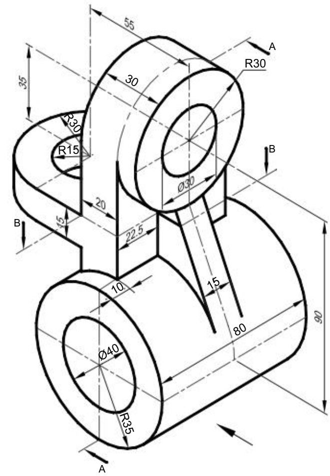


Fig. E7.60

7.26 AUXILIARY VIEWS

Some objects have their surfaces inclined to the regular planes of projections. The regular views of the orthographic projections will show the true shape and size of those surfaces which are perpendicular to the line of sight. None of them will show true shape of the inclined surface. In order to show the true shape and size of these inclined surfaces, it must be projected on a plane parallel to it. This plane is called an auxiliary plane and the view obtained is called auxiliary view. This can be seen in Fig. 7.61(a) and (b).

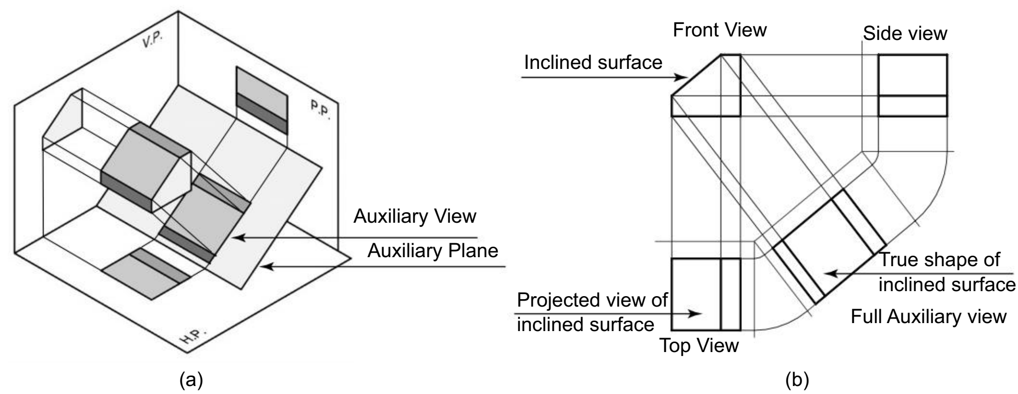


Fig. 7.61 **(a)** *Pictorial view* **(b)** *Orthographic views*

7.27 FULL AND PARTIAL AUXILIARY VIEWS

A full auxiliary view shows all the features of the object, including true shape and size of the inclined surface. Usually, the auxiliary view is drawn to get the features of the inclined surfaces which are not clear otherwise. The details of the other surfaces which do not add any further clarity are usually left out and the view is termed as a partial auxiliary view. Figure 7.61(b) shows the full auxiliary view whereas Fig. 7.62(b) shows the partial auxiliary view.

7.28 PRIMARY AUXILIARY VIEWS

Primary auxiliary views are those projected through a regular front, top or side view. This view is projected from the edge view of the inclined surface, which establishes the true length. The true width of the inclined surface may be transferred from the fold lines of the top or side views. Hidden lines are generally not shown in the auxiliary views unless they are essential to clarify certain features. The following steps may be used to prepare a primary auxiliary view:

1. Label each corner of the inclined surface in all the views.
2. Draw an auxiliary fold line parallel to the edge view of the slanted surface. This line may be at any convenient distance from the edge view.
3. Carefully project all the points one by one from edge view to primary view.
4. Measure the distance of each point from the adjacent view and transfer to the auxiliary view.

Consider the following problems.

Problem 7.24 *Pictorial view of an object is shown in Fig. 7.62(a). Using first angle projection, draw its (a) front view, (b) top view, (c) side view and (d) partial auxiliary view showing true shape of the inclined surface.*

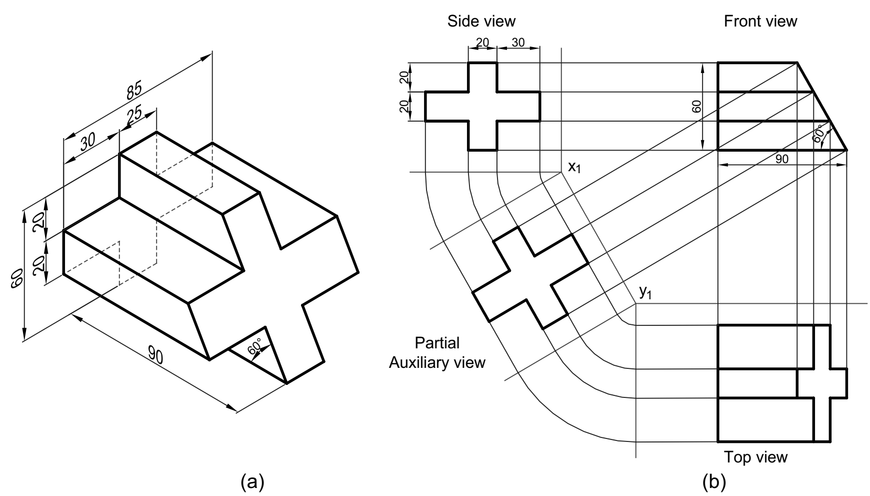


Fig. 7.62 **(a)** *Pictorial view* **(b)** *Orthographic views*

Problem 7.25 *Pictorial view of an object is shown in Fig. 7.63(a). Using first angle projection, draw its (a) front view, (b) top view, (c) partial auxiliary view showing true shape of the inclined surface.*

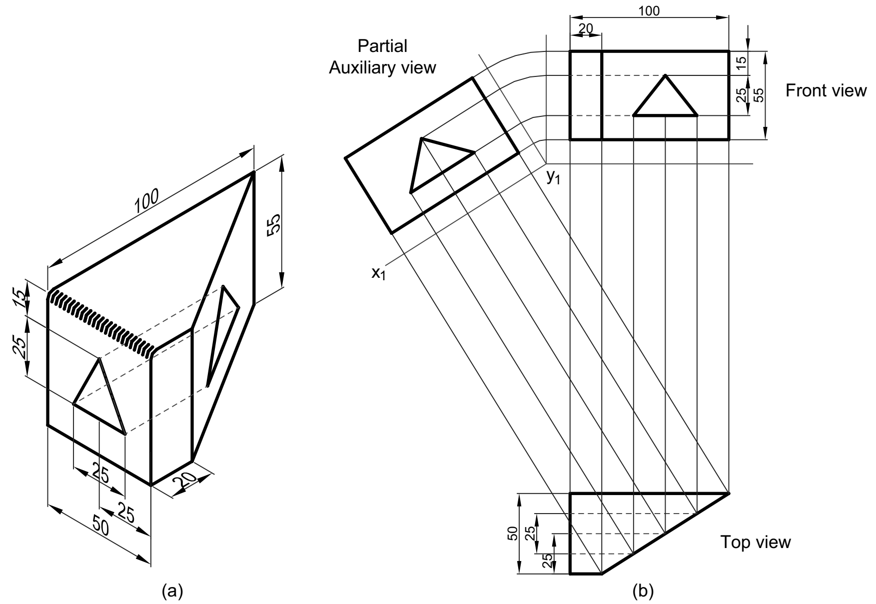


Fig. 7.63 **(a)** *Pictorial view* **(b)** *Orthographic views*

Problem 7.26 *Orthographic view of an object is shown in Fig. 7.64(a). Draw the true shape of the inclined surface on an auxiliary plane.*

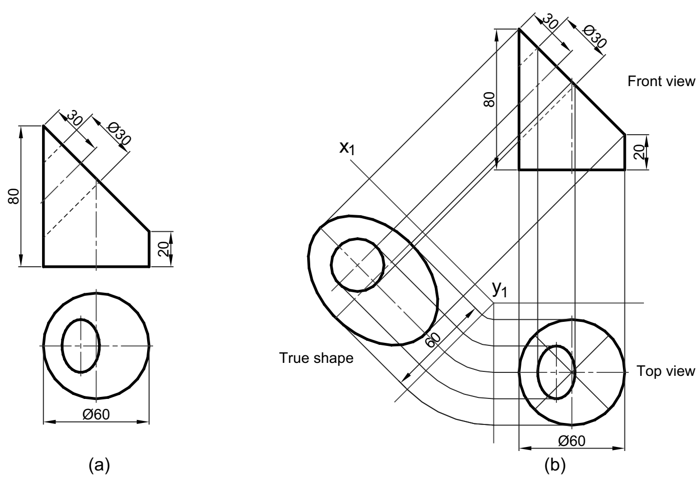


Fig. 7.64 **(a)** *Pictorial view* **(b)** *Orthographic views*

Problem 7.27 *Pictorial view of an angle plate is shown in Fig. 7.65(a). Draw its (a) front view, (b) partial side view showing true shape of the vertical surface and (c) partial auxiliary view showing true shape of the inclined surface.*

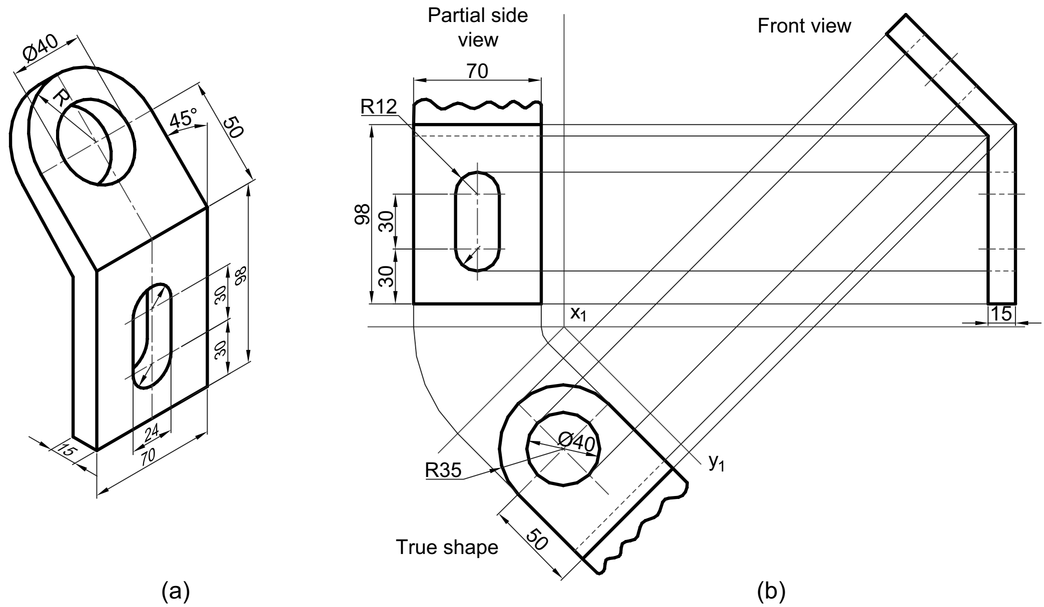


Fig. 7.65 **(a)** *Pictorial view* **(b)** *Orthographic views*

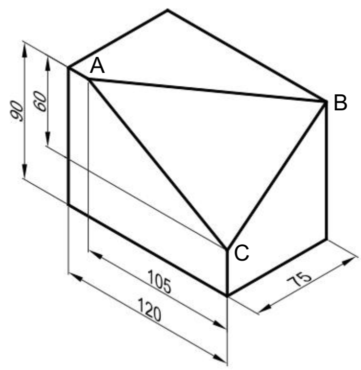
7.29 SECONDARY AUXILIARY VIEWS

Some objects may contain surfaces inclined to both the principal planes called *oblique planes*. The inclined surfaces of these planes do not provide an edge view in any of the principal views. In order to obtain the true shape of the inclined surface, a secondary auxiliary view is needed. The following steps may be used to prepare a secondary auxiliary view:

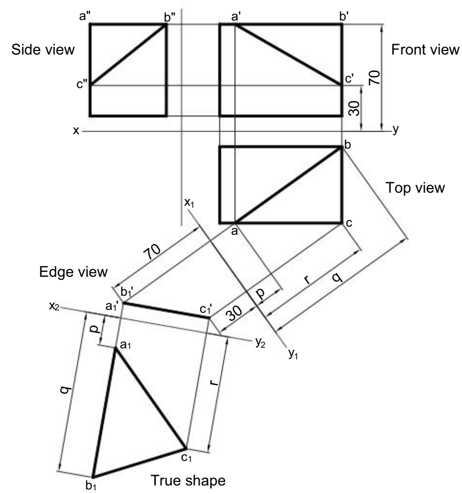
1. Locate an element in one principal view that is true in length.
2. Label the corners of the inclined surface and draw a fold line perpendicular to line of the true length elements.
3. Draw a primary auxiliary view that results in the inclined surface appearing as an edge view or line.
4. Now, draw a folded line parallel to this edge view.
5. Project points from the edge perpendicular to the secondary fold line to get points for the secondary auxiliary view.

Consider the following problems.

Problem 7.28 *Pictorial view of an object is shown in Fig. 7.66(a). Draw the true shape of the oblique surface on an auxiliary plane.*



(a)



(b)

Fig. 7.66 **(a)** *Pictorial view* **(b)** *Orthographic views*

EXERCISE 7C

7.1 Pictorial view of some objects is shown in Figs. E7.61 to E7.63. Draw their projections and determine the true shape of the surface inclined to the H.P.

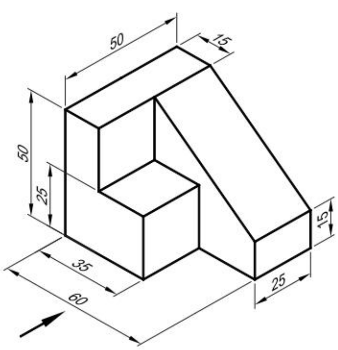


Fig. E7.61

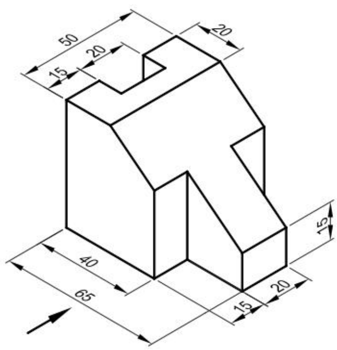


Fig. E7.62

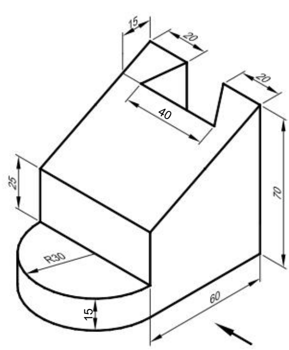


Fig. E7.63

7.2 Pictorial view of some objects is shown in Figs. E7.64 to E7.66. Draw their projections and determine the true shape of the inclined surface.

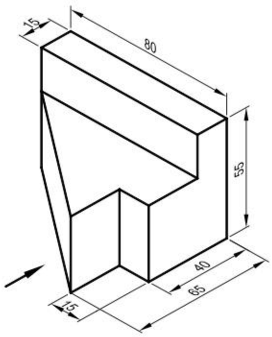


Fig. E7.64

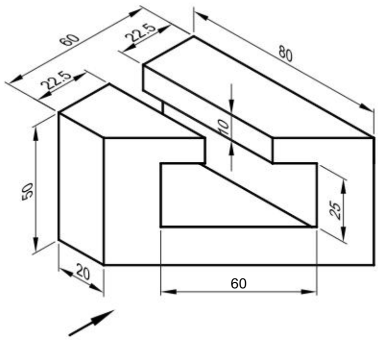


Fig. E7.65

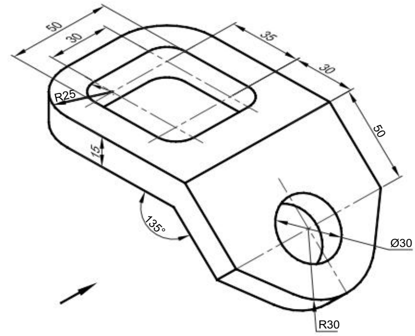


Fig. E7.66

7.3 Pictorial view of some objects is shown in Figs. E7.67 and E7.68. Draw their projections and determine the true shape of the oblique surface (surface inclined to both H.P. and V.P.).

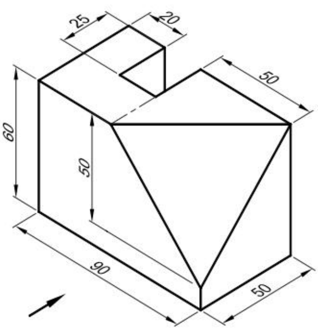


Fig. 7.67

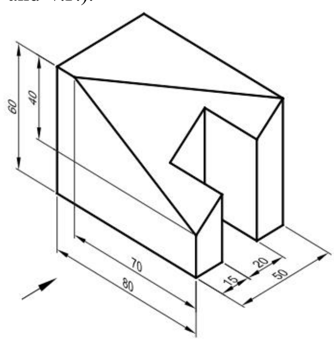


Fig. 7.68

7.4 Orthographic views of some objects are shown in Figs. E7.69 and E7.70. Determine the true shapes of their inclined surfaces.

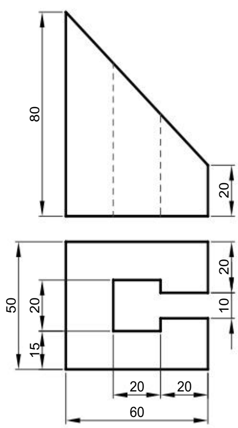


Fig. E7.69

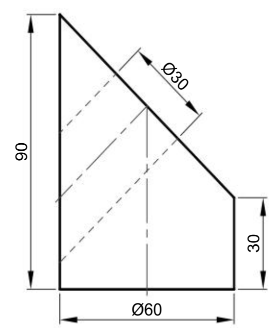


Fig. E7.70

VIVA-VOICE QUESTIONS

7.1 What do you mean by projection? Give its classification.

7.2 Differentiate between a pictorial view and multi-view.

7.3 What is an orthographic projection?

7.4 What is a multi-view projection? How it differs from axonometric projection?

7.5 How a solid or an object should be placed on the planes to obtain multi-views. Explain it with the help of necessary sketches.

7.6 Define vertical, horizontal and profile plane.

7.7 Define elevation, plan and end view.

7.8 Differentiate between first angle and third angle projection.

7.9 Give the symbolic representation of first and third angle projection.

7.10 What is the criterion for selection of the face of an object suitable for front view, while drawing multi-views?

7.11 Explain reference arrows method for representation of multi-views as suggested by BIS. Where this method is beneficial?

7.12 State the advantages of a sectional view.

7.13 State the advantages of a half-sectional view.

7.14 To obtain a half-sectional view, how much portion of the object is actually removed?

7.15 Can a cutting plane line ever be omitted in sectional drawings? If yes, when?

7.16 Name any five items that would not be cut in a sectional view though the cutting plane line may cut them longitudinally.

7.17 With the help of a suitable example show the difference between a revolved and a removed section.

7.18 State the advantages of auxiliary views.

7.19 Describe a situation when a secondary auxiliary view would be necessary.

MULTIPLE-CHOICE QUESTIONS

7.1 An object shown by more than one views in a drawing is called
(a) perspective drawing (b) isometric drawing
(c) oblique drawing (d) multi-view drawing

7.2 Which of the following describes the theory of orthographic projections?
(a) Projectors are parallel to each other and perpendicular to the plane of projection.
(b) Projectors are parallel to each other and parallel to the plane of projection.
(c) Projectors are parallel to each other and oblique to the plane of projection.
(d) Projectors are perpendicular to each other and parallel to the plane of projection.

7.3 In orthographic projections, the visual rays are assumed to
(a) diverge from station point
(b) converge from station point
(c) be parallel
(d) None of these

7.4 In orthographic projections, the *xy* is known as
(a) horizontal line (b) horizontal trace
(c) reference line (d) All of these

7.5 Which is **not** a principal view?
(a) front (b) bottom
(c) auxiliary (d) left side

7.6 The front view of an object is projected on the
(a) horizontal plane (b) vertical plane
(c) profile plane (d) auxiliary plane

7.7 The top view of an object is projected on the
(a) horizontal plane (b) vertical plane
(c) profile plane (d) auxiliary plane

7.8 The side view of an object is obtained on the
(a) horizontal plane (b) vertical plane
(c) profile plane (d) auxiliary plane

7.9 True shape of the inclined surface of an object can be obtained on the
(a) horizontal plane (b) vertical plane
(c) profile plane (d) auxiliary plane

7.10 Normal planes in a three-views drawing will appear as
(a) foreshortened in each view
(b) one line and two surfaces
(c) two lines and one surface
(d) three lines

7.11 Inclined planes in a three-views drawing will appear as
(a) foreshortened in each view
(b) one line and two surfaces
(c) two lines and one surface
(d) three lines

7.12 Oblique planes in a three-views drawing will appear as
(a) foreshortened in each view
(b) one line and two surfaces
(c) two lines and one surface
(d) three lines

7.13 Principal planes in an orthographic projections are
(a) front, top, profile
(b) front, top, side
(c) normal, perpendicular, profile
(d) vertical, horizontal, profile

7.14 In first angle projection method, the relative positions of the object, plane and observers are
(a) object is placed in between
(b) plane is placed in between
(c) observer is placed in between
(d) may be placed in any order

7.15 The top view of an object should be drawn exactly
(a) below or above the front view
(b) right or left of the front view
(c) below or right of the front view
(d) above or left of the front view

7.16 In first angle projection system, the right hand side view of an object is drawn exactly
(a) above of the front view
(b) below of the front view
(c) left of the front view
(d) right of the front view

7.17 In orthographic views, the height dimension on an object is seen in
(a) front and top (b) front and side
(c) top and left side (d) front, top and side

7.18 Minimum number of orthographic views necessary to show length, depth and height of an object are
(a) two (b) three (c) four (d) six

7.19 In orthographic views, the depth dimension on an object is seen in
(a) front and top (b) front and side
(c) top and left side (d) front, top and side

7.20 For orthographic projections, B.I.S. recommends the following projections
(a) First-angle projection
(b) Second-angle projection
(c) Third-angle projection
(d) Fourth-angle projection

7.21 Second angle projections is not used because
(a) top view is above *xy*
(b) front view is above *xy*
(c) views overlap each other
(d) views are foreshortened

7.22 The symbol for indicating the angle of projection shows two views of the frustum of a
(a) square pyramid (b) triangular pyramid
(c) cone (d) Any of these

7.23 For the object shown in Fig. M7.1, select the correct front view.

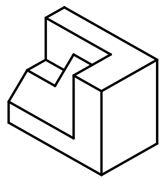
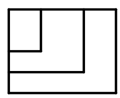
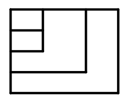
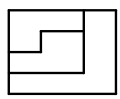
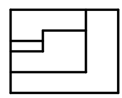


Fig. M7.1

(a)  (b) 

(c)  (d) 

7.24 For the object shown in Fig. M7.2, select the correct front view.

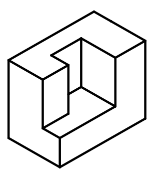
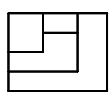
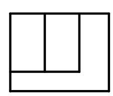
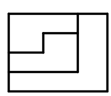
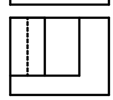


Fig. M7.2

(a)  (b) 

(c)  (d) 

7.25 Objects that are symmetric can be shown effectively using this type of section
(a) quarter section (b) half section
(c) full section (d) symmetric section

7.26 In which of the following type of sections, one quarter of the object is removed?
(a) revolved section (b) removed section
(c) quarter section (d) half section

7.27 Which of the following types of sections is limited by a break line?
(a) revolved section (b) removed section
(c) broken-out section (d) half section

7.28 This type of section is not in direct projection from the view containing the cutting plane
(a) revolved section (b) removed section
(c) broken-out section (d) full section

Answers to multiple-choice questions

7.1 (d), 7.2 (a), 7.3 (c), 7.4 (c), 7.5 (c), 7.6 (b), 7.7 (a), 7.8 (c), 7.9 (d), 7.10 (c), 7.11 (b), 7.12 (a), 7.13 (d), 7.14 (a), 7.15 (a), 7.16 (c), 7.17 (b), 7.18 (a), 7.19 (c), 7.20 (a), 7.21 (c), 7.22 (c), 7.23 (c), 7.24 (b),7.25 (b), 7.26 (d), 7.27 (c), 7.28 (a)

Chapter

8 PROJECTIONS OF POINTS

8.1 INTRODUCTION

A point is defined as a geometrical element that has no dimension. In engineering drawing, a point is represented by a dot. This chapter deals with the projections of points.

8.2 LOCATION OF A POINT

The orthographic projections are obtained on two principal planes (also known as reference planes) having negligible thickness, namely vertical plane (V.P.) and horizontal plane (H.P.) as shown in Fig. 7.4(a) and (b). The principal planes are perpendicular to each other and divide the space into four quadrants. A point lying in the space can be defined in one of the following positions with respect to the principal planes.

1. Above the H.P. and in front of the V.P.
2. Above the H.P. and behind the V.P.
3. Below the H.P. and behind the V.P.
4. Below the H.P. and in front of the V.P.
5. On the H.P. and in front of the V.P.
6. Above the H.P. and on the V.P.
7. On the H.P. and behind the V.P.
8. Below the H.P. and on the V.P.
9. On both the H.P. and the V.P.

8.3 CONVENTIONAL REPRESENTATION

The actual position of a point is designated by capital letters such as *A*, *B*, *C*, *P*, *Q*, *R*, etc. Its front view of is drawn on the vertical plane, the top view is drawn on the horizontal plane and the side view is drawn on the profile plane. The line of intersection of the principal planes is known as a *reference line* or *xy line*. The reference line should be drawn as continuous narrow line.

It is customary to rotate the H.P. in a clockwise direction about the reference line *xy* through 90°, such that it becomes co-planer with the V.P. Similarly, the profile plane is rotated about the reference line x_1y_1 through 90°, such that it also becomes co-planer with the V.P. This makes the front, the top and the side views coplanar when drawn on the sheet. The conventions used to represent the projections of a point are as follows:

1. The front view is represented by small letters with dashes such as a', b', c', p', q', r', etc.
2. The top view is represented by small letters without dashes such as a, b, c, p, q, r, etc.

3. The side view is represented by small letters with double dashes such as a'', b'', c'', p'', q'', r'', etc.

The lines connecting the front, the top and the side views of a point are called *projection lines* or *projectors*. They are drawn as thin continuous line. The projector connecting the front and the top views of a point is always perpendicular to the xy. Similarly, the projector connecting the front and the side views of a point is always perpendicular to the x_1y_1.

8.4 POINT ABOVE THE H.P. AND IN FRONT OF THE V.P.

A point situated above the H.P. and in front of V.P., lies in the first angle as shown in Fig. 8.1(a).

Problem 8.1 *Draw the front and the top views of a point A, lying 70 mm above the H.P. and 50 mm in front of the V.P. Also draw the side view.*

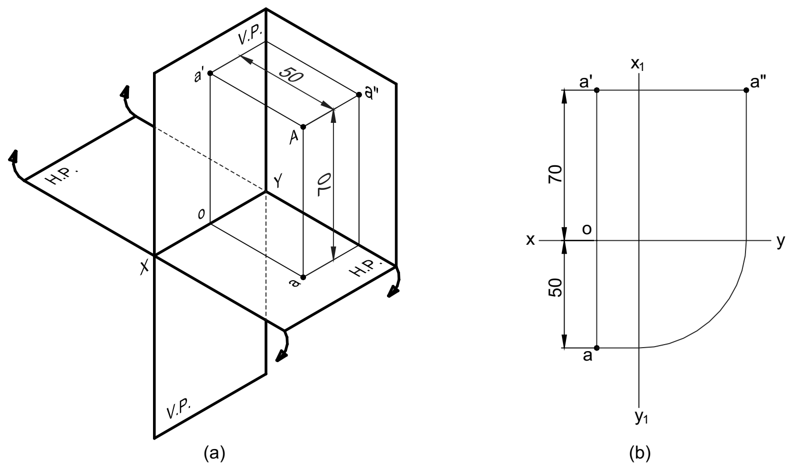


Fig. 8.1 *Point above the H.P. and in front of the V.P.* **(a)** *3-D view* **(b)** *Projections*

Visualisation Figure 8.1(a) shows point A lying in the first angle which is 70 mm above the H.P. and 50 mm in front of the V.P. Its front view (a') is obtained on the V.P., 70 mm above the xy and the top view (a) is obtained on the H.P., 50 mm in front of xy.

Now rotate the H.P. in clockwise direction about the xy through 90°, such that it becomes co-planer with the V.P. The final projections of the point as obtained are shown in Fig. 8.1(b). The front view (a') is 70 mm above the xy and the top view (a) is 50 mm below the xy. The projector connecting the front and the top views ($a'a$) is perpendicular to xy.

Construction Refer to Fig. 8.1(b).

1. Draw a horizontal line xy to represent the reference line.
2. Draw a vertical projector from any point on the xy.
3. Mark a' on the projector, 70 mm above the xy to represent the front view.
4. Mark a on the projector, 50 mm below the xy to represent the top view.
5. Locate a new reference line x_1y_1. Mark a'' on the projector, at 70 mm above the xy and 50 mm away from the x_1y_1. Point a'' represents the side view.

8.5 POINT ABOVE THE H.P. AND BEHIND THE V.P.

A point situated above the H.P. and behind the V.P., lies in the second angle as shown in Fig. 8.2(a).

Problem 8.2 *Draw the projections of a point B, lying 70 mm above the H.P. and 50 mm behind the V.P.*

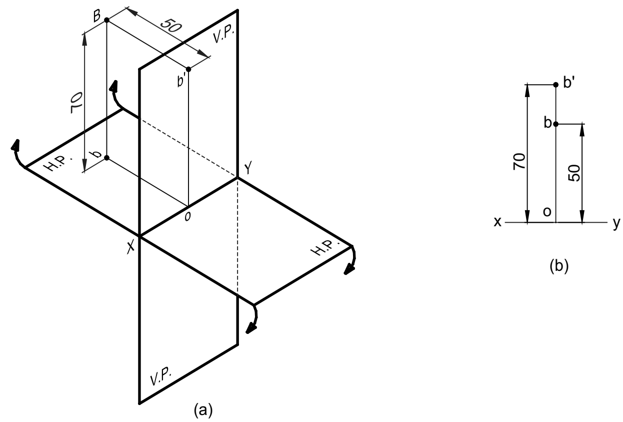


Fig. 8.2 *Point above the H.P. and behind the V.P.* **(a)** *3-D view* **(b)** *Projections*

Visualisation Figure 8.2(a) shows point *B* lying in the second angle which is 70 mm above the H.P. and 50 mm behind the V.P. Its front view (*b′*) is obtained on the V.P., 70 mm above the *xy* and the top view (*b*) is obtained on the H.P., 50 mm behind the *xy*.

Now rotate the H.P. in clockwise direction about the *xy* through 90°, such that it becomes co-planer with the V.P. The final projections of the point as obtained are shown in Fig. 8.2(b). The front view (*b′*) is 70 mm above the *xy* and the top view (*b*) is 50 mm above the *xy*. The projector connecting the front and the top views (*b′b*) is perpendicular to *xy*.

Construction Refer to Fig. 8.2(b).

1. Draw a horizontal line *xy* to represent the reference line.
2. Draw a vertical projector from any point on the *xy*.
3. Mark *b′* on the projector, 70 mm above the *xy* to represent the front view.
4. Mark *b* on the projector, 50 mm above the *xy* to represent the top view.

8.6 POINT BELOW THE H.P. AND BEHIND THE V.P.

A point situated below the H.P. and behind the V.P., lies in the third angle as shown in Fig. 8.3(a).

Problem 8.3 *Draw the projections of a point C, lying 70 mm below the H.P. and 50 mm behind the V.P.*

Visualisation Figure 8.3(a) shows point *C* lying in the third angle which is 70 mm below the H.P. and 50 mm behind the V.P. Its front view (*c′*) is obtained on the V.P., 70 mm below the *xy* and the top view (*c*) is obtained on the H.P., 50 mm behind *xy*.

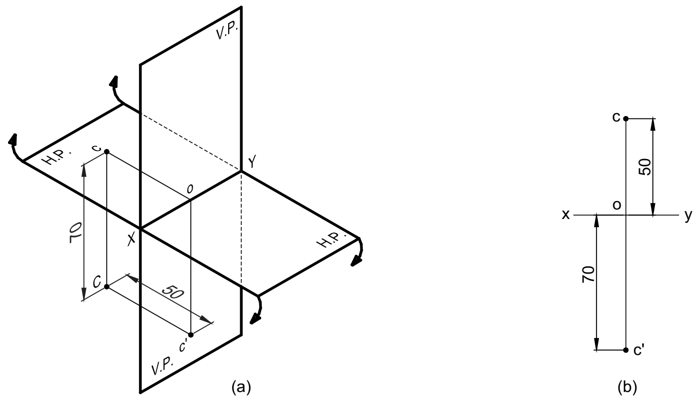


Fig. 8.3 *Point below the H.P. and behind the V.P.* **(a)** *3-D view* **(b)** *Projections*

Now rotate the H.P. in a clockwise direction about the *xy* through 90°, such that it becomes co-planer with the V.P. The final projections of the point as obtained are shown in Fig. 8.3(b). The front view (c') is 70 mm below the *xy* and the top view (c) is 50 mm above the *xy*. The projector connecting the front and the top views ($c'c$) is perpendicular to *xy*.

Construction Refer to Fig. 8.3(b).

1. Draw a horizontal line *xy* to represent the reference line.
2. Draw a vertical projector from any point on the *xy*.
3. Mark c' on the projector, 70 mm below the *xy* to represent the front view.
4. Mark c on the projector, 50 mm above the *xy* to represent the top view.

8.7 POINT BELOW THE H.P. AND IN FRONT OF THE V.P.

A point situated below the H.P. and in front of the V.P., lies in the fourth angle as shown in Fig. 8.4(a).

Problem 8.4 *Draw the projections of a point D, lying 70 mm below the H.P. and 50 mm in front of the V.P.*

Visualisation Figure 8.4(a) shows point *D* lying in the fourth angle which is 70 mm below the H.P. and 50 mm in front of the V.P. Its front view (d') is obtained on the V.P., 70 mm below the *xy* and the top view (d) is obtained on the H.P., 50 mm in front of *xy*.

Now rotate the H.P. in a clockwise direction about the *xy* through 90°, such that it becomes co-planer with the V.P. The final projections of the point as obtained are shown in Fig. 8.4(b). The front view (d') is 70 mm below the *xy* and the top view (d) is 50 mm below the *xy*. The projector connecting the front and the top views ($d'd$) is perpendicular to *xy*.

Construction Refer to Fig. 8.4(b).

1. Draw a horizontal line *xy* to represent the reference line.
2. Draw a vertical projector from any point on the *xy*.
3. Mark d' on the projector, 70 mm below the *xy* to represent the front view.
4. Mark d on the projector, 50 mm below the *xy* to represent the top view.

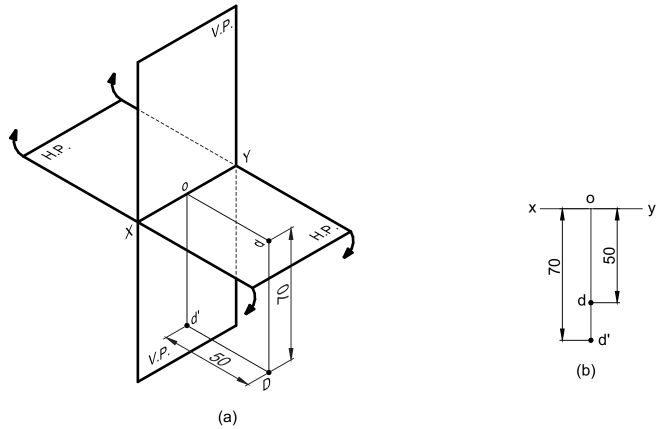


Fig. 8.4 *Point below the H.P. and in front of the V.P.* **(a)** *3-D view* **(b)** *Projections*

8.8 POINT ON THE H.P. AND IN FRONT OF THE V.P.

A point situated on the H.P. and in front of the V.P. is shown in Fig. 8.5(a).

Problem 8.5 *Draw the projections of a point E, lying on the H.P. and 50 mm in front of the V.P.*

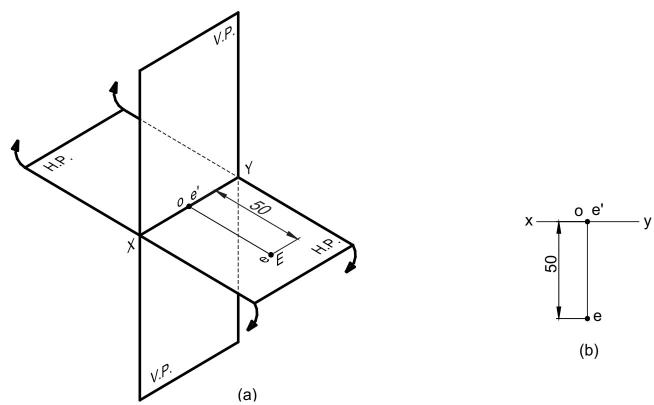


Fig. 8.5 *Point on the H.P. and in front of the V.P.* **(a)** *3-D view* **(b)** *Projections*

Visualisation Figure 8.5(a) shows point *E* lying in the desired position. Its front view (e') is obtained on the *xy* and the top view (e) is obtained 50 mm in front of the *xy*. Now rotate the H.P. in clockwise direction about the *xy* through 90°, such that it becomes co-planer with the V.P. The final projections of the point as obtained are shown in Fig. 8.5(b).

Construction Refer to Fig. 8.5(b).

1. Draw a horizontal line *xy* to represent the reference line.

2. Draw a vertical projector from any point on the *xy*.
3. Mark e' at the intersection of projector and the *xy* to represent the front view.
4. Mark *e* on the projector, 50 mm below the *xy* to represent the top view.

8.9 POINT ABOVE THE H.P. AND ON THE V.P.

A point situated above the H.P. and on the V.P. is shown in Fig. 8.6(a).

Problem 8.6 *Draw the projections of a point F, lying 70 mm above the H.P. and on the V.P.*

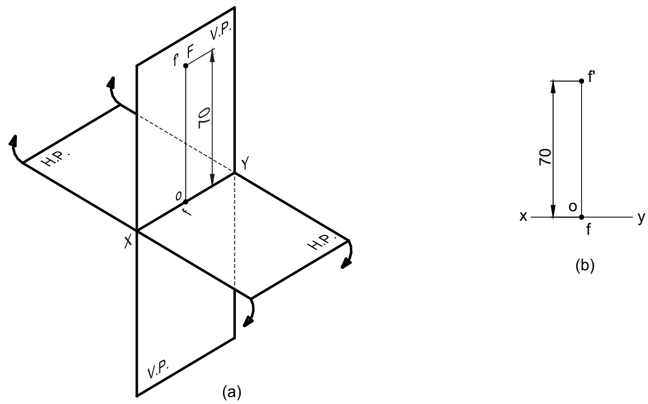


Fig. 8.6 *Point above the H.P. and on the V.P.* **(a)** *3-D view* **(b)** *Projections*

Visualisation Figure 8.6(a) shows point *F* lying in the desired position. Its front view (f') is obtained on *xy* and the top view (*f*) is obtained 50 mm in front of *xy*. Now rotate the H.P. in clockwise direction about the *xy* through 90°, such that it becomes co-planer with the V.P. The final projections of the point as obtained are shown in Fig. 8.6(b).

Construction Refer to Fig. 8.6(b).

1. Draw a horizontal line *xy* to represent the reference line.
2. Draw a vertical projector from any point on the *xy*.
3. Mark f' on the projector, 70 mm above the *xy* to represent the front view.
4. Mark *f* at the intersection of projector and the *xy* to represent the top view.

8.10 POINT ON THE H.P. AND BEHIND THE V.P.

A point situated on the H.P. and behind the V.P. is shown in Fig. 8.7(a).

Problem 8.7 *Draw the projections of a point G, lying on the H.P. and 50 mm behind the V.P.*

Construction Refer to Fig. 8.7(b).

1. Draw a horizontal line *xy* to represent the reference line.

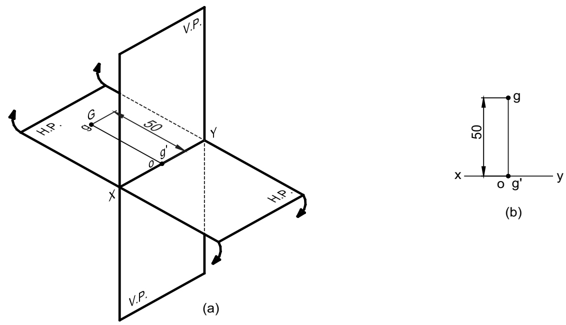


Fig. 8.7 *Point on the H.P. and in front of the V.P.* **(a)** *3-D view* **(b)** *Projections*

2. Draw a vertical projector from any point on the *xy*.
3. Mark g' at the intersection of projector and the *xy* to represent the front view.
4. Mark *g* on the projector, 50 mm above the *xy* to represent the top view.

8.11 POINT BELOW THE H.P. AND ON V.P.

A point situated below the H.P. and on the V.P. is shown in Fig. 8.8(a).

Problem 8.8 *Draw the projections of a point H, lying 70 mm above the H.P. and on the V.P.*

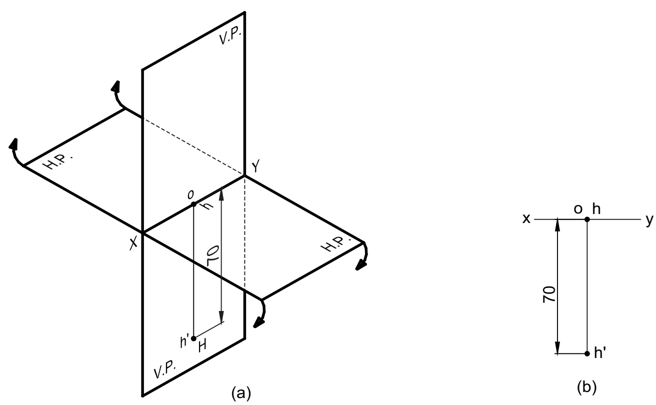


Fig. 8.8 *Point above the H.P. and on the V.P.* **(a)** *3-D view* **(b)** *Projections*

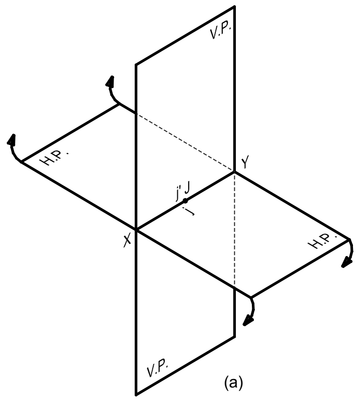
Construction Refer to Fig. 8.8(b).

1. Draw a horizontal line *xy* to represent the reference line.
2. Draw a vertical projector from any point on the *xy*.
3. Mark h' on the projector, 70 mm below the *xy* to represent the front view.
4. Mark *h* at the intersection of projector and the *xy* to represent the top view.

8.12 POINT ON BOTH H.P. AND V.P.

A point situated on both the H.P. and the V.P. is shown in Fig. 8.9(a).

Problem 8.9 *Draw the projections of a point J, lying on both the H.P. and the V.P.*



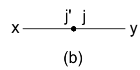


Fig. 8.9 *Point on both the H.P. and the V.P.* **(a)** *3-D view* **(b)** *Projections*

Construction Refer to Fig. 8.9(b).

1. Draw a horizontal line *xy* to represent the reference line.
2. Mark coinciding points *j′* and *j* on the *xy* to represent the front and the top views.

8.13 SUMMARY

The projections obtained in Problems 8.1 to 8.9 can be summarised in Table 8.1.

1. (a) If the point is *h* mm above the H.P. then the front view is *h* mm above the *xy*,
   (b) If the point is on the H.P. then the front view is *on* the *xy*.
   (c) If the point is *h* mm below the H.P. then the front view is *h* mm below the *xy*.
2. (a) If the point is *v* mm in front of the V.P. then the top view is *v* mm below the *xy*,
   (b) If the point is on the V.P. then the top view is on the *xy*.
   (c) If the point is *v* mm behind the V.P. then the top view is *v* mm above the *xy*.
   The vice versa of the above statements is also true.

8.14 MISCELLANEOUS PROBLEMS

Problem 8.10 *Draw the projections of the following points on a common reference line keeping the distance between their projectors 30 mm apart.*

(a) Point A is 20 mm below the H.P. and 50 mm in front of the V.P.
(b) Point B is in the H.P. and 40 mm behind the V.P.

(c) Point C is 30 mm in front of the V.P. and in the H.P.
(d) Point D is 50 mm above the H.P. and 30 mm behind the V.P.
(e) Point E is 20 mm below the H.P. and 50 mm behind the V.P.
(f) Point F is in the V.P. and 50 mm below the H.P.

Solution

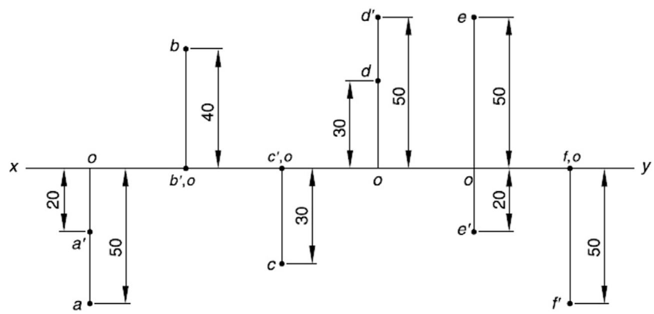


Fig. 8.10

Problem 8.11 *Projection of various points is given in Fig. 8.12. State the position of each point with respect to the planes of projection.*

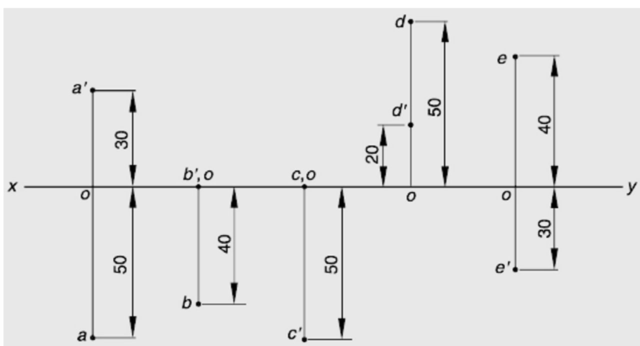


Fig. 8.11

Solution

(a) Point A is 30 mm above the H.P. and 50 mm in front of the V.P.
(b) Point B is in the H.P. and 40 mm in front of the V.P.
(c) Point C is 50 mm below the H.P. and in the V.P.
(d) Point D is 20 mm above the H.P. and 50 mm behind the V.P.
(e) Point E is 30 mm below the H.P. and 40 mm behind the V.P.

EXERCISE 8A

8.1 A point is 30 mm from the H.P. and 50 mm from the V.P. Draw its projections keeping it in all possible positions.

8.2 Draw the projections of the following points on a common reference line keeping the distance between their projectors 25 mm apart.

(a) Point *A* is 40 mm above the H.P. and 25 mm in front of the V.P.

(b) Point *B* is 40 mm above the H.P. and in the V.P.

(c) Point *C* is 25 mm in front of the V.P. and in the H.P.

(d) Point *D* is 25 mm above the H.P. and 30 mm behind the V.P.

(e) Point *E* is in the H.P. and 30 mm behind the V.P.

(f) Point *F* is 40 mm below the H.P. and 30 mm behind the V.P.

(g) Point *G* is 25 mm below the H.P. and 40 mm in front of the V.P.

(h) Point *H* is in the V.P. and 30 mm below the H.P.

8.3 Draw the projections of the following points on a common reference line keeping the distance between their projectors 30 mm apart.

(a) Point *P* is 35 mm below the H.P. and in the V.P.

(b) Point *Q* is 40 mm in front of the V.P. and 25 mm below the H.P.

(c) Point *R* is 45 mm above the H.P. and 20 mm behind the V.P.

(d) Point *S* is 30 mm below the H.P. and 45 mm behind the V.P.

(e) Point *T* is both in H.P. and V.P.

8.4 Projection of various points is given in Fig. E8.1. State the position of each point with respect to the planes of projection.

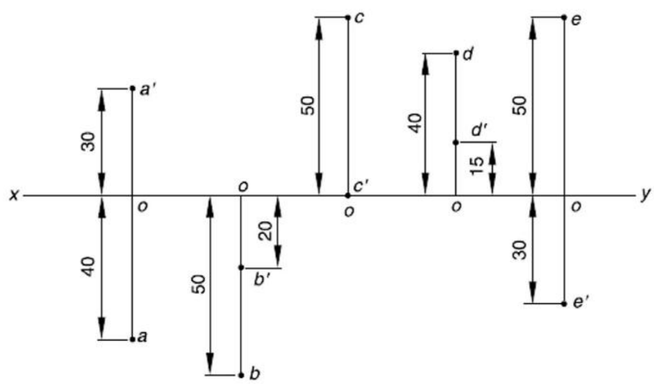


Fig. E8.1

VIVA-VOCE QUESTIONS

8.1 If both the views of a point coincide with each other and lie below the reference line, state the angle in which the point lies.

8.2 State the similarities and dissimilarities in the projections of points which lie in the second angle and the fourth angle.

8.3 State the position of the point, the front view of which lies 50 mm below the reference line and the top view 30 mm above the front view.

8.4 State the position of the point, the top view of which lies 50 mm above the reference line and the front view 30 mm below the top view.

8.5 If the front view of a point lies above the reference line, state the possible angles in which the point may lie.

8.6 If the top view of a point lies above the reference line, state the possible angles in which the point may lie.

8.7 If the front view of a point lies below the reference line, state the possible angles in which the point may lie.

8.8 If the top view of a point lies below the reference line, state the possible angles in which the point may lie.

8.9 State the relationship between front view and top view of a point.

8.10 State the position of the point if its both views lie on the reference line.

8.11 State the position of the point, the top view of which lies on the reference line and the front view 50 mm below it.

8.12 State the position of the point, the front view of which lies on the reference line and the top view 50 mm below it.

8.13 State the position of the point, the top view of which lies on the reference line and the front view 45 mm above it.

8.14 State the position of the point, the front view of which lies on the reference line and the top view 35 mm above it.

MULTIPLE-CHOICE QUESTIONS

8.1 The line joining the front and top views of a point is called
(a) reference line (b) projector
(c) connector (d) locus

8.2 A point lying in the H.P., has its top view above *xy*. Its front view is
(a) on *xy* (b) above *xy*
(c) below *xy* (d) Any of these

8.3 A point whose elevation and plan are above *xy*, is situated in
(a) first angle (b) second angle
(c) third angle (d) fourth angle

8.4 A point whose elevation is above the reference line, is probably situated in the
(a) first angle (b) second angle
(c) vertical plane (d) Any of these

8.5 A point is 20 mm below H.P. and 30 mm behind V.P. Its top view is
(a) 20 mm below *xy* (b) 30 mm below *xy*
(c) 20 mm above *xy* (d) 30 mm above *xy*

8.6 The front view of a point is 50 mm above the reference line and the top view is 20 mm below the front view. The point lies in
(a) first angle (b) second angle
(c) third angle (d) fourth angle

8.7 If both the front and the top views of a point lie on the opposite side of the reference line, the point is situated in which of the following angles?
(a) first or second (b) first or third
(c) second or fourth (d) third or fourth

8.8 If both the front and the top views of a point lie on the same side of the reference line, the point is situated in which of the following angles?
(a) first or second (b) first or third
(c) second or fourth (d) third or fourth

8.9 If top view of a point is situated 60 mm below the reference line and its front view is 20 mm above the top view, the point lies in
(a) first angle (b) second angle
(c) third angle (d) fourth angle

8.10 The front view of a point is 40 mm above *xy* and the top view is 50 mm below *xy*. The point is
(a) 40 mm above H.P.
(b) 40 mm below H.P.
(c) 50 mm above H.P.
(d) 50 mm below H.P.

8.11 State the position of a point the front view of which lies on the reference line and the top view is 40 mm above it.
(a) 40 mm above H.P. and in the V.P.
(b) 40 mm behind V.P. and in the H.P.
(c) 40 mm below H.P. and in the V.P.
(d) 40 mm in front of V.P. and in the H.P.

8.12 State the position of a point the top view of which lies on the reference line and the front view is 30 mm below it.
(a) 30 mm above H.P. and in the V.P.
(b) 30 mm behind V.P. and in the H.P.
(c) 30 mm below H.P. and in the V.P.
(d) 30 mm in front of V.P. and in the H.P.

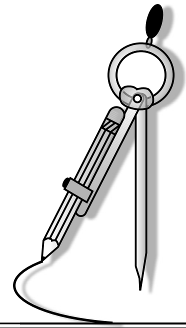
Answers to multiple-choice questions

8.1 (b), 8.2 (a), 8.3 (b), 8.4 (d), 8.5 (d), 8.6 (b), 8.7 (b), 8.8 (c), 8.9 (d), 8.10 (a), 8.11 (b), 8.12 (c)

Chapter

9

PROJECTIONS OF STRAIGHT LINES



9.1 INTRODUCTION

A straight line is defined as the locus of a point which moves unidirectionally. The straight line can also be defined as the shortest distance between two points. The projections of straight lines can be drawn by joining the respective projections of its end points. The word 'line' is used in text for straight lines for the sake of simplicity. The actual length of the line is commonly called true length and is denoted by T.L.

9.2 ORIENTATION OF A STRAIGHT LINE

A straight line may be in one of the following positions.

1. Line parallel to both horizontal plane (H.P.) and vertical plane (V.P.).
2. Line perpendicular to H.P. (and parallel to V.P.).
3. Line perpendicular to V.P. (and parallel to H.P.).
4. Line inclined to H.P. and parallel to V.P.
5. Line inclined to V.P. and parallel to H.P.
6. Line situated on H.P.
7. Line situated in V.P.
8. Line situated on both H.P. and V.P. (i.e., on the reference line, *xy*).
9. Line inclined to both the reference planes.
   (a) Line inclined to both H.P. and V.P. such that $\theta + \phi < 90°$.
   (b) Line inclined to both H.P. and V.P. such that $\theta + \phi = 90°$.

Projections of a straight line lying in the first angle shall have its front view above *xy* and the top view below *xy*. A clear concept of orthographic projections and projections of points is required to understand the projections of straight lines.

9.3 TRACE OF A STRAIGHT LINE

The points of intersection of a straight line (extended if necessary) with the reference plane are called traces of that line.

1. Horizontal trace The point at which the line (extended if necessary) intersects the H.P. is known as horizontal trace and is denoted by either H.T. or letter *h*. The front view of the horizontal trace lies on *xy* and is denoted by *h′*.

2. Vertical trace The point at which the line (extended if necessary) intersects the V.P. is known as vertical trace and is denoted by either V.T. or letter v'. The top view of the vertical trace lies on xy and is denoted by v.

9.4 LINE PARALLEL TO BOTH H.P. AND V.P.

It is the basic position of any line. Both the front and top views will be of true lengths.

Problem 9.1 *A 50 mm long line PQ is parallel to both the H.P. and the V.P. It is 25 mm in front of the V.P. and 60 mm above the H.P. Draw its projections and determine the traces.*

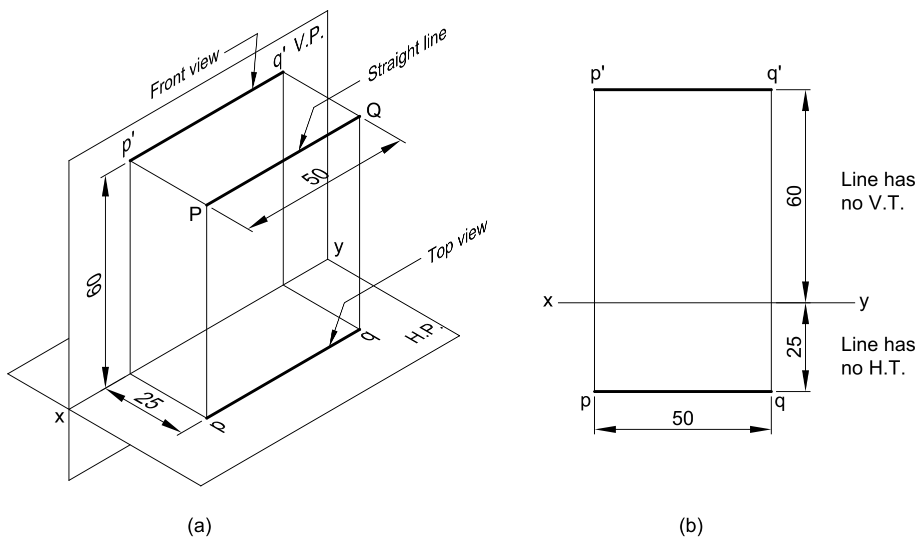


Fig. 9.1 *Line parallel to both H.P. and V.P.* **(a)** *Pictorial view* **(b)** *Orthographic views*

Visualisation Figure 9.1(a) shows the pictorial view of a straight line PQ which is 25 mm in front of the V.P. and 60 mm above the H.P. The front view $p'q'$ of the line shows the true length and is parallel to xy. The top view pq of the line also shows the true length and is parallel to xy.

Construction Refer to Fig. 9.1(b).

1. Draw a reference line xy.
2. Mark point p' 60 mm above xy and point p 25 mm below xy.
3. Draw a 50 mm long line $p'q'$ parallel to xy to represent the front view.
4. Draw another 50 mm long line pq parallel to xy to represent the top view.
5. **H.T.** The front view $p'q'$ shall never meet xy, even though it is extended in both directions. Therefore, it has no H.T.
6. **V.T.** The top view pq shall never meet xy, even though it is extended in both directions. Therefore, it has no V.T.

9.5 LINE PERPENDICULAR TO H.P.

A line perpendicular to the H.P. is always parallel to the V.P. The front view will be of true length lying perpendicular to *xy* whereas the top view will be a point.

Problem 9.2 *A 60 mm long line PQ has its end P 20 mm above H.P. The line is perpendicular to the H.P. and 40 mm in front of the V.P. Draw its projections and locate the traces.*

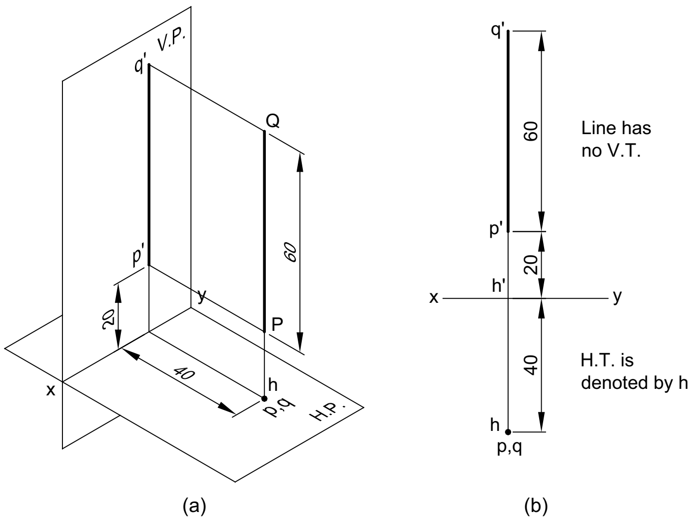


Fig. 9.2 *Line perpendicular to H.P.* **(a)** *Pictorial view* **(b)** *Orthographic views*

Visualisation Figure 9.2(a) shows the pictorial view of a straight line *PQ* whose end *P* is 20 mm above the H.P. and 40 mm in front of the V.P. As the line is perpendicular to the H.P., the end *Q* shall lie 80 mm above the H.P. and 40 mm in front of the V.P.

Construction Refer to Fig. 9.2(b).

1. Draw a reference line *xy*.
2. Mark point p' 20 mm above *xy* and point *p* 40 mm below *xy*.
3. Draw a 60 mm long line $p'q'$ perpendicular to *xy* to represent the front view.
4. Mark point *q* to coincide point *p*. The point represents the top view of the line.
5. **H.T.** Extend the front view $p'q'$ to meet *xy* at h'. Project h' to meet *pq* at point *h*. The point *h* coincides with the top view and denotes H.T.
6. **V.T.** As the line is parallel to the V.P. It shall never meet the V.P. even though extended in both directions. Therefore, it has no V.T.

9.6 LINE PERPENDICULAR TO V.P.

A line perpendicular to the V.P. is always parallel to the H.P. The top view will be of true length lying perpendicular to *xy* whereas the front view will be a point.

Problem 9.3 *A 60 mm long line PQ has its end P 20 mm in front of the V.P. The line is perpendicular to the V.P. and 40 mm above the H.P. Draw the projections of the line and determine its traces.*

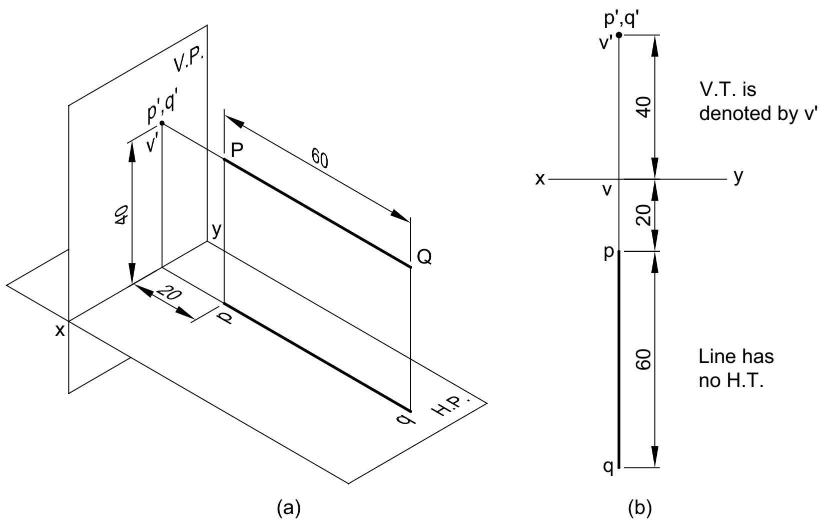


Fig. 9.3 *Line perpendicular to V.P.* **(a)** *Pictorial view* **(b)** *Orthographic views*

Visualisation Figure 9.3(a) shows the pictorial view of a straight line *PQ* whose end *P* is 40 mm above the H.P. and 20 mm in front of the V.P. As the line is perpendicular to the V.P., the end *Q* will be 80 mm in front of the V.P. and 40 mm above the H.P.

Construction Refer to Fig. 9.3(b).

1. Draw a reference line *xy*.
2. Mark point *p′* 40 mm above *xy* and point *p* 20 mm below *xy*.
3. Mark point *q′* to coincide point *p′*. The point represents the front view of the line.
4. Draw a 60 mm long line *pq* perpendicular to *xy* to represent the top view.
5. **H.T.** As the line *PQ* is parallel to H.P., it shall never meet the H.P. even though extended in both directions. Therefore, it has no H.T.
6. **V.T.** Extend the top view *pq* to meet *xy* at *v*. Project *v* to meet *p′q′* at point *v′*. The point *h* coincides with the front view and denotes V.T.

9.7 LINE INCLINED TO H.P. AND PARALLEL TO V.P.

When a line is inclined to the H.P. and parallel to the V.P. its front view will be of true length, inclined to *xy*. The top view will be of projected length (smaller than the true length) and lying parallel to *xy*.

Problem 9.4 *A 80 mm long line PQ has end P 20 mm above H.P. and 40 mm in front of the V.P. The line is inclined at 30° to the H.P. and is parallel to the V.P. Draw the projections of the line and determine its traces.*

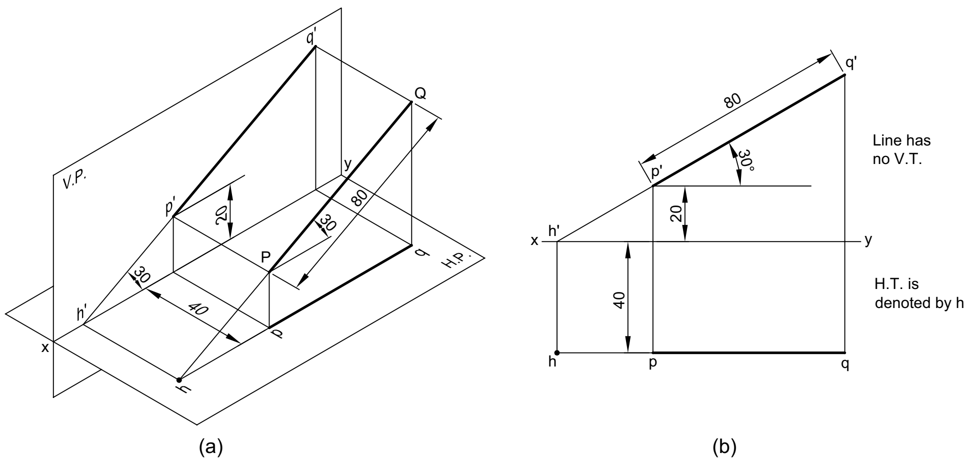


Fig. 9.4 *Line inclined to H.P.* **(a)** *Pictorial view* **(b)** *Orthographic views*

Visualisation Figure 9.4(a) shows the pictorial view of a straight line *PQ* whose end *P* is 20 mm above the H.P. and 40 mm in front of the V.P. The line is inclined at 30° to the H.P. and parallel to the V.P. As the line is parallel to the V.P., its front view will be of true length and inclined at 30° to *xy*. The top view will be of projected length and parallel to *xy*.

Construction Refer to Fig. 9.4(b).

1. Draw a reference line *xy*.
2. Mark point *p′* 20 mm above *xy* and point *p* 40 mm below *xy*.
3. Draw an 80 mm long line *p′q′* inclined at 30° to *xy* to represent the front view.
4. Project point *q′* on *xy* to meet the horizontal line from point *p* at point *q*. Join *pq* to represent the top view.
5. **H.T.** Extend line *p′q′* to meet *xy* at point *h′*. Project point *h′* to meet *pq* produced at point *h*. The point *h* denotes the H.T.
6. **V.T.** As the line is parallel to the V.P., it will never meet the V.P. even though extended in both directions. Therefore, it has no V.T.

9.8 LINE INCLINED TO V.P. AND PARALLEL TO H.P.

When a line is inclined to the V.P. and parallel to the H.P., its top view will be of true length which is inclined to *xy*. The front view will be of projected length (smaller than the true length of the line) parallel to *xy*.

Problem 9.5 *An 80 mm long line PQ is inclined at 30° to the V.P. and is parallel to the H.P. The end P of the line is 20 mm above the H.P. and 40 mm in front of the V.P. Draw the projections of the line and determine its traces.*

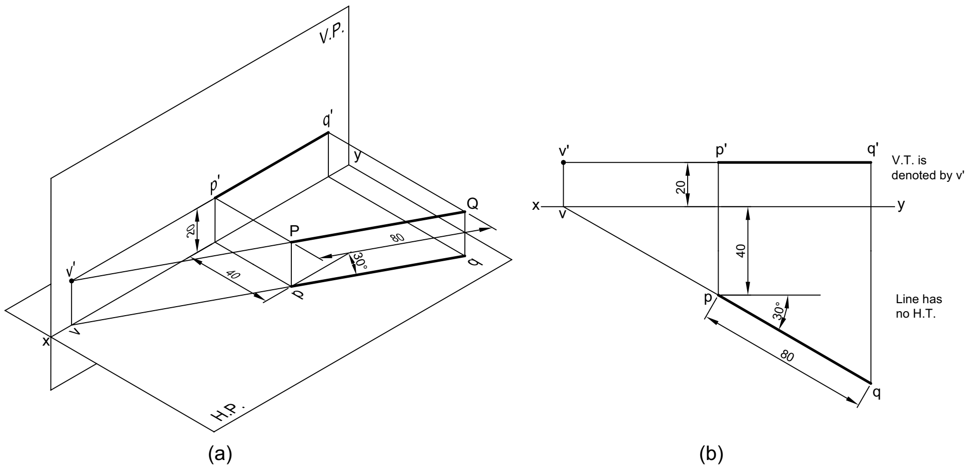


Fig. 9.5 *Line inclined to V.P.* **(a)** *Pictorial view* **(b)** *Orthographic views*

Visualisation Figure 9.5(a) shows the pictorial view of a straight line *PQ* whose end *P* is 20 mm above the H.P. and 40 mm in front of the V.P. The line is inclined at 30° to the V.P. and parallel to the H.P. As the line is parallel to the H.P., its top view will be of true length and inclined at 30° to *xy*. The front view will be of projected length lying parallel to *xy*.

Construction Refer to Fig. 9.5(b).

1. Draw a reference line *xy*.
2. Mark point p' 20 mm above *xy* and point *p* 40 mm below *xy*.
3. Draw an 80 mm long line *pq* inclined at 30° to *xy* to represent the top view.
4. Project point *q* on *xy* to meet the horizontal line from point p' at point q'. Join $p'q'$ to represent the front view.
5. **H.T.** As the line is parallel to the H.P., it shall never meet the H.P. even though extended in both directions. Therefore, it has no H.T.
6. **V.T.** Extend line *pq* to meet *xy* at point *v*. Project point *v* to meet $p'q'$ produced at point v'. The point v' denotes the V.T.

9.9 LINE SITUATED ON H.P.

When a line is situated on the H.P., its top view will be of true length. The front view will be of projected length lying on the reference line.

Problem 9.6 *A 60 mm long line PQ lying on the H.P. is inclined at 30° to the V.P. Its end P is 20 mm in front of the V.P. Draw the projections of the line and determine its traces.*

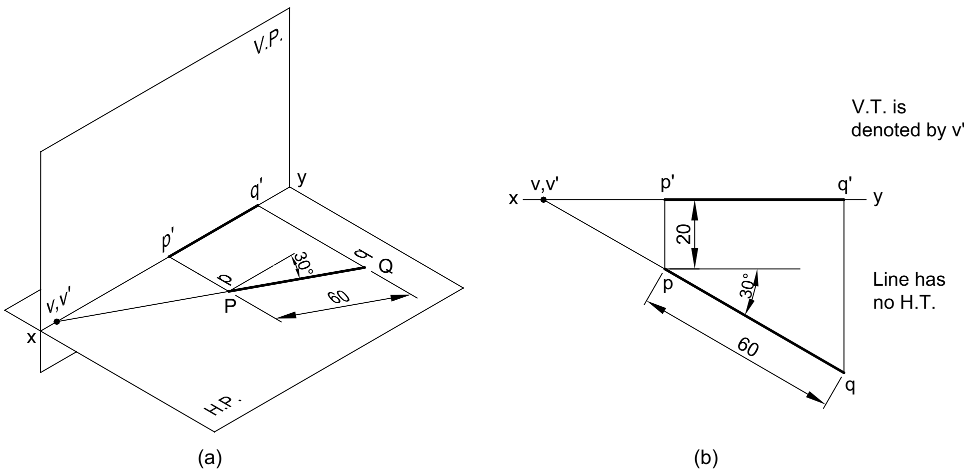


Fig. 9.6 *Line on the H.P.* **(a)** *Pictorial view* **(b)** *Orthographic views*

Visualisation Figure 9.6(a) shows the pictorial view of a straight line *PQ* whose end *P* is on the H.P. and 20 mm in front of the V.P. The line is inclined at 30° to the V.P. As the line is on the H.P., the top view will be of true length and inclined at 30° to *xy*. The front view will be of projected length lying on *xy*.

Construction Refer to Fig. 9.6(b).

1. Draw a reference line *xy*.
2. Mark point *p′* on *xy* and point *p* 20 mm below *xy*.
3. Draw a 60 mm long line *pq* inclined at 30° to *xy* to represent the top view.
4. Project point *q* on *xy* to meet the horizontal line from point *p′* at point *q′*. Join *p′q′* to represent the front view.
5. **H.T.** As the line is on the H.P., it shall never meet the H.P. at a point. Therefore, it has no H.T.
6. **V.T.** Extend line *pq* to meet *xy* at point *v*. Project point *v* to meet *p′q′* produced at point *v′*. The point *v′* coincides *v* and denotes the V.T.

9.10 LINE SITUATED IN THE V.P.

When a line is situated in the V.P. its front view will be of true length. The top view will be of projected length lying on the reference line.

Problem 9.7 *Draw the projections of a 70 mm long line PQ, situated in the V.P. and inclined at 30° to the H.P. The end P of the line is 25 mm above the H.P. Also, determine the traces of the line.*

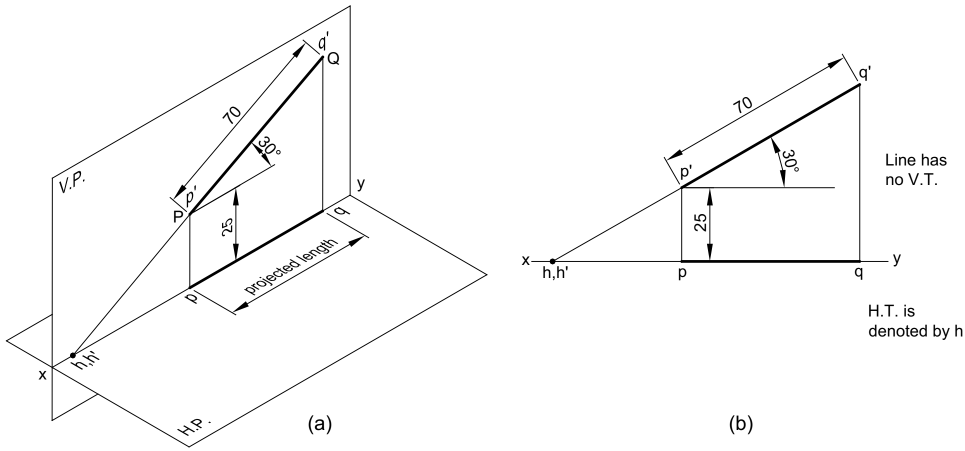


Fig. 9.7 *Line in the V.P.* **(a)** *Pictorial view* **(b)** *Orthographic views*

Visualisation Figure 9.7(a) shows the pictorial view of a straight line *PQ* whose end *P* is 25 mm above the H.P. and in the V.P. The line is inclined at 30° to the H.P. As the line is on the V.P., its front view is shall be of true length and inclined at 30° to *xy*. The top view shall be of projected length lying on *xy*.

Construction Refer to Fig. 9.7(b).

1. Draw a reference line *xy*.
2. Mark point *p′* 25 mm above *xy* and point *p* on *xy*.
3. Draw a 70 mm long line *p′q′* inclined at 30º to *xy* to represent the front view.
4. Project point *q′* on *xy* to meet the horizontal line from point *p* at point *q*. Join *pq* to represent the top view.
5. **H.T.** Extend line *p′q′* to meet *xy* at point *h′*. Project point *h′* to meet *pq* produced at point *h*. The point *h* coincides *h′* and denotes the H.T.
6. **V.T.** As the line is on the V.P., it will never meet the V.P. at a point. Therefore, it has no V.T.

9.11 LINE SITUATED BOTH IN H.P. AND V.P.

A line situated both on the H.P. and the V.P. lies on the reference line. The front and the top views will coincide on *xy*.

Problem 9.8 *Draw the projections of a 60 mm long line PQ, which is situated both on the H.P. and the V.P. Also, determine the traces of the line.*

Visualisation Figure 9.8(a) shows the pictorial view of a straight line *PQ* lying on the reference line. Hence, it is situated both on the H.P. and the V.P. The front and top views of the line are lies on *xy* and show the true lengths.

Construction Refer to Fig. 9.8(b).

1. Draw a reference line *xy*.
2. Mark points *p′* and *p* coinciding on *xy*.

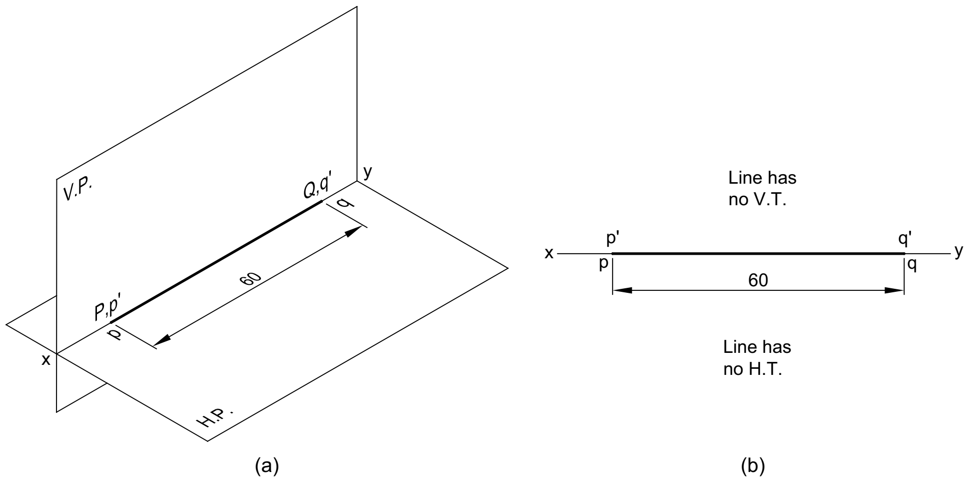


Fig. 9.8 *Line situated both on the H.P. and V.P.* **(a)** *Pictorial view* **(b)** *Orthographic views*

3. Mark points q' and q on xy at a distance of 60 mm from points p and p'.
4. **Traces:** As the line is parallel to both H.P. and V.P. it has neither H.T. nor V.T.

9.12 SUMMARY

Table 9.1

S. No.	*Orientation/ Position of line*	*Front view or elevation*	*Top view or plan*	*Horizontal Trace*	*Vertical Trace*
1.	Line parallel to both H.P. and V.P.	True length, parallel to xy	True length, parallel to xy	Does not exist	Does not exist
2.	Line perpendicular to H.P.	True length, perpendicular to xy	Point	Coincides with top view	Does not exist
3.	Line perpendicular to V.P.	Point	True length, perpendicular to xy	Does not exist	Coincides with front view
4.	Line inclined at θ to H.P. and parallel to V.P.	True length, inclined at θ to xy	Shorter than the true length, parallel to xy	Exists	Does not exist
5.	Line inclined at ϕ to V.P. and parallel to H.P.	Shorter than the true length, parallel to xy	True length, inclined at ϕ to xy	Does not exist	Exists
6.	Line situated in H.P. and inclined at ϕ to V.P.	Shorter than the true length, lying on xy	True length, inclined at ϕ to xy	Does not exist	Exists on xy
7.	Line situated in V.P. and inclined at θ to V.P.	True length, inclined at θ to xy	Shorter than the true length, lying on xy	Exists on xy	Does not exist
8.	Line situated both in H.P. and V.P.	Both front and top views are true length and coincide on xy		Does not exist	Does not exist

Table 9.1 summerises the various orientations of the straight lines which can be concluded as follows.

1. When a line is parallel to the V.P., its front view has true length and the top view is parallel to *xy*. When a line is parallel to the H.P., its top view has true length and the front view is parallel to *xy*.
2. If front view of a line is parallel to *xy* then the line is parallel to the H.P. and its top view provides the true length. If top view of a line is parallel to *xy* then the line is parallel to the V.P. and its front view provides the true length.
3. If the front view has true length then the line is parallel to the H.P. and the top view is parallel to *xy*. If the top view has true length then the line is parallel to the V.P. and the front view is parallel to *xy*.
4. If a line is parallel to the H.P. then it has no H.T. If line is parallel to the V.P. it has no V.T.
5. If a line has no H.T. then it is parallel to the H.P. If a line has no V.T. it is parallel to the VP.
6. If front view is a point then the line is perpendicular to V.P. and if top view is a point then the line is perpendicular to the H.P.

9.13 MISCELLANEOUS PROBLEMS

Problem 9.9 *A 50 mm long line AB has its end A, 30 mm above the H.P. and 20 mm in front of the V.P. The front view of the line is a point. Draw its projections.*

Interpretation As the front view of the line is a point, the line is perpendicular to the V.P.

Construction Refer to Fig. 9.9.

1. Draw a reference line *xy*.
2. Mark point *a′* 30 mm above *xy* and point *a* 20 mm below *xy*.
3. As front view of the line is a point, point *b′* shall coincide with point *a′*.
4. Draw 50 mm long line *ab*, perpendicular to *xy* to represent the top view.

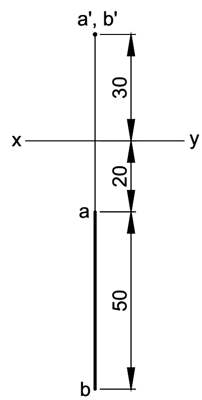


Fig. 9.9

Problem 9.10 *A 60 mm long line AB is parallel to and 20 mm in front of the V.P. The ends A and B of the line are 10 mm and 50 mm above the H.P., respectively. Draw the projections of the line and determine its inclination with the H.P. Also, locate the traces of the line.*

Construction Refer to Fig. 9.10.

1. Draw a reference line *xy*. Mark point *a′* 10 mm above *xy* and point *a* 20 mm below *xy*.
2. Draw a horizontal line 50 mm above *xy* as the locus of *b′*. Draw an arc with centre *a′* and radius 60 mm to meet the locus of *b′* at point *b′*. Join *a′b′* to represent the front view. Determine its inclination with *xy* as the inclination of line *AB* with H.P. Here $\theta = 42°$.
3. Draw the horizontal line from point *a* to meet projector from point *b′* at point *b*. Join *ab* to represent the top view.

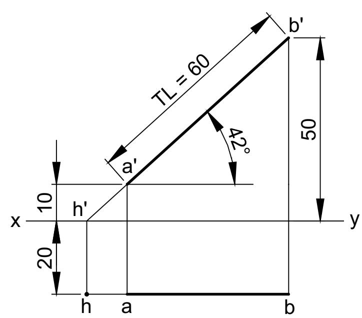


Fig. 9.10

4. **Traces** Extend line $a'b'$ to meet xy at point h'. Project point h' to meet ab produced at point h. The point h denotes the H.T. As the line is parallel to the V.P., it has no V.T.

Result Line does not have V.T. The H.T. (h) is 20 mm below xy. Inclination of the line with the H.P. $\theta = 42°$.

Problem 9.11 *An 80 mm long line MN has its end M 15 mm in front of the V.P. The distance between the ends projector is 50 mm. The front view is parallel to and 20 mm above reference line. Draw the projections of the line and determine its inclination with the V.P. Also, locate the traces.*

Interpretation Front view of a line is parallel to xy, therefore,

1. The line is parallel to the H.P.
2. The top view of the line has true length.
3. The front view has projected length equal to the distance between the projectors.

Construction Refer to Fig. 9.11.

1. Draw a reference line xy. Mark point m' 20 mm above xy and point m 15 mm below xy.
2. Draw a 50 mm long line $m'n'$ parallel to xy.
3. Draw an arc with centre m and radius 80 mm to meet projector from point n' at point n. Join mn to represent the top view. Determine its inclination with xy as the inclination of line MN with the V.P. Here $\phi = 51°$.
4. **Traces** Extend line mn to meet xy at point v. Project point v to meet $m'n'$ produced at point v'. The point v' denotes the V.T. As the line is parallel to the H.P., it has no H.T.

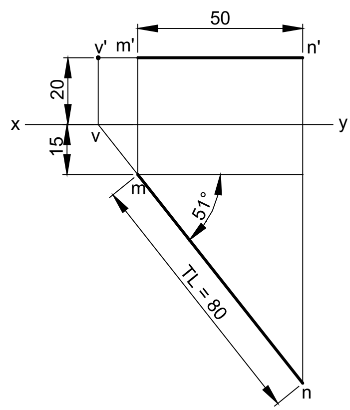


Fig. 9.11

Result Line does not have H.T. The V.T. (v') is 20 mm above xy. Inclination of the line with the V.P. $\phi = 51°$.

Problem 9.12 *The top view of a line measures 60 mm. The line is parallel to the V.P. and inclined at 45° to the H.P. One end of the line is 25 mm in front of the V.P. and lies on the H.P. Draw its projections and determine the true length.*

Interpretation Let the line be PQ parallel to the V.P. The front view has true length and the top view is parallel to xy.

Construction Refer to Fig. 9.12.

1. Draw a reference line xy. Mark point p' on xy and point p 25 mm below xy.
2. Draw a 60 mm long line pq parallel to xy. This represents the top view.
3. Draw line from point p', inclined at 45° to xy to meet the projector from point q at point q'. Join $p'q'$ to represent the front view. Measure length of $p'q'$ as true length of line PQ. Here T.L. = 85 mm.

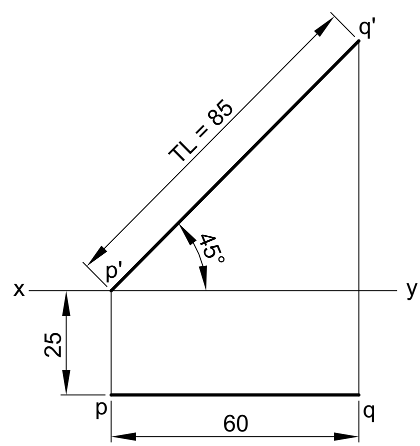


Fig. 9.12

Result True length of line PQ is $p'q'$ = 85 mm.

Problem 9.13 *A 70 mm long line PQ does not have H.T. and V.T. One end of the line is 30 mm in front of the V.P. and 20 mm above the H.P. Draw its projections.*

Interpretation As the line *PQ* does not have H.T. and V.T., it is parallel to both H.P. and V.P.

Construction Refer to Fig. 9.13.

1. Draw a reference line *xy*. Mark point *p′* 20 mm above *xy* and point *p* 30 mm below *xy*.
2. Draw a 70 mm long line *p′q′* parallel to *xy* to represent the front view.
3. Also, draw a 70 mm long line *pq* parallel to *xy* to represent the top view.

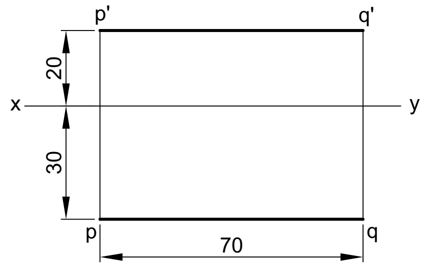


Fig. 9.13

Problem 9.14 *A 60 mm long line PQ is inclined at 45° to the V.P. The V.T. is 25 mm above reference line and the H.T. does not exist. Draw the projections of the line when one end of the line is 10 mm in front of the V.P.*

Interpretation As H.T. does not exist, the line *PQ* is parallel to the H.P. The front view will be parallel to *xy*. Since V.T. is 25 mm above *xy*, *p′ q′* and *v′* all will be 25 mm above *xy*.

Construction Refer to Fig. 9.14.

1. Draw a reference line *xy*. Mark point *v′* 25 mm above *xy* and point *v* on *xy*.
2. Draw a line from point *v* inclined at 45° to *xy*. Mark point *p* on it 10 mm below *xy*. Produce *vp* and mark on it another point *q* such that *pq* is 60 mm. Line *pq* represents the top view.
3. Project points *p* and *q* to meet a horizontal line from *v′* at points *p′* and *q′*, respectively. Join *p′q′* to represent the front view.

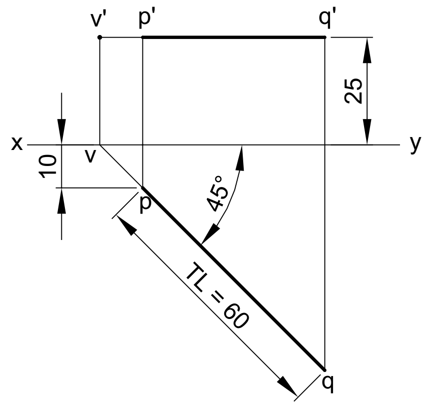


Fig. 9.14

EXERCISE 9A

9.1 A 60 mm long line *AB* is parallel to both the H.P. and the V.P. It is 20 mm above the H.P. and 30 mm in front of the V.P. Draw its projections.

9.2 A 60 mm long line *PQ* has its end A 25 mm above the H.P. and 40 mm in front of the V.P. Draw the projections of the line when it is parallel to both the reference planes.

9.3 A 55 mm long line *PQ* is perpendicular to the H.P. and 25 mm in front of the V.P. Draw its projections when one end of the line is 15 mm above the H.P.

9.4 A 50 mm long line *AB* is perpendicular to the V.P. and 40 mm above the H.P. One end of the line is 10 mm in front of the V.P. Draw its projections and locate the traces.

9.5 A 70 mm long line *PQ* is perpendicular to the H.P. and 30 mm in front of the V.P. End *P* of the line is on the H.P. Draw its projections and locate the traces.

9.6 A 60 mm long line lying in the V.P. is perpendicular to the H.P. Draw its projections and locate the traces when one end of the line is 15 mm above the H.P.

9.7 A 50 mm long line situated in the V.P. is parallel to the H.P. The line is at a distance of 40 mm from the H.P. Draw its projections and locate the traces.

9.8 A 70 mm long line is parallel to and 50 mm in front of the V.P. Draw its projections when the line is situated in the H.P.

9.9 A 70 mm long line lies both on the H.P. and the V.P. Draw its projections.

9.10 A 70 mm long line *MN* is inclined at 45° to the H.P. and parallel to the V.P. The end *M* is 15 mm above the H.P. and 25 mm in front of the V.P. Draw its projections and locate the traces.

9.11 An 80 mm long line is inclined at 60° to the V.P. and parallel to the H.P. One end of the line is 30 mm above the H.P. and 10 mm in front of the V.P. Draw its projections and locate the traces.

9.12 A 70 mm long line *CD* lying in the V.P. is inclined at 60° to the H.P. One end of the line 15 mm above the H.P. Draw its projections and locate the traces.

9.13 A 70 mm long line *PQ* lying in the H.P. is inclined at 45° to the V.P. End *P* of the line is in the V.P. Draw its projections and locate the traces.

9.14 A 70 mm long line has an end 40 mm above the H.P. and 20 mm in front of the V.P. The front view of the line is a point. Draw its projections and determine true inclination with the reference planes.

9.15 Two points *P* and *Q* lying in the V.P. are 90 mm apart. The horizontal distance between the points is 60 mm. If the point *P* is 15 mm above the H.P., find the height of the point *Q* above the H.P. and the inclination of the line joining *P* and *Q* with the H.P.

9.16 An 80 mm long line is parallel to and 20 mm above the H.P. One end of the line is in the V.P. whereas the other end is 40 mm in front of the V.P. Draw its projections and determine the true inclination of the line with the V.P.

9.17 A 75 mm long line is parallel to and 40 mm in front of the V.P. The ends of the line are 25 mm and 50 mm above the H.P. Draw its projections and determine the true inclination of the line with the H.P.

9.18 A 70 mm long line *AB* is parallel to the V.P. and inclined to the H.P. The end *A* is 20 mm above the H.P. and 30 mm in front of the V.P. The top view of the line measures 45 mm. Draw its projections.

9.19 An 80 mm long line has an end 15 mm above the H.P. and 25 mm in front of the V.P. The line is parallel to the V.P. and its top view measures 40 mm. Draw the projections of the line and find its inclination with the H.P.

9.20 The front view of an 80 mm long line *PQ* measures 50 mm. The line lies in the H.P. such that one end is 30 mm in front of the V.P. Draw the projections of the line and find its inclination with the V.P.

9.21 The top view of an 80 mm long line *AB* measures 55 mm. The line is in the V.P. and its one end being 20 mm above the H.P. Draw its projections and find inclination with the H.P.

9.22 A line *PQ* is parallel to the V.P. and inclined at 30° to the H.P. End *P* is 20 mm from both the reference planes and the top view measures 70 mm. Draw the projections of the line and determine its true length.

9.23 The front view of a line *PQ* parallel to the V.P. and inclined 60° to the H.P. is 50 mm. one end of the line is 20 mm in front of the V.P. and 25 mm above the H.P. Draw its projections and determine true length of the line.

9.24 A 65 mm long line *PQ* has end *P* 20 mm in front of the V.P. and 30 mm above the H.P. Draw its projections such that the line does not have traces.

9.25 An 80 mm long line *PQ* has an end 5 mm in front of the V.P. The V.T. of the line is 30 mm above the H.P. and the front view of the line is a point. Draw its projections.

9.26 The front view a line perpendicular to the H.P. measures 70 mm. The H.T. of the line is 25 mm below the reference line. Draw its projections.

9.27 A 70 mm long line is inclined at 30° to the H.P. The H.T. of the line lies 15 mm below the reference line and V.T. of the line does not exist. Draw its projections when one end of the line is 25 mm above the H.P.

9.28 A 75 mm long line is inclined at 45° to the V.P. Its V.T. is 30 mm above the reference plane and H.T. does not exist. One end of the line is 20 mm in front of the V.P. Draw its projections.

9.29 The front view of a line inclined at 60° to the V.P. measures 60 mm. One end of the line is 20 mm in front of the V.P. The V.T. is 15 mm above the reference line and H.T. does not exist. Draw its projections.

9.30 An electric switch and a bulb fixed on a wall are 5 m apart. The distance between them measured parallel to the floor is 4 metres. If the switch is 1 m above the floor, find the height of the bulb and inclination of line joining the two with the floor.

9.14 LINE IN FIRST ANGLE AND INCLINED TO BOTH THE REFERENCE PLANES

9.14.1 Line Inclined to Both the Reference Planes where $\theta + \phi < 90°$

When a line is inclined at θ with H.P. and ϕ with V.P., then both the front and the top views shall be smaller than the true length. The front and the top views shall also appear to be inclined at an apparent angles of α and β with the H.P. and the V.P., respectively. The angle α shall be greater than θ while angle β shall be greater than ϕ. Thus, apparent inclinations would be greater than true inclination.

Problem 9.15 *A 70 mm long line PQ, has its end P 20 mm above the H.P. and 30 mm in front of the V.P. The line is inclined at 45° to the H.P. and 30° to the V.P. Draw its projections.*

Visualisation

1. **Figures 9.15(a) and (b)** Consider a line PQ_1, 70 mm long has its end P 20 mm above the H.P. and 30 mm in front of the V.P. The line is inclined at $\theta = 45°$ to the H.P. and parallel to the V.P.
   (a) Draw the front view $p'q_1'$ of 70 mm (true length) inclined at $\theta = 45°$ to xy.
   (b) Draw the top view pq_1 parallel to xy.
   Now keeping the position of the end P fixed and the inclination $\theta = 45°$ with H.P. as constant, turn the line to make its inclined ϕ to the V.P. While turning the line following observations shall be made.
   (a) Since the inclination with the H.P. (θ) is constant, the length of the top view shall remain same. Therefore, the point q_1 shall move to the new position q along an arc (known as locus of q) drawn with centre p and radius pq_1.
   (b) The height of Q_1 from the H.P. shall remain constant (say h). In the front view q_1' shall move along the line ab parallel to xy. The line ab is called the locus of q'.
2. **Figures 9.15(c) and 9.15(d)** Consider a line PQ_2, 70 mm long has its end P 20 mm above the H.P. and 30 mm in front of the V.P. The line is inclined $\phi = 30°$ to the V.P. and parallel to the H.P.
   (a) Draw the top view pq_2 of 70 mm (true length) inclined at $\phi = 30°$ to xy.
   (b) Draw the front view $p'q_1'$ parallel to xy.
   Now keeping the position of the end P fixed and the inclination $\phi = 30°$ with V.P. as constant, turn the line to make it inclined θ to the H.P. While turning the line following observations shall be made.
   (a) Since the inclination with the V.P. (ϕ) is constant, the length of the front view shall remain same. Therefore, the point q_2' shall move to the new position q' along an arc (known as locus of q') drawn with centre p' and radius pq_1.
   (b) The distance of the end Q_2 from the V.P. shall remain constant (say d). In the top view q_2 shall move along the line cd parallel to xy. The line cd is called the locus of point q.
3. Step 1 states that point q' lies on the locus line ab and point q lies on the arc. Step 2 states that point q' lies on the arc and point q lies on the locus line cd. On combining both the steps, i.e., Figs. 9.15(b) and (d) points q' and q can be located, as shown in Fig. 9.15(e). It may be noted that the projector joining qq' shall perpendicular be to xy.

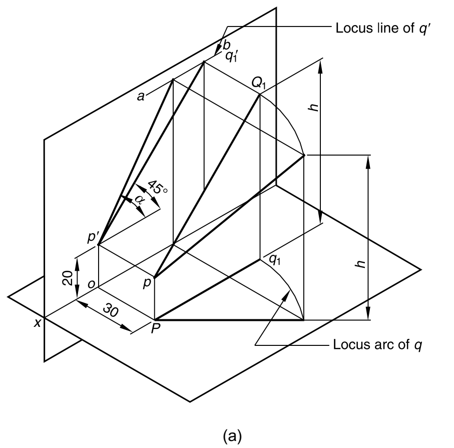


Fig. 9.15(a)

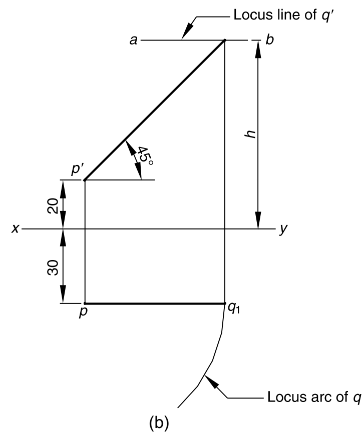


Fig. 9.15(b)

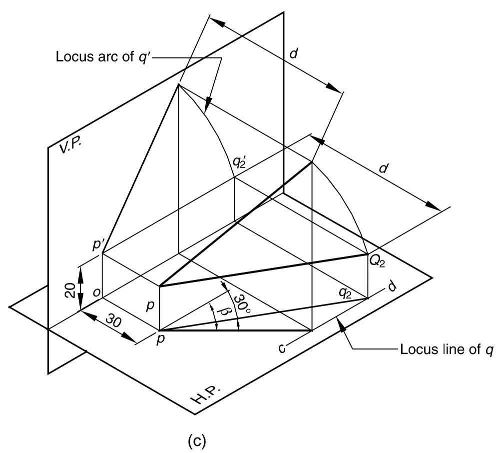


Fig. 9.15(c)

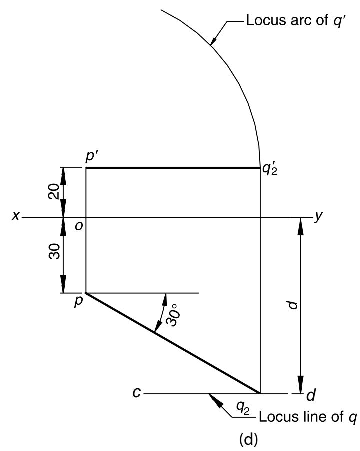


Fig. 9.15(d)

Construction Refer to Fig. 9.15(e).

1. Draw a reference line *xy*.
2. On a vertical projector, mark point p' 20 mm above *xy* and point p 30 mm below *xy*.
3. Draw a 70 mm long line $p'q_1'$ inclined at $\theta = 45^\circ$ to *xy*.
4. Draw another 70 mm long line pq_2, inclined at $\phi = 30^\circ$ to *xy*.
5. Project point q_1' to meet the horizontal line from point p at point q_1.
6. Draw an arc with centre p and radius pq_1 to meet the horizontal line from point q_2 at point q. Join pq to represent the top view.
7. Project point q_2 to meet the horizontal line from point p' at point q_2'.
8. Draw an arc with centre p' and radius $p'q_2'$ to meet the horizontal line from point q_1' at point q'. Join $p'q'$ to represent the front view.
9. Join qq' and ensure that the line is perpendicular to *xy*, to represent projector of the end Q.

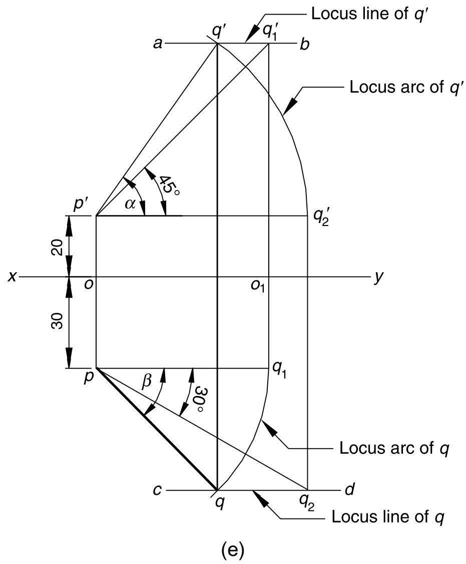


Fig. 9.15(e)

Conclusions

1. α is known as the apparent inclination with the H.P. The apparent inclination in the front view is greater than the actual inclination of the line with the H.P., i.e., $\alpha > \theta$.
2. β is known as the apparent inclination with the V.P. The apparent inclination in the top view is greater than the actual inclination of the line with the V.P., i.e., $\beta > \phi$.
3. Neither the front view nor the top view represents the true length of the line PQ.

Notation used

In the above problem, the notations used can be summarised as:

PQ	Actual line in space
θ	true inclination of line with the H.P.
ϕ	true inclination of line with the V.P.
PQ_1	Line assumed parallel to the V.P. and inclined θ to the H.P.
$p'q_1'$	front view of the line PQ_1, representing true length and inclination with the H.P.
pq_1	top view of the line PQ_1, representing length equivalent to top view
PQ_2	Line assumed parallel to the H.P. and inclined ϕ to the V.P.
$p'q_2'$	front view of the line PQ_1, representing length equivalent to front view.
pq_2	top view of the line PQ_1, representing true length and inclination with the V.P.
$p'q'$	Final front view of the line PQ
pq	Final top view of the line PQ
α	apparent angle made by the front view $p'q'$ with xy
β	apparent angle made by the top view pq with xy

9.14.2 Determination of True Length and Inclinations with Reference Planes

This is the reverse of Problem 9.15. Here projections of a line are given and one needs to determine the true length and true inclinations of the line. For this, turn the front and top views about a point and make it parallel to the reference line *xy*. Consider the following problem.

Problem 9.16 *A straight line PQ has its end P 20 mm above the H.P. and 30 mm in front of the V.P. and the end Q is 80 mm above the H.P. and 70 mm in front of the V.P. If the end projectors are 60 mm apart, draw the projections of the line. Determine its true length and true inclinations with the reference planes.*

Construction Refer to Fig. 9.16.

Draw the projections of the line.

1. Draw a reference line *xy*. Mark points *o* and o_1 on it such that they are 60 mm apart.
2. On the vertical projector through *o*, mark point p' 20 mm above *xy* and point *p* 30 mm below *xy*.
3. On the vertical projector through o_1, mark point q' 80 mm above *xy* and point *q* 70 mm below *xy*.
4. Join $p'q'$ and *pq* to represent the front and the top views of the line, respectively.
   Find true length (T.L.) and inclination (θ) of line with H.P.
5. Draw an arc with centre *p* and radius *pq* to meet the horizontal line from point *p* at point q_1.
6. Project point q_1 to meet horizontal line *ab* through point q' at point q_1'.
7. Join $p'q_1'$. The length $p'q_1'$ represents the true length of *PQ*. The inclination of $p'q_1'$ with *xy* represents true inclination of *PQ* with H.P. Here, T.L. = 94 mm and θ = 40°.
   Find true length (T.L.) and inclination (ϕ) of line with V.P.
8. Draw an arc with centre p' and radius $p'q'$ to meet the horizontal line from point p' at point q_2'.
9. Project point q_2' to meet horizontal line *cd* through point *q* at point q_2.
10. Join pq_2. The length pq_2 represents the true length of *PQ*. The inclination of pq_2 with *xy* represents true inclination of *PQ* with V.P. Here, ϕ = 25°. Ensure that the length pq_2 is equal to the length $p'q_1'$.

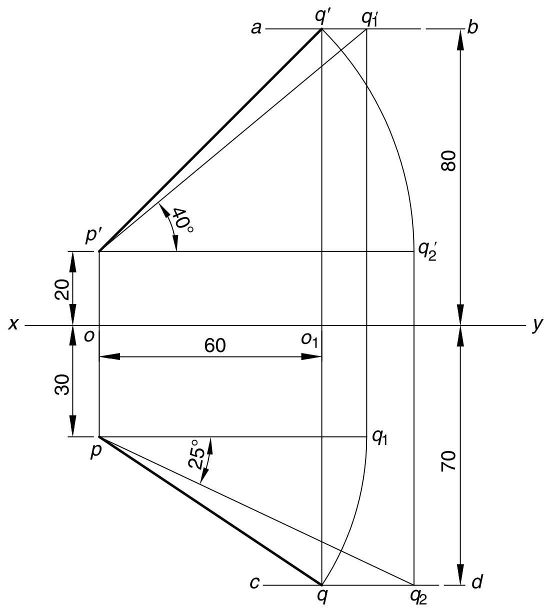


Fig. 9.16

Result True length, $p'q_1' = pq_2$ = 94 mm. Inclination with the H.P. θ = 40°. Inclination with the V.P. ϕ = 25°.

9.14.3 Trapezoid Method

The true length and true inclination of a line whose projections are given can also be determined using trapezoid method. Consider Problem 9.16 that needs to be solved using trapezoid method.

Problem 9.17 *A straight line PQ has its end P 20 mm above the H.P. and 30 mm in front of the V.P. and the end Q is 80 mm above the H.P. and 70 mm in front of the V.P. If the end projectors are 60 mm apart, draw the projections of the line. Determine its true length and true inclinations with the reference planes by trapezoid method.*

Construction Refer to Fig. 9.17.

1. Draw a reference line xy. Mark points o and o_1 on xy 60 mm apart.
2. On the vertical projector through point o, mark point p' 20 mm above the xy and point p 30 mm below the xy.
3. On the vertical projector through point o_1, mark point q' 80 mm above the xy and point q 70 mm below the xy.
4. Join $p'q'$ and pq to represent the front and top views of line PQ, respectively.
5. Draw perpendiculars $p'P_1$ and $q'Q_1$ to the line $p'q'$ such that $p'P_1 = op$ and $q'Q_1 = oq$.
6. Join P_1Q_1. Measure its length as true length and its inclination with $p'q'$ as true inclination with V.P. Here, T.L. = 94 mm and $\phi = 25°$.
7. Draw perpendiculars pP_2 and qQ_2 to the line pq such that $pP_2 = op'$ and $qQ_2 = oq'$.
8. Join P_2Q_2. Measure its inclination with pq as true inclination with H.P. Here, $\theta = 40°$. Measure length of P_2Q_2 and ensure it is equal to true length obtained in Step 6.
9. Produce P_1Q_1 to meet with front view $p'q'$ produced at V.T. to represent vertical trace.
10. Produce P_2Q_2 to meet with top view pq produced at H.T. to represent horizontal trace.

Result True length, $p'q_1' = pq_2 = 94$ mm. Inclination with the H.P. $\theta = 40°$. Inclination with the V.P. $\phi = 25°$.

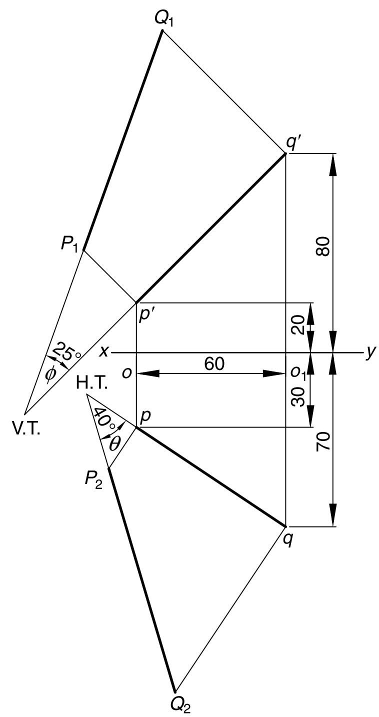


Fig. 9.17

9.14.4 Traces of a Line where $\theta + \phi < 90°$

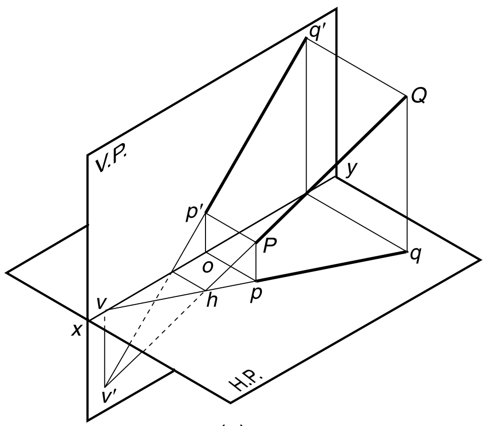
When a line is inclined to both the reference planes, it will meet both the H.P. and the V.P. Therefore, the line will have both the H.T. and the V.T. Consider the following problem.

Problem 9.18 *Locate the traces of a straight line PQ, kept in the first angle for the following cases.*

(a) *End P is 20 mm above the H.P. and 30 mm in front of the V.P. and the end Q is 80 mm above the H.P. and 60 mm in front of the V.P. The end projectors are 60 mm apart.*

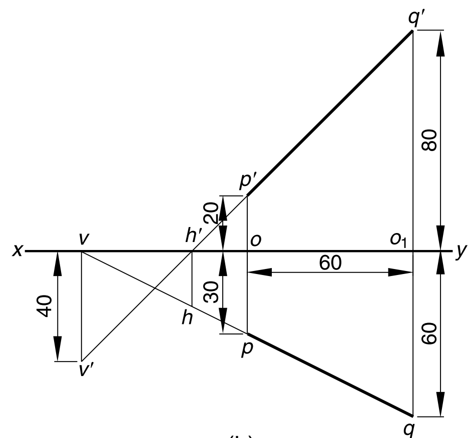
(b) *End P is 30 mm above the H.P. and 20 mm in front of the V.P. and the end Q is 60 mm above the H.P. and 80 mm in front of the V.P. The end projectors are 60 mm apart.*

(c) *End P is 20 mm above the H.P. and 30 mm in front of the V.P. and the end Q is 60 mm above the H.P. and 10 mm in front of the V.P. The end projectors are 60 mm apart.*



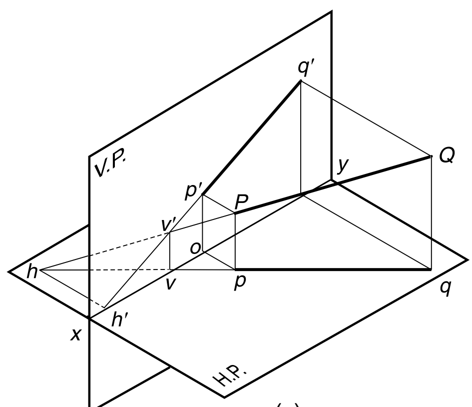
(a)

Fig. 9.18(a)



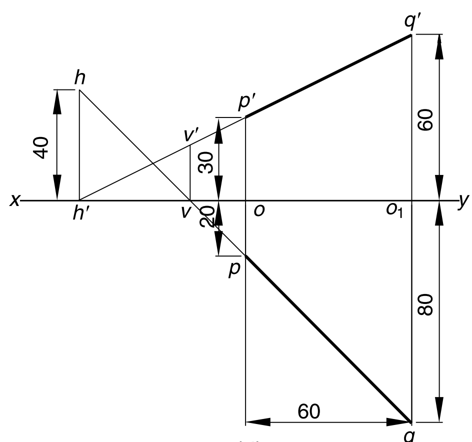
(b)

Fig. 9.18(b)



(c)

Fig. 9.18(c)



(d)

Fig. 9.18(d)

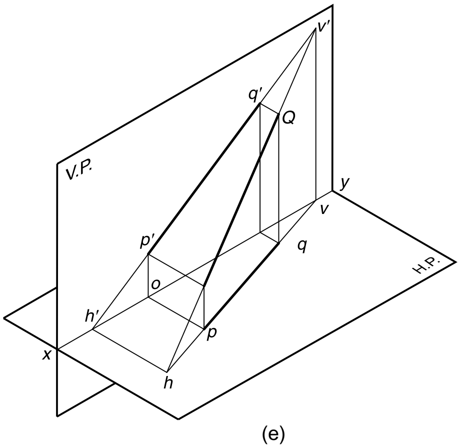


Fig. 9.18(e)

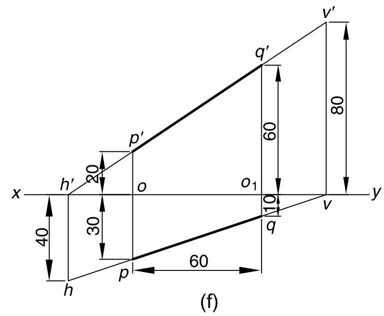


Fig. 9.18(f)

Visualisation

Case (a): Figure 9.18(a) shows pictorial view of a line situated in the space in given position with respect to the reference plane and Fig. 9.18(b) shows the projections of the line.

Case (b): Figure 9.18(c) shows pictorial view of a line situated in the space in given position with respect to the reference plane and Fig. 9.18(d) shows the projections of the line.

Case (c): Figure 9.18(e) shows pictorial view of a line situated in the space in given position with respect to the reference plane and Fig. 9.18(f) shows the projections of the line.

Construction Case (a)–Fig. 9.18(b), Case (b)–Fig. 9.18(d) and Case (c)–Fig. 9.18(f).

1. Draw a reference line *xy*. Mark points *o* and o_1 on *xy* such that they are 60 mm apart.
2. On the vertical projector through point *o*, mark points *p′* and *p* as the front and the top views of point *P*.
3. Similarly, on the vertical projector through point o_1, mark points *q′* and *q* as the front and the top views of point *Q*.
4. Join *p′q′* and *pq* to represent the front and the top views of the line *PQ*.
5. Produce the front view *p′q′* to meet *xy* at a point *h′*. Draw a vertical projector through point *h′* to meet the top view *pq*, produced if required, at point *h*. The point h represents the H.T.
6. Produce the top view *pq* to meet *xy* at a point *v*. Draw a vertical projector through point *v* to meet the front view *p′q′*, produced if necessary, at point *v′*. The point *v′* represents the V.T.
7. Measure the distance of *h* and *v′* from *xy*.

Results

(a) H.T. (*h*) is 20 mm below *xy* and V.T. (*v′*) is 40 mm below *xy*.
(b) H.T. (*h*) is 40 mm above *xy* and V.T. (*v′*) is 20 mm below *xy*.
(c) H.T. (*h*) is 40 mm below *xy* and V.T. (*v′*) is 80 mm above *xy*.

Conclusion The H.T. and V.T. may lie either on the same side or on the opposite sides of *xy*.

9.14.5 Projections of a Line Where $\theta + \phi = 90°$

A profile plane is perpendicular to both H.P. and V.P. When the sum of inclinations of a line with the H.P. and the V.P. is 90° (i.e. $\theta + \phi = 90°$), the line shall be parallel to the profile plane. In such a case, both the front and the top views shall be perpendicular to *xy* and shorter than the true length. In other words, both the apparent angles α and β shall be 90° each. Consider the following problem.

Problem 9.19 *A 100 mm long line PQ has its end P 10 mm above the H.P. and 70 mm in front of the V.P. The line is inclined at 60° to the H.P. and 30° to the V.P. Draw its projections.*

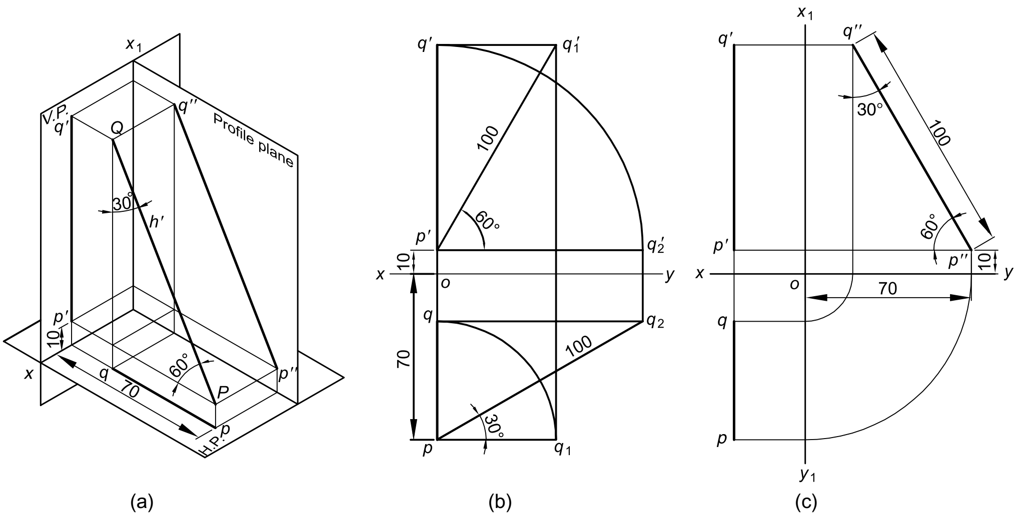


Fig. 9.19 **(a)** *Position of line* **(b)** *Method 1* **(c)** *Method 2*

Visualisation Figure 9.19(a) shows the pictorial view of a line *PQ* kept on the reference planes. The problem can be solved by two methods as follows:

Method 1 Change of position of line. (Similar to Problem 9.15)

Construction Refer to Fig. 9.19(b).

1. Draw a reference line *xy*.
2. On a vertical projector, mark point p' 10 mm above *xy* and point *p* 70 mm below *xy*.
3. Draw 100 mm long $p'q_1'$ inclined at $\theta = 60°$ to *xy*.
4. Draw another 100 mm long pq_2 inclined at $\phi = 30°$ to *xy*.
5. Project point q_1' to meet the horizontal line from point *p* at point q_1. Draw an arc with centre *p* and radius pq_1 to meet the horizontal line from point q_2 at point *q*. Join *pq* to represent the top view. It may be observed that *pq* is perpendicular to *xy*.
6. Project point q_2 to meet the horizontal line from point p' at point q_2'. Draw an arc with centre p' and radius $p'q_2'$ to meet the horizontal line from point q_1' at point q'. Join $p'q'$ to represent the front view. It may be observed that *pq* is perpendicular to *xy*.

Method 2 Use of profile plane.

Visualisation Consider the line is kept in the space as shown in Fig. 9.19(a). Let a profile plane exist on the right side of the line. The line is parallel to the profile plane such that it is inclined 60° to the H.P. and 30° with the V.P. It can be clearly seen that for such line, the front and the top views will appear as a line perpendicular to *xy*.

Construction Refer to Fig. 9.19(c).

1. Draw a reference line *xy* and x_1y_1, perpendicular to each other as shown.
2. Mark point p'' such that it is 10 mm above *xy* and 70 mm right to x_1y_1.
3. Draw $p''q''$ inclined at 60° to *xy*. The line is also inclined at 30° to x_1y_1.
4. Draw horizontal lines from points p'' and q'', and locate points p' and q' such that they lay on a line perpendicular to *xy*. Join $p'q'$ to represent the front view of the line.
5. Draw vertical projectors from p'' and q'' to meet *xy*. Rotate the obtained points keeping *o* as the centre to meet x_1y_1. Draw horizontal lines from them, known as locus lines for the top view.
6. Draw vertical projectors from p' and q' to meet the locus lines of the top view at points *p* and *q*. Join *pq* to represent the top view.

Conclusion When a line is inclined to the reference planes such that $\theta + \phi = 90°$ then the apparent angles $\alpha = \beta = 90°$, i.e., final front and top views are perpendicular to *xy*.

9.14.6 Traces of a Line Parallel to the Profile Plane

When a line is parallel to the profile plane, its traces can be conveniently located by the trapezoid method (refer to Section 9.14.3), as explained below. The traces can also be located with the help of a side view.

Problem 9.20 *The front and the top views of a straight line PQ lie on a common projector. Locate its traces when*

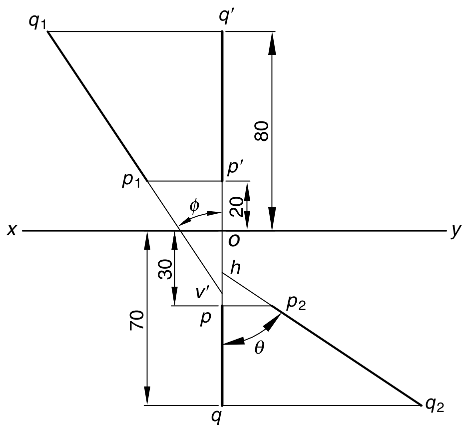
(a) The end P is 20 mm above the H.P. and 30 mm in front of the V.P. and the end Q is 80 mm above the H.P. and 70 mm in front of the V.P.

(b) The end P is 80 mm above the H.P. and 20 mm in front of the V.P. and the end Q is 30 mm above the H.P. and 70 mm in front of the V.P.

Visualisation As ends of the line lies on the common projector the line is parallel to the profile plane, i.e., $\theta + \phi = 90$. First draw the projections of the line and then determine the traces of the line in each of the cases.

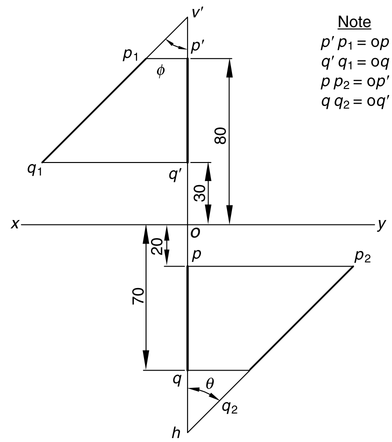
Construction Refer to Figs. 9.20(a) and (b).

1. Draw a reference line *xy*.
2. Mark points p', p, q' and q on a common projector as stated in the problem. Join $p'q'$ and pq to represent the front and the top views of line *PQ*, respectively.
3. Draw perpendicular lines $p'p_1$ and $q'q_1$ to the line $p'q'$ such that $p'p_1 = op$ and $q'q_1 = oq$. Points p_1 and q_1 should lie on the same side of $p'q'$, because ends *p* and *q* lies on the same side of *xy*.
4. Join p_1q_1 and produce it to meet front view $p'q'$ produced at point v'. Point v' denotes the V.T. of the line. The angle made by p_1q_1 with $p'q'$ represents the true inclination (ϕ) of the line *PQ* with the V.P.



(a)

Fig. 9.20(a)



(b)

Fig. 9.20(b)

5. Draw perpendicular lines pp_2 and qq_2 to the line pq such that $pp_2 = op'$ and $qq_2 = oq'$. Points p_2 and q_2 should lie on the same side of pq, because ends p' and q' lies on the same side of xy.
6. Join p_2q_2 and produce it to meet top view pq produced at point h. Point h denotes the H.T. of the line. The angle made by p_2q_2 with pq represents the true inclination (θ) of line PQ with H.P.

Result

(a) H.T. (h) is 17 mm below xy and V.T. (v') is 25 mm below xy.
(b) H.T. (h) is 100 mm below xy and V.T. (v') is 100 mm above xy.

Conclusion When $\theta + \phi = 90°$ then the traces (H.T. and V.T.) lie on the same projector that of the line PQ itself.

9.15 MISCELLANEOUS PROBLEMS

Problem 9.21 *An 80 mm long line AB is inclined at 30° to the H.P. and 45° to the V.P. The end A is 20 mm above the H.P. and lying in the V.P. Draw the projections of the line.*

Given Data	*Interpretation*
AB is 80 mm long	$a'b_1' = ab_2 = 80$ mm
End A is 20 mm above the H.P.	Point a′ is 20 mm above xy
End A lies in the V.P.	Point a is on xy
Line is inclined at 30° to the H.P. ($\theta = 30°$)	$a'b_1'$ is inclined at 30° to xy
Line is inclined at 45° to the V.P. ($\phi = 45°$)	ab_2 is inclined at 45° to xy

Construction Refer to Fig. 9.21.

1. Draw a reference line *xy*. Mark point a' 20 mm above *xy* and point *a* on *xy*.
2. Draw an 80 mm long line $a'b_1'$ inclined at 30° to *xy*.
3. Draw another 80 mm long line ab_2 inclined at 45° to *xy*.
4. Project b_1' to meet horizontal line from point *a* at point b_1. Draw an arc with centre *a* and radius ab_1 to meet the horizontal line from point b_2 at point *b*. Join *ab* to represent the top view.
5. Project b_2 to meet horizontal line from point a' at point b_2'. Draw an arc with centre a' and radius $a'b_2'$ to meet the horizontal line from point b_1' at point b'. Join $a'b'$ to represent the front view.
6. Join $b'b$ and ensure that it is perpendicular to *xy*, representing projector of the end *B*.

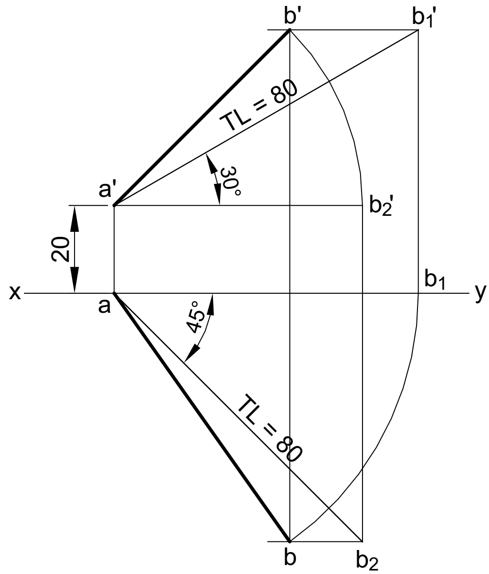


Fig. 9.21

Problem 9.22 *A 100 mm long line PQ is inclined at 30° to the H.P. and 45° to the V.P. Its mid-point is 35 above the H.P. and 50 mm in front of the V.P. Draw its projections.*

Given Data	*Interpretation*
PQ = 100 mm and M is the mid-point	$p_1'm' = m'q_1'$ = 50 mm; $p_2m = mq_2$ = 50 mm
Mid-point M is 35 mm above the H.P.	m' is 35 mm above xy
Mid-point M is 50 mm in front of V.P.	m is 50 mm below xy
Line is inclined at 30° to the H.P. (θ = 30°)	$p'm'q_1'$ is inclined at 30° to xy
Line is inclined at 45° to the V.P. (ϕ = 45°)	p_2mq_2 is inclined at 45° to xy

Construction Refer to Fig. 9.22.

1. Draw a reference line *xy*. On a vertical projector, mark point m' 35 mm above *xy* and point *m* 50 mm below *xy*.
2. Draw a 50 mm long line $m'q_1'$ inclined at 30° to *xy*. Produce it such that $p_1'q_1'$ = 100 mm.
3. Draw another 50 mm line mq_2 inclined at 45° to *xy*. Produce it such that p_2q_2 = 100 mm.
4. Project points p_1' and q_1' to meet horizontal line through point *m* at points p_1 and q_1 respectively. Draw an arc with centre *m* and radius mp_1 or

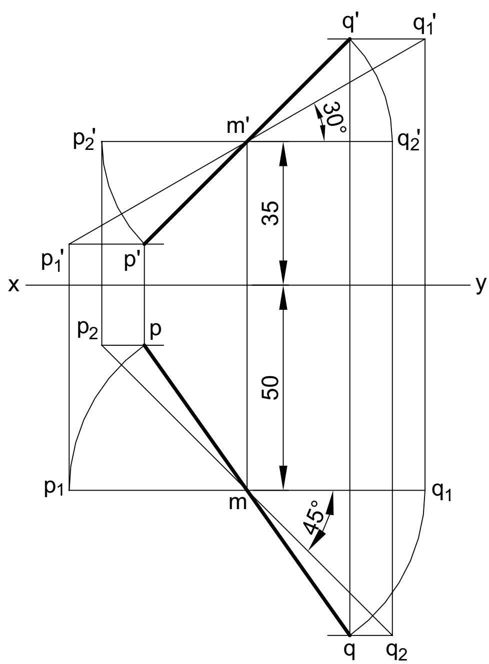


Fig. 9.22

mq_1 to meet the horizontal lines from points p_2 and q_2 at points p and q, respectively. Join pmq to represent the top view.

5. Project points p_2 and q_2 to meet the horizontal line through point m' at points p_2' and q_2' respectively. Draw an arc with centre m' and radius $m'p_2'$ or $m'q_2'$ to meet the horizontal lines through points p_1' and q_1' at points p' and q', respectively. Join $p'm'q'$ to represent front view.
6. Join $p'p$ and $q'q$ to ensure that they represent projector of the ends P and Q respectively.

Problem 9.23 *A 75 mm long line PQ lying in the first angle has its end P on the H.P. and end Q in the V.P. The line is inclined at 45° to the H.P. and 30° to the V.P. Draw its projections.*

Given Data	*Interpretation*
PQ is 75 mm long	$p'q_1' = pq_2 = 75$ mm
End *P* is in the H.P.	Points p' and q_2' are on *xy*
End *Q* is in the V.P.	Points q and q_2 are on *xy*
Line is inclined at 45° to the H.P. ($\theta = 45°$)	$p'q_1'$ is inclined at 45° to *xy*
Line is inclined at 30° to the V.P. ($\phi = 30°$)	p_2q_2 is inclined at 30° to *xy*

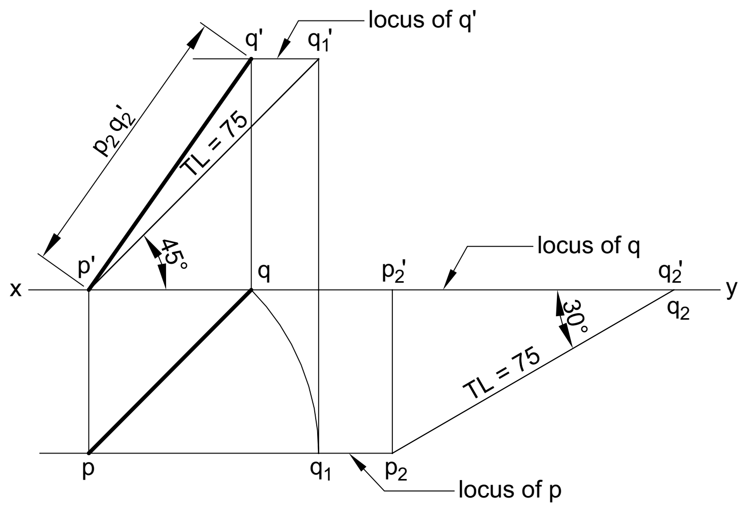


Fig. 9.23

Construction Refer to Fig. 9.23.

1. Draw a reference line *xy*. Mark points q_2 and q_2' coinciding each other on *xy*.
2. Draw a 75 mm long line p_2q_2 inclined at 30° to *xy*.
3. Projector p_2 to meet *xy* at point p_2'. Join $p_2'q_2'$. The length $p_2'q_2'$ represents the length of the front view of line *PQ*.
4. Draw a horizontal line from p_2 as the locus of point *p*.
5. Mark a point p' anywhere on the *xy*. Project point p' to meet the locus of *p* at point *p*.
6. Draw 75 mm long $p'q_1'$ inclined at 45° to *xy*.

7. Projector q_1' to meet horizontal line from point p at point q_1. Draw an arc with centre p and radius pq_1 to meet the horizontal line from point q_2 at point q. Point q lie on xy. Join pq to represent the top view.
8. Draw an arc with centre p' and radius $p_2'q_2'$ to meet the horizontal line from point q_1' at point q'. Join $p'q'$ to represent the front view.
9. Join $q'q$ and ensure that it is perpendicular to xy, representing projector of the end Q.

Problem 9.24 *A 70 mm long line PQ has its end P 20 mm above the H.P. and 40 mm in front of the V.P. The other end Q is 60 mm above the H.P. and 10 mm in front of the V.P. Draw the projections of PQ and determine its inclinations with the reference planes.*

Given Data	*Interpretation*
PQ is 70 mm long	$p'q_1' = pq_2 = 70$ mm
End P is 20 mm above the H.P.	Point p' is 20 mm above xy
End P is 40 mm in front of the V.P.	Point p is 40 mm below xy
End Q is 60 mm above the H.P.	Points q' and q_1' are 60 mm above xy
End Q is 10 mm in front of the V.P.	Points q and q_2 are 10 mm below xy

Construction Refer to Fig. 9.24.

1. Draw a reference line xy. On a projector, mark point p' 20 mm above xy and point p 40 mm below xy.
2. Draw a line ab parallel to and 60 mm above xy as the locus of point q'.
3. Draw another line cd parallel to and 10 mm below xy as the locus of point q.
4. Draw an arc with centre p' and radius 70 mm to meet ab at point q_1'. Join $p'q_1'$ to represent true inclination of line with the H.P. Here $\theta = 35°$.
5. Draw an arc with centre p and radius 70 mm to meet cd at point q_2. Join pq_2 to represent true inclination of line with the V.P. Here $\phi = 25°$.
6. Project q_1' to meet horizontal line from point p at point q_1. Draw an arc with centre p and radius pq_1 to meet cd at point q. Join pq to represent the top view.
7. Project q_2 to meet horizontal line from point p' at point q_2'. Draw an arc with centre p' and radius $p'q_2'$ to meet ab at point q'. Join $p'q'$ to represent the front view.
8. Join $q'q$ and ensure that it is perpendicular to xy, representing projector of the end Q.

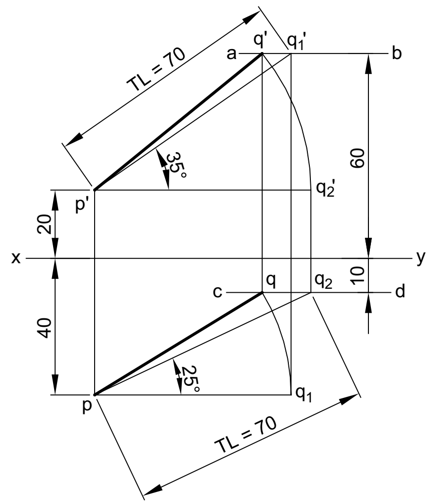


Fig. 9.24

Result Inclination with the H.P., $\theta = 35°$. Inclination with the V.P., $\phi = 25°$

Problem 9.25 *The top view of 75 mm long line PQ measures 50 mm. The end P is 15 mm above the H.P. and 50 mm in front of the V.P. The end Q is 20 mm in front of the V.P. and above the H.P. Draw the projections of PQ and determine its inclinations with the reference planes.*

Given Data	*Interpretation*
PQ is 75 mm long	$p'q_1' = pq_2 = 75$ mm
Top view measures 50 mm	$pq = pq_1 = 50$ mm
End *P* is 15 mm above the H.P.	Point p' is 15 mm above *xy*
End *P* is 50 mm in front of the V.P.	Point *p* is 50 mm below *xy*
End *Q* is 20 mm in front of the V.P.	Points *q* and q_2 are 20 mm below *xy*

Construction Refer to Fig. 9.25.

1. Draw a reference line *xy*. On a projector, mark point p' 15 mm above *xy* and point *p* 50 mm below *xy*.
2. Draw a line *cd* parallel to and 20 mm below *xy* as the locus of point *q*.
3. Draw an arc with centre *p* and radius 75 mm to meet *cd* at point q_2. Join pq_2 to represent true inclination of line with the V.P. Here $\phi = 24°$.
4. Draw another arc with centre *p* and radius 50 mm to meet *cd* at point *q*. Join *pq* to represent the top view.
5. Draw an arc with centre *p* and radius *pq* to meet horizontal line from point *p* at point q_1. Join
6. Draw an arc with centre p' and radius 75 to meet projector of point q_1 at point q_1'. Join $p'q_1'$ to represent true inclination of line with the H.P. Here $\theta = 48°$.
7. Project q_2 to meet horizontal line from point p' at point q_2'. Draw an arc with centre p' and radius $p'q_2'$ to meet horizontal line from point q_1' at point q'. Join $p'q'$ to represent the front view.
8. Join $q'q$ and ensure that it is perpendicular to *xy*, representing projector of the end *Q*.

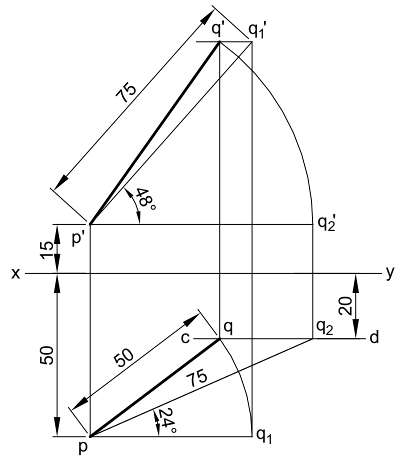


Fig. 9.25

Result Inclination with the H.P., $\theta = 48°$. Inclination with the V.P., $\phi = 24°$

Problem 9.26 *The front and top views of 75 mm long line PQ measures 50 mm and 60 mm, respectively. If the end P of the line is 35 mm above the H.P. and 15 in front of the V.P., draw its projections and locate the traces. Determine the true inclinations of the line PQ with the H.P. and the V.P.*

Given Data	*Interpretation*
PQ is 75 mm long	$p'q_1' = pq_2 = 75$ mm
Front view measures 50 mm	$p'q' = p'q_2' = 50$ mm
Top view measures 60 mm	$pq = pq_1 = 60$ mm
End *P* is 35 mm above the H.P.	Point p' is 35 mm above *xy*
End *P* is 15 mm in front of the V.P.	Point *p* is 15 mm below *xy*

Construction Refer to Fig. 9.26.

1. Draw a reference line *xy*. On a projector, mark point p' 35 mm above xy and point *p* 15 mm below *xy*.

2. Draw a 50 mm long line $p'q_2'$ parallel to xy. Draw another 60 mm long line pq_1 parallel to xy.
3. Draw an arc with centre p' and radius 75 mm to meet projector of q_1 at point q_1'. Join $p'q_1'$ to represent true inclination of line with the H.P. Here $\theta = 37°$.
4. Draw an arc with centre p and radius 75 mm to meet projector of q_2' at point q_2. Join pq_2 to represent true inclination of line with the V.P. Here $\phi = 48°$.
5. Draw an arc with centre p' and radius $p'q_2'$ (50 mm) to meet horizontal line from point q_1' at point q'. Join $p'q'$ to represent the front view.
6. Draw an arc with centre p and radius pq_1 (60 mm) to meet horizontal line from point q_2 at point q. Join pq to represent the top view.
7. Join $q'q$ and ensure that it is perpendicular to xy, representing projector of the end Q.
8. Produce $p'q'$ to meet xy at a point h'. Draw a vertical projector through point h' to meet the pq produced at point h. The point h represents the H.T. Here h is 28 mm above xy.

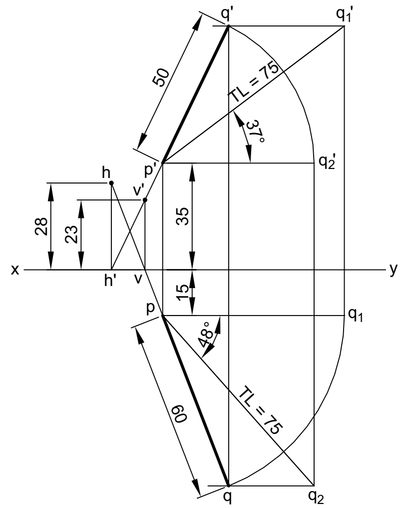


Fig. 9.26

9. Produce pq to meet xy at a point v. Draw a vertical projector through point v to meet $p'q'$, produced at point v'. Point v' represents the V.T. Here, point v' is 23 mm above xy.

Result Inclination with the H.P., $\theta = 37°$. Inclination with the V.P., $\phi = 48°$. H.T. (h) is 28 mm above xy. V.T. (v') is 23 mm above xy.

Problem 9.27 *The front and top views of a straight line PQ measures 50 mm and 65 mm, respectively. The point P is on the H.P. and 20 mm in front of the V.P. The front view of the line is inclined at 45° to the reference line. Determine the true length of PQ and its true inclinations with the reference planes. Also, locate the trace.*

Given Data	*Interpretation*
Front view measures 50 mm	$p'q' = p'q_2' = 50$ mm
Top view measures 65 mm	$pq = pq_1 = 65$ mm
End P is in the H.P.	Point p' is on xy
End P is 20 mm in front of the V.P.	Point p is 20 mm below xy
Front view is inclined 45° to xy ($\alpha = 45°$)	$p'q'$ is inclined at 45° to xy

Construction Refer to Fig. 9.27.

1. Draw a reference line xy. Mark point p' on xy and point p 20 mm below xy.
2. Draw a 50 mm long line $p'q'$ inclined at $\alpha = 45°$ to xy. Line $p'q'$ represents the front view.
3. Draw an arc with centre p and radius 65 mm to meet the projector from point q' at point q. Join pq to represent the top view.

4. Draw an arc with centre p' and radius $p'q'$ to meet the horizontal line from p' at point q_2'. Project q_2' to meet the horizontal line from point q at point q_2. Join pq_2 and measure its length as true length. Also, measure the inclination of pq_2 with xy as true inclination of line with the V.P. Here T.L. = 74 mm and $\phi = 47°$.
5. Draw another arc with centre p and radius pq to meet the horizontal line through p at point q_1. Project q_1 to meet the horizontal line from point q' at point q_1'. Join $p'q_1'$ and ensure that its length is equal to that of pq_2. Measure the inclination of $p'q_1'$ with xy as true inclination of line with the H.P. Here $\theta = 28°$.
6. Line $p'q'$ meets xy at point h'. Project point h' to meet line pq at point h. The point h represents the H.T. Here h is 20 mm below xy, coinciding with point p.
7. Produce line pq to meet xy at point v. Project point v to meet line $p'q'$ produced at point v'. The point v' represents the V.T. Here v' is 13 mm below xy.

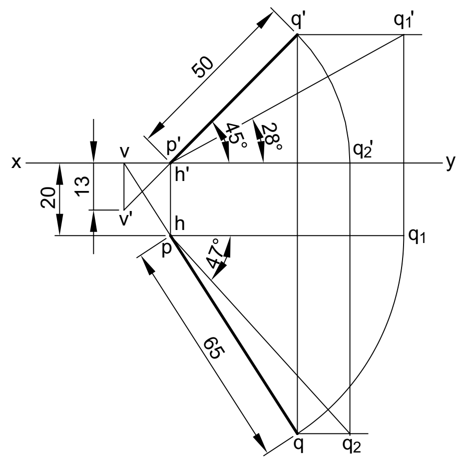


Fig. 9.27

Result True length, $p'q_1' = pq_2 = 74$ mm. Inclination with the H.P., $\theta = 28°$. Inclination with the V.P., $\phi = 47°$. H.T. (h') is 20 mm below xy. V.T. (v) is 13 mm below xy.

Problem 9.28 *A 70 mm long line PQ is inclined at 45° to the V.P. Its end P lies on the H.P. and 15 mm in front of the V.P. The top view of the line measures 60 mm. Draw the projections of the line PQ and determine its inclination with the H.P.*

Given Data	*Interpretation*
PQ is 70 mm long	$p'q_1' = pq_2 = 70$ mm
Line is inclined at 45° to the V.P. ($\phi = 45°$)	pq_2 is inclined at 45° to xy
End P is on the H.P.	Point p' is on xy line
End P is 15 mm in front of the V.P.	Point p is 15 mm below xy
Top view mesures 60 mm	$pq = pq_1 = 60$ mm

Construction Refer to Fig. 9.28.

1. Draw a reference line xy. Mark point p' on xy and point p 15 mm below xy.
2. Draw a 70 mm long line pq_2 inclined at $\phi = 45°$ to xy.
3. Draw an arc with centre p and radius 60 mm to meet the horizontal line through point q_2 at point q. Join pq to represent the top view.
4. Draw an arc with centre p and radius pq to meet the horizontal line from point p at point q_1. Draw another arc with

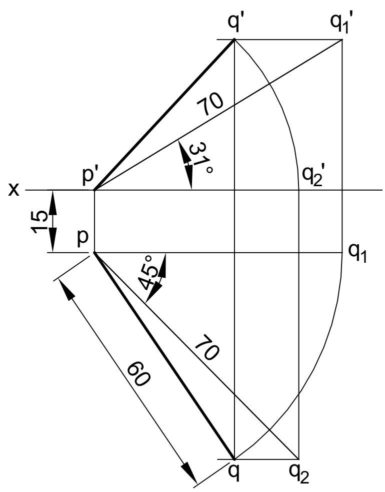


Fig. 9.28

centre p' and radius 70 mm to meet the projector of point q_1 at point q_1'. Join $p'q_1'$ to represent the true inclination of line with the H.P. Here $\theta = 31°$

5. Draw a vertical line from point q_2 to meet the horizontal line from point p' at point q_2'. Draw an arc with centre p' and radius $p'q_2'$ to meet the horizontal line from point q_1' at point q'. Join $p'q'$ to represent the front view.
6. Join $q'q$ and ensure that it is perpendicular to xy, representing projector of the end Q.

Result Inclination with the H.P., $\theta = 31°$.

Problem 9.29 *A 70 mm long line PQ is inclined at 30º to the H.P. The end P is 15 mm in front of the V.P. and 25 mm above the H.P. The front view of the line measures 45 mm. Draw the projections of the line PQ and determine its true angle of inclination with the V.P.*

Given Data	*Interpretation*
PQ is 70 mm long	$p'q_1' = pq_2 = 70$ mm
Line is inclined at 30º to the H.P. ($\theta = 30º$)	$p'q_1'$ is inclined at 30º to xy
End *P* is 25 mm above the H.P.	Point p' is 25 mm above xy
End *P* is 15 mm in front of the V.P.	Point p is 15 mm below xy
Front view measures 45 mm	$p'q' = p'q_2' = 45$ mm

Construction Refer to Fig. 9.29.

1. Draw a reference line xy. Mark point p' 25 mm above xy and point p 15 mm below xy.
2. Draw a 70 mm long line $p'q_1'$ inclined at $\theta = 30°$ to xy.
3. Draw an arc with centre p' and radius 45 mm to meet the horizontal line from point q_1' at point q'. Join $p'q'$ to represent the front view.
4. Draw an arc with centre p' and radius $p'q'$ to meet the horizontal line from point p' at point q_2'. Draw another arc with centre p and radius 70 mm to meet the projector from point q_2' at point q_2. Join pq_2 and measure its inclination with xy to represent inclination of line *PQ* with V.P. Here $\phi = 50º$.
5. Project point q_1' to meet the horizontal line from point p at point q_1. Draw an arc with centre p and radius pq_1 to meet the horizontal line from point q_2 at point q. Join pq to represent the top view.
6. Join $q'q$ and ensure that it is perpendicular to xy, representing projector of the end Q.

Result Inclination with the V.P., $\phi = 50º$.

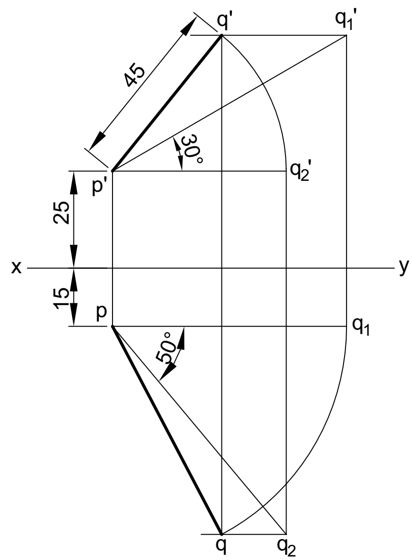


Fig. 9.29

Problem 9.30 *The front and top views of an 80 mm long line PQ measures 70 mm and 60 mm, respectively. The end P is on the H.P. and the end Q is in the V.P. Draw the projections of line PQ and determine its inclinations with the H.P. and the V.P. Also, locate the traces.*

Given Data	*Interpretation*
PQ is 80 mm long	$p'q_1' = pq_2 = 80$ mm
Front view measures 70 mm	$p'q' = p'q_2' = 70$ mm
Top view measures 60 mm	$pq = pq_1 = 60$ mm
End *P* is on the H.P.	Point p' is on *xy*
End *Q* is in the V.P.	Point *q* is on *xy*

Construction Refer to Fig. 9.30.

1. Draw a reference line *xy*. Mark point p' on *xy*.
2. Draw a 60 mm long line $p'q_1''$ Draw an arc with centre p' and radius 80 mm to meet the projector from point q_1'' at point q_1'. Join $p'q_1'$. Measure the inclination of $p'q_1'$ with *xy* as inclination of the line with the H.P. Here $\theta = 41°$.
3. Draw an arc with centre p' and radius 70 mm to meet the horizontal line from point q_1' at point q'. Join $p'q'$ to represent the front view.
4. Project point q' to meet *xy* at point *q* (because *q* lies on *xy*).
5. From point *q*, draw an arc of radius 60 mm to meet the projector from point p' at point *p*. Join *pq* to represents the top view.
6. Draw an arc with centre p' and radius $p'q'$ to meet the horizontal line from point p' at point q_2. Join pq_2. Measure the inclination of pq_2 with *xy* as inclination of the line with the V.P. Here $\phi = 29°$. Also, ensure that pq_2 is 80 mm long.
7. Line $p'q'$ meets *xy* at point h'. Project point h' to meet line *pq* at point *h*. The point *h* represents the H.T. Here, *h* is 39 mm below *xy*, coinciding with point *p*.
8. Similarly, line *pq* meets *xy* at point *v*. Project point *v* to meet line $p'q'$ at point v'. The point v' represents the V.T. Here, v' is 53 mm above *xy*, coinciding with point q'.

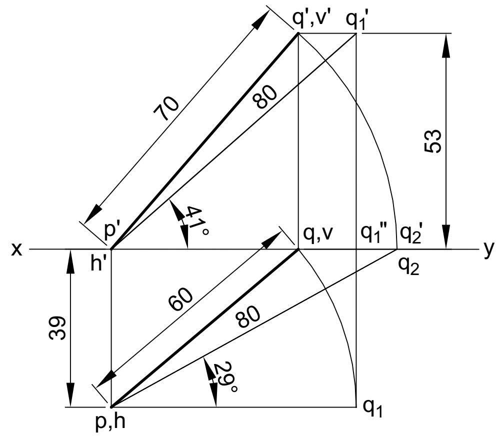


Fig. 9.30

Result Inclination with the H.P., $\theta = 41°$. Inclination with the V.P., $\phi = 29°$. H.T. (*h*) is 39 mm below *xy*. V.T. (v') is 53 mm above *xy*.

Problem 9.31 *The distance between the end projectors of a line PQ is 50 mm. The end P is 50 mm in front of the V.P. and 25 mm above the H.P. The end Q is 10 mm in front of the V.P. and above the H.P. The line is inclined at 30° to the V.P. Draw the projections of line PQ. Determine its true length and true angle of inclination with the H.P.*

Given Data	*Interpretation*
Distance between the end projectors is 50 mm	$oo_1 = 50$ mm
End *P* is 50 mm in front of the V.P.	Point *p* is 50 mm below *xy*
End *P* is 25 mm above the H.P.	Point p' is 25 mm above *xy*
End *Q* is 10 mm in front of the V.P.	Points *q* and q_2 are 10 mm below *xy*
Line is inclined at 30º to the V.P. ($\phi = 30º$)	pq_2 is inclined at 30º with *xy*

Construction Refer to Fig. 9.31.

1. Draw a reference line xy. Mark points o and o_1 on xy such that oo_1 = 50 mm.
2. On the projector through point o, mark point p' 25 mm above xy and point p 50 mm below xy.
3. On the projector through point o_1, mark point q 10 mm below xy. Join pq to represent the top view.
4. Draw a line from point p, inclined at 30° with xy to meet the horizontal line through point q at point q_2. Measure its length as true length of line PQ. Here, T.L. = 80 mm.
5. Project point q_2 to meet the horizontal line through point p' at point q_2'. Draw an arc with point centre p' and radius $p'q_2'$ to meet the projector from point q at point q'. Join $p'q'$ to represent the front view.
6. Draw an arc with p as centre and radius pq to meet the horizontal line through point p at point q_1. Project point q_1 to meet the horizontal line through point q' at point q_1'. Join $p'q_1'$. Measure its inclination with xy as inclination of the line with the H.P. Here θ = 37°.
7. Also ensure that lengths of $p'q_1'$ is equal to that of pq_2, representing true length of line PQ.

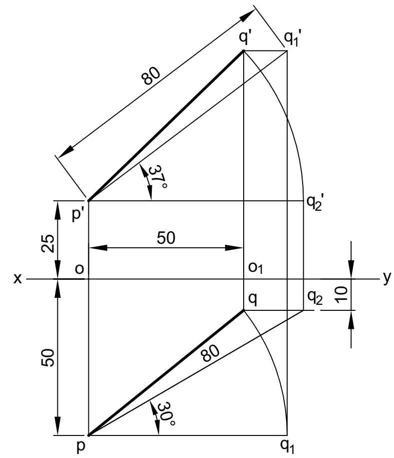


Fig. 9.31

Result True length $p'q_1' = pq_2$ = 80 mm. Inclination with the H.P., θ = 37º.

Problem 9.32 *An 80 mm long line PQ has its end P on the H.P. and 15 mm in front of the V.P. The line is inclined at 30° to the H.P. and its top view is inclined at 60° to the reference line. Draw the projections of line PQ and determine true angle of inclination with the V.P.*

Given Data	*Interpretation*
PQ is 80 mm long	$p'q_1' = pq_2$ = 80 mm
End P is on the H.P.	Point p' is on xy
End P is 15 mm in front of the V.P.	Point p is 15 mm below xy
Line is inclined at 30º to the H.P. (θ = 30º)	$p'q_1'$ is inclined at 30º to xy
Top view is inclined at 60º to the V.P. (β = 60º)	pq is inclined at 60º to xy

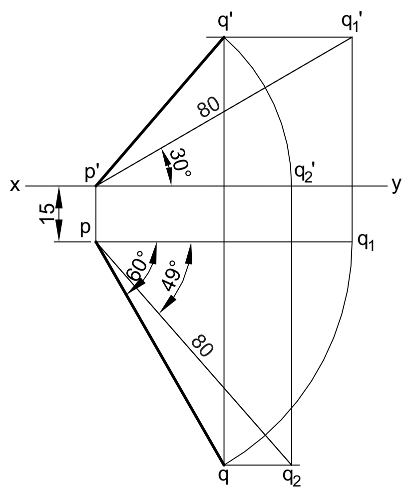


Fig. 9.32

Construction Refer to Fig. 9.32.

1. Draw a reference line xy. Mark point p' on xy and point p 15 mm below xy.
2. Draw an 80 mm long line $p'q_1'$, inclined at 30º to xy.
3. Project point q_1' to meet horizontal line from point p at point q_1.
4. Draw an arc with centre p and radius pq_1 to meet a line pq inclined at 60º to xy at point q. Join pq to represent the top view.
5. Draw another arc with centre p and radius 80 mm to meet the horizontal line from point q at point q_2. Measure its inclination with xy as the inclination of line with the V.P. Here ϕ = 49º.

6. Project point q_2 to meet the horizontal line through p' at point q_2'. Draw an arc with centre p' and radius $p'q_2'$ to meet the horizontal line through point q_1' at point q'. Join $p'q'$ to represent the front view.
7. Join $q'q$ and ensure that it is perpendicular to xy, representing projector of the end Q.

Result Inclination with the V.P., $\phi = 49°$.

Problem 9.33 *An 80 mm long line PQ has its end P 10 mm above the H.P. and 25 mm in front of the V.P. The line is inclined at 30° to the H.P. and 60° to the V.P. Draw its projections.*

Given Data	*Interpretation*
PQ is 80 mm long	$p'q_1' = pq_2 = 80$ mm
P is 10 mm above the H.P.	p' is 10 above xy
P is 25 mm in front of the V.P.	p is 25 mm below xy
Line is inclined at 30° to the H.P. ($\theta = 30°$)	$p'q_1'$ is inclined at 30° to xy
Line is inclined at 60° to the V.P. ($\phi = 60°$)	pq_2 is inclined at 60° to xy

Construction Refer to Fig. 9.33.

1. Draw a reference line xy. Mark point p' 10 mm above xy and point p 25 mm below xy.
2. Draw an 80 mm long line $p'q_1'$ inclined at 30° to xy.
3. Draw another 80 mm long line pq_2 inclined at 60° to xy.
4. Project q_1' to meet horizontal line from p at point q_1. Draw an arc with centre p and radius pq_1 to meet the horizontal line from point q_2 at point q. Join pq to represent the top view.
5. Project q_2 to meet horizontal line from p' at point q_2'. Draw an arc with centre p' and radius $p'q_2'$ to meet the horizontal line from q_1' at point q'. Join $p'q'$ to represent the front view.
6. Join $q'q$ and ensure that it that it is perpendicular to xy.
7. It may be noted that when $\theta + \phi = 90°$, both the front and the top views are perpendicular to xy. In other words, the apparent inclinations of the line with H.P. and V.P. are 90°, i.e., $\alpha = \beta = 90°$.

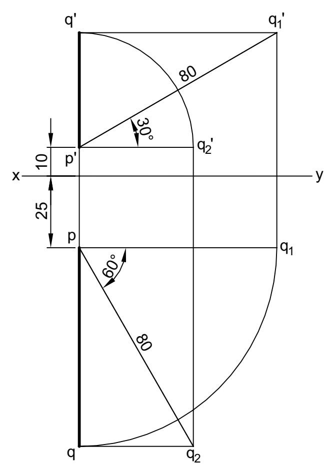


Fig. 9.33

Problem 9.34 *A 75 mm long line PQ has its end P 15 mm above the H.P. and 20 mm in front of the V.P. The front and top views are 45 mm and 60 mm long, respectively. Draw the projections of line PQ and determine its true inclinations with the reference planes.*

Given Data	*Interpretation*
PQ is 75 mm long (T.L. = 75 mm)	$p'q_1' = pq_2 = 75$ mm
P is 15 mm above the H.P.	Point p' is 15 above xy
P is 20 mm in front of the V.P.	Point p is 20 mm below xy
Front view is 45 mm long (F.V. = 45 mm)	$p'q' = p'q_2' = 45$ mm
Top view is 60 mm long (T.V. = 60 mm)	$pq = pq_1 = 60$ mm

Construction Refer to Fig. 9.34.

1. Draw a reference line xy. Mark point p' 15 mm above xy and point p 20 mm below xy.
2. Draw a 45 mm long line $p'q_2'$ parallel to xy.
3. Draw another 60 mm long line pq_1, parallel to xy.
4. Draw an arc with centre p' and radius 75 mm to meet the projector from point q_1 at point q_1'. Join $p'q_1'$. Measure its inclination with xy as true inclination of line with the H.P. Here $\theta = 37°$.
5. Draw an arc with centre p and radius 75 mm to meet the projector from point q_2' at point q_2. Join pq_2. Measure its inclination with xy as true inclination of line with the V.P. Here, $\phi = 53°$.
6. Draw an arc with centre p' and radius $p'q_2'$ to meet the horizontal line from point q_1' at point q'. Join $p'q'$ to represent the front view. Here front view is perpendicular to xy, i.e., $\alpha = 90°$.
7. Draw an arc with centre p and radius pq_1 to meet the horizontal line from point q_2 at point q. Join pq to represent the top view. Here top view is perpendicular to xy, i.e., $\beta = 90°$.
8. Join $q'q$ and ensure that it that it is perpendicular to xy.
9. It may be noted that, here $(F.V.)^2 + (T.V.)^2 = (T.L.)^2$, therefore both the front and the top views are perpendicular to xy, i.e., $\alpha = \beta = 90°$. Also, sum of the true incliantions of the line with H.P. and V.P. is 90°, i.e., $\theta + \phi = 90°$.

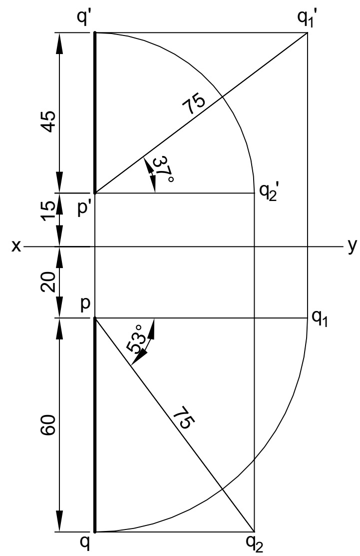


Fig. 9.34

Result Inclination with the H.P., $\theta = 37°$. Inclination with the V.P., $\phi = 53°$.

Problem 9.35 *An 80 mm long line PQ has its end P 15 mm above the H.P. and 50 mm in front of the V.P. while the end Q is in the V.P. Draw the projections of the line when the sum of its inclination with the H.P. and V.P. is 90°. Determine the true inclinations of line PQ with the reference planes and locate its traces.*

Given Data	*Interpretation*
PQ = 80 mm	$p'q_1' = pq_2$ = 80 mm
End P is 15 mm above the H.P.	p' is 15 above xy
End P is 50 mm in front of the V.P.	p is 50 mm below xy
End Q is in the V.P.	q is on xy
$\theta + \phi = 90°$	$\alpha = \beta = 90°$, $p'q'$ and pq are perpendicular to xy

Construction Refer to Fig. 9.35.

1. Draw a reference line xy. On a vertical projector, mark point p' 15 mm above xy, point p 50 mm below xy and point q on xy. Join pq to represent the top view.
2. Draw an arc with centre p and radius 80 mm to meet the horizontal line from point q at point q_2. Join pq_2. Measure its inclination with xy as true inclination of line with the V.P. Here, $\phi = 39°$.

3. Draw an arc with centre p and radius pq to meet the horizontal line from p at point q_1. Draw another arc with centre p' and radius 80 mm to meet the projector from point q_1 at point q_1'. Join $p'q_1'$. Measure its inclination with xy as the true inclination of line with the H.P. Here, $\theta = 51°$.
4. Project point q_2 to meet the horizontal line from point p' at point q_2'. Draw an arc with centre p' and radius $p'q_2'$ to meet the horizontal line from point q_1' at point q. Join $p'q'$ to represent front view. Here $p'q'$ is perpendicular to xy, i.e., $\alpha = 90°$.
5. Draw lines $p'a$ and $q'b$ perpendicular to $p'q'$ on the same side of $p'q'$ such that $p'a = op$, $q'b = oq$. Join ab and produce it to meet line $p'q'$ at point v'. The point v' represents the V.T. Here v' is 77 mm above xy and coincides with q'.

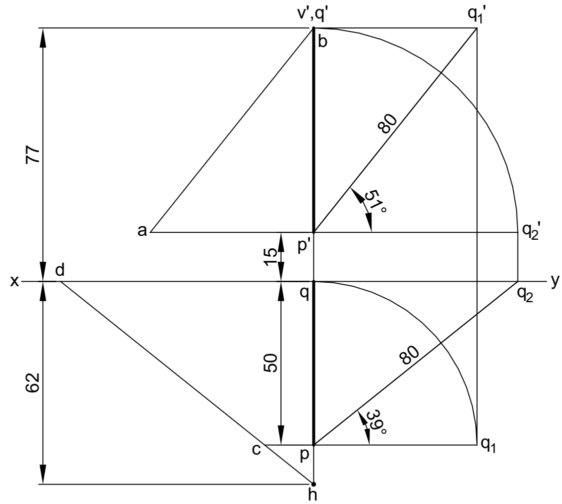


Fig. 9.35

6. Draw lines pc and qd perpendicular to pq on the same side of pq such that $pc = op'$, $qd = oq'$. Join cd and produce it to meet line pq produced at point h. Point h represents the H.T. Here h is 62 mm below xy.

Result Inclination with the H.P., $\theta = 51°$. Inclination with the V.P., $\phi = 39°$. V.T. (v') is 77 mm above xy. H.T. (h) is 62 mm below xy.

Problem 9.36 *A line PQ inclined at an angle of 30° to the H.P. has ends P and Q 30 mm and 65 mm in front of the V.P., respectively. The length of the top view is 60 mm and its H.T. is 15 mm in front of the V.P. Draw the projections of the line PQ and determine its true length and the V.T.*

Given Data	*Interpretation*
Line is inclined at 30° to H.P. ($\theta = 30°$)	$h'q_h'$ is inclined at 30° to xy
End P is 30 mm in front of the V.P.	Point p is 30 mm below xy
End Q is 65 mm in front of the V.P.	Points q and q_2 are 65 mm below xy
Top view is 60 mm	pq = 60 mm
H.T. is 15 mm in front of the V.P.	h is 15 mm below xy, $h'h$ = 15 mm

Construction Refer to Fig. 9.36.

1. Draw a reference line xy. Mark point p 30 mm below xy.
2. Draw a horizontal line 65 mm below xy as the loci of q and q_2. Draw an arc with centre p and radius 60 mm to meet the locus of q at point q. Join pq to represent the top view.
3. Draw a horizontal line 15 mm below xy as the locus of h. Produce pq to meet the locus of h at point h. Point h represents the H.T. Project point h to meet xy at point h'.
4. Draw an arc with centre h and radius hq to meet the horizontal line from point h at point q_h. Draw

a line from point h' inclined at 30º to xy to meet the projector from point q_h at point q_h'. Join $h'q_h'$.

5. Project point q to meet the horizontal line from q_h' at point q'. Join $h'q'$. Project point p to meet line $h'q'$ at point p'. Join $p'q'$ to represent the front view.
6. Draw an arc with centre p' and radius $p'q'$ to meet the horizontal line from point p' at point q_2'. Project point q_2' to meet the horizontal line from point q at point q_2. Join pq_2. Measure pq_2 as true length and its inclination with xy as true inclination with the V.P. Here, T.L. = 69 mm and $\phi = 30º$.
7. Produce $a'b'$ to meet xy at point v. Project v to meet $p'q'$ produced at point v'. Point v' represents the V.T. Here v' is 15 mm below xy.

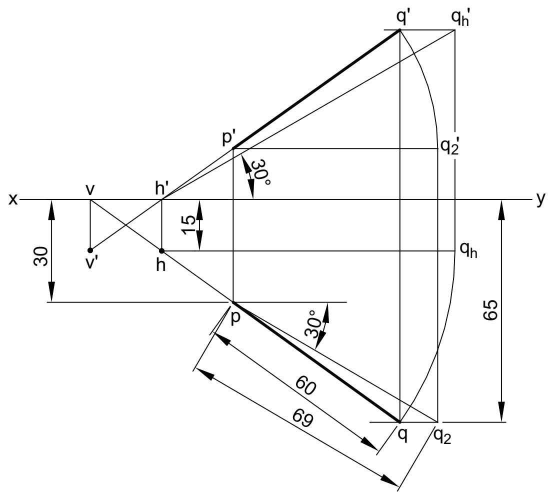


Fig. 9.36

Result True length $p'q_1' = 69$ mm. Inclination with the V.P., $\phi = 30º$. V.T. (v') is 15 mm below xy.

Problem 9.37 *A line PQ inclined at 30° to the V.P. has the end P 15 above the H.P. Its front view measures 70 mm and is inclined at 45° to reference line. The V.T. of the line is 25 below the H.P. Draw the projections of the line PQ and determine its true length and the H.T.*

Given Data	*Interpretation*
Line is inclined at 30º to the V.P. ($\phi = 30º$)	pq_2 is inclined at 30º to xy
End P is 15 mm above the H.P.	Point p' is 15 mm above xy
Front view measures 70 mm	$p'q' = p'q_2' = 70$ mm
Front view is inclined at 45º to xy ($\alpha = 45º$)	$p'q'$ inclined at 45º to xy
V.T. is 25 mm below the H.P.	v' is 25 mm below xy, $vv' = 25$ mm

Construction Refer to Fig. 9.37.

1. Draw a reference line xy. Mark point p' 15 mm above xy.
2. Draw a 70 mm long line $p'q'$ inclined at 45º to xy. This represents the front view.
3. Draw a horizontal line 25 mm below xy as the locus of v'. Produce $a'b'$ to meet the locus of v' at point v'. Point v' represents the V.T. Project v' to meet xy at point v.
4. Draw an arc with centre v' and radius $v'q'$ to meet the horizontal line from v' at point q_v'. Draw a line from point v, inclined at 30º to xy to meet the projector from point q_v' at point q_v. Join vq_v.
5. Project point q' to meet the horizontal line from point q_v at point q. Join vq. Project p' to meet vq at point p. Join pq to represent the top view.
6. Draw an arc with centre p and radius pq to meet the horizontal line from point p at point q_1. Project point q_1 to meet the horizontal line from point q' at point q_1'. Join $p'q_1'$. Measure $p'q_1'$ as true length and its inclination with xy as true inclination with the H.P. Here, T.L. = 81 mm and $\theta = 38°$.

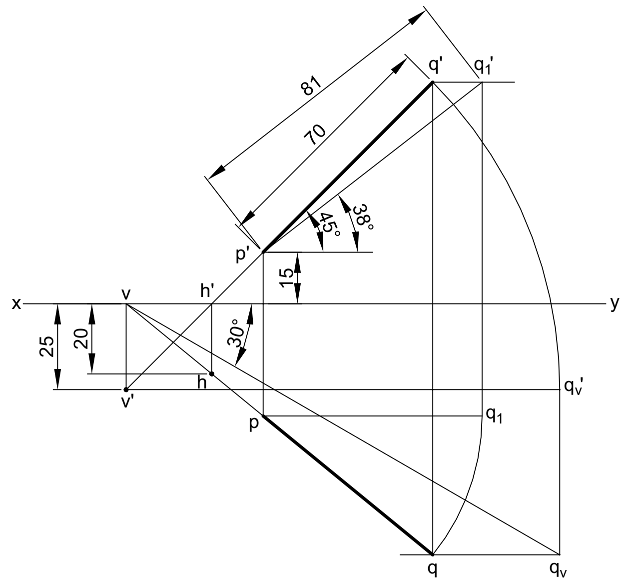


Fig. 9.37

7. Produce $p'q'$ to meet xy at h'. Project h' to meet pq produced at point h. Point h represents the H.T. Here h is 20 mm below xy.

Result True length $p'q_1' = 81$ mm. Inclination with the H.P., $\theta = 38°$. H.T. (h) is 20 mm below xy.

Problem 9.38 *A line PQ measures 70 mm. Projector through its V.T. and the end P are 40 mm apart. The end P is 30 mm above the H.P. and 40 mm in front of the V.P. The V.T. is 10 mm above the H.P. Draw the projections of the line PQ determine its inclinations with the H.P. and the V.P. Also, locate the H.T.*

Given Data	*Interpretation*
PQ measures 70 mm	$p_1'q_1' = pq_2 = 70$ mm
Projector through V.T. and end P are 40 mm apart	$vo = 40$ mm
End P is 30 mm above the H.P.	Point p' is 30 mm above xy
End P is 40 mm in front of the V.P.	Point p is 40 mm below xy
V.T. is 10 mm above the H.P.	v' is 10 mm above xy, $vv' = 10$ mm

Construction Refer to Fig. 9.38.

1. Draw a reference line xy. Mark points v and o on xy such that $vo = 40$ mm.
2. On the projector through o, mark point p' 30 mm above xy and point p 40 mm below xy. On the projector through v, mark point v' 10 mm above xy. Join $v'p'$ and vp. They represent the front and the top views of an imaginary line VP, an extended part of line PQ.
3. Draw an arc with centre v and radius vp to meet the horizontal line from point v at point p_1. Draw a vertical projector from p_1 to meet the horizontal line from p' at point p_1'. Join $v'p_1'$.

4. Produce $v'p_1'$ to a point q_1' such that $p_1'q_1'$ measures 70 mm. Measure the inclination of the line $p_1'q_1'$ with xy as true inclination of line PQ with the H.P. Here $\theta = 19^\circ$.
5. Draw a horizontal line from point q_1' to meet $v'p'$ produced at point q'. Join $p'q'$ to represent the front view.
6. Produce vp to meet the projector of point q' at point q. Join pq to represent the top view.
7. Draw an arc with centre p' and radius $p'q'$ to meet the horizontal line from p' at point q_2'. Project q_2' to meet the horizontal line from point q at point q_2. Join pq_2. Its length must be equal to true length 70 mm. Measure the inclination of the line pq_2 with xy as true inclination of line PQ with the V.P. Here $\phi = 42^\circ$.
8. Produce $p'q'$ to meet xy at point h'. Project h' to meet pq produced at point h. Point h represents the H.T. Here h is 30 mm above xy.

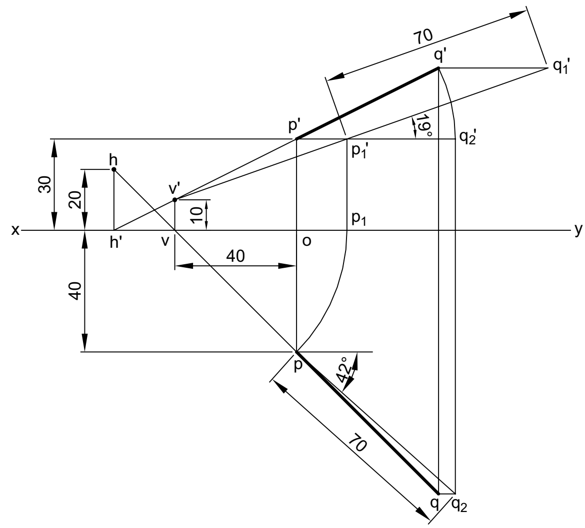


Fig. 9.38

Result Inclination with the H.P., $\theta = 19^\circ$. Inclination with the V.P., $\phi = 42^\circ$. H.T. (h) is 30 mm above xy.

Problem 9.39 *The front view of a line PQ is inclined at 30° to the reference line. The H.T. of the line is 30 mm in front of the V.P. whereas the V.T. is 20 mm below the H.P. One end of the line is 15 mm above the H.P. and the other end of the line is 100 mm in front of the VP. Draw the projections of the line PQ and determine its true length and true angles of inclination with the reference planes.*

Given Data	*Interpretation*
Front view is inclined at 30º to xy ($\alpha = 30^\circ$)	$p'q'$ is inclined at 30º to xy
H.T. is 30 mm in front of the V.P.	h is 30 mm below xy, $h'h$ = 30 mm
V.T. is 20 mm below the H.P.	v' is 20 mm below xy, vv' = 20 mm
End P is 15 mm above the H.P.	Point p' is 15 mm above xy
End Q is 100 mm in front of the V.P.	Point q is 100 mm below xy

Construction Refer to Fig. 9.39.

1. Draw a reference line xy. Mark a point h' on xy and point h 30 mm below xy. Point h represents H.T.
2. Draw horizontal lines 15 mm above xy as the locus of p' and 20 mm below xy as the locus of v'. Draw a line from point h', inclined at 30º to xy to meet locus of p' at point p' and locus of v' at point v'. Point v' represents V.T. Project point v' to meet xy at point v.
3. Join vh and produce it to meet projector of point p' at point p and a horizontal line 100 mm below xy at point q. Join pq to represent the top view.

4. Project point q to meet $v'h'p'$ produced at point q'. Join $p'q'$ to represent the front view.
5. Draw an arc with centre p and radius pq to meet the horizontal line from point p at point q_1. Project point q_1 to meet horizontal line from point q' at point q_1'. Join $p'q_1'$. Measure $p'q_1'$ as the true length of PQ and its inclination with xy as true inclination with the H.P. Here, T.L. = 79 mm and θ = 24°.
6. Draw an arc with centre p' and radius $p'q'$ to meet the horizontal line from point p' at point q_2'. Project point q_2' to meet horizontal line from point q at point q_2. Join pq_2. The length of pq_2 should be equal to that of $p'q_1'$ (T.L.) and its inclination with xy represents true inclination with the V.P. Here, ϕ = 37°.

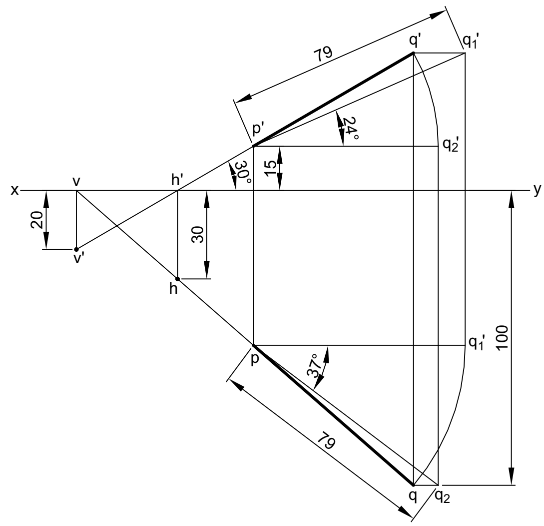


Fig. 9.39

Result True length $p'q_1' = pq_2$ = 79 mm. Inclination with the H.P., θ = 24°. Inclination with the V.P., ϕ = 37°.

Problem 9.40 *The end projectors of a line PQ are 50 mm apart, while those drawn for its H.T. and V.T. are 90 mm apart. The H.T. is 40 mm in front of the V.P. and the V.T. is 80 mm above the H.P. Draw the projections of PQ, if its end P is 10 mm above the H.P. Also, determine its true length and inclinations with the reference planes.*

Given Data	*Interpretation*
End projectors are 50 mm apart	oo_1 = 50 mm
Projectors of H.T. and V.T. are 90 mm apart	$h'v$ = 90 mm
H.T. is 40 mm in front of the V.P.	h is 40 mm below xy, $h'h$ = 40 mm
V.T. is 80 mm above the H.P.	v' is 80 mm above xy, vv' = 80 mm
End P is 10 mm above the H.P.	p' is 10 mm above xy

Construction Refer to Fig. 9.40.

1. Draw a reference line xy. Mark points h' and v on xy such that $h'v$ = 90 mm.
2. On the projector through h', mark point h 40 mm below xy. Similarly, on the projector through v, mark point v' 80 mm above xy. Join $h'v'$ and hv.
3. On line $h'v'$ mark point p' such that it is 10 mm above xy. Draw a vertical projector through p' to meet xy at point o and line hv at point p.

4. Mark point o_1 on xy such that oo_1 = 50 mm. Draw a vertical projector through o_1 to meet line $h'v'$ at point q' and line hv at point q. Lines $p'q'$ and pq represents the front and the top views, respectively.
5. Draw an arc with centre p and radius pq to meet the horizontal line from point p at point q_1. Project point q_1 to meet horizontal line from point q' at point q_1'. Join $p'q_1'$. Measure $p'q_1'$ as the true length of PQ and its inclination with xy as true inclination with the H.P. Here, T.L. = 70 mm and θ = 39°.
6. Draw an arc with centre p' and radius $p'q'$ to meet the horizontal line from point p' at point q_2'. Project point q_2' to meet horizontal line from point q at point q_2. Join pq_2. The length of pq_2 should be equal to that of $p'q_1'$ (T.L.) and its inclination with xy represents true inclination with the V.P. Here, ϕ = 18°.

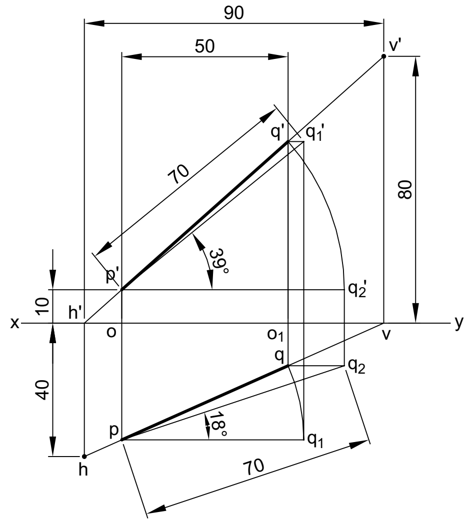


Fig. 9.40

Result True length $p'q_1'$ = pq_2 = 70 mm. Inclination with the H.P., θ = 39°. Inclination with the V.P., ϕ = 18°.

Problem 9.41 *A 100 mm long line PQ has the end P 50 mm in front of the V.P. The H.T. is 60 mm in front of the V.P. and V.T. is 80 mm above the H.P. The distance between the H.T. and the V.T. is 130 mm. Draw the projections of the line PQ and determine its inclinations with the H.P. and the V.P.*

Given Data	*Interpretation*
PQ is 100 mm long	$p'q_1'$ = pq_2 = 100 mm
P is 50 mm in front of the V.P.	p' is 50 mm below xy
H.T. is 60 mm in front of the V.P.	h is 60 mm below xy, $h'h$ = 60 mm
V.T. is 80 mm above the H.P.	v' is 80 mm above xy, vv' = 80 mm
Distance between the H.T. and V.T. is 130 mm	$h'v$ = 130 mm

Construction Refer to Fig. 9.41.

1. Draw a reference line xy. Mark points h' and v on xy such that $h'v$ = 130 mm.
2. On the vertical projector through h', mark point h 60 mm below xy. Similarly, on the vertical projector through v, mark point v' 80 mm above xy. Join $h'v'$ and hv.
3. On line hv, mark point p such that it is 50 mm below xy. Project point p to meet line $h'v'$ at point p'.
4. Draw an arc with centre p' radius $p'v'$ to meet the horizontal line through p' at a point q_v'. Project q_v' to meet the horizontal line from v at point q_v. Join pq_v. Measure its inclination with xy as true inclination with the V.P. Here, ϕ = 21°.

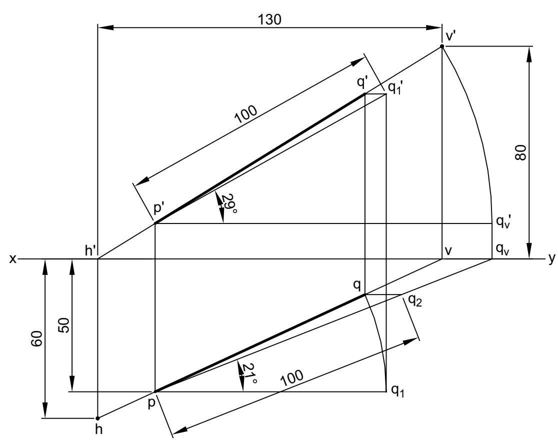


Fig. 9.41

5. Mark point q_2 on line pq_v such that pq_2 = 100 mm. Draw a horizontal line from q_2 to meet line pv at point q. Join pq to represent the top view.
6. Project point q to meet line $p'v'$ at point q'. Join $p'q'$ to represent the front view.
7. Draw an arc with centre p and radius pq to meet the horizontal line from point p at point q_1. Project q_1 to meet the horizontal line from point q' at a point q_1'. Join $p'q_1'$. Measure its inclination with xy as true inclination with the H.P. Here θ = 29°. Also, ensure $p'q_1'$ is 100 mm long.

Result Inclination with the H.P., θ = 29°. Inclination with the V.P., ϕ = 21°.

Problem 9.42 *Two straight lines PQ and QR make an angle of 120° between them in their front and top views. PQ is 60 mm long and is parallel to and 15 mm from both the H.P. and the V.P. Determine the true angle between PQ and QR, if point R is 50 mm above H.P.*

Given Data	*Interpretation*
Angle between *PQ* and *QR* in front and top views is 120°	Angle $p'q'r'$ = angle pqr = 120°
PQ is 60 mm long and parallel to both H.P. and V.P.	$p'q'$ = pq = 60 mm
PQ is and 15 mm from both H.P. and V.P.	$p'q'$ and pq are 15 mm from xy
R is 50 mm above H.P.	r' is 50 mm above xy

Construction Refer to Fig. 9.42.

1. Draw a reference line xy. Mark point p' at 15 mm above xy and point p at 15 mm below xy.
2. Draw 60 mm long lines $p'q'$ and pq, parallel to xy.
3. Draw a line from point q', inclined at 120° to xy such that it meets the horizontal line at 50 mm above xy at point r'. Join $q'r'$ and $p'r'$.
4. Draw a line from point q, inclined at 120° to xy such that it meets the projector from point r' at a point r. Join qr and pr.
5. As lines pq and $p'q'$ are parallel to xy, they represent the true length of side PQ. Here, PQ = 60 mm.
6. Draw an arc with centre p and radius pr to meet the horizontal line from p at point r_1. Project point r_1 to meet horizontal lines from point r' at point r_1'. Join $p'r_1'$ to represent the true length of line PR. Here, $PR = p'r_1'$ = 94 mm.
7. Draw an arc with centre q and radius qr, to meet the horizontal line from q at point r_2. Project point r_2 to meet horizontal lines from point r' at point r_2'. Join $q'r_2'$ to represent the true length of line QR. Here, $QR = q'r_2'$ = 53 mm.

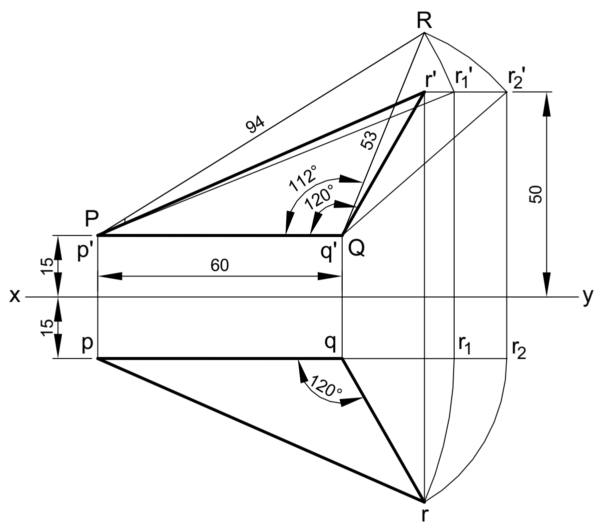
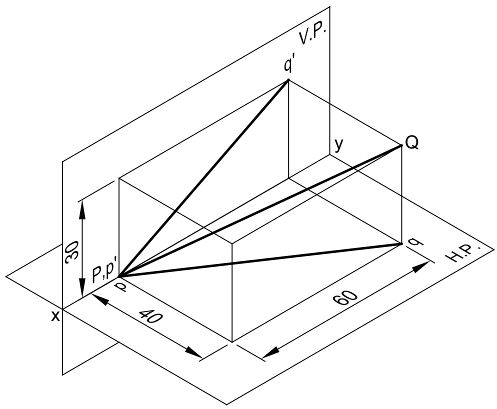


Fig. 9.42

8. Draw actual triangle *PQR* taking true lengths, i.e., 60 mm, 94 mm and 53 mm. Measure the included angle *PQR* as the actual angle between sides *PQ* and *QR*. Here, it is 112°.

Result True length $PQ = p'q'$ = 60 mm. True length $PR = p'r_1'$ = 94 mm. True length $QR = q'r_2'$ = 53 mm. True angle *PQR* = 112°.

Problem 9.43 *Find graphically the length of the largest rod that can be kept inside a hollow cuboid of 60 mm × 40 mm × 30 mm.*



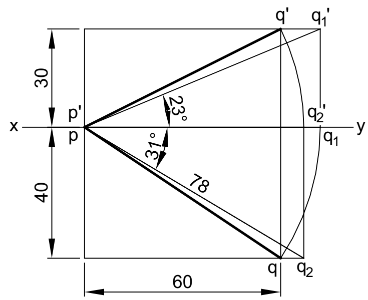


Fig. 9.43 **(a)** *Pictorial view* **(b)** *Projections*

Visualization Figure 9.43(a) shows the pictorial of the given cuboid. The largest rod that can be kept inside should have its end touching diagonally opposite corners of the cuboid. Therefore, it is required to determine the distance between the diagonally opposite corners.

Construction Refer to Fig. 9.43(b).

1. Draw a reference line *xy*. Mark points p' and p coinciding each other on *xy*.
2. Draw a projector on *xy* at a distance of 60 mm from $p'p$. On the projector mark point q' 30 mm above *xy* and point q 40 mm below *xy*. Join $p'q'$ and pq to represent the front and the top views, respectively.
3. Draw an arc with centre p and radius pq, to meet the horizontal line from point p at point q_1. Project point q_1 to meet horizontal lines from q' at q_1'. Join $p'q_1'$ to represent the true length. Here, T.L. = 78 mm.
4. Draw an arc with centre p' and radius $p'q'$, to meet the horizontal line from point p' at point q_2'. Project q_2' to meet horizontal lines from q at q_2. Join pq_2 and ensure that it is equal to $p'q_2'$, i.e., 78 mm.

Result Length of rod $PQ = p'q_1' = pq_2$ = 78 mm.

Problem 9.44 *A room is 6 m × 5 m × 4 m high. An electric bulb B is above the centre of the longer wall and 1 m below the ceiling. The bulb B is 50 cm away from the longer wall. The switch S for the light is 1.25 m above the floor on the centre of the adjacent wall. Determine graphically, the shortest distance between the bulb B and the switch S.*

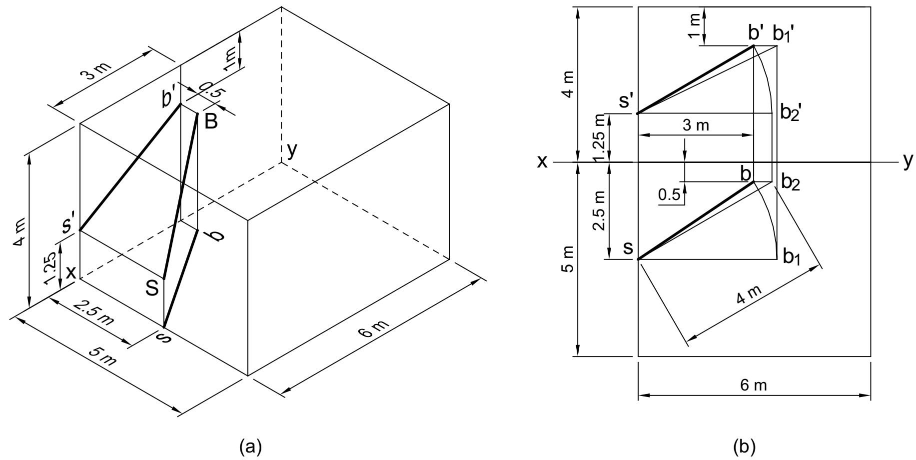


Fig. 9.44

Visualization Figure 9.44(a) shows the pictorial view of the given room. Points S and B represents switch and bulb respectively.

Construction Refer to Fig. 9.44(b).

1. Take scale 1:50. Draw a reference line *xy*. Mark point *s′* 1.25 m above *xy* and point *s* 2.5 m below *xy*.
2. Draw a projector 3 m away from *s′s*. Mark on the projector, point *b′* 3 m above *xy* and point b 0.5 m below *xy*. Join *s′b′* and *sb* to represent the front and the top views of the line joining the switch and the bulb.
3. Draw an arc with centre *s* and radius *sb*, to meet the horizontal line from point *s* at point b_1. Project b_1 to meet horizontal lines from *b′* at point b_1'. Join $s'b_1'$ to represent the true length. Here, T.L. = 4 m.
4. Draw an arc with centre *s′* and radius *s′b′*, to meet the horizontal line from point *s′* at point b_2'. Project b_2' to meet horizontal lines from *b* at point b_2. Join sb_2 and ensure its length is equal to $a'b_2'$, i.e., 4 m.

Result Distance between the bulb and the switch is 4 m.

Problem 9.45 *Three wires AB, CD and EF are tied at points A, C, E on a vertical pole 14 m long at height 12 m, 10 m and 8 m respectively from the ground. The lower ends of the wires are tied to hooks at point B, D and F on the ground level all of which lie at the corners of an equilateral triangle of 7.5 m side. If the pole is situated at the centre of the triangle, determine the length of each rope and their inclination with the ground. (Use scale 1:100)*

Construction Refer to Fig. 9.45.

1. Take scale 1:100. Draw a reference line *xy*.
2. Draw 14 mm long $o'o''$ as the front view of the pole and mark *o* to represent the top view of the pole.
3. On the pole mark point a' 12 m above *xy*, c' 10 m above *xy* and e' 8 mabove *xy*.
4. Draw an equilateral triangle *bdf* of 7.5 m sides with centre *o*. Draw radial lines *ab*, *cd* and *ef* to represent the top view of the ropes.
5. Project *b*, *d* and *f* on *xy* and obtain points $b'd'$ and f'. Join $a'b'$, $c'd'$ and $e'f'$ to represent the front view of the ropes.
6. As the top view *ab* is parallel to *xy*, line $a'b'$ represents the true length of rope *AB* and its inclination with *xy* line is the true inclination with the H.P. Here T.L. of rope *AB* is 12.8 m and θ_{ab} = 70°.
7. Draw an arc with centre *o* and radius *cd* or *ef* to meet horizontal line from point *o* at point o_1. Draw a vertical projector from o_1 to meet *xy* at point o_1'.
8. Join $c'o_1'$ to represent the true length of rope *CD* and its inclination with *xy* as the true inclination with the H.P. Here T.L. of rope *CD* is 10.9 m and θ_{ab} = 67°.
9. Join $e'o_1'$ to represent the true length of rope *EF* and its inclination with *xy* as the true inclination with the H.P. Here T.L. of rope *EF* is 9.1 m and θ_{ab} = 62°.

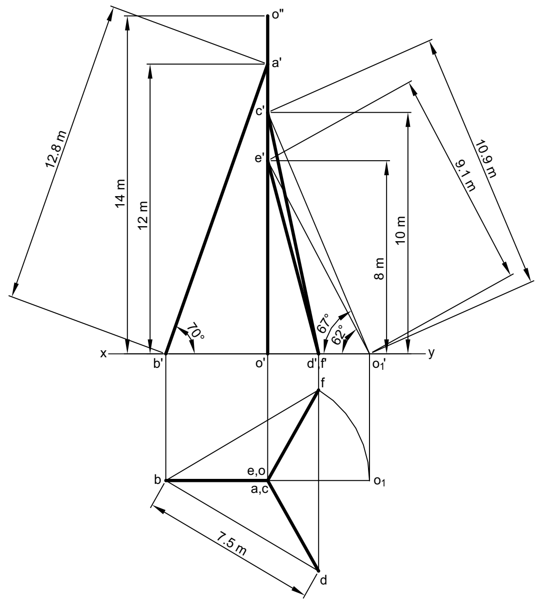


Fig. 9.45

EXERCISE 9B

9.1 An 80 mm long line *PQ* is inclined at 45° to the H.P. and 30° to the V.P. The end *P* is in the H.P. and 40 mm in front of the V.P. Draw its projections.

9.2 An 80 mm long line *PQ* has the end *Q* lying both in the H.P. and V.P. The line is inclined at 30° to H.P. and 45° to the V.P. Draw its projections.

9.3 A 100 mm long line *PQ* is inclined at 45° to H.P. and 30° to the V.P. Its mid-point is 50 above the H.P. and 40 mm in front of the V.P. Draw its projections.

9.4 A 120 mm long line *PQ* is inclined at 45° to the H.P. and 30° to the V.P. A point *M* lies on *PQ* at a distance of 40 mm from the end *P*. The front and top views of point *M* is 40 mm above *xy* and the top view is 35 mm below *xy*. Draw the projections of *PQ*.

9.5 A 100 mm long straight line *PQ* lying in the first angle. It is inclined at 30° to the H.P. and 45° to the V.P. The point *R* divides the line in the ratio of 1:3 and is situated 25 mm above the H.P. and 30 mm in front of the V.P. Draw the projections of line *PQ*.

9.6 An 80 mm long line *PQ* lying in the first angle, has the end *P* on the H.P. and end *Q* in the V.P. The line is inclined at 45° to the H.P. and 30° to the V.P. Draw its projections.

9.7 A straight line *PQ* has its end *P* 15 mm above the H.P. and 60 mm in front of the V.P. The end *Q* is 45 mm above the H.P. and 10 mm in front of the V.P. If the end projectors of the line are 60 mm apart, draw its projections. Determine the true length and true inclinations of the line with the principal planes.

9.8 A 90 mm long line *PQ* has the end *P* 20 mm above the H.P. and 35 mm in front of the V.P. The end *Q* is 80 mm above the H.P. and 60 mm in front of the V.P. Draw the projections of *PQ* and determine its true inclinations with the principal planes.

9.9 A line *PQ* has its end projectors 50 mm apart. The end *P* is 20 mm above the H.P. and 15 mm in front of the V.P. while the end *Q* is 60 mm above the H.P. and 70 mm in front of the V.P. Draw the projections of the line and determine its true length and inclinations with the principal planes. Also, locate its traces.

9.10 A 75 mm long line *PQ* is inclined at 30° to the H.P. Its end *P* is 20 mm above the H.P. and on the V.P. End *Q* is 60 mm in front of the V.P. Draw the projections of the line and locate its traces.

9.11 A line *PQ*, 75 mm long is inclined at an angle of 30° to the V.P. The end *P* is on the H.P. and 30 mm in front of the V.P. The end *Q* is 50 mm above the H.P. Draw the projections of the line and locate its traces.

9.12 A 75 mm long line *PQ* is inclined at 45° to the H.P. The end *P* is 15 mm above the H.P. and 25 mm in front of the V.P. A vertical plane containing line *PQ* is inclined at 45° to the V.P. Draw the projections of the line and determine its inclination with V.P. Also locate the traces.

9.13 A 75 mm long line *PQ* has the end *P* 25 mm above the H.P. and 10 mm in front of the V.P. The length of the front and top views are 60 mm. Draw the projections of the line and locate its traces.

9.14 A line *PQ* has its end *P* 10 mm above the H.P. and 15 mm in front of the V.P. The lengths of its front and top views are 50 mm and 60 mm respectively. If the top view of the line is inclined at 30° to the reference line, draw its projections. Determine the true length and true inclinations of the line with the principal planes. Also, locate the traces.

9.15 A line *PQ* inclined at 30° to the H.P. has the end *P* 20 mm above the H.P. and 10 mm in front of the V.P. The front view of the line measures 70 mm and is inclined at 60° to the reference line. Draw the projections of the line and determine its true length and inclinations with the principal planes. Also, locate its traces.

9.16 A line *PQ* inclined at 45° to the V.P. has its front view 60 mm long. The end *P* is 10 mm from both the principal planes while the end *Q* is 45 mm above the H.P. Draw the projections of the line and determine its true length and inclinations with the principal planes. Also, locate the traces.

9.17 A line *PQ* has the end projectors 50 mm apart. The front and top views of the line are 55 mm and 65 mm long respectively. If the end *P* is 15 mm above the H.P. and 25 mm in front of the V.P., draw the projections of the line. Determine its true length and inclinations with the principal planes. Also, locate the traces.

9.18 A line *PQ* inclined at 45° to the H.P. has its top view 60 mm long. The end *P* is 15 mm above the H.P. and 20 mm in front of the V.P. whereas the end *Q* is equidistant from both the principal planes. Draw the projections of the line and determine its true length and inclination with the V.P. Also, locate its traces.

9.19 A 120 mm long line *PQ* has the ends *P* and *Q*, 10 mm and 60 mm below the H.P. respectively. The end projectors are 50 mm apart. The mid-point of *PQ* is 60 mm in front of the V.P. Draw the projections of *PQ* and determine its inclinations with the reference planes.

9.20 A line *PQ* is inclined at 30° to the H.P. The end *P* is 15 mm in front of the V.P. and the mid-point *O* of the line is 40 mm above the H.P. The front view measures 60 mm and is inclined at 45° with the reference line. Draw the projections of the line and determine its true length and inclination with V.P. Also, locate its traces.

9.21 An 80 mm long line *PQ* is inclined at 30° to the H.P. and 45° to the V.P. End *P* is on the H.P. and end *Q* is in the V.P. Draw projections of the line and locate its traces.

9.22 A 90 mm long line *PQ* is inclined at 30° to the H.P. and 45° to the V.P. The end *P* is 10 mm above the H.P. and 100 mm in front of the V.P. Draw the projections of the line and determine distance of end *P* from the V.P. Also, locate the traces.

9.23 An 80 mm long line *PQ* is inclined at 30° to the H.P. and 45° to the V.P. The end *P* is 15 mm in front of the V.P. and the end *Q* is 80 mm above the H.P. Draw the projections of the line and determine distance of end *P* from the H.P. Locate its traces.

9.24 A line *PQ* is inclined at 30° to the H.P. and 45° to the V.P. The end *P* is 30 mm above the H.P. and end Q is 70 mm in front of the V.P. If the top view of the line measures 75 mm, draw its projections. Determine its true length, inclination with the V.P. and the traces.

9.25 An 80 mm long line PQ is inclined at 30° to the H.P. and 45° to the V.P. The end P is 15 mm above the H.P. and the end Q is 15 mm in front of the V.P. Draw projections of the line and locate its traces.

9.26 A 90 mm long line PQ has its end P 15 mm above the H.P. and 25 mm in front of the V.P. The line is inclined at 60° to the H.P. and 30° to the V.P. Draw its projections.

9.27 A 100 mm long line PQ has its end P 15 mm above the H.P. and 20 mm in front of the V.P. The front and top views are 80 mm and 60 mm long, respectively. Draw its projections and determine the true inclinations with the reference planes.

9.28 A 90 mm long line PQ has the end P on the H.P. and 70 mm in front of the V.P. The end Q is 10 mm in front of the V.P. Draw the projections of the line when the sum of its true inclinations with the H.P. and the V.P. is 90°. Determine the true inclinations with the reference planes and locate its traces.

Problems on traces

9.29 A 70 mm long line PQ is inclined at 45° to the V.P. and its top view measures 50 mm. The end P is 15 mm above the H.P. whereas the V.T. of the line is 20 mm below the H.P. Draw the projections of PQ and determine its inclination with the H.P. Also, locate its H.T.

9.30 The front view of a line PQ measures 60 mm is inclined at 45° to the reference line. The end P is 10 mm above the H.P. and V.T. of the line is 15 mm below the H.P. If the line PQ is inclined at 30° to the V.P., draw its projections. Determine its true length, inclination with the H.P. and locate its traces.

9.31 A line PQ is inclined at 45° to the H.P. and 30° to the V.P. End P is 75 mm in front of the V.P. and V.T. is 15 above the H.P. The top view measures 75 mm. Draw the projections of the line and determine its inclination with the H.P. Also, locate its H.T.

9.32 The front view of a line AB is inclined at 30° to the reference line. The H.T. of the line is 45 mm in front of the V.P. and V.T. is 30 mm below the H.P. The end A is 12 mm above the H.P. and end B is 105 mm in front of the V.P. Draw the projections of the line and find its true length and inclinations with the reference planes.

9.33 The distance between end projectors of a line PQ is 60 mm and between its traces is 90 mm. H.T. of the line is 40 mm in front of the V.P. and V.T. is 60 mm above the H.P. The end P lies 10 mm above the H.P. Draw the projections of the line and determine its true length. Also determine the true inclinations with the reference planes.

9.34 End projectors of a line PQ are 40 mm apart while those drawn for its H.T. and V.T. are 65 mm apart. The H.T. is 30 mm in front of the V.P. and V.T. is 45 mm above the H.P. Draw the projections of PQ, if the end P is 10 mm above the H.P. Also, determine its true length and inclinations with the reference planes.

Applications

9.35 A classroom of size 3 m × 5 m × 4 m high has two hooks A and B nailed on the adjacent walls. The hook A is in the middle of the longer wall, 1 m above the floor and is projected 0.3 m from the surface of the wall. The hook B is 2.5 m above the floor, 1.5 m from the other longer wall and is projected 0.3 m from the surface of the wall. Find the shortest distance between the projected ends of the hooks.

9.36 A TV table 0.8 m × 0.8 m × 0.5 m high is kept in the corner of a leaving room 4.5 m × 3.5 m × 4 m high. A 0.5 m hook is hanging in the centre of the roof. Find the distance between the end of the hook and the nearest corner of the table.

9.37 A 20 m high wireless aerial tower is tied at top by two guy ropes having angles of depression of 30° and 45°. The other end of guy ropes is connected to two poles at heights of 5 m and 2.5 m respectively. The two poles are 12 m apart. Draw projections of the arrangement and determine lengths of guy ropes.

9.16 PROJECTIONS OF A LINE IN DIFFERENT ANGLES

This part deals with the projections of straight lines inclined to one or both the reference planes when the ends of the line lie in different angles.

Problem 9.46 *A 75 mm long line PQ is inclined at 30° to the V.P. and parallel to the H.P. Draw its projections when whole line lies in the same angle and the end P is (a) 25 mm in front of the V.P. and 40 mm above the H.P., (b) 25 mm behind the V.P. and 40 mm above the H.P., (c) 25 mm behind the V.P. and 40 mm below the H.P., and (d) 25 mm in front of the V.P. and 40 mm below the H.P.*

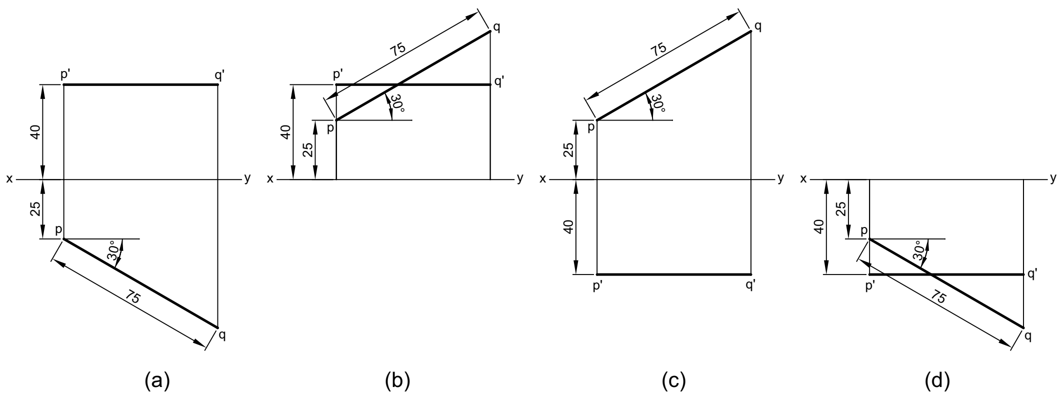


Fig. 9.46 *Projections of line PQ lying in* **(a)** *First angle* **(b)** *Second angle* **(c)** *Third angle* **(d)** *Fourth angle*

Construction Refer to Fig. 9.46(a) to (d).

1. Draw a reference line *xy*. Mark the front and top views of point *P* as points *p′* and *p*, respectively.
2. Draw a 75 mm long line *pq* inclined 30° to *xy* and situated on one side of *xy*, to represent the top view.
3. Project point *q* to meet the horizontal line from point *p′* at point *q′*. Join *p′q′* to represent the front view.

Problem 9.47 *A 90 mm long line PQ is inclined at 30° to the H.P. and 45° to the V.P. The end P is 15 mm above the H.P. and 25 mm in front of the V.P. Draw its projections. Consider the end Q is in the (a) first angle, (b) second angle, (c) third angle and (d) fourth angle.*

Construction Refer to Fig. 9.47(a) to (d).

1. Draw a reference line *xy*. Mark point *p′* 15 mm above *xy* and point *p* 25 mm below *xy*.
2. Draw a 90 mm long line $p'q_1'$ inclined at 30° to *xy*. For cases (a) and (b), q_1' should lie above *xy* whereas for cases (c) and (d), q_1' should lie below *xy*.
3. Draw another 90 mm long line pq_2 inclined at 45° to *xy*. For cases (a) and (d), q_2 should lie above *xy* whereas for cases (b) and (c), q_2 should lie below *xy*.
4. Project q_1' to meet the horizontal line from *p* at point q_1. Draw an arc with centre *p* and radius pq_1 to meet the horizontal line from point q_2 at point *q*. Join *pq* to represent the top view.
5. Project q_2 to meet the horizontal line from *p′* at point q_2'. Draw an arc with centre *p′* and radius $p'q_2'$ to meet the horizontal line from q_1' at point *q′*. Join *p′q′* to represent the front view.

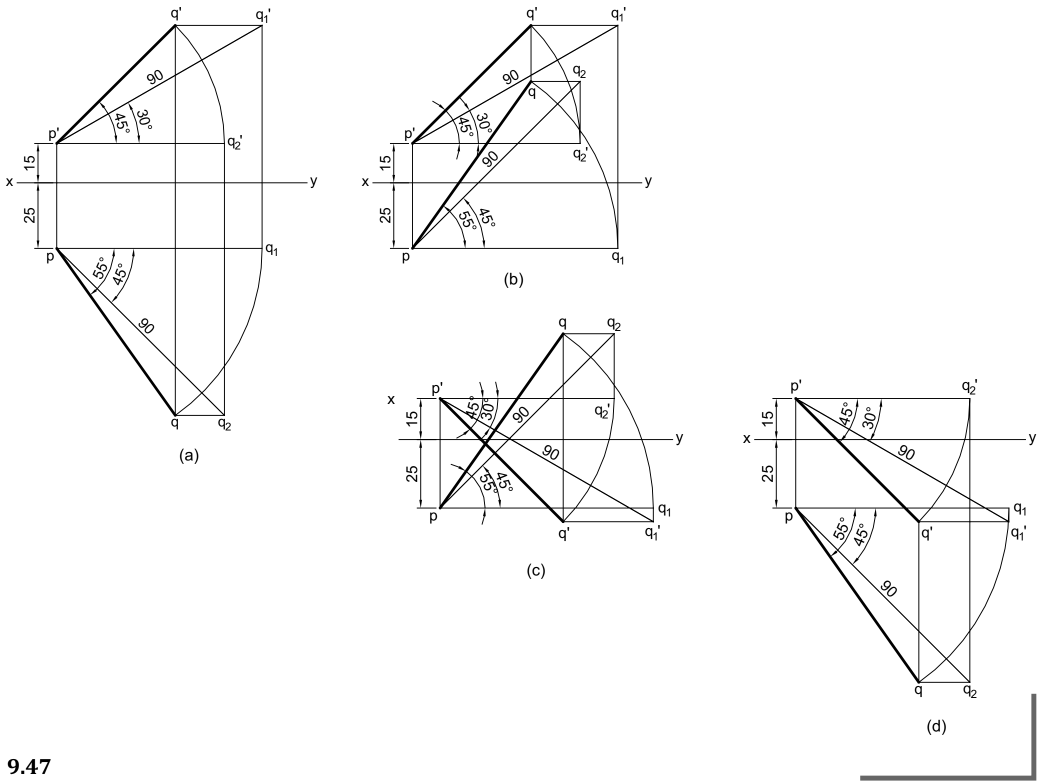


Fig. 9.47

Conclusions

1. When the end Q lies in the first angle, points q' and q_1' are above xy and points q and q_2 are below xy.
2. When the end Q lies in the second angle, points q', q_1', q and q_2 are above xy.
3. When the end Q lies in the third angle, points q' and q_1' are below xy and points q and q_2 are above xy.
4. When the end Q lies in the fourth angle, points q', q_1', q and q_2 are below xy.

9.17 MISCELLANEOUS PROBLEMS

Problem 9.48 *The front view of a line PQ is 70 mm long. The line is parallel to the H.P. and inclined at 45° to the V.P. The end P is 20 mm behind the V.P. and 30 mm below the H.P. whereas the the end Q is in the fourth angle. Draw its projections and determine the traces.*

Construction Refer to Fig. 9.48.

1. Draw a reference line xy. Mark point p' 30 mm below xy and point p 20 mm above xy.
2. Draw 70 mm long line $p'q'$ parallel to xy.
3. Draw line pq inclined at 45° to xy such that it meets the projector from point q' at a point q, below xy. The line pq represents the top view.
4. Mark v at the intersection of line pq with xy. Project v to meet $p'q'$ at point v'. The point v' represents the V.T. As the line is parallel to H.P., H.T. does not exist.

Result V.T. (v') is 30 mm below xy. H.T. does not exist.

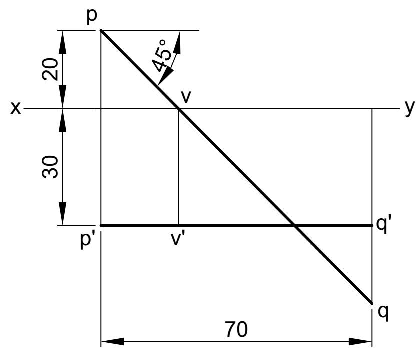


Fig. 9.48

Problem 9.49 *A 90 mm long line AB has 60 mm long front view parallel to xy. The end P is 25 mm from the V.P, 40 mm from the H.P. and lies in the second angle. The end Q lies in the first angle. Draw its projections and determine the true inclination with V.P. and the traces.*

Construction Refer to Fig. 9.49.

1. Draw a reference line xy. As point P lies in the second angle, mark point p' 40 mm above xy and point p 25 mm above xy.
2. Draw 60 mm long line $p'q'$ parallel to xy, to represent the front view.
3. Draw an arc with centre p and radius 90 mm to meet the projector from point q' at point q, below xy. Join pq to represent the top view.
4. Mark point v at the intersection of line pq with xy. Project point v to meet line $p'q'$ at point v'. Point v' represents the V.T. As the front view is parallel to xy, H.T. does not exist.

Result V.T. (v') is 40 mm above xy. H.T. does not exist.

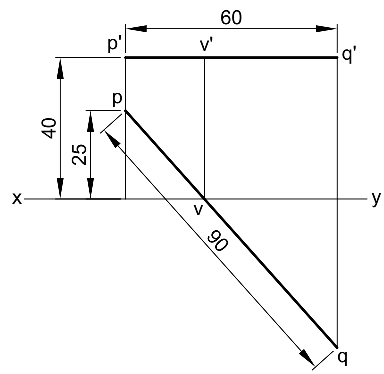


Fig. 9.49

Problem 9.50 *The end P of a 100 mm long line PQ is in the V.P. and 30 mm above the H.P. The end Q is below the H.P. and behind the V.P. The line is inclined at 45° to the H.P. and 30° to the V.P. Draw the projections of PQ and locate its H.T. and V.T.*

Given Data	*Interpretation*
Line PQ is 100 mm long	$p'q_1' = pq_2 = 100$ mm
End P is in the V.P.	p is on xy
End P is 30 mm above the H.P.	p' is 30 mm above xy
The line is inclined at 45° to the H.P. ($\theta = 45°$)	$p'q_1'$ is inclined at 45° to xy
The line is inclined at 30° to the V.P. ($\phi = 30°$)	pq_2 is inclined at 30° to xy

Construction Refer to Fig. 9.50.

1. Draw the reference line xy. Mark p' 30 mm above xy and p on xy.
2. Draw $p'q_1'$ 100 mm long making 30° with xy such that q_1' is below xy. Draw a horizontal line through q_1' as the locus of q'.
3. Draw pq_2 100 mm long making 60° with xy such that q_2 is above xy. Draw a horizontal line through q_2 as the locus of q.
4. Projector q_1' to meet the horizontal line from point p at point q_1. Draw an arc with centre p and radius pq_1 to meet the locus of q at point q. Join pq to represent the top view.
5. Project q_2 to meet the horizontal line from point p' at point q_2'. Draw an arc with centre p' and radius $p'q_2'$ to meet the locus of q' at point q'. Join $p'q'$ to represent the front view.

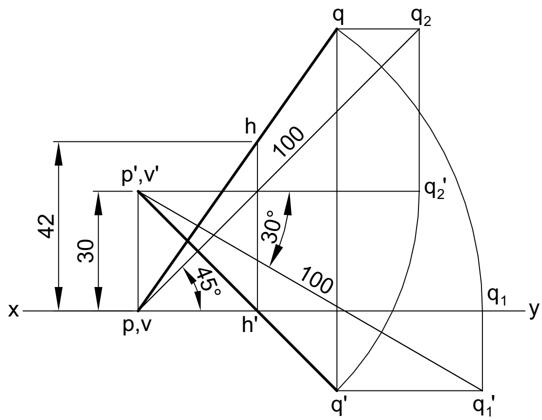


Fig. 9.50

6. Join $q'q$ and ensure that it is perpendicular to xy, representing projector of the end Q.
7. Line $p'q'$ meets xy at point h'. Project h' to meet pq at point h. Point h represents the H.T.
8. Line pq meets xy at point v. Project v to meet $p'q'$ at point v'. Point v' represents the V.T.

Result H.T. (h) is 42 mm above xy. V.T. (v') is 30 mm above xy.

Problem 9.51 *A 100 mm long line PQ is inclined at 30° to the H.P. and 45° to the V.P. A point M lies on the line at a distance of 60 mm from the end P and has the front view 15 mm above the xy and top view 25 mm below its front view. Draw the projections of the line and determine the traces.*

Given Data	*Interpretation*
Line PQ is 100 mm long, M is at 60 mm from p	$p_1'm' = p_2m = 60$ mm; $m'q_1' = mq_2 = 40$ mm
Line is inclined at 30° to the H.P. ($\theta = 30°$)	$p_1'm'q_1'$ is inclined at 30° to xy
Line is inclined at 45° to the V.P. ($\phi = 45°$)	p_2mq_2 is inclined at 45° to xy
Front view of M is 15 mm above xy	m' is 15 mm above xy
Top view of M is 20 mm below xy	mm' is 25 mm or m is 10 mm below xy

Construction Refer to Fig. 9.51.

1. Draw a reference line xy. Mark point m' 15 mm above xy and point m 25 below m'.
2. Draw a 40 mm long line $m'q_1'$ inclined at 30° to xy. Produce it such that $p_1'q_1' = 100$ mm.
3. Draw another 40 mm line mq_2 inclined at 45° to xy. Produce it such that $p_2q_2 = 100$ mm.
4. Project points p_1' and q_1' to meet the horizontal line through point m at points p_1 and q_1 respectively. Draw an arc with centre m and radii mp_1 and mq_1 to meet the horizontal lines from points p_1' and q_1' at points p and q, respectively. Join pmq to represent the top view.
5. Project points p_2 and q_2 to meet the horizontal line through point m' at points p_2' and q_2' respectively. Draw an arc with centre m' and radii $m'p_2'$ and $m'q_2'$ to meet the horizontal lines from points p_2 and q_2 at points p' and q', respectively. Join $p'm'q'$ to represent the front view.

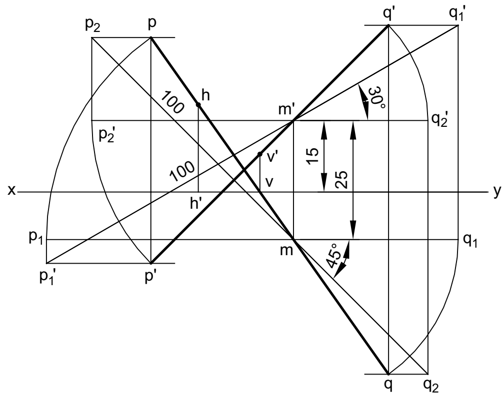


Fig. 9.51

6. Join $p'p$ and $q'q$ and ensure that they are perpendicular to xy.
7. Line $p'q'$ meets xy at point h'. Project h' to meet pq at point h. Point h represents the H.T.
8. Line pq meets xy at point v. Project v to meet $p'q'$ at point v'. Point v' represents the V.T.

Result H.T. (h) is 18 mm above xy. V.T. (v') is 8 mm above xy.

Problem 9.52 *The end P of a line PQ is 20 mm above the H.P. and 30 mm in front of the V.P. The end Q is 15 mm below the H.P. and 45 mm behind the V.P. If the end projectors are 50 mm apart, draw the projections of PQ and determine the true length, traces and inclination with the reference planes.*

Given Data	*Interpretation*
End P is 20 mm above the H.P.	p' is 20 mm above xy
End P is 30 mm in front of the V.P.	p is 30 mm below xy
End Q is 15 mm below the H.P.	q' is 15 mm below xy
End Q is 45 mm behind the V.P.	q is 45 mm above xy
End projectors are 50 mm apart	Distance between $p'p$ and $q'q$ is 50 mm

Construction Refer to Fig. 9.52.

1. Draw a reference line xy. Mark point p' 20 mm above xy and point p 30 mm below xy.
2. Draw a vertical projector at a distance 50 mm from pp'. On the projector, mark point q' 15 mm below xy and point q 45 mm above xy.
3. Join $p'q'$ and pq to represent the front and the top views, respectively.
4. Draw an arc with centre p and radius pq to meet the horizontal line from point p at point q_1. Project point q_1 to meet the

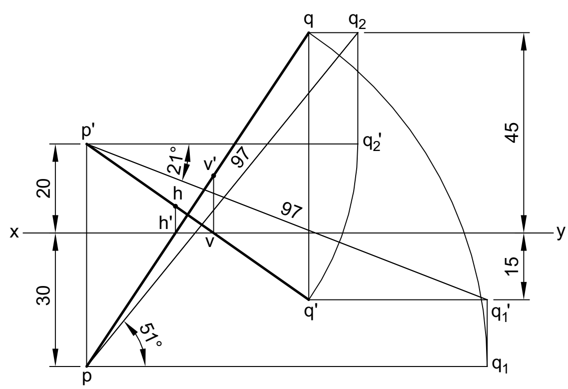


Fig. 9.52

horizontal line from point q' at point q_1'. Join $p'q_1'$. Measure its length as true length and its inclination with xy as true inclination with the H.P. Here, T.L. = 97 mm and $\theta = 21°$.

5. Draw an arc with centre p' and radius $p'q'$ to meet the horizontal line from point p' at point q_2'. Project point q_2' to meet the horizontal line from point q at point q_2. Join pq_2. Measure its length as true length (97 mm) and its inclination with xy is true inclination with the V.P. Here, $\phi = 51°$.
6. Line $p'q'$ meets xy at point h'. Project h' to meet pq at point h. Point h represents the H.T.
7. Line pq meets xy at point v. Project v to meet $p'q'$ at point v'. Point v' represents the V.T.

Result True length, $p'q_1' = pq_2 = 97$ mm. Inclination with the H.P., $\theta = 21°$. Inclination with the V.P., $\phi = 51°$. H.T. (h) is 6 mm above xy. V.T. (v') is 13 mm above xy.

Problem 9.53 *An 80 mm long line PQ has the end P 20 mm from the reference planes and lies in the third angle. The end Q is 40 mm from both the reference planes and lies in the second angle. Draw its projections and find the true inclination with the reference planes.*

Given Data	*Interpretation*
Line *PQ* is 80 mm long	$p'q_1' = pq_2 = 80$ mm
End *P* is 20 mm from H.P., lying in third angle	Point p' is 20 mm below xy
End *P* is 20 mm from V.P., lying in third angle	Point p is 20 mm above xy
End *Q* is 40 mm from H.P., lying in fourth angle	Points q' and q_1' are 40 mm above xy
End *Q* is 40 mm from V.P., lying in fourth angle	Points q and q_2 are 40 mm above xy

Construction Refer to Fig. 9.53.

1. Draw a reference line xy. Mark point p' 20 mm below xy and point a 20 mm above xy.
2. Draw a line 40 mm above xy as the locus of points q', q_1', q and q_2.
3. Draw an arc with centre p' and radius 80 mm to meet the locus of q_1' at point q_1'. Join pq_1' and measure its inclination as true inclination with the H.P. Here $\theta = 49°$.
4. Draw another arc with centre p and radius 80 mm to meet the locus of q_2 at point q_2. Join pq_2 and measure its inclination as true inclination with the V.P. Here $\phi = 14°$.

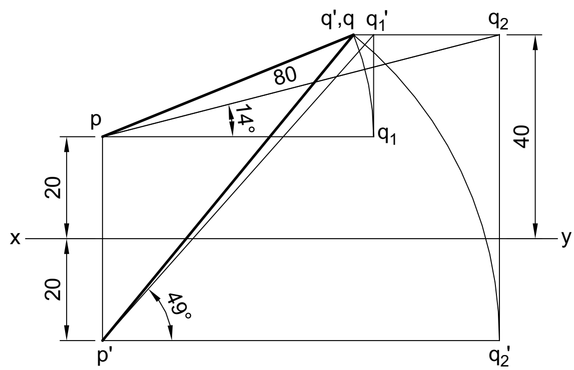


Fig. 9.53

5. Projector q_1' to meet the horizontal line from point p at point q_1. Draw an arc with centre p and radius pq_1 to meet the locus of q at point q. Join pq to represent the top view.
6. Project q_2 to meet the horizontal line from point p' at point q_2'. Draw an arc with centre p' and radius $p'q_2'$ to meet the locus of q' at point q'. Join $p'q'$ to represent the front view.
7. Join $q'q$ and ensure that it is perpendicular to xy, representing projector of the end Q.

Result Inclination with the H.P., $\theta = 49°$. Inclination with the V.P., $\phi = 14°$.

Problem 9.54 *A 120 mm long line PQ has the end projectors 50 mm apart. Ends P and Q are 10 mm and 60 mm below the H.P., respectively. The mid-point of PQ lies in the V.P. Draw the projections of the line and find its true inclinations with both reference planes. Assume that the end P lies in the fourth quadrant.*

Given Data	*Interpretation*
Line *PQ* is 120 mm long, *M* is the mid-point	$p_1'm' = m'q_1' = p_2m = mq_2 = 60$ mm
End projectors are 50 mm apart	$p'p$ and $q'q$ are 50 mm apart along *xy*
End *P* is 10 mm below the H.P.	Point p' is 10 mm below *xy*
End *Q* is 60 mm below the H.P.	Point q' is 60 mm below *xy*
Mid-point *M* lies in the V.P.	Point *m* is on *xy*

Construction Refer to Fig. 9.54.

1. Draw a reference line *xy*. Mark points *o* and o_1 on it such that $oo_1 = 50$ mm.
2. On the projector though point *o*, mark point p' 10 mm below *xy*. On the projector though point o_1 mark point q' 60 mm below *xy*. Join $p'q'$ to represent the front view.
3. Mark m' as the mid-point of $p'q'$. Project point m' to meet *xy* at point *m*. Points m' and *m* represents the front and top view of the mid-point *M*.
4. Draw an arc with centre *m* and radius 60 mm to meet the horizontal lines from points p' and q' at points p_1' and q_1', respectively. Join 120 mm long line $p_1'q_1'$ and measure its inclinations with *xy* as true inclination of line *PQ* with the H.P. Here $\theta = 25°$.
5. Project p_1' and q_1' to meet horizontal line through m (i.e., on *xy*) at points p_1 and q_1, respectively. Draw an arc with centre *m* and radii mp_1 and mq_1 to meet the projectors from points p' and q' at points *p* and *q* respectively. Join *pmq* to represent the top view.
6. Draw an arc with centre m' and radii $m'p'$ and mq' to meet the horizontal line from point m' at points p_2' and q_2', respectively. Project p_2' and q_2' to meet the horizontal lines from points *p* and *q* at points p_2 and q_2, respectively. Join p_2q_2 and ensure that it equal to 120 mm. Measure inclination p_2q_2 with *xy* as the true inclination of line *PQ* with the V.P. Here $\phi = 54°$.

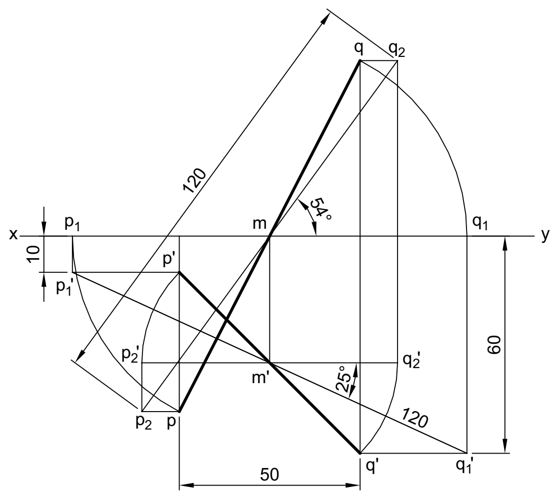


Fig. 9.54

Result Inclination with the H.P., $\theta = 25°$. Inclination with the V.P., $\phi = 54°$.

Problem 9.55 *The end P of 150 mm long line PQ is 50 mm behind the V.P. and 35 mm below the H.P. The other end Q is in the first quadrant. The line is inclined at 30° to the H.P. and has a point R in both the H.P. and the V.P. Draw the projections of the line and find its inclination with the V.P.*

Given Data	*Interpretation*
Line *PQ* is 150 mm long	$p'q_1' = pq_2 = 150$ mm
End *P* is 50 mm behind the V.P.	Point *p* is 50 mm above *xy*
End *P* is 35 mm below the H.P.	Point *p′* is 35 mm below *xy*
Line is inclined at 30° to the H.P. ($\theta = 30°$)	pq_1' is inclined at 30° to *xy*
Point *R* lies in both the H.P. and the V.P.	Points *r* and *r′* concides on *xy*.

Construction Refer to Fig. 9.55.

1. Draw a reference line *xy*. Mark *p′* 35 mm below *xy* and point *p* 50 mm above *xy*.
2. Draw a 150 mm long line $p'q_1'$ inclined at 30° to *xy*.
3. Mark point r_1' at the intersection of line $p'q_1'$ with *xy*. Project r_1' to meet horizontal line from point *p* at point p_1. Draw an arc with centre *p* and radius pr_1 to meet *xy* at point *r*. As point *R* lies in both the H.P. and the V.P., mark point *r′* coinciding with point *r* on *xy*.
4. Join *p′r′* and produce it to meet the horizontal line from point q_1' at point *q′*. Join *p′q′* to represent the front view.
5. Join *pr* and produce it to meet the projector of point *q′* at point *q*. Line *pq* represents the top view.
6. Draw an arc with centre *p′* and radius *p′q′* to meet the horizontal line from point *p′* at point q_2'. Draw a horizontal line from point *q* to meet the projector of q_2' at point q_2. Join pq_2 and ensure that it is 150 mm long. Measure its inclination with *xy* as the inclination of line with the V.P. Here $\phi = 46°$.

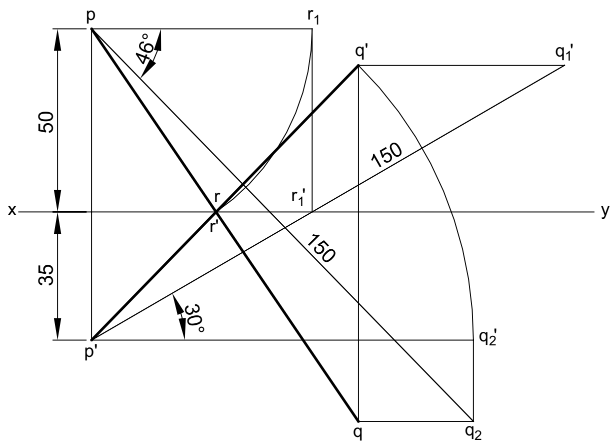


Fig. 9.55

Result Inclination with the V.P., $\phi = 46°$.

Problem 9.56 *A 100 mm long line PQ has its end point P 30 mm below the H.P. and 20 mm behind the V.P. The V.T. is 10 mm above the H.P. The projectors drawn through its V.T. and the end P are 40 mm apart. Draw the projections of the line. Determine H.T. of the line and inclinations with the reference planes.*

Given Data	*Interpretation*
Line PQ is 100 mm long	$p'q_1' = pq_2 = 100$ mm
End P is 30 mm below the H.P.	Point p' is 30 mm below xy
End P is 20 mm behind the V.P.	Point p is 20 mm above xy
V.T. is 10 mm above the H.P.	v' is 10 mm above xy, $vv' = 10$ mm
Projectors through V.T. and end P are 40 mm apart	vv' and pp' are 40 mm apart

Construction Refer to Fig. 9.56.

1. Draw a reference line xy. Mark point p' 30 mm below xy and point p 20 mm above xy.
2. Mark point v on xy at a distance of 40 mm from the projector $p'p$. Mark point v' 10 mm above v to represent the V.T.
3. Join $p'v'$ and pv and determine their true lengths and inclinations with the reference planes. (Draw an arc with centre p and radius pv to meet the horizontal line from p at point v_1. Project v_1 to meet the horizontal line from v' at point v_1'. Join $p'v_1'$. Draw another arc with centre p' and radius $p'v'$ to meet the horizontal line from p' at point v_2'. Project v_2' to meet horizontal line from v at point v_2. Join pv_2.) Here $\theta = 42°$ and $\phi = 19°$.

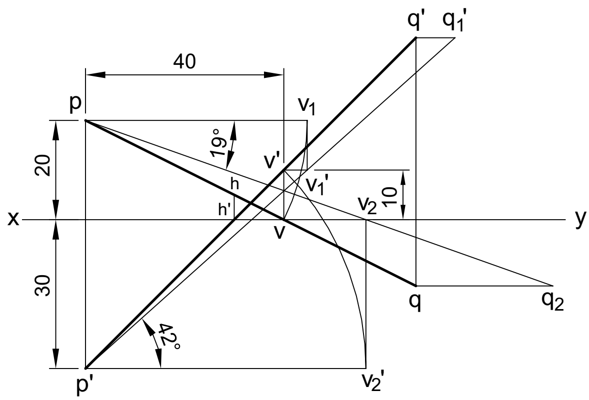


Fig. 9.56

4. Produce $p'v_1'$ to q_1' so that $p'q_1'$ is 100 mm long. Produce $p'v'$ to meet the horizontal line from q_1' at point q'. Join $p'q'$ to represent the front view.
5. Produce pv_2 to q_2 so that pq_2 is 100 mm long. Produce pv to meet the horizontal line from q_2 at point q. Join pq to represent the top view.
6. Line $p'q'$ meets xy at point h'. Project h' to meet pq at point h. Point h represents the H.T.

Result H.T. (h) is 5 mm above xy. Inclination with the H.P., $\theta = 42°$. Inclination with the V.P., $\phi = 19°$.

Problem 9.57 *A 75 mm long line PQ has the end P 50 mm below the H.P. and 20 mm behind the V.P. Its front and top views are 60 mm and 45 mm long, respectively. Draw its projections when end Q of the line is in the first quadrant. Determine the true inclinations of the line with the H.P. and the V.P.*

Given Data	*Interpretation*
Line PQ is 75 mm long	$p'q_1' = pq_2 = 75$ mm
End P is 50 mm below the H.P.	Point p' is 50 mm below xy
End P is 20 mm behind the V.P.	Point p is 20 mm above xy
Front view is 60 mm long	$p'q' = p'q_2' = 60$ mm
Top view is 45 mm long	$pq = pq_1 = 45$ mm

Visualisation Here $(F.V.)^2 + (T.V.)^2 = (T.L.)^2$, therefore both the front and the top views should be perpendicular to xy, i.e., $\alpha = \beta = 90°$. Also, sum of the true inclinations of the line with H.P. and V.P. should be 90°, i.e., $\theta + \phi = 90°$.

Construction Refer to Fig. 9.57.

1. Draw a reference line xy. Mark point p' 50 mm below xy and point p 20 mm above xy.
2. Draw a 60 mm long line $p'q_2'$, parallel to xy. Also draw a 45 mm long line pq_1, parallel to xy.
3. Draw an arc with centre p and radius 75 mm to meet the projector of q_2' at point q_2. Join pq_2 and measure its inclination with xy as inclination of line with the V.P. Here $\phi = 37°$.
4. Draw an arc with centre p' and radius 75 mm to meet the projector of q_1 at point q_1'. Join $p'q_1'$ and measure its inclination with xy as inclination of line with the H.P. Here $\theta = 53°$.
5. Draw an arc with centre p and radius pq_1 to meet horizontal line from q_2 at point q. Join pq to represent the top view.
6. Draw an arc with centre p' and radius $p'q_2'$ to meet the horizontal line from q_1' at point q'. Join $p'q'$ to represent the front view.

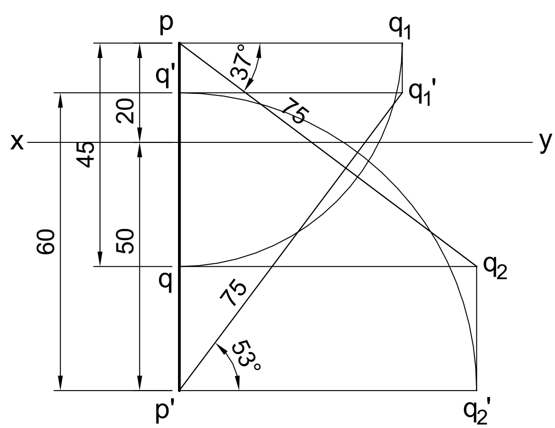


Fig. 9.57

Result Inclination with the H.P., $\theta = 53°$. Inclination with the V.P., $\phi = 37°$.

Problem 9.58 *Two oranges on a tree are respectively 3.5 m and 6.5 m above the ground and 2.5 m and 4.5 m from a 0.6 m thick wall but on the opposite side of it. The distance between the oranges along the wall is 6 m. Determine the true distance between the apples.*

Given Data	*Interpretation*
Orange P is 1.8 m above the ground	p' is 3.5 m above xy
Orange Q is 3 m above the ground	q' is 3 m above xy
Orange P is 1.2 m from a 0.3 m thick wall	p is 1.2 m below cd
Orange Q is 2.1 m from a 0.3 m thick wall (on opposite side)	q is 2.1 m above ab
The distance between the oranges along the wall is 2.7 m	$oo_1 = 2.7$ m

Construction Refer to Fig. 9.61.

1. Take scale 1:100. Draw a reference line xy. Mark points o and o_1 on xy such that $oo_1 = 6$ m.
2. Draw lines ab and cd parallel to and 0.3 m from xy, to represent 0.6 m thick wall.
3. On the projector through o mark point p' 3.5 m above xy and point p 2.5 m from the cd.
4. On the projector through o_1 mark point q' 6.5 m above xy and point q 4.5 m from the ab.

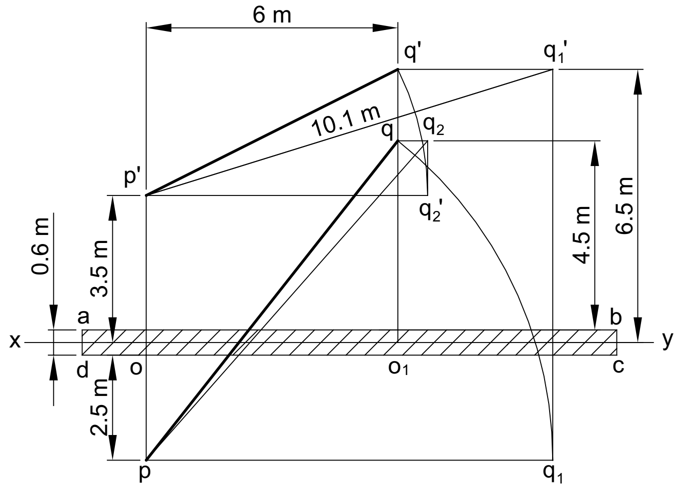


Fig. 9.61

5. Join $p'q'$ and pq to represent the front and the top views of the line joining two oranges.
6. Draw an arc with centre p and radius pq to meet the horizontal line from point p at point q_1. Project point q_1 to meet the horizontal line through point q' at point q_1'. Join $p'q_1'$. Measure its length as true distance between the oranges. Here, T.L. = 10.1 m.
7. Draw an arc with centre p' and radius $p'q'$ to meet the horizontal line from point p' at point q_2'. Project point q_2' to meet the horizontal line from point q at point q_2. Join pq_2. Ensure that its length is equal to that of $p'q_1'$.

Result True distance between the oranges, $p'q_1' = pq_2 = 10.1$ m.

EXERCISE 9C

9.1 A 90 mm long line PQ is inclined at 45° to the H.P. and 30° to the V.P. The end P is on the H.P. and 40 mm in front of the V.P. Draw the projections of the line when the end Q is in the fourth quadrant.

9.2 An 80 mm long line line CD is inclined at 30° to the H.P. and 45° to the V.P. The end point C is 15 mm above the H.P. and 10 mm in front of the V.P. whereas the other end point D lies in the second quadrant. Draw the projections of the line and determine its traces.

9.3 The front view of a line PQ measures 70 mm and is inlined at 30° to the reference line. The end P is on the H.P. and V.T. of the line is 15 mm below the H.P. The line is inclined at 45° to the V.P. Draw the projections of the line and find its true length, inclination with the H.P. and locate the H.T.

9.4 A 100 mm long line PQ is inclined at 45° to the H.P. and 30° to the V.P. The end P is 10 mm below the H.P. and 25 mm in front of the V.P. Draw the projections of the line and determine its traces, considering the point Q lies in the second quadrant.

9.5 A 120 mm long line PQ is inclined at 45° to the H.P. and 30° to the V.P. The mid-point of PQ is 20 mm above the H.P. and 40 mm in front of the V.P. Draw the projections of the line and determine its traces.

9.6 A point Q lies on the reference line, divides 120 mm long line PR in the ratio of 2:3. The end P is in the second quadrant and the end Q is in the fourth quadrant. The line is inclined at 45° to the H.P. and 30° to the V.P. Draw its projections and locate its H.T. and V.T.

9.7 An 80 mm long line PQ has the end P 10 mm away from both the reference planes and lies in the first quadrant. The line is inclined at 60° with the H.P. and its top view is inclined at 45° to the reference line. Draw its projections considering the end Q is in the third quadrant. Also, determine the inclination of the line with the V.P.

9.8 The end P of a line PQ is on the H.P. and 25 mm behind the V.P. The end Q is in the V.P. and 50 mm above the H.P. The distance between the end projectors is 75 mm. Draw the projections of PQ and determine its true length.

9.9 An 80 mm long line PQ is inclined at 60° to the H.P. and 30° to the V.P. The end P lies 20 mm below the H.P. and 30 mm behind the V.P. whereas the end Q lies in the first quadrant. Draw the projections of PQ and determine the traces.

9.10 An 80 mm long line AB has the end A 20 mm above the H.P. and the end B 15 mm in front of the V.P. The line is inclined at 30° to the H.P. and its top view is inclined at 60° to the reference line. Draw the projections of the line and determine its inclination with the V.P.

9.11 A straight line PQ is equally inclined to the H.P. and V.P. The end P is in front of the V.P. and 20 mm above H.P. whereas the end Q is behind the V.P. and 40 mm below H.P. A point R on the line is in V.P. and 10 mm below H.P. The distance between the end projectors is 60 mm. Draw the projections of the line and determine its inclinations with the reference planes.

9.12 The front and the top views of a 75 mm long line PQ measures 45 mm and 60 mm, respectively. Its end P is 15 mm below the H.P. and 20 mm behind the V.P. whereas the the end Q lies in the first quadrant. Draw the projections of PQ. Determine its inclination with the reference line and the traces.

9.13 The front view of a line PQ is inclined at 30° to the reference line. The H.T. of the line is 45 mm in front of the V.P. and VT is 30 mm below the H.P. The end P is 12 mm above the H.P. and the end Q is 105 mm in front of the V.P. Draw the projections of the line and find its true length and inclinations with the reference planes.

9.14 The end projectors of a line PQ are 90 mm apart. The H.T. and V.T. of the line coincide with each other on the reference line and lies at a distance of 35 mm from projectors of the end P, towards Q. The end P is 20 mm above the H.P. and Q is 45 mm behind the V.P. Draw the projections of the line and determine its true length and inclinations with reference planes.

9.15 A line PQ measures 100 mm. The projectors through its V.T. and the end P are 40 mm apart. The end P is 30 mm above H.P. and 20 mm in front of V.P. The V.T. is 10 mm below the H.P. Draw the projections of the line and determine H.T. and inclinations of line with the reference planes.

9.16 The distance between the end projectors of a line PQ is 70 mm and the projectors through the traces are 110 mm apart. The end P of the line is 10 mm above H.P. If the top and the front views of the line are inclined at 30° and 60° with reference planes, respectively. Draw the projections of the line and determine its traces, true length and inclinations with the reference planes.

9.17 A 75 mm long line PQ has the end P 15 mm above the H.P. and 30 mm in front of the V.P. The front and top views are inclined at 45° and 60° to the reference line, respectively. Draw the projections of PQ and determine its true inclinations with the reference planes.

VIVA-VOICE QUESTIONS

9.1 A straight line inclined at 30° to the H.P., is parallel to and 25 mm in front of the V.P. What is the position of its H.T. and V.T.?

9.2 A straight line inclined at 45° to the V.P., is parallel to and 40 mm above the H.P. What is the position of its H.T. and V.T.?

9.3 The front view of a line is parallel to xy and measures 30 mm. What is its true length, if the top view measures 65 mm?

9.4 The top view of a line is parallel to xy and measures 40 mm. What is its true length, if the front view measures 75 mm?

9.5 A line is inclined at 30° to the H.P. and 60° to the V.P. Which orthographic view of this line show its true length?

9.6 The distance between the end projectors of a line is zero. Which orthographic view of this line will show its true length?

9.7 A line is inclined to both the reference planes. State the positions of the front and top views of its H.T.

9.8 A line is inclined to both the reference planes. State the positions of the front and top views of its V.T.

9.9 The top view of a line is represented by a point on the reference line. State the position of the line.

9.10 A point on *xy* represents the front view of a straight line. What is the position of the line?

9.11 The top view of a line is 30 mm long. If the length of the line is extended by one third of its original length, what will be the measure of the new top view?

9.12 The front view of a line is 40 mm long. If the length of the line is reduced by one fourth of its original length, what will be the measure of the new front view?

9.13 One end of a line lies in the first angle and the other in the second angle. Which of the two views of the line will intersect the reference line?

9.14 One end of a line lies in the second angle and the other in the third angle. Which of the two views of the line will cross the reference line?

9.15 A line is inclined at an angle of 30° with H.P. What will be its inclination with V.P. if the distance between its end projectors is zero?

9.16 If the front view of a line lies in the reference line, state all the possible positions of the line.

9.17 If the top view of a line lies in the reference line, state all the possible positions of the line.

MULTIPLE-CHOICE QUESTIONS

9.1 If a line is parallel to both H.P. and V.P., its true length will be seen in
(a) front view
(b) top view
(c) side view
(d) Both front and top views

9.2 If the apparent and the true inclinations of a line with H.P. are equal, the line is
(a) parallel to horizontal plane
(b) parallel to vertical plane
(c) parallel to profile plane
(d) inclined to both reference planes

9.3 The point at which the line intersects the V.P., extended if necessary, is known as
(a) profile trace
(b) horizontal trace
(c) vertical trace
(d) auxiliary trace

9.4 If the front view of a line is parallel to the *xy* its true length is shown in
(a) front View
(b) top View
(c) side view
(d) Both front and top views

9.5 If top view of a line is a point, its front view is
(a) parallel to *xy* and of true length
(b) parallel to *xy* and of apparent length
(c) perpendicular to *xy* and of true length
(d) perpendicular to *xy* and of apparent length

9.6 Horizontal trace of a line exits when the line is
(a) parallel to horizontal plane
(b) inclined to horizontal plane
(c) perpendicular to vertical plane
(d) perpendicular to profile plane

9.7 If a line is inclined at 45° to the H.P. and 30° to the V.P., its front view is inclined at
(a) 30° to *xy*
(b) 45° to *xy*
(c) Between 30° and 45°
(d) Greater than 45°

9.8 If a line is inclined at 30° to the H.P. and 60° to the V.P., its front and top views are inclined at an angle of
(a) 30° and 60° to *xy* respectively
(b) 60° and 30° to *xy* respectively
(c) Both at 90° to *xy*
(d) Both greater than 30° but less than 90°

9.9 For a line situated in the first angle, which of the following statements is not correct?
(a) H.T. and V.T. may lie below *xy*
(b) H.T. lies below *xy* and V.T. lies above *xy*
(c) H.T. and V.T. may lie above *xy*
(d) H.T. lies above *xy* and V.T. lies below *xy*

9.10 A 90 mm long line *PQ*, inclined at 30° to the H.P. and 45° to the V.P. has end *P* 15 mm above H.P. and 25 mm in front of V.P. The other end *Q* will lie in
(a) first angle
(b) third angle
(c) second or fourth angle
(d) Any of these

9.11 If the front and top views of a line are inclined at 30° and 45° to the reference line, the true inclination of the line with H.P. will be
(a) 30°
(b) 45°
(c) less than 30°
(d) greater than 45°

9.12 If both the front and top views of a line are perpendicular to the reference line, the true inclination of the line with H.P. and V.P. may be respectively
(a) 15° and 75°
(b) 30° and 60°
(c) Both 45°
(d) Any of these

Answers to multiple-choice questions

9.1 (d), 9.2 (b), 9.3 (c), 9.4 (b), 9.5 (c), 9.6 (b), 9.7 (d), 9.8 (c), 9.9 (d), 9.10 (d), 9.11 (c), 9.12 (d)

Chapter

10

PROJECTIONS OF PLANES

10.1 INTRODUCTION

In this chapter, we deal with two-dimensional objects called *planes*. Planes have length, breadth and negligible thickness. Here only those planes are considered whose shape can be defined geometrically and are regular[1] in nature. Some of these are shown in Fig. 10.1.

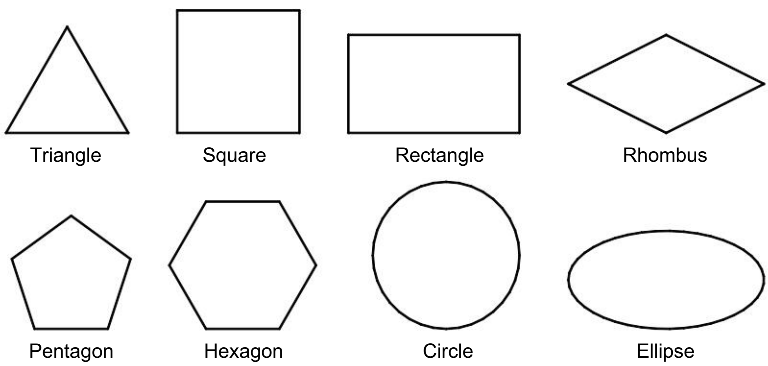


Fig. 10.1 *Planes*

10.2 ORIENTATION OF PLANES

The surface of a plane may be in one of the following positions as shown in Fig. 10.2(a) to (c) and Fig. 10.3(a) to (c).

1. Parallel to H.P. (and perpendicular to V.P.).
2. Parallel to V.P. (and perpendicular to H.P.).
3. Parallel to profile plane (i.e., perpendicular to both H.P. and V.P.).
4. Inclined to H.P. and perpendicular to V.P.
5. Inclined to V.P. and perpendicular to H.P.
6. Inclined to both H.P. and V.P.

[1]Readers are advised to refer to Chapter 3 for different methods of construction of regular polygons.

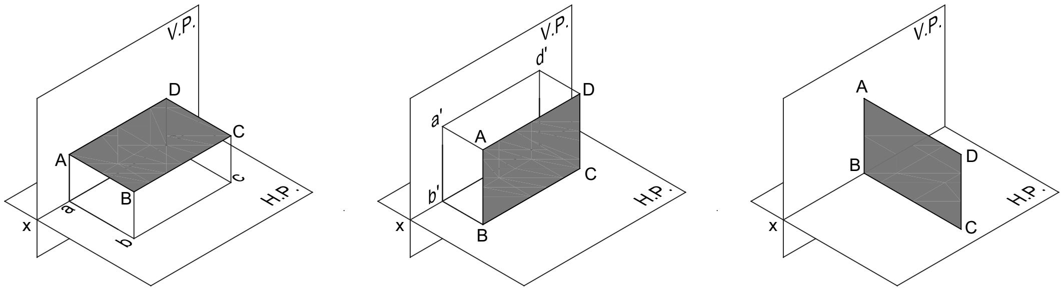


Fig. 10.2 *Plane* **(a)** *Parallel to H.P.* **(b)** *Parallel to V.P.* **(c)** *Parallel to profile plane*

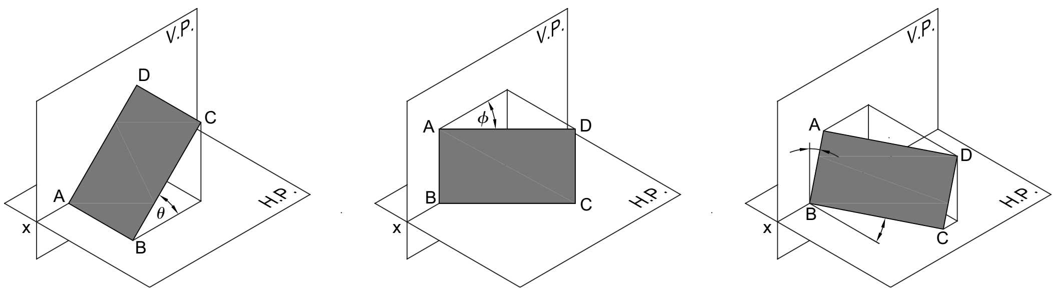


Fig. 10.3 *Plane* **(a)** *Inclined to H.P. and perpendicular to V.P.* **(b)** *Inclined to V.P. and perpendicular to H.P.* **(c)** *Inclined to both H.P. and V.P.*

10.3 PLANE PARALLEL TO H.P.

This is one of the basic positions of a plane. A plane parallel to the H.P. is always perpendicular to the V.P. The true shape and size of the plane can be viewed in the top view. Hence, first draw the top view and then project it to obtain a straight line representing the front view.

Problem 10.1 *A square plane of side 40 mm has its surface parallel to and 20 mm above the H.P. Draw its projections when (a) a side is parallel to the V.P., (b) one side is inclined at 30° to the V.P. and (c) all sides are equally inclined to the V.P.*

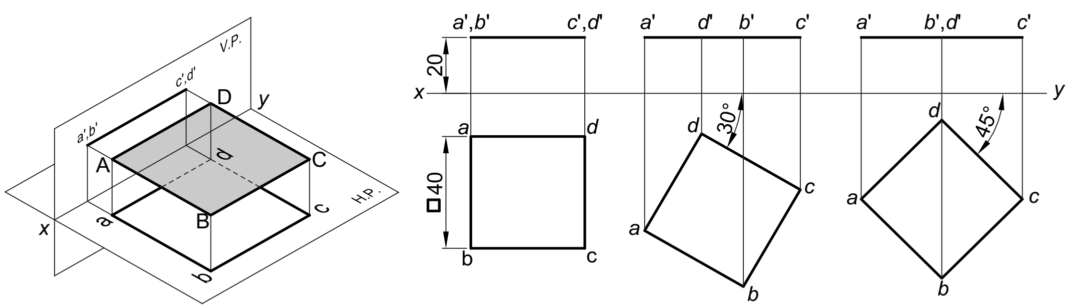


Fig. 10.4 **(a)** *3-D view in case (i)* **(b)** *Projections in cases (i), (ii) and (iii)*

Visualization Figure 10.4(a) shows a square plane *ABCD* with the surface parallel to and above the H.P. Side *AD* is parallel to the V.P. as desired in case (a). The top view shall be a square of side 40 mm. Therefore, first draw the top view and then project it to obtain a straight line representing the front view.

Construction Refer to Fig. 10.4(b).

Case (i) Draw a square *abcd* in the top view keeping ad parallel to *xy*. Project the corners from the top view and mark points a', b', c' and d' 20 mm above *xy*. Join $a'b'c'd'$ to represent the front view.

Case (ii) Draw a square *abcd* in the top view keeping *ab* inclined at 30° to *xy*. Project the corners from the top view and mark points a', b', c' and d' 20 mm above *xy*. Join $a'b'c'd'$ to represent the front view.

Case (iii) Draw the square *abcd* in the top view keeping *ab* inclined at 45° to *xy*. Project the corners from the top view and mark points a', b', c' and d' 20 mm above *xy*. Join $a'b'c'd'$ to represent the front view.

10.4 PLANE PARALLEL TO V.P.

This is another basic position of a plane. A plane parallel to the V.P. is always perpendicular to the H.P. The true shape and size of the plane can be viewed in the front view. Hence, first draw the front view and then project it to obtain a straight line representing the top view.

Problem 10.2 *A hexagonal plane of side 25 mm has its surface parallel to and 20 mm in front of V.P. Draw its projections, when a side is (a) parallel to the H.P., (b) perpendicular to the H.P., (c) inclined at 45° to the H.P.*

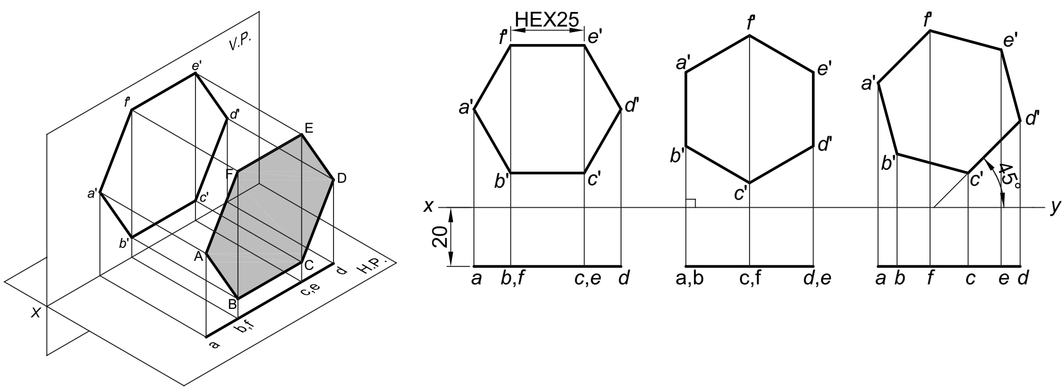


Fig. 10.5 **(a)** *3-D view in case (i)* **(b)** *Projections in cases (i), (ii) and (iii)*

Visualization Figure 10.5(a) shows a hexagonal plane *ABCDEF* with its surface parallel to and in front of V.P. Side *BC* is parallel to the H.P. as desired in case (i). The front view shall be a hexagon of side 25 mm. Therefore, first draw the front view and then project it to obtain a straight line representing the top view.

Construction Refer to Fig. 10.5(b).

Case (i) Draw a hexagon $a'b'c'd'e'f'$ in the front view keeping $b'c'$ parallel to *xy*. Project the corners from the front view and mark points *a*, *b*, *c*, *d*, *e* and *f* 20 mm below *xy*. Join *abcdef* to represent the top view.

Case (ii) Draw a hexagon $a'b'c'd'e'f'$ in the front view keeping $a'b'$ perpendicular to xy. Project the corners from the front view and mark points a, b, c, d, e and f 20 mm below xy. Join $abcdef$ to represent the top view.

Case (iii) Draw a hexagon $a'b'c'd'e'f'$ in the front view keeping $c'd'$ inclined at 45° to xy. Project the corners from the front view and mark points a, b, c, d, e and f 20 mm below xy. Join $abcdef$ to represent the top view.

10.5 PLANE PARALLEL TO PROFILE PLANE

A plane parallel to the profile plane has its surface perpendicular to both H.P. and V.P. The true shape and size of the plane shall be viewed on the profile plane, popularly known as end view or side view. The front and top views are projected from the side view.

Problem 10.3 *A triangular plane is in the form of an isosceles triangle of base side 30 mm and altitude 40 mm. Its surface is perpendicular to both H.P. and V.P. Draw its projections when the base side is parallel to the V.P.*

Visualization The plane ABC is perpendicular to both H.P. and V.P. and the side AB is parallel to the V.P., as shown in Fig. 10.6(a). Thus, the plane is parallel to the profile plane.

Construction Refer to Fig. 10.6(b).

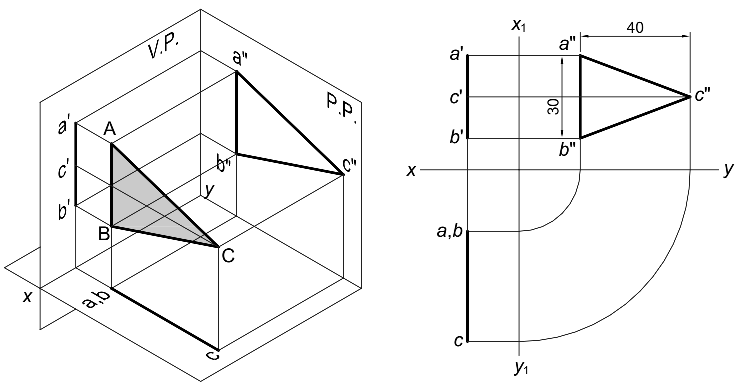


Fig. 10.6 **(a)** *3-D view* **(b)** *Projections*

1. Draw a triangle $a''b''c''$ as the side view keeping $a''b''$ perpendicular to xy.
2. Project the corners on x_1y_1 and obtain points a', b' and c' at some distance from x_1y_1. Join $a'b'c'$ to represent the front view.
3. Project a', b' and c' on xy and extend them to meet projectors coming from the side view to intersect at points a, b and c. Join abc to represent the top view.

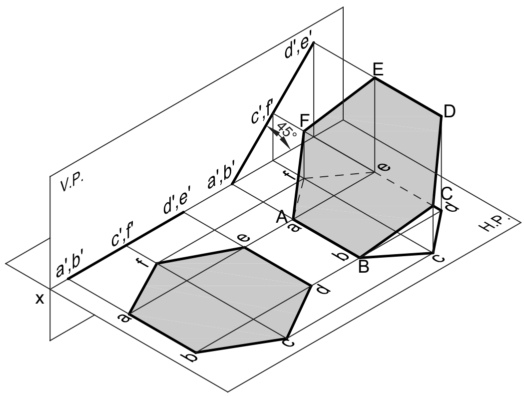
10.6 PLANE INCLINED TO H.P. AND PERPENDICULAR TO V.P.

When the surface of the plane is inclined at θ to the H.P. and perpendicular to the V.P., the projections are obtained in two stages. In the first stage, the plane is assumed to lie on the H.P. The true shape of the plane is viewed in the top view and a straight line lying on xy in the front view. In the second stage, the plane is tilted at θ to the H.P. The front view is redrawn inclined at θ to the xy. The final top view is obtained by joining the points of intersection of the vertical projectors of the corners from the front view with the horizontal projectors of the corners from the top view of the preceding stage.

Note 1 If the plane has a side on the H.P. (or parallel to the H.P. or on the ground), then keep an edge of the plane perpendicular to *xy* in the top view of the first stage.

Note 2 If the plane has a corner in the H.P. (or on the ground), then keep the line joining a corner and the centre of the plane parallel to *xy*.

Problem 10.4 *A hexagonal plane of side 30 mm has an edge on the H.P. The surface is inclined at 45° to the H.P. and perpendicular to the V.P. Draw its projections.*



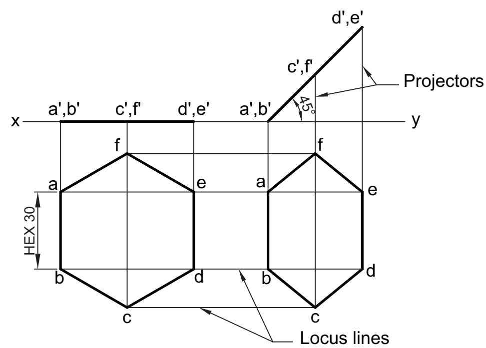


Fig. 10.7 **(a)** *3-D view* **(b)** *Projections*

Construction Refer to Fig. 10.7(b)

The plane has an edge on the H.P., so consider that initially the hexagonal plane is placed on the H.P. with side AB perpendicular to the V.P.

1. **First stage** Draw a hexagon *abcdef* keeping *ab* perpendicular to *xy* to represent the top view. Project the corners to *xy* and obtain *b′d′* as the front view.
2. **Second stage** Reproduce the front view of the first stage, keeping line *b′d′* inclined at 45° to *xy*. Obtain new points *a*, *b*, *c*, *d*, *e* and *f* in the top view by joining the points of intersection of the projectors drawn from the front view of the second stage with the corresponding horizontal locus lines drawn from the top view of the first stage. Join new *abcdef* to represent the final top view.

Problem 10.5 *A hexagonal plane of side 30 mm has a corner on the ground. Its surface is inclined at 45° to the H.P. and perpendicular to the V.P. Draw its projections when the diagonal through the corner in the H.P. is parallel to the V.P.*

Construction Refer to Fig. 10.8(b).

The plane has a corner on the H.P., so consider that the plane *ABCDEF* is placed on the H.P. with line joining the corner A and the centre of the plane parallel to the V.P.

1. **First stage** Draw a hexagon *abcdef* keeping *ao* parallel to *xy* to represent the top view. Project the corners to *xy* and obtain *a′d′* as the front view.
2. **Second stage** Reproduce the front view of first stage, keeping *a′* on *xy* and *a′d′* inclined at 45° to *xy*. Obtain new points *a*, *b*, *c*, *d*, *e* and *f* in the top view by joining the points of intersection of the

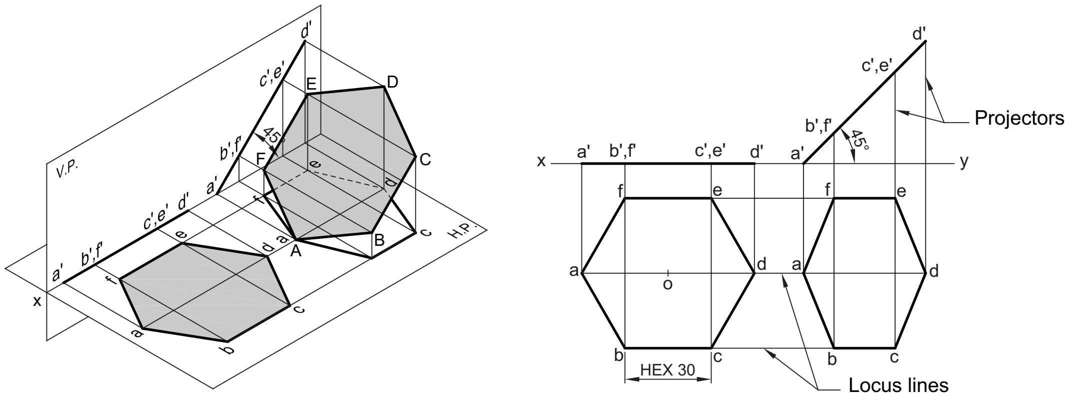


Fig. 10.8 **(a)** *3-D view* **(b)** *Projections*

projectors drawn from the front view of the second stage with the corresponding horizontal locus lines drawn from the top view of the first stage. Join new *abcdef* to represent the final top view.

Problem 10.6 *A circular plane of diameter 50 mm is resting on a point of the circumference on the H.P. The plane is inclined at 30° to the H.P. and its centre is 35 mm in front of the V.P. Draw its projections.*

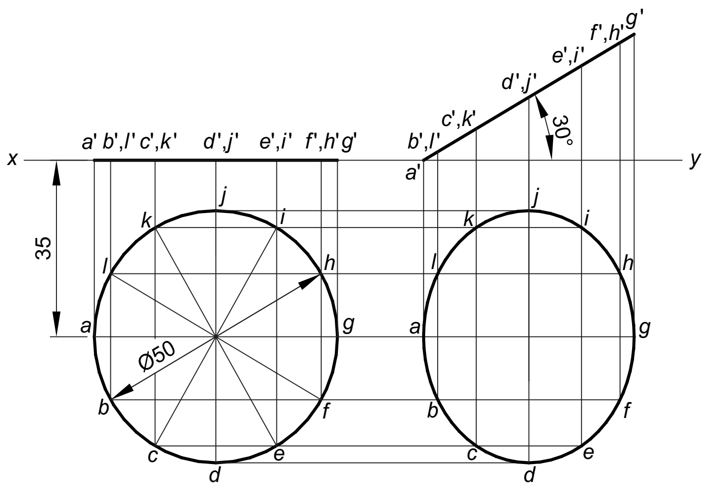


Fig. 10.9

Construction Refer to Fig. 10.9.

A circle has a point of the circumference on the H.P. so consider that initially the circle is placed on the H.P.

1. **First stage** Draw a circle of diameter 50 mm keeping centre 35 mm below *xy* to represent the top view. Divide the circle in 12 equal parts and project all the points to *xy* and obtain $a'g'$ as the front view.

2. **Second stage** Reproduce the front view of first stage keeping a' on xy and $a'g'$ inclined at 30° to the xy. Obtain points a, b, c, etc., in the new top view by joining the point of intersection of the vertical projectors drawn from front view of the second stage with the corresponding horizontal locus lines drawn from the top view of the first stage. Join new *abcdefghijkl* to represent the final top view.

Problem 10.7 *A rectangular plane of sides 70 mm and 35 mm has a shorter side on the H.P. The surface of the plane is inclined at 60° to the H.P. and perpendicular to the V.P. Draw its projections.*

Construction Refer to Fig. 10.10.

The plane has a shorter edge on the H.P., so consider that initially the plane *ABCD* is placed on the H.P. with side *AB* perpendicular to the V.P.

1. **First stage** Draw a rectangle *abcd* keeping *ab* perpendicular to *xy* to represent the top view. Project the corners to *xy* and obtain $b'c'$ as the front view.
2. **Second stage** Reproduce the front view of first stage keeping $b'c'$ inclined at 60° to *xy*. Obtain new points a, b, c and d in the top view by joining the points of intersection of the vertical projectors drawn from points a', b', c' and d' of the second stage with the corresponding horizontal locus lines drawn from points a, b, c and d of the first stage. Join new *abcd* to represent the final top view. The top view is a square of side 35 mm.

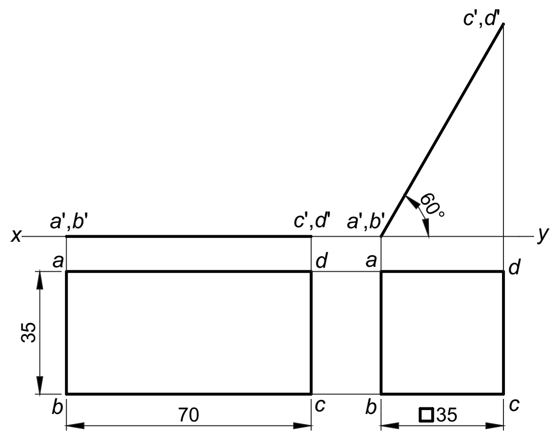


Fig. 10.10

Problem 10.8 *A rectangular plane of sides 70 mm and 35 mm is resting on a side on the H.P. The surface is inclined to the H.P. and perpendicular to the V.P. such that the top view appears as a square. Draw its projections and determine inclination of the plane with the H.P.*

Construction Refer to Fig. 10.10.

A rectangular plane inclined to the H.P. and perpendicular to the V.P. can appear as a square in the top view when the shorter edge of the plane is in the H.P. or parallel to H.P.

1. **First stage** Draw a rectangle *abcd* as the top view keeping 35 mm side *ab* perpendicular to *xy*. Project the corners to *xy* and obtain $b'c'$ as the front view.
2. **Second stage** Draw a square *abcd* of side 35 mm on the horizontal locus line of points a, b, c and d of the first stage. Project *ab* to meet *xy* at point $a'b'$. Draw an arc with centre a' and radius equal to length $a'd'$ of the first stage to meet the vertical projector of *cd* at point d'. Join $a'd'$. Measure the inclination of $a'd'$ with *xy* as inclination of the plane with the H.P. Here $\theta = 60°$.

Problem 10.9 *A rhombus of diagonals 70 mm and 45 mm is placed on an end of the major diagonal on the H.P. Its surface is inclined at 50° to the H.P. and perpendicular to the V.P. Draw its projections.*

Construction Refer to Fig. 10.11.

The plane has an end a of the major diagonal ac on the H.P., so consider that initially the rhombus $abcd$ is placed on the H.P. with ac parallel to xy.

1. **First stage** Draw a rhombus $abcd$ keeping ac parallel to xy to represent the top view. Project the corners to xy and obtain $a'c'$ as the front view.
2. **Second stage** Reproduce the front view of first stage keeping a' on xy and $a'c'$ inclined at 50° to xy. Obtain new points a, b, c and d in the top view by joining the points of intersection of the vertical projectors drawn from points a', b', c' and d' of the second stage with the corresponding horizontal locus lines drawn from points a, b, c and d of the first stage. Join new $abcd$ to represent the final top view. The top view is a square of diagonals 45 mm.

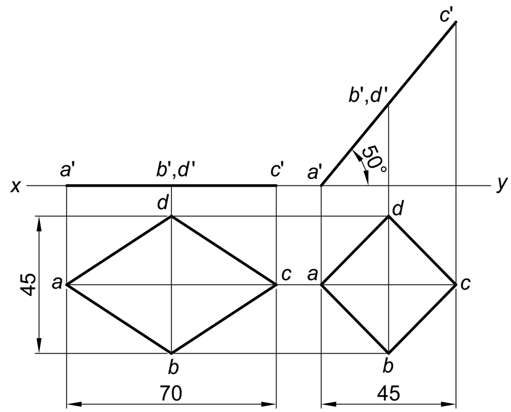


Fig. 10.11

Problem 10.10 *A rhombus of diagonals 70 mm and 45 mm is placed on an end of the major diagonal on the H.P. Its surface is inclined to the H.P. and perpendicular to the V.P. such that the top view appears as a square. Draw its projections and determine inclination of the rhombus with the H.P.*

Construction Refer to Fig. 10.11.

1. **First stage** Draw a rhombus $abcd$ as the top view keeping 70 mm diagonal ac parallel to xy. Project the corners to xy and obtain the front view $a'c'$.
2. **Second stage** Draw a square $abcd$ of diagonals 35 mm on the horizontal locus line through points a, b, c and d of the first stage. Project point a to meet xy at point a'. Project point c vertically upwards. Draw an arc with centre a' and radius equal to $a'c'$ of the first stage to meet the vertical projector of c at point c'. Join $a'c'$. Measure inclination of $a'c'$ with xy as inclination of the rhombus with the H.P. Here $\theta = 50°$.

Problem 10.11 *The top view of a plane whose surface is perpendicular to the V.P. and inclined at 45° to the H.P. is a circle of diameter 50 mm. Draw the projections of the plane and determine its true shape.*

Construction Refer to Fig. 10.12.

1. **First stage** Draw a circle of diameter 50 mm to represent the top view. Divide it into 12 equal parts and mark the divisions as a, b, c, etc. Project point a to meet xy at a'. Draw a line $a'g'$ inclined at 45° to xy to meet the vertical projector from point g at point g'. The line $a'g'$ represents the front view. Project points b, c, d, etc., to meet line $a'g'$ at points b', c', d', etc.
2. **Second stage** Reproduce the front view of the first stage keeping $a'g'$ on xy. Obtain new points a, b, c, etc., in the top view by joining the points of intersection of the vertical projectors drawn from points a', b', c', etc., of the second stage with the corresponding horizontal locus lines drawn from points a, b, c, etc., of the first stage. Join new $abcdefghijkl$ to represent the true shape of the plane.

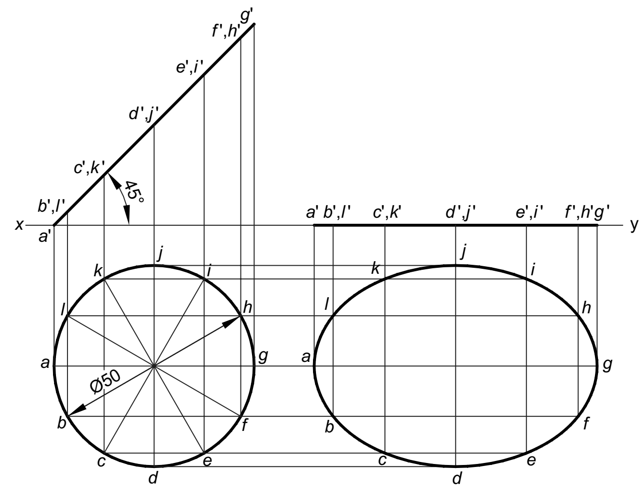


Fig. 10.12

Problem 10.12 *The top view of a plane is a regular pentagon of side 30 mm having one side inclined at 30° to the V.P. Its front view is a straight line inclined at 45° to the reference line. Draw the projections of the plane and determine its true shape.*

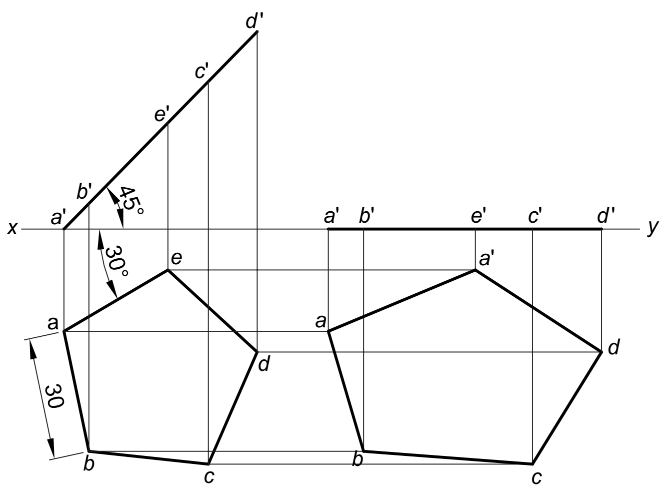


Fig. 10.13

Construction Refer to Fig. 10.13.

1. **First stage** Draw a pentagon of side 30 mm keeping *ae* inclined at 30° to *xy*. This is the top view. Project point *a* to meet *xy* at *a′*. Draw a line *a′d′* inclined at 45° to *xy* to meet the vertical projector from point *d* at point *d′*. The line *a′d′* represents the front view. Project points *b*, *c* and *e* to meet *a′d′* at points *b′*, *c′* and *e′*.

2. **Second stage** Reproduce front view of the first stage keeping *a′d′* on *xy*. Obtain new points *a*, *b*, *c*, *d* and *e* in the top view by joining the points of intersection of the vertical projectors drawn from points *a′*, *b′*, *c′*, *d′* and *e′* of the second stage with the corresponding horizontal locus lines drawn from points *a*, *b*, *c*, *d* and *e* of the first stage. Join new *abcde* to represent the true shape of the plane.

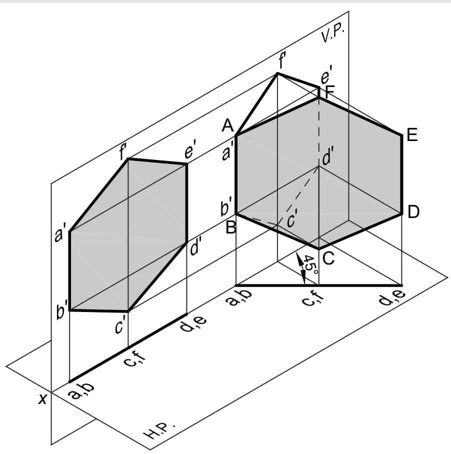
10.7 PLANE INCLINED TO V.P. AND PERPENDICULAR TO H.P.

When the surface of the plane is inclined at ϕ to the V.P. and perpendicular to the H.P., the projections are drawn in two stages. In the first stage, the plane is assumed to lie in the V.P. The true shape of the plane is viewed in the front view and a straight line lying on *xy* in the top view. In the second stage, the plane is tilted at ϕ to the V.P. The top view is redrawn inclined at ϕ to the *xy*. The final front view is obtained by joining the points of intersection of the vertical projectors of the corners from the top view with the horizontal projectors of the corners from the front view of the preceding stage.

Note 1 If the plane has a side parallel to the V.P. or in the V.P., then keep an edge of the plane perpendicular to *xy* in the front view of the first stage.

Note 2 If the plane has a corner in the V.P., then keep the line joining a corner and the centre of the plane parallel to *xy* in the front view of the first stage.

Problem 10.13 *A hexagonal plane of side 30 mm has an edge in the V.P. The surface of the plane is inclined at 45° to the V.P. and perpendicular to the H.P. Draw its projections.*



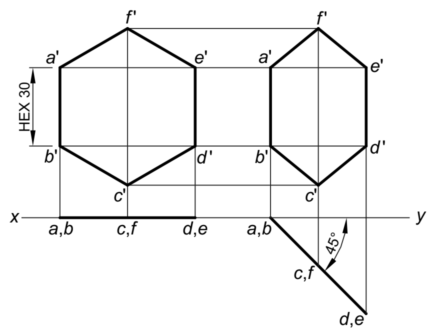


Fig. 10.14 **(a)** *3-D view* **(b)** *Projections*

Construction Refer to Fig. 10.14(b).

The plane has an edge in the V.P., so consider that initially the hexagonal plane ABCDEF is placed in the V.P. with side AB perpendicular to the H.P.

1. **First stage** Draw a hexagon *a′b′c′d′e′f′* keeping *a′b′* perpendicular to *xy* to represent the front view. Project the corners to *xy* and obtain *bd* as the top view.
2. **Second stage** Reproduce the top view of first stage keeping *ab* on *xy* and *bd* inclined at 45° to *xy*. Obtain new points *a′*, *b′*, *c′*, *d′*, *e′* and *f′* in the front view by joining the points of intersection

of the projectors drawn from the top view of the second stage with the corresponding horizontal locus lines drawn from the front view of the first stage. Join new $a'b'c'd'e'f'$ to represent the final front view.

Problem 10.14 *A hexagonal plane of side 30 mm has a corner in the V.P. The surface of the plane is inclined at 45° to the V.P. and perpendicular to the H.P. Draw its projections. Assume that the diagonal through the corner in the V.P. is parallel to the H.P.*

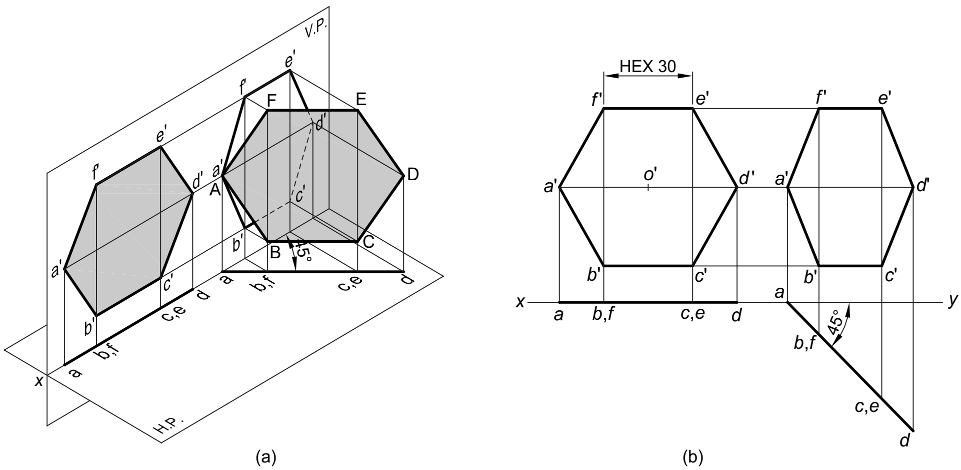


Fig. 10.15 **(a)** *3-D view* **(b)** *Projections*

Construction Refer to Fig. 10.15(b).

The plane has a corner in the V.P., so consider that the plane *ABCDEF* is placed in the V.P. with line joining the corner *A* and the centre of the plane parallel to the V.P.

1. **First stage** Draw a hexagon $a'b'c'd'e'f'$ keeping $a'o'$ parallel to *xy* to represent the front view. Project the corners to *xy* and obtain ad as the top view.
2. **Second stage** Reproduce the top view of first stage keeping *a* on *xy* and *ad* inclined at 45° to *xy*. Obtain new points a', b', c', d', e' and f' in the front view by joining the points of intersection of the projectors drawn from the top view of the second stage with the corresponding horizontal locus lines drawn from the front view of the first stage. Join new $a'b'c'd'e'f'$ to represent the final front view.

Problem 10.15 *A circular plane of diameter 50 mm is resting on a point of the circumference on the V.P. The plane is inclined at 30° to the V.P. and the centre is 35 mm above the H.P. Draw its projections.*

Construction Refer to Fig. 10.16.

A circle has a point of the circumference on the V.P. so consider that initially the circle is placed in the V.P.

1. **First stage** Draw a circle of diameter 50 mm keeping centre 35 mm above *xy* to represent the front view. Divide the circle in 12 equal parts. Project all the points to *xy* and obtain *ag* as the top view.
2. **Second stage** Reproduce the top view of first stage keeping *a* on *xy* and *ag* inclined at 30° to *xy*. Obtain points a', b', c', d', etc., in the new front view by joining the point of intersection of the vertical projectors drawn from the top view of the second stage with the corresponding horizontal

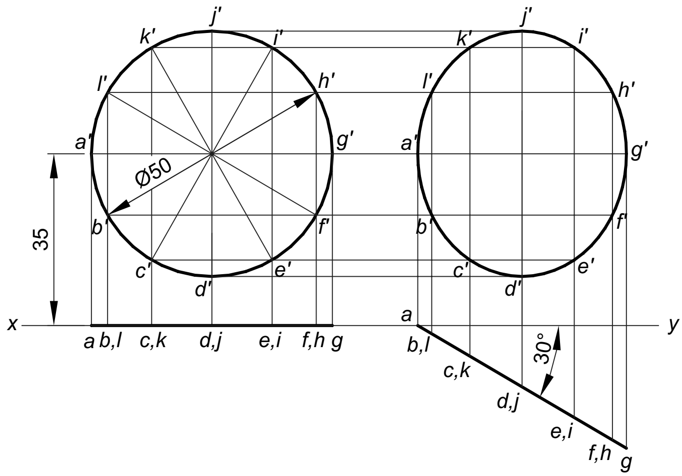


Fig. 10.16

locus lines drawn from the front view of the first stage. Join new *a′b′c′d′e′f′g′h′i′j′k′l′* to represent the final front view.

Problem 10.16 *An isosceles triangle of base 40 mm and altitude 54 mm has its base in the V.P. The surface of the plane is inclined at 50° to the V.P. and perpendicular to the H.P. Draw its projections.*

Construction Refer to Fig. 10.17.

An isosceles triangle has its base in the V.P., so consider that initially the triangle *ABC* is placed in the V.P. with base *AB* perpendicular to the H.P.

1. **First stage** Draw a triangle *a′b′c′* keeping *a′b′* perpendicular to *xy* to represent the front view. Project the corners to *xy* and obtain ac as the top view.
2. **Second stage** Reproduce the top view of first stage keeping *ab* on *xy* and *ac* inclined at 50° to *xy*. Obtain new points *a′*, *b′* and *c′* in the front view by joining the points of intersection of the vertical projectors from *a*, *b* and *c* of the second stage with the corresponding horizontal locus lines from *a′*, *b′* and *c′* of the first stage. Join *a′b′c′* to represent the final front view. Here, the front view is an equilateral triangle of side 40 mm.

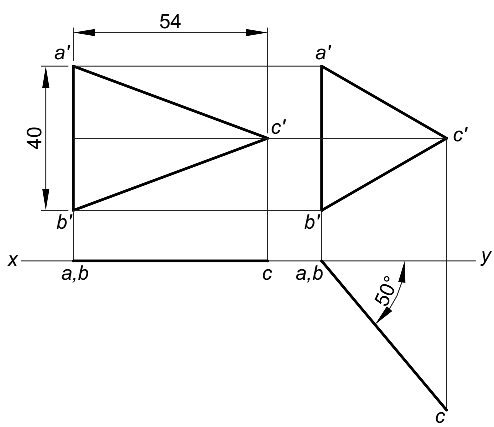


Fig. 10.17

Problem 10.17 *An isosceles triangle of base 40 mm and altitude 54 mm is placed in such a way that it appears as an equilateral triangle in the front view. Draw its projections and determine inclination of the triangle with the V.P.*

Construction Refer to Fig. 10.17.

An isosceles triangle can appear as an equilateral triangle of side 40 mm in the front view, when it is resting on its base in the V.P. and the surface is inclined to the V.P.

1. **First stage** Draw an isosceles triangle $a'b'c'$ keeping base $a'b'$ perpendicular to xy. Project all the corners to xy and obtain ac as the top view.
2. **Second stage** Draw an equilateral triangle $a'b'c'$ of side 40 mm on the horizontal locus lines of points a', b' and c' of the first stage. Project $a'b'$ to meet xy at ab. Draw an arc with centre a and radius equal to ac of the first stage to meet the vertical projector of c' at point c. Join ac. Measure inclination of ac with xy as inclination of the set-square with the V.P. Here $\phi = 50°$.

Problem 10.18 *A 60° set-square has the shortest edge of 40 mm lying in the V.P. The surface is inclined to the V.P. and perpendicular to the H.P. such that the front view appears as an isosceles triangle. Draw the projections of the set-square and determine its inclination with the V.P.*

Construction Refer to Fig. 10.18.

A 60° set-square inclined to the V.P. and perpendicular to the H.P. can appear as an isosceles triangle in the front view, when the shorter edge is in the V.P.

1. **First stage** Draw a right angled triangle $a'b'c'$ keeping 40 mm long $a'b'$ perpendicular to xy. Project the corners to xy and obtain ac as the top view.
2. **Second stage** Draw another right angled triangle $a'b'c'$ on the horizontal locus line from points a', b' and c' of the first stage such that length of $b'c'$ is equal to that of $a'b'$. Project $a'b'$ to meet xy at ab. Draw an arc with centre a and radius equal to ac of the first stage to meet the vertical projector of c' at point c. Join ac. Measure inclination of ac with xy as inclination of the set-square with the V.P. Here $\phi = 55°$.

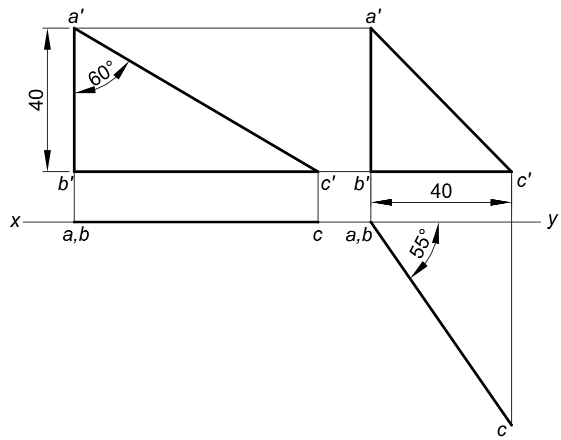


Fig. 10.18

Problem 10.19 *A square lamina ABCD of side 40 mm is suspended from a point O such that its surface is inclined at 30° to the V.P. The point O lies on the side AB 12 mm away from A. Draw its projections.*

Construction Refer to Fig. 10.19.

1. **First stage** Draw a square $a'b'c'd'$ keeping $a'd'$ parallel to xy. Mark a point o' on $a'd'$ at a distance 12 mm from end a' as the point of suspension. Also, mark the centre of the square g' to represent the centre of gravity.
2. **Second stage** Reproduce the front view of first stage such that $o'g'$ is perpendicular to xy. Project corners and obtain bd as the top view.
3. **Third stage** Reproduce the top view keeping bd inclined at 30° to xy. Obtain new points a', b', c' and d' in the front view by joining the points of intersection of the vertical projectors drawn from points a, b, c and d of the third stage with the corresponding horizontal locus lines drawn from points a', b', c' and d' of the second stage. Join new $a'b'c'd'$ to represent the final front view.

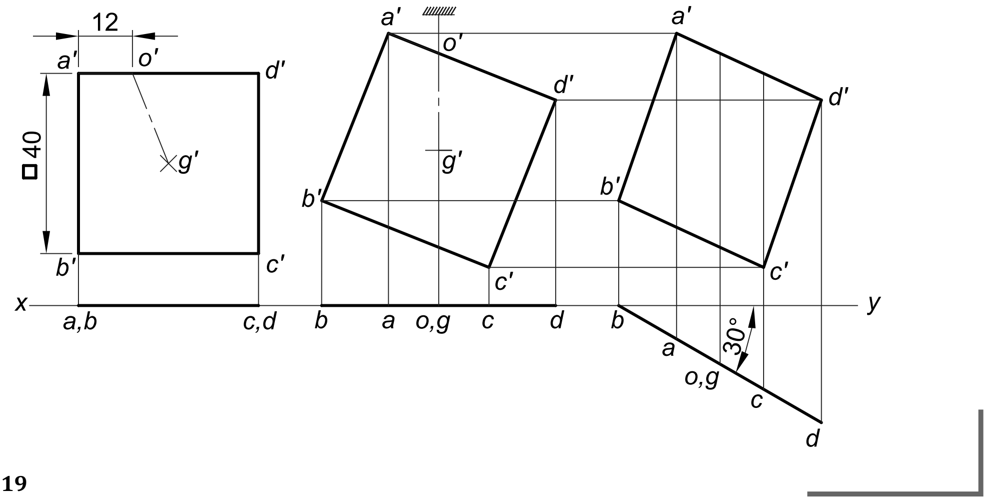


Fig. 10.19

10.8 TRACE OF A PLANE

A plane which is perpendicular or inclined to a reference plane will meet (extended if necessary) that reference plane in a line. This line is called *trace* of the plane. When the plane meets the H.P. then the trace is called *horizontal trace* or H.T. Similarly, when the plane meets the V.P. then the trace is called *vertical trace* or V.T.

10.8.1 Plane Parallel to H.P.

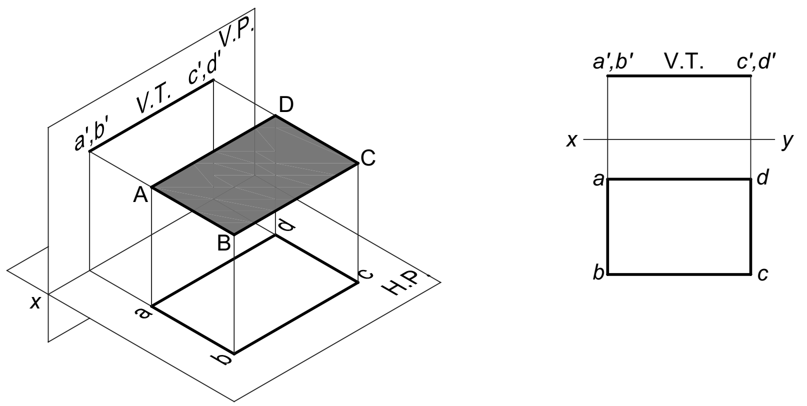


Fig. 10.20 *Trace of a plane parallel to the horizontal plane*

In Fig. 10.20, the plane *ABCD* is parallel to H.P.

1. On extending the plane it does not intersect the H.P., therefore, it has no H.T.
2. On extending the plane it intersects the V.P. on line *a′d′*. Therefore, *a′d′* represents the V.T. of the plane.

10.8.2 Plane Parallel to V.P.

In Fig. 10.21, the plane *ABCD* is parallel to V.P.

1. On extending the plane it does not intersect the V.P., therefore it has no V.T.
2. On extending the plane it intersects the H.P. at *ad*. Therefore, *ad* represents the H.T. of the plane.

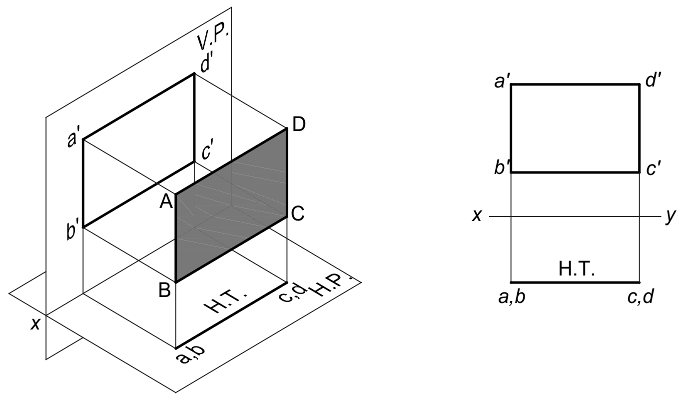


Fig. 10.21 *Trace of a plane parallel to the vertical plane*

10.8.3 Plane Parallel to the Profile Plane

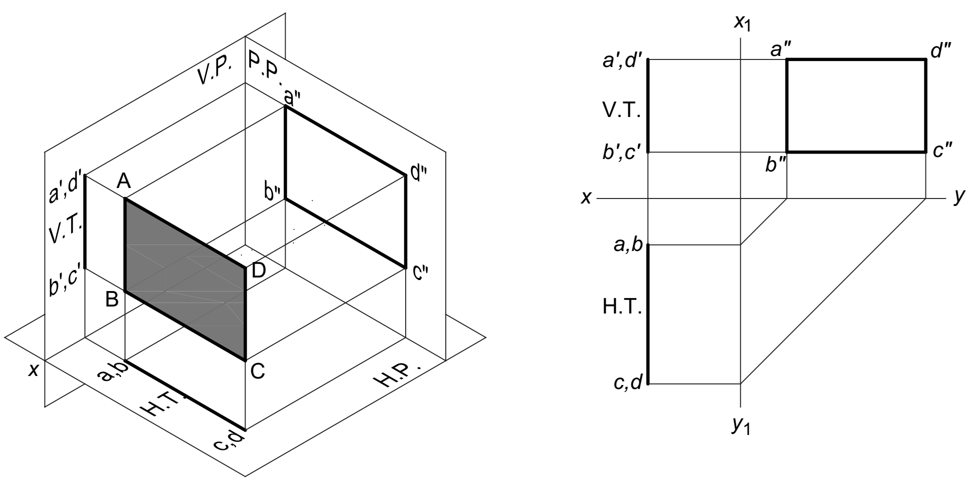


Fig. 10.22 *Trace of a plane parallel to the profile plane*

In Fig. 10.22, the plane *ABCD* is parallel to the profile plane.

1. On extending the plane it intersects the V.P. at $a'b'$. Therefore, $a'b'$ represents the V.T. of the plane.
2. On extending the plane it intersects the H.P. at *ad*. Therefore, *ad* represents the H.T. of the plane.

10.8.4 Plane Inclined to H.P. and Perpendicular to V.P.

In Fig. 10.23, the plane *ABCD* is perpendicular to the V.P. and inclined to the H.P.

1. On extending the plane it intersects the V.P. on line *a′d′*. Therefore, the line *a′d′* represents the V.T. of the plane.
2. On extending the plane, it intersects the H.P. on line 1-2. Therefore, the line 1-2 represents the H.T. of the plane. To draw the H.T. (a) Extend the front view to meet *xy* at point *h′* (b) Draw the projector from *h′* to meet horizontal lines from the top view at 1–2.

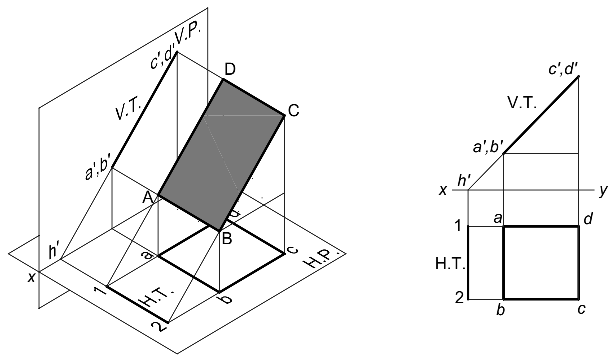


Fig. 10.23 *Trace of a plane inclined to H.P. and perpendicular to V.P.*

10.8.5 Plane Inclined to V.P. and Perpendicular to H.P.

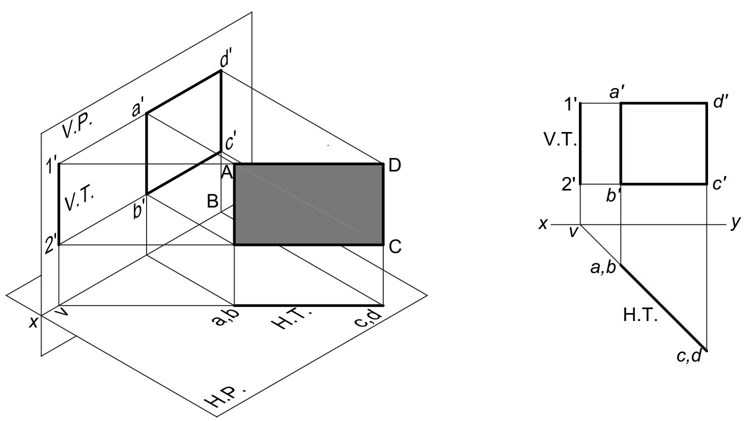


Fig. 10.24 *Trace of a plane inclined to V.P. and perpendicular to H.P.*

In Fig. 10.24, the plane *ABCD* is perpendicular to the H.P. and inclined to the V.P.

1. On extending the plane it intersects the H.P. on line *ad*. Therefore, the line ad represents the H.T. of the plane.
2. On extending the plane, it intersects the V.P. on line 1′-2′. Therefore, the line 1′–2′ represents the V.T. of the plane. To draw the V.T. (a) Extend the top view to meet *xy* at point *v* (b) Draw the projector from *v* to meet horizontal lines from the front view at 1′–2′.

Problem 10.20 *A rectangular plane of sides 40 mm and 60 mm has a corner on the H.P. and 20 mm in front of the V.P. The surface of the plane is parallel to the V.P. and all the sides are equally inclined to the H.P. Draw its projections and locate the traces.*

Construction Refer to Fig. 10.25.

1. Draw a rectangle $a'b'c'd'$ keeping corner a' on xy and edge $a'b'$ inclined at 45° to xy. This represents the front view.
2. Project the corners and obtain points a, b, c and d at a distance of 20 mm from xy. Join $abcd$. This is the top view.
3. As the plane is parallel to the V.P., it has no V.T.
4. The H.T. coincides with the top view of the plane, i.e., line db represents H.T.

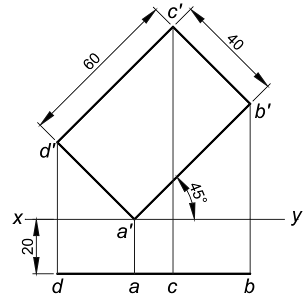


Fig. 10.25

Problem 10.21 *A thin hexagonal plate of side 30 mm has a side inclined at 45° to the V.P. Its V.T. is parallel to and 25 mm above the reference line. Draw its projections.*

Construction Refer to Fig. 10.26.

As V.T. is parallel to xy, the plane is parallel to H.P. As the V.T. is 25 mm above xy, the plane is 25 mm above H.P.

1. Draw a hexagon $abcdef$ keeping side ed inclined at 45° with xy to represent the top view.
2. Project the corners a, b, c, d, e and f, 25 mm above xy and obtain points a', b', c', d', e' and f'. Join $a'b'c'd'e'f'$ to represent the front view.

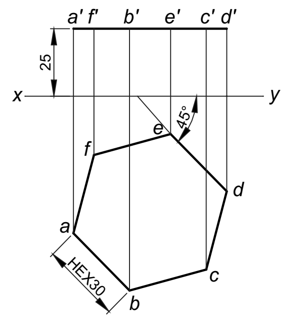


Fig. 10.26

10.9 SUMMARY

Table 10.1 summarises views and traces for the various orientations of the planes.

Table 10.1 *Summary for projections of planes*

S. No.	*Orientation of plane*	*Front view*	*Top view*	*H.T.*	*V.T.*
1.	Plane parallel to H.P.	A line parallel to xy	True shape of the plane	Does not exist	Coincides with front view
2.	Plane parallel to V.P.	True shape of the plane	A line parallel to xy	Coincides with top view	Does not exist
3.	Plane perpendicular to both H.P. and V.P.	A line perpendicular to xy	A line perpendicular to xy	Coincides with top view	Coincides with front view
4.	Plane inclined at θ to H.P. and perpendicular to V.P.	A line inclined at θ to xy	Projected shape of the plane	A straight line perpendicular to xy	Coincides with front view
5.	Plane inclined at ϕ to V.P. and perpendicular to H.P.	Projected shape of the plane	A line inclined at ϕ to xy	Coincides with top view	A straight line perpendicular to xy

EXERCISE 10A

Surface parallel or perpendicular to the reference planes

10.1 A hexagonal plane of side 25 mm has its surface parallel to and 20 mm above the H.P. Draw its projections, when a side is (a) parallel to V.P., (b) perpendicular to V.P., (c) inclined at 45° to V.P.

10.2 A circular plane of diameter 60 mm has its centre 20 mm above the H.P. and 35 mm in front of the V.P. The surface of the plane is parallel to the H.P. Draw its projections.

10.3 A composite plate of negligible thickness is made up of a rectangle with sides 60 mm and 40 mm and a semicircle on its longer side. The plate lies in the H.P. with one of the shorter sides parallel to the V.P. Draw its projections.

10.4 A pentagonal plane of side 35 mm has a corner on the H.P. and the side opposite to this corner is parallel to the H.P. The plane is parallel to and 20 mm in front of the V.P. Draw its projections and locate its traces.

10.5 A square *ABCD* of side 40 mm is suspended from a point *O*, lying on side *AB* at a distance 15 mm from corner *A*. The plane is parallel to and 25 mm in front of the V.P. Draw its projections and locate the traces.

10.6 A rectangular plane of sides 50 mm and 30 mm is perpendicular to both H.P. and V.P. The longer edges are parallel to the H.P. and nearest one is 20 mm above it. The shorter edge nearer to V.P. is 15 mm from it. Draw its projections.

10.7 A thin hexagonal plate of side 30 mm has one of the sides inclined at 45° to the VP. Its V.T. is parallel to and 25 mm above the reference line and the H.T. does not exist. Draw its projections.

10.8 An equilateral triangular plane of sides 60 mm has a side inclined at 45° to the H.P. Its H.T. is parallel to and 25 mm below the reference line. The V.T. does not exist. Draw its projections.

Surface inclined to H.P. and perpendicular to V.P.

10.9 A pentagonal plane of side 30 mm has an edge on the H.P. The surface of the plane is inclined at 45° to the H.P. and perpendicular to the V.P. Draw its projections and locate the traces.

10.10 A pentagonal plane of side 30 mm has a corner on the ground. The surface of the plane is inclined at 45° to the H.P. and perpendicular to the V.P. Draw its projections when the side opposite to the corner on which it is resting is parallel to H.P.

10.11 A circular plane of diameter 50 mm is resting on a point of the circumference on the H.P. The plane is inclined at 45° to the H.P. and the centre is 35 mm in front of the V.P. Draw its projections and locate the traces.

10.12 A trapezium *ABCD* having parallel sides *AB* = 50 mm, *CD* = 30 mm and height 40 mm is kept on its side *AB* in the V.P. The surface of the trapezium is inclined at 45° to the H.P. and perpendicular to the V.P. Draw its projections.

10.13 A square lamina of side 50 mm has a corner on the H.P. The diagonal through that corner is parallel to V.P. and inclined at 30° to the H.P. Draw its projections when the lamina is perpendicular to the V.P. Measure the distance of the topmost corner from the H.P.

10.14 A pentagonal plane of side 30 mm has a side on the H.P. and the surface perpendicular to the V.P. The corner opposite to that side is 40 mm above the H.P. Draw the projections of the plane and determine its inclination with the H.P.

10.15 A rectangular plate of sides 60 mm and 40 mm rest on a shorter edge on the H.P. with its surface perpendicular to the V.P. The centre of the plate is 20 mm above the H.P. and 30 mm in front of the V.P. Draw the projections of the plate and determine angle made by it with the H.P.

10.16 A hexagonal plate of side 30 mm has a centrally punched circular hole of diameter 30 mm. The plane is placed on a side in the H.P. such that the surface is perpendicular to the V.P. and inclined at 45° to the H.P. Draw its projections when the centre of the plate is 35 mm in front of the V.P.

10.17 A square plane of side 50 mm is placed on the H.P. The surface is inclined to the H.P. and perpendicular to the V.P. such that the top view appears as a rectangle of 50 mm and 25 mm sides. Draw its projections and determine inclination of the plane with the H.P.

10.18 A square plane of diagonal 70 mm is kept in such a way that its top view appears as a rhombus of 70 mm and 45 mm diagonals. Draw its projections and determine inclination of the plane with the H.P.

10.19 A 60° set-square has shortest edge of 50 mm in the H.P. The surface is inclined to the H.P. and perpendicular to the V.P. such that the top view appears as isosceles triangle. Draw its projections and determine inclination of the set-square with the H.P.

10.20 A circular plane of diameter 50 mm is placed on a point of the circumference in the H.P. Its surface is inclined to the H.P. and perpendicular to the V.P. such that top view appears as an ellipse of major and minor axes 50 mm and 30 mm, respectively. Draw the projections of the plane and determine its inclination with the H.P.

10.21 The top view of a plane is a circle of diameter 50 mm. Its front view is a straight line inclined at 45° to the reference line. Draw the projections of the plane and determine its true shape.

10.22 The top view of a plane whose surface is inclined at 45° to the H.P. and perpendicular to V.P. appears as a regular hexagon of side 30 mm with a side parallel to the reference line. Draw the projections of the plane and determine its true shape.

10.23 The top view of a plane is a square of side 40 mm having one side inclined at 30° to the V.P. Its front view is a straight line inclined at 45° to the reference line. Draw the projections of the plane and determine its true shape.

10.24 The top view of a triangular plane is an equilateral triangle of side 50 mm having one side parallel to the reference line. The front view of the plane is a 65 mm long straight line. Determine the true shape of the plane.

Surface inclined to V.P. and perpendicular to H.P.

10.25 A pentagonal plane of side 30 mm has an edge in the V.P. The surface of the plane is inclined at 45° to the V.P. and perpendicular to the H.P. Draw its projections and locate the traces.

10.26 A pentagonal plane of side 30 mm has a corner in the V.P. The surface of the plane is inclined at 45° to the V.P. and perpendicular to the H.P. Draw its projections when the side opposite to the corner in the V.P. is parallel to the V.P.

10.27 A hexagonal lamina of side 50 mm has a corner in the V.P. The diagonal through that corner is parallel to H.P. and inclined at 30° to the V.P. Draw its projections when the lamina is perpendicular to the H.P. Measure the distance of the topmost corner from the V.P.

10.28 A pentagonal plane of side 30 mm rests on an edge in the V.P. with its surface perpendicular to the H.P. The corner opposite to the edge on which it is resting, is 30 mm in front of the V.P. Draw the projections of the plane and determine its inclination with the V.P.

10.29 A hexagonal plane of side 30 mm has a centrally punched circular hole of 30 mm diameter. An edge of the plane is in the V.P. Its surface is perpendicular to the H.P. and inclined at 45° to the V.P. Draw its projections.

10.30 A circular lamina of diameter 50 mm has its centre 35 mm above the H.P. and 25 mm in front of the V.P. The surface of lamina is inclined at 45° to the V.P. and perpendicular to the H.P. Draw its projections.

10.31 A square plane of side 50 mm is placed in the V.P. The surface is inclined to the V.P. and perpendicular to the H.P. such that the front view appears as a rectangle of 50 mm and 25 mm sides. Draw its projections and determine inclination of the plane with the V.P.

10.32 A square plane of diagonal 60 mm is placed on the V.P. The surface is inclined to the V.P. and perpendicular to the H.P. such that front view appears as a rhombus of 60 mm and 46 mm diagonals. Draw its projections and determine inclination of the plane with the V.P.

10.33 A rectangular plane of side 35 mm and 70 mm is placed on a side in the V.P. The surface is inclined to the V.P. and perpendicular to the H.P. such that the front view appears as a square. Draw its projections and determine inclination of the plane with the V.P.

10.34 The major and the minor diagonals of a rhombus are 70 mm and 45 mm respectively. It is placed in such a way that its front view appears as a square. Draw its projections and determine inclination of the rhombus with the V.P.

10.35 A circular plate of diameter 50 mm is resting on a point of the rim in the V.P. Its surface is inclined to the V.P. and perpendicular to H.P. such that the front view appears as an ellipse of major and minor axes 50 mm and 30 mm, respectively. Draw the projections of the plate and determine its inclination with the V.P.

10.36 The front view of a plane whose surface is perpendicular to H.P. and inclined at 30° to the V.P. appears as a regular pentagon of side 30 mm with a side parallel to the reference line. Draw the projections of the plane and determine its true shape.

10.37 The front view of a plane is a straight line inclined at 30° with the reference line. Its top view is a regular hexagon of side 30 mm having an edge inclined at 45° with the reference line. Draw its projections and determine its true shape.

10.10 PLANE INCLINED TO BOTH THE REFERENCE PLANES

When the surface of a plane is inclined to both the reference planes, its projections are drawn in three stages. It is the extension of the problems done earlier in this chapter on projections of planes inclined to one of the reference planes.

10.10.1 An Element of the Plane in H.P.

Problem 10.22 *A hexagonal plane of side 30 mm has an edge on the H.P. Its surface is inclined at 45° to the H.P. and the edge on which the plane rests is inclined at 30° to the V.P. Draw its projections.*

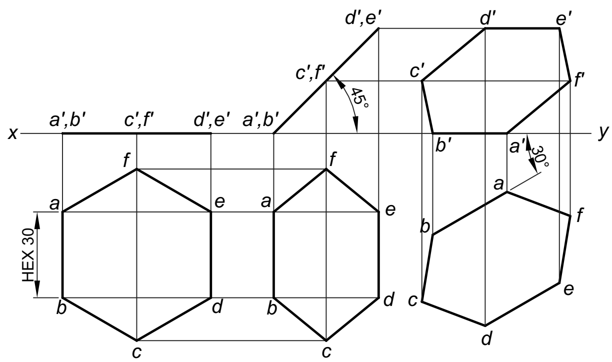


Fig. 10.27

Construction Refer to Fig. 10.27.

1. **First stage** Draw a hexagon *abcdef* keeping *ab* perpendicular to *xy* to represent the top view. Project the corners to *xy* and obtain *b′d′* as the front view.
2. **Second stage** Reproduce the front view of first stage keeping *a′b′* on *xy* and *b′d′* inclined at 45° to *xy*. Obtain points *a*, *b*, *c*, *d*, *e* and *f* of the top view by joining the points of intersection of the projectors from points *a′*, *b′*, *c′*, *d′*, *e′* and *f′* of the second stage with the corresponding locus lines from points *a*, *b*, *c*, *d*, *e* and *f* of the first stage. Join *abcdef*.
3. **Third stage** Reproduce the top view of the second stage keeping line *ab* inclined at 30° to *xy*. Obtain point *a′*, *b′*, *c′*, *d′*, *e′* and *f′* of the front view by joining the points of intersection of the projectors from points *a*, *b*, *c*, *d*, *e* and *f* of the third stage with the corresponding locus lines from points *a′*, *b′*, *c′*, *d′*, *e′* and *f′* of the second stage. Join *a′b′c′d′e′f′*.

Problem 10.23 *A hexagonal plane of side 30 mm has a corner on the ground. Its surface is inclined at 45° to the H.P. and the top view of the diagonal through the corner which is in the H.P. makes an angle of 60° with the V.P. Draw its projections.*

Construction Refer to Fig. 10.28.

1. **First stage** Draw a hexagon *abcdef* keeping *ao* parallel to *xy* to represent the top view. Project the corners to *xy* and obtain *a′d′* as the front view.

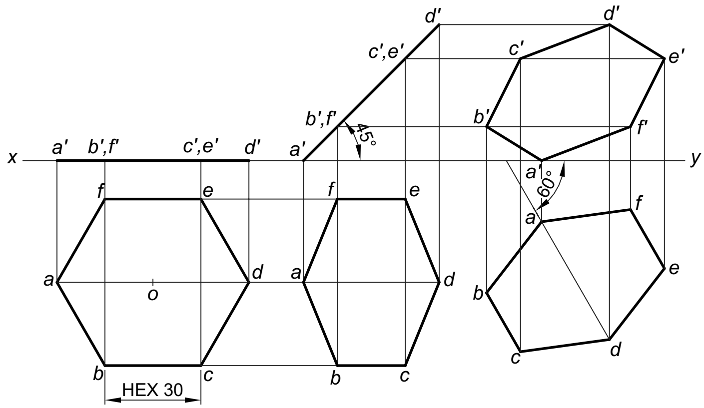


Fig. 10.28

2. **Second stage** Reproduce the front view of first stage keeping a' on xy and $a'd'$ inclined at 45° to xy. Obtain points a, b, c, d, e and f of the top view by joining the points of intersection of the projectors from points a', b', c', d', e' and f' of the second stage with the corresponding locus lines from points a, b, c, d, e and f of the first stage. Join *abcdef*.
3. **Third stage** Reproduce the top view of the second stage keeping line *ad* inclined at 60° to xy. Obtain point a', b', c', d', e' and f' of the front view by joining the points of intersection of the projectors from points a, b, c, d, e and f of the third stage with the corresponding locus lines from points a', b', c', d', e' and f' of the second stage. Join $a'b'c'd'e'f'$.

Problem 10.24 *A pentagonal plane of side 30 mm is resting on a corner in the H.P. The side opposite to the corner in the H.P. is parallel to and 35 mm above H.P. and inclined at 45° to the V.P. Draw its three principal views.*

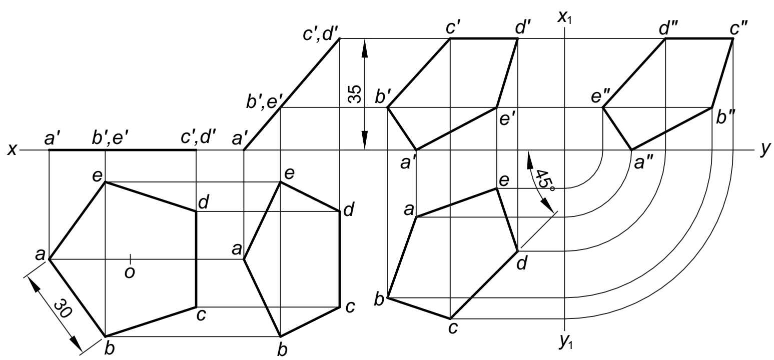


Fig. 10.29

Construction Refer to Fig. 10.29.

1. **First stage** Draw a pentagon *abcde* keeping *ao* parallel to xy to represent the top view. Project the corners to xy and obtain $a'c'$ as the front view.

2. **Second stage** Redraw the front view of first stage keeping a' on xy and $c'd'$ 35 mm above xy. Obtain points a, b, c, d and e of the top view by joining the points of intersection of the projectors from points a', b', c', d' and e' of the second stage with the corresponding locus lines from points a, b, c, d and e of the first stage. Join *abcde*.
3. **Third stage** Reproduce the top view of the second stage keeping *cd* inclined at 45° to xy. Obtain point a', b', c', d' and e' of the front view by joining the points of intersection of the projectors from points a, b, c, d and e of the third stage with the corresponding locus lines from points a', b', c', d' and e' of the second stage. Join $a'b'c'd'e'$.
4. **Side view** Draw x_1y_1. Project points a, b, c, d and e of the third stage on oy_1. Now taking centre o, transfer the points obtained on oy_1 to oy. Project all the points perpendicular to xy to meet the corresponding projectors (drawn perpendicular to x_1y_1) from points a', b', c', d' and e' of the third stage at points a'', b'', c'', d'' and e''. Join $a''b''c''d''e''$.

Problem 10.25 *The diagonals of a rhombus measure 100 mm and 40 mm. The longer diagonal is inclined at 30° to H.P. with an end in H.P. and the smaller diagonal is parallel to both the principal planes. Draw its projections.*

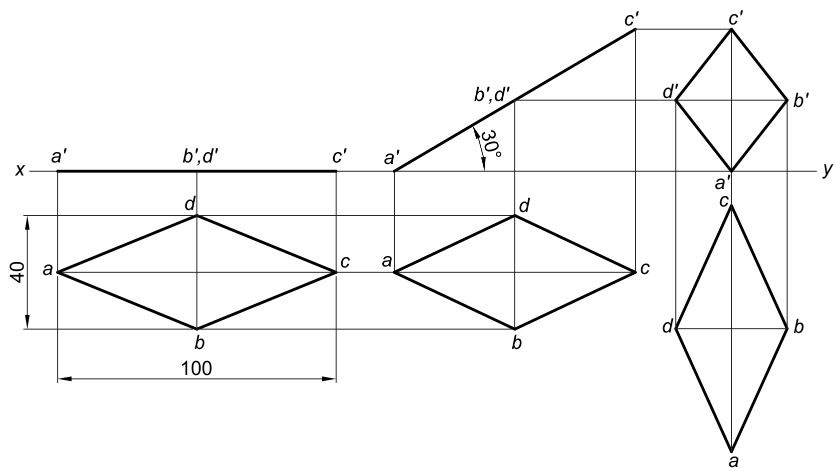


Fig. 10.30

Construction Refer to Fig. 10.30.

1. **First stage** Draw a rhombus *abcd* keeping diagonal *ac* parallel to xy and diagonal *bd* perpendicular to xy to represent the top view. Project all the corners to xy and obtain $a'c'$ as the front view.
2. **Second stage** Redraw the front view of the first stage keeping a' on xy and $a'c'$ inclined at 30° to xy. Obtain points a, b, c and d in the top view by joining the points of intersection of the projectors from points a', b', c' and d' of the second stage with the corresponding locus lines from points a, b, c and d of the first stage. Join *abcd*.
3. **Third stage** Reproduce the top view of the second stage keeping *bd* parallel to xy. Obtain points a', b', c' and d' in the front view by joining the points of intersection of the projectors from points a, b, c and d of the third stage with the corresponding locus lines from points a', b', c' and d' of the second stage. Join $a'b'c'd'$.

Problem 10.26 *A rectangular plane of edges 35 mm and 70 mm is resting on an edge in the H.P. The surface is inclined to the H.P. such that the top view appears as a square. Draw its projections when the edge resting on the H.P. is inclined at 30° to the V.P.*

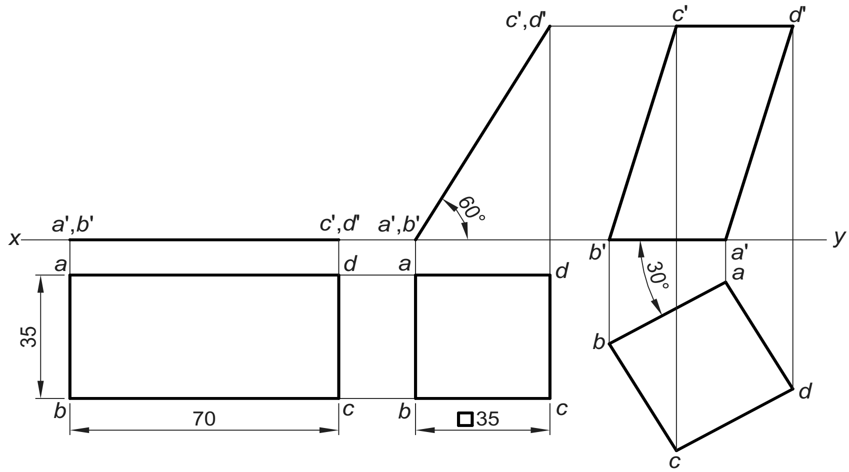


Fig. 10.31

Construction Refer to Fig. 10.31.

A rectangular plane inclined to the H.P. and perpendicular to the V.P. can appear as a square in the top view, when the shorter edge of the plane is in the H.P. or parallel to H.P.

1. **First stage** Draw a rectangle *abcd* keeping side *ab* perpendicular to *xy* to represent the top view. Project the corners to *xy* and obtain *b′c′* as the front view.
2. **Second stage** Draw a square *abcd* of 35 mm side on the locus line from points *a*, *b*, *c* and *d* of the first stage. Project point *a* to meet *xy* at *a′*. Draw an arc with centre *a′* and radius equal to *a′d′* of the first stage to meet the projector of point *d* at *d′*. Join *a′d′*.
3. **Third stage** Reproduce the top view of the second stage keeping *ab* inclined at 30° to *xy*. Obtain points *a′*, *b′*, *c′* and *d′* in the front view by joining the points of intersection of the projectors from points *a*, *b*, *c* and *d* of the third stage with the corresponding locus lines from points *a′*, *b′*, *c′* and *d′* of the second stage. Join *a′b′c′d′*.

Problem 10.27 *A thin square plate of side 40 mm stands on one of its corners in the H.P. and the opposite corner is raised so that one of the diagonals is twice of the other. If one of the diagonals is parallel to both the reference planes, draw its projections and determine the inclination of the plate with the H.P.*

Construction Refer to Fig. 10.32.

1. **First stage** Draw a square *abcd* keeping *ac* parallel to *xy* to represent the top view. Project all the corners and obtain *a′c′* as the front view.
2. **Second stage** Draw a rhombus *abcd* on the locus lines from points *a*, *b*, *c* and *d* such that diagonal *ac* is equal to half of *bd*. Project point *a* to meet *xy* at point *a′*. Draw an arc with centre *a′*

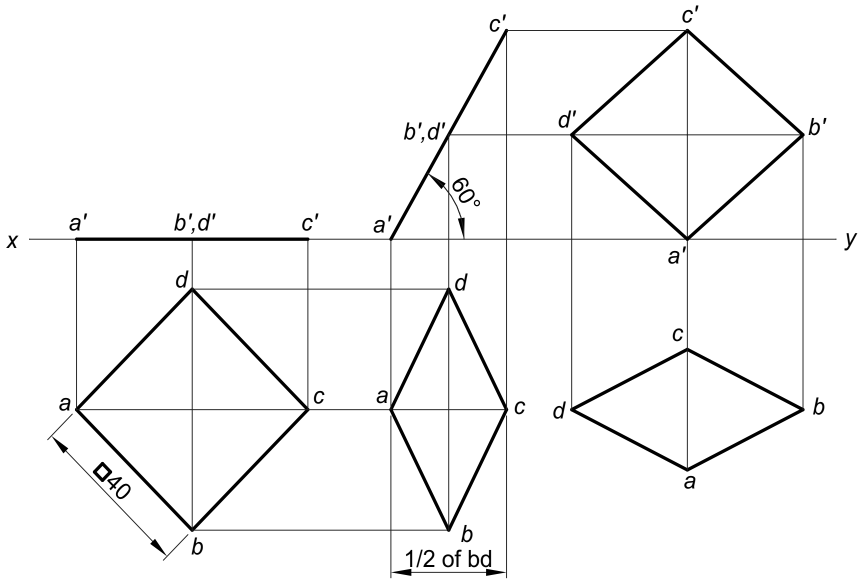


Fig. 10.32

and radius equal to $a'c'$ of the first stage to meet the projector of point c at c'. Join $a'c'$ and mark points b' and d' on it. Determine inclination of line $a'c'$ with xy as inclination of the surface with H.P. Here $\theta = 60°$.

3. **Third stage** Reproduce the top view of the second stage keeping diagonal *bd* parallel to *xy*. Obtain points a', b', c' and d' in the front view by joining the points of intersection of the projectors from points *a*, *b*, *c* and *d* of the third stage with the corresponding locus lines from points a', b', c' and d' of the second stage. Join $a'b'c'd'$.

Problem 10.28 *A rhombus of diagonals 60 mm and 40 mm having the longer diagonal parallel to the reference line represents the top view of a square lamina of diagonals 60 mm resting on a corner on the H.P. Draw the front view of the lamina and determine inclination of its surface with the H.P.*

Construction Refer to Fig. 10.33.

1. **First stage** Draw a square *abcd* of diagonal 60 mm keeping ac parallel to *xy* to represent the top view. Project all the corners and obtain $a'c'$ as the front view.
2. **Second stage** Draw a rhombus *abcd* on the locus lines drawn from points *a*, *b*, *c* and *d* of the first stage such that diagonal *ac* is 40 mm. Project point *a* to meet *xy* at point a'. Draw an arc with centre a' and radius equal to $a'c'$ of the first stage to meet the projector of point c at c'. Join $a'c'$. Mark points b' and d' on line $a'c'$. Measure the inclination of line $a'c'$ with xy as inclination of the surface with H.P. Here $\theta = 48°$.
3. **Third stage** Reproduce the top view of the second stage keeping diagonal *bd* parallel to *xy*. Obtain points a', b', c' and d' in the front view by joining the points of intersection of the projectors from points *a*, *b*, *c* and *d* of the third stage with the corresponding locus lines from points a', b', c' and d' of the second stage. Join $a'b'c'd'$.

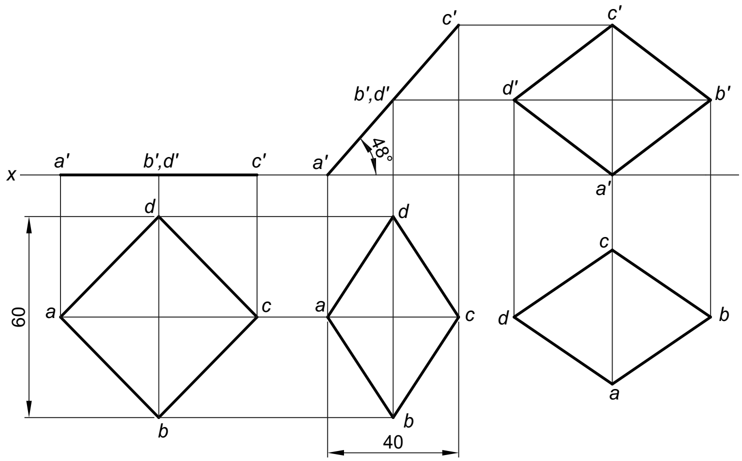


Fig. 10.33

10.10.2 An Element of the Plane in V.P.

Problem 10.29 *A hexagonal plane of side 30 mm has an edge in the V.P. The surface of the plane is inclined at 45° to the V.P. and the edge on which it rests is inclined at 30° to the H.P. Draw its projections.*

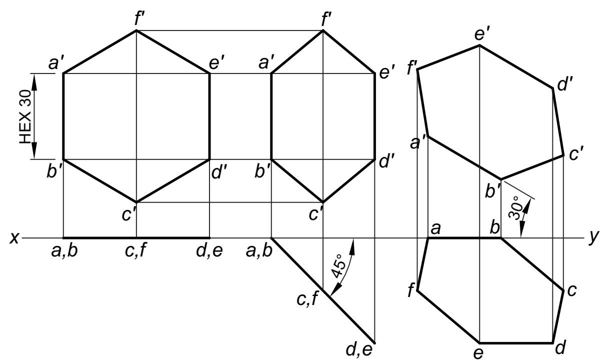


Fig. 10.34

Construction Refer to Fig. 10.34.

1. **First stage** Draw the hexagon *a′b′c′d′e′f′* keeping *a′b′* perpendicular to *xy* to represent the front view. Project the corners to *xy* and obtain *bd* as the top view.
2. **Second stage** Reproduce the top view of first stage keeping *ab* on *xy* and *ae* inclined at 45° to *xy*. Obtain points *a′*, *b′*, *c′*, *d′*, *e′* and *f′* of the front view by joining the points of intersection of the vertical projectors drawn from points *a*, *b*, *c*, *d*, *e* and *f* of the second stage with the corresponding locus lines drawn from *a′*, *b′*, *c′*, *d′*, *e′* and *f′* of the first stage. Join *a′b′c′d′e′f′*.

3. **Third stage** Reproduce the front view of the second stage keeping *a′b′* inclined at 30° to *xy*. Obtain points *a*, *b*, *c*, *d*, *e* and *f* in the top view by joining the points of intersection of the projectors from points *a′*, *b′*, *c′*, *d′*, *e′* and *f′* of the third stage with the corresponding locus lines from points *a*, *b*, *c*, *d*, *e* and *f* of the second stage. Join *abcdef*.

Problem 10.30 *A hexagonal plane of side 30 mm has a corner in the V.P. The surface of the plane is inclined at 45° to the V.P. and perpendicular to the H.P. The front view of the diagonal passing through that corner is inclined at 60° to the H.P. Draw its three principal views.*

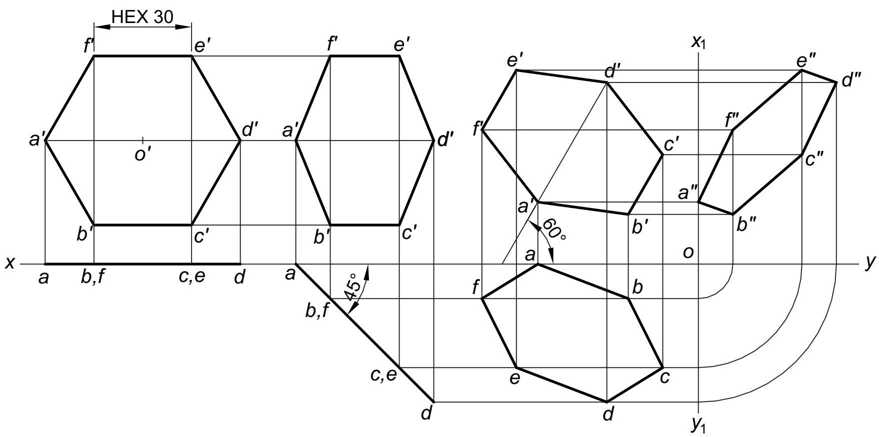


Fig. 10.35

Construction Refer to Fig. 10.35.

1. **First stage** Draw the hexagon *a′b′c′d′e′f′* keeping *a′o′* parallel to *xy* to represent the front view. Project the corners to *xy* and obtain *ad* as the top view.
2. **Second stage** Reproduce the top view of first stage keeping *a* on *xy* and *ad* inclined at 45° to *xy*. Obtain points *a′*, *b′*, *c′*, *d′*, *e′* and *f′* in the front view by joining the points of intersection of the projectors from points *a*, *b*, *c*, *d*, *e* and *f* of the second stage with the corresponding locus lines from points *a′*, *b′*, *c′*, *d′*, *e′* and *f′* of the first stage. Join *a′b′c′d′e′f′*.
3. **Third stage** Reproduce the front view of the second stage keeping line *a′d′* inclined at 60° to *xy*. Obtain points *a*, *b*, *c*, *d*, *e* and *f* in the top view by joining the points of intersection of the projectors from points *a′*, *b′*, *c′*, *d′*, *e′* and *f′* of the third stage with the corresponding locus lines from points *a*, *b*, *c*, *d*, *e* and *f* of the second stage. Join *abcdef*.
4. **Side view** Draw x_1y_1. Project points *a*, *b*, *c*, *d*, *e* and *f* of the third stage on oy_1. Now taking centre *o*, transfer the points obtained on oy_1 to *oy*. Project all the points perpendicular to *xy* to meet the corresponding projectors (drawn perpendicular to x_1y_1) from points *a′*, *b′*, *c′*, *d′*, *e′* and *f′* of the third stage at points *a″*, *b″*, *c″*, *d″*, *e″* and *f″*. Join *a″b″c″d″e″f″*.

Problem 10.31 *A semi-circular plane of diameter 70 mm has its straight edge on the V.P. and inclined at 30° to the H.P. Draw the projection of the plane when its surface is inclined at 45° to the V.P.*

Construction Refer to Fig. 10.36.

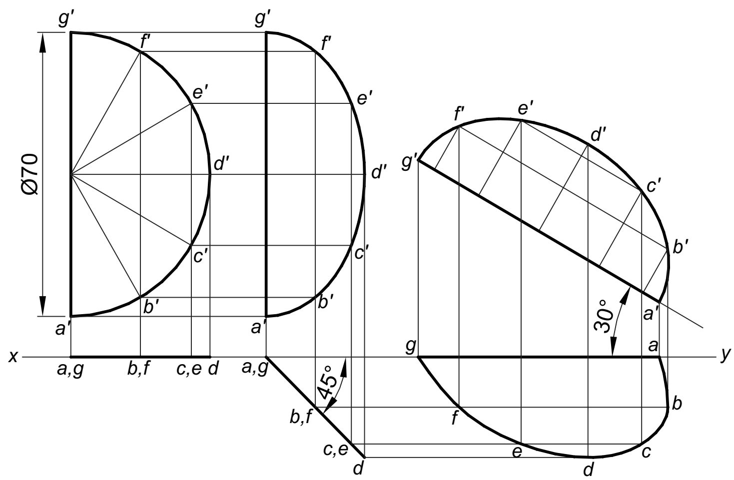


Fig. 10.36

1. **First stage** Draw a semi-circle $a'b'c'd'e'f'g'$ keeping $a'g'$ perpendicular to the xy to represent the front view. Divide the semicircle in six equal parts and project all these points to xy to obtain ad as the top view.
2. **Second stage** Reproduce the top view of first stage keeping ag on xy and ad inclined at 45° to xy. Obtain points a', b', c', d', e', f' and g' in the front view by joining the points of intersection of the projectors from points a, b, c, d, e, f and g of the second stage with the locus lines from points a', b', c', d', e', f' and g' of first stage. Join $a'b'c'd'e'f'g'$.
3. **Third stage** Reproduce the front view of the second stage keeping $a'g'$ inclined at 30° to xy. Obtain points a, b, c, d, e, f and g in the top view by joining the points of intersection of projectors from points a', b', c', d', e', f' and g' of the third stage with the locus lines from points a, b, c, d, e, f and g of the second stage. Join $abcdefg$.

Problem 10.32 *A 30-60 set-square has its 75 mm long hypotenuse in the V.P. and inclined at 30° to the H.P. The surface is inclined at 45° to the V.P. Draw three views of the set-square.*

Construction Refer to Fig. 10.37.

1. **First stage** Draw a right angled triangle $a'b'c'$ such that angle $a'b'c'$ = 60°, angle $b'a'c'$ = 30° and $a'b'$ perpendicular to xy. This is the front view of the set-square. Project all the corners to xy and obtain ac as the top view.
2. **Second stage** Reproduce the top view of first stage keeping ab on xy and *ac inclined at* 45° to xy. Obtain points a', b' and c' in the front view by joining the points of intersection of the projectors from points a, b and c of the second stage with the locus lines from points a', b' and c' of first stage. Join $a'b'c'$.
3. **Third stage** Reproduce the front view of the second stage keeping $a'b'$ inclined at 30° to xy. Obtain points a, b and c in the top view by joining the points of intersection of projectors from points a', b' and c' of the third stage with the locus lines from points a, b and c of the second stage. Join abc.

4. **Side view** Draw x_1y_1. Project points a, b and c of the third stage on oy_1. Now taking centre o, transfer the points obtained on oy_1 to oy. Project all the points perpendicular to xy to meet the corresponding projectors (drawn perpendicular to x_1y_1) from points a', b' and c' of the third stage at points a'', b'' and c''. Join $a''b''c''$.

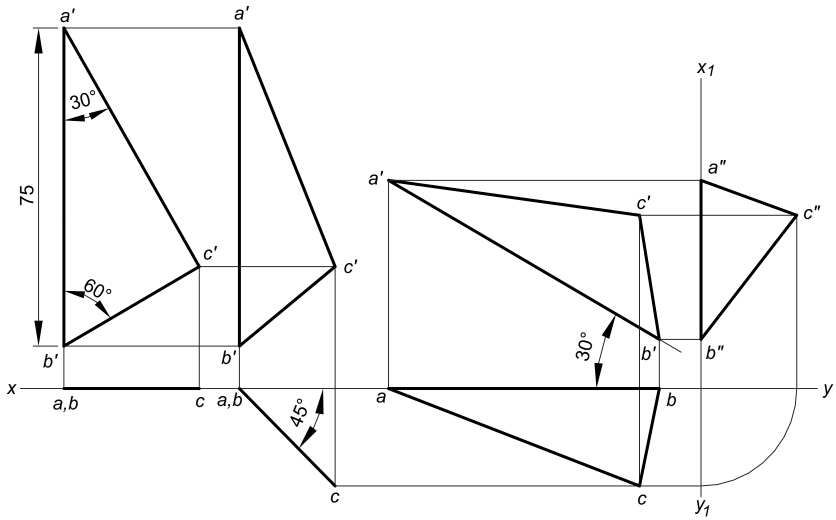


Fig. 10.37

Problem 10.33 *A pentagon ABCDE of side 30 mm has its side AB in the V.P. and inclined at 30° to the H.P. and the corner B is 15 mm above the H.P. and the corner D is 30 mm in front of the V.P. Draw the projections of the plane and find its inclination with the V.P.*

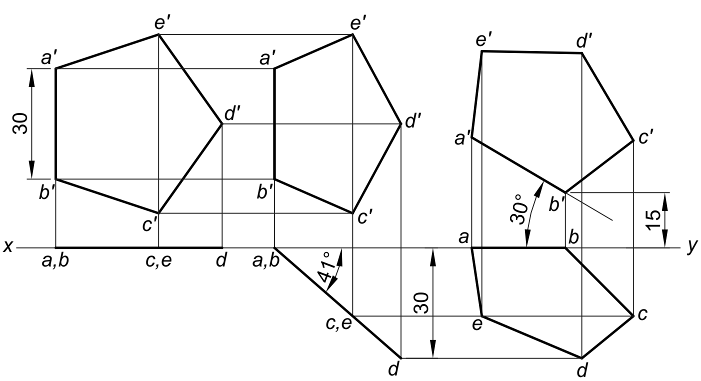


Fig. 10.38

Construction Refer to Fig. 10.38.

1. **First stage** Draw a pentagon $a'b'c'd'e'$ keeping side $a'b'$ perpendicular to xy to represent the front view. Project all the corners to xy and obtain ad as the top view.

2. **Second stage** Reproduce the top view of first stage keeping *ab* on *xy* and *d* 30 mm below *xy*. Measure inclination of *ad* with *xy* as inclination of plane with V.P. Here $\phi = 41°$. Obtain points *a′*, *b′*, *c′*, *d′* and *e′* of the front view by joining the points of intersection of the projectors from points *a*, *b*, *c*, *d* and *e* of the second stage with the locus lines from points *a′*, *b′*, *c′*, *d′* and *e′* of the first stage. Join *a′b′c′d′e′*.
3. **Third stage** Reproduce the front view of the second stage keeping *b′* is 15 mm above *xy* and *a′b′* is inclined at 30° to *xy*. Obtain points *a*, *b*, *c*, *d* and *e* for the top view by joining the points of intersection of projectors from points *a′*, *b′*, *c′*, *d′* and *e′* of the third stage with the locus lines from points *a*, *b*, *c*, *d* and *e* of the second stage. Join *abcde*.

Problem 10.34 *A circular lamina of diameter 60 mm has a centrally punched square hole of side 30 mm. Draw its projections when a diagonal of the hole is parallel to the V.P. and inclined at 30° to the H.P. while the other is inclined at 45° to the V.P.*

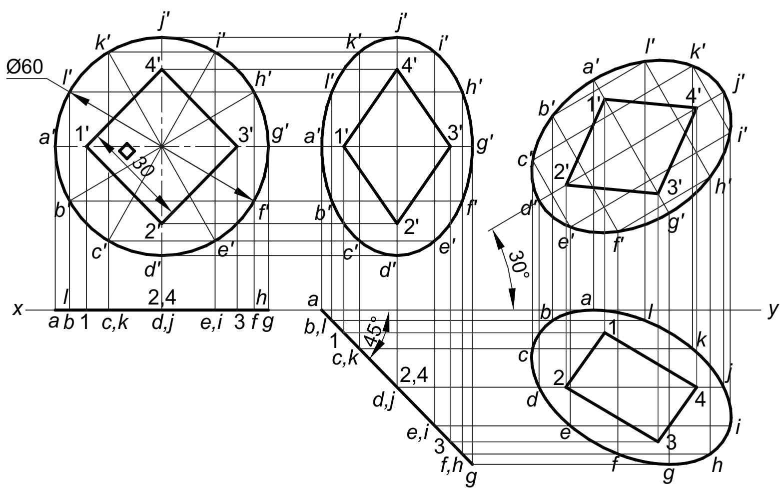


Fig. 10.39

Construction Refer to Fig. 10.39.

A lamina has a diagonal of the hole parallel to the V.P. so assume that initially the lamina is with diagonal 2′4′ parallel to the V.P.

1. **First stage** Draw the circular lamina *a′b′c′d′e′f′g′h′i′j′k′l′* containing square hole 1′2′3′4′ keeping 2′4′ perpendicular to *xy*. This is the front view. Divide the circle in 12 equal parts and project all the points to *xy* and obtain *ag* as the top view.
2. **Second stage** Reproduce the top view of first stage keeping *ag* inclined at 45° to *xy*. Obtain points *a′*, *b′*, *c′*, etc., of the front view by joining the points of intersection of the projectors from points *a*, *b*, *c*, etc., of the second stage with the locus lines from points *a′*, *b′*, *c′*, etc., of first stage. Join *a′b′c′d′e′f′g′h′i′j′k′l′* and 1′2′3′4′.
3. **Third stage** Reproduce the front view of the second stage keeping *d′*2′4′*g′* inclined at 30° to *xy*. Obtain points *a*, *b*, *c*, etc., of the top view by joining the points of intersection of projectors from points *a′*, *b′*, *c′*, etc., of the third stage with the locus lines from points *a*, *b*, *c*, etc., of the second stage. Join *abcdefghijkl* and 1–2–3–4.

Problem 10.35 *A plate having shape of an isosceles triangle has base 40 mm and altitude 54 mm. It is so placed that in the front view it is seen as an equilateral triangle of side 40 mm having a side inclined at 45° to the reference line. Draw its top view.*

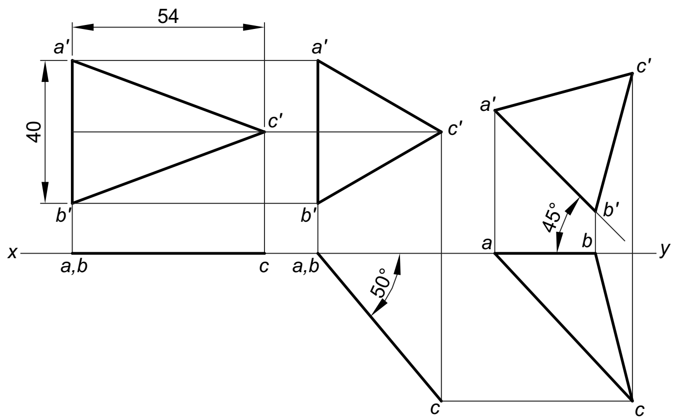


Fig. 10.40

Construction Refer to Fig. 10.40.

An isosceles triangle can appear as an equilateral triangle in the front view, when the base of the triangle is in the V.P. and the surface is inclined to the V.P.

1. **First stage** Draw an isosceles triangle *a′b′c′* keeping 40 mm base *a′b′* perpendicular to *xy*. This is the front view. Project all the corners to *xy* and obtain *ac* as the top view.
2. **Second stage** Draw an equilateral triangle *a′b′c′* of side 40 mm on the locus lines from points *a′*, *b′* and *c′* of the first stage. Project *a′b′* to meet *xy* at *ab*. Draw an arc with centre *a* and radius equal to length *ac* of the first stage to meet the projector of point *c′* at *c*. Join *ac* and measure its inclination with *xy*. Here $\phi = 50°$.
3. **Third stage** Reproduce the front view of the second stage keeping *a′b′* inclined at 45° to *xy*. Obtain points *a*, *b* and *c* of the top view by joining the points of intersection of projectors from points *a′*, *b′* and *c′* of the third stage with the locus lines from points *a*, *b* and *c* of the second stage. Join *abc*.

Problem 10.36 *A thin circular plate of diameter 60 mm appears in the front view as an ellipse of major and minor axes 60 mm and 40 mm respectively. Draw its projections when one of the diameters is parallel to both the reference planes.*

Construction Refer to Fig. 10.41.

1. **First stage** Draw a circle *a′d′g′j′* to represent the front view. Divide it into 12 equal parts. Project all the points to *xy* and obtain *ag* as the top view.
2. **Second stage** Draw locus line from points *a′* and *g′* of the first stage. Mark on it points *a′* and *g′* such that they are 40 mm apart (equal to the length of the minor axis). Project *a′* to meet *xy* at point *a*. Draw an arc with centre *a* and radius equal to length *ag* of the first stage to meet the projector of point *g′* at *g*. Join *ag*. Mark other points on line *ag* so that it becomes the copy of *ag* of the first stage. Obtain points *a′*, *b′*, *c′*, *d′*, etc., of the front view by joining the points of intersection of the

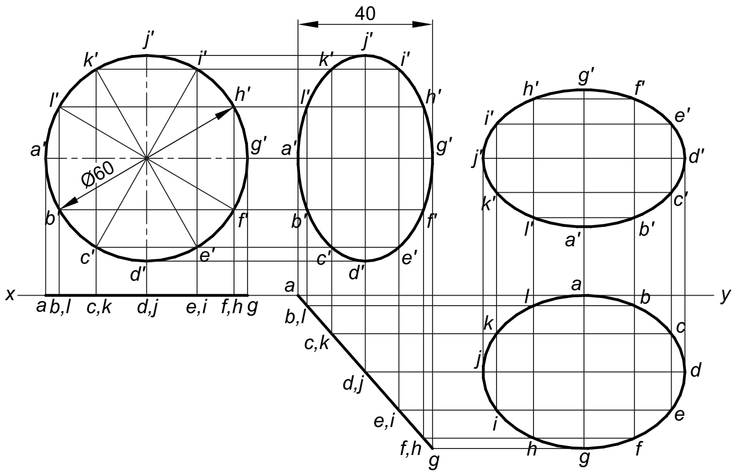


Fig. 10.41

projectors from points *a*, *b*, *c*, *d*, etc., of the second stage with the locus lines from points *a′*, *b′*, *c′*, *d′*, etc., of the first stage. Join *a′b′c′d′e′f′g′h′i′j′k′l′*.

3. **Third stage** Reproduce the front view of the second stage keeping *dj* parallel to *xy*. Obtain points *a*, *b*, *c*, *d*, etc., for the top view by joining the points of intersection of the projectors from points *a′*, *b′*, *c′*, *d′* of the third stage with the locus lines from points *a*, *b*, *c*, *d*, etc., of the second stage. Join *abcdefghijkl*.

10.10.3 Condition of Apparent Angle

Section 10.10.1 shows the element of the plane that has to be inclined in the third stage is parallel to the H.P. In case an element of the plane that has to be inclined in the third stage is inclined to the H.P. then there is a need to determine the apparent angle called β. The following problem illustrates the projection of plane under such circumstances.

Problem 10.37 *A hexagonal plane ABCDEF of side 30 mm has its corner A in the H.P. The surface of the plane is inclined at 45° to the H.P. and the diagonal containing corner A is inclined at 30° to the V.P. Draw its projections.*

Construction Refer to Fig. 10.42.

1. **First stage** Draw a hexagon *abcdef* keeping *ad* parallel to *xy* to represent the top view. Project all the corners to *xy* and obtain *a′d′* as the front view.
2. **Second stage** Reproduce the front view of first stage keeping *a′* on *xy* and *a′d′* inclined at 45° to *xy*. Obtain points *a*, *b*, *c*, *d*, *e* and *f* of the top view by joining the points of intersection of the projectors from points *a′*, *b′*, *c′*, *d′*, *e′* and *f′* of the second stage with the locus lines from points *a*, *b*, *c*, *d*, *e* and *f* of first stage. Join *abcdef*.

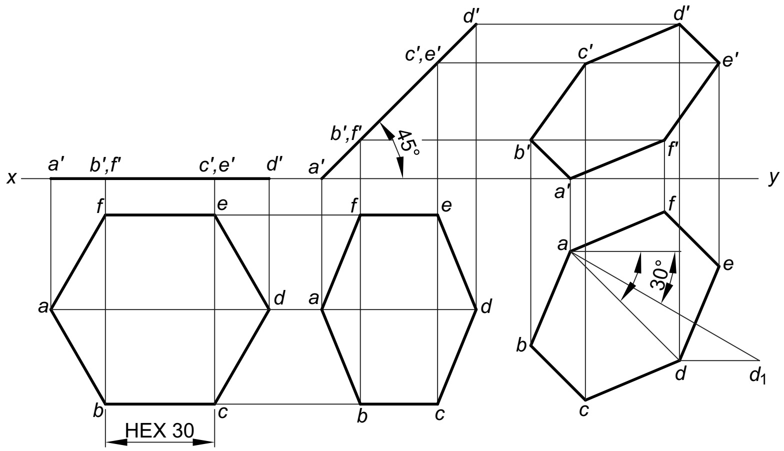


Fig. 10.42

(The diagonal ad which is inclined at 45º to H.P. is to be further inclined at 30º to the V.P. Hence, there is a need to determine the apparent angle β.)

3. **Third stage** Draw a line ad_1 equal to true length of the diagonal (*ad* of first stage) inclined at 30° to *xy*. Draw an arc with centre *a* and radius equal to the projected length of the diagonal (*ad* of second stage), to meet locus line from d_1 at point *d*. Join *ad*. The line *ad* is inclined at β angle with *xy*. Reproduce the top view of the second stage keeping *ad* inclined at β angle to *xy*. Obtain points *a*′, *b*′, *c*′, *d*′, *e*′ and *f*′ of the front view by joining the points of intersection of projectors from points *a*, *b*, *c*, *d*, *e* and *f* of the third stage with the locus lines from points *a*′, *b*′, *c*′, *d*′, *e*′ and *f*′ of the second stage. Join *a*′*b*′*c*′*d*′*e*′*f*′.

Section 10.10.2 shows the element of the plane that has to be inclined in the third stage is parallel to the V.P. In case an element of the plane that has to be inclined in the third stage is inclined to the V.P. then there is a need to determine the apparent angle called α. The following problem illustrates the projection of plane under such circumstances.

Problem 10.38 *A rectangular plate of sides 70 mm and 40 mm rests on its shorter side in the V.P. and the surface makes 45° with the V.P. The longer side of the plane is inclined at 30° to the H.P. Draw its projections.*

Construction Refer to Fig. 10.43.

1. **First stage** Draw a rectangle *a*′*b*′*c*′*d*′ keeping shorter edge *a*′*b*′ perpendicular to *xy* to represent the front view. Project all the corners to *xy* and obtain *ac* as the top view.
2. **Second stage** Reproduce the top view of first stage keeping *a* on *xy* and *ac* at 45° to *xy*. Obtain point *a*′, *b*′, *c*′ and *d*′ of the front view by joining the points of intersection of the projectors from points *a*, *b*, *c* and *d* of the second stage with the locus lines from points *a*′, *b*′, *c*′ and *d*′ of the first stage. Join *a*′*b*′*c*′*d*′.

   (The longer side *bc* which is inclined at 45º to V.P. is to be further inclined at 30º to the H.P. Hence, there is a need to determine the apparent angle α.)

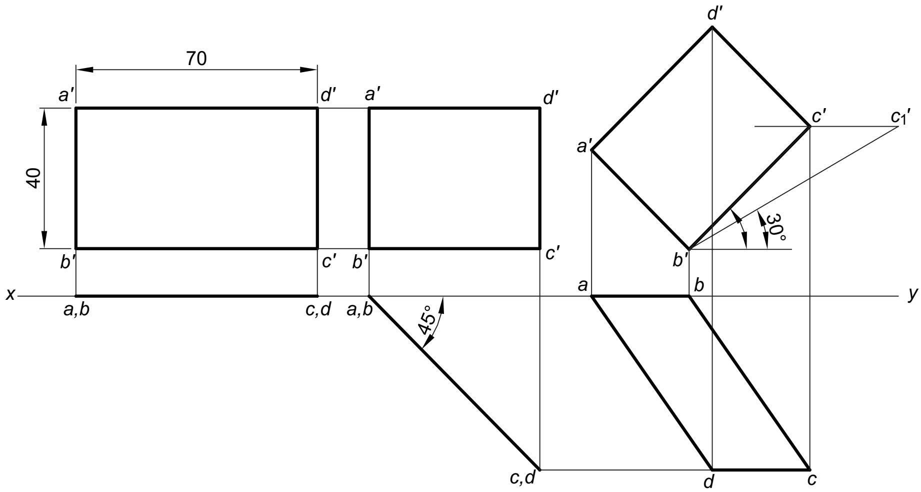


Fig. 10.43

3. **Third stage** Draw a line $b'c_1'$ equal to true length (70 mm) inclined at 30° to *xy*. Draw an arc with centre b' and radius equal to projected length of longer edge ($b'c'$ of second stage), to intersect the locus line from c_1' at point c'. The line $b'c'$ is now inclined at angle α to *xy*. Reproduce the front view of the second stage keeping $b'c'$ inclined at α to *xy*. Obtain points *a*, *b*, *c* and *d* for the top view by joining the points of intersection of projectors from points a', b', c' and d' of the third stage with the locus lines from points *a*, *b*, *c* and *d* of the second stage. Join *abcd*.

Problem 10.39 *Draw the projections of a circular plane of diameter 50 mm, resting on a point A of the circumference on the H.P. such that its surface is inclined at 45° to the H.P., and (a) the top view of the diameter through point A is inclined at 30° to the V.P., (b) the diameter through point A is inclined at 30° to the V.P.*

Construction Refer to Fig. 10.44.

1. **First stage** Draw a circle of diameter 50 mm to represent the top view. Divide it into 12 equal parts. Project all the points to *xy* and obtain $a'g'$ as the front view.
2. **Second stage** Reproduce the front view of the first stage keeping $a'g'$ inclined at 45° to *xy*. Obtain points *a*, *b*, *c*, *d*, etc., of the top view by joining the points of intersection of the projectors from points a', b', c', d', etc., of the second stage with the locus lines from points *a*, *b*, *c*, *d*, etc., of the first stage. Join *a*, *b*, *c*, *d*, etc., with a smooth curve.
3. **Third stage (when top view of the diameter through point A is inclined at 30° to the V.P.)** Reproduce the top view of the second stage keeping *ag* inclined at 30° to *xy*. Obtain points a', b', c', d', etc., in the front view by joining the points of intersection of the projectors from points *a*, *b*, *c*, *d*, etc., of the third stage with the locus lines from points a', b', c', d', etc., of the second stage. Join a', b', c', d', etc., with a smooth curve.
4. **Fourth stage (when the diameter through point A is inclined at 30° to the V.P.)** The diameter *AG* is inclined at 45° to H.P. and 30° with the V.P., therefore its top view shall be inclined at apparent angle β. Draw a line ag_1 equal to true length of the diameter (50 mm) inclined at 30° to *xy*.

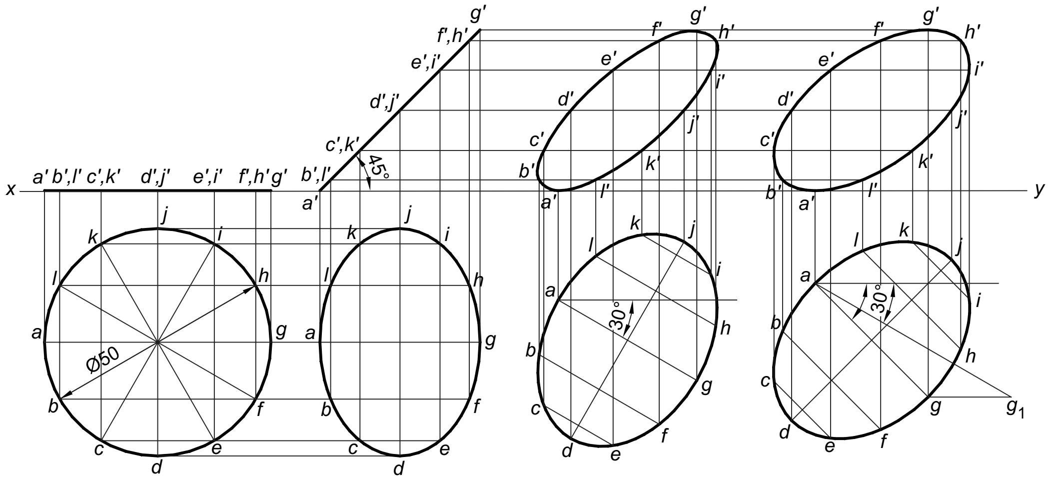


Fig. 10.44

Draw an arc with centre a and radius equal to ag of second stage, to meet locus line from point g_1 at g. Join ag. The line ag makes angle β with xy. Reproduce the top view of the second stage keeping ag inclined at β to xy. Obtain points a', b', c', d', etc., of the front view by joining the points of intersection of the projectors from points a, b, c, d, etc., of the fourth stage with the locus lines from points a', b', c', d', etc., of the second stage. Join a', b', c', d', etc., with a smooth curve.

10.10.4 Plane Inclined at θ to H.P. and ϕ to V.P. such that $\theta + \phi = 90°$

When a given plane is inclined at θ to the H.P. and ϕ to the V.P. with the condition $\theta + \phi = 90°$, the projections can be obtained by determining angle β. It is found that the value of β is always 90° in such case. Alternatively, the projections can be obtained on a profile plane on which the view appears to be a straight line. This can be projected to give the front and top views. The following problem illustrates both of these methods.

Problem 10.40 *A circular plane of diameter 70 mm has one of the ends of the diameter in the H.P. while the other end is in the V.P. The plane is inclined at 30° to the H.P. and 60° to the V.P. Draw its projections.*

Method 1: By change of position

Construction Refer to Fig. 10.45.

1. **First stage** Assume plane is placed on the H.P. with end A of diameter AB on the left-hand side. Draw a circle $adgj$ as the top view. Divide the circumference of the circle into 12 equal parts. Project all the points to xy and obtain $a'g'$ as the front view.
2. **Second stage** Reproduce the front view of first stage keeping a' on xy and $a'g'$ inclined at 30° to xy. Obtain points a, b, c, d, etc., of the top view by joining the points of intersection of the

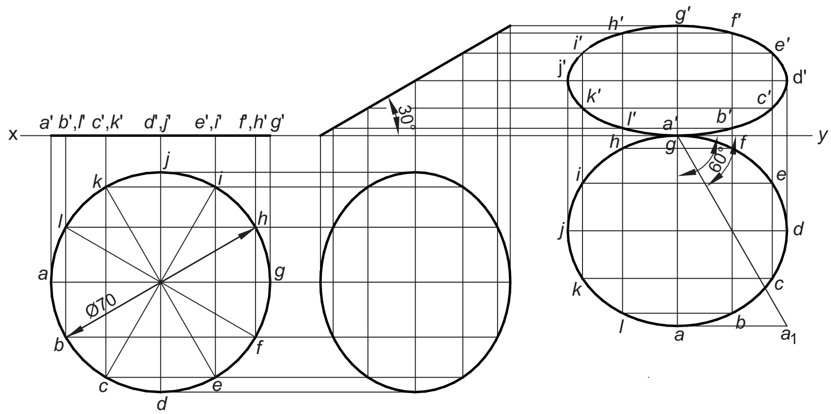


Fig. 10.45 *Projections by change of position method*

projectors from points a', b', c', d', etc., from the second stage with the locus lines from points a, b, c, d, etc., of the first stage. Join points a, b, c, d, etc., using a smooth curve.

We know that if a line is inclined to both H.P. and V.P. where sum of their inclinations is 90° (i.e., $\theta + \phi = 90°$) then the apparent angle $\beta = 90°$.

3. **Third stage** Reproduce the top view of the second stage keeping g on xy and ag perpendicular to xy. Obtain points a', b', c', d', etc., of the front view by joining the points of intersection of the projectors from points a, b, c, d, etc., of the third stage with the locus lines from points a', b', c', d', etc., of the second stage. Join $a'b'c'd'e'f'g'h'j'k'l'$ with a smooth curve.

Method 2: By profile plane

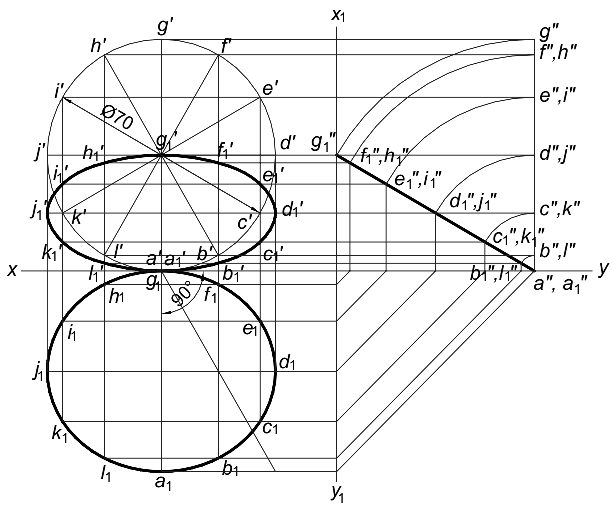


Fig. 10.46 *Projections by profile plane method*

Construction Refer to Fig. 10.46.

1. Draw a reference lines xy. Consider the circular plane is parallel to V.P.
2. Draw a circle $a'd'g'j'$ as the front view. Divide the circle into 12 equal parts. Divide the circumference of the circle into 12 equal parts.
3. Project all the points horizontally to obtain vertical line $a''g''$ as the side view.
4. Now the circular plane is turned such that it makes 30° with the H.P. and 60° with the V.P. Therefore, reproduce the side view such that g_1'' is on xy and $a_1''g_1''$ is inclined at 30° to the xy. This is the final side view.
5. Draw another reference line x_1y_1 passing through a_1'' perpendicular to xy.
6. Obtain points a_1', b_1', c_1', d_1', etc., as the final front view by joining the points of intersection of the vertical projectors from points a', b', c', d', etc., of the front view with the horizontal locus lines drawn from points a_1'', b_1'', c_1'', d_1'', etc., of the final side view. Join a_1', b_1', c_1', d_1', etc., with a smooth curve. This is the final front view.
7. Obtain points a_1, b_1, c_1, d_1, etc., of the top view by joining the points of intersection of the vertical projectors from points a_1', b_1', c_1', d_1', etc., with the locus lines from points a_1'', b_1'', c_1'', d_1'', etc., of the side views. Join $a_1b_1c_1d_1e_1f_1g_1h_1i_1j_1k_1l_1$ by a smooth curve representing top view.

EXERCISE 10B

An element of the plane in H.P.

10.1 A pentagonal plane of side 30 mm has an edge on the H.P. Its surface is inclined at 45° to the H.P. and the edge on which the plane rests is inclined at 30° to the V.P. Draw its projections.

10.2 A hexagonal lamina of side 30 mm has a corner on the ground. Its surface is inclined at 30° to the H.P. A plane containing the centre of the lamina and the corner lying in the H.P. is inclined at 45° with the V.P. Draw its three views.

10.3 Draw the projections of a triangular lamina of side 50 mm which has a side parallel to the H.P. and inclined at 60° to the V.P. The surface is inclined at 30° to the H.P.

10.4 A square lamina *ABCD* of side 30 mm has a corner *A* on the H.P. Its diagonal *AC* is inclined at 45° to the H.P. while the diagonal *BD* is parallel to the H.P. and inclined at 30° to the V.P. Draw its projections.

10.5 A thin pentagonal plate of side 30 mm is placed on a side in the H.P. inclined at 30° to the V.P. The corner opposite to that side is 50 mm above the H.P. Draw its projections.

10.6 A semi-circular plane of diameter 70 mm has its straight edge on the H.P. and inclined at 45° to the V.P. Draw the projection of the plane when its surface is inclined at 30° to the H.P.

10.7 A 30-60 set-square is kept on its 75 mm long hypotenuse in the H.P. inclined at 30° to the V.P. Draw its projections when the set-square itself is inclined at 45° to the H.P.

10.8 A square lamina of diagonal 60 mm is parallel to H.P. and inclined at 30° to the V.P. The other diagonal of the lamina is inclined at 45° to the H.P. Draw its projections.

10.9 A triangular plane *ABC* of side 40 mm has its side *AB* in the H.P. and inclined at 45° to the V.P. the corner *A* is 10 mm in front of the V.P. and the corner *C* is 25 mm above the H.P. Draw the projections of the plane and find its inclination with the H.P.

10.10 A thin hexagonal piece of metal sheet of side 30 mm has a centrally punched hole of diameter 30 mm. It is placed on a corner in the H.P. such that the surface is inclined at 30° to the H.P. The top view of the diagonal through the corner in the H.P. is inclined at 45° to the V.P. Draw its projections.

10.11 A square plane is placed on a side in the H.P. The surface is inclined to the H.P. such that the top view appears as a rectangle of sides 50 mm and 25 mm. Draw its projections when the side resting on the H.P. is inclined at 30° to the V.P.

10.12 The top view of a square plane of diagonal 80 mm appears as a rhombus of 80 mm and 50 mm diagonals. One of the corners of the plane is in the H.P. Draw its projections when one of the diagonals is parallel to both the principal plane.

10.13 A rhombus *ABCD* of diagonals 100 mm and 60 mm. It is placed on a corner *A* on the ground such that the top view appears as a square. Draw its projections when the diagonal *BD* is parallel to both H.P. and the V.P.

10.14 A lamina is in the shape of a rhombus of major and the minor diagonals 70 mm and 45 mm long respectively. It is placed on a corner in the H.P. such that top view appears as a square having diagonals equally inclined to the V.P. Draw its projections.

10.15 A 50 mm side equilateral triangle is placed on a side in the H.P. inclined at 30° to the V.P. The surface is inclined to the H.P. such that the top view appears a right angled triangle. Draw its projections and determine inclination of the plane with the H.P.

10.16 A circular plane of diameter 50 mm is placed on a point *A* of the circumference in the H.P. Its surface is inclined to the H.P. such that top view appears as an ellipse of major and minor axes 50 mm and 30 mm, respectively. Draw the projections of the plane when the top view of the diameter through point *A* is inclined at 45° to the V.P.

10.17 An isosceles triangle of base 60 mm and altitude 40 mm having the base parallel to the reference line represents the top view of an equilateral triangle of side 60 mm resting on a side on the H.P. Draw the front view of the triangle and determine inclination of its surface with the H.P.

An element of the plane in V.P.

10.18 A pentagonal plane of side 30 mm has an edge in the V.P. Its surface is inclined at 45° to the V.P. and the edge on which the plane rests is inclined at 30° to the H.P. Draw the projections.

10.19 A hexagonal plane of side 30 mm is lying in such way that the one of its sides touches both the reference planes. If the surface of the plane is inclined at 45° to the V.P., draw its projections.

10.20 Draw the projections of a triangular lamina of side 50 mm which has a side parallel to the V.P. and inclined at 45° to the H.P. The surface is inclined at 60° to the V.P.

10.21 A square lamina *ABCD* of side 40 mm has its corner *A* in the H.P. Its diagonal *AC* is inclined at 45° to the H.P. while the diagonal *BD* is parallel to the H.P. and inclined at 30° to the V.P. Draw its projections.

10.22 A rectangular plane of edges 40 mm and 70 mm is resting on an edge in the V.P. The surface is inclined to the V.P. such that the front view appears as a square. Draw its projections when the edge resting on the V.P. is inclined at 45° to the H.P.

10.23 A thin square plate of side 40 mm stands on a corner in the V.P. such that front view appears as a rhombus of diagonals in the ratio of 1:2. Draw its projection when the longer diagonal is parallel to both the principal planes.

10.24 The front view of an isosceles triangle of base 50 mm and altitude 70 mm is an equilateral triangle whose one side is inclined at 45° to the reference line. Draw its projections.

10.25 An equilateral triangle of side 60 mm is resting on a side in the V.P. such that the front view appears as a right angled triangle of hypotenuse 60 mm inclined at 30° to the reference line. Draw its projections.

10.26 The parallel sides of a trapezium *ABCD* are 70 mm and 40 mm long and 60 mm apart. It is kept on the 70 mm long side *AB* in the V.P. such that the front view appears as another trapezium of same parallel sides but 30 mm apart. Draw the projections of the trapezium when *AB* is inclined at 45° to the H.P.

10.27 A circular plane of diameter 60 mm has a point of its circumference in the V.P. Its surface is inclined to V.P. such that the front view appears as an ellipse having minor and major axes in the ratio of 1:2. Draw its projections when the major axis is inclined at 30° with the H.P.

Condition of apparent angle

10.28 A rectangular plate of sides 70 mm and 40 mm rests on its shorter side in the H.P. and the surface makes 45° with the V.P. The longer side of the plane is inclined at 30° to the V.P. Draw its projections.

10.29 A hexagonal plane of side 30 mm has its corner *A* in the V.P. The surface of the plane is inclined at 45° to the V.P. and the diagonal containing corner *A* is inclined at 30° to the H.P. Draw its projections.

10.30 A hexagonal plane of side 30 mm has a corner in the V.P. The surface of the plane is inclined at 45° to the V.P. Draw its projections when, (a) the top view of the diameter through the corner in the V.P. is inclined at 30° to the H.P., and (b) the diagonal through the corner in the V.P. is inclined at 30° to the H.P.

10.31 A circular plane of diameter 70 mm has one of the ends of the diameter in the H.P. while the other end is in the V.P. The plane is inclined at 60° to the H.P. and 30° to the V.P. Draw its projections.

10.32 A hexagonal plane of side 30 mm has a side on the H.P. and the side opposite to that in the V.P. The plane is inclined at 30° to the H.P. and 60° to the V.P. Draw its projections.

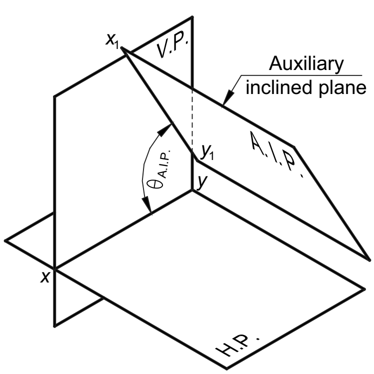
10.34 A hexagonal plane of side 30 mm has one of the corners in the H.P. and the corner opposite to it is in the V.P. The plane is inclined at 60° to the H.P. and 30° to the V.P. Draw its projections.

10.35 A pentagonal plane of side 30 mm is resting on a side in the H.P. and parallel to the V.P. The corner opposite to the side in the H.P., is in the V.P. and 25 mm above the H.P. Draw the projections of the plane and determine its inclination.

10.36 A pentagonal lamina of side 30 mm is resting on a side in the H.P. which is parallel to and 25 mm in front of the V.P. The lamina is tilted such that its highest corner is in the V.P. Draw the projections of the lamina.

10.11 AUXILIARY PLANE METHOD

Projections of planes whose surface are inclined to the H.P., V.P. or both can also be drawn using the auxiliary plane method. In this method, instead of changing the position of the views with respect to the reference line, the position of the reference line is changed and auxiliary views are projected. Auxiliary planes used for projecting the auxiliary views are of two types.



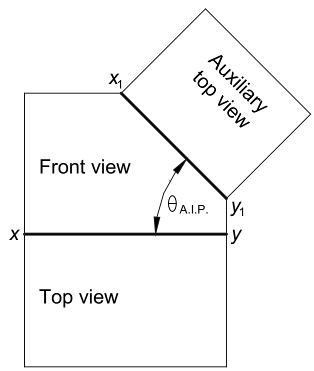


Fig. 10.47

1. **Auxiliary inclined plane (A.I.P.)** Figure 10.47(a) shows an auxiliary inclined plane. It is defined as a plane perpendicular to the V.P. It is represented by a line x_1y_1 which is inclined at $\theta_{A.I.P.}$ with the front view as shown in Fig. 10.47(b). Projection from the front view on an A.I.P. is called *auxiliary top view*.
2. **Auxiliary vertical plane (A.V.P.)** Figure 10.48(a) shows an auxiliary vertical plane. It is defined as a plane perpendicular to the H.P. It is represented by a line x_1y_1 which is inclined at $\phi_{A.V.P.}$ with the top view as shown in Fig. 10.48(b). Projection from the top view on the A.V.P. is called *auxiliary front view*.

Note

(i) Lines representing the auxiliary plane should not overlap the views.

(ii) Projectors should always be perpendicular to the x_1y_1 (representing the auxiliary plane) and rules of orthographic projections should be strictly followed.

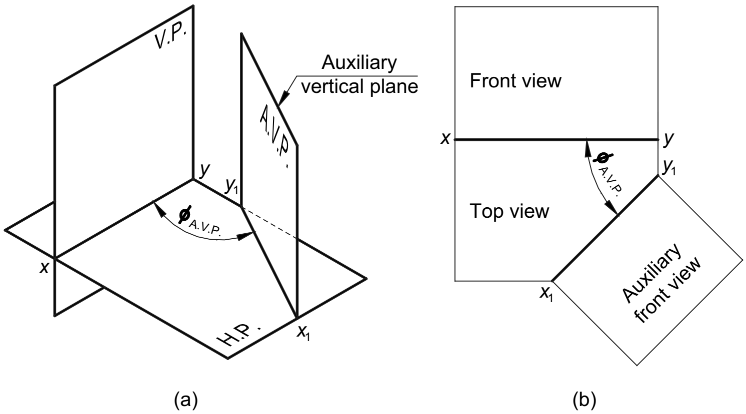


Fig. 10.48 (a) *and* **(b)**

10.11.1 Auxiliary Top View

Projection of an object on an auxiliary inclined plane (A.I.P.) is called an *auxiliary top view*. An A.I.P. is a plane perpendicular to the V.P. whose front view is a line. In this case, a new reference line representing A.I.P. is drawn whose V.T. is inclined at a given angle $\phi_{A.I.P.}$ to xy. All the points of the front view are projected perpendicular on the new reference line. The distance of the points from the new reference plane for the auxiliary top view are taken that of the corresponding points in the top view from the previous reference plane. Let us consider problems done earlier and solve them using auxiliary plane method.

Problem 10.41 *A hexagonal plane of side 30 mm has an edge on the H.P. The surface is inclined at 45° to the H.P. and perpendicular to the V.P. Draw its projections.*

Construction Refer to Fig. 10.49.

1. Draw a hexagon *abcdef* keeping *ab* perpendicular to xy to represent the top view.
   Project the corners to xy and obtain $b'd'$ as the front view.
2. Draw x_1y_1 inclined at 45° to $b'd'$ and passing through $a'b'$. Project a', b', c', d', e' and f' on x_1y_1.
   On the projectors mark points a_1, b_1, c_1, d_1, e_1 and f_1 such that their distance from x_1y_1 is equal to the distance of points a, b, c, d, e and f from xy, respectively.
   Join $a_1b_1c_1d_1e_1f_1$. Here $a'b'c'd'e'f'$ and $a_1b_1c_1d_1e_1f_1$ represent the front and the top views.

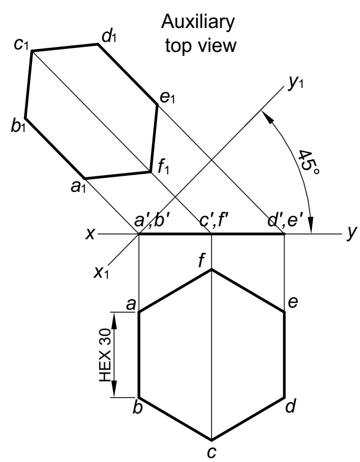


Fig. 10.49

10.11.2 Auxiliary Front View

Projection of an object on an auxiliary vertical plane (A.V.P.) is called *auxiliary front view*. An A.V.P. is a plane perpendicular to the H.P. whose top view is a line. In this case, a new reference line representing A.V.P. is drawn whose H.T. is inclined at a given angle $\theta_{A.V.P.}$ to the *xy*. All the points of the top view are projected perpendicular on the new reference line. The distance of the points from the new reference plane for the auxiliary front view are taken that of the corresponding points in the front view from the previous reference plane. Let us consider problems done earlier and solve them using auxiliary plane method.

Problem 10.42 *A hexagonal plane of side 30 mm has a corner in the V.P. The surface of the plane is inclined at 45° to the V.P. and perpendicular to the H.P. Draw its projections.*

Construction Refer to Fig. 10.50.

1. Draw a hexagon $a'b'c'd'e'f'$ keeping $a'o'$ parallel to xy to represent the front view. Project the corners to xy and obtain ad as the top view ad.
2. Draw x_1y_1 inclined at 45° to ad and passing through corner a. Project a, b, c, d, e and f on x_1y_1.
3. On the projectors, mark points a_1', b_1', c_1', d_1', e_1' and f_1' such that their distance from x_1y_1 is equal to the distance of points a', b', c', d', e' and f' from xy, respectively. Join $a_1'b_1'c_1'd_1'e_1'f_1'$. Here $abcdef$ and $a_1'b_1'c_1'd_1'e_1'f_1'$ represent the top and the front views.

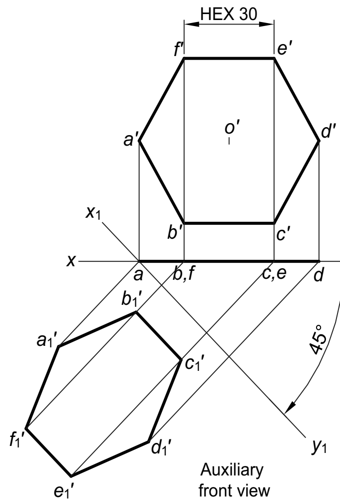


Fig. 10.50

10.11.3 Auxiliary Front and Top Views

When the surface of a plane is inclined to both the reference planes, then it is required to obtain auxiliary front and the top views to represent the final projections. Consider the following problems.

Problem 10.43 *A hexagonal plane of side 30 mm has an edge on the H.P. Its surface is inclined at 45° to the H.P. and the edge on which the plane rests is inclined at 30° to the V.P. Draw its projections.*

Construction Refer to Fig. 10.51.

1. Follow Steps of constructions of Problem 10.41 and obtain $a_1b_1c_1d_1e_1f_1$ as the auxiliary top view.
2. Draw x_2y_2 inclined at 30° to the a_1b_1. Project points a_1, b_1, c_1, d_1, e_1 and f_1 on x_2y_2.
3. On the projectors mark points a_1', b_1', c_1', d_1', e_1' and f_1' such that their distance from x_2y_2 line is equal to the distance of points a', b', c', d', e' and f' from x_1y_1, respectively. Join $a_1'b_1'c_1'd_1'e_1'f_1'$ to represent the auxiliary front view.

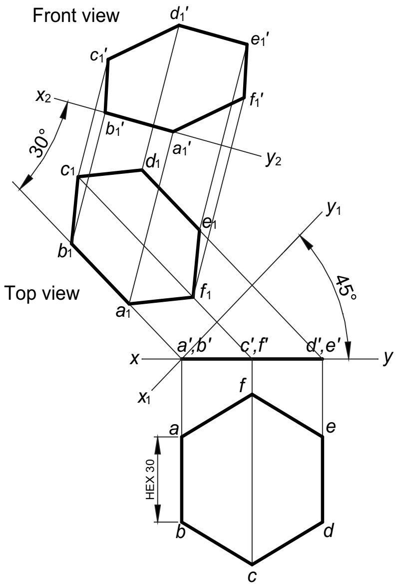


Fig. 10.51

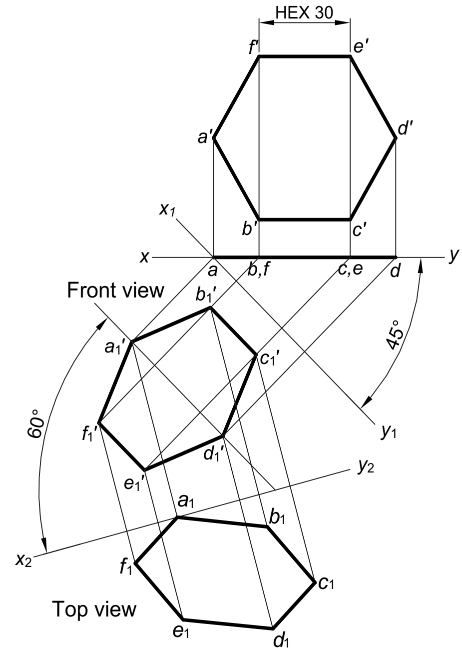


Fig. 10.52

Problem 10.44 *A hexagonal plane of side 30 mm has a corner in the V.P. The surface of the plane is inclined at 45° to the V.P. and perpendicular to the H.P. The front view of the diagonal passing through that corner is inclined at 60° to the H.P. Draw its projections.*

Construction Refer to Fig. 10.52.

1. Follow Steps of constructions of Problem 10.42 and obtain $a_1'b_1'c_1'd_1'e_1'f_1'$ as the auxiliary front view.
2. Draw x_2y_2 inclined at 60° to the $a_1'd_1'$. Project points a_1', b_1', c_1', d_1', e_1' and f_1' on x_2y_2.
3. On the projectors, mark points a_1, b_1, c_1, d_1, e_1 and f_1 such that their distance from x_2y_2 line is equal to the distance of points a, b, c, d, e and f from x_1y_1, respectively. Join $a_1b_1c_1d_1e_1f_1$.to represent the auxiliary top view.

10.12 ALTERNATIVE AUXILIARY PLANE METHOD

It can be observed that in the auxiliary plane method, the reference line in between the final projections is inclined at an angle. The alternative auxiliary plane targets to keep reference line in between the final projections as horizontal. Let us consider the problems solved earlier in this chapter using alternative auxiliary plane method.

Problem 10.45 *A hexagonal plane of side 30 mm has an edge on the H.P. Its surface is inclined at 45° to the H.P. and the edge on which the plane rests is inclined at 30° to the V.P. Draw its projections.*

Construction Refer to Fig. 10.53.

1. Draw a reference line xy. Draw another line x_2y_2 inclined at 30° to xy.
2. Draw a hexagon *abcdef* keeping *ab* on x_2y_2.
3. Draw another reference line x_1y_1 perpendicular to *ab* to represent A.V.P.
4. Project points *a*, *b*, *c*, *d*, *e* and *f* on x_1y_1 and obtain $a_0'd_0'$.
5. Reproduce $a_0'd_0'$ and obtain $a'd'$ inclined at 45° to x_1y_1.
6. Mark points a_1, b_1, c_1, d_1, e_1 and f_1 as the intersection of the projectors from points a', b', c', d', e' and f' (drawn perpendicular to x_1y_1 line) with the projectors from points *a*, *b*, *c*, *d*, *e* and *f* (drawn perpendicular to x_2y_2 line). Join $a_1b_1c_1d_1e_1f_1$ to represent the final top view.

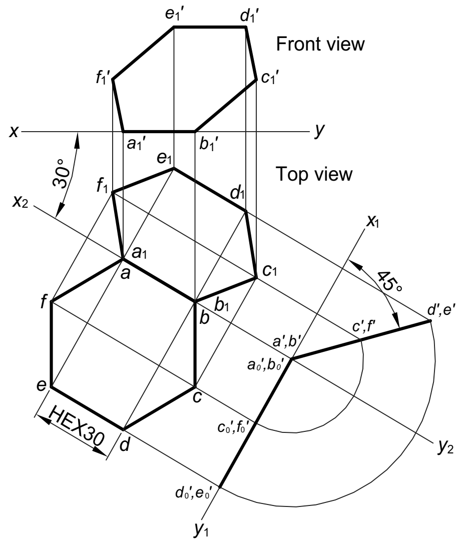


Fig. 10.53

7. Project points a_1, b_1, c_1, d_1, e_1 and f_1 on xy and produce. Mark points a_1', b_1', c_1', d_1', e_1' and f_1' on the respective projectors such that their distance from xy is equal to the distance of points a', b', c', d', e' and f' from x_1y_1. Join $a_1'b_1'c_1'd_1'e_1'f_1'$ to represent the final front view.

Problem 10.46 *A rectangular plane of edges 70 mm and 35 mm is resting on an edge in the H.P. The surface is inclined to the H.P. such that the top view appears as a square. Draw its projections when the edge resting on the H.P. is inclined at 30° to the V.P.*

Construction Refer to Fig. 10.54.

1. Draw a reference line xy. Draw another line x_2y_2 inclined at 30° to xy.
2. Draw a rectangle *abcd* keeping 35 mm long side *ab* on x_2y_2.
3. Draw another reference line x_1y_1 perpendicular to *ab* to represent A.V.P.
4. Project points *a*, *b*, *c* and *d* on x_1y_1 and obtain $a_0'd_0'$.
5. As top view should appear as a square, draw a square $a_1b_1c_1d_1$ keeping side a_1b_1 coinciding with *ab*. This represents the final top view.

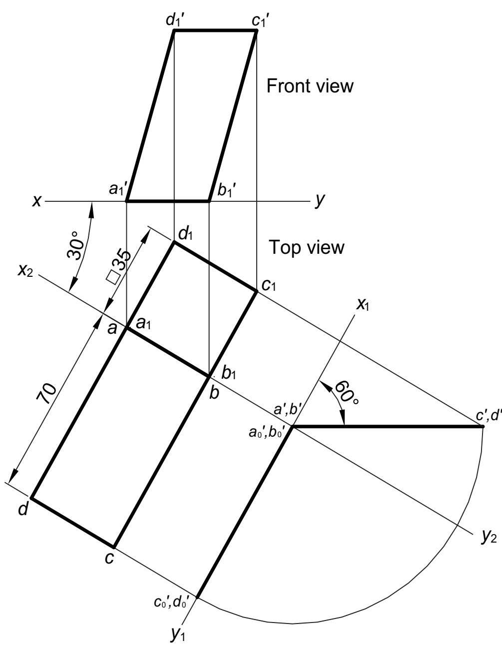


Fig. 10.54

6. Draw an arc with centre a_0' and radius $a_0'd_0'$ to meet the projector from c_1d_1 drawn perpendicular to x_1y_1 at $a'd'$. Measure its inclination with H.P. Here $\theta = 60°$.
7. Project points a_1, b_1, c_1 and d_1 on xy and produce. Mark points a_1', b_1', c_1' and d_1' on the respective projectors such that their distance from xy is equal to the distance of points a', b', c' and d' from x_1y_1. Join $a_1'b_1'c_1'd_1'$ to represent the final front view.

Problem 10.47 *A thin isosceles triangular plate of base 60 mm and altitude 80 mm has its base in the H.P. and inclined at 45° to the V.P. Its surface is inclined to H.P. such that the top view is an equilateral triangle. Draw its projections and determine the inclination of the plate with the H.P.*

Construction Refer to Fig. 10.55.

1. Draw a reference line xy. Draw another line x_2y_2 inclined at 45° to xy.
2. Draw a triangle abc keeping 60 mm long base ab on x_2y_2.
3. Draw another reference line x_1y_1 perpendicular to ab to represent A.V.P.
4. Project points a, b and c on x_1y_1 and obtain $a_0'c_0'$.
5. As top view should appear as an equilateral triangle, draw an equilateral triangle $a_1b_1c_1$ keeping side a_1b_1 coinciding with ab. This represents the final top view.
6. Draw an arc with centre a_0' and radius $a_0'c_0'$ to meet the projector from point c_1 drawn perpendicular to x_1y_1 at point c'. Measure its inclination with H.P. Here $\theta = 49°$.
7. Project points a_1, b_1 and c_1 on xy and produce. Mark points a_1', b_1' and c_1' on the respective projectors such that their distance from xy is equal to the distance of points a', b' and c' from x_1y_1. Join $a_1'b_1'c_1'$ to represent the final front view.

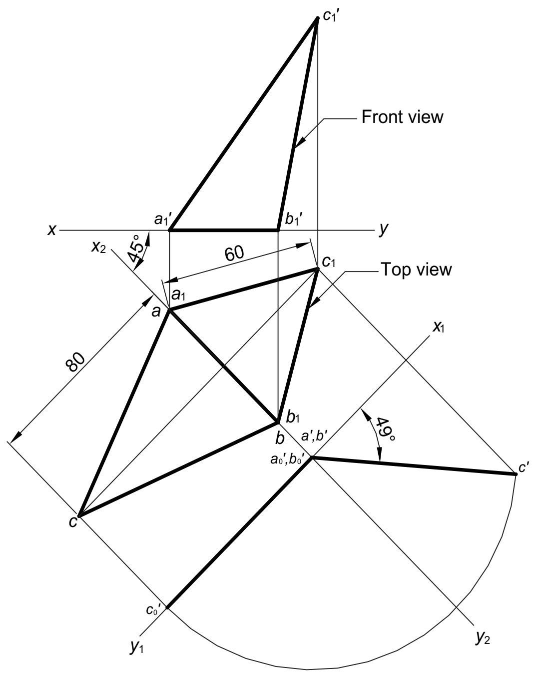


Fig. 10.55

Problem 10.48 *An isosceles triangular plane ABC of base 60 mm and altitude 50 mm has its base in the H.P. and inclined at 30° to the V.P. The corners A and C are in the V.P. Draw its projections and determine the inclination of the plane with the H.P.*

Construction Refer to Fig. 10.56.

1. Draw a reference line xy. Draw another line x_2y_2 inclined at 30° to xy.
2. Draw a triangle abc keeping 60 mm long base ab on x_2y_2 and point a on xy.
3. Draw another reference line x_1y_1 perpendicular to ab to represent A.V.P.
4. Project points a, b and c on x_1y_1 and obtain $a_0'c_0'$.

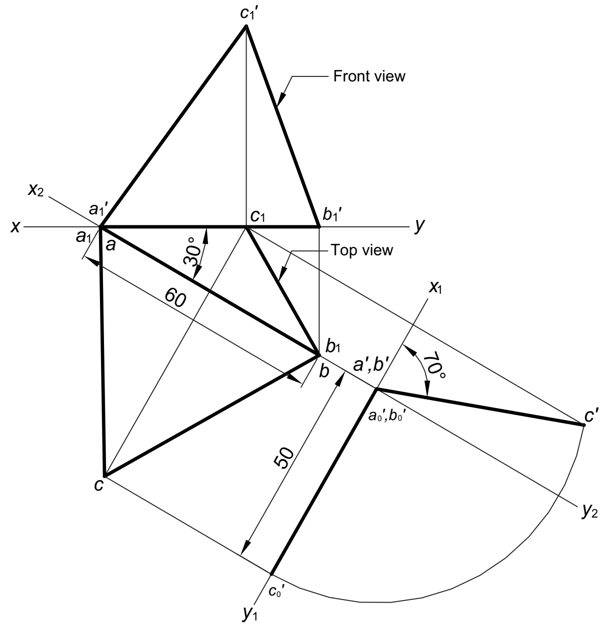


Fig. 10.56

5. It is given that the corner c should be in the V.P. Therefore, draw a_1b_1 coinciding with ab. Project point c on x_2y_2 and produce to meet xy at point c_1. Join $a_1b_1c_1$ to represent the final top view.
6. Draw an arc with centre a_0' and radius $a_0'c_0'$ to meet the projector from point c_1 drawn perpendicular to x_1y_1 at point c'. Measure its inclination with H.P. Here $\theta = 70°$.
7. Project points a_1, b_1 and c_1 on xy and produce. Mark points a_1', b_1' and c_1' on the respective projectors such that their distance from xy is equal to the distance of points a', b' and c' from x_1y_1. Join $a_1'b_1'c_1'$ to represent the final front view.

Problem 10.49 *A pentagonal cardboard ABCDE of side 30 mm is resting on the H.P. on side AB which is inclined at 30° to the V.P. Corner A is 28 mm in front of the V.P. The surface of the cardboard is tilted such that the corner D touches the V.P. Draw its projections and determine the inclination of the cardboard with H.P.*

Construction Refer to Fig. 10.57.

1. Draw a reference line xy. Draw another line x_2y_2 inclined at 30° to xy.
2. Draw a pentagon *abcde* keeping 30 mm long base *ab* on x_2y_2 and point a 28 mm below xy.
3. Draw another reference line x_1y_1 perpendicular to *ab* to represent A.V.P.

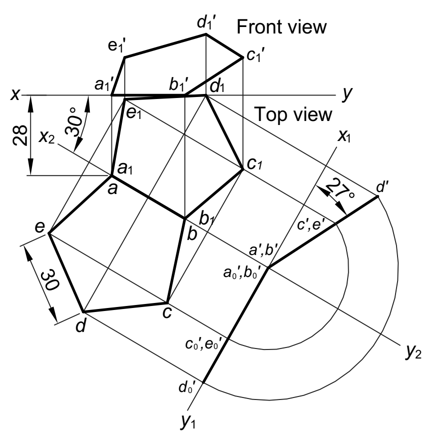


Fig. 10.57

4. Project points a, b, c, d and e on x_1y_1 and obtain $a_0'd_0'$.
5. It is given that the corner d should be in the V.P. Therefore, draw a_1b_1 coinciding with ab. Project point d on x_2y_2 and produce to meet xy at point d_1.
6. Draw an arc with centre a_0' and radius $a_0'd_0'$ to meet the projector from point d_1 drawn perpendicular to x_1y_1 at point d'. Measure its inclination with H.P. Here $\theta = 27°$.
7. Draw an arc with centre a_0' and radius $a_0'c_0'$ to meet $a'd'$ at points c' and e'. Mark points c_1 and e_1 as the intersection of the projectors (perpendicular to x_1y_1) from points c' and e' with the projectors (perpendicular to x_2y_2) from points c and e. Join $a_1b_1c_1d_1e_1$ to represent the final top view.
8. Project points a_1, b_1, c_1, d_1 and e_1 on xy and produce. Mark points a_1', b_1', c_1', d_1' and e_1' on the respective projectors such that their distance from xy is equal to the distance of points a', b', c', d' and e' from x_1y_1. Join $a_1'b_1'c_1'd_1'e_1'$ to represent the final front view.

Problem 10.50 *A hexagonal plane ABCDEF of side 35 mm is placed on the H.P. on its side AB which is inclined at 45° to the V.P. The surface of the plane is inclined in such a way that a side adjacent to the AB is parallel to the V.P. Draw the projections of the cardboard and determine its inclination with the H.P.*

Construction Refer to Fig. 10.58.

1. Draw a reference line xy. Draw another line x_2y_2 inclined at 45° to xy.
2. Draw a hexagon *abcdef* keeping 35 mm long base ab on x_2y_2.
3. Draw another reference line x_1y_1 perpendicular to ab to represent A.V.P.
4. Project points a, b, c, d, e and f on x_1y_1 and obtain $a_0'e_0'$.
5. It is given that a side adjacent to the ab is parallel to the V.P. Therefore, draw a_1b_1 coinciding with ab. Draw a line from point b_1 parallel to xy to meet projector from point c (perpendicular to x_2y_2) at point c_1.
6. Draw an arc with centre a_0' and radius $a_0'c_0'$ to meet the projector from point c_1 drawn perpendicular to x_1y_1 at point c'. Produce $a'c'$ such that $a'e'$ is the reproduction of $a_0'e_0'$. Measure its inclination as the inclination of the surface with H.P. Here $\theta = 55°$.
7. Mark points d_1, e_1 and f_1 as the intersection of the projectors from points d', e' and f' (drawn perpendicular to x_1y_1 line) with the projectors from points d, e and f (drawn perpendicular to x_2y_2 line). Join $a_1b_1c_1d_1e_1f_1$ to represent the final top view.
8. Project points a_1, b_1, c_1, d_1, e_1 and f_1 on xy and produce. Mark points a_1', b_1', c_1', d_1', e_1' and f_1' on the respective projectors such that their distance from xy is equal to the distance of points a', b', c', d', e' and f' from x_1y_1. Join $a_1'b_1'c_1'd_1'e_1'f_1'$ to represent the final front view.

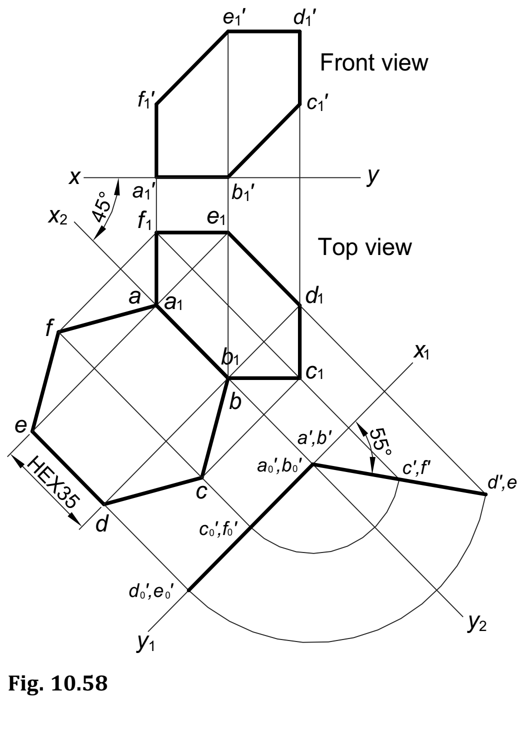


Fig. 10.58

10.13 TRUE SHAPE OF PLANE

When the projections of a plane is given and it is required to determine its true shape, first obtain the edge view of the plane. Thereafter project this edge view on an auxiliary plane parallel to it. Consider the following problems.

Problem 10.51 *A triangular plane PQR has its vertices P, Q and R at 40 mm, 60 mm and 20 mm above the H.P. respectively, and 25 mm, 65 mm and 10 mm in front of the V.P. respectively. If the projectors of P and Q are 40 mm apart and those of Q and R are 50 mm apart, determine the true shape of the plane PQR.*

Construction Refer to Fig. 10.59(a).

1. Draw triangles $p'q'r'$ and pqr to represent the front and the top views, respectively of plane *PQR*.
2. Through any corner, say p', draw a line parallel to xy to meet $r'q'$ at point m'. Project m' to meet rq at point m. Join pm which represents the true length of pm.
3. Draw a reference line x_1y_1 perpendicular to pm.
4. Project points p, q and r on x_1y_1 and produce. Mark points p_1', q_1' and r_1' on the respective projectors such that their distance from x_1y_1 is equal to the distance of points p', q' and r' from xy. Join $p_1'q_1'r_1'$ which is a line known as edge view.
5. Draw another reference line x_2y_2 parallel to the edge view $p_1'q_1'r_1'$.
6. Project points p_1', q_1' and r_1' on x_2y_2 and produce. Mark points p_1, q_1 and r_1 on the respective projectors such that their distance from x_2y_2 is equal to the distance of points p, q and r from x_1y_1. Join $p_1q_1r_1$ to represent the true shape of the plane.

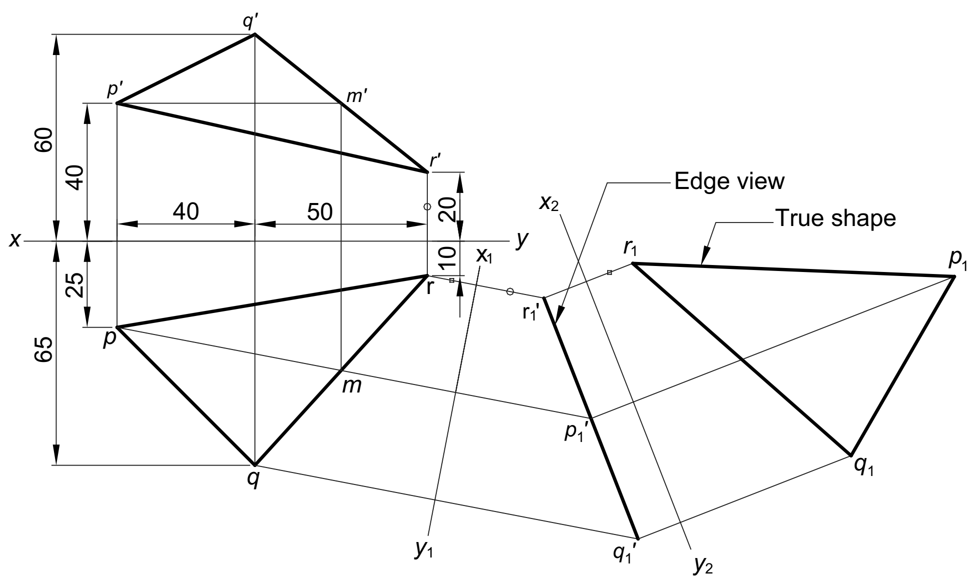


Fig. 10.59(a)

Alternate method Refer to Fig. 10.59(b).

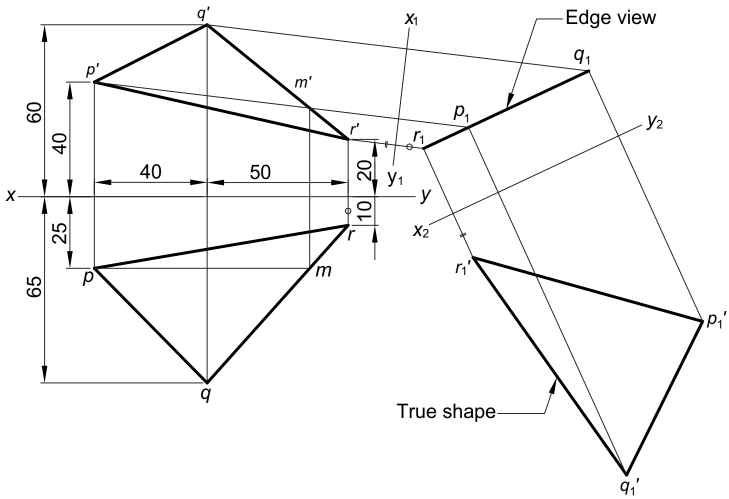


Fig. 10.59(b)

1. Draw triangles $p'q'r'$ and pqr to represent the front and the top views, respectively of plane pqr.
2. Through any corner, say p, draw a line parallel to xy to meet rq at point m. Project m to meet $r'q'$ at point m'. Join $p'm'$ which represents the true length of pm.
3. Draw a reference line x_1y_1 perpendicular to $p'm'$.
4. Project points p', q' and r' on x_1y_1 and produce. Mark points p_1, q_1 and r_1 on the respective projectors such that their distance from x_1y_1 is equal to the distance of points p, q and r from xy. Join $p_1q_1r_1$ which is a line known as edge view.
5. Draw another reference line x_2y_2 parallel to the edge view $p_1q_1r_1$.
6. Project points p_1, q_1 and r_1 on x_2y_2 and produce. Mark points p_1', q_1' and r_1' on the respective projectors such that their distance from x_2y_2 is equal to the distance of points p', q' and r' from x_1y_1. Join $p_1'q_1'r_1'$ to represent the true shape of the plane.

Problem 10.52 *A triangular plane PQR whose vertices P, Q and R are 80 mm, 60 mm and 20 mm above the H.P. respectively, has its top view as an isosceles triangle pqr of 50 mm base pq and altitude 60 mm having corner p on the reference line and side pq inclined at 45° to the reference line. Determine the true shape of the plane and its inclinations with the reference planes.*

Construction Refer to Fig. 10.60.

1. Draw an isosceles triangle pqr with pq = 50 mm and inclined at 45° to xy as the top view. Project corners p, q, and r and obtain $p'q'r'$ as the front view.
2. Through any corner, say q', draw a line parallel to xy to meet $p'r'$ at point m'. Project m' to meet pq at point m. Join qm which represents the true length of qm.
3. Draw a reference line x_1y_1 perpendicular to qm. Project points p, q and r on x_1y_1 and produce. Mark points p_1', q_1' and r_1' on the respective projectors such that their distance from x_1y_1 is equal to the distance of points p', q' and r' from xy. Join $p_1'q_1'r_1'$ which is an edge view.

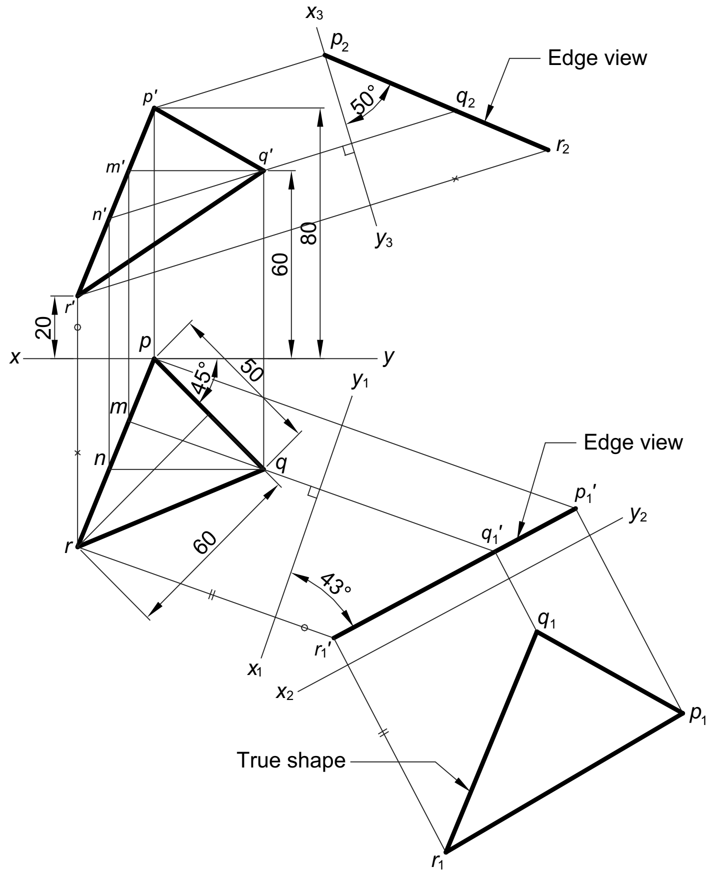


Fig. 10.60

Measure angle between x_1y_1 and the $p_1'q_1'r_1'$ as the inclination of the plane with H.P. Here, $\theta = 43°$.

4. Draw another reference line x_2y_2 parallel to $p_1'q_1'r_1'$. Project points p_1', q_1' and r_1' on x_2y_2 and produce. Mark points p_1, q_1 and r_1 on the respective projectors such that their distance from x_2y_2 line is equal to the distance of points p, q and r from x_1y_1. Join $p_1q_1r_1$ to represent the true shape of the plane.
5. Through any corner say q, draw a line parallel to xy to meet pq at point n. Project n to meet $p'r'$ at point n'. Join $q'n'$ which represents the true length of qn.
6. Draw a reference line x_3y_3 perpendicular to $q'n'$. Project points p', q' and r' on x_3y_3 and produce. Mark points p_2, q_2 and r_2 on the respective projectors such that their distance from x_3y_3 is equal to the distance of points p, q and r from xy. Join $p_2q_2r_2$ which is another edge view.

   Measure the angle between x_3y_3 and the edge view $p_2q_2r_2$ as the inclination of the plane with V.P. Here, $\phi = 50°$.

Problem 10.53 *A plane PQRS has its top view as a square of side 50 mm with pq making 30° to xy. The corners P, Q, R and S of the plane are 10 mm, 40 mm, 60 mm and 30 mm respectively from H.P. Determine the true shape of the plane.*

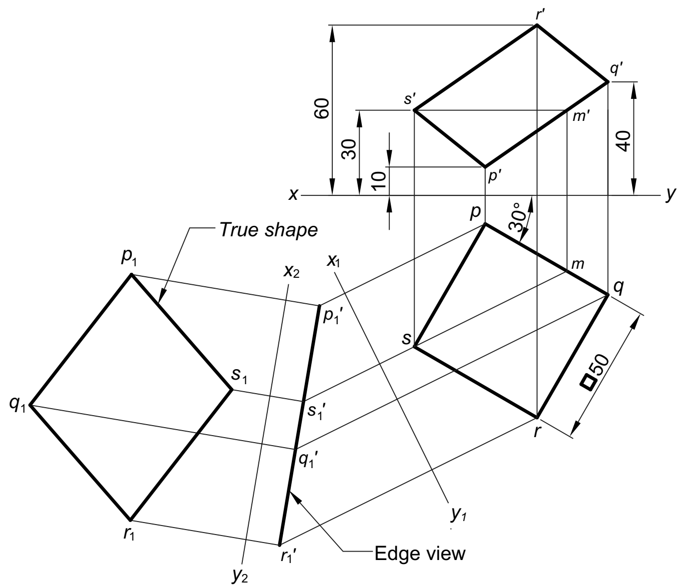


Fig. 10.61

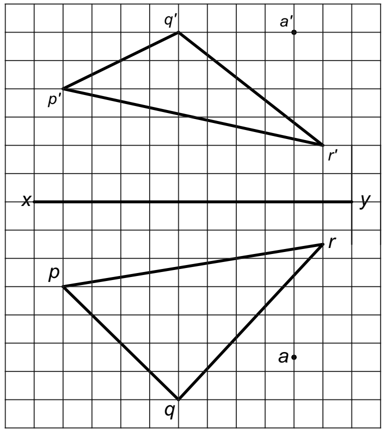
Construction Refer to Fig. 10.61.

1. Draw a square *pqrs* as the top view and project to obtain $p'q'r's'$ as the front view.
2. Through any corner, say s', draw a line parallel to *xy* to meet $p'q'$ at point m'. Project m' to meet *pq* at point *m*. Join *sm* which represents the true length of *sm*.
3. Draw a reference line x_1y_1 perpendicular to *sm*. Project points *p*, *q*, *r* and *s* on x_1y_1 and produce. Mark points p_1', q_1', r_1' and s_1' on the respective projectors such that their distance from x_1y_1 is equal to distance of points p', q', r' and s' from *xy*. Join $p_1'q_1'r_1's_1'$ to get the edge view.
4. Draw another reference line x_2y_2 parallel to $p_1'q_1'r_1's_1'$. Project points p_1', q_1', r_1' and s_1' on x_2y_2 and produce. Mark points p_1, q_1, r_1 and s_1 on the respective projectors such that their distance from x_2y_2 is equal to the distance of points *p*, *q*, *r* and *s* from x_1y_1. Join $p_1q_1r_1s_1$ to represent the true shape of the plane.

10.14 DISTANCE OF A POINT FROM THE PLANE

Problem 10.54 *Figure 10.62(a) is drawn on a 10 mm grid. It shows the projections of a triangular plane PQR and a point A. Determine the distance of the point A from the plane PQR.*

Construction Refer to Fig. 10.62(b).



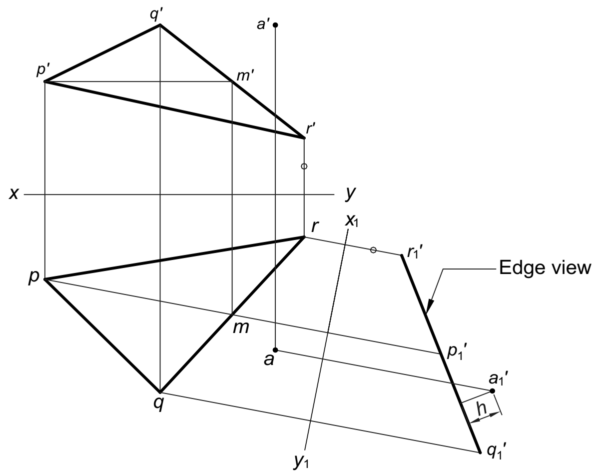


Fig. 10.62 **(a)** *Projections of plane* **(b)** *Edge view and distance of point a from the plane*

1. Draw triangles $p'q'r'$ and pqr to represent the front and the top views respectively of plane PQR.
2. Through any corner, say p', draw a line parallel to xy to meet $r'q'$ at point m'. Project m' to meet rq at point m. Join pm which represents the true length of pm.
3. Draw a reference line x_1y_1 perpendicular to pm. Project points p, q, r and a on x_1y_1 and produce. Mark points p_1', q_1', r_1' and a_1' on the respective projectors such that their distance from x_1y_1 is equal to the distance of points p', q', r' and a' from xy. Join $p_1'q_1'r_1'$ to get an edge view.
4. Draw a perpendicular from point a_1' on $p_1'q_1'r_1'$ and measure its distance h. Here h = 12 mm.

10.15 LOCATE A POINT

Problem 10.55 *Figure 10.63(a) is drawn on a 10 mm grid. It shows the front and top views of points P, Q and R. It also shows the top view of a point S. Locate the front view of the point S so that it lies in a plane containing points P, Q and R.*

Construction Refer to Fig. 10.63(b).

1. Mark points p, p', q, q', r, r' and s as given. Join them to form triangles $p'q'r'$ and pqr.
2. Through any corner, say p', draw a line parallel to xy to meet $r'q'$ at point m'. Project m' to meet rq at point m. Join pm which represents the true length of pm.
3. Draw a reference line x_1y_1 perpendicular to pm. Project points p, q and r on x_1y_1 and produce. Mark points p_1', q_1' and r_1' on the respective projectors such that their distance from x_1y_1 is equal to the distance of points p', q' and r' from xy. Join $p_1'q_1'r_1'$ to get an edge view.
4. Project point s on x_1y_1 and produce to meet the edge view of the plane $p_1'q_1'r_1'$ at point s_1'.
5. Project point s on xy and produce. Mark point s' on the projector such that its distance from xy is equal to the distance of point s_1' from x_1y_1. Point s' represents the front view of point s.

Alternative method Refer to Fig. 10.63(c).

1. Mark points p, p', q, q', r, r' and s as given. Join them to form triangles $p'q'r'$ and pqr.

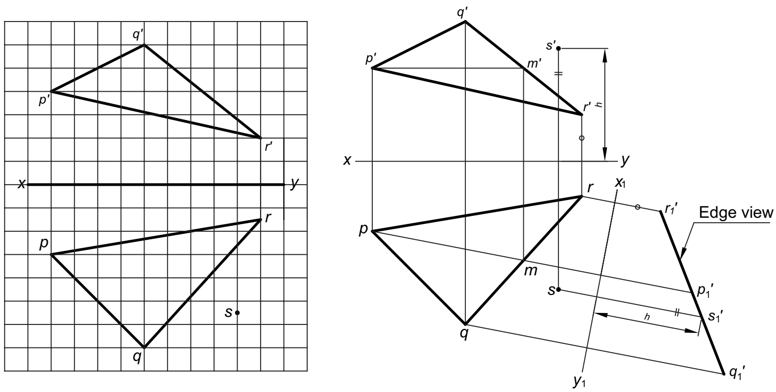


Fig. 10.63 **(a)** *Projections of plane* **(b)** *Edge view and location of point s* **(c)** *Location of points*

2. Join *ps* to intersect *qr* at point *m*.
3. Project point *m* to meet *q′r′* at point *m′*.
4. Produce line *p′m′* to meet the projector from point *s* at point *s′*. Point *s′* represents the front view of point *s*.

Problem 10.56 *Figure 10.64(a) is drawn on a 10 mm grid. It shows the front and top views of a plane PQR and point A. It also shows the top view of a point B and the front view of a point C. If the plane ABC is parallel to the plane PQR, locate the front view of point B and the top view of point C.*

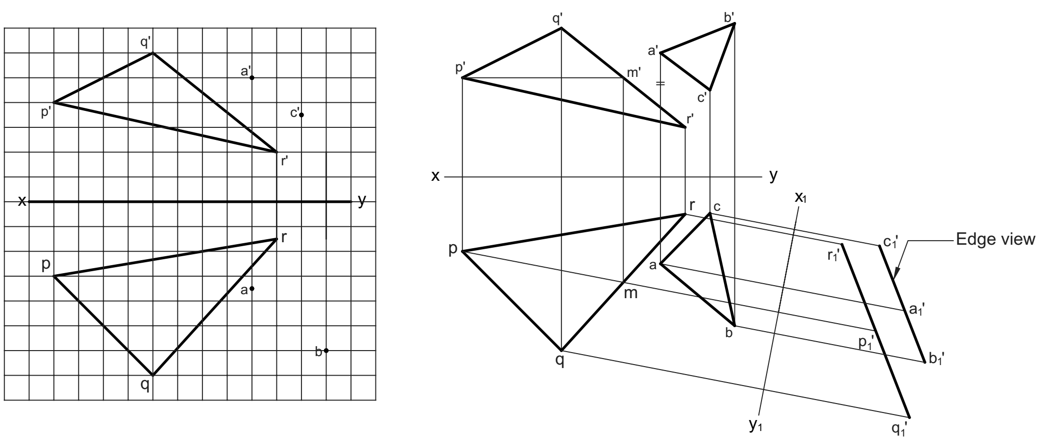


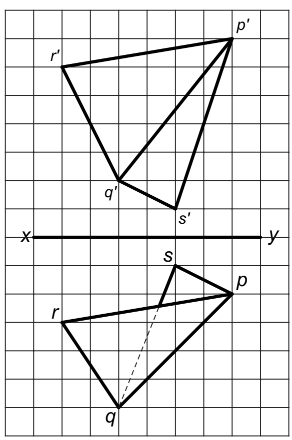
Fig. 10.64 **(a)** *Projections of planes* **(b)** *Edge view and projections of plane abc*

Construction Refer to Fig. 10.64(b).

1. Draw the given figure. Obtain the edge view $p_1'q_1'r_1'$.
2. Project point a on x_1y_1 and produce. Mark point a_1' on the projectors such that its distance from x_1y_1 is equal to the distance of point a' from xy.
3. Draw a line passing through point a_1' parallel to $p_1'q_1'r_1'$ to represent an edge view of the plane *ABC*.
4. Project point b on x_1y_1 and produce to meet the edge view of the plane *ABC* at point b_1'.
5. Project point b on xy and produce. Mark point b' on the projector such that its distance from xy is equal to the distance of point b_1' from x_1y_1. Point b' represents the front view of point b.
6. Mark point c_1' on the edge view of the plane *ABC* such that its distance from x_1y_1 is equal to the distance of point c' from xy.
7. Mark point c as the intersection of the projector of point c' on xy and projector of point c_1' on x_1y_1. Point c represents the top view of point c.
8. Join $a'b'c'$ and abc to represent the front and the top views of the plane *ABC*.

10.16 ANGLE BETWEEN TWO INTERSECTING PLANES

Problem 10.57 *Figure 10.65(a) is drawn on a 10 mm grid. It shows the front and top views of planes PQR and PQS intersecting at a common line PQ. Determine the angle between the two planes.*



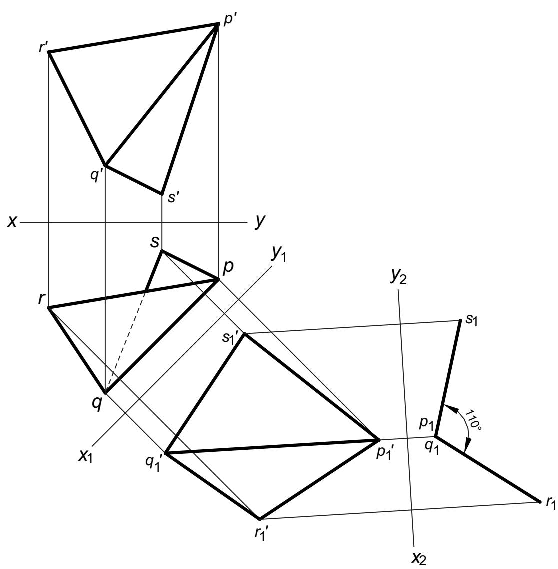


Fig. 10.65 **(a)** *Projections of plane* **(b)** *edge view and location of point A*

Construction Refer to Fig. 10.65(b).

1. Redraw the given figure to represent planes *PQR and PQS*.

2. Draw x_1y_1 parallel to pq. Project points p, q, r and s on x_1y_1 and produce. Mark points p_1', q_1', r_1' and s_1' on the respective projectors such that their distance from x_1y_1 is equal to the distance of points p', q', r' and s' from xy. Join $p_1'q_1'r_1'$ and $p_1'q_1's_1'$.
3. Draw x_2y_2 perpendicular to $p_1'q_1'$. Project points p_1', q_1', r_1' and s_1' on x_2y_2 and produce. Mark points p_1, q_1, r_1 and s_1 on the respective projectors such that their distance from x_2y_2 is equal to the distance of points p, q, r and s from x_1y_1. Join $p_1q_1r_1$ and $p_1q_1s_1$ to represent the edge view of the given planes.
4. Measure the included angle between $p_1q_1r_1$ and $p_1q_1s_1$. Here it is 110°.

EXERCISE 10C

10.1 A pentagonal plane of side 35 mm is resting on one of its edge in the H.P. with its surface perpendicular to the V.P. The corner opposite to the edge on which it is resting, is 40 mm above the H.P. Draw its projections. Also, project another front view on an A.V.P. which is inclined at 45° to the V.P.

10.2 A circular plate of diameter 60 mm has its surface on the H.P. Project its auxiliary top view on a reference line inclined at 30° with xy. From this top view project another front view on an A.V.P. inclined at 45° to the top view of the diameter appearing as the major axis.

10.3 A triangular plane ABC has sides AB = 80 mm, BC = 75 mm and CA = 65 mm. Its top view is a right angled triangle with ab inclined at 60° to the reference line. Draw projections of the plane and determine its inclination with H.P.

10.4 An isosceles triangle ABC of base AB 60 mm and altitude 80 mm has its base in the V.P. and inclined at 30° to the H.P. The corner A is 15 mm above the H.P. and corner C is in the H.P. Draw projections of the plane.

10.5 An isosceles triangular plane ABC of base 70 mm and altitude 80 mm, has its base in the H.P. and inclined at 45° to the V.P. The corner A and C are in the V.P. Draw its projections and determine the inclination of the plane with H.P.

10.6 A triangular plane ABC of base AB 70 mm and altitude 100 mm. The base is on the H.P. and inclined at 30° to the V.P. The plane is inclined to the H.P. in such a way that AC lies on a plane perpendicular to both the H.P. and the V.P. Draw the projections of the plane and find its inclination with the H.P.

10.7 A pentagon $ABCDE$ of side 35 mm is kept with side AB in the H.P. and inclined at 45° with the V.P. Pentagon is inclined to H.P. such that the adjacent side AE is in the V.P. Draw projections of the pentagon.

10.8 A rectangular plane of edges 40 mm and 70 mm is resting on an edge in the H.P. Its top view appears as a square with a diagonal is parallel to the reference line. Draw its projections and find true inclination of the plane with the H.P. and the V.P.

10.9 Figures E10.1 to E10.3 are drawn on a 10 mm grid. They show the front and the top views of planes

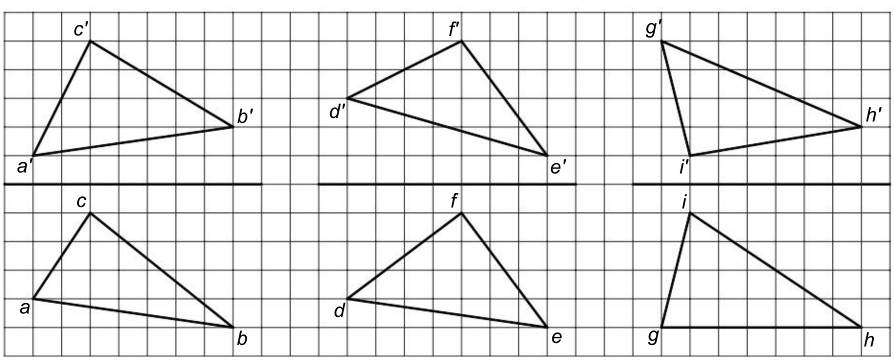


Fig. E10.1 **Fig. E10.2** **Fig. E10.3**

ABC, *DEF* and *GHI*. Show edge view in each case and determine inclinations of the plane with H.P. and V.P.

10.10 Figures E10.1 to E10.3 are drawn on a 10 mm grid. They show the front and the top views of planes *ABC*, *DEF* and *GHI*. Determine the true shape of the planes.

10.11 A triangular plane *PQR* has its vertices *P*, *Q* and *R* at 70 mm, 20 mm and 50 mm above the H.P. respectively, and 25 mm, 70 mm and 15 mm in front of the V.P. respectively. If the projectors of *P* and *Q* are 50 mm apart and those of *Q* and *R* are 40 mm apart, determine the true shape of the plane *PQR*.

10.12 Front and top views of a thin plate of negligible thickness appears as circles of 50 mm diameter. Determine its true shape.

10.13 A plane *PQRS* has its front view as a square of side 50 mm with side *PQ* inclined at 30° to *xy*. The corners *P*, *Q*, *R* and *S* of the plane are 20 mm, 50 mm, 65 mm and 35 mm respectively from the V.P. Determine the true shape of the plane.

10.14 A plane *PQRS* has its top view as a square of side 40 mm such that a side is parallel to the reference line, while its front view is a rectangle of sides 40 mm and 60 mm. Determine the true shape of the plane.

10.15 A circular plate of diameter 70 mm has a hexagonal hole of side 25 mm punched centrally. The plate is resting on the H.P. with its surfaces inclined at 30° to the H.P. and the vertical plane containing the diameter through the point on the H.P. is inclined at 45° to the V.P. Draw the projections of the plate when two parallel sides of the hole are perpendicular to the diameter of the circular plate passing through the point on which it rests.

10.16 Figures E10.4 to E10.6 are drawn on a 10 mm grid. They show the front and the top views of planes *ABC*, *DEF* and *GHI*. The figure also shows the projections of points *P*, *Q* an *R*. Determine the distance of the point *P*, *Q* and *R* from planes *ABC*, *DEF* and *GHI*, respectively.

10.17 Figure E10.7 is drawn on a 10 mm grid. It shows the front and top views of points *A*, *B* and *C*. It also

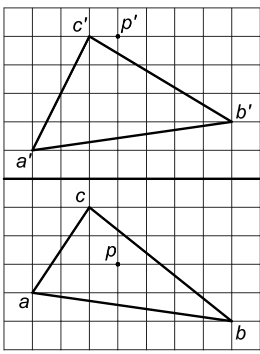


Fig. E10.4

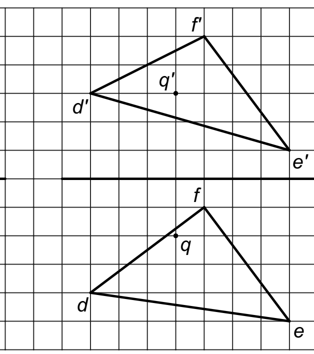


Fig. E10.5

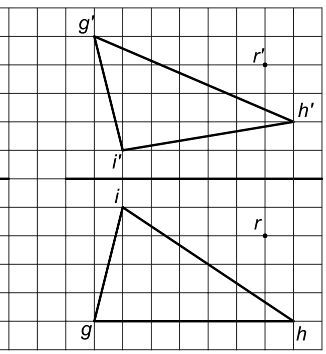


Fig. E10.6

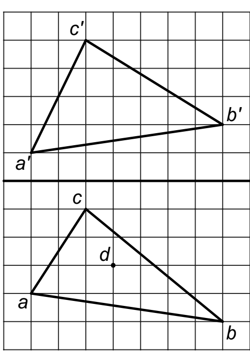


Fig. E10.7

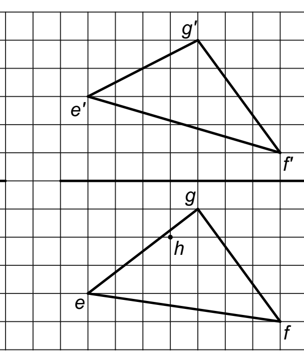


Fig. E10.8

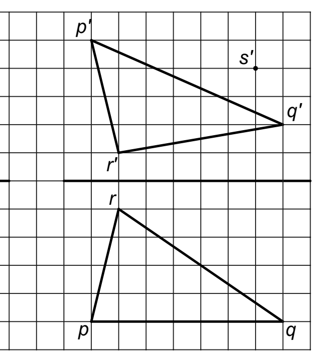


Fig. E10.9

shows the top view of a point *D*. Locate the front view of the point *D* so that it lies in a plane containing points *A*, *B* and *C*.

10.18 Figures E10.8 is drawn on a 10 mm grid. It shows the front and top views of a plane *EFG*. It also shows the top view of a point *H*. Locate the front view of the point *H* so that it lies at a distance of 10 mm from the in the plane *EFG*.

10.19 Figure E10.9 is drawn on a 10 mm grid. It shows the front and top views of a plane *PQR*. It also shows the front view of a point *S*. Locate the top view of the point *S* so that it lies at a distance of 10 mm from the in the plane *ABC*.

VIVA-VOCE QUESTIONS

10.1 If the top view of a plane is a straight line, will its front view always be the true shape?

10.2 If the front view of a plane lies in the reference line, will its top view always be the true shape?

10.3 The projections of a plane lying in the H.P. are drawn. What will be the change in the shape, size and position of the front view if the surface of the plane is inclined at 30° to the H.P.?

10.4 The projections of a plane parallel to V.P. are drawn. What will be the change in the shape, size and position of the top view if the surface of the plane is inclined at 45° to the V.P.?

10.5 A rectangular plane 60 mm long and 30 mm wide is parallel to and 20 mm above the H.P. What will be the shape and position of its front view if the longer side is inclined at 30° to the V.P.?

10.6 The top view of a plane is a circle and the front view is a line inclined at 60° to *XY*. What is the true shape of the plane?

10.7 The surface of a hexagonal plane is perpendicular to both H.P. and V.P. Which orthographic view will show the true shape?

10.8 The true shape of a pentagonal plane is seen in the side view. What will be the shapes of its front and top views?

10.9 Define the position of a plane rhombus such that its top view appears as a square.

10.10 Define the position of an elliptical plane such that its front view appears as a circle.

MULTIPLE-CHOICE QUESTIONS

10.1 If a thin set-square is kept perpendicular to both the horizontal and vertical planes, its true shape is seen in
- (a) horizontal plane
- (b) vertical plane
- (c) auxiliary inclined plane
- (d) profile plane

10.2 Planes which are inclined to both the horizontal and vertical planes are called
- (a) oblique planes
- (b) profile planes
- (c) auxiliary planes
- (d) None of these

10.3 If a thin rectangular plate of 60 mm and 30 mm sides is inclined at an angle of 60° to the H.P., its top view may be
- (a) square of 60 mm sides
- (b) square of 30 mm sides
- (c) rectangle of 60 mm and 45 mm sides
- (d) rectangle of 45 mm and 30 mm sides

10.4 In multi-view orthographic projection, the front view of a circular plane may be
- (a) circle
- (b) ellipse
- (c) straight line
- (d) Any of these

10.5 If both front and top views of a plane are straight lines the true shape will lie on
- (a) profile plane
- (b) horizontal plane
- (c) vertical plane
- (d) Any of these

10.6 If a circular plane is inclined at 30° with the H.P. and 60° with the V.P. its side view will be
- (a) ellipse
- (b) straight line

(c) circle
(d) true shape

10.7 The front view of an elliptical plane may be
(a) ellipse
(b) circle
(c) straight line
(d) Any of these

10.8 If the top view of a plane is a rhombus the object may be
(a) square
(b) parallelogram
(c) octagon
(d) Any of these

10.9 The trace of a hexagonal plane may be
(a) straight line
(b) point
(c) hexagon
(d) equilateral triangle

10.10 A 60° set-square has its shortest edge in the V.P. The surface is perpendicular to the H.P. and inclined to the V.P. Its front view may appear as an
(a) equilateral triangle
(b) isosceles triangle
(c) obtuse angled triangle
(d) Any of these

10.11 A 60° set-square has its shortest edge in the H.P. and the surface is perpendicular to the V.P. Its top view is
(a) isosceles triangle
(b) right angled triangle
(c) straight line
(d) Any of these

10.12 If a hexagonal plane is inclined to H.P. and perpendicular to V.P., its front view is
(a) line
(b) regular hexagon
(c) irregular hexagon
(d) None of these

10.13 If both the principal views of a plane object are ellipse of the same size, the side view is
(a) horizontal line
(b) vertical line
(c) inclined line
(d) ellipse

Answers to multiple-choice questions

10.1 (d), 10.2 (a), 10.3 (b), 10.4 (d), 10.5 (a), 10.6 (b), 10.7 (d), 10.8 (a), 10.9 (a), 10.10 (b), 10.11 (d), 10.12 (a), 10.13 (c)

Chapter

11

PROJECTIONS OF SOLIDS

11.1 INTRODUCTION

This chapter deals with the orthographic projections of three-dimensional objects called *solids*. However, only those solids are considered the shape of which can be defined geometrically and are regular in nature. The basic concepts of orthographic projections discussed in earlier chapters shall also apply here.

11.2 CLASSIFICATION OF SOLIDS

Solids are usually classified as given below:

- Solid
  - Right solid
    - Polyhedron
      - Regular polyhedron
        - Tetrahedron
        - Cube
        - Octahedron
        - Dodecahedron
        - Icosahedron
      - Prism
        - Triangular
        - Square
        - Pentagonal
        - Hexagonal
        - etc.
      - Pyramid
        - Triangular
        - Square
        - Pentagonal
        - Hexagonal
        - etc.
    - Solid of revolution
      - Cylinder
      - Cone
      - Sphere
  - Oblique solid

Fig. 11.1 *Classification of regular solids*

11.2.1 Polyhedron

A polyhedron is a solid bounded by planes called faces, which meet in straight lines called *edges*. *A regular polyhedron* has all the faces equal and regular as shown in Fig. 11.2.

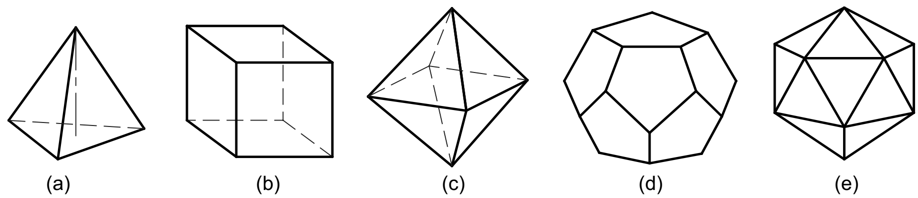


Fig. 11.2 *Regular polyhedron* **(a)** *Tetrahedron* **(b)** *Cube* **(c)** *Octahedron* **(d)** *Dodecahedron* **(e)** *Icosahedron*

1. Tetrahedron It has four equal equilateral triangular faces.

2. Cube It has six equal square faces.

3. Octahedron It has eight equal equilateral triangular faces.

4. Dodecahedron It has 12 equal pentagonal faces.

5. Icosahedron It has 20 equal equilateral triangular faces.

11.2.2 Prism

A prism is a polyhedron with two *n*-sided polygonal bases which are parallel and congruent, and lateral faces are rectangles. All cross-sections parallel to the bases are congruent with the bases. An imaginary line that joins the centre of the bases is called an axis. *A right and regular prism* has regular polygonal bases, axis perpendicular to the bases and all the faces are equal rectangles, as shown in Fig. 11.3. Prisms are named according to the shape of their base, so a prism with a triangular base is called a *triangular prism*; a square base is called a *square prism* and so on.

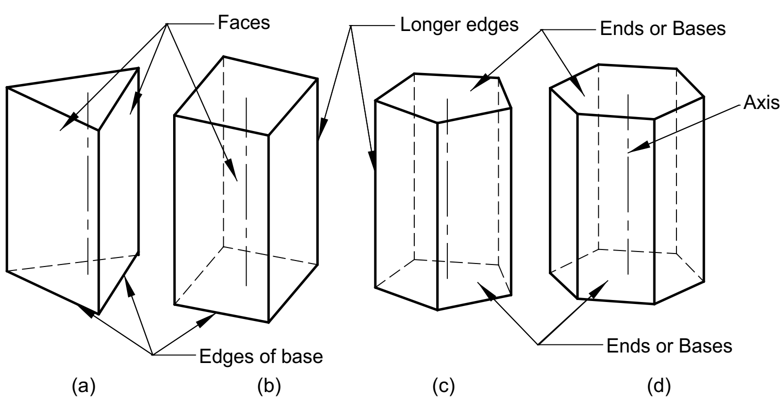


Fig. 11.3 *Prisms* **(a)** *Triangular* **(b)** *Square* **(c)** *Pentagonal* **(d)** *Hexagonal*

11.2.3 Pyramid

A pyramid is a polyhedron with n-sided polygonal base and lateral faces are triangles meeting at a point called the vertex or apex. An imaginary line that joins the apex with the centre of the base is known as the axis. *A right and regular pyramid* has a regular polygon base, axis perpendicular to the base and all the faces are equal isosceles triangles, as shown in Fig. 11.4. Pyramids are named according to the shape of their base, so a pyramid with a triangular base is called a *triangular pyramid*; a square base is called a *square pyramid* and so on. The centre of gravity of pyramids lies on the axis at one-fourth of its height from the base.

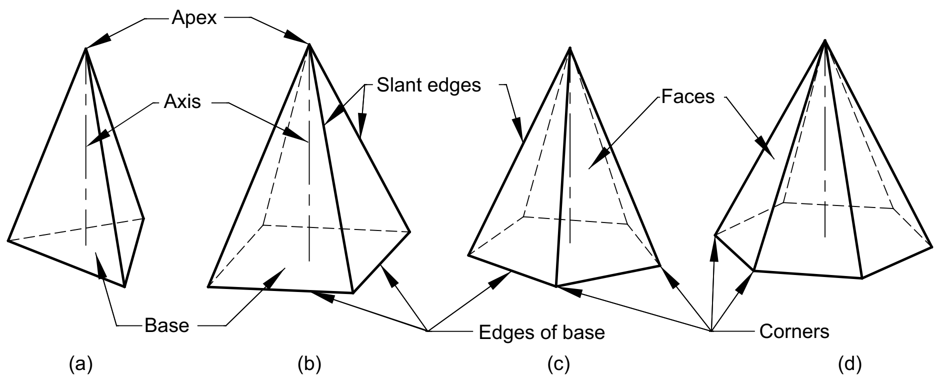


Fig. 11.4 *Pyramids* **(a)** *Triangular* **(b)** *Square* **(c)** *Pentagonal* **(d)** *Hexagonal*

11.2.4 Solid of Revolution

These solids are obtained by revolving a plane figure like rectangle, triangle or a semi-circle about a fixed line.

1. Cylinder A cylinder is a solid of revolution obtained by revolving a rectangle about one of its fixed side called an axis. It can be imagined as a prism of infinite number of lateral faces. Any line on the surface of a cylinder is called its *generator*. Thus, a cylinder has an infinite number of generators. *A right cylinder* has all the generators and the axis perpendicular to the base, as shown in Fig. 11.5(a).

2. Cone A cone is obtained by revolving a triangle about its fixed side called an *axis*. A cone can be imagined as a pyramid with infinite number of lateral faces. Any line on the surface of a cone is called its generator. Thus, a cone has an infinite number of generators. *A right cone* has all generators of equal length and the axis perpendicular to the base, as shown in Fig. 11.5(b).

3. Sphere A sphere is obtained by revolving a semi-circle around its diameter, as shown in Fig. 11.5(c).

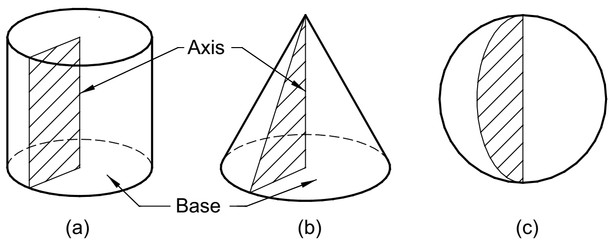


Fig. 11.5 *Solids of revolution* **(a)** *Cylinder* **(b)** *Cone* **(c)** *Sphere*

11.2.5 Oblique Solid

An oblique solid such as oblique prism, pyramid, cylinder or cone has its axis inclined to its base as shown in Fig. 11.6. The faces of an oblique prism are parallelograms of different sizes. The faces of an oblique pyramid are triangles of different sizes. The generators in an oblique cylinder have equal lengths whereas those in an oblique cone have unequal lengths.

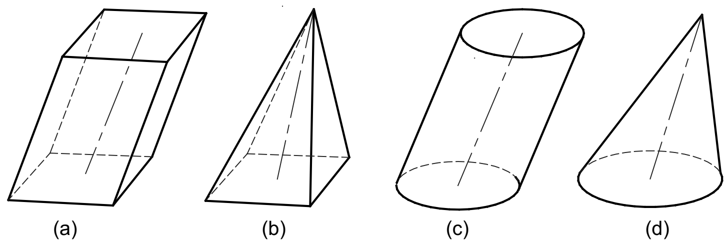


Fig. 11.6 *Oblique solids* **(a)** *Square prism* **(b)** *Square pyramid* **(c)** *Cylinder* **(d)** *Cone*

11.2.6 Frustum of Pyramid and Cone

When a regular pyramid or a cone is cut by a plane parallel to its base and the portion of the solid containing apex is removed, the remaining portion of the solid is called the *frustum* of that pyramid or cone, as shown in Fig. 11.7.

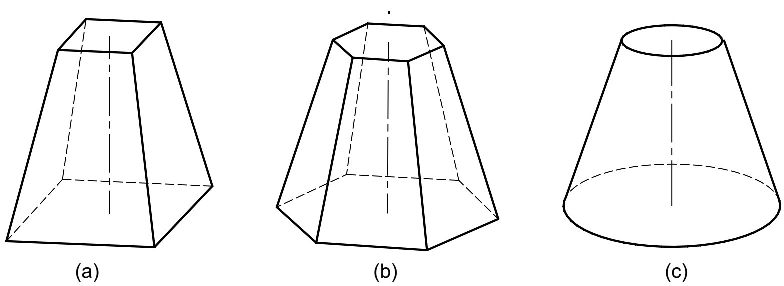
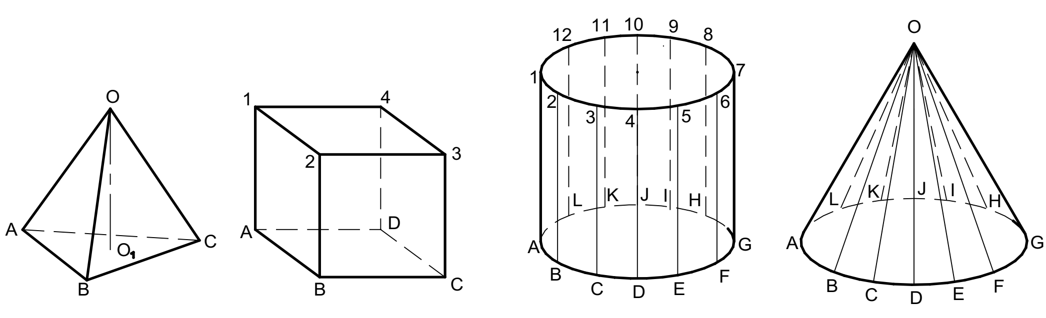


Fig. 11.7 *Frustums of* **(a)** *Square pyramid* **(b)** *Hexagonal pyramid* **(c)** *Cone*

11.3 RECOMMENDED METHOD OF LABELLING

It is recommended to label the corners of the solids in a manner as shown in Fig. 11.8.



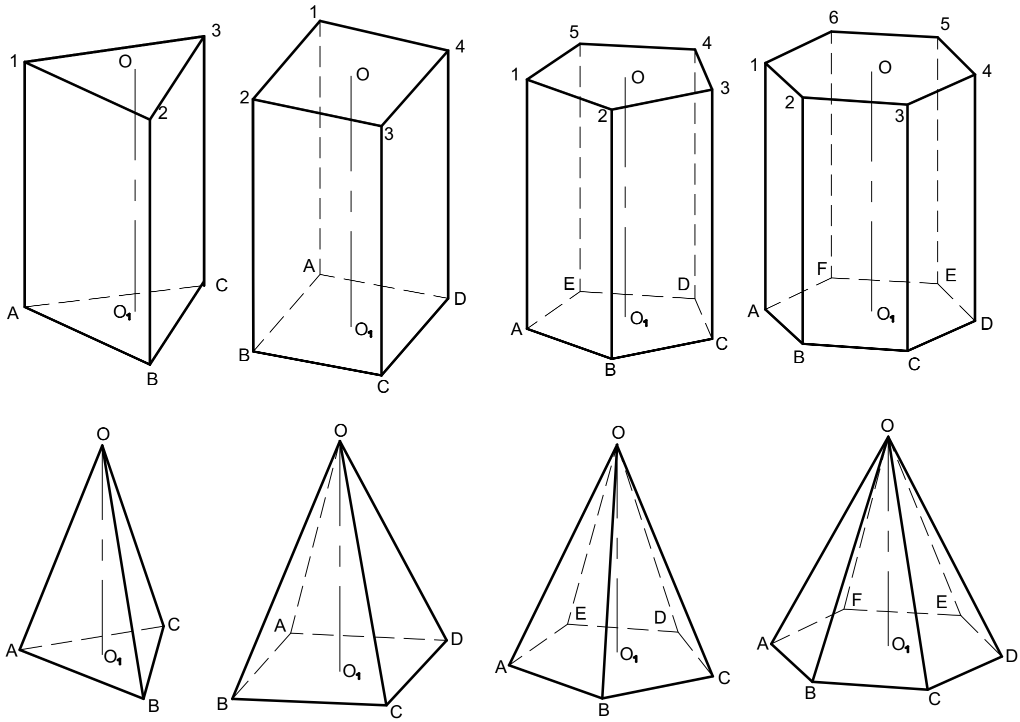


Fig. 11.8 *Suggested method of labelling*

11.4 ORIENTATION OF SOLID

The solid may be in one of the following positions:

1. Axis perpendicular to the H.P.
2. Axis perpendicular to the V.P.
3. Axis parallel to both the H.P. and the V.P. (i.e., perpendicular to the profile plane)
4. Axis inclined to the H.P. and parallel to the V.P.
5. Axis inclined to the V.P. and parallel to the H.P.
6. Axis inclined to both the H.P. and the V.P.

11.5 AXIS PERPENDICULAR TO H.P.

This is one of the basic positions of the solid. It is evident that if the axis of a right solid is perpendicular to the H.P., its base will be parallel to the H.P. The true shape and size of the base can be viewed in the top view. Therefore, first obtain the top view of the solid and then project it to obtain the front view.

Problem 11.1 *A square pyramid of base side 40 mm and axis 60 mm is resting on its base on the H.P. Draw its projections when (a) a side of the base is parallel to the V.P., (b) a side of the base is inclined at 30° to the V.P., (c) all the sides of the base are equally inclined to the V.P.*

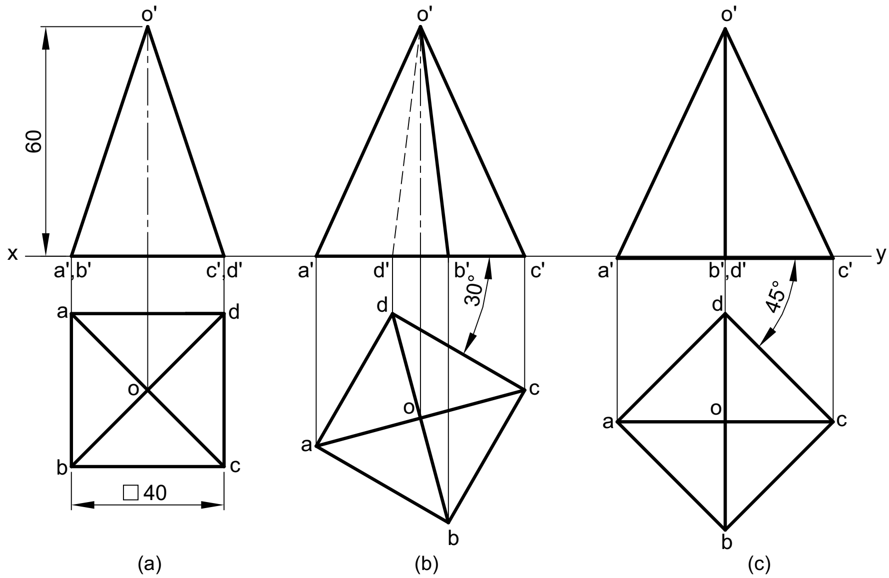


Fig. 11.9

Construction

1. **Side of the base parallel to V.P. (Fig. 11.9(a))** Draw a square *abcd* keeping *ad* parallel to the *xy*. Locate centre *o* and join it with all the corners *a*, *b*, *c* and *d*. This represents the top view. Project points *a*, *b*, *c* and *d* on *xy* and obtain points *a′*, *b′*, *c′* and *d′*. Project point *o*, 60 mm above *xy* and mark it as *o′*. Join *o′a′b′* and *o′c′d′*. This is the required front view.
2. **A side of the base inclined at 30° to V.P. (Fig. 11.9(b))** Draw a square *abcd* keeping *dc* inclined at 30° to the *xy*. Locate centre *o* and join it with the corners *a*, *b*, *c* and *d*. This represents the top view. Project *a*, *b*, *c* and *d* on *xy* to obtain *a′*, *b′*, *c′* and *d′*. Project point *o*, 60 mm above *xy* and mark it as *o′*. Join *a′o′*, *b′o′*, *c′o′* and *d′o′*. This is the required front view. It may be noted that *d′o′* is not visible and should be shown using dashed narrow line.
3. **All the sides of the base equally inclined to V.P. (Fig. 11.9(c))** Draw a square *abcd* keeping sides inclined at 45° to the *xy*. Locate centre *o* and join it with the corners *a*, *b*, *c* and *d*. This represents the top view. Project points *a*, *b*, *c* and *d* on *xy* and obtain points *a′*, *b′*, *c′* and *d′*. Project point *o*, 60 mm above the *xy* and mark it as *o′*. Join *a′o′*, *b′d′o′* and *c′o′*. This is the required front view.

Problem 11.2 *A square prism of base side 40 mm and axis 60 mm is resting on its base on the ground. Draw its projections when (a) a face is perpendicular to the V.P., (b) a face is inclined at 30° to the V.P., (c) all the faces are equally inclined to the V.P.*

Construction

1. **Face perpendicular to the V.P. (Fig. 11.10(a))** Draw a square *abcd* keeping *ab* perpendicular to *xy*. This represents the top view. Project all the corners of the top view on *xy* and obtain points *a′*, *b′*, *c′* and *d′*. Mark points 1′, 2′, 3′ and 4′, 60 mm above *xy*. Join all the edges and obtain the required front view.

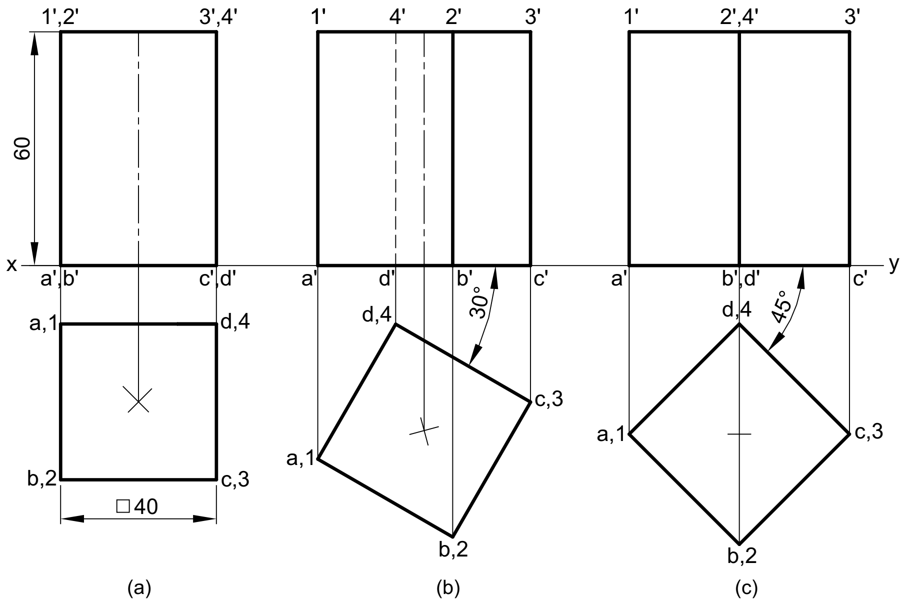


Fig. 11.10

2. **Face inclined at 30° to the V.P. (Fig. 11.10(b))** Draw a square *abcd* keeping *cd* inclined at 30° to *xy*. This represents the top view. Project all the corners of the top view on *xy* and obtain points *a*′, *b*′, *c*′ and *d*′. Mark points 1′, 2′, 3′ and 4′, 60 mm above *xy*. Join all the edges and obtain the required front view. Edge D4 is not visible in the front view so *d*′4′ should be shown using dashed narrow line.
3. **All the faces equally inclined to the V.P. (Fig. 11.10(c))** Draw a square *abcd* keeping sides inclined at 45° to *xy*. This represents the top view. Project all the corners of the top view on *xy* and obtain *a*′, *b*′, *c*′ and *d*′. Mark points 1′, 2′, 3′ and 4′, 60 mm above *xy*. Join all the edges and obtain the required front view.

11.6 AXIS PERPENDICULAR TO V.P.

This is another basic position of the solid. It is evident that if the axis of a right solid is perpendicular to V.P., its base will be parallel to the V.P. The true shape and size of the base can be viewed in the front view. Therefore, first obtain the front view of the solid and then project it to obtain the top view.

Problem 11.3 *A pentagonal prism of base side 30 mm and axis 60 mm has one of its bases in the V.P. Draw its projections when (a) a rectangular face is parallel to and 15 mm above the H.P., (b) a face is perpendicular to the H.P., (c) a face is inclined at 45° to the H.P.*

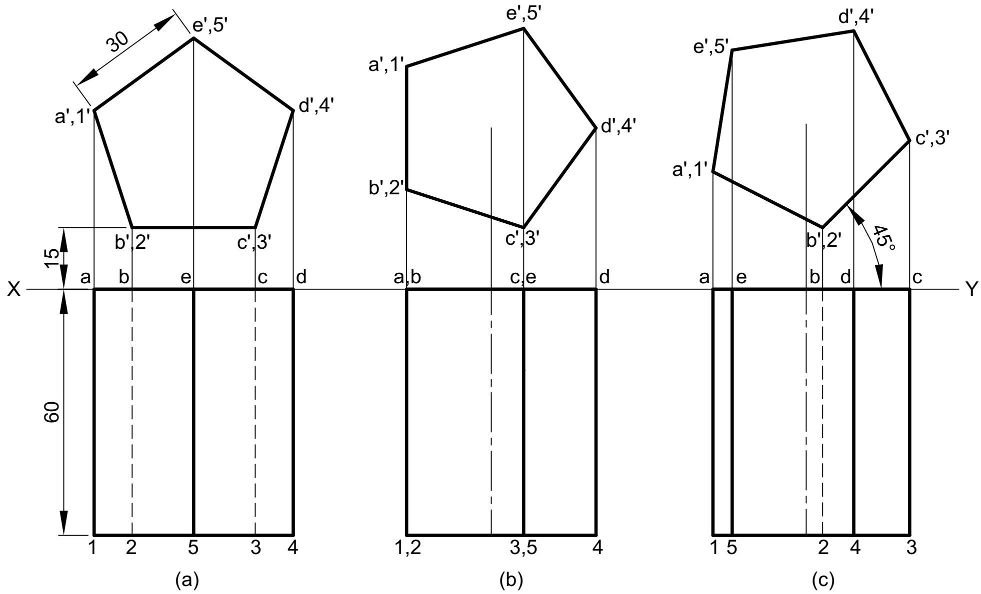


Fig. 11.11

Construction

1. **A rectangular face parallel to and 15 mm above H.P. (Fig. 11.11(a))** Draw a pentagon $a'b'c'd'e'$ keeping $b'c'$ parallel to and 15 mm above xy to represent the front view. Project all corners from the front view on the xy and obtain points a, b, c, d and e. Mark points 1, 2, 3, 4 and 5, 60 mm below xy. Join all the edges and obtain the required top view. Edges $b2$ and $c3$ are not visible and should be drawn using dashed narrow lines.
2. **A face perpendicular to the H.P. (Fig. 11.11(b))** Draw a pentagon $a'b'c'd'e'$ keeping $a'b'$ perpendicular to xy to represent the front view. Project all corners from the front view on the xy and obtain points a, b, c, d and e. Mark points 1, 2, 3, 4 and 5, 60 mm below xy. Join all the edges and obtain the required top view.
3. **A face inclined at 45° to the H.P. (Fig. 11.11(c))** Draw a pentagon $a'b'c'd'e'$ keeping $b'c'$ inclined at 45° to the xy to represent the front view. Project all corners from the front view on the xy and obtain points a, b, c, d and e. Mark points 1, 2, 3, 4 and 5, 60 mm below xy. Join all the edges and obtain the required top view. Edge $b2$ is not visible and should be drawn using dashed narrow line.

11.7 AXIS PARALLEL TO BOTH H.P. AND V.P.

It is evident that if the axis of right solids is parallel to both H.P. and V.P., the base of the solid will be perpendicular to the reference planes and parallel to the profile plane. The true shape and size of the base can be viewed in the side view. Therefore, first obtain the side view of the solid and then project it to obtain the front and the top views.

Problem 11.4 *A pentagonal prism of base side 30 mm and axis 60 mm is resting on one of its rectangular faces on the H.P. with axis parallel to the V.P. Draw its projections.*

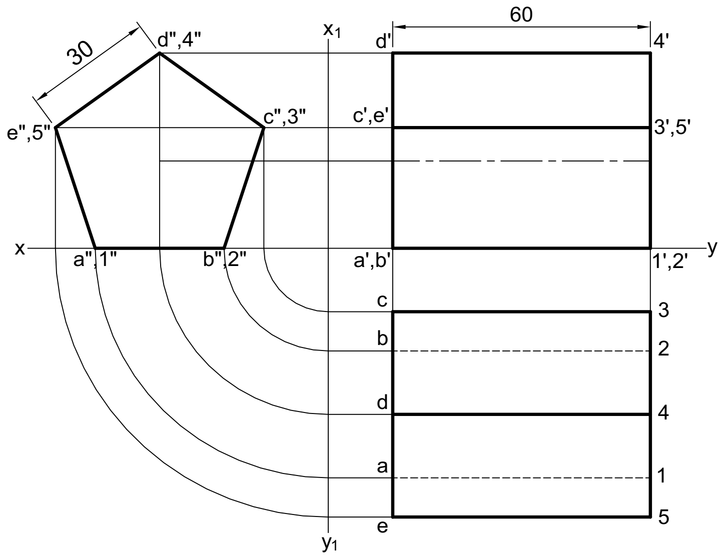


Fig. 11.12

Construction Refer to Fig. 11.12.

1. As the axis is parallel to both the planes, the true shape and size of the base is seen in the side view. Therefore, draw a pentagon $a''b''c''d''e''$ keeping $a''b''$ on xy.
2. Project the corners from the side view on x_1y_1 and produce to obtain the front view $a'1'4'd'$.
3. Project the corners from the side view on xy, rotate it through 90° and then extend them parallel to xy. Project the front view to meet them and obtain e53c as the required top view. Edges $a1$ and $b2$ are not visible and should be drawn using dashed narrow lines.

11.8 MISCELLANEOUS PROBLEMS

Problem 11.5 *A hexagonal prism of base edge 30 mm and axis 70 mm has its axis parallel to and 50 mm above the H.P. Its base is parallel to the V.P. and an edge of the base is inclined at 45° to the H.P. Draw its projections.*

Construction Refer to Fig. 11.13.

1. Draw a hexagon $a'b'c'd'e'f'$ keeping o_1' 50 mm above xy and side $c'd'$ inclined at 45° to xy to represent the front view. Project all corners and obtain $a14d$ as the top view.

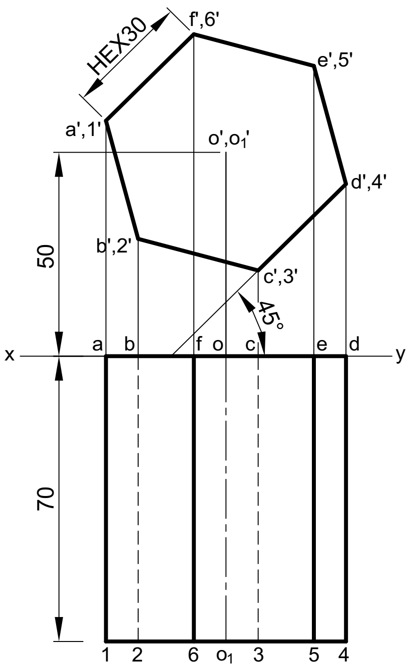


Fig. 11.13

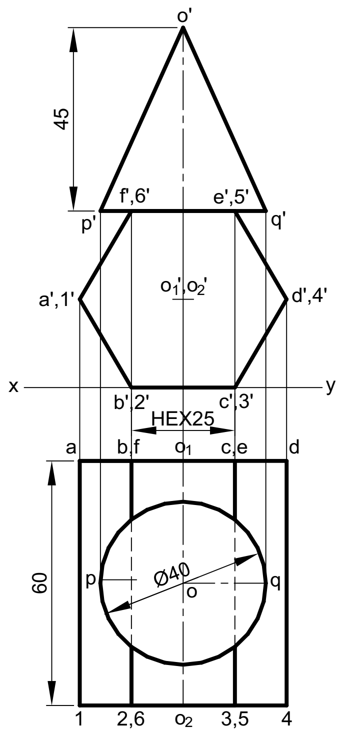


Fig. 11.14

Problem 11.6 *A hexagonal prism of base side 25 mm and axis 60 mm is resting on one of its rectangular faces on the H.P. with the axis perpendicular to the V.P. A right circular cone of base 40 mm diameter and axis 45 mm is placed centrally on the top of the prism. Draw the projections of the composite solid.*

Construction Refer to Fig. 11.14.

1. Draw a hexagon *a′b′c′d′e′f′* keeping *b′c′* on *xy* to represent the front view. Project the corners of the hexagon and obtain a14*d* as its top view.
2. Locate *o* as the mid-point of the top face *f*65*e*. With centre *o* draw a circle of diameter 40 mm. This is the top view of the cone. Project the ends of the circle and obtain *p′q′o′* as the front view of the cone.

Problem 11.7 *A tetrahedron of edge 65 mm has a face in the V.P. and an edge of that face is perpendicular to the H.P. Draw its projections.*

Construction Refer to Fig. 11.15.

1. Draw equilateral triangle *a′b′c′* keeping *a′b′* perpendicular to *xy*. Locate *d′* as its centroid. Join *d′* with the corners *a′*, *b′* and *c′*. This is the front view.
2. Project *a′*, *b′* and *c′* on *xy* and obtain points *a*, *b* and *c*.

3. As $c'd'$ is parallel to xy, cd should be of true length. Therefore, draw an arc with centre c and radius 65 mm to meet the projector of point d' at point d. Join the edges and obtain the required top view.

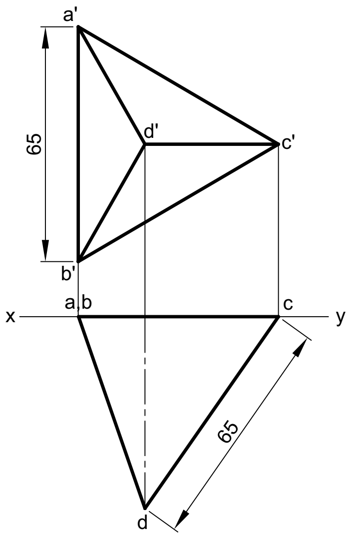


Fig. 11.15

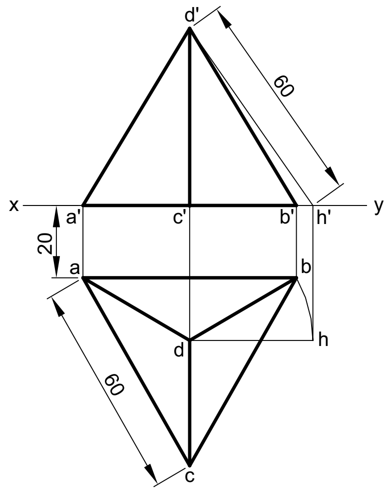


Fig. 11.16

Problem 11.8 *A tetrahedron of edge 60 mm is resting on a face on the H.P. such that an edge is parallel to and 20 mm in front of the V.P. Draw its projections.*

Construction Refer to Fig. 11.16.

1. Draw equilateral triangle *abc* keeping *ab* parallel to and 20 mm in front of the *xy*. Locate *d* as the centroid of the triangle. Join d with the corners *a*, *b* and *c*. This is the top view.
2. Project *a*, *b* and *c* on *xy* and obtain points a', b' and c'.
3. Draw an arc with centre *d* and radius *db* to meet the horizontal line through *d* at point *h*. Project point *h* on *xy* and obtain point h'.
4. Draw an arc with centre h' and radius 60 mm to meet the projector of point *d* at point d'. Join the edges and obtain the required front view.

EXERCISE 11A

Axis perpendicular to H.P.

11.1 Draw the projections of a pentagonal pyramid, base side 30 mm and axis 60 mm, rests on its base on the H.P. with a side of the base (a) parallel to the V.P., (b) perpendicular to the V.P. and (c) inclined at 30° to the V.P.

11.2 Draw the projections of a hexagonal prism, base side 30 mm and axis 70 mm, resting on its base on the H.P. such that a side of the base is (a) parallel to the V.P., (b) perpendicular to the V.P., and (c) inclined at 45° to the V.P.

11.3 A cone of base diameter 50 mm and axis 60 mm has its base parallel to and 10 mm above the H.P. while the axis is parallel to and 40 mm in front of the V.P. Draw its projections.

11.4 A square prism of base side 45 mm and axis 65 mm has its axis parallel to and 55 mm in front of V.P. An edge of its base is parallel to the H.P. and inclined at 30° to the V.P. Draw its projections.

11.5 Draw the projections of a frustum of a cone of base diameter 60 mm, top diameter 30 mm and height 75 mm resting on its base on the H.P.

11.6 Draw the projections of the frustum of a square pyramid base edge 50 mm, top edge 25 mm and height 60 mm resting on its base on the H.P. with a side of base inclined at 30° to the V.P.

11.7 A cube of edge 45 mm is resting on the H.P. with a vertical face inclined at 30° to the V.P. Draw its projections.

11.8 An octahedron of edge 40 mm rests on an apex on the H.P. Its axis perpendicular to H.P. and a horizontal edge is inclined at 30° to the V.P. Draw its projections.

11.9 A square slab of base side 60 mm and thickness 20 mm is resting on its base on the ground with an edge inclined at 30° to the V.P. A cone of base diameter 50 mm and axis 60 mm is placed centrally over the slab such that axes of the solids coincide. Draw its projections.

11.10 A cone of base diameter 40 mm and axis 55 mm is resting centrally on a hexagonal slab of side 25 mm and thickness 15 mm. Draw the projections of the arrangement when one of the sides of the base of the slab is inclined at 45° to the V.P.

11.11 A hexagonal prism of base side 30 mm and axis 50 mm is resting on its base on the H.P. with a side of base perpendicular to the V.P. A tetrahedron is placed on the top of the prism such that the three corners of the tetrahedron coincide with the alternate corners of the prism. Draw the projections of the arrangement.

Axis perpendicular to V.P.

11.12 Draw the projections of a square prism, base side 40 mm and axis 65 mm, having its base in the V.P. such that (a) a rectangular face is parallel to the H.P., (b) two faces are inclined at 30° to the H.P., and (c) all faces are equally inclined to the H.P.

11.13 Draw the projections of a hexagonal pyramid, base side 30 mm and axis 60 mm, resting on its base on the V.P. with a side of base (a) parallel to the H.P., (b) perpendicular to the H.P., and (c) inclined at 45° to the H.P.

11.14 A cylinder of base diameter 50 mm and axis 65 mm has its axis 40 mm above H.P. and perpendicular to V.P. Draw its projections when one of the bases being 10 mm in front of the V.P.

11.15 A cone of base diameter 50 mm and axis 65 mm has its apex in the V.P. Its axis is parallel to and 40 mm above the H.P. Draw its projections.

11.16 A tetrahedron of edge 60 mm is resting on one face on the V.P. such that one of the edges is parallel to and 20 mm above H.P. Draw its projections.

11.17 Draw the projections of a triangular prism of base edge 50 mm and axis 65 mm is placed on one of its rectangular faces on the H.P. such that the axis is perpendicular to the V.P.

11.18 A cube of edge 40 mm is resting on one of its edges on the H.P. with the faces containing the resting edge equally inclined to H.P. and the two vertical faces parallel to the V.P. Draw its projections.

11.19 A pentagonal pyramid of base side 25 mm and axis 60 mm rests on an edge of the base on the H.P. with axis perpendicular to V.P. Draw its projections when the base is 15 mm in front of V.P.

11.20 A circular disc of diameter 60 mm and thickness 20 mm is resting on its base on the H.P. A hexagonal prism of side 25 mm and axis 80 mm is surmounted centrally on the disc such that a face of the prism lies on the top of the disc and the axis of the prism is perpendicular to the V.P. Draw the projections of the arrangement.

Axis parallel to both H.P. and V.P.

11.21. A hexagonal prism of base side 30 mm and axis 70 mm, rests on a rectangular face on the H.P. such that the axis is parallel to and 45 mm in front of the V.P. Draw its projections.

11.9 INITIAL POSITION OF THE SOLID

Following are the general rules to decide the initial position of the solid when it is inclined to one or both the reference planes:

S. No.	*Required final position of the solid*	*Assume initial position of the solid*
1.(a)	Axis inclined to H.P. and parallel to the V.P. with an edge of the base in the H.P.	Base in H.P. with an edge of the base perpendicular to the reference line.
(b)	Axis inclined to H.P. and parallel to the V.P. with a corner of the base in the H.P.	Base in H.P. keeping the line joining the corner and centre of the base parallel to the reference line.
2.(a)	Axis inclined to V.P. and parallel to the H.P. with an edge of the base in the V.P.	Base in V.P. with an edge of the base perpendicular to the reference line.
(b)	Axis inclined to V.P. and parallel to the H.P. with a corner of the base in V.P.	Base in V.P. keeping the line joining the corner and centre of the base parallel to the reference line.
3.	Pyramid having one of its triangular faces in H.P. and axis parallel to V.P.	Base in H.P. with an edge of the base perpendicular to the reference line.
4.	Pyramid having one of its triangular faces in V.P. and axis parallel to H.P.	Base of the pyramid in V.P. with an edge of the base perpendicular to the reference line.
5.	Pyramid lying on one of its slant edges in H.P. and axis parallel to V.P.	Base in H.P. keeping the line joining the corner and centre of the base parallel to the reference line.
6.	Pyramid lying on one of its slant edges in V.P. and axis parallel to H.P.	Base in V.P. keeping the line joining the corner and centre of the base parallel to the reference line.
7.	Cone lying on one of its generators in H.P. or inclined to the H.P. and parallel to V.P.	Base of the cone in the H.P.
8.	Cone lying on one of its generators in V.P. or inclined to the V.P. and parallel to H.P.	Base of the cone in the V.P.

11.10 IDENTIFY VISIBLE AND HIDDEN LINES

General rule adopted to identify and distinguish the visible and the hidden lines in the orthographic views of the solid are as follows:

1. Outlines of an object are always visible, the outer edges of any view should be shown with continuous lines (i.e., dashed narrow lines should never be used.)
2. The edges or faces in a view that are towards the observer (away from xy) are visible. The corresponding edges or faces in the other view should be drawn using continuous lines.
3. The edges or faces in a view that are away from the observer (towards xy) are not visible. The corresponding edges or faces in the other view should be drawn using dashed narrow lines.
4. Two continuous lines never cross each other inside. Similarly, two hidden lines never cross each other.
5. When two lines representing the edges cross each other, one of them must be dashed narrow line.

These rules are applicable for only single solid. They do not apply for the solids with a hole or combination of solids.

11.11 AXIS INCLINED TO H.P. AND PARALLEL TO V.P.

When the axis of a right solid is inclined to the H.P. and parallel to the V.P., then the projections are drawn in two stages. Consider the following problems.

Problem 11.9 *A pentagonal prism of base edge 30 mm and axis 60 mm rests on an edge of its base in the H.P. Its axis is parallel to V.P. and inclined at 45° to the H.P. Draw its projections.*

Construction Refer to Fig. 11.17.

1. **First stage** Draw a pentagon *abcde* keeping side *cd* perpendicular to *xy*. This represents the top view. Project all the corners and obtain *a′d′*4′1′ to represent the front view.
2. **Second stage** Reproduce the front view of the first stage keeping *c′d′* on *xy* and *c′*3′ inclined at 45° to it. Obtain *a*, *b*, *c*, *d*, *e*, 1, 2, 3, 4 and 5 in the top view as the intersecting points of the projectors from the front view of the second stage with the corresponding locus lines from the top view of the first stage.
3. Join the outlines ab, *b*2, 2-3, 3-4, 4-5, 5*e* and *ae* using continuous lines. The corner 1′ is towards the observer, therefore join 1*a*, 1-2 and 1-5 using continuous lines. The edge *c′d′* is on *xy*, therefore join *cd*, *cb*, *c*3, *de* and *d*4 using dashed narrow lines.

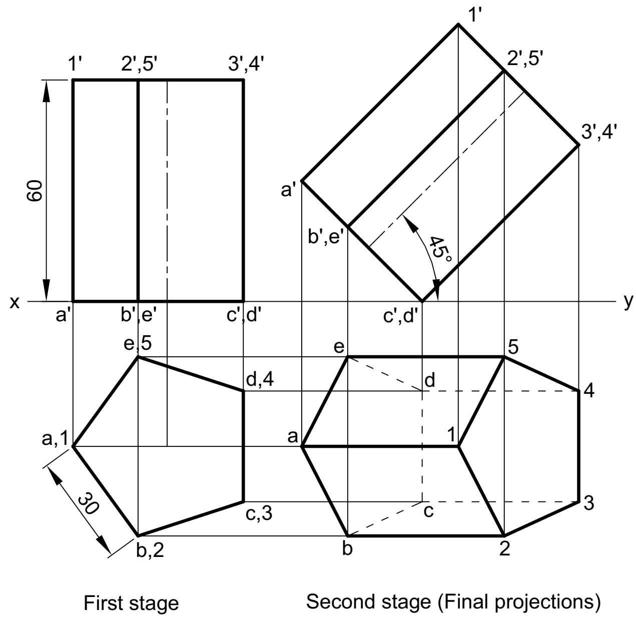


Fig. 11.17

Problem 11.10 *A pentagonal prism of base side 30 mm and axis 70 mm has a corners on the H.P. and the axis is inclined at 45° to the H.P. Draw its projection when the plane containing the resting corner and the axis is parallel to the V.P.*

Construction Refer to Fig. 11.18.

1. **First stage** Draw a pentagon *abcde* keeping side *ab* perpendicular to *xy*. This represents the top view. Project the corners and obtain *a′d′*4′1′ to represent the front view.
2. **Second stage** Reproduce the front view of the first keeping *d′* on *xy* and the axis is inclined at 45° to *xy*. Obtain *a*, *b*, *c*, *d*, *e*, 1, 2, 3, 4 and 5 in the top view as the intersecting points of the projectors from the front view of the second stage with the corresponding locus lines from the top view of the first stage.

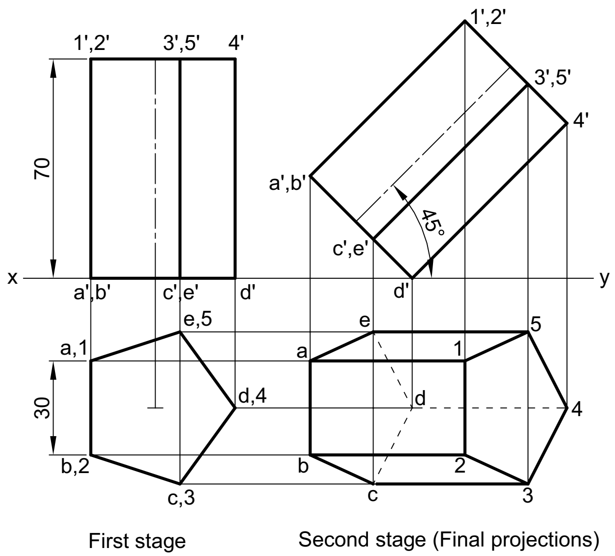


Fig. 11.18

3. Join the outlines *ab*, *bc*, *c*3, 3-4, 4-5, *e*5 and *ae* using continuous lines. Then edge 1′2′ is towards the observer, therefore join 1-2, 1*a*, 1-5, 2*b* and 2-3 using continuous lines. The corner *d′* is on *xy*, therefore join *dc*, *de* and *d*4 using dashed narrow lines.

Problem 11.11 *A hexagonal pyramid of base side 30 mm and axis 60 mm has an edge of its base on the ground. Its axis is inclined at 30° to the ground and parallel to the V.P. Draw its projections.*

Construction Refer to Fig. 11.19.

1. **First stage** Draw a hexagon *abcdef* keeping side de perpendicular to *xy*. Join the corners of the hexagon with the centroid *o*. This represents the top view. Project the corners and obtain *b′d′o′* to represent the front view.

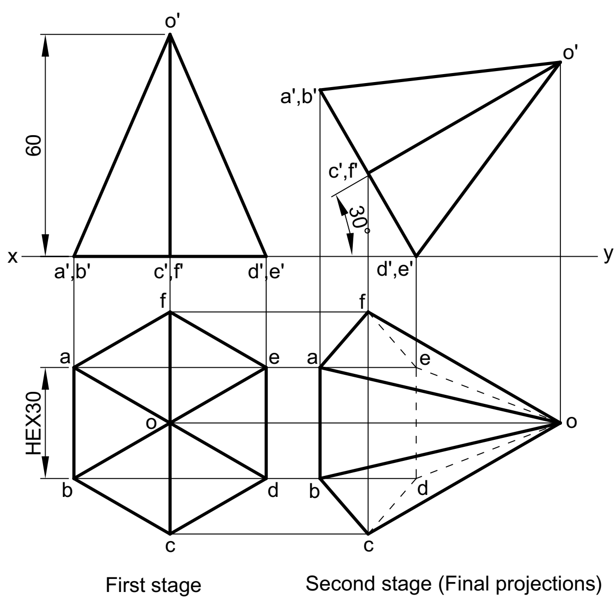


Fig. 11.19

2. **Second stage** Reproduce the front view of first stage keeping *d′e′* on *xy* and *d′a′* at 45° to *xy*. Obtain *a*, *b*, *c*, *d*, *e*, *f*, 1, 2, 3, 4, 5 and 6 in the top view as the intersecting points of the projectors from the front view of the second stage with the corresponding locus lines from the top view of the first stage.
3. Join the outlines *ab*, *bc*, *co*, *fo* and *af* using continuous lines. The face *a′b′o′* is towards the observer, therefore join *ao* and *bo* using continuous lines. The corner/edge *d′e′* is towards *xy*, therefore join *de*, *dc*, *do*, *ef* and *eo* using using dashed narrow lines.

Problem 11.12 *A hexagonal pyramid of base edge 30 mm and axis 60 mm, has a triangular face on the ground and the axis parallel to the V.P. Draw its projections.*

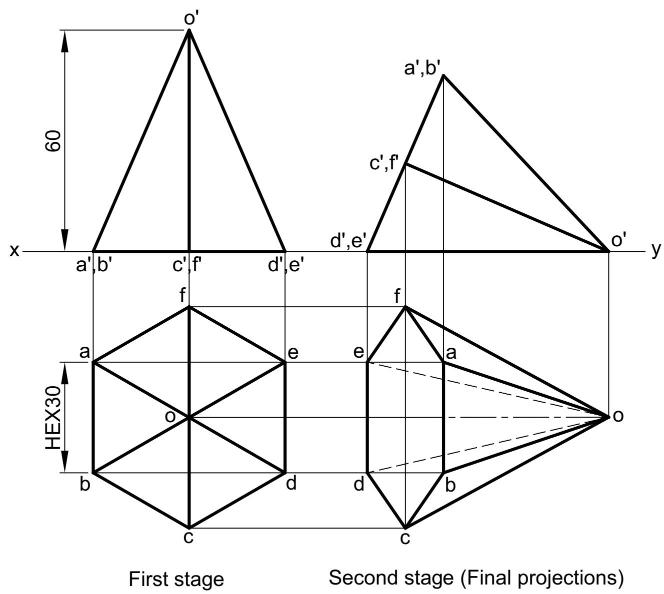


Fig. 11.20

Construction Refer to Fig. 11.20.

1. **First stage** Draw a hexagon *abcdef* keeping side de perpendicular to *xy*. Join the corners of the hexagon with the centroid *o*. This represents the top view. Project all the corners and obtain *b′d′o′* to represent the front view.
2. **Second stage** Reproduce the front view of the first stage keeping line *e′d′o′* on *xy*. Obtain *a*, *b*, *c*, *d*, *e*, *f* and *o* in the top view as the intersecting points of the projectors from the front view of the second stage with the corresponding locus lines from the top view of the first stage.
3. The face *o′d′e′* is on *xy* (away from observer), therefore join *od* and *oe* using dashed narrow lines. Join the remaining edges using continuous lines.

Problem 11.13 *A hexagonal pyramid of base edge 30 mm and axis 60 mm, is lying on a slant edge on the ground with the axis parallel to the V.P. Draw its projections when the face containing the resting edge are equally inclined to the H.P.*

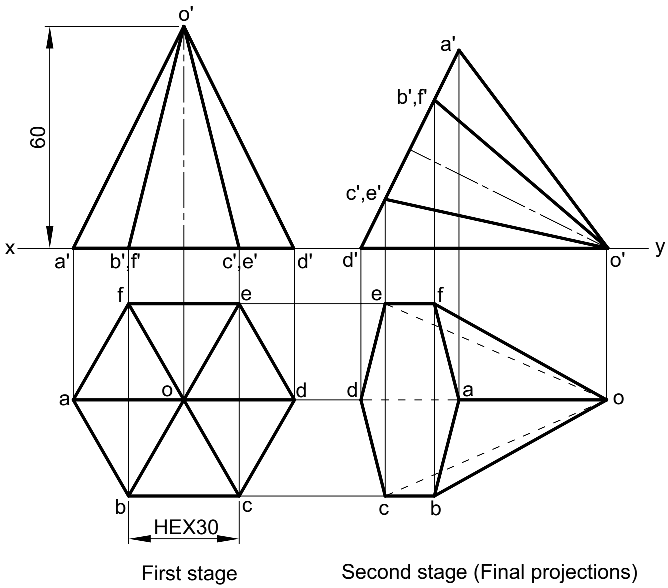


Fig. 11.21

Construction Refer to Fig. 11.21.

1. **First stage** Draw a hexagon *abcdef* keeping diagonal ad parallel to *xy*. Join the corners of the hexagon with the centroid *o*. This represents the top view. Project all the corners and obtain *a′d′o′* to represent the front view.
2. **Second stage** Reproduce the front view of the first stage keeping slant edge *o′d′* on *xy*. Obtain *a*, *b*, *c*, *d*, *e*, *f* and *o* in the top view as the intersecting points of the projectors from the front view of the second stage with the corresponding locus lines from the top view of the first stage.
3. Join the outlines using continuous lines. The corner *a′* is towards observer, therefore join *ao*, *ab* and *af* using continuous lines. Join *oe* and *oc* using dashed narrow lines because they are intersecting the continuous lines *ab* and *af*.

Problem 11.14 *A cylinder of base diameter 50 mm and axis 70 mm has a generator in the V.P. and inclined at 45° to the H.P. Draw its projections.*

Construction Refer to Fig. 11.22.

1. **First stage** Draw a circle *abcdefghijkl* touching *xy* and divide into 12 equal parts to represent the top view. Project the points of the top view and obtain *a′g′*7′1′ to represent the front view.
2. **Second stage** Reproduce the front view of first stage keeping generator *j′*10′ at 45° to the *xy*. Obtain *a*, *b*, *c*, *d*, etc., and 1, 2, 3, 4, etc., in the top view as the intersecting points of the projectors from the front view of the second stage with the corresponding locus lines from the top view of the first stage. Join visible edges with continuous lines and hidden edges with dashed narrow lines.

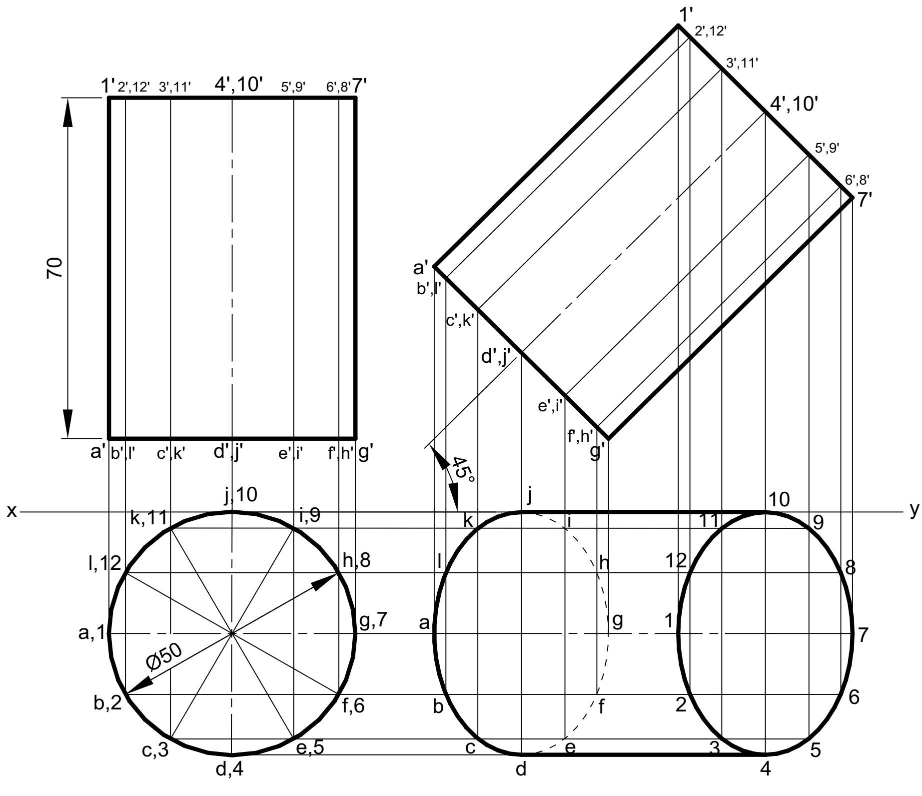


Fig. 11.22

Problem 11.15 *A pentagonal pyramid of base side 30 mm and axis 60 mm has an edge of base parallel to H.P. Its axis is parallel to V.P. and inclined at 45° to the H.P. Draw its projections when the apex lies in the H.P.*

Construction Refer to Fig. 11.23.

1. **First stage** Draw a pentagon *abcde* keeping *cd* perpendicular to *xy*. Join the corners of the pentagon with the centroid *o*. This represents the top view. Project the corners and obtain *a′d′o′* to represent the front view.
2. **Second stage** Reproduce the front view of the first stage keeping apex *o′* on *xy* and the axis inclined at 45° to *xy*. Obtain points *a*, *b*, *c*, *d*, *e* and *o* in the top view as the intersecting points of the projectors from the front view of the second stage with the corresponding locus lines from the top view of the first stage.
3. Join the outlines using continuous lines. The corner *a′* is towards observer, therefore join *ab*, *ae* and *ao* using continuous lines. The face *o′c′d′* is towards *xy*, therefore join *oc* and *od* using dashed narrow lines.

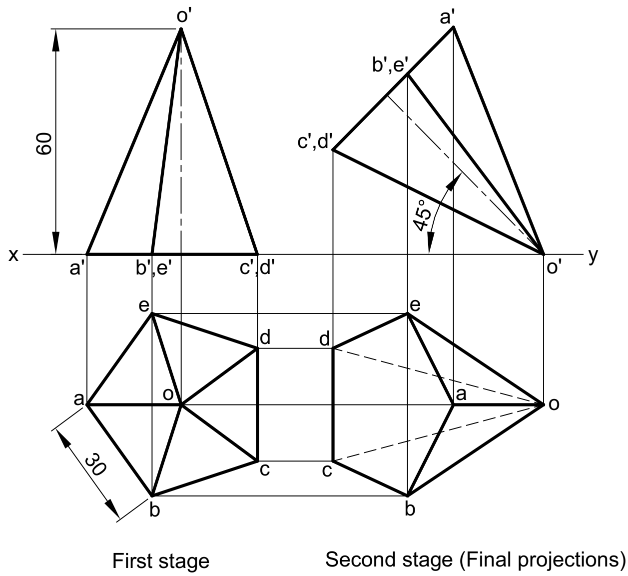


Fig. 11.23

Problem 11.16 *Draw projections of a cube of edge 45 mm, resting on one of its corners in the H.P. with a solid diagonal vertical.*

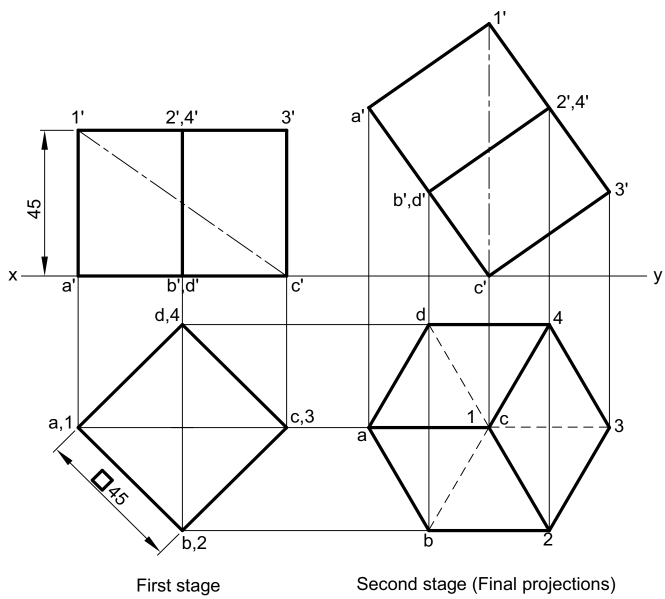


Fig. 11.24

Construction Refer to Fig. 11.24.

1. **First stage** Draw a square *abcd* keeping diagonal ac parallel to *xy* to represent the top view. Project the corners and obtain $a'c'3'1'$ to represent the front view. Mark $c'1'$ as one of the solid diagonals.
2. **Second stage** Reproduce the front view of the first stage keeping c' on *xy* and $c'1'$ perpendicular to *xy*. Obtain *a*, *b*, *c*, *d*, 1, 2, 3 and 4 in the top view as the intersecting points of the projectors from the front view of the second stage with the corresponding locus lines from the top view of the first stage. Join visible edges with continuous lines and hidden edges with dashed narrow lines.

Problem 11.17 *A tetrahedron of edge 70 mm has an edge on the ground and the faces containing that edge are equally inclined to the H.P. Draw its projections when the edge lying on the ground is perpendicular to the V.P.*

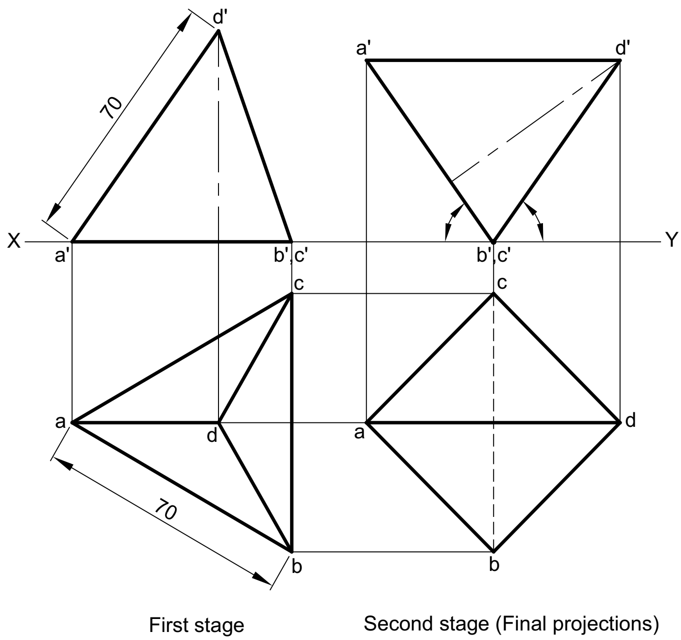


Fig. 11.25

Construction Refer to Fig. 11.25.

1. **First stage** Draw an equilateral triangle *abc* keeping *bc* perpendicular to *xy*. Join the corners of the triangle with the centroid *o*. This represents the top view. Project *a*, *b* and *c* on *xy* and obtain points a', b' and c'. As ad is parallel to *xy*, its front view $a'd'$ is of true length. Therefore, draw an arc with centre d' and radius 70 mm to meet the projector of point *d* at point d'. Join and obtain $a'b'd'$ to represent the front view.
2. **Second stage** Reproduce the front view of the first stage keeping $b'c'$ on *xy* and the inclination of faces $a'b'c'$ and $b'c'd'$ with *xy* equal. Obtain points *a*, *b*, *c* and *d* in the top view as the intersecting points of the projectors from the front view of the second stage with the corresponding locus lines from the top view of the first stage. Join visible edges with continuous lines and hidden edges with dashed narrow lines.

Problem 11.18 *A hexagonal pyramid of base side 25 mm and axis 60 mm is freely suspended from one of the corners of the base. Draw its projections when its axis is parallel to the V.P.*

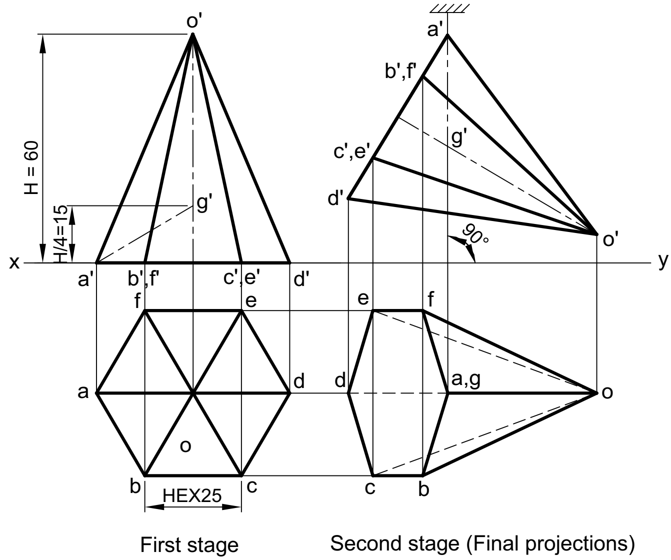


Fig. 11.26

Note

1. Centre of gravity (g') of a pyramid and a cone lies on its axis at a height ¼ of the axis length from the base of the solid.
2. On suspending the solid, line joining the point of suspension and centre of gravity (g') becomes perpendicular to the ground.

Construction Refer to Fig. 11.26.

1. **First stage** Draw a hexagon *abcdef* keeping diagonal ad parallel to *xy*. Join the corners of the hexagon with the centroid *o*. This represents the top view. Project all the corners and obtain $a'd'o'$ to represent the front view. Mark point g' on the axis at a height one-fourth of the axis length (=15 mm) from the base. Let the pyramid is to be hanged from corner *a*. Join $a'g'$.
2. **Second stage** Reproduce the front view of the first stage keeping $a'g'$ perpendicular to *xy*. Obtain *a*, *b*, *c*, *d*, *e*, *f* and *o* in the top view as the intersecting points of the projectors from the front view of the second stage with the corresponding locus lines from the top view of the first stage. Join visible edges with continuous lines and hidden edges with dashed narrow lines.

11.12 AXIS INCLINED TO V.P. AND PARALLEL TO H.P.

When the axis of a right solid is inclined (at ϕ) to the V.P. and parallel to the H.P., then the projections are drawn in two stages. Consider the following problems.

Problem 11.19 *A hexagonal prism of base edge 30 mm and axis 70 mm has an edge of its base in the V.P. such that the axis is inclined at 30° to the V.P. and parallel to the H.P. Draw its projections.*

Construction Refer to Fig. 11.27.

1. **First stage** Draw a hexagon *a′b′c′d′e′f′* keeping side *d′e′* perpendicular to *xy*, to represent the front view. Project the corners and obtain *ad*41 to represent the top view.
2. **Second stage** Reproduce the top view of the first stage keeping *ed* on *xy* and *d*4 inclined at 30° to *xy*. Obtain *a′*, *b′*, *c′*, *d′*, *e′*, *f′*, 1′, 2′, 3′, 4′, 5′ and 6′ in the front view as the intersecting points of the projectors from the top view of the second stage with the corresponding locus lines from the front view of the first stage.
3. Join the outlines *a′b′*, *b′c′*, *c′*3′, 3′4′, 4′5′, 5′6′, 6′*f′* and *a′f′* using continuous lines. The edge 1-2 is towards the observer, therefore join 1′2′, 1′*a′*, 1′6′, 2′*b′* and 2′3′ using continuous lines. The edge *de* is towards *xy*, therefore join *d′e′*, *d′c′*, *d′*4′, *e′f′* and *e′*5′ using dashed narrow lines.

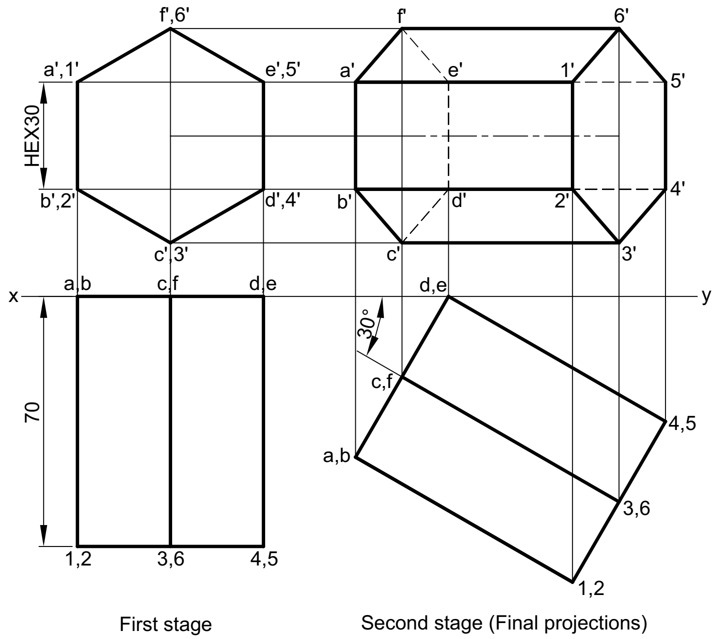


Fig. 11.27

Problem 11.20 *A pentagonal pyramid of base side 30 mm and axis 55 mm has a triangular face in the V.P. and the base edge contained by that triangular face is perpendicular to the H.P. Draw its projections.*

Construction Refer to Fig. 11.28.

1. **First stage** Draw a pentagon *a′b′c′d′e′* keeping *c′d′* perpendicular to *xy*. Join the corners of the pentagon with the centroid *o′*. This represents the front view. Project all the corners and obtain ado to represent the top view.
2. **Second stage** Reproduce the top view of the first stage keeping *cdo* on *xy*. Obtain *a′*, *b′*, *c′*, *d′*, *e′* and *o′* in the front view as the intersecting points of the projectors from the top view of the second stage with the corresponding locus lines from the front view of the first stage.
3. Join the outlines *b′c′*, *c′d′*, *d′e′*, *e′o′* and *o′b′* using continuous lines. The corner a is towards the observer, therefore join *a′b′*, *a′e′* and *a′o′* using continuous lines. The face *cdo* is on *xy*, therefore join *c′o′* and *d′o′* using dashed narrow lines.

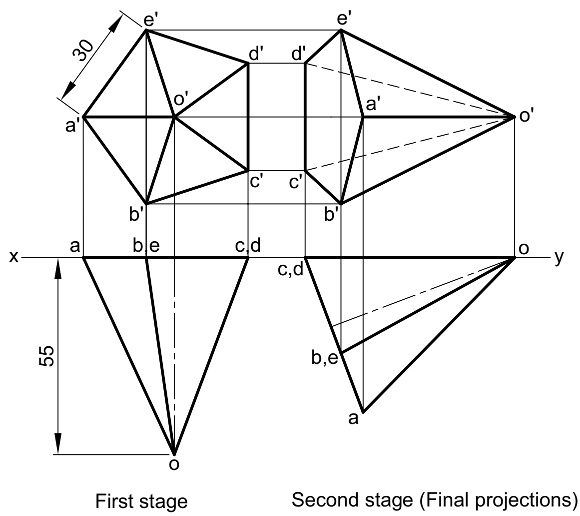


Fig. 11.28

Problem 11.21 *A cone of base diameter 50 mm and axis 60 mm has a generator in the V.P. and the axis parallel to the H.P. Draw its projections.*

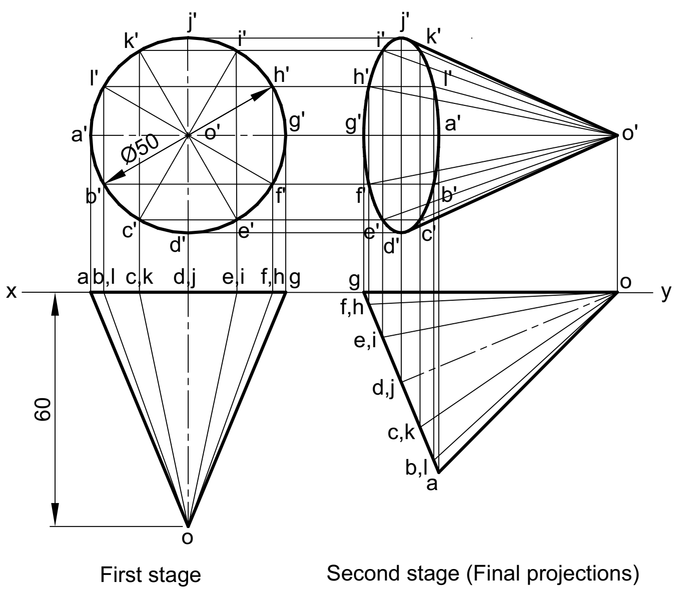


Fig. 11.29

Construction Refer to Fig. 11.29.

1. **First stage** Draw a circle *adgj* and divide into 12 equal parts to represent the front view. Project the front view and obtain *aog* to represent the top view.
2. **Second stage** Reproduce the top view of the first stage keeping generator *og* on *xy*. Obtain points a', b', c', d', e', f', g', h', i', j', k', l' and o' in the front view as the intersecting points of the projectors from the top view of the second stage with the corresponding locus lines from the front view of the first stage. Join visible edges with continuous lines.

Problem 11.22 *A pentagonal prism of base side 30 mm and axis 60 mm has one of its rectangular faces on the H.P. and the axis inclined at 60° to the V.P. Draw its projections.*

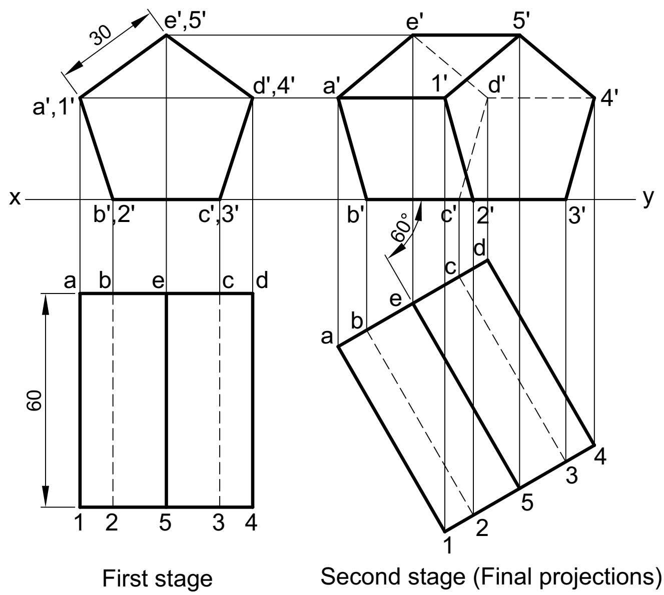


Fig. 11.30

Construction Refer to Fig. 11.30.

1. **First stage** Draw a pentagon $a'b'c'd'e'$ keeping $b'c'$ on *xy* to represent the front view. Project the corners and obtain *ad*41 to represent the top view.
2. **Second stage** Reproduce the top view of the first stage keeping *e*5 inclined at 60° to *xy*. Obtain a', b', c', d', e', $1'$, $2'$, $3'$, $4'$ and $5'$ in the front view as the intersecting points of the projectors from the top view of the second stage with the corresponding locus lines from the front view of the first stage.
3 Join the outlines using continuous lines. The corner 1 is towards the observer, therefore join $1'a'$, $1'2'$ and $1'5'$ using continuous lines. The corner d is towards *xy*, therefore join $d'c'$, $d'e'$ and $d'4'$ using dashed narrow lines.

11.13 MISCELLANEOUS PROBLEMS

Problem 11.23 *Draw three views of the frustum of a hexagonal pyramid of base edge 35 mm, top edge 20 mm and axis 45 mm, has one of its faces on the H.P. and axis parallel to V.P.*

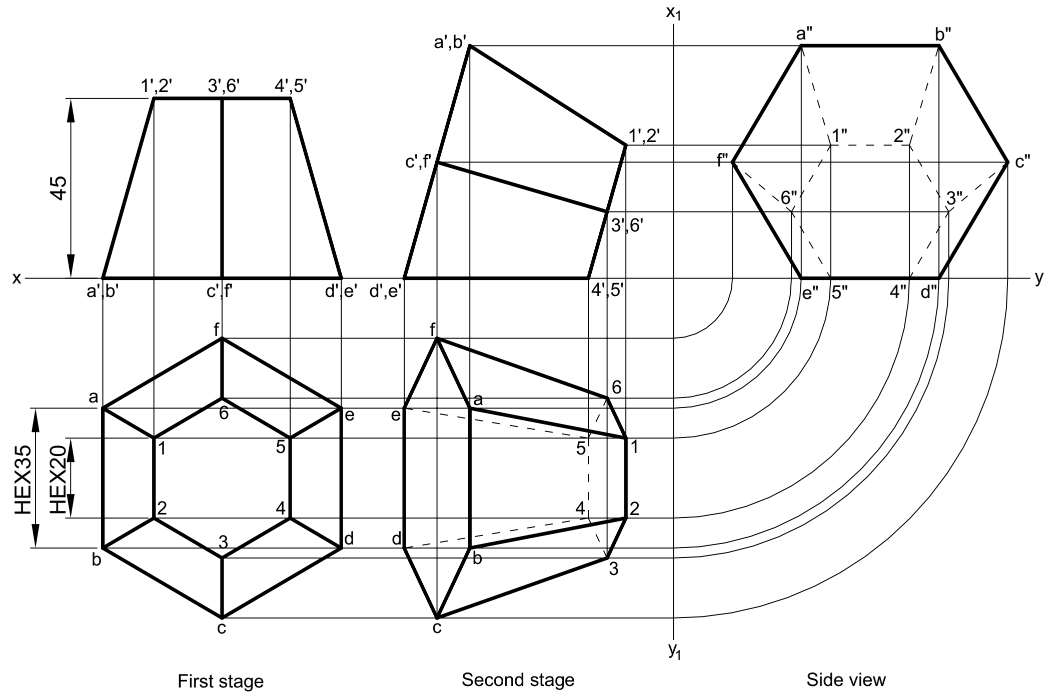
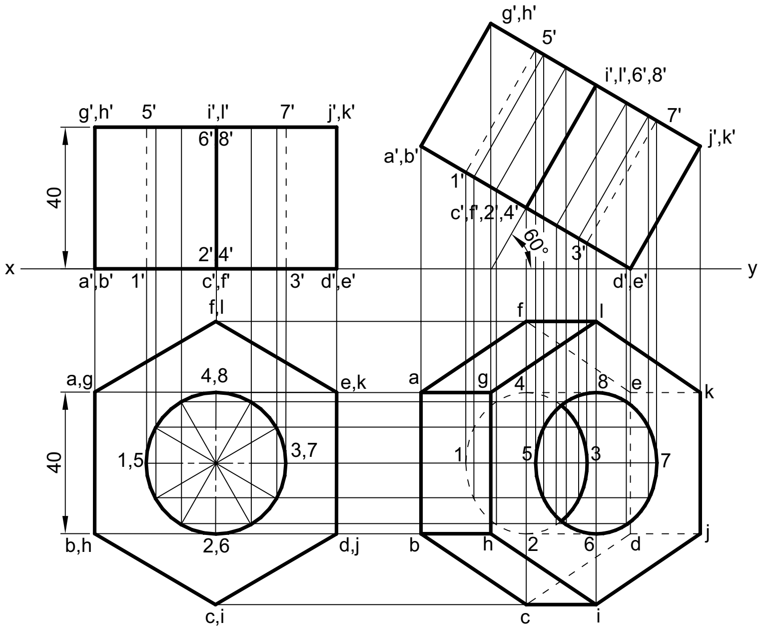


Fig. 11.31

Construction Refer to Fig. 11.31.

1. **First stage** Draw two concentric hexagons *abcdef* and 123456 with sides *de* and 45 perpendicular to *xy*. Join 1*a*, 2*b*, 3*c*, 4*d*, 5*e* and 6*f*. This represents the top view. Project all the corners and obtain *a′d′*4′1′ to represent the front view.
2. **Second stage** Reproduce the front view of the first stage keeping *d′e′*5′4′ on *xy*. Obtain *a*, *b*, *c*, *d*, *e*, *f*, 1, 2, 3, 4, 5 and 6 in the top view as the intersecting points of the projectors from the front view of the second stage with the corresponding locus lines from the top view of the first stage. Join visible edges with continuous lines and hidden edges with dashed narrow lines.
3. **Side view** Draw lines parallel to *xy* from the top view to meet x_1y_1. Thereafter, rotate them through 90° and project vertically to meet the horizontal projectors from the front view at points *a″*, *b″*, *c″*, *d″*, *e″*, 1″, 2″, 3″, 4″ and 5″. Join the points as shown. This is the required side view.

Problem 11.24 *A hexagonal prism, base side 40 mm and axis 40 mm has a centrally drilled circular hole of diameter 40 mm. Draw its projections when the prism is resting on an edge of its base on the H.P. and the axis inclined at 60° to the H.P. and parallel to the V.P.*



First stage Second stage (Final projections)

Fig. 11.32

Construction Refer to Fig. 11.32.

1. **First stage** Draw a hexagon *abcdef* keeping *de* perpendicular to *xy*. Draw a circle 1234 concentric to the hexagon to represent the hole. This represents the top view. Project all the corners and obtain $a'd'j'g'$ to represent the front view.
2. **Second stage** Reproduce the front view of the first stage keeping $d'e'$ on *xy* and line $d'j'$ is inclined at 60° to *xy*. Obtain *a*, *b*, *c*, etc., and 1, 2, 3, etc., in the top view as the intersecting points of the projectors from the front view of the second stage with the corresponding locus lines from the top view of the first stage. Join visible edges with continuous lines and hidden edges with dashed narrow lines.

Problem 11.25 *A hexagonal pyramid of base side 25 mm and height 50 mm is resting centrally on the flat surface of a cylindrical disc of base diameter 60 mm and thickness 20 mm, such that one of the edges of the base of the pyramid is parallel to the V.P. Draw the projections of the combined solid when their common axis is inclined at 45° to the H.P. and parallel to the V.P.*

Construction Refer to Fig. 11.33.

1. **First stage** Draw a circle of diameter 60 mm with centre *o*. Draw hexagon *abcdef* concentric with the circle keeping side *ef* parallel to *xy*. This represents the top view. Project the corners and obtain $1'7'19'o'13'1'$ to represent the front view.
2. **Second stage** Reproduce the front view of first stage keeping $7'$ is on *xy* and $7'1'$ at 45° to *xy*. Obtain *a*, *b*, *c*, etc., and 1, 2, 3 etc., in the top view as the intersecting points of the projectors from the front view of the second stage with the corresponding locus lines from the top view of the first stage. Join visible edges with continuous lines and hidden edges with dashed narrow lines.

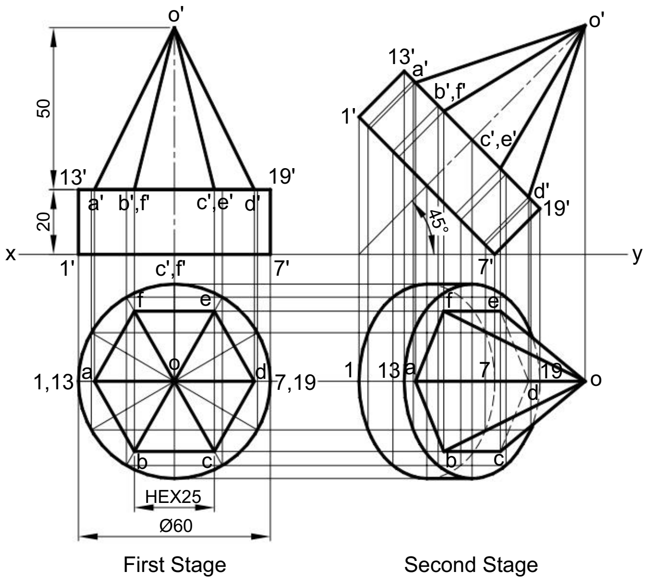


Fig. 11.33

EXERCISE 11B

Axis inclined to H.P. and parallel to V.P.

11.1 A square prism of base edge 35 mm and axis 60 mm is resting on an edge of its base such that the axis is parallel to V.P. and inclined at 45° to the H.P. Draw its projections.

11.2 A pentagonal pyramid of base edge 30 mm and axis 60 mm is resting on a corner of its base on the H.P. Draw its projections when its axis is parallel to the V.P. and inclined at 60° to the H.P.

11.3 A cone of base diameter 50 mm and height 60 mm is resting on a point of its base circle on the H.P. with its axis parallel to the V.P. and inclined at 45° to the H.P. Draw its projections.

11.4 A pentagonal prism, base side 30 mm and axis 80 mm, stands on a corner of its base on the H.P. with the longer edge through that corner inclined at 45° to the H.P. and the face opposite to this slant edge is perpendicular to the V.P. Draw its projections.

11.5 A square pyramid of base edge 40 mm and axis 60 mm is resting on a triangular face on the V.P. Draw its projections when the axis is parallel to and 30 mm above the H.P.

11.6 A hexagonal pyramid of base side 30 mm and axis 60 mm is lying on one of its triangular faces in the H.P. Draw its three views.

11.7 A cone of base diameter 50 mm and axis 65 mm is lying on one of its generators on the H.P. Draw its projections.

11.8 Draw three views of a square pyramid of base side 40 mm and axis 60 mm, which is resting on a corner of its base on the H.P. and the slant edge containing that corner is vertical.

11.9 A cylinder of base diameter 50 mm and axis 70 mm is resting on a point of its base circle on H.P. Draw its projections when its generators are inclined at 45° to the H.P.

11.10 A pentagonal prism of base side 25 mm and axis 60 mm is resting on one of its sides of the base on the H.P. and the plane of the base is inclined at 60° to the H.P. Draw its projections.

11.11 A pentagonal pyramid of base side 30 mm and axis 60 mm is resting on a corner of its base on the H.P. such that the highest edge of the base is 25 mm above the H.P. Draw its projections.

11.12 A pentagonal prism of base side 30 mm and axis 70 mm is resting on a corner of its base on the H.P. such that the highest edge of the prism is parallel to and 80 mm above H.P. Draw its projections.

11.13 A cone of base diameter 50 mm and axis 70 mm is resting on a point of its base circle on the H.P. and the highest point of the base circle is 25 mm above the H.P. and 40 mm in front of the V.P. Draw its projections.

11.14 A pentagonal prism, of base side 30 mm and axis 70 mm is resting on one of its rectangular faces in the V.P. with axis inclined at 60° to the H.P. Draw its projections.

11.15 A frustum of a pentagonal pyramid has base edge 40 mm, top edge 25 mm and height 60 mm is resting on an edge of its base on the H.P. Draw its projections when the axis is inclined at 45° to the H.P. and parallel to the V.P.

11.16 A cube of edge 40 mm is resting on a corner on the H.P. and one of its solid diagonal is parallel to both the reference planes. Draw its three views.

11.17 A tetrahedron of side 60 mm is resting on an side on the H.P. such that a face perpendicular to V.P. is inclined at 45° to the H.P. Draw its projections.

11.18 An octahedron of edge 40 mm is resting on a face on the H.P. and the axis is parallel to the V.P. Draw its projections.

11.19 A square slab of side 60 mm and thickness 25 mm has a coaxial hole of 40 mm diameter. Draw its projections when it is resting on a corner in the H.P. and the axis is inclined at 30° to the H.P.

11.20 A pentagonal pyramid of base side 30 mm and height 70 mm is resting axially on the flat surface of a cylindrical block of base diameter 60 mm and thickness 20 mm, such that an edge of the base of the pyramid is parallel to the V.P. Draw the projections of the solids when the axis is inclined at 45° to H.P. and parallel to the V.P.

Axis inclined to V.P. and parallel to H.P.

11.21 A hexagonal pyramid, base side 30 mm and axis 60 mm, rests on an edge of its base on the V.P. with a triangular face containing that edge inclined at 45° to the V.P. and perpendicular to the H.P. Draw its projections.

11.22 A hexagonal prism of base side 30 mm and axis 70 mm is resting on a rectangular face on the H.P. and a longer edge is making 60° with the V.P. Draw its projections.

11.23 A cone of base diameter 50 mm and axis 60 mm has its axis parallel to H.P. and inclined at 30° to the V.P. Draw its projections.

11.24 A triangular prism of base side 40 mm and axis 65 mm is resting on a rectangular face on the H.P. Draw its projections when its axis is inclined at 30° to the V.P.

11.25 A cylinder of base diameter 50 mm and axis 65 mm is resting on one of its generators on the H.P. Draw its projections when the axis is inclined at 30° to the V.P.

11.26 A hexagonal pyramid of base side 30 mm and axis 60 mm has an edge of its base in the V.P. and the apex is 50 mm away from both the reference planes. Draw its projections when the axis is parallel to the H.P.

11.27 A cube of edge 40 mm is resting on a corner on the V.P. and the solid diagonal through that corner is perpendicular to the V.P. Draw its projections.

11.28 Draw the projections of a tetrahedron of edge 60 mm having an edge in the V.P. and another edge parallel to both the reference planes.

11.14 AXIS INCLINED TO BOTH H.P. AND V.P.

When the axis of a solid is inclined to both the reference planes, the projections are drawn in three stages. It is an extension of the problems on projections of solids inclined to one of the reference planes.

11.14.1 An Element of the Solid in the H.P.

Problem 11.26 *A square prism of base edge 35 mm and axis 60 mm is resting on an edge of its base on the H.P. and the axis inclined at 45° to the H.P. If the edge resting on the H.P. is inclined at 30° to the V.P., draw its projections.*

Construction Refer to Fig. 11.34.

1. **First stage** Draw a square *abcd* keeping *cd* perpendicular to *xy*, to represent the top view. Project the corners and obtain *b′c′*3′2′ as the front view.
2. **Second stage** Redraw the front view of first stage keeping *c′d′* on *xy* and *c′d′*3′4′ inclined at 45° to *xy*. Obtain points *a*, *b*, *c*, *d*, 1, 2, 3 and 4 in the top view as the intersecting points of the projectors from the front view of the second stage with the corresponding locus lines from the top view of the first stage. Join the points and obtain *ab*34 as the top view. (The edge *c′d′* is on *xy*, therefore join *cd* using dashed narrow lines.)
3. **Third stage** Reproduce the top view of the second stage keeping *b*3 and *a*4 inclined at 60° to *xy*. Thus, the edge *cd* is inclined at 30° to the V.P. Obtain *a′*, *b′*, *c′*, *d′*, 1′, 2′, 3′ and 4′ in the front view as the intersecting points of the projectors from the top view of the third stage with the corresponding locus lines from the front view of the second stage. Join the points and obtain *b′c′d′*4′1′2′ as the required front view. (The corner *a* is towards *xy*, therefore join *a′b′*, *a′d′* and *a′*1′ using dashed narrow lines.)

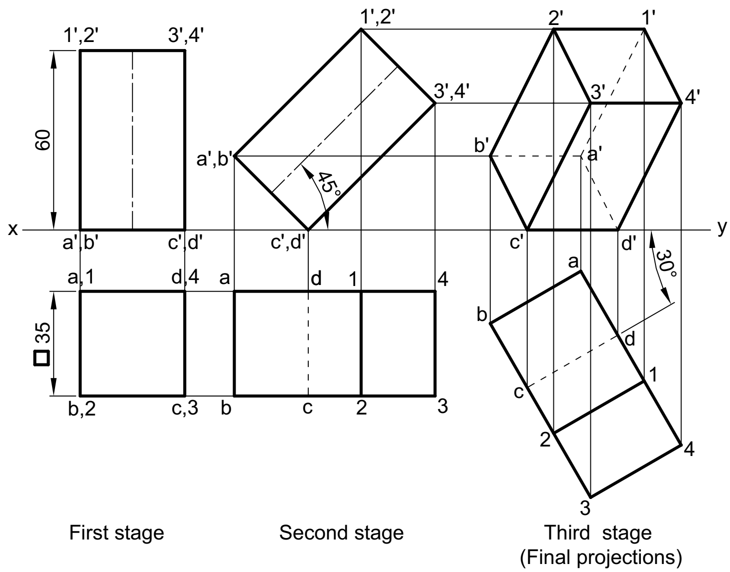


Fig. 11.34

Problem 11.27 *A pentagonal prism of base side 30 mm and height 60 mm rests on one of its base side on the H.P. inclined at 30° to the V.P. Its axis is inclined at 45° to the H.P. Draw its projections.*

Construction Refer to Fig. 11.35.

1. **First stage** Draw a pentagon *abcde* keeping *cd* perpendicular to *xy* to represent the top view. Project the corners and obtain *a′c′*3′1′ as the front view.
2. **Second stage** Redraw the front view of first stage keeping *c′d′* on *xy* and *c′d′*3′4′ inclined at 45° to *xy*. Obtain *a*, *b*, *c*, *d*, *e*, 1, 2, 3, 4 and 5 in the top view as the intersecting points of the projectors from the front view of the second stage with the corresponding locus lines from the top view of the first stage. Join the points and obtain *ab*2345*e* as the top view. (The edge *cd* is on *xy*, therefore join *cd*, *cb*, *c*3, *de* and *d*4 using dashed narrow lines.)
3. **Third stage** Reproduce the top view of the second stage keeping *cd* inclined at 30° to *xy*. Obtain *a′*, *b′*, *c′*, *d′*, *e′*, 1′, 2′, 3′, 4′ and 5′ in the front view as the intersecting points of the projectors from the top view of the third stage with the corresponding locus lines from the front view of the second stage. Join the points and obtain *b′c′d′*4′5′1′2′ as the required front view. (The edge *ae* is towards *xy*, therefore join *a′e′*, *a′b′*, *a′*1′, *e′d′* and *e′*5′ using dashed narrow lines.)

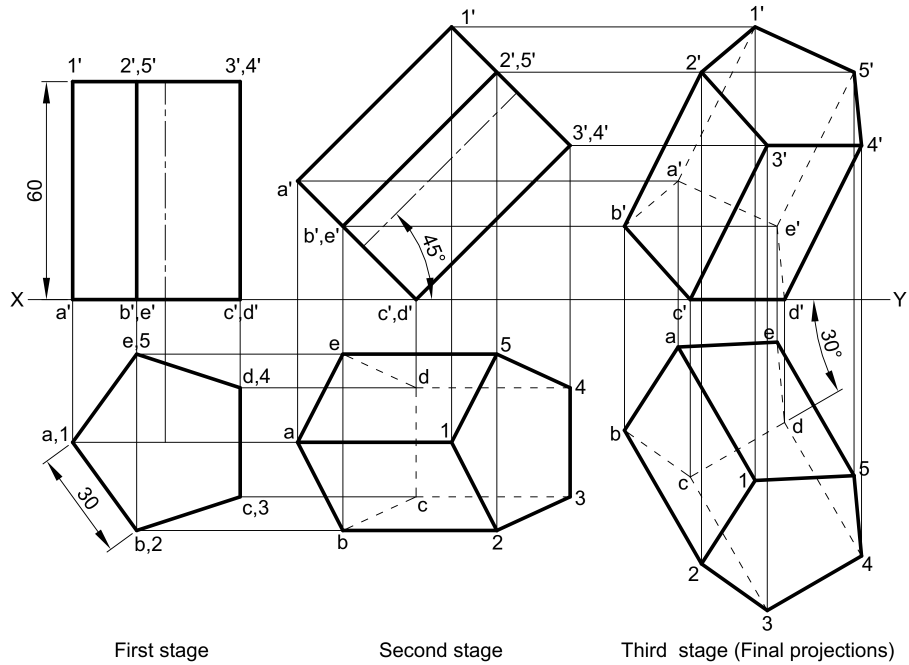


Fig. 11.35

Problem 11.28 *A square pyramid of base side 40 mm and axis 55 mm is resting on one of its triangular faces on the H.P. A vertical plane containing the axis is inclined at 45° to the V.P. Draw its projections.*

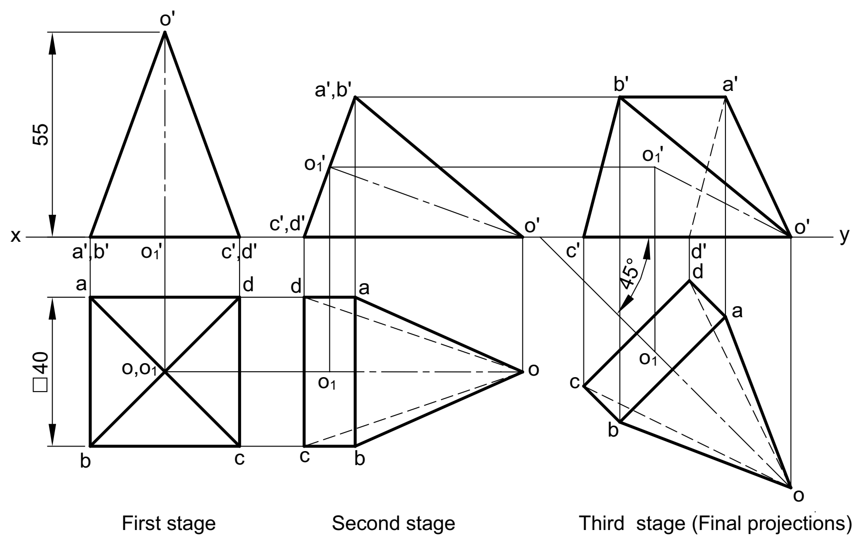


Fig. 11.36

Construction Refer to Fig. 11.36.

1. **First stage** Draw a square *abcd* keeping *cd* perpendicular to *xy*. Join the corners of the square with the centroid *o*. This represents the top view. Project the corners and obtain *b′c′o′* as the front view.
2. **Second stage** Reproduce the front view of the first stage keeping *c′d′o′* on *x*y. Obtain *a*, *b*, *c*, *d* and *o* in the top view as the intersecting points of the projectors from the front view of the second stage with the corresponding locus lines from the top view of the first stage. Join the points and obtain *adcbo* as the top view.
3. **Third stage** Reproduce the top view of the second stage keeping vertical plane containing the axis oo_1 inclined at 45° to *xy*. Obtain *a′*, *b′*, *c′*, *d′* and *o′* in the front view as the intersecting points of the projectors from the top view of the third stage with the corresponding locus lines from the front view of the second stage. Join the points and obtain *a′b′c′d′o′* as the required front view.

Problem 11.29 *A hexagonal pyramid of base side 30 mm and axis 60 mm, has an edge of its base on the ground inclined at 45° to the V.P. and the axis is inclined at 30° to the H.P. Draw its projections.*

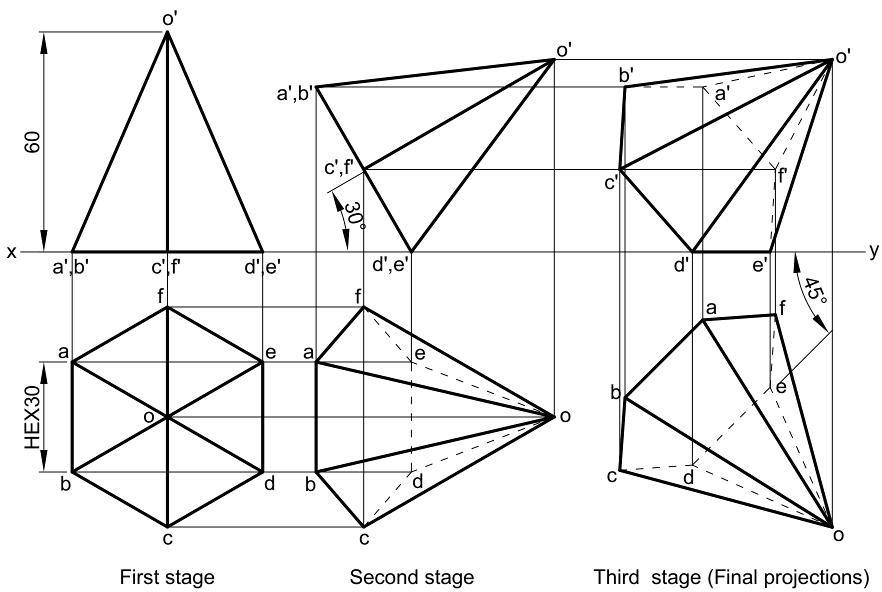


Fig. 11.37

Construction Refer to Fig. 11.37.

1. **First stage** Draw a hexagon *abcdef* keeping de perpendicular to *xy*. Join the corners of the hexagon with the centroid *o*. This represents the top view. Project the corners and obtain *b′d′o′* as the front view.
2. **Second stage** Reproduce the front view of the first stage keeping *d′e′* on *xy* and *b′d′* inclined at 60° to *xy*. Thus, the axis is inclined at 30° to *xy*. Obtain *a*, *b*, *c*, *d*, *e*, *f* and *o* in the top view as the intersecting points of the projectors from the front view of the second stage with the corresponding locus lines from the top view of the first stage. Join the points and obtain *abcof* as the top view. (The edge *d′e′* is on *xy*, therefore join *de*, *dc*, *do*, *ef* and *eo* using dashed narrow lines.)

3. **Third stage** Reproduce the top view of the second stage keeping de inclined at 45° to *xy*. Obtain *a*′, *b*′, *c*′, *d*′, *e*′, *f*′ and *o*′ in the front view as the intersecting points of the projectors from the top view of the third stage with the corresponding locus lines from the front view of the second stage. Join the points and obtain *b*′*c*′*d*′*e*′*o*′ as the required front view. (The edge *af* is towards *xy*, therefore join *a*′*f*′, *a*′*b*′, *a*′*o*′, *f*′*e*′ and *f*′*o*′ using dashed narrow lines.)

Problem 11.30 *A hexagonal pyramid of base side 30 mm and axis 50 mm, rests on one of its base corners on the ground with axis inclined at 45° to the H.P. Draw its projections when a vertical plane containing the axis and the corner that lies in the H.P. is inclined at 30° to the V.P.*

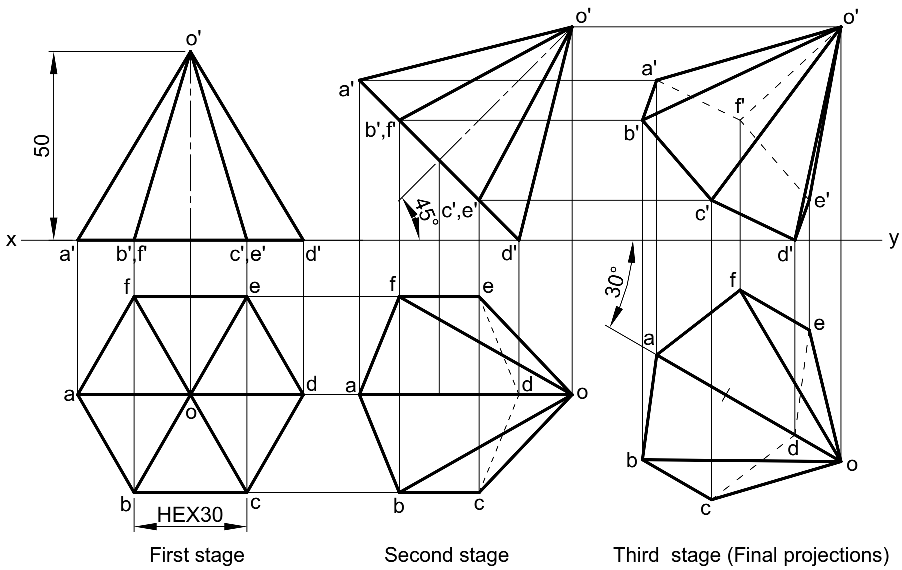


Fig. 11.38

Construction Refer to Fig. 11.38.

1. **First stage** Draw a hexagon *abcdef* keeping ad parallel to *xy*. Join the corners of the hexagon with the centroid *o*. This represents the top view. Project the corners and obtain *a*′*d*′*o*′ as the front view.
2. **Second stage** Reproduce the front view of first stage keeping *d*′ on *xy* and *a*′*d*′ inclined at 45° to *xy*. Obtain *a*, *b*, *c*, *d*, *e*, *f* and *o* in the top view as the intersecting points of the projectors from the front view of the second stage with the corresponding locus lines from the top view of the first stage. Join the points and obtain *abcoef* as the top view.
3. **Third stage** Reproduce the top view of the second stage keeping *ao* inclined at 30° to *xy*. Obtain *a*′, *b*′, *c*′, *d*′, *e*′, *f*′ and *o*′ in the front view as the intersecting points of the projectors from the top view of the third stage with the corresponding locus lines from the front view of the second stage. Join the points and obtain *a*′*b*′*c*′*d*′*e*′*o*′ as the required front view.

Problem 11.31 *A hexagonal pyramid of base side 30 mm and axis 60 mm has one of its slant edges on the H.P. and inclined at 45° to the V.P. Draw its projections when the base is visible.*

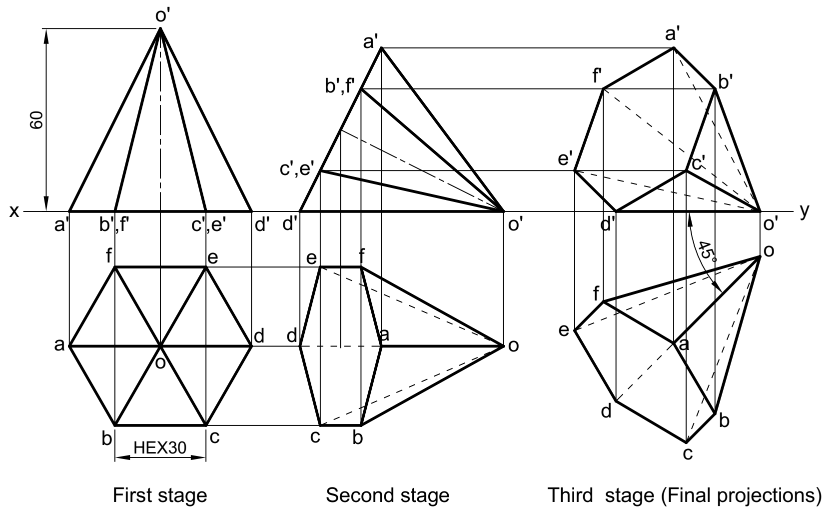


Fig. 11.39

Construction Refer to Fig. 11.39.

1. **First stage** Draw a hexagon *abcdef* keeping ad parallel to *xy*. Join the corners of the hexagon with the centroid *o*. This represents the top view. Project the corners and obtain *a′d′o′* as the front view.
2. **Second stage** Reproduce the front view of the first stage keeping *d′o′* on *xy*. Obtain *a*, *b*, *c*, *d*, *e*, *f* and *o* in the top view as the intersecting points of the projectors from the front view of the second stage with the corresponding locus lines from the top view of the first stage. Join the points and obtain *bcdefo* as the top view. (Join the outlines using continuous lines. The corner *a′* is towards observer, therefore join *a′o′*, *a′b′* and *a′f′* using continuous lines.)
3. **Third stage** Reproduce the top view of the second stage keeping *oad* inclined at 45° to *xy*. The base *abcdef* should be farther to the *x*y than the apex *o*, so that the base is visible. Obtain *a′*, *b′*, *c′*, *d′*, *e′*, *f′* and *o′* in the front view as the intersecting points of the projectors from the top view of the third stage with the corresponding locus lines from the front view of the second stage. Join the points and obtain *b′a′f′e′d′o′* as the required front view. (Join the outlines and the edges of the base using continuous lines. The corner *c* is towards observer, therefore join *c*'*o′* using continuous lines.)

Problem 11.32 *A hexagonal pyramid of base side 30 mm and axis 60 mm rests on an edge of base on the H.P. with the triangular face containing that edge perpendicular to the H.P. and parallel to the V.P. Draw its projections so that the base is visible.*

Construction Refer to Fig. 11.40.

1. **First stage** Draw a hexagon *abcdef* keeping de perpendicular to *xy*. Join the corners of the hexagon with the centroid *o*. This represents the top view. Project the corners and obtain *b′d′o′* as the front view.

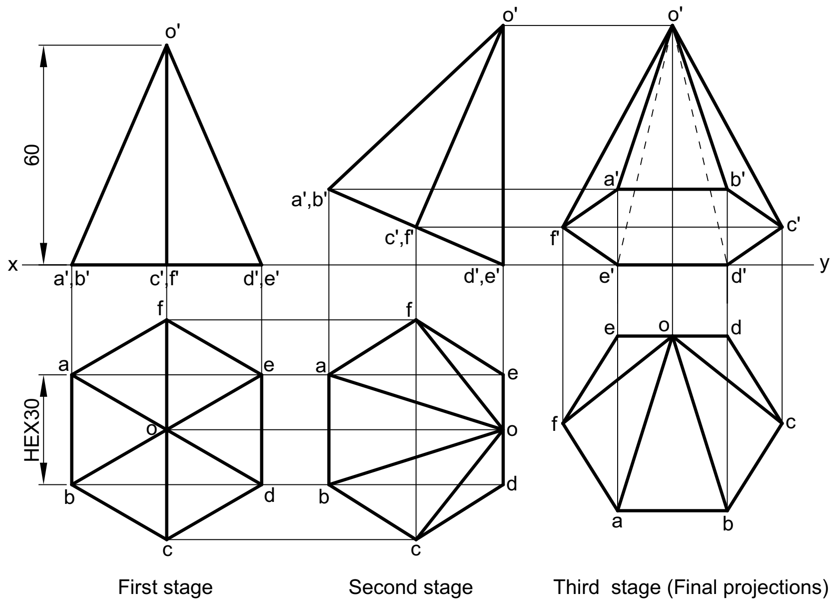


Fig. 11.40

2. **Second stage** Reproduce the front view of the first stage keeping *d′e′* on *xy* and *d′e′o′* perpendicular to *xy*. Obtain *a*, *b*, *c*, *d*, *e*, *f* and *o* in the top view as the intersecting points of the projectors from the front view of the second stage with the corresponding locus lines from the top view of the first stage. Join the points and obtain *abcdoef* as the top view.
3. **Third stage** Reproduce the top view of the second stage keeping de parallel to *xy*. The base abcdef should be farther to *xy* than the apex *o*, so that the base is visible. Obtain *a′*, *b′*, *c′*, *d′*, *e′*, *f′* and *o′* in the front view as the intersecting points of the projectors from the top view of the third stage with the corresponding locus lines from the front view of the second stage. Join the points and obtain *c′d′e′f′o′* as the required front view. (Join the outlines and the edges of the base using continuous lines. As the lines *e′o′* and *d′o′* intersect the base edges, join them using dashed narrow lines.)

Problem 11.33 *A pentagonal pyramid of base side 30 mm and axis 60 mm rests on a corner of its base on the H.P. such that its apex is 55 mm above the ground. A vertical plane containing the corner of the base that lies on the H.P. and the axis is inclined at 30° to the V.P. Draw its projections.*

Construction Refer to Fig. 11.41.

1. **First stage** Draw a pentagon *abcde* keeping *ab* perpendicular to *xy*. Join the corners of the pentagon with the centroid *o*. This represents the top view. The line *od* is parallel to *xy*. Project the corners and obtain *b′d′o′* as the front view.
2. **Second stage** Reproduce the front view of the first stage keeping *d′* on *xy* and *o′* 55 mm above *xy* (For this, mark *d′* on *xy*, then draw an arc with centre *d′* and radius *d′o′* to meet a horizontal line 55 mm above *xy*). Obtain *a*, *b*, *c*, *d*, *e* and *o* in the top view as the intersecting points of the projectors from the front view of the second stage with the corresponding locus lines from the top view of the first stage. Join the points and obtain *abcoe* as the top view.

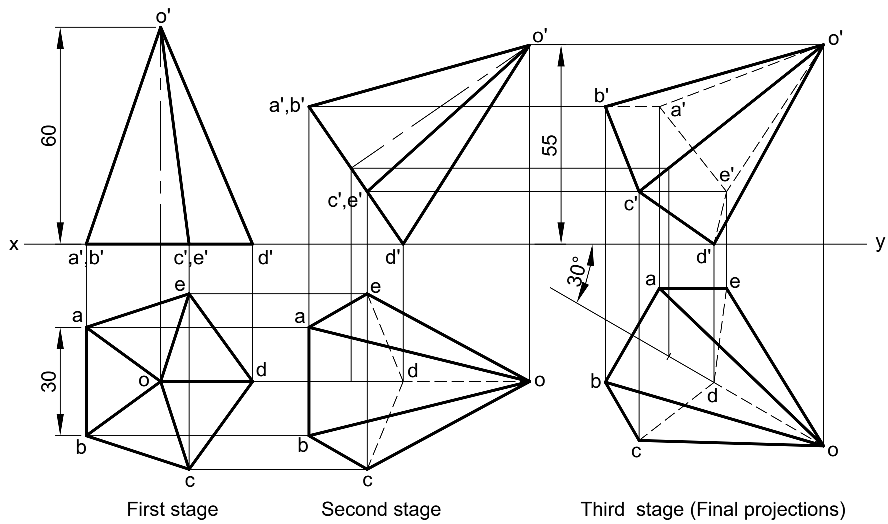


Fig. 11.41

3. **Third stage** Redraw the top view of second stage keeping plane containing *do* inclined at 30° to *xy*. Obtain *a′*, *b′*, *c′*, *d′*, *e′* and *o′* in the front view as the intersecting points of the projectors from the top view of the third stage with the corresponding locus lines from the front view of the second stage. Join the points and obtain *b′c′d′o′* as the required front view.

Problem 11.34 *A pentagonal pyramid of base side 30 mm and axis 60 mm rests on an edge of its base on the ground so that the highest point of the base is 20 mm above the ground. Draw its projections when a vertical plane containing the axis is inclined at 30° to the V.P.*

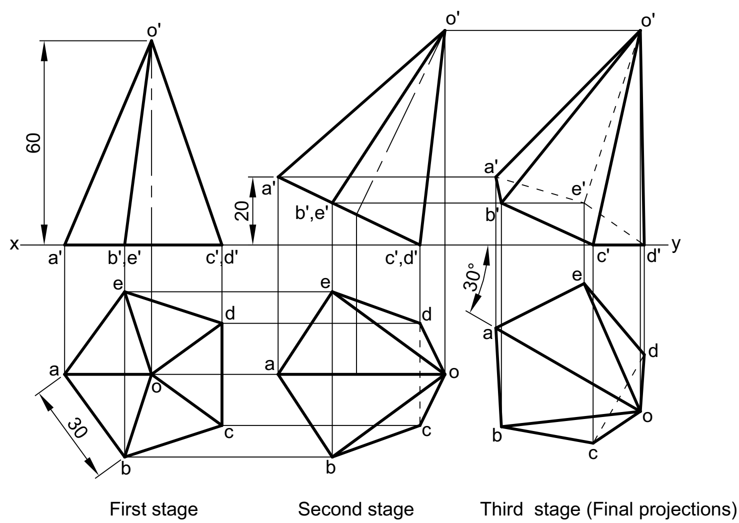


Fig. 11.42

Construction Refer to Fig. 11.42.

1. **First stage** Draw a pentagon *abcde* keeping *cd* perpendicular to *xy*. Join the corners of the pentagon with the centroid *o*. This represents the top view. Project the corners and obtain *a′c′o′* as the front view.
2. **Second stage** Reproduce the front view of the first stage keeping *c′d′* on *xy* and *a′* 20 mm above *xy* (For this, mark a point *c′d′* on *xy*, then draw an arc with centre *c′d′* and radius *a′c′* to meet the horizontal line 20 mm above *xy*). Obtain *a*, *b*, *c*, *d*, *e* and *o* in the top view as the intersecting points of the projectors from the front view of the second stage with the corresponding locus lines from the top view of the first stage. Join the points and obtain *abcode* as the top view.
3. **Third stage** Reproduce the top view of the second stage keeping vertical plane containing *ao* inclined at 30° to *xy*. Obtain *a′*, *b′*, *c′*, *d′*, *e′* and *o′* in the front view as the intersecting points of the projectors from the top view of the third stage with the corresponding locus lines from the front view of the second stage. Join the points and obtain *a′b′c′d′o′* as the required front view.

Problem 11.35 *A pentagonal pyramid of base side 30 mm and axis 60 mm is held on a corner of its base on the ground and the triangular face opposite to it is horizontal. Draw the projections of the pyramid when the edge of the base contained by this triangular face is inclined at 60° to the V.P. and the apex is towards the observer.*

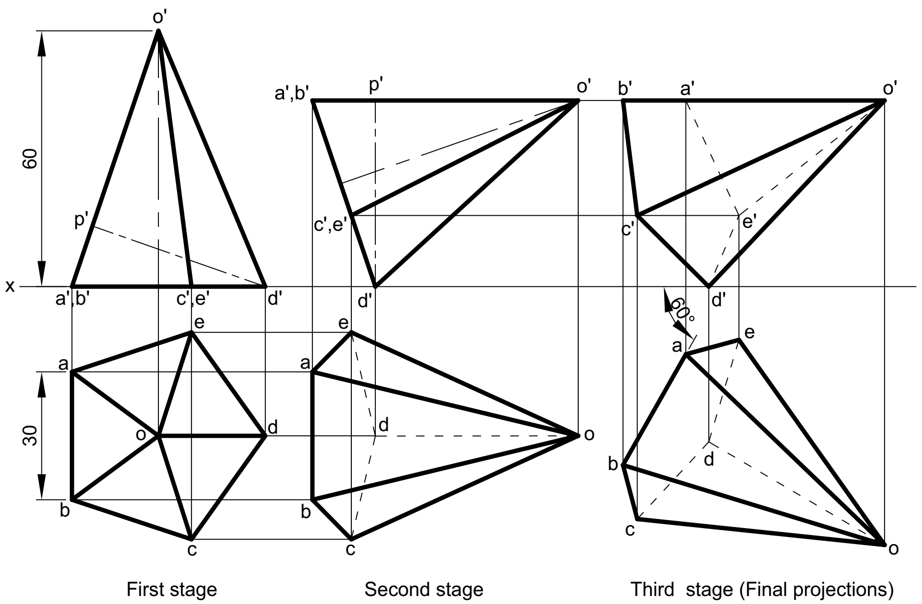


Fig. 11.43

Construction Refer to Fig. 11.43.

1. **First stage** Draw a pentagon abcde keeping *ab* perpendicular to *xy*. Join the corners of the pentagon with the centroid *o*. This represents the top view. The line *od* is parallel to *xy*. Project the corners and obtain *b′d′o′* as the front view. Draw a perpendicular line *d′p′* from point *d′* on line *a′o′*.

2. **Second stage** Reproduce the front view of the first stage keeping d' on xy and $p'd'$ perpendicular to xy. Thus, $a'b'o'$ is parallel to xy. Obtain a, b, c, d, e and o in the top view as the intersecting points of the projectors from the front view of the second stage with the corresponding locus lines from the top view of the first stage. Join the points and obtain *abcoe* as the top view.
3. **Third stage** Reproduce the top view of the second stage keeping ab inclined at 60° to xy and apex towards the observer. Obtain a', b', c', d', e' and o' in the front view as the intersecting points of the projectors from the top view of the third stage with the corresponding locus lines from the front view of the second stage. Join the points and obtain $a'b'c'd'o'$ as the required front view.

Problem 11.36 *A cylinder of base diameter 50 mm and axis 65 mm rests on a point of its base circle on the H.P. Draw its projections when the axis is inclined at 30° to the H.P. and top view of the axis is perpendicular to the V.P.*

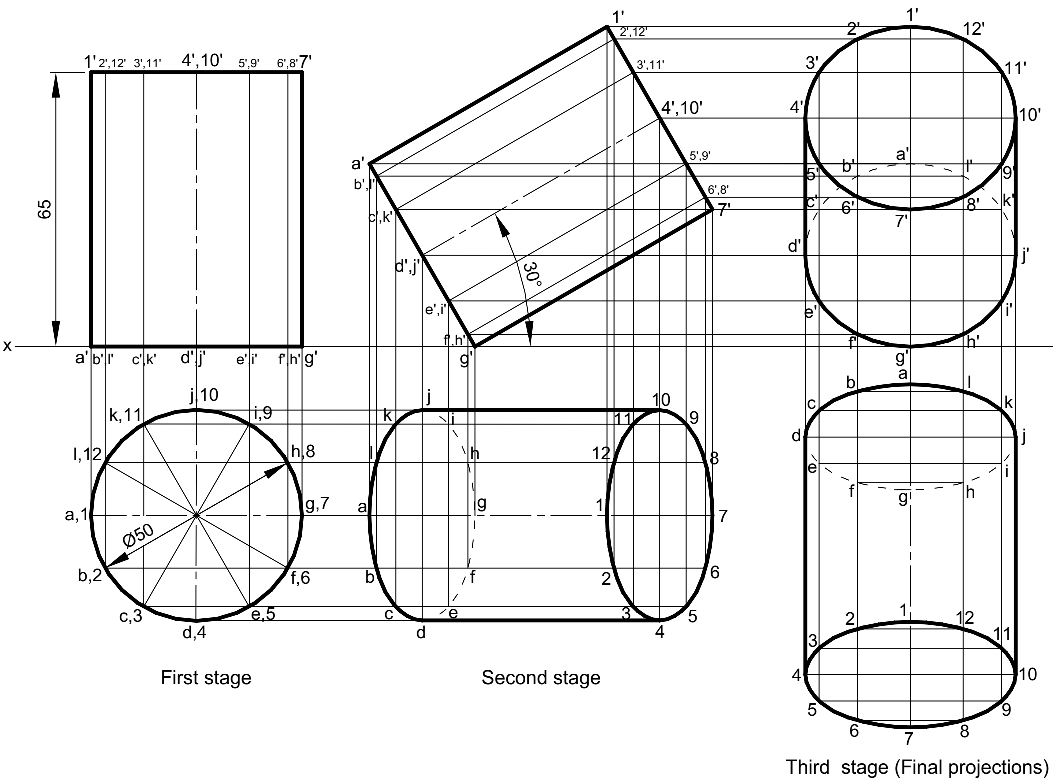


Fig. 11.44

Construction Refer to Fig. 11.44.

1. **First stage** Draw a circle *adgj* and divide it into 12 equal parts to represent the top view. Project the top view and obtain $a'g'7'1'$ as the front view.

2. **Second stage** Reproduce the front view of first stage keeping g' on xy and $g'7'$ inclined at 30° to xy. Obtain a, b, c, etc., and 1, 2, 3, etc., in the top view as the intersecting points of the projectors from the front view of the second stage with the corresponding locus lines from the top view of the first stage. Join the points and obtain a-d-4-7-10-j as the top view.
3. **Third stage** Reproduce the top view of the second stage keeping $ag17$ perpendicular to xy. Obtain a', b', c', etc., and $1'$, $2'$, $3'$, etc., in the front view as the intersecting points of the projectors from the top view of the third stage with the corresponding locus lines from the front view of the second stage. Join the points and obtain the required front view.

Problem 11.37 *Draw the projections of a cube of edge 40 mm resting on one of its corners on the H.P. with a solid diagonal perpendicular to the V.P.*

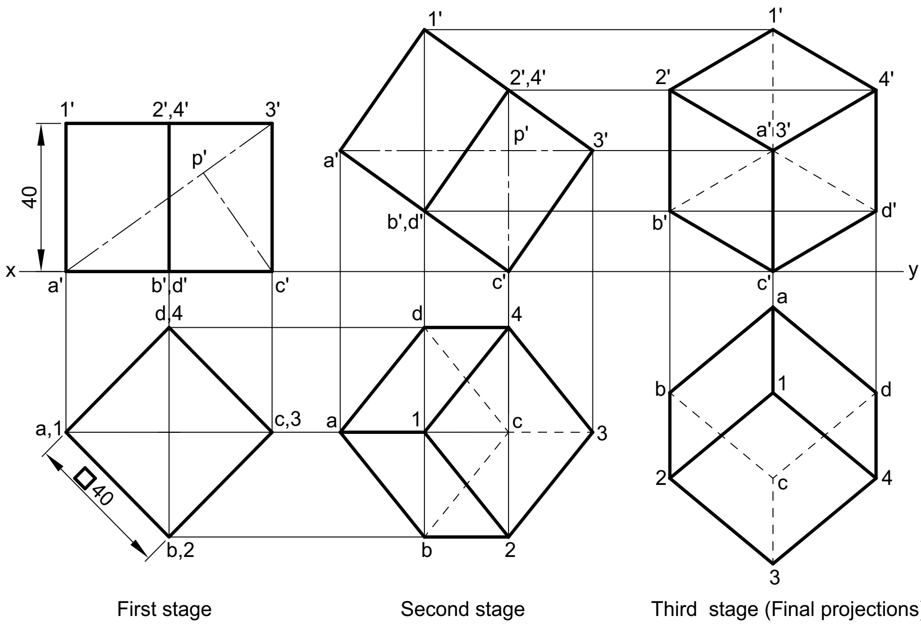


Fig. 11.45

Construction Refer to Fig. 11.45.

1. **First stage** Draw a square $abcd$ keeping ab inclined at 45° to xy. This is the top view. Project the corners and obtain $a'c'3'1'$ as the front view. Mark a diagonal $a'3'$. Draw a perpendicular line $c'p'$ from corner c' on the diagonal $a'3'$.
2. **Second stage** Reproduce the front view of the first stage keeping c' on xy, $c'p'$ perpendicular to xy and $a'3'$ is parallel to xy. Obtain a, b, c, d, 1, 2, 3 and 4 in the top view as the intersecting points of the projectors from the front view of the second stage with the corresponding locus lines from the top view of the first stage. Join the points and obtain a-b-2-3-4-d as the top view.
3. **Third stage** Reproduce the top view of the second stage keeping solid diagonal $a3$ perpendicular to xy. Obtain a', b', c', d', $1'$, $2'$, $3'$ and $4'$ in the front view as the intersecting points of the projectors from the top view of the third stage with the corresponding locus lines from the front view of the second stage. Join the points and obtain $1'2'b'c'd'4'$ as the required front view.

Problem 11.38 *A square pyramid of base side 40 mm and axis 60 mm is freely suspended from a corner of its base. Draw its projections when the axis as a vertical plane is inclined at 45° to the V.P.*

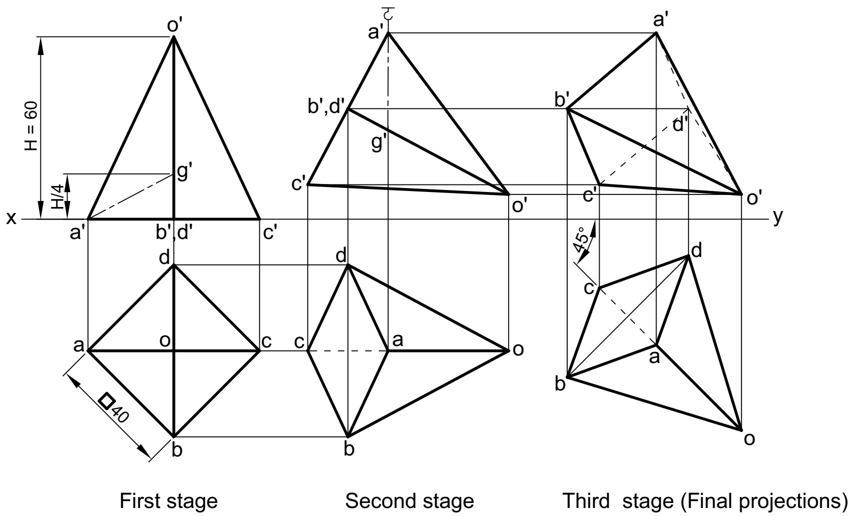


Fig. 11.46

Construction Refer to Fig. 11.46.

1. **First stage** Draw a square *abcd* keeping ac parallel to *xy*. Join the corners of the square with the centroid *o*. This represents the top view. Project the corners and obtain *a′c′o′* as the front view. Mark centre of gravity (*g′*) on the axis in the front view at a height *h*/4 (= 15 mm) from *xy*.
2. **Second stage** Reproduce the front view of the first stage keeping *a′* at the highest position and *a′g′* perpendicular to *xy*. Obtain *a*, *b*, *c*, *d* and *o* in the top view as the intersecting points of the projectors from the front view of the second stage with the corresponding locus lines from the top view of the first stage. Join the points and obtain *bcdo* as the top view.
3. **Third stage** Reproduce the top view of the second stage keeping *cao* inclined at 45° to *xy*. Obtain *a′*, *b′*, *c′*, *d′* and *o′* in the front view as the intersecting points of the projectors from the top view of the third stage with the corresponding locus lines from the front view of the second stage. Join the points and obtain *a′b′c′o′* as the required front view.

Problem 11.39 *A cone of base diameter 50 mm and axis 60 mm is freely suspended from the mid-point of a generator. Draw its projections when the top view of that generator is inclined at 45° to the reference line and apex is nearer to the observer.*

Construction Refer to Fig. 11.47.

1. **First stage** Draw a circle *adgj* to represent the top view. Divide it into 12 equal parts. Project the points and obtain *a′g′o′* as the front view. Mark *p′* as the centre of the generator *a′o′*. Also mark *q′* as the centre of gravity on the axis at a height *h*/4 (= 15 mm) from *xy*.
2. **Second stage** Reproduce the front view of the first stage keeping *p′q′* perpendicular to *xy*. Obtain *a*, *b*, *c*, *d*, etc., and *o* in the top view as the intersecting points of the projectors from the front view

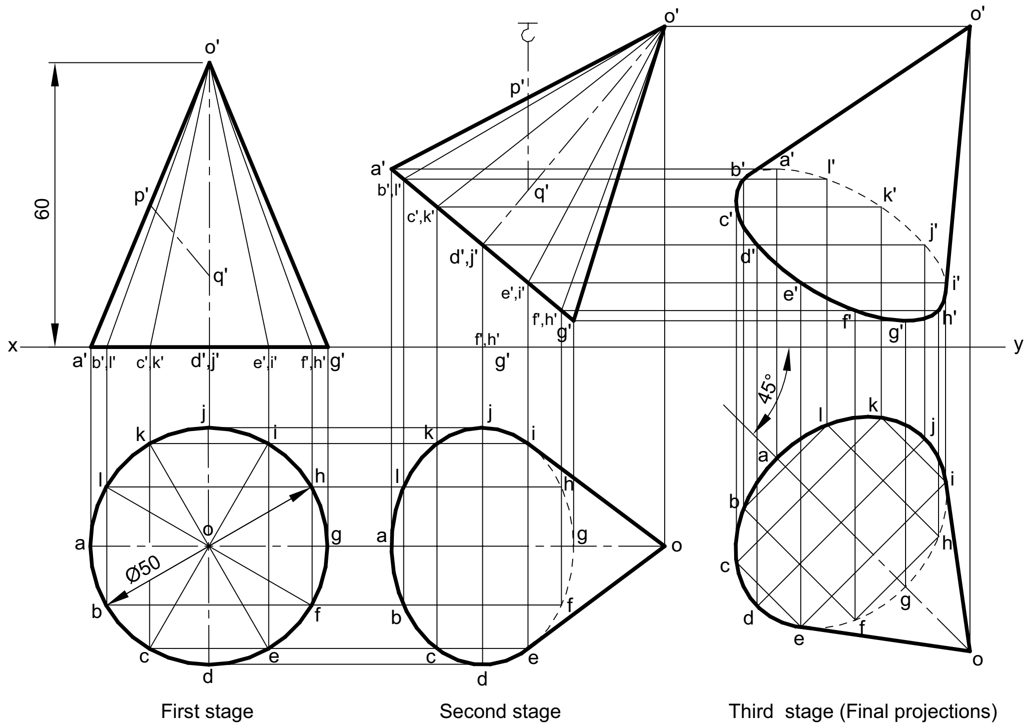


Fig. 11.47

of the second stage with the corresponding locus lines from the top view of the first stage. Join the points and obtain *adoj* as the top view.

3. **Third stage** Reproduce the top view of the second stage keeping *ago* inclined at at 45° to *xy*. The apex *o* should be farther to *xy* than the base, so that the apex is nearer the observer. Obtain a', b', c', d', etc., and o' in the front view as the intersecting points of the projectors from the top view of the third stage with the corresponding locus lines from the front view of the second stage. Join the points and obtain $b'c'd'e'f'g'h'i'o'$ as the required front view.

Problem 11.40 *A square pyramid of base diagonal 50 mm and axis 60 mm is titled until the top view of the base appears as a rhombus having one of the diagonal twice of the other. Draw its projections when the axis as a vertical plane is inclined at 45° to the V.P.*

Construction Refer to Fig. 11.48.

1. **First stage** Draw a square *abcd* keeping *ac* parallel to *xy*. Join the corners of the square with centroid *o*. This represents the top view. Project the corners and obtain $a'c'o'$ as the front view.
2. **Second stage** Mark points *a*, *b*, *c* and *d* on the locus line from points *a*, *b*, *c* and *d* of the first stage such that diagonal *ac* is 25 mm. Project *c* on *xy* and obtain point c'. Draw an arc with centre c' and radius equal to length $a'c'$ of first stage to meet the projector of point *a* at point a'. Join $a'c'$ and reproduce the front view of the first stage here. Project o' and join points to obtain *abod* as the new top view.

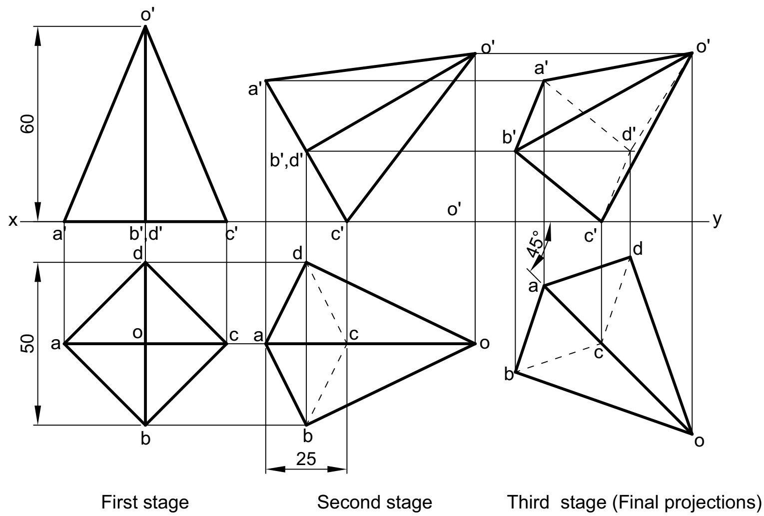


Fig. 11.48

3. **Third stage** Reproduce the top view of the second stage keeping *aco* inclined at 45° to *xy*. Obtain *a*′, *b*′, *c*′, *d*′ and *o*′ in the front view as the intersecting points of the projectors from the top view of the third stage with the corresponding locus lines from the front view of the second stage. Join the points and obtain *a*′*b*′*c*′*o*′ as the required front view.

Problem 11.41 *A tetrahedron of edge 70 mm is resting on one of its edges in the H.P. which is inclined at 45° to the V.P. and a face contained by that edge is inclined at 30° to the H.P. Draw its projections.*

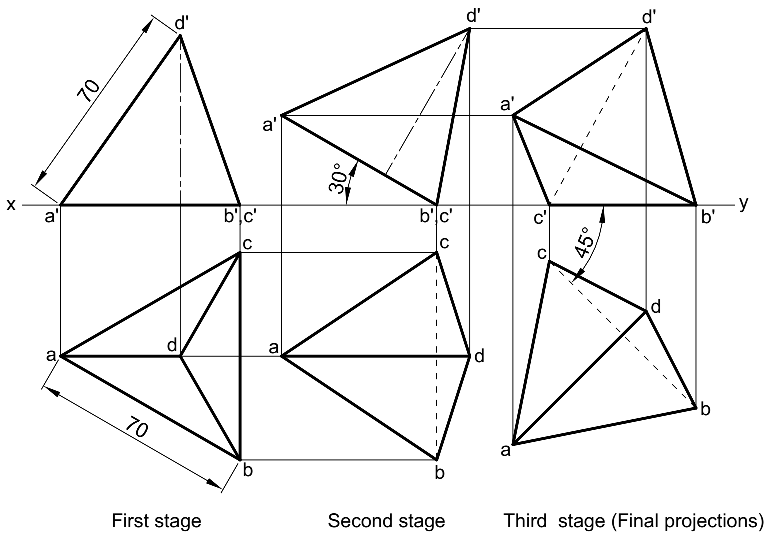


Fig. 11.49

Construction Refer to Fig. 11.49.

1. **First stage** Draw a triangle *abc* keeping *bc* perpendicular to *xy*. Join the corners of the triangle with the centroid *d*. This represents the top view. Project the corners and obtain *a′b′d′* as the front view.
2. **Second stage** Redraw the front view of first stage keeping *b′c′* on *xy* and *a′b′c′* inclined at 30° to *xy*. Obtain points *a*, *b*, *c* and *d* in the top view as the intersecting points of the projectors from the front view of the second stage with the corresponding locus lines from the top view of the first stage. Join the points and obtain *abdc* as the top view.
3. **Third stage** Reproduce the top view of the second stage keeping *cb* inclined at 45° to *xy*. Obtain *a′*, *b′*, *c′* and *d′* in the front view as the intersecting points of the projectors from the top view of the third stage with the corresponding locus lines from the front view of the second stage. Join the points and obtain *a′c′b′d′* as the required front view.

Problem 11.42 *A frustum of a pentagonal pyramid of base edge 40 mm, top edge 20 mm and axis 60 mm is resting on its face on the H.P. Draw its projections when an edge of its base is parallel to the V.P. and the small base is towards the observer.*

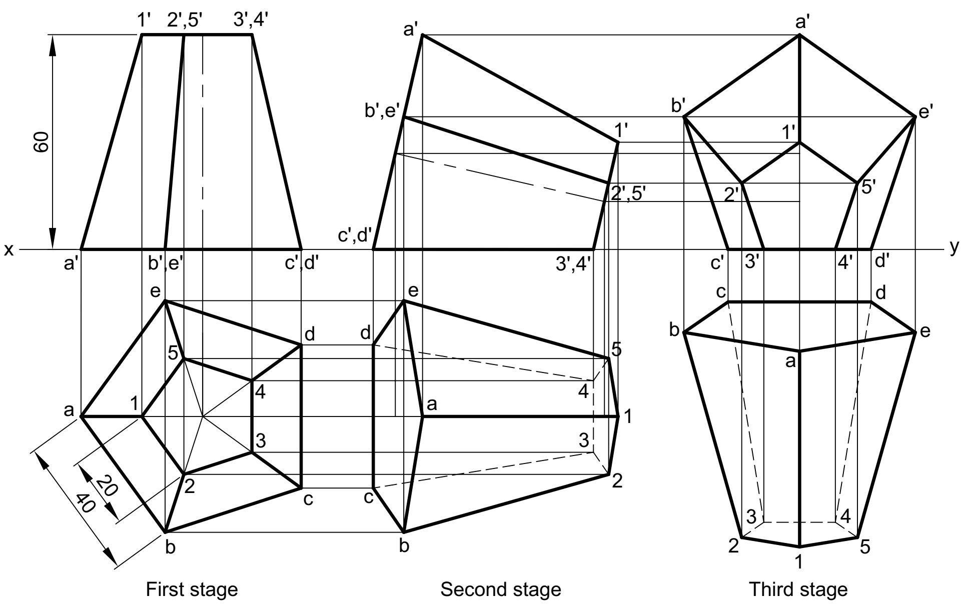


Fig. 11.50

Construction Refer to Fig. 11.50.

1. **First stage** Draw two concentric pentagons *abcdef* and 12345 keeping *cd* perpendicular to *xy*. Join *a*1, *b*2, *c*3, *d*4 and *e*5. This is the top view. Project the corners and obtain *a′c′*3′1′ as the front view.
2. **Second stage** Reproduce the front view of the first stage keeping *c′d′*4′3′ on *xy*. Obtain *a*, *b*, *c*, *d*, *e*, 1, 2, 3, 4 and 5 in the top view as the intersecting points of the projectors from the front view of the second stage with the corresponding locus lines from the top view of the first stage. Join the points and obtain *bcde*512 as the top view.

3. **Third stage** Reproduce the top view of the second stage keeping *cd* parallel and nearer to *xy*. Thus, the base 12345 is nearer to the observer. Obtain *a′*, *b′*, *c′*, *d′*, *e′*, 1′, 2′, 3′, 4′ and 5′ in the front view as the intersecting points of the projectors from the top view of the third stage with the corresponding locus lines from the front view of the second stage. Join the points and obtain *a′b′c′d′e′* as the required front view.

11.14.2 An Element of the Solid in the V.P.

Problem 11.43 *A pentagonal prism of base side 30 mm and axis 60 mm has an edge of its base in the V.P. and inclined at 45° to the H.P. Its axis is inclined at 30° to the V.P. Draw its projections.*

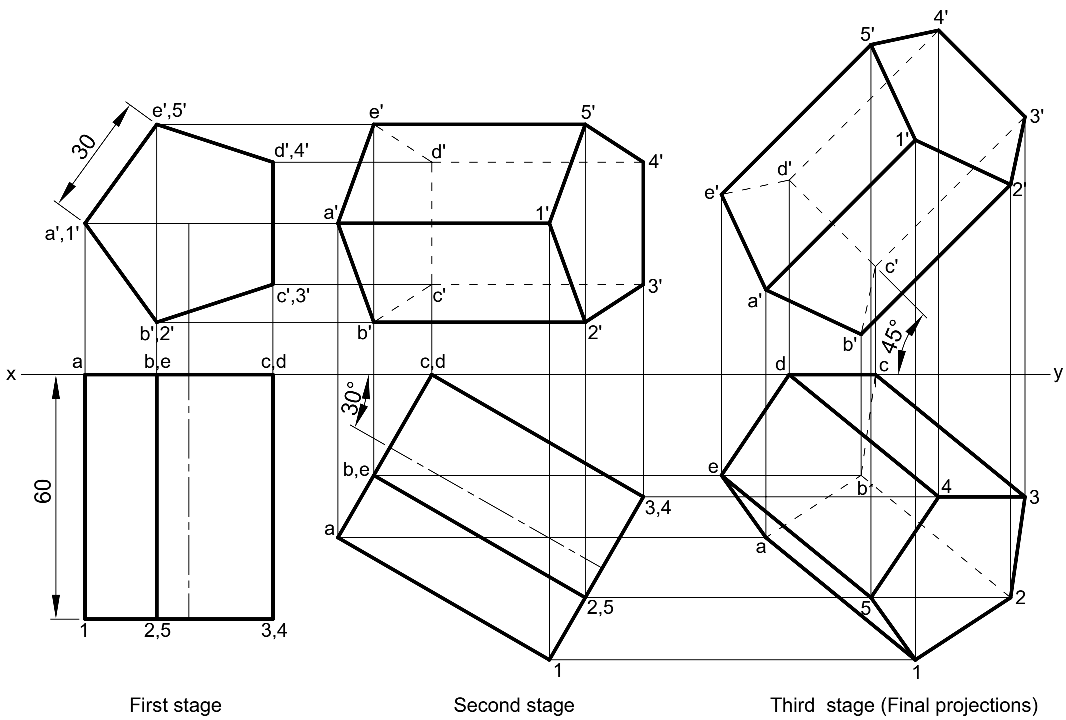


Fig. 11.51

Construction Refer to Fig. 11.51.

1. **First stage** Draw a pentagon *a′b′c′d′e′* keeping *c′d′* perpendicular to *xy*. This is the front view. Project the corners and obtain *ad*41 as the top view.
2. **Second stage** Reproduce the top view of first stage keeping *cd* on *xy* and *cd*34 inclined at 30° to *xy*. Obtain *a′*, *b′*, *c′*, *d′*, *e′*, 1′, 2′, 3′, 4′ and 5′ in the front view as the intersecting points of the projectors from the top view of the second stage with the corresponding locus lines from the front view of the first stage. Join the points and obtain *a′b′*2′3′4′5′*e′* as the front view. (The edge *cd* is on *xy*, therefore join *c′d′*, *c′b′*, *c′*3′, *d′e′* and *d′*4′ using dashed narrow lines.)

3. **Third stage** Reproduce the front view of the second stage keeping $c'd'$ inclined at 45° to *xy*. Obtain *a*, *b*, *c*, *d*, *e*, 1, 2, 3, 4 and 5 in the top view as the intersecting points of the projectors from the front view of the third stage with the corresponding locus lines from the top view of the second stage. Join these points and obtain *a*123*cde* as the required top view. (The corner b' is towards *xy*, therefore join *ba*, *bc* and *b*2 using dashed narrow lines.)

Problem 11.44 *A hexagonal prism of base edge 30 mm and axis 70 mm has an edge of its base in the V.P. and inclined at 60° to the H.P. Draw its projections, when the edge of the other base farthest away from V.P. is at a distance of 85 mm from the V.P.*

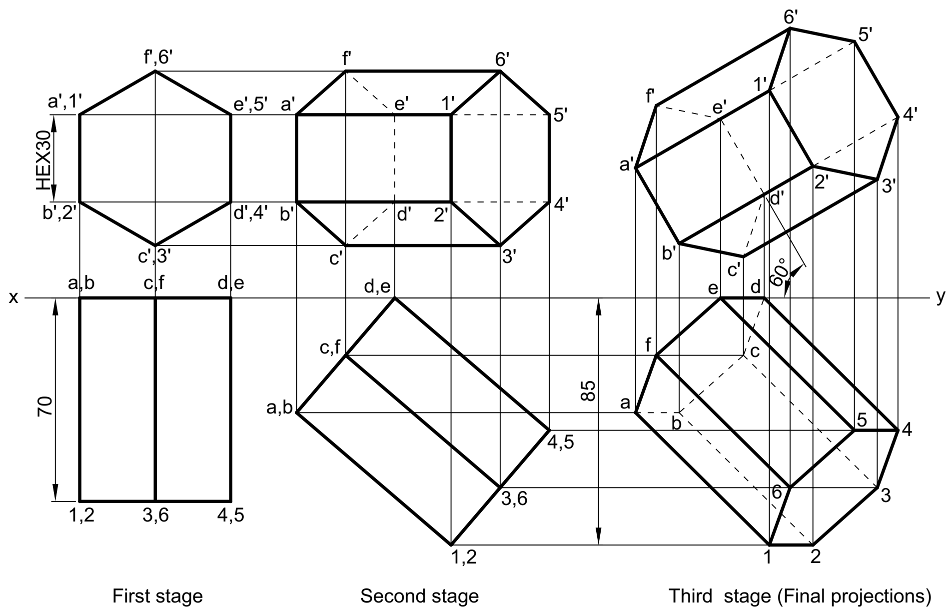


Fig. 11.52

Construction Refer to Fig. 11.52.

1. **First stage** Draw a hexagon $a'b'c'd'e'f'$ keeping $d'e'$ perpendicular to *xy*. This is the front view. Project the corners and obtain *ae*51 to represent the top view.
2. **Second stage** Reproduce the top view of the first stage keeping *de* on *xy* and 1-2 at a distance of 85 mm from *xy* (For this, mark point *de* on *xy*, then draw an arc with centre *de* and radius *d*1 to meet a horizontal line 85 mm from *xy*.) There could be two possible inclinations of the solid, one of them is presented over here. Obtain a', b', c', d', e', f', $1'$, $2'$, $3'$, $4'$, $5'$ and $6'$ in the front view as the intersecting points of the projectors from the top view of the second stage with the corresponding locus lines from the front view of the first stage. Join the points and obtain $a'b'c'3'4'5'6'f'$ as the front view.
3. **Third stage** Reproduce the front view of second stage keeping $d'e'$ inclined at 60° to *xy*. Obtain *a*, *b*, *c*, *d*, *e*, *f*, 1, 2, 3, 4, 5 and 6 in the top view as the intersecting points of the projectors from the front view of the third stage with the corresponding locus lines from the top view of the second stage. Join the points and obtain *a*1234*def* as the required top view. (The edge $b'c'$ is towards *xy*, therefore join *bc*, *ba*, *b*2, *cd* and *c*3 using dashed narrow lines.)

Problem 11.45 *A square pyramid of base side 40 mm and axis 60 mm has a corner of its base in the V.P. A slant edge contained by that corner is inclined at 45° to the V.P. Draw its projections when a plane containing the slant edge and the axis is inclined at 45° to the H.P.*

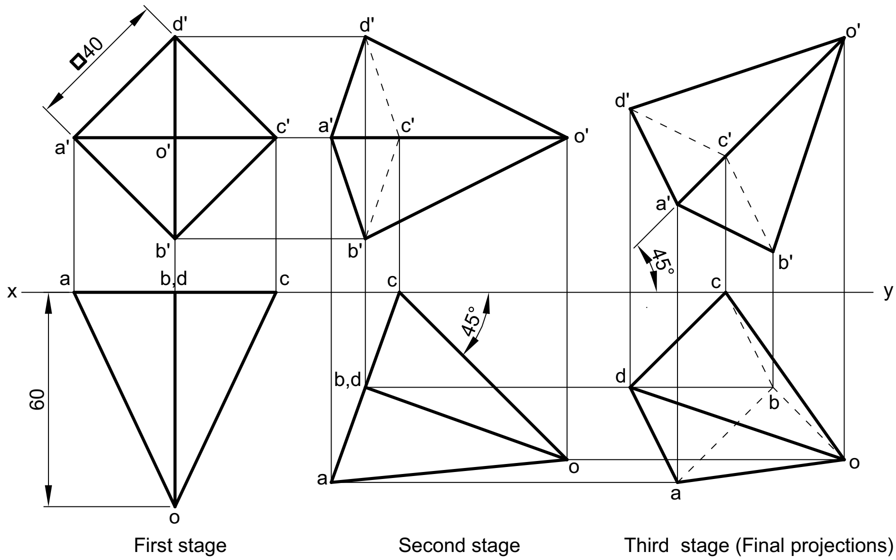


Fig. 11.53

Construction Refer to Fig. 11.53.

1. **First stage** Draw a square $a'b'c'd'$ keeping $a'c'$ parallel to xy. Join the corners of the square with the centroid o. This represents the front view. Project the corners and obtain aco as its top view.
2. **Second stage** Reproduce the top view of the first stage keeping c on xy and the slant edge oc inclined at 45° to xy. Obtain a', b', c', d' and o' in the front view as the intersecting points of the projectors from the top view of the second stage with the corresponding locus lines from the front view of the first stage. Join the points and obtain $a'b'o'd'$ as the front view.
3. **Third stage** Reproduce the front view of the second stage keeping $a'c'o'$ inclined at 45° to xy. Obtain a, b, c, d and o in the top view as the intersecting points of the projectors from the front view of the third stage with the corresponding locus lines from the top view of the second stage. Join the points and obtain $aocd$ as the final top view. (The corner $b'c'$ is towards xy, therefore join ba, bc and bo using dashed narrow lines.)

Problem 11.46 *A cone of base diameter 50 mm and axis 60 mm has one of its generators in the V.P. and inclined at 30° to the H.P. Draw its projections when the apex is 15 mm above the H.P.*

Construction Refer to Fig. 11.54.

1. **First stage** Draw circle $a'd'g'j'$ and divide into 12 equal parts to represent the front view. Project the front view and obtain ago as the top view.
2. **Second stage** Reproduce the top view of the first stage keeping og on xy. Obtain a', b', c', d', e', f', g', h', i', j', k', l' and o' in the front view as the intersecting points of the projectors from the top

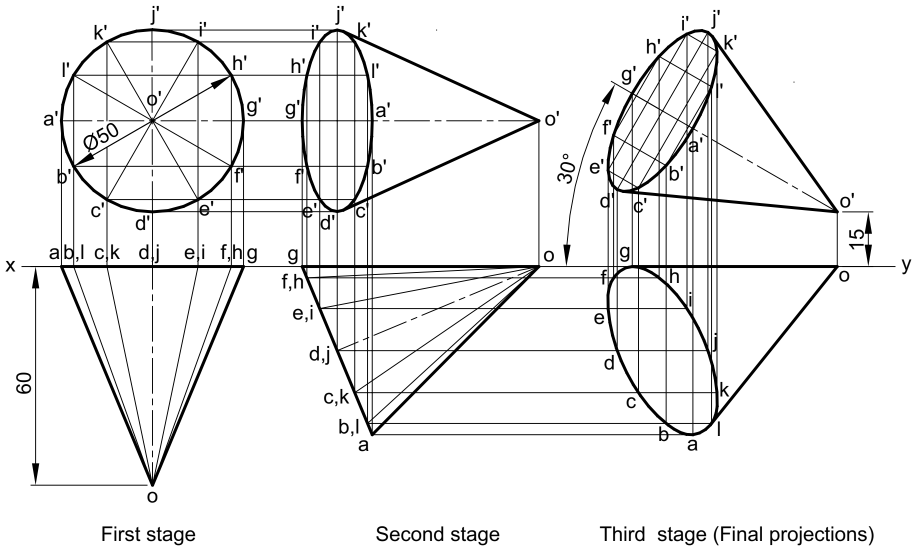


Fig. 11.54

view of the second stage with the corresponding locus lines from the front view of the first stage. Join these points and obtain the front view.

3. **Third stage** Reproduce the front view of the second stage keeping o' 15 mm above xy and $o'a'g'$ inclined at 30° to xy. Obtain a, b, c, d, e, f, g, h, i, j, k, l and o in the top view as the intersecting points of the projectors from the front view of the third stage with the corresponding locus lines from the top view of the second stage. Join the points and obtain the final top view.

11.14.3 Condition of Apparent Angle

Problem 11.47 *A hexagonal prism of base edge 30 mm and axis 60 mm rests on one of its base edges on the H.P. such that the axis is inclined at 30° to H.P. and 45° to the V.P. Draw its projections.*

Construction Refer to Fig. 11.55.

1. **First stage** Draw a hexagon *abcdef* keeping de perpendicular to xy. This represents the top view. Project the top view and obtain $b'd'4'2'$ as the front view.
2. **Second stage** Reproduce the front view of first stage keeping $d'e'$ on xy and $d'4'$ inclined at 30° to xy. Obtain a, b, c, d, e, f, 1, 2, 3, 4, 5 and 6 in the top view as the intersecting points of the projectors from the front view of the second stage with the corresponding locus lines from the top view of the first stage. Join the points and obtain *abc*3456*f* as the top view.
   When a line is inclined at θ to the H.P. and ϕ to the V.P., its top view is inclined at β to xy.
3. **Third stage** Draw a line o_1o_3, 60 mm (true length of the axis) inclined at 45° to xy. Draw an arc with centre o_1 and radius equal to the length of the axis o_1o_2 in the top view of the second stage to meet horizontal line from o_3 at point o_2. Line o_1o_2 is inclined at β to xy. Reproduce the top view

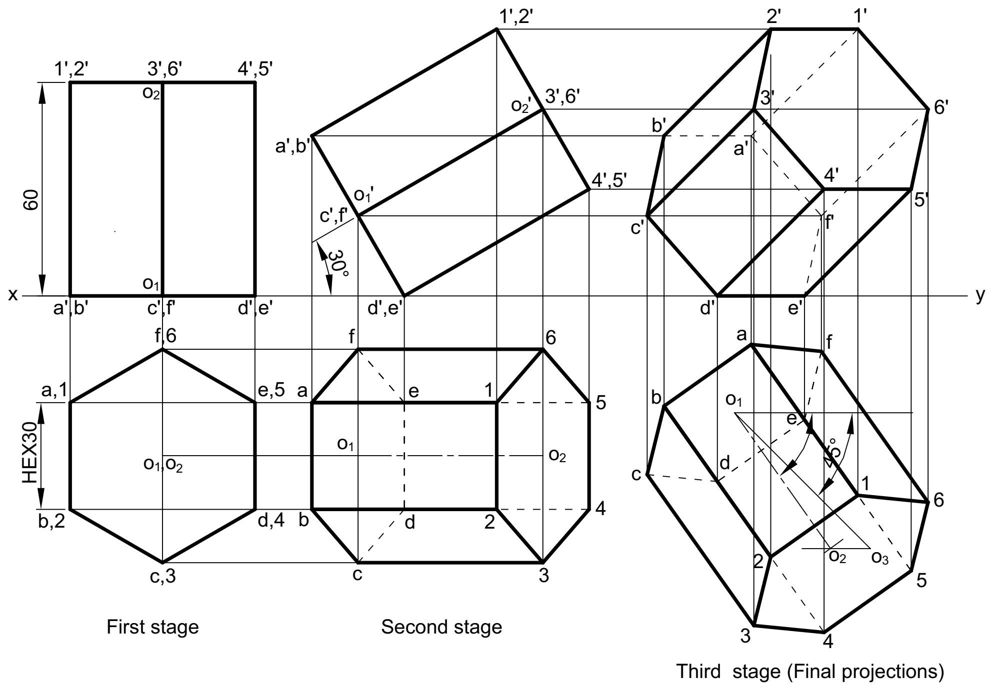


Fig. 11.55

of the second stage keeping o_1o_2 at β to xy. Obtain a', b', c', d', e', f', $1'$, $2'$, $3'$, $4'$, $5'$ and $6'$ in the front view as the intersecting points of the projectors from the top view of the third stage with the corresponding locus lines from the front view of the second stage. Join the points and obtain $b'c'd'e'5'6'1'2'$ as the front view.

Problem 11.48 *Draw the projections of a cone of base diameter 50 mm and axis 60 mm resting on a point of the base circle on the ground with axis inclined at 30° to the H.P. and (a) 45° to the V.P., and (b) the top view of the axis inclined at 45° to the V.P.*

Construction Refer to Fig. 11.56.

1. **First stage** Draw circle *adgj* and divide into 12 equal parts to represent the top view and project it to obtain $a'g'o'$ as the front view.
2. **Second stage** Reproduce the front view of first stage keeping g' on xy and $g'a'$ inclined at 60° to xy. Obtain a, b, c, etc., in the top view as the intersecting points of the projectors from the front view of the second stage with the corresponding locus lines from the top view of the first stage. Join the points and obtain *adoj* as the top view.

Case (a): The axis is inclined at 45° with the V.P.

3. **Third stage** Determine the apparent angle β. For this, draw a line o_1o_2, 60 mm long (true length of the axis) inclined at 45° to xy. Draw an arc with o_1 as the centre and radius equal to the top view

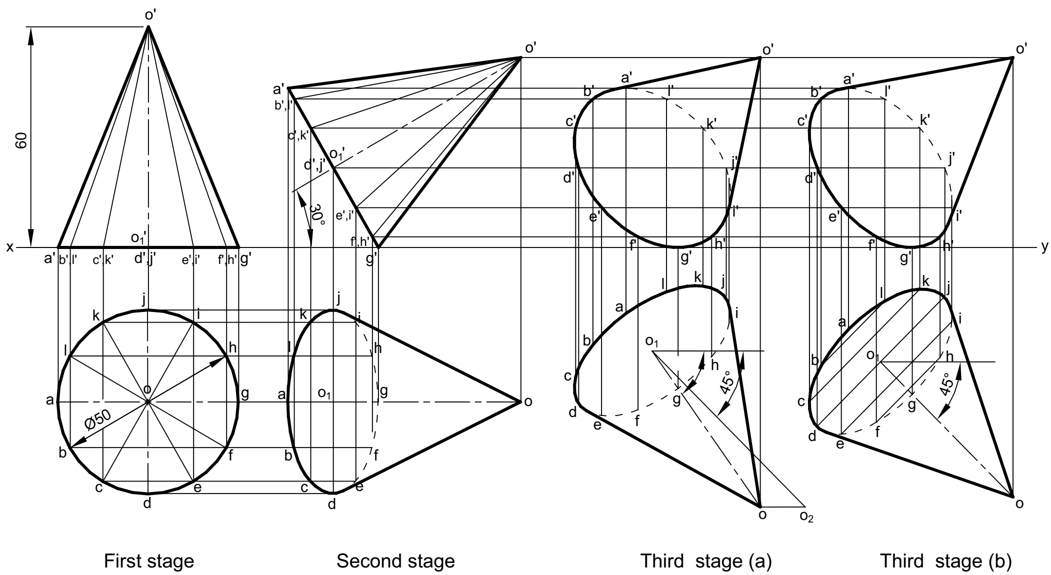


Fig. 11.56

of the axis (o_1o in the second stage) to meet horizontal line from o_2 at point o. Reproduce the top view of the second stage keeping oo_1 at β angle to xy. Obtain a', b', c', etc., in the front view as the intersecting points of the projectors from the top view of the third stage with the corresponding locus lines from the front view of the second stage. Join the points and obtain the required front view.

Case (b): The top view of the axis is inclined at 45° with the V.P.

4. **Fourth stage** Reproduce the top view of the second stage keeping oo_1 inclined at 45° to xy. Obtain a', b', c', etc., in the front view as the intersecting points of the projectors from the top view of the third stage with the corresponding locus lines from the front view of the second stage. Join the points and obtain the required front view.

Problem 11.49 *A square pyramid of base side 40 mm and axis 50 mm has a triangular face on the ground and the axis inclined at 45° to the V.P. Draw its projections.*

Construction Refer to Fig. 11.57.

1. **First stage** Draw a square *abcd* keeping *cd* perpendicular to xy. Join the corners of the square with the centroid o. This represents the top view. Project the corners and obtain $b'c'o'$ as the front view.
2. **Second stage** Reproduce the front view of the first stage keeping $c'd'o'$ on xy. Obtain a, b, c, d and o in the top view as the intersecting points of the projectors from the front view of the second stage with the corresponding locus lines from the top view of the first stage. Join the points and obtain *adcbo* as the top view.

   Here the axis already inclined to the H.P. is also inclined at 45° to the V.P. Therefore, determine the apparent angle β.

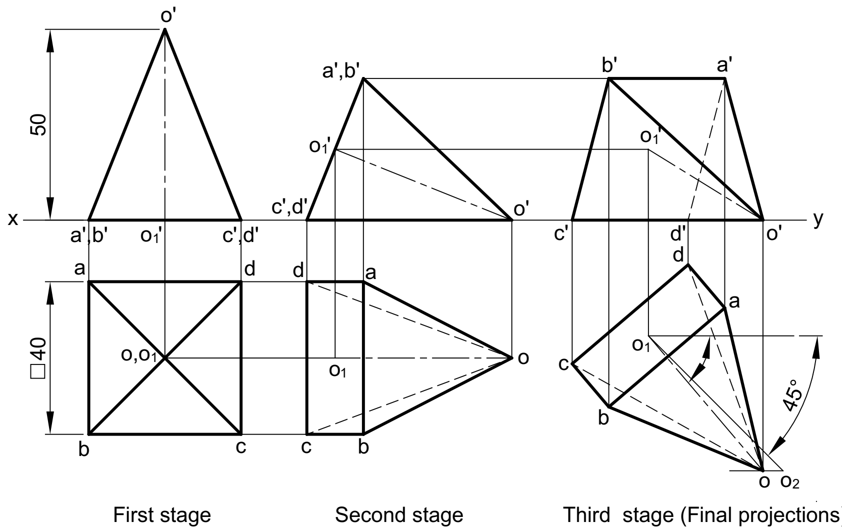


Fig. 11.57

3. **Third stage** Draw a line o_1o_2, 50 mm long (true length of the axis) inclined at 45° to *xy*. Draw an arc with centre o_1 and radius equal to the length of the axis oo_1 in the top view of the second stage to meet horizontal line from o_2 at point *o*. Line oo_1 is inclined at β to *xy*. Reproduce the top view of the second stage keeping oo_1 at β to *xy*. Obtain a', b', c', d' and o' in the front view as the intersecting points of the projectors from the top view of the third stage with the corresponding locus lines from the front view of the second stage. Join the points and obtain $a'b'c'd'o'$ as the required front view.

Problem 11.50 *A pentagonal prism of base side 25 mm and axis 60 mm has its axis inclined at 60° to the H.P. and 30° to the V.P. The farthest shorter edge is parallel to and 90 mm above the H.P. while the nearest corner is 10 mm in front of the V.P. Draw its projections.*

Construction Refer to Fig. 11.58.

1. **First stage** Draw a pentagon *abcde* keeping *ab* perpendicular to *xy*. This represents the top view. Project the points and obtain $b'd'4'2'$ as the front view.
2. **Second stage** Mark a point $1'2'$ 90 mm above *xy*. Reproduce the front view of the first stage keeping $1'2'a'b'$ at 60° to *xy*. Obtain *a*, *b*, *c*, *d*, *e*, 1, 2, 3, 4 and 5 in the top view as the intersecting points of the projectors from the front view of the second stage with the corresponding locus lines from the top view of the first stage. Join the points and obtain *abc*345*e* as the top view.
   Here axis is inclined 60° to H.P. and 30° to V.P. Therefore, β angle is equal to 90°.
3. **Third stage** Reproduce the top view of the second stage such that point 4 is 10 mm below *xy* and the axis o_1o_2 is perpendicular to *xy*. Obtain a', b', c', d', e', $1'$, $2'$, $3'$, $4'$ and $5'$ in the front view as the intersecting points of the projectors from the top view of the third stage with the corresponding locus lines from the front view of the second stage. Join the points and obtain $1'2'3'c'd'e'5'$ as the front view.

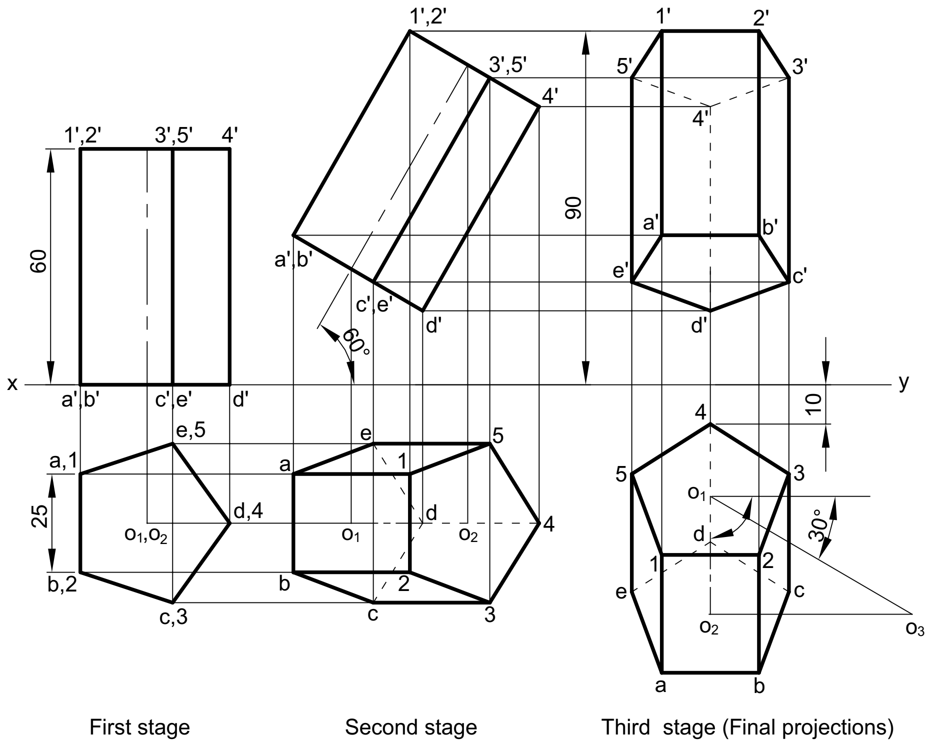


Fig. 11.58

Problem 11.51 *A cylinder of base diameter 50 mm and axis 70 mm has a point of its base circle in the V.P. Its axis is inclined at 30° to the V.P. and 45° to the H.P. Draw its projections.*

Construction Refer to Fig. 11.59.

1. **First stage** Draw circle $a'd'g'j'$ and divide into 12 equal parts to represent the front view and project it to obtain $a17g$ as its top view.
2. **Second stage** Reproduce the top view of the first stage keeping g on xy and $g7$ inclined at 30° to xy. Obtain a', b', c', d', etc., and 1′, 2′, 3′, 4′, etc., in the front view as the intersecting points of the projectors from the top view of the second stage with the corresponding locus lines from the front view of the first stage. Join these points and obtain $a'd'4'7'10'j'$ as the top view.
   When a line is inclined at θ to the H.P. and ϕ to the V.P., its front view is inclined at α to xy.
3. **Third stage** Therefore, draw a line $o_1'o_3'$, 70 mm (true length of the axis) inclined at 45° to xy. Draw an arc with o_1' as the centre and radius equal to the front view of the axis ($o_1'o_2'$ from the second stage) to meet horizontal line from o_3' at point o_2'. Reproduce the front view of the second stage keeping $o_1'o_2'$ at α to xy. Obtain a, b, c, d, etc., and 1, 2, 3, 4, etc., in the top view as the intersecting points of the projectors from the front view of the third stage with the horizontal projectors drawn from the top view of the second stage. Join the points and obtain the required top view.

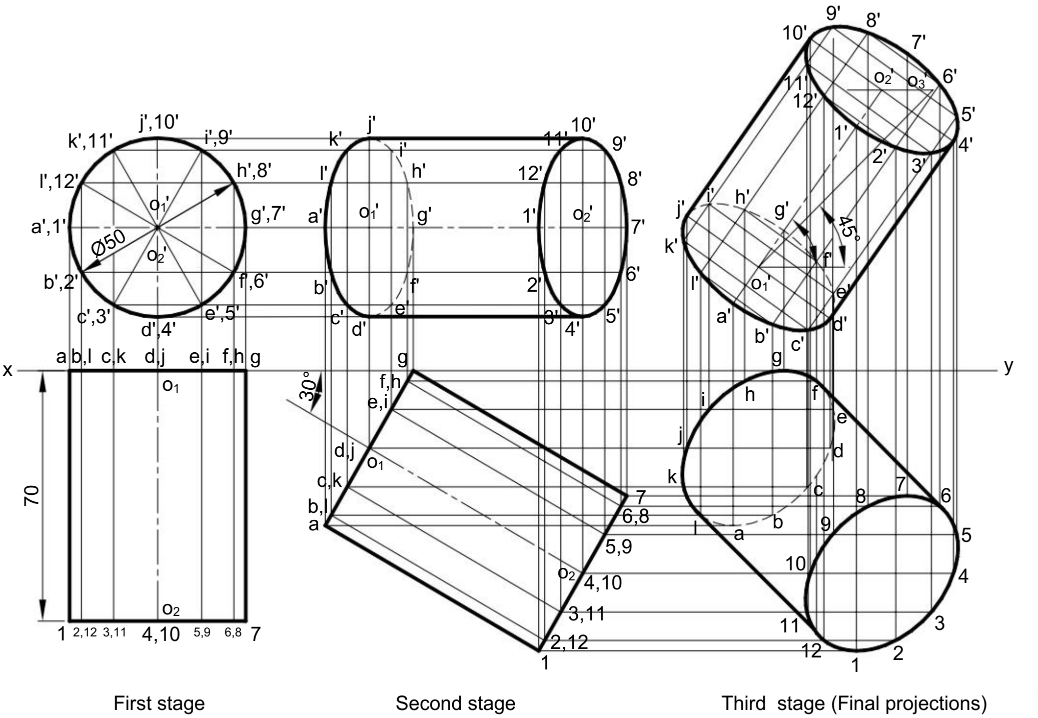


Fig. 11.59

EXERCISE 11C

An element of the solid in the H.P.

11.1 A pentagonal pyramid of base side 30 mm and axis 70 mm has one of the corners on the ground and its axis inclined at 45° to the H.P. A vertical plane containing the axis and that corner is inclined at 30° to the V.P. Draw its projections.

11.2 A hexagonal prism of base side 30 mm and axis 70 mm has an edge of the base parallel to the H.P. and inclined at 45° to the V.P. Draw its projections when its axis is inclined at 60° to the H.P.

11.3 Draw the projections of a cone of base diameter 50 mm and axis 70 mm, resting on a point of its base circle on the ground such that its axis is inclined at 30° to the H.P. and the top view of the axis is inclined at 45° to the V.P.

11.4 Draw the projections of a hexagonal pyramid of base side 30 mm and axis 65 mm, standing on an edge of the base in the H.P. inclined at 45° to the V.P. The slant face containing that edge is inclined at 60° to the H.P.

11.5 A triangular prism of base side 50 mm and axis 70 mm is resting on a corner of its base on the H.P. with a longer edge containing that corner inclined at 45° to the H.P. The vertical plane containing that edge and the axis is inclined at 30° to the V.P. Draw its projections.

11.6 A hexagonal pyramid of base edge 30 mm and axis 65 mm has a triangular face on the H.P. and the edge of the base containing that face makes an angle of 30° with the V.P. Draw its projections.

11.7 A hexagonal prism of base side 30 mm and axis 70 mm is resting on one of the edges of its base in the H.P. inclined at 60° to the V.P. The base is inclined at 60° to the H.P. Draw its projections.

11.8 A pentagonal pyramid of base side 30 mm and axis 60 mm has one of the edges of the base in the H.P. and parallel to the V.P. The solid is tilted in such a manner that the highest point of the base is 40 mm above H.P. Draw its projections.

11.9 A square pyramid of base side 50 mm and axis 60 mm has one of its triangular faces on the H.P. and a slant edge containing that face is parallel to the V.P. Draw its projections.

11.10 A square pyramid of base side 40 mm and height 70 mm has one of its triangular faces parallel to and 25 mm above H.P. and the shortest side of that face is inclined at 60° to the V.P. Draw its projections when the base is visible.

11.11 A cone of base diameter 60 mm and axis 80 mm rests on a point of its base circle on the ground with a generator normal to the H.P. Draw its projections when plan of the axis is inclined at 30° to the V.P.

11.12 A hexagonal pyramid of base side 30 mm and axis 75 mm has one of its triangular faces perpendicular to H.P. and inclined at 45° to the V.P. Draw its projections when the base side of the vertical face is on the H.P.

11.13 A cone of base diameter 50 mm and axis 70 mm is resting on one of its generators on the ground and inclined at 30° to the V.P. Draw its projections when the apex is nearer to V.P. than the base.

11.14 A hexagonal pyramid of base side 30 mm and axis 75 mm has one of its slant edges on the H.P. and vertical plane containing that edge and the axis is inclined at 30° to the V.P. Draw its projections when apex is 15 mm in front of the V.P.

11.15 A cube of side 50 mm has a corner in the H.P. Draw its projections when a solid diagonal is parallel to the H.P. and inclined at 30° to the V.P.

11.16 Draw the projections of a hexagonal prism of base side 25 mm and axis 65 mm, resting on the ground on one of its corners with a solid diagonal perpendicular to the V.P.

11.17 A tetrahedron of edge 70 mm has an edge in the H.P. and inclined at 45° to the V.P. A face containing that edge is perpendicular to the H.P. Draw its projections.

11.18 Draw the projections of a pentagonal prism of base side 25 mm and axis 65 mm, hanging from the roof of a building through its corner in such a way that a vertical plane containing that corner is inclined at 45° to the V.P.

11.19 A hexagonal pyramid of base side 30 mm and axis 70 mm is suspended freely from one of its base corners such that a vertical plane containing the axis is inclined at 45° to the V.P. Draw its projections.

11.20 A cone of base diameter 50 mm and axis 65 mm is freely suspended from a point of its rim such that the top view of the axis is perpendicular to V.P. and the apex is towards the observer. Draw its projections.

11.21 A square pyramid of base side 50 mm and axis 60 mm is freely suspended by a string tied at the mid-point of a side of its base. Draw its projections when a vertical plane containing the axis is inclined at 45° to the V.P.

11.22 A hexagonal pyramid of base edge 30 mm and slant edge 75 mm is resting on an edge of base on the ground in such a way that the edge of the base on which it rests is inclined at 45° to the V.P. and the base itself is inclined at 60° to the H.P. Draw its projections.

11.23 A square pyramid whose faces are isosceles triangle of base 50 mm and altitude 70 mm is resting on one of its faces on the H.P. such that the top view of the axis is inclined at 30° to the V.P. Draw its projections.

11.24 A pentagonal prism of base side 30 mm and height 60 mm rests on one of its base side on the H.P. such that the rectangular face contained by that edge appears as a square of 30 mm side in the top view. Draw its projections when the edge on the H.P. is inclined at 30° to the V.P.

11.25 A square pyramid of base edge 40 mm and axis 60 mm is resting on an edge of base on the H.P. such that the triangular face contained by that edge appears as an equilateral triangle in the top view. Draw its projections when the edge on the H.P. is inclined at 30° to the V.P.

11.26 A hexagonal pyramid of base side 30 mm and axis 60 mm rests on an edge of base on the H.P. such that the face contained by that edge appears as a straight line parallel to the reference line in the top view. Draw its projections.

[*Hints: Solution same as Fig. 11.39*]

11.27 A tetrahedron *ABCV* of edge 60 mm has an edge *AB* on the ground and the face *ABV* is inclined to H.P. such that the plan of *ABV* is a right angled triangle. Draw its projections when the edge *AB* is inclined at 30° to the V.P.

11.28 A bucket is in the form of the frustum of a cone has bottom diameter 75 mm, top diameter 30 mm and height 80 mm. The bucket is filled with water and then tilted through 45°. Draw its projections showing water surface in both views. The axis of the bucket is parallel to the V.P.

11.29 A crucible has top diameter 100 mm and bottom diameter 50 mm and height 80 mm. It is completely filled with molten metal and then tilted to pour the metal such that its axis is inclined at 45° to its initial vertical position. Show the surface of the metal in both the views. Wall thickness of the crucible is to be ignored.

An element of the solid in the V.P.

11.30 A hexagonal prism of base side 30 mm and axis 75 mm has an edge of the base on the V.P. and inclined at 30° to the H.P. The rectangular face containing that edge is inclined at 45° to the V.P. Draw its projections.

11.31 A pentagonal pyramid of base edge 30 mm and axis 70 mm has an edge of its base in the V.P. and parallel to the H.P. such that the triangular face contained by that edge is inclined at 45° to the V.P. Draw its projections.

11.32 A pentagonal pyramid of base side 30 mm and axis 70 mm has a triangular face in the V.P. and a slant edge containing that face is parallel to H.P. Draw its projections.

11.33 A square pyramid of base side 40 mm and axis 75 mm has a triangular face in the V.P. and an inclined plane containing the axis is inclined at 45° to the H.P. Draw its projections when its base is closer to the H.P. than the apex.

Condition of apparent angle

11.34 A square prism of base side 40 mm and axis 65 mm is resting on an edge of its base on the H.P. with axis inclined at 45° to the H.P. and 30° to the V.P. Draw its projections.

11.35 Draw the projections of a cone of base diameter 30 mm and axis 65 mm when it is resting on one of its generators on the ground and (a) that generator is inclined at 30° to the V.P., (b) the axis is inclined at 30° to the V.P.

11.36 Draw the projections of a pentagonal prism of base side 30 mm and axis 60 mm, resting on one of its edges of the base on the ground with the axis inclined at 30° to the H.P. and (a) 60° to the V.P., (b) top view of the axis is inclined at 60° to the V.P.

11.37 A cone of base side 60 mm and axis 75 mm is placed with its apex on the reference line and the axis is inclined at 30° to the H.P. and 60° to the V.P. Draw its projections.

11.15 AUXILIARY PLANE METHOD

The projections of solids inclined to one or both the reference planes using change of position method has been described earlier. Auxiliary plane method can also be used for the purpose. In this method, instead of changing the position of the views with respect to the reference line, a new reference plane is considered over which auxiliary view are projected. Auxiliary planes used for projecting the auxiliary views are of two types, namely; auxiliary inclined plane (A.I.P.) and auxiliary vertical plane (A.V.P.).

11.15.1 Auxiliary Top View

Projection of an object on an auxiliary inclined plane (A.I.P.) is called an *auxiliary top view*. An A.I.P. is a plane perpendicular to the V.P. whose front view is a line. In this case, a new reference line representing A.I.P. is drawn whose V.T. is inclined at a given angle $\theta_{\text{A.I.P.}}$ to the *xy*. All the points of the front view are projected perpendicular on the new reference line. The distance of the points from the new reference plane for the auxiliary top view are taken that of the corresponding points in the top view from the previous reference plane. Let us consider problems done earlier and solve them using auxiliary plane method.

Problem 11.52 *A pentagonal prism of base edge 30 mm and axis 70 mm rests on an edge in the H.P. Its axis is parallel to V.P. and inclined at 45° to the H.P. Draw its projections.*

Construction Refer to Fig. 11.60.

1. Draw a pentagon *abcde* keeping *cd* perpendicular to *xy*. This is the top view. Project the points and obtain $a'd'4'1'$ as the front view.
2. Draw a new reference line x_1y_1 inclined at 45° to the axis passing through $3'4'$ (considering edge 3-4 is in the H.P.).
3. Project all the points of the front view on x_1y_1 and produce them. Mark points a_1, b_1, c_1, d_1, e_1, 1_1, 2_1, 3_1, 4_1 and 5_1 on the respective projectors such that their distance from x_1y_1 is equal to the distance of points *a*, *b*, *c*, *d*, *e*, 1, 2, 3, 4 and 5 from *xy*, respectively.
4. Join the outer edges and edges connecting a_1 using continuous lines. The edge $3'4'$ is on x_1y_1, therefore join 3_14_1, 3_12_1, 3_1c_1, 4_15_1 and 4_1d_1 using dashed narrow lines. This gives $1_12_1b_1c_1d_1e_15_1$ as the required auxiliary top view.

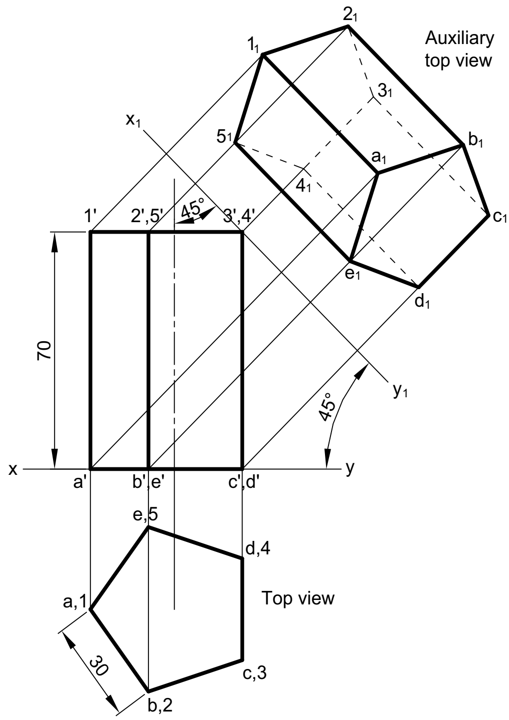


Fig. 11.60

Problem 11.53 *A hexagonal pyramid of base edge 30 mm and axis 60 mm, has a triangular face on the ground and the axis parallel to the V.P. Draw its projections.*

Construction Refer to Fig. 11.61.

1. Draw a hexagon *abcdef* keeping *de* perpendicular to *xy*. Join the corners of the hexagon with the centroid *o*. This represents the top view. Project all the corners and obtain $b'd'o'$ as the front view.
2. Draw a new reference line x_1y_1 coinciding with $o'd'e'$ (considering face *ODE* on H.P.).
3. Project all points of the front view on x_1y_1 and produce them. Mark points a_1, b_1, c_1, d_1, e_1, f_1 and o_1 on the respective projectors such that their distance from x_1y_1 is equal to the distance of *a*, *b*, *c*, *d*, *e*, *f* and *o* from *xy*.
4. Join the outer edges and edges connecting a_1 and b_1 using continuous lines. The face $o'd'e'$ is on x_1y_1, therefore join o_1d_1 and o_1e_1 using dashed narrow lines. This gives $c_1d_1e_1f_1o_1$ as the required auxiliary top view.

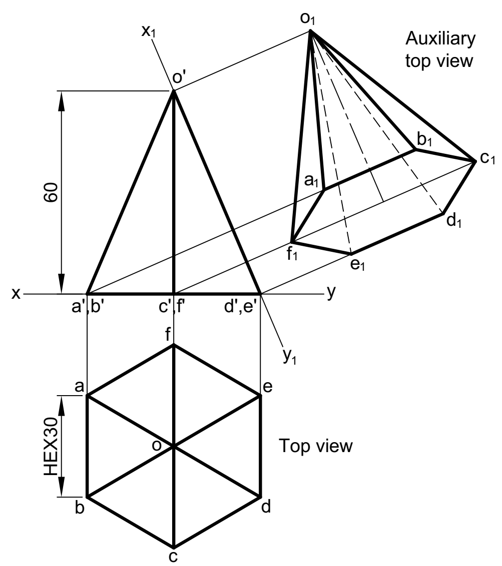


Fig. 11.61

Problem 11.54 *A hexagonal pyramid of base side 30 mm and axis 70 mm is resting on its base on the H.P. with a side of the base parallel to the V.P. Draw its projections. Project an auxiliary top view on a plane inclined at 30° to the reference line.*

Construction Refer to Fig. 11.62.

1. Draw a hexagon *abcdef* keeping *ef* parallel to *xy*. Join the corners of the hexagon with the centroid *o*. This represents the top view. Project all the corners and obtain $b'd'o'$ as the front view. Draw a new reference line x_1y_1 inclined at 30° to *xy*.
2. Project all points of the front view on x_1y_1 and produce them. Mark points a_1, b_1, c_1, d_1, e_1, f_1 and o_1 on the respective projectors such that their distance from x_1y_1 is equal to the distances of points *a*, *b*, *c*, *d*, *e*, *f* and *o* from *xy*.
3. Join the outer edges and edges connecting a_1 using continuous lines. Join the other edges c_1o_1, d_1o_1 and e_1o_1 using dashed narrow lines. This gives $b_1c_1d_1e_1f_1o_1$ as the required auxiliary top view.

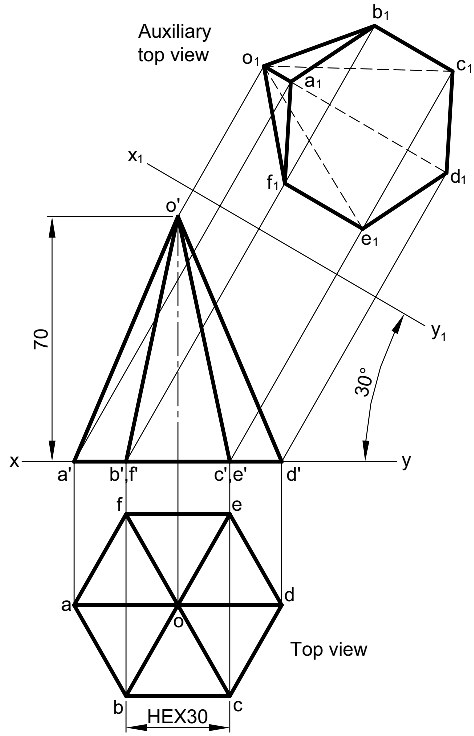


Fig. 11.62

Problem 11.55 *Draw the projections of the frustum of a pentagonal pyramid of base edge 40 mm, top edge 20 mm and axis 60 mm, resting on its base on the H.P. with a side of base perpendicular to the V.P. Project another top view on an auxiliary inclined plane parallel to the line showing true length of slant edge.*

Construction Refer to Fig. 11.63.

1. Draw two concentric pentagons keeping *ab* perpendicular to *xy*. Join *a*1, *b*2, *c*3, *d*4 and *e*5. This is the top view. Project the points and obtain $a'd'4'1'$ as front view.
2. As *d*4 is parallel to *xy*, $d'4'$ represents true length in the front view. Draw a new reference line x_1y_1 parallel to $d'4'$.
3. Project all points of the front view on x_1y_1 and produce them. Mark points a_1, b_1, c_1, d_1, e_1, 1_1, 2_1, 3_1, 4_1 and 5_1 on the respective projectors such that their distance from x_1y_1 is equal to the distance of points *a*, *b*, *c*, *d*, *e*, 1, 2, 3, 4 and 5 from *xy*.
4. Join the outer edges and edges connecting a_1 and b_1 using continuous lines. The edge $4'd'$ is towards x_1y_1, therefore join 4_1d_1, 4_15_1 and 4_13_1 using dashed narrow lines. This gives $1_12_13_1c_1d_1e_15_11_1$ as the required auxiliary top view.

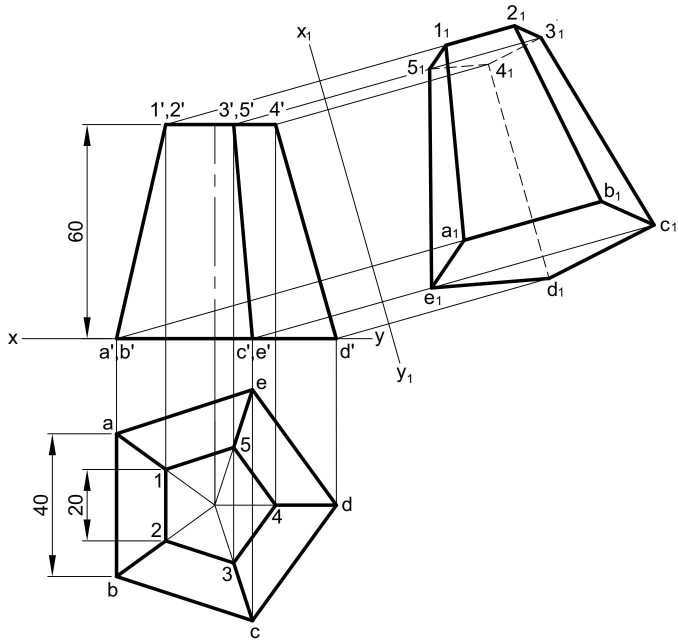


Fig. 11.63

11.15.2 Auxiliary Front View

Projection of an object on an auxiliary vertical plane (A.V.P.) is called auxiliary front view. An A.V.P. is a plane perpendicular to the H.P. whose top view is a line. In this case, a new reference line representing A.V.P. is drawn whose H.T. is inclined at a given angle $\phi_{A.V.P.}$ to the xy. All the points of the top view are projected perpendicular on the new reference line. The distance of the points from the new reference plane for the auxiliary front view are taken that of the corresponding points in the front view from the previous reference plane. Let us consider problems done earlier and solve them using auxiliary plane method.

Problem 11.56 *A hexagonal prism of base edge 30 mm and axis 70 mm has an edge of its base in the V.P. such that the axis is inclined at 30° to the V.P. and parallel to the H.P. Draw its projections.*

Construction Refer to Fig. 11.64.

1. Draw a hexagon $a'b'c'd'e'f'$ keeping $d'e'$ perpendicular to xy. This is the front view. Project the points and obtain $ad41$ as the top view.
2. Draw a new reference line x_1y_1 passing through 4-5 and inclined at 60° to xy. Thus, x_1y_1 is inclined at 30° to the axis.
3. Project all the points of the front view on x_1y_1 and produce them. Mark points a_1', b_1', c_1', d_1', e_1', f_1', $1_1'$, $2_1'$, $3_1'$, $4_1'$, $5_1'$ and $6_1'$ on the projectors such that their distance from x_1y_1 is equal to the distance of points a', b', c', d', e', f', $1'$, $2'$, $3'$, $4'$, $5'$ and $6'$ from xy, respectively.
4. Join outer edges and edges connecting $a1'$ and $b1'$ using continuous lines. Join edges connecting $4_1'$ and $5_1'$ using dashed narrow lines. This gives $1_1'2_1'3_1'c_1'd_1'e_1'f_1'6_1'$ as the required auxiliary front view.

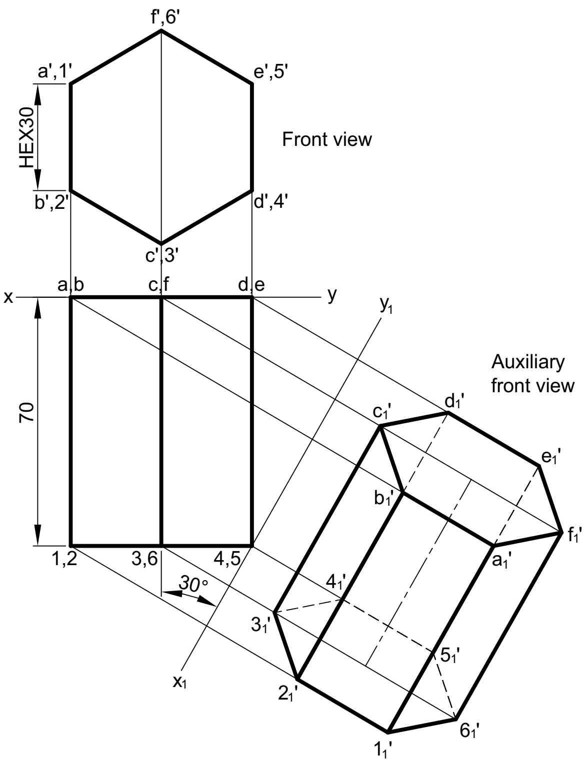


Fig. 11.64

Problem 11.57 *A pentagonal prism of base edge 30 mm and axis 60 mm is resting on an edge of its base in the H.P. and the axis inclined at 30° to the H.P. Draw its projections. Also, draw an auxiliary front view on an A.I.P. inclined at 45° to the resting edge.*

Construction Refer to Fig. 11.65.

1. **First stage** Draw a pentagon *abcde* keeping *cd* perpendicular to *xy*. This is the top view. Project the corners and obtain $a'c'3'1'$ as the front view.
2. **Second stage** Reproduce the front view of first stage keeping $c'd'$ on *xy* and $c'd'$ inclined at 30° to *xy*. Obtain *a*, *b*, *c*, *d*, *e*, 1, 2, 3, 4 and 5 as the intersecting points of the vertical projectors drawn from the front view of the second stage with the horizontal projectors drawn from the top view of the first stage. Join the points and obtain *ab*2345*e* as the top view.
3. **Auxiliary front view** Draw a new reference line x_1y_1 inclined at 45° to *cd*. Project all the points of the top view on x_1y_1 and produce them. Mark points a_1', b_1', c_1', d_1', e_1', $1_1'$, $2_1'$, $3_1'$, $4_1'$ and $5_1'$ on the respective projectors such that their distance from x_1y_1 is equal to the distance of points a', b', c', d', e', $1'$, $2'$, $3'$, $4'$ and $5'$ from *xy*.
4. Join outer edges and edges connecting e_1' using continuous lines. Join edges connecting $2_1'$ and $3_1'$ using dashed narrow lines. This gives $a_1'b_1'c_1'd_1'4_1'5_1'1_1'$ as the required auxiliary front view.

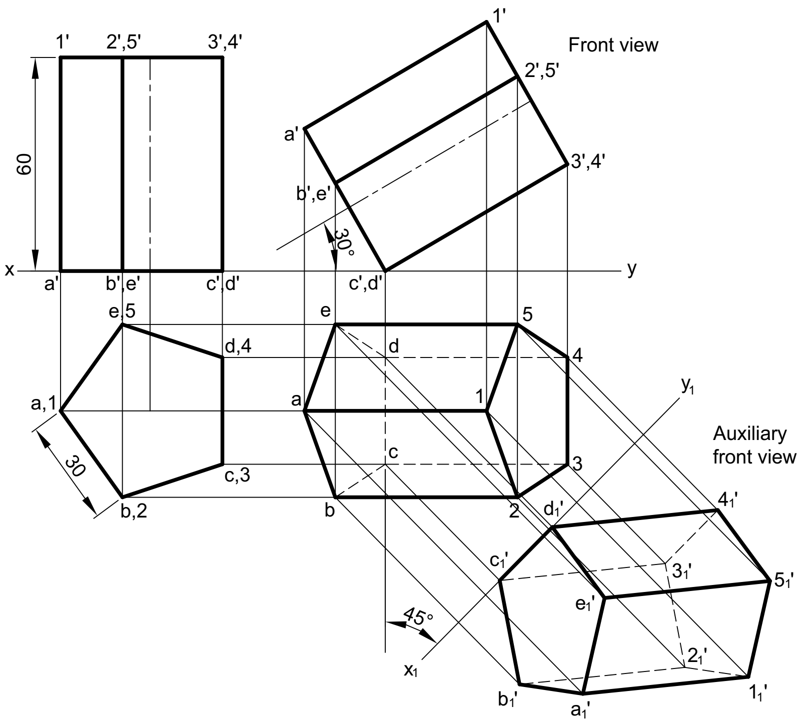


Fig. 11.65

Problem 11.58 *A square pyramid of base side 40 mm and axis 60 mm is resting on a corner of its base in the H.P. such that highest point of the base is 40 mm above the H.P. and the axis is parallel to the V.P. Draw its projections. Also, draw an auxiliary front view on a plane inclined at 30° to the top view of the axis.*

Construction Refer to Fig. 11.66.

1. **First stage** Draw a square *abcd* keeping ac parallel to *xy*. Join the corners of the square with the centroid *o*. This represents the top view. Project all the corners and obtain $a'c'o'$ as the front view.
2. **Second stage** Reproduce the front view of the first stage keeping c' on *xy* and a' 40 mm above *xy*. Obtain *a*, *b*, *c*, *d* and *o* as the intersecting points of the vertical projectors drawn from the front view of the second stage with the horizontal projectors drawn from the top view of the first stage. Join the points and obtain *abod* as the top view.
3. **Auxiliary front view** Draw a new reference line x_1y_1 inclined at 30° to *aco*. Project all the points of the top view on x_1y_1 and produce them. Mark points a_1', b_1', c_1', d_1' and o_1' on the respective projectors such that their distance from x_1y_1 is equal to the distance of points a', b', c', d' and o' from *xy*.
4. Join outer edges and edges connecting d_1' using continuous lines. Join edge $b_1'o_1'$ using dashed narrow lines. This gives $a_1'b_1'c_1'o_1'$ as the required auxiliary front view.

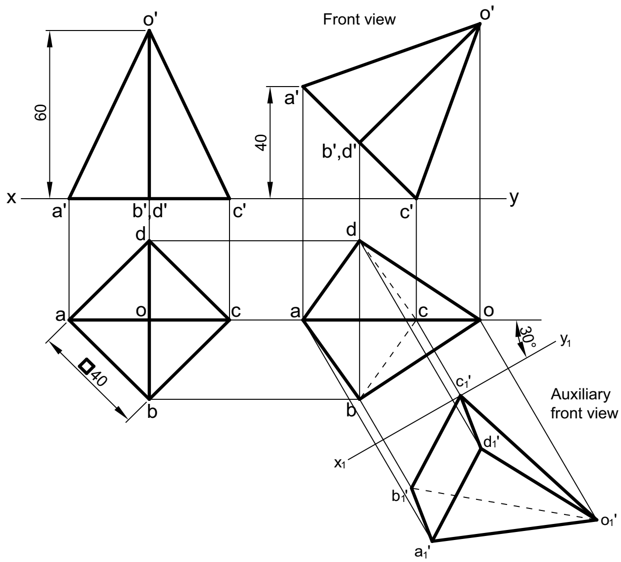


Fig. 11.66

11.15.3 Auxiliary Front and Top Views

When the solid is inclined to both the reference planes, then it is required to obtain auxiliary front and the top views to represent the final projections. Consider the following problems.

Problem 11.59 *A hexagonal pyramid of base side 30 mm and axis 60 mm has an edge of its base on the ground and the axis inclined at 30° to the H.P. The edge of the base on which it rests is inclined at 45° to the V.P. Draw its projections.*

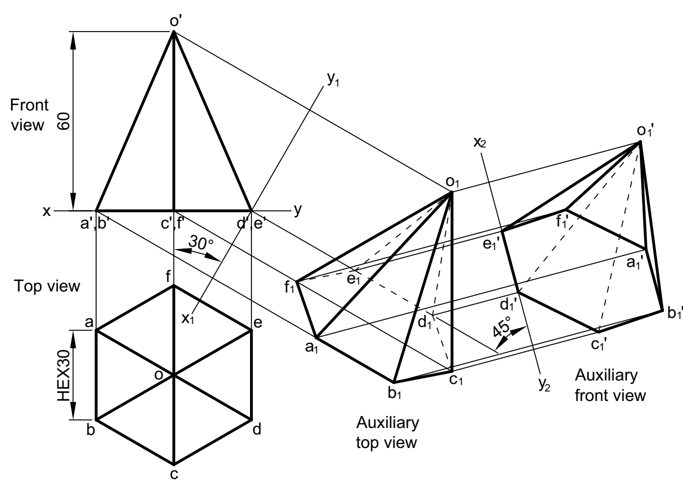


Fig. 11.67

Construction Refer to Fig. 11.67.

1. Draw a hexagon *abcdef* keeping de perpendicular to *xy*. Join the corners of the hexagon with the centroid *o*. This represents the top view. Project the corners and obtain $b'd'o'$ as the front view.
2. **Auxiliary top view** Draw reference line x_1y_1 passing through $d'e'$ inclined at 30° to $o'c'f'$. Project all the points of the front view on x_1y_1 and produce them. Mark points a_1, b_1, c_1, d_1, e_1, f_1 and o_1 on the respective projectors such that their distances from x_1y_1 is equal to the distances of points *a*, *b*, *c*, *d*, *e*, *f* and *o* from *xy*. Join the points and obtain $a_1b_1c_1o_1f_1$ as the auxiliary top view.
3. **Auxiliary front view** Draw another reference line x_2y_2 inclined at 45° to d_1e_1. Project all the points of the auxiliary top view on x_2y_2 and produce them. Mark points a_1', b_1', c_1', d_1', e_1', f_1' and o_1' on the respective projectors such that their distances from x_2y_2 is equal to the distances of points a', b', c', d', e', f' and o' respectively from x_1y_1. Join the points and obtain $b_1'c_1'd_1'e_1'o_1'$ as the auxiliary front view.

Problem 11.60 *A hexagonal pyramid of base side 30 mm and axis 60 mm has one of its slant edges on the H.P. and inclined at 30° to the V.P. Draw its projections when the base is visible.*

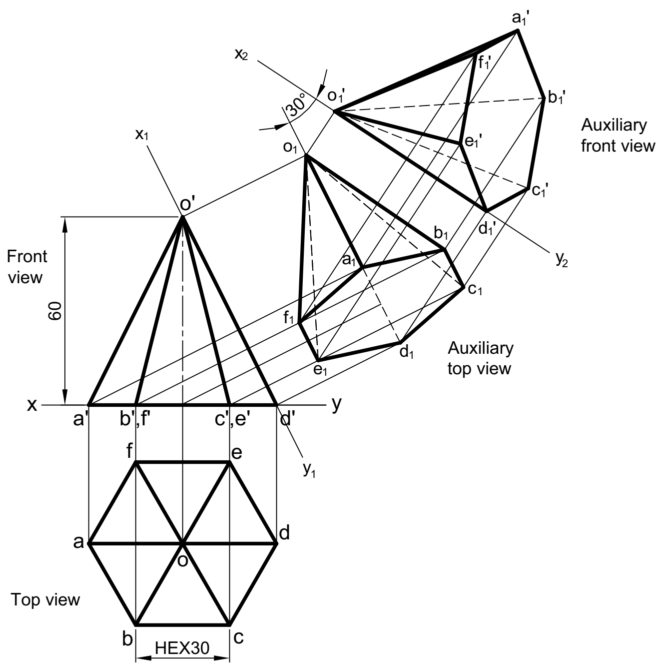


Fig. 11.68

Construction Refer to Fig. 11.68.

1. Draw a hexagon *abcdef* keeping *ad* parallel to *xy*. Join the corners of the hexagon with the centroid *o*. This represents the top view. Project the corners and obtain $a'd'o'$ as the front view.
2. **Auxiliary top view** Draw reference line x_1y_1 passing through $o'd'$.

Project all the points of the front view on x_1y_1 and produce them. Mark points a_1, b_1, c_1, d_1, e_1, f_1 and o_1 on the respective projectors such that their distance from x_1y_1 is equal to the distance of points a, b, c, d, e, f and o from xy. Join the points and obtain $b_1c_1d_1e_1f_1o_1$ as the auxiliary top view.

3. **Auxiliary front view** Draw another reference line x_2y_2 inclined at 30° to $o_1a_1d_1$. Project all the points of the auxiliary top view on x_2y_2 and produce them. Mark points a_1', b_1', c_1', d_1', e_1', f_1' and o_1' on the respective projectors such that their distance from x_2y_2 is equal to the distance of points a', b', c', d', e', f' and o' respectively from x_1y_1. Join the points and obtain $a_1'b_1'c_1'd_1'o_1'$ as the auxiliary front view.

11.16 PROJECTIONS OF SPHERES

A sphere occurring in any orientation in a space appears as a circle of diameter equal to the diameter of the sphere.

Problem 11.61 *Two spheres, each of 70 mm diameters, touching each other are resting on the H.P. Draw their projections when the line joining their centres is inclined at 45° to the V.P. Also show the point of their contact in the front view.*

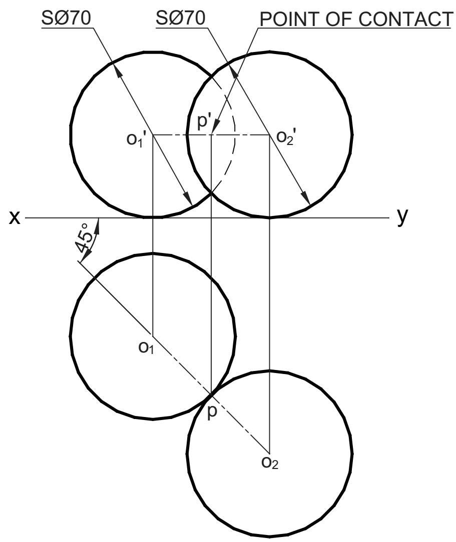


Fig. 11.69

Construction Refer to Fig. 11.69.

1. Draw two circles each of diameters 70 mm with centres o_1 and o_2 such that o_1o_2 is 70 mm and inclined at 45° to the V.P. This represents the top view.
2. Project the centres and obtain o_1' and o_2', 35 mm above xy. Draw circles each of 70 mm diameters with centres o_1' and o_2'. This is the required front view. It may be noted that sphere with centre o_2' is fully visible, while the sphere with centre o_1' is partially visible.
3. Locate point p on o_1o_2 where spheres touch each other. Project point p and obtain p' on $o_1'o_2'$. This p' represents the point of contact in the front view.

Problem 11.62 *Two spheres of diameters 70 mm and 30 mm are placed on the H.P. touching each other. Draw their projections when the line joining their centres is parallel to the V.P. Also show the point of contact of spheres in the top view.*

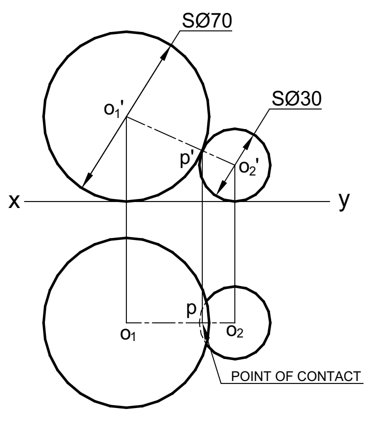


Fig. 11.70

Construction Refer to Fig. 11.70.

1. Locate centre o_1' 35 mm above xy and draw a circle of diameter 70 mm. This is the front view of first sphere. Project the centre and mark centre o_1 in the top view. Draw the circle of diameter 70 mm with centre o_1. This is the top view of first sphere.

2. Mark centre o_2' 50 mm $(r_1' + r_2')$ mm from centre o_1' and 15 mm above *xy*. Draw a circle of diameter 15 mm with centre o_2'. This is the front view of second sphere. Project the centre o_2' to meet the horizontal line from centre o_1 at centre o_2. Draw circle of diameter 30 mm with centre o_2. This is the top view of the second sphere.
3. Mark point p' on $o_1'o_2'$ in the front view at the point of contact of the spheres. Project p' on o_1o_2 and obtain p as the point of contact in the top view.

Problem 11.63 *Three spheres of radii 40 mm, 25 mm and 15 mm are placed on the H.P., each touching the other two. Draw their projections when the line joining the centres of the two bigger spheres is parallel to the V.P.*

Construction Refer to Fig. 11.71.

1. Mark centres a', 40 mm above *xy* and b', 25 mm above *xy*, such that a' and b' are 65 mm $(r_a + r_b)$ apart. Draw circles of radii 40 mm and 25 mm with centres a' and b' respectively. This is the front view of spheres *a* and *b*.
2. Project the centres and obtain points *a* and *b* on a line parallel to *xy*. Draw circles of radii 40 mm and 25 mm with centres *a* and *b* respectively. This is the top view of spheres *a* and *b*.
3. Assume sphere *c* is in contact with sphere *a*. Locate c_1' at a height of 15 mm from *xy* and 55 mm $(r_a + r_c)$ from a'. Project the centre c_1' and obtain point c_1 on the line *ab* produced. Draw circles of radius 15 mm with centres c_1' and c_1.

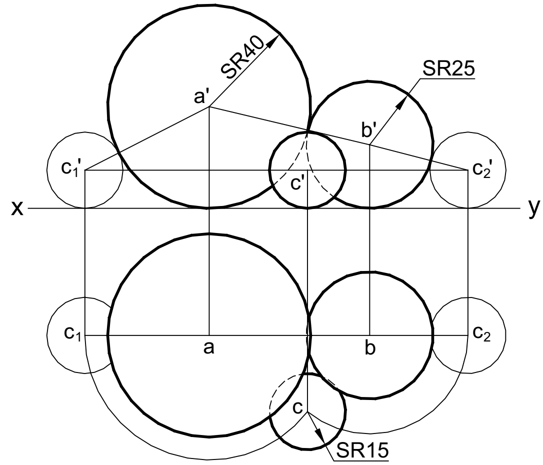


Fig. 11.71

4. Assume sphere *c* is in contact with sphere *b*. Locate c_2' at a height of 15 mm from *xy* and 40 mm $(r_b + r_c)$ from b'. Project the centre c_2' and obtain point c_2 on the line *ab* produced. Draw circles of radius 15 mm with centres c_2' and c_2.
5. Draw arcs with centres *a* and *b* of radii ac_1 and bc_2 respectively, to meet each other at point *c*. This is the final centre of sphere *c* in the top view.
6. Project point *c* to meet line locus line $c_1'c_2'$ at point c'. With centres *c* and c' draw circles of radius 15 mm. This gives the required front and top views of sphere *c*.

Problem 11.64 *Three equal spheres of diameters 70 mm are placed on the H.P., each touching the other two and line joining the centres of two of them is perpendicular to the V.P. A fourth sphere of 50 mm diameter is placed over the three spheres such that it is in contact with all of them. Draw the projections of the arrangement and determine the distance of the centre of the fourth sphere above the H.P.*

Construction Refer to Fig. 11.72.

1. Draw an equilateral triangle *abc* with side *ab* perpendicular to *xy*. Draw three circles each of 70 mm diameter with centres *a*, *b* and *c*. This is the top view of three equal size spheres.
2. Mark *d* as the centroid of the triangle *abc*. Draw a circle of 50 mm diameter with centre *d*. This is the top view of the fourth sphere.

3. Project points *a*, *b* and *c*, 35 mm above *xy* and obtain *a*′, *b*′ and *c*′. Draw circles of 70 mm diameter with the centres *a*′, *b*′ and *c*′. This is the front view of three equal size spheres.
4. As *cd* is parallel to *xy*, *c*′*d*′ represents the true length. Therefore, draw an arc with centre *c*′ of radius 60 mm (35 mm + 25 mm) to meet the projector of point *d* at point *d*′. Draw circle of 50 mm diameter with the centre *d*′. This is the front view of the fourth sphere.
5. Measure the distance of *d*′ from *xy*. Here the centre of the fourth sphere is 79 mm above H.P.

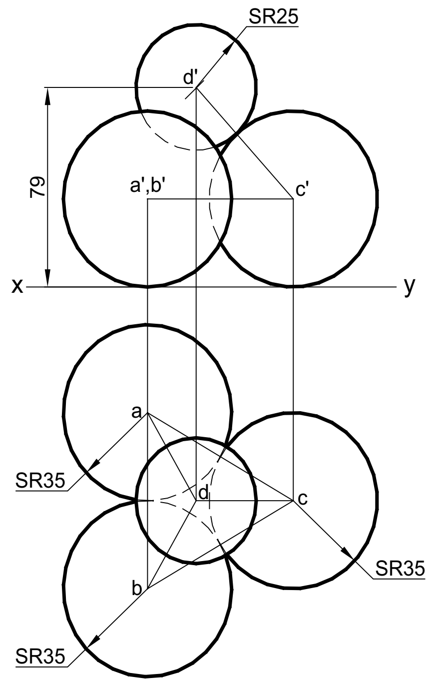


Fig. 11.72

Problem 11.65 *A hemispherical bowl of diameter 150 mm is resting on its curved surface on the H.P. Three equal spheres of diameter 50 mm are placed in the bowl, each touching the other two such that the line joining the centres of two of them is perpendicular to V.P. Draw the projections of the arrangement, neglecting thickness of the bowl.*

Construction Refer to Fig. 11.73.

1. Mark centre *o*′, 75 mm above *xy* and draw a semicircle. Project it and obtain a circle of diameter 75 mm with centre *o*. This represents the front and top views of the hemispherical bowl.
2. Draw an equilateral triangle *abc* of 50 mm side keeping *ab* perpendicular to *xy* and *o* as the centroid. Draw circles of diameter 50 mm with centres *a*, *b* and *c*. This represents the top view of the three equal spheres.
3. As *oc* is parallel to *xy*, *o*′*c*′ gives the true length. Draw an arc with centre *o*′ and radius 50 mm (75 mm – 25 mm) to meet the projector of point *c* at point *c*′.
4. Project *a* and *b* to meet a horizontal line from point *c*′ at points *a*′ and *b*′. Draw circles of 50 mm diameter with centres *a*′, *b*′ and *c*′. This represents the front view of the three equal spheres.

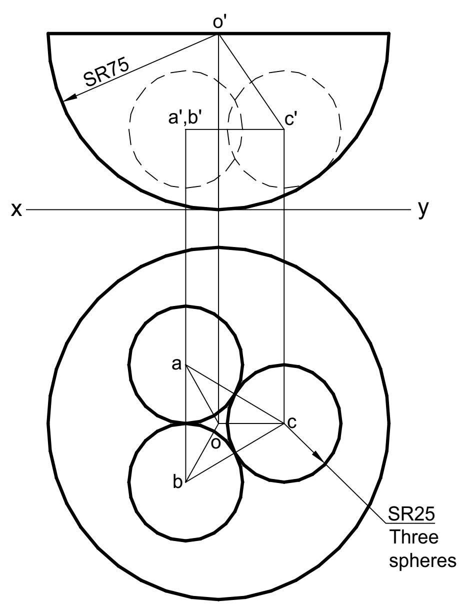


Fig. 11.73

Problem 11.66 *A small portion of the sphere of diameter 70 mm is cut by a plane such that a flat surface of diameter 50 mm is formed. It is kept on a point of this flat on the H.P. making an angle of 30° with the H.P. Draw its projections.*

Construction Refer to Fig. 11.74.

1. **First stage** Draw the front view of the sphere of diameter 70 mm and cut the bottom portion to obtain the flat surface of diameter 50 mm.
2. Obtain the top view of the truncated sphere as concentric circles. Divide the inner circle into 12 equal parts. Project them and obtain points in the front view.
3. **Second stage** Reproduce the front view keeping flat surface inclined at 30° to the H.P.
4. Project the points of the flat surface to meet the horizontal line from the points of the top view of the first stage and obtain an ellipse. Also project centre *o′* to obtain centre *o* on its locus. Draw circle of diameter 70 mm to represent the outer boundary of the sphere.

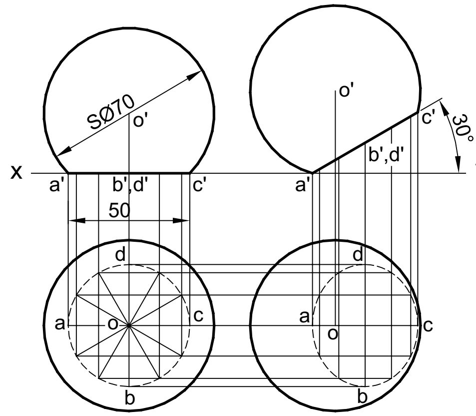


Fig. 11.74

Problem 11.67 *Three equal cones of base diameter 50 mm and axis 70 mm are resting on their base on the H.P., each touching the other two. A sphere of diameter 40 mm is placed centrally between them. Draw three views of the arrangement and determine the distance of the centre of the sphere above the H.P.*

Construction Refer to Fig. 11.75.

1. Draw a triangle *abc* of side 50 mm keeping *ab* perpendicular to *xy*. Draw circles each of diameter 50 mm with centres *a*, *b* and *c*. This represents top view of the cones. Project the points and obtain front view for the cones.
2. Mark centre *o* as the centroid of the triangle *abc*. Draw a circle with centre *o* of diameter 40 mm to represent the top view of the sphere.
3. As *oc* is parallel to *xy*, *o′c′* gives the true length. Draw a line *mn* parallel to and 20 mm from the end generators of the cone *c* to meet projector of *o* at point *o′*.
4. Draw a circle with centre *o′* of diameter 40 mm to represent the front view of the sphere.
5. Measure the distance of *o′* from *xy*. Here the centre of the sphere is 49 mm above H.P.

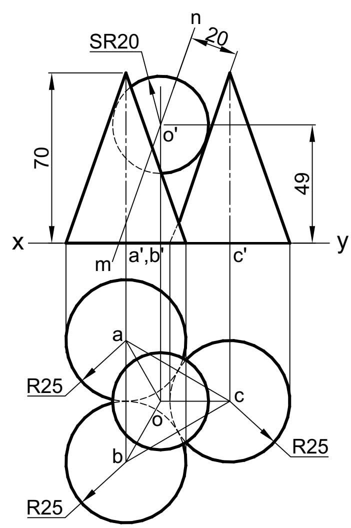


Fig. 11.75

Problem 11.68 *Six equal spheres are resting on the H.P., each touching the other two spheres and a vertical face of a hexagonal prism of base side 30 mm and axis 60 mm. Determine the diameter of the spheres and draw the projections of the arrangement when a side of the base of the prism is perpendicular to the V.P.*

Construction Refer to Fig. 11.76.

1. Draw hexagon *abcdef* keeping *de* perpendicular to *xy*. This is the top view of the prism. Project the points and obtain its front view.

2. Mark o as the centroid of the hexagon. Join od and oe and produce them, say up to points m and n respectively.
3. Bisect angle mde and ned to intersect each other at o_1. Join oo_1 to meet de at point p. Length po_1 represents the radius of the spheres. Here it is 26 mm.
4. Project o_1 and obtain o_1', 26 mm above xy. Draw circles with centres o_1 and o_1' each of radius 26 mm to represent the top and front views of the sphere 1.
5. Draw a circle with centre o of radius oo_1. Let the perpendicular bisectors of the edges of the base of the prism meet this circle at points o_2, o_3, o_4, o_5 and o_6. Draw circles of radius 26 mm with each centre to represent the top view of other spheres.
6. Project points o_2, o_3, o_4, o_5 and o_6 to obtain o_2', o_3', o_4', o_5' and o_6' on a horizontal line through o_1'. Draw circles of radius 26 mm with each centre to represent the front view of other spheres.

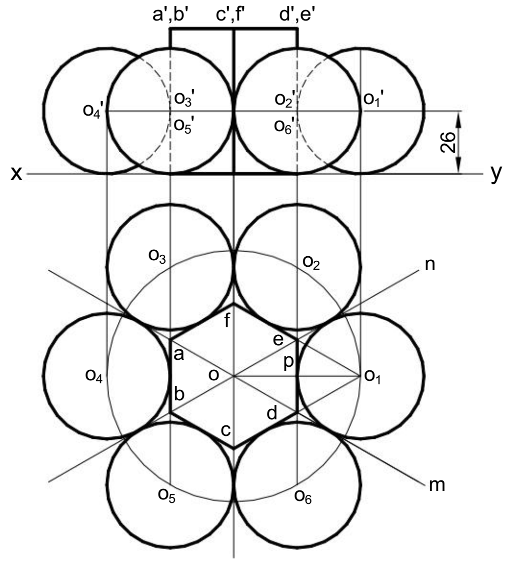


Fig. 11.76

Problem 11.69 *Four equal spheres are resting on the H.P., each touching the other two spheres and a triangular face of a square pyramid of base side 40 mm and axis 70 mm. Determine the diameters of the spheres and draw the projections of the arrangement when a side of the base of the pyramid is perpendicular to the V.P.*

Construction Refer to Fig. 11.77.

1. Draw square $abcd$ keeping ab perpendicular to xy. Join the corners of the square with its centroid o. This is the top view of the pyramid. Project the points and obtain $b'c'o'$ as its front view.
2. Produce oc and od to mark points m and n respectively.
3. Bisect angle mcd and angle ndc to intersect each other at point p. Join op to meet edge cd at point e.
4. Project point p vertical, to meet line xy at p''. Extend it to locate point p' such that $c'p'' = ep = p''p'$. Join $o''p'$.
5. Draw bisector of angle $o'c'p''$ as $c'q'$. Let $c'q'$ intersect $c'p'$ at point o_1'. Project o_1' vertical to meet line ep at point o_1. The points o_1 and o_1' represents to centres for sphere 1.

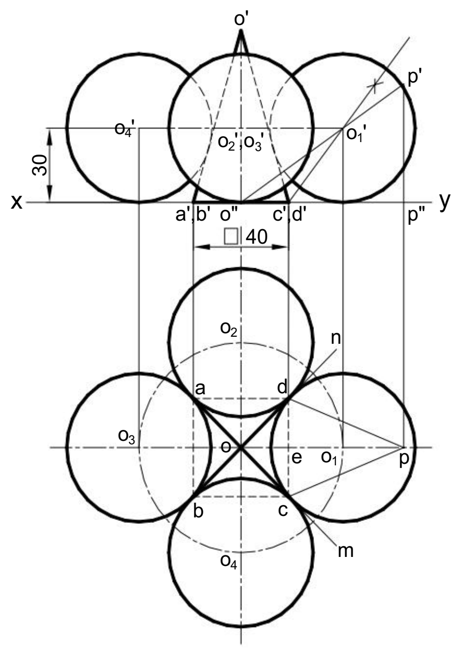


Fig. 11.77

6. Draw circles with centres o_1 and o_1' of radius co_1 to represent top and front views of sphere 1.
7. Draw a circle with centre o of radius oo_1. Let the perpendicular bisectors of the edges of the base of the pyramid meet this circle at points o_2, o_3 and o_4. Draw circles with each centre of radius co_1 to represent the top view of other spheres.
8. Project points o_2, o_3 and o_4 to obtain o_2', o_3' and o_4' on a horizontal line through o_1'. Draw circles with each centre of radius co_1 to represent front view of other spheres.

EXERCISE 11D

Auxiliary plane method

11.1 A pentagonal prism of base edge 30 mm and axis 70 mm has an edge of its base in the H.P. Its axis is parallel to the V.P. and inclined at 30° to the H.P. Draw its projections using auxiliary plane method.

11.2 A square prism of base side 30 mm and axis 70 mm is resting on its base in the V.P. with a diagonal of the base perpendicular to the H.P. Draw its projections. Also, draw its projections on an auxiliary vertical plane passing through a corner and inclined at 45° to the V.P.

11.3 A hexagonal pyramid of base side 30 mm and axis 70 mm is resting on a corner of its base in the H.P. such that the apex is 50 mm above the H.P. Project a front view on an A.V.P. making an angle of 30° with the reference line.

11.4 Draw projections of a cone of base diameter 50 mm and axis 65 mm is resting on a generator in the H.P. Project another front view on a plane perpendicular to the top view of the axis of the cone.

11.5 Draw projections of a pentagonal prism of base edge 30 mm axis 70 mm is lying on an edge of its base in the H.P. with the highest point of its lower base 25 mm above the H.P. Project another front view on a reference line making an angle of 60° with the top view of the axis.

11.6 A pentagonal pyramid of base side 30 mm and axis 70 mm is resting on an edge of its base in the H.P. inclined at 45° to the V.P. Draw its projections when the face containing this edge is vertical. Use auxiliary plane method.

Projections of spheres

11.7 Draw the projections of a hemisphere resting on a point with its flat face making 30° to H.P.

11.8 Two spheres of diameters 60 mm are resting on H.P. touching each other. Draw its projections when the line joining their centres is inclined at 60° to V.P. and parallel to H.P. Show the point of contact in the front view.

11.9 Two spheres of diameters 60 mm and 30 mm are placed on H.P. touching each other. Draw its projections when the line joining their centres is parallel to V.P.

11.10 Two spheres of diameters 70 mm and 25 mm are placed on H.P. touching each other. Draw its projections when the line joining their centres is inclined at 30° to V.P. Show point of contact of spheres in the top view.

11.11 Three equal spheres of diameters 70 mm are placed on H.P., each touching other the other two and the line joining the centres of two of them is parallel to V.P. A fourth sphere of same diameter is placed over the three spheres so as to form a pile. Draw the three views of the arrangement and determine the distance of fourth sphere from the H.P.

11.12 Three spheres of diameters 70 mm, 50 mm and 30 mm are placed on H.P., each touching the other two. Draw its projections when the line joining the centres of the two spheres is parallel to V.P.

11.13 Three equal spheres of diameters 60 mm are placed on H.P., each touching other the other two and the line joining the centres of two of them is inclined at 45° to V.P. A fourth sphere of same diameter is placed over the three spheres so as to form a pile. Draw the projections of the arrangement and determine the distance of fourth sphere from the H.P.

11.14 A hemispherical bowl of diameter 120 mm is resting on its curved surface on the ground. Three equal spheres of diameter 40 mm are placed in the bowl, each touching the other two such that the line joining the centres of two of them is parallel to the H.P. and perpendicular to V.P. Draw the projections of the arrangement and determine the height of the spheres from the ground.

11.15 A lamp shed is in the shape of truncated sphere of diameter 70 mm with flat circular top diameter 25 mm and bottom diameter 40 mm and parallel to each other. Draw its projections if the line passing through the centres of the circles and the centre of the sphere is parallel to V.P. and inclined at 30° to the H.P.

11.16 Four equal spheres are resting on the H.P., each touching the other two spheres and a vertical face of the square prism of base side 30 mm. Determine the diameter of the spheres and draw the projections of the arrangement when a side of the base of the prism is perpendicular to the V.P.

11.17 Five equal spheres are resting on the H.P., each touching the other two spheres and a vertical face of a pentagonal prism of base side 30 mm and axis 70 mm. Determine the diameter of the spheres and draw the projections of the arrangement when a side of the base of the prism is parallel to the V.P.

11.18 Five equal spheres are resting on the H.P., each touching the other two spheres and a triangular face of a pentagonal pyramid of base side 30 mm and axis 65 mm. Determine the diameter of the spheres and draw the projections of the arrangement when a side of the base of the pyramid is perpendicular to the V.P.

11.19 Six equal spheres are resting on the H.P., each touching the other two spheres and a triangular face of a hexagonal pyramid of base side 30 mm and axis 70 mm. Determine the diameter of the spheres and draw the projections of the arrangement when a side of the base of the pyramid is parallel to the V.P.

11.20 Three equal cones of base diameter 50 mm and axis 70 mm are resting on its base the H.P., each touching the other two. A sphere of diameter 40 mm is placed centrally between them. Draw the projections of the arrangement when a plane containing the axes of two cones is perpendicular to the V.P.

VIVA-VOCE QUESTIONS

11.1 Differentiate between a triangular pyramid and a tetrahedron.

11.2 State the shape and number of faces in dodecahedron and icosahedron.

11.3 Define cylinder and cone in terms of surface of revolution.

11.4 What do you understand by a right regular solid?

11.5 Differentiate between frustum of a pyramid and a truncated pyramid.

11.6 A cube is resting on one of its corners in the H.P. with a solid diagonal vertical. What will be the outer shape of its top view?

11.7 A cube is resting on one of its corners in the H.P. with a solid diagonal perpendicular to the V.P. What will be the outer shape of its front view?

11.8 What is the difference between the top view of a hexagonal prism and that of a hexagonal pyramid when both solids rest on their bases on the H.P. with similar orientation?

11.9 State the position of a tetrahedron so as to get a square as the outer shape in its top view.

11.10 State the location of the centroid of a square pyramid.

MULTIPLE-CHOICE QUESTIONS

11.1 Among the following solids, a regular polyhedron is
(a) square prism
(b) square pyramid
(c) cube
(d) sphere

11.2 A solid having minimum number of faces is
(a) tetrahedron
(b) triangular prism
(c) square pyramid
(d) cube

11.3 A pyramid is cut by a plane parallel to its base removing the apex, the remaining part is known as
(a) truncated
(b) frustum
(c) sectioned
(d) prism

11.4 Number of faces in a dodecahedron are
(a) 4
(b) 8
(c) 12
(d) 20

11.5 If three orthographic views of a sphere containing a cylindrical hole are drawn, the maximum number of circles that may appear altogether
(a) 1
(b) 3
(c) 4
(d) 6

11.6 An orthographic view of a hemisphere may appear as
(a) circle
(b) ellipse
(c) parabola
(d) hyperbola

11.7 The number of stages that are necessary to get the orthographic views of a solid having its axis inclined to both the reference planes
(a) one
(b) two
(c) three
(d) four

11.8 A tetrahedron is resting on its face on the H.P. with a side parallel to the V.P. Its front view will be
(a) equilateral triangle
(b) isosceles triangle
(c) scalene triangle
(d) right-angled triangle

11.9 A square pyramid is resting on a face in the V.P. The number of dotted lines will appear in the front view
(a) one
(b) two
(c) three
(d) four

11.10 The solid will have two dotted lines in the top view when it is resting on its face in the H.P.
(a) square pyramid
(b) pentagonal pyramid
(c) hexagonal pyramid
(d) All of these

11.11 A cube is resting on H.P. with a solid diagonal perpendicular to it. The top view will appear as
(a) square
(b) rectangle
(c) irregular hexagon
(d) regular hexagon

11.12 A right circular cone resting on a point of its base circle on the H.P. with axis inclined at 30° to the H.P. and 45° to the V.P. The angle between the reference line and top view of the axis is
(a) 30°
(b) between 30° and 45°
(c) 45°
(d) more than 45°

11.13 A right circular cone resting on a generator in the H.P. and axis inclined at 45° to the V.P. The angle between the reference line and top view of the axis is
(a) less than 45°
(b) 45°
(c) more than 45°
(d) Any of these

11.14 A cylinder rests on a point of its base circle on the H.P. with axis inclined at 30° to the H.P. and 60° to the V.P. The inclination of the top view of the axis with the reference line is
(a) 30°
(b) 60°
(c) 90°
(d) None of these

Answers to multiple-choice questions

11.1 (c), 11.2 (b), 11.3 (b), 11.4 (c), 11.5 (c), 11.6 (a), 11.7 (c), 11.8 (b), 11.9 (b), 11.10 (d), 11.11 (d), 11.12 (d), 11.13 (c), 11.14 (c)

Chapter

12 SECTIONS OF SOLIDS

12.1 INTRODUCTION

The orthographic views of a solid may contain a number of dotted lines. These lines indicate the presence of hidden details which may lie behind or somewhere in the middle of the object. The interpretation of the object's shape becomes difficult with increasing number of such lines. As a remedy, it becomes obligatory to draw sectional views for a better and easier interpretation of the internal features. The present chapter describes the methods of obtaining sectional views and other related drawing.

The object is considered to be cut by a plane called a section plane or a cutting plane. The portion of the object, which falls between the section plane and the observer, is assumed to be removed. Thus the internal details become visible. The projections of the remaining object are termed as sectional views. It is always convenient to start by drawing the orthographic views of the uncut object. Then these are modified into sectional views. The cut surface which is common to the object and the section plane is shaded with parallel lines called *hatching* to differentiate it from other surfaces. If the section plane is inclined to the plane of projection, the cut surface does not show its true shape. In such cases, it is generally required to determine the true shape of the cut surface popularly called the true shape of section.

12.2 TERMINOLOGY

The following terms are frequently used in this chapter:

1. **Section plane** It is an imaginary plane which cuts the given object to show the internal details. This plane is represented by its trace.
2. **Cut surface** It is the surface created due to cutting the object by section plane. It is shown by hatching lines.
3. **Hatching lines** These are used to indicate the cut surface. These are represented by continuous lines drawn at 45º to the reference line, parallel to each other at a uniform spacing of 2 to 3 mm. For details, refer to Section 7.19.
4. **Apparent section** It is the projection of cut surface when the section plane is not parallel to the plane of projection.
5. **True shape of section** The projection of the cut surface on a plane parallel to the section plane is known as true shape of section. It shows actual shape and size of the cut surface.

12.3 TYPES OF SECTION PLANES

Section planes are of the following types:

1. Section plane perpendicular to V.P.
   (a) Horizontal section plane
   (b) Auxiliary inclined plane (A.I.P.)
2. Section plane perpendicular to H.P.
   (a) Plane parallel to V.P.
   (b) Auxiliary vertical plane (A.V.P.)
3. Profile section plane, i.e., a plane perpendicular to both the H.P. and the V.P.
4. Oblique section plane, i.e., inclined to both H.P. and V.P. (not considered in current study)

12.3.1 Section Plane Perpendicular to V.P.

A section plane that is perpendicular to the V.P. may either be parallel or inclined to the H.P.

1. Horizontal section plane It is a plane parallel to the H.P. as shown in Fig. 12.1(a). Its front view is a straight line parallel to *xy*. This line also represents the vertical trace (V.T.) of the section plane as shown in Fig. 12.1(b). This plane has no horizontal trace. When an object is cut by a horizontal section plane, the sectional top view of the object gives the true shape of the section.

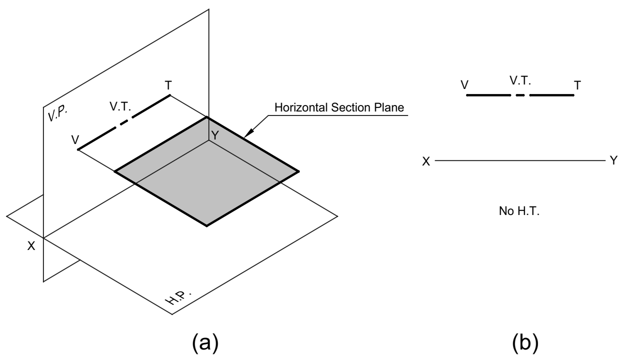


Fig. 12.1 *Horizontal section plane* **(a)** *Pictorial view* **(b)** *Orthographic view*

2. Auxiliary inclined plane (A.I.P.) It is a plane perpendicular to the V.P. and inclined to the H.P. as shown in Fig. 12.2(a). Its front view is a straight line inclined at θ to *xy*. This line also represents the vertical trace (V.T.) of the section plane as shown in Fig. 12.2(b). The horizontal trace (H.T.) of this plane is a line perpendicular to *xy*. When an object is cut by an auxiliary inclined plane, the true shape of section is obtained by projecting the apparent section of the object on another plane parallel to the section plane.

12.3.2 Section Plane Perpendicular to H.P.

A section plane that is perpendicular to the H.P. may either be parallel or inclined to the V.P.

1. Section plane parallel to V.P. It is a plane perpendicular to the H.P and parallel to the V.P. as shown in Fig. 12.3(a). Its top view is a straight line parallel to *xy*. This line also represents the horizontal trace

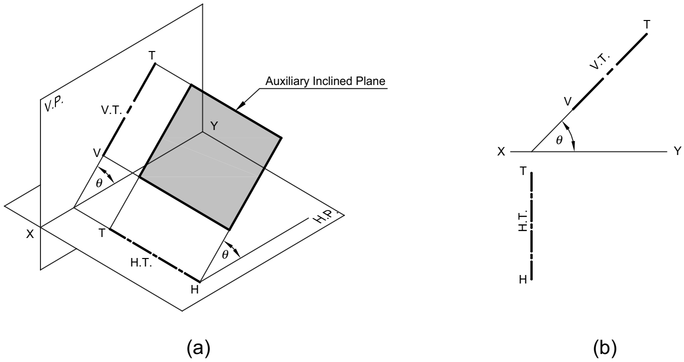


Fig. 12.2 *Auxiliary inclined plane* **(a)** *Pictorial view* **(b)** *Orthographic view*

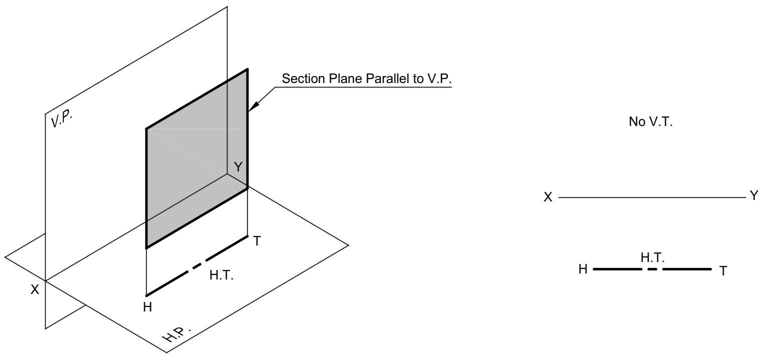


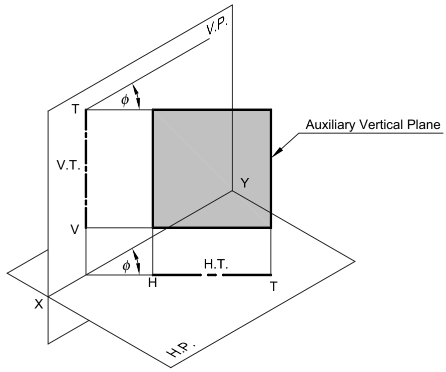
Fig. 12.3 *Section plane parallel to V.P.* **(a)** *Picorial view* **(b)** *Orthographic view*

(H.T.) of the section plane as shown in Fig. 12.3(b). This plane has no vertical trace. When an object is cut by a section plane parallel to the V.P., the sectional front view of the object gives the true shape of section.

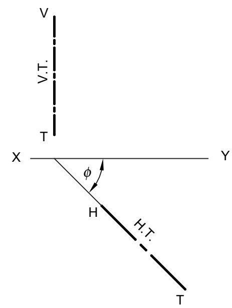
2. Auxiliary vertical plane (A.V.P.) It is a plane perpendicular to the H.P. and inclined to the V.P. as shown in Fig. 12.4(a). Its top view is a straight line inclined at ϕ to xy. This line also represents the horizontal trace (H.T.) of the section plane as shown in Fig. 12.4(b). The vertical trace (V.T.) of the section plane is a line perpendicular to xy. When an object is cut by an auxiliary vertical plane, the true shape of section is obtained by projecting the apparent section of the object on another plane parallel to the section plane.

12.3.3 Profile Section Plane

It is a plane perpendicular to both the H.P. and the V.P. as shown in Fig. 12.5(a). Both of its top and front views are straight lines perpendicular to xy. These lines also represent H.T. and V.T. respectively as shown in Fig. 12.5(b). The sectional side view gives the true shape of section.

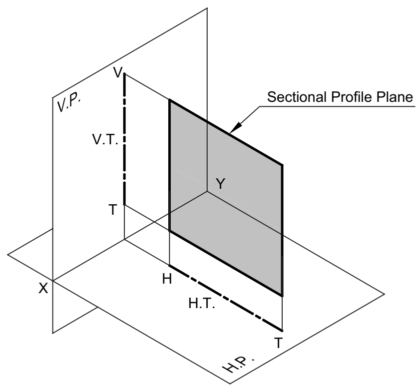


(a)

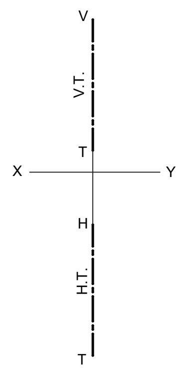


(b)

Fig. 12.4 *Auxiliary vertical plane* **(a)** *Pictorial view* **(b)** *Orthographic view*



(a)



(b)

Fig. 12.5 *Profile section plane* **(a)** *Pictorial view* **(b)** *Orthographic view*

Note

1. In case where two or more positions of section planes are possible to draw which satisfies the given conditions then as far as possible, the section plane through which minimum portion of the solid is cut away is selected.
2. The portion of the solid lying between the observer and the section plane is assumed to be removed. The retained part of the solid is drawn with dark continuous lines and removed part of the solid with thin continuous lines.
3. The hatching lines (section lines) are used to indicate cut surface in the sectional view. Usually, the hatching lines are inclined at 45º to the principal planes. If the boundary of the object itself is inclined at 45º to the reference line, hatching lines may drawn at 45º to the boundary or at 30º or 60º to the reference line. For details, refer to Section 7.19.

12.4 SECTION BY A PLANE PERPENDICULAR TO V.P.

Problem 12.1 *A triangular prism, base side 50 mm and axis 50 mm is lying on one of its rectangular faces on the H.P. with its axis perpendicular to the V.P. It is cut by a section plane parallel to and 20 mm above H.P. Draw its front view and sectional top view.*

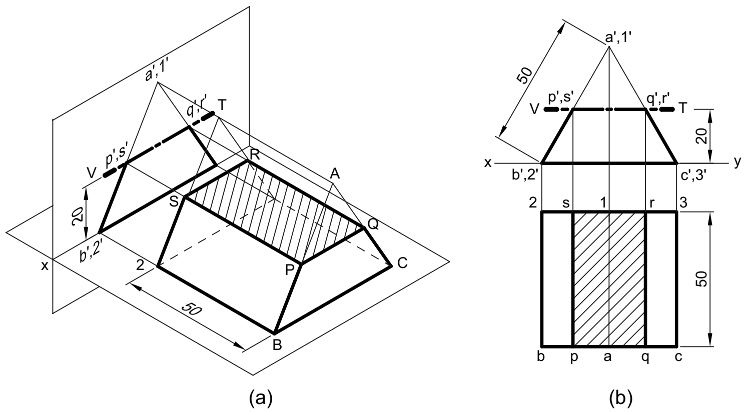


Fig. 12.6 *Section of prism by horizontal plane* **(a)** *Pictorial view* **(b)** *Orthographic view*

Figure 12.6(a) shows the pictorial view of a triangular prism cut by a horizontal section plane.

Construction Refer to Fig. 12.6(b).

1. **Projections** Draw an equilateral triangle $a'b'c'$ to represent the front view. Project all the corners to obtain $bc32$ as the top view.
2. **Cutting plane** Draw V.T. of the section plane parallel to and 20 mm above xy. Let V.T. cut the edges $a'b'$ at p', $a'c'$ at q', $1'3'$ at r' and $1'2'$ at s'.
3. **Sectional top view** Project points p', q', r' and s' to meet the top view at points p, q, r and s. Join $pqrs$ and hatch the enclosed space. (Hatching lines should be inclined at 45° to xy).
4. As the section plane is parallel to the H.P., $pqrs$ represents the true shape of the section.

Problem 12.2 *A triangular prism of base side 50 mm and axis 50 mm lies on one of its rectangular faces on the H.P. with its axis inclined at 30° to the V.P. It is cut by a horizontal section plane at a distance of 5 mm from the axis. Draw its front view and sectional top view.*

Construction Refer to Fig. 12.7.

1. **First stage** Draw a triangle $a'b'c'$ keeping side $b'c'$ on xy to represent the front view. Project all the corners and obtain $bc32$ as the top view.
2. **Second stage** Reproduce the top view of the first stage keeping a1 inclined at 30° to xy. Project the top view to meet locus lines from the front view of the first stage and obtain $1'a'c'2'$ as the new front view.
3. **Cutting plane** Draw V.T. of the section plane parallel to and 5 mm above the axis of the prism. Let V.T. cut $a'b'$ at p', $a'c'$ at q', $1'3'$ at r', and $1'2'$ at s'.

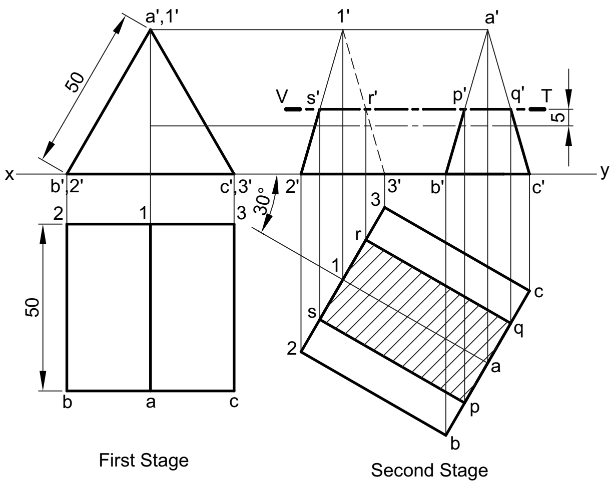


Fig. 12.7

4. **Sectional top view** Project points p', q', r' and s' to meet *ab*, *ac*, 1-3 and 1-2 at points *p*, *q*, *r* and *s*. Join *pqrs* and hatch the enclosed space.

Problem 12.3 *A pentagonal pyramid of base side 30 mm and axis 60 mm is resting on its base in the H.P. with an edge of the base parallel to the V.P. A horizontal section plane cuts the pyramid bisecting the axis. Draw its front view and sectional top view.*

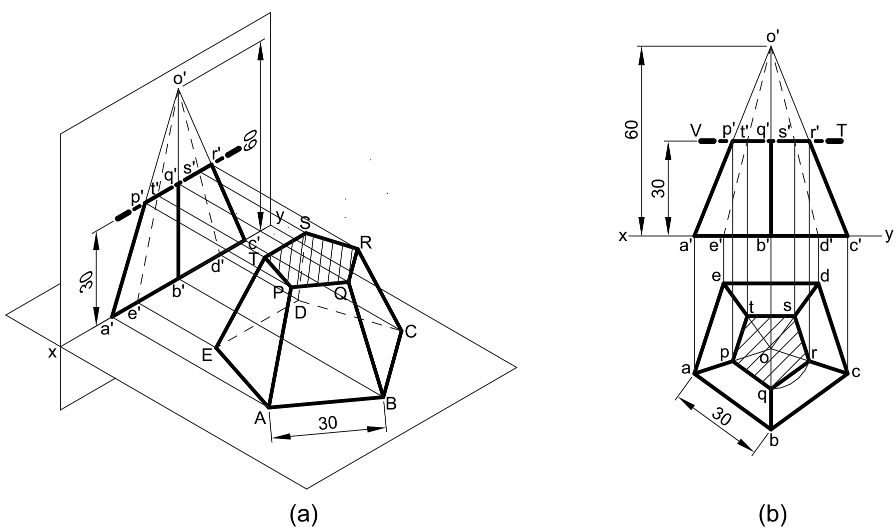


Fig. 12.8 *Section of pyramid by horizontal plane* **(a)** *Pictorial view* **(b)** *Orthographic view*

Figure 12.8(a) shows the pictorial view of a pentagonal pyramid kept on its base on H.P. which is cut by a horizontal section plane.

Construction Refer to Fig. 12.8(b).

1. **Projections** Draw a pentagon *abcde* keeping side de parallel to *xy*. Join all the corners of the pentagon with centroid *o*. This is the top view. Project all the corners and obtain *a′c′o′* as the front view.
2. **Cutting plane** Draw V.T. of the section plane parallel to and 30 mm above *xy*. Let V.T. cut the slant edges *o′a′* at *p′*, *o′b′* at *q′*, *o′c′* at *r′*, *o′d′* at *s′* and *o′e′* at *t′*.
3. **Sectional top view** Project *p′*, *r′*, *s′* and *t′* on their respective edges *oa*, *oc*, *od* and *oe* and obtain points *p*, *r*, *s* and *t*. Point *q′* cannot be projected directly on *ob*. However, it is known that the sectional top view should be a regular pentagon. Therefore, draw an arc with centre *o* and radius *ro* to meet *ob* at point *q*. Join *pqrst* and hatch the enclosed portion.
   As the section plane is parallel to the H.P., *pqrst* represents the true shape of the section.

Problem 12.4 *A square pyramid of base side 40 mm and axis 60 mm is resting on its base on the H.P. with a side of base parallel to the V.P. Draw its sectional views and true shape of the section, if it is cut by a section plane perpendicular to the V.P., bisecting the axis and is (a) parallel to the H.P., (b) inclined at 45° to the H.P. (c) inclined at 60° to the H.P.*

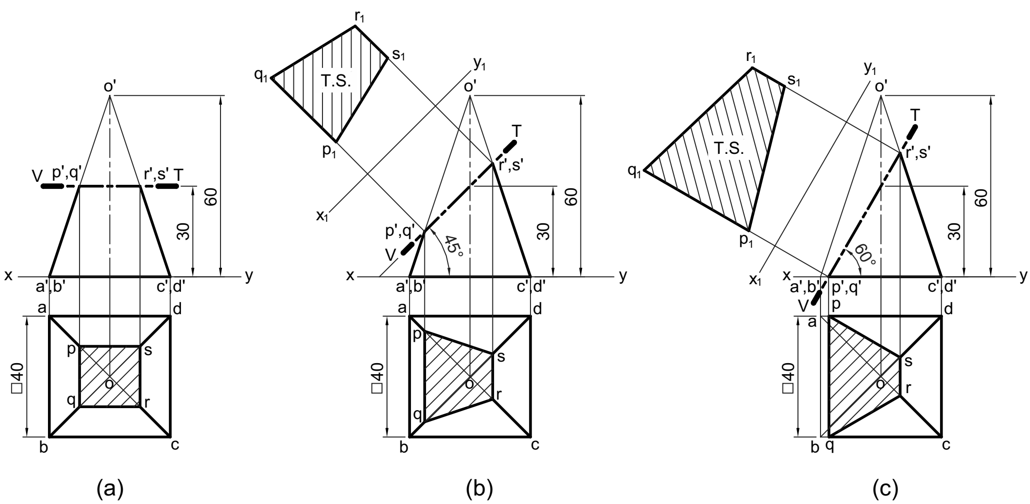


Fig. 12.9 *Section of a pyramid by a section plane* **(a)** *Parallel to H.P.,* **(b)** *Inclined at 45° to H.P. and* **(c)** *Inclined at 60° to H.P.*

Construction

Projections Draw a square *abcd* keeping side ad parallel to *xy* and join the corners with centroid *o*. This represents the top view. Project the corners and obtain *b′c′o′* as the front view.

(a) Section plane parallel to the H.P. Refer to Fig. 12.9(a).

1. **Cutting plane** Draw V.T. of the section plane parallel to *xy* bisecting the axis. Let V.T. cut the edges *o′a′* at *p′*, *o′b′* at *q′*, *o′c′* at *r′* and *o′d′* at *s′*.
2. **Sectional top view** Project *p′*, *q′*, *r′* and *s′* to meet their respective edges *oa*, *ob*, *oc* and *od* in the top view at points *p*, *q*, *r* and *s*. Join *pqrs* and hatch the enclosed space.
   As the section plane is parallel to *xy*, *pqrs* represents the true shape of the section.

(b) Section plane inclined at 45° to the H.P. Refer to Fig. 12.9(b).

1. **Cutting plane** Draw V.T. of the section plane inclined at 45° to xy bisecting the axis. Let V.T. cut the edges $o'a'$ at p', $o'b'$ at q', $o'c'$ at r' and $o'd'$ at s'.
2. **Sectional top view** Project p', q', r' and s' to meet their respective edges oa, ob, oc and od in the top view at points p, q, r and s. Join $pqrs$ and hatch the enclosed space.
3. **True shape** Draw x_1y_1 parallel to V.T. Project points p', q', r' and s' on x_1y_1. Locate p_1, q_1, r_1 and s_1 on the projectors such that their distances from x_1y_1 are equal to the distances of p, q, r and s from xy respectively. Join $p_1q_1r_1s_1$ and hatch the enclosed space to get the true shape of section.

(c) Section plane inclined at 60° to the H.P. Refer to Fig. 12.9(c).

1. **Cutting plane** Draw V.T. of the section plane inclined at 60° to xy bisecting the axis. Let V.T. cut the edges $a'd'$ at p', $b'c'$ at q', $o'c'$ at r' and $o'd'$ at s'.
2. **Sectional top view** Project p', q', r' and s' to meet their respective edges ad, bc, oc and od in the top view at points p, q, r and s. Join $pqrs$ and hatch the enclosed space.
3. **True shape** Draw x_1y_1 parallel to V.T. Project points p', q', r' and s' on x_1y_1. Locate p_1, q_1, r_1 and s_1 on the projectors such that their distances from x_1y_1 are equal to the distances of p, q, r and s from xy respectively. Join $p_1q_1r_1s_1$ and hatch the enclosed space to get the true shape of section.

Problem 12.5 *A square pyramid of base side 40 mm and axis 60 mm is resting on its base on the H.P. with all the sides of the base equally inclined to the V.P. Draw its sectional views and true shape of the section, if it is cut by a section plane perpendicular to the V.P., bisecting the axis and is (a) parallel to the H.P., (b) inclined at 45° to the H.P. and (c) inclined at 60° to the H.P.*

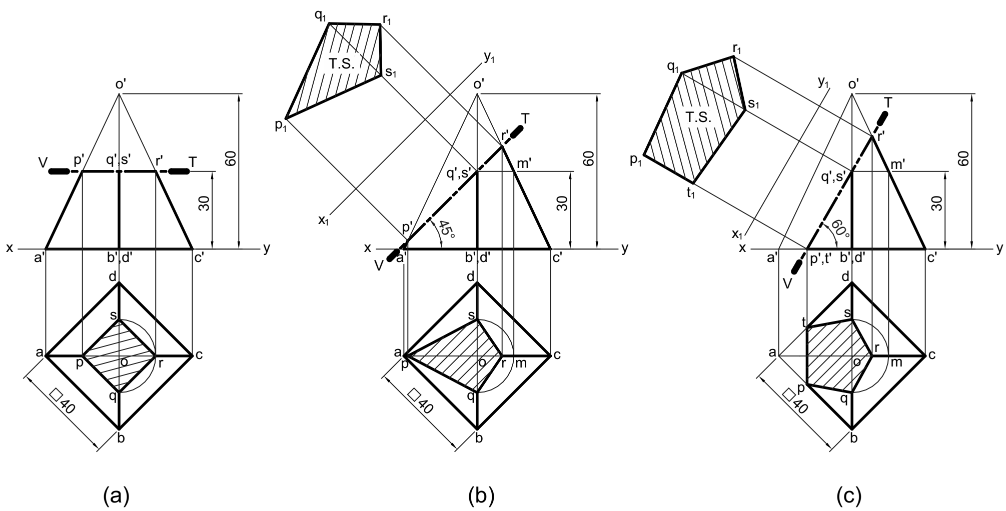


Fig. 12.10 *Section of a pyramid by a section plane* **(a)** *Parallel to H.P.* **(b)** *Inclined at 45° to H.P. and* **(c)** *Inclined at 60° to HP.*

Construction

Projections Draw a square $abcd$ keeping sides inclined at 45° to xy. Join the corners with centroid o. This represents the top view. Project all the corners and obtain $a'c'o'$ as the front view.

(a) Section plane parallel to the H.P. Refer to Fig. 12.10(a).

1. **Cutting plane** Draw V.T. of the section plane parallel to xy bisecting the axis. Let V.T. cut the edges $o'a'$ at p', $o'b'$ at q', $o'c'$ at r' and $o'd'$ at s'.
2. **Sectional top view** Project p' and r' to meet oa and oc at p and r respectively.
   Points q' and s' cannot be projected directly on ob and od. However, it is known that the sectional top view should be a square. Therefore, draw an arc with centre o and radius ro to meet ob and od at points q and s respectively. Join $pqrs$ and hatch the enclosed space. As the section plane is parallel to xy, $pqrs$ represents the true shape of the section.

(b) Section plane inclined at 45° to the H.P. Refer to Fig. 12.10(b).

1. **Cutting plane** Draw V.T. of the section plane inclined at 45° bisecting the axis. Let V.T. cut the edges $o'a'$ at p', $o'b'$ at q', $o'c'$ at r' and $o'd'$ at s'.
2. **Sectional top view** Project p' and r' to meet oa and oc at points p and r, respectively. Draw a horizontal line from $q's'$ to meet $o'c'$ at m'. Project m' to meet oc at m. Draw an arc with centre o and radius om to meet ob and od at points q and s respectively. Join $pqrs$ and hatch the enclosed space.
3. **True shape** Draw x_1y_1 parallel to V.T. Project p', q', r' and s' on x_1y_1. Locate p_1, q_1, r_1 and s_1 on their respective projectors such that their distances from x_1y_1 are equal to the distances of p, q, r and s from xy respectively. Join $p_1q_1r_1s_1$ and hatch the enclosed space to get the true shape of section.

(c) Section plane inclined at 60° to the H.P. Refer to Fig. 12.10(c).

1. **Cutting plane** Draw V.T. of the section plane inclined at 60° bisecting the axis. Let V.T. cut the edges $a'b'$ at p', $o'b'$ at q', $o'c'$ at r', $o'd'$ at s' and $a'd'$ at t'.
2. **Sectional top view** Project p', r' and t' to meet ab, oc and ad at p, r and t respectively. Draw a horizontal line from $q's'$ to meet $o'c'$ at m'. Project m' to meet oc at m. Draw an arc with centre o and radius om to meet ob and od at points q and s respectively. Join $pqrst$ and hatch the enclosed space.
3. **True shape** Draw x_1y_1 parallel to V.T. Project p', q', r', s' and t' on x_1y_1. Locate p_1, q_1, r_1, s_1 and t_1 on their respective projectors such that their distances from x_1y_1 are equal to the distances of p, q, r, s and t from xy respectively. Join $p_1q_1r_1s_1t_1$ and hatch the enclosed space to get the true shape of section.

Problem 12.6 *A pentagonal pyramid of base side 30 mm and axis 60 mm is resting on its base on the H.P. with an edge of the base parallel to the V.P. It is cut by a section plane perpendicular to the V.P., inclined at 60° to the H.P. and bisecting the axis. Draw its front view and sectional top view and true shape of the section.*

Figure 12.11(a) shows the pictorial view of a pentagonal pyramid kept on the H.P. which is cut by an A.I.P. making 60º to the H.P.

Construction Refer to Fig. 12.11(b).

1. **Projections** Draw a pentagon $abcde$ keeping side ed parallel to xy. Join the corners with centroid o. This represents the top view. Project all the corners and obtain $a'c'o'$ as the front view.
2. **Cutting plane** Draw V.T. of the section plane inclined at 60° to xy bisecting the axis. Let V.T. cut the edges $a'b'$ at p', $o'b'$ at q', $o'c'$ at r', $o'd'$ at s', $o'e'$ at t' and $a'e'$ at u'.
3. **Sectional top view** Project p', r', s', t' and u' to meet their respective edges ab, oc, od, oe and ae in the top view at points p, r, s, t and u.

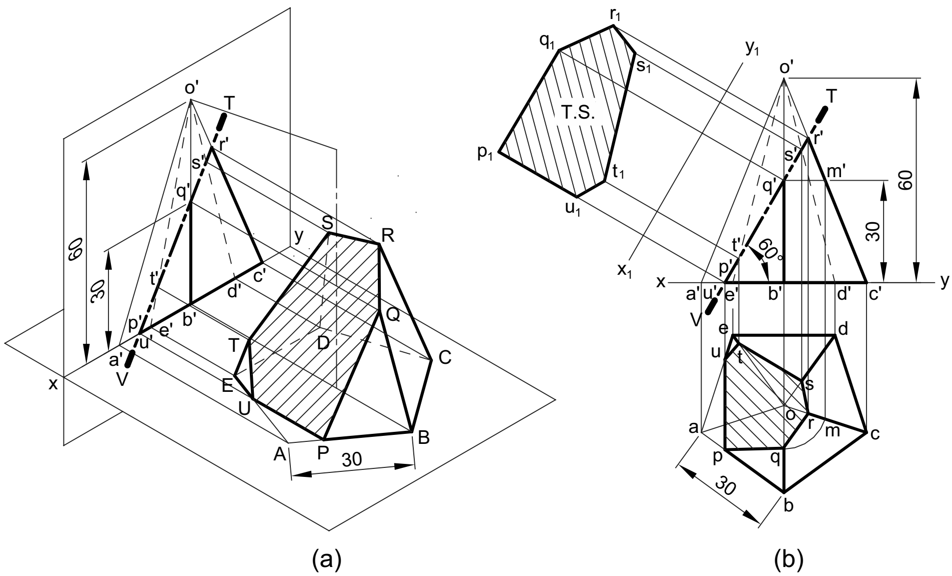


Fig. 12.11 *Section of pyramid by A.I.P.* **(a)** *Pictorial view* **(b)** *Orthographic view*

Draw a horizontal line from q' to meet $o'c'$ at m'. Project m' to meet oc at m. Draw an arc with centre o and radius om to meet ob at point q. Join $pqrstu$ and hatch the enclosed space to represent the sectional top view.

4. **True shape** Draw x_1y_1 parallel to V.T. Project p', q', r', s', t' and u' on x_1y_1. Locate p_1, q_1, r_1, s_1, t_1 and u_1 on their respective projectors such that their distances from x_1y_1 are equal to distances of p, q, r, s, t and u from xy respectively. Join $p_1q_1r_1s_1t_1u_1$ and hatch the enclosed space to get the true shape of section.

Problem 12.7 *A pentagonal pyramid of base side 30 mm and axis 60 mm is resting on a triangular face on the H.P. with its axis parallel to the V.P. It is cut by a horizontal section plane passing through the centroid of the pyramid. Draw its projections.*

Construction Refer to Fig. 12.12.

1. **First stage** Draw a pentagon *abcde* keeping side *cd* perpendicular to *xy*. Join the corners of the pentagon with the centroid *o*. This represents the top view. Project all the corners and obtain $d'c'o'$ as the front view.
2. **Second stage** Reproduce the front view of the first stage such that line $c'd'o'$ representing the triangular face is on *xy*. Obtain points *a*, *b*, *c*, *d*, *e* and *o* in the top view as the intersecting points of the projectors from the front view of the second stage with the corresponding locus lines from the top view of the first stage.
3. **Cutting plane** Mark centroid m' on the axis at 1/4th of its distance from the base. Draw V.T. of the section plane parallel to *xy* and passing through m'. Let V.T. cut the edges $c'b'$ at p', $o'b'$ at q', $o'a'$ at r', $o'e'$ at s' and $d'e'$ at t'.
4. **Sectional top view** Project p', q', r', s' and t' to meet *cb*, *ob*, *oa*, *oe* and *de* at points *p*, *q*, *r*, *s* and *t* respectively. Join *pqrst* and hatch the enclosed space.

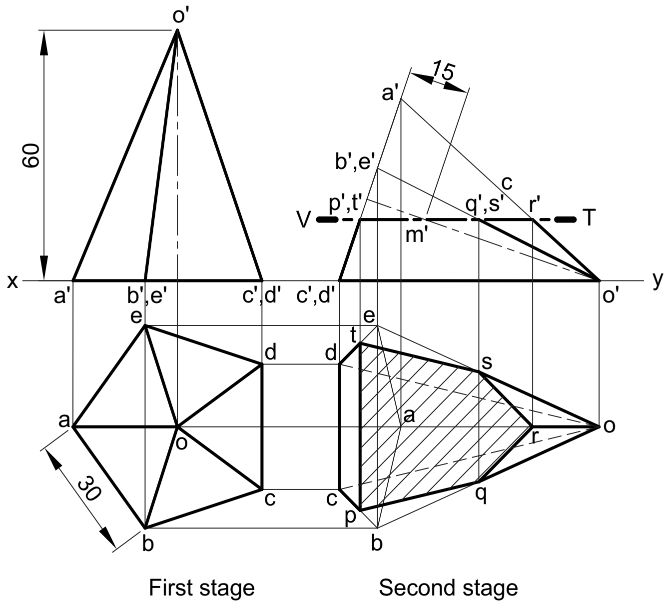


Fig. 12.12

Problem 12.8 *A pentagonal pyramid of base side 30 mm and axis 60 mm is on a triangular face in the H.P. with its axis parallel to the V.P. It is cut by an A.I.P. inclined at 60° to the H.P. and passing through the highest point of the base. Draw its sectional top view and true shape of the section.*

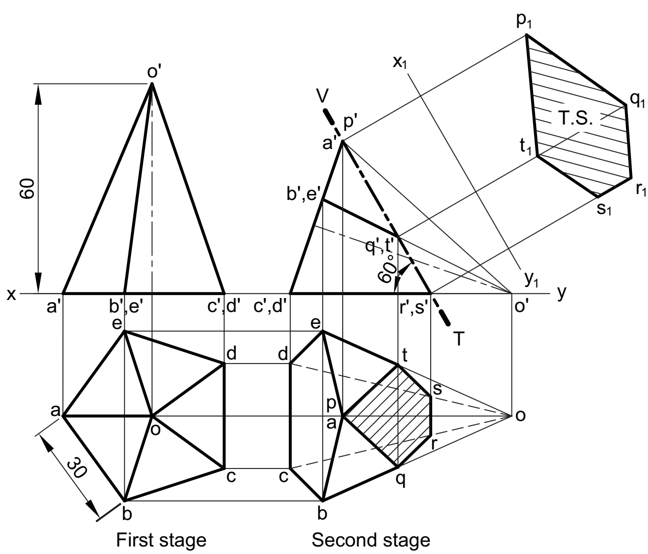


Fig. 12.13

Construction Refer to Fig. 12.13.

1. **First stage** Draw a pentagon *abcde* keeping side *cd* perpendicular to *x*y. Join the corners of the pentagon with the centroid *o*. This represents the top view. Project all the corners and obtain $a'c'o'$ as the front view.
2. **Second stage** Reproduce the front view of the first stage such that line $c'd'o'$ representing the triangular face is on *xy*. Obtain points *a*, *b*, *c*, *d*, *e* and *o* in the top view as the intersecting points of the projectors from the front view of the second stage with the corresponding locus lines from the top view of the first stage.
3. **Cutting plane** Draw V.T. of the section plane inclined at 60° to *x*y and passing through a'. Let V.T. cut the edges $o'b'$ at q', $o'c'$ at r', $o'd'$ at s' and $o'e'$ at t'.
4. **Sectional top view** Project q', r', s' and t' to meet *ob*, *oc*, *od* and *oe* at points *q*, *r*, *s* and *t* respectively. Join *pqrst* and hatch the enclosed space. This represents the required sectional top view.
5. **True shape** Draw x_1y_1 parallel to V.T. Project p', q', r', s' and t' on x_1y_1. Locate p_1, q_1, r_1, s_1 and t_1 on the projectors such that their distances from x_1y_1 are equal to distances of points *p*, *q*, *r*, *s* and *t* from *xy*, respectively. Join $p_1q_1r_1s_1t_1$ and hatch the enclosed space to get the true shape of section.

Problem 12.9 *A cone of base diameter 50 mm and axis 60 mm is resting on its base on the H.P. It is cut by an A.I.P. inclined at 45° to the H.P. and passing through a point on the axis, 20 mm above the base. Draw its sectional top view and obtain true shape of the section.*

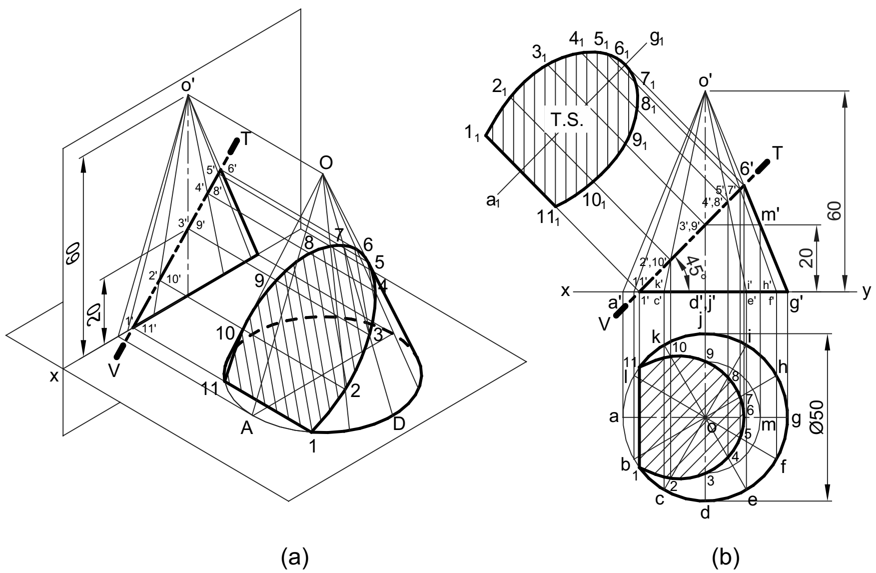


Fig. 12.14 *Section of cone by A.I.P.* **(a)** *Pictorial view* **(b)** *Orthographic view*

Figure 12.14(a) shows the pictorial view of a cone cut by an A.I.P.

Construction Refer to Fig. 12.14(b).

1. **Projections** Draw a circle *adgj* to represent the top view. Divide the circle into 12 equal parts and project to obtain $a'g'o'$ as the front view.
2. **Cutting plane** Draw V.T. of the section plane inclined at 45° to *xy* and passing through a point $3'$ lying on the axis at a height of 20 mm from the base.

3. **Sectional top view** Let V.T. cut the base at 1′ and 11′ while the generators *o′c′* at 2′, *o′d′* at 3′, *o′e′* at 4′, *o′f′* at 5′, *o′g′* at 6′, *o′h′* at 7′, *o′i′* at 8′, *o′j′* at 9′ and *o′k′* at 10′. Project 1′, 2′, 4′, 5′, 6′, 7′, 8′, 10′ and 11′ to meet in the top view at points 1, 2, 4, 5, 6, 7, 8, 10 and 11.
4. Points 3′ and 9′ cannot be projected directly on *od* and *oj*. For this draw a horizontal line from 3′ to meet *o′g′* at *m′*. Project *m′* to meet *og* at *m*. Draw an arc with centre *o* and radius *om* to meet *od* and *oj* at points 3 and 9 respectively.
5. Join 1-2-3-4-5-6-7-8-9-10-11 and hatch the enclosed space.
6. **True shape** Draw a_1g_1 parallel to V.T. Project 1′, 2′, 3′, 4′, 5′, 6′, 7′, 8′, 9′, 10′ and 11′ on a_1g_1. Locate 1_1, 2_1, 3_1, 4_1, 5_1, 6_1, 7_1, 8_1, 9_1, 10_1 and 11_1 on the projectors such that their distances from a_1g_1 are equal to distances of points 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 and 11 from *ag*, respectively. Join $1_12_13_14_15_16_17_18_19_110_111_1$ and hatch the enclosed space to get the true shape of section.

Problem 12.10 *A cone of base diameter 50 mm and axis 60 mm lies on one of its generators on the H.P. such that its axis is parallel to the V.P. The cone is cut by a horizontal section plane whose V.T. trisects the axis from the base. Draw its sectional views.*

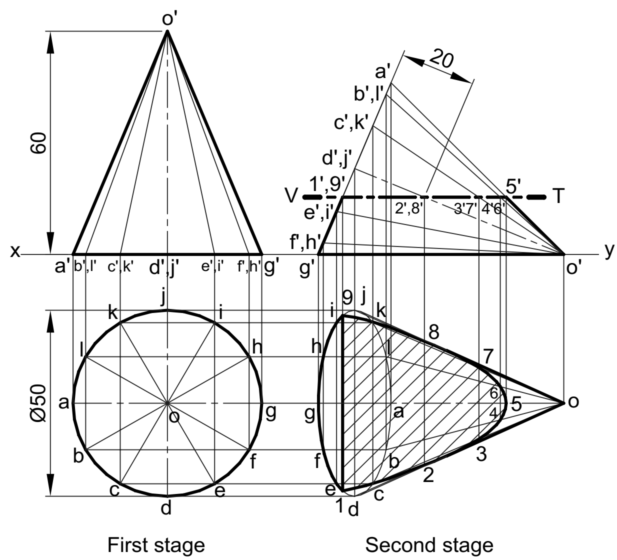


Fig. 12.15

Construction Refer to Fig. 12.15.

1. **First stage** Draw a circle *adgj* to represent the top view. Divide the circle into 12 equal parts and project to obtain *a′g′o′* as the front view.
2. **Second stage** Reproduce front view of the first stage keeping generator *g′o′* on *xy*. Project the front view to meet locus lines from the top view of the first stage and obtain *gdoj* as the new top view.
3. **Cutting plane** Draw V.T. of the section plane parallel to *xy* and passing through a point on the axis lying 20 mm from the base. Let V.T. cut the base at 1′ and 9′, and the generators *o′d′* at 2′, *o′c′* at 3′, *o′b′* at 4′, *o′a′* at 5′, *o′l′* at 6′, *o′k′* at 7′ and *o′j′* at 8′.

4. **Sectional top view** Project points $1'$, $2'$, $3'$, $4'$, $5'$, $6'$, 7, $8'$ and $9'$ to meet in the top view at points 1, 2, 3, 4, 5, 6, 7, 8 and 9. Join 1-2-3-4-5-6-7-8-9 and hatch the enclosed space.

Problem 12.11 *A cone of base diameter 50 mm and axis 60 mm long is resting on its base on the H.P. It is cut by an A.I.P. bisecting the axis inclined at 45° to the H.P. Draw its sectional top view and true shape of the section.*

Construction Refer to Fig. 12.16.

1. **Projections** Draw a circle *adgj* to represent the top view. Divide the circle into 12 equal parts and project to obtain $a'g'o'$ as the front view.
2. **Cutting plane** Draw V.T. of the section plane inclined at 45° to *xy* and bisecting the axis. Let the V.T. cut the generators $o'a'$ at $1'$, $o'b'$ at $2'$, $o'c'$ at $3'$, $o'd'$ at $4'$, $o'e'$ at $5'$, $o'f'$ at $6'$, $o'g'$ at $7'$, $o'h'$ at $8'$, $o'i'$ at $9'$, $o'j'$ at $10'$, $o'k'$ at $11'$ and $o'l'$ at $12'$.
3. **Sectional top view** Project $1'$, $2'$, $3'$, $5'$, $6'$, $7'$, $8'$, $9'$, $11'$ and $12'$ to meet in the top view at points 1, 2, 3, 5, 6, 7, 8, 9, 11 and 12.
4. Draw a horizontal line from $4'$ and $10'$ to meet $o'g'$ at m'. Project m' to meet *og* at *m*. Draw an arc with centre *o* and radius *om* to meet *od* and *oj* at points 4 and 10 respectively.

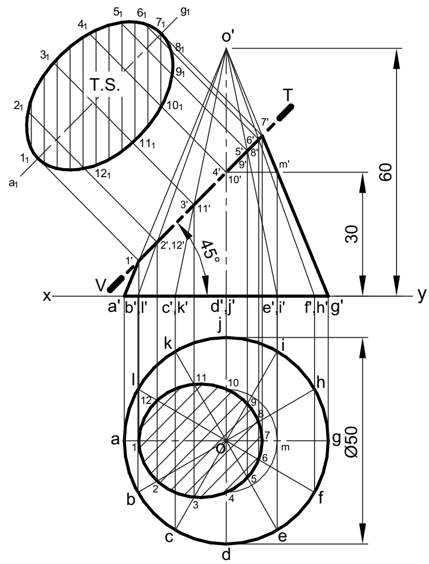


Fig. 12.16

5. Join 1-2-3-4-5-6-7-8-9-10-11-12 and hatch the enclosed space.
6. **True shape** Draw a_1g_1 parallel to V.T. Project $1'$, $2'$, $3'$, $4'$, $5'$, $6'$, $7'$, $8'$, $9'$, $10'$, $11'$ and $12'$ on a_1g_1. Locate points 1_1, 2_1, 3_1, 4_1, 5_1, 6_1, 7_1, 8_1, 9_1, 10_1, 11_1 and 12_1 on the projectors such that their distances from a_1g_1 are equal to distances of points 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12 from *ag*, respectively. Join $1_12_13_14_15_16_17_18_19_110_111_112_1$ and hatch the enclosed space to get the true shape of section.

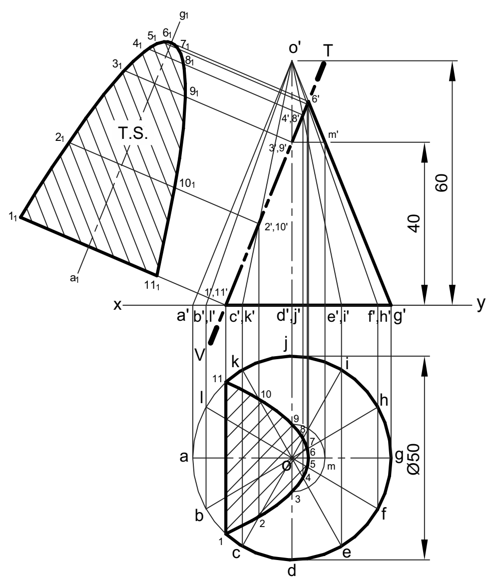


Fig. 12.17

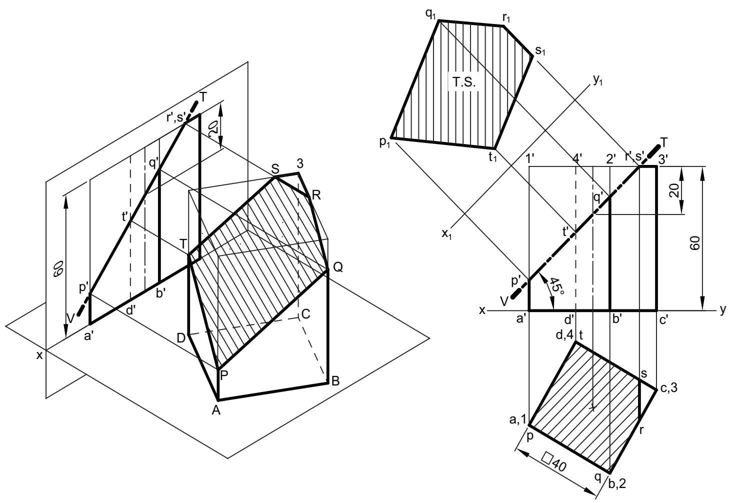
Problem 12.12 *A cone of base diameter 50 mm and axis 60 mm is resting on its base on the H.P. It is cut by an A.I.P. parallel to one of the extreme generators passing through a point on the axis 40 mm from its base. Draw its sectional top view and obtain true shape of the section.*

Construction Refer to Fig. 12.17.

1. **Projections** Draw a circle *adgj* to represent the top view. Divide the circle into 12 equal parts and project to obtain $a'g'o'$ as the front view.

2. **Cutting plane** Draw V.T. of the section plane parallel to $o'a'$ passing through a point on the axis 40 mm from the base. Let V.T. cut the base at $1'$ and $11'$ and generators $o'c'$ at $2'$, $o'd'$ at $3'$, $o'e'$ at $4'$, $o'f'$ at $5'$, $o'g'$ at $6'$, $o'h'$ at $7'$, $o'i'$ at $8'$, $o'j'$ at $9'$, $o'k'$ at $10'$.
3. **Sectional top view** Project points $1'$, $2'$, $4'$, $5'$, $6'$, $7'$, $8'$, $10'$ and $11'$ to meet the top view at points 1, 2, 4, 5, 6, 7, 8, 10 and 11.
4. Draw a horizontal line from $3'9'$ to meet $o'g'$ at m'. Project m' to meet og at m. Draw an arc with centre o and radius om to meet od and oj at points 3 and 9 respectively.
5. Join 1-2-3-4-5-6-7-8-9-10-11 and hatch the enclosed space.
6. **True shape** Draw a_1g_1 parallel to V.T. Project points $1'$, $2'$, $3'$, $4'$, $5'$, $6'$, $7'$, $8'$, $9'$, 10' and $11'$ on a_1g_1. Locate points 1_1, 2_1, 3_1, 4_1, 5_1, 6_1, 7_1, 8_1, 9_1, 10_1 and 11_1 on the projectors such that their distances from a_1g_1 are equal to distances of points 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 and 11 from ag respectively. Join $1_12_13_14_15_16_17_18_19_110_111_1$ and hatch the enclosed space to get the true shape of section.

Problem 12.13 *A square prism of base side 40 mm and axis 60 mm rests on its base on H.P. such that one of the vertical faces is inclined at 30° to the V.P. A section plane perpendicular to V.P., inclined at 45° to H.P. passing through the axis at a point 20 mm from its top end cuts the prism. Draw its front view, sectional top view and true shape of section.*



(a) (b)

Fig. 12.18 *Section of prism by A.I.P.* **(a)** *Pictorial view* **(b)** *Orthographic view*

Figure 12.18(a) shows the pictorial view of a square prism cut by an A.I.P. The V.T. of the section plane is inclined at 45° to the reference line.

Construction Refer to Fig. 12.18(b).

1. **Projections** Draw a square *abcd* keeping side *cd* inclined at 30° to *xy* to represent the top view. Project all the corners and obtain $a'c'3'1'$ as the front view.

2. **Cutting plane** Draw V.T. of the section plane inclined at 45° to xy passing through a point of the axis 20 mm from its top end. Let V.T. cut the edges $a'1'$ at p', $b'2'$ at q', $2'3'$ at r', $3'4'$ at s' and $d'4'$ at t'.
3. **Sectional top view** Project points p', q', r', s' and t' to meet the top view at points p, q, r, s and t. Join $pqrst$ and hatch the enclosed space.
4. **True shape** Draw x_1y_1 parallel to V.T. Project points p', q', r', s' and t' on x_1y_1. Locate p_1, q_1, r_1, s_1 and t_1 on the projectors such that their distances from x_1y_1 are equal to distances of points p, q, r, s, and t from xy, respectively. Join $p_1q_1r_1s_1t_1$ and hatch the enclosed space to get the true shape of section.

Problem 12.14 *A cylinder of base diameter 50 mm and axis 60 mm is resting on its base on the H.P. It is cut by a section plane perpendicular to V.P., the V.T. of which cuts the axis at a point 40 mm from the bottom face and inclined at 45° to the reference line. Draw its front view, sectional top view and true shape of the section.*

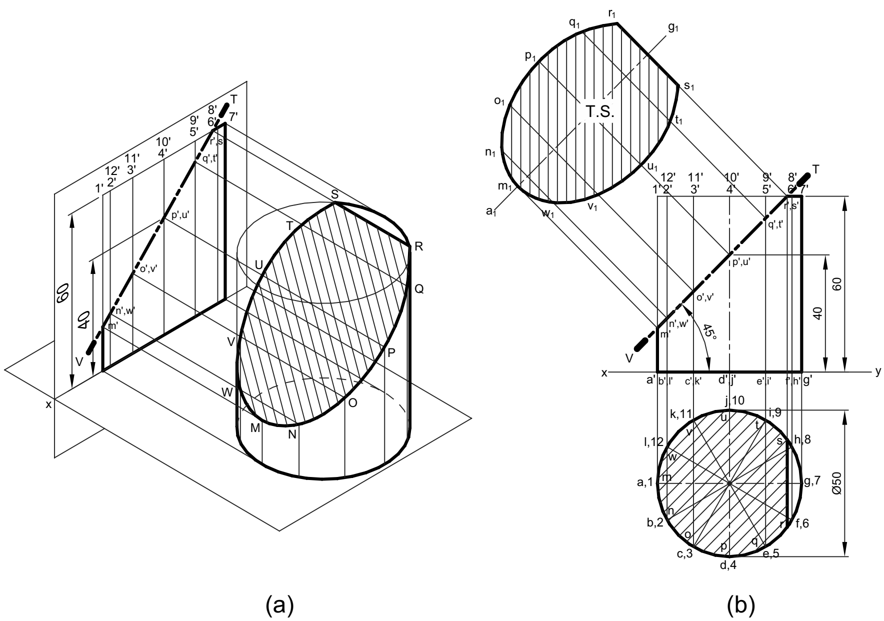


Fig. 12.19 *Section of cylinder by A.I.P.* **(a)** *Pictorial view* **(b)** *Orthographic view*

Figure 12.19(a) shows pictorial view of a cylinder cut by an A.I.P.

Construction Refer to Fig. 12.19.

1. **Projections** Draw a circle *adgj* to represent the top view. Divide the circle into 12 equal parts and project to obtain $a'g'7'1'$ as the front view.
2. **Cutting plane** Draw V.T. of the section plane inclined at 60° to xy passing through a point on the axis 40 mm above the base. Let V.T. cut the generators and the rim at m', n', o', p', q', r', s', t', u', v' and w'.
3. **Sectional top view** Project m', n', o', p', q', r', s', t', u', v' and w' in the top view to get m, n, o, p, q, r, s, t, u, v and w. Join *mnopqrstuvw* and hatch the enclosed space.

4. **True shape** Draw a_1g_1 parallel to V.T. Project points m', n', o', p', q', r', s', t', u', v' and w' on a_1g_1. Locate m_1, n_1, o_1, p_1, q_1, r_1, s_1, t_1, u_1, v_1 and w_1 on the projectors such that their distances from a_1g_1 are equal to distances of points *m*, *n*, *o*, *p*, *q*, *r*, *s*, *t*, *u*, *v* and *w* from *ag*. Join $m_1n_1o_1p_1q_1r_1s_1t_1u_1v_1w_1$ and hatch the enclosed space to get the true shape of section.

Problem 12.15 *A pentagonal prism of base side 30 mm and axis 70 mm has an edge of its base on the H.P. The axis is parallel to the V.P. and inclined at 60° to the H.P. It is cut by an A.I.P. inclined at 60° to the H.P. and passing through the highest corner of the prism. Draw its sectional top view and true shape of the section.*

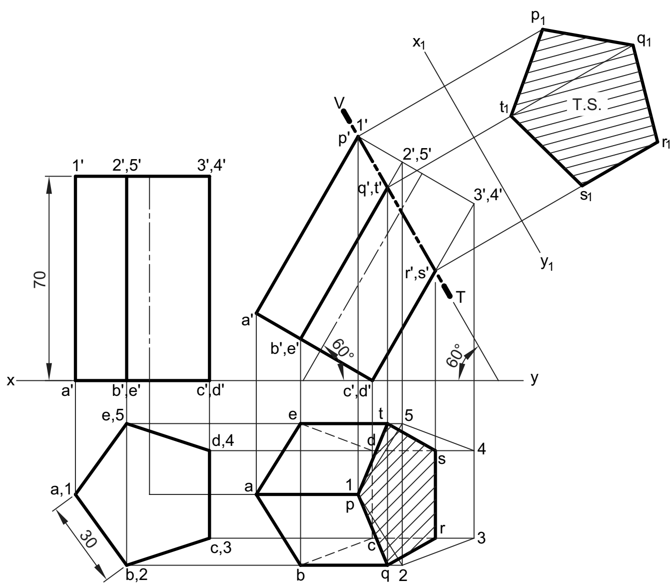


Fig. 12.20

Construction Refer to Fig. 12.20.

1. **First stage** Draw a pentagon *abcde* keeping side *cd* perpendicular to *xy*. This represents the top view. Project all the corners and obtain $a'd'4'1'$ as the front view.
2. **Second stage** Reproduce the front view of the first stage keeping $c'd'$ on *xy* and $c'3'$ inclined at 60° to *xy*. Project this front view to meet the locus lines from the top view of the first stage and obtain ab2345e as the new top view.
3. **Cutting plane** Draw V.T. of the section plane inclined at 60° to *xy* and passing through the highest corner $1'$. Let V.T. cut the edges $a'1'$ at p', $b'2'$ at q', $c'3'$ at r', $d'4'$ at s' and $e'5'$ at t'.
4. **Sectional top view** Project points p', q', r', s' and t' to meet corresponding edges *a*1, *b*2, *c*3, *d*4 and *e*5 at points *p*, *q*, *r*, *s* and *t*. Join *pqrst* and hatch the enclosed space.
5. **True shape** Draw x_1y_1 parallel to V.T. Project p', q', r', s' and t' on x_1y_1. Locate points p_1, q_1, r_1, s_1 and t_1 on the projectors such that their distances from x_1y_1 are equal to distances of points *p*, *q*, *r*, *s* and *t* from *xy*, respectively. Join $p_1q_1r_1s_1t_1$ and hatch the enclosed space to get the true shape of section.

Problem 12.16 *A hexagonal prism of base side 30 mm and axis 70 mm is resting on a face on the H.P. with axis parallel to the V.P. It is cut by a plane whose V.T. is inclined at 30° to the reference line and passes through a point on the axis 20 mm from one of its ends. Draw its sectional top view and obtain true shape of the section.*

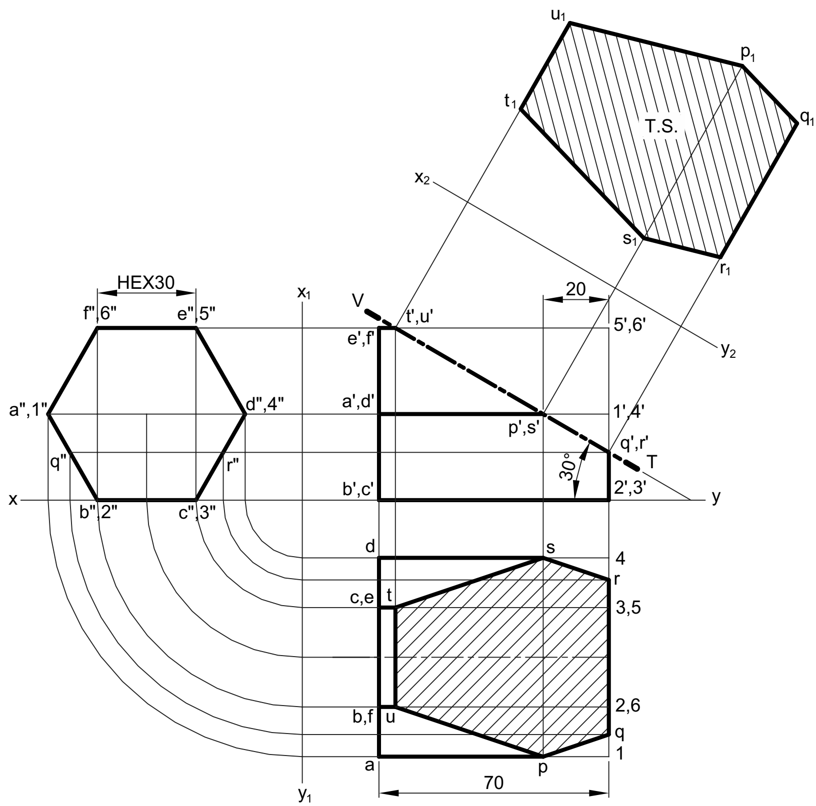


Fig. 12.21

Construction Refer to Fig. 12.21.

1. **Projections** Draw a hexagon $a''b''c''d''e''f''$ keeping side $b''c''$ on xy. This is the side view. Project this view and obtain $b'2'6'f'$ as its front view and $a14d$ as the top view.
2. **Cutting plane** Draw V.T. of the section plane inclined at 30° to xy and passing through a point on the axis lying at 20 mm from the right end. Let V.T. cut the edges $a'1'$ at p', $1'2'$ at q', $3'4'$ at r', $d'4'$ at s', $e'5'$ at t' and $f'6'$ at u'.
3. **Sectional top view** Project p', s', t' and u' to meet in the top view at points p, s, t and u respectively. Points q' and r' cannot be projected directly. Therefore, draw horizontal lines from q' and r' to meet side view at points q'' and r''. Project q'' and r'' perpendicular on xy and then rotate them through 90° and then draw horizontal lines from them such that they meet 1–2 at q and 3–4 at r. Join $pqrstu$ and hatch the enclosed space.

4. **True shape** Draw x_2y_2 parallel to V.T. Project p', q', r', s', t' and u' on x_2y_2. Locate points p_1, q_1, r_1, s_1, t_1 and u_1 on the projectors such that their distances from x_2y_2 are equal to distances of points p, q, r, s, t and u from xy, respectively. Join $p_1q_1r_1s_1t_1u_1$ and hatch the enclosed space to get the true shape of section.

Problem 12.17 *A cylinder of base diameter 50 mm and axis 70 mm is lying on a generator on the H.P. with its axis parallel to the V.P. It is cut by an A.I.P. inclined at 30° to the H.P. passing through a point on the axis 30 mm from one of its ends. Draw its sectional top view and obtain true shape of the section.*

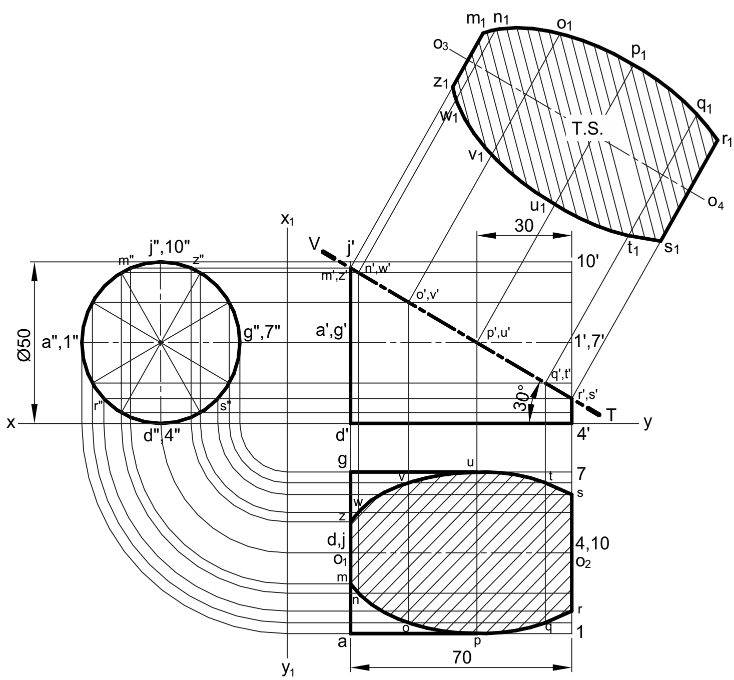


Fig. 12.22

Construction Refer to Fig. 12.22.

1. **Projections** Draw a circle $a''d''g''j''$ keeping d'' on xy. This is the side view. Project this view and obtain $d'4'10'j'$ as the front view and $a17g$ as the top view.
2. **Cutting plane** Draw V.T. of the section plane inclined at 30° to xy and passing through a point on the axis 30 mm from an end to cut the ends and generators.
3. **Sectional top view** Project n', o', p', q', t', u', v' and w' to get n, o, p, q, t, u, v and w in the top view. Points m', r', s' and z' cannot be projected directly. For this, draw horizontal lines from m', r', s' and z' to meet the side view at m'', r'', s'' and z''. Project m'', r'', s'' and z'' perpendicular on xy and then rotate them through 90° and then draw horizontal lines from them such that they meet aj at m, 1–4 at r, 4–7 at s and gj at z. Join $mnopqrstuvwz$ and hatch the enclosed space.

4. **True shape** Draw o_3o_4 parallel to V.T. Project m', n', o', p', q', r', s', t', u', v', w' and z' on o_3o_4. Locate m_1, n_1, o_1, p_1, q_1 r_1, s_1, t_1, u_1, v_1, w_1 and z_1 on the projectors such that their distances from o_3o_4 are equal to distances of *m, n, o, p, q, r, s, t, u, v, w* and *z* from o_1o_2. Join $m_1n_1o_1p_1q_1r_1s_1t_1u_1v_1w_1z_1$ and hatch the enclosed space to get the true shape of section.

Problem 12.18 *A sphere of diameter 60 mm is kept on the H.P. It is cut by an A.I.P. inclined at 45° to the H.P. The section plane passes through a point at a distance of 15 mm from the centre of the sphere and above it. Draw its front view, sectional top view and true shape of the section.*

Construction Refer to Fig. 12.23.

1. **Projections** Draw a circle with centre *o* and diameter 60 mm as the top view. Project the top view and draw another circle with centre o' and 60 mm diameter as the front view.
2. **Cutting plane** Draw an arc with centre o' and radius 15 mm. Draw V.T. of the section plane inclined at 45° to *xy* and tangential to the arc. Let it cut the circle at points a' and g'.
3. **True shape** Draw a_1g_1 of length equal to *ag* and parallel to V.T. Draw a circle with diameter a_1g_1. Hatch the enclosed space to get the true shape of section. Divide this circle into 12 equal parts and label as a_1, b_1, c_1, d_1, e_1, f_1, g_1, h_1, i_1, j_1, k_1 and l_1.
4. **Sectional top view** Project a_1, b_1, c_1, d_1, e_1, f_1, g_1, h_1, i_1, j_1, k_1 and l_1 on V.T. and locate points a', b', c', d', e', f', g', h', i', j', k' and l' respectively. Project a', b', c', d', e', f', g', h', i', j', k' and l' to the top view. Obtain points *a, b, c, d, e, f, g, h, i, j, k* and *l* on the projectors such that their distances from x_2y_2 are equal to the distances of points a_1, b_1, c_1, d_1, e_1, f_1, g_1, h_1, i_1, j_1, k_1 and l_1 from x_1y_1. Hatch the enclosed space.

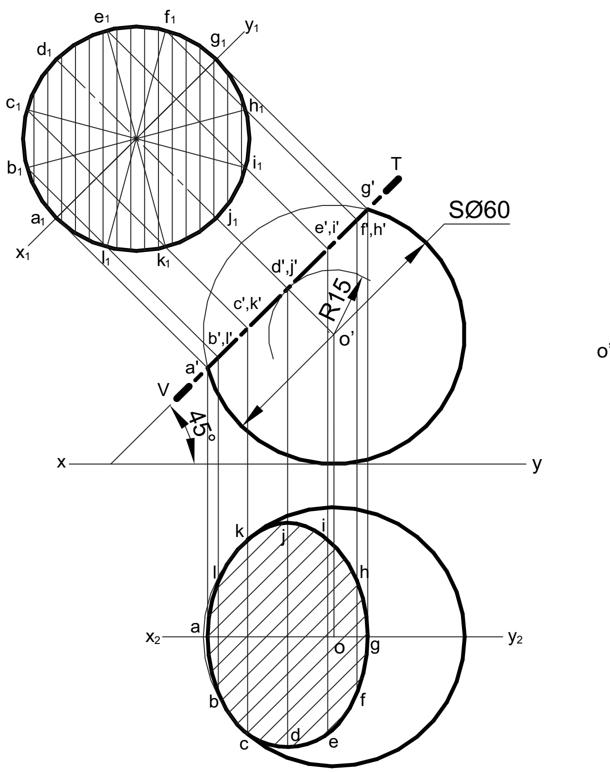


Fig. 12.23

Note Since true shape of the section for the sphere is a circle, it is easier to draw the true shape first and then project it to obtain the sectional top view.

12.5 SECTION BY A PLANE PERPENDICULAR TO H.P.

Problem 12.19 *A square prism of base side 40 mm and axis 60 mm rests on its base on the H.P. such that one of its rectangular faces is inclined at 30° to the V.P. It is cut by a section plane parallel to V.P., bisecting a face of the prism which is inclined at 30° to the V.P. Draw its sectional front view and top view.*

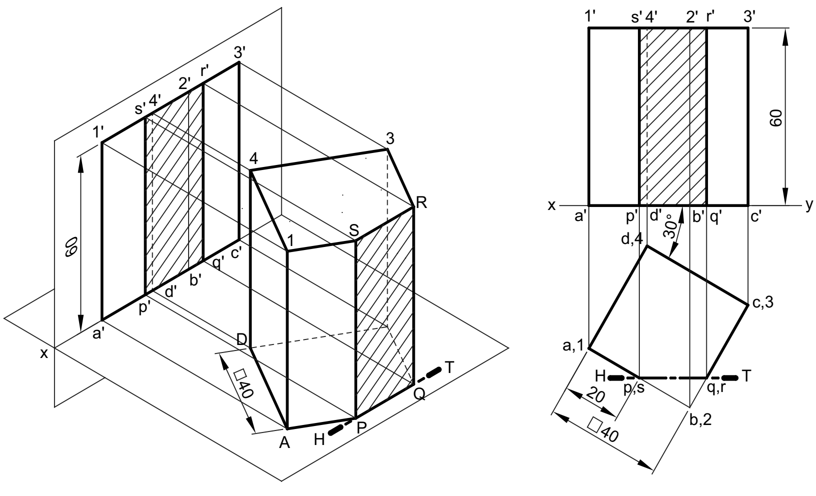


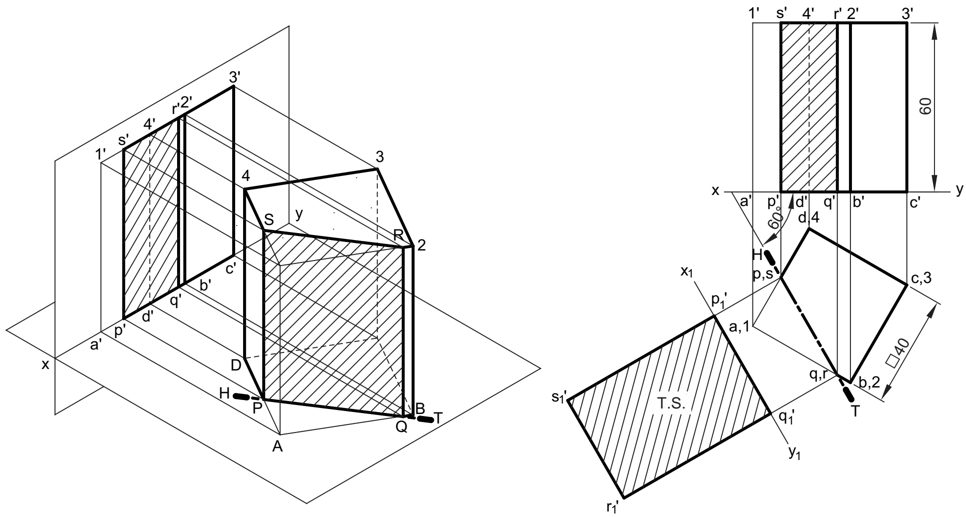
Fig. 12.24 *Section of prism by plane parallel to V.P.* **(a)** *Pictorial view* **(b)** *Orthographic view*

Figure 12.24(a) shows the pictorial view of the square prism cut by a section plane parallel to the V.P.

Construction Refer to Fig. 12.24(b).

1. **Projections** Draw a square keeping side *cd* inclined at 30° to *xy* to represent the top view. Project all the corners and obtain $a'c'3'1'$ as the front view.
2. **Cutting plane** Draw H.T. of the section plane parallel to *xy* passing through mid-point of side *ab*. Let H.T. cut the edges *a*b at *p*, *bc* at *q*, 3–2 at *r* and 2–1 at *s*.
3. **Sectional front view** Project points *p*, *q*, *r* and *s* to meet corresponding edges $a'b'$, $b'c'$, $3'2'$ and $2'1'$ at points p', q', r' and s'. Join $p'q'r's'$ and hatch the enclosed space. As the section plane is parallel to the V.P., $p'q'r's'$ represents the true shape of the section.

Problem 12.20 *A square prism of base side 40 mm and axis 60 mm rests on its base on the H.P. such that one of its rectangular faces is inclined at 30° to the V.P. It is cut by a section plane perpendicular to H.P. and inclined at 60° to V.P. passing through the prism such that a face which is inclined at 60° to the V.P. is bisected. Draw its sectional front view, top view and true shape of section.*



(a) (b)

Fig. 12.25 *Section of prism by A.V.P.* **(a)** *Pictorial view* **(b)** *Orthographic view*

Figure 12.25(a) shows the pictorial view of the square prism cut by an A.V.P inclined at 60° to the V.P.

Construction Refer to Fig. 12.25(b).

1. **Projections** Draw a square keeping side *cd* inclined at 30° to *x*y to represent the top view. Project all the corners and obtain *a′c′*3′1′ as the front view.
2. **Cutting plane** Draw H.T. of the section plane inclined at 60° to *xy* and passing through mid-point of side *ad*. Let H.T. cut the edges ad at *p*, *ab* at *q*, 2–1 at *r* and 1–4 at *s*.
3. **Sectional front view** Project points *p*, *q*, *r* and *s* to meet corresponding edges *a′d′*, *a′b′*, 2′1′ and 1′4′ at points *p′*, *q′*, *r′* and *s′*. Join *p′q′r′s′* and hatch the enclosed space.
4. **True shape** Draw x_1y_1 parallel to H.T. Project *p*, *q*, *r* and *s* on x_1y_1. Locate p_1', q_1', r_1' and s_1' on the projectors such that their distances from x_1y_1 are equal to distances of points *p′*, *q′*, *r′* and *s′* from *xy*, respectively. Join $p_1'q_1'r_1's_1'$ and hatch the enclosed space to get the true shape of section.

Problem 12.21 *A pentagonal prism of base side 30 mm and axis 60 mm lies on one of its rectangular faces on the H.P. with its axis inclined at 45° to the V.P. A vertical section plane parallel to the V.P. cuts the prism at a distance of 20 mm from one of the end faces. Draw its sectional front view and top view.*

Construction Refer to Fig. 12.26.

1. **First stage** Draw a pentagon *a′b′c′d′e′* keeping *b′c′* on *xy* to represent the front view. Project all the corners and obtain *ad*41 as the top view.
2. **Second stage** Reproduce the top view of the first stage keeping *e*5 inclined at 45° to *xy*. Project the top view to meet locus lines from the front view of the first stage and obtain *a′b′*3′4′5′e′ as the new front view.
3. **Cutting plane** Draw H.T. of the section plane parallel to *xy* such that it passes through a point on the axis lying at 20 mm from lower end of axis. Let H.T. cut the edges *a*1 at *p*, *b*2 at *q*, *c*3 at *r*, *cd* at *s*, *de* at *t* and *e*5 at *u*.

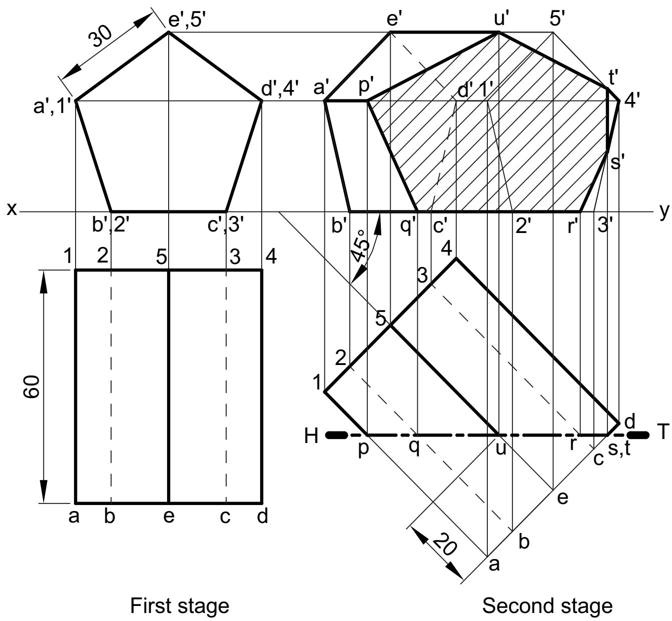


Fig. 12.26

4. **Sectional front view** Project points p, q, r, s, t and u to meet $a'1'$, $b'2'$, $c'3'$, $c'd'$, $d'e'$ and $e'5'$ at p', q', r', s', t' and u'. Join $p'q'r's't'u'$ and hatch the enclosed space.
   As the section plane is parallel to the V.P., $p'q'r's't'u'$ represents the true shape of the section.

Problem 12.22 *A hexagonal prism of base side 30 mm and axis 70 mm has an edge of its base on the H.P. with axis parallel to the V.P. and inclined at 45° to the H.P. It is cut by an A.V.P. inclined at 45° to the V.P. and passing through a point on the axis, 15 mm from the top end. Draw its sectional front view and obtain true shape of the section.*

Construction Refer to Fig. 12.27.

1. **First stage** Draw a hexagon *abcdef* keeping *de* perpendicular to *xy*. This is the top view. Project all the corners and obtain *ad*41 as the front view.
2. **Second stage** Reproduce the front view of the first stage keeping $d'e'$ on *xy* and $d'4'$ inclined at 45° to *xy*. Project the front view to meet locus lines from the top view of the first stage and obtain *abc*3456*f* as the new top view.
3. **Cutting plane** Mark point m' on the axis at a point lying 15 mm from the upper end. Project m' to meet axis in the top view at point m. Draw H.T. of the section plane inclined at 45° to *xy* and passing through m. Let H.T. cut the edges $c3$ at p, $d4$ at q, $e5$ at r, 5-6 at s, 1-2 at t and $b2$ at u.
4. **Sectional front view** Project p, q, r, s, t and u to meet $c'3'$, $d'4'$, $e'5'$, $5'6'$, $1'2'$ and $b'2'$ at points p', q', r', s', t' and u'. Join $p'q'r's't'u'$ and hatch the enclosed space.
5. **True shape** Draw x_1y_1 parallel to H.T. Project p, q, r, s, t and u on x_1y_1. Locate points p_1', q_1', r_1', s_1', t_1 and u_1' on the projectors such that their distances from x_1y_1 are equal to distances of points p', q', r', s', t' and u' from *xy*, respectively. Join $p_1'q_1'r_1's_1't_1'u_1'$ and hatch the enclosed space to get the true shape of section.

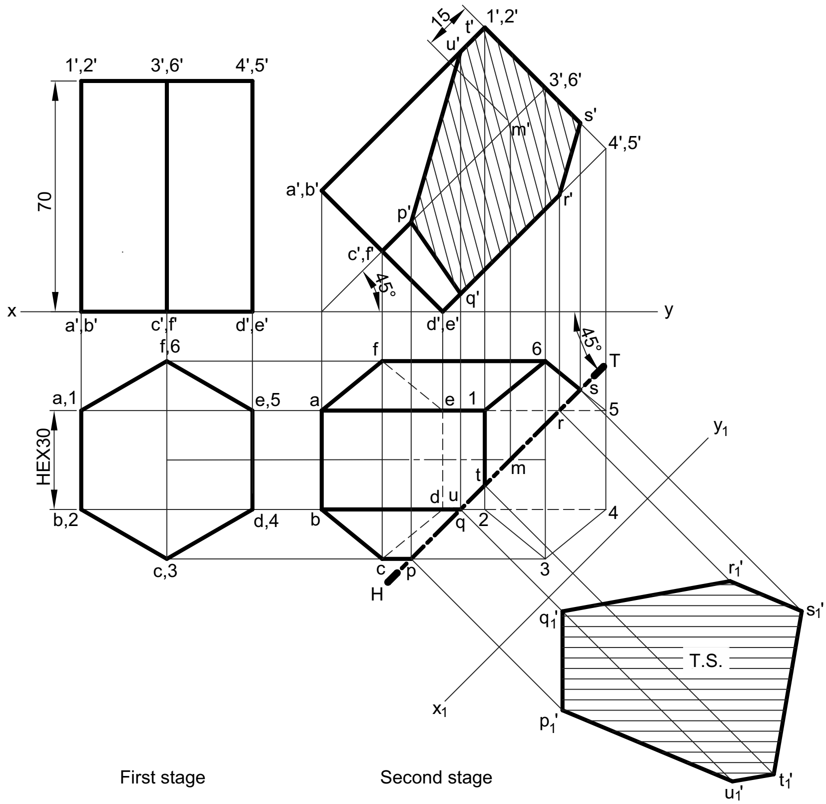


Fig. 12.27

Problem 12.23 *A hexagonal prism of base side 30 mm and axis 70 mm is resting on a face on the ground with axis parallel to the V.P. It is cut by an A.V.P. which is inclined at 45° to the V.P. and passes through a point 25 mm on the axis from one of its ends. Draw its sectional front view and obtain true shape of the section.*

Construction Refer to Fig. 12.28.

1. **Projections** Draw a hexagon $a''b''c''d''e''f''$ keeping side $b''c''$ on xy. This is the side view. Project the end view and obtain $b'2'6'f'$ as its front view and $a14d$ as the top view.
2. **Cutting plane** Draw H.T. of the section plane inclined at 45° to xy and passing through a point on the axis lying at 25 mm from an end. Let H.T. cut the edges $a1$ at p, $b2$ at q, $c3$ at r, 3-4 at s, 4-5 at t, $e5$ at u and $f6$ at v.
3. **Sectional front view** Project p, q, r, u and v to meet $a'1'$, $b'2'$, $c'3'$, $e'5'$ and $f'6'$ at points p', q', r', u' and v', respectively. Points s and t cannot be projected directly. Therefore, draw horizontal lines from s and t to meet x_1y_1 and then rotate them through 90° and then draw vertical lines from them such that they meet $3''4''$ and $4''5''$ at points s'' and t''. Draw horizontal lines from s'' and t'' to meet front view at points s' and t'. Join $p'q'r's't'u'v'$ and hatch the enclosed space.

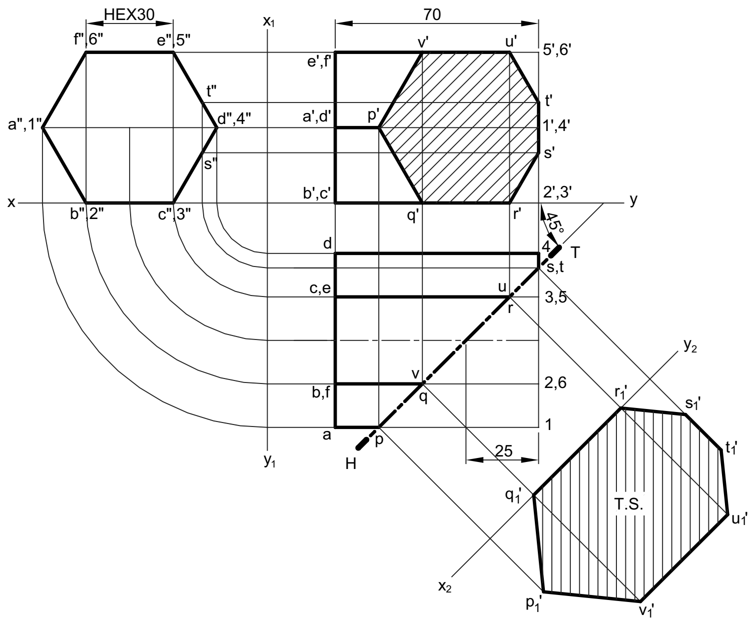


Fig. 12.28

4. **True shape** Draw x_2y_2 parallel to H.T. Project p, q, r, s, t, u and v on x_2y_2. Locate points p_1', q_1', r_1', s_1', t_1', u_1' and v_1' on the projectors such that their distances from x_2y_2 are equal to distances of p', q', r', s', t', u' and v' from xy, respectively. Join $p_1'q_1'r_1's_1't_1'u_1'v_1'$ and hatch the enclosed space to get the true shape of section.

Problem 12.24 *A pentagonal pyramid of base side 30 mm and axis 60 mm is resting on its base on the H.P. with a side of base parallel to V.P. and nearer to it. It is cut by a section plane parallel to the V.P. and 12 mm in front of the axis of the pyramid. Draw its sectional front view and top view.*

Figure 12.29(a) shows the pictorial view of a pentagonal pyramid kept on H.P. cut by a section plane parallel to the V.P.

Construction Refer to Fig. 12.29(b).

1. **Projections** Draw a pentagon *abcde* keeping side *de* parallel to *xy*. Join all the corners with centroid *o*. This is the top view. Project the corners and obtain $a'o'c'$ as the front view.
2. **Cutting plane** Draw H.T. of the section plane parallel to *xy* and lying at a distance of 12 mm from centroid *o*. Let H.T. cut the edges *ab* at *p*, *ob* at *q* and *bc* at *r*.
3. **Sectional front view** Project *p* and *r* to meet $a'b'$ and $b'c'$ at points p' and r'.
   Point *q* cannot be directly projected to $o'b'$. Therefore, draw an arc with centre *o* and radius *oq* to meet *oc* at point *m*. Project *m* to meet $o'c'$ at m'. Draw horizontal line from m' to meet $o'b'$ at q'. Join $p'q'r'$ and hatch the enclosed space. As the section plane is parallel to the V.P., $p'q'r'$ represents the true shape of the section.

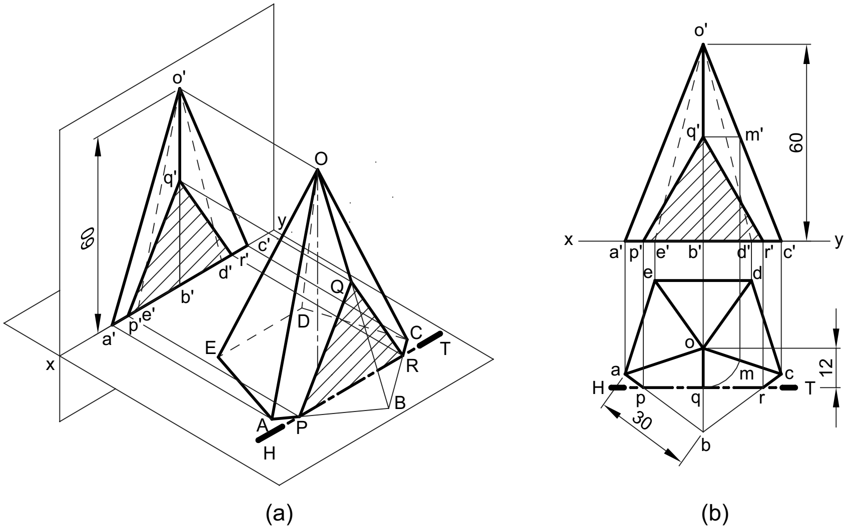


Fig. 12.29 *Section of pyramid by plane parallel to V.P.* **(a)** *Pictorial view* **(b)** *Orthographic view*

Problem 12.25 *A pentagonal pyramid of base side 30 mm and axis 60 mm is resting on its base on the H.P. with an edge of the base nearer the V.P., parallel to it. A vertical section plane inclined at 45° to the V.P. cuts the pyramid at a distance of 8 mm from the axis. Draw its sectional front view, top view and true shape of the section.*

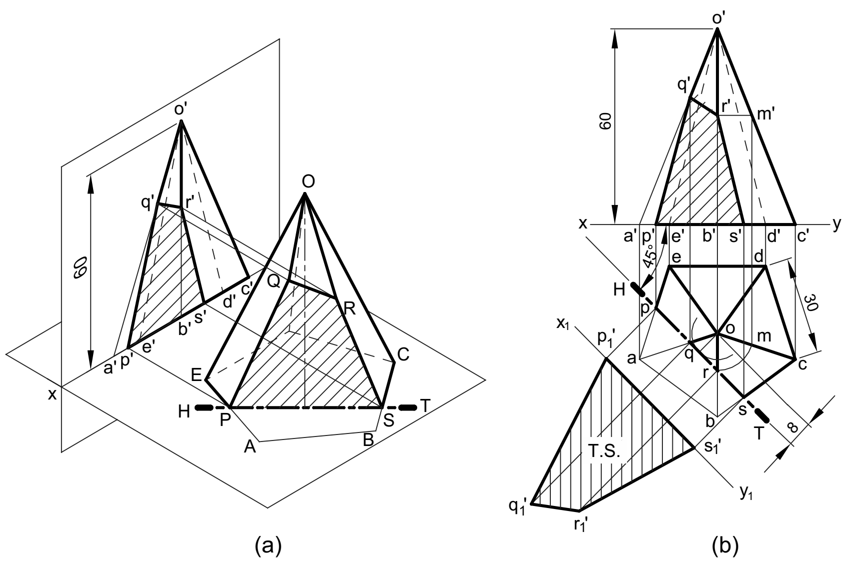


Fig. 12.30 *Section of pyramid by A.V.P.* **(a)** *Pictorial view* **(b)** *Orthographic view*

Figure 12.30(a) shows the pictorial view of a pentagonal pyramid kept on H.P. cut by A.V.P. inclined at 45° to the V.P.

Construction Refer to Fig. 12.30(b).

1. **Projections** Draw a pentagon *abcde* keeping side *de* parallel to *xy*. Join all the corners with centroid *o*. This is the top view. Project the corners and obtain $a'o'c'$ as the front view.
2. **Cutting plane** Draw an arc/circle with centre *o* and radius 8 mm. Draw H.T. of the section plane tangent to the arc and inclined at 45° to *xy*. Therefore, H.T. lies at a distance of 8 mm from the axis. Let H.T. cut the edges *ae* at *p*, *oa* at *q*, *ob* at *r* and *bc* at *s*.
3. **Sectional front view** Project *p*, *q* and *s* to meet $a'e'$, $o'a'$ and $b'c'$ at points p', q' and s'. Point *r* cannot be directly projected to $o'b'$. Therefore, draw an arc with centre *o* and radius *or* to meet *oc* at point *m*. Project *m* to meet $o'c'$ at m'. Draw horizontal line from m' to meet $o'b'$ at r'. Join $p'q'r's'$ and hatch the enclosed space.
4. **True shape** Draw x_1y_1 parallel to H.T. Project *p*, *q*, *r* and *s* on x_1y_1. Locate points p_1', q_1', r_1' and s_1' on the projectors such that their distances from x_1y_1 are equal to distances of points p', q', r' and s' from *xy*, respectively. Join $p_1'q_1'r_1's_1'$ and hatch the portion enclosed space.

Problem 12.26 *A cone of base diameter 50 mm and axis 60 mm is resting on its base on the H.P. It is cut by a section plane whose H.T. is inclined at 60° to the reference line and passes through a point 15 mm away from the axis. Draw its sectional front view and obtain true shape of the section.*

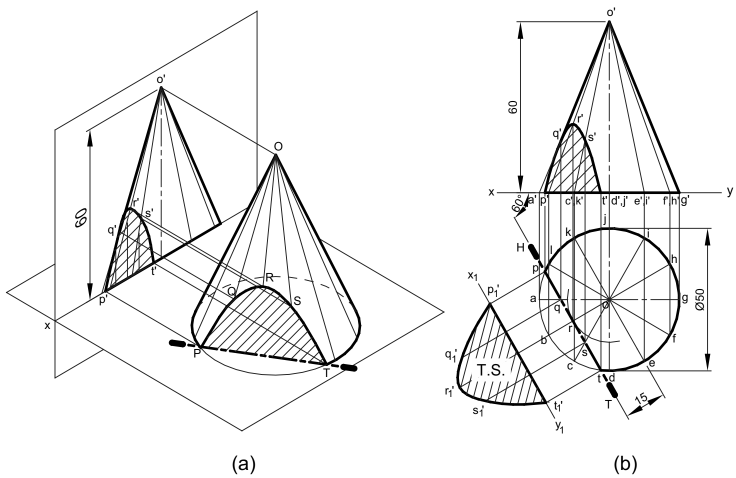


Fig. 12.31 *Section of cone by A.V.P.* **(a)** *Pictorial view* **(b)** *Orthographic view*

Figure 12.31(a) shows the pictorial view of a cone kept on H.P. cut by A.V.P. inclined at 60° to the V.P.

Construction Refer to Fig. 12.31(b).

1. **Projections** Draw a circle *adgj* to represent the top view. Divide the circle into 12 equal parts and project to obtain $a'g'o'$ as the front view.
2. **Cutting plane** Draw an arc with centre *o* and radius 15 mm. Draw H.T. of the section plane tangent to this arc inclined at 60° to *xy*. Thus, H.T. lies at a distance of 15 mm from the axis. Let H.T. cut the base and the generators al at *p*, *oa* at *q*, *ob* at *r*, *oc* at *s* and *cd* at *t*.

3. **Sectional front view** Project p, q, r, s and t to meet $a'1'$, $o'a'$, $o'b'$, $o'c'$, and $c'd'$ at points p', q', r', s' and t'. Join $p'q'r's't'$ and hatch the enclosed space.
4. **True shape** Draw x_1y_1 parallel to H.T. Project p, q, r, s and t on x_1y_1. Locate points p_1', q_1', r_1', s_1' and t_1' on the projectors such that their distances from x_1y_1 are equal to distances of points p', q', r', s' and t' from xy, respectively. Join $p_1'q_1'r_1's_1't_1'$ and hatch the portion enclosed space to get the true shape of section.

Problem 12.27 *A pentagonal pyramid of base side 30 mm and axis 60 mm is resting on a triangular face on the H.P. with its axis parallel to the V.P. It is cut by a plane whose H.T. is inclined at 30° to the reference line and bisects the axis such that the apex is removed. Draw its sectional front view and obtain true shape of the section.*

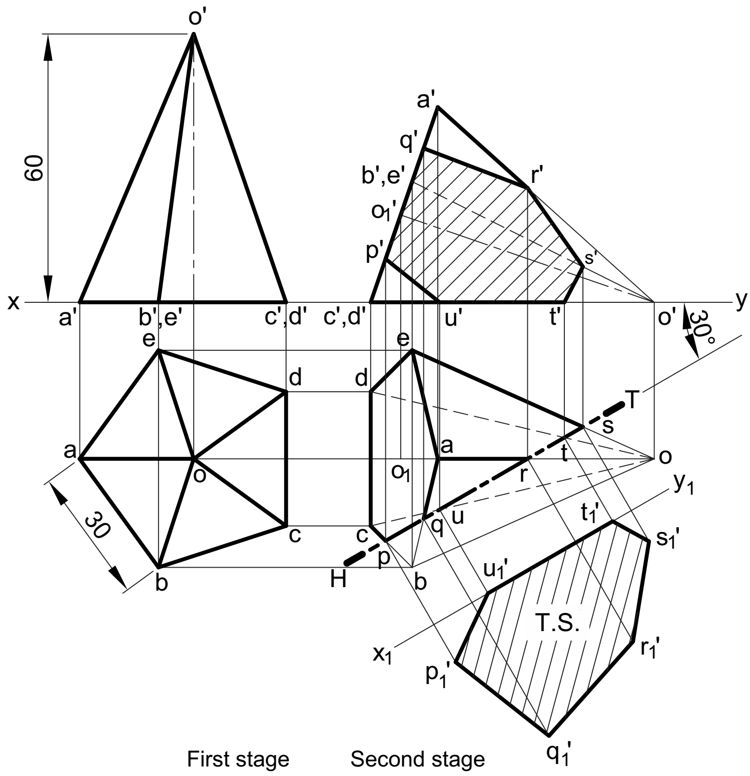


Fig. 12.32

Construction Refer to Fig. 12.32.

1. **First stage** Draw a pentagon *abcde* keeping side *cd* perpendicular to *xy*. Join the corners of the pentagon with the centroid *o*. This represents the top view. Project all the corners and obtain $a'o'c'$ as the front view.

2. **Second stage** Reproduce the front view of the first stage such that line $c'd'o'$ representing the triangular face is on xy. Obtain points a, b, c, d, e and o in the top view as the intersecting points of the projectors from the front view of the second stage with the corresponding locus lines from the top view of the first stage.
3. **Cutting plane** Draw H.T. of the section plane inclined at 30° to xy and passing through the mid-point of oo_1. Let H.T. cut the edges bc at p, ab at q, oa at r, oe at s, od at t and oc at u.
4. **Sectional front view** Project p, q, r, s, t and u to meet $b'c'$, $a'b'$, $o'a'$, $o'e'$, $o'd'$ and $o'c'$ at points p', q', r', s', t' and u'. Join $p'q'r's't'u'$ and hatch the enclosed space.
5. **True shape** Draw x_1y_1 parallel to H.T. Project p, q, r, s, t and u on x_1y_1. Locate points p_1', q_1', r_1', s_1', t_1' and u_1' on the projectors such that their distances from x_1y_1 are equal to distances of points p', q', r', s', t' and u' from xy, respectively. Join $p_1'q_1'r_1's_1't_1'u_1'$ and hatch the enclosed space to get the true shape of section.

Problem 12.28 *A pentagonal pyramid of base side 30 mm and axis 60 mm is resting on a triangular face on the H.P. with its axis parallel to the V.P. It is cut by a vertical section plane inclined at 60° to the V.P. and passing through the centre of the base. Draw its sectional front view and obtain true shape of the section retaining the apex.*

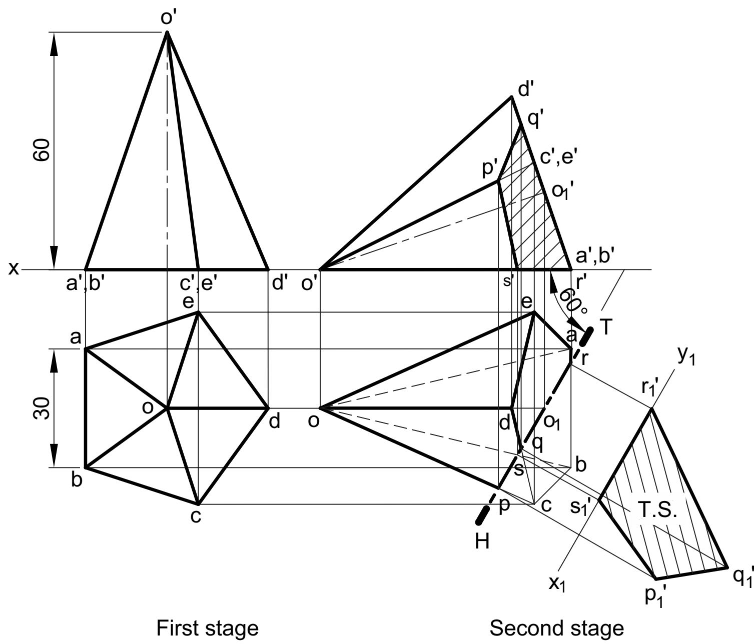


Fig. 12.33

Construction Refer to Fig. 12.33.

1. **First stage** Draw a pentagon $abcde$ keeping side ab perpendicular to xy. Join the corners of the pentagon with the centroid o. This represents the top view. Project all the corners and obtain $d'o'd'$ as the front view.

2. **Second stage** Reproduce the front view of the first stage such that line $a'b'o'$ representing the triangular face is on xy. Obtain points a, b, c, d, e and o in the top view as the intersecting points of the projectors from the front view of the second stage with the corresponding locus lines from the top view of the first stage.
3. **Cutting plane** Draw H.T. of the section plane inclined at 60° to xy and passing through end point of the axis o_1. Let H.T. cut the edges oc at p, cd at q, ab at r and ob at s.
4. **Sectional front view** Project p, q, r and s to meet $o'c'$, $c'd'$, $a'b'$ and $o'b'$ at points p', q', r' and s'. Join $p'q'r's'$ and hatch the enclosed space.
5. **True shape** Draw x_1y_1 parallel to H.T. Project p, q, r and s on x_1y_1. Locate points p_1', q_1', r_1', and s_1' on the projectors such that their distances from x_1y_1 are equal to distances of points p', q', r' and s' from xy, respectively. Join $p_1'q_1'r_1's_1'$ and hatch the enclosed space to get the true shape of section.

Problem 12.29 *A sphere of diameter 60 mm is kept on the H.P. It is cut by an A.V.P. inclined at 45° to V.P. The section plane passes through the sphere at a distance of 10 mm from the centre of the sphere and in front of it. Draw its sectional front view, top view and true shape of the section.*

Construction Refer to Fig. 12.34.

1. **Projections** Draw a circle with centre o and diameter 60 mm as the top view. Project the top view and obtain another circle with centre o' and diameter 60 mm as the front view.
2. **Cutting plane** Draw an arc with centre o and radius 10 mm. Draw H.T. of the section plane inclined at 45° to xy and tangential to the arc. Let it cut the circle at points a and g.
3. **True shape** Draw a line $a_1'g_1'$ parallel and equal to H.T. Draw a circle with diameter $a_1'g_1'$. Hatch the enclosed space to represent the true shape.
   Divide this circle into 12 equal parts and name the divisions as a_1', b_1', c_1', d_1', e_1', f_1', g_1', h_1', i_1', j_1', k_1' and l_1'.
4. **Sectional front view** Project a_1', b_1', c_1', d_1', e_1', f_1', g_1', h_1', i_1', j_1', k_1' and l_1' on H.T. and locate points a, b, c, d, e, f, g, h, i, j, k and l, respectively.
   Project points a, b, c, d, e, f, g, h, i, j, k and l to the front view. Obtain points a', b', c', d', e', f', g', h', i', j', k' and l' on the projectors such that their distances from x_2y_2 are equal to distances of points a_1', b_1', c_1', d_1', e_1', f_1', g_1', h_1', i_1', j_1', k_1' and l_1' from $a_1'g_1'$. Hatch the enclosed space.

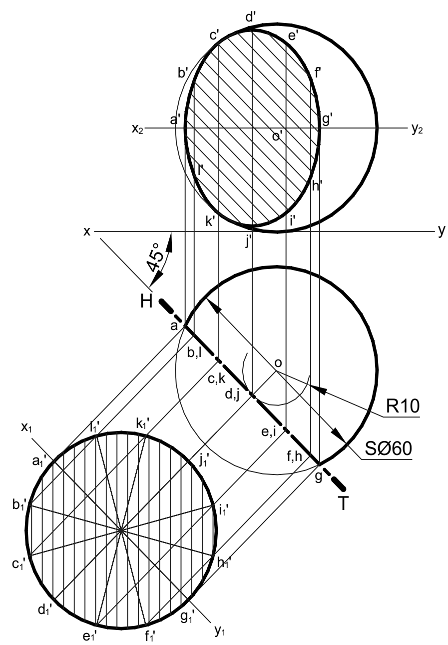


Fig. 12.34

12.6 SECTION BY A PLANE PERPENDICULAR TO BOTH H.P. AND V.P.

Problem 12.30 *A square prism of base side 40 mm and axis 60 mm rests on its base on the H.P. such that one of its rectangular faces is inclined at 30° to the V.P. It is cut by a section plane perpendicular to both the H.P. and the V.P., passing through one of the vertical edges. Draw its front view, top view and sectional side view.*

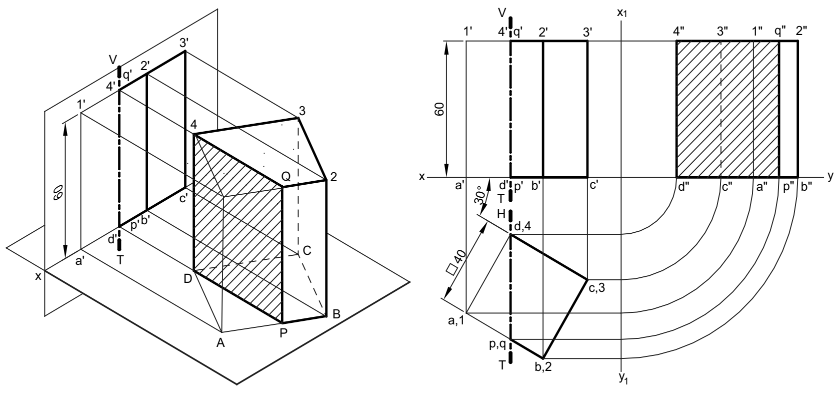


Fig. 12.35 *Section of prism by a plane parallel to both H.P. and V.P.* **(a)** *Pictorial view* **(b)** *Orthographic View*

Figure 12.35(a) shows the pictorial view of a square prism cut by a sectional profile plane.

Construction Refer to Fig. 12.35(b).

1. **Projections** Draw a square *abcd* keeping side *cd* inclined at 30° to *xy* to represent the top view. Project all the corners and obtain *a′c′*3′1′ as the front view. Also obtain *b″*2″4″*d″* as the side view.
2. **Cutting plane** Draw H.T. and V.T. of the section plane perpendicular to *xy* and passing through edge *d*4. Let H.T. cut the edges *ab* at *p* and 1-2 at *q*.
3. **Sectional side view** Project points *p* and *q* to the front view and obtain *p′* and *q′*. Also project these points to the side view and obtain *p″* and *q″*. Join *d″p″q″*4″ and hatch the enclosed space. As the section plane is parallel to the profile plane, *d″p″q″*4″ represents the true shape of the section.

Problem 12.31 *A pentagonal pyramid of base side 35 mm and axis 60 mm is resting on the H.P. on an edge of its base such that axis is inclined 45° to the H.P. and parallel to the V.P. It is cut by a section plane perpendicular to both the principal planes and passes through the edge on which the pyramid is resting. Draw the front view, top view and sectional side view.*

Construction Refer to Fig. 12.36.

1. **First stage** Draw a pentagon *abcde* keeping *ab* perpendicular to *xy* and join the corners with centroid *o*. This is the top view. Project the corners and obtain *a′d′o′* as the front view.

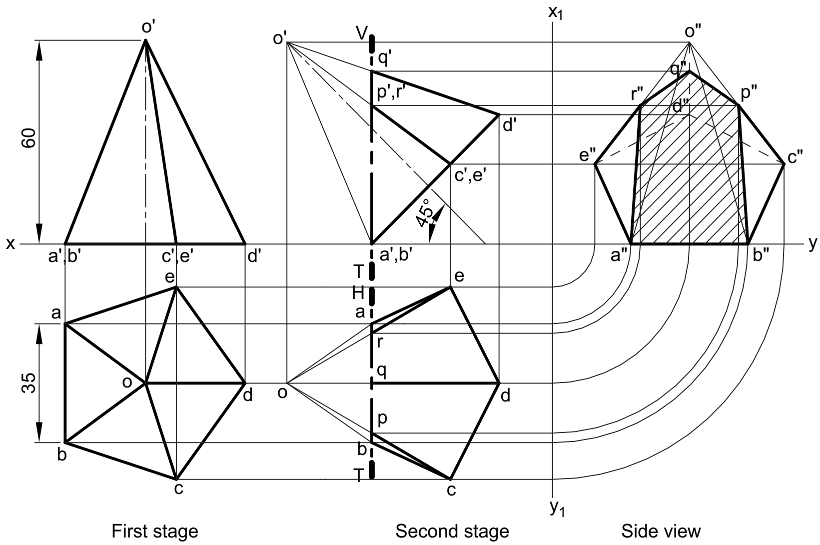


Fig. 12.36

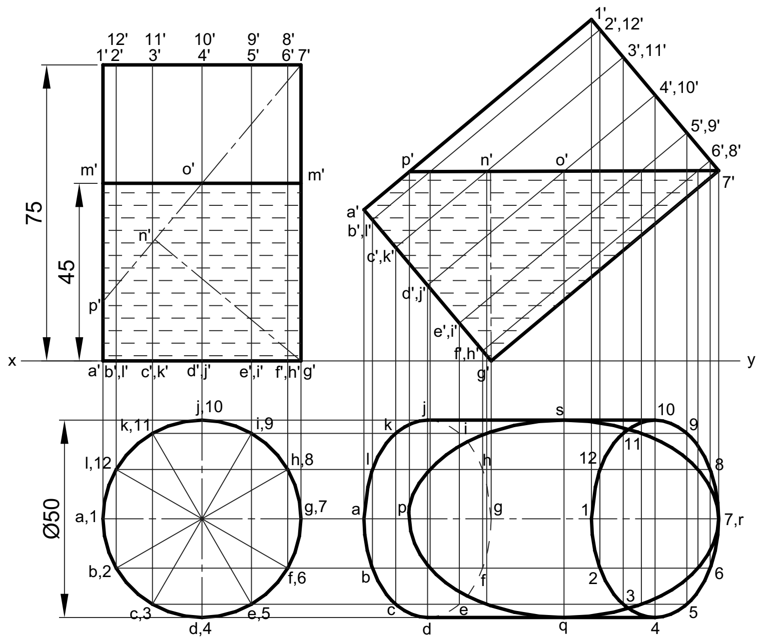
2. **Second stage** Reproduce the front view such that $a'b'$ is on xy and axis $o'o_1'$ is inclined at 45° to xy. Project the front view to meet locus lines from the top view of the first stage and obtain *obcdea* as the new top view.
3. **Side view** Obtain $o''e''a''b''c''$ as the side view by taking projectors from the front and top views.
4. **Cutting plane** Draw H.T. and V.T. of the section plane perpendicular to xy and passing through edge $a'b'$ and ab. Let H.T. cut the edges oc at p, od at q and oe at r.
5. **Sectional side view** Project p, q and r to mark p', q' and r' in the front view. Also project p, q and r to meet their respective edges in the side view at points p'', q'' and r''. Join $a''b''p''q''r''$ and hatch the enclosed space.

12.7 MISCELLANEOUS PROBLEMS

Problem 12.32 *A thin cylindrical glass vessel of base diameter 50 mm and height 75 mm, resting on the H.P. contains water up to 45 mm from its base. The vessel is then tilted so that water is just at the point of tricking out. Draw the projections of the glass in its tilted position showing clearly the water surface.*

Construction Refer to Fig. 12.37.

1. **First stage** Draw a circle *adgj* to represent the top view. Divide the circle into 12 equal parts and project to obtain $a'g'7'1'$ as the front view. Draw horizontal line $m'm'$, 45 mm from the base showing initial level of water. Locate o' as the mid-point of $m'm'$. Join $7'o'$ and produce it to meet $a'1'$ at p'. The inclination of line $p'7'$ with xy represents the angle through glass vessel should be tilted. Draw $g'n'$ perpendicular to $p'7'$



First stage Second stage

Fig. 12.37

2. **Second stage** Reproduce the front view of the first stage keeping g' on xy, $g'n'$ perpendicular to xy and $p'7'$ parallel to xy. Project the front view to meet locus lines from the top view of the first stage and obtain a-d-4-7-10-j as the new top view.
3. **Water surface** Mark the points of intersection of $p'7'$ with the generators of the cylinder. Project them to meet their respective generators in the top view and obtain ellipse *pqrs*. This is the required position of water surface in the top view when the glass vessel is tilted.

Note Hatching is not done in the ellipse *pqrs* because it does not represent the cut surface.

Problem 12.33 *A cylinder of base diameter 60 mm and axis 70 mm long is resting on its base in the H.P. It is cut by two auxiliary inclined planes which make angles of 60° and 45° to the H.P. and pass through the top end of the axis. Draw its sectional top view and true shape of the section.*

Construction Refer to Fig. 12.38.

1. **Projections** Draw a circle *adgj* to represent the top view. Divide the circle into 12 equal parts and project to obtain $a'g'7'1'$ as the front view.
2. **Cutting plane** Draw V_1T_1 and V_2T_2 of the section planes inclined at 60° and 45° to xy, respectively. V_1T_1 and V_2T_2 should pass through $4'$ and $10'$.
3. **Sectional top view** As the cutting planes cut all the generators. Hatch the circle in the top view showing sectional top view. It may be noted that a line 4-10 will be visible in the top view.

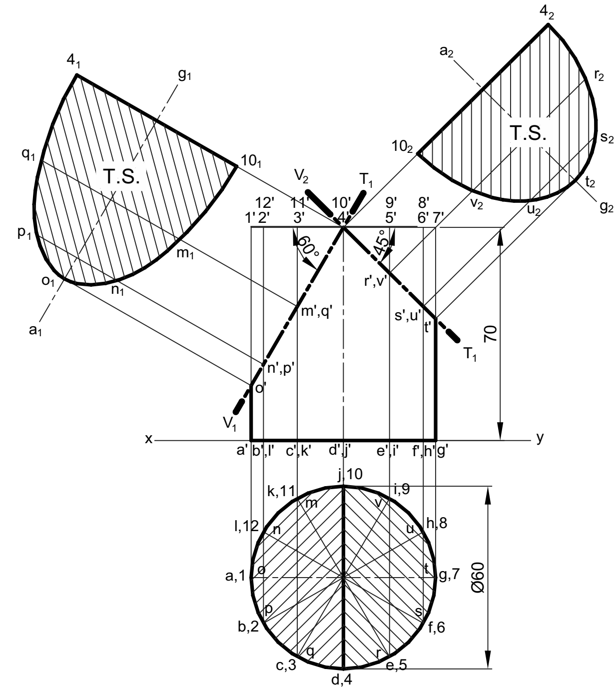


Fig. 12.38

4. **True shape 1** Draw a_1g_1 parallel to V_1T_1. Project points 10, m', n', o', p', q' and 4 on a_1g_1. Locate 10_1, m_1, n_1, o_1, p_1, q_1 and 4_1 on the projectors such that their distances from a_1g_1 are equal to distances of 10, m, n, o, p, q and 4 from ag. Join $10_1m_1n_1o_1p_1q_14_1$ and hatch the enclosed space to get the true shape of section.
5. **True shape 2** Draw a_2g_2 parallel to V_2T_2. Project points 4, r, s, t, u, v and 10 on a_2g_2. Locate 4_2, r_2, s_2, t_2, u_2, v_2 and 10_2 on the projectors such that their distances from a_2g_2 are equal to distance of 4, r, s, t, u, v and 10 from xy. Join $4_2r_2s_2t_2u_2v_210_2$ and hatch the enclosed space to get the true shape of section.

Problem 12.34 *A cone of base diameter 50 mm and axis 60 mm is resting on its base in the H.P. It is cut by a horizontal plane and an A.I.P. inclined at 45° to the H.P. Both the planes meet at a point on the axis 35 mm above the base. Draw its sectional top view and obtain true shape of the section.*

Construction Refer to Fig. 12.39.

1. **Projections** Draw a circle *adgj* to represent the top view. Divide the circle into 12 equal parts and project to obtain $a'g'o'$ as the front view.
2. **Cutting plane** Mark a point 4′ on the axis at a height of 35 mm from the base. Draw V4′ of the section plane inclined at 45° to *xy* and 4′T of the section plane parallel to *xy*.

3. **Sectional top view** Let V.T. cut the generators $o'a'$ at $1'$, $o'b'$ at $2'$, $o'c'$ at $3'$, $o'd'$ at $4'$, $o'e'$ at $5'$, $o'f'$ at $6'$, $o'g'$ at $7'$, $o'h'$ at $8'$, $o'i'$ at $9'$, $o'j'$ at $10'$, $o'k'$ at $11'$ and $o'l'$ at $12'$. Project $1'$, $2'$, $3'$, $5'$, $6'$, $7'$, $8'$, $9'$, $11'$ and $12'$ to meet in the top view at points 1, 2, 3, 5, 6, 7, 8, 9, 11 and 12. The horizontal line from $4'10'$ meets $o'g'$ at $7'$. Project $7'$ to meet og at 7. Draw an arc with centre o and radius $o7$ to meet od and oj at points 4 and 10 respectively. Join 1-2-3-4-5-6-7-8-9-10-11-12 and hatch the enclosed space.
4. **True shape** For sectional plane $4'$T, enclosed space 4-5-6-7-8-9-10 is the true shape. For sectional plane V$4'$ Draw a_1g_1 parallel to it. Project points $10'$, $11'$, $12'$, $1'$, $2'$, $3'$ and $4'$ on a_1g_1. Locate 10_1, 11_1, 12_1, 1_1, 2_1, 3_1 and 4_1 on the projectors such that their distances from a_1g_1 line are equal to distances of points 10, 11, 12, 1, 2, 3 and 4 from ag, respectively. Join $10_111_112_11_12_13_14_1$ and hatch the enclosed space to get the true shape of section.

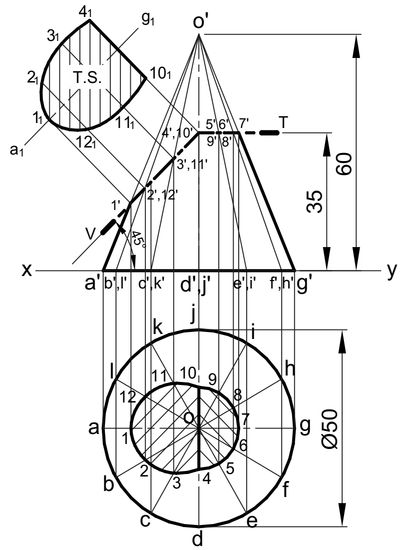


Fig. 12.39

Problem 12.35 *A composite solid of height 60 mm is made of half cylinder (base diameter 60 mm) and half hexagonal prism (base side 30 mm). It is resting on the H.P. on its base such that an edge of the hexagon is perpendicular to V.P. The solid is cut by an A.I.P. inclined at 45° to H.P. and passing through a point on the axis 15 mm below the top face. Draw its sectional top view and obtain true shape of the section.*

Construction Refer to Fig. 12.40.

1. **Projections** Draw a semi-hexagon *abcd* and a semi-circle *defghia* meeting on diameter line *ad*. This is the top view. Project the top view and obtain $c'g'7'3'$ as the front view.
2. **Cutting plane** Draw V.T. of the section plane inclined at 45° to *xy* and passing through a point on the axis lying at a point 15 mm from the top end. Let V.T. cut the edges and generators $a'1'$ at m', $1'2'$ at n', $3'4'$ at o', $d'4'$ at p', $e'5'$ at q', $f'6'$ at r', $g'7'$ at s', $h'8'$ at t' and $i'9'$ at u'.
3. **Sectional top view** Project m', n', o', p', q', r', s', t' and u' to meet in the top view at points m, n, o, p, q, r, s, t and u. Join *mnopqrstu* and hatch the enclosed space.
4. **True shape** Draw k_1g_1 parallel to V.T. Project points m', n', o', p', q', r', s', t' and u' on k_1g_1.

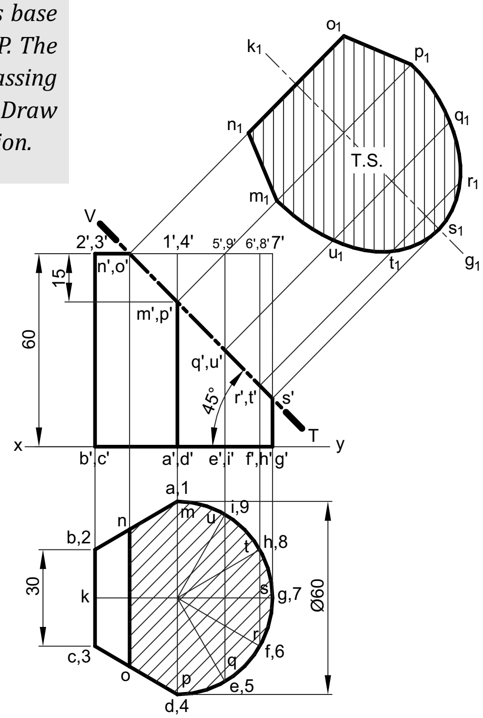


Fig. 12.40

Locate m_1, n_1, o_1, p_1, q_1, r_1, s_1, t_1 and u_1 on the projectors such that their distances from k_1g_1 are equal to distances of points *m*, *n*, *o*, *p*, *q*, *r*, *s*, *t* and *u* from *kg*, respectively. Join $m_1n_1o_1p_1q_1r_1s_1t_1u_1$ and hatch the enclosed space to get the true shape of section.

Problem 12.36 *A square pyramid of base side 50 mm and axis 75 mm, rests on one of its triangular faces on the ground. The top view of the axis is inclined at 30° to the V.P. It is cut by a horizontal section plane, the V.T. of which intersects the axis at a point 20 mm from the base. Draw its front view and sectional top view.*

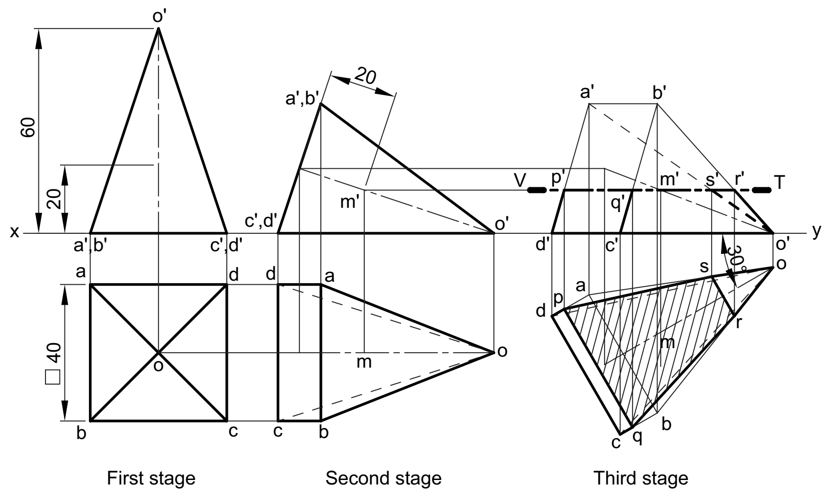


Fig. 12.41

Construction Refer to Fig. 12.41.

1. **First stage** Draw a square *abcd* keeping side *cd* perpendicular to *xy*. Join the corners of the square with the centroid *o*. This represents the top view. Project all the corners and obtain *a′o′c′* as the front view.
2. **Second stage** Reproduce the front view of the first stage such that line *c′d′o′* representing the triangular face is on *xy*. Obtain points *a*, *b*, *c*, *d* and *o* in the top view as the intersecting points of the projectors from the front view of the second stage with the corresponding locus lines from the top view of the first stage.
3. **Third stage** Reproduce the top view of the second stage such that axis is inclined at 30° to *xy*. Obtain points *a′*, *b′*, *c′*, *d′* and *o′* in the front view as the intersecting points of the projectors from the top view of the third stage with the corresponding locus lines from the front view of the second stage.
4. **Cutting plane** Draw V.T. of the section plane parallel to *xy* and passing through point *m′*. Let V.T. cut the edges *a′d′* at *p′*, *b′c′* at *q′*, *o′b′* at *r′* and *o′a′* at *s′*.
5. **Sectional top view** Project *p′*, *q′*, *r′* and *s′* to meet *ad*, *bc*, *ob* and *oa* at *p, q, r and s.* Join *pqrs* and hatch the enclosed space.

Problem 12.37 *A solid is composed of a cone of base diameter 60 mm and height 60 mm placed centrally on its base over a square plate of side 100 mm and thickness 30 mm with a base side parallel to the V.P. It is cut by an A.I.P. inclined at 45° to the H.P. and bisecting the axis of the cone. Draw the sectional top view and true shape of the section.*

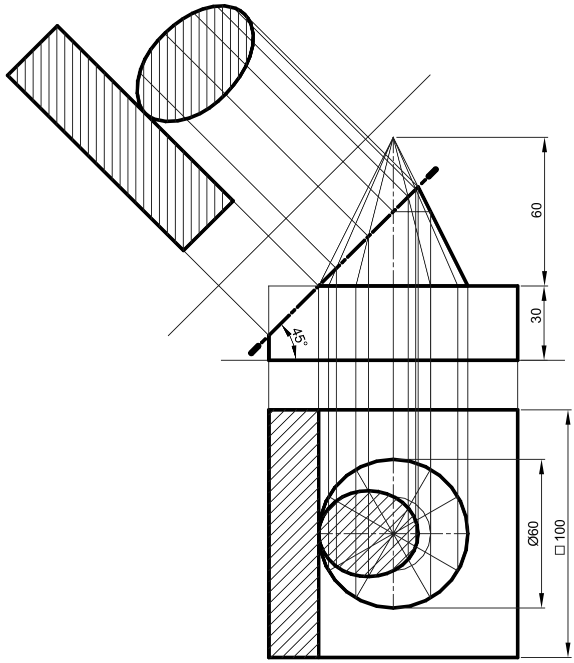


Fig. 12.42

Problem 12.38 *A cylindrical disc of base diameter 100 mm and height 45 mm is lying on one of its ends on the H.P. A hemisphere of diameter 120 mm is kept centrally on its flat face on the disc. The composed solid is cut by an A.I.P. inclined at 45° to the H.P. and is passing through the lowest point of the extreme generator of the disc. Draw the sectional top view and true shape of the section.*

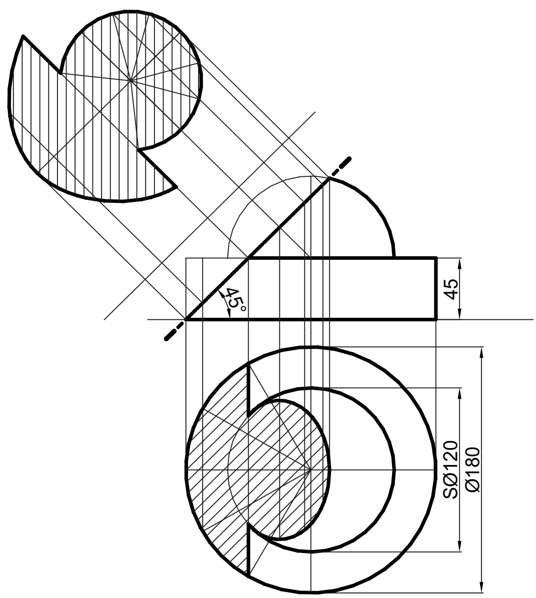


Fig. 12.43

EXERCISE 12A

12.1 A hexagonal prism of base side 30 mm and axis 70 mm is resting on a corner of its base on the H.P. and axis inclined at 60° to the H.P. and parallel to the V.P. It is cut by a horizontal section plane which divides the prism into two equal halves. Draw its sectional top view.

12.2 A pentagonal prism of base side 30 mm and axis 60 mm is resting on a face on the H.P. with its axis inclined at 30° to the V.P. It is cut by a horizontal section plane passing through a point 10 mm below the top longer edge. Draw its sectional top view.

12.3 A square pyramid of base side 45 mm and axis 70 mm is resting on its base in the H.P. with a side of base parallel to the V.P. Draw its sectional top view, sectional side view and true shape of the section when it is cut by a section plane perpendicular to the V.P., bisecting the axis and is inclined at (a) 45° to the H.P. and (b) 60° to the H.P.

12.4 A square pyramid of base side 45 mm and axis 70 mm is resting on its base in the H.P. with all the sides of the base equally inclined to the V.P. Draw its sectional top view, sectional side view and true shape of the section when it is cut by a section plane perpendicular to the V.P., bisecting the axis and is inclined at (a) 45° to the H.P. and (b) 60° to the H.P.

12.5 A cone of base diameter 50 mm and axis 60 mm is resting on its base in the H.P. Draw its sectional views and true shape of the section, if it is cut by a section plane perpendicular to the V.P., bisecting the axis and is (a) parallel to the H.P., (b) inclined at 45° to the H.P. and (c) inclined at 60° to the H.P.

12.6 A hexagonal pyramid of base side 30 mm and axis 70 mm rests on a triangular face in the H.P. with its axis parallel to the V.P. It is cut by an A.I.P. inclined at 60° to the H.P. bisecting the axis. Draw its sectional top view and true shape of the section.

12.7 A pentagonal pyramid of base edge 30 mm and axis 60 mm has its base on the H.P. and an edge of the base perpendicular to V.P. It is cut by a section plane, perpendicular to the V.P. and inclined at 45° to the H.P. and intersecting the axis at a point 25 mm above the base. Draw the sectional top view, sectional side view and true shape of the section.

12.8 A hexagonal pyramid of base edge 30 mm and axis 70 mm has its base on the H.P. and an edge of the base perpendicular to V.P. It is cut by a section plane, perpendicular to the V.P. and inclined at 60° to the H.P. bisecting the axis. Draw its sectional top view, sectional side view and true shape of the section.

12.9 A cone of base diameter 60 mm and axis 70 mm long is resting on its base on the H.P. It is cut by an A.I.P. parallel to one of its extreme generators and passing through a point on the axis 20 mm below the axis. Draw its sectional top view and obtain true shape of the section.

12.10 A cube of 40 mm edges is resting on one of its faces on the H.P. such that a vertical face is inclined at 30° to the V.P. It is cut by an A.I.P. inclined at 30° to the H.P. and passing through a point on the axis 30 mm above the H.P. Draw its sectional front view, sectional top view, true shape of section and auxiliary top view on a plane parallel to the section plane.

12.11 A cylinder of diameter 50 mm and axis 70 mm is resting on its base on the H.P. It is cut by a section plane inclined at 45° to the H.P. and perpendicular to V.P. such that the plane bisects the axis. Draw its front view, sectional top view and another top view on an A.I.P. parallel to the section plane.

12.12 A cylinder of base diameter 50 mm and axis 70 mm is resting on its base on the H.P. It is cut by a section plane perpendicular to the V.P., the V.T. of which cuts the axis at a point 20 mm from the top end of the axis and inclined at 45° to the reference line. Draw its front view, sectional top view and another top view on an A.I.P. parallel to the section plane.

12.13 A cone of base diameter 60 mm and axis 70 mm long rests on one of its generators in the H.P. with its axis parallel to the V.P. It is cut by an A.I.P. inclined at 60° to the H.P. bisecting the axis. Draw its sectional top view and true shape of the section.

12.14 A hemisphere of diameter 70 mm rests on its flat face on the H.P. It is cut by an A.I.P. inclined at 30° to H.P. and passing through a point 12 mm from the centre of the sphere such that larger portion of the hemisphere is retained. Draw its sectional top view and obtain true shape of the section.

12.15 A pentagonal pyramid of base side 30 mm and height 70 mm rests on one of its corners on the H.P. Its axis is inclined at 45° to the H.P. and maintains a distance of 40 mm from V.P. A sectional plane parallel to and

50 mm away from the V.P. cuts the pyramid. Draw its top view and sectional front view.

12.16 A hexagonal prism of base side 30 mm and axis 60 mm is resting on its base on the H.P. It is cut buy a section plane parallel to the V.P. and 10 mm in front of the axis of the prism. Draw its top view and sectional front view.

12.17 A hexagonal pyramid of base edge 30 mm and axis 60 mm has its base on the H.P. and a base edge parallel to the V.P. A vertical section plane, inclined at 45° to the V.P., cuts the pyramid at a distance of 6 mm from the axis. Draw the sectional front view and an auxiliary front view on a plane parallel to the section plane.

12.18 A cone of base diameter 60 mm and axis 70 mm long rests on its base in the H.P. It is cut by a plane whose H.T. makes an angle of 30° with the reference line and passes through a point 10 mm from the axis. Draw its sectional front view and obtain the true shape of the section.

12.19 A square prism of base side 40 mm and axis 60 mm is resting on its base on the H.P. with a side perpendicular to the V.P. It is cut by an A.V.P. making 30° to the V.P. and contains the axis of the prism. Draw its top view, sectional front view and the true shape of the section.

12.20 A cube of 40 mm edges is resting on the H.P. on one of its faces with a vertical face inclined at 30° to the V.P. It is cut by a section plane perpendicular to the H.P., parallel to the V.P. and passing through a point 10 mm away from the axis. Draw its sectional front view and top view.

12.21 A hexagonal prism of base side 30 mm and axis 70 mm has its face on the H.P. and the axis parallel to V.P. It is cut by a plane, the H.T. of which makes an angle of 45° with the reference line bisecting the axis. Draw the sectional front view and true shape of the section.

12.22 A cylinder of base diameter 50 mm and axis 60 mm rests on its base on H.P. It is cut by an A.V.P. making 45° with the reference line at a distance 15 mm from the axis. Draw its sectional front view, top view and true shape of the section.

12.23 A hexagonal pyramid of base side 25 mm and axis 70 mm rests on a triangular face on the H.P. with its axis parallel to the V.P. It is cut by a plane whose H.T. makes an angle of 30° with the reference line and bisects the axis such that the apex is removed. Draw its sectional front view and obtain true shape of the section.

12.24 A cone of base diameter 50 mm and axis 70 mm long rests on a generator in the H.P. with its axis parallel to the V.P. It is cut by a plane whose H.T. makes an angle of 30° with the reference line and bisects the axis such that the apex is (a) removed and (b) retained. Draw its sectional front view and obtain true shape of the section.

12.25 A cylinder of base diameter 50 mm and axis 60 mm rests on its base on the H.P. It is cut by a section plane perpendicular to both the H.P. and the V.P. such that its distance from the axis is 18 mm. Draw its front, top and sectional end views.

12.26 A cylinder of base diameter 60 mm and axis 70 mm long is lying on a generator on the H.P. with its axis parallel to the V.P. A vertical section plane, the H.T. of which is inclined at 30° to the V.P. and passes through a point 25 mm on the axis from one of its ends cut the cylinder. Draw its sectional front view and obtain true shape of the section.

12.27 A pentagonal pyramid of base side 30 mm and axis 70 mm is resting on its base in the H.P. with an edge of the base parallel to the V.P. A horizontal section plane cuts the pyramid at a distance of 30 mm from the base. It is further cut by an A.I.P. passing through the same point and one of the extreme corners of the base. Draw its sectional views and determine true shape of section.

12.28 A cylinder of diameter 50 mm and axis 75 mm is resting on its generator on the H.P. with its axis parallel to the V.P. It is cut by two A.I.P.s which pass through a common point 25 mm from one end of the topmost generator and touch the opposite end points of the bottom most generator. Draw its sectional top view and determine the true shape of the section.

12.29 A thin glass vessel of base diameter 60 mm and height 75 mm is completely filled with water. It is then tilted on the rim of its base circle such that base is inclined at 30° to the H.P. In the process some water from it is drained out. Draw the projections of the cylindrical vessel showing remaining water in it.

12.30 A hollow square prism of 5 mm thick walls rests on its base on the H.P. with a vertical face inclined at 30° to the V.P. It is cut by an A.I.P. inclined at 45° to the H.P. and passing through a point 10 mm below the top end of the axis. Taking external edge of base as 40 mm and axis 60 mm long, draw its sectional top view, sectional side view and true shape of the section.

12.31 A hollow cylinder of outside diameter 60 mm, axis 65 mm and thickness 8 mm is resting on its base on the H.P. An A.I.P. inclined at 30° to the H.P. and passing through a point on the axis 12 mm from its top end cuts the cylinder. Draw its sectional top view, sectional side view and true shape of the section.

12.8 ANTI-SECTION

We have learnt so far that if the position of the section plane is given we can draw the sectional views and also determine the true shape of the section. However if the true shape of the cut surface is given it is also possible to derive the position of the section plane. In other words, it is the reverse of the problems discussed earlier in this chapter.

12.8.1 Anti-section of Prism and Cube

A prism cut by a section plane may have the true shape of the cut surface as a triangle, square, rectangle, rhombus, trapezium, etc. Let us consider the following problems.

Problem 12.39 *A square prism of base side 40 mm and axis 70 mm rests on one of its bases on the H.P. with edges of the base equally inclined to the V.P. It is cut by an A.I.P. such that the true shape of the section is the largest possible triangle. Draw front view, sectional top view and true shape of the section.*

Construction Refer to Fig. 12.44.

1. **Projections** Draw a square *abcd* keeping sides inclined at 45° to *xy*. This is the top view. Project all the corners and obtain $a'c'3'1'$ as the front view.
2. **Cutting plane** To obtain a triangle the section plane should intersect three faces of the prism. Therefore, draw V.T. to pass through points 2′, 4′ and *c*′. This will give the largest isosceles triangle.
3. **Sectional top view** Projecting points 2′, 4′ and *c*′ to the top view we get apparent section as 2-4-*c*. As edges of the enclosed space are inclined at 45° to *xy*, hatch space with lines inclined at 30°/60° to *xy*.
4. **True shape** Draw x_1y_1 parallel to V.T. Project points 2′, 4′ and *c*′ on x_1y_1. Locate points 2_1, 4_1 and c_1 on the projectors such that their distances from x_1y_1 are equal to distances of points 2, 4 and *c* from *xy*. Join $2_14_1c_1$ and hatch the enclosed space to get the true shape of section.

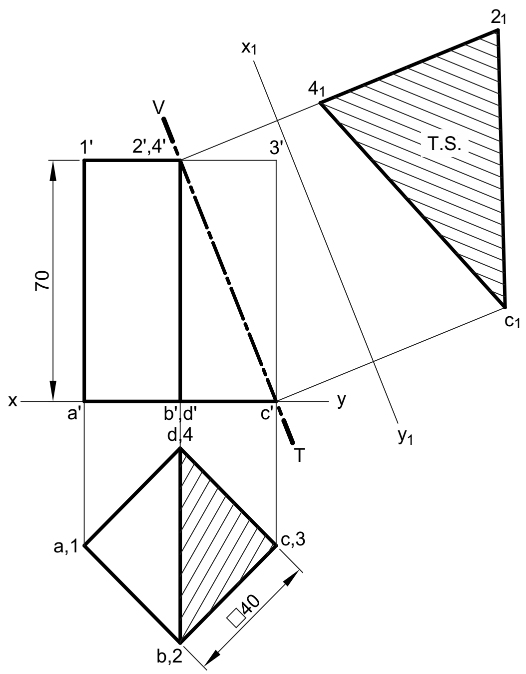


Fig. 12.44

Problem 12.40 *A pentagonal prism of base side 30 mm and axis 60 mm is kept on a base on the H.P. with a rectangular face perpendicular to V.P. It is cut by an A.I.P. such that the true shape of the section is an isosceles triangle of largest base and altitude 50 mm. Draw the sectional top view and true shape of the section.*

Construction Refer to Fig. 12.45.

1. **Projections** Draw a pentagon *abcde* keeping side *ab* perpendicular to *xy*. This is the top view. Project all the corners and obtain $a'd'4'1'$ as the front view.
2. **Cutting plane** For the longest base the A.I.P. should pass through 3-5. For the altitude of 50 mm, the V.T. should cut an intercept of 50 mm in the front view. Therefore, draw an arc with centre $3'$ and radius 50 mm to meet $d'4'$ at point p'. Draw V.T. of the section plane joining $3'p'$.
3. **Sectional top view** Project points $3'$, $5'$ and p' to the top view and obtain 3-5-*p* as apparent section. Hatch the enclosed space.
4. **True shape** Draw x_1y_1 parallel to V.T. Project points $3'$, $5'$, p' on x_1y_1. Locate points 3_1, 5_1 and p_1 on the projectors such that their distances from x_1y_1 are equal to distances of points 3, 5 and *p* from *xy*, respectively. Join $3_14_15_1$ and hatch the enclosed space to get the true shape of section.

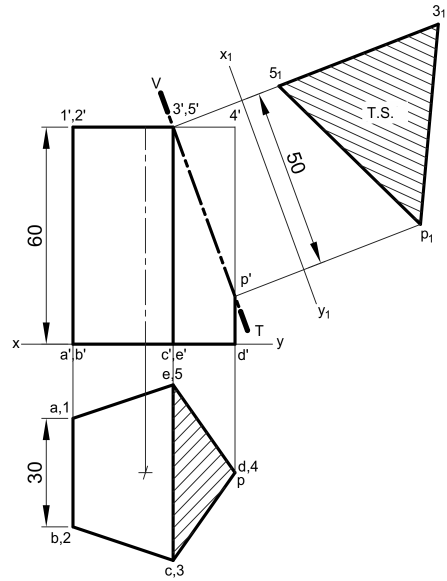


Fig. 12.45

Problem 12.41 *The true section of a square prism cut by an auxiliary inclined plane is a rhombus of 80 mm and 50 mm long diagonals. The A.I.P. passes through one of the longer edges of the prism at a height of 10 mm from its base. Draw the projections of the truncated prism and project the true shape of the section. Also, determine the inclination of the A.I.P. with the H.P.*

Visualisation To obtain a rhombus, the section plane should intersect four faces of the prism. To get minor diagonal of rhombus as 50 mm the diagonal of the prism should be 50 mm. For the major diagonal of 80 mm the V.T. of the section plane should cut an intercept of 80 mm in the front view.

Construction Refer to Fig. 12.46.

1. Draw a square *abcd* of 50 mm diagonals keeping sides inclined at 45° to *xy*. This is the top view. Project the corners on *xy*. The height of the prism is unknown.
2. As V.T. of the section plane should pass through a point 10 mm above the H.P., mark point p' 10 mm above *xy* on the locus edge $a'1'$.
3. Draw an arc with centre p' and radius 80 mm to meet locus edge $c'3'$ at point $3'$. Draw V.T. to pass through $p'3'$. Obtain $a'c'3'1'$ as the front view of the prism with height equal to $c'3'$.
4. Determine inclination of V.T. from *xy* as the inclination of A.I.P. with H.P. Here $\theta = 51°$.
5. **Sectional top view** Let V.T. cut the edges $a'1'$ at p', $b'2'$ at q', $c'3'$ at r' and $d'4'$ at s'. Project points p', q', r' and s' to the top view and obtain *pqrs* as apparent section. Hatch the enclosed space.
6. **True shape** Draw a_1c_1 parallel to V.T. Project p', q', r' and s' on a_1c_1. Locate points p_1, q_1, r_1 and s_1 on the projectors such that their distances from a_1c_1 are equal to distances of points *p*, *q*, *r* and *s* from *ac*. Join $p_1q_1r_1s_1$ and hatch the enclosed space to get the true shape of section.

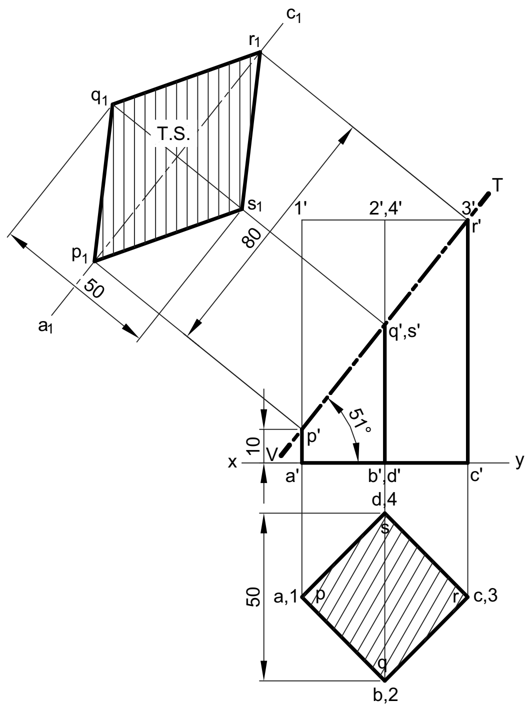


Fig. 12.46

> **Problem 12.42** *A cube of sides 45 mm rests on the H.P. on its square base. It is cut by a section plane such that the true shape of the section is a trapezium whose one of the parallel sides is equal to the diagonal of the face of the cube and the other is half of the former. Draw the sectional top view and true shape of the section. Determine the inclination of the section plane.*

Visualisation To obtain a trapezium, all the vertical faces of the cube should be equally inclined to the V.P. and the section plane should intersect its four faces. As one of the parallel sides of the trapezium is equal to diagonal of a square face, the V.T. of the section plane should pass through diagonal 2′4′. As other parallel side of the trapezium is half of the 2′4′, V.T. should pass through mid-point of sides $b'c'$ and dc'.

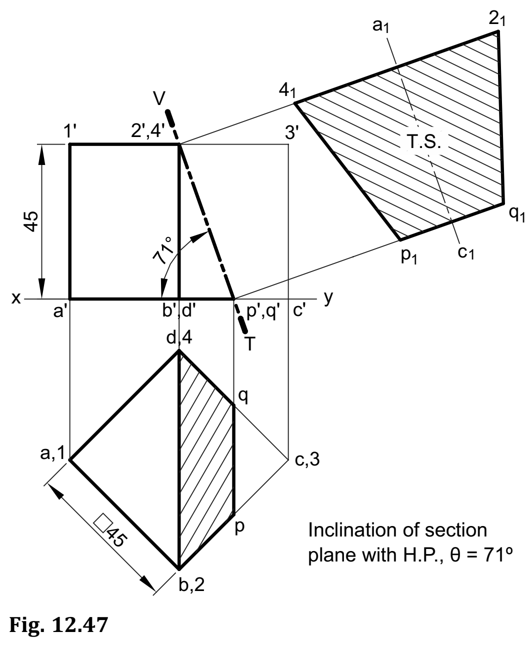


Fig. 12.47

Construction Refer to Fig. 12.47.

1. **Projections** Draw a square *abcd* keeping all sides inclined at 45° to *xy*. This is the top view. Project all the corners and obtain $a'c'3'1'$ as the front view.
2. **Cutting plane** Mark mid-points of side *bc* and *cd* as points *p* and *q*. Project them to obtain points p' and q' on $b'c'$ and $c'd'$ respectively.
3. Draw V.T. to pass through $2'$, $4'$, p' and q'. Determine inclination of V.T. with *xy* as inclination of A.I.P. with H.P. Here $\theta = 71°$.
4. **Sectional top view** Obtain apparent section as $2pq4$. Hatch the enclosed space with lines inclined at 30°/60° with *xy*.
5. **True shape** Draw a_1c_1 parallel to V.T. Project points $2'$, p', q' and $4'$ on a_1c_1. Locate points 2_1, p_1, q_1 and 4_1 on the projectors such that their distances from a_1c_1 are equal to distances of points 2, *p*, *q* and 4 from *ac*. Join $2_1p_1q_1 4_1$ and hatch the enclosed space to get the true shape of section.

Problem 12.43 *A cube of 45 mm long edges rests on the H.P. with vertical faces equally inclined to the V.P. It is cut by a section plane perpendicular to V.P. so that the true shape of the section is a regular hexagon. Draw the sectional top view and true shape of the section. Also, determine the inclination of the section plane with the H.P.*

Visualisation To obtain a hexagon, the section plane should intersect all the six faces of the cube.

Construction Refer to Fig. 12.48.

1. **Projections** Draw a square *abcd* keeping all sides inclined at 45° to *xy*. This is the top view. Project all the corners and obtain $a'c'3'1'$ as the front view.
2. **Cutting plane** Draw V.T. to pass through mid-points of $1'2'$ and $b'c'$. Determine inclination of V.T. with *xy* as inclination of the section plane with H.P. Here $\theta = 55°$.
3. **Sectional top view** Let V.T. cut the edges $1'2'$ at p', $b'2'$ at q', $b'c'$ at r', $c'd'$ at s', $d'4'$ at t' and $4'1'$ at u'. Project p', q', r', s', t' and u' to meet their respective edges in the top view and obtain *pqrstu* as apparent section. Hatch the enclosed space with lines inclined at 30°/60° with *xy*.
4. **True shape** Draw a_1c_1 parallel to V.T. Project points p', q', r', s', t' and u' on a_1c_1. Locate points p_1, q_1, r_1, s_1, t_1 and u_1 on the projectors such that their distances from a_1c_1 are equal to distances of points *p*, *q*, *r*, *s*, *t* and *u* from *ac*. Join $p_1q_1r_1s_1t_1u_1$ and hatch the enclosed space to get the true shape of section.

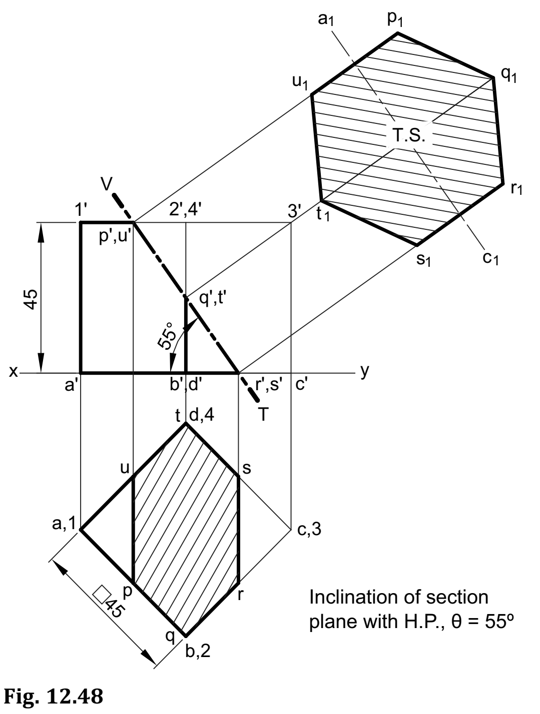


Fig. 12.48

12.8.2 Anti-section of Cylinder

A cylinder cut by a section plane may have true shape of the cut surface as a rectangle, ellipse, etc. Let us consider the following problems.

Problem 12.44 *A cylinder of base diameter 50 mm and axis 60 mm is kept on a base on the H.P. It is cut by an A.V.P. inclined at 30° to the V.P. such that true shape of the section is a rectangle of smaller side 30 mm. Draw its top view, sectional front view and true shape of the section. Determine the distance of the section plane from the axis of the cylinder.*

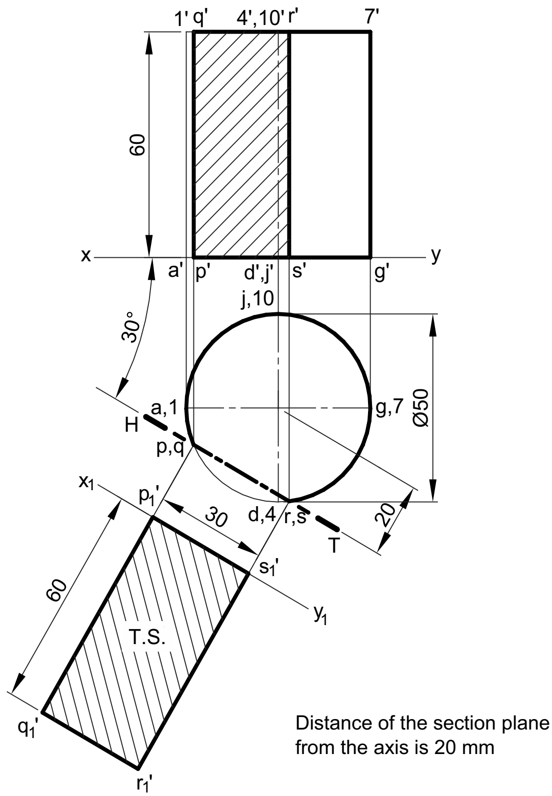


Fig. 12.49

Construction Refer to Fig. 12.49.

1. **Projections** Draw a circle *adgj* of 50 mm diameter as the top view and project to obtain $a'g'7'1'$ as its front view.
2. **Cutting plane** Draw H.T. of the section plane inclined at 30° to *xy* which cuts the circle to have a chord of 30 mm. Measure the distance of H.T. from the centre of the circle as distance of the sectional plane with the axis. Here it is at 20 mm.

3. **Sectional front view** Let H.T. cut the circle at points p, q, r and s. Project p, q, r and s to the front view and obtain points p', q', r' and s'. Join $p'q'r's'$ and hatch the enclosed space.
4. **True shape** Draw x_1y_1 parallel to H.T. Project p, q, r and s on x_1y_1. Locate p_1', q_1', r_1' and s_1' on the projectors such that their distances from x_1y_1 are equal to distances of points p', q', r' and s' from xy. Join $p_1'q_1'r_1's_1'$ and hatch the enclosed space to get the true shape of section.

Problem 12.45 *A cylinder is cut by an auxiliary inclined plane such that the true shape of the section is an ellipse of major and minor axes as 70 mm and 50 mm respectively. The smallest generator of the truncated cylinder is 15 mm. Draw its projections and obtain the true shape of the section. Determine the inclination of the section plane.*

Visualisation To obtain an ellipse, an A.I.P. should intersect all the generators of the cylinder kept on the H.P. To obtain minor axis as 50 mm the cylinder should have its base diameter 50 mm. To obtain major axis as 70 mm the V.T. of the section plane should cut an intercept of 70 mm in the front view.

Construction Refer to Fig. 12.50.

1. **Top view** Draw a circle *adgj* of 50 mm diameter to represent the top view. Divide it into 12 equal parts. Project the circle on xy as the locus of generators. The height of the cylinder is unknown.
2. **Cutting plane** Mark m' 15 mm above xy on the projector of point a. Draw an arc with centre m' and radius 70 mm to meet projector of 7 at point 7′. Draw V.T. of the section plane to connect m'7′. Determine inclination of V.T. with xy as inclination of the section plane with H.P. Here $\theta = 44°$.
3. **Front view** Obtain $a'g'$7′1 as the front view having axis equivalent to length g'7′.
4. **Sectional top view** Let V.T. cut the generators a'1′ at m', b'2′ at n', c'3′ at o', d'4′ at p', e'5′ at q', f'6′ at r', g'7′ at s', h'8′ at t', i'9′ at u', j'10′ at v', k'11′ at w' and l′12′ at z'. Project intersecting point to obtain m, n, o, p, q, r, s, t, u, v, w and z in the top view. Join *mnopqrstuvwz* and hatch the enclosed space.
5. **True shape** Draw a_1g_1 parallel to V.T. Project m', n', o', p', q', r', s', t', u', v', w' and z' on a_1g_1. Locate m_1, n_1, o_1, p_1, q_1, r_1, s_1, t_1, u_1, v_1, w_1 and z_1 on the projectors such that their distances from a_1g_1 are equal to distances of m', n', o', p', q', r', s', t', u', v', w' and z' from ag. Join $m_1n_1o_1p_1q_1r_1s_1t_1u_1v_1w_1z_1$ and hatch the enclosed space to get the true shape of section.

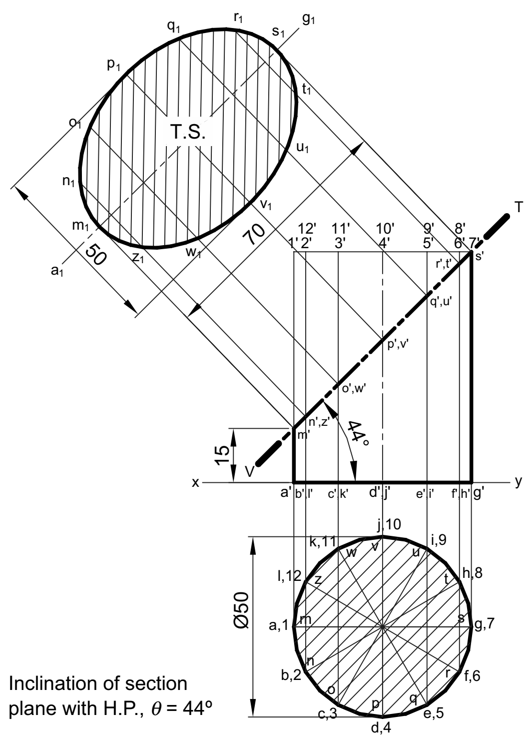


Fig. 12.50

Problem 12.46 *A cylinder of base diameter 50 mm and axis 70 mm has its axis parallel to the H.P. and inclined at 30° to the V.P. It is cut by a vertical section plane such that true shape of the section is an ellipse of major axis 60 mm. Draw its top view, sectional front view and true shape of the section.*

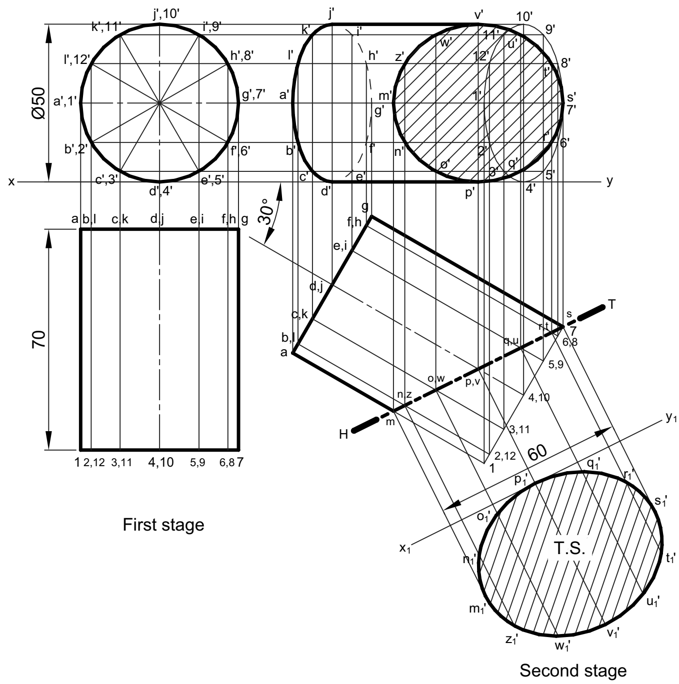


Fig. 12.51

Construction Refer to Fig. 12.51.

1. **First stage** Draw a circle $a'd'g'j'$ to represent the front view. Divide it into 12 equal parts and project to obtain $ag71$ as the top view.
2. **Second stage** Reproduce the top view of the first stage such that axis is inclined at 30° to xy. Project the top view to meet locus lines from the front view of the first stage and obtain $a'd'4'7'10'j'$ as the new front view.
3. **Cutting plane** Draw an arc keeping point 7 as the centre and radius 60 mm to meet generator $a1$ at point m. Draw H.T. of the section plane to connect $m7$. Let H.T. cut the generators $a1$ at m, $b2$ at n, $c3$ at o, $d4$ at p, $e5$ at q, $f6$ at r, $g7$ at s, $h8$ at t, $i9$ at u, $j10$ at v, $k11$ at w and $l12$ at z.
4. **Sectional front view** Project m, n, o, p, q, r, s, t, u, v, w and z on the respective generators in the front view and obtain m', n', o', p', q', r', s', t', u', v', w' and z'. Join $m'n'o'p'q'r's't'u'v'w'z'$ and hatch the enclosed space.
5. **True shape** Draw x_1y_1 parallel to H.T. Project m, n, o, p, q, r, s, t, u, v, w and z on x_1y_1. Locate m_1', n_1', o_1', p_1', q_1', r_1', s_1', t_1', u_1', v_1', w_1' and z_1' on the projectors such that their distances from x_1y_1 are equal to distances of m', n', o', p', q', r', s', t', u', v', w' and z' from xy, respectively. Join $m_1'n_1'o_1'p_1'q_1'r_1's_1't_1'u_1'v_1'w_1'z_1'$ and hatch the enclosed space to get the true shape of section.

12.8.3 Anti-section of Pyramid and Tetrahedron

A pyramid cut by a section plane may have true shape of the cut surface as a trapezium, triangle, etc. Let us consider the following problems.

Problem 12.47 *A square pyramid of base side 40 mm and axis 60 mm is resting on its base on the ground with all the edges of the base equally inclined to the V.P. It is cut by an A.I.P. such that true shape of the section is an isosceles triangle of largest base and smallest altitude. Draw the sectional top view and true shape of the section.*

Visualisation To obtain a triangle in the true shape the square pyramid should be kept with edges of the base equally inclined to *xy* and the section plane should intersect three faces. As the isosceles triangle has largest base, the V.T. of the section plane should pass through the base diagonal. As the triangle has smallest altitude, V.T. should shortest which is possible when it is perpendicular to *c′o′*.

Construction Refer to Fig. 12.52.

1. **Projections** Draw a square *abcd* keeping sides inclined at 45° to *xy*. Join the corners with centroid *o*. This represents the top view. Project all the corners and obtain *a′c′o′* as the front view.
2. **Cutting plane** Draw V.T. of the section plane from *b′d′* perpendicular on *o′c′* to obtain p′ on *o′c′*. This is the shortest possible V.T.
3. **Sectional top view** Project *p′* to meet *oc* at point *p*. Join *bpd* and hatch the enclosed space.
4. **True shape** Draw a_1c_1 parallel to V.T. Project points *b′*, *p′* and *d′* on a_1c_1. Locate points b_1, p_1 and d_1 on the projectors such that their distances from a_1c_1 are equal to distances of points *b*, *p* and *d* from *ac*. Join $b_1p_1d_1$ and hatch the enclosed space to get the true shape of section.

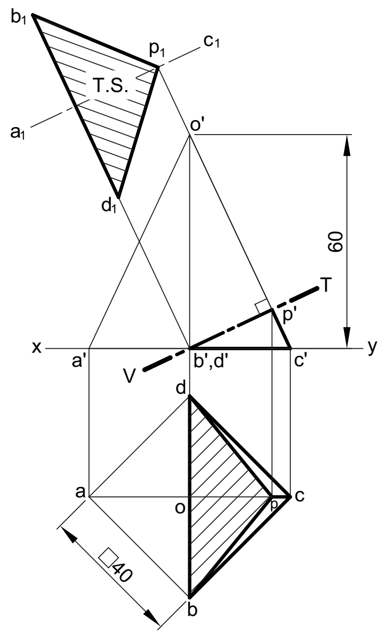


Fig. 12.52

Problem 12.48 *A square pyramid of base side 40 mm and axis 60 mm is resting on its base on the ground with all the edges of the base equally inclined to the V.P. It is cut by a section plane perpendicular to both H.P. and V.P. such that the true shape of the section is an isosceles triangle of altitude 45 mm. Draw its front view, top view and sectional side view.*

Visualisation To obtain a triangle, square pyramid should be kept with edges of the base equally inclined to *xy* and the section plane should intersect three faces. As the isosceles triangle has altitude of 45 mm, the intercept of V.T. in the front view should be 45 mm long.

Construction Refer to Fig. 12.53.

1. **Projections** Draw a square *abcd* keeping sides inclined at 45° to *xy*. Join the corners with centroid *o*. This represents the top view. Project all the corners and obtain *a′c′o′* as the front view. Also, obtain *b″o″d″* as the side view.

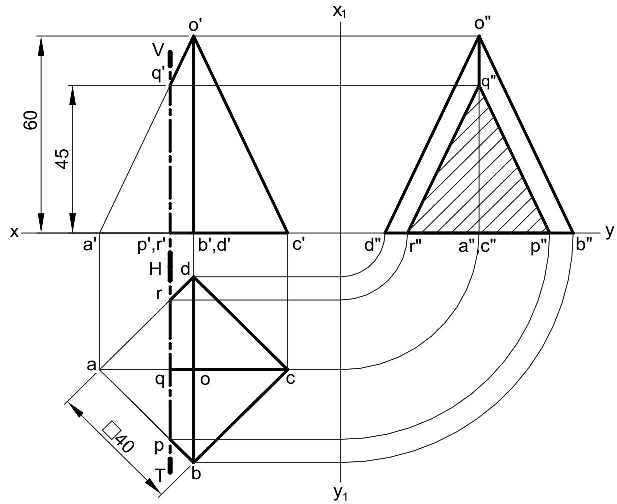


Fig. 12.53

2. **Cutting plane** Draw a horizontal line 45 mm above *xy* to intersect *a′o′* at *q′*. Draw V.T. of the section plane perpendicular from point *q′* to meet *xy* at *p′r′*. Also, draw H.T. of the section plane and locate points *p*, *q* and *r*.
3. **Sectional side view** Project *q′* from the front view to interset *a″o″* at *q″*. Project *p* and *r* from the top view to intersect *a″b″* and *a″d″* at *p″* and *r″*. Join *p″q″r″* and hatch the enclosed space.

Problem 12.49 *A square pyramid of base edge 40 mm and height 60 mm rests on its base on the ground. It is cut by an A.I.P. such that true shape of the section is a trapezium of parallel sides 10 mm and 30 mm. Draw the sectional top view and project the true shape of the section. Determine the inclination of the section plane.*

Visualisation To obtain a trapezium, the square pyramid should be kept with an edge of the base parallel to the V.P. the section plane should intersect four faces. As the trapezium has parallel sides of 10 mm and 30 mm, the length of intercept of two opposite faces should be 10 mm and 30 mm long.

Construction Refer to Fig. 12.54.

1. **Projections** Draw a square *abcd* keeping side *ad* parallel to *xy* and join the corners with centroid *o*. This represents the top view. Project the corners and obtain *b′c′o′* as the front view.
2. **Sectional top view** Mark points *p* and *q* on edges *oa* and *ob* respectively such that *pq* is 10 mm long and perpendicular to the *xy*. Similarly, mark points *r* and *s* on edges *oc* and *od* respectively such that *rs* is 30 mm long and perpendicular to the *xy*. Join *pqrs* and hatch the enclosed space.
3. **Cutting plane** Project *p*, *q*, *r* and *s* on *o′a′*, *o′b′*, *o′c′* and *o′d′* to obtain points *p′*, *q′*, *r′* and *s′*. Draw V.T. of the section plane connecting *p′*, *q′*, *r′* and *s′*. Determine inclination of V.T. with *xy* as inclination of the section plane with H.P. Here $\theta = 56°$.

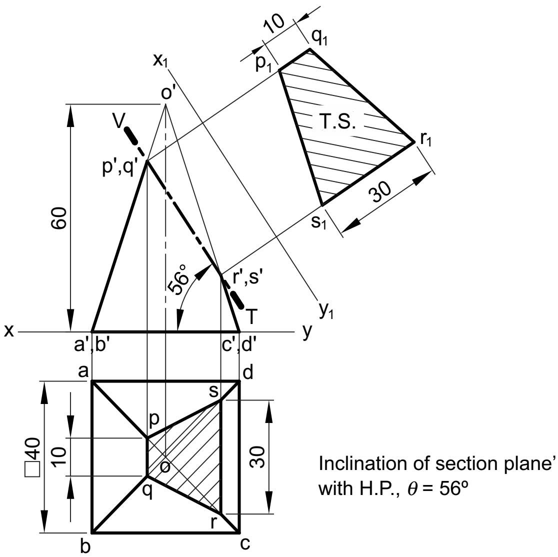


Fig. 12.54

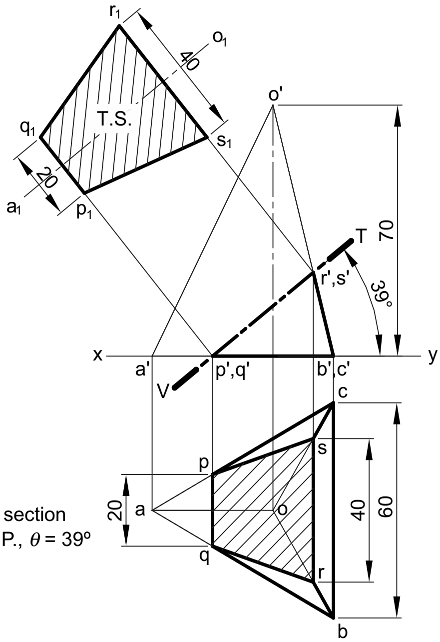
4. **True shape** Draw x_1y_1 parallel to V.T. Project points p', q', r' and s' on x_1y_1. Locate points p_1, q_1, r_1 and s_1 on the projectors such that their distances from x_1y_1 are equal to distances of points p, q, r and s from xy. Join $p_1q_1r_1s_1$ and hatch the enclosed space to get the true shape of section.

Problem 12.50 *A triangular pyramid of base edge 60 mm and axis 70 mm rests on H.P. with a face perpendicular to the V.P. An A.I.P. cuts the pyramid in such a manner that the true shape of the section is a trapezium of parallel sides 20 mm and 40 mm. Draw the true shape of the section and determine the inclination of the section plane with the horizontal plane.*

Visualisation To obtain a trapazium, the section plane should intersect all the faces and base of the triangular pyramid. As the trapezium has parallel sides of 20 mm and 40 mm, the length of intercept of the base and a face should be 20 mm and 40 mm long.

Construction Refer to Fig. 12.55.

1. **Projections** Draw a triangle *abc* keeping side *bc* perpendicular to *xy* and join the corners with centroid *o*. This represents the top view. Project the corners and obtain $a'b'o'$ as the front view.
2. **Sectional top view** Mark points *p* and *q* on edges *ac* and *ab* respectively such that *pq* is 20 mm long and perpendicular to the *xy*. Similarly, mark points *r* and *s* on edges *ob* and *oc* respectively such that *rs* is 40 mm long and perpendicular to the *xy*. Join *pqrs* and hatch the enclosed space.
3. **Cutting plane** Project *p*, *q*, *r* and *s* on $a'c'$, $a'b'$, $o'b'$ and $o'c'$ to obtain points p', q', r' and s'. Draw V.T. of the section plane connecting p', q', r' and s'. Determine inclination of V.T. with *xy* as inclination of the section plane with H.P. Here $\theta = 39°$.
4. **True shape** Draw a_1o_1 parallel to V.T. Project points p', q', r' and s' on a_1o_1. Locate points p_1, q_1, r_1 and s_1 on the projectors such that their distances from a_1o_1 are equal to distances of points p, q, r and s from *ao*. Join $p_1q_1r_1s_1$ and hatch the enclosed space to get the true shape of section.



Inclination of section plane with H.P., θ = 39°

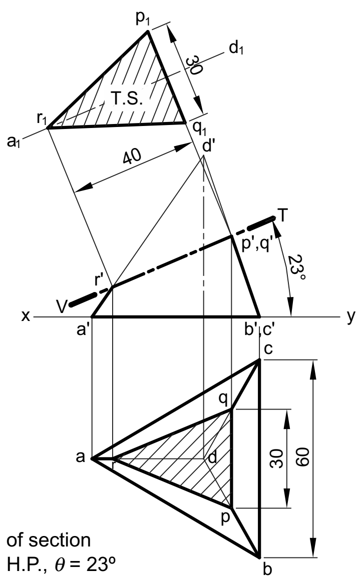
Fig. 12.55

Problem 12.51 *A tetrahedron of 60 mm long edges is lying on the H.P. on one of its faces with an edge perpendicular to the V.P. It is cut by an A.I.P. so that the true shape of the section is an isosceles triangle of base 30 mm and altitude 40 mm. Find the inclination of the section plane with the H.P. and draw the front view, sectional top view and true shape of the section.*

Visualisation To obtain a triangle, the section plane should intersect three faces of the tetrahedron. As the triangle has base 30 mm, the length of intercept on *obc* should be 30 mm. As the triangle has altitude 40 mm, V.T. should cut an intercept of 40 mm in the front view.

Construction Refer to Fig. 12.56.

1. **Projections** Draw a triangle *abc* keeping side *bc* perpendicular to *xy* and join the corners with centroid *o*. This represents the top view. Project the corners and obtain *a′b′d′* as the front view. Side *a′d′* should be 60 mm long.
2. **Cutting plane** In the top view mark points *p* and *q* on edges *db* and *dc* respectively such that *pq* is 30



Inclination of section plane with H.P., θ = 23°

Fig. 12.56

mm long and perpendicular to the xy line. Project points p and q on edges $d'b'$ and $d'c'$ to obtain p' and q' coinciding each other.

3. Draw an arc with centre p' and radius 40 mm to cut $a'd'$ at point r'. Draw V.T. of the section plane to connect p', q' and r'. Determine inclination of V.T. with xy as inclination of the section plane with H.P. Here $\theta = 23°$.
4. **Sectional top view** Project r' on ad to obtain point r. Join pqr and hatch the enclosed space.
5. **True shape** Draw a_1d_1 parallel to V.T. Project points p', q' and r' on a_1d_1. Locate points p_1, q_1 and r_1 on the projectors such that their distances from a_1d_1 are equal to distances of points p, q and r from ad. Join $p_1q_1r_1$ and hatch the enclosed space to get the true shape of section.

Problem 12.52 *A tetrahedron of 60 mm long edges is resting on the H.P. on one of its faces with an edge perpendicular to V.P. It is cut by an A.I.P. so that true shape of the section is a square. Set the required cutting plane and draw the sectional top view and obtain true shape of the section.*

Visualisation To obtain a square, the section plane should intersect all the four faces of the tetrahedron. Moreover, the section plane should pass through mid-points of the edges.

Construction Refer to Fig. 12.57.

1. **Projections** Draw a triangle abc keeping side bc perpendicular to xy and join the corners with centroid o. This represents the top view. Project the corners and obtain $a'b'd'$ as the front view. Side $a'd'$ should be 60 mm long.
2. **Cutting plane** Draw V.T. of the section plane passing through mid-points p', q', r', and s' of the edges $a'c'$, $a'b'$, $b'd'$ and $c'd'$ respectively.
3. **Sectional top view** Project p', q', r', and s' on ac, ab, bd and cd to obtain points p, q, r and s. Hatch the enclosed space.
4. **True shape** Draw a_1d_1 parallel to V.T. Project points p', q', r' and s' on a_1d_1. Locate points p_1, q_1, r_1 and s_1 on the projectors such that their distances from a_1d_1 are equal to distances of points p, q, r and s from ad. Join $p_1q_1r_1s_1$ and hatch the enclosed space to get the true shape of section.

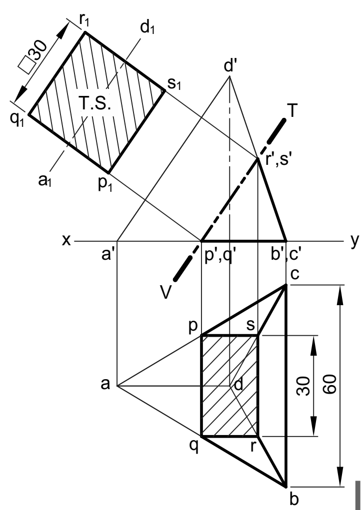


Fig. 12.57

12.8.4 Anti-section of Cone

A cone cut by a section plane may have true shape of the cut surface as a triangle, parabola, hyperbola, ellipse, etc. Let us consider the following problems.

Problem 12.53 *A cone of base diameter 50 mm and axis 60 mm rests on its base on the H.P. It is cut by an A.I.P. passing through the mid-point of the axis such that true shape of the section is an ellipse of largest major axis. Draw the projections of the truncated cone and true shape of the section.*

Visualisation To obtain an ellipse, the section plane should intersect all the generators of the cone. To obtain the largest major axis, the V.T. of the section plane should be longest.

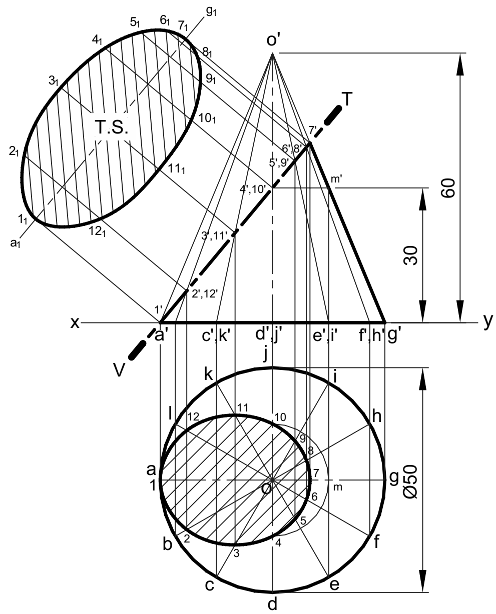


Fig. 12.58

Construction Refer to Fig. 12.58.

1. **Projections** Draw a circle *adgj* to represent the top view. Divide the circle into 12 equal parts and project to obtain $a'g'o'$ as the front view.
2. **Cutting plane** Draw V.T. of the section plane connecting a' with mid-point of the axis.
3. Obtain the sectional top view and true shape of the section. (Refer to Problem 12.11)

Problem 12.54 *A cone of base diameter 50 mm and axis 60 mm rests on its base in the H.P. It is cut by a section plane perpendicular to V.P. such that the true shape of the section is a parabola of base 40 mm. Draw its three views and obtain the true shape of the section.*

Visualisation To get a parabola V.T. of the section plane should be parallel to an end generator. As the parabola has a base of 40 mm, V.T. should make an intercept of 40 mm in the top view.

Construction Refer to Fig. 12.59.

1. **Projections** Draw a circle *adgj* to represent the top view. Divide the circle into 12 equal parts and project to obtain $a'g'o'$ and $d''o''j''$ as its front and side views respectively.
2. **Cutting plane** Draw a chord of the base circle 1-11 of 40 mm, perpendicular to *xy*. Project point 1 on $b'c'$ to obtain $1'$. Draw V.T. of the section plane parallel to $a'o'$ passing through $1'$.
3. Obtain the sectional top view and true shape of the section. (Refer to Problem 12.12)
4. Also obtain the sectional side view as shown.

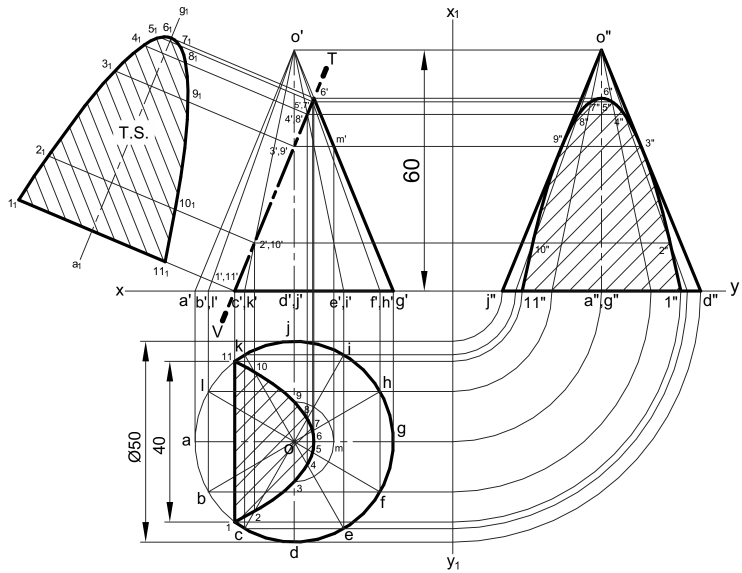


Fig. 12.59

Problem 12.55 *A cone of base diameter 50 mm and axis 60 mm is resting on its base on the ground. It is cut by a plane such that the true shape of the section is a rectangular hyperbola of base 40 mm and seen in the front view. Draw the sectional front view and find the distance of the section plane from the axis of the cone.*

Visualisation To obtain a rectangular hyperbola in the front view, the section plane should be parallel to the V.P. As the hyperbola has base 40 mm, the intercept of H.T. should be 40 mm long.

Construction Refer to Fig. 12.60.

1. **Projections** Draw a circle *adgj* to represent the top view. Divide the circle into 12 equal parts and project to obtain $a'g'o'$ as the front view.
2. **Cutting plane** Draw H.T. parallel to *xy* cutting a chord of 40 mm. Determine its distance from centre *o* as the distance of section plane from the axis. Here it is 15 mm.
3. **Sectional front view** Let H.T. cut *bc* at *p*, *oc* at *q*, *od* at *r*, *oe* at *s*, *ef* at *t*. Project *p*, *q*, *s* and *t* to meet $b'c'$, $o'c'$, $o'e'$ and $e'f'$ at p', q', s' and t'.
4. Point r' cannot be obtained directly on $o'd'$. Therefore, draw an arc with centre *o* and radius *or* to meet *og* at *m*. Project *m* to meet $o'g'$ at m'. Draw a horizontal line from m' to meet $o'd'$ at r'. Join $p'q'r's't'$ and hatch the enclosed space.

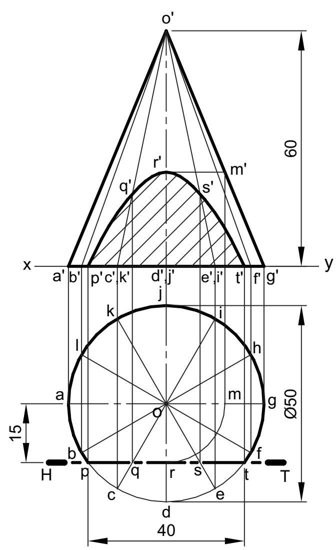


Fig. 12.60

Problem 12.56 *A cone of base diameter 50 mm and axis 60 mm rests on its base on the H.P. It is cut by an A.I.P. such that the true shape of the section is a hyperbola of base 30 mm and altitude 40 mm. Draw its sectional top view and true shape of the section.*

Visualisation To obtain a hyperbola the inclination of the section plane should be less than the half of the apex angle. As the hyperbola has a base 40 mm, the section plane should make an intercept of 40 mm in the top view. As the altitude is 50 mm, V.T. of the section plane should make an intercept of 50 mm in the front view.

Construction Refer to Fig. 12.61.

1. **Projections** Draw a circle *adgj* to represent the top view. Divide the circle into 12 equal parts and project to obtain $a'g'o'$ as the front view.
2. **Cutting plane** Draw a 30 mm long chord pt, perpendicular to *xy*. Project *p* and *t* it on *xy* and obtain p' and t'. Draw an arc with centre p' and radius 40 mm to meet $a'o'$ at r'. Draw V.T. of the section plane connecting $p'r'$.
3. **Sectional top view** Let V.T. cut $b'c'$ at p', $b'o'$ at q', $a'o'$ at r', $l'o'$ at s' and $k'l'$ at t'. Project p', q', r', s' and t' to meet their respective generators at *p*, *q*, *r*, *s* and *t*. Join *pqrst* and hatch the enclosed space.
4. **True shape** Draw a_1g_1 parallel to V.T. Project points p', q', r', s' and t' on a_1g_1. Locate points p_1, q_1, r_1, s_1 and t_1 on the projectors such that their distances from a_1g_1 are equal to distances of points *p*, *q*, *r*, *s* and *t* from *ag*. Join $p_1q_1r_1s_1t_1$ and hatch the enclosed space to get the true shape of section.

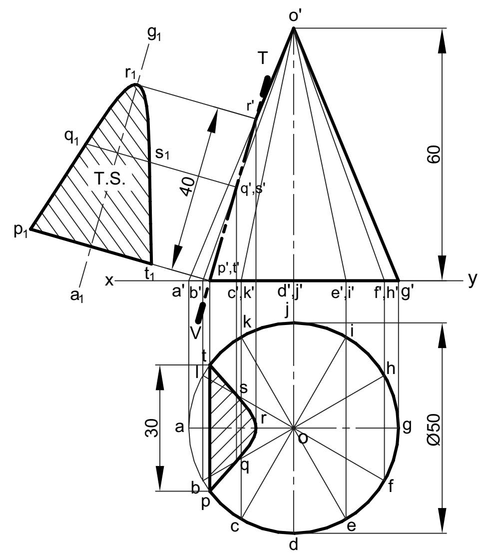


Fig. 12.61

Problem 12.57 *A cone of base diameter 50 mm is cut by an auxiliary inclined plane and its larger portion is retained. The true shape of the section obtained by cutting the cone is an isosceles triangle of base 40 mm and altitude 55 mm. Draw the projections of the cone when it is kept on its cut surface on the ground.*

Visualisation To get an isosceles triangle, the V.T. of the section plane should pass through the apex of the cone. For triangle of base 50 mm the section plane should make an intercept of 50 mm. For altitude of 75 mm the V.T. should be 75 mm long.

Construction Refer to Fig. 12.62.

1. **First stage** Draw a circle *adgj* to represent the top view. Divide the circle into 12 equal parts. Project the circle on *xy*. The height of the cone is unknown.
2. **Sectional top view** Draw a chord to the circle *pq* of 40 mm and perpendicular to *xy*. Join *pqo* and hatch the enclosed space.
3. **Cutting plane** Project *p* and *q* on *xy* to obtain points p' and q'. Draw an arc with centre p' and radius 55 mm to meet the projector of the centre o at point o'. Draw V.T. to connect p', q' and o'.

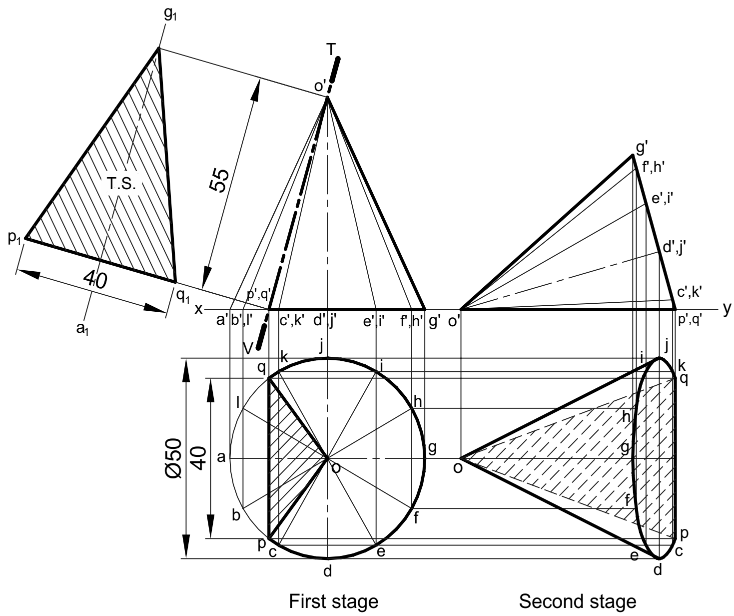


Fig. 12.62

4. **True shape** Draw a_1g_1 parallel to V.T. Project p', q' and o' on a_1g_1. Locate p_1, q_1 and o_1 on the projectors such that their distances from a_1g_1 are equal to distances of p, q and o from ag. Join $p_1q_1o_1$ and hatch the enclosed space to get the true shape of section.
5. **Second stage** Reproduce $p'o'g'$ keeping $p'o'$ on xy. Project all the points from this front view to meet the corresponding locus lines from the top view of the first stage. Join all the points and obtain the new top view as shown.

Problem 12.58 *A cone of base diameter 50 mm and axis 50 mm rests on its base on the ground. It is cut by an A.I.P. such that the true shape of the section is an isosceles triangle of vertex angle 40°. Set the required cutting plane and find its inclination with the H.P. Draw the sectional top view and true shape of the section.*

Construction Refer to Fig. 12.63.

1. **Projections** Draw a circle *abcd* to represent the top view. Project it to obtain $a'c'o'$ as its front view.
2. **Altitude of triangle** Determine the slant height $x = a'o'$ from the front view. Draw an isosceles triangle *efg* of side equal to $a'o'$ and the included angle of 40° as shown in Fig. 12.63(a). Determine the altitude h.
3. **Cutting plane** Draw an arc with centre o' and radius h to meet $a'c'$ at point p' and q'. Draw V.T. of the section plane to connect $p'q'$ and o'.
4. **Sectional top view** Project p' and q' to meet circle at points at p and q. Join poq and hatch the enclosed space.

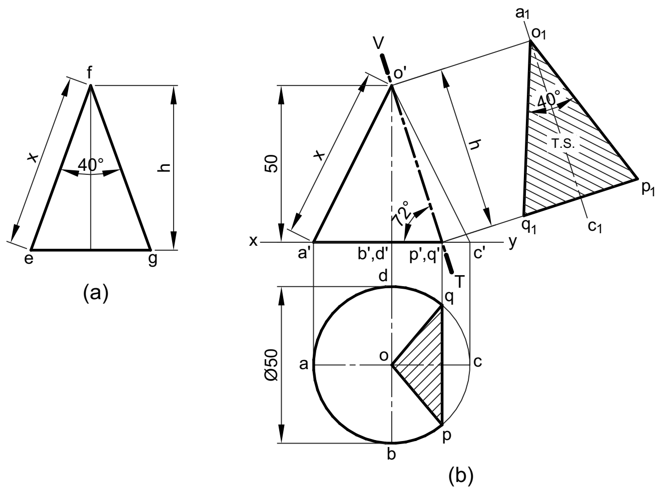


Fig. 12.63 **(a)** *Determine altitude of triangle* **(b)** *Sectional top view and true shape*

5. **True shape** Draw a_1c_1 parallel to V.T. Project p', q' and o' on a_1c_1. Locate p_1, q_1 and o_1 on the projectors such that their distances from a_1g_1 are equal to distances of p, q and o from ac. Join $p_1q_1o_1$ and hatch the enclosed space to get the true shape of section.

12.8.5 Anti-section of Sphere

The true shape of section of sphere is always a circle. Let us consider the following problem.

Problem 12.59 *A sphere of diameter 60 mm is kept on the H.P. It is cut by an A.V.P. in such a way that the front view of the cut surface is an ellipse having major and minor axes 50 mm and 40 mm respectively. Draw the top view, sectional front view and true shape of the section.*

Construction Refer to Fig. 12.64.

1. **Determine inclination of A.V.P.** Draw a line $p'q'$ equal to minor axis and parallel to xy. Project $p'q'$ on xy and obtain pq equal to major axis as shown in Fig 12.64(a). Determine inclination of pq with V.P. as inclination of auxiliary vertical plane with V.P., i.e., ϕ.
2. **Projections** Draw a circle with centre o and diameter 60 mm as the top view. Project the top view and obtain another circle with centre o' and diameter 60 mm as the front view.

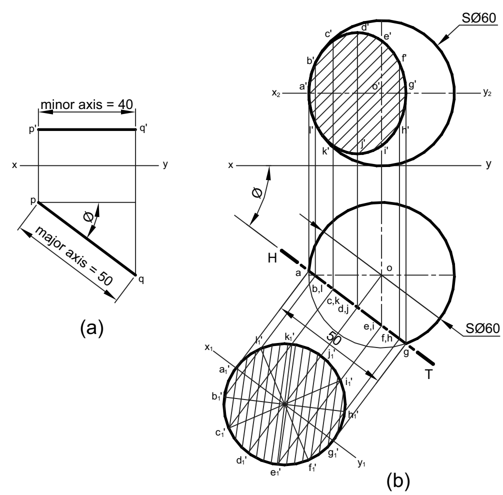


Fig. 12.64 **(a)** *Determine inclination with V.P.* **(b)** *Sectional front view and true shape*

3. **Cutting plane** Draw a chord of length *pq* having its inclination ϕ. This represents the H.T. of the section plane.
4. Obtain true shape and sectional front view. (Refer to Problem 12.29)

EXERCISE 12B

12.1 A triangular prism of base side 40 mm is lying on one of its rectangular faces on the H.P. with its axis perpendicular to the V.P. It is cut by a horizontal section plane such that the true shape of the section is a rectangle of sides 30 mm and 60 mm. Draw its front view and sectional top view. (Hint: Refer to Problem 12.1)

12.2 A triangular prism of base side 40 mm and axis 60 mm is lying on a base on the H.P. with a side of the base perpendicular to the V.P. It is cut by an A.I.P. such that true shape of the section is an isosceles triangle of base 40 mm and altitude 60 mm. Draw its front view, sectional top view and true shape of the section.

12.3 A triangular prism of base edge 50 mm and height 40 mm stands on a base on the ground with one of its rectangular faces perpendicular to the V.P. It is cut by an A.I.P. such that the true shape of the section is a trapezium of 10 mm and 30 mm parallel sides. Draw its projections and obtain true shape of section.

12.4 A square prism of base side 40 mm is kept on its base on the H.P. such that one of its rectangular faces makes an angle of 30° with V.P. It is cut by a section plane parallel to V.P. such that the true shape of the section is a rectangle of sides 30 mm and 60 mm. Draw its sectional front view and top view.
(Hint: Refer to Problem 12.19)

12.5 A square prism of base side 40 mm and axis 60 mm is lying on a base on the H.P. with edges of the base equally inclined to the V.P. It is cut by an A.I.P. in such a manner that the true shape of the section is the rhombus of major diagonal 75 mm. Draw its front view, sectional top view and true shape of the section. (Hint: Refer to Problem 12.41)

12.6 A square prism of axis 60 mm is resting on its base on the H.P. with all the rectangular faces equally inclined to the V.P. It is cut by an A.I.P. such that true shape of the section is a rhombus having diagonals 80 mm and 45 mm. Draw the front view, sectional top view and true shape of the section. Determine the inclination of the A.I.P. with the H.P.

1.7 A pentagonal prism of base side 30 mm and axis 60 mm is resting on a base in the H.P. with an edge of the base perpendicular to the V.P. It is cut by an A.I.P. in such a way that the true shape of the section is a trapezium of one of its parallel sides 30 mm, another side maximum possible and altitude 50 mm. Draw the projections and true shape of the section.

12.8 A cube of 50 mm long edge lies on one of its faces on the H.P. It is cut by an A.I.P. producing a largest rhombus. Draw the projections, true shape of the section and determine the inclination of the section plane with the H.P.

12.9 A cube of 50 mm long edge is lying on one of its faces on the H.P. It is cut by an A.I.P. producing a largest equilateral triangle. Draw the projections, true shape of the section and determine the inclination of the section plane with the H.P. (Hint: Refer to Problem 12.39)

12.10 A cube resting on a face on the H.P. is cut by an A.I.P. such that true shape of the section is a regular hexagon of side 30 mm. Draw the front view and sectional top view of the cube. Also, project the true shape of the section. (Hint: The cube should have 60 mm diagonal.)

12.11 A square prism is resting on one of its bases on the H.P. with an edge of the base perpendicular to the V.P. It is cut by an A.I.P. in such that the true shape of the section is a rectangle of sides 60 mm and 40 mm. The minimum height of one of the side faces of cut prism is 15 mm. Draw its projections and true shape of the section.

12.12 A cylinder of base diameter 50 mm and axis 60 mm is kept on the H.P. on its base. It is cut by an A.I.P. such that the true shape of the section is the largest possible ellipse. Draw its front view, sectional top view and true shape of the section.

12.13 A square prism of base side 40 mm and axis 70 mm lying rests its base on the H.P. with edges of the base equally inclined to the V.P. Draw front view, sectional top view and true shape of the section when it is cut by an A.I.P. such that the true shape of section is (a) largest equilateral triangle, (b) largest rhombus, (c) largest rectangle, (d) isosceles triangle of base 40 mm and altitude 60 mm and (e) equilateral triangle of 45 mm side.

12.14 A pentagonal prism of base side 30 mm and axis 70 mm is kept on its base on the H.P. a rectangular face perpendicular to V.P. Draw front view, sectional top view and true shape of the section when it is cut by an A.I.P. such that the true shape of section is (a) largest trapezium, (b) largest isosceles triangle, (c) largest equilateral triangle and (d) trapezium with parallel sides 30 mm and 45 mm and altitude 60 mm.

12.15 A cylinder of base diameter 50 mm and axis 70 mm rests on its base in the H.P. It is cut by an auxiliary inclined plane such that the true shape of the section is a semi-ellipse which has 60 mm long semi-major axis. Draw its projections. Also, determine true shape of section and inclination of the cutting plane with H.P.

12.16 A cylinder of diameter 50 mm stands on its base on the H.P. It is cut by an A.V.P. inclined at 30° to the V.P. such that true shape of the section is a rectangle of sides 60 mm and 30 mm sides. Draw the projections and true shape of the section. Determine the distance of the section plane from the axis of the cylinder.

12.17 A pentagonal pyramid of base side 30 mm and axis 60 mm is kept on the H.P. on its base with a side of the base perpendicular to the V.P. It is cut by an A.I.P. such that true shape of the section is a trapezium having one of the parallel sides as 20 mm and other parallel side being largest possible. Draw its front view, sectional top view and true shape of the section.

12.18 A square pyramid of base side 40 mm and axis 60 mm is resting on its base on the ground with all the edges of the base equally inclined to the V.P. It is cut by an A.I.P. such that true shape of the section is an equilateral triangle of largest side. Draw the sectional top view and true shape of the section.

12.19 A square pyramid of base side 40 mm and axis 60 mm is resting on its base on the ground with all the edges of the base equally inclined to the V.P. It is cut by a section plane parallel to the V.P. such that the true shape of the section is an isosceles triangle of altitude 45 mm. Draw its sectional front view and top view.

12.20 A square pyramid of base side 40 mm and axis 60 mm rests on its base on the ground. It is cut by a section plane parallel to the V.P. such that the true shape of the section is a trapezium having one of its parallel sides 10 mm long. Draw its sectional front view and top view.

12.21 A tetrahedron of 60 mm long edges is lying on the H.P. on one of its faces with an edge perpendicular to the V.P. It is cut by an A.I.P. so that the true shape of the section is an isosceles triangle of base 50 mm and altitude 40 mm. Find the inclination of the section plane with the H.P. and draw the front view, sectional top view and true shape of the section.

12.22 A tetrahedron when cut by an A.I.P. produces the true shape of the section as a square of side 30 mm. Draw its front view, sectional top view and obtain true shape of the section. Also determine the inclination of A.I.P. with the H.P. (Hint: Refer to Problem 12.52)

12.23 A hexagonal pyramid of base 30 mm side and axis 60 mm long rests on its base in the H.P. with a side of base parallel to V.P. It is cut by an A.I.P. such that true shape of the section is an isosceles triangle of largest base and smallest altitude. Draw the sectional top view and true shape of the section.

12.24 A tetrahedron of 60 mm edges is resting on the H.P. on one of its faces with an edge perpendicular to the V.P. It is cut by an A.I.P. so that true shape of the section is a rectangle of smaller side 20 mm. Set the required cutting plane and draw the front view, sectional top view and true shape of the section.

12.25 A cone of base diameter 50 mm and axis 60 mm is resting on its base on the H.P. It is cut by an A.I.P. passing through the lower end of one of the extreme generators in such a way that the true shape of the section is an ellipse of smallest major axis. Draw its projections and obtain true shape of the section.

12.26 A cone of base diameter 50 mm and axis 60 mm rests on its base in the H.P. It is cut by an A.I.P. such that the true shape of the section is a parabola of altitude 50 mm. Draw its three views and obtain the true shape of the section.

12.27 A cone of base diameter 50 mm and axis 60 mm rests on its base in the H.P. It is cut by a plane such that the true shape of the section is a rectangular hyperbola of altitude 30 mm in the front view. Draw the sectional front view and find the distance of the section plane from the axis of the cone.

12.28 A cone of base diameter 50 mm and axis 45 mm rests on its base in the H.P. Draw the sectional top view when it is cut by a plane such that the true shape of the section is an isosceles triangle of (a) base 40 mm (b) altitude 50 mm. Also, obtain the true shape of section.

12.29 A cone of base diameter 60 mm is cut by a profile plane such that the true shape of the section is a rectangular hyperbola of axis 25 mm. Draw the three sectional views.

12.30 A cone of base diameter 60 mm is cut by a profile plane such that the true shape of the section is a hyperbola of base 50 mm. Draw the three sectional views.

12.31 A sphere of diameter 60 mm is kept on the H.P. It is cut by an A.I.P. in such a way that the top view of the cut surface is an ellipse having major and minor axes 50 mm and 30 mm respectively. Draw the front view, sectional top view and true shape of the section.

VIVA-VOCE QUESTIONS

12.1 State the relationship of an auxiliary vertical plane with the reference planes.

12.2 Define an auxiliary inclined plane, auxiliary vertical plane and a profile plane.

12.3 How can the true shape of section be obtained when a solid is cut by an AIP?

12.4 How can the true shape of section be obtained when a solid is cut by an AVP?

12.5 A solid is cut by a profile plane. Which orthographic view is likely to show the true shape of section?

12.6 How would you locate the section plane which cuts a cone to get an isosceles triangle as true shape of section?

12.7 How would you locate the section plane which cuts a square pyramid to get a trapezium as true shape of section?

12.8 How would you locate the section plane which cuts a cube to get an equilateral triangle of largest possible side as true shape of section?

MULTIPLE-CHOICE QUESTIONS

12.1 Which of the following views provide clarity and reveal internal features of a part?
(a) Section views
(b) Oblique views
(c) Auxiliary views
(d) Pictorial views

12.2 A cube is resting on a face in the H.P. with vertical faces equally inclined to the V.P. It is cut by an A.I.P. The true shape of section view is
(a) triangle
(b) rhombus
(c) texagon
(d) Any of these

12.3 A cone is cut by a section plane parallel to the profile plane. Its true shape of section is seen in
(a) front view
(b) top view
(c) side view
(d) auxiliary view

12.4 A square pyramid resting on its base in the H.P. and a side of base parallel to V.P. It is cut by an A.I.P. Its true shape will be
(a) square
(b) rectangle
(c) trapezium
(d) parallelogram

12.5 A square pyramid 50 mm side resting on its base in the H.P. is cut by a horizontal section plane bisecting its axis. Its true shape of section is
(a) square of 25 mm side
(b) trapezium with parallel sides 25 mm & 50 mm
(c) square of 50 mm side
(d) triangle of base 50 mm side

12.6 A square pyramid 45 mm side and axis 60 mm long, resting on its base in the H.P. is cut by a horizontal section plane passing through a point on the axis 20 mm below the apex. Its true shape of section is a square of side

(a) 15 mm
(b) 30 mm
(c) 40 mm
(d) 45 mm

12.7 A triangular prism resting on a rectangular face in the H.P. It is cut by a horizontal plane. Its sectional top view is
(a) equilateral triangle
(b) isosceles triangle
(c) rectangle
(d) None of these

12.8 A cone resting on its base on the H.P. is cut by a section plane parallel to V.P. has its sectional front view
(a) ellipse
(b) parabola
(c) hyperbola
(d) semi-circle

12.9 A cube is resting on a face in the H.P. with vertical faces equally inclined to the V.P. It is cut by an A.I.P. passing through the solid diagonal. The true shape of section view is
(a) square
(b) rectangle
(c) hexagon
(d) rhombus

12.10 A cylinder of 50 mm diameter and axis 120 mm long is lying on its generator in H.P. It is cut by a vertical section plane to get largest ellipse as the true shape of section. The major axis of this ellipse will be
(a) 50 mm
(b) between 50 mm and 120 mm
(c) 120 mm
(d) 130 mm

12.11 A cylinder of 60 mm diameter and axis 80 mm long is lying on its generator in H.P. It is cut by a section plane to get an ellipse as the true shape of section. The minor axis of this ellipse will be
(a) 60 mm
(b) 80 mm
(c) 100 mm
(d) None of these

12.12 If a polyhedron is cut by any section plane, the true shape of section is a closed figure made up of
(a) straight lines
(b) curves
(c) combination of lines and curves
(d) Any of these

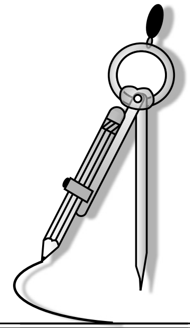
Answers to multiple-choice questions

12.1 (a), 12.2 (d), 12.3 (c), 12.4 (c), 12.5 (a), 12.6 (a), 12.7 (c), 12.8 (c), 12.9 (d), 12.10 (d), 12.11 (a), 12.12 (a)

Chapter

13

DEVELOPMENT OF SURFACES



13.1 INTRODUCTION

The development of surface is the shape of a plain sheet that by proper folding could be converted into the desired object. In engineering practice, a large number of objects like milk can, funnel, bucket, measuring flask, duct of air conditioner, hopper, chimney, tray, storage tank, boiler shell etc. shown in Fig. 13.1, are made of metal sheets. The fabrication of these objects can be planned in an economic way if the accurate shape and size of metal sheet is known. This chapter deals with proper layout planning of the surface of the object on a single plane called the development of surfaces.

(a) (b) (c) (d) (e) (f) (g) (h)

Fig. 13.1 *Metal sheet used for making* **(a)** *Milk can* **(b)** *Funnel* **(c)** *Bucket* **(d)** *Measuring flask* **(e)** *Duct of air conditioner* **(f)** *Hopper* **(g)** *Chimney* **(h)** *Tray*

13.2 CLASSIFICATION OF SURFACES

Surfaces of various geometrical objects may be classified as the following:

1. **Plane surface** Objects like prism, pyramid, cube and polyhedron are bounded by plane surfaces.
2. **Singly curved surface** Objects like cylinder and cone are bounded by singly curved surfaces.
3. **Doubly curved surface** Objects like sphere, paraboloid, ellipsoid and hyperboloid are bounded by doubly curved surfaces.

13.3 METHODS OF DEVELOPMENT

Development methods may be classified as the following:

1. **Parallel line method** This method is adopted in the development of prisms and cylinders, in which all the edges/generators of lateral surface are parallel in each other.

2. **Radial line method** This method is adopted in the development of pyramids and cones in which the apex is taken as centre and the slant edge or generator as radius of its development.
3. **Triangulation method** This method is generally applied in the development of transition pieces and oblique objects.
4. **Approximation method** This method is adopted in the development of doubly curved surface like that of a sphere since the exact development of such surface is not possible.

13.4 DEVELOPMENT OF PRISMS

Prisms are developed by parallel line method. In this method, first of all the front view and top view of the prism are drawn. Two parallel lines called stretch out lines are drawn from the ends of the prism in a direction perpendicular to the axis. The length of these lines is same as the perimeter of the base of the prism. The faces of the prism are marked between the stretched outlines which represent the development of the lateral surface.

Problem 13.1 *A square prism of base side 30 mm and axis 60 mm is resting on its base on the H.P. with a rectangular face parallel to the V.P. Draw the development of the prism.*

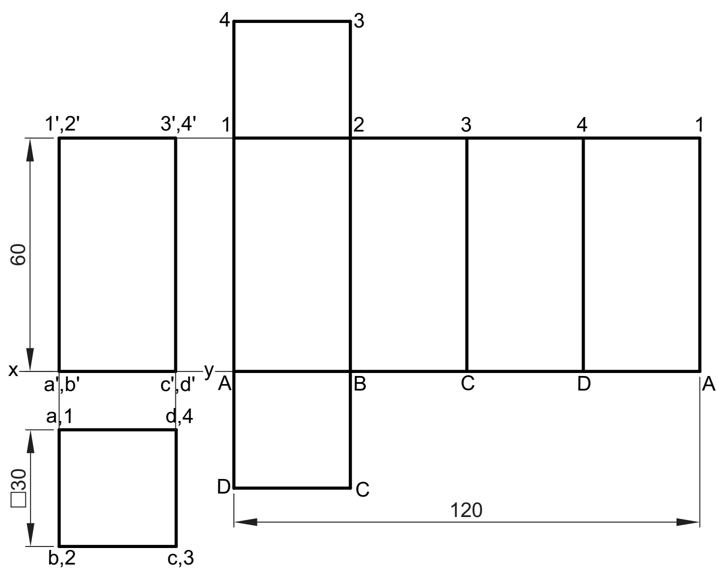


Fig. 13.2

Visualisation The fabrication of prism shall require (a) four rectangular faces of sides 60 mm and 30 mm and (b) two square bases of side 30 mm.

Construction Refer to Fig. 13.2.

1. Draw a square *abcd* keeping ad parallel to *xy* to represent the top view. Project all the points to obtain rectangle $a'd'4'1'$ as the front view.
2. Stretch out lines 1-1 and A-A from the front view, equal to the perimeter of the base (120 mm).

3. Divide 1-1 and A-A in four equal parts and name their intermediate points as 2, 3, 4 and B, C, D respectively. Join vertical edges 1A, 2B, 3C and 4D in the development.
4. Attach squares 1234 and ABCD to AB and 12 respectively as the bases of the prism. This gives the complete development of the prism.

Note:

1. In the development of the lateral surfaces of the closed objects, the first and the last edge shall have the same name.
2. Usually, development of the lateral surface of the object is drawn and the bases are omitted. They can easily be added whenever required.

Problem 13.2 *A pentagonal prism of base side 30 mm and axis 70 mm is resting on its base on the H.P. with a rectangular face parallel to the V.P. It is cut by an auxiliary inclined plane (A.I.P.) whose V.T. is inclined at 45° to the reference line and passes through the mid-point of the axis. Draw the development of the lateral surface of the truncated prism.*

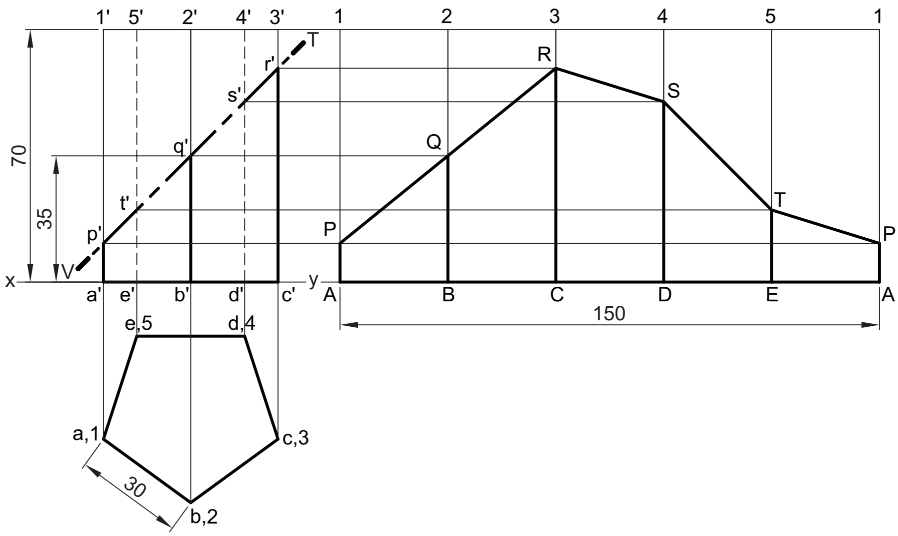


Fig. 13.3

Construction Refer to Fig. 13.3.

1. **Projections** Draw a pentagon *abcde* keeping de parallel to *xy* to represent the top view. Project all points to obtain $a'c'3'1'$ as the front view.
2. **Cutting plane** Draw V.T. of the section plane inclined at 45° to *xy* such that it bisects the axis. Let V.T. cut the edges $a'1'$ at p', $b'2'$ at q', $c'3'$ at r', $d'4'$ at s' and $e'5'$ at t'.
3. **Development** Consider the seam along $a'1'$. Stretch out lines 1-1 and A-A from the front view equal to the perimeter of the base (150 mm). Divide 1-1 and A-A in five equal parts and name their intermediate points as 2, 3, 4, 5 and *B*, *C*, *D*, *E* respectively. Join 1*A*, 2*B*, 3*C*, 4*D* and 5*E*.
4. Draw horizontal lines from points p', q', r', s' and t' to meet corresponding edges A1, B2, C3, D4 and E5 at points *P*, *Q*, *R*, *S* and *T* respectively. Join each of *PQ*, *QR*, *RS*, *ST*, *TP* with straight lines.
5. Dark the portion of the development that is retained after truncating the prism.

Problem 13.3 *A hexagonal prism of base side 30 mm and axis 70 mm is resting on its base on the ground with a side of base inclined at 45° to the V.P. It is cut by an auxiliary inclined plane inclined at 45° to the H.P. and passes through a point 15 mm below the top end of the axis. Draw the development of the lateral surface of the truncated prism.*

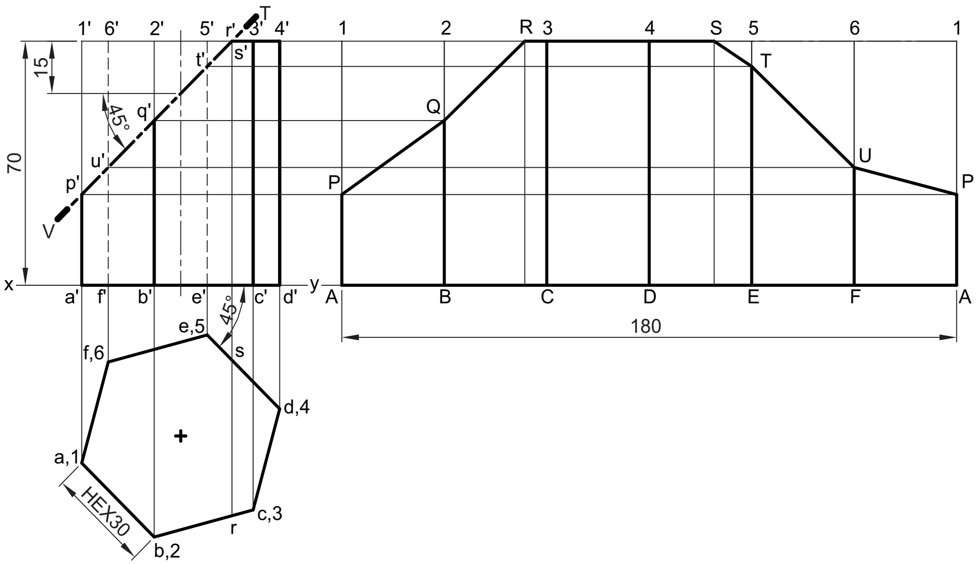


Fig. 13.4

Construction Refer to Fig. 13.4.

1. **Projections** Draw a hexagon *abcdef* keeping *ed* inclined at 45° with *xy* to represent the top view. Project all points to obtain *a′d′*4′1′ as the front view.
2. **Cutting plane** Draw V.T. of the cutting plane inclined at 45° to *xy* such that it passes through a point 15 mm below the top end of the axis. Let V.T. cut the edges *a*′1′ at *p*′, *b*′2′ at *q*′, 2′3′ at *r*′, 4′5′ at *s*′, *e*′5′ at *t*′ and *f*′6′ at *u*′.
3. **Development** Consider seam at *a*′1′. Stretch out lines 1-1 and A-A from the front view equal to the perimeter of the base (180 mm). Divide 1-1 and A-A in six equal parts and name their intermediate points as 2, 3, 4, 5, 6 and *B*, *C*, *D*, *E*, *F* respectively. Join 1*A*, 2*B*, 3*C*, 4*D*, 5*E* and 6*F* in the development.
4. Draw horizontal lines from points *p*′, *q*′, *t*′ and *u*′ to meet their corresponding edges *A*1, *B*2, *E*5, *F*6 at points *P*, *Q*, *T*, *U*, respectively.
5. Project *r*′*s*′ to meet the corresponding edges in the top view at points *r* and *s* respectively. Mark points *R* and *S* on the development such that distance 2*R* = 2*r*, and 4*S* = 4*s*. Join each of *PQ*, *QR*, *RS*, *ST*, *TU* and *UP* with straight lines.
6. Dark the portion of the development that is retained after truncating the prism.

Problem 13.4 *A hexagonal prism of base side 30 mm and height 70 mm, is resting on its base on the H.P. with a side of the base perpendicular to the V.P. The prism has a cylindrical hole of diameter 40 mm, drilled centrally such that the axis of hole is perpendicular to the V.P. Draw the development of the lateral surface of the prism.*

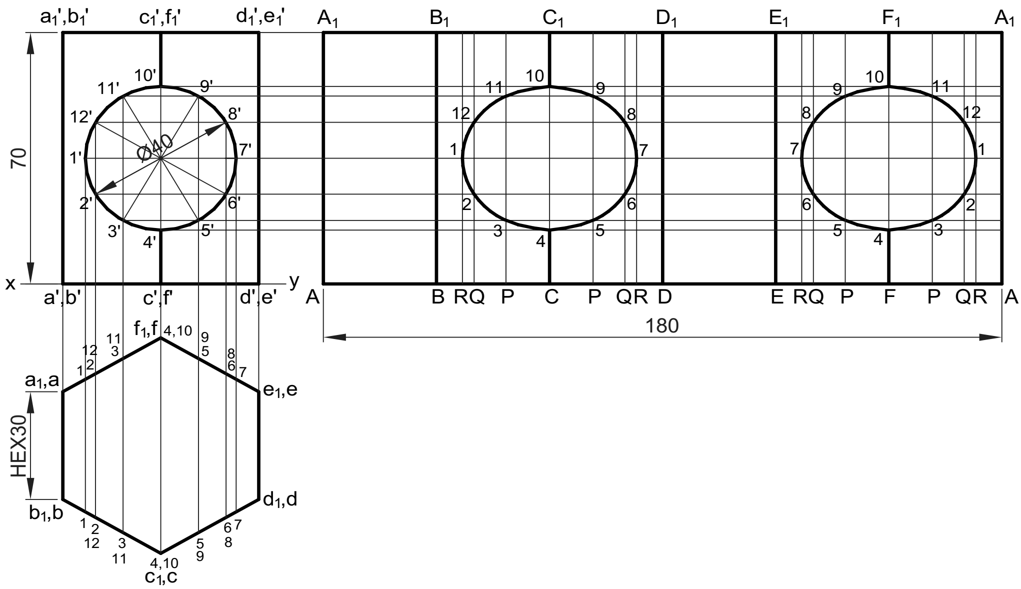


Fig. 13.5

Construction Refer to Fig. 13.5.

1. Draw a hexagon *abcdef* keeping *ab* perpendicular to *xy* to represent the top view. Project all the points to obtain $b'd'd_1'b_1'$ as the front view.
2. Draw a circle of 40 mm diameter in the front view and divide it into 12 equal parts. Project all the points to the top view.
3. Consider seam at $a'1'$. Stretch out lines A_1-A_1 and A-A from the front view equal to the perimeter of the hexagon. Divide A_1-A_1 and A-A in six equal parts and name their intermediate points as B_1, C_1, D_1, E_1, F_1 and B, C, D, E, F respectively. Join A_1A, B_1B, C_1C, D_1D, E_1E and F_1F.
4. In the development, mark points P, Q and R such that $CP = c3$, $PQ = 3\text{-}2$, $QR = 2\text{-}1$ etc. Draw vertical lines from P, Q and R.
5. Draw horizontal lines from points $1'$, $2'$, $3'$,…, etc., to meet corresponding vertical lines in the development at points 1, 2, 3,…, etc. Join them to obtain the required development as shown.

Problem 13.5 *Figure 13.6(a) shows the front view of a truncated hexagonal prism of base side 30 mm and axis 90 mm. The prism is resting on the H.P. with an edge of the base parallel to the VP. Draw the development of its lateral surface.*

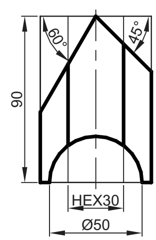


Fig. 13.6(a)

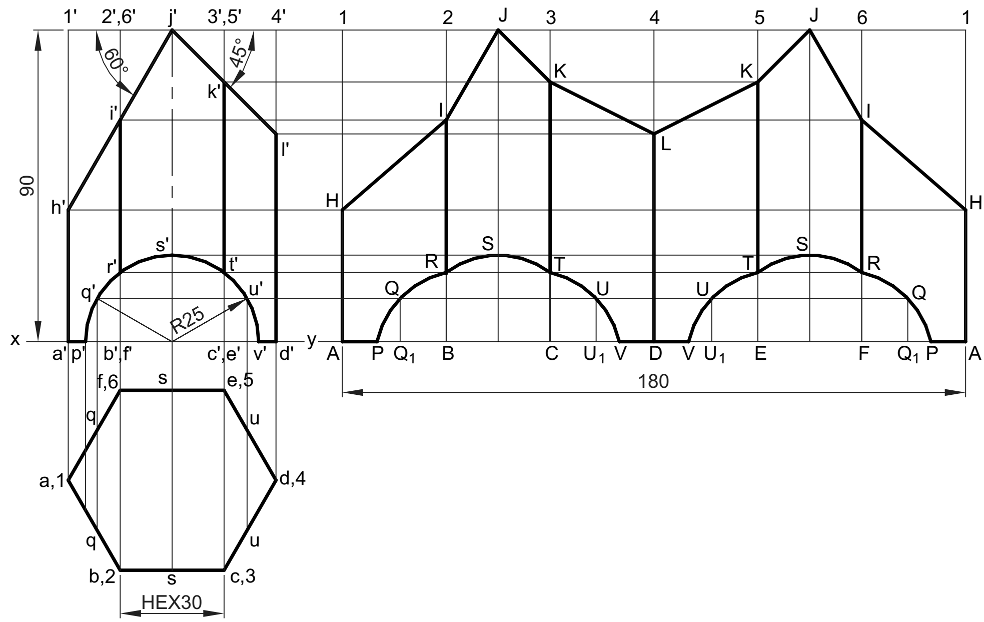


Fig. 13.6(b)

Construction Refer to Fig. 13.6(b).

1. Draw a hexagon *abcdef* keeping *ef* parallel to *xy* to represent the top view. Project all points to obtain $a'd'4'1'$ as the front view.
2. Draw the cutting planes as given in Fig. 13.6(a). Name the point of intersection of prism with cutting plane as h', i', j', k', l' and those with the cutting arc as p', q', r', s', t', u', v'.
3. Stretch out lines 1-1 and A-A from the front view equal to the perimeter of the base. Draw 1*A*, 2*B*, 3*C*, 4*D*, 5*E* and 6*F* to complete the development of the uncut prism.
4. Project points q' and u' to obtain points *q* and u in the top view. In the development, mark points Q_1 and U_1 such that $AQ_1 = aq$ and $DU_1 = du$. Draw vertical lines from Q_1 and U_1 as the locus of points *Q* and *U*.
5. Draw horizontal lines from points h', i', j', k', l', and p', q', r', s', t', u', v' to meet their corresponding locus lines at *H*, *I*, *J*, *K*, *L* and *P*, *Q*, *R*, *S*, *T*, *U*, *V*. Join them to obtain the development as shown.

Problem 13.6 *The pentagonal prism of side 30 mm rests on its base on the H.P. It is cut by A.I.P. inclined at 30° to the H.P. and passes through the top end of the axis. Also, a cylindrical hole of diameter 40 mm is drilled perpendicular to the V.P. the axis of which is 5 mm away from the axis of the prism as shown in Fig. 13.7(a). Draw the development of its lateral surface.*

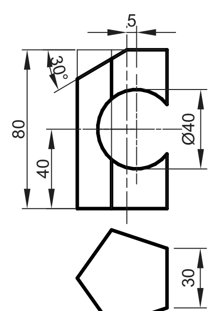


Fig. 13.7(a)

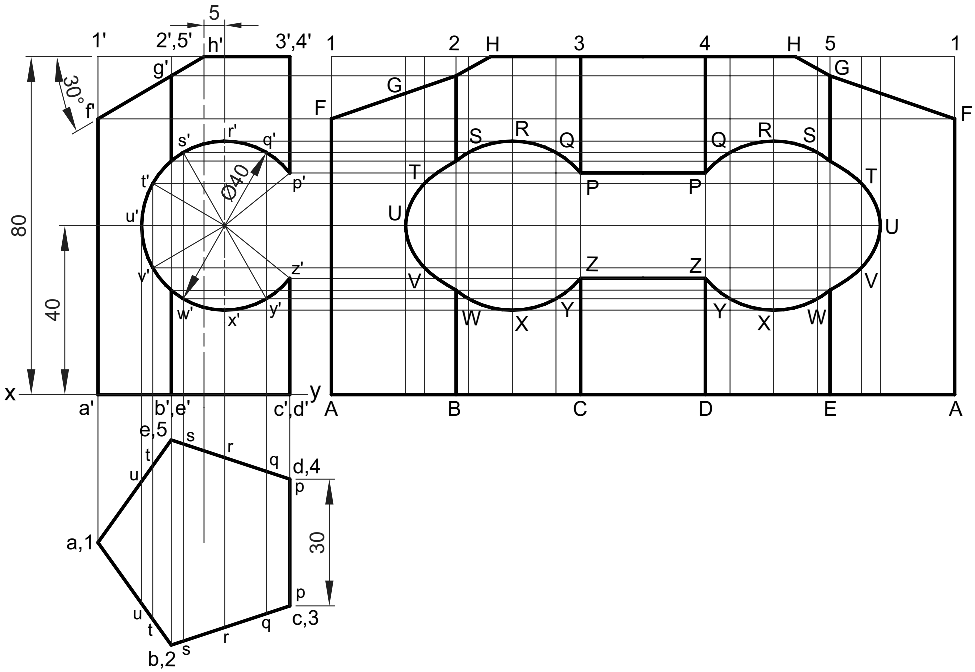


Fig. 13.7(b)

Construction Refer to Fig. 13.7(b).

1. Draw a pentagon *abcde* keeping *cd* perpendicular to *xy* to represent the top view. Project all points to obtain $a'c'3'1'$ as the front view.
2. Draw the cutting planes as given in Fig. 13.7(a). Name the point of intersection of prism with cutting plane as f', g' and h' and those with the cutting arc as p', q', r', s', t', u', v', w', x', y', z'.
3. Stretch out lines 1-1 and A-A and draw 1*A*, 2*B*, 3*C*, 4*D* and 5*E* to complete the development of the uncut prism.
4. Project points q', r', s', t' and u' to obtain points *q*, *r*, *s*, *t* and *u* in the top view. In the development, draw vertical lines as locus of points *Q*, *R*, *S*, *T* and *U* such that their distances from *B* is equal to *bq*, *br*, *bs*, *bt* and *bu*, respectively.
5. Draw horizontal lines from points f', g', h' and p', q', r', s', t', u', v', w', x', y', z' to meet their corresponding locus lines at *F*, *G*, *H* and *P*, *Q*, *R*, *S*, *T*, *U*, *V*, *W*, *X*, *Y*, *Z*. Join them to obtain the development as shown.

13.5 DEVELOPMENT OF CYLINDERS

Cylinders are also developed by parallel line method in a way similar to the prisms. Here the length of stretch line is equal to the circumference of the base circle of the cylinder.

Problem 13.7 *A cylinder of base diameter 50 mm and axis 70 mm is resting on ground with its axis vertical. It is cut by a section plane perpendicular to the V.P., inclined at 45° to the H.P., passing through the top of a generator and cuts all the other generators. Draw the development of its lateral surface.*

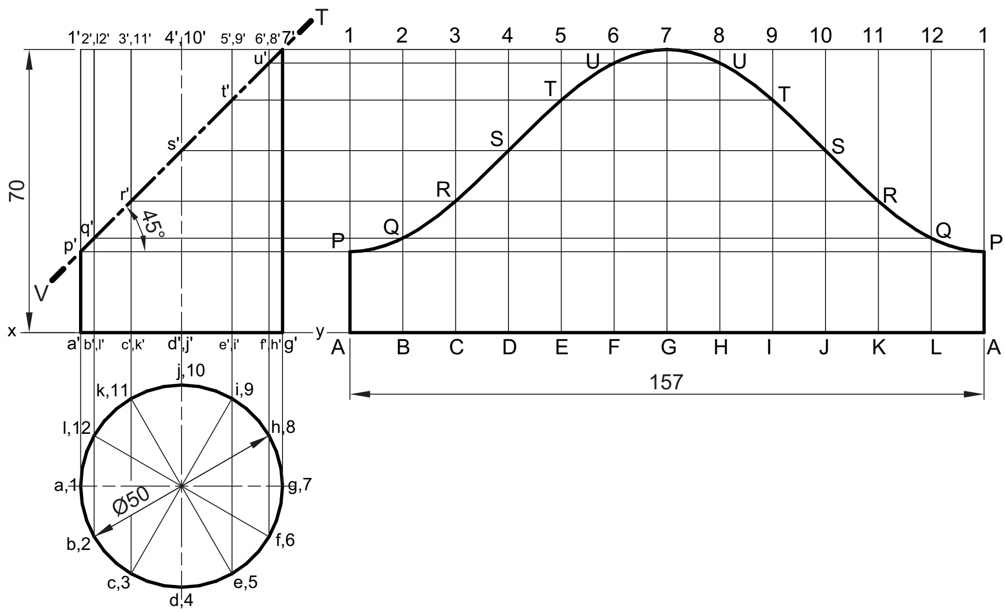


Fig. 13.8

Construction Refer to Fig. 13.8.

1. **Projections** Draw a circle *adgj* to represent the top view and divide it into 12 equal parts. Project all the points to obtain $a'g'7'1'$ as the front view.
2. **Cutting plane** Draw V.T. of the cutting plane inclined at 45° to *xy* such that it passes through 7′. Let V.T. cut the generators $a'1'$ at p', $b'2'$ at q', $c'3'$ at r', $d'4'$ at s', etc., as shown.
3. **Development** Consider seam at $a'1'$. Stretch out lines 1-1 and A-A through the front view equal to the perimeter of the cylinder (157 mm). Divide 1-1 and A-A into 12 equal parts and join them to represent generators *B*2, *C*3, *D*4, *E*5, *F*6, *G*7, *H*8, *I*9, *J*10, *K*11 and *L*12.
4. Draw horizontal lines from points p', q', r', s', etc., to meet their corresponding generators *A*1, *B*2, *C*3, *D*4, etc., at points *P*, *Q*, *R*, *S*, etc., respectively.
5. Join *PQRSTU7UTSRQP* with a continuous smooth curve.
6. Dark the portion of the development that is retained after truncating the cylinder.

Problem 13.8 *A square hole of side 25 mm is cut in a cylindrical drum of diameter 50 mm and height 70 mm. The faces of the hole are inclined at 45° to the H.P. and axis intersects with that of the drum at right angles. Draw the development of its lateral surface.*

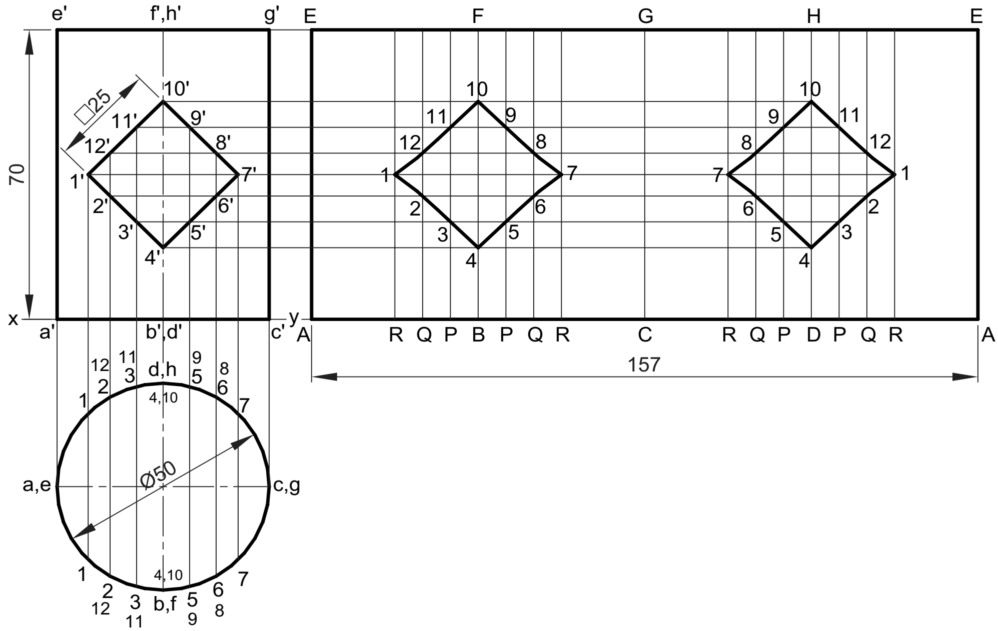


Fig. 13.9

Construction Refer to Fig. 13.9.

1. Draw a circle *abc*d to represent the top view. Project all the points to obtain $a'c'g'e'$ as the front view.
2. Draw a square $1'4'7'10'$ such that the centre is 35 mm above xy and all the sides are inclined at 45° to *xy*. On the edges of the square consider some more points as $2'$, $3'$, $5'$, $6'$, $8'$, $9'$, $11'$ and $12'$.
3. Stretch out lines A-A and E-E equal to the perimeter of the cylinder. Divide 1-1 and A-A into 4 equal parts and join all the generators.
4. Project all the points of the square to obtain points 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12 in the top view.
5. In the development, mark points P, Q and R such that BP = b3, PQ = 3-2, QR = 2-1 etc. Draw vertical lines from P, Q and R.
6. Draw horizontal lines from points $1'$, $2'$, $3'$, $4'$, $5'$, $6'$, $7'$, $8'$, $9'$, $10'$, $11'$ and $12'$ to meet their corresponding locus lines or generators in the development at points 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12 respectively.
7. Join all the points with smooth curves. Darken the portion of the development that is retained after truncating the cylinder.

Note In the development of the cylinder, the cutting lines converges to form arc. Therefore, in the development 1-4, 4-7, 7-10 and 10-1 are arcs of circles.

Problem 13.9 *Figure 13.10(a) shows the front view of a truncated cylinder of diameter 50 mm resting on its base on the H.P. Draw the development of its lateral surface.*

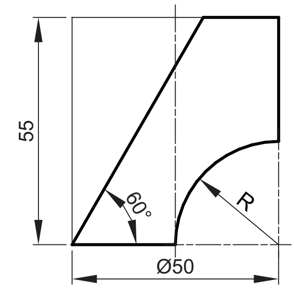


Fig. 13.10(a)

Construction Refer to Fig. 13.10(b).

1. Draw a circle *adgj* to represent the top view and divide it into 12 equal parts. Project all the points to obtain $a'g'7'1'$ as the front view.
2. Draw the cutting planes as given in Fig. 13.10(a). Name the point of intersection of generators with cutting plane appearing as line as p', q', r', s' and those with the cutting arc as t', u', v', w'.
3. **Development** Consider seam at $a'1'$. Stretch out lines 1-1 and A-A through the front view equal to the perimeter of the cylinder (157 mm). Divide 1-1 and A-A into 12 equal parts and join them to represent generators *B*2, *C*3, *D*4, *E*5, *F*6, *G*7, *H*8, *I*9, *J*10, *K*11 and *L*12. Also, obtain generator in the development corresponding to point t'.
4. Project s' to obtain *s* in the top view. In the development, mark points *S* such that 4*S* = 10-*S* = *ds*.
5. Draw horizontal lines from points p', q', r', t', u', v', w' to meet their corresponding generators at points *P*, *Q*, *R*, *T*, *U*, *V* and *W* respectively.
6. Join *APQRS* and *DTUVW* with smooth curves. Dark the portion of the development that is retained after truncating the cylinder.

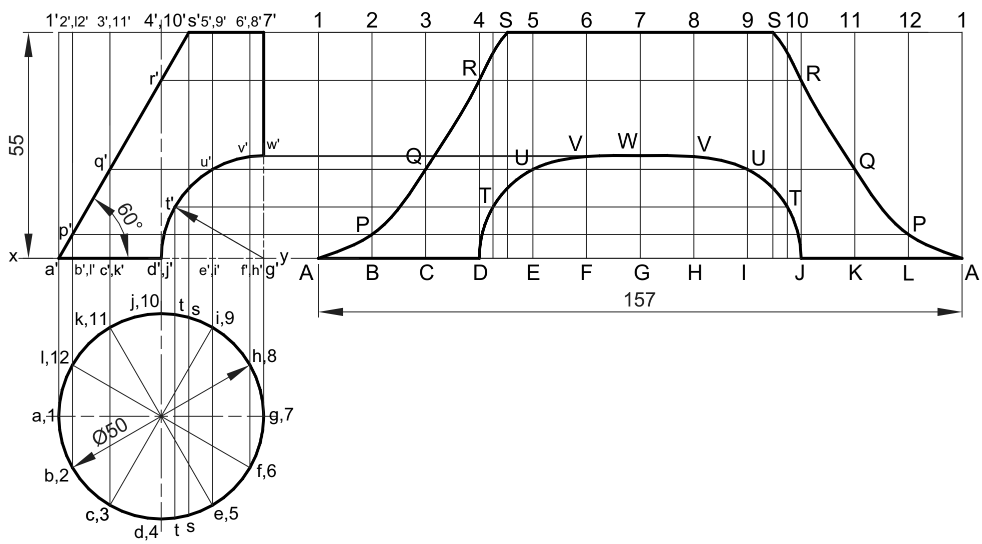


Fig. 13.10(b)

Problem 13.10 *A cylindrical drum of base diameter 50 mm and axis 70 mm is resting on its base on the H.P. A square hole of side 40 mm is cut through the drum such that one of the faces of the square hole is inclined at 30° to the H.P. The axis of the hole is perpendicular to the V.P. and is 10 mm away from the axis of the cylinder. Draw the development of the retained cylinder.*

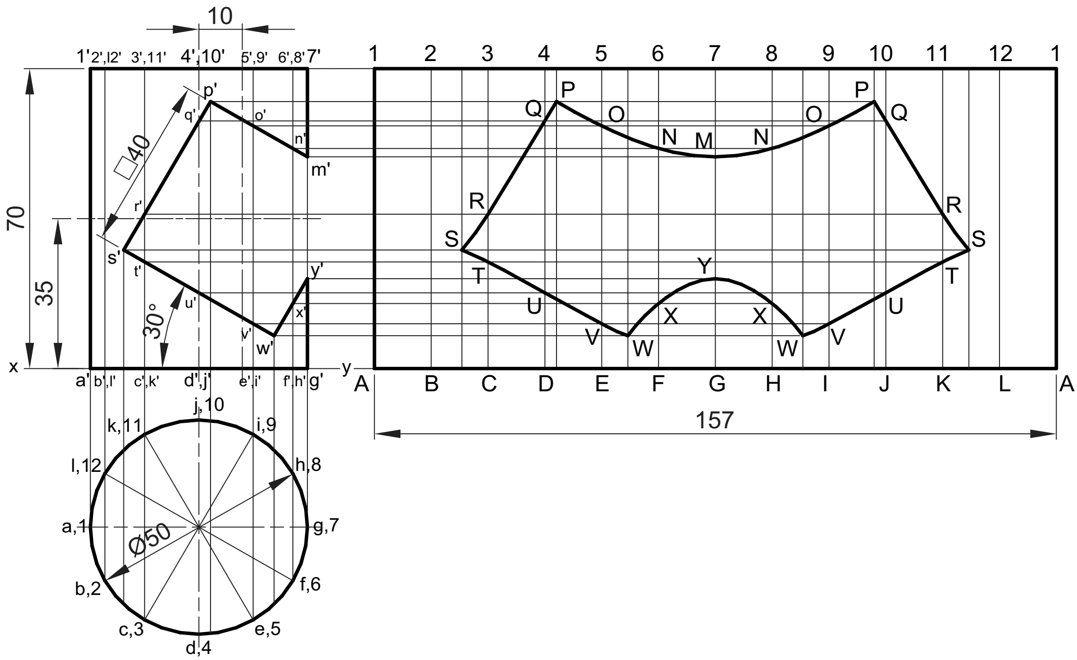


Fig. 13.11

Construction Refer to Fig. 13.11.

1. Draw a circle *adgj* to represent the top view and divide it into 12 equal parts. Project all the points to obtain $a'g'7'1'$ as the front view.
2. Draw a square of side 40 mm keeping $s'w'$ inclined at 30° to the horizontal as shown in Fig. 13.11. Name the point of intersection of generators with cutting plane as m', n', o', q', r', t', u', v', x' and y'.
3. Consider seam at $a'1'$. Stretch out lines 1-1 and A-A through the front view equal to the perimeter of the cylinder (157 mm). Divide 1-1 and A-A into 12 equal parts and join them to represent generators *B*2, *C*3, *D*4, *E*5, *F*6, *G*7, *H*8, *I*9, *J*10, *K*11 and *L*12.
4. Draw generators from the key points p', s' and w'. Project them to the top view. Obtain the corresponding generators in the development.
5. Draw horizontal lines from m', n', o', p', q', r', s', t', u', v', w', x' and y' to meet their corresponding generators at *M*, *N*, *O*, *P*, *Q*, *R*, *S*, *T*, *U*, *V*, *W*, *X* and *Y*.
6. Join *MNOP*, *PQRS*, *STUVW* and *WXY* with smooth curves. Dark the portion of the development that is retained after truncating the cylinder.

Problem 13.11 *An object is composed of truncated half-cylinder and half-prism whose projections are given in Fig. 13.12(a). Draw the development of its lateral surface.*

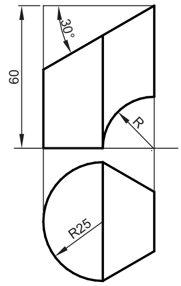


Fig. 13.12(a)

Construction Refer to Fig. 13.12(b).

1. Draw the front and the top views of the solid as given. Draw generators for the half-cylinder.
2. Consider seam at $a'1'$. Stretch out 1-1 and A-A from the front view and step off $AD = ad$ (arc), $DE = de$, $EF = ef$, $FG = fg$, $GA = ga$ (arc), i.e., perimeter of the top view.
3. Draw the cutting plane and name its points at the intersection as p', q', r', s', t', u' and v'.
4. Project t' and u' to obtain points t and u in the top view. In the development, mark points T_1 and U_1 such that $DT_1 = dt$ and $T_1\ U_1 = tu$. Draw vertial lines from T_1 and U_1.
7. Draw horizontal lines from p', q', r', s', t', u' and v' to meet the corresponding generators at points P, Q, R, T, U and V respectively.
5. Join $PQRS$ and $DTUV$ with smooth curves. Also join $S5$, 56 and VV with straight lines. Dark the portion of the development that is retained after truncating the cylinder.

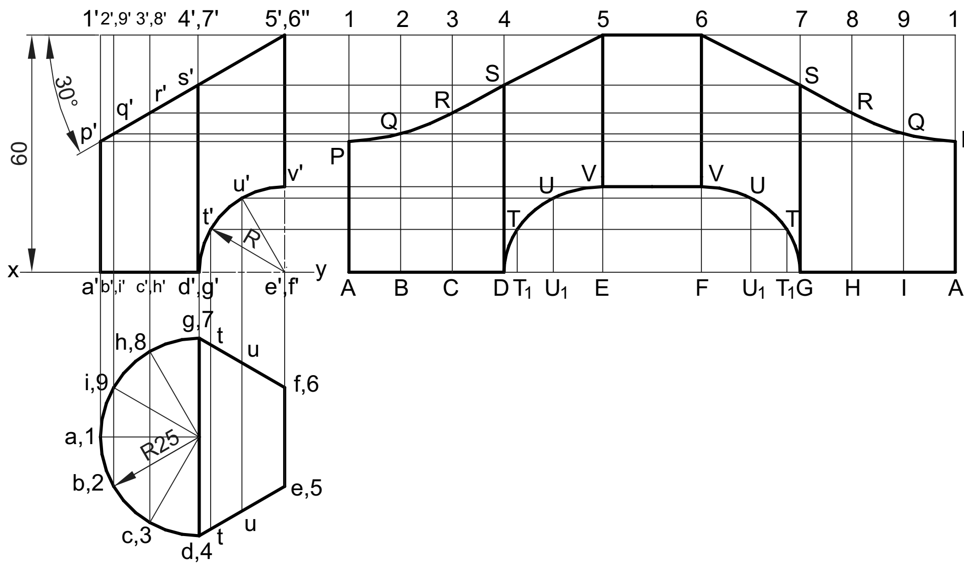


Fig. 13.12(b)

13.6 DEVELOPMENT OF CONES

Development of lateral surface of cone is obtained by radial line method. In this method, the development is in the form of sector of a circle, the radius of which is equal to the slant height of the cone. The subtended angle θ of this sector is calculated as $\theta = \frac{r}{R} \times 360^{\circ}$, where r is the radius of the base circle and R is the slant height of the cone. Alternatively, the arc length of this sector can directly be transferred from the top view of the base circle. It is done by taking 1/12th of the arc length from the base circle and then marking 12 times in the development. Although approximate, it is one of the most convenient and preferred methods.

Problem 13.12 *A cone of base diameter 50 mm and axis 60 mm is resting on its base on the H.P. Draw the development of its lateral surface.*

Calculation of θ

Slant height of cone

$$R = o'g' = \sqrt{r^2 + h^2} = \sqrt{25^2 + 60^2} = 65 \text{ mm}$$

Subtended angle

$$\theta = \frac{r}{R} \times 360^\circ = \frac{25}{65} \times 360^\circ = 138^\circ \quad \text{(approx.)}$$

Construction Refer to Fig. 13.13.

1. Draw a circle *adgj* as the top view and divide it into 12 equal parts. Project all the points and obtain $a'o'g'$ as the front view.
2. The end generators $o'a'$ and $o'g'$ gives the true length of the generators because their top views are parallel to *xy*. Therefore, mark *OA* parallel to $o'g'$.
3. Determine the subtended angle θ of the development. Here it is 138°.
4. Draw a sector *A-O-A* with included angle θ. Divide the sector into 12 equal parts and mark the generators as *OB*, *OC*, *OD*, etc. This is the required development of the cone.

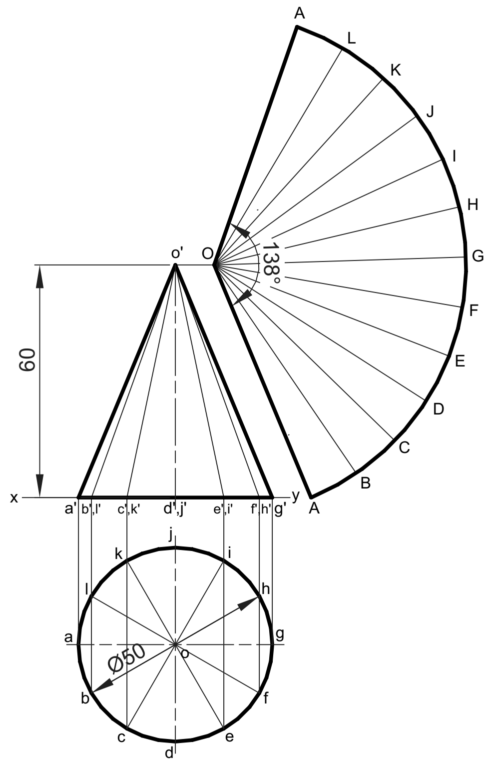


Fig. 13.13

Problem 13.13 *A cone of base diameter 50 mm and axis 60 mm is resting on its base on the H.P. A section plane perpendicular to V.P. and inclined at 45° to H.P., bisects the axis of the cone. Draw the development of its lateral surface.*

Calculation of θ

Slant height of cone $R = o'g' = \sqrt{r^2 + h^2} = \sqrt{25^2 + 60^2} = 65$ mm

Subtended angle $\theta = \dfrac{r}{R} \times 360^\circ = \dfrac{25}{65} \times 360^\circ = 138^\circ$ (approx.)

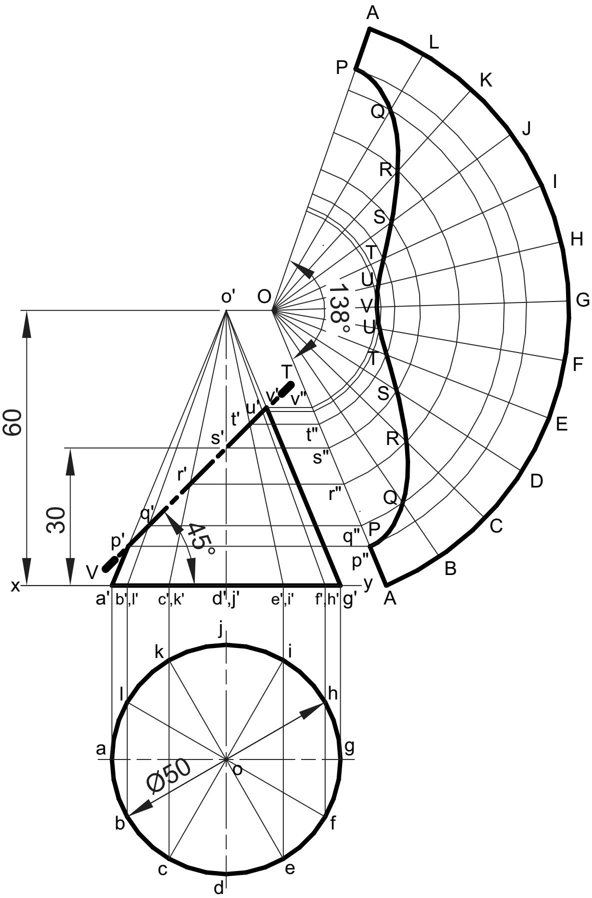


Fig. 13.14

Construction Refer to Fig. 13.14.

1. Draw a circle *adgj* as the top view and divide it into 12 equal parts. Project all the points and obtain $a'o'g'$ as the front view.
2. Draw V.T. of the cutting plane inclined at 45° to *xy* such that it passes through mid-point of the axis. Let V.T. cut the generators $o'a'$ at p', $o'b'$ at q', $o'c'$ at r', $o'd'$ at s', etc., as shown.
3. Determine the subtended angle θ as 138°. Draw a sector *A-O-A* with included angle θ. Divide the sector into 12 equal parts and mark the generators as *OB*, *OC*, *OD*, etc.
4. Draw the horizontal lines from points p', q', r', etc., to meet *OA* in the development at points p'', q'', r'', etc. Draw arcs with centre *O* and radii Op'', Oq'', Or'', etc., to meet the corresponding generators at points *P*, *Q*, *R*, etc.
5. Join all the points obtained in the development with smooth curves. Darken the portion of the development that is retained after truncating the cone.

Problem 13.14 *A cone of base diameter 50 mm and axis 60 mm is resting on its base on the H.P. Draw the development of its lateral surface when it is cut by an auxiliary inclined plane inclined at 60° to the H.P. and bisecting the axis.*

Calculation of θ

Slant height of cone

$$R = o'g' = \sqrt{r^2 + h^2} = \sqrt{25^2 + 60^2} = 65 \text{ mm}$$

Subtended angle

$$\theta = \frac{r}{R} \times 360^\circ = \frac{25}{65} \times 360^\circ = 138^\circ \text{ (approx.)}$$

Construction Refer to Fig. 13.15.

1. Draw a circle *adgj* as the top view and divide it into 12 equal parts. Project all the points and obtain $a'o'g'$ as the front view.
2. Draw V.T. of the cutting plane inclined at 60° to *xy* such that it passes through mid-point of the axis. Let V.T. cut the generators $c'o'$ at q', $d'o'$ at r′, $e'o'$ at s', $f'o'$ at t', $g'o'$ at u' and base circle at p'.
3. Determine the subtended angle θ as 138°. Draw a sector *A-O-A* with included angle θ. Divide the sector into 12 equal parts and mark the generators as *OB*, *OC*, *OD*,…, etc.
4. Draw the horizontal lines from points q', r', s', t and u' to meet line *OA* in the development at points q'', r'', s'', t'' and u'', respectively. Draw arcs with centre *O* and radii Oq'', Or'', Os'', Ot'' and Ou'' to meet the corresponding generators at points *Q*, *R*, *S*, *T* and *U*, respectively.

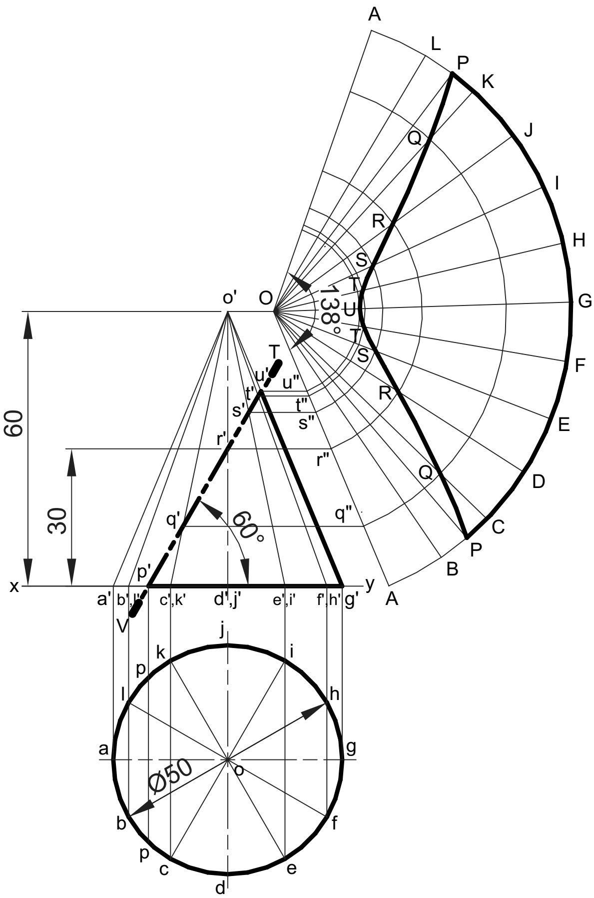


Fig. 13.15

5. Project point p' to meet the circle in the top view at point *p*. Locate point *P* in the development such that *BP* = *LP* = *bp*.
6. Join all the points obtained in the development with smooth curves. Darken the portion of the development that is retained after truncating the cone.

Problem 13.15 *The frustum of a cone of base diameter 60 mm, top diameter 20 mm and height 50 mm, is resting on its base on the H.P. It is cut by an A.I.P. inclined at 30° to the H.P., the H.T. of which is tangential to the base circle. Draw the development of the lateral surface of the retained frustum.*

Calculation of θ

$$\Delta a'g'o' \approx \Delta 1'1'o'$$

$$\therefore \quad \frac{60}{o'd'} = \frac{20}{o'd'-50} \Rightarrow o'd' = 75 \text{ mm}$$

Slant height of cone

$$R = o'g' = \sqrt{r^2 + h^2} = \sqrt{30^2 + 75^2} = 80.8 \text{ mm}$$

Subtended angle

$$\theta = \frac{r}{R} \times 360^\circ = \frac{30}{80.8} \times 360^\circ = 134^\circ \quad \text{(approx.)}$$

Construction Refer to Fig. 13.16.

1. Draw two concentric circles of 60 mm and 20 mm diameters as the top view. Divide each of them into 12 equal parts. Project all the points and obtain $a'g'2'1'$ as the front view. Produce $a'1'$ and $g'2'$ to meet at apex o'.
2. Draw V.T. of the cutting plane inclined at 30° to *xy* such that it passes through a'. Let V.T. cut the generators $b'o'$ at p', $c'o'$ at q', $d'o'$ at r', $e'o'$ at s', $f'o'$ at t' and $g'o'$ at u'.
3. Determine the subtended angle θ as 134°. Draw a sector *A-O-A* with included angle θ. Divide the sector into 12 equal parts and mark the generators as OB, OC, OD, etc.
4. Draw the horizontal lines from $1'$ to meet *OA* at point 1. Draw an arc 1-1 with centre *O* and radius *O*-1.
5. Draw the horizontal lines from p', q', r', s', t and u′ to meet *OA* at points p'', q'', r'', s'', t'' and u'', respectively. Draw arcs with centre *O* and radii Op'', Oq'', Or'', Os'', Ot'' and Ou'' to meet the corresponding generators at points *P*, *Q*, *R*, *S*, *T* and *U*, respectively.
6. Join *APQRSTUTSRQPA* with smooth curves.

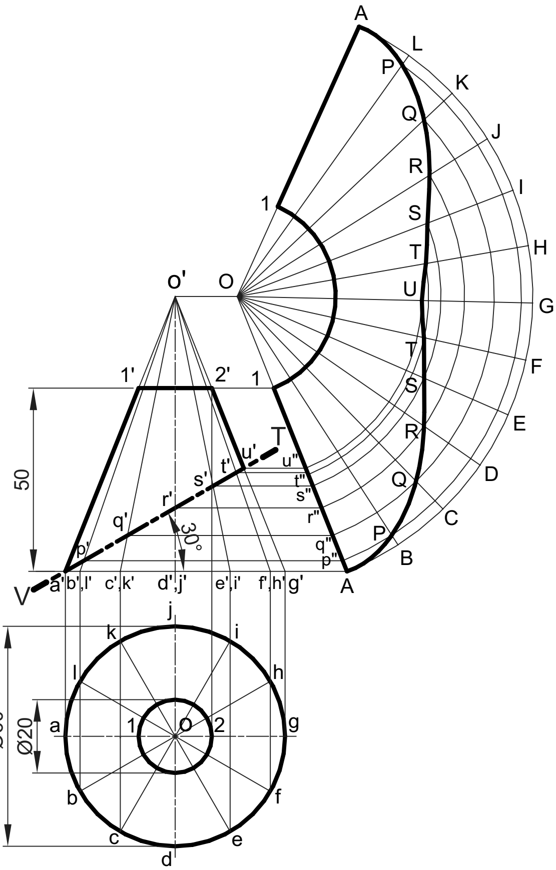


Fig. 13.16

Problem 13.16 *A cone of base diameter 50 mm and axis 60 mm is resting on its base on the H.P. A square hole of diagonal 25 mm is drilled through it such that axis of the hole intersects the axis of the cone at a height of 20 mm from the base. The faces of the hole are equally inclined to the H.P. Draw the development of its lateral surface.*

Calculation of θ

Slant height of cone $\quad R = o'3' = \sqrt{r^2 + h^2} = \sqrt{25^2 + 60^2} = 65 \text{ mm}$

Subtended angle

$$\theta = \frac{r}{R} \times 360^\circ = \frac{25}{65} \times 360^\circ = 138^\circ \text{ (approx.)}$$

Construction Refer to Fig. 13.17.

1. Draw a circle 1-2-3-4 as the top view. Project all the points and obtain $1'o'3'$ as the front view.
2. Draw a square $a'd'g'j'$ of diagonal 25 mm such that its centre is 20 mm above the base. The sides of the square should be inclined at 45° to the horizontal. On the edges of the square mark randomly more points as b', c', e', f', h', i', k' and l', which may not be equidistant.
3. Draw generator through the critical points a' and g'. Also draw generators through points b', c', e', f', h', i', k' and l'. Project them to obtain a, b, c,… etc., in the top view.
4. Determine the subtended angle θ as 138°. Draw a sector 1-O-1 with included angle θ.
5. Mark the generators in the development as OA_1, OB_1, OC_1, etc., such that $2C_1 = 2E_1$ = arc $1c$, $1B_1$ = $1F_1$ = arc $2b$, $1C_1 = 1G_1$ = arc $2c$, etc.
6. Draw horizontal lines from the points a', b', c', e', f', g', h', i', k' and l' to meet OA at points a'', b'', c'', etc. Draw arcs with centre O and radii Oa'', Ob'', Oc'', etc., to meet the corresponding generators at points A, B, C, etc.
7. Join $ABCD$, $DEFG$, $GHIJ$, $JKLA$ with smooth curves.

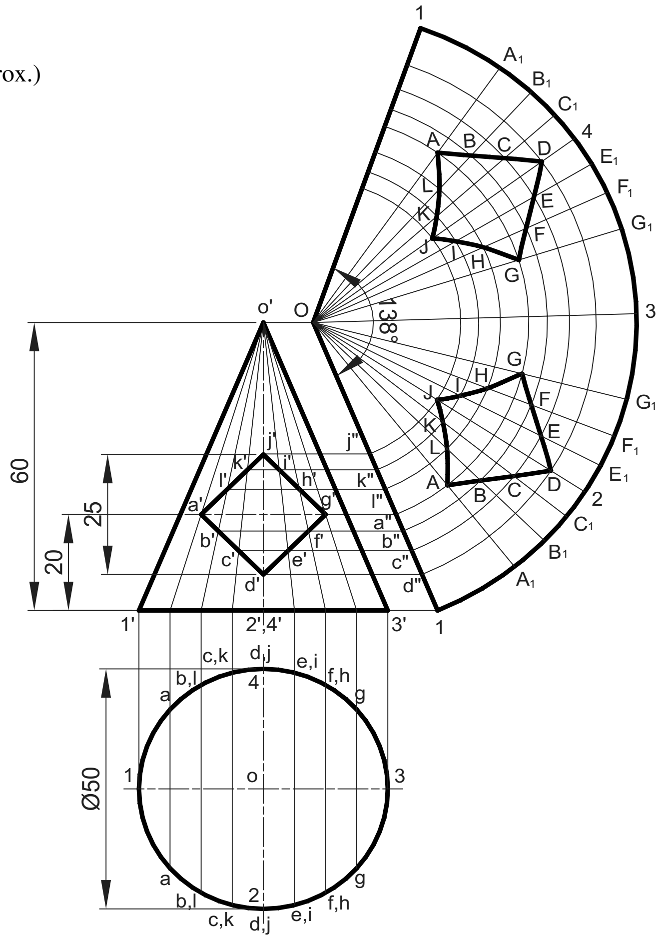


Fig. 13.17

Problem 13.17 *A cone of base diameter 50 mm and axis 60 mm stands on its base on the H.P. An auxiliary vertical plane whose H.T. is inclined at 45° to the V.P., cuts the cone at a distance of 10 mm from the axis. Draw the sectional front view and develop the lateral surface.*

Calculation of θ

Slant height of cone $R = o'g' = \sqrt{r^2 + h^2} = \sqrt{25^2 + 60^2} = 65$ mm

Subtended angle

$$\theta = \frac{r}{R} \times 360^\circ = \frac{25}{65} \times 360^\circ = 138^\circ \text{ (approx.)}$$

Construction Refer to Fig. 13.18.

1. Draw a circle *adgj* as the top view and divide it into 12 equal parts. Project all the points and obtain $a'o'g'$ as the front view.
2. Draw an arc with centre *o* and radius 10 mm. Draw H.T. of the cutting plane tangential to the arc, inclined at 45° to *xy*. Let H.T. cut the generators *od* at *q*, *oe* at *r*, *of* at *s*, *og* at *t* and base circle at points *p* and *u*.
3. Project *p*, *r*, *s*, *t* and *u* to meet their respective generators in the front view at points p', r', s', t' and u'.
4. Point *q* cannot be projected directly. Draw an arc with centre *o* and radius *oq* to meet *oa* at point q_1. Project q_1 to meet $o'a'$ at q_1'. Draw horizontal line from q_1' to meet $o'd'$ at point q'. Hatch the area enclosed by $p'q'r's't'u'$.
5. Determine the subtended angle θ as 138°. Draw a sector *A-O-A* with included angle θ. Divide the sector into 12 equal parts and mark the generators as *OB*, *OC*, *OD*, etc.
6. Draw horizontal lines from points q', r', s' and t' up to *OA* and thereafter rotate them keeping *O* as the centre to meet their respective generators at points *Q*, *R*, *S* and *T*.
7. Mark points *P* and *U* in the development such that *DP* = *dP* and *GU* = *gu*.
8. Join *PQRSTU* with smooth curve as shown.

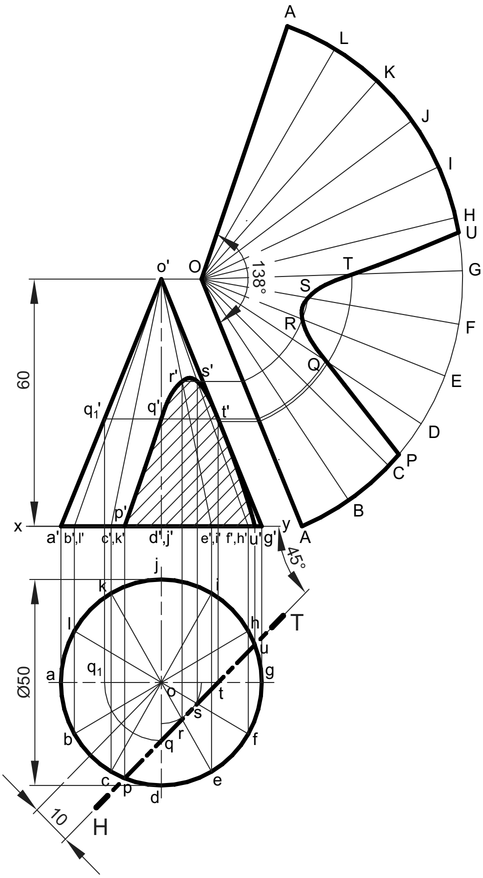


Fig. 13.18

13.7 DEVELOPMENT OF PYRAMIDS

Development of lateral surface of a pyramid consists of a series of isosceles triangles. It is obtained by radial line method, similar to that of the cone. However, for the isosceles triangles the true length of the slant edge may or may not be available in the front view. In case true length of the slant edge is not available, one needs to first determine the true length of the slant edge. The following problems illustrate the development of the lateral surface of the pyramids.

Problem 13.18 *Draw the development of the lateral surface of a square pyramid of base side 40 mm and axis 60 mm, resting on its base on the H.P. such that (a) all the sides of the base are equally inclined to the V.P., and (b) a side of the base is parallel to the V.P.*

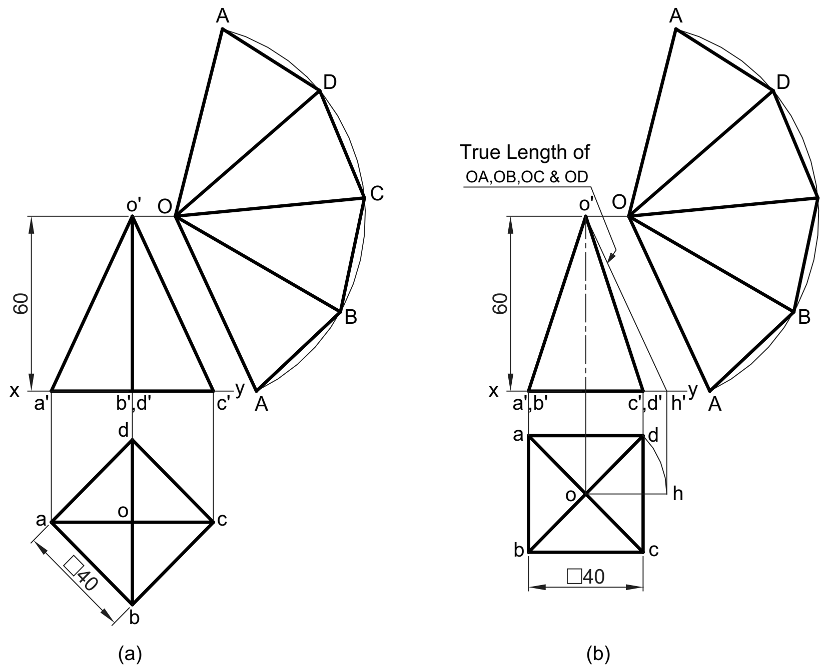


Fig. 13.19

Construction

(a) When all the sides of the base are equally inclined to the VP. (Refer to Fig. 13.19(a)).

1. Draw a square *abcd* keeping *ab* inclined at 45° to *xy*. Also, draw the diagonal lines of the square. This represents the top view. Project all the corners to obtain $a'o'c'$ as the front view.
2. Consider seam at $o'a'$. Slant edges $o'a'$ and $o'c'$ in the front view represents the true length because their top views are parallel to *xy*. Therefore, draw a line *O*A parallel and equal to $o'c'$.
3. Draw an arc with centre *O* and radius *OA*. Step off a distance of 40 mm on the arc to obtain *B*, *C*, *D* and *A*. Thus, $AB = BC = CD = DA = 40$ mm.
4. Join the base sides *AB*, *BC*, *CD*, *DA* and slant edges *OA*, *OB*, *OC*, *OD*, *OA*.

(b) When a side of the base is parallel to the VP. (Refer to Fig. 13.19(b)).

1. Draw a square *abcd* keeping ad parallel to *xy*. Also, draw the diagonal lines of the square. This represents the top view. Project all the corners to obtain triangle $a'o'c'$ as the front view.
2. As slant edges *oa*, *ob*, *oc* and *od* are inclined to *xy*. Therefore, slant edges $o'a'$, $o'b'$, $o'c'$ and $o'd'$ do not represent the true lengths. To determine the true length of the slant edges.
   i. Draw an arc *dh* with centre *o* and radius *od* to meet the horizontal line from *o* at point *h*.
   ii. Project point *h* to meet *xy* at point h'. Join $o'h'$ to represent the true length of slant edges.
3. Consider seam at $o'a'$. Draw line *OA* parallel and equal to $o'h'$. Draw an arc with centre *O* and radius *OA*. Step off a distance of 40 mm on the arc to obtain *B*, *C*, *D* and *A*. Thus, $AB = BC = CD = DA = 40$ mm.
4. Join the base sides *AB*, *BC*, *CD*, *DA* and slant edges *OA*, *OB*, *OC*, *OD*, *OA*.

It can be observed that as the objects in case (a) and (b) are same, their developments are also same.

Problem 13.19 *A square pyramid of base side 40 mm and axis 60 mm is resting on its base on the H.P. such that all the sides of the base are equally inclined to the V.P. It is cut by a section plane perpendicular to the V.P. and inclined at 60° to the H.P., bisecting the axis. Draw the development of its lateral surface.*

Construction Refer to Fig. 13.20.

1. Draw a square *abcd* keeping *ab* inclined at 45° to *xy*. Also, draw the diagonal lines of the square. This represents the top view. Project all the corners to obtain *a′o′c′* as the front view.
2. Draw V.T. of the cutting plane inclined at 60° to *xy* such that it passes through mid-point of the axis. Let V.T. cut *a′b′* and *a′d′* at *p′*, *o′b′* and *o′d′* at *q′*, *o′c′* at *r′*.
3. Consider seam at *o′a′*. Draw a line *OA* parallel and equal to *o′c′*. Draw an arc with centre *O* and radius *OA*. Step off a distance of 40 mm on the arc to obtain *B*, *C*, *D* and *A*. Thus, *AB* = *BC* = *CD* = *DA* = 40 mm. Join the base sides *AB*, *BC*, *CD*, *DA* and slant edges *OA*, *OB*, *OC*, *OD*, *OA*.
4. Draw the horizontal lines from points *q′* and *r′* to meet line *OA* in the development at points *q″* and *r″*, respectively. Draw arcs with centre *O* and radii *Oq″* and *Or″* to meet the corresponding generators at points *Q* and *R*, respectively.
5. Project point *p′* to meet the square in the top view at point *p*. Locate point *P* in the development such that *AP* = *ap*.
6. Join *PQRQP* with straight lines. Darken the portion of the development that is retained after truncating the cone.

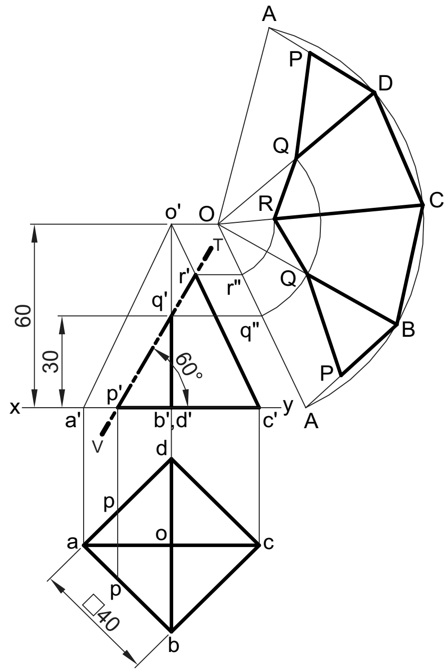


Fig. 13.20

Problem 13.20 *A square pyramid of base side 40 mm and axis 60 mm is resting on its base on the H.P. such that a side of the base is parallel to the V.P. It is cut by a section plane perpendicular to the V.P. and inclined at 45° to the H.P., bisecting the axis. Draw the development of its lateral surface.*

Construction Refer to Fig. 13.21.

1. Draw a square *abcd* keeping ad parallel to *xy*. Also, draw the diagonal lines of the square. This represents the top view. Project all the corners to obtain *a′o′c′* as the front view.
2. Draw V.T. of the cutting plane inclined at 45° to *xy* such that it passes through mid-point of the axis. Let V.T. cut the *o′a′* and *o′b′* at *p′*, *o′c′* and *o′d′* at *q′*.

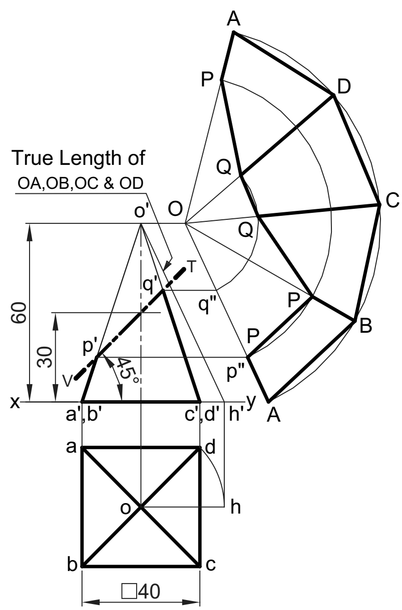


Fig. 13.21

3. Draw an arc *dh* with centre *o* and radius *od* to meet the horizontal line through *o* at point *h*. Project point *h* to meet *xy* at point *h′*. Join *o′h′* to represent the true length of slant edges.
4. Consider seam at *o′a′*. Draw line *OA* parallel and equal to *o′h′*. Draw an arc with centre *O* and radius *OA*. Step off a distance of 40 mm on the arc to obtain *B*, *C*, *D* and *A*. Join the base sides *AB*, *BC*, *CD*, *DA* and slant edges *OA*, *OB*, *OC*, *OD*, *OA*.
5. Draw the horizontal lines from points *p′* and *q′* to meet line *OA* at points *p″* and *q″*, respectively. Draw arcs with centre *O* and radii *Op″* and *Oq″* to meet the corresponding generators at points *P* and *Q*, respectively.
6. Join *PPQQP* with straight lines. Darken the portion of the development that is retained after truncating the cone.

Problem 13.21 *A pentagonal pyramid of base side 30 mm and axis 60 mm, rests on its base on the H.P. with a side of the base parallel to the V.P. It is cut by two section plane which meet at a height of 20 mm from the base. One of the section planes is horizontal, while the other is an auxiliary inclined plane whose V.T. is inclined at 45° to the H.P. Draw the development of the lateral surface of the solid when apex is removed.*

Construction Refer to Fig. 13.22.

1. Draw a pentagon *abcde* keeping *de* parallel to *xy*. Join the corners with centroid *o*. This represents the top view. Project all the corners to obtain *a′o′c′* as the front view.
2. Draw V.T. of the cutting plane such that *p′q′* is parallel to *xy* and *q′r′* inclined at 45° to *xy*.
3. Draw an arc *ch* with centre *o* and radius *oc* to meet the horizontal line through centre *o* at point *h*. Project *h* to meet *xy* at point *h′*. Join *o′h′* to represent the true length of the slant edges.
4. Consider seam at *o′a′*. Draw line *OA* parallel and equal to *o′h′*. Draw an arc with centre *O* and radius *OA*. Step off a distance of 30 mm on the arc to obtain *B*, *C*, *D*, *E* and *A*. Join the base sides *AB*, *BC*, *CD*, *DE*, *EA* and slant edges *OA*, *OB*, *OC*, *OD*, *OE*, *OA*.
5. Draw horizontal lines from *p′*, *q′* and *t′* to meet *OA* at point *p″*. Draw horizontal lines from *r′* and *s′* to meet *OA* at *r″* and *s″*, respectively. Draw arcs with centre *O* and radii *Op″*, *Or″* and *Os″* to meet *OA* at *P*, *OB* at *Q*, *OC* at *R*, *OD* at *S*, *OE* at *T*. Also, mark point *Q* as the mid-point of chord *T*M.
6. Join *PQRSQTP* with straight lines to obtain the development as shown.

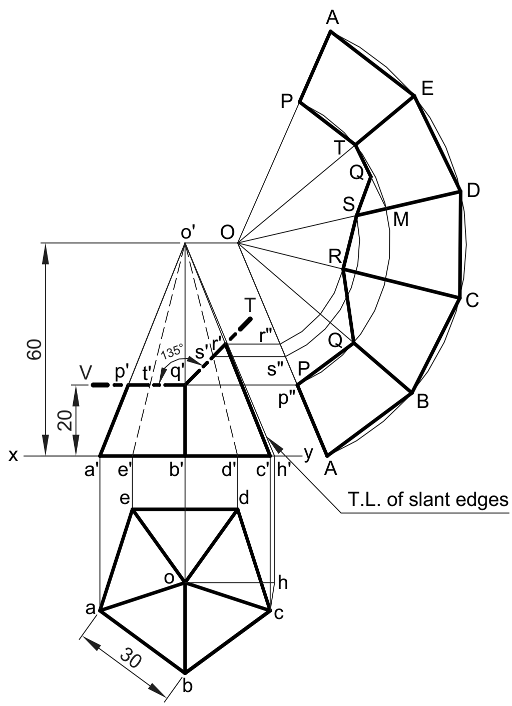


Fig. 13.22

Problem 13.22 *A hexagonal pyramid of base side 30 mm and axis 60 mm, rests on its base on the H.P. with a side of the base parallel to the V.P. It is cut by planes perpendicular to V.P. to obtain the front view as shown in Fig. 13.23(a). Draw the development of the lateral surface of the retained solid.*

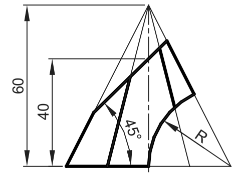


Fig. 13.23(a)

Construction Refer to Fig. 13.23(b).

1. Draw a hexagon *abcdef* keeping *ef* parallel to *xy*. Join all the corners with centroid *o*. This represents the top view. Project all the corners to obtain $a'o'd'$ as the front view.
2. Draw a curve $p'q'r'$ and line $s't'u'v'$ in the front view to represent the V.T. of the cutting planes.
3. Line $o'd'$ represents true length of the slant edges. Therefore, draw line *OA* parallel and equal to $o'd'$. Draw an arc with centre *O* and radius *OA*. Step off a distance of 30 mm on the arc to obtain *B*, *C*, *D*, *E*, *F* and *A*. Join the base sides and slant edges in the development.
4. Draw horizontal lines from points q', r', s', t', u' and v' to meet *OA* and thereafter rotate them keeping *O* as the centre to meet the corresponding generators at points *Q*, *R*, *S*, *T*, *U* and *V*.
5. Point p' lies at the mid of base edges *bc* and *ef*. Therefore, in the development mark point *P* at the mid of *BC* and *EF*.
6. Join *PQRQP* with a smooth curve whereas *STUVUTS* with straight lines as shown.

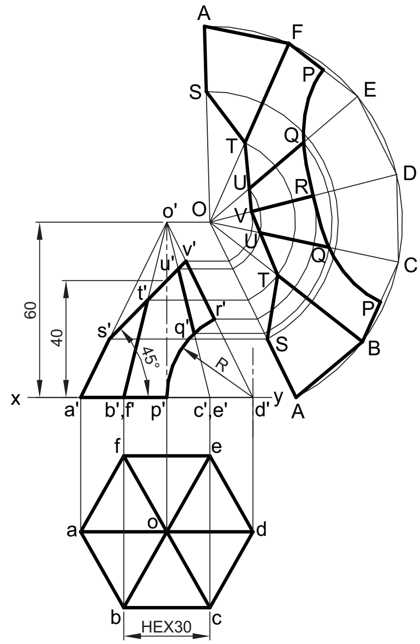


Fig. 13.23(b)

Problem 13.23 *The frustum of a square pyramid of base side 40 mm, top side 15 mm and height 40 mm, rests on its base on the H.P. with all the sides of the base equally inclined to the V.P. A rectangular slot of sides 30 mm and 15 mm is cut through it. Figure 13.24(a) shows the front view of the solid. Draw the development of its lateral surface.*

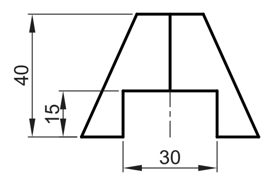


Fig. 13.24(a)

Construction Refer to Fig. 13.24(b).

1. Draw two concentric squares *abcd* and 1234. Join *a*1, *b*2, *c*3 and *d*4. This represents the top view. Project all the corners to obtain trapezium $a'1'3'c'$ as the front view. Produce $a'1'$ and $c'3'$ to meet each other at point o'.
2. Draw line $p'q'r's't'$ in the front view to represent the V.T. of the cutting planes.
3. Line $o'3'c'$ represents true length of the slant edges. Therefore, draw line *O*1*A* parallel and equal to $o'3'c'$. Draw arcs with centre *O* and radii *OA* and *O*-1. Step off a distance of 40 mm on the arc *AA* to obtain *B*, *C*, *D*. Similarly, step off a distance of 15 mm on the arc 1-1 to obtain 2, 3, 4. Join 1234, *ABCD*, 1*A*, 2*B*, 3*C* and 4*D*.

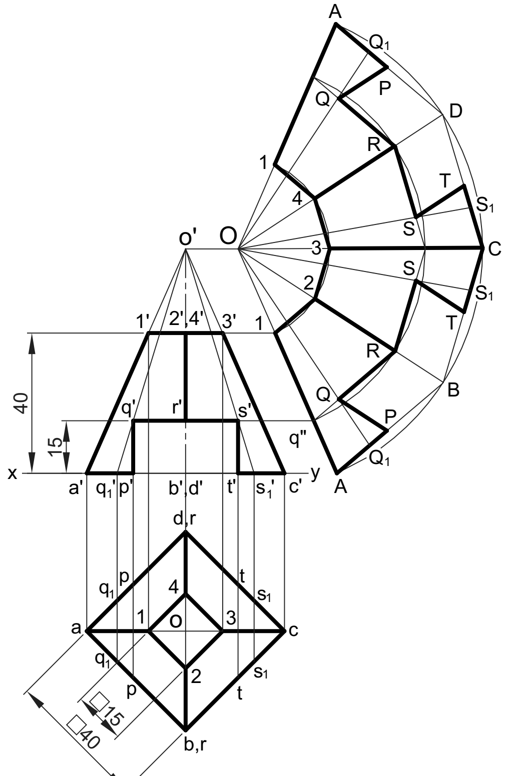


Fig. 13.24(b)

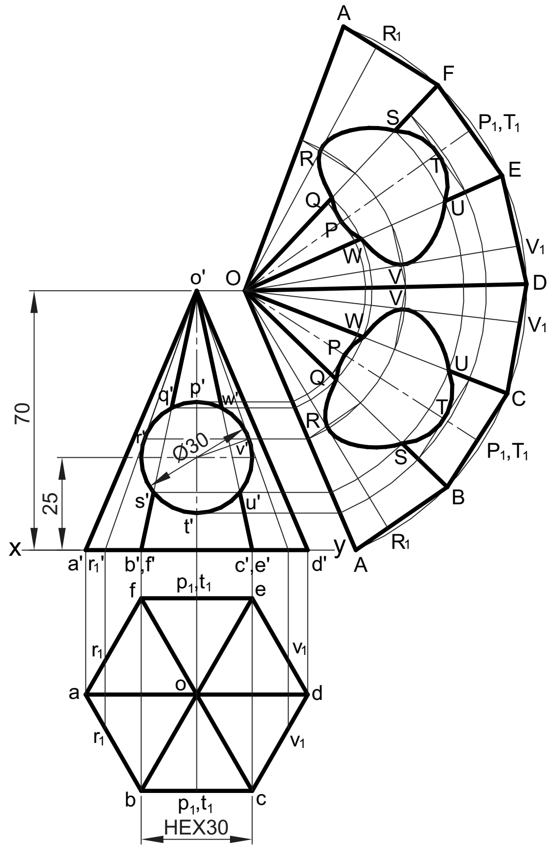


Fig. 13.25

4. Draw generators $o'q_1'$ and $o's_1'$ passing through q' and s' respectively. Project q_1', p', t', s_1' to meet at points q_1, p, t and s_1 in the top view. Transfer these points in the development and obtain points Q_1, P, T and S_1. Join OQ_1 and OS_1 to represent the locus of points Q and S respectively.
5. Draw horizontal lines from q', r' and s' to meet OA at q''. Draw an arc with centre O and radius Oq'' to meet corresponding generators and edges at points Q, R and S. It may be noted that points P, Q, S and T lies on the chords. Join $PQRST$ with lines as shown.

Problem 13.24 *A hexagonal pyramid of base side 30 mm and axis 70 mm is resting on the ground with a side of base parallel to the V.P. A circular hole of diameter 30 mm is cut through the faces of the pyramid such that axes of the hole and the pyramid intersects at right angle and 25 mm above the base. Draw the development of its lateral surface.*

Construction Refer to Fig. 13.25.

1. Draw a hexagon *abcdef* along with diagonal lines as the top view. Project all the corners to obtain $a'o'd'$ as the front view. Draw AOA as the development of the lateral surface.

2. Draw a circle of diameter 30 mm having centre 25 above the base in the front view.
3. Draw generators tangential to the circle at points r' and v'. Project the generators and obtain r_1 and v_1 in the top view. Obtain corresponding generators OR_1 and OV_1 in the development.
4. Draw horizontal lines from p', q', r', s', t', u', v', q' up to OA, and thereafter rotate them keeping centre O to meet their corresponding generators at points P, Q, R, S, T, U, V and W. It may be noted that points P, R, T, V lies on the chords of the faces and not on the arcs.
5. Join the points with curve as shown to obtain the required development.

Problem 13.25 *A solid is composed of half-pyramid and half-cone. It is cut by an auxiliary inclined plane whose V.T. is inclined at 45° to the H.P. and bisecting the axis as shown in Fig. 13.26(a). Draw the development of the lateral surface of the retained solid.*

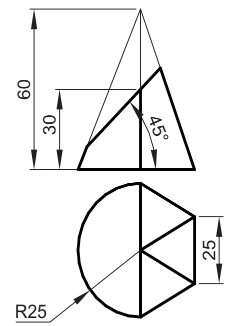


Fig. 13.26(a)

Construction Refer to Fig. 13.26(b).

1. Draw front and top views of the solid as given. Draw generator of the half-cone.
2. Name the points of intersection of V.T. in the front view as p', q', r', s' and t'.
3. Draw a line OA of length equal to the true length. Draw an arc AA with centre O and radius Step off arcs of ab, bc, cd, de, ef, fg, gh, hi and ia on the arc AA to obtain points B, C, D, E, F, G, H and I. Join OB, OC, OD, etc.
4. Draw the horizontal lines from points p', q', r', s' and t' to meet OA at points p'', q'', r'', s'' and t''. Draw arcs with centre O and radii Op'', Oq'', Or'', Os'' and Ot'' to meet the corresponding generators at points P, Q, R, S and T.
5. Join all the points with curves and lines as shown to obtain the required development.

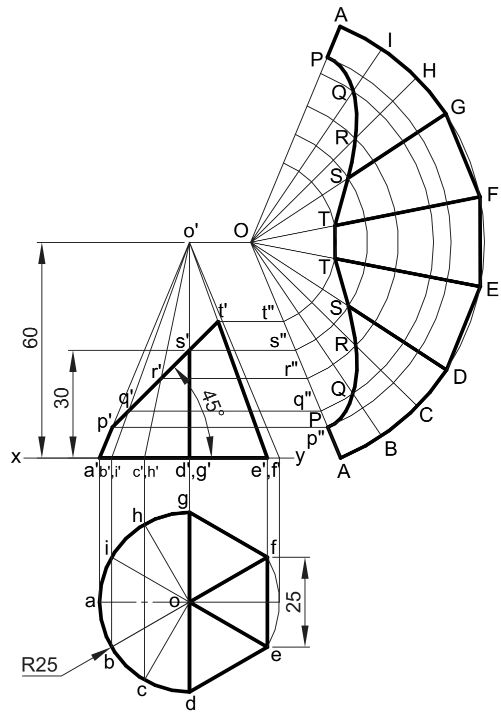


Fig. 13.26(b)

13.8 DEVELOPMENT OF SPHERES

Spheres have doubly curved surfaces, whose exact development cannot be obtained. The spherical surfaces can be divided into smaller parts in the form of zones or Lunes. Figures 13.27(a) and (b) show a zone and a Lune of the sphere, respectively. The following problems illustrate development of spheres.

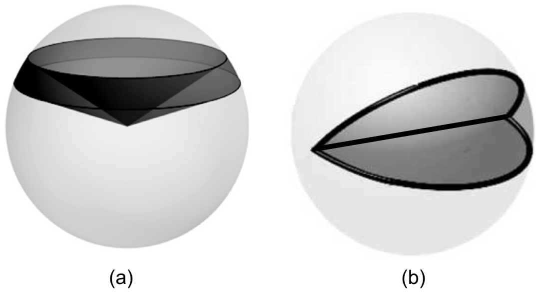


Fig. 13.27 *Sphere* **(a)** *Zone* **(b)** *Lune*

Problem 13.26 *Draw the development of a sphere of diameter 50 mm using zone method.*

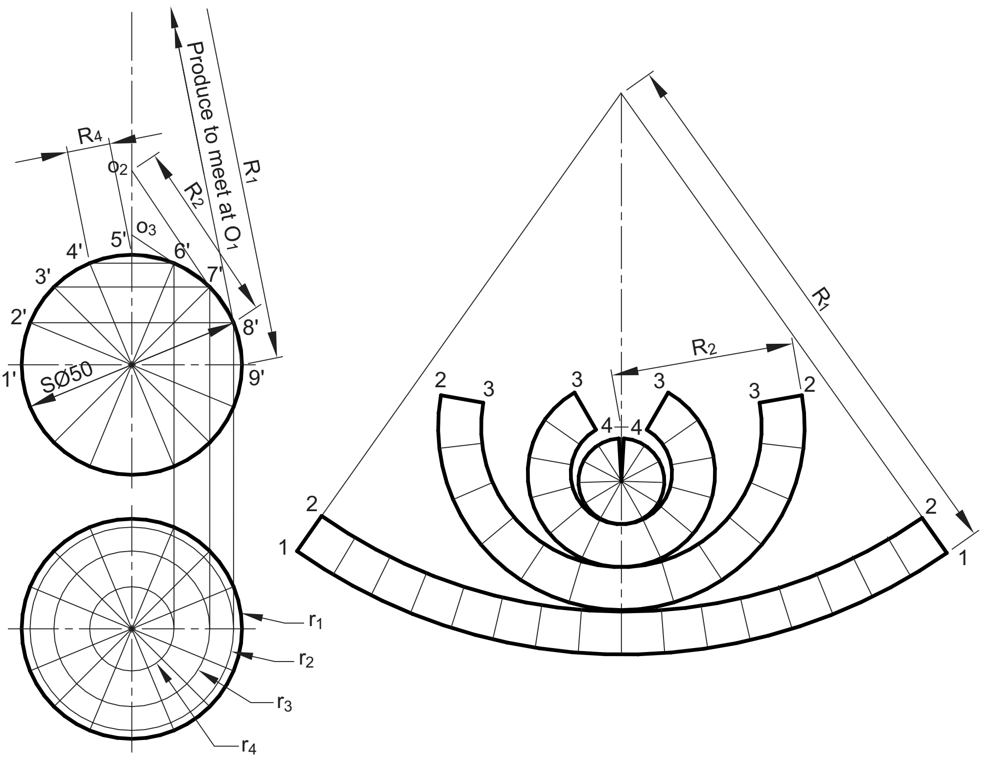


Fig. 13.28

Construction Refer to Fig. 13.28.

In zone method, the development of the one hemisphere is drawn. Two similar developments are needed for the whole sphere.

1. Draw circles of 50 mm diameters to represent the front and top views of the sphere.
2. Divide the upper half hemisphere of the front view into 8 equal parts. Name the points as 1′, 2′, 3′, etc. Draw a series of parallel lines connecting 1′9′, 2′8′, 3′7′ and 4′6′. These lines slices the upper hemisphere into a cone (4′5′6′) and three frustums of cones (1′9′8′2′, 2′8′7′3′, 3′7′6′4′).
3. Project 8′, 7′ and 6′ to the top view and draw concentric circles having radii r_2, r_3 and r_4. Produce 9′8′, 8′7′ and 7′6′ to meet the vertical line at O_1, O_2, and O_3. The distances 9′O_1, 8′O_2, 7′O_3 and 5′6′ be defined as R_1, R_2, R_3 and R_4.
4. Draw arcs 1-1 and 2-2 with centre O_1 and radii R_1 and (R_1-8′9′) to subtend an angle of $(25/R_1) \times 360°$. This is the development of the frustum 1′9′8′2′.
5. Similarly, draw arcs 2-2 and 3-3 with centre O_2 and radii R_2 and (R_2-7′8′) to subtend an angle of $(r_2/R_2) \times 360°$. Also, draw arcs 3-3 and 4-4 with centre O_3 and radii R_3 and (R_3-6′7′) to subtend an angle of $(r_3/R_3) \times 360°$. Finally, draw arc 4-4 with centre O_4 and radii R_4 to subtend an angle of $(r_4/R_4) \times 360°$.

Problem 13.27 *Draw the development of a sphere of diameter 50 mm using Lune method.*

Construction Refer to Fig. 13.29.

In Lune method, the development of the one-eighth of sphere is drawn. Eight similar developments are needed for the whole sphere.

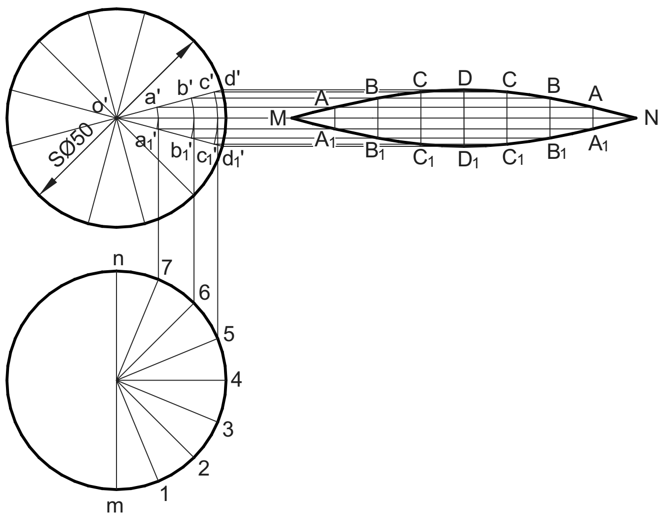


Fig. 13.29

1. Draw circles of 50 mm diameters to represent the front and top views of the sphere.
2. Divide the sphere in the front view into 12 equal parts called Lune. Consider Lune $d'o'd_1'$ whose development is to be drawn.
3. Divide the semicircle *mn* in top view, into 8 equal parts and name the points as 1, 2, 3, 4, 5, 6, 7. Project them in the front view to get 1′, 2′, 3′ and 4′.
4. Draw arcs with centre o' and radii $o'1'$, $o'2'$, $o'3'$, etc., to meet $o'd'$ and $o'd_1'$ at points a', b', c' and a_1', b_1', c_1' respectively.
5. Draw stretch out lines from lune $d'o'd_1'$ of length equal to arc *mn* (78.5 mm). Divide *mn* into 8 equal parts and draw vertical lines through each point.
6. Draw stretch out lines from a', b', c', a_1', b_1', c_1' to obtain A', B', C', A_1', B_1', C_1', etc.
7. Join the points as shown to represent the development of the Lune.

13.9 DEVELOPMENT OF TRANSITION PIECES

Transition pieces are used to connect two hollow objects having different cross sections such as a piece connecting two ducts one with a square section and other with a circular section together as shown in

Fig. 13.30. The development of transition pieces is obtained by triangulation method. The method consists of dividing the lateral surface into small triangles and obtain the true shape of each triangle. These triangles are then assembled in a single plane adjacent to each other which gives the final development. Consider the following problems.

Fig. 13.30 *Transition pieces*

Problem 13.28 *Draw the development of a 75 mm long transition piece that connects a square section of side 25 mm with a square of side 60 mm. The transition piece rests on the H.P. on 60 mm side square base such that 60 mm square section has an edge parallel to the V.P. while 25 mm square has an edge inclined at 45° to the V.P.*

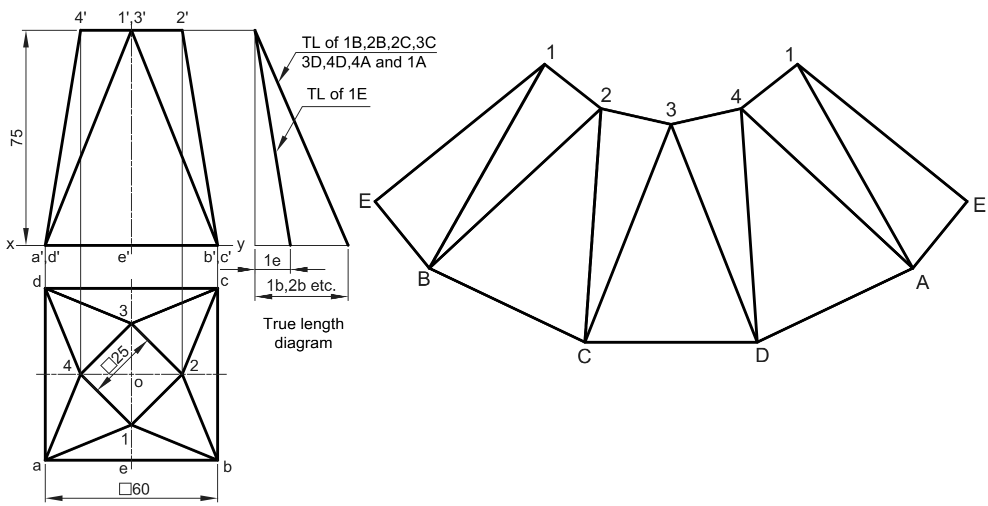


Fig. 13.31

Construction Refer to Fig. 13.31.

1. **Projections** Draw squares *abcd* and 1234 keeping common centre *o*, *ab* parallel to *xy* and 1-2 inclined at 45° to *xy*. Project the corners and obtain trapezium *a′b′*2′4′ as the front view.
2. **True length diagram** Draw a triangle of height 75 mm and base 1*e*. The hypotenuse of this triangle represents the true length of 1*E*.
3. Similarly, draw another triangle of height 75 mm and base 1*b*. The hypotenuse of this triangle represents the true length of lateral edges 1*B*, 2*B*, 2*C*, etc.
4. **Development** Consider seam at 1′*e*′. Draw a line 1*E* in the development equal to *TL* of 1*E*.
5. Locate point *B* such that *EB* = 30 mm (½ of base edge) and 1*B* = *TL* of 1*B*.
6. Locate point 2 such that 2*B* = *TL* of 2*B* and distance 1-2 = 25 mm (top edge).
7. Locate point *C* such that *BC* = 60 mm (base edge) and 2*C* = *TL* of 2*C*.

8. Locate point 3 such that 3*C* = *TL* of 3*C* and distance 2-3 = 25 mm.
9. Locate point *D* such that *CD* = 60 mm (base edge) and 3*D* = *TL* of 3*D*.
10. Locate point 4 such that 4*D* = *TL* of 4*D* and distance 3-4 = 25 mm.
11. Locate point *A* such that *DA* = 60 mm (base edge) and 4*A* = *TL* of 4*A*.
12. Locate point 1 such that 1*A* = *TL* of 1*A* and distance 4-1 = 25 mm.
13. Locate point *E* such that *AE* = 30 mm (½ of base edge) and 1*E* = *TL* of 1*E*.

Problem 13.29 *Draw the development of a 75 mm long transition piece that connects a circular section of diameter 40 mm with a square section of side 70 mm. Both sections of the transition piece are coaxial and it is resting on its square base on the H.P.*

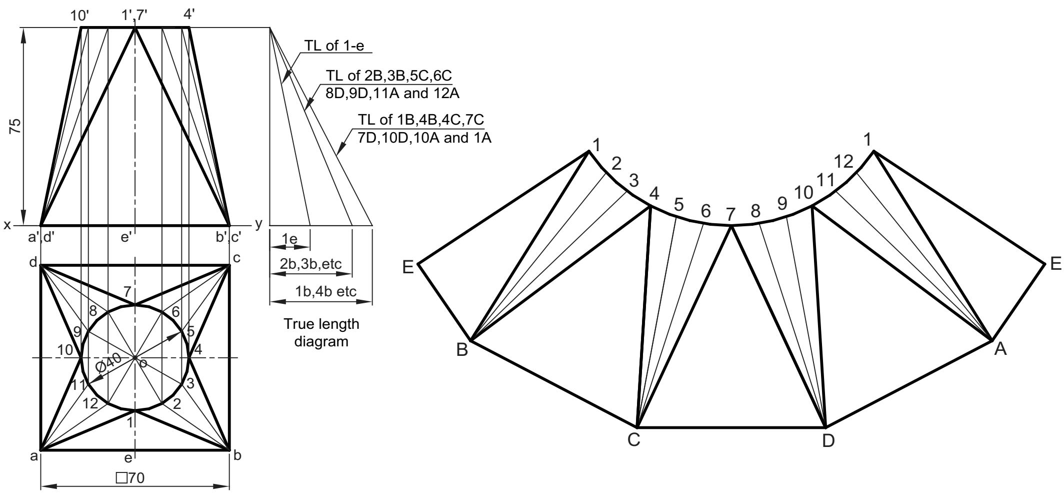


Fig. 13.32

Construction Refer to Fig. 13.32.

1. **Projections** Draw square *abcd* and circle 1-4-7-10 keeping common centre *o* and *ab* parallel to *xy*. Project the corners and obtain trapezium *a*′*b*′4′10′ as the front view.
2. **True length diagram** Draw a triangle of height 75 mm and base 1*e*. The hypotenuse of this triangle represents the true length of 1*E*.
3. Similarly, draw another triangle of height 75 mm and base 1*b*. The hypotenuse of this triangle represents the true length of lateral edges 1*B*, 4*B*, 4*C*, etc.
4. Similarly, draw another triangle of height 75 mm and base 2*b*. The hypotenuse of this triangle represents the true length of lateral edges 2*B*, 3*B*, 5*C*, etc.
5. **Development** Consider seam at 1′*e*′. Draw a line 1*E* in the development equal to *TL* of 1*E*.
6. Locate point *B* such that *EB* = 35 mm (½ of base edge) and 1*B* = *TL* of 1*B*.
7. Locate point 2 such that 2*B* = *TL* of 2*B* and 1-2 = length of arc 1-2 in the top view.
8. Locate point 3 such that 3*B* = *TL* of 3*B* and 2-3 = length of arc 2-3 in the top view.
9. Locate point 4 such that 4*B* = *TL* of 4*B* and 3-4 = length of arc 3-4 in the top view.
10. Locate point *C* such that *BC* = 70 mm (base edge) and 4*C* = *TL* of 4*C*.
11. Proceed to locate points 5, 6, 7, D, 8, 9, 10, A, 11, 12, 1, *E*.
12. Join the points as shown to represent the required development of the transition piece.

Problem 13.30 *Draw the development of the lateral surface of a transition piece shown in Fig. 13.33(a), used to connect two rectangular ducts of different cross sections in an air-conditioner unit.*

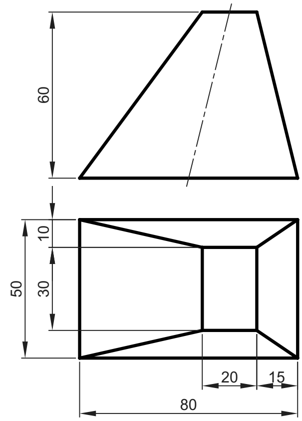


Fig. 13.33(a)

Construction Refer to Fig. 13.33(b).

1. Draw the given top and the front views of the transition piece. Consider seam at $5'e'$, label all the corners.
2. Determine the true length (*TL*) of *e*5 and slant edges *a*1, *b*2, *c*3, *d*4 as shown.
3. In the top view, draw a diagonal for each face. Determine the true lengths of the diagonals *e*2, *b*3, *c*4, *d*1, *a*5 as shown.
4. Draw a line *E*5 equal to *TL* of *E*5 in the development.
5. Now draw triangles *E*52, *E*2B, *B*23, *B*3C, *C*34, *C*4*D*, *D*41, *D*1*A*, *A*15 and *A*5*E* with the help of combination of the true lengths of the base, slant edges and diagonals.
6. Join these points as shown to obtain the required development.

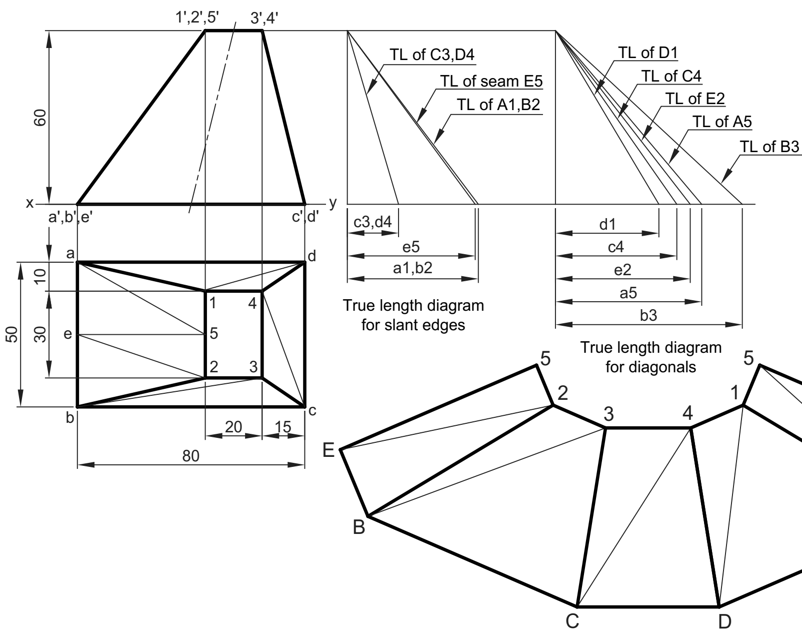


Fig. 13.33(b)

13.10 DEVELOPMENT OF TRAY

Lateral surface of the tray can be considered as the frustum of a pyramid and can be developed by attaching a base to it. This needs a long strip and a base. On folding this requires more number of joints and also the material is not used to an optimum level. Henceforth the trays are developed in a manner as illustrated in the following solved problem.

Problem 13.31 *A tray is made of sheet metal whose orthographic views are shown in Fig. 13.34(a). Draw the development of the surface of the entire tray.*

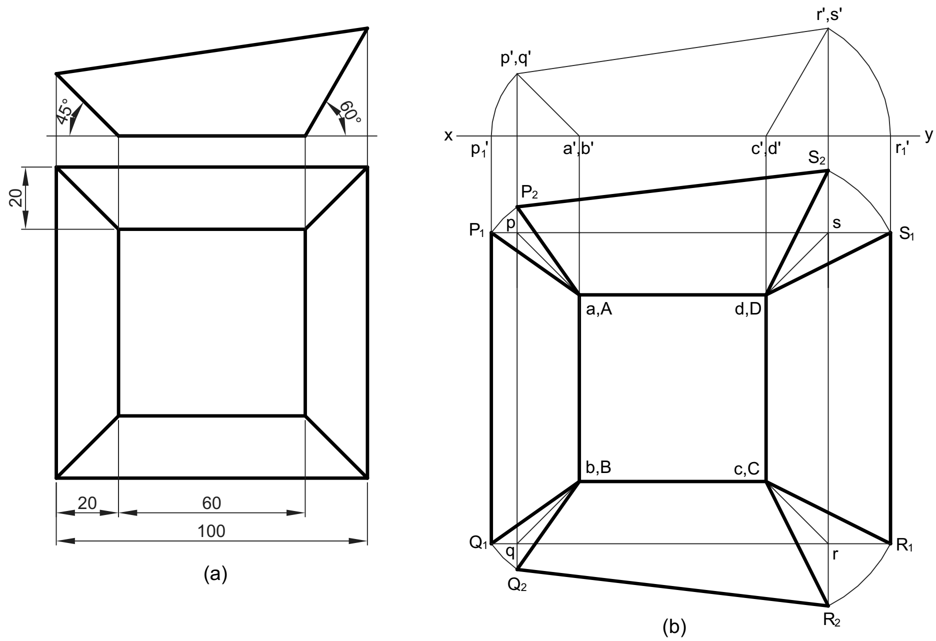


Fig. 13.34 *Tray* **(a)** *Projections* **(b)** *Development*

Construction Refer to Fig. 13.34(b).

1. Draw the given top and the front views of the tray. Label the corners.
2. The true shape of base *ABCD* is same as *abcd* in the top view. Unfold the faces one by one.
3. Draw an arc with centre a' and radius $a'p'$ to meet xy at point p_1'. Project p_1' to meet horizontal line from points p and q at points P_1 and Q_1, respectively. Join aP_1 and bQ_1 to represent the true length of edges *AP* and *BQ*, respectively. Join P_1Q_1. The ABQ_1P_1 represents the true shape of the face *abpq*.
4. Similarly, draw an arc with centre c' and radius $c'r'$ to meet xy at point r_1'. Project r_1' to meet horizontal line from points r and s at points R_1 and S_1, respectively. Join rR_1 and sS_1 to represent the true length of edges *CR* and *DS*, respectively. Join R_1S_1. The CDS_1R_1 represents the true shape of the face *cdsr*.
5. Draw an arc with centre b and radius BQ_1 to meet vertical line through q at point Q_2. Similarly, draw an arc with centre c and radius CR_1 to meet vertical line through r at point R_2. Join Q_2R_2. The BCR_2Q_2 represents the true shape of the face *bcrq*.
6. Similarly, draw an arc with centre a and radius AP_1 to meet vertical line through p at point P_2. Similarly, draw an arc with centre d and radius DS_1 to meet vertical line through s at point S_2. Join P_2S_2. The ADS_2P_2 represents the true shape of the face *adsp*.

13.11 DEVELOPMENT OF OBLIQUE OBJECTS

All oblique objects, whether prism pyramid cylinder or cone, have their axes inclined to their base as shown in Fig. 11.5. The lateral edges of an oblique prism are of equal lengths and faces are parallelograms of different sizes. The generators of an oblique cylinder are also of equal lengths. The lateral edges of oblique pyramid are of unequal lengths and faces are triangles of different sizes. The generators of an oblique cone are also of unequal lengths. Following problems illustrate the development of oblique objects.

Problem 13.32 *An oblique pentagonal prism of base side 30 mm has 70 mm long axis inclined at 60° to the base. The prism rests on its base on the H.P. such that axis and an edge of the base are parallel to the V.P. Draw the development of its lateral surface.*

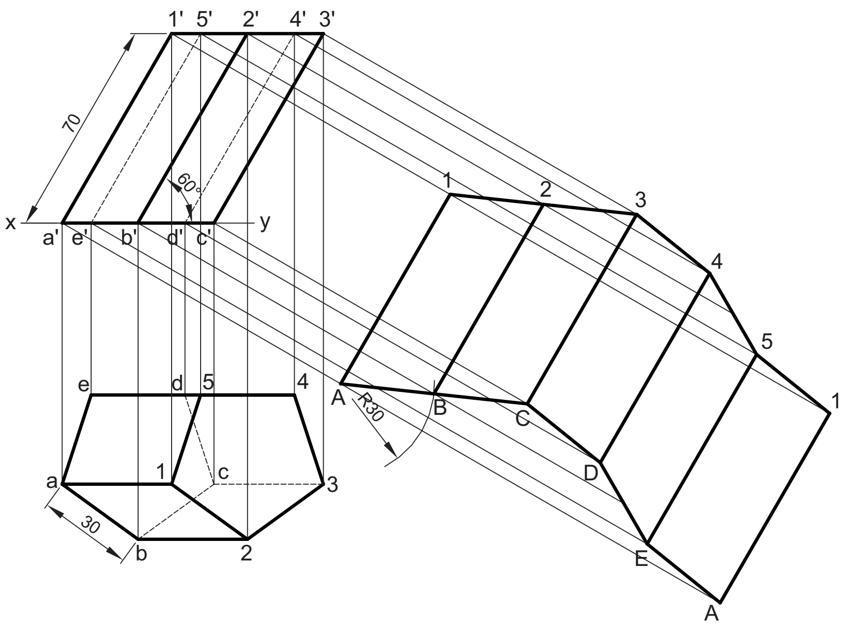


Fig. 13.35

Construction Refer to Fig. 13.35.

1. Draw a pentagon *abcde* keeping *de* parallel to the *xy* to represent top view of the lower base. Project all the corners to obtain *a′c′*. Draw a parallelogram *a′c′*3′1′ to represent the front view. Now project the top base 1′3′ to obtain its corresponding top view. Complete the front and the top views and label the corners as shown.
2. Consider the seam at 1′*a′*. Stretch out lines *a′A*, *b′B*, *c′C*, *d′D*, *e′E*, 1′1, 2′2, 3′3, 4′4 and 5′5 perpendicular to the lateral edges of the front view.
3. Draw *A*1 parallel to *a*1. Starting from point *A*, step off arcs of radius 30 mm to meet the stretch out lines in succession at points *B*, *C*, *D*, *E* and *A*. Join *AB*, *BC*, *CD*, *DE*, *EA*.

4. Similarly, starting from point 1, step off arcs of radius 30 mm to meet the stretch out lines in succession at points 2, 3, 4, 5 and 1. Join 1-2, 2-3, 3-4, 4-5 and 5-1.
5. Join the lateral edges *A*1, *B*2, *C*3, *D*4 and *E*5, which are of equal lengths and parallel to each other.

Problem 13.33 *An oblique cylinder of base diameter 50 mm has 70 mm long axis inclined at 60° to the base. The cylinder rests on its base on the H.P. Draw the development of its lateral surface.*

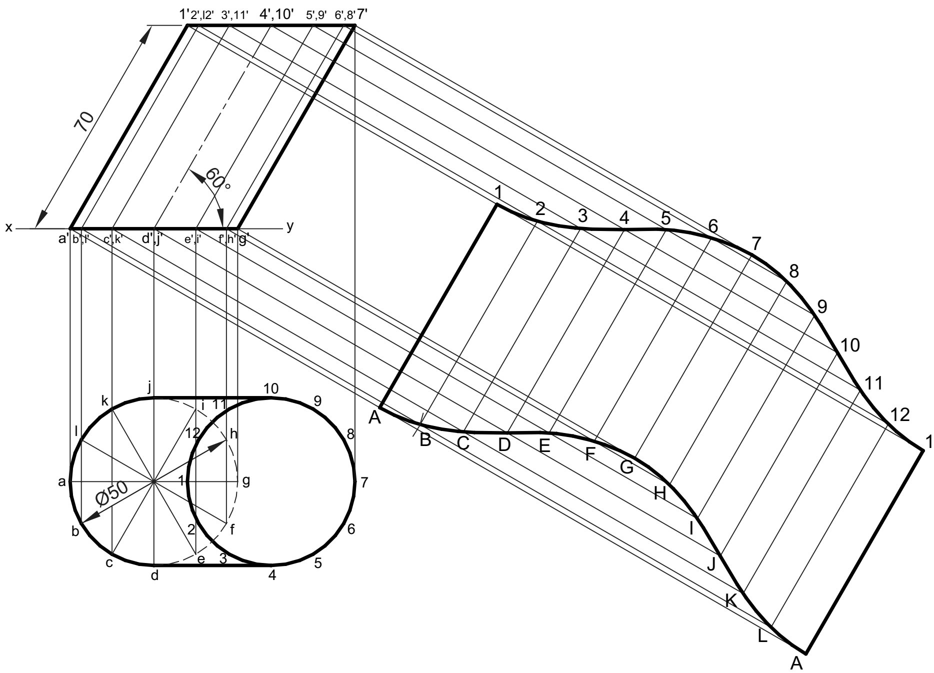


Fig. 13.36

Construction Refer to Fig. 13.36.

1. Draw a circle *adgj* to represent top view of the lower base. Project to obtain $a'g'$. Draw a parallelogram $a'g'7'1'$ to represent the front view. Now project the top base $1'7'$ to obtain its corresponding top view. Complete the front and top views and label the generators as shown.
2. Consider the seam at $1'a'$. Stretch out lines $a'A$, $b'B$, $c'C$, $d'D$, etc., and $1'1$, $2'2$, $3'3$, $4'4$, etc., perpendicular to the generators of the front view.
3. Draw *A*1 parallel to *a*1. Starting from point *A*, step off arcs of radius equal to 1/12 of the circumference of the base circle, to meet the stretch out lines in succession at points *B*, *C*, *D*, etc. Join *ABCDEFGHIJKL* with smooth curve.
4. Similarly, starting from point 1, step off arcs of radius 1/12 of the circumference of the base circle, to meet the stretch out lines in succession at points 2, 3, 4, etc. Join all the points with smooth curve. Join *A*1 to represent the required development.

Problem 13.34 *An oblique hexagonal pyramid of base side 30 mm has 75 mm long axis inclined at 60° to the base. The pyramid is resting on its base on the H.P. such that axis is parallel to the V.P. and an edge of the base is perpendicular to the V.P. Draw the development of its lateral surface.*

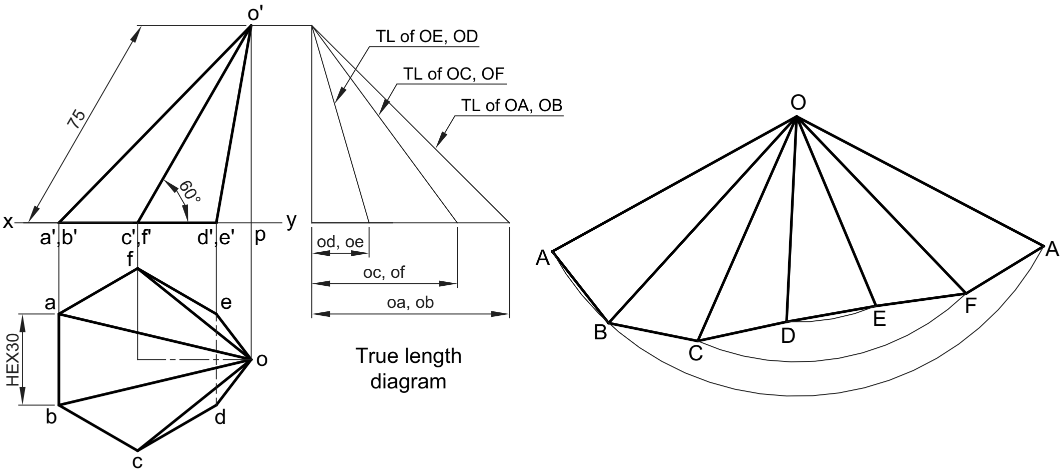


Fig. 13.37

Construction Refer to Fig. 13.37.

1. Draw a hexagon *abcdef* keeping *ab* perpendicular *xy*. Project all the corners to obtain *a′e′* on *xy*. Draw a triangle *a′e′o′* keeping 75 mm long axis inclined at 60° to *xy*. This represents the front view. Project apex *o′* to meet the horizontal line through the centre of the hexagon at point *o*. Join all the slant edges in the top view.
2. Draw a triangle of height *o′p* and base length *oa*, *ob*, *oc*, *od*, *oe* and *of* to obtain true length of *OA*, *OB*, *OC*, *OD*, *OE* and *OF* respectively.
3. Consider the seam at *o′a′*. Now draw triangles *OAB* taking *OA* and *OB* from true length diagram and base *AB* 30 mm long. Proceed to draw triangles *OBC*, *OCD*, *ODE*, *OEF* and *OFA* in the similar manner to obtain the required development.

Problem 13.35 *An oblique cone of base diameter 50 mm has 70 mm long axis inclined at 45° to the base. The cone is resting on its base on the H.P. Draw the development of its lateral surface.*

Construction Refer to Fig. 13.38.

1. Draw a circle *adgj*. Project the circle to obtain *a′g′* on *xy*. Draw a triangle *a′g′o′* keeping 75 mm long axis inclined at 45° to *xy*. This represents the front view. Project the apex *o′* to meet the horizontal line through the centre of the circle at point *o*. Draw tangential lines from o to meet the circle. This represents the top view.
2. Draw a triangle of height *o′p* and base length *oa*, *ob*, *oc*, *od*, etc., to obtain true length of *OA*, *OB*, *OC*, *OD*, etc., respectively.
3. Consider the seam at *o′a′*. Draw a line *OA* equal to its true length.
4. Locate point *B* such that *OB* = *TL* of *OB* and *AB* = length of arc *ab* in the top view.
5. Proceed to, locate points *C*, *D*, *E*, etc., and join them to obtain the required development.

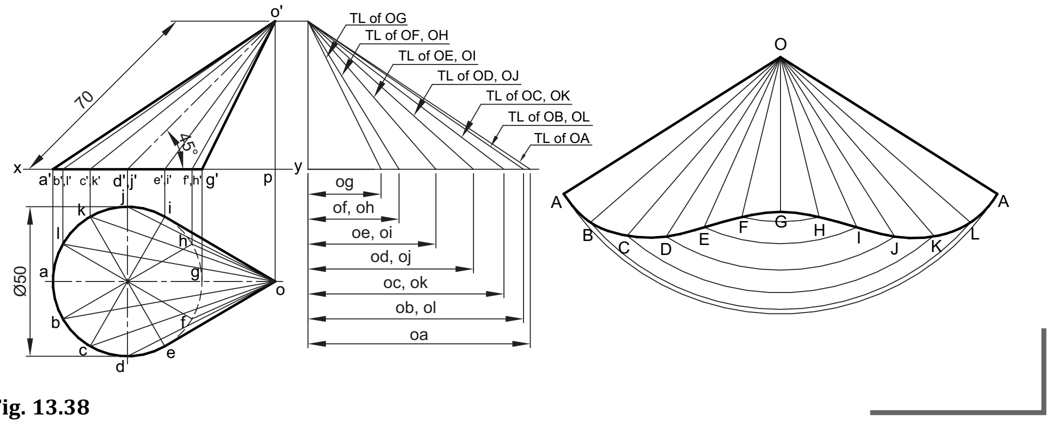


Fig. 13.38

13.12 APPLICATIONS

As discussed earlier, a large number of objects such as funnel, milk can, pipe joint, hopper, etc., are made of metal sheets. The fabrication of these objects can be planned in an economic way if the accurate shape and size of metal sheet is known. The following problems illustrate the development of some of these objects.

Problem 13.36 *A vertical funnel is made by joining a frustum of cone of top diameter 60 mm, base diameter 30 mm and axis 25 mm long with another frustum of cone of top diameter 30 mm, base diameter 15 mm and axis 50 mm long. The lowest end of the funnel is bevelled off by an auxiliary inclined plane, inclined at 30° to the H.P. Draw the development of the funnel.*

Construction Refer to Fig. 13.39.

1. Draw the front view of the funnel.
2. Full top view is not essential. Just draw a semi-circle and divide it into six parts. Project the points to the front view and obtain generators.

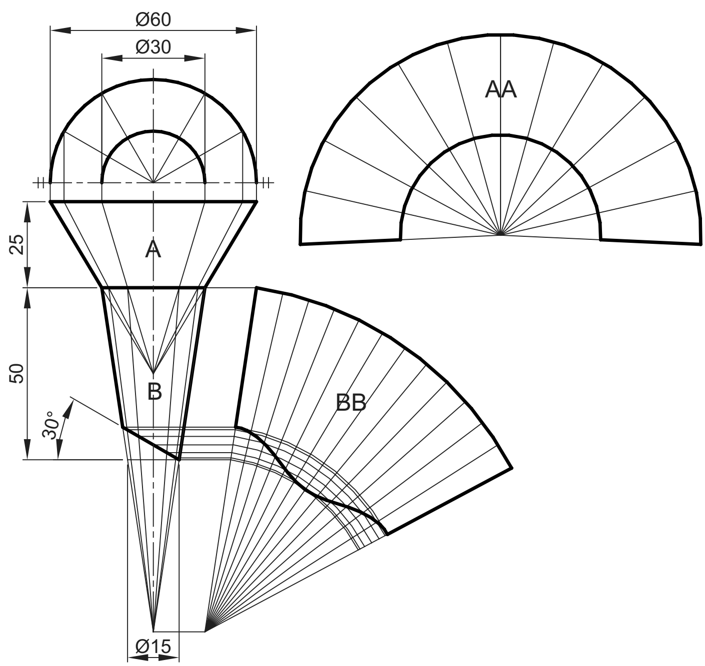


Fig. 13.39

3. A keen observation shows that the funnel is formed from two pieces. Part *A* is the frustum of a cone whose development is shown by *AA*.
4. Part *B* is a truncated cone whose development is shown by *BB*. These two developed pieces *AA* and *BB* represents the required development of the funnel.

Problem 13.37 *Draw the shape of metal sheet required to prepare a milk-can whose front view is shown in Fig. 13.40(a).*

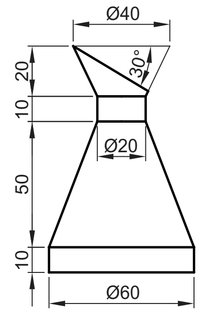


Fig. 13.40(a)

Construction Refer to Fig. 13.40(b).

1. Draw the front view of the given milk can. Divide the front view in four parts namely *A*, *B*, *C* and *D*.
2. Part *A* is a cylinder and its development is a rectangle of length 188 mm shown by *AA*.
3. Part *B* is a frustum of a cone. Determine the slant height of the cone as 81 mm the subtended angle as 134°. Draw its development as *BB*.
4. Part *C* is cylinder and its development is a rectangle of length 63 mm shown by *CC*.
5. Part *D* is a frustum of a cone truncated at its base. To mark generators, draw a semicircle of 40 mm diameter. Divide the semi-circle into six equal parts and project them to the front view. Follow the construction of Problem 13.15 and obtain *DD* as its development.

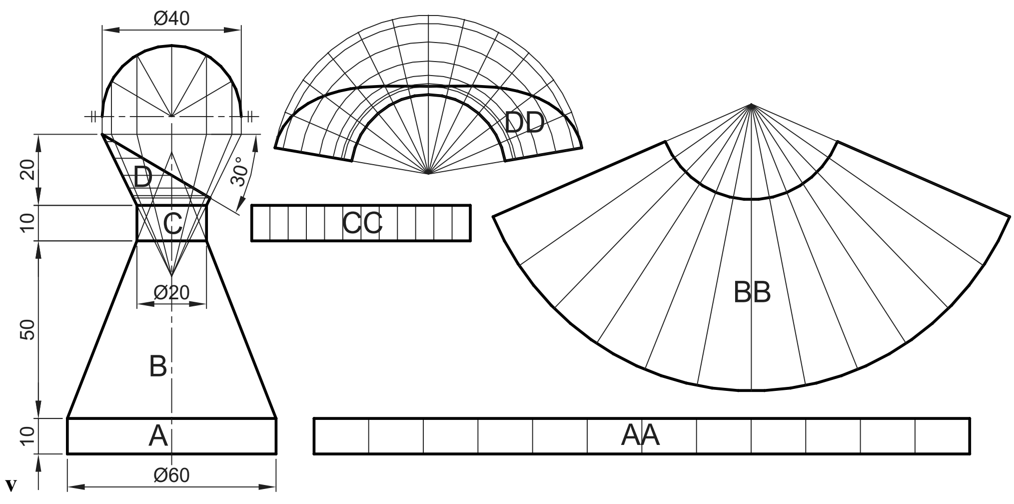


Fig. 13.40(b)

Problem 13.38 *Draw the shape of metal sheet required to prepare a pipe joint whose front view is shown in Fig. 13.41(a).*

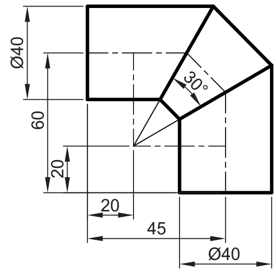


Fig. 13.41(a)

Construction Refer to Fig. 13.41(b).

1. Draw the front view of the given pipe joint. Divide the front view in three parts namely *A*, *B* and *C*.
2. Part *A* is a truncated cylinder. To mark generators, draw a semicircle of 40 mm diameter. Divide the semi-circle into six equal six parts and project them to the front view. Follow the construction of Problem 13.7 and obtain AA as its development.
3. Part *B* is also a cylinder truncated at both ends. Obtain its development as *BB*.
4. Parts *C* is similar to part *A*. So its development is also be similar to *AA*.

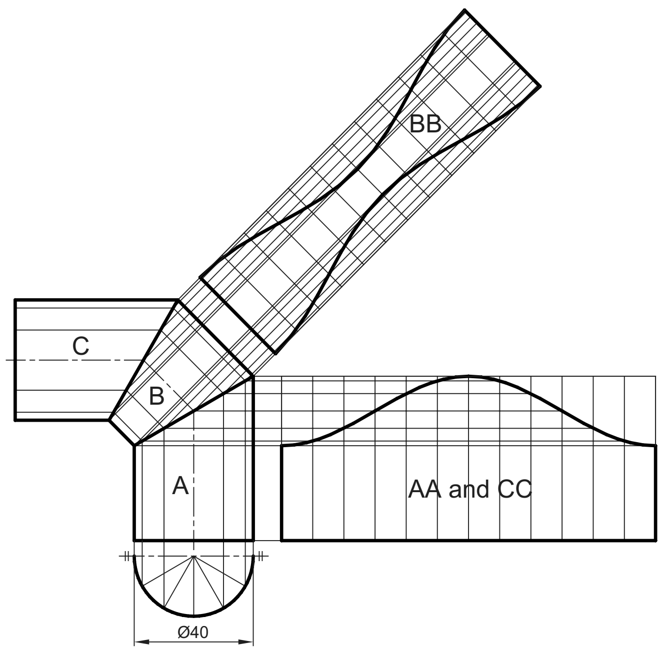


Fig. 13.41(b)

Problem 13.39 *Draw the shape of metal sheet required to prepare a hopper whose front view is shown in Fig. 13.42(a).*

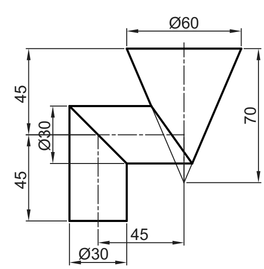


Fig. 13.42(a)

Construction Refer to Fig. 13.42(b).

1. Draw the front view of the given hopper. Divide the front view in three parts namely *A*, *B* and *C*.
2. Part *A* is a truncated cylinder. To mark generators, draw a semicircle of 30 mm diameter. Divide the semi-circle into six equal six parts and

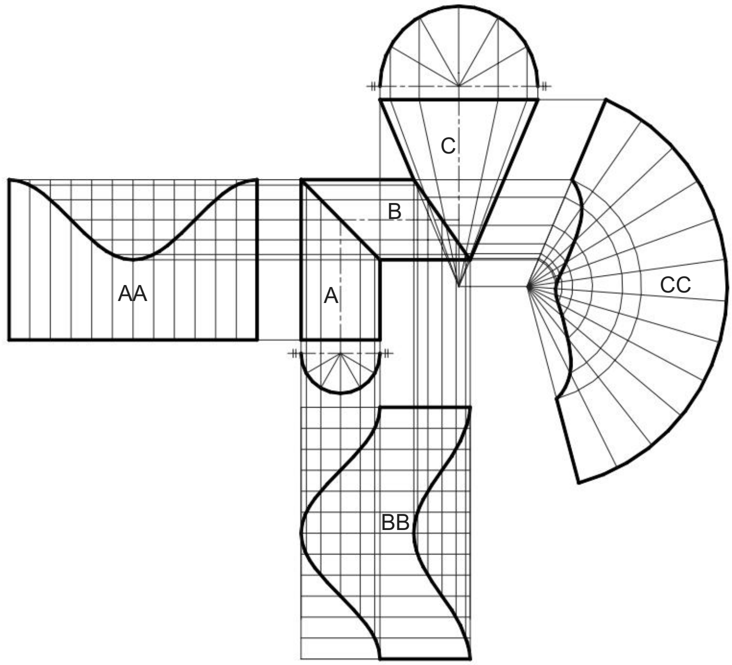


Fig. 13.42(b)

project them to the front view. Follow the construction of Problem 13.7 and obtain *AA* as its development.

3. Part *B* is also a cylinder truncated at both ends. Obtain its development as *BB*.
4. Part *C* is a truncated cone. To mark generators, draw a semicircle of 60 mm diameter. Divide the semi-circle into six equal parts and project them to the front view. Follow the construction of Problem 13.13 and obtain CC as its development.

EXERCISE 13A

Prisms and cylinders

13.1 A hexagonal prism of base side 25 mm and axis 50 mm is resting on its base on the H.P. with a rectangular face parallel to the V.P. Draw the development of the prism.

13.2 A pentagonal prism of base side 30 mm and axis 60 mm is resting on its base on the H.P. with a rectangular face parallel to the V.P. It is cut by a section plane perpendicular to the V.P., inclined at 30° to the H.P. and passing through a point on the axis, 25 mm from one of the bases. Draw the development of its lateral surface.

13.3 A square prism of base side 40 mm and axis 60 mm stands on its base on the H.P. with vertical faces equally inclined to V.P. A circular hole of 50 mm diameter is drilled centrally through the prism such that the axis of the hole is perpendicular to V.P. Draw the development of the lateral surface of the prism.

13.4 A hexagonal prism of base side 30 mm and axis 70 mm rests on its base on the H.P. with a vertical face parallel to V.P. A circular hole of diameter 50 mm is drilled centrally through the prism such that the axis of the hole and the axis of the prism are perpendicular bisectors of each other. Draw the development of the lateral surface of the retained prism.

13.5 A cylinder of base diameter 50 mm and axis 60 mm is resting on its base on the H.P. It is cut by a section plane perpendicular to V.P., the V.T. of which cuts the axis at a point 40 mm from the bottom face and inclined at 45° to the reference line. Draw the development of its lateral surface.

13.6 A cylinder of base diameter 50 mm and axis 70 mm is resting on its base on the H.P. A circular hole of diameter 25 mm is drilled through the cylinder such that the axes of the hole and the cylinder are the perpendicular bisectors. Draw the development of its lateral surface.

13.7 A cylinder of base diameter 50 mm and axis 70 mm contains a square hole of 30 mm side. The faces of the square hole are equally inclined to the base of the cylinder and the axis of the hole bisects the axis of the cylinder at right angle. Draw the development of its lateral surface.

13.8 A hexagonal hole of side 25 mm is cut in a cylindrical drum of diameter 50 mm and height 70 mm. One of the faces of the hole is perpendicular to the H.P. and the axis intersects with that of the drum at right angles. Draw the development of its lateral surface.

Cones and pyramids

13.9 A cone of base diameter 50 mm and axis 60 mm is resting on its base on the H.P. It is cut by a section plane perpendicular to the V.P. and inclined at 30° to the H.P. such that the H.T. is tangential to the base circle. Draw the development of its lateral surface.

13.10 A right cone of base diameter 50 mm and axis 60 mm is resting on its base on the H.P. It is cut by an auxiliary inclined plane parallel to and 10 mm away from the extreme generator. Draw the development of the lateral surface of the remaining solid.

13.11 An isosceles triangle of base 50 mm and altitude 60 mm with a circular hole of diameter 25 mm at a height of 20 mm from the base is the front view of a truncated cone. Draw the development of its lateral surface.

13.12 A cone of base diameter 50 mm and axis 60 mm rests on its base on the H.P. A section plane parallel to the V.P. cuts the cone at a distance of 10 mm from the axis. Draw the sectional front view and develop the lateral surface.

13.13 A square pyramid of base side 40 mm and axis 60 mm is resting on its base on the H.P. such that all the sides of the base are equally inclined to the V.P. It is cut by a section plane perpendicular to the V.P. and inclined at 45° to the H.P., bisecting the axis. Draw the development of its lateral surface.

13.14 A square pyramid of base side 40 mm and axis 60 mm is resting on its base on the H.P. such that a side of the base is parallel to the V.P. It is cut by a section plane perpendicular to the V.P. and inclined at 60° to the H.P., bisecting the axis. Draw the development of its lateral surface.

13.15 A hexagonal pyramid of base side 40 mm and axis 60 mm is resting on its base on the H.P. with an edge of the base perpendicular to the V.P. It is cut by a an auxiliary inclined plane whose V.T. is inclined at 60° to the H.P. bisecting the axis. Draw the development of its lateral surface.

13.16 A square pyramid, base side 30 mm and axis 60 mm is resting on its base on the H.P. such that all the sides of the base are equally inclined to the V.P. A square hole of side 20 mm is drilled through it such that axis of the hole and the pyramid intersect at right angle and 20 mm above the base. Draw the development of its lateral surface.

13.17 A square pyramid, base side 30 mm and axis 70 mm is resting on its base on the H.P. such that all the sides of the base are equally inclined to the V.P. A circular hole of diameter 30 mm is cut through the faces of the pyramid such that axis of the hole and pyramid intersect at right angle and 20 mm above the base. Draw the development of its lateral surface.

13.18 A pentagonal pyramid edge of base side 30 mm and axis 60 mm, is resting on its base in the H.P. such that an edge of the base is parallel and nearer the V.P. An auxiliary vertical plane, whose H.T. is inclined at 45° to the V.P., cuts the pyramid at a distance of 8 mm from the axis. Draw the sectional front view and develop the lateral surface of the retained pyramid.

13.19 A tetrahedron of side 50 mm is resting on one of its faces on the ground with an edge of the resting face parallel to the V.P. Draw the projections and the development of the tetrahedron.

Spheres

13.20 Draw development of the surface of a hemisphere of diameter 60 mm by zone method.

13.21 Draw development of the surface of a sphere of diameter 60 mm by Lune method.

Transition pieces and tray

13.22 A 70 mm long transition piece connects a square opening of side 25 mm with a coaxial rectangular opening of sides 80 mm and 60 mm. The transition piece is placed on its rectangular base on the H.P. with its longer edge parallel to the V.P. and the square opening has its edges equally inclined to V.P. Draw the development of its lateral surface.

13.23 Draw the development of a 75 mm long transition piece that connects a square section of side 70 mm with a coaxial hexagonal section of side 25 mm. The transition piece rests on its square base on the H.P. and one side of each section is parallel to the V.P.

13.24 Draw the development of a 60 mm long transition piece connecting a circular section of diameter 40 mm with a coaxial hexagonal section of side 40 mm. The transition piece rests on its hexagonal section in the H.P. with an edge perpendicular to the V.P.

13.25 Draw the development of a 45 mm long transition piece which connects a cylindrical pipe of diameter 35 mm with a coaxial duct of square section of side 50 mm. The transition piece rests on its square section on the H.P. such that an edge of square section is parallel to the V.P.

13.26 A tray is made of sheet metal whose orthographic views are shown in Fig. E13.1. Draw the development of the surface of the entire tray.

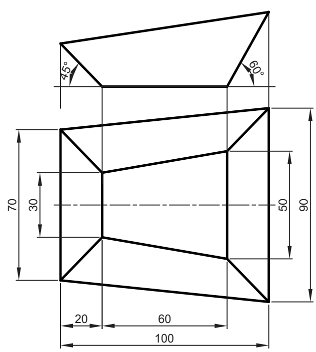


Fig. E13.1

Oblique objects

13.27 An oblique square prism of base side 30 mm has 60 mm long axis inclined at 75° to the base. The prism rests on its base on the H.P. such that axis and an edge of the base are parallel to the V.P. Draw the development of its lateral surface.

13.28 An oblique cylinder of base diameter 50 mm has 60 mm long axis inclined at 75° to the base. The cylinder, rests on its base on the H.P. Draw the development of its lateral surface.

13.29 An oblique pentagonal pyramid of base side 30 mm has 70 mm long axis inclined at 75° to the base. The pyramid is resting on its base on the H.P. such that axis and an edge of the base are parallel to the V.P. Draw the development of its lateral surface.

13.30 An oblique cone of base diameter 50 mm has 70 mm long axis is inclined at 60° to the base. The cone is resting on its base on the H.P. Draw the development of its lateral surface.

Applications

13.31 Draw the development of cylindrical steel chimney erected on a roof *ABC* as shown in Fig. E13.2.

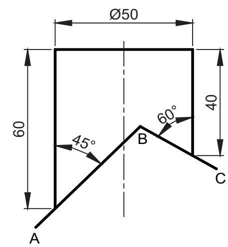


Fig. E13.2

13.32 Draw the shape of metal sheet required to prepare a milk-can whose front view is shown in Fig. E13.3.

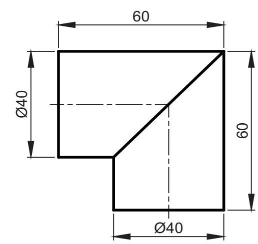


Fig. E13.3

13.33 Draw the shape of metal sheet required to prepare a pipe joint whose front view is shown in Fig. E13.4.

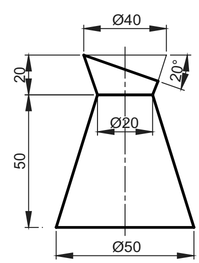


Fig. E13.4

13.13 ANTI-DEVELOPMENT

The reverse of a development is called anti-development. Here the front and top views of an object are obtained when its development is given. Consider the following problems.

Problem 13.40 *A pentagonal prism of base side 30 mm and axis 75 mm stands on its base on the ground such that one of its rectangular faces is parallel to and nearer the V.P. A thread is wound around the prism, starting from the corner of the lower base farthest away from the V.P. to the corresponding corner of the upper base. Find the minimum length of the thread and show it on the front view of the prism.*

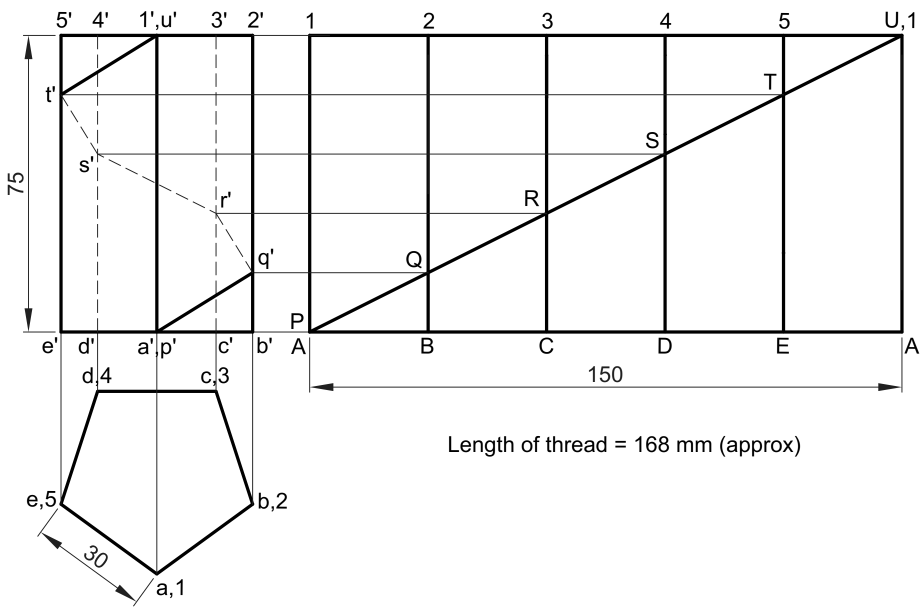


Fig. 13.43

Construction Refer to Fig. 13.43.

1. Draw a pentagon *abcde* keeping *cd* parallel to *xy* to represent the top view. Project all the points to obtain rectangle $e'b'2'f'$ as the front view.
2. Stretch out lines 1-1 and *A*-*A* from the front view, equal to the perimeter of the base. Mark vertical edges 1*A*, 2*B*, 3*C*, 4*D* and 5*E* to complete the development of the prism.
3. Represent the thread by a straight line *PU*. Let it cut the vertical edges *B*2 at *Q*, *C*3 at *R*, *D*4 at *S* and *E*5 at *T*.
4. Draw horizontal lines from *P*, *Q*, *R*, *S*, *T* and *U* to meet $a'1'$ at p', $b'2'$ at q', $c'3'$ at r', $d'4'$ at s', $e'5'$ at t' and $a'1'$ at u'.

5. Join points $p'q'r's't'u'$ as shown. Only $p'q'$ and $t'u'$ are visible because they lie on the faces of the prism which is towards observer.

Problem 13.41 *A hexagonal prism made of metal sheet with cut marks on its faces has its development as shown in Fig. 13.44(a). Draw the projections of the hexagonal prism formed by wrapping the metal sheet such that the axis is perpendicular to the H.P. and a face is parallel to the V.P. Also, show cut marks in the front view.*

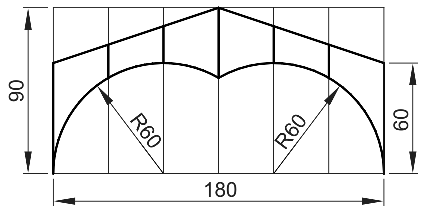


Fig. 13.44(a)

Visualization Wrapping of the metal sheet will result into a hexagonal prism of base side 30 mm and axis 90 mm.

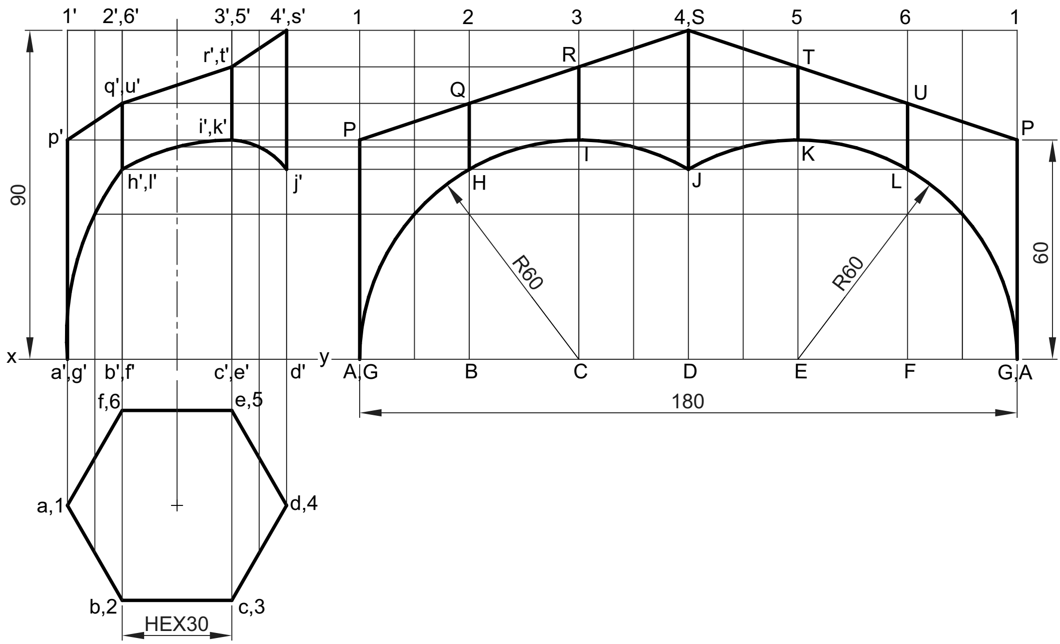


Fig. 13.44(b)

Construction Refer to Fig. 13.44(b).

1. Draw a hexagon *abcdef* keeping *ef* parallel to *xy* to represent the top view. Project all points to obtain $a'd'4'1'$ as the front view.
2. Stretch out lines 1-1 and *A-A* from the front view equal to the perimeter of the base. Draw 1*A*, 2*B*, 3*C*, 4*D*, 5*E* and 6*F* to complete the development of the uncut prism.
3. Draw given cut lines on the development. Let the intersection of cut marks with vertical edges occurs at points *P*, *Q*, *R*, *S*, *T*, *U*, *P* and *G*, *H*, *I*, *J*, *K*, *L*, *G*.
4. Draw horizontal lines from points *P*, *Q*, *R*, *S*, *T*, *U* and *G*, *H*, *I*, *J*, *K*, *L* to meet the corresponding edges in the front view at points p', q', r', s', t', u' and g', h', i', j', k', l' respectively.

5. Addition of generators on the development and the faces will help to define the cut profile in a better manner. Join the points as shown to obtain the required projections.

Problem 13.42 *A semi-circle of 150 mm diameter with a largest circle inscribed in it, is the development of the lateral surface of a cylinder. Draw the projections of the cylinder with the corresponding circle marked on it.*

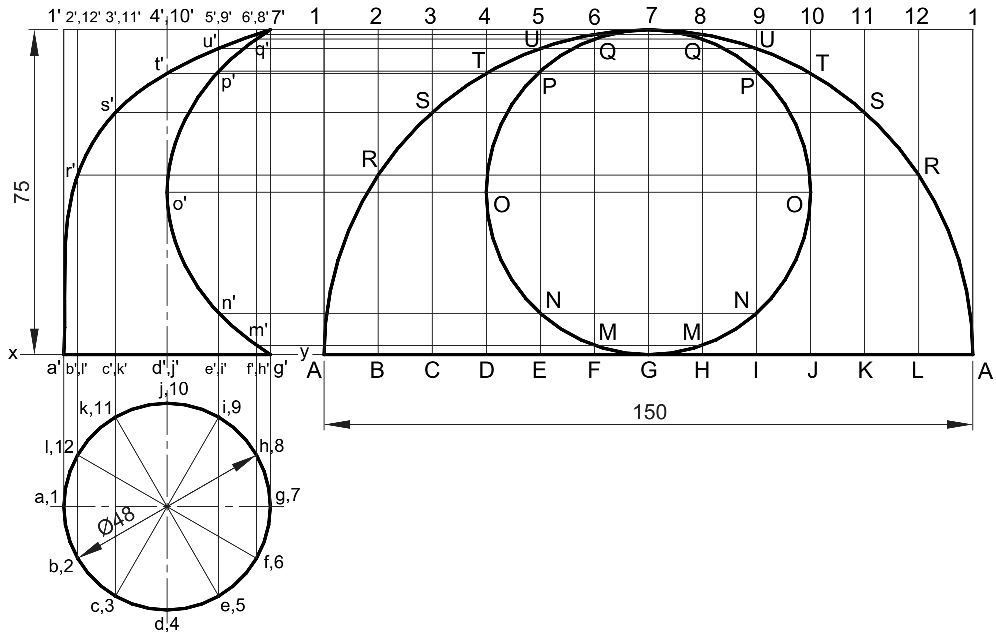


Fig. 13.45

Visualization The rectangle to enclose a semi-circle of 150 mm diameter should be atleast 150 mm long and 75 mm high. Wrapping the rectangle will result into a cylinder of base diameter 48 mm and axis 75 mm.

Construction Refer to Fig. 13.45.

1. Draw a circle *adgj* of diameter 48 mm to represent the top view and divide it into 12 equal parts. Project all the points to obtain $a'g'7'1'$ as the front view.
2. Consider seam at $1'a'$. Stretch out lines 1-1 and *A-A* from the front view equal to the perimeter of the cylinder (150 mm). Divide 1-1 and *A-A* into 12 equal parts and join them to represent generators.
3. Draw a semi-circle with centre *G* and radius 75 mm. Also, draw a circle of diameter *G*7, 75 mm long. This represents the required development.
4. Let the generators meet the semicircle at points *A*, *R*, *S*, *T*, *U*, *V*, 7, and with the circle at *G*, *M*, *N*, *O*, *P*, *Q*, 7. Draw horizontal lines from points *R*, *S*, *T*, *U*, *V* and *M*, *N*, *O*, *P*, *Q,* to meet the corresponding generators in the front view at points r', s', t', u', v', etc., and m', n', o', p', q', etc.
5. Join $a'r's't'u'v'7'$ and $g'm'n'o'p'q'7'$ by smooth curves as shown and obtain the required front view.

Problem 13.43 *A parabola of 150 mm base and 72 mm height is just enclosed in a thin rectangular metal sheet which is then wrapped to form a cylinder. Draw the projections of the cylinder with the corresponding parabola marked on it.*

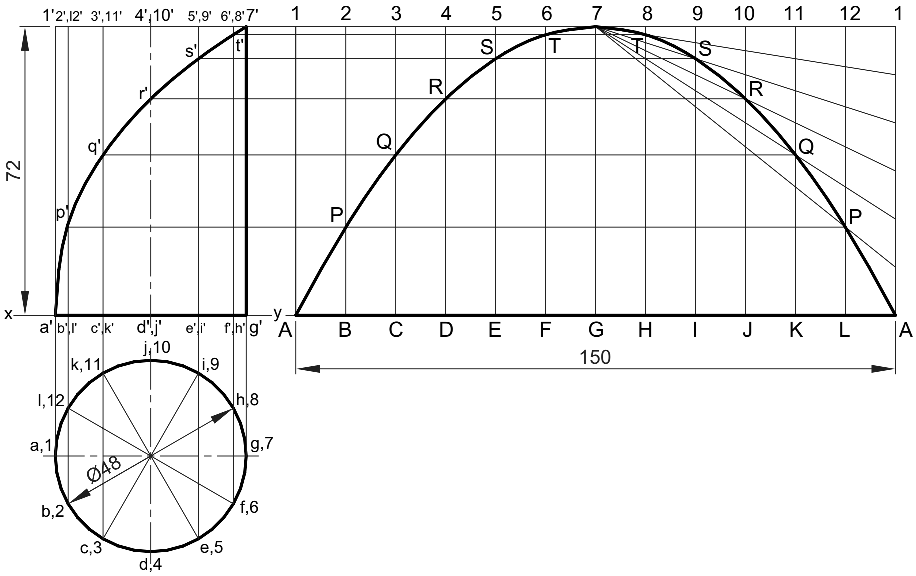


Fig. 13.46

Visualization The rectangle to enclose the parabola should be atleast 150 mm long and 72 mm high. Wrapping the rectangle will result into a cylinder of base diameter 48 mm and axis 72 mm.

Construction Refer to Fig. 13.46.

1. Draw a circle *adgj* of diameter 48 mm to represent the top view and divide it into 12 equal parts. Project all the points to obtain $a'g'7'1'$ as the front view.
2. Consider seam at $1'a'$. Stretch out lines 1-1 and *A-A* from the front view equal to the perimeter of the cylinder (150 mm). Divide 1-1 and *A-A* into 12 equal parts and join them to represent generators.
3. Inscribe a parabola in the development (refer Problem 5.12). For this, divide *A*1 into 6 equal parts. Join the parts with point 7 to intersect generators at points *P*, *Q*, *R*, *S* and *T*. Join *APQRST*7.
4. Draw horizontal lines from points *P*, *Q*, *R*, *S* and *T* to meet the corresponding generators in the front view at points p', q', r', s' and t'.
5. Join $a'q'r's't'7'$ by smooth curve as shown to represent the parabola in the front view.

Problem 13.44 *A rhombus with largest side is made in a thin rectangular plate of sides 150 mm and 70 mm. The plate is then wrapped to form a cylinder. Draw the projections of the cylinder with the rhombus marked on it.*

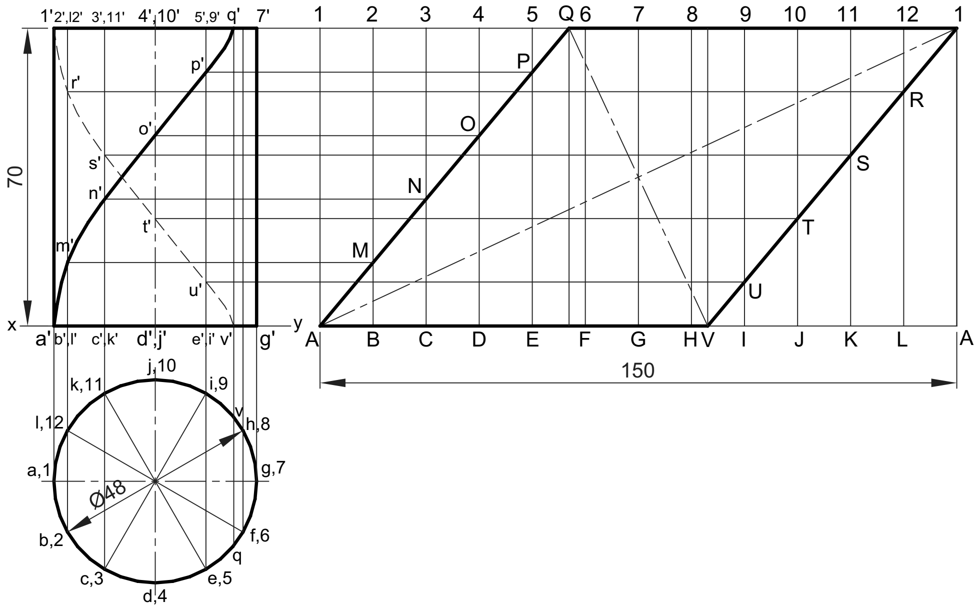


Fig. 13.47

Visualization A rhombus with largest side will have largest diagonals. Wrapping the rectangle will result into a cylinder of base diameter 48 mm and axis 70 mm.

Construction Refer to Fig. 13.47.

1. Draw a circle *adgj* of diameter 48 mm to represent the top view and divide it into 12 equal parts. Project all the points to obtain $a'g'7'1'$ as the front view.
2. Consider seam at $1'a'$. Stretch out lines 1-1 and *A-A* from the front view equal to the perimeter of the cylinder (150 mm). Divide 1-1 and *A-A* into 12 equal parts and join them to represent generators.
3. Join *A*-1 to represent one of the diagonals of the rhombus. Draw *QV* as the perpendicular bisector of diagonal *A*-1, meeting 1-1 at point *Q* and *A-A* at point *V*. Join *AQ1V* to represent the largest rhombus.
4. Let the generators meet the sides of the rhombus at points *M*, *N*, *O*, *P* and *R*, *S*, *T*, *U*. Draw horizontal lines from points *M*, *N*, *O*, *P*, *R*, *S*, *T* and *U* to meet the corresponding generators in the front view at points m', n', o', p', q', r', s', t' and u'.
5. Mark points q and v on the top view such that $q6 = Q6$ and $hv = HV$. Project q and v to meet bases in the front view at points q' and v'.
6. Join $a'm'n'o'p'q'$ and $1'r's't'u'v'$ by smooth curves as shown to represent the rhombus in the front view.

Problem 13.45 *A cone of base diameter 60 mm and axis 75 mm is resting on its base on the H.P. Draw the projections of the cone and show on it, the shortest path traced by a point, start moving from a point on the circumference of the base, moving around the cone and returns to the same point.*

Calculation of θ

Slant height of cone

$$R = o'g' = \sqrt{r^2 + h^2} = \sqrt{30^2 + 75^2} = 81 \text{ mm}$$

Subtended angle

$$\theta = \frac{r}{R} \times 360^{\circ} = \frac{25}{81} \times 360^{\circ} = 134^{\circ}$$

Construction Refer to Fig. 13.48.

1. Draw a circle *adgj* to represent the top view. Divide it into 12 equal parts. Project all the points and obtain $a'o'g'$ as the front view. Consider seam at $o'a'$.
2. Determine the subtended angle θ as 134°. Draw a sector *A-O-A* with included angle θ. Divide sector into 12 equal parts and mark the generators as *OB*, *OC*, *OD*, etc.
3. Join *A-A* on the development to represent the shortest path. Let the line meets the generators at points *P*, *Q*, *R*, *S*, *T*, *U* and *V* as shown.
4. Draw arcs with centre *O* and radii *OQ*, *OR*, *OS*, *OT*, *OU* and *OV* to meet line *OA* at points q'', r'', s'', t'', u'' and v''. Draw horizontal lines from q'', r'', s'', t'', u'' and v'' to meet their corresponding generators at points q', r', s', t', u' and v'. Join $p'q'r's't'u'v'$ to represent the shortest path in the front view.
5. Project points q', r', t', u' and v' on the respective generators in the top view to obtain points *q*, *r*, *t*, *u* and *v*. Draw horizontal line from point s' to meet $o'g'$ at s_1'. Project s_1' to meet *og* at point s_1. Draw an arc with centre *o* and radius os_1 to meet *od* and *oj* at point *s*.
6. Join *pqrstuvutsrqp* by a smooth curve to represent the shortest path in the top view.

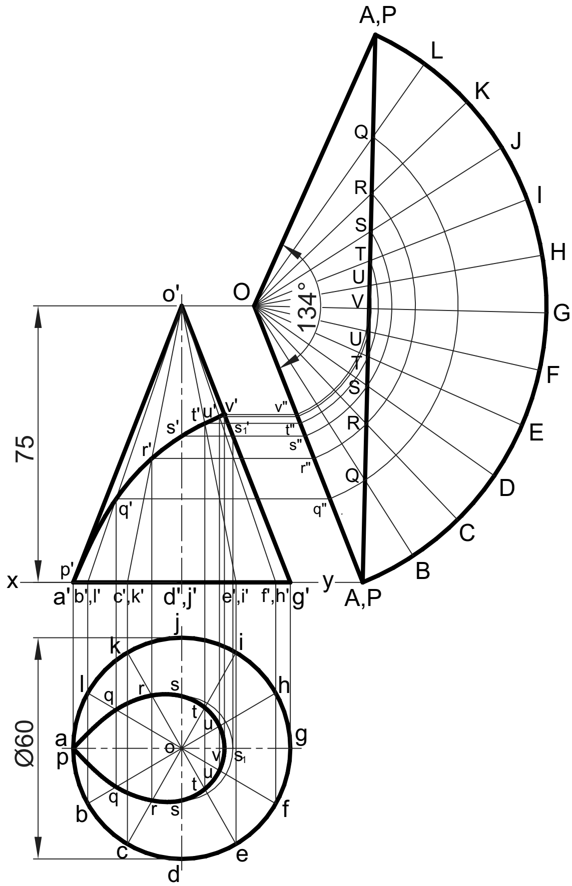


Fig. 13.48

Problem 13.46 *A circular hole of largest size is made in a semi-circular metal sheet of diameter 100 mm. The sheet is wrapped to form a cone resting on its base on the H.P. Draw the projections of the cone with the circle marked on it.*

Radius of base circle of the cone $r = \text{Slant height} \times \frac{\theta}{360} = 50 \times \frac{180^{\circ}}{360^{\circ}} = 25$ mm

Visualization Wrapping a semi-circle of 100 mm diameter will result into a cone of base diameter 50 mm and generator 50 mm.

Construction Refer to Fig. 13.49.

1. Draw a circle *adgj* of 50 mm diameter to represent the top view. Divide it into 12 equal parts. Project all the points to obtain $a'o'g'$ as the front view, where $a'o'$ is 50 mm.
2. Consider seam at $o'a'$. Draw a semicircle *AOA* to represent the development of the cone. Divide it into 12 equal parts and mark the generators as *OB*, *OC*, *OD*,…, etc.
3. Draw a circle of 50 mm diameter on the development to represent the required circular hole. Mark the points of intersection of the generators with the circular hole as 1, 2, 3, etc.
4. Draw arcs with centre *O* and radii *O*1, *O*2, *O*3, etc., to meet *OA* at points 1″, 2″, 3″, etc. Draw horizontal lines from 1″, 2″, 3″, etc., to meet their corresponding generators in the front view at points 1′, 2′, 3′, etc. Join the points to obtain the mark of the circular hole in the front view.
5. Project points 1′, 2′, 4′, 5′, 6′, 7′, 8′, 10′ and 11′ to meet their respective generators in the top view at points 1, 2, 4, 5, 6, 7, 8, 10 and 11.
6. Draw horizontal line points 3′ and 9′ to meet $o'a'$ at point p'. Project p' to meet *oa* at point *p*. Draw an arc with centre *o* and radius *op* to meet *od* and *oj* at points 3 and 9 respectively.
7. Join all the points in the top view as shown to represent the circular hole.

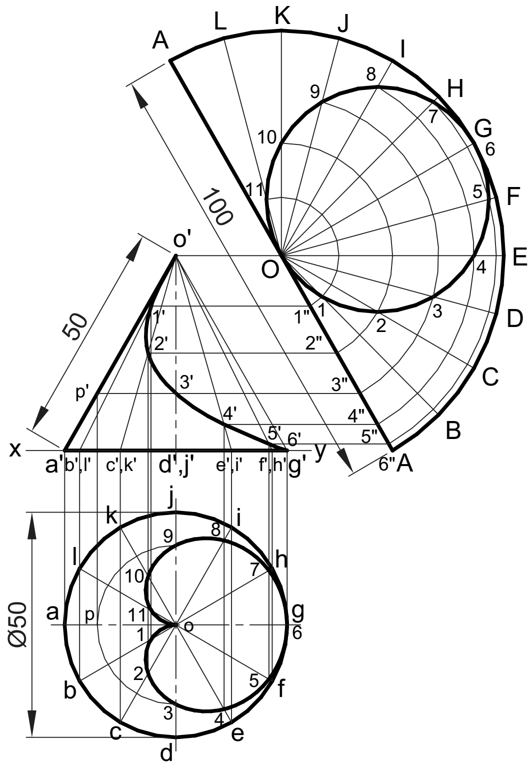


Fig. 13.49

Problem 13.47 *A square hole is made in a semi-circular plate of diameter 100 mm. The centreline of the semi-circle is one of the diagonals of the square hole. The plate is folded to form a cone resting on its base on the H.P. Draw the three views of the cone with the square hole marked on it.*

Radius of base circle of the cone $r = \text{Slant height} \times \dfrac{\theta}{360} = 50 \times \dfrac{180^\circ}{360^\circ} = 25\text{ mm}$

Visualization Wrapping a semi-circle of 100 mm diameter will result into a cone of base diameter 50 mm and generator 50 mm.

Construction Refer to Fig. 13.50.

1. Draw a circle *adgj* of 50 mm diameter to represent the top view. Divide it into 12 equal parts. Project all the points to obtain $a'o'g'$ as the front view, where $a'o'$ is 50 mm.

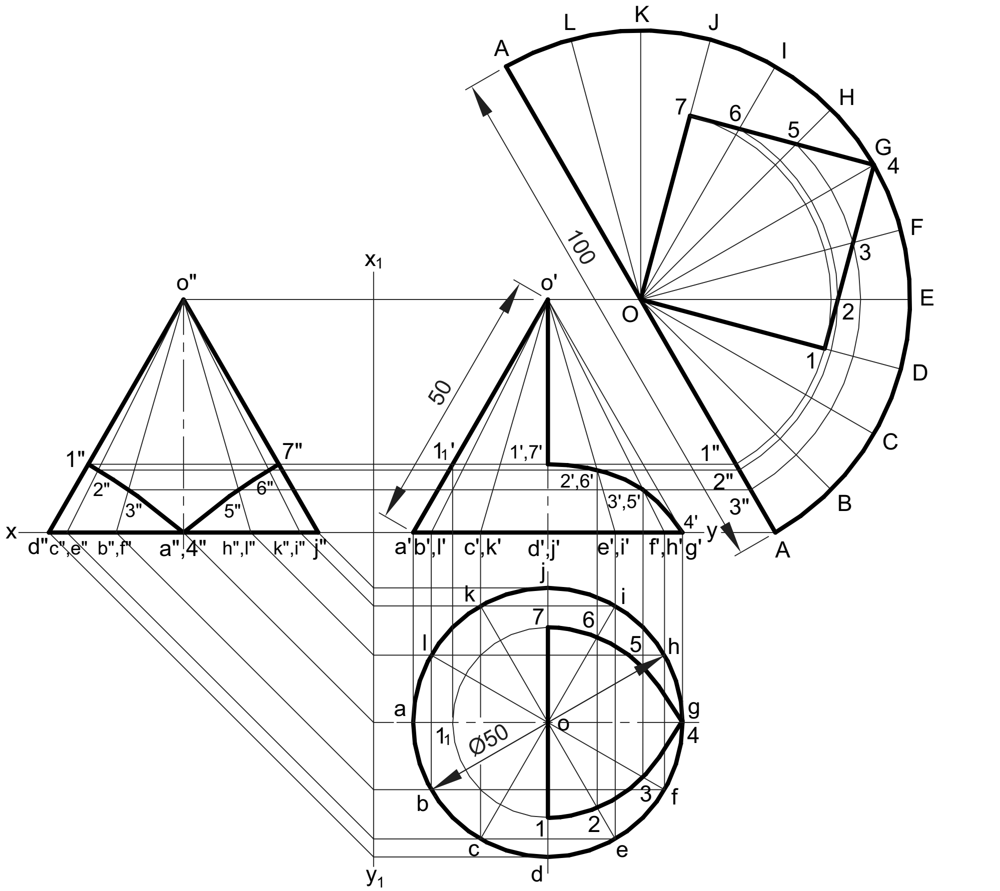


Fig. 13.50

2. Project the front and the top views to obtain $d''o''j''$ as the side view.
3. Consider seam at $o'a'$. Draw a semicircle AOA to represent the development of the cone. Divide it into 12 equal parts and mark the generators as OB, OC, OD, etc.
4. Draw a square 0-1-4-7 on the development such that generator OG represents one of the diagonals of the square. Mark the points of intersection of the generators with sides of the square as 1, 2, 3, etc.
5. Draw arcs with centre O and radii $O1$, $O2$, $O3$, etc., to meet OA at points $1''$, $2''$, $3''$, etc. Draw horizontal lines from $1''$, $2''$, $3''$, etc., to meet their corresponding generators in the front view at points $1'$, $2'$, $3'$, etc. Join the points to obtain the mark of the square in the front view.
6. Project points $2'$, $3'$, $5'$ and $6'$ to the top view on their respective generators and mark as 2, 3, 5 and 6. Draw a horizontal line from $1'$ and $7'$ to meet $o'a'$ at point $1_1'$. Project $1_1'$ to meet oa at point 1_1. Draw an arc with centre o and radius $o1_1$ to meet od and oj at points 1 and 7 respectively.
7. Join the points 1-2-3-4 and 7-6-5-4 by smooth curves to represent the square in the top view.
8. Project the front and the top views to obtain the required side view.

Problem 13.48 *A square hole of 35 mm side is cut centrally into a sector having included angle of 135° and radius 60 mm. One of the diagonals of the hole lies on the centreline of the sector. The sector is folded to form a cone resting on its base on the H.P. Draw the projections of the cone with the square hole marked on it.*

Diameter of base circle of the cone

$$d = \text{Slant height} \times \frac{2\theta}{360} = 60 \times \frac{2 \times 135^\circ}{360^\circ} = 45 \text{ mm}$$

Visualization Wrapping a sector having included angle of 135° and radius 60 mm will result into a cone of base diameter 45 mm and generator 60 mm.

Construction Refer to Fig. 13.51.

1. Draw a circle *adgj* of 45 mm diameter to represent the top view. Divide it into 12 equal parts. Project all the points to obtain $a'o'g'$ as the front view, where $a'o'$ is 60 mm.
2. Consider seam at $o'a'$. Draw a sector *AOA* to represent the development of the cone. Divide it into 12 equal parts and mark the generators as *OB*, *OC*, *OD*, etc.
3. Draw a square 1-5-9-5 on the development such that diagonal 1-9 lies on the mid of *OG*. Mark the points of intersection of the generators with sides of the square as 1, 2, 3, etc.
4. Draw generators *OP* and *OQ* to pass through corner 5 of the square. Transfer the points to the top view and obtain *p* and *q*. Project *p* and *q* to obtain p' and q' in the front view. Join $o'p'$ and $o'q'$.

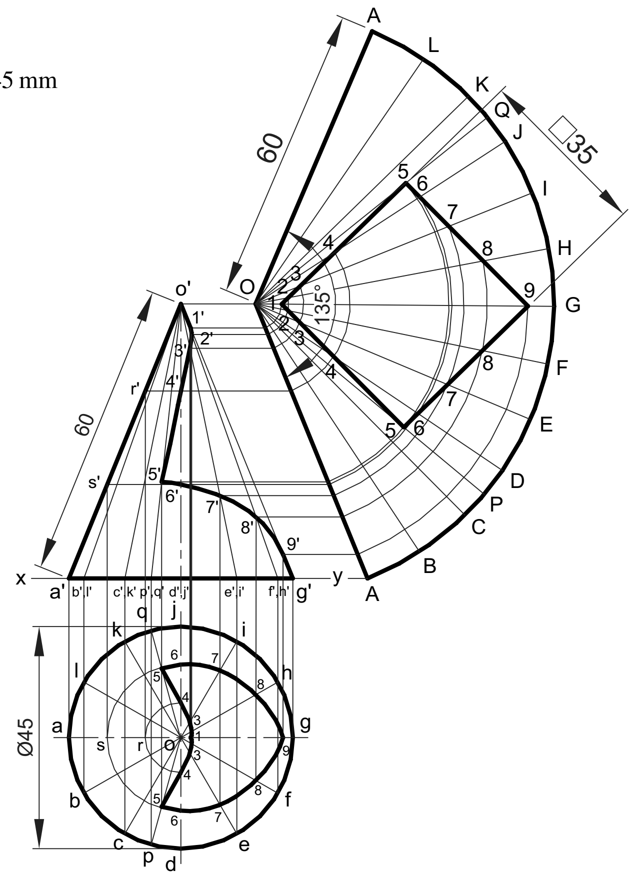


Fig. 13.51

5. Draw arcs with centre *O* and radii *O*1, *O*2, *O*3, etc., to meet *OA*. Draw horizontal lines from the points obtained on *OA* to meet their corresponding generators in the front view at points $1'$, $2'$, $3'$, etc. Join the points to obtain the mark of the square in the front view.
6. Project points $1'$ $2'$, $3'$, $5'$, $7'$, $8'$ and $9'$ to the top view on their respective generators and obtain 1, 2, 3, 5, 7, 8 and 9. Draw a horizontal line from $4'$ and $6'$ to meet $o'a'$ at point r' and s'. Project r' and s' to meet *oa* at point *r* and *s*. Draw arcs with centre *o* and radii *or* and *os* to meet *od* and *oj* at points 4 and 6 respectively.
7. Join the points 1-2-3-4-5 and 5-6-7-8-9 by smooth curves to represent the square in the top view.

Problem 13.49 *The frustum of a square pyramid has its base side 40 mm, top side 20 mm and height 60 mm. It is placed on its base on the H.P. with an edge of the base perpendicular to the V.P. A wire connects the mid-point of bottom edge of the front face to the mid-point of the top edge of the opposite face by passing over the surfaces of the frustum by the shortest distance. Draw the projections of the frustum and show path of the wire in the front view, top view and the development.*

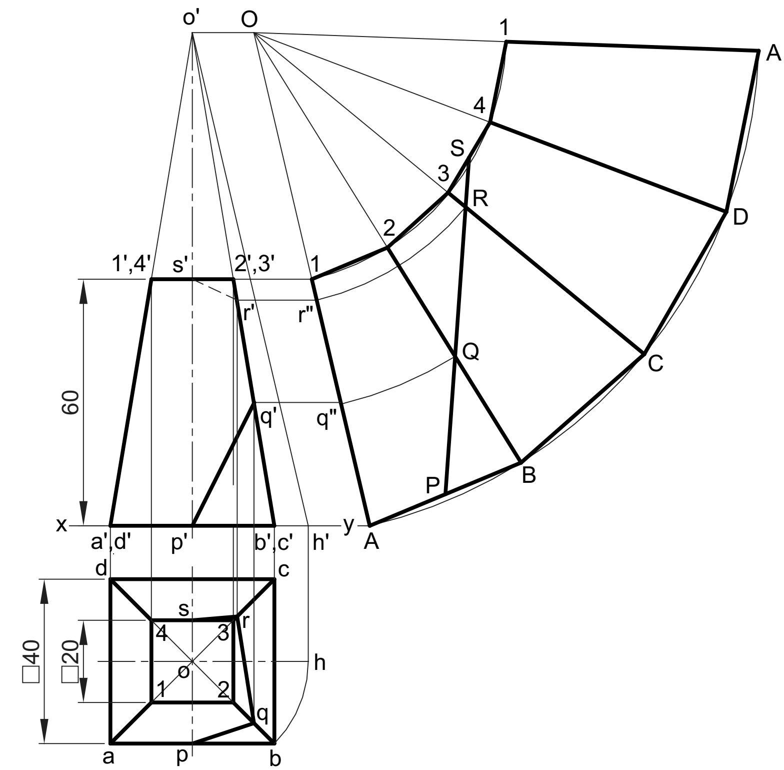


Fig. 13.52

Construction Refer to Fig. 13.52.

1. Draw squares *abcd* and 1234 keeping sides *ab* and 1-2 parallel to *xy*. Join *a*1, *b*2, *c*3 and *d*4. This represents the top view. Project all the corners to obtain trapezium $a'b'2'1'$ as the front view.
2. Produce $a'1'$ and $b'2'$ to meet each other on the centre line at o'. Draw an arc *bh* with centre *o* and radius *ob* to meet the horizontal line through *o* at point *h*. Project point *h* to meet *xy* at point h'. Join $o'h'$ to represent the true length.
3. Consider seam at $1'a'$. Draw line *OA* parallel and equal to $o'h'$. Draw an arc with centre *O* and radius *OA*. Step off a distance of 40 mm on the arc to obtain *B*, *C*, *D* and *A*. Join the base sides *AB*, *BC*, *CD*, *DA*.
4. Draw another arc with centre *O* and radius $o'1'$ to meet slant edges at points 1, 2, 3 and 4. Join top sides 1-2-3-4-1 with straight lines. Also, join the slant edges *A*1, *B*2, *C*3 and *D*4.

5. Mark *P* and *S* as the mid-points of *AB* and 3-4. Proceed to mark *p* and *s* as the mid-points of *ab* and 3-4 in the top view. Also, mark *p′* and *s′* as the mid-points of *a′b′* and 3′-4′ in the front view.
6. Join *PS*. This represents the wire in the development with shortest distance. Let *PS* cut the edges 2-*B* and 3-*C* at *Q* and *R* respectively.
7. Draw arcs with centre *O* and radii *OQ* and *OR* to meet *OA* at points *q″* and *r″*, respectively. Draw the horizontal lines from points *q″* and *r″* to meet line *o′b′* and *o′c′* at points *q′* and *r′* respectively.
8. Join *p′q′r′s′* to represent wire in the front view. It may be noted that *r′s′* lies on the face that is not visible and therefore should be drawn as hidden line.
9. Project *q′* and *r′* to meet *ob* and *oc* at points *q* and *r*. Join *pqrs* to represent wire in the top view.

Problem 13.50 *A pentagonal pyramid of base side 30 mm and axis 60 mm has its base on the ground. An edge of the base is nearer and parallel to the V.P. A thread is wound around its slant faces starting at the farthest corner of the base from the V.P. and back to it. Find geometrically the shortest length of the thread required and show the projections of the shortest length of the thread in the front and top views of the pyramid.*

Construction Refer to Fig. 13.53.

1. Draw a pentagon *abcde* keeping *de* parallel to *xy*. Join the corners with centroid *o*. This represents the top view. Project all the corners to obtain *a′o′c′* as the front view.
2. Draw an arc *bh* with centre *o* and radius *ob* to meet the horizontal line through centre *o* at point *h*. Project *h* to meet *xy* at point *h′*. Join *o′h′* to represent the true length of the slant edges.
3. Consider seam at *o′a′*. Draw line *OA* parallel and equal to *o′h′*. Draw an arc with centre *O* and radius *OA*. Step off a distance of 30 mm on the arc to obtain *B*, *C*, *D*, *E* and *A*. Join the base sides *AB*, *BC*, *CD*, *DE*, *EA* and slant edges *OA*, *OB*, *OC*, *OD*, *OE*, *OA*.
4. Join *A*-*A* on the development to represent the shortest length of thread. Here it is 120 mm long. Let the line meets *OB*, *OC*, *OD* and *OE* at points *P*, *Q*, *R* and *S*.
5. Draw arcs with centre *O* and radii *OP*, *OQ*, *OR* and *OS* to meet *OA* at points *p″* and *q″*. Draw horizontal lines from *p″* and *q″* to meet their corresponding generators at points *p′*, *q′*, *r′* and *s′*.
6. Join *a′p′q′r′s′a′* to represent the shortest length of thread in the front view. It may be

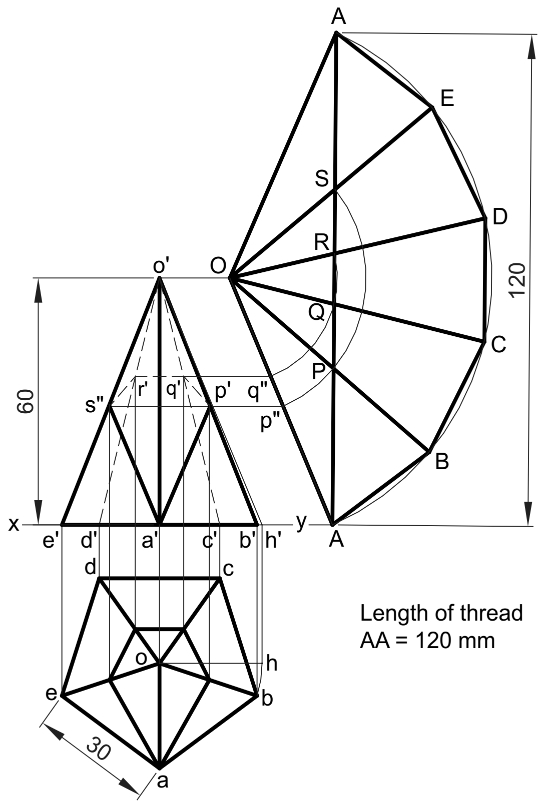


Fig. 13.53

noted that only $a'p'$ and $a's'$ are visible and to be drawn with wide continuous lines. The line joining $p'q'r's'$ should be drawn as hidden lines.

7. Project p', q', r' and s' to meet *ob*, *oc*, *od* and *oe* at points *p*, *q*, *r* and *s*. Join *apqrsa* to represent the shortest length of thread in the top view.

EXERCISE 13 B

1.1 A pentagonal prism of height 60 mm is resting on its base on the H.P. with a side of the base parallel to the V.P. It is cut by planes perpendicular to V.P. such that the development of the prism is an isosceles triangle of base 140 mm and altitude 60 mm. Draw the projections of the truncated prism considering seam of the prism is farthest away from the V.P.

1.2 A hexagonal prism of base side 30 mm and axis 70 mm stands on its base on the ground. A thread is wound around the prism, starting from the corner of the lower base to the corresponding corner of the upper base. Find the minimum length of the thread and show it on the front view of the prism.

1.3 A semicircle of radius 75 mm is just enclosed in a thin rectangular metal sheet. The sheet is wrapped to form a cylinder. Draw the projections of the cylinder with the corresponding semicircle marked on it.

1.4 A circle of largest possible diameter is marked on a rectangle of sides 150 mm and 60 mm. This represents the development of the lateral surface of a cylinder. Draw the projections of the cylinder with the corresponding circle marked on it.

1.5 An ellipse of axes 150 mm and 70 mm is marked on a thin rectangular metal sheet of side 150 mm and 70 mm. The sheet is wrapped to form a cylinder. Draw the projections of the cylinder with the corresponding ellipse marked on it.

1.6 A chord of largest possible size is marked on the sector of a circle having included angle of 135° and radius 60 mm. The sector is the development of a cone. Draw the projections of the cone with the chord marked.

1.7 A rhombus of largest possible size is made on the surface of a semi-circular metal sheet having diameter 100 mm. The sheet is wrapped to form a cone resting on its base on the H.P. Draw the projections of the cone with the rhombus marked on it.

1.8 A circle of largest possible size is made on the surface of a semi-circular metal sheet having diameter 100 mm. The sheet is wrapped to form a cone resting on its base on the H.P. such that seam is farthest away from the V.P. Draw the projections of the cone with the circle marked on it.

1.9 A plate is in the form of a sector of a circle, having included angle of 135° and radius 60 mm. A largest possible rhombus is marked on the plate and then the plate is wrapped to form a cone. Draw the projections of the cone with the corresponding rhombus marked on it.

1.10 An equilateral triangle of largest size is marked on the surface of a semi-circular plate of diameter 100 mm. The plate is folded to form a cone. Draw the projections of the cone with the corresponding equilateral triangle marked on it.

1.11 A half cone of base diameter 50 mm and axis 70 mm is resting on its half base on the H.P. with the flat face parallel and nearer to the V.P. A string of shortest possible length is wound around its surface from one point of the base circle and brought back to the same point. Draw the projections of the cone with path of string marked on it.

1.12 A hexagonal pyramid of base side 30 mm and axis 70 mm is resting on its base on the H.P. with a side of the base parallel to the V.P. Draw the projections of the pyramid and show on it, the shortest path traced by a point, start moving from a point on the circumference of the base, moving around the pyramid and returns to the same point.

1.13 The frustum of a square pyramid has its base side 40 mm, top side 20 mm and height 70 mm. It is placed on its base on the H.P. with an edge of the base parallel to the V.P. A wire connects the mid-point of bottom edge of the front face to the mid-point of the top edge of the same face by moving over the surfaces of the frustum by the shortest distance. Draw the projections of the frustum and show path of the wire in the front view, top view and the development.

VIVA-VOCE QUESTIONS

1.1 Differentiate between singly curved surface and doubly curved surface.

1.2 Name the method used for obtaining the developments of prisms and cylinders.

1.3 Name the method used for obtaining the developments of pyramids and cones.

1.4 Name two common methods of getting the development of spheres.

1.5 What precaution should be taken while obtaining the development of pyramids?

1.6 What are the dimensions of the cone if its development is a semicircle of 120 mm diameter?

1.7 State a few practical applications of the development of surfaces.

MULTIPLE-CHOICE QUESTIONS

1.1 The development of surface of an oblique solid is obtained by
(a) radial line (b) parallel line
(c) triangulation (d) approximation

1.2 Methods for the development of surfaces are
(a) parallel line method
(b) radical line method
(c) triangular method
(d) All of them

1.3 Development of sphere is done by
(a) zone or lune method
(b) parallel line or radial line method
(c) triangulation method
(d) Any of these methods

1.4 The nature of lateral surface of a cylinder is
(a) plane surface
(b) singly curved surface
(c) doubly curved surface
(d) singly or doubly curved surface

1.5 The nature of surface of a sphere is
(a) plane surface
(b) singly curved surface
(c) doubly curved surface
(d) singly or doubly curved surface

1.6 If a semicirclular thin sheet is folded to form a cone, then the front view of the cone appears as,
(a) equilateral triangle (b) isosceles triangle
(c) rectangle (d) semicircle

1.7 Sector of a circle of radius 60 mm and angle 120° represents development of the lateral surface of a cone. The top view of the cone is a circle of diameter
(a) 20 mm (b) 40 mm
(c) 60 mm (d) 80 mm

1.8 If the front view of a cone is represented by an equilateral triangle of 60 mm side. The area of its lateral surface is
(a) 30 π (b) 60 π
(c) 90 π (d) 120 π

1.9 The development of suface of a tetrahedron of 60 mm edge can be represented by an equilateral triangle of side
(a) 60 mm (b) 90 mm
(c) 120 mm (d) None of these

1.10 The development of suface of a tetrahedron of 60 mm edge can be represented by a parallelogram of adjacent sides
(a) 60 mm and 90 mm
(b) 60 mm and 120 mm
(c) 90 mm and 120 mm
(d) None of these

1.11 A rectangle of 120 mm X 60 mm represents the development of the lateral surface of
(a) a square prism of side 30 mm
(b) a hexagonal prism of side 20 mm
(c) a cylinder of diameter $120/\pi$
(d) All of these

1.12 A string is wound around a hexagonal prism of base 20 mm side and axis 50 mm long, to connect opposite ends of the same longer edge. The minimum length of string required is
(a) 110 mm (b) 120 mm
(c) 130 mm (d) 140 mm

Answers to multiple-choice questions

13.1 (c), 13.2 (a), 13.3 (a), 13.4 (b), 13.5 (c), 13.6 (a), 13.7 (b), 13.8 (b), 13.9 (c), 13.10 (b), 13.11 (d), 13.12 (c)

Chapter

14

INTERSECTION OF SURFACES

14.1 INTRODUCTION

When a solid penetrates into another solid, it is known as *interpenetration of solids*. Due to such interpenetration, their lateral surfaces intersect to produce closed loops which may either be made of straight lines or curves. These loops are known as *lines or curves of intersection.*

Since two plane surfaces intersect in a straight line, the intersection of prism with prism, or pyramid with pyramid, or prism with pyramid, results in a polygon. Similarly, if any one or both of the two solids have curved surfaces, it will result in a closed curve. In both the cases, the term "curve of intersection" is frequently used. It is important to note that the points lying on the curve of intersection are always common to the surfaces of both the solids.

14.2 ENGINEERING APPLICATIONS

Mainly, the curve of intersection is needed in fabrication of items like funnels, chimneys, hoppers, towers, storage tanks, vessels, ducts of air-conditioners, etc. In fabrication of such items, first of all, the curve of intersection is obtained and then their developments are prepared.

14.3 METHODS OF DETERMINING THE CURVES OF INTERSECTION

The curve of intersection is a line or curve common to the surfaces of the penetrating solids. It is composed of points at which the lines of one surface intersect with those of the other surface. Following methods are used to obtain the curves of intersection:

1. **Line or generator method** In this method, convenient number of lines are drawn on the lateral surface of one of the solids in the region of interest. The points of intersection of these lines with the edges or generators of the other solid are then marked. These points are then transferred to their corresponding positions in the other views. The projected points are joined in proper sequence to get the curve of intersection.

2. **Cutting plane method** In this method, the intersecting solids are considered to be cut by a number of section planes, one by one. These planes may either be auxiliary inclined planes or sometimes auxiliary vertical planes. The planes are so selected that they produce sections of regular shapes in both the solids. The points common to both of these intersecting solids falling on each of the section planes are obtained. These points are then transferred to their corresponding positions in the other views. The projected points are joined in proper sequence to get the curve of intersection.

Both of these methods can be better understood with the help of examples. A thorough knowledge of "Projections of Solids" and "Sections of Solids" will help in understanding this topic easily.

14.4 TYPES OF INTERPENETRATING SOLIDS

The interpenetrating solids may be similar or like solids. Examples are given below.

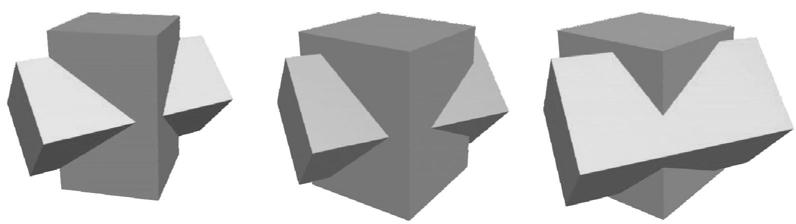
1. Prism intersecting prism
2. Pyramid intersecting pyramid
3. Cylinder intersecting cylinder
4. Cone intersecting cone
5. Sphere intersecting sphere

They may also be dissimilar or unlike solids. Examples are given below.

1. Prism intersecting cylinder
2. Cone intersecting cylinder
3. Cone intersecting prism
4. Sphere intersecting cylinder

14.5 INTERSECTION OF PRISM BY ANOTHER SOLID

Generally, the line method is convenient for this combination of intersection. The intersection points can be marked at the points where the edges of one prism intersect with the other, and when these points are joined by following a proper sequence, a closed polygon is formed.



(a) (b) (c)

Fig. 14.1

Problem 14.1 *A square prism, of base side 50 mm, is resting on its base on the H.P. It is completely penetrated by another square prism of base side 40 mm, such that the axes of both the prisms intersect each other at right angles and faces of both the prisms are equally inclined to the V.P. Draw the projections of the combination and show the lines of intersection.*

Construction Refer to Fig. 14.2.

1. Draw TV of the vertical prism, assuming a suitable height (say 100 mm). Project FV and SV and label them as shown.
2. Draw SV of the horizontal prism, assuming its suitable length (say 100 mm). Project FV and TV and label them as shown.
3. The faces of the vertical prisms are seen as lines in the TV. First locate the points of intersection in the TV of the edges 1-1, 2-2, 3-3 and 4-4 of the horizontal prism with the faces of the vertical prism on left side as p_1, p_2, p_3 and p_4 and on right side as q_1, q_2, q_3 and q_4.

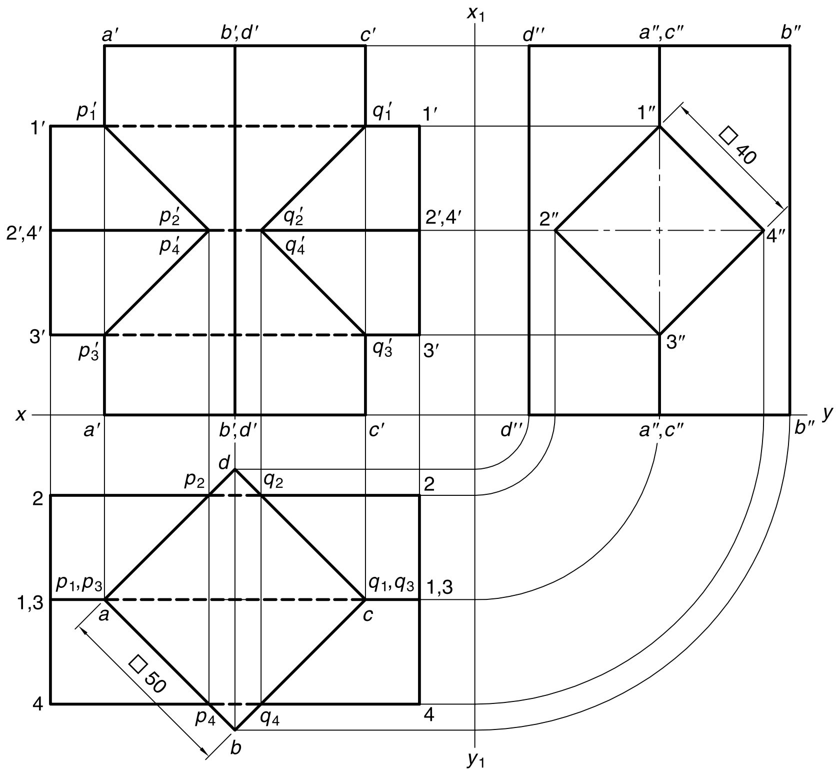


Fig. 14.2

4. Project points p_1, p_2, p_3 and p_4 to FV to meet their corresponding edges $1'1'$, $2'2'$, $3'3'$ and $4'4'$ at points p_1', p_2', p_3' and p_4'. Similarly project q_1, q_2, q_3 and q_4 to FV and obtain points q_1', q_2', q_3' and q_4'.
5. Draw the lines $p_1'p_2'$, $p_2'p_3'$ $q_1'q_2'$, $q_2'q_3'$. Lines $p_1'p_4'$, $p_3'p_4'$ $q_1'q_4'$, $q_3'q_4'$ coincide with front lines. These lines show the line of intersection.
6. Show the portion of horizontal prism which is inside the vertical prism by dotted lines in both the FV and TV.

Problem 14.2 *A square prism of base side 60 mm, is resting on its base on the H.P. It is completely penetrated by another square prism of base side 40 mm, such that their axes are 10 mm apart. The axis of the penetrating prism is parallel to both H.P. and V.P., while the faces of both the prisms are equally inclined to the V.P. Draw the projections of the combination and show lines of intersection.*

Construction Refer to Fig. 14.3.

1. Draw a square *abcd* to represents TV of the vertical prism. Assuming its suitable height (say 100 mm) project FV and SV and label them as shown.

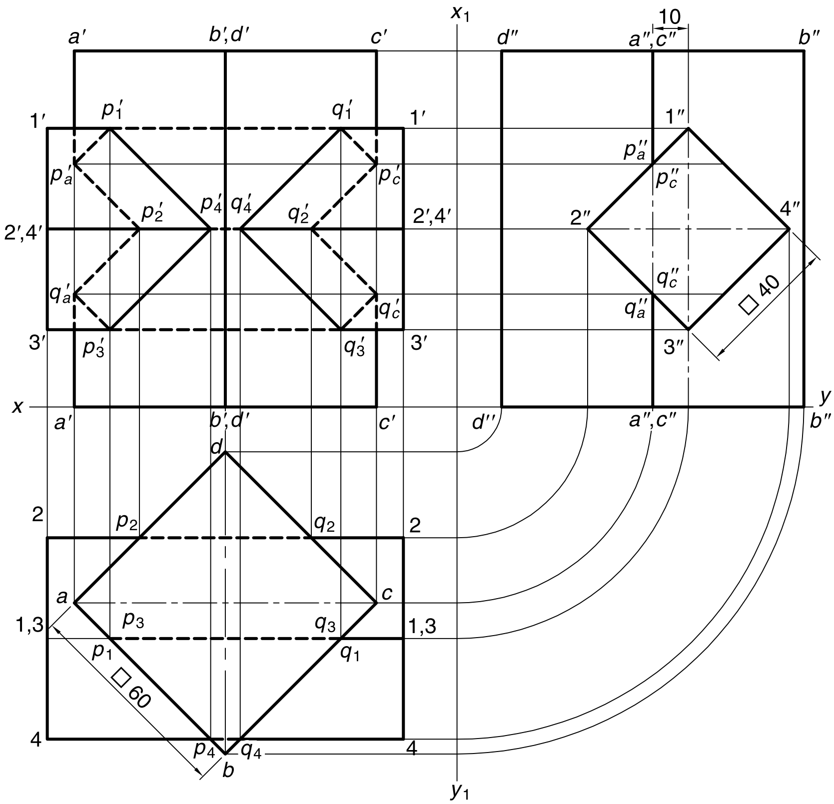


Fig. 14.3

2. Draw another square 1″2″3″4″ keeping centre 10 mm away from the axis of the vertical prism to represent the SV of the horizontal prism. Assuming its suitable length (say 100 mm), project FV and TV and label them as shown.
3. The faces of the vertical prism are seen as lines in the TV. First locate the points of intersection in the TV of the edges 1-1, 2-2, 3-3 and 4-4 and of the horizontal prism with the faces of the vertical prism on left side as p_1, p_2, p_3 and p_4 and on right side as q_1, q_2, q_3 and q_4.
4. Project points p_1, p_2, p_3 and p_4 to FV to meet their corresponding edges 1′1′, 2′2′, 3′3′ and 4′4′ at points p_1', p_2', p_3' and p_4'. Similarly, project q_1, q_2, q_3 and q_4 to FV and obtain points q_1', q_2', q_3' and q_4'.
5. In the SV, the vertical edges $a''a''$ and $c''c''$ intersects with the faces 1″2″ and 2″3″ of the horizontal prism at p_a'', q_a'', p_c'' and q_c''. Project these points to FV upto vertical edges $a'a'$ and $c'c'$.
6. Join lines $p_1'p_a'p_2'q_a'p_3'$ and $q_1'p_c'q_2'q_c'q_3'$ by dotted lines and then $p_1'p_4'p_3'$ and $q_1'q_4'q_3'$ by full lines as shown. These lines show the lines of intersection.
7. Show the portion of horizontal prism which is inside the vertical prism by dotted lines in both the FV and TV.

Problem 14.3 *A square prism of base side 60 mm is resting on its base on H.P. with a face inclined at 30° to V.P. It is completely penetrated by another square prism of base side 45 mm and faces of which are equally inclined to V.P. The axes of both the prisms intersect each other at right angles. Draw the projections of the combination and show lines of intersection.*

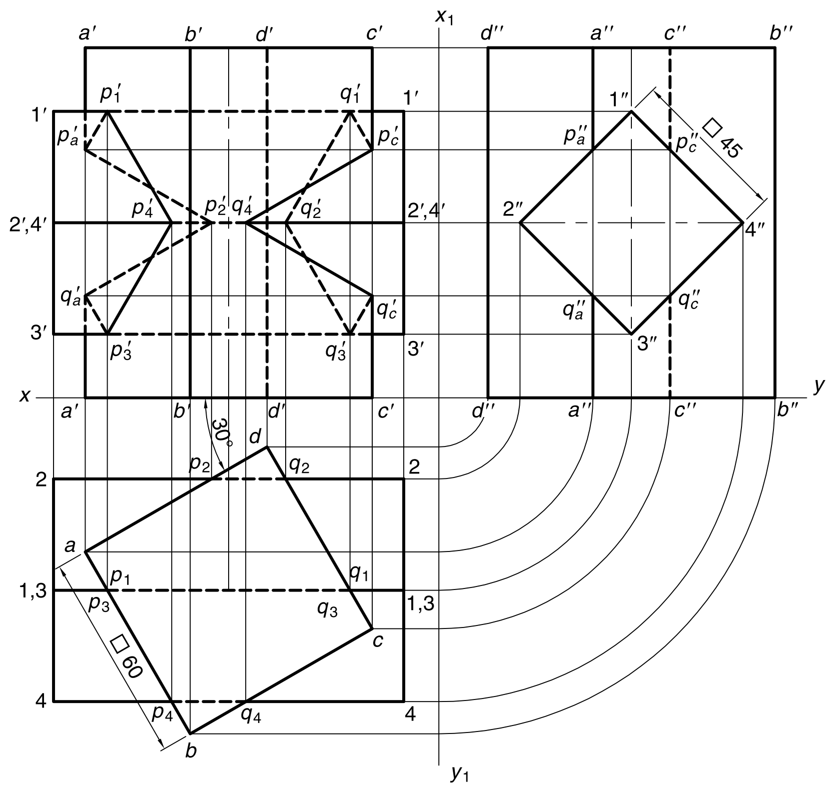


Fig. 14.4

Construction Refer to Fig. 14.4.

1. Draw a square *abcd* to represent TV of the vertical prism. Assuming its suitable height (say 100 mm), project FV and SV and label them as shown.
2. Draw another square 1″2″3″4″ keeping centre at the mid-point of the axis of the vertical prism to represent the SV of the horizontal prism. Assuming its suitable length (say 100 mm), project FV and TV and label them as shown.
3. The faces of the vertical prism are seen as lines in the TV. First locate the points of intersection in the TV of the edges 1-1, 2-2, 3-3 and 4-4 of the horizontal prism with the faces of the vertical prism on left side as p_1, p_2, p_3 and p_4 and on right side as q_1, q_2, q_3 and q_4.
4. Project points p_1, p_2, p_3 and p_4 to FV to meet their corresponding edges 1′1′, 2′2′, 3′3′ and 4′4′ at points p_1', p_2', p_3' and p_4'. Similarly, project q_1, q_2, q_3 and q_4 to FV and obtain points q_1', q_2', q_3' and q_4'.

5. In the SV, the vertical edges $a''a''$ and $c''c''$ intersect with the faces 1″2″, 2″3″, 1″4″ and 3″4″ of the horizontal prism at p''_a, q''_a, p''_c and q''_c. Project these points to FV up to vertical edges $a'a'$ and $c'c'$.
6. Join lines $p'_1p'_ap'_2q'_ap'_3$ and $p'_cq'_1q'_2q'_3q'_c$ by dotted lines and then $p'_1p'_4p'_3$ and $p'_cq'_4q'_c$ by full lines as shown. These lines show the line of intersection.
7. Show the portion of horizontal prism which is inside the vertical prism by dotted lines in both the FV and TV.

Problem 14.4 *A square prism of base side 60 mm is resting on its base on H.P. with a face inclined at 30° to V.P. It is completely penetrated by a square prism of base side 50 mm, axis of which is parallel to both the principal planes and faces equally inclined to the V.P. The axes of the prisms are 18 mm apart. Draw the projections of the combination and show lines of intersection.*

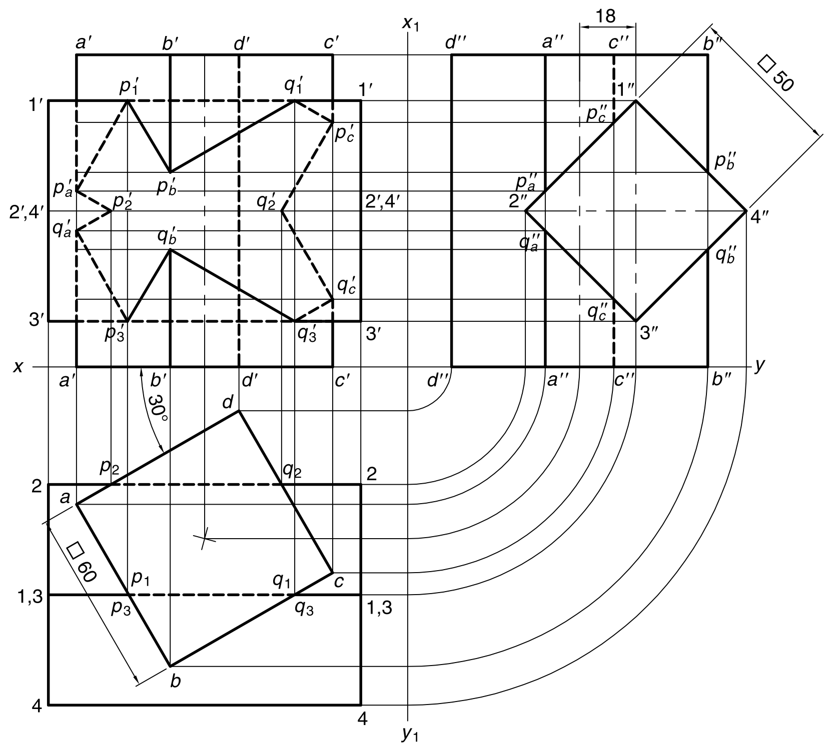


Fig. 14.5

Construction Refer to Fig. 14.5.

1. Draw a square *abcd* to represent TV of the vertical prism. Assuming its suitable height (say 100 mm), project FV and SV and label them as shown.
2. Draw another square 1″2″3″4″ keeping centre 18 mm away from the axis of the vertical prism to represent the SV of the horizontal prism. Assuming its suitable length (say 100 mm), project FV and TV and label them as shown.

3. The faces of the vertical prism are seen as lines in the TV. First locate the points of intersection in the TV of the edges 1-1, 2-2, and 3-3 of the horizontal prism with the faces of the vertical prism on left side as p_1, p_2, and p_3, and on right side as q_1, q_2, and q_3.
4. Project points p_1, p_2, and p_3 to FV to meet their corresponding edges $1'1'$, $2'2'$, and $3'3'$ at points p_1', p_2' and p_3'. Similarly, project q_1, q_2, and q_3 to FV and obtain points q_1', q_2' and q_3'.
5. In the SV, the vertical edges $a''\,a''$ and $c''c''$ intersect with the faces $1''2''$ and $2''3''$ of the horizontal prism at p_a'', q_a'', p_c'' and q_c''. Also, the vertical edge $b''b''$ intersects with the faces $1''4''$ and $3''4''$ of the horizontal prism at p_b'' and q_b''. Project these points to FV upto vertical edges $a'a'$, $c'c'$ and $b'b'$.
6. Join lines $p_1'p_a'p_2'q_a'p_3'$ and $q_1'p_c'q_2'q_c'q_3'$ by dotted lines and then $p_1'p_b'q_1'$ and $q_3'q_b'q_3'$ by full lines as shown. These lines show the line of intersection.
7. Show the portion of horizontal prism which is inside the vertical prism by dotted lines in both the FV and TV.

Problem 14.5 *A square prism of base side 60 mm is resting on its base on the H.P. with a face inclined at 30° to the V.P. It is completely penetrated by a horizontal cylinder of base diameter 60 mm such that axes of the prism and the cylinder intersect each other at right angles. Draw the projections of the combination and show the curves of intersection.*

Construction Refer to Fig. 14.6(a).

1. Draw a square *abcd* to represent TV of the prism. Assuming its suitable height (say 100 mm), project FV and SV and label them as shown.

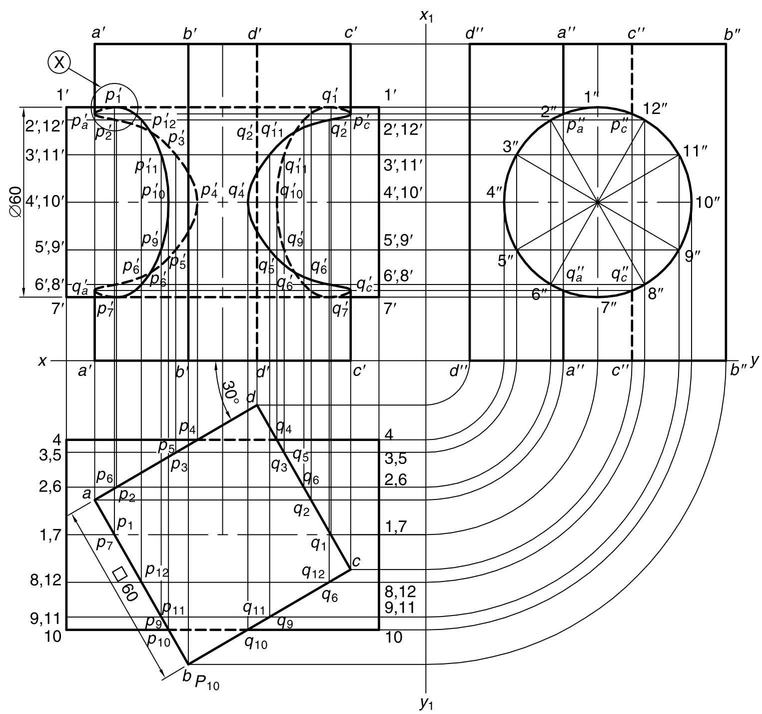


Fig. 14.6(a)

2. Draw a circle 1″4″7″10″ keeping centre at the mid-point of the axis of the vertical prism to represent the SV of the cylinder. Assuming its suitable length (say 100 mm), project FV and TV and label them as shown.
3. The faces of the prism are seen as lines in the TV. First locate the points of intersection in the TV of the generators 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 of the cylinder with the faces of the prism on left side as p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}, p_{11}, p_{12} and on right side as q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9, q_{10}, q_{11}, q_{12}.
4. Project points p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}, p_{11}, p_{12} to FV to meet their corresponding generators 1′, 2′, 3′, 4′, 5′, 6′, 7′, 8′, 9′, 10′, 11′, 12′ at points p_1', p_2', p_3', p_4', p_5', p_6', p_7', p_8', p_9', p_{10}', p_{11}', p_{12}'. Similarly project q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9, q_{10}, q_{11}, q_{12} to FV and obtain points q_1', q_2', q_3', q_4', q_5', q_6', q_7', q_8', q_9', q_{10}', q_{11}', q_{12}'.
5. In the SV, the vertical edges $a''a''$ and $c''c''$ intersect with the curved surface 1″2″ and 2″3″ of the cylinder at p_a'', q_a'', p_c'' and q_c''. Project these points to FV up to vertical edges $a'a'$ and $c'c'$.
6. Join points $p_1'p_2'p_3'p_4'p_5'q_a'p_7'$ and $p_c'q_1'q_2'q_2'q_2'q_2'q_2'q_2'q_c'$ by dotted lines and then $p_1'p_{12}'p_{11}'p_{10}'p_9'p_8'p_7'$ and $p_c'q_{12}'q_{11}'q_{10}'q_9'q_8'q_7'$ by full lines as shown. These lines show the curve of intersection.
7. Show the portion of the cylinder which is inside the prism by dotted lines in both the FV and TV. The portion marked as x in the front view is magnified and shown in Fig. 14.6(b).

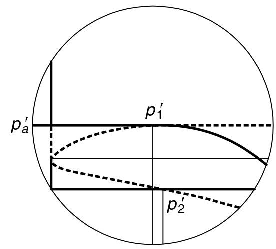
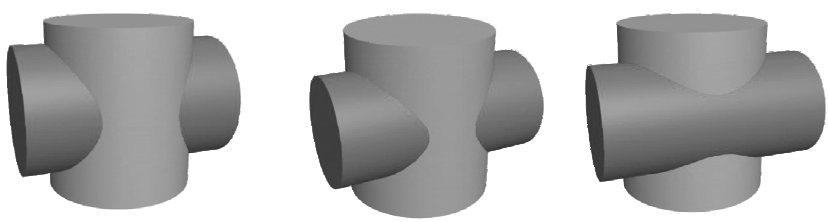


Fig. 14.6(b) *Detail of Fig. 14.6(a) at X*

14.6 INTERSECTION OF CYLINDER BY ANOTHER SOLID

When a cylinder is penetrated by any other solid, the intersection of their surfaces will be along a curve or curves. As no edges are present in a cylinder, a number of generators are required to obtain points of intersection and, thereby, the curve of intersection. Sometimes it is required to have some more critical intersection points, commonly known as key points, where the curve makes change in direction. To obtain a smooth curve of intersection, all the point of intersection and critical intersection points are joined together in a proper sequence to solve such problems. Here Problems 14.6 to 14.8 are solved by generator method and Problem 14.9 by cutting plane method. The cutting plane method is more convenient.



(a) (b) (c)

Fig. 14.7

Problem 14.6 *A cylinder of base diameter 70 mm is resting on its base on the H.P. It is penetrated by another cylinder of base diameter 60 mm, such that their axes intersect each other at right angles. Draw the projections of the combination and show the curves of intersection.*

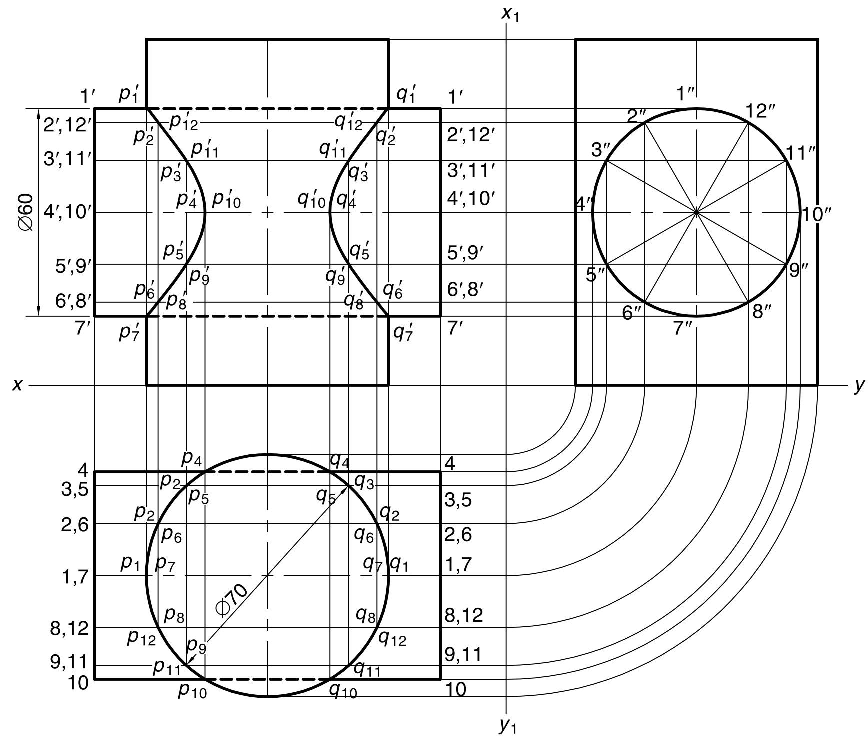


Fig. 14.8

Construction Refer to Fig. 14.8.

1. Draw a circle of diameter 70 mm to represent TV of the vertical cylinder. Assuming its suitable height (say 100 mm), project FV and SV.
2. Draw a circle 1″2″3″4″ of diameter 60 mm keeping centre at the mid-point of the axis of the vertical cylinder to represent the SV of the horizontal cylinder. Assuming suitable length (say 100 mm), project FV and TV of the horizontal cylinder. Mark 12 generators in all the three views of this cylinder as shown.
3. Generators 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 intersect the vertical cylinder at p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}, p_{11}, p_{12} on the left side and q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9, q_{10}, q_{11}, q_{12} on the right side in the TV.
4. Project points p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}, p_{11}, p_{12} and q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9, q_{10}, q_{11}, q_{12} to FV to obtain p_1', p_2', p_3', p_4', p_5', p_6', p_7', p_8', p_9', p_{10}', p_{11}', p_{12}' and q_1', q_2', q_3', q_4', q_5', q_6', q_7', q_8', q_9', q_{10}', q_{11}', q_{12}' on their corresponding generators.
5. Join them to obtain the curve of intersection as shown.

6. Join $p'_1q'_1$, $p'_7q'_7$, p_4q_4 and $p_{10}q_{10}$ by dotted lines to represent the hidden portion of the penetrating horizontal cylinder.

Problem 14.7 *A vertical cylinder of base diameter 70 mm is resting on its base on the H.P. It is penetrated by another cylinder of base diameter 50 mm, the axis of which is parallel to both the principal planes. The two axes are 8 mm apart. Draw the projections of the combination and show the curves of intersection.*

Construction Refer to Fig. 14.9(a).

1. Draw a circle of diameter 70 mm to represent TV of the vertical cylinder. Assuming its suitable height (say 100 mm), project FV and SV.

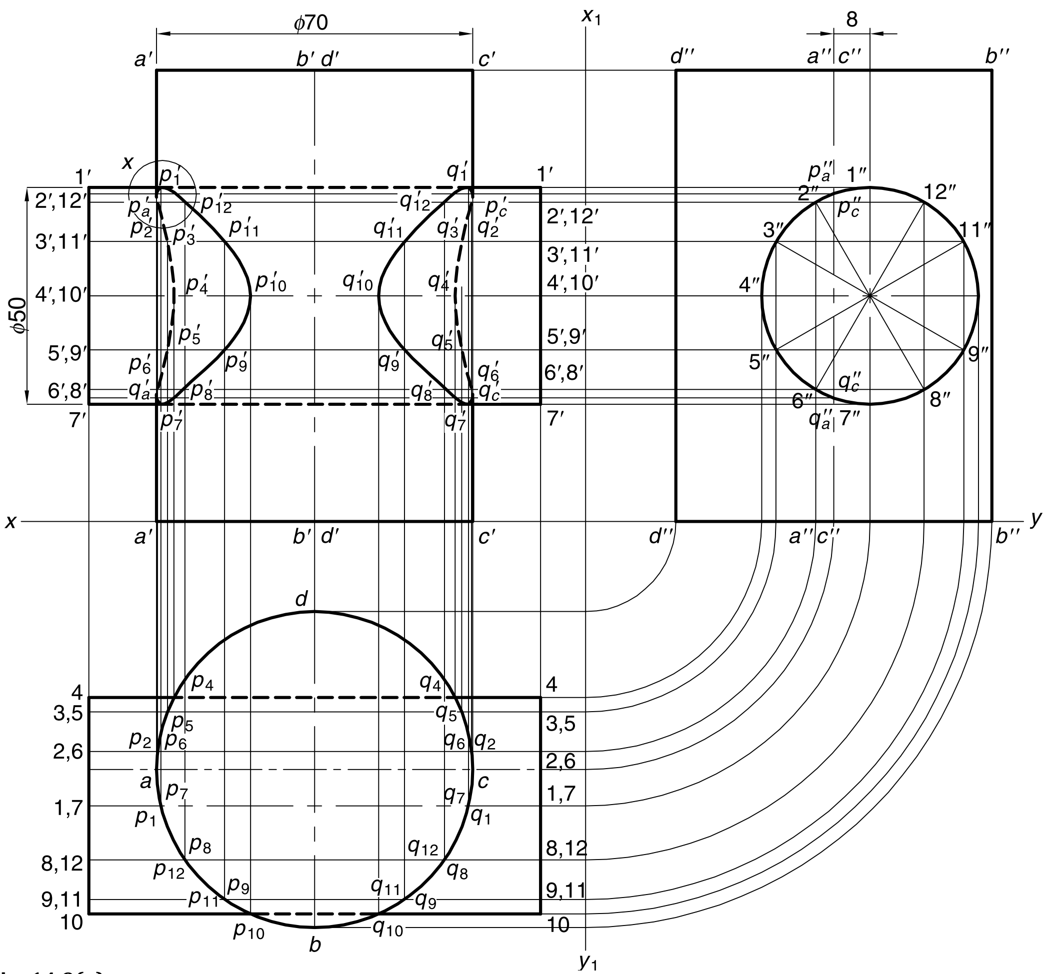


Fig. 14.9(a)

2. Draw another circle 1″2″3″4″ of diameter 50 mm keeping centre 8 mm away from the axis of the vertical cylinder to represent the SV of the horizontal cylinder. Assuming suitable length (say 100 mm), project FV and TV of the horizontal cylinder. Mark 12 generators in all the three views of this cylinder as shown.
3. Generators 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 intersect the vertical cylinder at p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}, p_{11}, p_{12} on the left side and q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9, q_{10}, q_{11}, q_{12} on the right side in the TV.
4. Project points p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}, p_{11}, p_{12} and q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9, q_{10}, q_{11}, q_{12} on their corresponding generators in the FV to obtain points p_1', p_2', p_3', p_4', p_5', p_6', p_7', p_8', p_9', p_{10}', p_{11}', p_{12}' and q_1', q_2', q_3', q_4', q_5', q_6', q_7', q_8', q_9', q_{10}', q_{11}', q_{12}'.
5. Mark points p_a'', p_c'', q_a'', q_c'' at the intersection of the generators $a''a''$ and $c''c''$ with the circle in the SV. Project them to obtain p_1', p_c', q_a', q_c' in the FV.
6. Join points p_1', p_{a}', p_2', p_3', p_4', p_5', q_a', p_7' and q_1', p_c', q_2', q_3', q_4', q_5', q_6', q_c', q_7' by dotted lines and then p_1', p_{12}', p_{11}', p_{10}', p_9', q_8', p_7' and $q_1'q_{12}'q_{11}'q_{10}'q_9'q_8'q_7'$ by full lines as shown. These lines show the curve of intersection in FV.
7. Join $p_1'q_1'$, $q_7'q_7'$, p_4q_4 and $p_{10}q_{10}$ by dotted lines to represent the hidden portion of the penetrating horizontal cylinder. The portion marked as x in the front view is magnified and shown in Fig. 14.9(b).

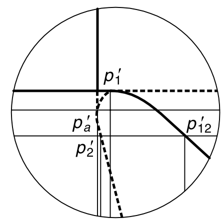


Fig. 14.9(b) *Detail of Fig. 14.9(a) at x*

Problem 14.8 *A cylinder of base diameter 70 mm is resting on its base on the H.P. It is penetrated by another cylinder of base diameter 50 mm, the axis of which is parallel to both the principal planes. The two axes are 14 mm apart. Draw the projections of the combination and show the curves of intersection.*

Construction Refer to Fig. 14.10(a).

1. Draw a circle of diameter 70 mm to represent TV of the vertical cylinder. Assuming its suitable height (say 100 mm), project FV and SV.
2. Draw another circle 1″2″3″4″ of diameter 50 mm keeping centre 14 mm away from the axis of the vertical cylinder to represent the SV of the horizontal cylinder. Assuming suitable length (say 100 mm), project FV and TV of the horizontal cylinder. Mark 12 generators in all the three views of this cylinder as shown.
3. Generators 1, 2, 3, 4, 5, 6, 7, 8, 9, 11 and 12 intersect the vertical cylinder at p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{11}, p_{12} on the left side and q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9, q_{11}, q_{12} on the right side in the TV.
4. Project points p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{11}, p_{12} and q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9, q_{11}, q_{12} to their corresponding generators in the FV to obtain points p_1', p_2', p_3', p_4', p_5', p_6', p_7', p_8', p_9', p_{11}', p_{12}' and q_1', q_2', q_3', q_4', q_5', q_6', q_7', q_8', q_9', q_{11}', q_{12}'.
5. Mark points p_a'', q_c'', q_a'', q_c'' at the intersection of the generators $a''a''$ and $c''c''$ with the circle in the SV. Project them to obtain p_a', q_c', q_a', q_c' in the FV.
6. Join points $p_1'p_2'p_a'p_3'p_4'p_5'q_a'p_6'p_7'$ and q_1', q_2', p_c', q_3', q_4', q_5', q_c', q_6', q_7' by dotted lines and then p_1', p_{12}', p_{11}', p_b', q_{11}', q_{12}', q_1' and $p_7'\,p_8'\,p_9'\,p_b'\,q_9'\,q_8'\,q_7'$ by full lines as shown. These lines show the curve of intersection in FV.

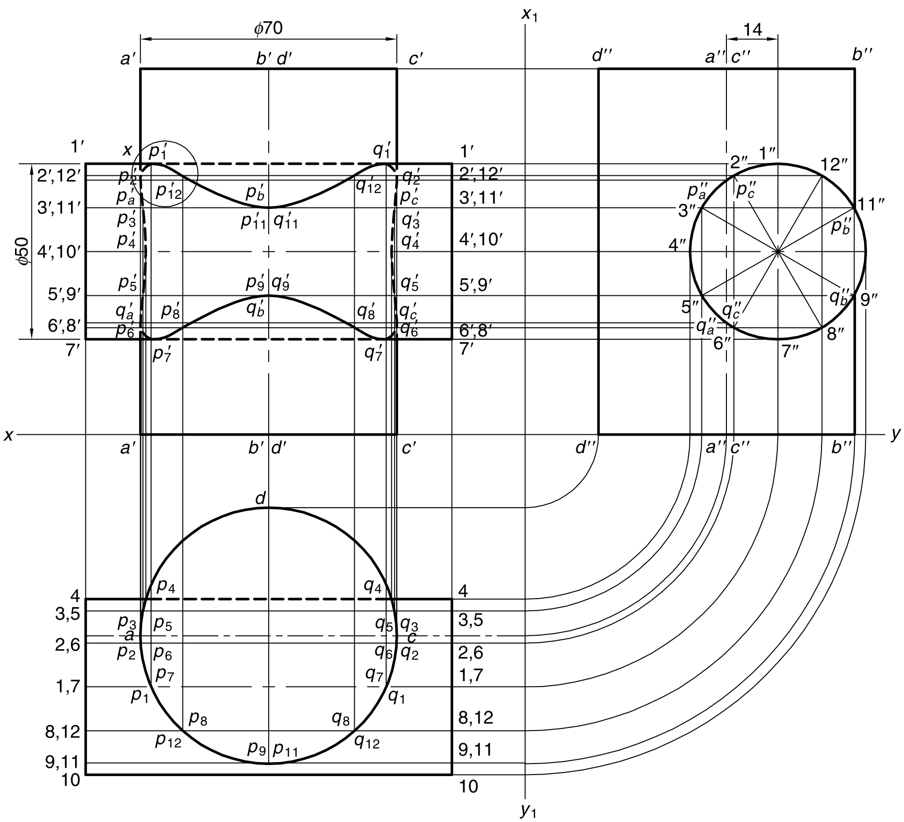


Fig. 14.10 (a)

7. Join $p_1'q_1'$, $p_7'q_7'$ and p_4q_4 by dotted lines to represent the hidden portion of the penetrating horizontal cylinder. The portion marked as x in the front view is magnified and shown in Fig. 14.10(b).

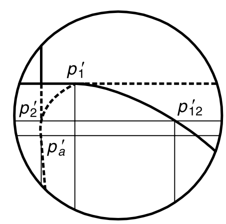


Fig. 14.10(b) *Detail of Fig. 14.10(a) at x*

Problem 14.9 *A cylinder of base diameter 70 mm is resting on its base in the H.P. It is penetrated by a square prism of base side 30 mm, the axis of which is parallel to both the principal planes and faces equally inclined to the H.P. The axes of the cylinder and prism are 10 mm apart. Draw the projections of the combination and show the curves of intersection.*

Construction Refer to Fig. 14.11(a).

1. Draw a circle of diameter 70 mm to represent TV of the cylinder. Assuming its suitable height, (say 100 mm), project FV and SV.
2. Draw a square 1″2″3″4″ to represent the side view of the prism and obtain its FV and TV.
3. Mark points p_a'', p_c'', q_a'', q_c'' at the intersection of the generators $a''a''$ and $c''c''$ of the cylinder with 1″2″ and 2″3″. Some additional points may also be taken on the sides of the square as shown in the side view.

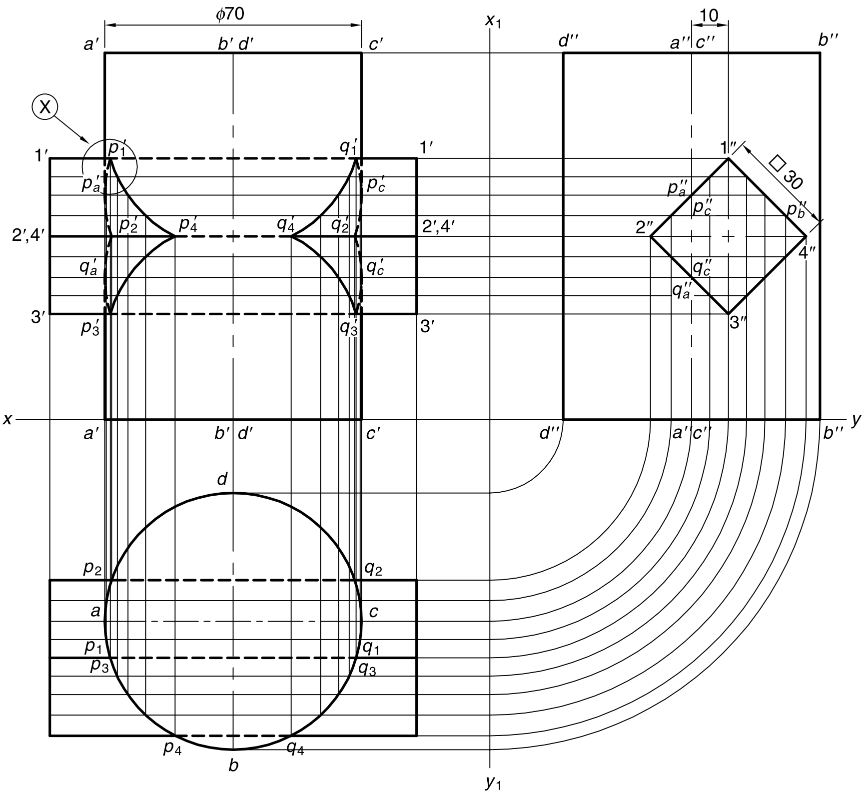


Fig. 14.11(a)

4. Transfer all these points in the FV and TV of the prism.
5. Consider a number of horizontal section planes, passing one by one through the points as referred in Steps 3 and 4 above, and obtain the common points between section of the prism (which is a rectangle) and section of the cylinder (which is a circle) in the TV.
6. Transfer these points to their corresponding positions in the FV.
7. Join points $p_1' p_a' p_2' q_a' p_3'$ and $q_1' p_c' q_2' q_c' q_3'$ by dotted curve. Also, join $p_1' p_4' p_3'$ and $q_1' q_4' q_3'$ by continuous curve to represent the curve of intersection.
8. Join $p_1' q_1'$, $p_4' q_4'$, $p_3' q_3'$, $p_2 q_2$, $p_1 q_1$ and $p_4 q_4$ by dotted lines to show the portion of the prism falling inside the cylinder.
9. In this figure, more points were taken on the square in the SV and projected to the corresponding FV and TV to obtain a smooth profile of the curve of intersection. The portion marked as x in the front view is magnified and shown in Fig. 14.11(b).

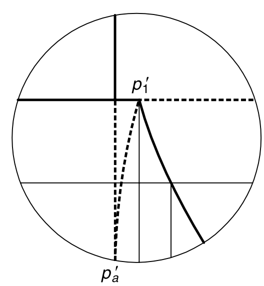


Fig. 14.11(b) *Detail of Fig. 14.11(a) at x*

14.7 INTERSECTION OF PYRAMID BY ANOTHER SOLID

Problem 14.10 *A square pyramid of base side 70 mm and axis 100 mm, is resting on its base on the H.P. with all the sides of the base equally inclined to V.P. A square prism of base side 30 mm, having its axis parallel to both the principal planes, penetrates the pyramid. The axes of the solids intersect each other at 30 mm above the base of the pyramid. Draw their projections showing the curves of intersection when the faces of the prism are equally inclined to the H.P.*

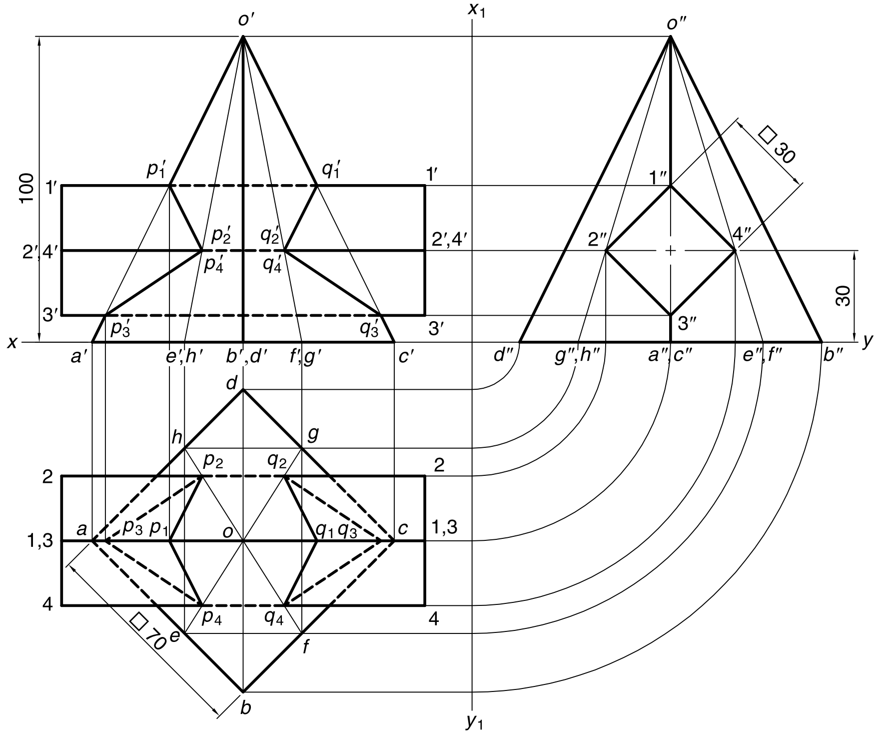


Fig. 14.12(a)

Construction (Generator Method) Refer to Fig. 14.12(a).

1. Draw three views of the square pyramid as usual.
2. Draw a square 1″2″3″4″ of side 30 mm, keeping its centre 30 mm above *xy* to represent the SV of the penetrating prism as shown.
3. Project this square in FV and TV to represent the square prism in these views.
4. Draw lines *o*″2″ and *o*″4″. Produce them up to *g*″*h*″ and *e*″*f*″ respectively. Locate their positions in the FV and TV.
5. In the top view, mark points p_2 and q_2 as the points of intersection of edge 2-2 with the line *oh* and *og* respectively. Similarly, mark points p_4 and q_4 as the points of intersection of edge 4-4 with the line *oe* and *of*, respectively.

6. Project points p_2, p_4, q_2 and q_4 to get p_2', p_4', q_2' and q_4' in the FV.
7. Mark points p_1' and q_1' as points of intersection of $o'a'$ and $o'c'$ with $1'$-$1'$. Similarly, mark points p_3' and q_3' as point of intersection of $o'a'$ and $o'c'$ with $3'$-$3'$.
8. Project these points in the TV and find p_1, q_1, p_3 and q_3.
9. Join points $p_2p_3p_4$ and $q_2q_3q_4$ by dotted lines and also, $p_2p_1p_4$ and $q_2q_1q_4$ by continuous lines to represent the curve of intersection in the TV.
10. Join $p_1'p_2'p_3'$ and $q_1'q_2'q_3'$ by continuous lines to represent the curve of intersection in the FV.

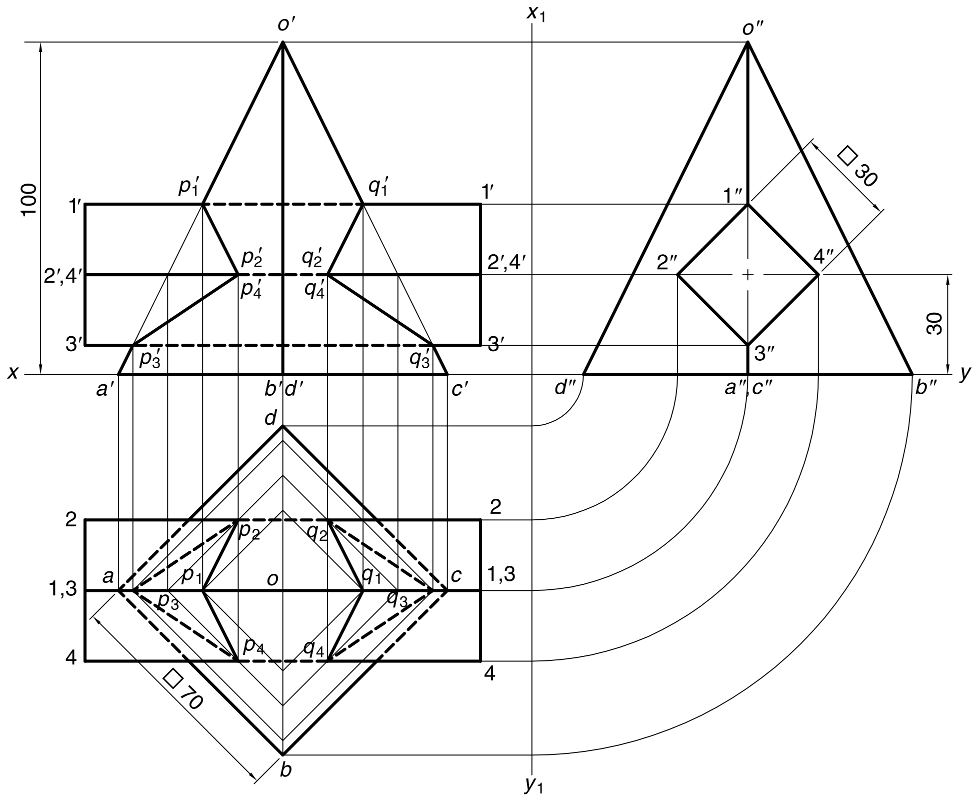


Fig. 14.12(b)

Construction (Cutting Plane Method) Refer to Fig. 14.12(b).

1. Draw three views of the square pyramid as usual.
2. Draw a square 1″2″3″4″ of side 30 mm, keeping its centre 30 mm above *xy* to represent the SV of the penetrating prism as shown.
3. Project this square in FV and TV to represent the square prism in these views.
4. Consider three horizontal section planes, passing one by one through 1′, 2′4′, 3′, and obtain the sections of the pyramid as squares in the top view.
5. Mark points p_1, q_1, p_2, q_2, p_3, q_3, p_4 and q_4 which are common to the squares obtained in step 4 with the edges 1-1, 2-2, 3-3 and 4-4 of the prism in the TV.

6. Project these points to their corresponding section planes in the FV.
7. Join points $p_2p_3p_4$ and $q_2q_3q_4$ by dotted lines and also, $p_2p_1p_4$ and $q_2q_1q_4$ by continuous lines to represent the curve of intersection in the TV.
8. Join $p_1'p_2'p_3'$ and $q_1'q_2'q_3'$ by continuous lines to represent the curve of intersection in the FV.

Problem 14.11 *A square pyramid of base side 70 mm and axis 100 mm is resting on its base on the H.P. with the sides of the base equally inclined to V.P. It is penetrated by a square prism of base side 30 mm having its axis parallel to both the principal planes and 30 mm above the H.P. The base edges of the prism are equally inclined to the H.P. Draw the projections of the combination showing curves of intersection when the axes of the solids are 8 mm apart.*

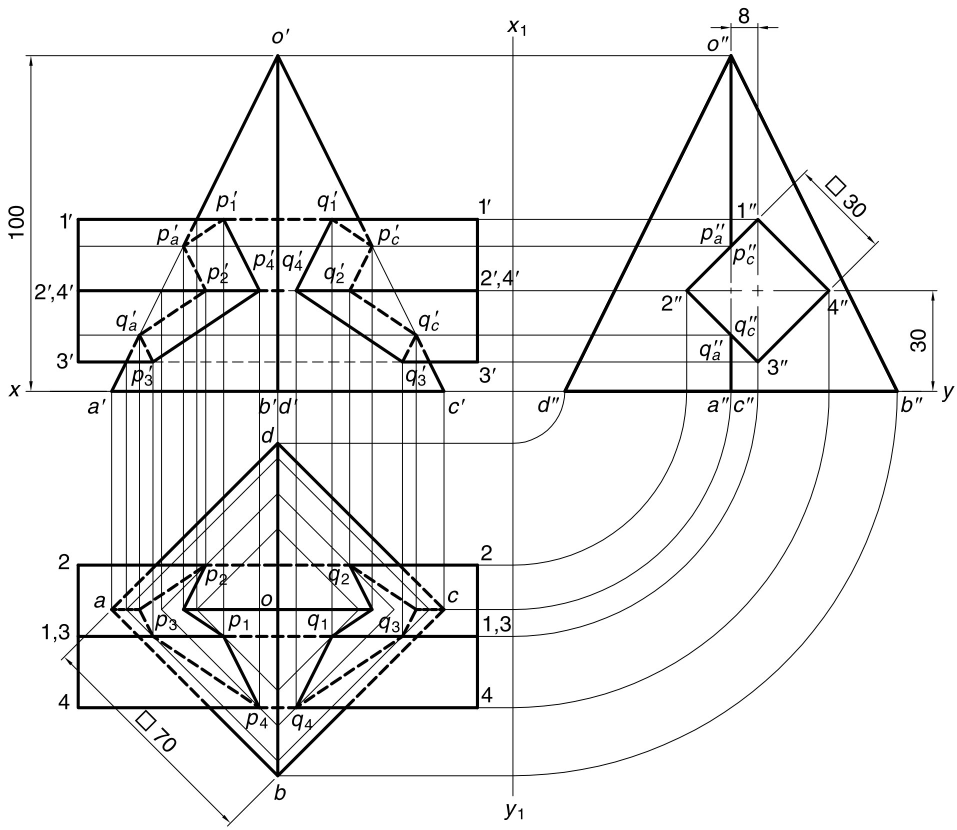


Fig. 14.13

Construction (Cutting Plane Method) Refer to Fig. 14.13.

1. Draw three views of the square pyramid as usual.
2. Draw a square 1″2″3″4″ of side 30 mm, keeping its centre 30 mm above *xy* and 8 mm away from the axis of the pyramid to represent SV of the penetrating prism.
3. Project this square in FV and TV to represent the square prism in these views.

4. Consider three horizontal section planes, passing one by one through 1′, 2′4′, 3′ and obtain the sections of the pyramid as squares in the top view.
5. Mark points p_1, q_1, p_2, q_2, p_3, q_3, p_4 and q_4 which are common to the squares obtained in Step 4 with the edges 1-1, 2-2, 3-3 and 4-4 of the prism in the TV.
6. Project these points to their corresponding section planes in the FV.
7. Also, mark points p_a'', q_a'', p_c'', q_c'' as points of intersection of $o''a''$ and $o''c''$ with the square 1″2″3″4″ in the SV.
8. Project these points to obtain p_a', q_a', p_c', and q_c' in the FV and p_a, q_a, p_c and q_c in the TV.
9. Join points $p_2q_ap_3p_4$ and $q_2q_cq_3q_4$ by dotted lines and also, $p_2p_ap_1p_4$ and $q_2p_cq_1q_4$ by continuous lines to represent the curve of intersection in the TV.
10. Join $p_1'p_a'p_2'q_a'p_3'$ and $q_1'p_c'q_2'q_c'q_3'$ by dotted lines. Also, join $p_1'p_4'p_3'$ and $q_1'q_4'q_3'$ by continuous lines to represent the curve of intersection in the FV.

Problem 14.12 *A square pyramid of base side 70 mm and axis 100 mm, is resting on its base on the H.P. with all the sides of the base equally inclined to the V.P. It is penetrated by a cylinder of base diameter 50 mm. The axis of the cylinder is parallel to both the principal planes and intersects the axis of the pyramid at a point 30 mm above the H.P. Draw the projections of the combination and show the curves of intersection.*

Construction Refer to Fig. 14.14.

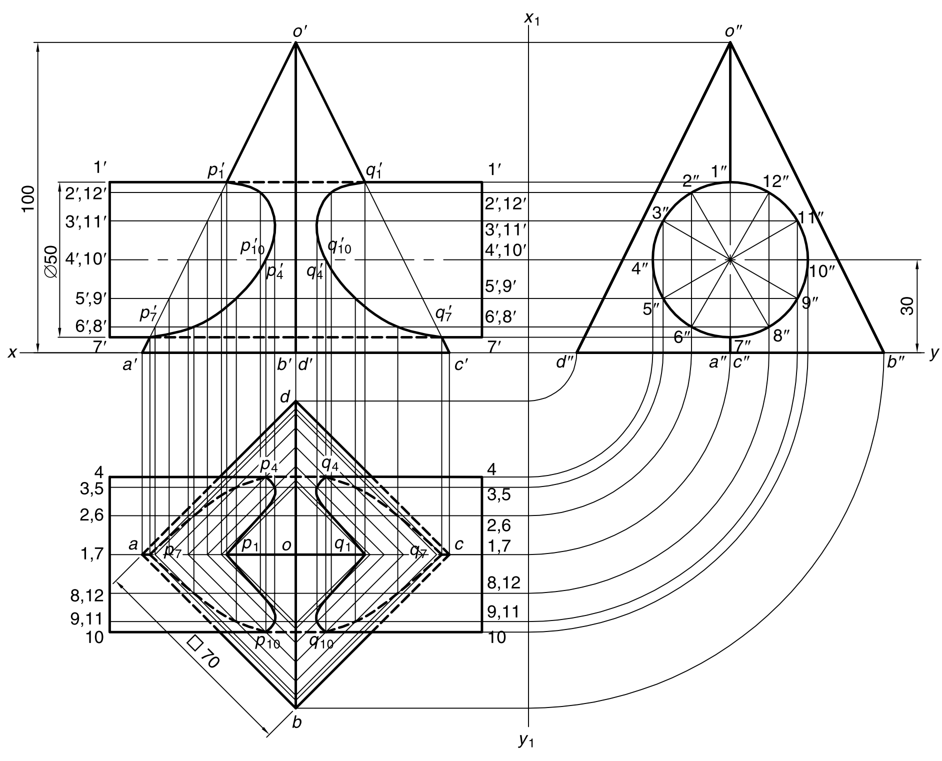


Fig. 14.14

1. Draw three views of the square pyramid as usual.
2. Draw a circle of diameter 50 mm, keeping its centre 30 mm above the base of the pyramid on its axis in the SV Project it to obtain FV and TV of the cylinder.
3. Mark 12 generators in all the three views of the cylinder.
4. Consider seven horizontal section planes, passing one by one through 1′, 2′12′, 3′11′, 4′10′, 5′9′, 6′8′ and 7′ to obtain the common points between section of square pyramid (which are squares) and section of cylinder (which are rectangles) in the TV.
5. Join these points in both the TV and FV to obtain the curve of intersection as shown.

14.8 INTERSECTION OF CONE BY ANOTHER SOLID

Problem 14.13 *A cone of base diameter 80 mm and axis 100 mm, is resting on its base on the H.P. It is completely penetrated by a cylinder of base diameter 40 mm. The axes of the solids intersect each other at right angles, 30 mm above the base of the cone. Draw the projections of the combination and show curves of intersection.*

Construction (Generator Method) Refer to Fig. 14.15(a).

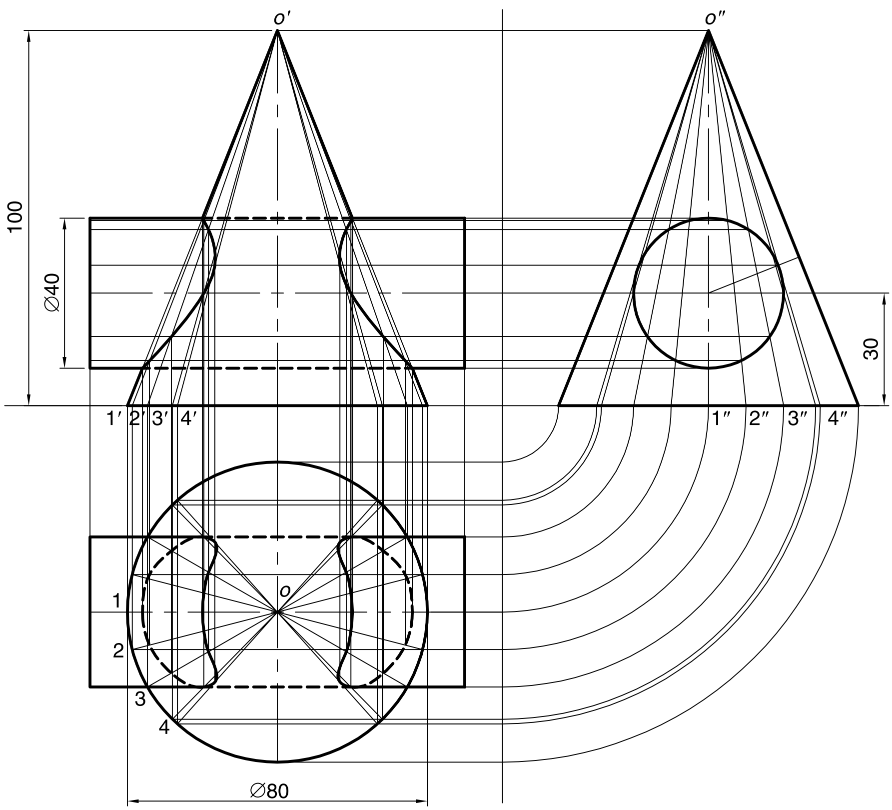


Fig. 14.15(a)

1. Draw three views of the cone.
2. Draw a circle of diameter 40 mm, keeping its centre 30 mm above the base of the cone to represent the SV of the penetrating cylinder.
3. Project the circle in FV and TV to represent the cylinder in these views.
4. Draw a generator $o''4''$ tangent to the circle on the right-hand side. Also, draw generators $o''1''$, $o''2''$, $o''3''$. Draw similar generators on the left-hand side. Locate their positions in the FV and TV.
5. Project the points of intersection of these generators with the circle from the SV to FV and TV by usual method.
6. Join all the points by proper lines in FV and TV as shown to represent the curve of intersection.

Construction (Cutting plane Method) Refer to Fig. 14.15(b).

1. Draw three views of the cone as usual.
2. Draw a circle of diameter 40 mm, keeping its centre 30 mm above the base of the cone on its axis in the SV. Project it to obtain FV and TV of the cylinder.

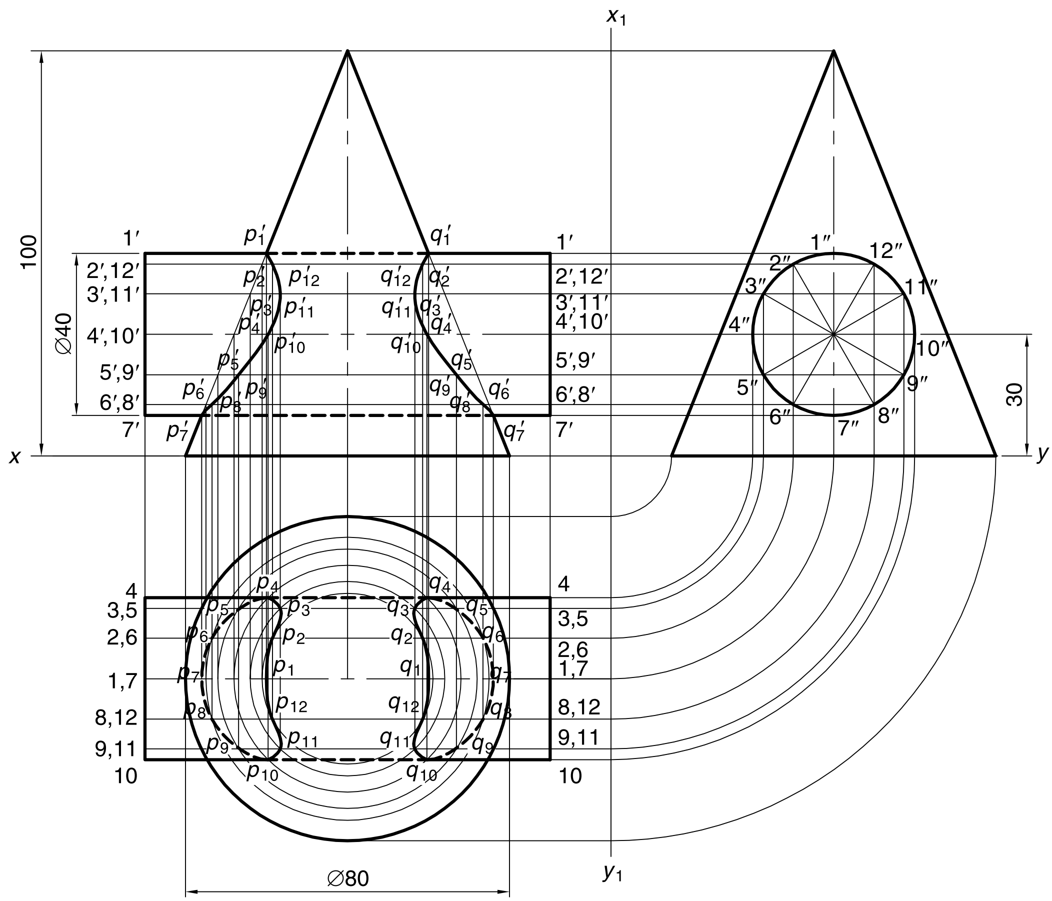


Fig. 14.15(b)

3. Mark 12 generators in all the three views of the cylinder.
4. Consider seven horizontal section planes, passing one by one through 1′, 2′12′,3′11′, 4′10′, 5′9′, 6′8′ and 7′, and obtain the common points between section of cone (which are circles) and section of cylinder (which are rectangles) in the TV.
5. Transfer these points to their corresponding positions in the FV.
6. Join these points in both the TV and FV to obtain the curve of intersection as shown.

Problem 14.14 *A cone of base diameter 100 mm and axis 110 mm, is resting on its base on the H.P. It is completely penetrated by a cylinder of base diameter 45 mm whose axis is parallel to both the principal planes and 30 mm above the H.P. The axes of the solids are 10 mm apart. Draw the projections of the combination and show curves of intersection.*

Construction (Cutting Plane Method) Refer to Fig. 14.16.

1. Draw three views of the cone as usual.

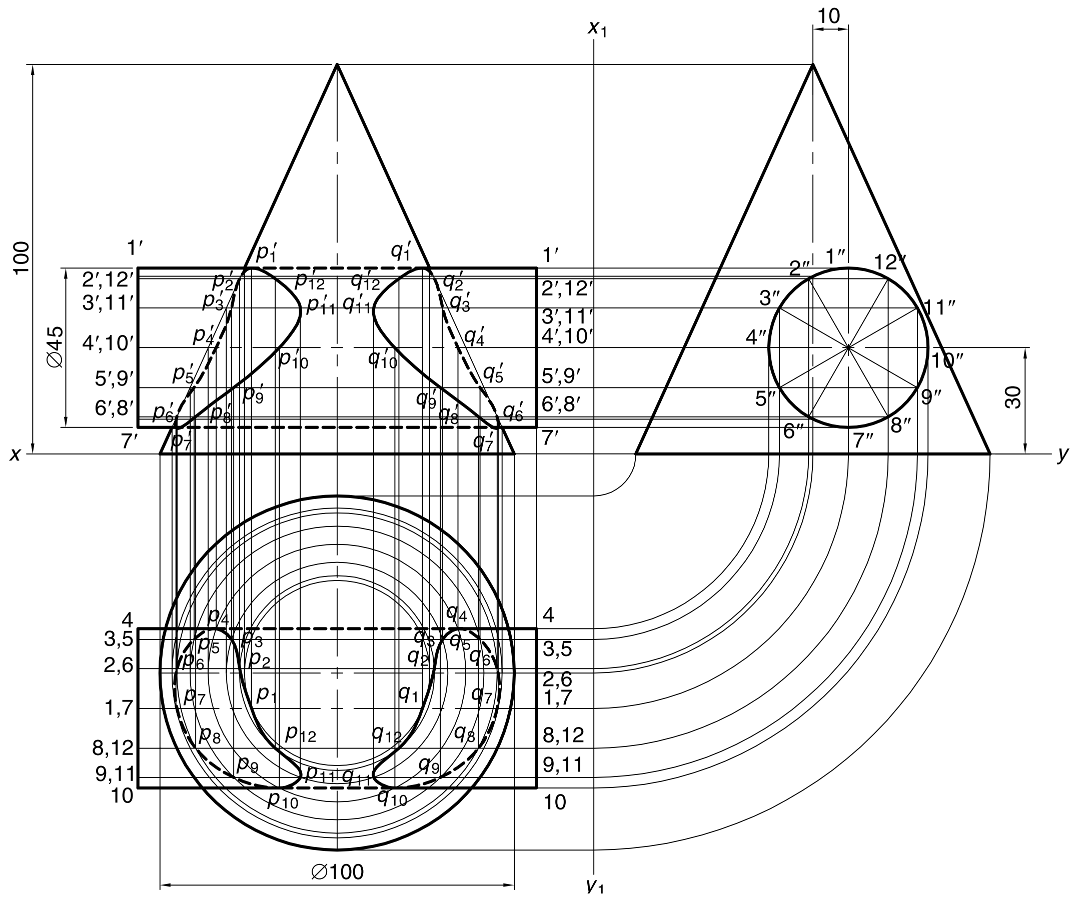


Fig. 14.16

2. Draw a circle of diameter 45 mm, keeping its centre 30 mm above the base and 10 mm away towards right hand side from the axis of the cone in the SV. Project it to obtain FV and TV of the cylinder.
3. Mark 12 generators in all the three views of the cylinder.
4. Consider seven horizontal section planes, passing one by one through 1′, 2′12′, 3′11′, 4′10′, 5′9′, 6′8′ and 7′ and obtain the common points between section of cone (which are circles) and section of cylinder (which are rectangles) in the TV.
5. Transfer these points to their corresponding positions in the FV.
6. Join these points in both the TV and FV to obtain the curve of intersection as shown.

Problem 14.15 *A cone of base diameter 100 mm and axis 110 mm, is resting on its base on the H.P. It is completely penetrated by a cylinder of base diameter 40 mm whose axis is parallel to both the principal planes and 35 mm above the H.P. The axes of the solids are 10 mm apart. Draw the projections of the combination and show curves of intersection.*

Construction Refer to Fig. 14.17.

1. Draw three views of the cone as usual.

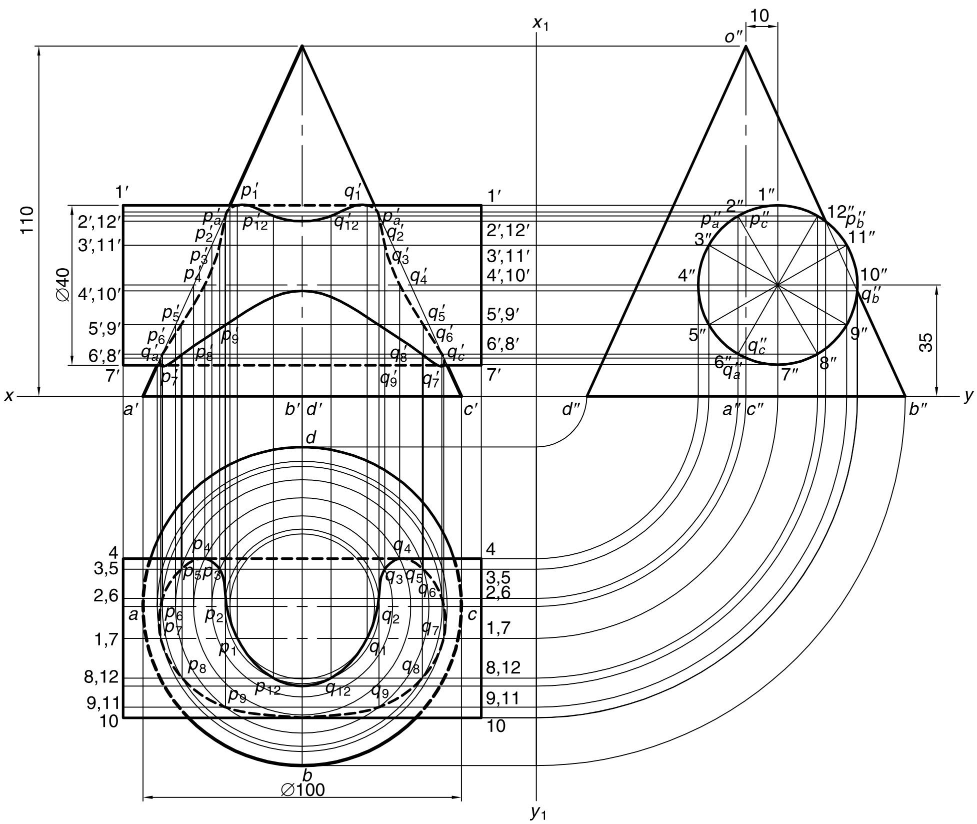


Fig. 14.17

2. Draw a circle of diameter 40 mm, keeping its centre 35 mm above the base and 10 mm away towards right-hand side from the axis of the cone in the SV. Project it to obtain FV and TV of the cylinder.
5. Mark 12 generators in all the three views of the cylinder.
4. Consider seven horizontal section planes, passing one by one through 1′, 2′12′, 3′11′, 4′10′, 5′9′, 6′8′ and 7′ and obtain the common points between section of cone (which are circles) and section of cylinder (which are rectangles) in the TV.
5. Transfer these points to their corresponding positions in the FV.
6. Take two extra cutting planes through points p_b'' and q_b'' where the circle meets the extreme right generator in the SV. Allow two additional section planes through these points and obtain corresponding points in FV and TV in the same way as in Steps 4 and 5 above.
7. Join all these points in both the TV and FV to obtain the curve of intersection as shown.

Problem 14.16 *A cone of base diameter 80 mm and axis 100 mm, is resting on its base on the H.P. It is completely penetrated by a cylinder of base diameter 50 mm such that both the solids envelop an imaginary common sphere and their axes intersect each other. Draw the projections of the combination and show curves of intersection.*

Construction Refer to Fig. 14.18.

1. Draw three views of the cone as usual.

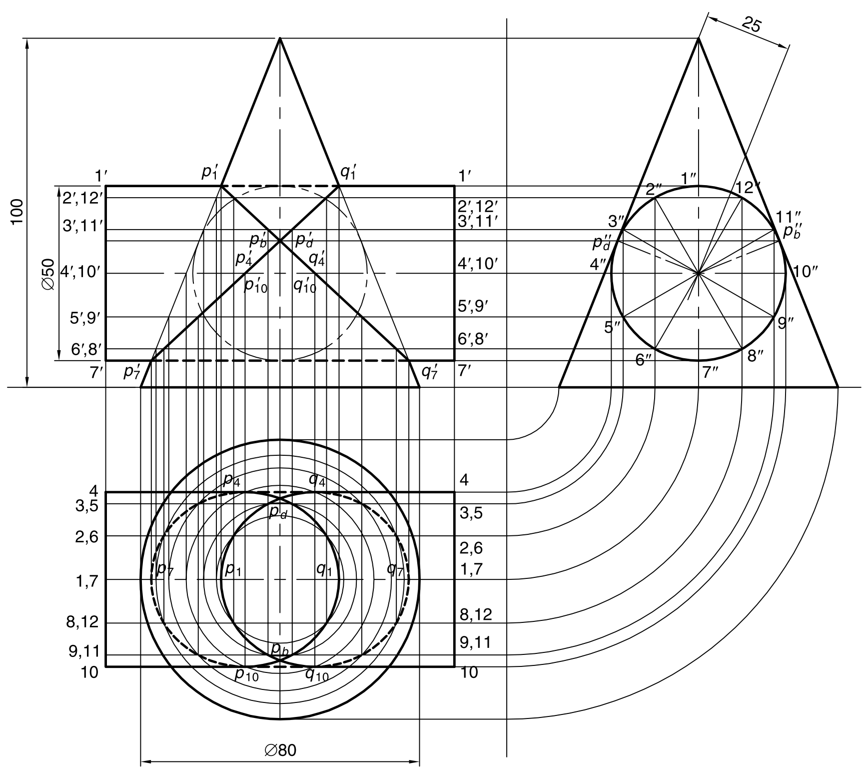


Fig. 14.18

2. In the SV, draw a line parallel to one of the extreme generators of the cone at a distance of 25 mm to obtain a point of intersection with the axis of the cone. Taking this point as centre, draw a circle of diameter 50 mm to represent the SV of the penetrating cylinder. Project it to obtain FV and TV of the cylinder.
3. Mark 12 generators in all the three views of the cylinder.
4. Consider seven horizontal section planes, passing one by one through 1′, 2′12′, 3′11′, 4′10′, 5′9′, 6′8′ and 7′, and obtain the common points between section of cone (which are circles) and section of cylinder (which are rectangles) in the TV.
5. Transfer these points to their corresponding positions in the FV.
6. Take an extra cutting plane through points p_b'' and p_d'' where the circle meets the extreme generators in the SV and obtain corresponding points in FV and TV in the same way as in Steps 4 and 5 above.
7. Join all these points in both the TV and FV to obtain the curve of intersection as shown.

Problem 14.17 *A cone of base diameter 100 mm and axis 110 mm, is resting on its base in the H.P. It is completely penetrated by a square prism of base edge 40 mm and axis 120 mm, intersecting the axis of the cone at right angles. The axis of the prism is parallel to both the principal planes and 35 mm above the H.P. and the faces are equally inclined to the H.P. Draw the projections of the combination and show curves of intersection.*

Construction Refer to Fig. 14.19.

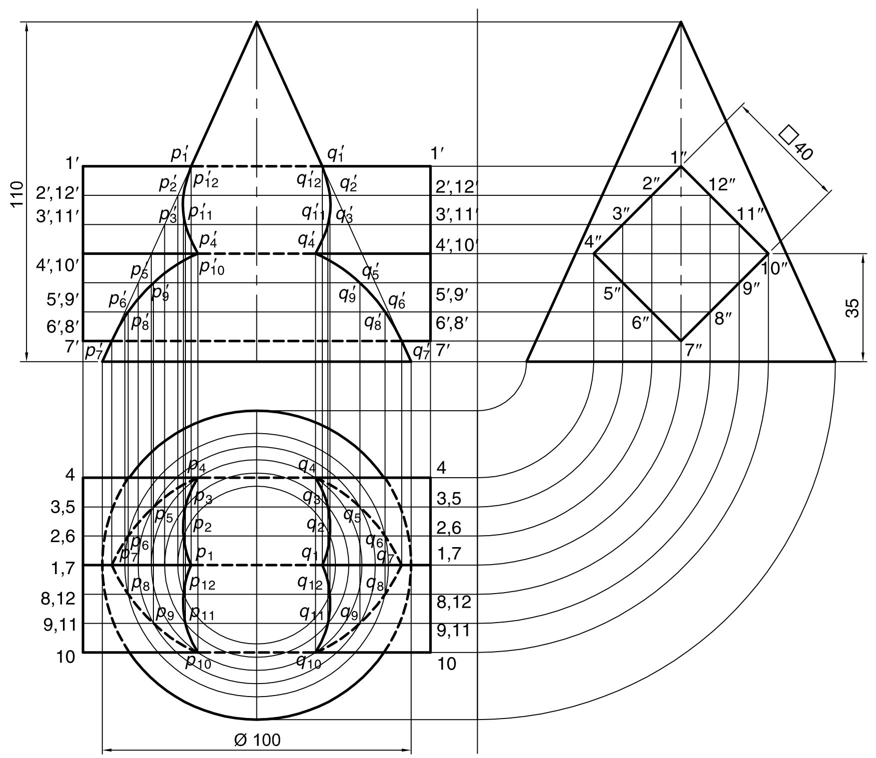


Fig. 14.19

1. Draw three views of the cone as usual.
2. Draw a square 1″4″7″10″ to represent the side view of the prism and obtain its FV and TV.
3. Mark points 2″, 3″, 5″, 6″, 8″, 9″, 11″, 12″ as shown and transfer to FV and TV.
4. Consider seven horizontal section planes, passing one by one through 1′, 2′12′, 3′11′, 4′10′, 5′9′, 6′8′ and 7′ and obtain the common points between section of cone (which are circles) and section of prism (which are rectangles) in the TV.
5. Transfer these points to their corresponding positions in the FV.
6. Join these points in both the TV and FV to obtain the curve of intersection.
7. Show the portion of the prism which is inside the cone by dotted lines in both the FV and TV.

14.9 WHEN AXES INTERSECT AT AN ANGLE OTHER THAN RIGHT ANGLE

Problem 14.18 *A vertical square prism of base edge 50 mm and axis 100 mm, is resting on its base in the H.P. It is penetrated by another square prism of base edge 30 mm and axis 120 mm long axis. The axes of both the prisms are parallel to the V.P. and bisect each other at 60°. The faces of both the prisms are equally inclined to the V.P. Draw their projection and show the lines of intersection.*

Construction Refer to Fig. 14.20.

1. Draw the projections of both the prisms as shown. Draw a square 1″2″3″4″ to represent the axial view of the penetrating prism.
2. The faces of the vertical prism are seen as lines in the TV. First locate the points of intersection of the edges 1-1, 2-2, 3-3 and 4-4 of the inclined prism with the faces of the vertical prism on left side as p_1, p_2, p_3 and p_4, and on the right side as q_1, q_2, q_3 and q_4 in the TV.
3. Project points p_1, p_2, p_3 and p_4 to FV to meet their corresponding edges 1′1′, 2′2′, 3′3′ and 4′4′ at points p_1', p_2', p_3', and p_4'. Similarly project q_1, q_2, q_3 and q_4 to FV and obtain points q_1', q_2', q_3', and q_4'.
4. Draw lines $p_1'p_2'$, $p_2'p_3'$, $q_1'q_2'$, $q_2'q_3'$. Lines $p_1'p_4'$, $p_3'p_4'$, $q_1'q_4'$, $q_3'q_4'$ coincide with the above lines. These lines show the line of intersection.

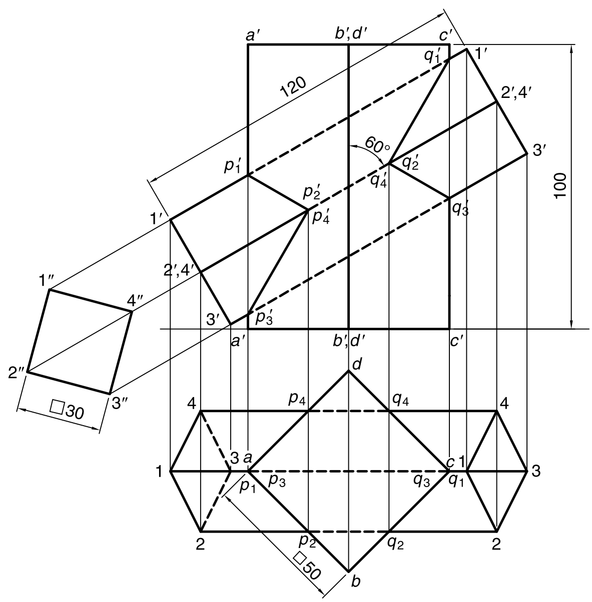


Fig. 14.20

5. Show the portion of inclined prism which is inside the vertical prism by dotted lines in both the FV and TV.

Problem 14.19 *A vertical cylinder of base diameter 80 mm and axis 130 mm, is resting on its base in the H.P. It is penetrated by another cylinder of base diameter 50 mm and axis 150 mm. The axes of both the cylinders are parallel to the V.P. and bisect each other at an angle of 60 degrees. Draw their projection and show the curves of intersection.*

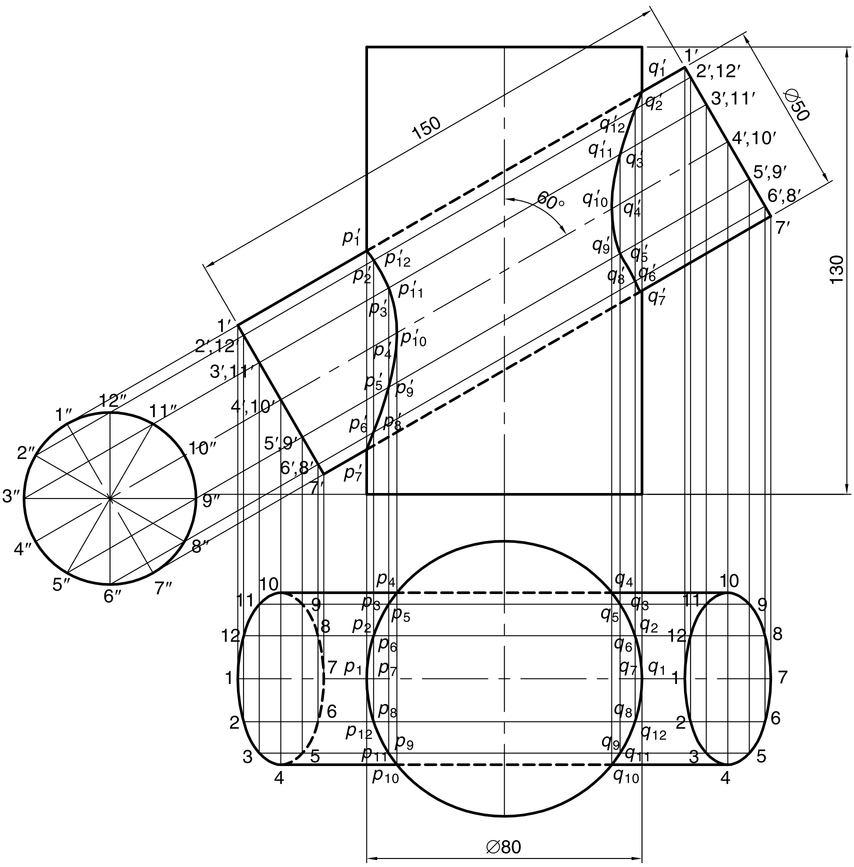


Fig. 14.21

Construction Refer to Fig. 14.21.

1. Draw the projections of both the cylinders as shown. Draw a circle with a 50 mm diameter to represent the axial view of the penetrating cylinder.
2. First locate the points of intersection of the generators 1-1, 2-2, 3-3, etc., of the inclined cylinder with the base circle of the vertical cylinder on the left side as p_1, p_2, p_3 etc., and on the right side as q_1, q_2, q_3 etc., in the TV.
3. Project points p_1, p_2, p_3, etc., to FV to meet their corresponding, generators 1′1′, 2′2′, 3′3′, etc., at points p_1', p_2', p_3' etc. Similarly, project q_1, q_2, q_3, etc., to FV and obtain points q_1', q_2', q_3', etc.

4. Draw curves joining points p_1', p_2', p_3', p_4' etc., and q_1', q_2', q_3', q_4', etc. These curves show the curves of intersection.
5. Show the portion of inclined cylinder which is inside the vertical cylinder by dotted lines in both the FV and TV.

Problem 14.20 *A cone of base diameter 80 mm and axis 100 mm, is resting on its base in the H.P. It is penetrated by a cylinder of base diameter 50 mm and axis 120 mm. The axis of the cylinder is perpendicular to the H.P. and 10 mm away from the axis of the cone. Draw their projections and show the curve of intersection.*

Construction Refer to Fig. 14.22.

1. Draw the projections of the cone and cylinder and label them as shown.
2, Mark points p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}, p_{11} and p_{12} as the intersection of cylinder with the generators of the cone.
3. Project these points to obtain p_1', p_2', p_3', p_4', p_5', p_6', p_7', p_8', p_9', p_{10}', p_{11}' and p_{12}' in the FV.
4. Join these points to get the required curve.

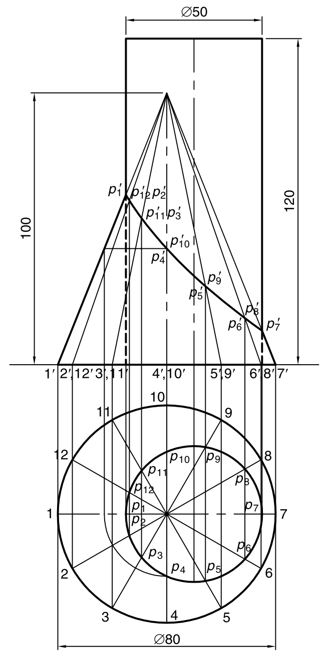


Fig. 14.22

14.10 INTERSECTION OF SPHERE BY ANOTHER SOLID

Problem 14.21 *A sphere of diameter 80 mm is penetrated by a square prism of base side 40 mm. The faces of the prism are equally inclined to V.P. while the axis is perpendicular to H.P. and passes through the centre of the sphere. Draw the projections of the combination and show the curves of intersection.*

Construction Refer to Fig. 14.23.

1. Draw circles of diameter 80 mm to represent FV and TV of the sphere.
2. Draw a square 1-4-7-10 of side 50 mm, concentric to the circle, to represent the TV of the penetrating prism and obtain its FV.
3. Consider seven vertical section planes, passing one by one through points 10, 11-9, 12-8, 1-7, 2-6, 3-5 and 4.
4. The sections of sphere and prism by these section planes are circles and rectangles in the FV.
5. Join points common to both the corresponding sections to obtain the curve of intersection as shown.

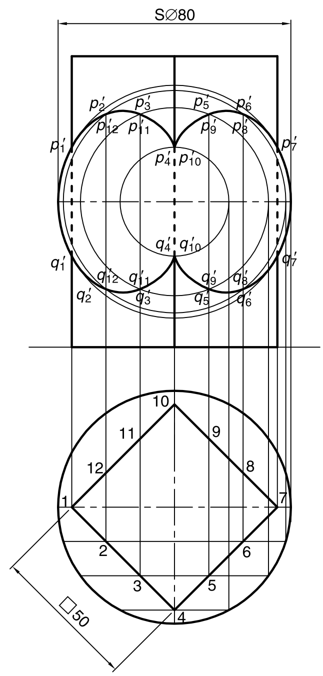


Fig. 14.23

14.11 MISCELLANEOUS PROBLEMS

Problem 14.22 *A hexagonal prism of base side 40 mm and axis 100 mm, is resting on its base on H.P. with a side of base parallel to V.P. It is penetrated by a horizontal cylinder of base diameter 50 mm and axis 100 mm such that their axes bisect each other at right angles. Draw three views of the combination and show the curves of intersection.*

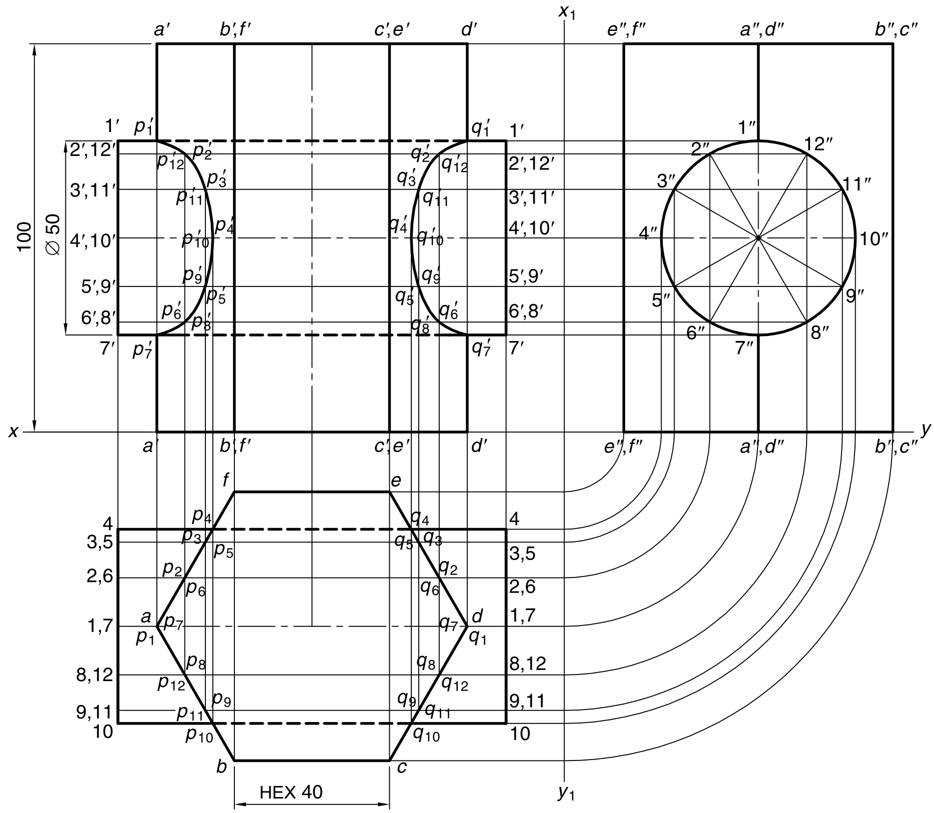


Fig. 14.24

Construction Refer to Fig. 14.24.

1. Draw a hexagon *abcdef* of side 40 mm to represent TV of the prism. Project its FV and SV with 100 mm long axis and label them.
2. Draw SV of the cylinder which is a circle of diameter 50 mm and the centre of which bisects the axis of the prism at right angles. Project this circle to get the FV and TV and label them as shown.
3. The faces of the prism are seen as lines in the TV. First locate the points of intersection in the TV of the generators 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12 of the cylinder with the faces of the prism on left side as p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}, p_{11}, p_{12} and on right side as q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9, q_{10}, q_{11}, q_{12}.

4. Project points $p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}, p_{11}, p_{12}$ to FV to meet their corresponding generators 1′, 2′, 3′, 4′, 5′, 6′, 7′, 8′, 9′, 10′, 11′, 12′ at points $p_1', p_2', p_3', p_4', p_5', p_6', p_7', p_8', p_9', p_{10}', p_{11}', p_{12}'$. Similarly, project $q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9, q_{10}, q_{11}, q_{12}$ to FV and obtain points $q_1', q_2', q_3', q_4', q_5', q_6', q_7', q_8', q_9', q_{10}', q_{11}', q_{12}'$.
5. Join points $p_1' p_2' p_3' p_4' p_5' p_6' p_7'$ and $q_1' q_2' q_3' q_4' q_5' q_6' q_7'$ by continuous lines as shown. These lines show the curves of intersection.

Problem 14.23 *A square pyramid of base side 70 mm and axis 100 mm, is resting on its base on the H.P. with a side of base inclined at 30° to the V.P. A square prism of base side 30 mm having its axis parallel to both the principal planes is penetrated through it. The axes of the solids intersect each other 30 mm above the base of the pyramid. Draw the projections showing the curves of intersection when rectangular faces of the prism are equally inclined to the H.P.*

Construction Refer to Fig. 14.25.

1. Draw three views of the square pyramid and the penetrating prism as usual.

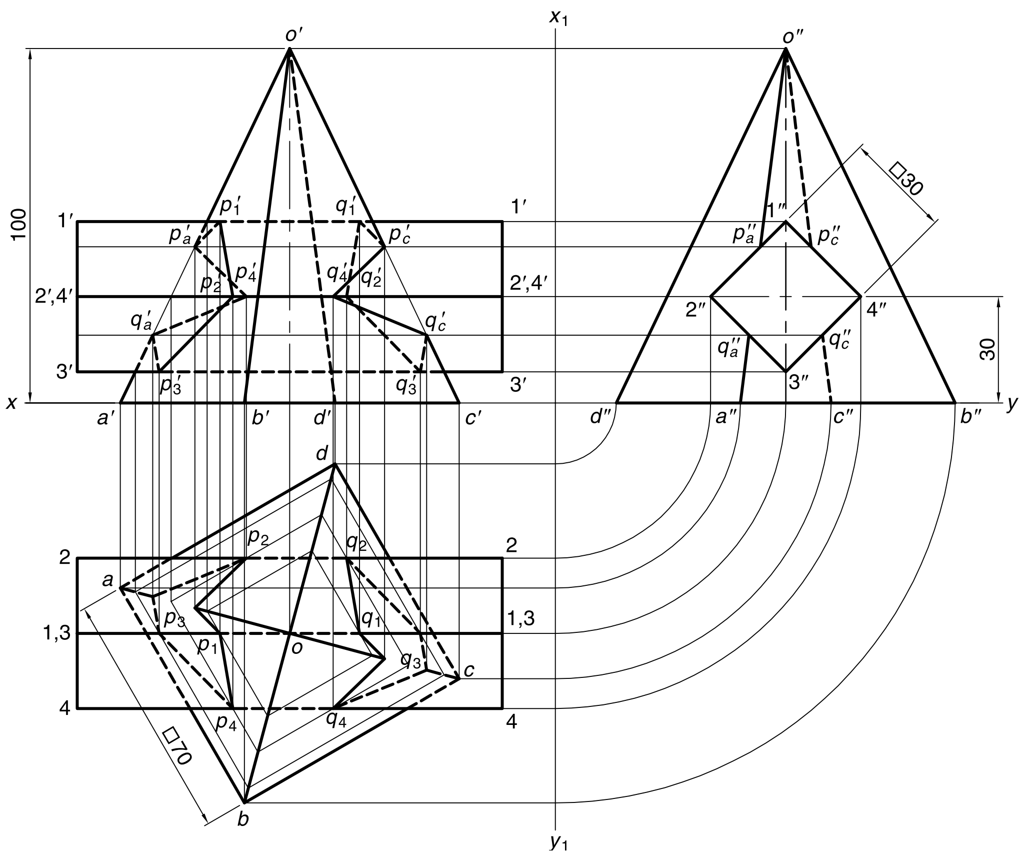


Fig. 14.25

2. Consider three horizontal section planes, passing one by one through 1′, 2′4′, 3′ and obtain the sections of the pyramid as squares in the top view.
3. Mark points which are common to the squares obtained in Step 2 with the edges of the prism in the TV and project to their corresponding positions in the FV.
4. Also, mark points of intersection of $o''a''$ and $o''c''$ with the square 1″2″3″4″ in the SV and obtain p_a', q_a', p_c' and q_c' in the FV and p_a, q_a, p_c and q_c in the TV.
5. Join points as shown to obtain curve of intersection in the FV and TV.

Problem 14.24 *A cylinder of base diameter 70 mm is resting on its base on the H.P. It is completely penetrated by a cone of base diameter 80 mm and axis 100 mm. The axes of the two solids are parallel to the V.P. and bisect each other at right angles. Draw three views of the combination and show curves of intersection.*

Construction Refer to Fig. 14.26.

1. Draw three views of the given cylinder and the given cone in their respective positions as shown.

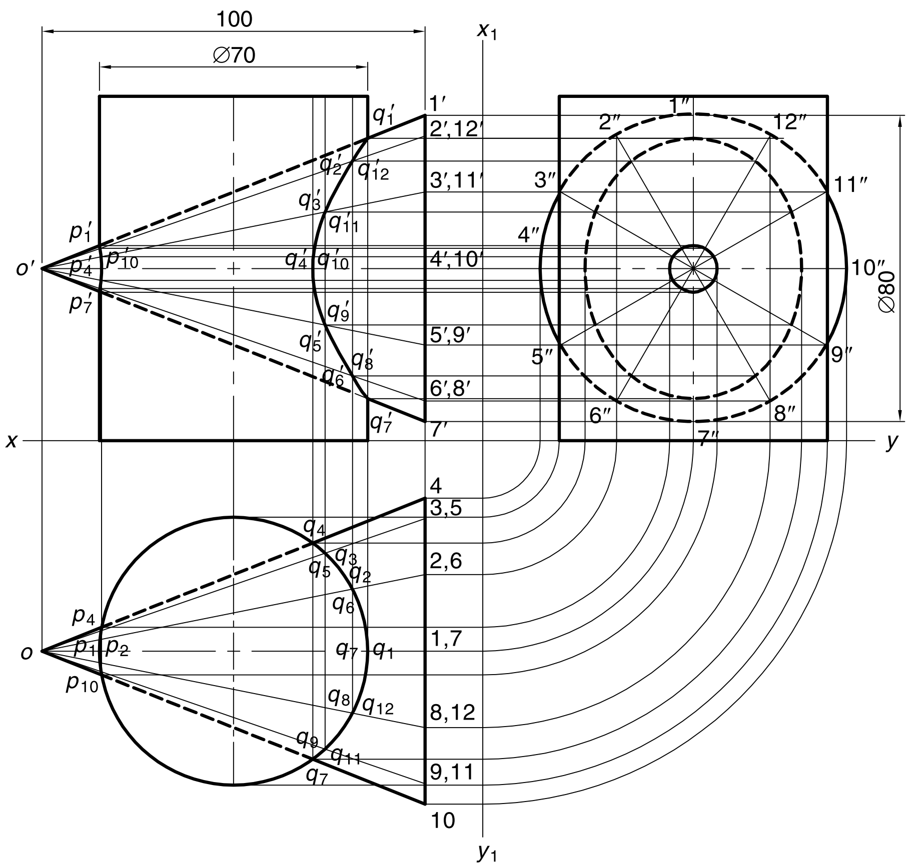


Fig. 14.26

2. Mark 12 generators on the cone in all the three views and label them.
3. Consider seven section planes, one by one, all perpendicular to V.P., passing through the apex and generators of the cone.
4. All these planes cut the cylinder and give the same circle in the top view. The sections of cone by these planes are isosceles triangles in the top view.
5. Mark the points of intersection of these triangles with the base circle of the cylinder in the top view.
6. Transfer these points to FV and SV at their respective positions and join them to get the desired curve of intersection in FV and SV.

EXERCISES

14.1 A hexagonal prism of base side 40 mm and axis 100 mm is resting on its base on theH.P. with a side of the base parallel to the V.P. It is penetrated by a square prism of base side 35 mm and axis 100 mm such that the axes of both the prisms intersect each other at right angles. The faces of the square prism is equally inclined to the H.P. Draw the projections of the combination and show the lines of intersection.

14.2 A pentagonal prism of base side 45 mm and axis 100 mm is resting on its base on the H.P. with a side of the base parallel to the V.P. It is penetrated by a square prism of base side 35 mm and axis 100 mm such that the axes of both the prisms bisect each other at right angles. The faces of the square prism are equally inclined to the H.P. Draw the projections of the combination and show the lines of intersection.

14.3 A square prism of base side 50 mm is resting on its base on H.P. with the faces equally inclined to the V.P. It is completely penetrated by a horizontal cylinder of base diameter 50 mm such that their axes of bisect each other at right angles. Assuming suitable lengths of both the solids draw their projections and show the curves of intersection.

14.4 A square prism of base side 60 mm and axis 100 mm is resting on its base onH.P. with the faces equally inclined to the V.P. It is completely penetrated by a hexagonal prism of base side 30 mm and axis 100 mm having a face parallel to H.P. The axes of the prisms bisect each other at right angles. Draw their projections and show the curves of intersection.

14.5 A triangular prism of base side 80 mm and axis 100 mm, is resting on its base on theH.P. with a side of the base parallel to the V.P. It is penetrated by another triangular prism of base side 40 mm and axis 100 mm having a face parallel to the H.P. The axes of the prisms bisect each other at right angles. Draw the projections of the combination and show the lines of intersection.

14.6 A square prism of base side 60 mm and axis 100 mm is resting on its base on theH.P. with the faces equally inclined to the V.P. It is penetrated by another square prism of the same dimensions having its axis parallel to both the reference planes and 15 mm away from the axis of the first prism. Draw the projections of the combination and show lines of intersection when the faces of the penetrating prism are equally inclined to the H.P.

14.7 A square prism of base side60 mm and axis 100 mm is resting on its base on theH.P. with a face inclined at 30º to the V.P. It is penetrated by a horizontal square prism of base side 45 mm and axis 100 mm. The axis of the penetrating prism is 15 mm away from that of the former and a face inclined at 30º to the H.P. Draw three views of the combination and show lines of intersection.

14.8 A square prism base side 50 mm and axis 90 mm, rests on its base on the ground with a face inclined at 30º to the V.P. It is penetrated by a horizontal cylinder of diameter 40 mm. Their axes bisect each other at right angles. Draw three views of the combination and show the curves of intersection.

14.9 A triangular prism of base side 60 mm and axis 100 mm is resting on its base on the H.P. with a nearer face parallel to the V.P. It is penetrated by a cylinder of base diameter 50 mm and axis 90 mm. The axis of the cylinder is parallel to both the reference planes and 15 mm away from the axis of the prism towards the observer. Draw the projections of the combination and show the curves of intersection.

14.10 A cylinder resting on its base on the H.P. is penetrated by another cylinder with their axes bisecting at right angles. Draw the projections of the combination and show the curves of intersection. Consider the base diameters of the vertical and the penetrating cylinders as 60 mm and 50 mm respectively.

14.11 A vertical cylinder of base diameter 60 mm rests on its base on the H.P. It is penetrated by a horizontal cylinder of same diameter such that their axes bisect each other at right angles. Draw their three views and show the curves of intersection.

14.12 A cylinder of base diameter 70 mm is resting on its base on the H.P. It is penetrated by another cylinder of base diameter 50 mm, the axis of which is parallel to both the principal planes. The two axes are 10 mm apart. Draw the projections of the combination and show the curves of intersection.

14.13 A cylinder of base diameter 60 mm rests on its base on the ground. It is penetrated by another cylinder of base diameter 40 mm, the axis of which is parallel to both the principal planes. The axes of the two cylinders are 8 mm apart. Draw the projections of the combination and show the curves of intersection.

14.14 A vertical cylinder of base diameter 60 mm rests on its base on the ground. It is penetrated by another cylinder of base diameter 50 mm, the axis of which is parallel to both the principal planes. The axes of the two cylinders are 12 mm apart. Draw the projections of the combination and show the curves of intersection.

14.15 A cylinder of base diameter 70 mm is resting on its base in the H.P. It is penetrated by a square prism of base side 30 mm, the axis of which is parallel to both the principal planes and faces equally inclined to the V.P. The axes of the cylinder and prism bisect each other. Draw the projections of the combination and show the curves of intersection.

14.16 A cylinder of base diameter 70 mm is resting on its base in the H.P. It is penetrated by a hexagonal prism of base side 30 mm, the axis of which is parallel to both the principal planes and a face parallel to the H.P. The axes of the cylinder and prism bisect each other. Draw the projections of the combination and show the curves of intersection.

14.17 A cylinder of base diameter 60 mm is resting on its base in the H.P. It is penetrated by a square prism of base side 30 mm, the axis of which is parallel to both the principal planes and faces equally inclined to the H.P. The axes of the cylinder and prism are 15 mm apart. Draw the projections of the combination and show the curves of intersection.

14.18 A cylinder of base diameter 70 mm is resting on its base in the H.P. It is penetrated by a triangular prism of base side 55 mm, the axis of which is parallel to both the principal planes and a face parallel to and 30 mm above H.P. Draw the projections of the combination and show the curves of intersection.

14.19 A hexagonal pyramid of base side 45 mm and axis 100 mm is resting on its base on the H.P. with a side of the base parallel to the V.P. It is penetrated by a horizontal square prism of base side 30 mm and axis 100 mm. The axis of the prism is 30 mm above the H.P. parallel to V.P. and intersects the axis of the pyramid. The faces of the prism is equally inclined to the H.P. Draw the projections of the combination and show the lines of intersection.

14.20 A pentagonal pyramid of base side 50 mm and axis 100 mm is resting on its base on the H.P. with a side of the base parallel to the V.P. It is penetrated by a square prism of base side 30 mm such that its axis is parallel to both the principal planes and intersects the axis of the pyramid 30 mm above the H.P. Draw the projections of the combination and show the lines of intersection if the faces of the prism are equally inclined to the H.P.

14.21 A square pyramid of base side 50 mm and axis 100 mm is resting on its base on H.P. with all the edges of the base equally inclined to the V.P. It is penetrated by a horizontal cylinder of base diameter 40 mm such that its axis intersects the axis of the pyramid 30 mm above the H.P. Draw their three views and show the curves of intersection.

14.22 A square pyramid of base side 60 mm and axis 100 mm is resting on its base on H.P. with the edges of the base equally inclined to the V.P. It is completely penetrated by a hexagonal prism of base side 25 mm and axis 100 mm having a face parallel to H.P. The axis of the prism meets the axis of the pyramid at right angles 30 mm above the ground. Draw their projections and show the curves of intersection.

14.23 A triangular pyramid of base side 80 mm and axis 80 mm is resting on its base on the H.P. with a side of the base parallel to the V.P. It is penetrated by a triangular prism of base side 30 mm and axis 100 mm having a face parallel to and 15 mm above the H.P. The axis of the prism meets the axis of the pyramid at right angles. Draw the projections of the combination and show the lines of intersection.

14.24 A square pyramid of base side 60 mm and axis 90 mm is resting on its base on theH.P. with the edges of the base equally inclined to the V.P. It is penetrated by a square prism of base side 30 mm and axis 100 mm. The axis of the prism is parallel to both the reference planes and 15 mm away from the axis of the pyramid and 30 mm above the H.P. Draw the projections of the combination and show the lines of intersection when the faces of the penetrating prism are equally inclined to the H.P.

14.25 A square pyramid side of base side 60 mm and axis 100 mm is resting on its base on the H.P. with an edge of the base inclined at 30º to the V.P. A horizontal square prism of base side 35 mm and axis 100 mm with faces equally inclined to the H.P. penetrates it such that its axis meets at a point 30 mm above the base. Draw three views of the combination and show the lines of intersection.

14.26 A square pyramid side of base side 70 mm and axis 90 mm with an edge of the base inclined at 30º to the V.P. It is penetrated by a cylinder of base diameter 45 mm. Their axes meet at right angles, 30 mm above the base of the pyramid. Draw their projections when the axes of the solids are on a plane parallel to the V.P. Also show the curves of intersection.

14.27 A cone resting on its base on the H.P. is penetrated by a cylinder. The axes of the solids intersect each other at right angles, 30 mm above the base of the cone. Draw the projections of the combination and show curves of intersection. Take the base diameter of the cone as 80 mm, axis 100 mm and the base diameter of the cylinder as 45 mm.

14.28 A cone of base diameter 80 mm and axis 90 mm rests on its base on the H.P. It is penetrated by a horizontal cylinder of base diameter 45 mm such that their axes intersect at right angles at such a point that the cylinder meets the extreme generators of the cone tangentially in the side view. Draw their three views and show the curves of intersection.

14.29 A cone of base diameter 80 mm and axis 100 mm rests on its base on the H.P. It is penetrated by a cylinder of base diameter 40 mm, the axis of which is parallel to both the principal planes and meets the axis of the cone at a distance 25 mm above the base. The two axes are 6 mm apart. Draw the projections of the combination and show the curves of intersection.

14.30 A cone of base diameter 80 mm and axis 100 mm rests on its base on the H.P. It is penetrated by a cylinder of base diameter 50 mm, the axis of which is parallel to both the principal planes and meets the axis of the cone at a distance 30 mm above the base. The two axes are 10 mm apart. Draw the projections of the combination and show the curves of intersection.

14.31 A cone of base diameter 80 mm and axis 90 mm is resting on its base in the H.P. It is penetrated by a square prism of base side 30 mm, the axis of which is parallel to both the principal planes and 60 mm below the apex and faces equally inclined to the H.P. Draw the projections of the combination and show the curves of intersection.

14.32 A cone of base diameter 70 mm and axis 80 mm is resting on its base in the H.P. It is penetrated by a triangular prism of base side 45 mm, the axis of which is parallel to both the principal planes and a face parallel to and 10 mm above H.P. Draw the projections of the combination and show the curves of intersection.

14.33 A cone of base diameter 80 mm and axis 100 mm is resting on its base in the H.P. It is penetrated by a hexagonal prism of base side 25 mm, the axis of which is parallel to V.P. and a face parallel to theH.P. and 5 mm above the base of the cone. Draw the projections of the combination and show the curves of intersection.

14.34 A cone of base diameter 80 mm and axis 100 mm is resting on its base in the H.P. It is penetrated by a square prism of base side 30 mm, the axis of which is parallel to both the principal planes and faces equally inclined to the H.P. The axis of the prism is 15 mm away from that of the cone and 70 mm below the apex. Draw the projections of the combination and show the curves of intersection.

14.35 A cone of base diameter 60 mm and axis 100 mm is resting on the H.P. It is penetrated by a square prism of base side 40 mm such that its axis is parallel to and 10 mm away from that of the cone. The faces of the prism are equally inclined to the V.P. The plane containing the two axes is parallel to the V.P. Draw the projections of the combination and show curves of intersection.

14.36 A cone of base diameter 90 mm and axis 110 mm is resting on its base on the H.P. It is completely penetrated by another cone of base diameter 80 mm and axis 120 mm, the axis of which is parallel to both the principal planes and 35 mm above the H.P.

The axes of the solids intersect each other at right angles. Draw the projections of the combination and show the curves of intersection.

14.37 A sphere of diameter 80 mm is penetrated by a square prism of base side 30 mm. The faces of the prism are equally inclined to the V.P. while the axis is perpendicular to the H.P. and passes through a point 12 mm away from the centre of the sphere. Draw the projections of the combination and show curves of intersection.

14.38 A sphere of diameter 80 mm is penetrated by a cylinder of base diameter 50 mm. The axis of the cylinder is perpendicular to the H.P. and passes through a point 12 mm away from the centre of the sphere. Draw the projections of the combination and show curves of intersection.

VIVA-VOCE QUESTIONS

14.1 What do you mean by key points? What is its significance in intersection of surfaces?

14.2 Name the methods of determining the curves of intersection.

14.3 Describe the conditions in which the curves of intersection between cylinder and cylinder is represented by straight lines.

14.4 Describe the conditions in which the curves of intersection between a cone and a cylinder is represented by straight lines.

MULTIPLE-CHOICE QUESTIONS

14.1 When two prisms intersect at right angle, the curve of intersection is made up of
(a) circular arc
(b) elliptical arc
(c) curved line
(d) straight line

14.2 When two cylinders of equal diameters envelope a common sphere, the curve of intersection is made up of
(a) parabola
(b) semi-circle
(c) straight line
(d) None of these

14.3 The line of intersection between cylinder and cone, unless they envelope a common sphere, is made up of
(a) straight line
(b) curved line
(c) circular arcs
(d) parabolic curve

14.4 Which of the following method is **not** used for obtaining curves of intersection?
(a) Line method
(b) Curve method
(c) Generator method
(d) Cutting plane method

14.5 The study of intersection of surfaces helps in
(a) sheet metal work
(b) building drawing
(c) architectural drawing
(d) All of these

14.6 The intersection of a cone by a plane results in
(a) conic section
(b) cycloid
(c) helix
(d) None of these

14.7 The curve of intersection of any solid with a line is
(a) a point
(b) a line
(c) a closed loop
(d) None of these

14.8 When a cylinder penetrates into a vertical cone with their axes parallel to each other, the top view of the curve of intersection is
(a) a circle
(b) an ellipse
(c) a parabola
(d) a cycloid

14.9 When a vertical cylinder is penetrated by a horizontal cylinder, the top view of the curve of intersection is
(a) circular arc
(b) elliptical arc
(c) closed loop
(d) None of these

14.10 A cone resting on its base in the H.P. is penetrated by a horizontal cylinder. The top view of the curve of intersection results in
(a) circular arc
(b) elliptical arc
(c) closed loop
(d) None of these

14.11 The curve of intersection of a vertical cylinder with an auxiliary vertical plane is
(a) a point
(b) a straight line
(c) a curved line
(d) a closed loop

14.12 The curve of intersection of a vertical cone with an auxiliary vertical plane is
(a) straight line
(b) elliptical curve
(c) parabolic curve
(d) hyperbolic curve

14.13 The points at which the curve of intersection changes its nature are known as
(a) arbitrary points
(b) key points
(c) crucial points
(d) intersection points

Answers to multiple-choice questions

14.1 (d), 14.2 (c), 14.3 (b), 14.4 (b), 14.5 (a), 14.6 (a), 14.7 (a), 14.8 (a), 14.9 (a), 14.10 (c), 14.11 (b), 14.12 (d), 14.13 (b)

Chapter

15

ISOMETRIC PROJECTIONS

15.1 INTRODUCTION

Isometric projection is used to create a pictorial drawing of an object. It is defined as *a single-view parallel projection obtained by keeping the object in such a position that all the three mutually perpendicular geometrical axes are equally inclined to the plane of projection*. The projectors follow the rules of orthographic projections, i.e. projectors are parallel to each other and perpendicular to the plane of projection.

A multiview drawing requires two or more orthographic projections to define the exact shape of a three-dimensional object. Each orthographic view is a two-dimensional drawing showing only two out of three dimensions of the object. Consequently, no single view contains sufficient information to completely define the shape of the object. All orthographic views must be correlated together to interpret the object. These views can only be correctly interpreted and visualized by those persons who have a good knowledge of the principles of projections and a great practice to read multiviews to interpret the actual shape. Thus, it is necessary to draw a pictorial view of one kind or the other so as to enable a common man to understand. The isometric projection is most popular as it provides the overall view of the object at the first sight.

15.2 AXONOMETRIC PROJECTION

An axonometric projection is a type of single-view parallel projection used to create a pictorial drawing of an object. The object is placed in such a position that the three mutually perpendicular faces are visible from a single direction. Because an object can be placed in unlimited number of positions relative to the axonometric plane, an infinite number of axonometric views can be drawn. For practical reasons, these positions are recognized to classify the axonometric projections into the following divisions.

1. Trimetric projection Trimetric projection is an axonometric projection of an object so placed that no two axes make equal angle with the plane of projection as shown in Fig. 15.1(a). In other words, each of the three axes and the lines parallel to them, respectively, have different ratios of foreshortening when projected to the plane of projection. Since the three axes are foreshortened differently, three different trimetric scales must be prepared and used. Trimetric perspective is seldom used and is found in only a few video games.

2. Dimetric projection Dimetric projection is an axonometric projection of an object so placed that two of its axes make equal angle with the plane of projection and the third axis makes either a smaller or a greater angle. In other words, two axes making equal angle with the plane of projection and the lines parallel to them are foreshortened equally while the third axis is foreshortened in a different ratio (See Fig. 15.1(b)). Since the three axes are foreshortened in two different manners, two different dimetric scales must be prepared and used.

3. Isometric projection Isometric projection is an axonometric projection of an object so placed that all three axes make equal angle with the plane of projection. In other words, all the three axes and the lines parallel to them, respectively, have the same ratios of foreshortening when projected to the plane of projection (See Fig. 15.1(c)). Since the three axes are foreshortened equally, single isometric scale is prepared and used.

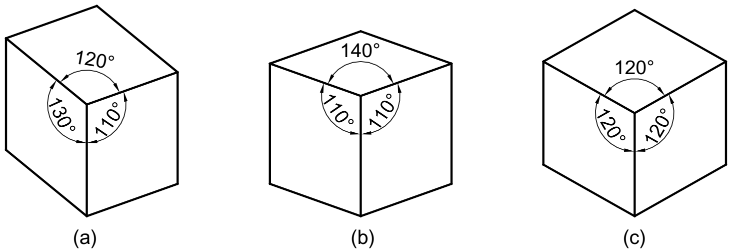


Fig. 15.1 *Axonometric projection* **(a)** *Trimetric* **(b)** *Dimetric* **(c)** *Isometric*

Thus, isometric projection is the simplest of these because the principal axes make equal angles with the plane of projection and are foreshortened equally. It is widely used in industries because of its ease of construction, the ability to measure and compare dimensions parallel to the isometric axes directly, and to combine isometrics from different sources into composite projections. Engineers and technologists often use pictorial drawings in one form or another, to supplement and clarify machine and structural details that would be otherwise difficult to visualize through multiviews. As the three-dimensional pictorial shows the entire details of an object in a single view, it enhances the ability to visualize and conveys technical information quickly.

15.3 PRINCIPLE OF ISOMETRIC PROJECTION

The term 'isometric' comes from the Greek language which means 'equal measure', reflecting that the scale along each axis of the projection is the same. The isometric projection can be visualized is by considering a view of a cube with one of the solid diagonals perpendicular to the vertical plane and the three axes equally inclined to the vertical plane as shown in Fig. 15.2. The final front view is the isometric projection of the cube.

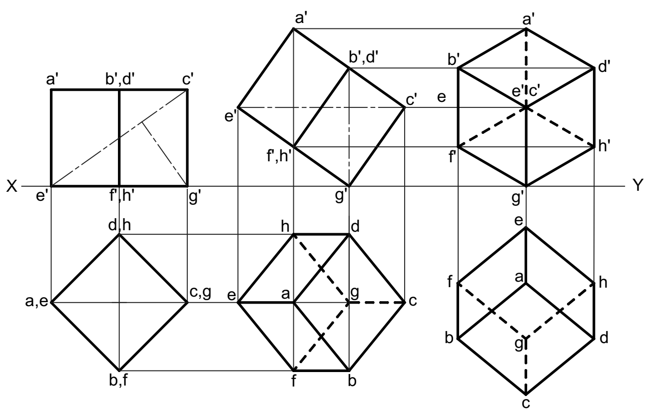


Fig. 15.2 *Cube rests on the H.P. with a solid diagonal perpendicular to the V.P.*

Figure 15.3(a) shows the front view when hidden lines are removed. It gives the realistic view of the cube. The corners are renamed in capital letters. A keen study of this view reveals the following information.

1. The outer boundary *ABFGHDA* is a regular hexagon.
2. All the faces of the cube which are actually square in shape appear as rhombus.
3. The three lines *CB*, *CD* and *CG* meeting at *C*, represent the three mutually perpendicular edges of the cube.
   (a) They make equal angles of 120° with each other.
   (b) They are equal in length but smaller than the true length of the edge of the cube.
   (c) The line *CG* is vertical, and the other lines *CB* and *CD* make 30° with the horizontal.
4. All other lines representing the edges of the cube are parallel to one or the other of the above three lines, i.e., *CB*, *CD* and *CG*, and are equally foreshortened.
5. The diagonal *BD* of the top face *ABCD* is parallel to V.P., and hence shows its true length.

A comparison of the rhombus *ABCD* of the front view with the square face of the cube (represented by *A′BC′D*) is shown in Fig. 15.3(b).

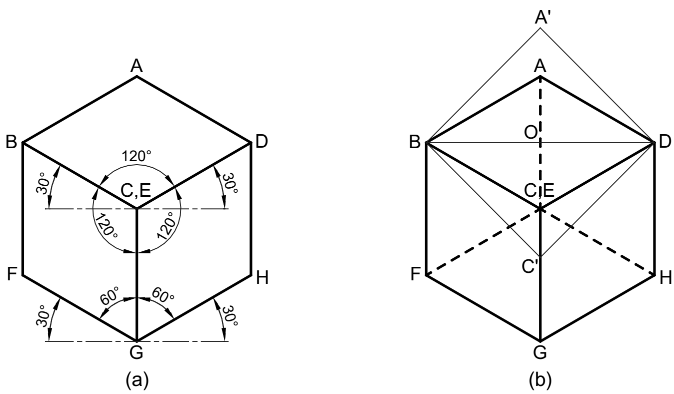


Fig. 15.3 **(a)** *Analysis of the axes and included angles* **(b)** *Comparison of faces with actual cube*

15.4 TERMINOLOGY

Referring to Fig. 15.3 (a), the important terms used in isometric projections are as follows:

1. **Isometric axes** The three lines *CB*, *CD* and *C*G, meeting at point *C* and inclined at an angle of 120° with each other, are called isometric axes.
2. **Isometric lines** The lines parallel to the isometric axes are called isometric lines. Here lines *AB*, *BF*, *FG*, *GH*, *DH* and *AD* are isometric lines.
3. **Non-isometric lines** The lines which are not parallel to isometric axes are known as non-isometric lines. Here diagonals *BD*, *AC*, *CF*, *BG*, etc., are non-isometric lines.
4. **Isometric plane** The plane representing any face of the cube as well as other plane parallel to it is called an isometric plane. Here, *ABCD*, *BCGF*, *CGHD*, etc., are isometric planes.
5. **Non-isometric plane** The plane which is not parallel to isometric planes are known as non-isometric planes. Here, the plane *ABGH*, *CDEF*, *AFH*, *CFH*, etc., are non-isometric planes.
6. **Isometric scale** It is the scale which is used to convert the true length into isometric length. Mathematically, Isometric length = 0.816 × True length

15.5 CONSTRUCTION OF AN ISOMETRIC SCALE

Referring to Fig. 15.3(b), all the edges of the cube are equally foreshortened. Therefore, the square faces are seen as rhombuses in the isometric projection. The foreshortening of the edge can be calculated as follows:

In triangle ABO, $\dfrac{BA}{BO} = \dfrac{1}{\cos 30^\circ} = \dfrac{2}{\sqrt{3}}$

In triangle $A'BO$, $\dfrac{BA'}{BO} = \dfrac{1}{\cos 45^\circ} = \dfrac{\sqrt{2}}{1}$

Therefore, $\dfrac{\text{Isometric length}}{\text{True length}} = \dfrac{BA}{BA'} = \dfrac{2}{\sqrt{3}} \times \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{\sqrt{3}} = \dfrac{9}{11}$ (approx) or 0.816 (approx)

This reduction of the true length can be obtained either by multiplying it by a factor 0.816 or by taking the measurement with the help of an isometric scale. Figure 15.4(a) shows the conventional isometric scale. The steps of construction are as follows:

1. Draw a horizontal line *bo*.
2. Draw lines *ba′* and *ba* inclined at 45° and 30° with line *bo*, respectively.
3. Mark off the true scale on the line *ba′* as 0′, 10′, 20′, 30′, etc.
4. Draw vertical lines from points 0′, 10′, 20′, 30′, etc., to meet line *ba* at points 0, 10, 20, 30, etc. The marked divisions of *ba* represent the isometric lengths.

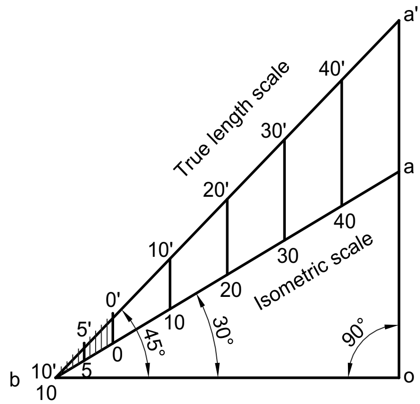


Fig. 15.4(a) *Conventional isometric scale*

15.5.1 Simplified Form of Isometric Scale

The isometric scale can also be constructed in a simplified way by using the principles of plane or diagonal scales taking R.F. = $\sqrt{2}:\sqrt{3}$. Figures 15.4(b) and (c) show plano-isometric and diagono-isometric scales respectively, for a maximum length of 11 cm. The length of scale can be calculated as $L_s = \dfrac{\sqrt{2}}{\sqrt{3}} \times 11$ = 8.98 cm ≈ 9 cm (approx). The steps of construction are self-explanatory.

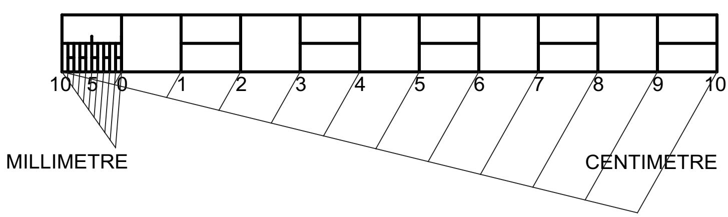


Fig. 15.4(b) *Plano-isometric scale*

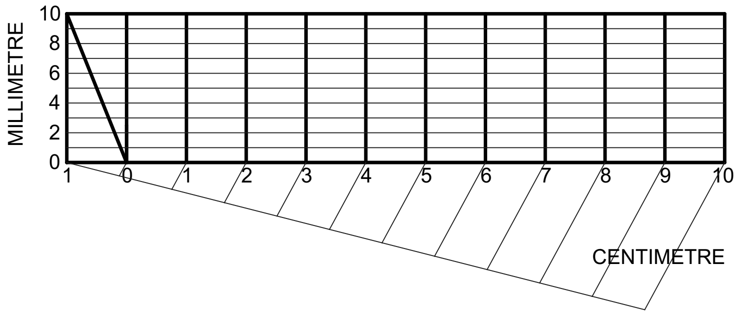


Fig. 15.4(c) *Diagono-isometric scale*

15.6 CHARACTERISTICS OF PRINCIPAL LINES IN ISOMETRIC PROJECTION

The following are the characteristics of the principal lines in an isometric projection:

1. All lines that are parallel on the object are parallel on the isometric projection.
2. Vertical line on the object remains vertical in the isometric projection.
3. The horizontal lines on the object are drawn at an angle of 30° with the horizontal.
4. The lines parallel to the principal lines known as isometric lines are equally foreshortened.
5. The lines which are not parallel to principal lines known as non-isometric lines are not equally foreshortened. For example, diagonals *BD* and *AC* are of equal lengths in front view but are of different lengths in the isometric projection. The non-isometric lines are drawn by locating positions of their ends on isometric planes.

15.7 ISOMETRIC PROJECTION AND ISOMETRIC VIEW

In an isometric projection, a scale factor of 0.816 is used to prepare the drawing whereas in an isometric view the true length is used. Thus, the isometric view of an object is larger than the isometric projection. Because of ease of construction and advantage of measuring the dimensions directly from the drawing, it has become a general practice to use the true lengths instead of isometric lengths.

Figure 15.5(a) shows the orthographic views of a cuboid. Figure 15.5 (b) shows its isometric projection whereas Fig. 15.5 (c) shows its isometric view. Thus, isometric projection looks smaller in size than the isometric view. It may be noted that, if it is desired to draw isometric view an object containing some spherical feature it is general practice to draw the isometric projection only.

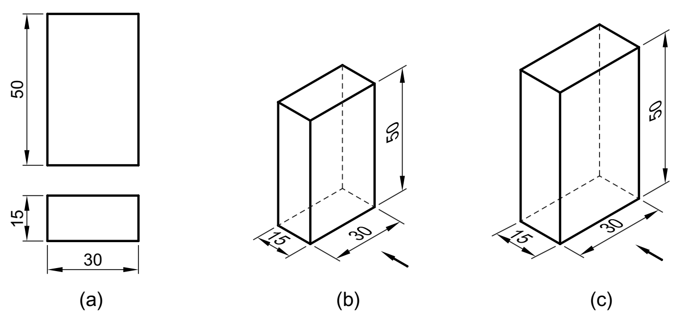


Fig. 15.5 **(a)** *Orthographic projection* **(b)** *Isometric projection* **(c)** *Isometric view*

15.8 DIMENSIONING ON ISOMETRIC PROJECTION

Following points should be remembered while dimensioning an isometric projection or view:

1. While dimensioning isometric projection or isometric view, the true length should be written for the dimension values.
2. As far as possible all extension lines and dimension lines must be isometric lines, lying in isometric planes.
3. It is usual practice to avoid the hidden lines unless they are essential to make the drawing clear.
4. Centre lines of the circular features should be drawn parallel to the isometric axis.
5. Dimensions for the circular feature should lie on the plane in which it appears to a greater extent.

15.9 ISOMETRIC VIEW OF PLANES

To draw the isometric view of a plane surface of a lamina, draw a suitable principal plane on which it lies. Figures 15.6(a) and (b) show the vertical principal planes whereas Fig. 15.6(c) and (d) show the horizontal principal planes. The arrow shows the direction for viewing the front view.

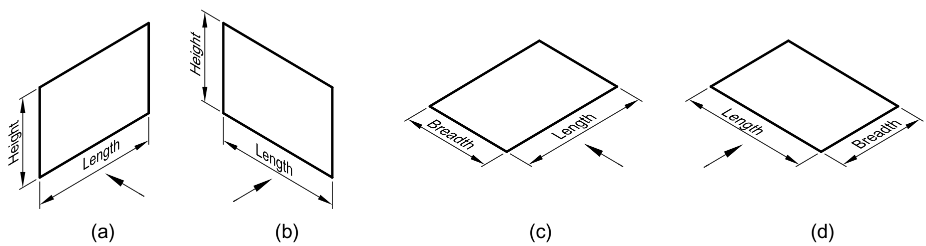


Fig. 15.6 **(a)** *and* **(b)** *Vertical planes* **(c)** *and* **(d)** *Horizontal planes*

When the plane is having its surface parallel to the V.P., then the vertical isometric plane is used to draw the isometric view or projection as shown in Fig. 15.6(a) and (b). Height is taken on the vertical axis and the length is taken on the axis inclined at 30° to the horizontal.

When the plane is having its surface parallel to the H.P., then horizontal isometric plane is used to draw the isometric view or projection as shown in Fig. 15.6(c) and (d). Length and breadth are taken on the different axes each inclined at 30° to the horizontal.

Problem 15.1 *Draw the isometric view of a square of side 40 mm kept in (a) vertical position and (b) horizontal position.*

Construction

1. **When the square is kept vertical (Fig. 15.7(a) and (b)).**
   (a) Draw a 40 mm long vertical line *AD*.
   (b) Draw another 40 mm long line *AB* inclined at 30° to the horizontal.
   (c) Complete the rhombus *ABCD* which is the required isometric view.
2. **When the square is kept horizontal (Fig. 15.7(c))**
   (a) Draw 40 mm long lines *AD* and *AB*, each inclined at 30° to the horizontal such that the included angle between them is 120°.
   (b) Join *ABCD* to represent the isometric view of the square.

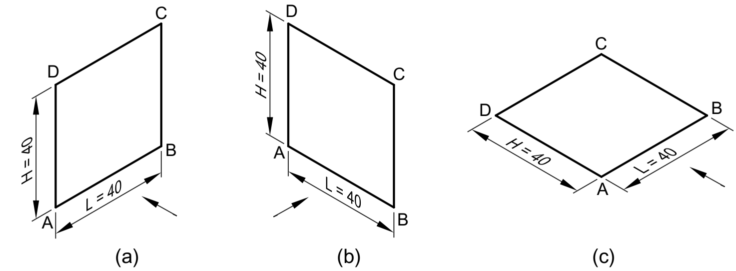


Fig. 15.7 *Isometric view of a square in* **(a)** *and* **(b)** *Vertical position* **(c)** *Horizontal position*

Problem 15.2 *Draw the isometric view of a triangle ABC whose front view is shown in Fig. 15.8(a).*

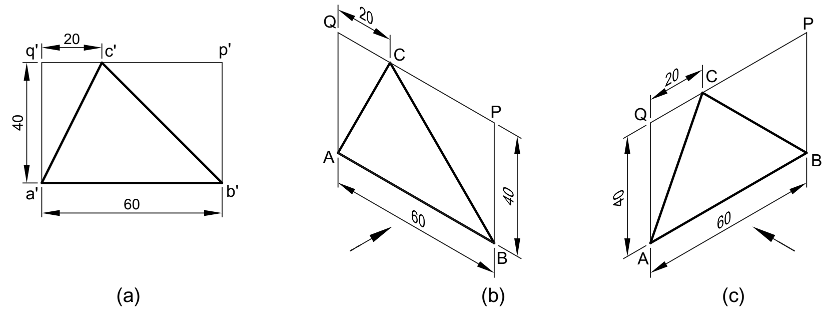


Fig. 15.8 *Triangle* **(a)** *Front view* **(b)** *and* **(c)** *Isometric views*

Construction Refer to Fig. 15.8(b) and (c).

Side *a′b′* is the isometric line while *b′c′* and *c′a′* are non-isometric lines.

1. Enclose the triangle in a rectangle *a′b′p′q′*.
2. Draw isometric view of the rectangle *a′b′p′q′*. For this,
   (a) Draw a vertical line *AQ* of length equal to *a′q′*.
   (b) Draw another line *AB* of length equal to *a′b′* and inclined at 30° to the horizontal.
   (c) Complete the parallelogram *ABPQ*.
3. Mark a point *C* on the side *PQ* such that *QC* = *q′c′*.
4. Join *ABC* to represent the isometric view of the triangle.

Problem 15.3 *Draw the isometric view of a quadrilateral ABCD whose top view is shown in Fig. 15.9(a).*

Construction Refer to Fig. 15.9(b) and (c).

1. Draw the given quadrilateral *abcd* and enclose it in a rectangle *abpq*.
2. Draw isometric view *ABPQ* of the rectangle *abpq* with sides inclined at 30° to the horizontal as shown in Fig. 15.9(b) or (c).
3. Mark a point *C* on line *PQ* such that length *QC* = *qc*.
4. Draw *SD* = *sd* and *RD* = *rd* and is parallel to isometric sides *AB* and *AQ*, respectively.
5. Join *ABCD* to represent the isometric view of the quadrilateral.

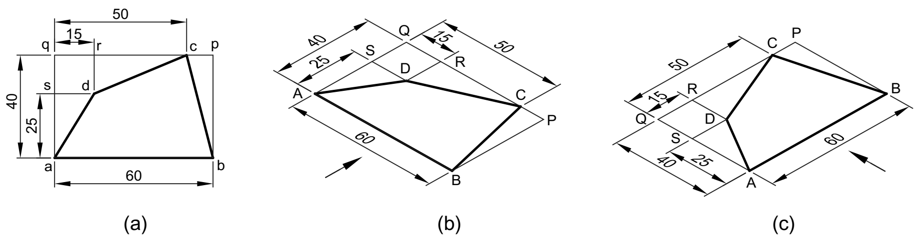


Fig. 15.9 *Quadrilateral* **(a)** *Top view* **(b)** *and* **(c)** *Isometric views*

Problem 15.4 *Draw the isometric view of a hexagon of side 30 mm whose surface is parallel to the H.P. and a side parallel to the V.P.*

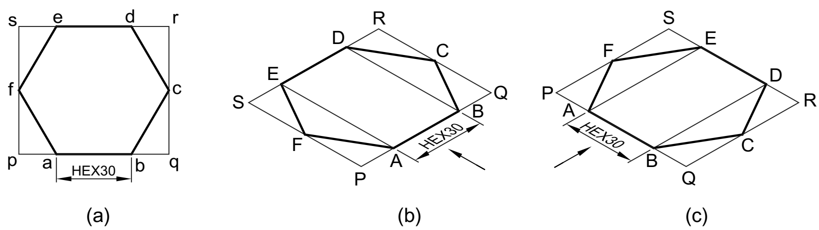


Fig. 15.10 *Hexagon* **(a)** *Orthographic view* **(b)** *and* **(c)** *Isometric views*

Construction Refer to Fig. 15.10(b) and (c).

1. Draw a hexagon *abcdef* and enclose it in a rectangle *pqrs*.
2. Draw isometric view *PQRS* of the rectangle *pqrs* keeping sides inclined at 30° to the horizontal as shown in Fig. 15.10(b) or (c).
3. Mark points *A* and *B* on side *PQ* such that *PA* = *pa* and *PB* = *pb*.
4. Mark point *C* on side *QR* such that *QC* = *qc*.
5. Mark points *D* and *E* on side *RS* such that *RD* = *rd* and *SE* = *se*.
6. Mark point *F* on side *PS* such that *PF* = *pf*.
7. Join *ABCDEF* to represent the isometric view of the hexagon.

Problem 15.5 *Draw the isometric view of a hexagonal plane of side 40 mm whose surface is parallel to the V.P. and a side perpendicular to the H.P.*

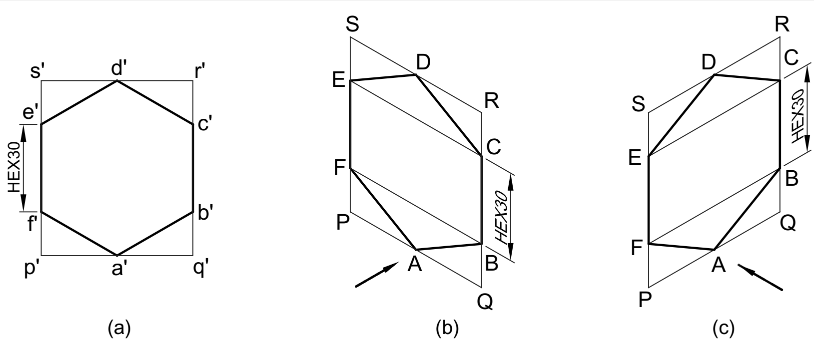


Fig. 15.11 *Hexagon* **(a)** *Orthographic view* **(b)** *and* **(c)** *Isometric views*

Construction Refer to Fig. 15.11(b) and (c).

1. Draw a hexagon *a′b′c′d′e′f′* as the front view keeping side *e′f′* vertical. Enclose the hexagon in a rectangle *p′q′r′s′*.
2. Draw the isometric view of the rectangle *p′q′r′s′* as *PQRS* keeping *PQ* inclined at 30° to the horizontal and *QR* vertical.
3. Mark point *A* on side *PQ*, points *B* and *C* on side *QR*, point *D* on side *RS*, and points *E* and *F* on side *RS*.
4. Join *ABCDEF* to represent the isometric view of the hexagon.

Problem 15.6 *Draw the isometric view of a circle of diameter 60 mm on all the three principal planes using coordinate method.*

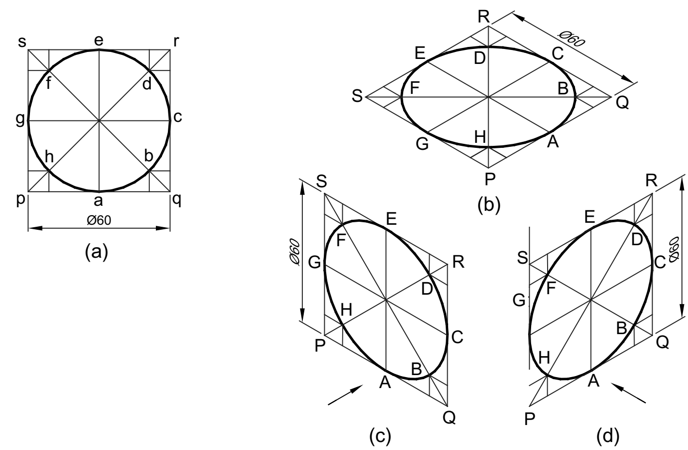


Fig. 15.12 *Circle* **(a)** *Orthographic view* **(b)** *and* **(c)** *Isometric views on different isometric planes*

Construction Refer to Fig. 15.12(b) and (d).

1. Draw a circle of 60 mm diameter to represent the orthographic view.
2. Enclose the circle in a square *pqrs* as shown in Fig. 15.12(a). Let the circle touch the sides of the square tangentially at points *a*, *c*, *e* and *g*.
3. Draw the diagonals of the square which meet the circle at points *b*, *d*, *f* and *h*.
4. Draw rhombus *PQRS* to represent the isometric view of the square *pqrs*.
5. Mark points *A*, *C*, *E* and *G* as the mid-point of the sides *PQ*, *QR*, *RS* and *PS* respectively.
6. Draw another rhombus *BDFH* of side *bd* and obtain points *B*, *D*, *F* and *H*.
7. Join *ABCDEFGH* to represent the isometric view of the circle.

15.10 FOUR CENTRE METHOD TO DRAW ELLIPSE AND ELLIPTICAL ARCS

A circle on an isometric plane appears as an ellipse in the isometric projection, as shown in Fig. 15.13(a) to (c). This ellipse can be drawn with ease using four centre method. Consider the following problem.

Problem 15.7 *Draw the isometric view of a circular lamina of diameter 50 mm on all the three principal planes using four centre method.*

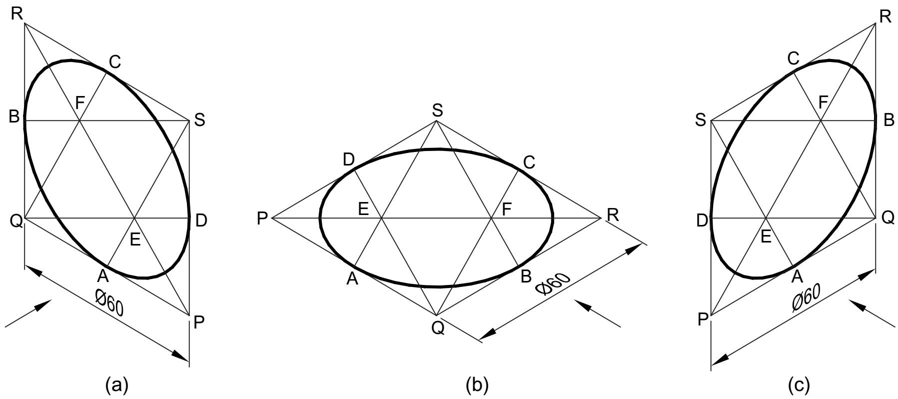


Fig. 15.13 **(a)** *to* **(c)** *Isometric views of a circle on different isometric planes*

Construction Refer to Fig. 15.13(a) to (c).

1. Draw rhombus *PQRS* with sides equal to the diameter of the circle to represent an isometric plane.
2. Mark *A*, *B*, *C* and *D* as the mid-points of the sides *PQ*, *QR*, *RS* and *PS* respectively.
3. Join end of the minor diagonal *Q* to meet mid-points *C* and *D*. Similarly, join end of the minor diagonal *S* to meet mid-points *A* and *B*.
4. Let lines *QD* and *AS* meet at point *E*. Lets lines *QC* and *BS* meet at point *F*. Then *Q*, *E*, *S* and *F* are the four centres for construction of the ellipse.
5. Draw arc *CD* with centre *Q* and radius *QC*. Draw arc *AB* with centre *S* and radius *BS*.
6. Draw arc *AD* with centre *E* and radius *AE*. Draw arc *BC* with centre *F* and radius *BF*.
7. This gives the required ellipse in the isometric view representing the circle.

Problem 15.8 *Draw the isometric view of a semi-circular plane of 60 mm diameter when (a) it is kept on the horizontal plane, and (b) when it is kept on the vertical plane.*

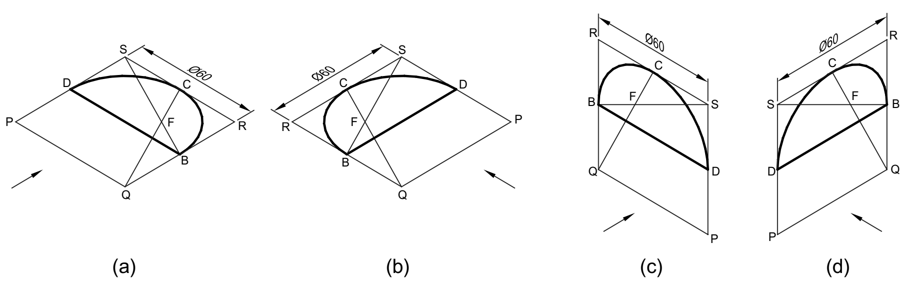


Fig. 15.14 *Isometric views of a semi-circular plane having surface parallel to the* **(a)** *and* **(b)** *H.P.* **(c)** *and* **(d)** *V.P.*

Construction Refer to Fig. 15.14(a) to (d).

1. Draw a rhombus *ABCD* of side equal to diameter of the semi-circle to represents the isometric view of the isometric plane.
2. Obtain four centres *Q*, *E*, *S* and *F* in each case as explained in Problem 15.7.
3. Use centres *Q* and *F*, draw half ellipse and join the end points *BD*.

15.11 ISOMETRIC VIEW OF RIGHT SOLIDS

The isometric view of a square prism, rectangular prism, etc., have their edges parallel to the isometric axes. The following points should be kept in mind:

1. The isometric view should be drawn such that maximum possible details are visible.
2. For every outer corner of the solid, at least three lines for the edges must converge. Of these at least two must be for the visible edges.
3. It is usual practice to avoid the hidden lines unless they are essential to make the drawing clear. However, it is advisable to check every corner so that no line for a visible edge is left out.
4. Two lines showing visible edge will never intersect each other.

Problem 15.9 *Draw the isometric view of a square prism of base side 40 mm and axis 60 mm resting on the H.P. on the (a) base with axis perpendicular to the H.P., (b) rectangular face with axis perpendicular to the V.P., and (c) rectangular face with axis parallel to the V.P.*

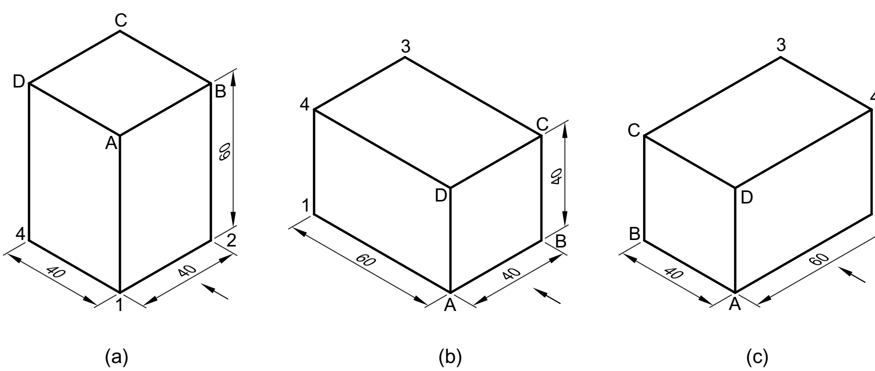


Fig. 15.15 **(a)** *to* **(c)** *Isometric views of a square prism in different orientations*

Construction

Case (a) (Fig. 15.15(a)) Draw rhombus *ABCD* such that *AB* and *AD* are inclined at 30° to the horizontal. Draw 60 mm long vertical lines *A*1, *B*2 and *D*4. Join 4-1-2.

Cases (b) and (c) (Fig. 15.15(b) and (c)) Draw rhombus *ABCD* such that *AB* is inclined at 30° to the horizontal and *AD* is vertical. Draw 60 mm long lines *A*1, *C*3 and *D*4 inclined at 30° to the horizontal. Join 1-4-3. If the object is viewed along the face *ABCD* the axis would be perpendicular to the V.P. and if the object is viewed along the rectangular face *AD*41 the axis would be parallel to the V.P.

15.12 ISOMETRIC VIEW OF SOLID CONTAINING NON-ISOMETRIC LINES

The inclined lines of an object are represented by non-isometric lines. These are drawn by one of the following methods:

1. **Box method** In the box method, the object is assumed to be enclosed in a rectangular box and both the isometric and non-isometric lines are drawn by locating the corresponding points of contact with the surfaces and edge of the box.
2. **Offset method** In the offset method, the lines parallel to the isometric axes are drawn from every corner or the reference point of an end to obtain the corner or the reference point at the other end.

Problem 15.10 *Draw the isometric view of a hexagonal prism of base side 30 and axis 70 mm. The prism is resting on its base on the H.P. with an edge of the base parallel to the V.P.*

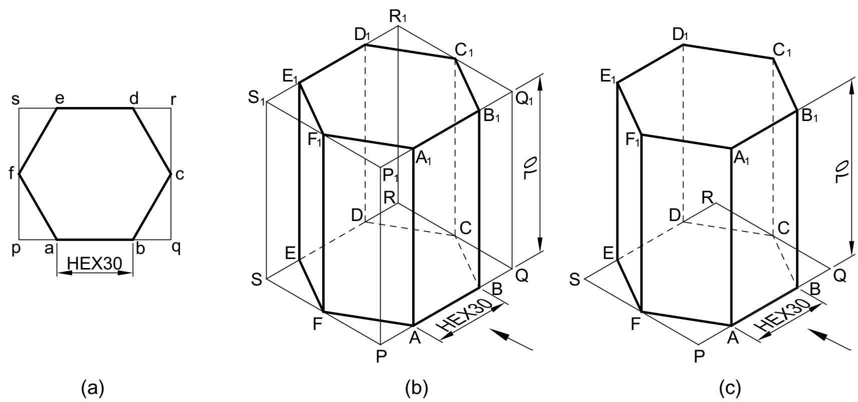


Fig. 15.16 *Hexagonal prism* **(a)** *Top view* **(b)** *and* **(c)** *Isometric views*

Construction

1. Draw a hexagon *abcdef* to represent the top view of the prism. Enclose the hexagon into a rectangle *pqrs* as shown in Fig. 15.16(a).
2. Draw isometric view *PQRS* of the rectangle *pqrs* keeping sides inclined at 30° to the horizontal.

Box method Refer to Fig. 15.16(b).

1. Erect 70 mm long vertical lines PP_1, QQ_1, RR_1 and SS_1. Join $P_1Q_1R_1S_1$.
2. Mark points A, B, C, D, E and F in the isometric view such that $PA = pa$, $PB = pb$, $QC = qc$, $RD = rd$, $SE = se$ and $PF = pf$.
3. Also, mark points A_1, B_1, C_1, D_1, E_1 and F_1 such that $P_1A_1 = pa$, $P_1B_1 = pb$, $Q_1C_1 = qc$, $R_1D_1 = rd$, $S_1E_1 = se$ and $P_1F_1 = pf$.
4. Join all the corners as shown to obtain the required isometric view.

Offset method Refer to Fig. 15.16(c).

1. Mark points A, B, C, D, E and F in the isometric view such that $PA = pa$, $PB = pb$, $QC = qc$, $RD = rd$, $SE = se$ and $PF = pf$.
2. Draw 70 mm long vertical lines AA_1, BB_1, CC_1, DD_1, EE_1 and FF_1.
3. Join $A_1B_1C_1D_1E_1F_1$ to obtain the required isometric view.

Problem 15.11 *Draw an isometric projection of a pentagonal prism of base side 35 mm and axis 60 mm. The prism rests on its base on the H.P. with an edge of the base parallel to the V.P.*

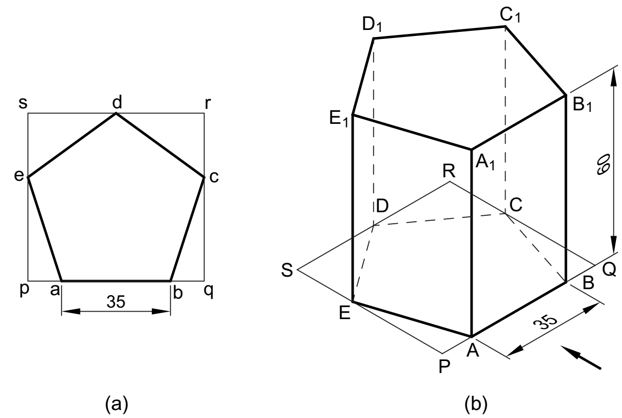


Fig. 15.17 *Pentagonal prism* **(a)** *Top view on a scale factor of 0.816* **(b)** *Isometric projection*

Construction Refer to Fig. 15.17(b).

1. Use isometric scale for all measurements because it is desired to draw isometric projection.
2. Draw a pentagon *abcde* of side 35 × 0.816 mm to represent the top view of the prism. Enclose the pentagon into a rectangle *pqrs* as shown in Fig. 15.17(a).
3. Draw isometric view *PQRS* of the rectangle *pqrs* keeping sides inclined at 30° to the horizontal.
4. Mark points *A*, *B*, *C*, *D* and *E* in the isometric view such that *PA* = *pa*, *PB* = *pb*, *QC* = *qc*, *SD* = *sd* and *PE* = *pe*.
5. Draw 60 × 0.816 mm long vertical lines AA_1, BB_1, CC_1, DD_1 and EE_1.
6. Join $A_1B_1C_1D_1E_1$ to obtain the required isometric view. Dotted lines may be drawn for clarity. Dimension the figure indicating true lengths.

Note: In isometric projection, figures are drawn on a scale factor of 0.816 (isometric scale). The dimension figures show true lengths and not the isometric lengths.

Problem 15.12 *Draw the isometric view of a cylinder of base diameter 50 mm and axis 60 mm. The axis of the cylinder is perpendicular to the (a) H.P., (b) V.P.*

Construction

Using offset method Refer to Fig. 15.18(a) and (b).

1. Draw a rhombus *ABCD* of side 50 mm. In case (a) *AB* and *BC* are inclined at 30° to the horizontal whereas in case (b) *AB* is inclined at 30º to the horizontal and *AD* is vertical.
2. Inscribe ellipse in the rhombus *ABCD* using four centre method, refer to Problem 15.7.
3. Draw 60 mm long lines BB_1, EE_1, DD_1 and FF_1 along the isometric axis as shown.
4. Draw another ellipse using four centre method where centre-points are B_1, E_1, D_1 and F_1.
5. Draw common tangents to connect both the ellipse.
6. Erase the inner half of the ellipse (lower half in case (a), rear half in case (b)) to obtain the required isometric view.

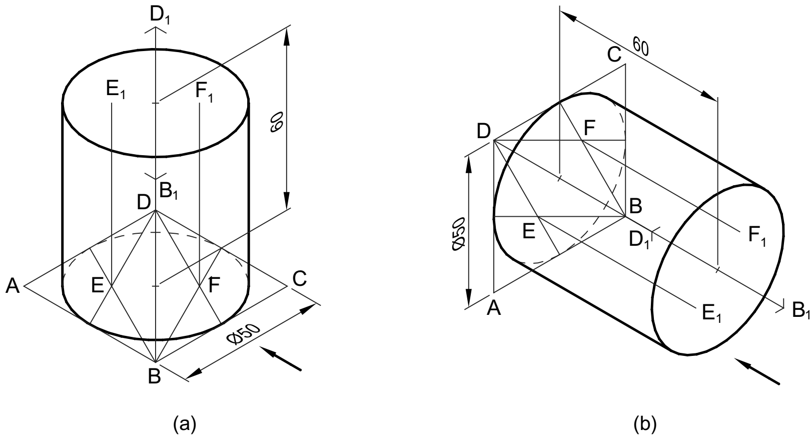


Fig. 15.18 *Isometric view of a cylinder having axis perpendicular to the* **(a)** *H.P.* **(b)** *V.P.*

Problem 15.13 *Draw the isometric view of a pentagonal pyramid of base side 30 mm and axis 50 mm. The pyramid is kept on its base on the (a) H.P. (b) V.P.*

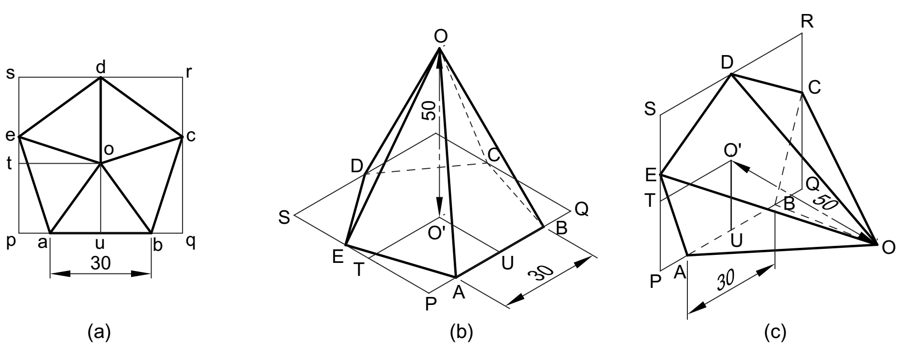


Fig. 15.19 *Pentagonal pyramid* **(a)** *Top view* **(b)** *and* **(c)** *Isometric view*

Construction

Using offset method Refer to Fig. 15.19(b) and (c).

1. Draw a pentagon *abcde* and join all the corners with its centroid *o*. Enclose the pentagon into a rectangle *pqrs* as shown in Fig. 15.17(a).
2. Draw a parallelogram *PQRS* of side lengths *pq* and *ps*. In case (a), *PQ* and *PS* are inclined at 30° to the horizontal whereas in case (b), *PQ* is inclined at 30° to the horizontal and *PS* is vertical.
3. Mark points *A*, *B*, *C*, *D* and *E* on the edges of the rhombus such that *PA* = *pa*, *PB* = *pb*, *QC* = *qc*, *SD* = *sd* and *PE* = *pe*.
4. Mark points *T* and *U* on *PS* and *PQ* respectively such that *PT* = *pt* and *PU* = *pu*. Mark point *O′* such that *TO′* = to and *UO′* = *uo*, where *TO′* and *UO′* are along the isometric axis.
5. Draw line *OO′* 50 mm long along the isometric axis, in case (a) *OO′* is vertical and in case (b) *OO′* is inclined at 30° to the horizontal.
6. Join points *A*, *B*, *C*, *D* and *E* with apex *O* to obtain the required isometric view.

Problem 15.14 *Draw the isometric projection of a cone of base diameter 50 mm and axis 60 mm. The cone has its base on the (a) H.P. (b) V.P.*

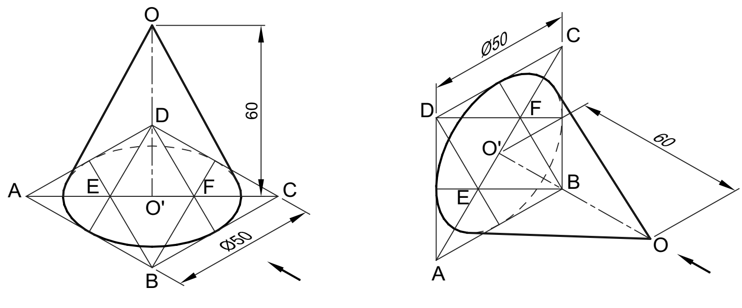


Fig. 15.20 *Isometric projection of a cone having its base on the* **(a)** *H.P.* **(b)** *V.P.*

Construction

Using offset method Refer to Fig. 15.20(a) and (b).

1. Use isometric scale for all measurements because it is desired to draw isometric projection.
2. Draw a rhombus *ABCD* of side 50 × 0.816 mm. In case (a), *AB* and *BC* are inclined at 30° to the horizontal whereas in case (b), *AB* is inclined at 30° to the horizontal and *AD* is vertical.
3. Inscribe ellipse in the rhombus *ABCD* using four centre method, refer to Problem 15.7.
4. Mark point *O′* as the mid-point of the rhombus *ABCD*.
5. Draw line *OO′*, 60 × 0.816 mm long along the isometric axis, in case (a), *OO′* is vertical and in case (b), *OO′* is inclined at 30° to the horizontal.
6. Draw two tangents from point *O* to the ellipse.
7. Erase the portion of the ellipse which is not visible to obtain the required isometric projection.

Problem 15.15 *A triangular prism of base edge 40 mm and axis 60 mm has an edge of its base on the H.P. The axis is parallel to the V.P. and inclined at 30° to the H.P. Draw the isometric view of the prism in the stated condition.*

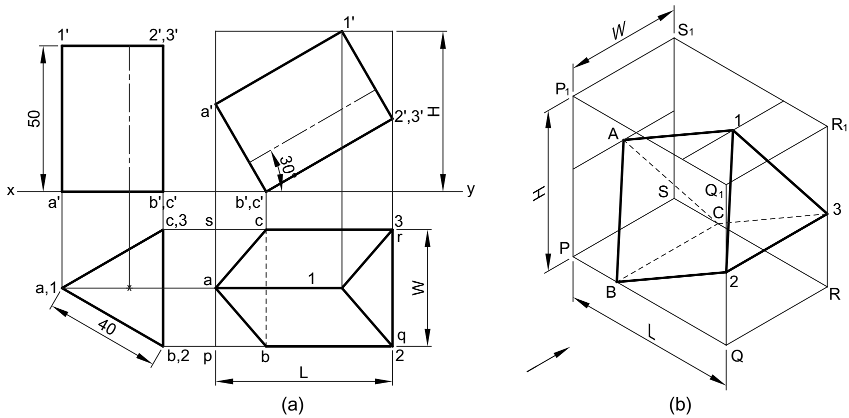


Fig. 15.21 **(a)** *Orthographic views* **(b)** *Isometric view*

Construction

Using box method Refer to Fig. 15.21.

1. **First stage** Draw a triangle *abc* keeping side *bc* perpendicular to *xy* to represent the top view. Project the corners to obtain *a′b′*2′1′ as the front view.
2. **Second stage** Reproduce the front view of the first stage keeping *b′* on *xy* and *b′*2′ inclined at 30° to *xy*. Project the points of the front view to meet the corresponding points from the top view of the first stage and obtain *ab*23*c* as the new top view.
3. Draw a rectangle *pqrs* to enclose the top view. Similarly, draw a rectangle to enclose the front view. Let these rectangle represents a cuboid of size $L \times W \times H$.
4. **Isometric view** Draw a rhombus *PQRS* of sides $L \times W$. Project the corners to a height *H* and obtain rhombus $P_1Q_1R_1S_1$.
5. Mark point *A*, *B*, *C*, 1, 2 and 3 inside the cuboid $PQRSP_1Q_1R_1S_1$ to obtain the required isometric view.

Problem 15.16 *A hexagonal pyramid of base side 30 mm and axis 60 mm long has an edge of its base on the H.P. Its axis is inclined at 30° to the ground and parallel to the V.P. Draw the isometric view of the pyramid in the stated condition.*

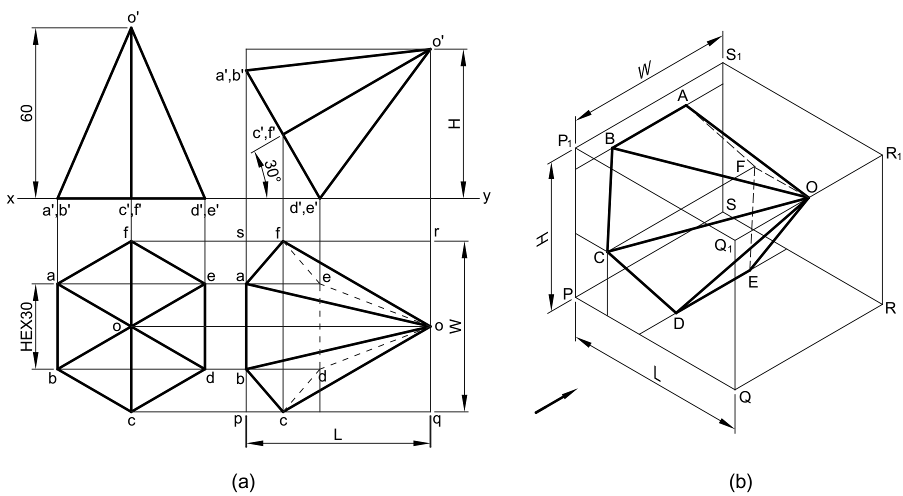


Fig. 15.22 **(a)** *Orthographic views* **(b)** *Isometric view*

Construction

Using box method Refer to Fig. 15.22.

1. **First stage** Draw a hexagon *abcdef* keeping side *de* perpendicular to *xy*. Join the corners with the centroid *o*. This represents the top view. Project the corners and obtain *b′d′o′* as the front view.
2. **Second stage** Reproduce the front view keeping *d′e′* on *xy* and *d′a′* inclined at 60° to *xy*. Project the points of the front view to meet the corresponding points from the top view of the first stage and obtain *abcof* as the new top view.
3. Draw a rectangle *pqrs* to enclose the top view. Similarly, draw a rectangle to enclose the front view. Let these rectangle represents a cuboid of size $L \times W \times H$.

4. **Isometric view** Draw a parallelogram *PQRS* of sides $L \times W$. Project the corners to a height H and obtain parallelogram $P_1Q_1R_1S_1$.
5. Mark point *A*, *B*, *C*, *D*, *E*, *F* and *O* inside the cuboid $PQRSP_1Q_1R_1S_1$ to obtain the required isometric view.

15.13 ISOMETRIC VIEW OF TRUNCATED SOLID

Problem 15.17 *Draw the isometric projection of the frustum of a hexagonal pyramid of base side 40 mm, top side 25 mm and height 70 mm. The frustum rests on the base on the H.P.*

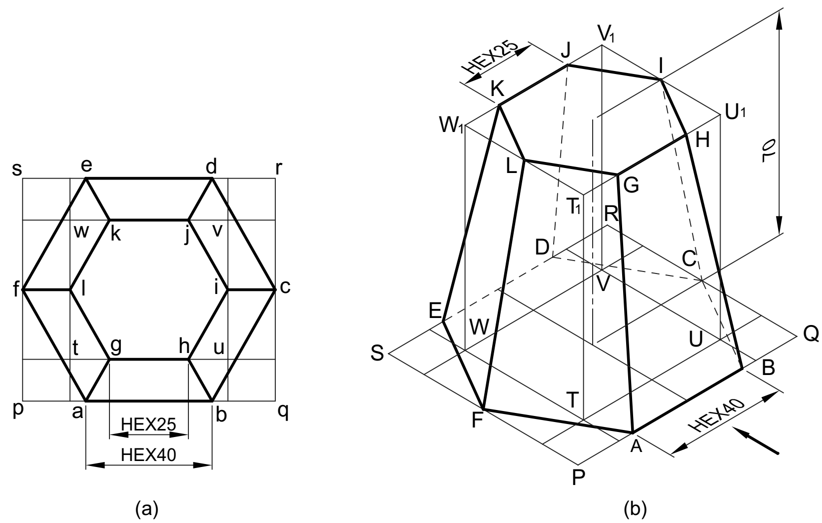


Fig. 15.23 *Frustum of a hexagonal pyramid* **(a)** *Top view on a scale factor* **(b)** *Isometric projection or view*

Construction Refer to Fig. 15.23.

1. Draw two concentric hexagons *abcdef* of side 40×0.816 mm and *ghijkl* of side 25×0.816 mm. Enclose the hexagons into rectangles *pqrs* and *tuvw* as shown in Fig. 15.23(a).
2. Draw parallelograms *PQRS* and *TUVW* having common centre and sides inclined at 30° to the horizontal.
3. Mark points *A*, *B*, *C*, *D*, *E* and *F* on the edges of the rhombus *PQRS*. Join *ABCDEF* to represent the hexagon of the lower base.
4. Project rhombus *TUVW* to a height of 70×0.816 mm to obtain $T_1U_1V_1W_1$.
5. Mark points *G*, *H*, *I*, *J*, *K* and *L* on the edges of the rhombus $T_1U_1V_1W_1$. Join *GHIJKL* to represent the hexagon for the upper base.
6. Join *AG*, *BH*, *CI*, *DJ*, *EK*, *FL* to represent the slant edges. Show visible edges of the frustum with dark lines and dimension the figure as shown.

Problem 15.18 *Draw the isometric projection of the frustum of a cone of base diameter 60 mm, top diameter 30 mm and height 55 mm.*

Construction

Using offset method Refer to Fig. 15.24.

1. Draw a rhombus *ABCD* of side 60 × 0.816 mm. Let *O* is the centre of the rhombus.
2. Inscribe an ellipse in the rhombus *ABCD* using four centre method, refer to Problem 15.7.
3. Mark axis OO_1 of 55 × 0.816 mm. Describe another rhombus *EFGH* about centre O_1 of side 25 × 0.816 mm.
4. Inscribe another ellipse in the rhombus *EFGH* using four centre method.
5. Darken the visible edges and dimension the figure as shown.

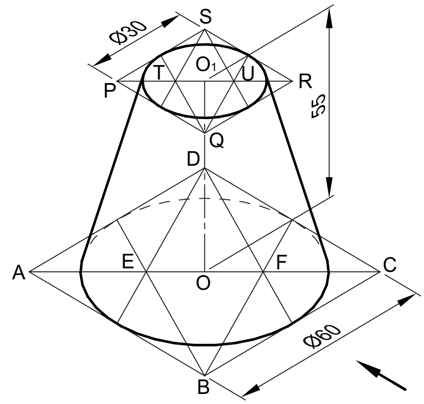


Fig. 15.24 *Isometric projection of the frustum of a cone*

Problem 15.19 *Draw the isometric view of a sphere of diameter 60 mm truncated by a horizontal plane at a height of 20 mm from the centre plane.*

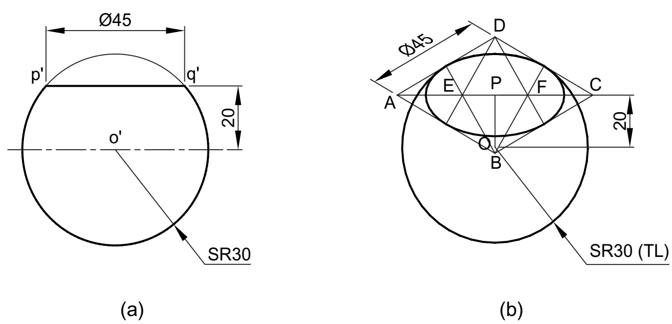


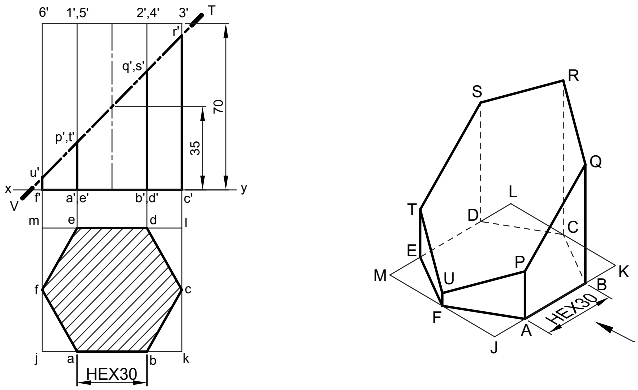
Fig. 15.25 *Sphere cut by a horizontal plane* **(a)** *Front view* **(b)** *Isometric projection*

Construction Refer to Fig. 15.25.

It is a usual practice to use a scale factor of 0.816 when an object contains spherical feature.

1. Draw a circle of diameter 60 mm with centre *o′*. Draw a chord *p′q′* to the circle at a distance of 20 mm from the centre, as shown in Fig. 15.25(a). The figure represents the front view of the truncated sphere. Measure length *p′q′* as 45 mm, representing diameter of the circle at the cut surface.
2. Draw a circle of true radius 30 mm with centre *O*, to represent the isometric projection of the whole sphere.
3. Mark a point *P* at a height of 20 × 0.816 mm from centre *O*. Point *P* represents centre of the circle at the cut surface. Describe about point *P*, a rhombus *ABCD* of side *p′q′* × 0.816 mm.
4. Inscribe ellipse in the rhombus *ABCD* using four centre method. The ellipse touches the sphere tangentially at two points.
5. Erase the part of the sphere lying above the tangent point to obtain the required isometric view or projection.

Problem 15.20 *A hexagonal prism of base side 30 mm and axis 70 mm is resting on its base on the H.P. with a side of the base parallel to the V.P. It is cut by an A.I.P. inclined at 45° to the H.P. and bisecting the axis. Draw its isometric view.*



(a) (b)

Fig. 15.26 *Hexagonal prism cut by an A.I.P.* **(a)** *Orthographic projections* **(b)** *Isometric view*

Construction Refer to Fig. 15.26.

1. Draw a hexagon *abcdef* to represent the top view. Project the top view to obtain *f′c′*3′6′ as the front view.
2. Draw VT of the section plane passing thought the mid-point of the axis and inclined at 45° to the *xy*, as shown in Fig. 15.26(a). Let it cut the edges *a′*1′ at *p′*, *b′*2′ at *q′*, *c′*3′ at *r′*, *d′*4′ at *s′*, *e′*5′ at *t′* and *f′*6′ at *u′*.
3. Draw a rectangle *jklm* to enclose the hexagon in the top view.
4. Draw isometric view *JKLM* of the rectangle *jklm* keeping sides inclined at 30° to the horizontal.
5. Mark points *A*, *B*, *C*, *D*, *E* and *F* in the isometric view such that *JA* = *ja*, *JB* = *jb*, *KC* = *kc*, *MD* = *md*, *ME* = *me* and *JF* = *jf*.
6. Erect vertical lines *AP*, *BQ*, *CR*, *DS*, *ET* and *FU* such that *AP* = *ap*, *BQ* = *bq*, *CR* = *cr*, *DS* = *ds*, *ET* = *et* and *FU* = *fu*.
7. Join *PQRSTU* and obtain the required isometric view.

Problem 15.21 *A triangular pyramid of base side 50 mm and axis 60 mm is resting on its base on the H.P. with an edge of the base parallel to the V.P. It is cut by an A.I.P. inclined at 45º to the H.P. and bisecting the axis. Draw its isometric view.*

Construction Refer to Fig. 15.27.

1. Draw a triangle *abc* keeping *ab* parallel to *xy*. Join the corners with centroid *o* to represent the top view. Project all the corners and obtain *a′b′o′* as the front view.
2. Draw VT of the section plane inclined at 45° to *xy* and passing though the mid-point of the axis. Let it cut the slant edges *a′o′* at *d′*, *b′o′* at *e′* and *c′o′* at *f′*. Project points *d′*, *e′* and *f′* to meet the respective edges in the top view and obtain points *d*, *e* and *f*.
3. Draw a rectangle *abpq* to enclose the triangle in the top view.

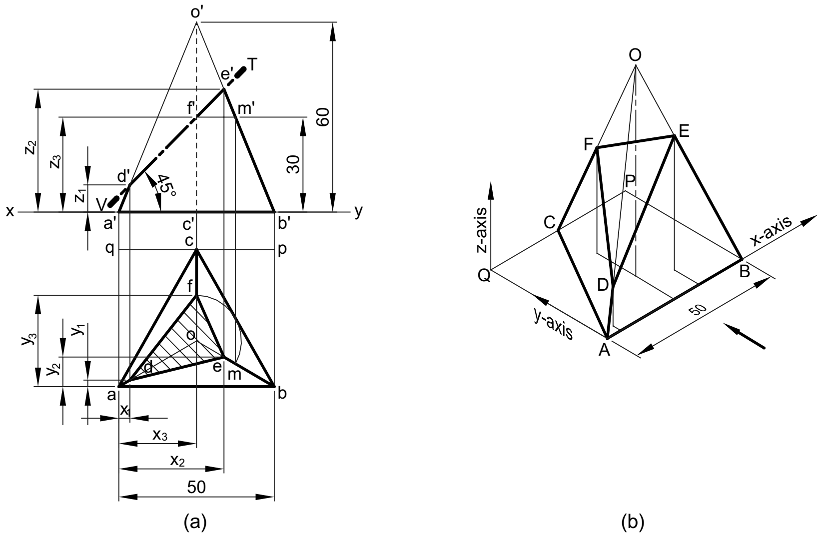


Fig. 15.27 *Triangular pyramid cut by an A.I.P.* **(a)** *Orthographic projections* **(b)** *Isometric view*

4. Draw isometric view *ABPQ* of the rectangle *abpq* keeping sides inclined at 30° to the horizontal. Mark *C* as the mid-point of *PQ*. Join *ABC* to represent the base of the pyramid.
5. Mark point *D* taking x_1, y_1, z_1 lengths from point *A* in *x*, *y* and *z*-directions, respectively.
6. Mark point *E* taking x_2, y_2, z_2 lengths from point *A* in *x*, *y* and *z*-directions, respectively.
7. Mark point *F* taking x_3, y_3, z_3 lengths from point *A* in *x*, *y* and *z*-directions, respectively.
8. Join *DEF*. Also join *AD*, *BE* and *CF* to obtain the required isometric view.

15.14 ISOMETRIC VIEW OF COMPOSITE SOLIDS

Problem 15.22 *A square pyramid of base side 25 mm and axis 40 mm rests centrally over a cylindrical block of base diameter 50 mm and thickness 20 mm. Draw the isometric projection of the arrangement.*

Construction Refer to Fig. 15.28.

1. Draw a line 1-2-O such that distance between 1-2 = 20 × 0.816 mm and 2-O = 40 × 0.816 mm.
2. Draw rhombus *ABCD* of side 50 × 0.816 mm side, keeping point 1 as its centre. Inscribe an ellipse in the rhombus *ABCD* using four centre method.
3. Project centre *B*, *D*, *E* and *F* for a length of 20 × 0.816 mm to obtain centres *G*, *I*, *J* and *H*. Draw another ellipse using four centre method, where

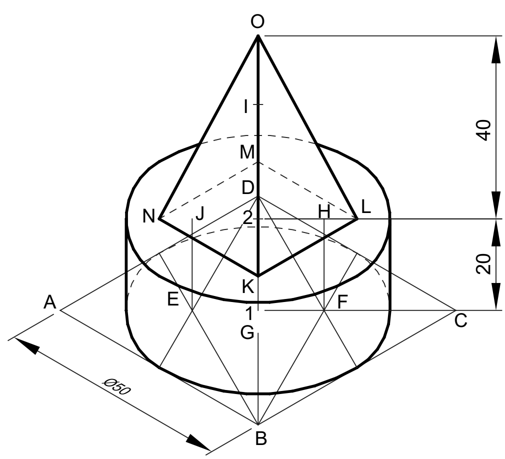


Fig. 15.28 *Isometric projection*

G, I, J and H are the centre points. (Alternatively, draw a 50×0.816 mm side rhombus keeping point 2 as its centre. Inscribe an ellipse inside this rhombus using four centre method.)

4. Draw rhombus $KLMN$ of side 25×0.816 mm keeping point 2 as its centre, to represent the base of the pyramid. Join OK, OL, OM and ON to represent the slant edges of the pyramid.
5. Darken the visible portion of the solid and dimension the figure.

Sometimes it is convenient to draw first the top surface and then project it to obtain the bottom surface. Consider the following problem.

Problem 15.23 *A hexagonal prism of base side 30 mm and axis 50 mm has an axially drilled square hole of sides 25 mm. One of the faces of the square hole is parallel to a face of the hexagon. Draw the isometric projection.*

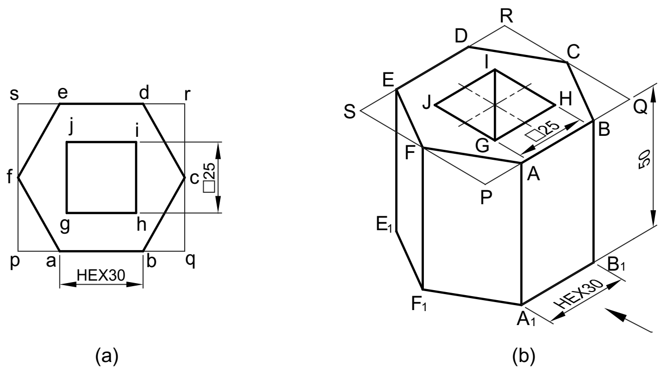


Fig. 15.29 *Hexagonal prism with a square hole* **(a)** *Top view* **(b)** *Isometric projection*

Construction Refer to Fig. 15.29.

1. Draw a hexagon *abcdef* of side 30×0.816 mm and a square *ghij* of side 25×0.816 mm such that they have a common centre. This represents the top view of the prism on a scale factor of 0.816. Enclose the hexagon into a rectangle *pqrs* as shown in Fig. 15.29(a).
2. Draw isometric projection $PQRS$ of the rectangle *pqrs* keeping sides inclined at 30° to the horizontal.
3. Mark points A, B, C, D, E and F in the isometric projection such that $PA = pa$, $PB = pb$, $QC = qc$, $RD = rd$, $SE = se$ and $PF = pf$. Also mark points G, H, I, J as shown.
4. Draw 50×0.816 mm long vertical lines AA_1, BB_1, EE_1 and FF_1, visible to the observer.
5. Join visible edges of the base, i.e., $E_1F_1A_1B_1$, and visible edge of the hole, i.e., IG.

Problem 15.24 *A square prism of base edge 50 mm and height 60 mm is resting on its base on the H.P. such that a face is parallel to the VP. A square hole of side 30 mm is made through the prism. The axis of the hole and the prism bisect each other at right angles. The faces of the hole are equally inclined to the ground. Draw the isometric projection of the object.*

Construction Refer to Fig. 15.30.

1. Draw a rectangle $a'b'2'1'$ of sides 50×0.816 mm and 60×0.816 mm. Draw a square $p'q'r's'$ of side 30×0.816 mm with edges inclined at 45° to the horizontal. The rectangle and square have a common centre. This represents the front view of the object on a scale factor of 0.816.

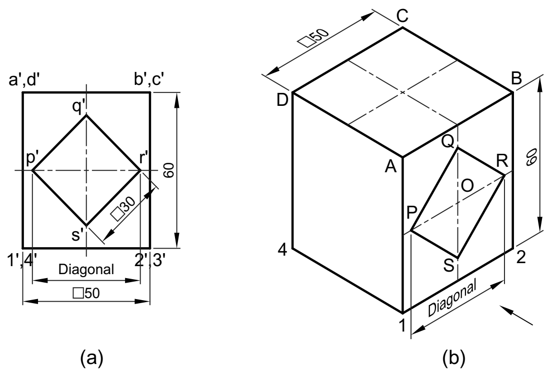


Fig. 15.30 *Square prism with a square hole* **(a)** *Front view on a scale factor* **(b)** *Isometric projection*

2. Draw rhombus *ABCD* of sides 50 × 0.816 mm long and inclined at 30° to the horizontal.
3. Draw 60 × 0.816 mm long vertical lines *A*1, *B*2 and *D*4, visible to the observer. Also, join visible edges of the base, i.e., 4-1-2.
4. Locate centre *O* of the face *AB*21.
5. Mark points *P*, *Q*, *R* and *S* at a distance of half the diagonal length of the square from the centre *O*, along the isometric axis as shown. Join *PQRS*.

Problem 15.25 *A hexagonal prism of base side 25 mm and axis 70 mm is placed centrally on its rectangular face over a cylindrical block of base diameter 80 mm and thickness 30 mm. Draw the isomeric projection of the arrangement.*

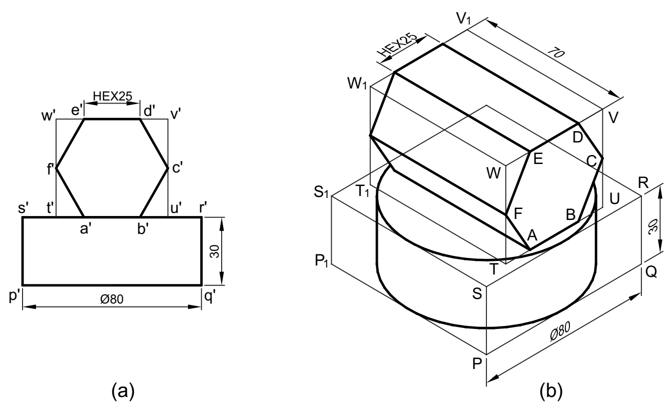


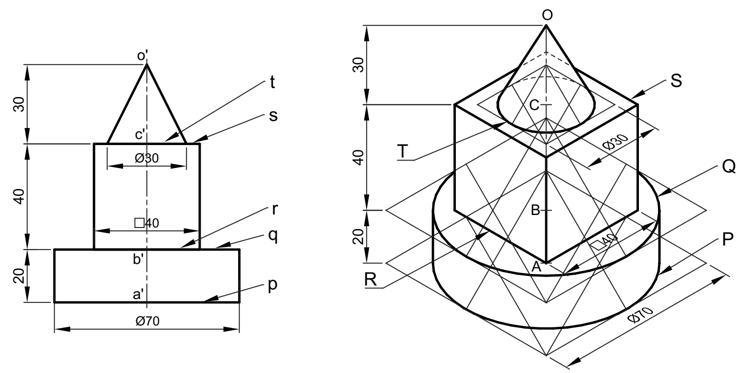
Fig. 15.31 *Hexagonal prism placed over a cylindrical block* **(a)** *Front view on a scale factor of 0.816* **(b)** *Isometric projection*

Construction Refer to Fig. 15.31.

1. Draw a rectangle $p'q'r's'$ of sides 80 × 0.816 mm and 30 × 0.816 mm to represent the front view of the cylindrical block.

2. Also, draw a hexagon $a'b'c'd'e'f'$ of side 20 × 0.816 mm placed centrally over the rectangle, representing the front view of the hexagonal prism. Enclose the hexagon in a rectangle $t'u'v'w'$.
3. Draw a block $PQRSP_1Q_1R_1S_1$. Inscribe ellipse on faces PQQ_1P_1 and RSS_1R_1. Connect both the ellipse with vertical tangents to represent the cylindrical block.
4. Draw TUU_1T_1 such that $TU = t'u'$, TT_1 = 70 × 0.816 mm and placed centrally over the rhombus RSS_1R_1. Project the points to obtain block $TUVWT_1U_1V_1W_1$.
5. Inscribe hexagon on the faces $TUVW$ and $T_1U_1V_1W_1$. Connect the hexagons to show the longer edges of the prism.

Problem 15.26 *A cone is placed centrally on the top of a cube of 40 mm side which is placed centrally over a cylindrical block. The cone has its base diameter 30 mm and axis 30 mm. The cylindrical block has its base diameter 70 mm and thickness 20 mm. Draw isometric projection of the arrangement.*



(a) (b)

Fig. 15.32 *Combination of solids* **(a)** *Front view* **(b)** *Isometric projection*

Construction Refer to Fig. 15.32.

Figure 15.32(a) shows the front view of the arrangement of the solids.

1. Draw a centre line $ABCO$ such that AB = 20 × 0.816 mm, BC = 40 × 0.816 mm and CO = 30 × 0.816 mm.
2. Draw a rhombus of side 70 × 0.816 mm keeping point A as its centre. Inscribe an ellipse P inside this rhombus using four centre method.
3. Draw another rhombus of side 70 × 0.816 mm keeping point B as its centre. Inscribe another ellipse Q inside this rhombus using four centre method. Connect the ellipses P and Q by tangent lines.
4. Draw rhombuses R and S of side 40 × 0.816 mm keeping point B and C as their centres, respectively. Join the corners with vertical lines.
5. Draw a rhombus of side 30 × 0.816 mm keeping point C as its centre. Inscribe an ellipse T inside this rhombus using four centre method. Draw tangent lines from point O to this ellipse.
6. Darken the visible portion of the solid and dimension the figure.

Problem 15.27 *A sphere of diameter 50 mm is surmounted centrally on the top of a square block of side 60 mm and thickness 20 mm. Draw the isometric view of the arrangement.*

Construction Refer to Fig. 15.33.

1. Draw a centre line 1-2-O such that distance between points 1-2 = (30 × 0.816) mm and 2-O = (25 × 0.816) mm.
2. Draw a rhombus *ABCD* of (30 × 0.816) mm keeping point 1 as its centre.
3. Draw another rhombus *EFGH* of (30 × 0.816) mm keeping point 2 as its centre.
4. Join *AE, BF*, *CG* and *DH* to represent the vertical edges of the square prism.
5. Draw a circle with *O* as the centre and radius 25 mm to represent the sphere.
6. Darken the visible portion of the solid and dimension the figure.

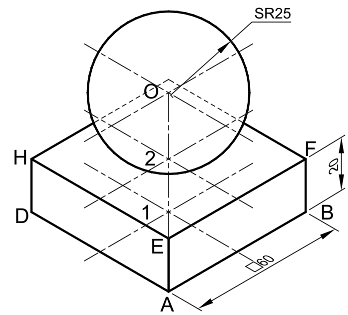


Fig. 15.33 *Isometric view*

Problem 15.28 *A sphere of 50 mm diameter is cut by a section plane at a distance 15 mm from its centre. It is surmounted over the frustum of a square pyramid of base side 60 mm, top side 40 mm and height 40 mm. Draw the isometric view of the arrangement.*

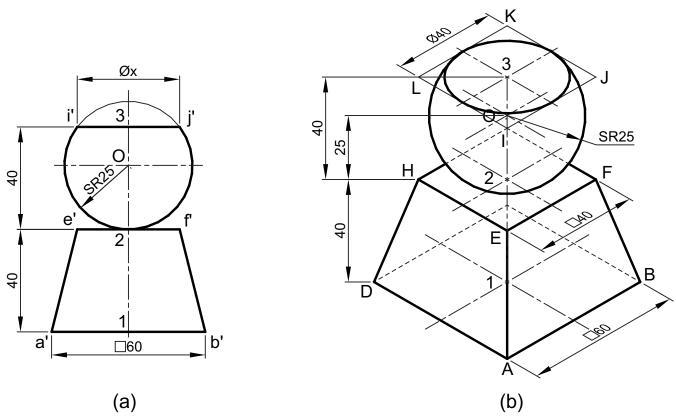


Fig. 15.34 *Truncated sphere surmounted over a frustum* **(a)** *Front view* **(b)** *Isometric projection*

Construction Refer to Fig. 15.34.

1. Draw a circle of 50 mm diameter and cut it by a horizontal section plane passing through a point 15 mm from the centre. Measure the length of the chord *i′j′* as 40 mm.
2. Draw a centre line 1-2-O-3 such that distance between 1-2 = 40 × 0.816 mm, 2-O = 25 × 0.816 mm and O-3 = 15 × 0.816 mm.
3. Draw a rhombus *ABCD* of side 60 × 0.816 mm keeping point 1 as its centre.
4. Draw another rhombus *EFGH* of side 40 × 0.816 mm keeping point 2 as its centre.
5. Join *AE*, *BF*, *CG* and *DH* to represent the slant edges of the frustum of the square pyramid.
6. Draw a circle with centre *O* and radius 25 mm to represent the sphere.

7. Draw another rhombus *IJKL* of side *i′j′* × 0.816 mm keeping point 3 as its centre.
8. Inscribe an ellipse in the rhombus *IJKL* using four centre method.
9. Darken the visible portion of the solid and dimension the figure.

Problem 15.29 *A hemisphere of 50 mm diameter rests centrally with is flat surface at the top, over the frustum of a cone of base diameter 80 mm, top diameter 60 mm and height 50 mm. Draw isometric projection of the arrangement.*

Construction Refer to Fig. 15.35.

1. Draw a centre line *ABC* such that *AB* = 50 × 0.816 mm and *B*C = 25 × 0.816 mm.
2. Draw a rhombus of side 80 × 0.816 mm keeping point *A* as its centre. Inscribe an ellipse *P* inside this rhombus using four centre method.
3. Draw another rhombus of side 60 × 0.816 mm keeping point *B* as its centre. Inscribe another ellipse *Q* inside this rhombus using four centre method.
4. Connect the ellipses drawn in Steps 2 and 3 by tangent lines.
5. Draw a semicircle *R* with *B* as the centre and radius 25 mm to represent the hemisphere.
6. Draw another rhombus of side 25 × 0.816 mm keeping point *C* as its centre. Inscribe an ellipse *S* inside the rhombus using four centre method.
7. Darken the visible portion of the solid and dimension the figure.

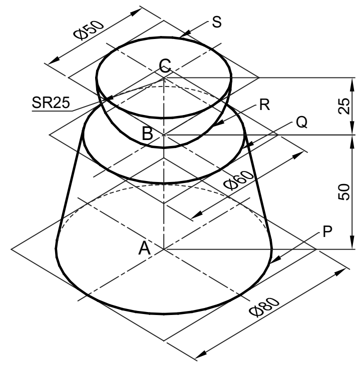


Fig. 15.35 *Isometric projection of a hemisphere resting centrally over the frustum of a cone*

Problem 15.30 *A hexagonal prism of base 30 mm side and height 40 mm is surmounted by a hemisphere such that the hemisphere is touching all the edges of the top base. Draw isomeric projection of the arrangement.*

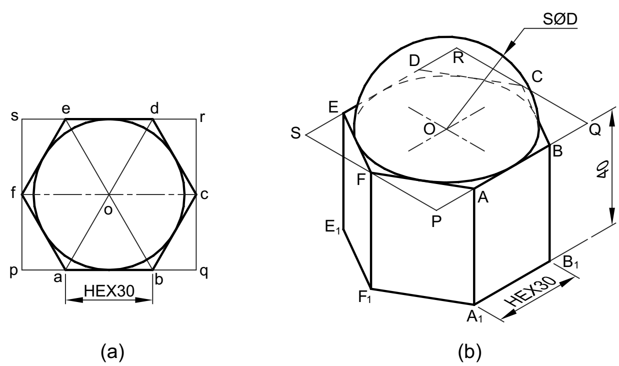


Fig. 15.36 *Hexagonal prism surmounted by a hemisphere* **(a)** *Top view* **(b)** *Isometric projection*

Construction Refer to Fig. 15.36.

1. Draw a hexagon *abcdef* of side 30 × 0.816 mm and inscribe a circle with centre *o*. This represents the top view on a scale factor of 0.816.

2. Enclose the hexagon in a rectangle *pqrs* as shown in Fig. 15.36(a). Also measure the diameter of the circle *S∅D* as the diameter of the sphere.
3. Draw a parallelogram *PQRS* to represent isometric view of the rectangle *pqrs*. Locate points *A*, *B*, *C*, *D*, *E* and *F* on the edges of the parallelogram to represent the hexagon.
4. Draw 40 × 0.816 mm long AA_1, BB_1, EE_1 and FF_1 to represent visible vertical edges of the prism. Join $E_1F_1A_1B_1$ to represent the base of the prism.
5. Draw an ellipse passing through mid-points of the edges *AB*, *BC*, *CD*, *DE*, *EF* and *AF*.
6. Locate centre *O* and draw a semicircle touching the ellipse.
7. Darken the visible portion of the solid and dimension the figure.

Problem 15.31 *Figure 15.37(a) shows the front view of a paper weight consisting of the frustum of a cone surmounted by a cut sphere. Draw its isometric view.*

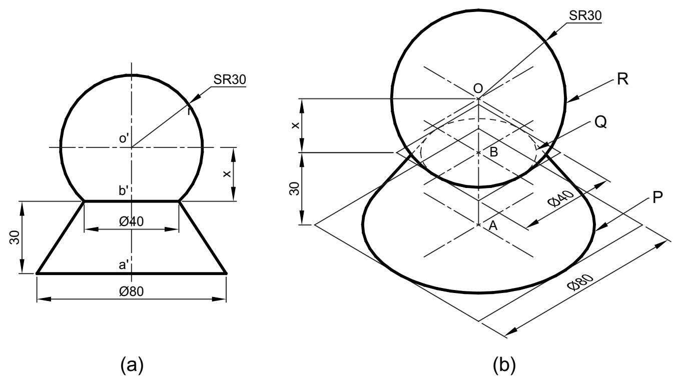


Fig. 15.37 *Paper weight* **(a)** *Front view* **(b)** *Isometric projection*

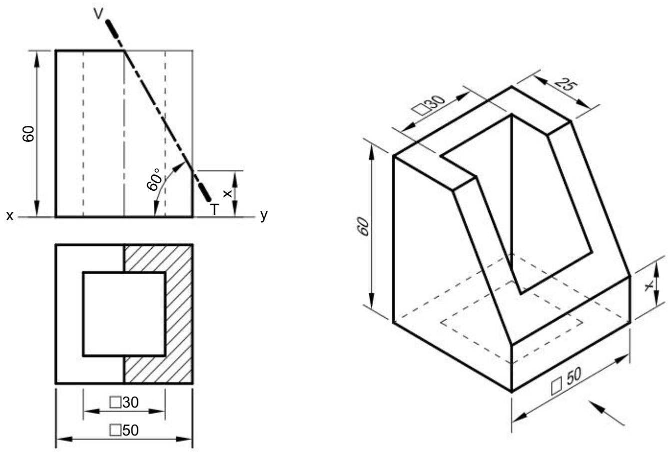
Construction Refer to Fig. 15.37.

1. Draw a circle with *o′* as the centre and radius 30 mm. Draw a chord of 40 mm to the circle and determine its distance from the centre as length *x*. Refer to Fig. 15.37(a).
2. Draw a centre line *ABO* such that *AB* = 30 × 0.816 mm and *BC* = *x* × 0.816 mm.
3. Draw a rhombus of side 80 × 0.816 mm keeping point *A* as its centre. Inscribe an ellipse *P* inside this rhombus using four centre method.
4. Draw another rhombus of side 40 × 0.816 mm keeping point *B* as its centre. Inscribe another ellipse *Q* inside this rhombus using four centre method.
5. Draw an arc with centre *O* and radius 30 mm to meet the ellipse *Q* as shown.
6. Darken the visible portion of the solid and dimension the figure.

Problem 15.32 *A hollow square prism of height 60 mm is resting on its base on the H.P. with an edge of the base parallel to the V.P. Outside and inside edges of the base are 50 mm and 30 mm respectively. It is cut by an A.I.P. inclined at 30° to the H.P. and passing though the topmost point of the axis. Draw its isometric projection.*

Construction Refer to Fig. 15.38.

1. Draw the front view of the arrangement as Fig. 15.38(a).
2. Draw the isometric projection of the solid as shown in Fig. 15.38(b).



(a) (b)

Fig. 15.38 *Hollow square prism cut by an A.I.P.* **(a)** *Orthographic projections* **(b)** *Isometric projection*

EXERCISE 15A

Plane surface

15.1 Draw the isometric view of a square of side 40 mm kept parallel to the H.P.

15.2 Draw the isometric view of a hexagon of side 30 mm whose surface is parallel to the V.P. and a side perpendicular to the H.P.

15.3 Draw isometric views of a triangle of sides 80 mm, 60 mm and 50 mm on all the three principal planes.

15.4 Draw the isometric view of a cube of side 50 mm. Also show in the view, circles of diameter 50 mm marked on all the visible faces of the cube.

15.5 Draw isometric view of a hexagonal plane of side 40 mm with a central hole of diameter 40 mm when the surface of the plane is parallel to the H.P.

15.6 Draw isometric view of a composite plane made up of a rectangle of sides 60 mm and 40 mm with a semicircle on its longer side.

Simple solids

15.7 Draw an isometric view of a pentagonal prism of base side 30 mm and axis 60 mm resting on its base in the H.P. with a face parallel and nearer to the V.P.

15.8 Draw an isometric view of a hexagonal prism of base side 30 mm and axis 70 mm, lying on a face on the H.P. with axis parallel to both H.P. and V.P.

15.9 Draw the isometric view of a cylinder of base diameter 50 mm and axis 60 mm lying on one of its generator on the H.P.

15.10 Draw an isometric view of a pentagonal pyramid of base side 30 mm and axis 60 mm resting on its base on the H.P. with a face parallel and nearer to the V.P.

15.11 A square prism of base edge 40 mm and axis 60 mm has an edge of its base on the H.P. The axis is parallel to the V.P. and inclined at 30° to the H.P. Draw its isometric view.

15.12 A pentagonal pyramid of base side 30 mm and axis 60 mm long is resting on a face on the H.P. with axis parallel to the V.P. Draw its isometric view in the stated condition.

Truncated solid

15.13 Draw isometric projection of the frustum of a pentagonal pyramid of base side 40 mm, top side 20 mm and height 35 mm resting on its base on the H.P.

15.14 A paper weight is in the form of a sphere of diameter 50 mm truncated by a horizontal plane at a distance of 40 mm from the topmost point of the sphere. Draw its isometric projection.

15.15 A pentagonal prism of base side 30 mm and axis 60 mm is resting on its base on the H.P. with a side of the base perpendicular to the V.P. It is cut by an A.I.P. inclined at 45° to the H.P. and passing through an edge of the top base. Draw its isometric view.

15.16 A triangular pyramid having a base 50 mm side and axis 65 mm long is resting on its base in the H.P. with a side of the base parallel to the V.P. It is cut by an A.I.P. inclined at 45° with the H.P. and bisecting the axis. Draw its isometric view.

15.17 A cone of base diameter 50 mm and axis 60 mm rests on its base on the H.P. It is cut by an A.I.P. inclined at 30° to H.P. and passes through the axis at a distance of 25 mm above the base. Draw the isometric view of the truncated cone.

Composite solids

15.18 A composite solid is made up of a rectangular block of 80 mm and 40 mm side and 30 mm height, and semi-cylinders of diameter 80 mm height 30 mm attached on two opposite flat vertical face of the block. Draw its isometric projection.

15.19 A cone of base diameter 30 mm and axis 50 mm rests centrally over a square prism of base side 50 mm and axis 30 mm. Draw the isometric projection of the arrangement.

15.20 The frustum of a cone of base diameter 60 mm, top diameter 40 mm and height 50 mm is surmounted centrally over a cylindrical block of diameter 80 mm and thickness 30 mm. Draw its isometric projection.

15.21 A hexagonal prism of base side 30 mm and axis 50 mm has an axially drilled circular hole of diameter 30 mm. Draw its isometric projection.

15.22 A pentagonal prism of base side 25 mm and axis 80 mm is placed centrally on its rectangular face over a square block of base side 60 mm and thickness 30 mm. Draw the isomeric projection of the arrangement.

15.23 A spherical ball of diameter 60 mm is placed centrally over a square block of side 60 mm and thickness 30 mm. Draw the isometric view of the arrangement.

15.24 A cube of 60 mm side has square holes of 30 mm side, cut through from all the six faces. The sides of the square holes are parallel to the edges of the cube. Draw the isometric view of the cube.

15.25 Draw the isometric view of a door-step having three steps of 20 cm tread and 15 cm rise. The steps measure 75 cm widthwise.

15.15 CONVERSION OF ORTHOGRAPHIC VIEWS INTO ISOMETRIC VIEWS

15.15.1 Objects Extruded in One Direction Only

Plane figure with holes or cut of certain shape, called *features*, when extruded gives a solid object. The orthographic projections of such objects show features on either front, top or side view and the remaining two views show rectangular boxes with straight lines running across. The isometric views of such objects are prepared by drawing the features on one of the isometric plane and thereafter extruding it to a mentioned depth. Consider the following problems.

Problem 15.33 *The front and top views of a casting are shown in Fig. 15.39(a). Draw its isometric view.*

Visualization The basic feature of the casting is seen in the top view while the front view shows the extruded thickness of 30 mm. As the object is symmetrical, side of length 60 mm can be plotted on either x or y-direction.

Construction Refer to Fig. 15.39.

1. Draw *ABCDEFGHIJKL* on the x-y plane to represent the top view, shown by shade in Fig. 15.39(b).
2. Extrude all the corners 30 mm in z-axis direction and join their end points.
3. Darken the visible edges of the object and dimension the view to obtain Fig. 15.39(c).

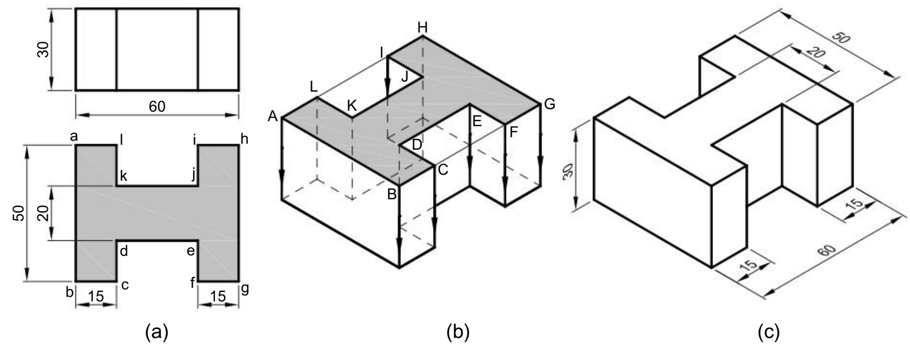


Fig. 15.39

Problem 15.34 *The front and top views of a casting are shown in Fig. 15.40(a). Draw its isometric view.*

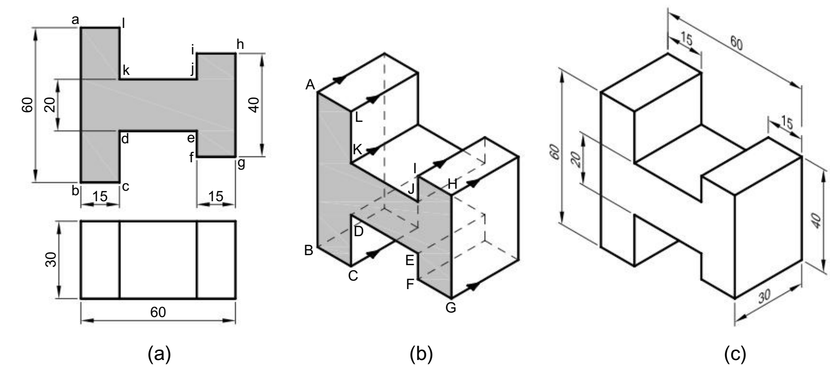


Fig. 15.40

Visualization The basic feature of the casting is seen in the front view while the top view shows the extruded thickness of 30 mm. To view major portion of the object in the isometric, the side of length 60 mm should be plotted along *y*-axis.

Construction Refer to Fig. 15.40.

1. Draw *ABCDEFGHIJKL* on the *y*-*z* plane to represent the front view, shown by shade in Fig. 15.40(b).
2. Extrude all the corners 30 mm in *x*-axis direction and join their end points.
3. Darken the visible edges of the object and dimension the view to obtain Fig. 15.40(c).

Problem 15.35 *The front and right-hand side views of an I-beam are shown in Fig. 15.41(a). Draw the isometric view of the beam.*

Visualization The basic feature of the casting is seen in the front view while the right-hand side view shows the extruded thickness of 60 mm. To obtain the right-hand side view visible in the isometric, the front view should be plotted on *y*-*z* plane.

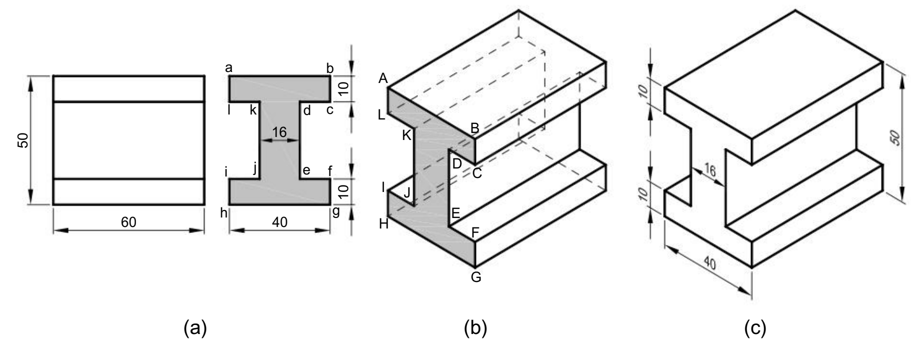


Fig. 15.41

Construction Refer to Fig. 15.41.

1. Draw *ABCDEFGHIJL* on the *y-z* plane to represent the front view, shown by shade in Fig. 15.41(b).
2. Extrude all the corners 60 mm in *x*-axis direction and join their end points.
3. Darken the visible edges of the object and dimension the view to obtain Fig. 15.41(c).

Problem 15.36 *The front and left-hand side views of a casting are shown in Fig. 15.42(a). Draw its isometric view.*

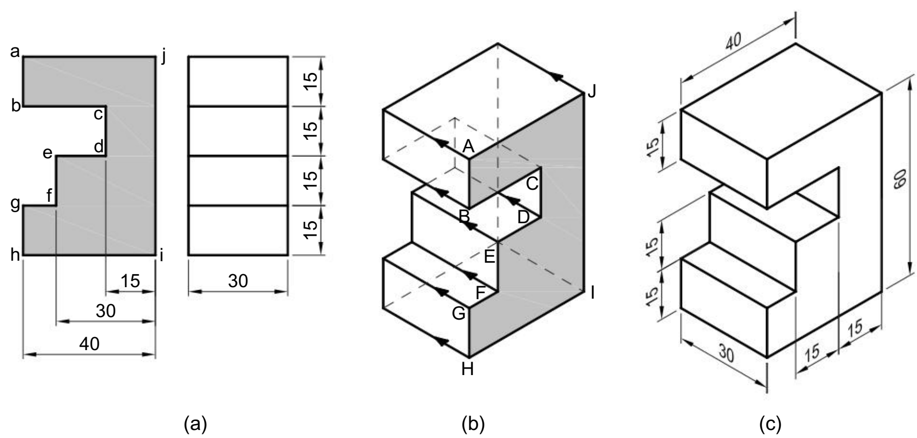


Fig. 15.42

Visualization The basic feature of the casting is seen in the front view while the left-hand side view shows the extruded thickness of 30 mm. To obtain the left-hand side view visible in the isometric, the front view should be plotted on the *x-z* plane.

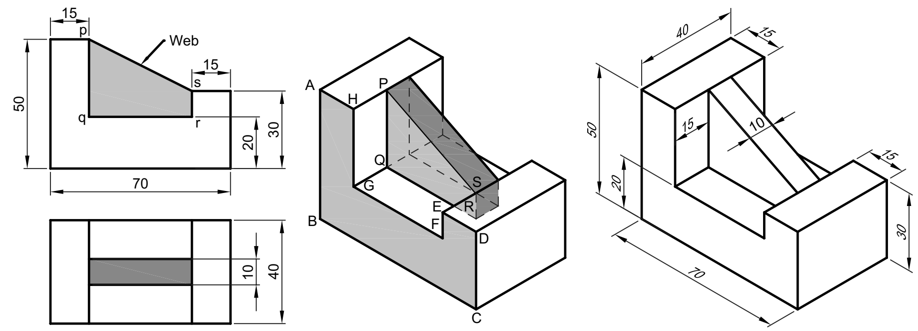
Construction Refer to Fig. 15.42

1. Draw *ABCDEFGHIJ* on the *x-z* plane to represent the front view, shown by shade in Fig. 15.42(b).
2. Extrude all the corners 30 mm in *y*-axis direction and join their end points.
3. Darken the visible edges of the object and dimension the view to obtain Fig. 15.42(c).

15.15.2 Isometric Projections of Extruded Object with Holes, Slots, Ribs and/or Webs

The objects extruded may have holes, slots, ribs and/or web in the perpendicular direction. The isometric projections of such objects can be drawn by carefully extruding the plane to obtain the basic solid and then placing the web or cutting the holes/slots at the specified location. Consider the following problems.

Problem 15.37 *The front and top views of a casting are shown in Fig. 15.43(a). Draw its isometric view.*



(a) (b) (c)

Fig. 15.43

Visualization The basic feature of the casting is seen in the front view while the top view shows the extruded thickness of 40 mm. The shaded portion in the front and the top view represents the web. To view major portion of the object in the isometric, the side of length 70 mm should be plotted along the *y*-axis.

Construction Refer to Fig. 15.43(b).

1. Draw *ABCDEFGH* on the isometric *y-z* plane corresponding to the front view, excluding web. Extrude all the points by 40 mm along the *x*-direction and join their end points.
2. Mark points *P*, *Q*, *R* and *S* on the extruded surface, 15 mm away from points *H*, *G*, *F* and *E* respectively, in *x*-direction. Join *PQRS*.
3. Extrude points *P*, *Q*, *R* and *S*, 10 mm in *x*-direction and join their end points.
4. Darken the visible edges of the object to obtain Fig. 15.43(c) and dimension it.

Problem 15.38 *The front and top views of a casting are shown in Fig. 15.44(a). Draw its isometric view.*

Visualization The basic feature of the casting is seen in the front view while the top view shows the extruded thickness of 40 mm. The front and the top views also show that there are two holes of 15 mm diameter on the base. As the object is symmetrical, the length of 120 mm can be plotted along either *x* or *y*-axis.

Construction Refer to Fig. 15.44(b).

1. Draw the front view without hole on the isometric *y-z* plane. Draw semi-ellipse by four centre method. Extrude all the points 40 mm towards the *x*-axis and join them.

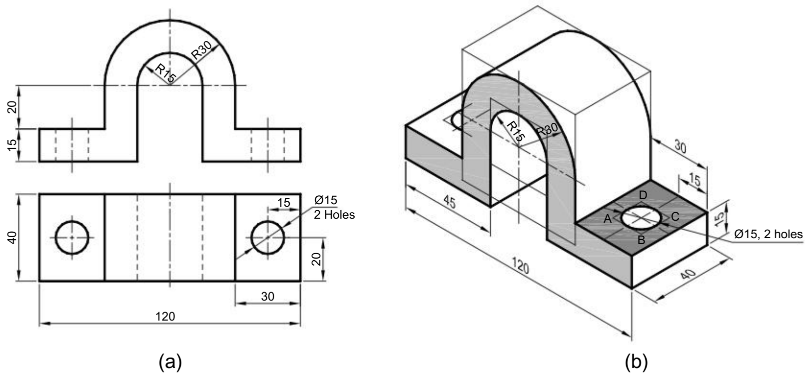


Fig. 15.44

2. On the extruded surfaces, draw rhombus *ABCD* of side 15 mm (hole diameter). Inscribe an ellipse in the rhombus using four centre method. Transfer all the centres 25 mm downward and ensure that the ellipse corresponding to the lower edge of the hole is not visible through the upper ellipse.
3. Proceed to the other side of the object and the draw the visible part of the edges of the drilled hole.
4. Darken the visible edges of the object and dimension the figure.

15.15.3 Isometric Projections of Angle Plates

Angle plate is composed of two plates of any arbitrary shape placed at an angle (usually at right angle) to each other. A careful observation may help in separating both the plates. The isometric projections of angle plates can also be drawn by extruding. Consider the following problems.

Problem 15.39 *The front and top views of an angle plate are shown in Fig. 15.45(a). Draw its isometric view.*

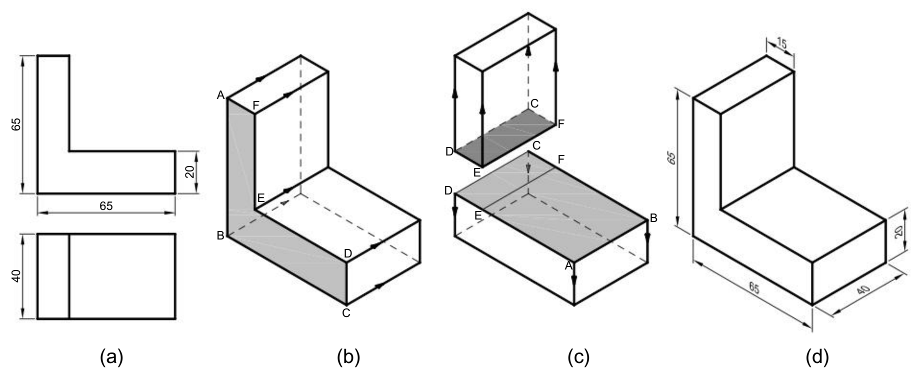


Fig. 15.45

Visualization The front view shows that angle plates are at right angle to each other and the top view shows that they are of rectangular cross-sectional area. To view major portion of the object in the isometric, the side of length 65 mm should be plotted along the *y*-axis.

Construction

Method 1

1. Draw *ABCDEF* on the *y-z* plane to represent the front view, shown by shade in Fig. 15.45(b).
2. Extrude all the corners, 40 mm in x-axis direction and join their end points.
3. Darken the visible edges of the object and dimension the figure to obtain Fig. 15.45(d).

Method 2

1. Draw the parallelogram *ABCD* on the x-y plane. Extrude *ABCD* 20 mm along the *z*-axis in downward direction.
2. On the surface of *ABCD* mark points *E* and *F*, as shown in Fig. 15.45(c). Extrude *CDEF* 20 mm along the *z*-axis in upward direction.
3. Finally darken the visible edges and dimension the figure to obtain Fig. 15.45(d).

Problem 15.40 *The front and right-hand side views of an angle plate are shown in Fig. 15.46(a). Draw its isometric view.*

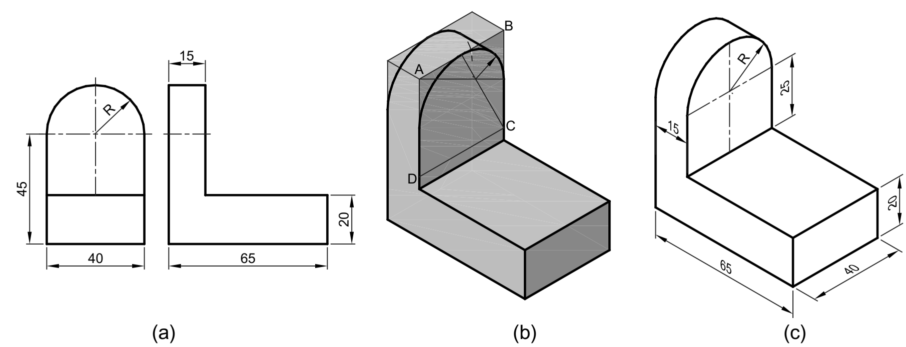


Fig. 15.46

Visualization The front view shows that angle plates are at right angle to each other and the side view shows that one of the plates has a semi-circular shape on the upper end. To view major portion of the object in the isometric, the side of length 65 mm should be plotted along the *y*-axis.

Construction

1. Draw the angle plate by any one of the methods explained in Problem 15.38, and obtain the figure similar to Fig. 15.45(d).
2. On the vertical surface of the angle plate, draw the rhombus *ABCD* of side 40 mm as shown in Fig. 15.46(b). Draw a semi-ellipse using four centre method inside the rhombus.
3. Extrude the points of the semi-ellipse 15 mm in the *y*-axis direction and obtain another semi-ellipse. Join both semi-ellipses with a tangent line.
4. Darken the visible edges of the object and dimension to obtain Fig. 15.46(c).

Problem 15.41 *The front and top views of a casting are shown in Fig. 15.47(a). Draw its isometric view.*

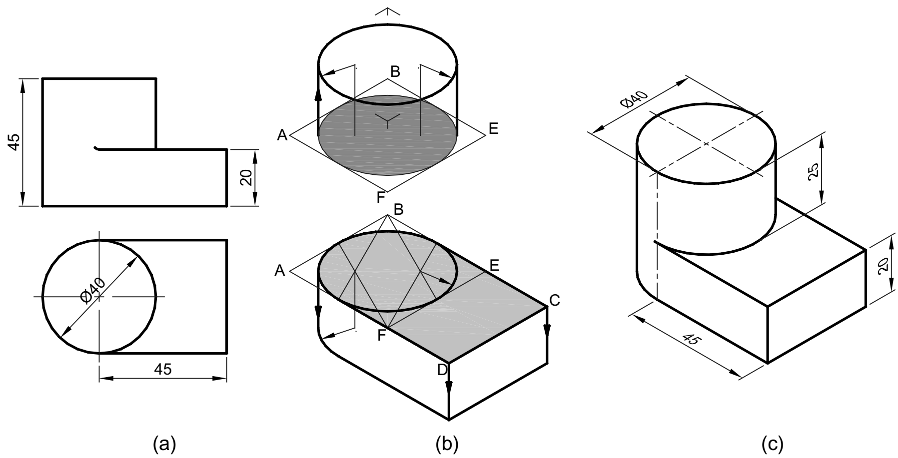


Fig. 15.47

Visualization The front view shows that angle plates are at right angle to each other and the top view shows that one of the plate is of cylindrical cross-sectional area. To view major portion of the object in the isometric, the length should be plotted along the *y*-axis.

Construction

1. Draw the parallelogram *ABCD* on the *x*-*y* plane. Also mark a rhombus *ABEF* and inscribe an ellipse, as shown in Fig. 15.47(b), to represent the cylinder at its end.
2. Extrude all the points 20 mm along the *z*-axis in downward direction and complete the bottom plate.
3. Extrude the points of the ellipse 25 mm along the *z*-axis in upward direction.
4. Join all the ellipses by tangent lines. Darken the visible edges and dimension the figure to obtain Fig. 15.47(c).

15.15.4 Isometric Projections of Angle Plates with Holes, Slots and/or Ribs

The orthographic view of angle plate gets complicated when slots are cut or ribs are extended. The isometric projections of such objects can be drawn by first drawing the angle plate and then drawing slots or rib in the given position. So there is a need first to observe the basic shape of angle plates and then to observe the holes drilled and/or ribs attached. Consider the following problem.

Problem 15.42 *The front and top views of an angle plate are shown in Fig. 15.48(a). Draw its isometric view.*

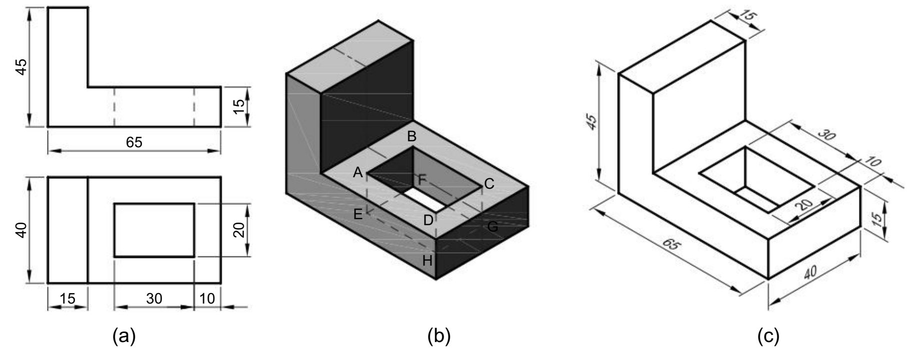


Fig. 15.48

Visualization The front view shows that the angle plates are at right angle to each other and the top view shows that they are of rectangular cross-sectional area. The top view also shows that a rectangular hole has been created on the base plate. To enable major portion of the object visible, the side of length 65 mm should be plotted along the *y*-axis.

Construction

1. Draw the isometric view of the angle plate using one of the methods explained in Problem 15.38.
2. Draw a parallelogram *ABCD* of sides 30 mm and 20 mm on the top surface of the horizontal plate to represent the rectangle of the slot, as shown in Fig. 15.48(b).
3. Extrude the points of the parallelogram *ABCD* 15 mm along the *z*-axis in downward direction to obtain *EFGH*.
4. Darken the portion of the parallelogram *EFGH* falls within parallelogram *ABCD* and dimension to obtain Fig. 15.48(c).

Problem 15.43 *The front and top views of an angle plate are shown in Fig. 15.49(a). Draw its isometric view.*

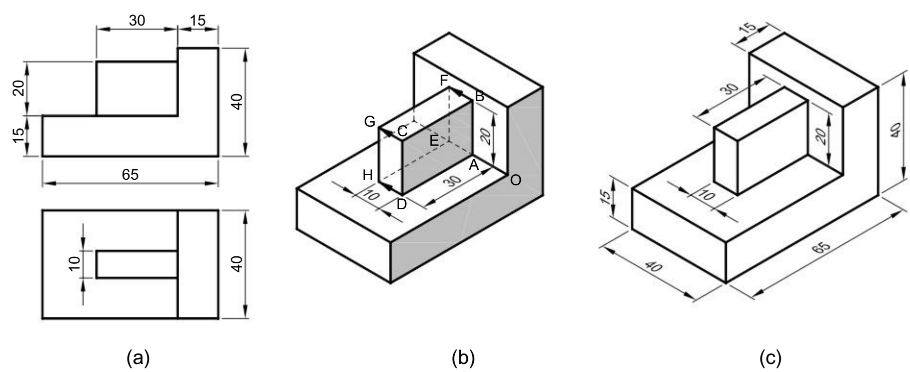


Fig. 15.49

Visualization The front view shows that the angle plates are at right angle to each other and the top view shows that they are of rectangular cross-sectional area. The views also show that there is a rectangular block of 30 mm × 20 mm × 10 mm placed in contact with both the plates of angle plate. To enable major portion of the object visible, the side of length 65 mm should be plotted along the *x*-axis.

Construction

1. Draw the isometric view of the angle plate using a method explained in Problem 15.38.
2. Locate a point *A* at a distance of 15 mm from point *O* in the y-axis direction. Draw a parallelogram *ABCD* of sides 30 mm and 20 mm to represent the rectangle, as shown in Fig. 15.49(b). Extrude all the points 10 mm in the y-axis direction and obtain *EFGH*.
3. Darken the visible edges of the rectangular block and dimension to obtain Fig. 15.49(c).

Problem 15.44 *The front and side views of an angle plate are shown in Fig. 15.50(a). Draw its isometric view.*

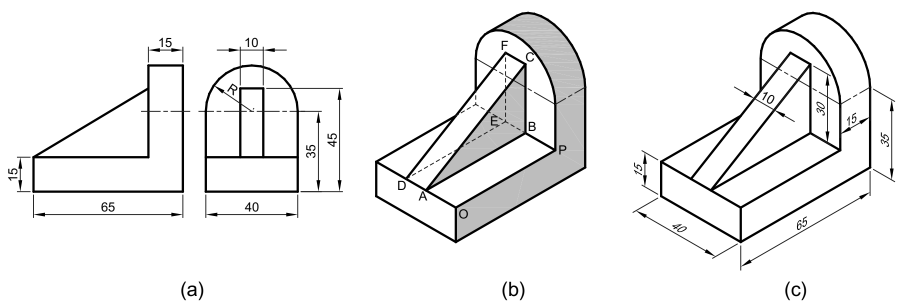


Fig. 15.50

Visualization The front view shows that the angle plates are at right angle to each other and the left hand side view shows that they are of rectangular cross-sectional area. The views also show that one of the plates has a semi-circular shape on the upper end. In addition to this, the view shows that there is a rib strengthening the angle plates. To enable major portion of the object visible, the side of length 65 mm should be plotted along the *x*-axis.

Construction

1. Draw the isometric view of the angle plate using a method explained in Problem 15.39.
2. Locate points *A* and *B* on the top surface of the horizontal plate at a distance of 15 mm from points *O* and *P*.
3. Draw a triangle *ABC* to represent the right-angled triangle of the rib, as shown in Fig. 15.50(b). Extrude all the points 10 mm in y-axis direction and obtain *DEF*.
4. Darken the visible edges of the rib and dimension to obtain Fig. 15.50(c).

15.16 MISCELLANEOUS PROBLEMS

Problem 15.45 *The front and top views of a casting are shown in Fig. 15.51(a). Draw its isometric view.*

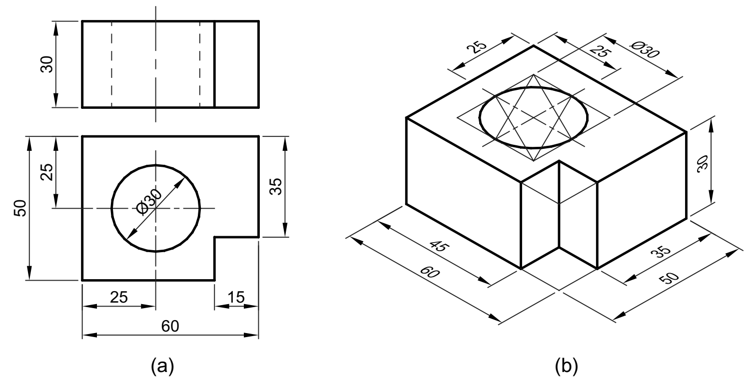


Fig. 15.51

Figure 15.51(b) shows the required isometric view. Construction lines are left intact for guidance.

Problem 15.46 *The front and left-hand side views of a casting are shown in Fig. 15.52(a). Draw its isometric view.*

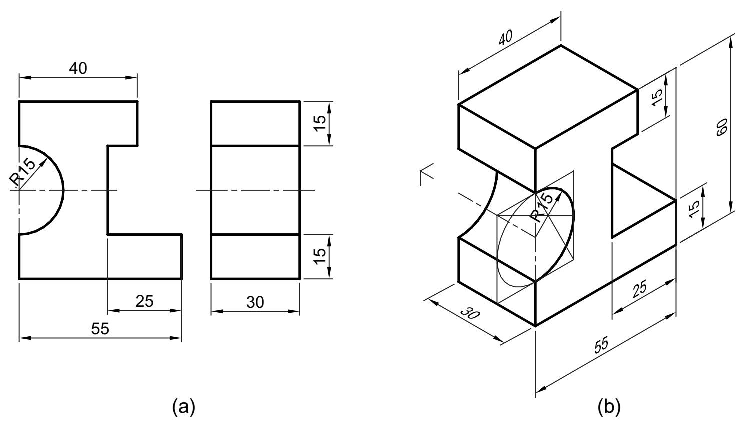


Fig. 15.52

Figure 15.52(b) shows the required isometric view. Construction lines are left intact for guidance.

Problem 15.47 *The front and right-hand side views of a casting are shown in Fig. 15.53(a). Draw its isometric view.*

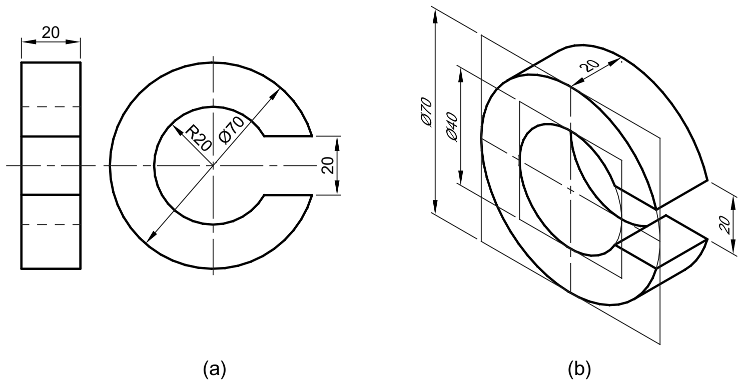


Fig. 15.53

Figure 15.53(b) shows the required isometric view. Construction lines are left intact for guidance.

Problem 15.48 *The front and top views of an object are shown in Fig. 15.54(a). Draw its isometric view.*

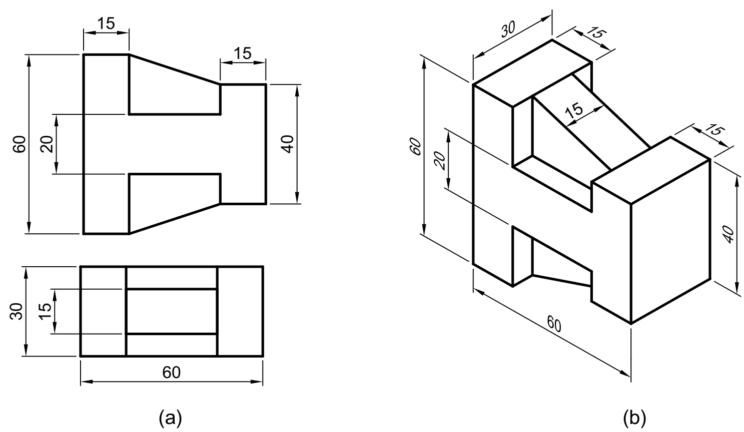


Fig. 15.54

Figure 15.54(b) shows the required isometric view.

Problem 15.49 *Two orthographic views of a block are shown in Fig. 15.55(a). Draw the isometric view of the block.*

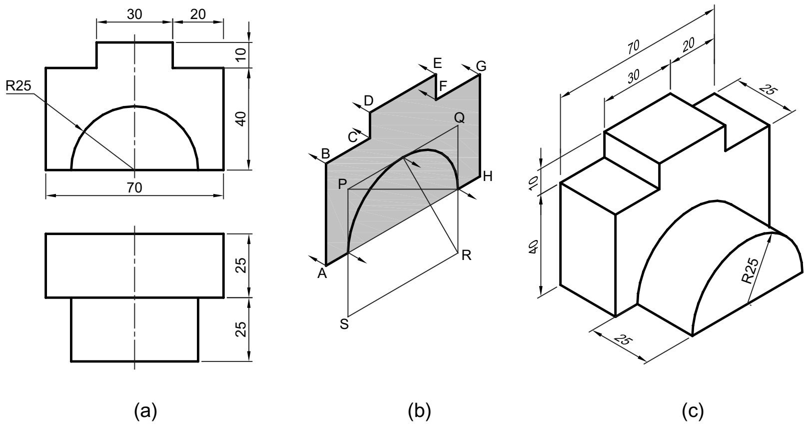


Fig. 15.55

Construction Refer to Fig. 15.55.

1. Draw the polygon *ABCDEFGH* on the *x-z* plane. Also, draw a rhombus *PQRS* and mark semi-ellipse, as shown in Fig. 15.55(b).
2. Extrude the points of the polygon 25 mm along the *y*-axis in a direction towards V.P. and complete the back plate.
3. Extrude the points of the semi-ellipse 25 mm along the *y*-axis in a direction away from the V.P. Join the semi-ellipses by tangent lines.
4. Darken the visible edges and dimension the figure to obtain Fig. 15.55(c).

Problem 15.50 *Two orthographic views of an object are shown in Fig. 15.56(a). Draw its isometric view.*

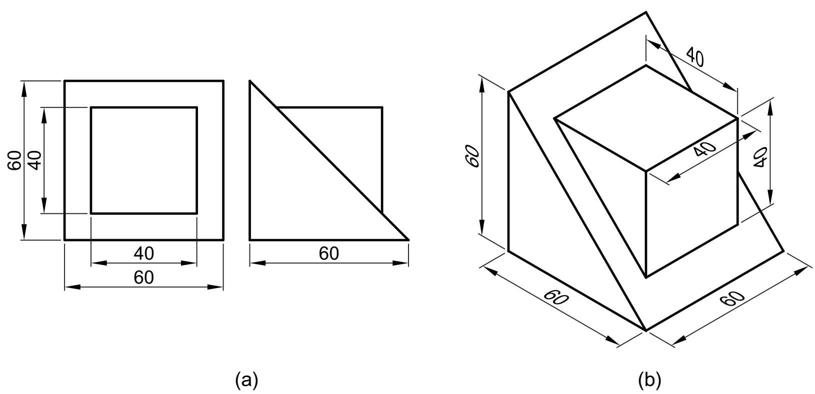


Fig. 15.56

Figure 15.56(b) shows the required isometric view.

Problem 15.51 *An object has its front, top and side views same and is shown in Fig. 15.57(a). Draw its isometric view.*

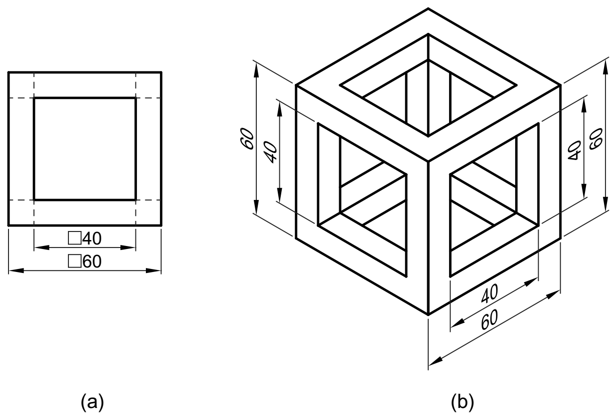


Fig. 15.57

Figure 15.57(b) shows the required isometric view.

Problem 15.52 *Three orthographic views of an object are shown in Fig. 15.58(a). Draw its isometric view.*

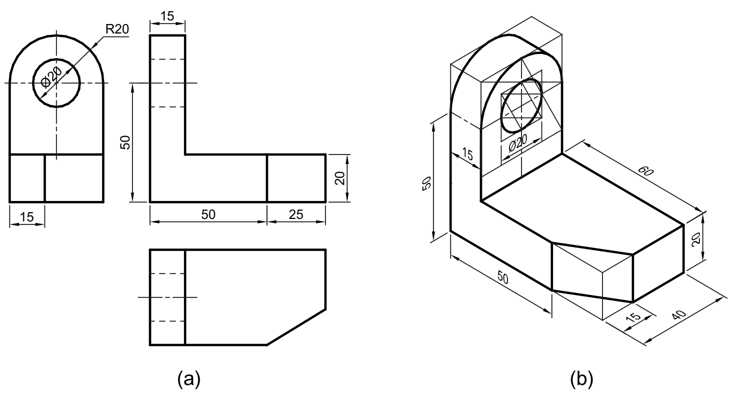


Fig. 15.58

Figure 15.58(b) shows the required isometric view. Length is taken on the left-hand side to visualise its major portion. Construction lines are left intact for guidance.

Problem 15.53 *The front and the top views of an object are shown in Fig. 15.59(a). Draw its isometric view.*

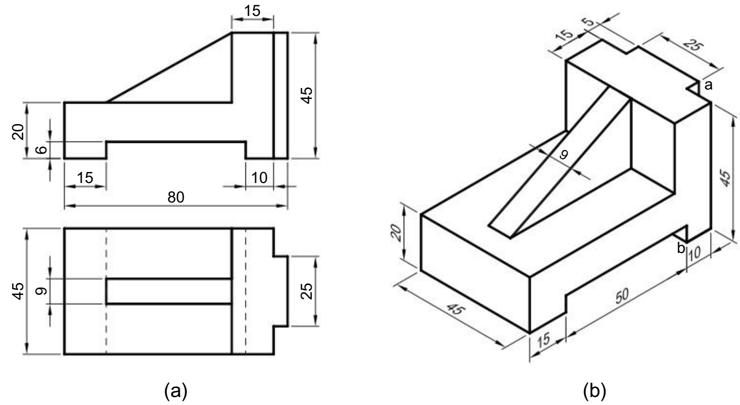


Fig. 15.59

Figure 15.59(b) shows the required isometric view. It may be noted that:

1. Length should be taken on the right-hand side as to visualize its major portion.
2. Draw all the three lines originating from both points *a* and *b* carefully.

Problem 15.54 *The front and top views of an object are shown in Fig. 15.60(a). Draw its isometric view.*

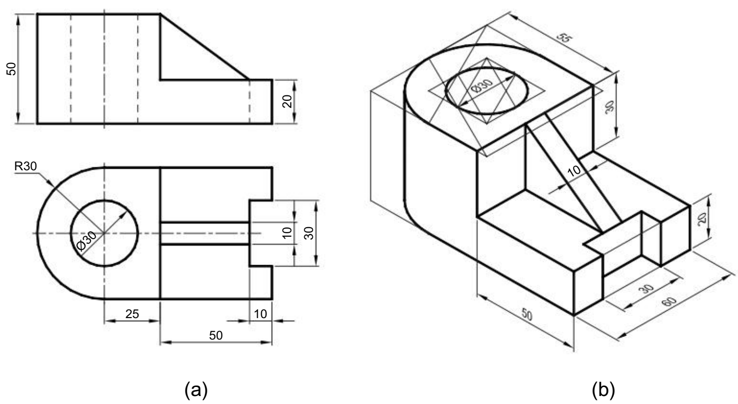


Fig. 15.60

Figure 15.60(b) shows the required isometric view. Construction lines are left intact for guidance.

Problem 15.55 *The front and top views of an object are shown in Fig. 15.61(a). Draw its isometric view.*

Figure 15.61(b) shows the required isometric view. Construction lines are left intact for guidance. It may be noted that:

1. All the three lines originating from point *a* should be drawn.
2. Lines for the visible lower edges of the circular hole should be drawn.

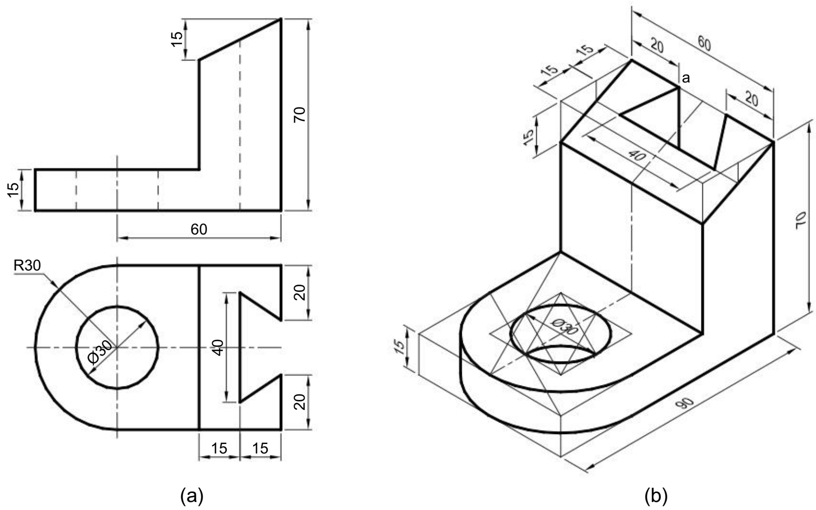


Fig. 15.61

Problem 15.56 *The front and top views of an object are shown in Fig. 15.62(a). Draw its isometric view.*

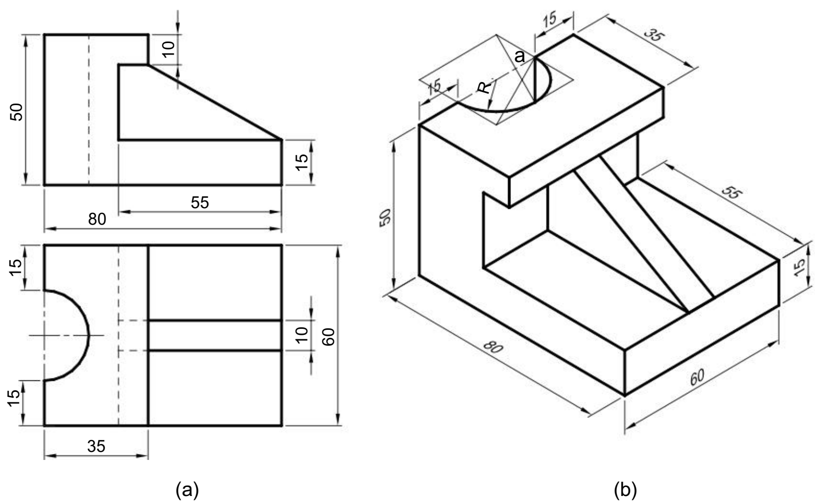
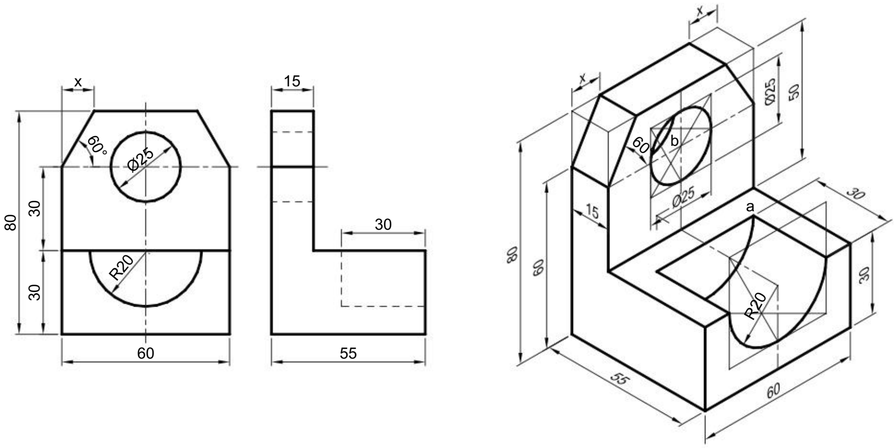


Fig. 15.62

Figure 15.62(b) shows the required isometric view. Construction lines are left intact for guidance. Two lines and a curve originating from point *a* should be drawn.

Problem 15.57 *The front and side views of an object are shown in Fig. 15.63(a). Draw its isometric view.*



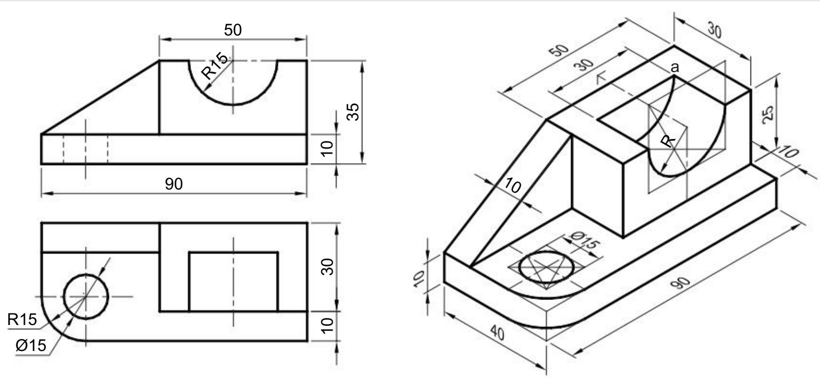
(a) (b)

Fig. 15.63

Figure 15.63(b) shows the required isometric view. Construction lines are left intact for guidance. It may be noted that:

1. Length *x* has to be determined in order to transfer it in the isometric view.
2. Two lines and a curve originating from point *a* should be drawn.
3. Lines for the visible edge of the circular hole should be drawn at point *b*.

Problem 15.58 *The front and top views of an object are shown in Fig. 15.64(a). Draw the isometric view of the object.*

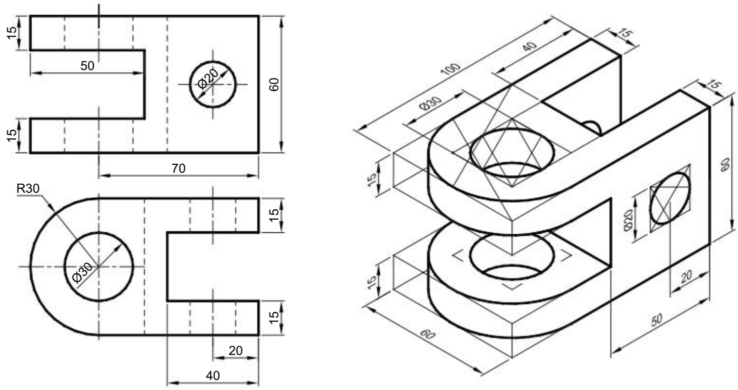


(a) (b)

Fig. 15.64

Figure 15.64(b) shows the required isometric view. Construction lines are left intact for guidance. Two lines and a curve originating from point *a*, need be drawn.

Problem 15.59 *The front and top views of an object are shown in Fig. 15.65(a). Draw its isometric view.*

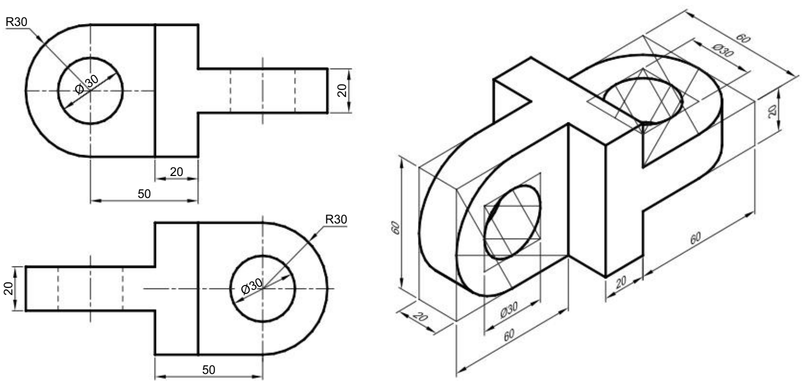


(a) (b)

Fig. 15.65

Figure 15.65(b) shows the required isometric view. Construction lines are left intact for guidance.

Problem 15.60 *The front and top views of an object are shown in Fig. 15.66(a). Draw its isometric view.*

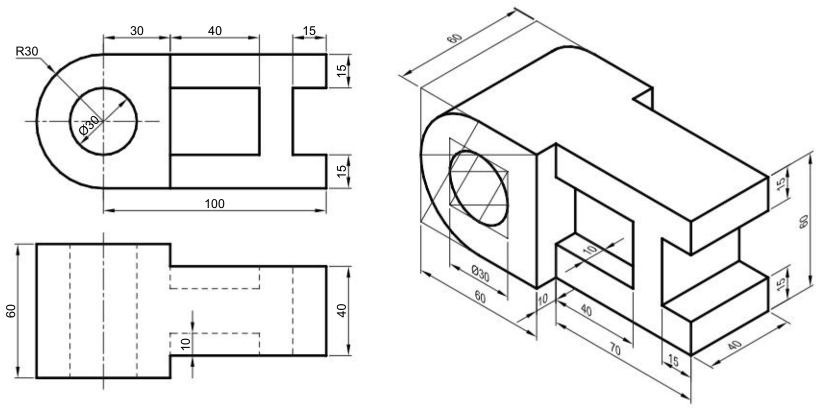


(a) (b)

Fig. 15.66

Figure 15.66(b) shows the required isometric view. Construction lines are left intact for guidance.

Problem 15.61 *The front and top views of an object are shown in Fig. 15.67(a). Draw its isometric view.*

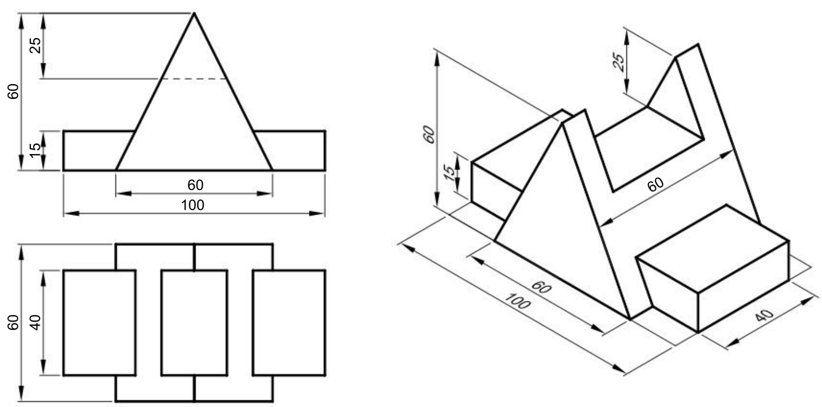


(a) (b)

Fig. 15.67

Figure 15.67(b) shows the required isometric view. Construction lines are left intact for guidance.

Problem 15.62 *The front and top views of an object are shown in Fig. 15.68(a). Draw its isometric view.*



(a) (b)

Fig. 15.68

Figure 15.68(b) shows the required isometric view.

EXERCISE 15B

Solids extruded in one direction

15.1 Figures E15.1–E15.12 shows the orthographic projections of an object extruded in single direction. Draw their isometric view.

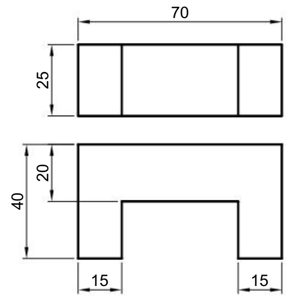


Fig. E15.1

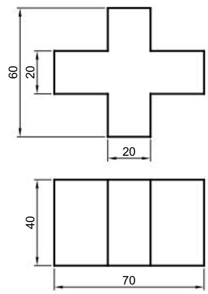


Fig. E15.2

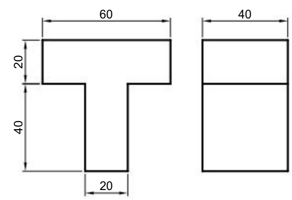


Fig. E15.3

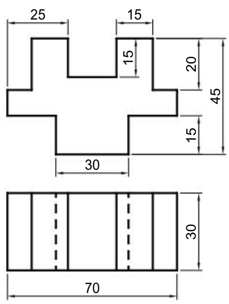


Fig. E15.4

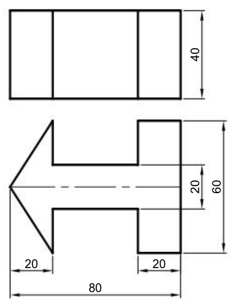


Fig. E15.5

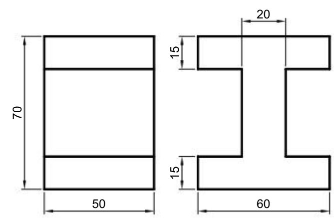


Fig. E15.6

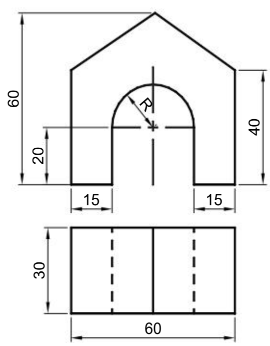


Fig. E15.7

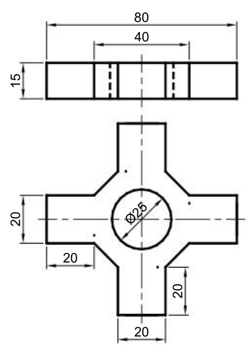


Fig. E15.8

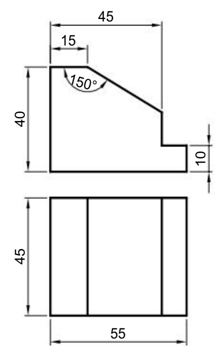


Fig. E15.9

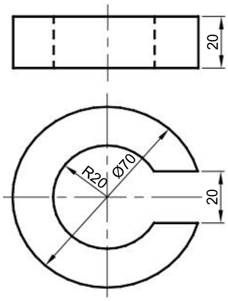


Fig. E15.10

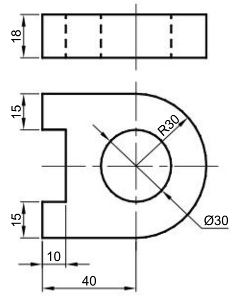


Fig. E15.11

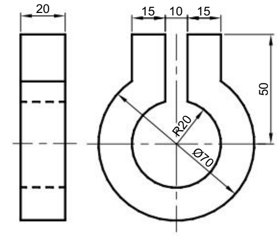


Fig. E15.12

15.2 Figures E15.13–E15.15 shows the orthographic projections of an object extruded in single direction and having hole, web or rib in the perpendicular direction. Draw their isometric view.

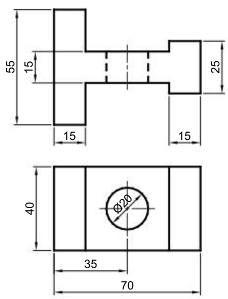


Fig. E15.13

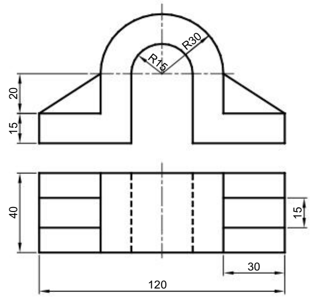


Fig. E15.14

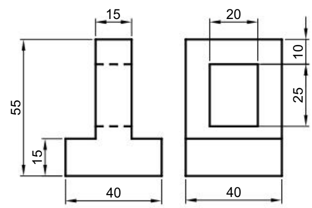


Fig. E15.15

Angle plates

15.3 Figures E15.16–E15.18 shows the orthographic projections (in first angle) of angle plates with certain additional features such as ribs, slots, holes etc. Draw their isometric view.

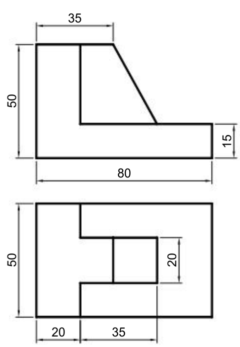


Fig. E15.16

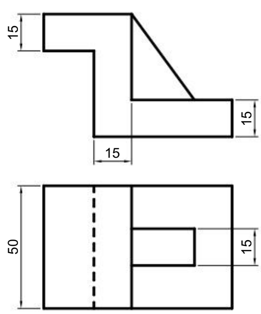


Fig. E15.17

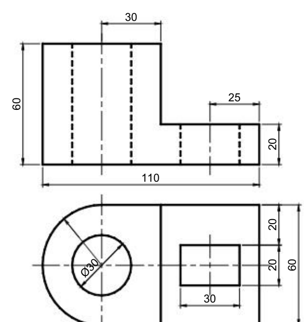


Fig. E15.18

Typical objects

15.4 Figures 15.19–15.26 shows the orthographic projections of objects in first angle projections. Draw their isometric view.

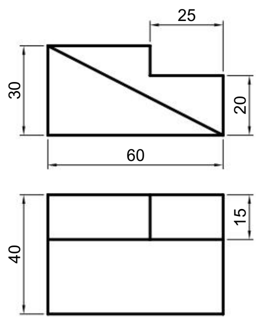


Fig. E15.19

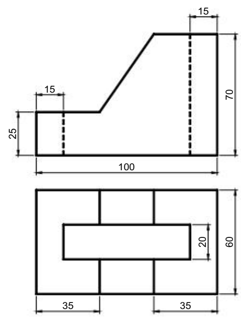


Fig. E15.20

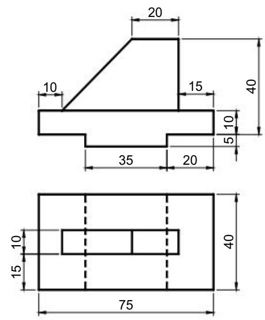


Fig. E15.21

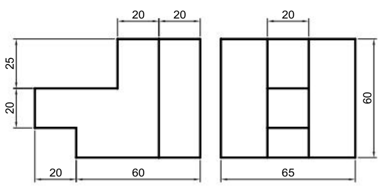


Fig. E15.22

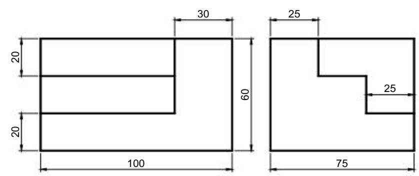


Fig. E15.23

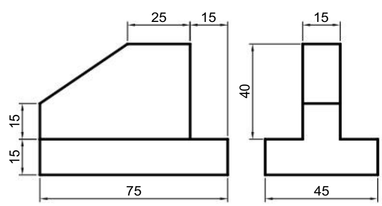


Fig. E15.24

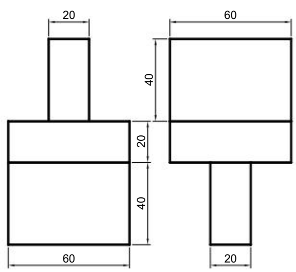


Fig. E15.25

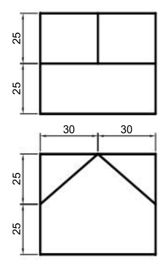


Fig. E15.26

15.17 FREE HAND SKETCHING OF ISOMETRIC VIEWS

Appendix A shows an isometric graph paper which helps in making free hand sketches of isometric projections with ease. The grid lines are marked along the three isometric axes and are 10 mm apart. Consider the following problem.

Problem 15.63 *The front, top and side views of an object are shown in Fig. 15.69(a). Draw its free hand isometric view.*

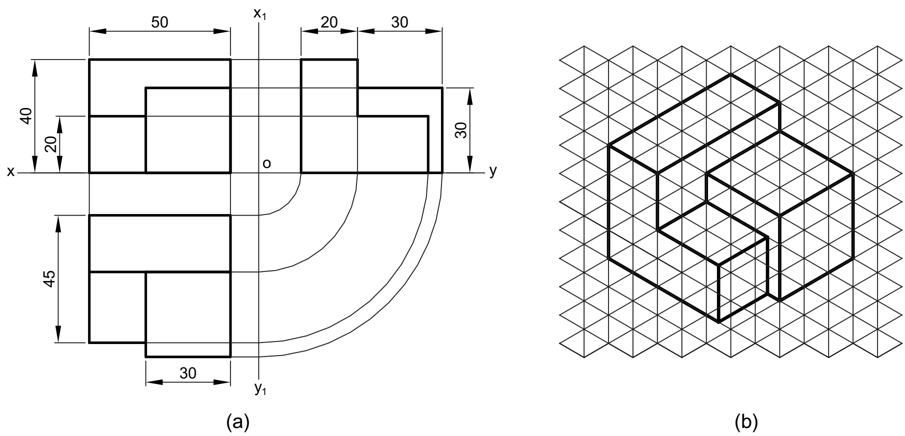


Fig. 15.69

Figure 15.69(b) shows the required isometric view.

15.18 MISSING VIEWS

Each orthographic view is a two-dimensional drawing showing only two out of three dimensions of an object. At least two orthographic views are required to define the exact shape of a three-dimensional object. When two orthographic views are given and it is required to draw the third view, one must correlate the given views to interpret the object accurately. Construction of free hand isometric view can be useful in interpretation of the features of an object. The following steps are sometimes helpful in drawing missing views.

1. Study the given orthographic views. A point in orthographic view may represent a corner or an edge. Similarly, a line may represent an edge or a surface.
2. Study surface by surface to create a mental image in the 3-D form. If necessary, create a free hand isometric view of the object to understand the features of the object accurately.
3. Project the missing view from the two orthographic views.

Consider the following problem.

Problem 15.64 *The front and top views of an object are shown in Fig. 15.70(a). Draw its side view.*

Construction Refer to Fig. 15.70(b).

1. Redraw the front and the top views to the scale.
2. Study surface by surface to create a mental image in the 3-D form. If necessary, draw free hand isometric view as shown in Fig. 15.36(b).
3. It is clear that the right-hand side view will show more features of the object that the left-hand side view. Draw the projectors from the front and top views as guidelines for the right-hand side view.
4. Draw the lines to obtain the side view as shown in Fig. 15.70(b).

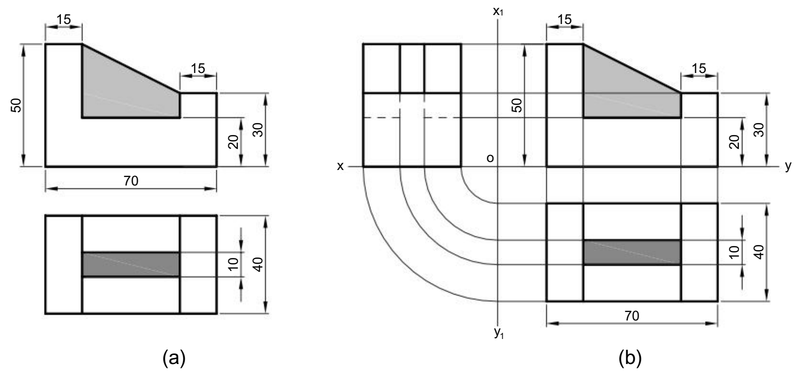


Fig. 15.70

EXERCISE 15C

15.1 The front and top views of an angle plate are shown in Fig. 15.49(a). Add a right hand side view.

15.2 The front and side views of an angle plate are shown in Fig. 15.50(a). Add a top view.

15.3 The front and top views of objects are given in Figs. E15.16–E15.18. Add right-hand side views to each pair of orthographic projections.

15.4 The front and top views of objects are given in Figs. E15.19–E15.21. Add suitable side views to each pair of orthographic projections.

15.5 The front and left-hand side views of objects are given in Figs. E15.22–E15.24. Add top views to each pair of orthographic projections.

VIVA-VOCE QUESTIONS

15.1 What is the R.F. of an isometric scale?

15.2 How would you construct an isometric scale?

15.3 Differentiate between isometric lines and non-isometric lines.

15.4 What is the relation among projectors in isometric projection?

15.5 State the relation between true length and isometric length?

15.6 Differentiate between isometric projection and isometric view.

15.7 Name the methods preferred for drawing ellipse in isometric projections.

15.8 Define isometric axes and isometric planes.

15.9 What are the principles of dimensioning in isometric projections?

15.10 What are the advantages of drawing isometric views?

MULTIPLE-CHOICE QUESTIONS

15.1 The projectors in isometric view are
- (a) converging
- (b) diverging
- (c) parallel to plane of projection
- (d) perpendicular to plane projection

15.2 Pictorial views drawn on isometric scale are called
(a) isometric drawing
(b) isometric projection
(c) isometric view
(d) Any of these

15.3 The exact value of R.F. of an isometric scale is
(a) 9/11
(b) 0.815
(c) 0.8165
(d) $\sqrt{2} / \sqrt{3}$

15.4 The angle that isometric lines make with each other is
(a) 45°
(b) 60°
(c) 90°
(d) 120°

15.5 A square in a regular multi-view projection appears in an isometric view as
(a) box
(b) square
(c) parallelogram
(d) rhombus

15.6 In comparison to an isometric projection, the appearance of an isometric view is
(a) larger
(b) smaller
(c) more accurate
(d) more realistic

15.7 On isometric plane, a circle appears as
(a) a spiral
(b) a circle
(c) an ellipse
(d) an involute

15.8 While making isometric projections the ellipse is preferably drawn by
(a) four centre method
(b) oblong method
(c) cncentric circles method
(d) parallelogram method

15.9 Isometric projections **cannot** be drawn by
(a) box method
(b) coordinate method
(c) offset method
(d) zone method

15.10 A sphere in isometric projection appears as a circle of diameter
(a) equal to the diameter of sphere
(b) 0.816 times the diameter of sphere
(c) less than 0.816 diameter of sphere
(d) greater than the diameter of sphere

15.11 The correct isometric view corresponding to the front and top views shown in Fig. M15.1 is

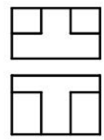
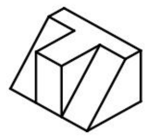
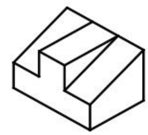
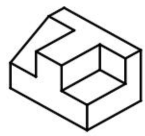
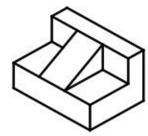


Fig. M15.1

(a) 
(b) 
(c) 
(d) 

15.12 The correct isometric view corresponding to the front and top views shown in Fig. M15.2 is

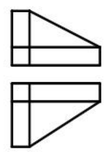
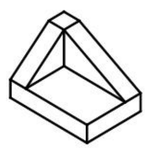
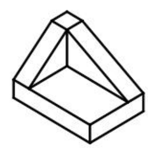
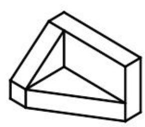
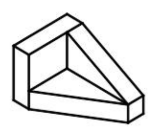


Fig. M15.2

(a) 
(b) 
(c) 
(d) 

15.13 The correct isometric view corresponding to the front view shown in Fig. M15.3 is

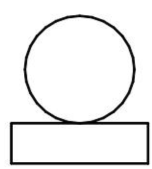
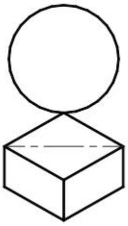
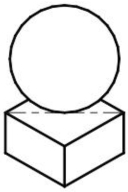
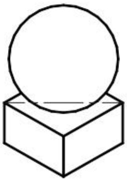
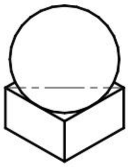


Fig. M15.3

(a) 

(b) 

(c) 

(d) 

15.14 The correct isometric view corresponding to the front view shown in Fig. M15.4 is

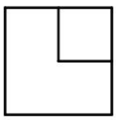
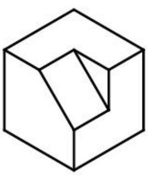
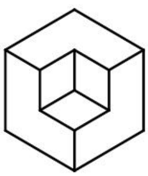
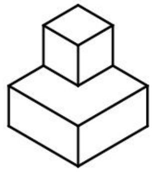
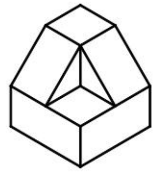


Fig. M15.4

(a) 

(b) 

(c) 

(d) 

15.15 The front and the top views of an object are same as shown in Fig. M15.5. The correct isometric view corresponding to the orthographic views is

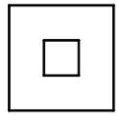
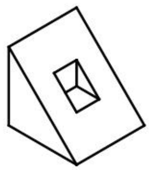
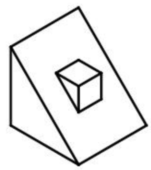
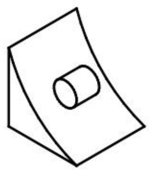


Fig. M15.5

(a) 

(b) 

(c) 

(d) All of these

Answers to multiple-choice questions

15.1 (d), 15.2 (b), 15.3 (d), 15.4 (d), 15.5 (d), 15.6 (a), 15.7 (c), 15.8 (a), 15.9 (d), 15.10 (a), 15.11 (c), 15.12 (b), 15.13 (c), 15.14 (b), 15.15 (d)

Chapter

16 OBLIQUE PROJECTIONS

16.1 INTRODUCTION

Oblique projection is defined as a pictorial projection in which projectors are parallel to each other and inclined to the plane of projection at any angle other than right angle.

In orthographic projections (both multi-view and axonometric), the projectors are parallel to each other and perpendicular to the plane of projection. Whereas in oblique projection, the projectors, although parallel to each other, are oblique to the plane of projection. (See Fig. 16.1). It may be seen that the face of an object parallel to the plane of projection will have the same appearance in both multi-view and oblique projections. To take this advantage, it is customary to have one of the faces of the object parallel to the plane of projection. This is the chief advantage of oblique projection over other forms of pictorial drawings.

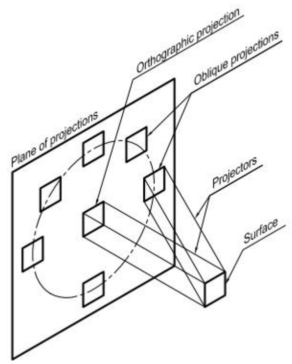


Fig. 16.1 *Orthographic and oblique projections*

16.2 TERMINOLOGY

The following terms are frequently used in oblique projections:

1. Receding axis In oblique projection, two axes are always in the front face at right angle to each other. The third axis represents an edge perpendicular to the plane of projection and may be at any convenient angle. This inclined line is called the receding axis (See Fig. 16.2).

2. Receding angles The angle made by the receding axis with the horizontal is known as receding angles. Ordinarily, this inclination of projection is 30º, 45º or 60º with horizontal, since these angles may be drawn with ease using set-squares. The angle that should be used in an oblique drawing depends upon the shape of the object and location of its significance. For example, a large angle is useful to obtain a better view of the rectangular recess on the top, while a small angle is useful to obtain a better view of the rectangular recess on the side (See Fig. 16.2).

3. Receding edge The projection of the edge of an object lying perpendicular to the plane of projection and drawn parallel to the receding axis in oblique projection, is commonly known as receding edge (See Fig. 16.3).

4. Receding plane The projection of the surface of an object lying perpendicular to the plane of projection and parallel or perpendicular to the ground, drawn in oblique projection, is commonly known as receding plane (See Fig. 16.3).

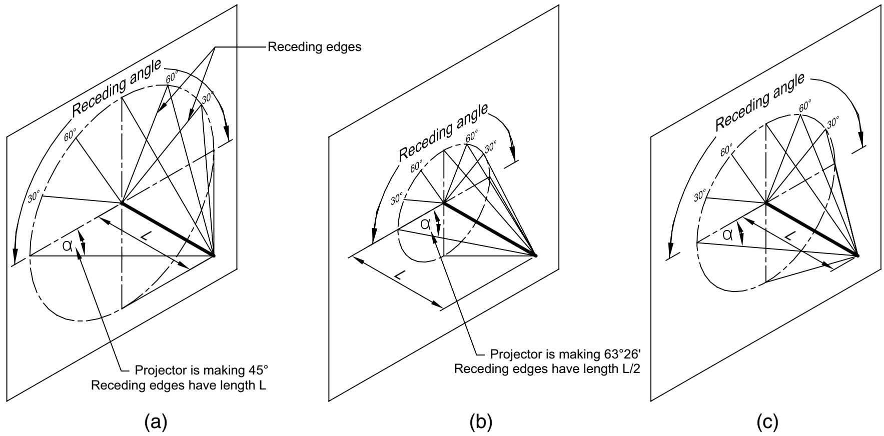


Fig. 16.2 **(a)** *Cavalier projection* **(b)** *Cabinet projection* **(c)** *General oblique*

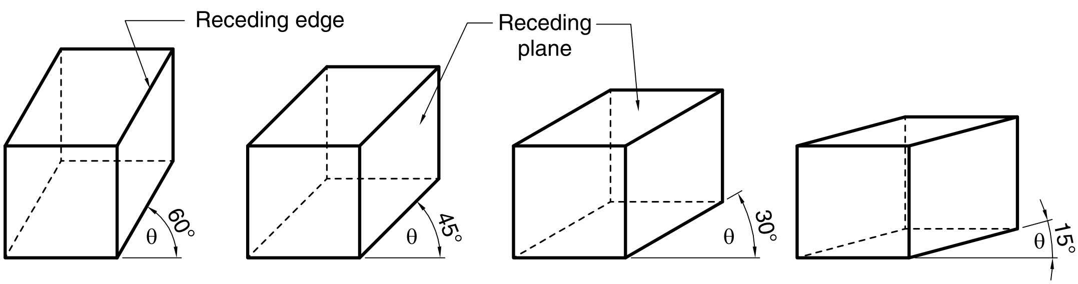


Fig. 16.3 *Cavalier projection of a cube with a few positions of receding axis*

16.3 DIRECTION OF PROJECTORS

The projectors may make any angle with the plane of projection, except an angle of 90º. The oblique projections distinguished from each other on the basis of the angle made by the projectors with the plane of projection are given below.

1. Cavalier projection
2. Cabinet projection
3. General oblique

1. Cavalier projection When the projectors make 45º with the plane of projection, the drawing is called a cavalier projection, (See Fig. 6.2(a)). This form of oblique drawing has an advantage that the lines which are perpendicular to the plane of projection also project to their true length as those which are parallel to the plane. The receding axis may make any angle with the horizontal. This angle must not be confused with the an angle of 45º made by the projecting line with the plane of projection. Figure 16.3 shows cavalier projection of a cube with different angles (θ) made by the receding axis with the horizontal. It may be seen that a receding angle of 30º gives a much better appearance than others.

In cavalier projection, all the three axes will project in their true length, and therefore the same scale may be used in all of them to make constructions. This is the distinguished feature of cavalier projection.

2. Cabinet projection When the projectors make 63º26' with the plane of projection, the drawing is called a cabinet projection. The tangent of this angle is 2, which means that a line perpendicular to the vertical plane is just twice as long as its projectors (See Fig. 6.2(b)). In other words, the line perpendicular to the vertical plane will have its projection length one-half of the actual line length. Again it should be mentioned that the angle which the receding axis makes with the horizontal may have any value just as in cavalier projection.

In a cabinet drawing, it must be remembered that all receding lines must be reduced to half size. Figure 16.4 shows cabinet projection of a cube with different angles made by the receding axis with the horizontal. It may be seen that a receding angle of 45º gives a much better appearance than others and hence should be preferred for cabinet projection.

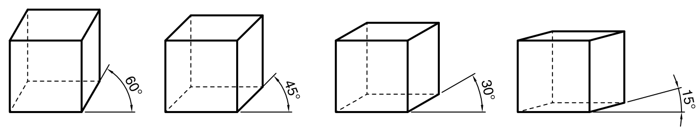


Fig. 16.4 *Cabinet projection of a cube with a few positions of receding axis*

3. General oblique When the projectors make angles other than 45º and 63º26' with the plane of projection as shown in Fig. 6.2(c), the drawing is called general oblique. Commonly, the line perpendicular to the vertical plane has its projection length varying from 0.5 to 0.75 of the actual line length. Figure 16.5 shows general projection of a cube having its projection length is 0.75 of the actual line length, and receding axis inclined at different angles with the horizontal.

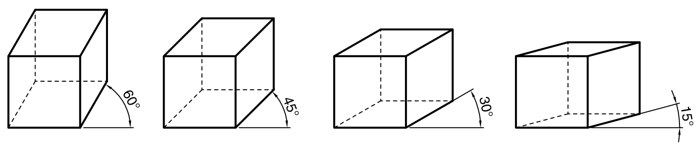


Fig. 16.5 *General oblique projection of a cube with a few positions of receding axis*

16.4 RULES FOR THE CHOICE OF POSITION OF AN OBJECT

The object should be in a position that reduces the distortion and labour to a minimum. This could be achieved when the face of an object showing the essential contours is placed parallel to the plane of projection (See Figs. 16.6(a) and (b)).

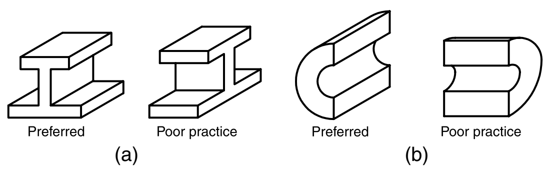


Fig. 16.6

16.5 DIMENSIONING OBLIQUE DRAWINGS

The principle of dimensioning, studied in connection with working drawings, applies in general to oblique projections, with the following additions:

1. Dimensions should be made to read from the bottom and right-hand side of the sheet.
2. Dimension lines, extension lines and arrowheads must lie in the same oblique plane to which they apply (See Fig. 16.7).
3. Dimensions must lie in oblique plane determined by the dimensioning lines and extension lines.
4. Numerals and letters may be made to lie in oblique planes in the same manner as shown for isometric.

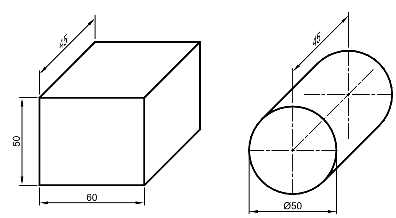


Fig. 16.7 *Preferred method for dimensioning*

5. As far as possible, the dimensions should be placed in the front face because this makes the dimensioning similar to that in the orthographic views.
6. Dimensions should be placed outside the outlines of the drawing except when greater clearness or directness of application results from placing the dimensions directly on the view.

16.6 ADVANTAGES OF OBLIQUE DRAWING

The distinct advantages of oblique drawings over other pictorial forms are given below.

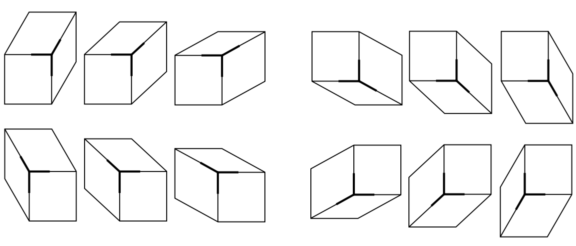
1. The front face of an object, or any face parallel to it, may be drawn like its true orthographic projection and hence circles may be drawn as true circles.
2. Distortion may be largely be overcome by a careful foreshortening of the scale on the receding axis.
3. Dimensioning is simpler since only one set of dimensions need be made in an oblique plane.
4. There is a greater range of choice of positions of the axes than in other forms, except trimetric.

16.7 OBLIQUE PROJECTIONS

16.7.1 Planes

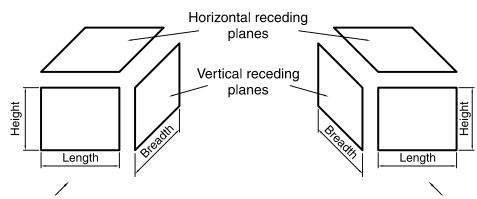
Figures 16.8(a) and (b) show the possible ways of drawing the oblique projection of a cube. Out of these methods, the one shown in Fig. 16.8(a) with receding lines upwards, is commonly used in practice.

To draw the oblique projection of a plane surface of lamina, draw a suitable principle plane on which it lies. Figures 16.9(a) and (b) show the vertical plane, horizontal receding planes and vertical receding planes.



(a) (b)

Fig. 16.8 **(a)** *Receding lines are upward* **(b)** *Receding lines are downward*



(a) (b)

Fig. 16.9

> **Problem 16.1** *Draw the cavalier and cabinet projections of a hexagonal plane of side 30 mm, having its surface perpendicular to both the principle planes. Consider receding axis makes 30° with the horizontal.*

Construction Refer to Figs. 16.10(b) and (c).

Let Fig. 16.10(a) show the right-hand side view of a hexagonal plane.

1. Draw the given hexagonal plane *abcdef* and enclose it in a rectangle *pqrs* of smallest size.
2. Draw a line *PQ* inclined at 30° to the horizontal. Its length should be equal to *pq* for the cavalier projection and half the length of *pq* for the cabinet projection.

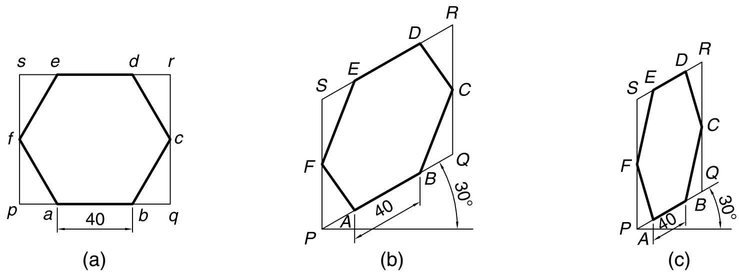


Fig. 16.10 **(a)** *Othographic view* **(b)** *Cavalier projection* **(c)** *Cabinet projection*

3. At ends P and Q, draw vertical lines PS and QR of lengths ps and qr. Join RS to complete the parallelogram $PQRS$. Thus, rectangle $pqrs$ is represented by parallelogram $PQRS$ in oblique projections.
4. Mark points A and B on PQ, C on QR, D and E on RS and F on PS. For cavalier projection, Fig. 16.10(b), take $PA = pa$, $PB = pb$, $QC = qc$, $RD = rd$, $RE = re$ and $PF = pf$. For cabinet projection, Fig. 16.10(c), take $PA = \frac{1}{2}\,pa$, $PB = \frac{1}{2}\,pb$, $QC = qc$, $RD = \frac{1}{2}\,rd$, $RE = \frac{1}{2}\,re$ and $PF = pf$.
5. Join $ABCDEF$, which represents the hexagonal plane.

Problem 16.2 *Draw the cavalier projection of a quadrilateral whose top view is shown in Fig. 16.11(a). Take receding axis inclination as 45° with horizontal.*

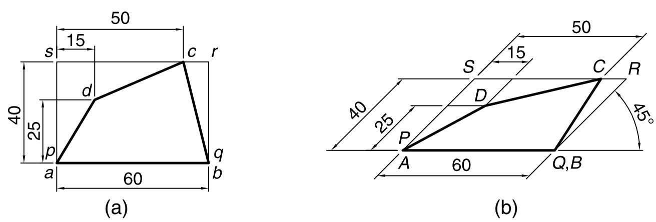


Fig. 16.11 **(a)** *Orthographic view* **(b)** *Cavalier projection*

Construction Refer to Fig. 16.11(b).

1. Draw the given quadrilateral $abcd$ and enclose it in a rectangle $pqrs$ of smallest size.
2. Draw a horizontal line PQ of length equal to pq.
3. At ends P and Q draw lines PS and QR inclined at 45° to line AB, of lengths ps and qr respectively.
4. Join RS to complete the parallelogram $PQRS$.
5. Mark points A and B at ends P and Q. Mark a point C on SR such that $RC = rc$. Also, mark point D as shown.
6. Join $ABCD$ to represent the quadrilateral in the cavalier projection.

Angles If an angle that is specified in degree lies in receding plane, it is necessary to convert the angle into linear measurement in order to draw the angle in linear measurement.

Problem 16.3 *Draw cavalier projection of a triangular plane, whose side view appears as shown in Fig. 16.12(a). Take receding axis inclination with horizontal as (a) 30° (b) 45°.*

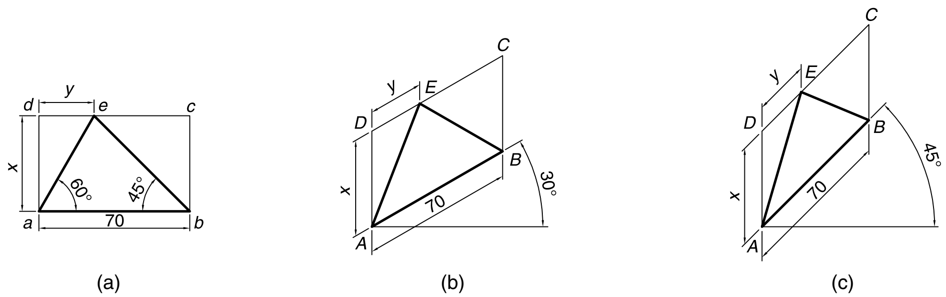


Fig. 16.12 **(a)** *Orthographic view* **(b)** *Receding angle at 30°* **(c)** *Receding angle at 45°*

Construction Refer to Figs 16.12(b) and (c).

1. Redraw the given triangular plane *abe* and enclose it in a rectangle *abcd* of smallest size as shown in Fig. 16.12(a).
2. Draw a line *AB* of length *ab*, inclined at an angle equal to that of receding axis.
3. At ends *A* and *B*, draw vertical lines *AD* and *BC* of lengths *ad* and *bc*. Join *CD* to complete the parallelogram *ABCD*.
4. Mark a point *E* on *CD* such that *CE* = *ce* and *DE* = *de*.
5. Join *ABC* to represent the required triangular plane.

Contours It is not always possible to place an object so that all of its significant contours are parallel to the plane of projection, specially, when a solid has two sets of contours in different planes. Offset method is used to construct contours including circles, arcs and other curved or irregular lines on receding planes.

Problem 16.4 *Draw the contour shown in Fig. 16.13(a) on the vertical receding planes of a cavalier projection. Consider receding angle as 30° to the horizontal.*

Construction Refer to Fig. 16.13(b). The points on the curves are located by coordinate method.

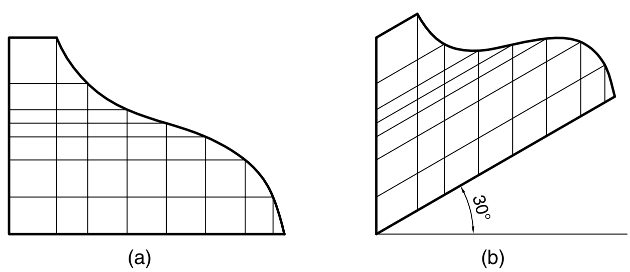


Fig. 16.13 **(a)** *Orthograpic view* **(b)** *Cavalier projection*

Problem 16.5 *Using offset method, draw a circular lamina of diameter 50 mm on both the receding planes of (a) a cavalier and (b) a cabinet. Consider receding angle as 30°.*

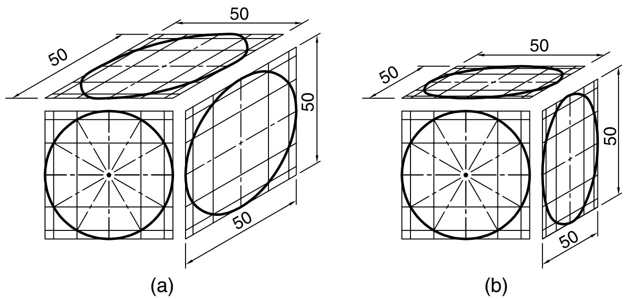


Fig. 16.14 **(a)** *Cavalier projection* **(b)** *Cabinet projection*

Construction Refer to Figs. 16.14(a) and (b).

The points on the curves are located by coordinate method.

Four-centre approximate method Four-centre ellipse method, by constructing ellipse with the compasses, could also be used to draw circular contours on receding planes of the cavalier projection.

Problem 16.6 *Using four-centre approximate method, draw cavalier projection of a circular disc having diameter of 50 mm on the receding planes. Consider receding angle as 45°.*

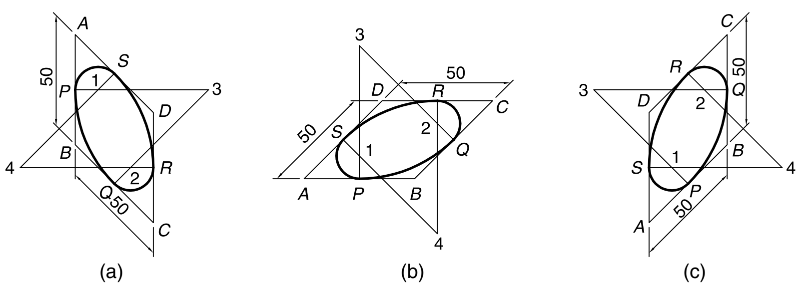


Fig. 16.15

Construction Refer to Figs. 16.15(a), (b) and (c).

1. Draw a rhombus *ABCD* of side 50 mm to represent the cavalier view of a square.
2. Mark *P*, *Q*, *R* and *S* as the mid-points of the sides *AB*, *BC*, *CD* and *DA* respectively.
3. At point *P*, draw a line *P*-1-3 perpendicular to *AB*. At point *Q*, draw a line *Q*-2-3 perpendicular to *BC*. At point *R*, draw a line *R*-2-4 perpendicular to *CD*. At point *S*, draw a line *S*-1-4 perpendicular to *DA*. It may be noted that points 1, 2, 3 and 4 are located at their intersections.
4. With centre 1 and radius *P*1, draw arc *PS*. With centre 2 and radius *Q*2, draw arc *QR*. With centre 3 and radius *P*3, draw arc *PQ*. With centre 4 and radius *R*4, draw arc *RS*.
5. The arcs join to form the ellipse, which represents the circle in the cavalier projection.

16.7.2 Basic Solids

Oblique projections of an object may be drawn by the following methods:

1. Box method In the box method, the object is assumed to be enclosed in a rectangular box and both the lines are drawn by locating the corresponding points of contact with the surfaces and edge of the box.

2. Offset method In the offset method, the lines parallel to the isometric axes are drawn from every corner or the reference point of an end to obtain the corner or the reference point at the other end.

Problem 16.7 *Draw the cavalier projection of a square prism having base side 40 mm and axis 60 mm resting on the H.P. (a) on its base and (b) on its rectangular faces.*

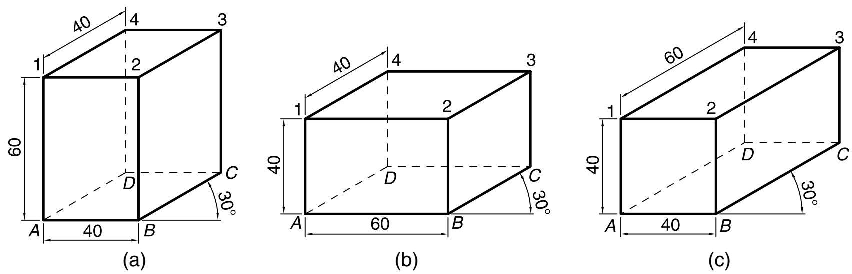


Fig. 16.16

Figure 16.16(a) shows the square prism is resting on its base in the H.P. Here, $AB = BC = 40$ mm and height $A1 = 60$ mm. Figure 16.16(b) shows the prism is resting on its rectangular face in the H.P. Here, $AD = A1 = 40$ mm and length $AB = 60$ mm. Figure 16.16(c) also shows the prism resting on its rectangular face in the H.P. Here, $AB = A1 = 40$ mm and width $AD = 60$ mm.

Construction Refer to Figs. 16.16(a), (b) and (c).

1. Draw a rhombus *ABCD* with its two sides *AB* and *CD* horizontal and other two sides *BC* and *DA* along the receding axis (preferred angle 30º for cavalier projection).
2. Draw *A*1, *B*2, *C*3 and *D*4 by offsetting corners *A*, *B*, *C* and *D* vertically. These lines show vertical edges of the prism.
3. Join 1-2-3-4.

Problem 16.8 *Draw the cabinet projection of a hexagonal prism, having base side 30 mm and axis 70 mm, (a) when the base is on the V.P. and an edge of the base is parallel to the H.P. and (b) when a rectangular face is parallel to the V.P.*

Construction Refer to Fig. 16.17(b) and (c).

1. Draw the top view of the prism as a hexagon *pqrstu* as shown in Fig. 16.17(a). Enclose this hexagon into a rectangle *abcd*.
2. Draw a box $ABCDA_1B_1C_1D_1$. For the cabinet projection, it is always preferred to have receding axis inclination as 45º and the receding length as half of the actual length. Therefore, for Fig. 16.17(b), $AA_1 = 35$ mm and dimensioned as 70 mm, while for Fig. 16.17(c), $BC = ½$ of *bc*.

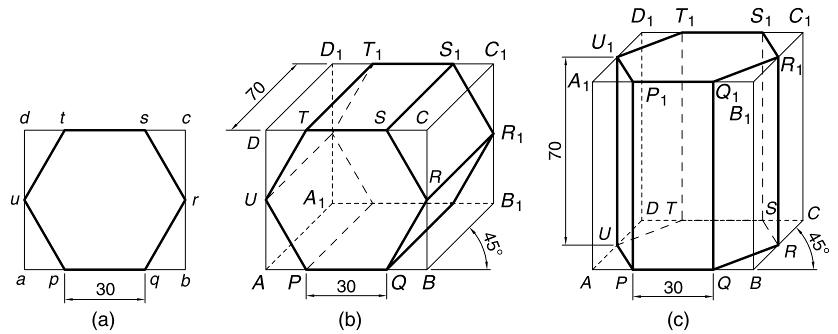


Fig. 16.17 **(a)** *Orthographic view* **(b)** *Cabinet projection* **(c)** *Cabinet projection*

3. Mark points P, Q, R, S, T and U in the cabinet projection such that $AP = ap$, $AQ = aq$, $BR = br$, $DT = dt$, $DS = ds$ and $DU = du$. Also, mark points P_1, Q_1, R_1, S_1, T_1 and U_1 such that $A_1P_1 = ap$, $A_1Q_1 = aq$, $B_1R_1 = br$, $D_1T_1 = dt$, $D_1S_1 = ds$ and $D_1U_1 = du$. For Fig. 16.17(c), $BR = ½\ br$, $DU = ½\ du$, $B_1R_1 = ½\ br$, $D_1U_1 = ½\ du$.
4. Join all the corners as shown and obtain cabinet projection of the prism.

Problem 16.9 *Draw the cavalier projection of a pentagonal prism, having base side 40 mm and axis 70 mm, (a) when the base is on the V.P. and an edge of the base is parallel to the H.P. and (b) when a rectangular face is parallel to the V.P.*

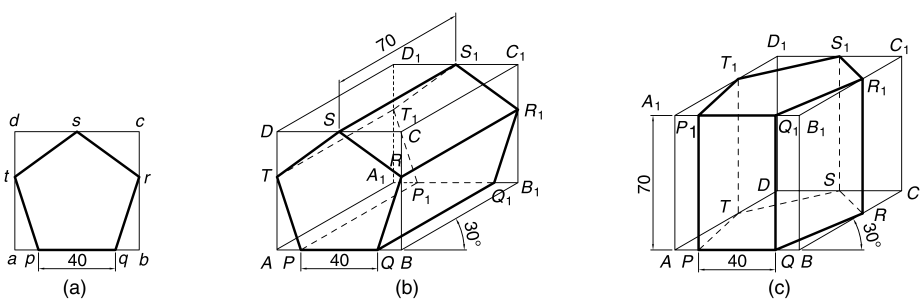


Fig. 16.18 **(a)** *Orthographic* **(b)** *Cavalier projection* **(c)** *Cavalier projection*

Construction Refer to Figs 16.18(b) and (c).

1. Draw a pentagon *pqrst* and enlose it in a rectangle *abcd*, as shown in Fig. 16.18(a).
2. Draw a box $ABCDA_1B_1C_1D_1$. For cavalier projection, it is always preferred to have receding axis inclination as 30º and the receding length as equal to the actual length. Therefore, for Fig. 16.18(b), AA_1 = 70 mm and for Fig. 16.18(c), $BC = bc$.
3. Mark points P, Q, R, S, T and U in the cabinet projection such that $AP = ap$, $AQ = aq$, $BR = br$, $DT = dt$, $DS = ds$ and $DU = du$. Also, mark points P_1, Q_1, R_1, S_1, T_1 and U_1 such that $A_1P_1 = ap$, $A_1Q_1 = aq$, $B_1R_1 = br$, $D_1T_1 = dt$, $D_1S_1 = ds$ and $D_1U_1 = du$.
4. Join all the corners as shown and obtain cavalier projection of the prism.

Problem 16.10 *Draw the cavalier projection of a cylinder having base diameter 50 mm and axis 65 mm when*

(a) the axis is perpendicular to the plane of projection,

(b) the base is on the H.P. and

(c) the axis is parallel to both H.P. and V.P.

Take receding angle as 45°.

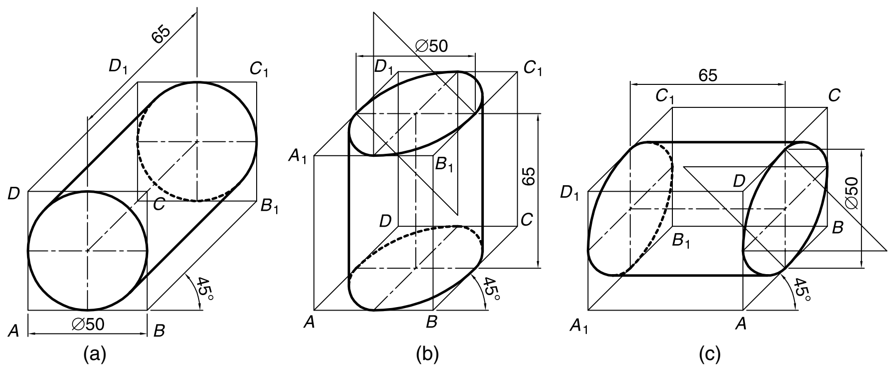


Fig. 16.19

Construction Refer to Figs 16.19(a), (b) and (c).

1. Draw a box $ABCDA_1B_1C_1D_1$ such that $AB = BC = 50$ mm and $AA_1 = 65$ mm.
2. For Fig. 16.19(a), draw circles on faces $ABCD$ and $A_1B_1C_1D_1$ and join them tangentially to represent the cylinder. For Figs. 16.19(b) and (c), draw an ellipse representing circle on the receding planes $ABCD$ and $A_1B_1C_1D_1$ using approximate four-centre method. Join the ellipse tangentially to represent the cylinder.

Problem 16.11 *Draw an oblique projection of a hexagonal pyramid, having base side 30 mm and axis 70 mm when its axis is vertical, using (a) cavalier method, with receding axis inclined at 30° to the horizontal (b) cabinet method with receding axis inclined at 45° to the horizontal.*

Construction Refer to Figs. 16.20(a) and (b).

1. Draw the top view of the prism as a hexagon *pqrstu* shown in Fig. 16.20(a). Enclose this hexagon into a rectangle *abcd*.
2. Draw the receding plane *ABCD*. For cavalier projection, Fig. 16.20(b), take $AB = ab$ and $BC = bc$. For cabinet projection, Fig. 16.20(c), take $AB = ab$ and $BC = ½$ of bc.
3. Mark points P, Q, R, S, T and U such that $AP = ap$, $AQ = aq$, $BR = br$, $DT = dt$, $DS = ds$ and $DU = du$. For Fig. 16.20(c), $BR = ½\ br$, $DU = ½\ du$. Locate the centre O_1 and project it 70 mm vertical to locate the apex O. Join apex with the corners P, Q, R, S, T and U as shown.

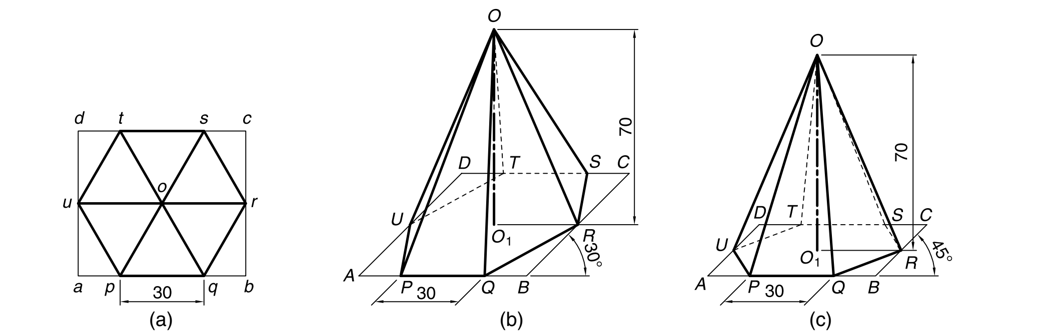


Fig. 16.20 **(a)** *Orthographic* **(b)** *Cavalier projection* **(c)** *Cabinet projection*

16.7.3 Truncated Solids

Problem 16.12 *The frustum of a hexagonal pyramid having base side 35 mm, top side 20 mm and axis 70 mm rests on an edge in the H.P. with an edge of the base perpendicular to the V.P. Draw its cabinet projection when (a) receding angle is 60° and (b) receding angle is 45°.*

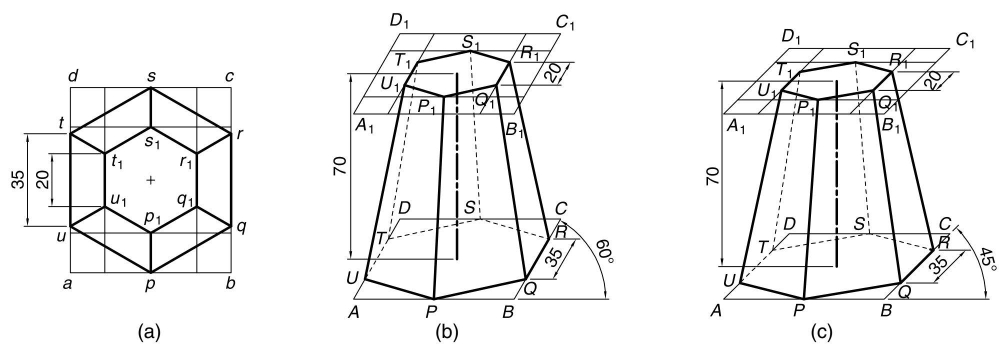


Fig. 16.21 **(a)** *Orthographic***(b)** *Cabinet projection* **(c)** *Cabinet projection*

Construction Refer to Figs. 16.21(b) and (c).

1. Draw the top view of the prism and enclose this figure in a rectangle *abcd*.
2. Draw a box $ABCDA_1B_1C_1D_1$ such that side $AB = ab$, $BC = bc$ and height AA_1=70 mm.
3. Mark points P, Q, R, S, T and U on the base $ABCD$ such that $AP = ap$, $BQ = ½\,bq$, $BR = ½\,br$, $DS = ds$, $DT = ½\,dt$, and $DU = ½\,du$. Similarly, mark P_1, Q_1, R_1, S_1, T_1 and U_1, using coordinate method, on the top face $A_1B_1C_1D_1$.
4. Join the points as shown.

Problem 16.13 *A triangular pyramid of base side 60 mm and axis 80 mm is resting on its base on the H.P. with an edge of the base parallel to the V.P. It is cut by an A.I.P. inclined at 45° to the H.P. and bisecting the axis. Draw its cavalier projection with receding axis inclined at 45° to the horizontal.*

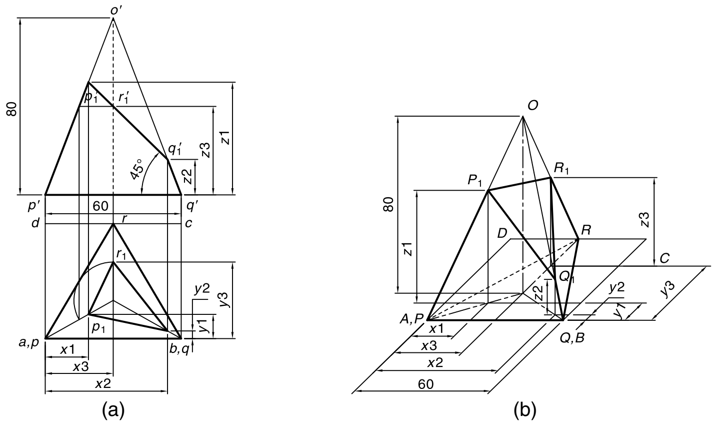


Fig. 16.22 **(a)** *Orthographic projection* **(b)** *Cavalier projection*

Construction Refer to Figs. 16.22(b).

1. Draw the orthographic projection of the prism as shown in Fig. 16.22(a) and enclose the top view in rectangles *abcd*.
2. Draw a rhombus *ABCD* with *AB* horizontal and *BC* inclined at 45º to the horizontal, such that sides *AB* = *ab* and *BC* = *bc*.
3. Mark a point *R* as the mid-point of *CD*.
4. From the orthographic projection, obtain the coordinates of points *P*, *Q* and *R* from one of the corners *A* as ($x1$, $y1$, $z1$) ($x2$, $y2$, $z2$) and ($x3$, $y3$, $z3$) respectively. Using coordinate method, obtain points P_1, Q_1 and R_1 in the oblique view.
5. Join the points as shown.

16.7.4 Composite and Hollow Solids

Problem 16.14 *A cone of base diameter 50 mm axis 70 mm is placed centrally on the top of a square block having base side 70 mm and height 30 mm. Draw its oblique projection.*

Construction First draw the oblique projection of the square box. Then construct the cone on the top surface of the box. Figure 16.23 shows the required oblique projection.

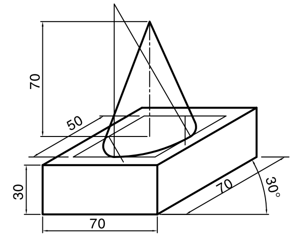


Fig. 16.23 *Cavalier projection*

Problem 16.15 *A cylinder of base diameter 60 mm and axis 75 mm has a coaxial through square hole of side 20 mm. Draw its cavalier projection when cylinder rests on its base in the horizontal plane. Consider receding angle as 45°.*

Construction First draw the oblique projection of the cylinder. Then construct the square on the top surface of the cylinder and extend it downward to represent the hole. Figure 16.24 shows the required oblique projection.

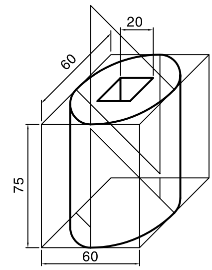


Fig. 16.24 *Cavalier projection*

16.7.5 Orthographic to Oblique Views

Problem 16.16 *Figure 16.25(a) shows the orthographic projections of an object. Draw its cavalier projection.*

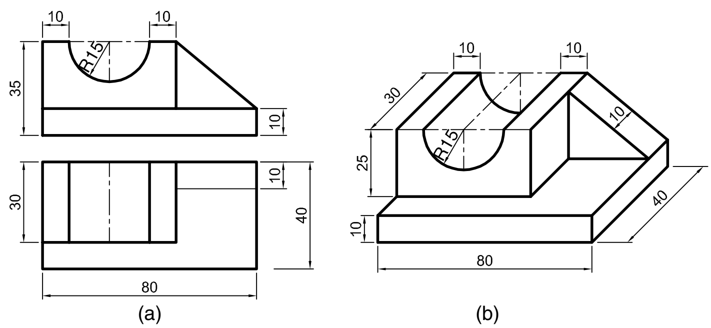


Fig. 16.25 **(a)** *Orthographic projection* **(b)** *Cavalier projection*

Problem 16.17 *Figure 16.26(a) shows the orthographic projections of an object. Draw its cavalier projection.*

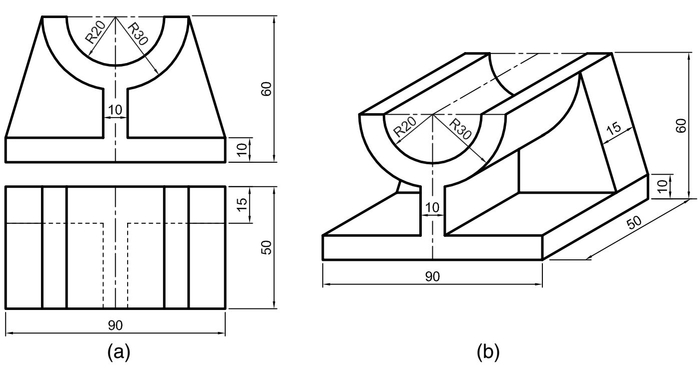


Fig. 16.26 **(a)** *Orthographic projection* **(b)** *Cavalier projection*

Problem 16.18 *Figure 16.27(a) shows the orthographic projections of an object. Draw its cavalier projection.*

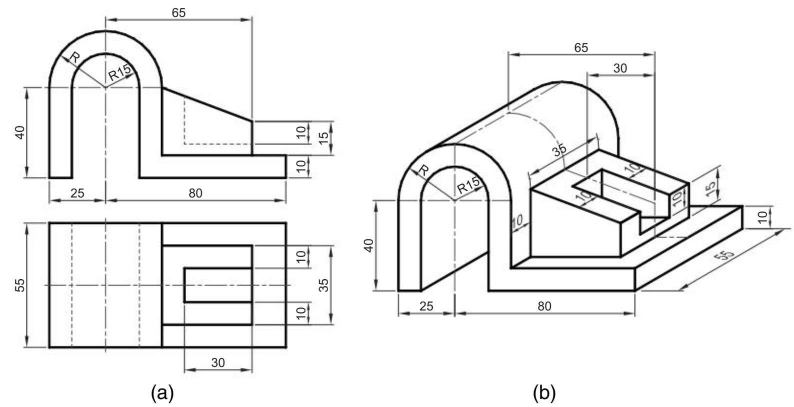


Fig. 16.27 **(a)** *Orthographic projection* **(b)** *Cavalier projection*

EXERCISE 16

16.1 The frustum of a pentagonal pyramid of base side 35 mm, top side 20 mm and axis 70 mm, rests on an edge in the H.P. with an edge of the base perpendicular to the V.P. Draw its cabinet projection when receding angle is 45º.

16.2 A cylinder of base diameter 50 mm and axis 60 mm is lying on one of its generator on the top of a square block of base side 70 mm and height 30 mm. Draw its oblique projection when the cylinder is placed centrally over the block.

16.3 Frustum of a cone of base diameter 60 mm and axis 50 mm is placed centrally on the top of a square block of base side 80 mm and height 40 mm. Draw its oblique projection.

16.4 A hexagonal plate of base side 30 mm and axis 20 mm has a coaxial through hole of diameter 30 mm. Draw its cavalier projection when plate rests on its base in the horizontal plane. Consider receding angle as 45º.

16.5 Figures E16.1 to E16.6 show the orthographic projections of an object drawn in first angle projections. Draw their cavalier projections.

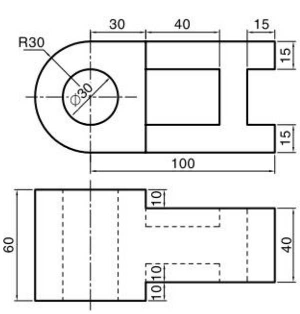


Fig. E16.1

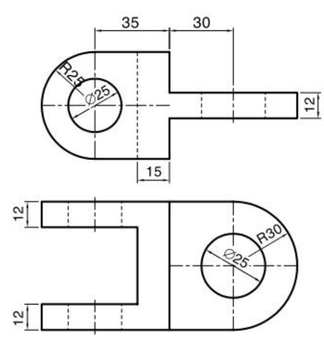


Fig. E16.2

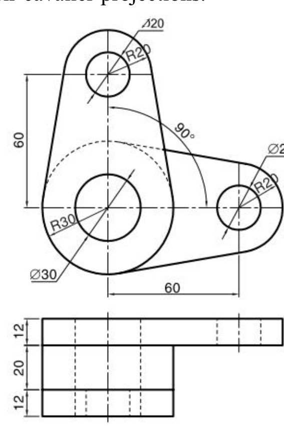


Fig. E16.3

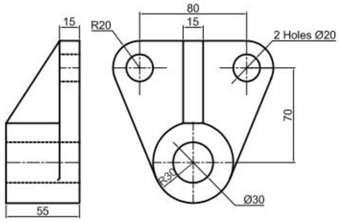


Fig. E16.4

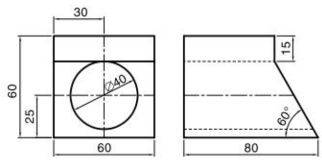


Fig. E16.5

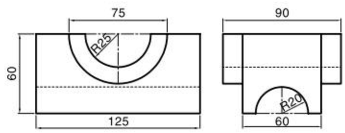


Fig. E16.6

VIVA-VOCE QUESTIONS

16.1 State the similarities and dissimilarities between isometric projections and oblique projections.

16.2 Differentiate between the orthographic projection and oblique projection.

16.3 Explain the terms receding axes, receding angles and receding planes.

16.4 Differentiate between cavalier projection and cabinet projection.

16.5 With the help of suitable examples explain the rules for selecting the position of an object in oblique projection.

16.6 What are the principles of dimensioning in oblique drawing?

16.7 What are the advantages of drawing oblique projection?

16.8 Name the method that is preferred for drawing ellipse in oblique projections.

MULTIPLE-CHOICE QUESTIONS

16.1 The face of an object containing circles, irregular shapes, etc., is kept parallel to the plane of projection is known as
- (a) isometric projection
- (b) perspective projection
- (c) oblique projection
- (d) None of these

16.2 In an oblique projection, the front surface of the object is kept at an angle eith respect to plane of projection is
- (a) perpendicular
- (b) parallel
- (c) 45°
- (d) either 30º or 60º

16.3 The drawings in which the receding lines are drawn to half the scale are called
- (a) isometric
- (b) cavalier
- (c) cabinet
- (d) perspective

16.4 The distortion in oblique projections can be decreased by
- (a) placing the projection in correct scale
- (b) reducing the length of the receding lines
- (c) placing the projection obliquely to the plane of projection
- (d) enlarging the dimensions parallel to the plane of projection

16.5 In oblique projections, the receding lines meet the plane of projection at an angle
- (a) 0º
- (b) 30º
- (c) 90º
- (d) less than 90º

16.6 To emphasise the features on the side of an object, the receding lines are drawn at the following angle to the plane of projection, which is
- (a) 45°
- (b) 60°
- (c) greater than 45°
- (d) less than 45°

16.7 In oblique projections, a semi circle parallel to the plane of projection appears as
- (a) semicircle
- (b) semi-ellipse
- (c) cycloid
- (d) partial ellipse

16.8 The projectors in oblique projections are
- (a) converging at plane of projection
- (b) parallel to plane of projection
- (c) inclined to plane of projection
- (d) perpendicular to plane of projection

16.9 In the cavalier projection, an angle at which the projectors meet the plane of projection is
- (a) 30º
- (b) 45º
- (c) 63º26'
- (d) None of these

16.10 In the cabinet projection, an angle at which the projectors meet the plane of projection is
- (a) 30º
- (b) 45º
- (c) 63º26'
- (d) None of these

16.11 In the general oblique projection, an angle at which the projectors meet the plane of projection is
- (a) 45º
- (b) 63º26'
- (c) 90º
- (d) None of these

16.12 While making cavalier projections, the ellipse is preferably drawn by
- (a) four-centre approximate method
- (b) oblong method
- (c) concentric circles method
- (d) parallelogram method

Answers to multiple-choice questions

16.1 (c), 16.2 (b), 16.3 (c), 16.4 (b), 16.5 (d), 16.6 (d), 16.7 (a), 16.8 (c), 16.9 (b), 16.10 (c), 16.11 (d), 16.12 (a)

Chapter

17

PERSPECTIVE PROJECTIONS

17.1 INTRODUCTION

Perspective projection is a three-dimensional representation of an object on a plane as it is perceived by the human eye from a particular point. It is a geometric method of obtaining images which are similar to the photographs taken by a camera.

The major difference between parallel projection, be it orthographic oblique or isometri*c*, and perspective projection lies in the fact that, in the later case, the point of sight is at a finite distance from the object. The projectors from the object therefore converge to the point of sight instead of being parallel to each other as in the former types of projection. Such drawing is also known as scenographic projection or central projection.

17.2 APPLICATIONS OF PERSPECTIVE

Perspective projections are extensively used by architects and civil engineers to show the appearance of proposed buildings, roads, railroad tracks and interior designs. Because the perspective projection shows an object as it actually appears to the human eyes, it is also used for producing sceneries and advertising drawings.

17.3 TYPES OF PERSPECTIVE

Depending upon the nature of the object, perspective drawings can be of one of the following types:

1. One-point perspective or parallel perspective
2. Two-point perspective or angular perspective
3. Three-point perspective or oblique perspective
4. Zero-point perspective
5. Infinite-point perspective
6. Aerial perspective or atmospheric perspective

1. One-point perspective (Parallel perspective) Objects made up of lines, either parallel or perpendicular with the viewer's line of sight, can be represented with one-point perspective. All elements which are parallel to the plane of projection are drawn as parallel lines, whereas those which are perpendicular to the plane of projection converge at a single point (See Fig. 17.1(a)). Such a point, where the lines of sight converge, is commonly known as a *vanishing point* and the plane of projection is known as *picture plane*. One-point

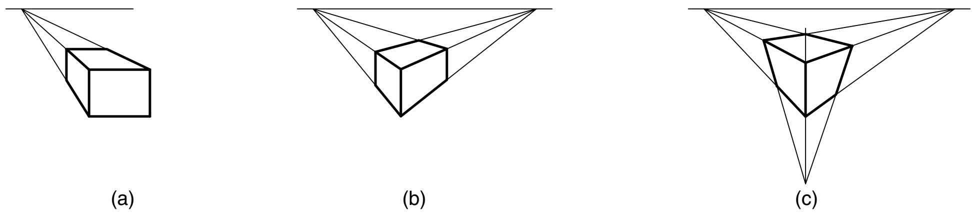


Fig. 17.1 **(a)** *One-point perspective* **(b)** *Two-point perspective* **(c)** *Three-point perspective*

perspective is also called *parallel perspective* because one face of the object is parallel to the plane of projection. It is generally used for roads, railroad tracks, or such buildings which are viewed directly from the front.

2. Two-point perspective (Angular perspective) Two-point perspective as shown in Fig. 17.1(b) is used when the object is positioned with all horizontal edges at an angle with the plane of projection and with all vertical edges parallel to it. When a house is viewed from the corner, one wall would converge at one point, the other wall would converge at another point opposite to it. Two-point perspective is also called *angular perspective* because the object is positioned at an angle with the plane of projection. It is generally used for buildings viewed from vertical edge though a corner.

3. Three-point perspective (Oblique perspective) Three-point perspective is generally used for buildings seen from above. In addition to the two vanishing points, one for each wall, there is another one for those walls receding into the ground (See Fig. 17.1(c)). Each of the three vanishing points corresponds with one of the three axes of the object. Three-point perspective is also called *oblique perspective* because it is constructed using multiple vanishing points. It is generally used for tall buildings.

4. Zero-point perspective Vanishing points exist when parallel lines are present in the object. A perspective without any vanishing points is called zero-point perspective. It occurs when the viewer is observing a non-linear scene like a mountain range, a landscape or a random arrangement of spherical objects. It may clearly be noted that although orthographic projections do not have any vanishing points, they are neither perspective nor equivalent to a zero-point perspective.

5. Infinite-point perspective Linear perspectives consisting of one-point, two-point, and three-point perspective are dependent on the structure of the object being viewed. However, it is possible to have an object which consists of infinite pairs of parallel lines which may not be parallel to any of the three principal axes. In such a case, a new distinct vanishing point is created for each pair. Since the ratio at which more distant objects decrease in size is constant, and the number of vanishing points is large, it is called infinite-point perspective.

6. Aerial perspective (Atmospheric perspective) It is the technique used in painting to create the illusion of depth or recession by depicting distant objects as paler, less detailed and bluer than near objects. As the distance between an object and a viewer increases, the contrast between the object and its background decreases. The contrast of any markings or details on the object also decreases. The colours of the object also become less saturated and shift towards blue. It is important to emphasize that this does not blur the outlines of the markings of objects.

17.4 CHARACTERISTIC FEATURES OF PERSPECTIVE PROJECTIONS

The characteristic features of perspective projections are given below.

1. Parallel lines no longer remain parallel.
2. Horizontal parallel lines converge to a single point on the distant horizon.
3. Objects become smaller as their distance from the observer increases.

17.5 TERMINOLOGY

The following terms are frequently used in perspective projections:

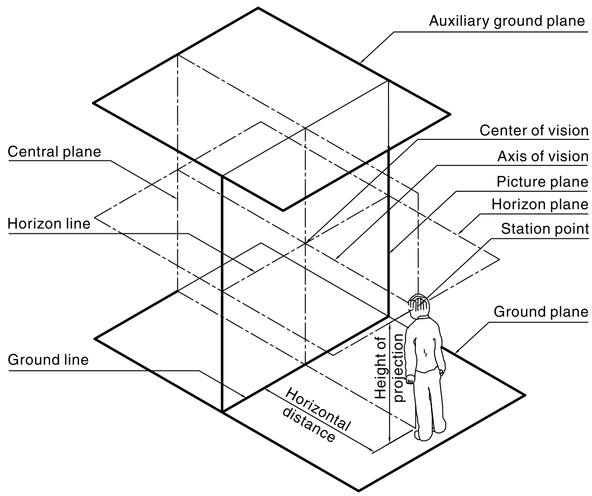


Fig. 17.2 *Terminology used in perspective projection*

1. Ground plane (GP) It is a flat natural surface or a horizontal plane on which the object is placed. The observer also stands on this plane to view the object from a convenient distance.

2. Station point (SP) Station point is the position of the observer's eye while viewing the object. It is the convergence point for light that creates the image.

3. Picture plane (PP) It is a two-dimensional surface in front of the observer and perpendicular to the ground plane. The perspective view of the object is obtained on this plane. It is analogous to the vertical plane used in the orthographic projections. This plane is also used to draw the front view of the perspective rays and sometimes that of the object.

4. Horizon plane (HP) It is an imaginary horizontal plane parallel to the ground plane passing through the observer's eye or the station point.

5. Auxiliary ground plane (AGP) It is also an imaginary horizontal plane parallel to the ground plane and placed above the horizon plane. This plane is used to draw the top view of the perspective rays and the object.

6. Central plane (CP) It is an imaginary vertical plane perpendicular to both the ground plane and the picture plane passing through the station point.

7. Ground line (GL) It is the line of intersection of the ground plane with the picture plane. This line is the horizontal trace of the picture plane on the ground plane.

8. Horizon line (HL) It is the line of intersection of the horizon plane with the picture plane. This line is the vertical trace of the horizon plane on the picture plane. It is the line where the sky meets the earth. All points vanish to this line. It is also called eye level.

9. Centre line (CL) It is the line of intersection of the central plane with the picture plane. This line is the vertical trace of the central plane on the picture plane.

10. Axis of vision (AV) It is a line drawn from the station point perpendicular to the picture plane. It is also known as the line of sight of viewpoint, central ray, axis of sight, perpendicular axis or principal visual ray.

11. Centre of vision It is the point of intersection of the axis of vision with the horizon line.

12. Visual rays These are the lines joining the different points on the object to the station point.

13. Vanishing point Parallel lines on the same axis converge at a point on the horizon line. This point of convergence is called the vanishing point.

14. Horizontal distance It is the distance between the station point and the picture plane.

15. Height of projection It is the distance of the station point above the ground plane.

17.6 THE MYTH OF PERSPECTIVES

One-point, two-point and three-point perspectives appear to embody different forms of calculated perspective. The methods required to generate these perspectives by hand are different. Mathematically, however, all three are identical. The difference is simply in the relative orientation of the rectilinear scene to the viewer. For example, the three images illustrating one-, two- and three-point perspective in the above section can be generated in two ways with identical results:

1. The "standard" way would be to alter the viewer's position in each perspective with a stationary cube
2. Identical to the above way is to simply rotate the cube in space in front of a stationary viewer.

17.7 METHODS OF DRAWING PERSPECTIVE VIEWS

There are two common methods of drawing perspective views of an object:

1. Visual ray method and its alternative method
2. Vanishing point method and its alternative method

The following examples illustrate the methods of drawing perspective projections.

Problem 17.1 *Draw a perspective view of a square plane of side 50 mm resting on the GP with one of its corners touching PP and a side right to the corner inclined at 60° to it. The station point is 70 mm in front of PP, 65 mm above GP and lies in a CP which is 35 mm towards right of the corner touching the PP.*

The problem can be solved in the following manner:

Construction Refer to Fig. 17.3(a).

1. Draw three horizontal lines to represent PP, HL and GL, as shown.
2. Construct a square *abcd* to represent top view of the square plane. Project it to GL and obtain $a'b'c'd'$ as its front view.
3. Mark points s and s' in the required position to represent the station points in top and front views respectively.
4. Join s with points a, b, c and d to represent top view of the perspective rays. Similarly, join s' with points a', b', c' and d' to obtain front view of the perspective rays.
5. Mark a_1, b_1, c_1 and d_1, the piercing points of top view of the perspective rays with PP.
6. Project these points to meet the front view of the corresponding perspective rays to get points A, B, C and D. Join them to obtain the required perspective view.

Construction Refer to Fig. 17.3(b).

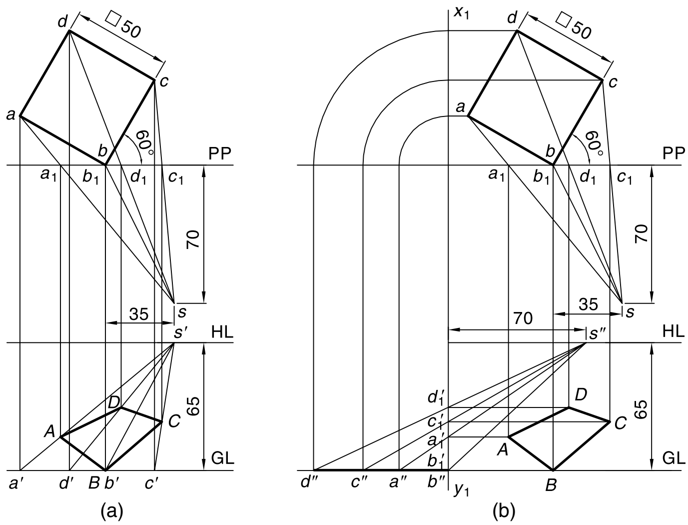


Fig. 17.3 **(a)** *Visual ray method* **(b)** *Visual ray method*

1. Draw three horizontal lines to represent PP, HL and GL, as shown. Also, draw a vertical line x_1y_1.
2. Construct a square *abcd* to represent top view of the square plane and obtain $a''b''c''d''$ as its side view.
3. Mark points s and s'', 70 mm away from PP and x_1y_1 respectively to represent the station points in top and side views.
4. Join s with points a, b, c and d to represent top view of the perspective rays. Similarly, join s'' with points a'', b'', c'' and d'' to obtain the side view of the perspective rays.
5. Mark a_1, b_1, c_1 and d_1, the piercing points of top view of the perspective rays with PP. Also, mark a_1', b_1', c_1' and d_1', the piercing points of the side view of the perspective rays with x_1y_1.
6. Draw vertical lines from a_1, b_1, c_1, d_1 and horizontal lines from a_1', b_1', c_1', d_1' to get points A, B, C and D. Join them to obtain the required perspective view.

Construction Refer to Fig. 17.3(c).

1. Draw three horizontal lines to represent PP, HL and GL, as shown.
2. Construct a square *abcd* to represent top view of the square plane.
3. Mark a point s in the required position to represent the station point in the top view.
4. Join s with points a, b, c and d to represent top view of the perspective rays. Mark a_1, b_1, c_1 and d_1, the piercing points of top view of these rays with PP.
5. Draw a line sv_1 parallel to edge bc to meet PP at v_1. Project v_1 on HL to get v_1'.
6. Extend line da to meet PP at p. Project points p and b to meet GL at p' and b' respectively.
7. Join $v_1'p'$ and $v_1'b'$. Project points a_1 and d_1 to meet $v_1'p'$ at points A and D respectively. Project points c_1 to meet $v_1'b'$ at point C.
8. Join points A, B, C and D to obtain the required perspective view.

Construction Refer to Fig. 17.3(d).

1. Draw three horizontal lines to represent PP, HL and GL, as shown.
2. Construct a square *abcd* to represent top view of the square plane.

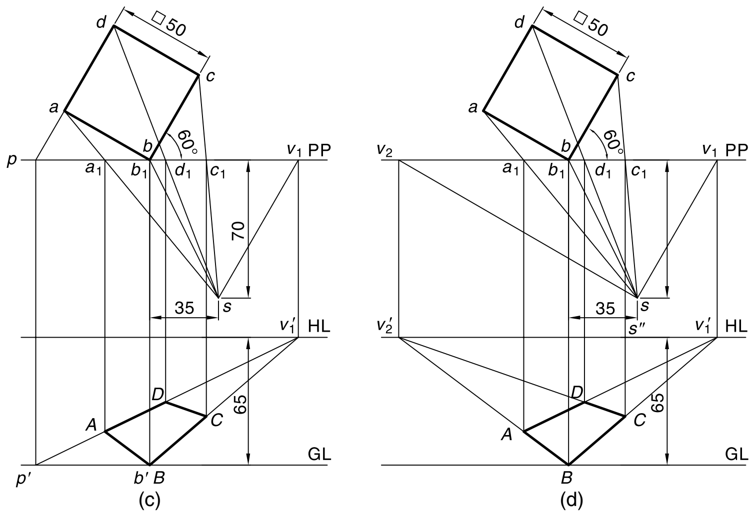


Fig. 17.3 **(c)** *Vanishing point method* **(d)** *Vanishing point method*

3. Mark a point s to represent the station point in the top view.
4. Join s with points a, b, c and d to represent top view of the perspective rays. Mark a_1, b_1, c_1 and d_1, the piercing points of top view of these rays with PP.
5. Draw a line sv_1 parallel to edge bc to meet PP at v_1. Project v_1 on HL to get v_1'.
6. Similarly, draw a line sv_2 perpendicular to sv_1 to meet PP at v_2. Project v_2 on HL to get v_2'.
7. Since b touches the PP, project it directly to GL to get B. Join $v_1'B$ and $v_2'B$.
8. Project a_1 and c_1 to $v_2'B$ and $v_1'B$ respectively to get points A and C.
9. Join $v_1'A$ and $v_2'C$, which meet each other at D. It may be noted that a projector drawn from d_1 also meets at d_1'.
10. Join points A, B, C and D to obtain the required perspective view.

Note: This is a problem of two-point perspective and all methods give the same result.

Problem 17.2 *A square plane of side 50 mm lies on the GP with an edge parallel to and 20 mm behind the PP. The station point is 60 mm in front of PP, 65 mm above GP and lies in a CP which is 55 mm towards right of the centre of the square plane. Draw its perspective view.*

Construction Refer to Fig. 17.4.

1. Draw three horizontal lines to represent PP, HL and GL, as shown.

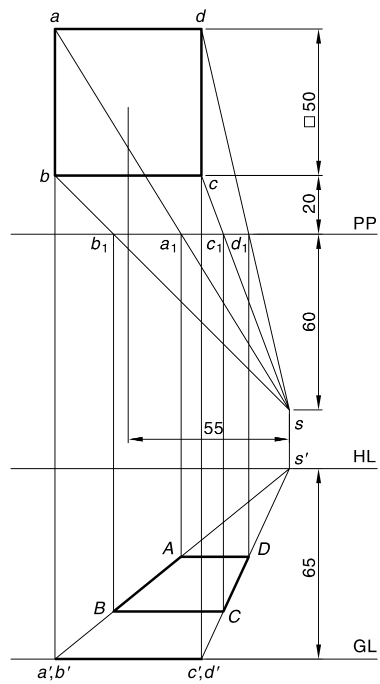


Fig. 17.4 *Visual ray method*

2. Construct a square *abcd* to represent top view of the square plane. Project it to GL and obtain $a'b'c'd'$ as its front view.
3. Mark top view of station point *s*, 60 mm below PP and 55 mm towards right of the centre of the square. Project point *s* to HL to obtain s', which represents its front view.
4. Join *s* with points *a*, *b*, *c* and *d* to represent top view of the perspective rays. Similarly, join s' with points a', b', c' and d' to obtain front view of the perspective rays.
5. Mark a_1, b_1, c_1 and d_1, the piercing points of top view of the perspective rays with PP.
6. Project these points to meet the front view of the corresponding perspective rays to get points *A*, *B*, *C* and *D*. Join them to obtain the required perspective view.

Note: As an edge of the base is parallel to PP, the problem is of one-point perspective.

Problem 17.3 *Draw a perspective view of a square plane of side 50 mm which stands vertically on the GP with an edge parallel to and 20 mm behind the PP. The surface of the plane is inclined at 45° to PP. The station point is 60 mm in front of PP, 65 mm above GP and lies in a CP which is 55 mm towards the right of the centre of the square plane..*

Construction Refer to Fig. 17.5.

1. Draw three horizontal lines to represent PP, HL and GL, as shown.
2. Draw a 50 mm long line *bc*, inclined at an angle of 45° with PP, to represent top view of the square plane.
3. Mark a point *s* in the required position to represent the station point in the top view.
4. Join *s* with points *a*, *b*, *c* and *d* to represent top view of the perspective rays. Mark a_1, b_1, c_1 and d_1, the piercing points of top view of these rays with PP.
5. Draw a line sv_1 parallel to edge *bc* to meet PP at v_1. Project v_1 on HL to get v_1'.
6. Extend line *ba* to meet PP at *p*. Project point *p* to meet GL at point p'. Also, mark point q' on projector pp', 50 mm above GL.
7. Join $v_1'p'$ and $v_1'q'$. Project points a_1, and d_1 to meet $v_1'p'$ at points *A* and *D*. Project points b_1 and c_1 to meet $v_1'q'$ at point *B* and *C*.
8. Join points *A*, *B*, *C* and *D* to obtain the required perspective view.

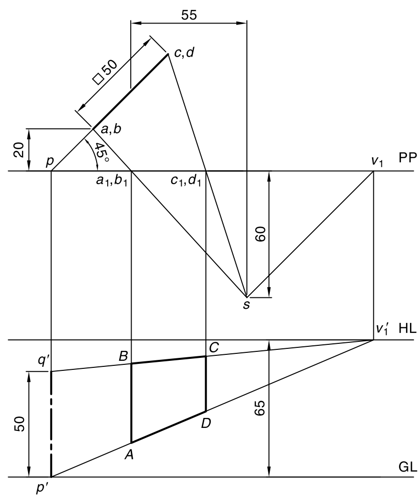


Fig. 17.5 *Vanishing point method*

Problem 17.4 *Draw a perspective view of a square prism of base side 40 mm and axis 60 mm, resting on its base in the GP with its axis that is 40 mm behind the PP and a vertical face right to the axis inclined at 60° to it. The station point is 50 mm in front of PP, 90 mm above GP and lies in a CP which is 50 mm towards right of the axis.*

Construction Refer to Fig. 17.6(a).

1. Draw three horizontal lines to represent PP, HL and GL, as shown.
2. Construct a square *abcd* to represent top view of the square prism. Project it to GL and obtain $a'c'3'1'$ as its front view.

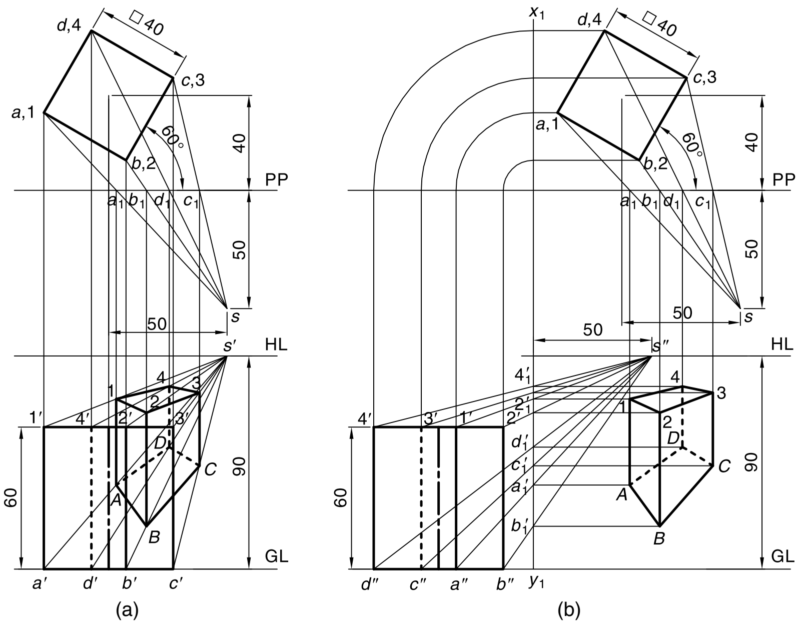


Fig. 17.6 **(a)** *Visual ray method* **(b)** *Visual ray method*

3. Mark points s and s' to represent the station points in top and front views respectively.
4. Join s with points a, b, c and d to represent top view of the perspective rays. Similarly, join s' with points a', b', c', d', 1′, 2′, 3′ and 4′ to obtain front view of the perspective rays.
5. Mark a_1, b_1, c_1 and d_1, the piercing points of top view of the perspective rays with PP.
6. Project these points to meet the front view of the corresponding perspective rays to get points A, B, C, D, 1, 2, 3 and 4. Join them to obtain the required perspective view.

Construction Refer to Fig. 17.6(b).

1. Draw three horizontal lines to represent PP, HL and GL, as shown. Also, draw a vertical line x_1y_1.
2. Construct a square $abcd$ to represent top view of the square prism and obtain $d''b''2''4''$ as its side view.
3. Mark points s and s'', 50 mm away from PP and x_1y_1 respectively to represent the station points in top and side views.
4. Join s with points a, b, c and d to represent top view of the perspective rays. Similarly, join s'' with points a'', b'', c'', d'', 1″, 2″, 3″ and 4″ to obtain the side view of the perspective rays.
5. Mark a_1, b_1, c_1 and d_1, the piercing points of top view of the perspective rays with PP. Also, mark a_1', b_1', c_1', d_1', $1_1'$, $2_1'$, $3_1'$ and $4_1'$, the piercing points of the side view of the perspective rays with x_1y_1.
6. Draw vertical lines from a_1, b_1, c_1, d_1 and horizontal lines from a_1', b_1', c_1', d_1' to get points A, B, C, D. Similarly, draw vertical lines from 1_1, 2_1, 3_1, 4_1 and horizontal lines from a_1', b_1', c_1', d_1' to get points 1, 2, 3, 4. Join them to obtain the required perspective view.

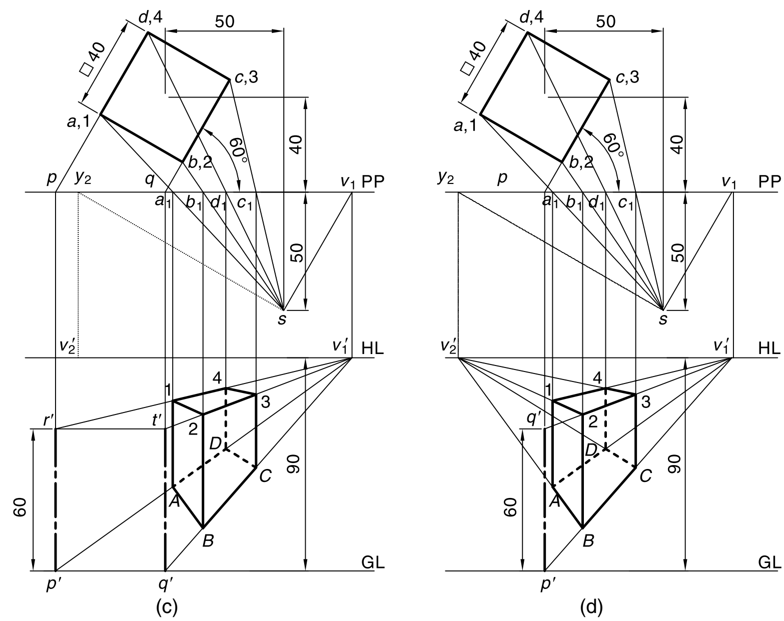


Fig. 17.6 **(c)** *Vanishing point method* **(d)** *Vanishing point method*

Construction Refer to Fig. 17.6(c).

1. Draw three horizontal lines to represent PP, HL and GL, as shown.
2. Construct a square *abcd* to represent top view of the square prism.
3. Mark a point s in the required position to represent the station point in the top view.
4. Join s with points a, b, c and d to represent top view of the perspective rays. Mark a_1, b_1, c_1 and d_1, the piercing points of top view of these rays with PP.
5. Draw a line sv_1 parallel to edge bc, to meet PP at v_1. Project v_1 on HL to get v_1'.
6. Extend lines da and cb to meet PP at p and q respectively. Project points p and q to meet GL at p' and q' respectively.
7. Mark r' and t' on the projectors of pp' and qq', 60 mm above GL.
8. Join $v_1'p'$, $v_1'q'$, $v_1'r'$ and $v_1't'$. Project point a_1 to meet $v_1'p'$ and $v_1'r'$ at points A and 1 respectively. Similarly, project point b_1 to meet $v_1'q'$ and $v_1't'$ at points B and 2. Project point c_1 to meet $v_1'q'$ and $v_1't'$ at points C and 3. Project point d_1 to meet $v_1'p'$ and $v_1'r'$ at points D and 4.
9. Join points A, B, C, D, 1, 2, 3 and 4 to obtain the required perspective view.

Construction Refer to Fig. 17.6(d).

1. Draw three horizontal lines to represent PP, HL and GL, as shown.
2. Construct a square *abcd* to represent top view of the square prism.
3. Mark a point s to represent the station point in the top view.
4. Join s with points a, b, c and d to represent top view of the perspective rays. Mark a_1, b_1, c_1 and d_1, the piercing points of top view of these rays with PP.
5. Draw a line sv_1 parallel to edge bc to meet PP at v_1. Project v_1 on HL to get v_1'.
6. Similarly, draw a line sv_2 perpendicular to sv_1 to meet PP at v_2. Project v_2 on HL to get v_2'.

7. Extend line cb to meet PP at p. Project points p to meet GL at p'. Mark q' on the projectors of pp', 60 mm above GL.
8. Join $v_1'p'$ and $v_1'q'$. Project point b_1 to meet $v_1'p'$ and $v_1'q'$ at points B and 2 respectively. Similarly, project point c_1 to meet $v_1'p'$ and $v_1'q'$ at points C and 3.
9. Join $v_2'B$, $v_2'2$, $v_2'C$ and $v_2'4$. Project point a_1 to meet $v_2'B$ and $v_2'2$ at points A and 1 respectively. Similarly, project point d_1 to meet $v_2'C$ and $v_2'3$ at points D and 4.
10. Join points A, B, C, D, 1, 2, 3 and 4 to obtain the required perspective view.

Problem 17.5 *Draw a perspective view of a square pyramid of base side 40 mm and axis 60 mm, resting on its base in the GP with its axis 40 mm behind the PP and an edge of the base right to the axis inclined at 60° to it. The station point is 50 mm in front of PP, 90 mm above GP and lies in a CP which is 50 mm towards the right of the axis.*

Construction Refer to Fig. 17.7(a).

1. Draw three horizontal lines to represent PP, HL and GL, as shown.
2. Construct a square $abcd$ to represent top view of the square pyramid. Project it to GL and obtain $a'c'o'$ as its front view.
3. Mark points s and s' to represent the station points in top and front views respectively.
4. Join s with points a, b, c, d and o to represent top view of the perspective rays. Similarly, join s' with points a', b', c', d' and o' to obtain front view of the perspective rays.
5. Mark a_1, b_1, c_1, d_1 and o_1, the piercing points of top view of the perspective rays with PP.
6. Project these points to meet the front view of the corresponding perspective rays to get points a_1', b_1', c_1', d_1' and o_1'. Join them to obtain the required perspective view.

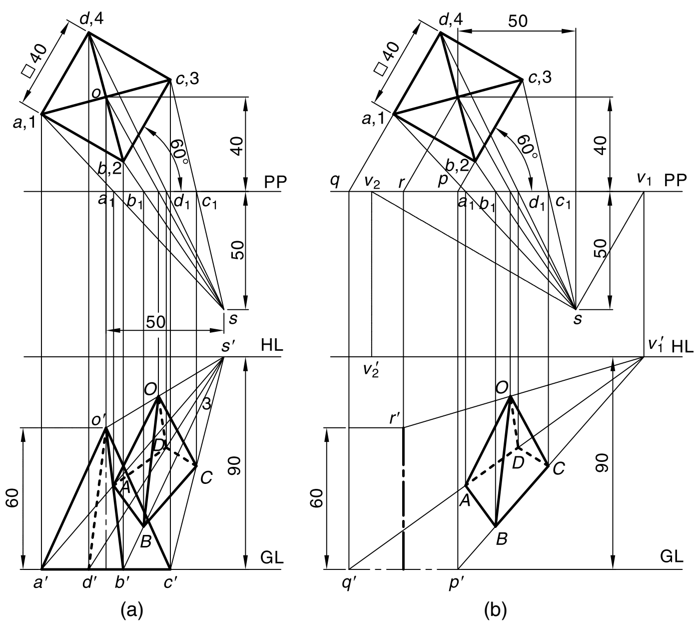


Fig. 17.7 **(a)** *Visual ray method* **(b)** *Vanishing point method*

Construction Refer to Fig. 17.7(b).

1. Draw three horizontal lines to represent PP, HL and GL, as shown.
2. Construct a square *abcd* with *o* as its centre to represent top view of the pyramid.
3. Mark a point *s* in the required position to represent the station point in the top view.
4. Join *s* with points *a*, *b*, *c* and *d* to represent top view of the perspective rays. Mark a_1, b_1, c_1 and d_1, the piercing points of top view of these rays with PP.
5. Draw a line sv_1 parallel to edge *bc* to meet PP at v_1. Project v_1 on HL to get v_1'.
6. Extend lines *cb* and *da* to meet PP at points *p* and *q* respectively. Project points *p* and *q* to meet GL at p' and q' respectively.
7. Also, draw a line from point *o* parallel to sv_1 or *cb* to meet PP at point *r*. Project point *r* to meet at a point r', 60 mm above GL.
8. Join $v_1'p'$, $v_1'q'$ and $v_1'r'$. Project points b_1 and c_1 to meet $v_1'p'$ at points *B* and *C* respectively. Similarly, project points a_1 and d_1 to meet $v_1'q'$ at points *A* and *D*. Project points o_1 to meet $v_1'r'$ at point *O*.
9. Join points *A*, *B*, *C*, *D* and *O* to obtain the required perspective view.

Problem 17.6 *A square prism of base side 40 mm and axis 60 mm, lies on its base in the GP with a face parallel to and 15 mm behind the PP. The station point lies in a CP which is 50 mm towards the right of the axis, 65 mm in front of the PP and 80 mm above GP. Draw its perspective view.*

Construction Refer to Fig. 17.8(a).

1. Draw three horizontal lines to represent PP, HL and GL, as shown.

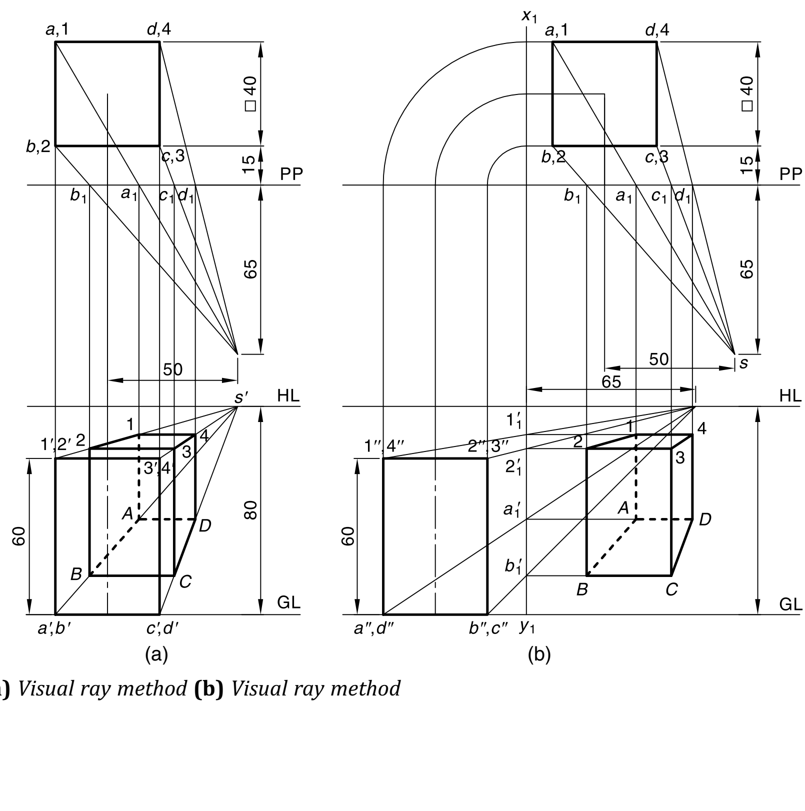


Fig. 17.8 **(a)** *Visual ray method* **(b)** *Visual ray method*

2. Construct a square *abcd* to represent top view of the square prism. Project it to GL and obtain $b'c'3'2'$ as its front view.
3. Mark points s and s' to represent the station points in top and front views.
4. Join s with points a, b, c and d to represent top view of the perspective rays. Similarly, join s' with points a', b', c', d' and $1'$, $2'$, $3'$, $4'$ to obtain front view of the perspective rays.
5. Mark a_1, b_1, c_1 and d_1, the piercing points of top view of the perspective rays with PP.
6. Project these points to meet the front view of the corresponding perspective rays to get points A, B, C, D and 1, 2, 3, 4. Join these points to obtain the required perspective view.

Construction Refer to Fig. 17.8(b).

1. Draw three horizontal lines to represent PP, HL and GL as shown. Also, draw a vertical line x_1y_1.
2. Construct a square *abcd* to represent top view of the square prism and obtain $a''b''2''1''$ as its side view.
3. Mark points s and s'', 65 mm away from PP and x_1y_1 respectively to represent the station points in top and side views.
4. Join s with points a, b, c and d to represent top view of the perspective rays. Similarly, join s'' with points a'', b'', c'', d'' and $1''$, $2''$, $3''$, $4''$ to obtain the side view of the perspective rays.
5. Mark a_1, b_1, c_1 and d_1, the piercing points of top view of the perspective rays with PP. Also, mark a_1', b_1', $1_1'$ and $2_1'$, the piercing points of the side view of the perspective rays with x_1y_1.
6. Draw vertical lines from a_1, b_1, c_1, d_1 and horizontal lines from a_1', b_1', $1_1'$, $2_1'$ to get points A, B, 1, 2 and D, C, 4, 3. Join them to obtain the required perspective view.

Problem 17.7 *A square pyramid of base side 40 mm and axis 60 mm rests on the GP with an edge of the base parallel to and 15 mm behind the PP. The station point is 90 mm above the GP and 75 mm in front of the PP and lies in a CP which is 40 mm towards the right of the axis of the pyramid. Draw its perspective projection.*

Construction Refer to Fig. 17.9(a).

1. Draw three horizontal lines to represent PP, HL and GL, as shown.
2. Construct a square *abcd* with diagonals meeting at centre o to represent top view of the square pyramid. Project it to GL and obtain $b'c'o'$ as its front view.
3. Mark points s and s' to represent the station points in top and front views respectively.
4. Join s with points a, b, c, d and o to represent top view of the perspective rays. Similarly, join s' with points a', b', c', d' and o' to obtain front view of the perspective rays.
5. Mark a_1, b_1, c_1, d_1 and o_1, the piercing points of top view of the perspective rays with PP.
6. Project these points to meet the front view of the corresponding perspective rays to get points A, B, C, D and O. Join them to obtain the required perspective view.

Construction Refer to Fig. 17.9(b).

1. Draw three horizontal lines to represent PP, HL and GL, as shown. Also, draw a vertical line x_1y_1.
2. Construct a square *abcd* with diagonals meeting at center o to represent top view of the square plane and obtain $a''b''o''$ as its side view.
3. Mark points s and s'', 70 mm away from PP and x_1y_1 respectively to represent the station points in top and side views.
4. Join s with points a, b, c, d and o to represent top view of the perspective rays. Similarly, join s'' with points a'', b'', c'', d'' and o'' to obtain the side view of the perspective rays.

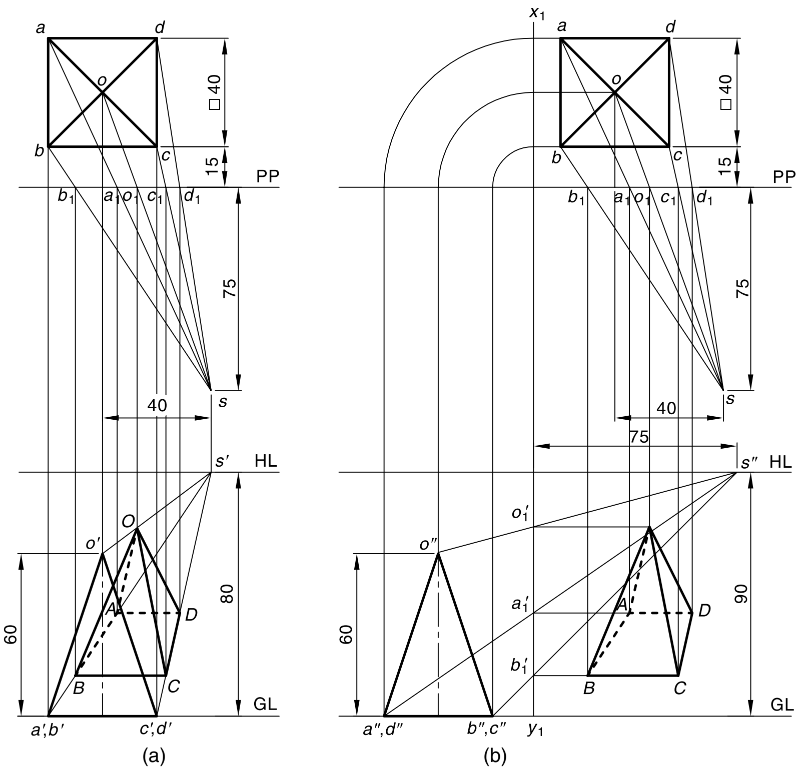


Fig. 17.9 **(a)** *Visual ray method* **(b)** *Visual ray method*

5. Mark a_1, b_1, c_1, d_1 and o_1, the piercing points of top view of the perspective rays with PP. Also, mark a_1', b_1' and o_1', the piercing points of the side view of the perspective rays with x_1y_1.
6. Draw vertical lines from a_1, b_1, c_1, d_1 and horizontal lines from a_1', b_1', o_1' to get points *A*, *B*, *C*, *D* and *O* as shown. Join them to obtain the required perspective view.

Problem 17.8 *The frustum of a square pyramid of base edge 50 mm, top edge 25 mm and height 40 mm rests on its base in the GP with an edge of the base parallel to and 15 mm behind the PP. The station point is 70 mm above the GP and 60 mm in front of the PP and lies in a CP which is 40 mm towards the right of the axis. Draw its perspective projection.*

Construction Refer to Fig. 17.10(a).

1. Draw three horizontal lines to represent PP, HL and GL, as shown.
2. Construct squares *abcd* and 1-2-3-4 to represent top view of the frustum. Project it to GL and obtain $b'c'3'2'$ as its front view.
3. Mark points *s* and *s′* to represent the station points in top and front views respectively.
4. Join s with points *a*, *b*, *c*, *d* and 1, 2, 3, 4 to represent top view of the perspective rays. Similarly, join *s′* with points *a′*, *b′*, *c′*, *d′* and 1′, 2′, 3′, 4′ to obtain front view of the perspective rays.

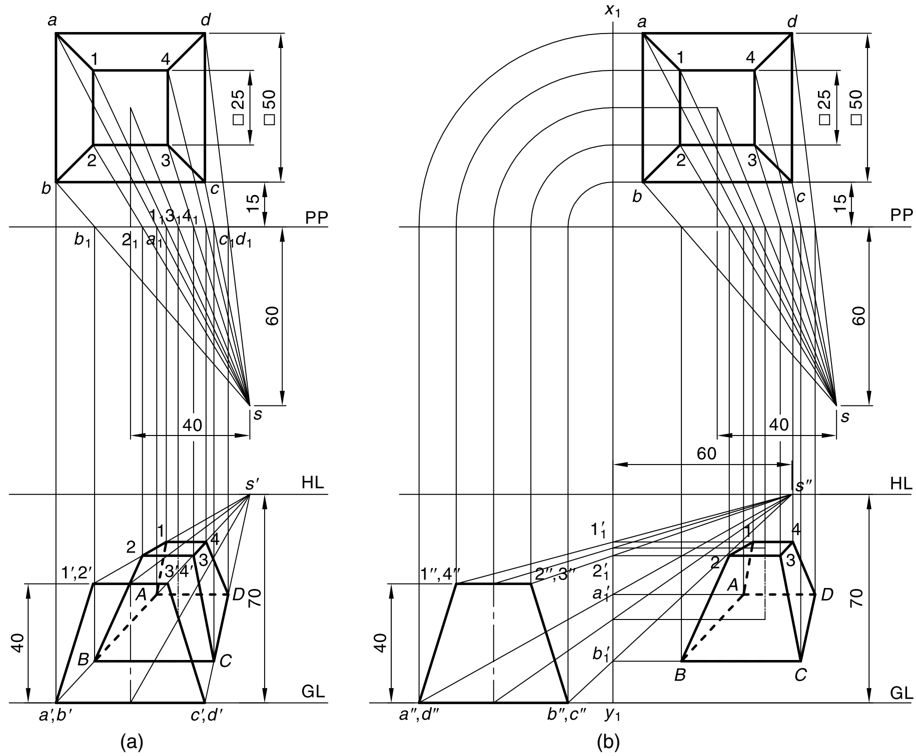


Fig. 17.10 **(a)** *Visual ray method* **(b)** *Visual ray method*

5. Mark a_1, b_1, c_1, d_1 and 1_1, 2_1, 3_1, 4_1, the piercing points of top view of the perspective rays with PP.
6. Project these points to meet the front view of the corresponding perspective rays to get points *A*, *B*, *C*, *D* and 1, 2, 3, 4. Join them to obtain the required perspective view.

Construction Refer to Fig. 17.10(b).

1. Draw three horizontal lines to represent PP, HL and GL, as shown. Also, draw a vertical line x_1y_1.
2. Construct squares *abcd* and 1-2-3-4 to represent top view of the frustum and obtain $a''b''2''1''$ as its side view.
3. Mark points *s* and s'', 70 mm away from PP and x_1y_1, respectively, to represent the station points in top and side views.
4. Join *s* with points *a*, *b*, *c*, *d* and 1, 2, 3, 4 to represent top view of the perspective rays. Similarly, join s'' with points a'', b'', c'', d'' and $1''$, $2''$, $3''$, $4''$ to obtain the side view of the perspective rays.
5. Mark a_1, b_1, c_1, d_1 and 1_1, 2_1, 3_1, 4_1, the piercing points of top view of the perspective rays with PP. Also, mark a_1', b_1', c_1', d_1' and $1_1'$, $2_1'$, $3_1'$, $4_1'$ the piercing points of the side view of the perspective rays with x_1y_1.
6. Draw vertical lines from a_1, b_1, c_1, d_1 and horizontal lines from a_1', b_1', c_1', d_1' and $1_1'$, $2_1'$, $3_1'$, $4_1'$ to get points *A*, *B*, *C*, *D* and 1, 2, 3, 4. Join these points to obtain the required perspective view.

Problem 17.9 *A square prism of base side 40 mm and axis 60 mm is resting on its rectangular face on the GP with axis inclined at 30° to PP. A side of base nearer to the PP is 20 mm behind it and 20 mm to the left of the station point. The station point is 80 mm in front of PP and 70 mm above GP. Draw its perspective view.*

Construction Refer to Fig. 17.11.

1. Draw three horizontal lines to represent PP, HL and GL, as shown.
2. Draw a rectangle *ab*21, with side *bc* 20 mm behind PP and the longer side *b*2 inclined at an angle of 30° with PP, to represent top view of the square prism.
3. Mark a point *s* in the required position to represent the station point in the top view.
4. Join *s* with points *a*, *b*, *c*, *d* and 1, 2, 3, 4 to represent top view of the perspective rays. Mark a_1, b_1, 1_1 and 2_1, the piercing points of top view of perspective rays with PP.
5. Draw a line sv_1 parallel to side *b*2 to meet PP at v_1. Project v_1 on HL to get v_1'.
6. Extend line 1*a* and 2*b* to meet PP at *p* and *q* respectively. Project points *p* and *q* to meet GL at point p' and q' respectively. Also, mark points r' and t' on projectors pp' and qq', 40 mm above GL.
7. Join $v_1'p'$, $v_1'q'$, $v_1'r'$ and $v_1't'$. Project point a_1 to meet $v_1'p'$ and $v_1'r'$ at points *A* and *D* respectively. Similarly, project point b_1 to meet $v_1'q'$ and $v_1't'$ at points *B* and *C*. Project point 1_1 to meet $v_1'p'$ and $v_1'r'$ at points 1 and 4 respectively. Project point 2_1 to meet $v_1'q'$ and $v_1't'$ at points 2 and 3.
8. Join points *A*, *B*, *C*, *D*, 1, 2, 3 and 4 to obtain the required perspective view.

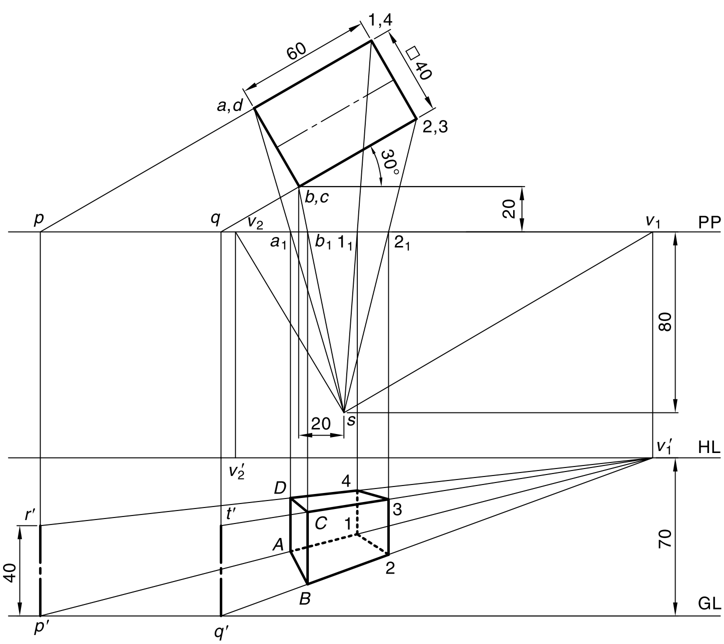


Fig. 17.11 *Vanishing point method*

17.8 MISCELLANEOUS PROBLEMS

Problem 17.10 *A hexagonal plane of side 30 mm lies on the GP with an edge parallel to and 10 mm behind the PP. The station point is 60 mm in front of PP, 75 mm above GP and lies in a CP which is 40 mm towards right of the center of the object. Draw its perspective view.*

Construction Refer to Fig. 17.12.

1. Draw three horizontal lines to represent PP, HL and GL as shown.
2. Construct a hexagon *abcdef* with 30 mm long side to represent top view of the plane. Project it to GL and obtain $a'd'$ as its front view.
3. Mark top view of station point *s*, 60 mm below PP and 40 mm towards right of the centre of the plane. Project point s to HL to obtain s' representing its front view.
4. Join s with points *a*, *b*, *c*, *d*, *e* and *f* to represent top view of the perspective rays. Similarly, join s' with points a', b', c', d', e' and f' to obtain front view of the perspective rays.
5. Mark a_1, b_1, c_1, d_1, e_1 and f_1, the piercing points of top view of the perspective rays with PP.
6. Project these points to meet the front view of the corresponding perspective rays to get points *A*, *B*, *C*, *D*, *E* and *F*. Join them to obtain the required perspective view.

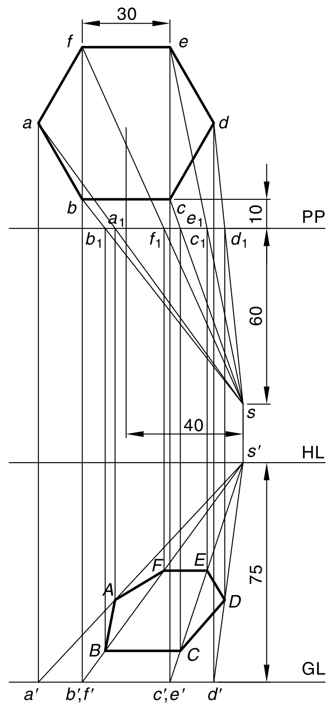


Fig. 17.12 *Visual ray method*

Problem 17.11 *A hexagonal plane of side 30 mm stands vertically on the GP on an edge and a corner 10 mm behind the PP. The surface of the plane makes an angle of 45° with the PP. The station point is 60 mm in front of the PP, 75 mm above the GP and lies in a CP which is 40 mm towards right of the centre of the plane. Draw its perspective view.*

Construction Refer to Fig. 17.13.

1. Draw three horizontal lines to represent PP, HL and GL, as shown.
2. Draw front view $a_0'b_0'c_0'd_0'e_0'f_0'$ and top view a_0d_0 of the hexagonal plane considering its surface parallel to PP.
3. Reproduce the top view as line *ad*, inclined at an angle of 60º with PP to represent top view of the hexagonal plane in the required position.
4. Mark a point *s* in the required position to represent the station point in the top view.
5. Join *s* with points *a*, *b*, *c*, *d*, *e* and *f* to represent top view of the perspective rays. Mark a_1, b_1, c_1 d_1, e_1 and f_1, the piercing points of top view of these rays with PP.
6. Draw a line sv_1 parallel to edge *ad* to meet PP at v_1. Project v_1 on HL to get v_1'.
7. Extend line *da* to meet PP at *p*. Project point *p* to meet GL at point p'. Also, mark point q' and r' on pp', where it meets horizontal lines drawn from *a* and *f* respectively.

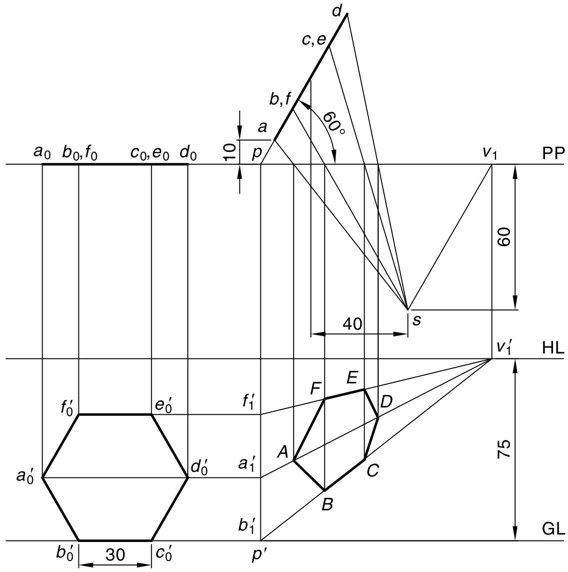


Fig. 17.13 *Vanishing point method*

8. Join $v_1'p'$, $v_1'q'$ and $v_1'r'$. Project points a_1 to meet $v_1'q'$ at point A. Similarly, project point b_1 to meet $v_1'p'$ and $v_1'r'$ at point B and F. Project point c_1 to meet $v_1'p'$ and $v_1'r'$ at points C and E. Project point d_1 to meet $v_1'q'$ at point D.
9. Join points A, B, C, D, E and F to obtain the required perspective view.

Problem 17.12 *Draw the perspective view of a circular plane of diameter 50 mm placed on GP with its centre 40 mm behind PP. The station point is 45 mm in front of the PP, 80 mm above GP and lies in the CP which is 35 mm to the right of the centre of the circle.*

Construction Refer to Fig. 17.14.

1. Draw three horizontal lines to represent PP, HL and GL, as shown.
2. Construct a circle with a 50 mm diameter to represent top view of the circular plane. Project it to GL and obtain $a'e'$ as its front view.
3. Mark top view of station point s, 45 mm below PP and 35 mm towards right of the centre of the square. Project point s to HL to obtain s' representing its front view.
4. Join s with points a, b, c, d, e, f, g and h to represent top view of the perspective rays. Similarly, join s' with points a', b', c' d', e', f', g' and h' to obtain front view of the perspective rays.
5. Mark a_1, b_1, c_1, d_1, e_1, f_1, g_1 and h_1, the piercing points of top view of the perspective rays with PP.

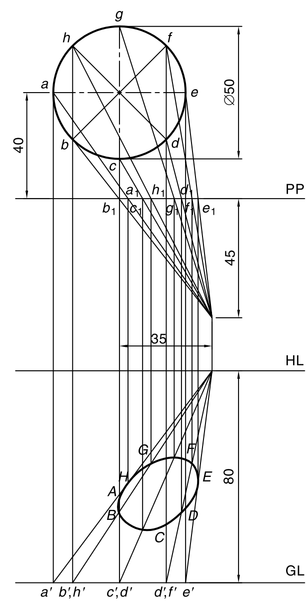


Fig. 17.14 *Visual ray method*

6. Project these points to meet the front view of the corresponding perspective rays to get points *A*, *B*, *C*, *D*, *E*, *F*, *G* and *H*. Join them to obtain the required perspective view.

Problem 17.13 *Draw the perspective view of a pentagonal plane of side 30 mm a side of which is inclined at 45° to the PP. It placed on GP with its centre 20 mm behind the PP. The station point is 70 mm in front of the PP, 65 mm above the GP and lies in a CP which is 50 mm to the right of the centre of the pentagon.*

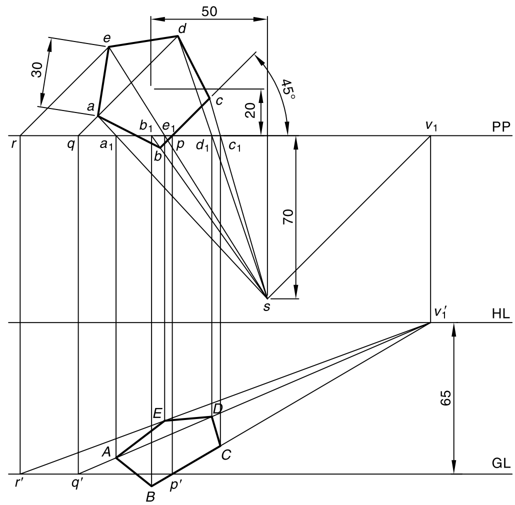


Fig. 17.15 *Vanishing point method*

Construction Refer to Fig. 17.15.

1. Draw three horizontal lines to represent PP, HL and GL, as shown.
2. Construct a pentagon *abcde* with side *bc* inclined at 45º to the PP, to represent top view of the pentagonal plane.
3. Mark a point *s* in the required position to represent the station point in the top view.
4. Join *s* with points *a*, *b*, *c*, *d* and *e* to represent top view of the perspective rays. Mark a_1, b_1, c_1, d_1 and e_1, the piercing points of top view of these rays with PP.
5. Draw a line sv_1 parallel to edge *bc* to meet PP at v_1. Project v_1 on HL to get v_1'.
6. Let line *bc* meet PP at point *p*. Project point *p* to meet GL at *p*′.
7. Extend line *da* (*da* is parallel to sv_1) to meet PP at *q*. Project point *q* to meet GL at *q*′.
8. Draw a line through point *f* parallel to sv_1 to meet PP at *r*. Project point *q* to meet GL at *r*′.
9. Join $v_1'p'$, $v_1'q'$ and $v_1'r'$. Project points b_1 and c_1 to meet $v_1'p'$ at points *B* and *C* respectively. Similarly, project points a_1 and d_1 to meet $v_1'q'$ at points *A* and *D*. Project point e_1 to meet $v_1'r'$ at points *E*.
10. Join points *A*, *B*, *C*, *D* and *E* to obtain the required perspective view.

Problem 17.14 *A hexagonal pyramid of base side 30 mm and axis 65 mm rests on the GP with an edge of the base parallel to and 10 mm behind the PP. The station point is 40 mm above the ground and 70 mm in front of the PP and 50 mm towards the right of the axis of the pyramid. Draw its perspective projection.*

Construction Refer to Fig. 17.16.

1. Draw three horizontal lines to represent PP, HL and GL, as shown. Also, draw a vertical line x_1y_1.
2. Construct a hexagon *abcdef* with diagonals meeting at centre *o* to represent top view of the hexagonal pyramid and obtain $b''o''f''$ as its side view.
3. Mark points *s* and *s*″, 70 mm away from PP and x_1y_1 respectively, to represent the station points in top and side views.
4. Join *s* with points *a*, *b*, *c*, *d*, *e*, *f* and *o* to represent top view of the perspective rays. Similarly, join *s*″ with points *a*″, *b*″, *c*″, *d*″, *e*″, *f*″ and *o*″ to obtain the side view of the perspective rays.

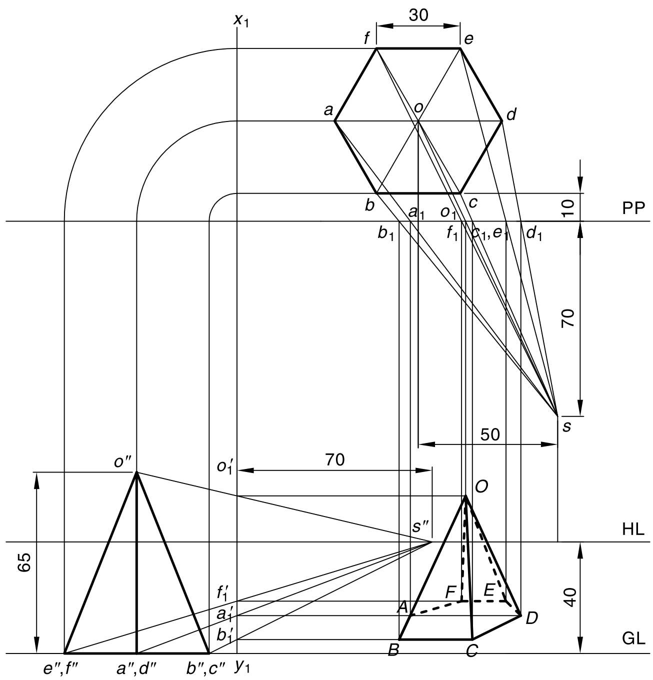


Fig. 17.16 *Visual ray method*

5. Mark a_1, b_1, c_1, d_1, e_1, f_1 and o_1, the piercing points of top view of the perspective rays with PP. Also, mark a_1', b_1', c_1' and o_1', the piercing points of the side view of the perspective rays with x_1y_1.
6. Draw vertical lines from a_1, b_1, c_1, d_1, e_1, f_1 and horizontal lines from a_1', b_1', f_1', o_1' to get points A, B, C, D, E, F and O as shown. Join them to obtain the required perspective view.

Problem 17.15 *A cylinder of base diameter 40 mm and axis 50 mm rests on the GP with its axis parallel to and 30 mm behind the PP. The station point is 80 mm above the ground, 50 mm in front of the PP and 30 mm towards the right of the axis. Draw its perspective projection.*

Construction Refer to Fig. 17.17.

1. Draw three horizontal lines to represent PP, HL and GL, as shown. Also, draw a vertical line x_1y_1.
2. Construct a circle *abcdefgh* to represent top view of the cylinder and obtain $c''3''7''g''$ as its side view.
3. Mark points s and s'', 50 mm away from PP and x_1y_1 respectively, to represent the station points in top and side views.
4. Join s with points a, b, c, d, e, f, g and h to represent top view of the perspective rays. Similarly, join s'' with points a'', b'', c'', d'', e'', f'', g'', h'' and $1''$, $2''$, $3''$, $4''$, $5''$, $6''$, $7''$, $8''$ to obtain the side view of the perspective rays.
5. Mark a_1, b_1, c_1, d_1, e_1, f_1, g_1 and h_1, the piercing points of top view of the perspective rays with PP. Also, mark a_1', b_1', c_1', f_1', g_1' and $1_1'$, $2_1'$, $3_1'$, $6_1'$, $7_1'$ (few points are not shown in the figure), the piercing points of the side view of the perspective rays with x_1y_1.

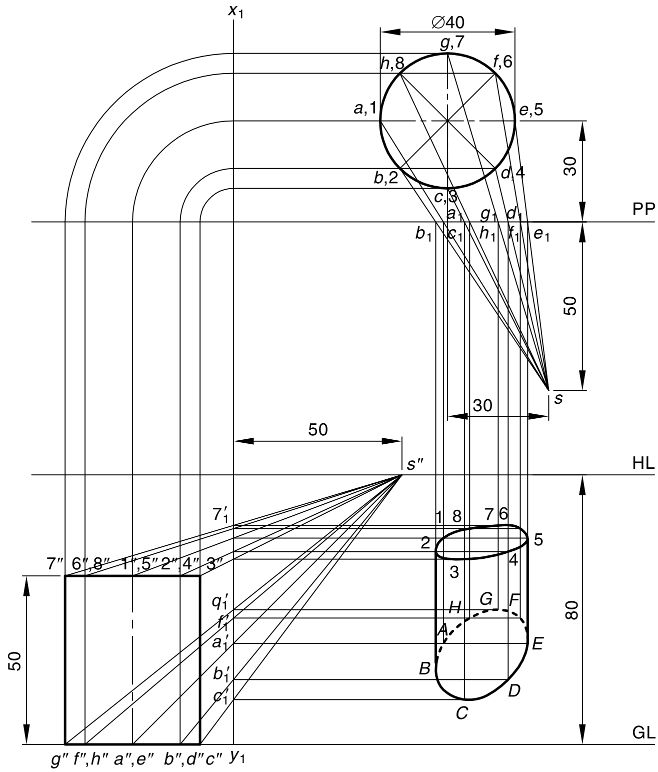


Fig. 17.17 *Visual ray method*

6. Draw vertical lines from a_1, b_1, c_1, d_1, e_1, f_1, g_1, h_1 and horizontal lines from a_1', b_1', c_1', f_1', g_1' and $1_1'$, $2_1'$, $3_1'$, $6_1'$, $7_1'$ to get points *A*, *B*, *C*, *D*, *E*, *F*, *G*, *H* and 1, 2, 3, 4, 5, 6, 7, 8 as shown. Join them to obtain the required perspective view.

Problem 17.16 *A pentagonal prism of base side 30 mm and axis 70 mm is resting on its face in the GP with the axis inclined at 60° to the PP. The station point is 90 mm in front of the PP, 100 mm above the GP and lies in the CP which is 70 mm rightwards to the corner nearer to the PP. Draw a perspective view when the corner nearer to the PP is 15 mm behind it.*

Construction Refer to Fig. 17.18.

1. Draw three horizontal lines to represent PP, HL and GL, as shown.
2. Draw front view $a_0'b_0'c_0'd_0'e_0'$ and top view $a_0d_04_01_0$ of the pentagonal prism, considering its rectangular face rests in the GP and bases parallel to PP.
3. Reproduce the top view as rectangle *ad*41 such that *d*4 is making at an angle of 60º with PP. This represents the top view of the pyramid in the required position.
4. Mark a point *s* in the required position to represent the station point in the top view.

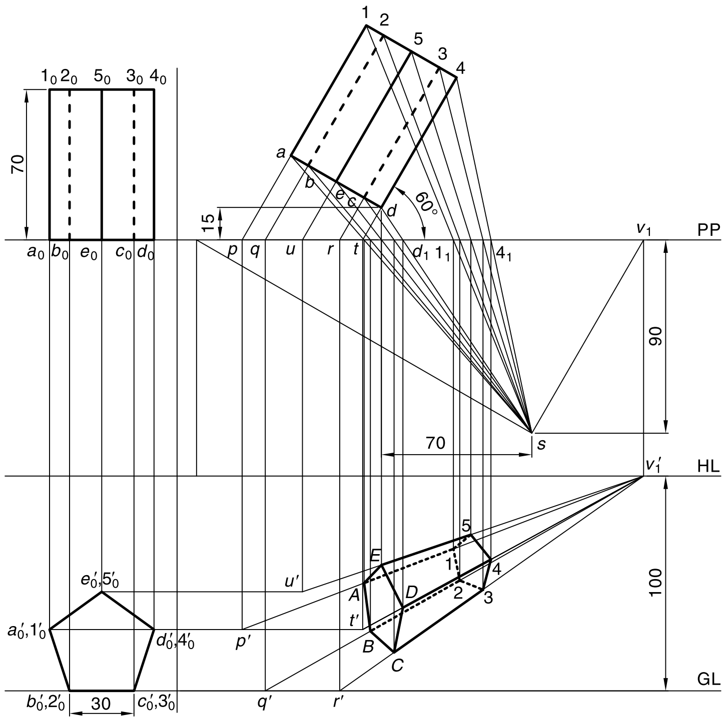


Fig. 17.18 *Vanishing point method*

5. Join s with points a, b, c, d, e and 1, 2, 3, 4, 5 to represent top view of the perspective rays. Mark a_1, b_1, c_1, d_1, e_1 and 1_1, 2_1, 3_1, 4_1, 5_1, (all point are not marked in the figure) the piercing points of top view of perspective rays with PP.
6. Draw a line sv_1 parallel to side $d4$ to meet PP at v_1. Project v_1 on HL to get v_1'.
7. Extend line $1a$, $2b$, $3c$, $4d$, $5e$ to meet PP at p, q, r, t and u respectively. Project them vertically to meet horizontal lines from points a_1', b_1', c_1', d_1', and e_1' at points p', q', r', t' and u' respectively.
8. Join $v_1'p'$, $v_1'q'$, $v_1'r'$, $v_1't'$ and $v_1'u'$. Project points a_1 and 1_1 to meet $v_1'p'$ at points A and 1 respectively. Similarly, project points b_1 and 2_1 to meet $v_1'q'$ at points B and 2. Project point points c_1 and 3_1 to meet $v_1'r'$ at points C and 3. Project point points d_1 and 4_1 to meet $v_1't'$ at points D and 4. Project point points e_1 and 5_1 to meet $v_1'u'$ at points E and 5. Join all the points to obtain the required perspective view as shown.

Problem 17.17 *A square pyramid of base edge 30 mm and axis 50 mm rests centrally over a hexagonal slab of base side 30 mm and thickness 25 mm. An edge of the base of the pyramid is parallel to that of the slab which is 10 mm behind the PP. The station point is 100 mm above the GP, 70 mm in front of the PP and lies in a CP which is 50 mm away from the axis of the combined solid. Draw a perspective view of the combination.*

Note Figure 17.19 shows the required perspective view.

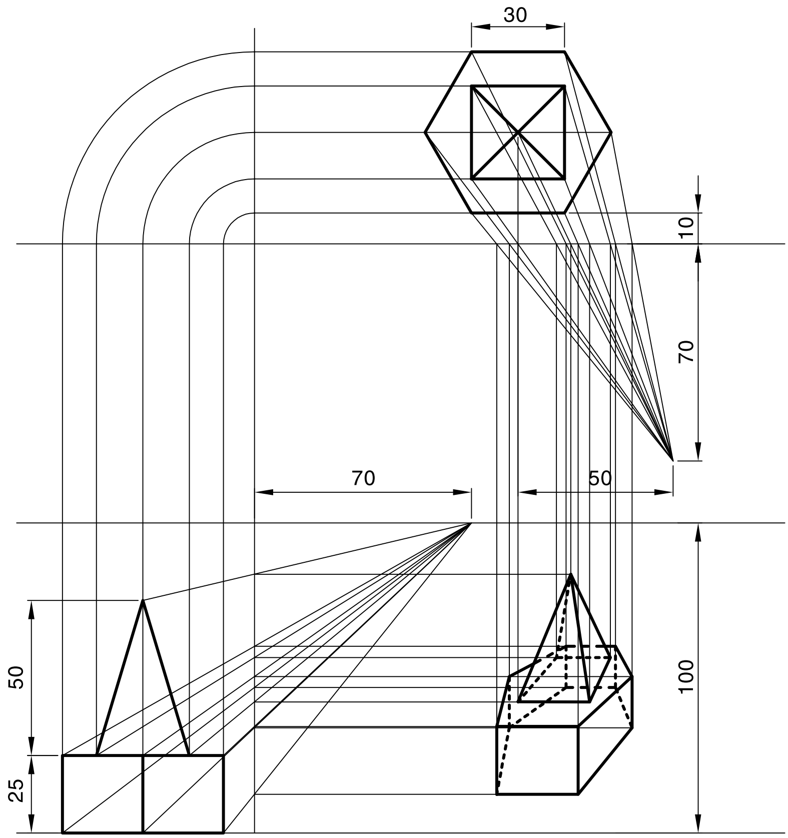


Fig. 17.19 *Visual ray method*

Problem 17.18 *A hollow rectangular slab of sides 60 mm and 50 mm and thickness 20 mm has a wall thickness of 10 mm. It is resting on its base on the GP such that the longer side makes an angle of 30° with the PP and the vertical edge nearer to the observer is 15 mm behind it. Draw its perspective view if the SP is 100 mm above the GP, 70 mm in front of the PP and lies in a CP which is 30 mm away from the corner nearest to the PP.*

Note Figure 17.20 shows the required perspective view.

Problem 17.19 *Draw a perspective view of the object whose orthographic and isometric views are shown in Figs 17.21(a) and (b) respectively. It is resting on the GP and 15 mm behind the PP. The SP is 100 mm above the GP, 75 mm in front of the PP and lies in a CP which is 25 mm towards the left of the extreme left end of the object.*

Note Figure 17.21(c) shows the required perspective view of the given object.

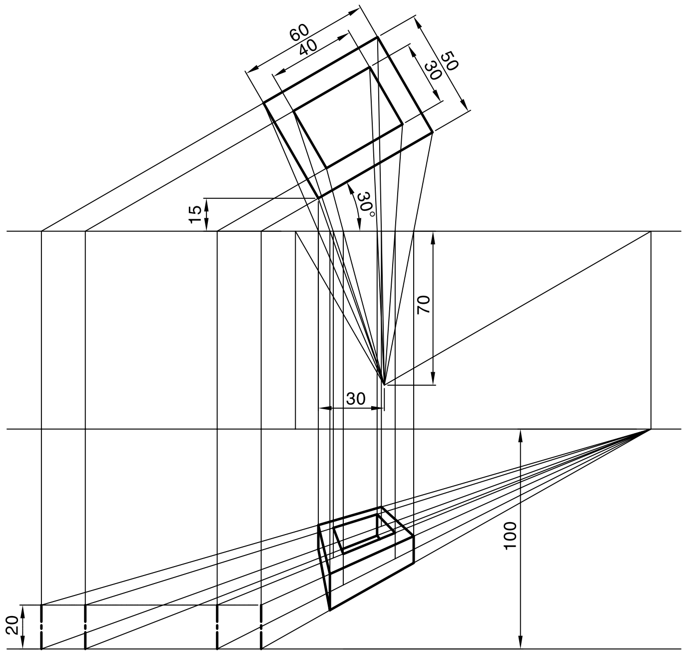


Fig. 17.20 *Vanishing point method*

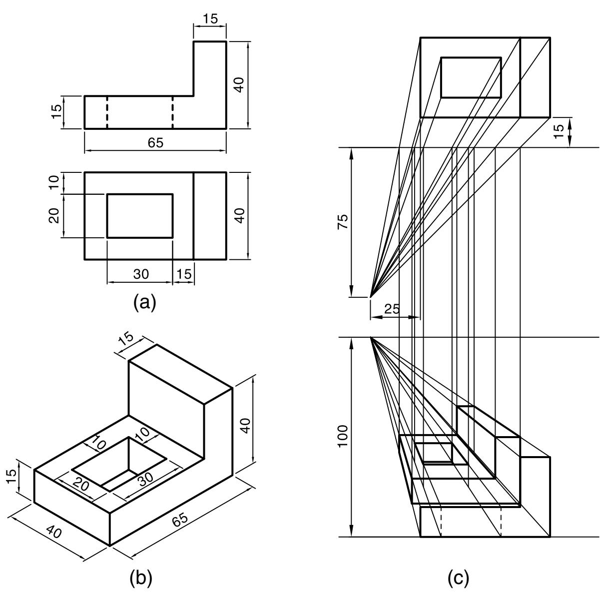


Fig. 17.21 **(a)** *Orthographic view* **(b)** *Isometric view* **(c)** *Perspective view*

EXERCISE 17

17.1 Draw a perspective view of a square plane of side 60 mm resting on the GP with one of its corners touching PP and a side right to the corner inclined at 30º to it. The station point is 50 mm in front of PP, 60 mm above GP and lies in a CP which is 40 mm towards right of the corner touching the PP.

17.2 A square plane of side 60 mm lies on the GP with the edge nearer to the observer lying in the PP. The station point is 50 mm in front of PP, 60 mm above GP and lies in a CP which is 50 mm towards right of the centre of the object. Draw its perspective view.

17.3 Draw a perspective view with a square plane of side 50 mm which stands vertically on the GP with an edge parallel to and 10 mm behind the PP. The surface of the plane is inclined at 30º to PP. The station point is 60 mm in front of PP, 65 mm above GP and lies in a CP which is 55 mm towards right of the centre of the plane.

17.4 A rectangular plane of sides 60 mm and 40 mm is lying in the GP with the longer side parallel to and 15 mm behind the PP. The station point is 50 mm in front of the PP, 60 mm above GP and lies in the CP passing through the centre of the object. Draw its perspective view.

17.5 A pentagonal plane of side 30 mm side lies on the GP with an edge parallel to and 20 mm behind the PP. The station point is 50 mm in front of PP, 65 mm above GP and lies in a CP which is 40 mm towards right of the centre of the object. Draw its perspective view.

17.6 A pentagonal plane of side 30 mm stands vertically on the GP on an edge and a corner touching the PP. The surface of the plane makes an angle of 30º with the PP. The station point is 60 mm in front of PP, 75 mm above GP and lies in a CP which is 40 mm towards right of the centre of the plane. Draw its perspective view.

17.7 Draw the perspective view of a pentagonal plane of side 30 mm having its surface perpendicular to the PP. It placed on GP with its centre 50 mm behind PP. The station point is 50 mm in front of the PP, 65 mm above GP and lies in a CP which is 50 mm to the right of the centre of the pentagon.

17.8 A hexagonal plane of side 40 mm has a centrally cut square hole of side 30 mm such that a side of the hole and a side of the hexagon are parallel PP. It lies on the GP with a nearer edge of the hexagon 10 mm behind the PP. The station point is 50 mm in front of PP, 70 mm above GP and lies in a CP which is 40 mm towards right of the centre of the object. Draw its perspective view.

17.9 A hexagonal plane of side 40 mm has a centrally cut square hole of side 30 mm such a hexagonal plane with a 40 mm side has a centrally cut hexagonal hole with 20 mm side with their corresponding sides parallel to each other. The plane stands vertically on the GP on an edge and a corner touching the PP. The surface of the plane makes an angle of 30º with the PP. The station point is 60 mm in front of PP, 75 mm above GP and lies in a CP which is 40 mm towards right of the centre of the plane. Draw its perspective view.

17.10 Draw a perspective view of a hexagonal prism of base side 40 mm and axis 60 mm resting on its base in the GP with a side of base parallel to and 10 mm behind the PP. The station point is 50 mm in front of PP, 75 mm above GP and lies in a CP which is 50 mm towards the right of the axis.

17.11 A composite plane is made up of a rectangle with 60 mm and 40 mm sides and a semicircle on its longer side. Draw its perspective view when it is lying in the GP. The longer side is perpendicular to PP and the shorter side is 10 mm behind it. The station point is 50 mm in front of the PP, 60 mm above the GP and lies in the CP which is 50 mm to the right of the centre of the semicircle.

17.12 Draw a perspective view of a square pyramid of base side 40 mm and axis 60 mm resting on its base in the GP with its axis 40 mm behind the PP and all the edges of the base equally inclined to it. The station point is 50 mm in front of PP, 75 mm above GP and lies in a CP which is 50 mm towards right of the axis.

17.13 A pentagonal prism of base side 40 mm and axis 60 mm lies on its base in the GP with a face parallel to and 15 mm behind the PP. The station point lies in a CP which is 50 mm towards right of the axis, 65 mm in front of PP and 80 mm above GP. Draw its perspective view.

17.14 A pentagonal pyramid of base side 40 mm and axis 60 mm rests on the GP with an edge of the base parallel to and 10 mm behind the PP. The station point is 75 mm above the GP and 60 mm in front of the PP and lies in a CP which is 40 mm towards the right of the axis of the pyramid. Draw its perspective projection.

17.15 A square prism of base side 40 mm and axis 60 mm is resting on its rectangular face on the GP with axis inclined at 45º to PP. A side of base nearer to the PP is 20 mm behind it and 20 mm to the left of the station point. The station point is 80 mm in front of PP and 70 mm above GP. Draw its perspective view.

17.16 A hexagonal prism of base side 30 mm and axis 70 mm is resting on its face in the GP with the axis inclined at 30º to the PP. The station point is 90 mm in front of PP, 100 mm above the GP and lies in the CP which is 70 mm rightwards to the corner nearer to the PP. Draw a perspective view when the corner nearer the observer touches the PP.

17.17 A cylinder of base diameter 40 mm and axis 50 mm rests on the GP with its axis parallel to and 30 mm behind the PP. The station point is 80 mm above the ground and 50 mm in front of the PP and lies in the CP which passes through the axis of the cylinder. Draw its perspective projection.

17.18 The frustum of a square pyramid of base edge 50 mm, top edge 25 mm and height 40 mm rests on its base in the GP with an edge of the base inclined at 30º to the PP and axis 40 mm behind it. The station point is 70 mm above the GP and 60 mm in front of the PP and lies in a CP which is 40 mm towards the right of the axis. Draw its perspective projection.

17.19 The frustum of a hexagonal pyramid of base edge 40 mm, top edge 20 mm and axis 40 mm rests on the GP with an edge of the base parallel to and 10 mm behind the PP. The station point is 40 mm above the ground and at a distance of 70 mm in front of the PP and 50 mm towards the right of the axis of the pyramid. Draw its perspective projection.

VIVA-VOCE QUESTIONS

17.1 What are the characteristics of perspective projections?

17.2 Name different types of perspectives and their fields of application.

17.3 State the limitations of perspective drawing.

17.4 State the alternative names of one-point, two-point, three-point and aerial perspectives.

17.5 Compare visual ray and vanishing point methods of drawing perspective views.

17.6 Compare the merits and demerits of perspective projections with isometric and oblique projections.

17.7 Define the following terms with reference to perspective projections: (a) Ground plane (b) Picture plane (c) Horizon plane (d) Central plane.

17.8 Define the following terms with reference to perspective projections (a) Ground line (b) Horizon line (c) Central line (d) Axis of vision.

17.9 What is a station point? What is its relation with the visual rays?

17.10 Make a line diagram and indicate the following: (a) Ground plane (b) Oicture plane (c) Horizon plane (d) Central plane (e) Ground line (f) Horizon line (g) Central line (h) Axis of vision

MULTIPLE-CHOICE QUESTIONS

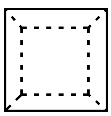
17.1 The type of pictorial projection generally used by the architects is
(a) orthographic
(b) oblique
(c) perspective
(d) isometric

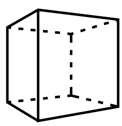
17.2 The form of drawing similar to the view of objects as perceived by human eye is
(a) perspective
(b) oblique
(c) axonometric
(d) isometric

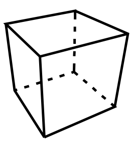
17.3 Perspective projections are drawn by
(a) single vanishing point method
(b) double vanishing point method
(c) triple vanishing point method
(d) All of these

17.4 Two point perspective is also known as
(a) parallel perspective
(b) angular perspective
(c) oblique perspective
(d) atmospheric perspective

17.5 One-point perspective view of a cube can be represented as

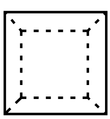
(a) 

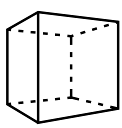
(b) 

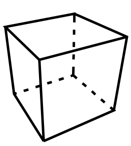
(c) 

(d) None of these

17.6 Two-point perspective view of a cube can be represented as

(a) 

(b) 

(c) 

(d) None of these

17.7 The illusion of depth in paintings is depicting by
(a) one-point perspective
(b) two-point perspective
(c) three-point perspective
(d) aerial perspective

17.8 As the distance of an object from the observer increases, its size in the perspective view
(a) remains constant
(b) increases
(c) decreases
(d) Any of these

17.9 The imaginary vertical plane passing through the observer's eye is called
(a) ground plane
(b) horizon plane
(c) central plane
(d) picture plane

17.10 The imaginary horizontal plane passing through the observer's eye is called
(a) ground plane
(b) horizon plane
(c) central plane
(d) picture plane

17.11 The line joining any point on the object to the station point is known as
(a) axis of vision
(b) visual ray
(c) centre line
(d) horizon line

17.12 Pictorial views are obtained by
(a) isometric projection
(b) oblique projection
(c) perspective projection
(d) All of these

Answers to multiple-choice questions

17.1 (c), 17.2 (a), 17.3 (d), 17.4 (b), 17.5 (a), 17.6 (b), 17.7 (d), 17.8 (c), 17.9 (c), 17.10 (b), 17.11 (b), 17.12 (d)

Code No: 09A10191

R09

SET-1

B. Tech I Year Examinations, December-January, 2011-2012
ENGINEERING DRAWING
(COMMON TO CE, BME, MCT, ETM, MIM)

Time: 3 hours **Max. Marks: 75**

Answer any five questions
All questions carry equal marks

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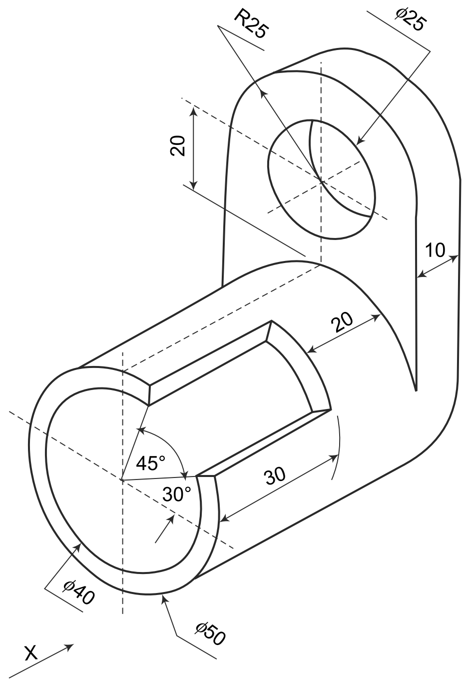
1. A circle having a 50 mm diameter rolls within a circle with a 150 mm diameter with internal contact. Draw the locus of a point lying on the circumference of the rolling circle for its complete turn. Name the curve. Also draw a tangent and a normal to the curve, at a point that is 40 mm from the centre of the bigger circle. [15]
2. A line PQ, inclined at 45° to the V.P., has a 60 mm long front view. The end P is 10 mm from both the principal planes while the end Q is 45 mm above the H.P. Draw the projections of the line and determine its true length and inclinations with the principal planes. Also, locate its traces. [15]
3. An equilateral triangular plane with a 60 mm side has a side inclined at 45° to the H.P. Its H.T. is parallel to and 25 mm below *xy* and its V.T. does not exist. Draw its projections. [15]
4. A square prism is resting on one of its bases on the H.P. with an edge of the base perpendicular to the V.P. It is cut by an A.I.P. such that the true shape of the section is a rectangle with 80 mm and 50 mm sides. The minimum height of one of the side faces of cut prism is 15 mm. Draw its projections and true shape of the section. [15]
5. A cone, diameter of base 90 mm and altitude 80 mm rests with its base on ground. A vertical cylinder of 40 mm diameter has its axis 5 mm in front of that of the cone and the axes are contained in a plane making an angle of 30 degrees with the VP. Draw the curves of penetration of the surface. [15]
6. A solid is in the form of a cylinder of base diameter 50 mm up to a height of 60 mm and thereafter tapers into a frustum of a cone of top diameter 30 mm. The total height of the solid is 90 mm. Draw the isometric projection of the solid. [15]
7. Draw the elevation, top view and side view of the object shown in figure. All dimensions are in mm. [15]



8. Draw the perspective view of a frustum of a square pyramid with 40 mm edges at the base, 30 mm at the top, and 50 mm in height. The frustum is resting on its base with its base edges equally inclined to the picture plane and one of the base corners touching it. The station point is 80 mm in front of the picture plane, 15 mm to the left of the axis of the frustum, and 60 mm above the ground plane. [15]

Code No: 09A10191

R09

SET-2

B. Tech I Year Examinations, December-January, 2011-2012
ENGINEERING DRAWING
(COMMON TO CE, BME, MCT, ETM, MIM)

Time: 3 hours **Max. Marks: 75**

Answer any five questions
All questions carry equal marks

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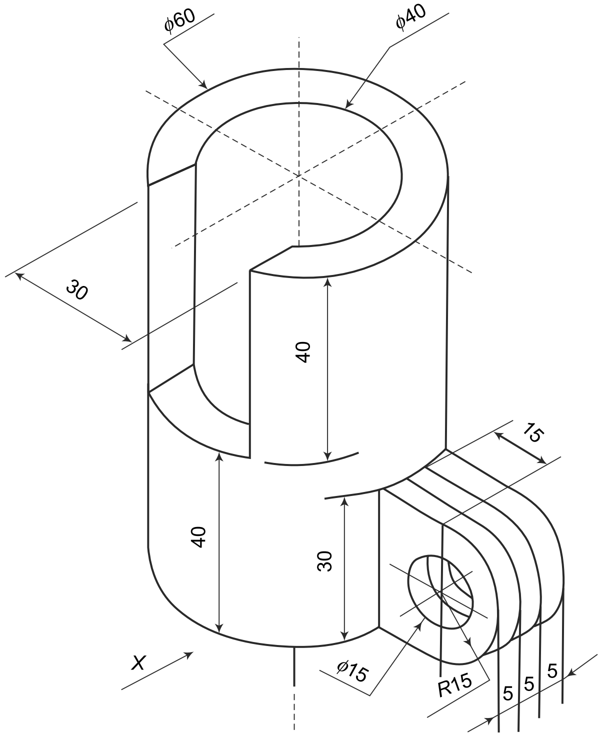
1. Construct two branches of a hyperbola when its transverse axis is 50 mm long and foci are 70 mm apart. Locate its directrix and determine the eccentricity. [15]
2. A 75 mm long line PQ is inclined at an angle of 30° to the H.P. The end P is 20 mm above the H.P. and on the V.P. The end Q is 60 mm in front of the V.P. Draw the projections of the line and locate its traces. [15]
3. A hexagonal pyramid, having base with a 30 mm side and a 75 mm long axis, has one of its base edge is on the H.P., and vertical plane containing this edge and the axis is inclined at 300 to the V.P. Draw its projections when apex is 15 mm in front of the V.P. [15]
4. A thin glass vessel, with a 60 mm base diameter and a 75 mm height, is completely filled with water. It is then tilted on the rim of its base circle such that the base makes an angle of 300 to the H.P. In the process, some water from it is drained out. Draw the projections of the cylindrical vessel showing remaining water in it. [15]
5. A vertical cone 80 mm diameter of base and axis 100 mm long is penetrated by a vertical cylinder of 60 mm diameter and 100 mm long such that the top circular end of the cylinder contains the apex of the cone and a plane perpendicular to both HP and VP containing the axes of both the solids and the axis of the cylinder is at a distance of 10 mm from the axis of the cone and is towards the observer. Draw the top and front view of the solids showing the curves of intersection. [15]
6. A masonry pillar is in the form of a frustum of a hexagonal pyramid. The pillar is of 2 m height and side of its base and top base are 0.5 m and 0.3 m respectively. Draw its isometric projection. [15]
7. Draw the elevation, top view and side view of the object shown in figure. All dimensions are in mm. [15]



8. A square pyramid 45 mm base edge and 50 mm axis rests on its base on the ground such that two parallel base edges recede at 30° to the right of PP with the nearest corner of base 10 mm behind PP. The station point is 45 mm in front of PP and 70 mm above ground and 10 mm to the right of the nearest corner. Draw the perspective projection of the solid. [15]

Code No: 09A10191

R09

SET-3

B. Tech I Year Examinations, December-January, 2011-2012
ENGINEERING DRAWING
(COMMON TO CE, BME, MCT, ETM, MIM)

Time: 3 hours **Max. Marks: 75**

Answer any five questions
All questions carry equal marks

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1. The distance between two stations is 130 km. a train covers this distance in 2.5 hours. Construct a plain scale to measure time upto a single minute. The RF of the scale is 1:260000. Find the distance covered by the train in 45 minutes. [15]

2. A 120 mm long line PQ, is inclined at 45° to the H.P. and 30° to the V.P. A point M lies on the line at a distance of 40 mm from P and its front view is 50 mm above the xy line and the top view is 35 mm below the xy line. Draw its projections and locate its traces. [15]

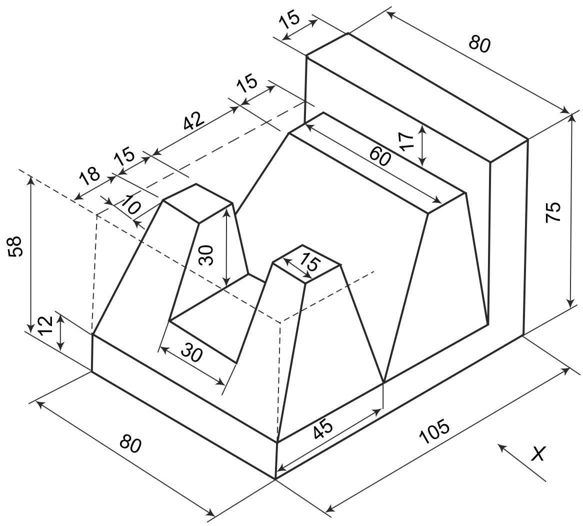
3. A square slab having base with a 60 mm side and 20 mm thickness is resting on its base on the ground with an edge inclined at 30° to the V.P. A cone of base with a 50 mm diameter and 60 mm axis is placed centrally over the slab such that the axes of the solids coincide. Draw the projections of the composite solid. [15]

4. A cylinder, with a 50 mm base diameter and a 70 mm long axis, is kept on the H.P. on its base. It is cut by an A.I.P. such that the true shape of the section is the largest possible ellipse. Draw its front view, sectional top view and true shape of the section. [15]

5. A vertical square prism with 50 mm sides and 100 mm length has its side faces equally inclined to the VP. It is completely penetrated by a horizontal cylinder 60 mm in diameter and 100 mm in length. The axes of the two solids bisect each other perpendicularly. Draw the projections showing curves of intersection when the plane containing the two axes is parallel to the VP. [15]

6. Draw isometric view of a cylinder of base diameter 55 mm and axis length 65 mm when the axis of the cylinder is (i) vertical (ii) horizontal. [15]

7. Draw the elevation, top view and side view of the object shown in figure. All dimensions are in mm. [15]



8. A hexagonal prism, side of base 30 mm and 65 mm long rests with its base on the ground. The nearest vertical edge is 10 mm to the left of the eye and 15 mm behind the PP. One of the faces containing the edge recedes at 45° to the PP, towards the left. The eye is 150 mm from the picture plane and is at a height of 80 mm. Draw the perspective view of the prism. [15]

Code No: 09A10191

R09

SET-4

B. Tech I Year Examinations, December-January, 2011-2012
ENGINEERING DRAWING
(COMMON TO CE, BME, MCT, ETM, MIM)

Time: 3 hours **Max. Marks: 75**

Answer any five questions
All questions carry equal marks

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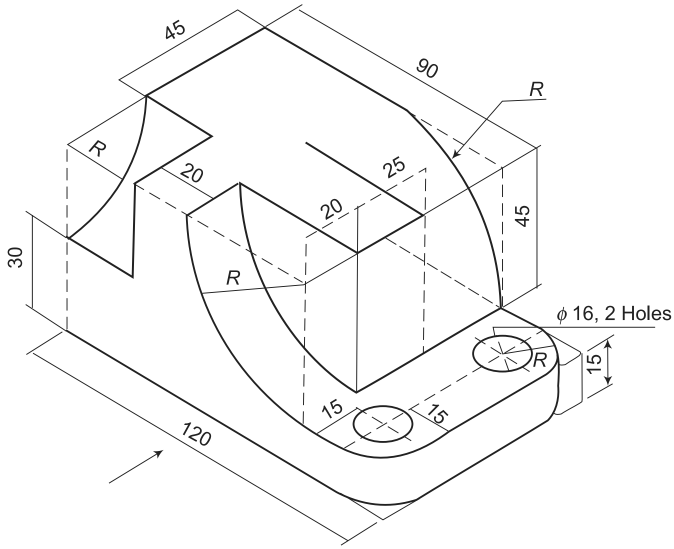
1. Draw the path that would be traced by an end of the string, when it is unwound from the circumference of the disc, which is in the form of a square having a 30 mm side surmounted by semicircles on opposite sides. [15]
2. An 80 mm long line PQ, has its end Q both in the H.P. and the V.P. The line is inclined at 45° to the H.P. and 30° to the V.P. Draw its projections and locate its traces. [15]
3. A hexagonal prism having base with a 30 mm side and 50 mm long axis is resting on its base on the H.P. with a side of base perpendicular to the V.P. A tetrahedron is placed on the top of the prism such that the three corners of the tetrahedron coincide with the alternate corners of the prism. Draw the projections of the arrangement. [15]
4. A hexagonal prism, having a base with a 25 mm side and a 70 mm long axis is resting on a corner of its base in the H.P. and axis inclined at 60° to the H.P. and parallel to the V.P. It is cut by a horizontal section plane which divides the prism into two equal halves. Draw its sectional top view. [15]
5. A cylinder of 60 mm diameter having its axis vertical is penetrated by another cylinder of 40 mm diameter. The axis of the penetrating cylinder is parallel to VP and bisects the axis of the vertical cylinder marking an angle of 60° with it. Draw the orthographic projections showing the curves of intersection. [15]
6. Draw an isometric view of a hexagonal prism having a base with 25 mm side and a 65 mm long axis, which is lying on its face in the H.P. with axis parallel to both H.P. and V.P. [15]
7. Draw the following views for the object shown in figure. All dimensions are in mm.
   (a) Front view
   (b) Top view
   (c) Left side view [15]



8. A rectangular prism of 110 × 70 × 40 mm size is lying on its 110 mm × 70 mm rectangular face on the ground plane with a vertical edge touching the PP and the end faces inclined at 50° with the PP. The station point is 80 mm in front of the PP, 65 mm above the ground plane and at 40 mm to the right of the vertical edge that touches the picture plane. Draw the perspective view of the prism. [15]

Code No: 09A10191

R09

SET-1

B. Tech I Year Examinations, May-June, 2012
ENGINEERING DRAWING
(Common to all Branches)

Time: 3 hours **Max. Marks: 75**

Answer any five questions
All questions carry equal marks

---

1. (a) The vertex of a hyperbola is 65 mm from its focus. Draw the curve if the eccentricity is 3/2. Draw a normal and a tangent at a point on the curve, 75 mm from the directrix.
   (b) Draw a cycloid given the diameter of a rolling circle as d = 30 mm. Draw a normal and tangent at any point on the curve. [7+8]

2. A 120 mm long straight line PQR, is inclined at 30° to the H.P. and 45° to the V.P. The point Q divides the line in the ratio of 1:3 and is situated 40 mm above the H.P. and 60 mm in front of the V.P. Draw its projections and locate its traces. [15]

3. A square ABCD of 50 mm side has its corner A in the H.P, its diagonal AC inclined at 30° to the H.P. and the diagonal BD inclined at 45° to the V.P. and parallel to the H.P. Draw its projections. [15]

4. A pentagonal prism of base 30 mm side and 60 mm height is resting on the base in HP such that one of the rectangular face is parallel to the VP. It is cut by a plane perpendicular to VP and 60 degrees inclined to HP and bisecting the axis of the solid. Draw development of lateral surface of the bottom part of the solid. [15]

5. A vertical cylinder 70 mm diameter is penetrated by a square prism of side 30 mm and its axis is parallel to both HP and VP. Rectangular faces of the prism are equally inclined to the VP. Axis of vertical cylinder intersecting the axis of the horizontal square prism. Draw the projections showing curves of intersection. [15]

6. Draw the isometric view of the block, two views of which are shown in Fig. 1. All dimensions are in mm. [15]

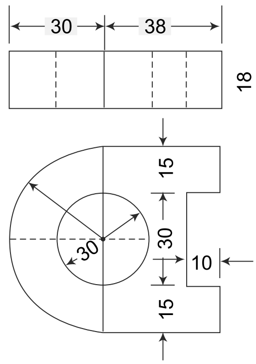


Fig. 1

7. Draw the front view, top view and left side view of the object shown in Fig. 2 (All dimensions are in mm). [15]

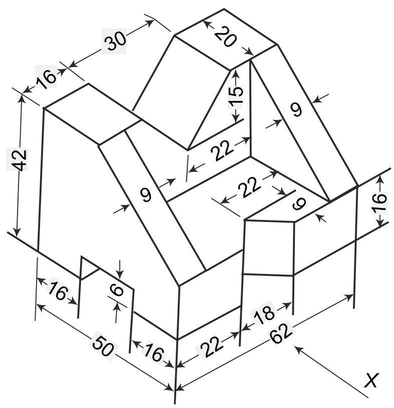


Fig. 2

8. A pentagonal pyramid of side of base 25 mm and height is 50 mm rest with an edge of the base touching the P.P. The station point is on the central line passing through the apex and 80 mm in front of P.P and 65 mm above the ground. Draw the perspective view of the solid. [15]

Code No: 09A10291

R09

SET-1

B. Tech I Year Examinations, December-January, 2011-2012

ENGINEERING DRAWING

(Common to Electrical & Electronics Engineering, Electronics & Instrumentation Engineering and Electronics & Computer Engineering)

Time: 3 hours **Max. Marks: 75**

Answer any five questions

All questions carry equal marks

---

1. (a) Draw a plain scale of RF 1:40 to read Metres and Decimetres and long enough to measure up to 8m. Show lengths of 4.3 m and 6.2 m on this scale.

   (b) Draw the hyperbola when the focus and the vertex are 25 mm apart. Consider eccentricity as 3/2. Draw a tangent and normal to the curve at a point that is 35 mm from the focus. [15]

2. A 75 mm long line PQ is inclined at 45° to the H.P. The end P is 15 mm above the H.P. and 25 mm in front of the V.P. A vertical plane containing line PQ makes an angle of 45° with the V.P. Draw the projections of the line and determine its inclination with V.P. Also, locate its traces. [15]

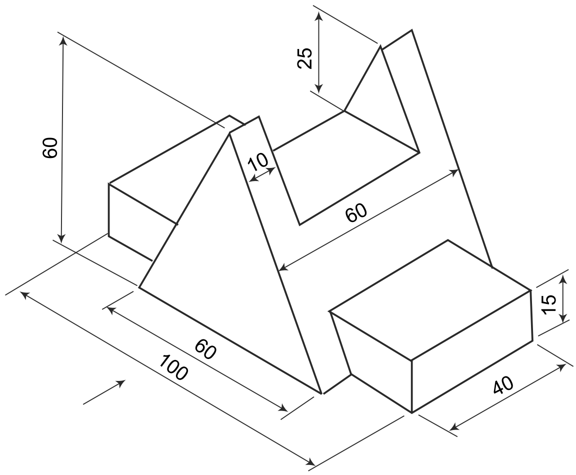
3. A square ABCD with a 40 mm side is suspended from a point O, which is on side AB, 15 mm from A. The plane is parallel to and 25 mm in front of the V.P. Draw its projections and locate the traces. [15]

4. A cylinder, with a 60 mm base diameter and a 70 mm long axis, is lying on a generator on the H.P with its axis parallel to the V.P. A vertical section plane, the H.T. of which makes an angle of 30° with the V.P. and passes through a point distant 25 mm on the axis from one of its ends, cuts the cylinder. Draw its sectional front view and obtain the true shape of the section. [15]

5. A vertical cylinder 80 mm diameter is penetrated by another cylinder of 40 mm diameter and its axis is parallel VP and inclined at 30 degrees to HP. Axis of vertical cylinder is intersecting the axis of inclined cylinder. Draw the projections showing curves of intersection. [15]

6. A vertical cylinder of base diameter 50 mm and height 70 mm is cut by a plane inclined at 55° to HP and perpendicular to VP, which meets the axis at a distance of 20 mm from top base. Draw the isometric view of the remaining portion of the cylinder. [15]

7. Draw the following views for the object shown in figure. All dimensions are in mm.
   (a) Front view
   (b) Top view
   (c) Left side view [15]



8. A cylinder measuring 50 mm in diameter and 100 mm in height stands on its base on the ground plane. The axis of the cylinder is 30 mm behind the picture plan and 10 mm on the right of the observer. The observer is 120 mm in front of the PPP and 30 mm above the ground plane. Draw the perspective projection of the cylinder. [15]

Code No: 09A10291

R09

SET-2

B. Tech I Year Examinations, December-January, 2011-2012
ENGINEERING DRAWING
(Common to Electrical & Electronics Engineering, Electronics & Instrumentation Engineering and Electronics & Computer Engineering)

Time: 3 hours **Max. Marks: 75**

Answer any five questions
All questions carry equal marks

---

1. (a) The R.F. of a scale is 1/400. Construct the scale to measure a maximum distance of 50 m and show a distance of 37.6 m on it. Name the scale and find length of the scale.

   (b) The major axis of an ellipse is 120 mm long and the foci are at a distance of 20 mm from its ends. Draw the ellipse using one-half of it by concentric circles method and the other half by rectangle method. [15]

2. A line PQ has its end projectors 50 mm apart. The end P is 20 mm above the H.P. and 15 mm in front of the V.P., while the end Q is 60 mm above the H.P. and 70 mm in front of the V.P. Draw the projections of the line and determine its true length and inclinations with the principal planes. Also locate its traces. [15]

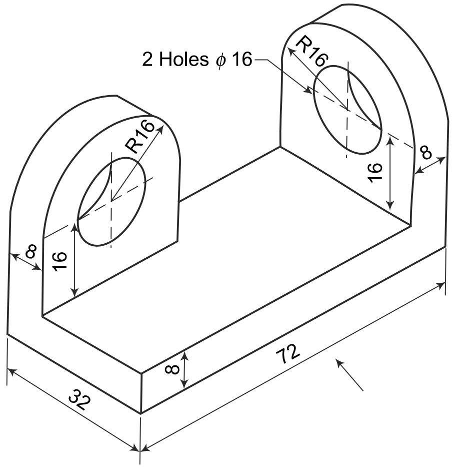
3. A pentagonal plane with a 35 mm long side has its corner on the H.P., and the side opposite to this corner is parallel to the H.P. The plane is parallel to and 20 mm in front of the V.P. Draw its projections and locate its traces. [15]

4. A square prism, with a base having a 40 mm side and a 70 mm axis, is lying on one of its bases on the H.P. with edges of the base equally inclined to the V.P. It is cut by an A.I.P. in such a manner that the true shape of the section is the rhombus with a 80 mm major diagonal. Draw front view, sectional top view and true shape of the section. [15]

5. A vertical cylinder 70 mm diameter is penetrated by another cylinder of diameter 40 mm with its axis is parallel to VP and 30 degrees inclined to HP. Axis of vertical cylinder is 10 mm away from the axis of inclined cylinder. Draw the projections showing curves of intersection. [15]

6. A square pyramid having a side of 50 mm base and 75 mm as axis height stands centrally on circular block of 100 mm diameter and 50 mm thick. The base edges of the pyramid are parallel to VP. Draw the isometric projection of the two objects. [15]

7. Draw the following views for the object shown in figure. All dimensions are in mm.
   (a) Front view
   (b) Top view
   (c) Left side view [15]



8. A rectangular prism with a base of 20 mm × 40 mm and an axis of 50 mm is resting on its base on the GP with its side faces equally inclined to the PPP and one vertical edge touching the PPP. The longer base edge is on the right and the station point is 50 mm in front of the PPP and 65 mm above the GP. The central plane is 10 mm on the left of the axis of the prism. Draw a perspective view of the prism. [15]

Code No: 09A10291

R09

SET-3

B. Tech I Year Examinations, December-January, 2011-2012
ENGINEERING DRAWING
(Common to Electrical & Electronics Engineering, Electronics & Instrumentation Engineering and Electronics & Computer Engineering)

Time: 3 hours **Max. Marks: 75**

Answer any five questions
All questions carry equal marks

---

1. (a) The distance between two stations by road is 200 km and it is represented on a certain map by a 5 cm long line. Find the R. F. and construct a diagonal scale showing a single kilometer and long enough to measure up to 600 km. Show a distance of 467 km on this scale.

   (b) Draw an ellipse when the distance of its focus from the directrix is 60 mm and eccentricity is 2/3. Draw tangent and normal to the curve at a point 40 mm from focus. [15]

2. A 120 mm long line PQ is inclined at 45° to the H.P. and 30° to the V.P. Its mid-point is 50 above the H.P. and 40 mm in front of the V.P. Draw its projections. [15]

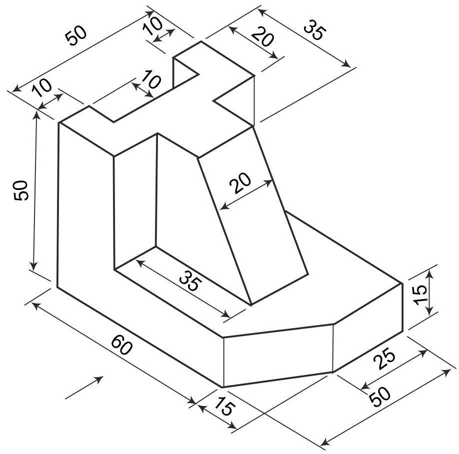
3. A pentagonal plane with a 30 mm long side is resting on one of its edge in the H.P., with its surface perpendicular to the V.P. The corner opposite to that edge is 40 mm above the H.P. Draw the projections of the plane and determine its inclination with the H.P. [15]

4. A hexagonal prism, having a base with a 20 mm side and 60 mm height is resting on the base in HP such that one of the rectangular faces is parallel to the VP. It is cut by a plane perpendicular to VP and 60 degrees inclined to HP and cutting the midpoint of the axis of the solid. Draw development of lateral surface of the bottom part of the solid. [15]

5. A cylinder of diameter 50 mm penetrates fully into a cone of base diameter 80 mm altitude 110 mm, which is resting on its base on HP. The axis of the cylinder intersects the axis of the cone at right angles at a height of 30 mm above the base of the cone. The axis of cylinder is parallel to both the planes. Draw the projections of the solids showing the curves of intersection. [15]

6. A pentagonal pyramid of base of side 30 mm rests on the top of a pentagonal prism of side 30 mm, with their sides coinciding with each other. The solid stands on HP with one of the sides of the base perpendicular to the VP. The height of prism = 40 mm. The height of pyramid = 50 mm. Draw the isometric projection of the solid. [15]

7. Draw the following views for the object shown in figure. All dimensions are in mm.
   (a) Front view
   (b) Top view
   (c) Left side view [15]



8. A circular plate of 30 mm diameter rests on one of the points of its rim on the ground plane (GP). It is parallel to and 25 mm behind the picture plane for the perspective projection (PPP). The station point is 50mm in front of the PPP and 60 mm above the GP. Draw a perspective projection of the plate if the CP is 10 mm to the left of the centre of the circular plate. [15]

Code No: 09A10291

R09

SET-4

B. Tech I Year Examinations, December-January, 2011-2012
ENGINEERING DRAWING
(Common to Electrical & Electronics Engineering, Electronics & Instrumentation Engineering and Electronics & Computer Engineering)

Time: 3 hours **Max. Marks: 75**

Answer any five questions
All questions carry equal marks

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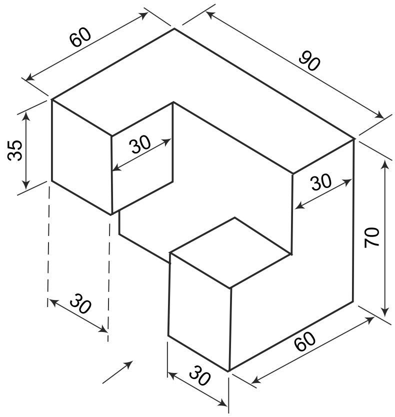
1. (a) Construct and name the scale of R.F. 1: 250 to show decimeter and long enough to measure up to 30 m. Indicate a distance of 28.9 m on it.
   (b) Construct an ellipse of major diameter 120 mm and minor diameter 80mm using concentric circle method. [15]
2. A 120 mm long line PQ is inclined at 45° to the H.P. and 30° to the V.P. A point M lies on the line at a distance of 40 mm from P and its front view is 50 mm above the *xy* line and the top view is 35 mm below the *xy* line. Draw its projections. [15]
3. A square lamina with a 50 mm side rests on the H.P., on one of its corners, such that the diagonal through that corner is parallel to the V.P. and inclined at 30° to the H.P. Draw its projections when the lamina is perpendicular to the V.P. Measure the distance of the topmost corner from the H.P. [15]
4. A square prism, having a base with a 30 mm side and 60 mm height is resting on the base in HP such that one of the rectangular faces is parallel to the VP. It is cut by a plane perpendicular to VP and 60 degrees inclined to HP and bisecting the axis of the solid. Draw development of lateral surface of the bottom part of the solid. [15]
5. A cylinder of diameter 44 mm pierces through a vertical cylinder of diameter 44 mm. The axis of the piercing cylinder is parallel to both the HP and VP. The axes are separated by distance of 6 mm, the axis of the horizontal cylinder being nearer to the observer. Draw the curves of intersection. [15]
6. A frustum of a cone 30 mm as top diameter, 50 mm as bottom diameter and 60 mm long is placed vertically on a square slab of side 70 mm and 30 mm thick, such that both the solids have the common axis. Draw the isometric projection of the combination of solids. [15]
7. Draw the following views for the object shown in figure. All dimensions are in mm.
   (a) Front view
   (b) Top view
   (c) Left side view [15]



8. A hexagonal prism of base edge 30 mm and axis 60 mm long rests on its base on the ground plane with two side faces inclined at 30° to the PPP and a vertical edge nearest to the PPP at 18 mm behind it. The station point is 55 mm in front of the picture plane PPP and 85 mm above the ground plane. The CP is 18 mm to the left of the axis of the prism. Draw the perspective projection of the prism. [15]

Code No: 09A10391

R09

B. Tech I Year Examinations, May-June, 2012
ENGINEERING DRAWING
(Common to all Branches)

Time: 3 hours **Max. Marks: 75**

Answer any five questions
All questions carry equal marks

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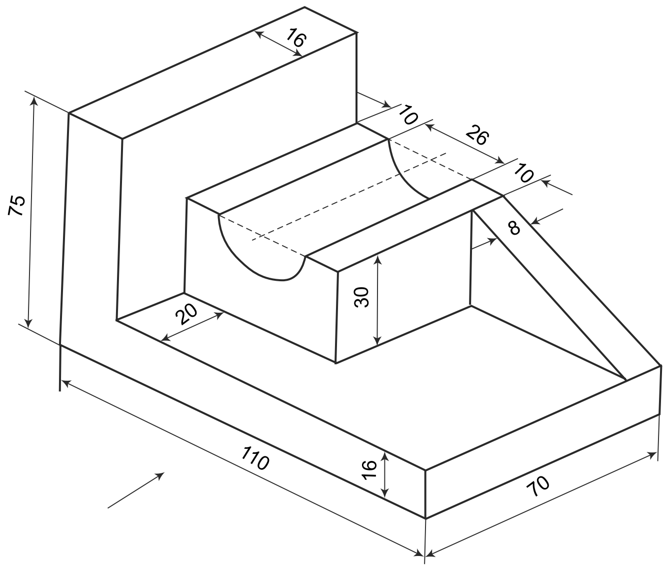
1. The distance between two stations is 100 km and on a road map it is shown by 30 cm. Draw a diagonal scale and indicate distances of 46.8 km and 32.4 km on it. [15]
2. A line of 100 mm long makes an angle 35° with H.P and 45° with V.P. Its mid Point is 20 mm above H.P and 15 mm in front of V.P. Draw the projections of the line. [15]
3. A hexagonal prism of base 25 mm side and axis 45 mm long, is positioned with one of its base edges on H.P. such that the axis is inclined at 300 to the H.P and 45° to V.P. Draw its projections. [15]
4. A cone of base 50 mm diameter and axis 60 mm long is resting on its base on H.P. A section plane perpendicular to V.P and H.P. cuts the cone at a distance of 10 mm from the axis. Draw the development of the cut solid. [15]
5. A vertical cylinder of 80 mm diameter is penetrated by another cylinder of 50 mm diameter. The axis of the penetrating cylinder is parallel to both H.P and V.P and is 10 mm away from that of the vertical cylinder. Draw the projections. [15]
6. A cylinder, with diameter of base 60 mm and axis 70 mm long, is resting on its base on H.P. A section plane, perpendicular to V.P and inclined at 45° to H.P, passes through the axis at a distance of 20 mm from its top end. Draw the isometric projection of the truncated cylinder. [15]
7. Draw the following views of the object shown in below figure. All dimensions are in mm.
   (i) Front view
   (ii) Top view
   (iii) Side view from right [15]



8. A pentagonal pyramid of side base 25 mm and height 50 mm, rests with an edge of the base touching the P.P. The station point is on the central line passing through the apex and 80 mm from P.P. and 60 mm above ground. Draw the perspective of the solid. [15]

Code No: 09A10491

R09

SET-1

B. Tech I Year Examinations, December-January, 2011-2012
ENGINEERING DRAWING
(Common to Electronics & Communication Engineering and Aeronautical Engineering)

Time: 3 hours **Max. Marks: 75**

Answer any five questions
All questions carry equal marks

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1. (a) Construct a diagonal scale of 1:25 to read metres, decimeters and centimeters and long enough to measure 4 m. Mark on it a distance of 2.47 m.

   (b) Draw a parabola in the parallelogram of sides 120 mm and 80 mm, take the longer side as horizontal base. Consider one of the included angles between the sides as 60 degrees. [15]

2. A 70 mm long line PQ is inclined at 45° to the H.P., and its top view measures 50 mm. The end P is 15 mm above the H.P. while the V.T. of the line is 20 mm below the H.P. Draw its projections and determine its inclination with the V.P. Also, locate its H.T. [15]

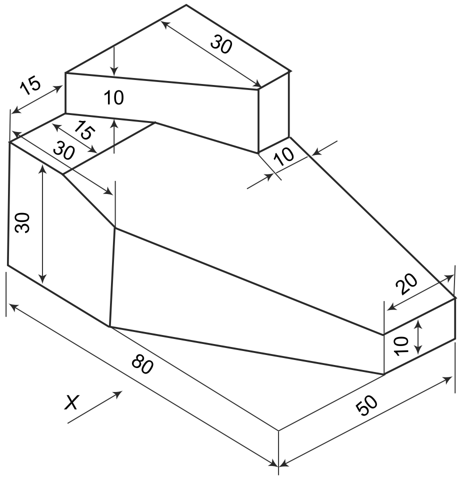
3. A square lamina with a 50 mm side rests on the H.P., on one of its corners, such that the diagonal through that corner is parallel to the V.P. and inclined at 30 to the H.P. Draw its projections when the lamina is perpendicular to the V.P. Measure the distance of the top most corner from the H.P. [15]

4. A square prism with a base having 40 mm sides and height 60 mm is kept on its base on the H.P. such that one of its rectangular faces makes an angle of 30° with V.P. It is cut by a section plane parallel to V.P. such that the true shape of the section is a rectangle with 30 mm and 60 mm sides. Draw its sectional front view and top view. [15]

5. A vertical cylinder 80 mm diameter is penetrated by another cylinder of the same size and its axis is parallel to both HP and VP. Axis of vertical cylinder is intersecting the axis of horizontal cylinder. Draw the projections showing curves of intersection. [15]

6. Draw an isometric projection of a frustum of the pentagonal pyramid with a 40 mm base side, 20 mm top side and 35 mm height resting on its base in the H.P. [15]

7. Draw the elevation, top view and side view of the object shown in figure. All dimensions are in mm. [15]



8. A rectangular prism of base 30 mm × 40 mm rests on the GP on its base with a corner of the base touching the PPP. The longer base edge is on the right and inclined at 30° to the PPP. The station point is 50 mm in front of the PPP and 75 mm above the GP. If the central plane is 20 mm on the left of the axis of the pyramid. Draw a perspective projection of the pyramid. [15]

Code No: 09A10491

R09

SET-2

B. Tech I Year Examinations, December-January, 2011-2012
ENGINEERING DRAWING
(Common to Electronics & Communication Engineering and Aeronautical Engineering)

Time: 3 hours **Max. Marks: 75**

Answer any five questions
All questions carry equal marks

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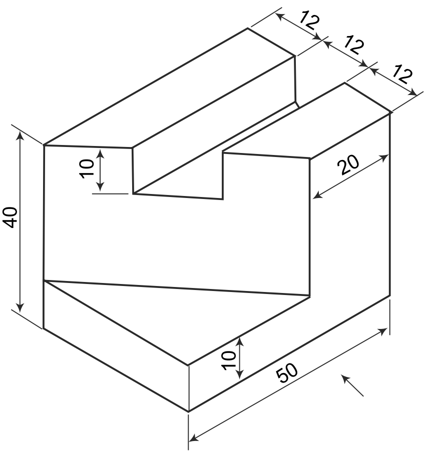
1. (a) A room of 1728 m^3 volume is shown by a cube of 4 cm side. Find the R.F. and construct a scale to measure up to 50 m. Also indicate a distance of 37.6 m on the scale.
   (b) Draw a parabola when span is 80 mm and rise is 30 mm using tangent method. [15]
2. A 120 mm long line PQ has its ends P and Q 10 mm and 60 mm below the H.P., respectively. The end projectors are 50 mm apart. The mid-point of PQ is 60 mm in front of the V.P. Draw the projections and find the angles with both the reference planes. [15]
3. An equilateral triangle with an 60 mm long edge rests on a corner in the V.P. such that the edge opposite to that corner is perpendicular to the H. P. The surface of the plane is inclined at 45 to the V.P. Draw its projections. [15]
4. A cylinder with a 50 mm base diameter and a 90 mm long axis, rests on its base in the H.P. It is cut by an auxiliary inclined plane such that the true shape of the section is a semi-ellipse which has a 70 mm long semi-major axis. Draw its projections. Also, determine true shape of section and inclination of the cutting plane with H.P. [15]
5. A horizontal cylinder 40 mm diameter and axis length 75 mm centrally penetrates vertical cylinder 50 mm as base diameter. Draw the plan and elevation, showing curves of intersection. Assume the axis of the horizontal cylinder is parallel to VP. [15]
6. A hexagonal prism with a 30 mm base and 45 mm axis has an axial hole with a 30 mm diameter. Draw its isometric projection. When its axis is perpendicular to H.P. and two rectangular faces are parallel to V.P. [15]
7. Draw the elevation, top view and side view of the object shown in figure. All dimensions are in mm. [15]



8. A circular plate of 60 mm diameter is lying on the GP with its centre 42 mm behind the PPP. The station point is 85 mm in front of the PPP and 60 mm above the GP. Draw the perspective projection of the plate if the CP is 35 mm to the left of the centre of the plate. [15]

Code No: 09A10491 | **R09** | **SET-3**

B. Tech I Year Examinations, December-January, 2011-2012
ENGINEERING DRAWING
(Common to Electronics & Communication Engineering and Aeronautical Engineering)

Time: 3 hours **Max. Marks: 75**

Answer any five questions
All questions carry equal marks

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1. a) An area of 400 cm^2 on a map represents an area of 25 m^2 on a field. Construct a scale to measure up to 5 km and capable to show a distance of 3.56 km. Indicate this distance on the scale.

   (b) Draw a parabola when span and rise are 100 mm and 80 mm respectively. Draw the curve using rectangle method. [15]

2. A line PQ is inclined at 30° to the H.P. The end P is 15 mm in front of the V.P. and the mid-point of the line is 40 mm above the H.P. The front view measures 60 mm and is inclined at 45° with the reference line. Draw the projections of the line and determine its true length and inclination with V.P. Also, locate its traces. [15]

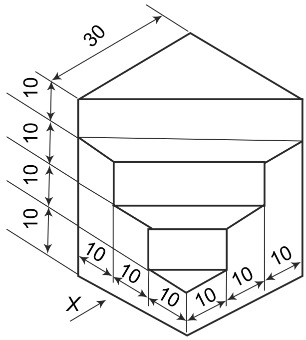
3. A pentagonal plane with a 25 mm side rests on the H.P., on one of its corners with its surface perpendicular to the V.P. and inclined at 30° to the H.P. Draw its projections when the side opposite to the corner on which it is resting is parallel to the H.P. [15]

4. A cone with base circle diameter 50 mm and 60 mm height is resting on the base in HP. It is cut by a plane perpendicular to VP and 60 degrees inclined to HP and bisecting the axis of the solid. Draw development of lateral surface of the bottom part of the solid. [15]

5. A horizontal cylinder of 50 mm diameter penetrates a vertical cylinder of 75 mm diameters resting on HP. The two axes are coplanar. The axis of the horizontal cylinder is 50 mm above the HP. Draw the projection showing the curves of intersection. [15]

6. A square prism, side of base 4 cm and 8 cm long rests centrally on a cylindrical slab 6 cm diameter and 3 cm thick. Draw the isometric projection of the solid. [15]

7 Draw the elevation, top view and side view of the object shown in figure. All dimensions are in mm. [15]



8. A rectangular pyramid of sides of 30 × 20 mm and height 35 mm rests with its base on ground such that one of the longer base edge is parallel to picture plane and 30 mm behind it. The station point is 50 mm in front of picture plane, 30 mm to the left of the axis of the pyramid and 50 mm above the ground. Draw the perspective view of the pyramid. [15]

Code No: 09A10491

R09

SET-4

B. Tech I Year Examinations, December-January, 2011-2012
ENGINEERING DRAWING
(Common to Electronics & Communication Engineering and Aeronautical Engineering)

Time: 3 hours **Max. Marks: 75**

Answer any five questions
All questions carry equal marks

---

1. (a) The distance between two points on a map is 15 cm. The real distance between them is 20 km. Draw a diagonal scale to measure up to 25 km and show a distance of 13.6 km on it.

   (b) Draw a path of a ball which is thrown from ground level which reaches a height of 30 m and a horizontal distance of 60 m before return to the ground. Name the curve. [15]

2. The front view of a line AB makes an angle of 30 with the xy line. The H.T. of the line is 45 mm in front of the V.P. while its V.T. is 30 mm below the H.P. The end A is 12 mm above the H.P. and end B is 105 mm in front of the V.P. Draw the projections of the line and find its true length, and inclinations with the H.P. and the V.P. [15]

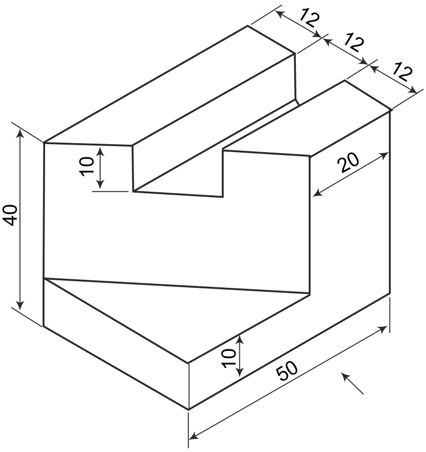
3. A thin hexagonal plane with a 25 mm side rests on a corner in the H.P., such that its surface is perpendicular to the H.P. and inclined at 45° to the V.P. Draw its projections when two sides of the plane are perpendicular to the H.P. [15]

4. A cone with base circle diameter 50 mm and height 60 mm is resting on the base in HP. It is cut by a plane perpendicular to VP and 45 degrees inclined to HP and cutting the axis of the solid 15 mm from top. Draw development of lateral surface of the bottom part of the solid. [15]

5. A vertical cylinder of 60 mm diameter and 80 mm height is penetrated by a horizontal cylinder 40 mm diameter and 80 mm long. The axis of the penetrating cylinder is parallel to VP and 6 mm in front of the axis of the vertical cylinder. Draw the projections and show the intersection curve. [15]

6. A cone of base diameter 30 mm and height 40 mm rests centrally over a cube of sides 50 mm. Draw the isometric projection of the combination of solids. [15]

7. Draw the elevation, top view and side view of the object shown in figure. All dimensions are in mm. [15]



8. Draw the perspective view of a square pyramid of base side 50 mm and height 80 mm resting on GP with the nearest edge of base parallel to PP and 30 mm behind it. The station point is situated at a distance of 120 mm from PP, 50 mm above GP and 80 mm to the right of the apex of the pyramid. [15]

Code No: 09A10591

R09

SET-1

B. Tech I Year Examinations, December/January, 2011-12
ENGINEERING DRAWING
(Computer Science & Engineering)

Time: 3 hours **Max. Marks: 75**

Answer any five questions
All questions carry equal marks

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1. a) Construct and name the scale of R.F. 1:250 to show decimeter and long enough to measure up to 30 m. Indicate a distance of 28.9 m on it.

   (b) Draw the locus of a point lying on the circumference of a circle having a 70 mm diameter, which rolls on a circle with a 140 mm diameter with internal contact for one complete rotation. [15]

2. A line PQ is inclined at 45° to the H.P. and 30° to the V.P., and its top view measures 75 mm. The end P is 75 in front of the V.P. while its V.T. is 15 mm above the H.P. Draw its projections and determine its inclination with the H.P. Also, locate its H.T. [15]

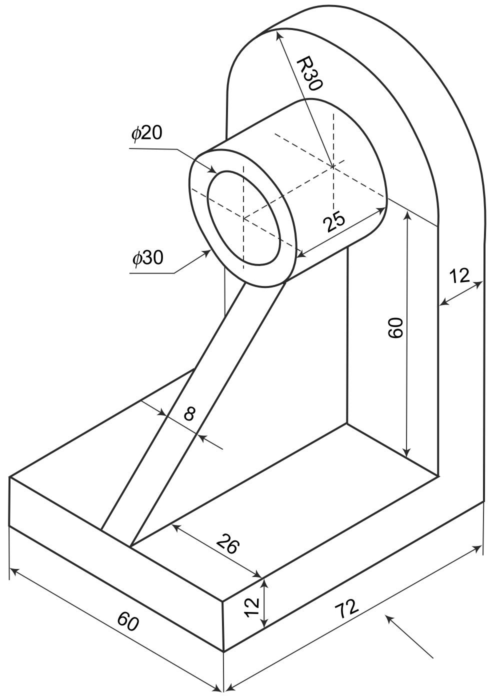
3. A hexagonal plane with a 30 mm side rests on one of its side on the H.P., such that surface is perpendicular to the V.P. and inclined at 45° to the H.P. Draw its projections when the plane lies in the first quadrant. [15]

4. A cone with base circle diameter 50 mm and 60 mm height is resting on the base in HP. It is cut by a plane perpendicular to VP and 60 degrees inclined to HP and bisecting the axis of the solid. Draw development of lateral surface of the bottom part of the solid. [15]

5. A cone of base diameter 80 mm and height 110 mm rests on the HP. It is penetrated by horizontal cylinder of diameter 50 mm. The axes of cone and cylinder intersect at a height of 25 mm above the base of the cone. Draw the projections of the curves of intersection between the solids. Axis of the cylinder is parallel to V.P. [15]

6. Draw the isometric projection of a square prism having a side of base 40 mm and altitude 50 mm surmounting a sphere of diameter 60 mm. [15]

7. Draw the elevation, top view and side view of the object shown in figure. All dimensions are in mm. [15]



8. A pentagonal pyramid of base side 25 mm and axis length 50 mm is resting on GP, on its base with a side of base parallel and 20 mm behind PP. The station point is 60 mm above GP and 80 mm in front of PP and lies in a central plane which is 35 mm to the right of the axis of the pyramid. Draw the perspective view of the pyramid. [15]

Code No: 09A10591

R09

SET-2

B. Tech I Year Examinations, December/January, 2011-12
ENGINEERING DRAWING
(Computer Science & Engineering)

Time: 3 hours **Max. Marks: 75**

Answer any five questions
All questions carry equal marks

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1. (a) The R.F.of a scale is 1/400. Construct the scale to measure a maximum distance of 50 m and show a distance of 37.6m on it. Name the scale and find length of the scale.

   (b) A circle having a 50 mm diameter rolls within a circle with a 150 mm diameter with internal contact. Draw the locus of a point lying on the circumference of the rolling circle for its complete turn. Name the curve. Also draw a tangent and a normal to the curve, at a point that is 40 mm from the centre of the bigger circle. [15]

2. A 90 mm long line PQ has the end P in the H.P. and 70 mm in front of the V.P., while the end Q is 10 mm in front of the V.P. Draw the projections of the line when the sum of its inclination with the H.P. and V.P. is 90°. Determine the true inclination with the reference planes and locate its traces. [15]

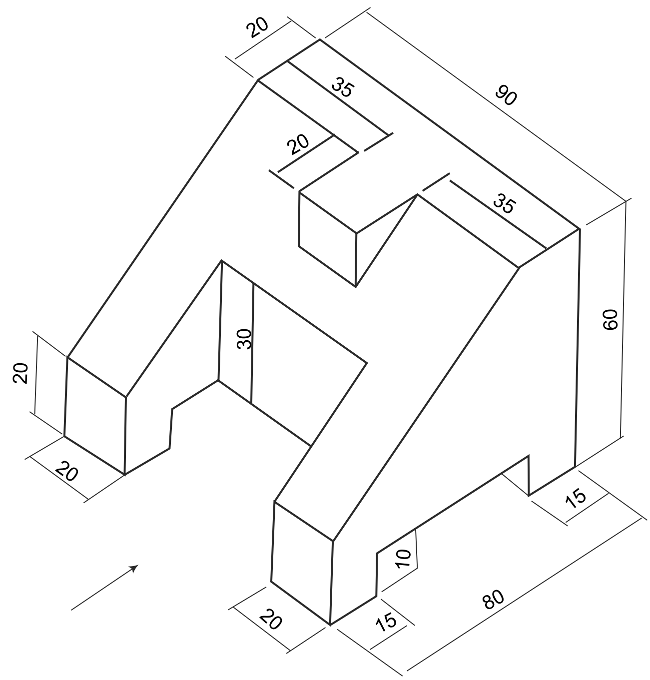
3. A equilateral triangular plane with a 60 mm side has a side inclined at 45° to the H.P. Its H.T. is parallel to and 25 mm below xy and its V.T. does not exist. Draw its projections. [15]

4. A square prism, having a base with a 40 mm side and a 60 mm axis, kept on its base on the H.P. with an edge of the base perpendicular to the V.P. It is cut by a horizontal section plane bisecting the axis. Draw its front view and sectional top view. [15]

5. A cone of base diameter 70 mm and altitude 80 mm is resting on HP on its base. It is penetrated by a cylinder of diameter 30 mm and the axis is parallel to both HP and VP. The axis of the cylinder is situated at a distance 20 mm above the base of the cone and 5 mm away from the axis of the cone and is towards the observer. Draw the curves of intersection of the solids. [15]

6. A sphere of diameter 45 mm rests centrally over a frustum of cone of base diameter 60 mm. top diameter 40 mm and height 60 mm. Draw isometric projections of the combination of solids. [15]

7. Draw the following views for the object shown in figure. All dimensions are in mm.
   (a) Front view
   (b) Top view
   (c) Left side view [15]



8. Draw the perspective projection of a cube of edge 30 mm kept with a face on the ground and two vertical faces perpendicular to picture plane. The front face of the cube is 20 mm behind PP. The station point is 60 mm in front of the PP and 60 mm above the ground. The nearest edge of the cube is 20 mm to the right of the station point. [15]

Code No: 09A10591

R09

SET-3

B. Tech I Year Examinations, December/January, 2011-12
ENGINEERING DRAWING
(Computer Science & Engineering)

Time: 3 hours **Max. Marks: 75**

Answer any five questions
All questions carry equal marks

---

1. (a) A cube of 5 cm sides represents a tank of 1000 m^3 volume. Find the R.F. and construct a scale to measure up to 35m and mark a distance of 27 m on it.

   (b) A fixed point is 90 mm from the fixed straight line. Draw the locus of a point P moving in such a way that its distance from the fixed point is twice its distance from the fixed straight line. Name the curve. [15]

2. A 90 mm long line PQ, has its end A 15 mm above the H.P. and 25 mm in front of the V.P. The line is inclined at 60° to the H.P. and 30° to the V.P. Draw its projections. [15]

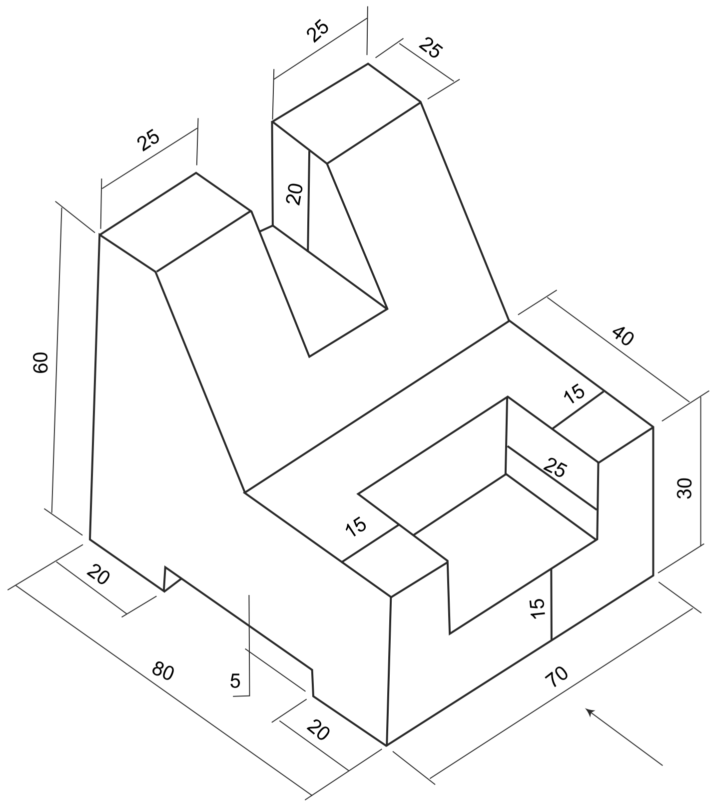
3. A square plane with a 40 mm side has one of its sides inclined at 300 to the H.P. The surface of the plane is perpendicular to both H. P. and V.P. Draw its projections and locate its traces. [15]

4. A hexagonal prism, having a base with a 30 mm side and a 60 mm axis, is resting on its base on the H.P. It is cut by a section plane parallel to the V.P. and 10 mm in front of the axis of the prism. Draw its top view and sectional front view. [15]

5. A cylinder of diameter 30 mm penetrates into a cylinder of diameter 60 mm. Their axes intersect each other at an angle of 60°. Draw the front view and top view of the solids showing the curves of intersection. [15]

6. A hexagonal pyramid of base side 30 mm and axis length 70 mm is resting on HP on its base with a side of base parallel to VP. It is cut by a plane inclined at 40° to HP and perpendicular to VP and bisects the axis. Draw the isometric view of the lower part of the pyramid. [15]

7. Draw the following views for the object shown in figure. All dimensions are in mm.
   (a) Front view
   (b) Top view
   (c) Left side view [15]



8. A cube of edge 5 cm rests with one face on the ground, the nearest vertical edge being 5 cm to the left of the station point and 2.5 cm behind the PP. A face containing the nearest vertical edge is inclined at 60° to the PP. The station point is 7.5 cm above the ground and 10 cm in front of the PP. Draw the perspective view of the cube. [15]

Code No: 09A10591

R09

SET-4

B. Tech I Year Examinations, December/January, 2011-12
ENGINEERING DRAWING
(Computer Science & Engineering)

Time: 3 hours **Max. Marks: 75**

Answer any five questions
All questions carry equal marks

---

1. (a) A line 1 cm long represents a length of 4 decametre. Draw a plain scale and mark a distance of 6.7 m on it. Find RF and length of the scale.

   (b) Draw the hyperbola when its vertex and its focus are at a distance of 40 mm and 25 mm respectively from the directrix. Plot at least six points. [15]

2. A line PQ has the end P at 10 mm above the H.P. and 15 mm in front of the V.P. The lengths of its front and top views are 60 mm and 50 mm respectively. If the top view of the line is inclined at 300 to the reference line, draw its projections. Determine its true length and inclination with the principal planes. Also, locate its traces. [15]

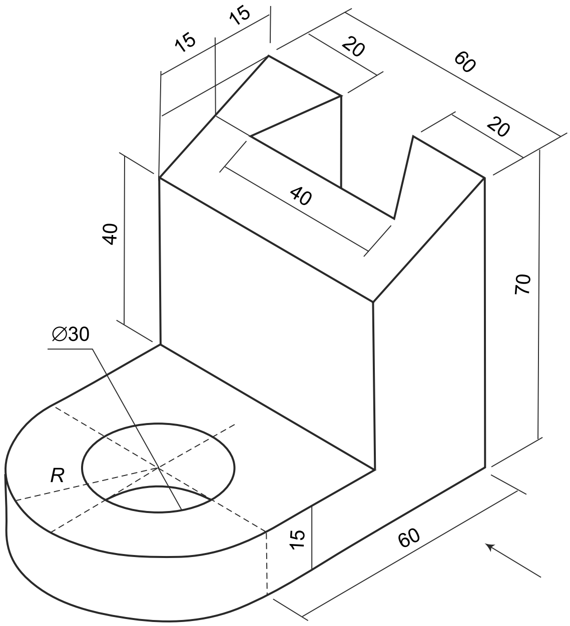
3. A rectangular plane with 50 mm and 30 mm sides is perpendicular to both H.P. and V.P. The longer edges are parallel to the H.P. and the nearest one is 20 mm above it. The shortest edge nearer to the V.P. is 15 mm from it. Draw its projections. [15]

4. A cube with 40 mm long edges is resting on the H.P. on one of its faces with a vertical face inclined at 30° to the V.P. It is cut by a section plane perpendicular to the H.P., parallel to the V.P. and passing through a point that is 10 mm away from the axis. Draw its sectional front view and sectional top view. [15]

5. A vertical cylinder 70 mm diameter is penetrated by another cylinder of the same size and its axis is parallel to both HP and VP. Axis of vertical cylinder is 10 mm away from the axis of horizontal cylinder. Draw the projections showing curves of intersection. [15]

6. A triangular pyramid having base with a 60 mm side and an 80 mm long axis is resting on its base in the H.P. with a side of base perpendicular to the V.P. It is cut by an A.I.P. making an angle of 450 with the H.P. and bisecting the axis. Draw its isometric view of the bottom portion. [15]

7. Draw the following views for the object shown in figure. All dimensions are in mm.
   (a) Front view
   (b) Top view
   (c) Left side view [15]



8. A cylinder 500 mm diameter and height 1000 mm stands on the ground with its circular base. The axis of the solid is 300 mm behind the PP and 100 mm to the right of the observer. The observer is 1200 mm in front of the PP and 300 mm above the ground. Draw the perspective projection of the cylinder to a suitable scale. [15]

Code No: 09A11291 | **R09** | **SET-1**

B. Tech I Year Examinations, December/January, 2011-12
ENGINEERING DRAWING
(Common to Information Technology, Bio Technology and Automobile Engineering)

Time: 3 hours **Max. Marks: 75**

Answer any five questions
All questions carry equal marks

---

1. (a) An area of 49 sq cm on a map represents an area of 16 m^2 on a field. Draw a scale long enough to measure 8 m. Mark a distance of 6 m 9 dm on the scale. Find RF and length of the scale.

   (b) Construct an ellipse of major diameter 120 mm and minor diameter 80 mm using concentric circle method for half of the curve and oblong method for the other half of the curve. [15]

2. A 90 mm straight long line PQ, has the end P 20 mm above the H.P. and 35 mm in front of the V.P. The other end Q is 80 mm above the H.P. and 60 mm in front of the V.P. Draw its projections and determine its true inclination with the principal planes. [15]

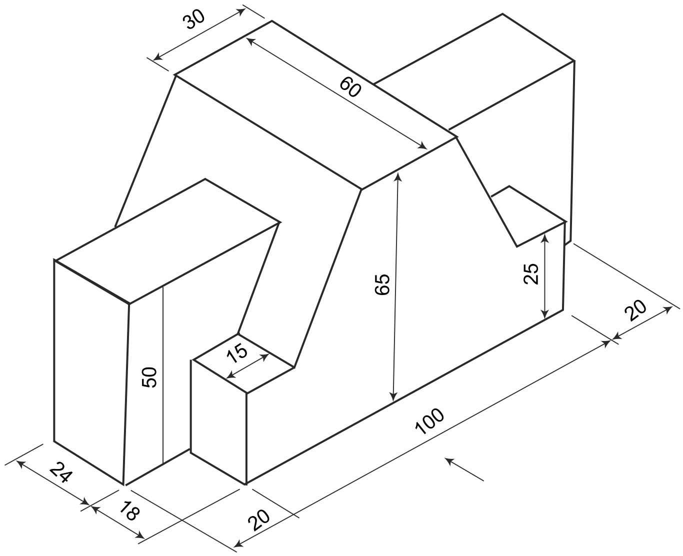
3. An equilateral triangle with a 60 mm long edge rests on a corner in the V.P. such that the edge opposite to that corner is perpendicular to the H.P. The surface of the plane is inclined at 450 to the V.P. Draw its projections. [15]

4. A pentagonal prism, having a base with a 30mm side and 60mm height is resting on the base in HP such that one of the rectangular faces is parallel to the VP. It is cut by a plane perpendicular to VP and 45 degrees inclined to HP and cutting the axis of the solid 10mm from the top. Draw development of lateral surface of the bottom part of the solid. [15]

5. A vertical cylinder 70mm diameter is penetrated by a square prism of side 30mm and its axis is parallel to VP and 30 degrees inclined to HP. Rectangular faces of the prism are equally inclined to the VP. Axis of vertical cylinder is intersecting with the axis of the horizontal square prism. Draw the projections showing curves of intersection. [15]

6. A frustum of a square pyramid of base side 40 mm and top base side 20 mm and height 50 mm is centrally placed on top of a circular slab of diameter 60 mm and thickness 40 mm. Draw the isometric projection of the solids. [15]

7. Draw the following views for the object shown in figure. All dimensions are in mm.
   (a) Front view
   (b) Top view
   (c) Left side view [15]



8. A rectangular pyramid, with the base measuring 40 mm × 25 mm and the axis 50 mm, rests with its base on the ground plane such that the longer base edge is parallel to the picture plane and 20 mm behind it. The station point is 65 mm in front of the picture plane, 35 mm to the left of the axis of the pyramid, and 65 mm above the ground plane. Draw the perspective projection of the pyramid. [15]

Code No: 09A11291 **R09** **SET-2**

B. Tech I Year Examinations, December/January, 2011-12
ENGINEERING DRAWING
(Common to Information Technology, Bio Technology and Automobile Engineering)

Time: 3 hours **Max. Marks: 75**

Answer any five questions
All questions carry equal marks

---

1. (a) A cube of 5 cm sides represents a tank of 1000 m^3 volume. Find the R.F. and construct a scale to measure up to 35 m and mark a distance of 27 m on it.

   (b) The major axis of an ellipse is 120 mm long and the foci are at a distance of 20 mm from its ends. Draw the ellipse using one-half of it by concentric circles method and the other half by rectangle method. [15]

2. A line PQ has its end projectors 50 mm apart. The end P is 20 mm above the H. P. and 15 mm in front of the V.P., while the end Q is 60 mm above the H. P. and 70 mm in front of the V. P. Draw the projections of the line and determine its true length and inclinations with the principal planes. Also locate its traces. [15]

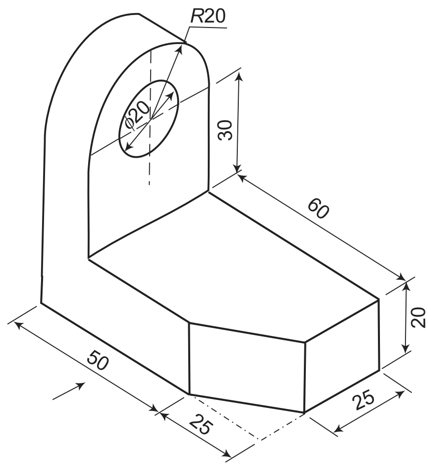
3. A pentagonal plane with a 25 mm side rests on the H.P., on one of its corners with its surface perpendicular to the V.P. and inclined at 30° to the H.P. Draw its projections when the side opposite to the corner on which it is resting is parallel to the H.P. [15]

4. A hexagonal pyramid, having a base with a 20 mm side and 50 mm height is resting on the base in HP such that one of the base sides is parallel to the VP. It is cut by a plane perpendicular to VP and 60 degrees inclined to HP and bisecting the axis of the solid. Draw development of lateral surface of the bottom part of the solid. [15]

5. A vertical square prism, with 50 mm sides at its base and 100 mm long axis, has two of its rectangular faces inclined at 30 degrees to the VP. A hole of 50 mm diameter is drilled in the prism. The axis of the hole is parallel to both the HP and the VP bisects the axis of the prism. Draw the projections showing the curves of intersection. [15]

6. A cone of base diameter 40 mm and height 50 mm rests centrally over a frustum of a pentagonal pyramid of base side 45 mm and top base side 35 mm and height 55 mm. Draw the isometric projections of the solids. [15]

7. Draw the following views for the object shown in figure. All dimensions are in mm.
   (a) Front view
   (b) Top view
   (c) Left side view [15]



8. Draw the perspective view of the frustum of a square pyramid with the edges at the base 40 mm, edges at the top 30 mm and height 50 mm. The frustum is resting on its base with its base edges equally inclined to the picture plane and one of the base corners touching it. The station point is 80 mm in front of the picture plane, 15 mm to the left of the axis of the frustum, and 60 mm above the ground plane. [15]

Code No: 09A11291

R09

SET-3

B. Tech I Year Examinations, December/January, 2011-12
ENGINEERING DRAWING
(Common to Information Technology, Bio Technology and Automobile Engineering)

Time: 3 hours **Max. Marks: 75**

Answer any five questions
All questions carry equal marks

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1. (a) Construct a diagonal scale of 1:25 to read metres, decimeters and centimeters and long enough to measure 4 m. Mark on it a distance of 2.47 m.

   (b) The focus of a hyperbola is 35 mm from its directrix. Draw the curve when eccentricity is 4/3. Draw a tangent and a normal to the curve at a point 30mm from the focus. [15]

2. A 75 mm long line PQ is inclined at an angle of 30° to the H.P. The end P is 20 mm above the H. P. and on the V.P. The end Q is 60 mm in front of the V.P. Draw the projections of the line and locates its traces. [15]

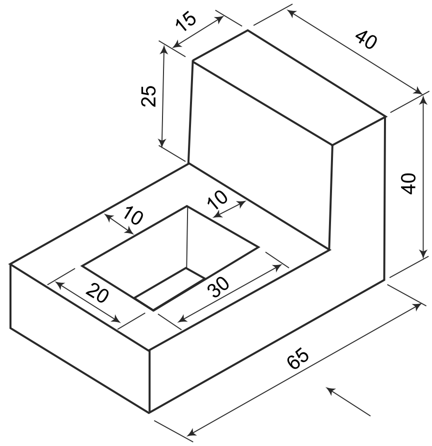
3. A rhombus with 60 mm and 40 mm long diagonals has a corner in the V.P. The surface of the plane is perpendicular to the H.P., and the front view appears as a square. Draw its projections and determine the inclination of the rhombus with the V.P. [15]

4. A square pyramid, having a base with a 30 mm side and 60 mm height is resting on the base in HP such that one of the base sides is parallel to the VP. It is cut by a plane perpendicular to VP and 45 degrees inclined to HP and cutting the axis of the solid 20mm from top. Draw development of lateral surface of the bottom part of the solid. [15]

5. A vertical cylinder, 80 mm in diameter and 100 mm in length, is completely penetrated by a horizontal square prism with 40 mm sides and 100 mm length. The axis of the prism is parallel to the VP, 8 mm in front of the axis of the cylinder, and 50 mm above the base of the cylinder. Draw the projections showing curves of intersection if the side faces of the prism are equally inclined to the HP. [15]

6. Draw the isometric projection of a hexagonal prism of side of base 30 mm and altitude 50 mm surmounting a square pyramid of side 30 mm and height 45 mm such that the axes of the two solids are collinear and at least one of the edges of the two solids are parallel. [15]

7. Draw the following views for the object shown in figure. All dimensions are in mm.
   (a) Front view
   (b) Top view
   (c) Left side view [15]



8. A pentagonal prism, with 30 mm edges at its base and the axis 60 mm long, rests on one of its rectangular faces on the GP, with its axis inclined at 30 degrees to the picture plane and one of the corners of the nearer base touching the PPP. The station point is 75 mm in front of the PPP and 50 mm above the GP. If the central plane is passing through the midpoint of the axis, draw the perspective view of the prism. [15]

Code No: 09A11291

R09

SET-4

B. Tech I Year Examinations, December/January, 2011-12
ENGINEERING DRAWING
(Common to Information Technology, Bio Technology and Automobile Engineering)

Time: 3 hours **Max. Marks: 75**

Answer any five questions
All questions carry equal marks

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1. (a) Construct a diagonal scale showing kilometer, hectometer and decameter in which a 2 cm long line represents 1 km and the scale is long enough to measure up to 7 km. Find R.F. and mark 4 km 5 hm 3 dm on it.

   (b) Draw the hyperbola when the focus and the vertex are 25 mm apart. Consider eccentricity as 3/2. Draw a tangent and normal to the curve at a point that is 35 mm from the focus. [15]

2. A 75 mm long line PQ is inclined at 45° to the H.P. The end P is 15 mm above the H.P. and 25 mm in front of the V.P. A vertical plane containing line PQ makes 45° with the V.P. Draw the projections of the line and determine its inclination with V.P. Also, locate its traces. [15]

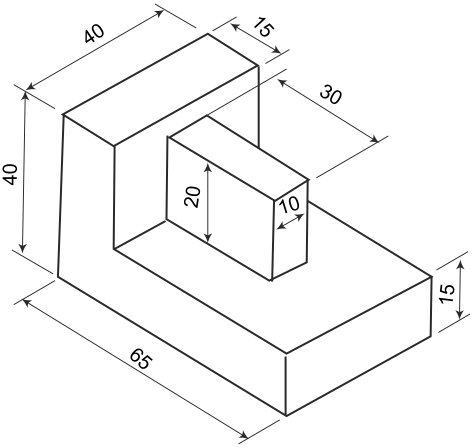
3. A hexagonal plane with a 30 mm side rests on one of its side on the H.P., such that its surface is perpendicular to the V.P. and inclined at 450 to the H.P. Draw its projections when the plane lies in the first quadrant. [15]

4. A square pyramid, having a base with a 40 mm side and a 60 mm long axis, is resting on its base on the ground with all the edges of the base equally inclined to the V.P. It is cut by an A.I.P. such that true shape of the section is an equilateral triangle of largest side. Draw the sectional top view and true shape of the section. [15]

5. A vertical square prism with 40 mm edges at its base and 80 mm height is standing on its base with an edge of its base inclined at 30° to the VP. It is penetrated by a horizontal cylinder, 40 mm in diameter, such that the axis of the cylinder is parallel to the VP, 10 mm in front of the axis of the prism, and 40 mm above the base of prism. Draw the projections showing the curves of intersection. [15]

6. The frustum of a cone with a 60 mm base diameter, 40 mm top diameter and 50 mm height is surmounted centrally over a cylindrical block with 80 mm diameter and 30 mm thickness. Draw its isometric projection. [15]

7. Draw the following views for the object shown in figure. All dimensions are in mm.
   (a) Front view
   (b) Top view
   (c) Left side view [15]



8. A cylinder measuring 50 mm in diameter and 100 mm in height stands on its base on the GP. The axis of the cylinder is 30 mm behind the picture plane and 10 mm on the right of the observer. The observer is 120 mm in front of the PPP and 130 mm above the ground plane. Draw the perspective projection of the cylinder. [15]

Code No: 09A10291-3

R09

B. Tech I Year Examinations, May-June, 2012
ENGINEERING DRAWING
(Common to all Branches)

Time: 3 hours **Max. Marks: 75**

Answer any five questions
All questions carry equal marks

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1. (a) A cricket ball is thrown vertically up, it reaches a maximum height of 15 meters and falls on the ground at a distance of 30 meters from point of projection of the ball till it reaches the ground. Draw the path followed by the cricket ball and name the curve.

   (b) Construct a scale of 1:5 to show decimeters and centimeters and long enough to measure up to 1 m. Show a distance of 6.3 dm on it. [15]

2. A 100 mm long line AB is parallel to and 40 mm above the H.P. Its two ends are 25 mm and 50 mm in front of the V.P. respectively. Draw its projections and find its inclination with the V.P and show traces. [15]

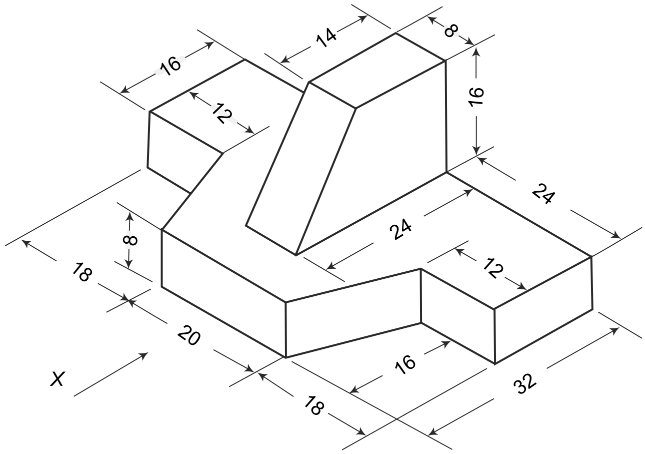
3. A square pyramid, having a base with a 40 mm side and a 75 mm long axis, has a triangular face in the V.P. and an auxiliary inclined plane passing through the axis makes an angle of 45° with the H.P. Draw its projections when its base is closer to the H.P. than to its apex. [15]

4. A cylinder, with a 50 mm diameter and a 70 mm long axis, is resting on its base on the H.P. It is cut by a section plane inclined at 45° to the H.P. and perpendicular to the V.P., such that the plane bisects the axis. Draw its front view, sectional top view and true shape of the section on an A.I.P. parallel to the section plane. [15]

5. A vertical cylinder of diameter 80 mm is completely penetrated by another cylinder of 60 mm diameter, their axis bisecting each other at right angles. Draw their projections showing curves of penetration, assuming the axis of the penetrating cylinder to be parallel to V.P. [15]

6. A sphere with a 50 mm diameter rests centrally over a cube with a 60 mm side. Draw its isometric view. [15]

7. Draw the following views of the block shown in figure. All dimensions are in mm.
   (a) Front view
   (b) Top view
   (c) Right side view [15]



8. Draw the perspective view of a straight line AB, 35 mm long parallel to both the picture plane and ground plane, and 7 mm above the ground plane, and 18 mm behind the picture plane. The station point is 50 mm in front of the picture plane, 36 mm above the ground plane and is contained by a central plane 16 mm to the left of end A. [15]

Code No: 09A10491

R09

B. Tech I Year Examinations, May-June, 2012
ENGINEERING DRAWING
(Common to Electronics & Communication Engineering and Aeronautical Engineering)

Time: 3 hours **Max. Marks: 75**

Answer any five questions
All questions carry equal marks

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1. (a) Construct a diagonal scale of R.F = 1/3000 to show meters, decimeters and centimeters and long enough to measure upto 300 m. Mark on it a distance of 246 meters.

   (b) Construct an ellipse. When the distance of the focus from the directrix is equal to 60 mm and eccentricity 2/3. Also draw a normal and a tangent to the curve at a point 35 mm from the focus. [15]

2. The ends of a line AB are on the same projector. The end A is 15 mm above the H.P. and 50 mm infront of the V.P. The end B is 40 mm above the H.P. and 10 mm infront of the V.P. Determine the true length and traces of line AB and its inclinations with the two planes. [15]

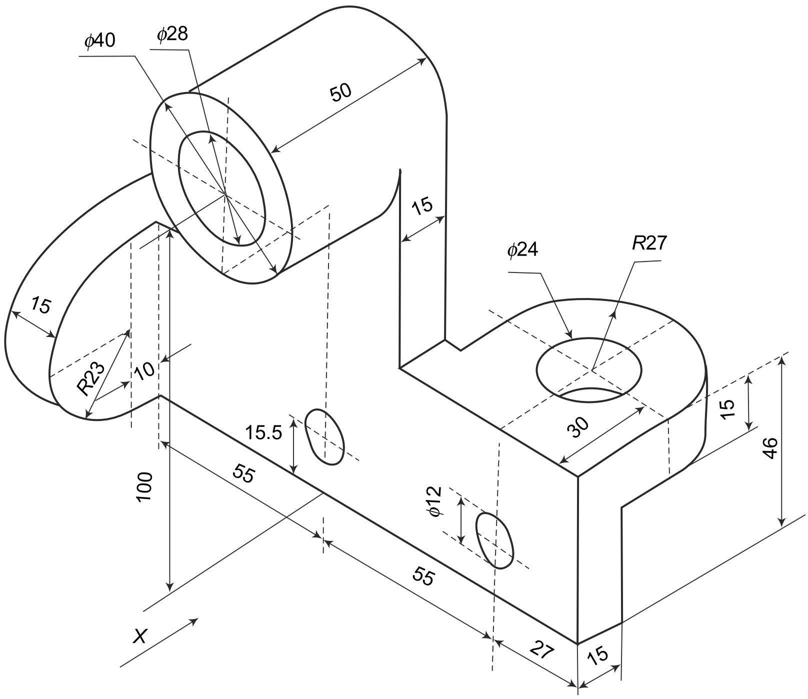
3. A hexagonal pyramid, base 25 mm side and axis 60 mm long, has one of its slant edges on the ground. The plane containing that edge and the axis is perpendicular to the H.P. and inclined at 30° to the V.P. Draw the projections when the base is nearer the V.P. than the apex. [15]

4. A cone, base 50 mm diameter and axis 80 mm long is resting on its base on the H.P. It is cut by a section plane perpendicular to the V.P, inclined at 30° to the H.P and cutting the midpoint of its axis. Draw its front view, Sectional top view and true shape of the section. [15]

5. A vertical cylinder of 70 mm diameter is completely penetrated by another cylinder of 50 mm diameter, their axes bisecting each other at right angles. Draw their projections ********************************parallel to the V.P. [15]

6. Draw the isometric projection of a pentagonal prism with side of base 25 mm and axis 70 mm long. The pyramid is resting on its base on H.P. with an edge of the base perpendicular to V.P. [15]

7. Draw the orthographic front view, top view and side view of the object whose isometric view is shown in figure below. (All dimensions are in mm) [15]



8. A rectangular pyramid, base 40 mm × 25 mm and axis 60 mm long, is placed on the ground plane on its base, with the longer edge of the base parallel to and 30 mm behind the picture plane. The central plane is 35 mm to the right of the apex and the station point is 30 mm front of the picture plane and 20 mm above the ground plane. Draw the perspective view of pyramid. [15]

Code No: 09A10591

R09

B. Tech I Year Examinations, May-June, 2012
ENGINEERING DRAWING
(Common to Computer Science & Engineering, Production Engineering (Mechanical))

Time: 3 hours **Max. Marks: 75**

Answer any five questions
All questions carry equal marks

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1. (a) Draw a parabola using 'tangent method' with its base equal to 180 mm and axis equal to 70 mm.
   (b) A room of 1728 m^3 volume is shown by a cube of 4 cm side. Find the R.F. and construct a scale to measure up to 50 m. Also indicate a distance of 37.6 m on the scale. [15]

2. The front view of a line PQ is 60 mm long and makes 450 with the reference line. The end P is 10 mm above the H.P. and the V.T. of the line is 15 mm below the H.P. If the line PQ is inclined at 300 to the V.P., draw its projections. Determine its true length, inclination with the H.P. and locate its traces. [15]

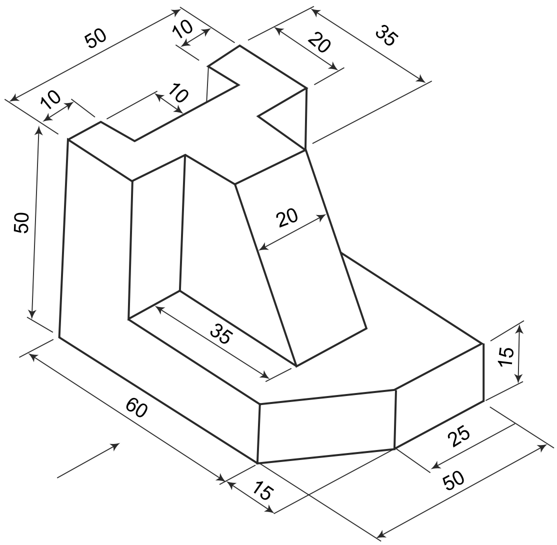
3. (a) A circular plane with a 60 mm diameter is resting on a point of its circumference on the V.P. The center is 40 mm above the H.P. and the surface is inclined at 45° to the V.P., and perpendicular to the H.P. Draw its projections.
   (b) A hexagonal pyramid, having base with a 30 mm side and 70 mm long axis, has a triangular face on the ground and axis parallel to V.P. Draw its projections. [15]

4. A square prism, having a base with a 40 mm side and a 60 mm long axis, rests on its base on the H.P. such that one of its rectangular faces makes an angle of 30° with the V.P. It is cut by a section plane perpendicular to the H.P. and inclined at 60° to the V.P. passing through the prism such that the face which makes 60° with the V.P. is bisected. Draw its sectional front view, top view and true shape of section. [15]

5. A vertical cylinder with an 80 mm base diameter and a 130 mm long axis, is resting on its base in the H.P. It is penetrated by another cylinder with a 50 mm base diameter and 150 mm long axis. The axes of both the cylinders are parallel to the V.P. and bisect each other at an angle of 300. Draw their projection and show the curves of intersection. [15]

6. A cone is placed centrally on the top of a cube with a 40 mm side which is placed centrally over a cylindrical block. The cone has a 30 mm base diameter and a 40 mm axis. The cylindrical block has an 80 mm base diameter and 10 mm thickness. Draw isometric projection of the arrangement. [15]

7. Draw the Front View, Top View and Right Side View for the following figure. (All dimensions are in mm) [15]



8. Draw a perspective view of a square prism having base with a 40 mm side and 60 mm long axis, resting on its base in the GP with its axis that is 40 mm behind the PP and a vertical face right to the axis inclined at 60° to it. The station point is 50 mm in front of PP, 90 mm above GP and lies in a CP which is 50 mm towards right of the axis. [15]

Code No: 09A10191

R09

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY, HYDERABAD
B.Tech I Year Examinations May/June-2013
ENGINEERING DRAWING
(Common to CE, CHEM, AE, CEE, ACE)

Time: 3 hours **Max. Marks: 75**

Answer any five questions
All questions carry equal marks

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1. (a) The distance between two towns is 300 km. A passenger train covers the distance in 1 hour. Construct a scale to measure off the distance covered by the train in a single minute upto 1 hour. Show on it the distance covered in 42 minutes.

   (b) Draw an involute of a circle of 50 mm diameter. Also draw a normal and a tangent at a point distant 100 mm from the centre of the circle.

2. A hexagonal plane 25 mm sides is having one of it's corners on the H.P. with it's surface inclined at 45° to the H.P. and diagonal through the corner on H.P is parallel to the V.P. Draw it's projections.

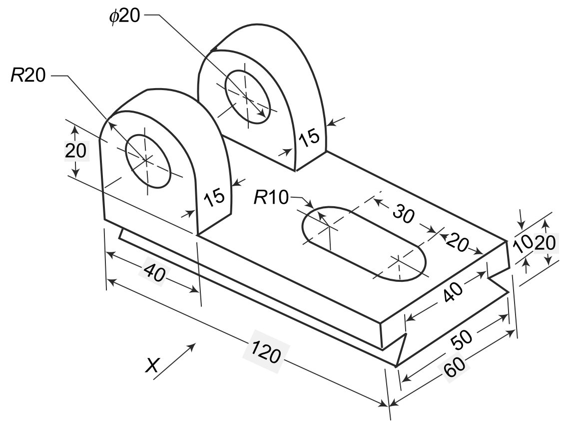
3. An equilateral triangular plate of side 40 mm is resting on VP on one of its sides with its surface inclined at 450 to VP and perpendicular to HP. Draw its projections.

4. A square prism, having a base with a 40 mm side and a 60 mm long axis, rests on its base on the H.P. such that one of its rectangular faces makes an angle of 30° with the V.P. It is cut by a section plane perpendicular to the H.P. and inclined at 60° to the V.P. passing through the prism such that the face which makes 60° with the V.P. is bisected. Draw its sectional front view, top view and true shape of section.

5. A horizontal cone of base diameter 100 mm and height 80 mm penetrates a vertical cylinder of base diameter 60 mm and height 100 mm such that both the axes intersect and are parallel to the VP. The apex of the cone is at the centre of an end generator of the cylinder. Axis of the cone is parallel to the base of the cylinder. Draw the views of the object and show the curves of interpenetration.

6. A pentagonal prism of base edge 40 mm and 60 mm long rests on its longer edge on the ground with the face opposite to this edge parallel to the ground. A cube of 25 mm edge rests on this face on one of its faces. Two adjacent base edges of the cube make equal inclinations to one of the longer edges of the face parallel to the ground. A sphere 40 mm diameter rests centrally on the top of the cube. Draw the isometric projections of the arrangement of the solids.

7. Draw the front, top and both side views of the isometric projection given in figure. All dimensions are in mm.



8. A cylinder of base diameter 50 mm and axis 70 mm long is resting on one of its generators on the ground such that it is parallel to the PP. One of the generators is touching the PP. The station point is along the central line of the object 60 mm away from the PP and 70 mm above the ground. Draw the perspective view of the object.

Code No: 09A10291

R09

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY, HYDERABAD

B.Tech I Year Examinations May/June-2013

ENGINEERING DRAWING

(Common to EEE, ECOMPE, EIE, ETM, ICE)

Time: 3 hours **Max. Marks: 75**

Answer any five questions

All questions carry equal marks

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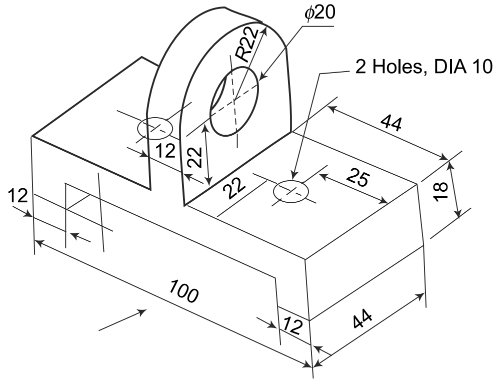
1. An inelastic string is wound around the circumference of a circular disc of 40 mm. diameter. Draw the curve traced out by the end of the string, when it is completely wound around the disc for one revolution keeping the string always in contact with the disc. Name the curve.
2. A line AB 80 mm long, is inclined at 40 degrees to H.P. Its one end is 10 mm above H.P and 8 mm. infront of V.P. Its front view measures 60mm. Draw the projections of the line AB and determine its inclination with V.P.
3. A pentagonal prism is resting on one of its corners of the base on H.P. The longer edge containing the corner is inclined at 450 to the H.P. The top view of the axis makes an angle of 300 to V.P. Draw the projections of the solid when the edge of the solid is 30 mm and height is 70 mm.
4. A pentagonal pyramid of 30mm side and height 70 mm is resting on its base on H.P such that one of the base edges is parallel to V.P. It is cut by a section plane perpendicular to V.P and inclined at 60 degrees to H.P and passes through a point 20mm below the apex. Draw the development of the lateral surface of the bottom part of the pyramid.
5. A vertical cylinder of 60 mm diameter height 100 mm is penetrated by a horizontal cylinder of 35 mm. dia and 100 mm. length, whose axes is parallel to V.P, such that their axes are separated by 5mm. Draw the curves of intersection. The axis of the horizontal cylinder is nearer to the observer
6. Draw the isometric projection of a cone of 3cm diameter, height 4cms placed centrally on the top face of truncated square pyramid of top face side 4 cm and bottom face side 5 cm with the height of 5 cm.
7. Draw three views for the component shown in Fig. 1



All dimensions are in mm

Fig. 1

8. A rectangular block of 20 mm × 30 mm × 60 mm is resting on the ground on one of its largest faces. One of its vertical edges is in the picture plane and the longer edge is inclined at an angle 300 to the picture plane. The station point is 30 mm infront of picture plane and 50 mm above the ground plane and passing through centre of block. Draw the perspective view of the block.

Code No: 09A10391

R09

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY, HYDERABAD

B.Tech I Year Examinations May/June-2013

ENGINEERING DRAWING

(Common to ME, MCT, MMT, AME, MEP, MSNT)

Time: 3 hours **Max. Marks: 75**

Answer any five questions

All questions carry equal marks

1.(a) A rectangular field of 25000 square metre is represented on a map by a rectangle of 5 cm × 4 cm sides. Calculate RF and draw a diagonal scale to read up to a single metre and long enough to measure up to 500 m. Mark a length of 362 m on the scale.

(b) Draw a hypocycloid generated by a rolling circle of 60 mm diameter for one complete revolution. The radius of the directing circle is 100 mm. Draw a tangent and normal to the hypocycloid at 50 mm from the centre of the directing circle.

2. A thin circular plate of 50 mm diameter is resting on point A of it's rim with the surface of the plate inclined at 45° to the H.P and the diameter through A inclined at 30° to the V.P. Draw the projection of the plate when it's centre is 40 mm above the H.P. and 40 mm in front of the V.P.

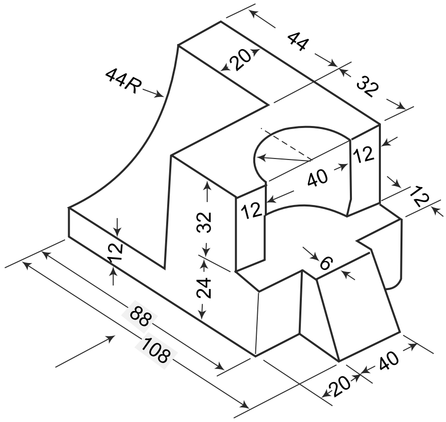
3. A pentagonal pyramid, side of base 25 mm and height 60 mm has one of its slant (triangular) faces on the H.P. and the edge of base contained by that slant edge makes an angle 30 degrees to the V.P. Draw its projections.

4. A cylinder of base diameter 65 mm and height 90 mm is cut by a plane inclined 45° to HP, perpendicular to VP and bisecting the axis of the cylinder. Draw the development of bottom part of the cylinder.

5. A cone penetrates a vertical cylinder of base diameter 60 mm and height 80 mm. The cone has its apex touching the ground and axis parallel to the VP. Its base has a diameter of 120 mm and height of 80 mm. The distance between the axes of the objects is 30 mm and the plane joining the axes of the objects is parallel to the VP. Draw the views of the objects showing the curves of interpenetration.

6. A square pyramid of 50 mm base edge and height 70 mm is resting on its base on the ground with one of the base edges being parallel to the VP. It is cut by a horizontal plane which intersects and cuts axis at a distance of 50 mm from the base. Another square pyramid whose base exactly coincides with the cut portion of the first pyramid and whose height is 50mm is placed on the first pyramid. Draw the isometric projection of the pyramids.

7. Draw the front, top and right side view of the isometric projection given in Figure. All dimensions are in mm.



8. A cone of base diameter 50 mm and height 70 mm is resting on a point on the circumference of the base. The axis is parallel to both the HP and the VP. One of the points on the base is touching the PP. The station point is 50 mm to the right of the centre of the axis of the object. The station point is 60 mm from the PP and 80 mm above the ground. Draw the perspective projection of the object.

Code No: 09A10491

R09

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY, HYDERABAD

B.Tech I Year Examinations May/June-2013

ENGINEERING DRAWING

ECE

Time: 3 hours **Max. Marks: 75**

Answer any five questions

All questions carry equal marks

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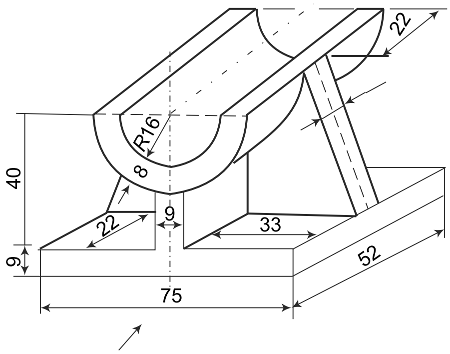
1. An area of 144 sq.cm. on a map represents an area of 36 sq.km. on the field. Find the R.F. of the scale for this map and draw a diagonal scale to show kilometres, hectametres and decameters and to measure upto 10 km. Indicate on this scale a distance of 7.56 km.
2. A line AB measures 75 mm and has end A 10 mm infront of V.P. and 15 mm above H.P. and the other end B, 55 mm infront of V.P and 50 mm above HP. Draw the projections of the line and find the inclinations of the line with both the reference planes. Also, draw the traces.
3. A square pyramid of 35 mm side and 60 mm height rests on one of its triangular faces on the H.P, such that the base edge is inclined at 40° to V.P. Draw the projections of pyramid. When the apex is nearer to the viewer?
4. A vertical cone of 50 mm diameter of base and height 65 mm is resting on its base in H.P and is cut by a section plane perpendicular to V.P and inclined at 60 degrees to H.P and passes through a point 25 mm above the base. Draw the development of the lateral surface upper portion of the cone.
5. A vertical cylinder of 60 mm diameter is penetrated by a horizontal square prism of side 30 mm and length 100 mm, the axis of which is parallel to V.P and all the edges of the square prism are equally inclined to H.P. Draw their projections showing the curves of intersection. Axes of both the solids intersect at a height of 30 mm from the base of the cylinder.
6. Draw the isometric projection of square prism of side 8 cm and height 12 cms when the axis is (a) vertical (b) horizontal.
7. Draw the three views for the component shown in Fig. 1.



All dimensions are in mm

Fig. 1

8. Draw the perspective view of cube of 40mm edge resting on ground on one its faces. It has one of its vertical edges in the pp and all vertical faces are equally inclined to the picture plane. The station point is 30 mm infront of pp, 60 mm above the ground plane and is contained by a central plane 15 mm to the left of the centre of the cube.

Code No: 09A10591

R09

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY, HYDERABAD
B.Tech I Year Examinations May/June-2013
ENGINEERING DRAWING
(CSE)

Time: 3 hours **Max. Marks: 75**

Answer any five questions
All questions carry equal marks

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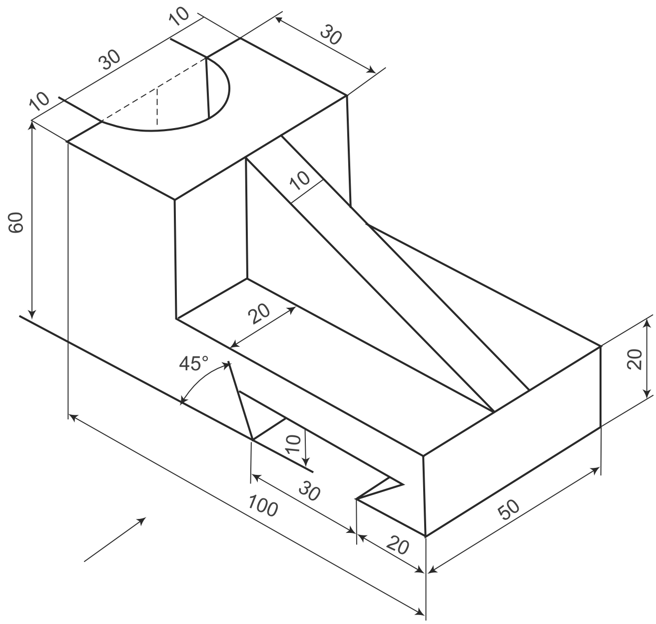
1. The major and minor axes of an ellipse are given as 12 cm and 7.5 cm respectively. Draw the normal and tangent from a point 2cm. above the axis. Draw the ellipse by arcs of circles method.
2. The top view of a line 70 mm long measures 60 mm, while the length of front view is 50mm. Its one end is 8 mm infront of V.P and 12 mm. above H.P. Draw the projections of the line and determine its inclinations with H.P and V.P.
3. A Hexagonal pyramid of base edge 25 mm. and height 70 mm rests on one of its base edges an HP such that the edge is inclined at 30° to V.P and its axis makes an angle of 45° to H.P. Draw the projections of the pyramid.
4. A cylinder of 50 mm diamet- and height 70 mm rests on its base on the ground. A slot of shape of an equilateral triangle of side 25 mm. is cut through the cone, so that its axis is perpendicular to V.P. and bisects the axis of the cylinder at right angles. Draw the development of the lateral surface of the cylinder with the slot.
5. A vertical cylinder of 60 mm diameter height 100 mm is penetrated by another cylinder of same size. The axis of the penetrating cylinder is parallel to both H.P and V.P and 6 mm away from the axis of the vertical cylinder and nearer to the viewer. Draw the projections showing the curves of intersection.
6. Draw the Isometric projection of a cylinder of 6cm diameter and height 6 cm when the axis is (a) vertical (b) horizontal.
7. Draw three views for the component shown in Fig. 1.



All dimensions are in mm.

Fig. 1

8. A square prism of side of base 40 mm and height 60mm rests with its base on the ground such that one of the base edges is inclined at 35° to the pp and one of the vertical edges is in pp. The station point is 30 mm infront of pp and 80 mm above the ground plane and lies in a central plane 4 0 mm to the right of the centre of the prism. Draw the perspective view.

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