Experiment: 5

TORSIONAL PENDULUM

<u>AIM</u>: To determine the rigidity modulus of the material of the given wire and moment of inertia of the irregular body torsional oscillations method.

<u>APPARATUS</u>: screw gauge, steel wire, rectangular plate, circular disc, irregular body, chuck nuts, stop clock, meter scale

<u>INTRODUCTION</u>: A rigid body suspended from one end of a wire whose other end is fixed to the rigid support is called a torsional pendulum. The rigid body (rod or disc) is turned in its own plane to twist the wire (within its elastic limit), so that, on being released, it executes torsional oscillations about the wire as axis. The angular acceleration of the disc is proportional to its angular displacement, and therefore, its motion is simple harmonic. Hence its time period is given by,

$$T = 2\pi\sqrt{\frac{I}{C}} \qquad \text{…………..} \ [1]$$

Where I = moment of inertia of the suspended body about the wire

C = couple per unit twist of the wire

But we have an expression for twisting couple per unit twist of the cylinder (wire) as,

$$C = \frac{\pi n r^4}{2l} \qquad \text{………….} \ [2]$$

Where l = length of the suspension wire; r = radius of the wire; n = rigidity modulus of the material of the suspension wire

Substituting [2] in [1] and squaring, we get an expression for rigidity modulus for the material of the suspension wire as,

$$n = \frac{8\pi l}{r^4}\left(\frac{I}{T^2}\right)_{avg} N/m^2$$

Hooke's law is the fundamental law of elasticity. It states that "when a material is within the elastic limit, the stress is proportional to the strain". In such a case, the ratio stress/strain is a constant, called **modulus of elasticity.**

Corresponding to the three types of strain, we have three types of elastic constants, viz., i) **Young's modulus**, corresponding to linear (or tensile) strain ii) **Bulk modulus** , corresponding to volume strain iii) **Rigidity modulus**, corresponding to shear strain

When the material is within the elastic limit, the Modulus of rigidity (n) is defined as the ratio of shearing stress to the shearing strain.

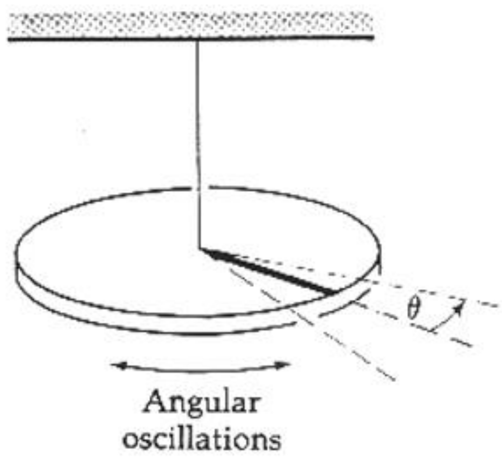
Rigidity modulus (n) = $\frac{\text{Shearing stress}}{\text{Shearing strain}}$

In this case, while there is a change in the shape of the body, there is no change in its volume. It takes place by the movement of contiguous layers of the body, one over the other.

Moment of Inertia: For linear or translational motion, an object's resistance to a change in its state of motion is called its inertia and it is measured in terms of its mass (kg). When a rigid, extended body is rotated, its resistance to a change in its state of rotation is called its rotational inertia, or moment of inertia (I). Moment of inertia of a body, in the case of rotational motion plays the same role as that of the mass of the body in the case of translatory motion. Moment of inertia depends on i) the amount of mass present in the body ii) the distribution of that mass about the chosen axis of rotation. Moment of inertia of the same body is different for different axes of rotation. Moment of inertia of a collection of discrete masses about an arbitrary axis of rotation is I = $\sum_{i=1}^{n} m_i r_i^2$ Where, r is the perpendicular distance from the axis of rotation to each mass.

Moment of inertia is the mass property of a rigid body that determines the torque (couple) needed for a desired angular acceleration about an axis of rotation. If the moment of inertia of a body about an axis of rotation is large, then more torque is required to be applied to change its state of rotation.

FIGURE:



FORMULA:

1. $\eta = \frac{8\pi l}{r^4}\left(\frac{I}{T^2}\right)_{avg}$

Where η is the rigidity modulus of the material of the wire in Nm^{-2}
l is the length of the pendulum wire in m
r is the radius of the wire in m
I is the moment of inertia of the body in kgm^2
T is the time period of oscillation in s

2. Moment of Inertia of the irregular body about an axis perpendicular to its plane

$$I_1 = \left(\frac{I}{T^2}\right)_{avg} \times T_1^2 \ kgm^2$$

Where T_1 is the time period of oscillation for the given irregular body in s

TABULAR COLUMN:

1. To find radius of the given material wire

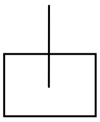
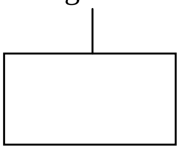
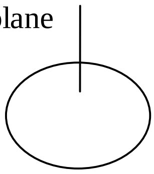
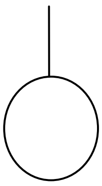
Least count of screw gauge (LC) = $\frac{\text{Pitch}}{\text{Total number of head scale divisions}}$ = 0.01 mm

Zero error (ZE) =……. div
Zero correction (ZC) =………div

Trial no.	PSR (mm)	HSD (div)	Total reading = PSR + (HSR±ZC)x LC (mm)
1			
2			
3			

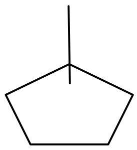
Mean diameter of the wire = d= ………mm
Radius of the wire r = d/2 = ………..mm = …….m

2. To determine the rigidity modulus of the material of the wire

Body	Axis	Time for 10 oscillations 't' (s)			Time Period T= t/10 (s)	Moment of Inertia (kgm^2) I	I/T^2 (kgm^2s^{-2})
		Trail 1	Trail 2	Mean			
Rectangular Plate Mass(M) = ……….kg Length (L) = ………. m Breadth(B)= ……….m	Perpendicular to its plane 					$\frac{M(L^2+B^2)}{12}$	
	Along the plane - perpendicular to length 					$\frac{ML^2}{12}$	
Circular disc Mass (M) = …….. kg Radius (R)= ……...m	Perpendicular to plane 					$\frac{MR^2}{2}$	
	Along the plane 					$\frac{MR^2}{4}$	

$\left(\frac{I}{T^2}\right)_{avg}$ = ……….. kg m^2s^{-2}

3. To find moment of inertia of irregular body

Body	Axis	Time for 10 oscillations 't' (s)			Time period T_1= t/10 (s)	Moment of Inertia about an axis perpendicular to its plane (kgm^2)
		Trail 1	Trail 2	Mean		
Irregular body	Perpendicular to its plane 					$I_1 = \left(\frac{I}{T^2}\right)_{avg} \times T_1^2$ = ………………

PROCEDURE:

Part A: To determine the rigidity modulus of the material of the given wire

1. A rectangular body is set in to torsional oscillations about an axis perpendicular to its plane. Time taken for 10 oscillations is noted and period is evaluated.
2. This activity is repeated by suspending it along the plane.
3. A similar activity is performed with a circular plate first by fixing the wire perpendicular to its plane and then by fixing the wire parallel to its plane and each time the period is found.
4. The dimensions of the plates such as mass, length, breadth, diameter, dimensions of the wire such as radius, length are recorded.
5. The moment of Inertia of both the plates about various axes are determined using the appropriate formula given in the tabular column and in each case I/T^2 is found out.
6. Rigidity modulus of the material of the wire is calculated using the formula

Part B: To determine the Moment of Inertia of the irregular body

1. The given irregular body is set into torsional oscillations about an axis perpendicular to its plane and the period of oscillation is found out.
2. The moment of inertia is calculated using the formula.

RESULT:

1. The rigidity modulus of the given material of the wire is η = …… N/m^2
2. Moment of Inertia of the given irregular body about an axis perpendicular to its plane = …… kgm^2

PROPORTIONAL ERROR CALCULATION:

For circular body moment of inertia about an axis perpendicular to the plane is given by

$$I = \frac{MR^2}{2}$$

Proportional error in determining I is, $\frac{\delta I}{I} = \frac{2\delta R}{R}$

Where,

δR is the least count of the meter scale and R is the radius of the circular plate.

Since rigidity modulus is calculated using $\eta = \frac{8\pi l}{r^4}\left(\frac{I}{T^2}\right)_{avg} \; N/m^2$

Proportional error in determining η is

$$\frac{\delta\eta}{\eta} = \frac{\delta l}{l} + \frac{4\delta r}{r} + \frac{\delta I}{I} + \frac{2\delta T}{T}$$

Where, δr is least count of screw gauge,
δl is least count of meter scale,
δT is least count of stop clock.

Result: The rigidity modulus of the given material of the wire $= \eta \pm \delta\eta$
= ……………

References:

1.Text book of physics-Duncan-Macmillan publn ,Page no 133
2.Properties of Matter –Mathur –Shyamlal Publn-Page no 44