

YOUNG'S MODULUS OF ELASTICITY

AIM: To determine Young's modulus of a beam by Single Cantilever method.

APPARATUS: Uniform bar with cantilever set up, microscope, slotted weights, screw gauge, vernier calipers

INTRODUCTION: When an elastic body is subjected to a particular stress, or force per unit area, it will respond with a particular strain, or deformation. If the stress is small enough, the body will return to its original shape after the stress is removed, exhibiting its elasticity. If the stress is greater, the material may be incapable of returning to its original shape, causing it to be permanently deformed. At some even greater value of stress, the material will break or fracture. The particular values of stress that cause these three situations differ for every material. Knowing these values, however, is vitally important.

There are different types of stress: tension or tensile stress, compression or compressive stress, shear stress, and hydraulic stress. The quantity for all types of stress, however, can be defined as follows:

$$\text{stress} = \frac{F}{A}$$

where F is the force applied and A is the cross-sectional area of the material. (In the case of hydraulic stress, A represents the surface area of the material.) Notice that the standard unit of stress is $[\text{N/m}^2]$.

The quantity strain (for tensile and compressive types of stress) can be defined as follows:

$$\text{strain} = \frac{\Delta L}{L}$$

where L is the original length of the material, and ΔL is the change in length that results after the stress is applied. Notice that strain is a dimensionless quantity. (In the cases of shear and hydraulic stress, strain is defined slightly differently.)

Hooke's and Elastic limit: Within the elastic limit of a solid material, the deformation (strain) produced by a force (stress) of any kind is proportional to the force. If the elastic limit is not exceeded, the material returns to its original shape and size after the force is removed, otherwise it remains deformed or stretched. The force at which the material exceeds its elastic limit is called 'limit of proportionality.'

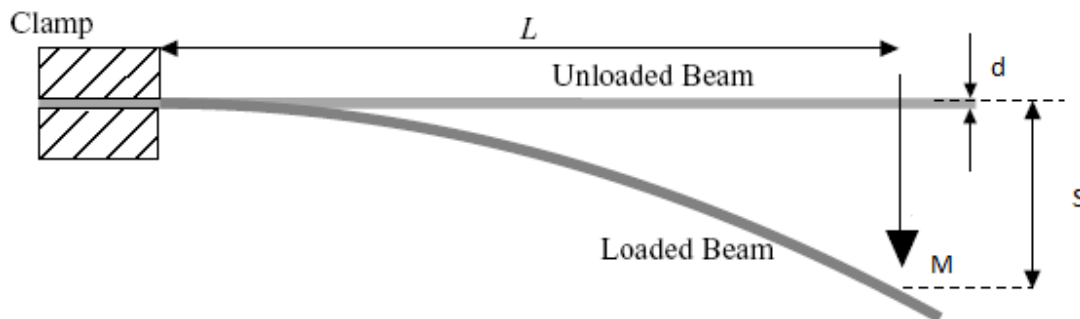
Young's modulus (E) describes tensile elasticity, or the tendency of an object to deform along an axis when opposing forces are applied along that axis; it is defined as the ratio of tensile stress to tensile strain. It is often referred to simply as the elastic modulus.

The shear modulus or modulus of rigidity (G or μ) describes an object's tendency to shear (the deformation of shape at constant volume) when acted upon by opposing forces; it is defined as shear stress over shear strain.

The bulk modulus (K) describes volumetric elasticity, or the tendency of an object to deform in all directions when uniformly loaded in all directions; it is defined as volumetric stress over volumetric strain, and is the inverse of compressibility. The bulk modulus is an extension of Young's modulus to three dimensions.

Cantilever is a bar of uniform cross section whose length is very much larger than thickness. The beam is clamped at one end and loaded at the free end. The beam bends under the action of couple produced by the load. Upper surface of the beam gets stretched and lower surface gets compressed. The extension is maximum in the upper most filaments and compression is maximum in the lowermost ones. The surface which does not get affected is known as neutral surface. If the bending is uniform, the longitudinal filaments get bent into circular arcs in planes parallel to the plane of symmetry (plane of bending). The line of intersection of plane of bending with neutral surface is called neutral axis.

FIGURE:



FORMULA:

$$Y = \frac{4gL^3}{bd^3} \left(\frac{M}{S} \right)_{avg}$$

Where Y is the Young's modulus of the material of the given beam in Nm^{-2}

M is the load added in kg

g is acceleration due to gravity in ms^{-2}

L is the length of the cantilever in m

b is breadth of the beam in m

d is thickness of the beam in m

S is the depression produced for the load M in m

TABULAR COLUMNS:

(I) To determine the breadth of the given beam (b) using Vernier Calipers

Least count = 1 M.S.D / Total no. of V.S.D

=..... cm

Trial No	MSR (cm)	CVD (div)	TR = MSR + (CVD x LC) (cm)
1.			
2.			
3.			

Mean 'b' =cm=.....m

(II) To determine the Thickness of the given beam (d) using screw gauge

$$\text{Pitch} = \frac{\text{Distance covered on pitch scale}}{\text{No. of rotations given to the head scale}}$$

$$\text{Least Count (L.C)} = \frac{\text{Pitch}}{\text{Total no. of head scale divisions}}$$

Zero error = div

Zero correction =div

Trial No.	PSR (mm)	HSD (div)	CHSD (div)	TR = PSR + (CHSD x LC) (mm)
1.				
2.				
3.				

Mean 'd' =mm =m

(III) To determine $\frac{M}{S}$ using travelling microscope

Least count = 1 M.S.D / Total no. of V.S.D=..... cm

Length of the cantilever L=.....cm

Load M (g)	TM reading for Load increasing			TM reading for Load decreasing			$R_1=(r_1+r_2)/2$ (cm)	Load M (g)	TM reading for Load increasing			TM reading for Load decreasing			$R_2=(r_3+r_4)/2$ (cm)	Depression for 60g S (R_1-R_2) cm	$\frac{M}{S} \left(\frac{g}{cm} \right)$
	MSR (cm)	CVD (div)	TR r_1 (cm)	MSR (cm)	CVD (div)	TR r_2 (cm)			MSR (cm)	CVD (div)	TR r_3 (cm)	MSR (cm)	CVD (div)	TR r_4 (cm)			
0								60									
20								80									
40								100									

$$\left(\frac{M}{S} \right)_{avg} = \dots \text{g/cm} = \dots \text{Kg/m}$$

PROCEDURE:

1. A pin is fixed vertically on the uniform bar at the loaded end.
2. Microscope is focused to the tip of the pin and the reading is recorded.
3. Add weights in step of 20g up to 100g and record readings. Similarly, decrease the load in steps of 20g and record the readings.
4. Calculate the mean depression for 60g.
5. Young's Modulus is calculated using the formula

$$Y = \frac{4gL^3}{bd^3} \left(\frac{M}{S} \right)_{avg}$$

CALCULATION :

$$Y = \frac{4gL^3}{bd^3} \left(\frac{M}{S} \right)_{avg}$$

$$g = 9.8 \text{ ms}^{-2}$$

$$L = \dots\dots\dots \text{m}$$

$$b = \dots\dots\dots \text{m}$$

$$d = \dots\dots\dots \text{m}$$

$$Y =$$

RESULT:

The Young's Modulus of the material of the given beam is determined using single cantilever method and its value is found to be $Y = \dots\dots\dots \text{N/m}^2$.

PROPORTIONAL ERROR CALCULATION:

Young's modulus of the material of the beam is given by, $Y = \frac{4gL^3}{bd^3} \left(\frac{M}{S} \right)_{avg}$

The quantities L, S, b, d, h are measured in this experiment. The maximum proportional error in Y is given by,

$$\frac{\delta Y}{Y} = \frac{3\delta L}{L} + \frac{\delta b}{b} + \frac{3\delta d}{d} + \frac{\delta S}{S}$$

Where, δL = least count of meter scale

δb = least count of vernier caliper

δd = least count of screw gauge

$\delta h = 2 \times \text{least count of travelling microscope}$ (since h is measured by taking difference in microscope readings)

Result: Young's modulus of the material of the beam = $Y \pm \delta Y$