DS-288 Numerical Methods
UE-201 Introduction to Scientific Computing
Due date: August 29, 2023 (Tuesday 11:59 PM)

Homework-1

Total 100 points | Weightage 10%

Read the following instructions carefully.

- Write your NAME and SR. NUMBER on the first page of the report(only one PDF for all questions in order). Start each question on a new page.
- Answers for all the questions and respective explanations (if required) should be mentioned in the report explicitly. In coding exercises, also give algorithm/background theory along with code to get full credit for that question.
- LaTeX is recommended for the report. Use Python/Matlab for coding. Give proper annotations and comments in code wherever required. Name code file according to question number (i.e., q1,q2,q4...).
- Put all codes and the report in a folder, with the folder named **DS288_SRNo_Name**, compress it into a zip file, and submit that zip file to the teams.
- Don't use any inbuilt functions for solving problems (i.e., np.linalg.solve); use a proper algorithm to get credits. Marks will be deducted if plagiarism is found in the report or codes. Late submissions won't be accepted.

1. Comment on the nature of the following functions(algebraic/transcendental) with proper reasoning. (x belongs to real numbers unless not specified explicitly)
   - $f_1(x) = x + \pi$
   - $f_2(x) = x + \sin \pi$
   - $f_3(x) = x!$ $\qquad x \in \mathbb{Z}^+$
   - $f_4(x) = \frac{1}{x+x^2}$ $\qquad x \neq 0, x \neq -1$
   - $f_5(x) = x + \sin x[1 + 2\cos 2x] - \sin 3x$

[10 points]

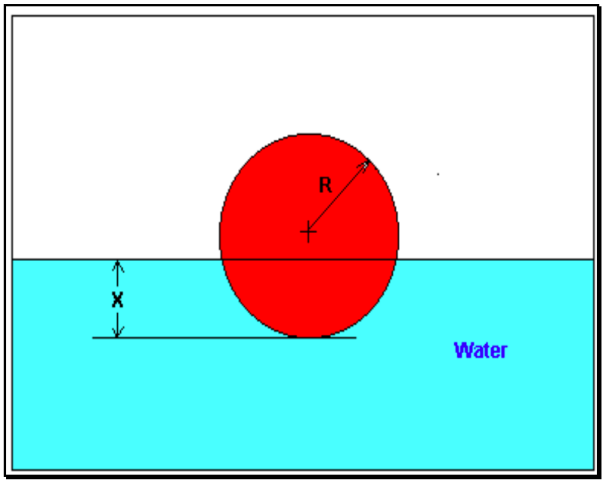

Figure 1: Ques2

2. You are working for 'ASTRAL PIPES COMPANY' that makes floats for water tanks. The floating ball has a specific gravity of 0.6 and has a radius of 5.5cm. You are asked to find the depth to which the ball is submerged when floating in water. The equation that gives the depth x to which the ball is submerged under water is given by

$$x^3 - 0.165x^2 + 3.993 \times 10^{-4} = 0$$

Here x is in SI units. To find the depth x to which the ball is submerged in water, use the **bisection method** of determining roots of equations. To estimate the root of the above equation, perform iterations until you reach a relative tolerance of 10^{-4} between successive iterations. Report minimum number of iterations required.

[20 points]

3. Using the **Newton's and Modified Newton's methods,** find the solution of $f(x) = 0$ for the functions listed. Iterate until you reach a relative tolerance of 10^{-6} between successive iterates with initial guess $x_0 = 0$. Report the root found and the number of iterations needed for each method. Give a plot of relative tolerance with number of iterations and compare the convergence of these methods. [$15 \times 2 = 30$ points]

   (a) $f(x) = (x + e^{-x^2} \cos x)^2$.

   (b) $f(x) = x - 2\log(1 + e^{-x})$.

4. Consider the following differential equation:

$$\frac{dP}{dt} = \alpha P - \beta P^2$$

This is the Logistic differential equation, an approximate population growth model. According to this model the rate of change of population ($\frac{dP}{dt}$) is positively proportional to its current population (αP), indicating more population leads to further reproduction and negatively proportional to the square of its current population($-\beta P^2$), indicating more population leads to more competition hence survival of fittest comes in action (P^2 comes from numbers of face of among P population, P choose 2, $\binom{P}{2}$).

This equation is analytically solvable using a variable separable method. On solving, one gets following evolution of population:

$$P(t) = \frac{\alpha/\beta}{1 + Ae^{-\alpha t}}$$

$$\text{where, } A = \frac{\alpha/\beta - P(0)}{P(0)}$$

Take $\alpha = k; \beta = 5 * 10^{-5}k$. A group of microbiologists observed the growth of Escherichia coli (E. coli), a common bacteria found in the intestines of humans and animals. According to observation, initially there were 1000 bacteria in a shell, and after 10hrs, there were 10,000 bacteria. They tried to model this as the logistic growth model. What will be a value of estimated k(under 10^{-4} relative tolerance)?

Use the **Secant method** with some appropriate initial guesses. Explain each step and the choices you made in detail. [40 points]