

BAYESIAN MULTI-OBJECTIVE OPTIMIZATION FOR QUANTITATIVE RISK ASSESSMENT IN MICROBIOLOGY

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Context

This study is a part of the PRIMA European project **ArtisanFood** that aims at controlling food-borne pathogens in artisanal fermented food of meat and dairy, origin produced in the Mediterranean region. As a participating country one of the main objective for France is to allow the continuation of the production of raw milk soft cheeses, which is today potentially at risk due to future European regulations. At the French national level this project is in collaboration with ANSES, CNIEL, CentraleSupélec - Université Paris-Saclay and organizations from the dairy industry.

Problem Statement

The primary goal is to establish efficient intervention strategies, in order to “economically” reduce the risk of *Haemolytic Uremic Syndrome (HUS)* caused by *Shiga-Toxin producing Escherichia coli (STEC)* present in raw-milk soft cheese.

Intervention strategies in cheese making:

- **Pre-harvest milk sorting:**
STEC and *E. coli* strains follow same fecal route!
- $p_{\text{test}}^{\text{milk}}$: Probability of testing a bulk tank of milk
- Farms with *E. coli* conc. $> l^{\text{sort}}$ are rejected
- C_{pre} : Cost of testing and rejecting bulk tank milk
- **Post-harvest cheese sampling:**
– $p_{\text{test}}^{\text{cheese}}$: Probability of testing a batch of cheese
- From a single batch n_{sample} cheeses are tested for presence of STEC
- C_{post} : Cost of testing and rejecting cheese batches

The aim is to find the **optimal** values of the process intervention parameters $\{p_{\text{test}}^{\text{milk}}, l^{\text{sort}}, p_{\text{test}}^{\text{cheese}}, n_{\text{sample}}\}$, that minimize the risk and the costs (C_{pre} & C_{post}).

Quantitative Risk Assessment (QRA)

QRA model proposed by Perrin et al. (2014)

- **Farm module + Pre-harvest step**
STEC conc. $Y_{\text{milk}}^{\text{STEC}}$ in farm milk is computed

$$Y_{\text{milk}}^{\text{STEC}} = Y_{\text{milk}}^{\text{EC}} \cdot (Y_{\text{feces}}^{\text{STEC}} / Y_{\text{feces}}^{\text{EC}})$$

- **Cheese module**
Evolution of STEC is modeled with ODEs

$$\frac{dy}{dt} = \mu^{\max}(t) \cdot y(t) \cdot \left(1 - \frac{y(t)}{y^{\max}}\right)$$

STEC cells form colonies inside cheese

- **No. of colonies (Poisson):** N^{colony}
- **Size of colonies (LogNormal):** Y^{colony}

- **Consumer module**

Batch risk is computed using a dose-response model:

$$\Gamma = \sum_s N_s^{\text{colony}} \cdot Y_s^{\text{colony}}$$

$$R^{\text{batch}} = \sum_{\text{age}} g_{\text{age}} \int_{\Gamma} P[\text{HUS}|\gamma, \text{age}] \cdot p(\gamma) d\gamma$$

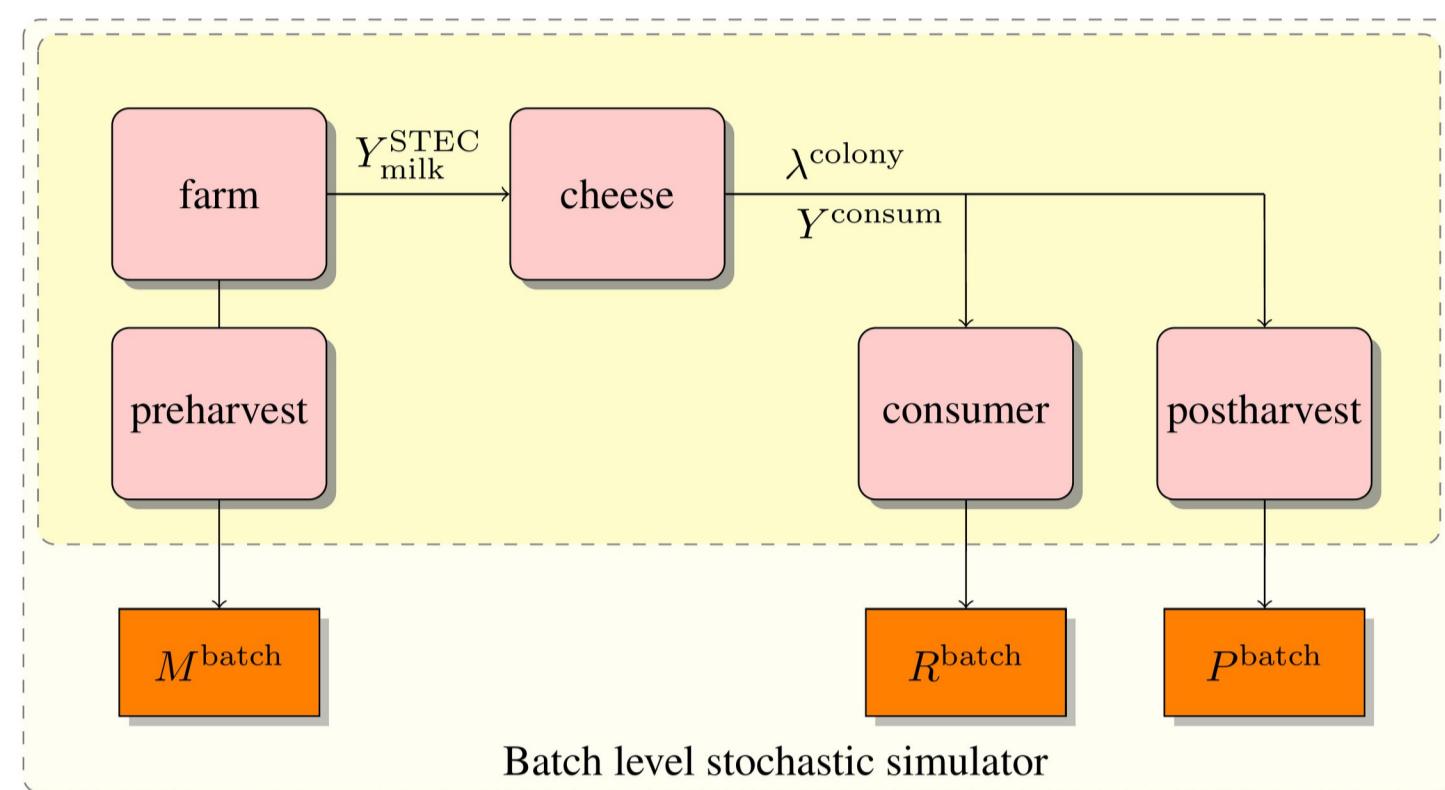
averaging over consumer age

- **Post-harvest module**

Proportion of rejected batches P^{batch}

$$P^{\text{batch}} = P[\Gamma > 0] = 1 - \exp(-K \cdot n_{\text{sample}})$$

Quantities of Interest (QoI)



Several batches are simulated to estimate

- $R_{\text{avg}} = \mathbb{E}[R^{\text{batch}} \cdot (1 - P^{\text{batch}} \cdot p_{\text{test}}^{\text{cheese}})]$
- $P_{\text{avg}} = \mathbb{E}[P^{\text{batch}} \cdot p_{\text{test}}^{\text{cheese}}]$
- $M_{\text{avg}} = \mathbb{E}[M^{\text{batch}}]$

QoIs are

- **Relative risk:** $f_1 = \frac{R_{\text{avg}}}{(1 - P_{\text{avg}}) \cdot K_1}$, where K_1 is baseline risk (no interventions)
- **Intervention cost:** $f_2 = C_{\text{pre}}(M_{\text{avg}}) + C_{\text{post}}(P_{\text{avg}})$

Bi-objective optimization

We consider the bi-objective optimization problem

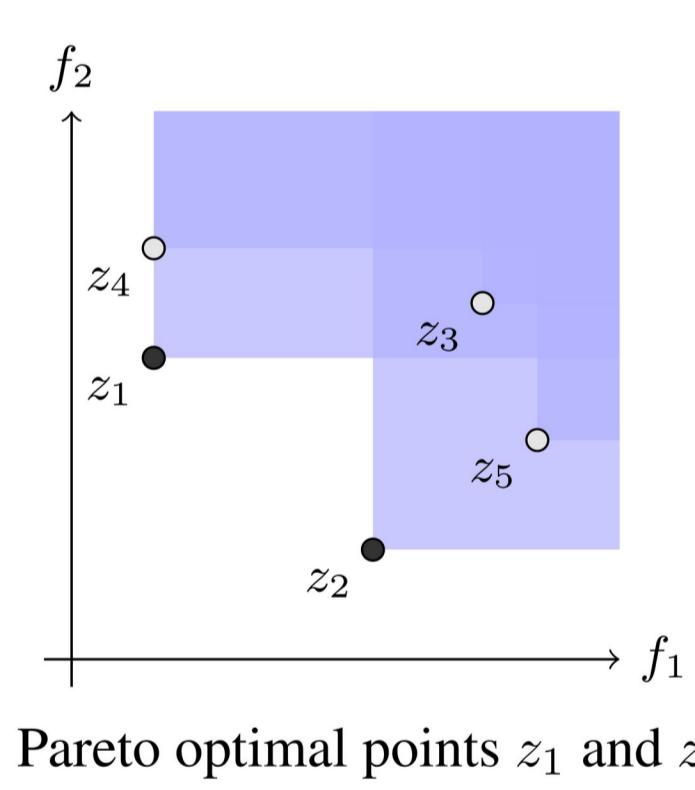
$$\min_{x \in \mathbb{X}} f(x)$$

where, $f = (f_1, f_2)$

- Not necessarily has a unique solution $x^{\text{opt}} \in \mathbb{X}$, in presence of conflicting objectives
- The solution set \mathcal{P} consists of Pareto optimal points

$$\mathcal{P} = \{x \in \mathbb{X} : \nexists x' \in \mathbb{X}, f(x') \prec f(x)\}$$

where $f' \prec f \implies f'_i \leq f_i, \forall i$, with at least one of the inequalities being strict



Pareto optimal points z_1 and z_2

- In stochastic setting, we assume additive noise: for $x_i \in \mathbb{X}$, we observe $Z_i = f(x_i) + \varepsilon_i$, $\varepsilon_i \sim \mathcal{N}(0, \Sigma)$
- The problem boils down to estimating \mathcal{P}

PALS

Optimization of the QRA simulator

- It is **stochastic** and computationally **expensive**
- Gradient based optimization is not feasible
- Thus we reside on Bayesian approaches

Pareto Active Learning for Stochastic simulators proposed by Zuluaga et al. (2013) and extended by Barracosa et al. (2021).

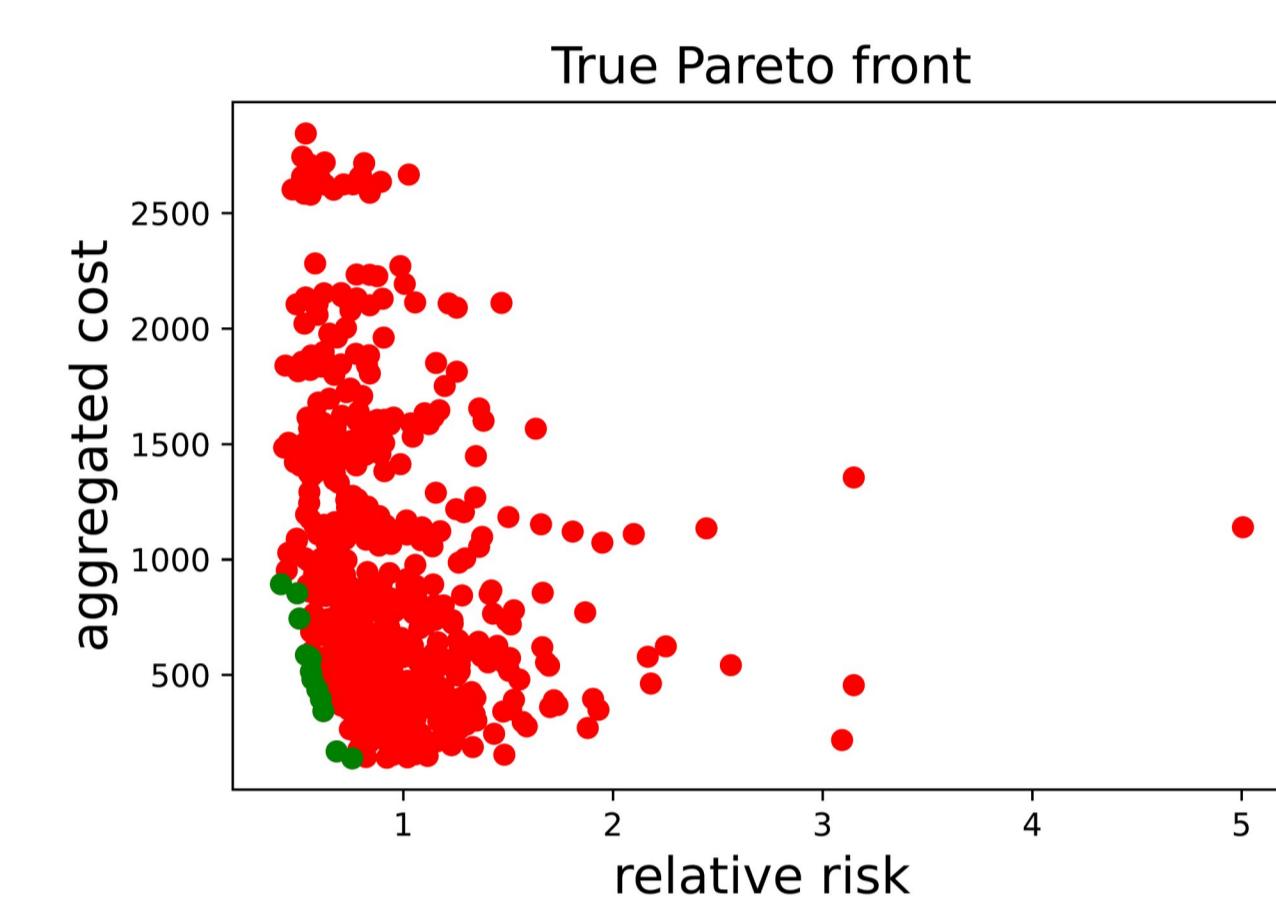
- It uses Gaussian process regression for approximating the simulator function
- Estimates \mathcal{P} by classifying each point in \mathbb{X} as **Pareto optimal**, **Non-Pareto optimal** (dominated) and **Unclassified**

Experimental results

- Minimizing f over the input space \mathbb{X}

- $n_{\text{sample}} \in \{5, 10, 20, 30, 50\}$
- $l^{\text{sort}} \in \{10, 20, 30, 40, 50\}$
- $p_{\text{test}}^{\text{milk}} \in \{1/10, 1/20, 1/30, 1/40, 1/50\}$
- $p_{\text{test}}^{\text{cheese}} \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$

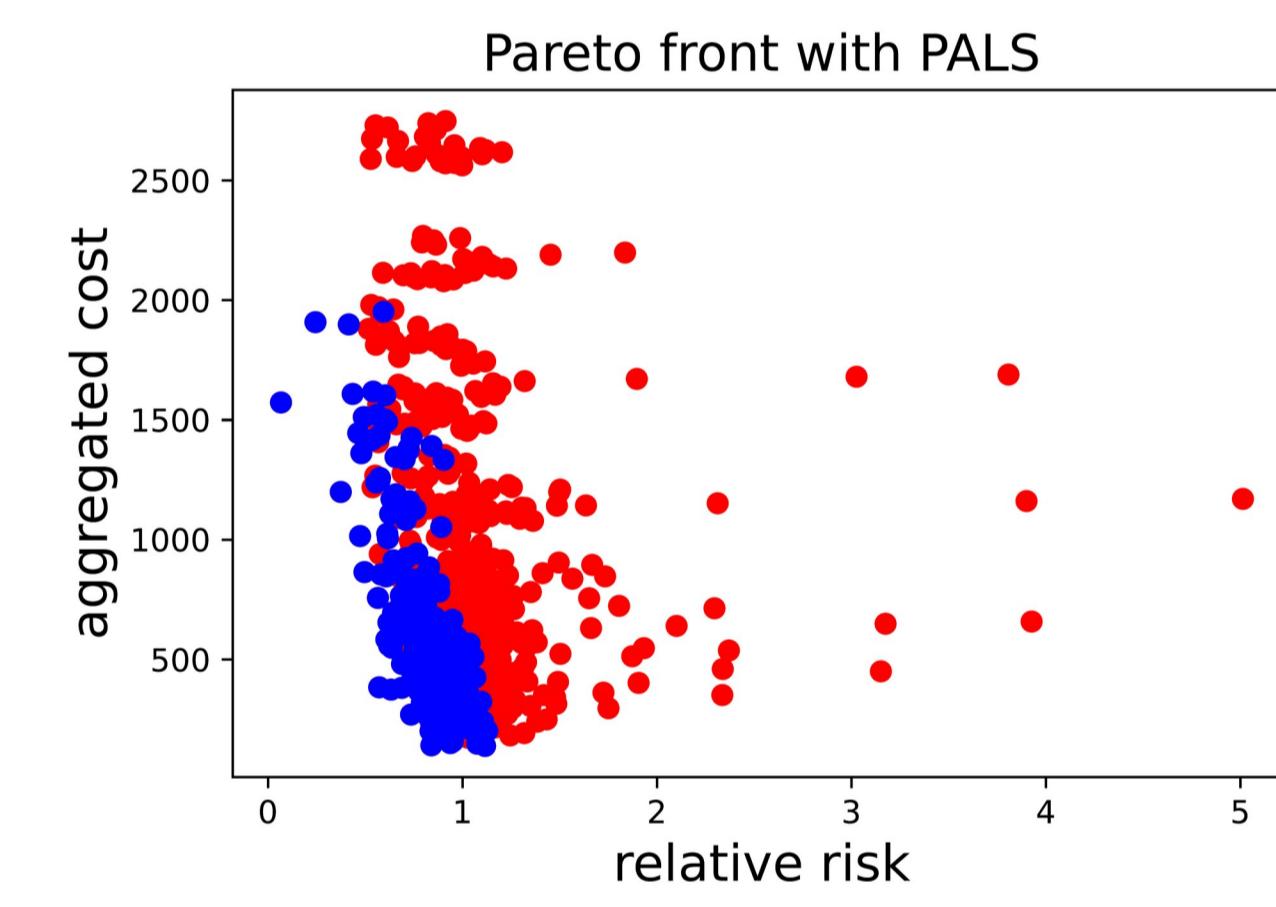
- **True Pareto front:** estimated using 5000 samples for each of $5 \times 5 \times 5 \times 5 = 625$ input points



- Pareto optimal (green) and dominated points (red)

- **Pareto front estimated using PALS**

- Initial design size = 60, budget = 40
- batch size per iteration = 300



- With PALS using significantly less (100×300) evaluations, the user can provide the following insights

- Most of the dominated (red) points are well classified
- The points corresponding to \mathcal{P} remain unclassified (blue)

References

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