DS-288 Numerical Methods
UE-201 Introduction to Scientific Computing
Due date: November 21, 2023 (Tuesday 11:59 PM)

Homework-5

Total 100 points | Weight 10%

Please read the following instruction carefully.

- Please write your NAME and SR. NUMBER on the report
- Answers for all the questions and respective explanations (if required), should be mentioned in the report explicitly.
- Put all codes and the report in a folder, with the folder named **DS288-YourName-HW4**, compress it into a zip file, and submit that zip file in the teams.
- Only the submitted zip file will be checked. Make sure it has all the required materials.
- Please write a script (.sh file or .m file) to run all the programs with one command.

Writing Programming with explanations (if any) for **All** Questions.

1. Consider the initial value problem

$$\frac{dy}{dx} = \frac{(x-y)}{2}, \quad y(0) = 1, \quad 0 \le x \le 3$$

Write a program to compare following numerical solutions with the exact solution

   (a) The Euler method for $h = 1,\ 0.5,\ 0.25$ and 0.125 [10]
   (b) The modified Euler method for $h = 1,\ 0.5,\ 0.25$ and 0.125 [10]
   (c) The Taylor method of order 4 (i,e, n=4) for $h = 1,\ 0.5,\ 0.25$ and 0.125 [10]
   (d) The Runge-Kutta method of order 4 (i,e, RK4) for $h = 1,\ 0.5,\ 0.25$ and 0.125 [10]
   (e) The explicit Adams-Bashforth four-step method for $h = 0.125$ [10]
   (f) The implicit Adams-Moulton three-step method for $h = 0.125$ [10]

2. Consider the second-order initial value problem [8]

$$y'' + 4y' + 5y = 0, \text{ with } y(0) = 3, \quad y'(0 = -5)$$

   (a) Write down the equivalent system of two first-order equations.
   (b) Use the Runga-Kutta method to solve the reformulated problem over $[0, 5]$ using $M = 50$ subintervals of width $h = 0.1$
   (c) Compare the numerical solution with the true solution:

$$y(x) = 3e^{-2x}\cos(x) + e^{-2x}\sin(x)$$

3. Solve the nonlinear system with the initial solution $\mathbf{x}^0 = (0.1, .0.1. -0.1)^t$ with the following methods [6+6]

$$\begin{aligned} 3x_1 - \cos(x_2x_3) - \frac{1}{2} &= 0 \\ x_1^2 - 81(x_2+0.1)^2 + \sin x_3 + 1.06 &= 0 \\ e^{-x_1x_2} + 20x_3 + \frac{10\pi - 3}{3} &= 0 \end{aligned}$$

   (a) Newton's method for first five iterations

   (b) Fixed-point method until accuracy $\epsilon = 10^{-5}$

   Hints: See book (Burden and Faires) Chapter-10 (Examples-1 and -2)

4. Solve the following boundary value problem [10+10]

$$\frac{d^2y}{dx^2} = \left(\frac{2x}{1+x^2}\right)\frac{dy}{dx} - \left(\frac{2}{1+x^2}\right)y + 1, \quad 0 < x < 4, \quad y(0) = 1.25, \quad y(4) = -0.95$$

   (a) Using the Shooting method for $h = 0.2$

   (b) Using finite-difference method for $h = 0.2$

   (c) Compare the numerical solutions with the true solution:

$$y(x) = 1.25 + 0.486089652x - 2.25x^2 + 2x\tan^{-1}(x) - \frac{1}{2}\ln(1+x^2) + \frac{1}{2}x^2\ln(1+x^2)$$