DS-288 Numerical Methods
UE-201 Introduction to Scientific Computing
Due date: September 12, 2023 (Sunday 11:59 PM)

Homework-2

Total 100 points | Weight 10%

Read the following instructions carefully.

- Write your NAME and SR. NUMBER on the first page of the report(only one PDF for all questions in order). Start each question on a new page.
- In coding exercises, also give algorithm/background theory along with code and discuss attached plots briefly to get full credit for that question.
- LaTeX is recommended for the report. Use Python/Matlab for coding. Give proper annotations and comments in code wherever required. Name code file according to question number (i.e., q1,q2,q4...).
- Put all codes and the report in a folder, with the folder named **DS288_LastFiveDigitsSRNo_Name**, compress it into a zip file, and submit that zip file to the teams.
- Don't use any inbuilt functions for solving problems (i.e., np.linalg.solve); use a proper algorithm to get credits. Marks will be deducted if plagiarism is found in the report or codes. Late submissions won't be accepted.

1. A few days back, we all witnessed huge success of the remarkable project Chandrayaan-3. The intricacies of space travel demand meticulous planning and accurate calculations. A minor deviation in the initial trajectory can lead to significant consequences, ranging from missed orbital insertion to catastrophic mission failure. To make such a mission successful, the team must know the precise location of the Moon with respect to Earth and make a trajectory plan according to that. Kepler's book Epitome of Copernican Astronomy (1621) introduced the oldest method to locate celestial bodies, around 70 years before Newton's law of gravity in Philosophiæ Naturalis Principia Mathematica.

   According to Kepler's equation:
   $$M = E - e \sin E$$
   Here M is the mean anomaly, the fraction of an elliptical orbit (angle) that has elapsed since the orbiting body passed periapsis (point in Moon's orbit when it is closest to the Earth), expressed in rad. E is eccentric anomaly, an angular hyperparameter and e is eccentricity of the Moon's orbit, that is 0.0549 (almost circular, plot ellipse with this much of e and

see yourself. Even Earth's orbit is more circular having $e \approx 0.0167$) with semi-major axis $a = 384,400$ km and semi-minor axis $b = 383,820$ km and having orbital period of roughly $T = 27.32$ days.

Chandrayaan-3 was launched on 14th July 2023 at 2:40 pm and entered lunar gravity on 5th Aug 2023 at 7:00 pm. From all these information one can calculate the mean anomaly(M). Further on solving Kepler's equation, one can get eccentric anomaly(E). If you have accurate E, you can get position of the Moon with respect to Earth using,

$$x = a(\cos E - e)$$
$$y = b \sin E$$

Explain can this be solve using fixed point iteration(FPI)? Using FPI find E with relative tolerance of 10^{-8} and then compute the coordinates of the Moon at the time of Lunar Orbit Injection (LOI). [Assume at time of launch Moon was at periapsis] [20]

2. a) Prove the Newton method has a quadratic order of convergence for simple root and linear order of convergence for root with multiplicity(m)>1. For m>1, show with little modification in Newton's method, one can achieve quadratic order of convergence.

   b) Prove the order of convergence of the Secant method is the Golden Ratio.
   Müller's method takes a similar approach but projects a parabola through three points. Compute root of $f(x) = \cos x - xe^x$ using the Secant method and Müller's method. Give number of iterations required to achieve relative tolerance of 10^{-6}. Compare convergence of both methods.
   For Secant method, use $x_0 = 0, x_1 = 1$ and for Müller's method use $x_0 = 0, x_1 = 0.5, x2 = 1$.
   [10+10]

3. Functions that exhibit rapid oscillations or fluctuations and have multiple roots in a small interval are often referred to as "oscillatory functions." These functions can be challenging to analyze and work with due to their rapid changes and the presence of multiple roots crossings in a narrow range. Highly oscillatory functions are encountered in various mathematics, physics, and engineering fields, such as wave propagation, signal processing, and quantum mechanics. Analyzing and approximating these functions can be complex, requiring specialized techniques to handle their oscillatory behavior and the presence of multiple roots. $f(x) = \frac{\sin 1/x}{x}$ is one of such functions (plot and observe its nature yourself).

   Consider $f(x) = x^2 + \cos(30e \cdot x), e \approx 2.718$. Design an efficient algorithm to compute its positive roots under relative tolerance of 10^{-6}. Give reasoning behind your algorithm clearly.

Explain all assumptions you made and all crucial steps of your approach. Your code will be evaluated based on how many roots it can locate and computation time it takes. [30]

Hint: You can use a combination of methods discussed so far. This function has less than 30 positive roots, and all lie between 0 and 1.

PS: Don't give such reasoning - the function plot suggests roots lie near these many points, so I chose these points as initial points.

4. Code for the Lagrange form of the fifth-degree polynomial that interpolates the population from 1930 to 1980 in Table 1. Plot this Lagrange polynomial from 1930 to 2030 (take population every 10 years). Estimate population of 1990, 2000, 2010 and 2020 from the above obtained polynomial. Give the mean square error of these estimates. Explain: Is this a reasonable way to estimate the population of INDIA in future years? [30]

Year	Population(in Cr.)
....	....
1930	28.5
1940	32.4
1950	37.6
1960	45.1
1970	55.8
1980	69.7
1990	87.1
2000	105.7
2010	124.1
2020	139.6

Table 1: Population of India (1930-2020)