Application Of Bilevel Programming To Model Predictive Control

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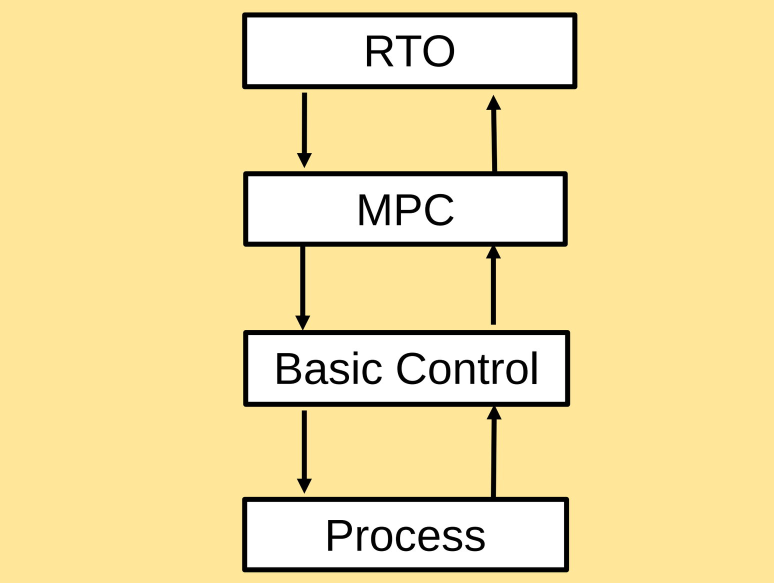
1. Background

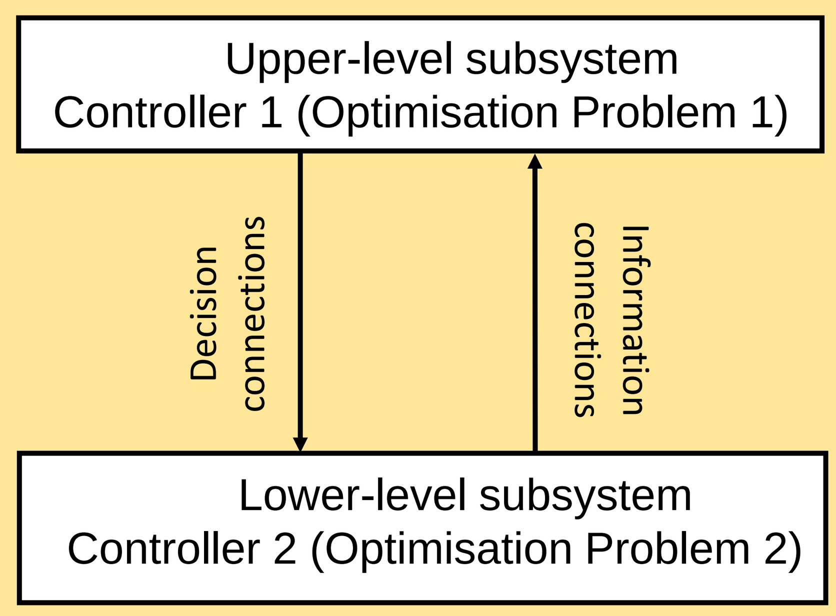
- In any large-scale system there exist many levels in the ***process control hierarchy***. The simplest hierarchical system is **the *Bilevel decision-making structure.***
- **Hierarchy** is characterized by *vertical decomposition*, *priority of action,* and *performance interdependence.*
- Earlier such problems were solved independently using the multi-objective technique.

Multilevel Structure



Process Control Hierarchy

Bilevel MPC Control Structure



Bilevel decision-making structure

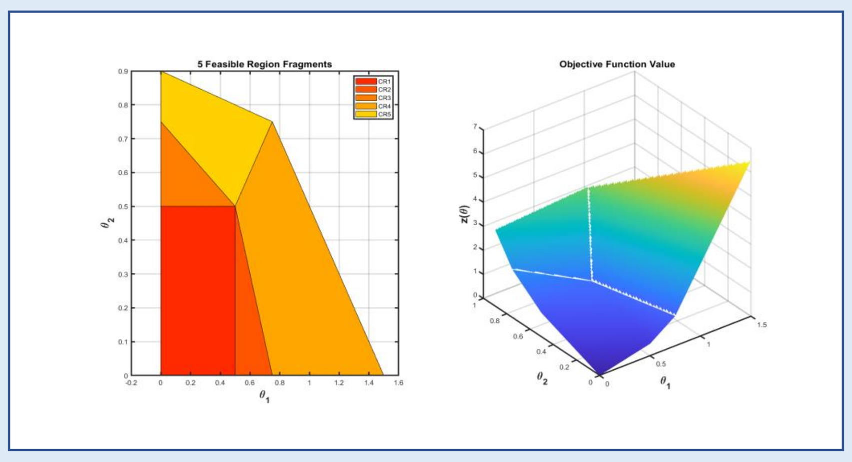
2. Motivation

- Sparse attempts have been made to solve **hierarchical control problems** using the multi-parametric technique.

Multi-parametric Bilevel Programming Algorithm used in the proposed work to **solve Bilevel multi-parametric/explicit MPC controller (Bilevel mp-MPC)** guarantees a **global optimal solution.**

Multi-parametric Bilevel Programming Algorithm

$$\min_{x,y} F(x,y) = -8x_1 - 4x_2 + 4y_1 - 40y_2 + 4y_3$$
$$\text{s.t.} \min_{y} f(x,y) = x_1 + 2x_2 + y_1 + y_2 + 2y_3$$
$$s.t.\ -y_1 + y_2 + y_3 \le 1$$
$$2x_1 - y_1 + 2y_2 - 0.5y_3 \le 1$$
$$2x_2 + 2y_1 - y_2 - 0.5y_3 \le 1$$
$$y \ge 0$$
$$x \ge 0$$



Five critical regions are obtained with y as an explicit function of x.

Step1:	Step 2:	Step3:	Step 4:
Inner Level solved multi-parametrically	*Substituting parametric solution in the outer level y*=f(x).*	*Solving five LP outer level problem*	*Substitution of optimized variables to get solutions to inner-level problems.*

3. Methodology

- *System considered:* **Temperature Control Laboratory (TCLab)**
- *Steps Taken:*
  - Modeling the system.
  - Reducing model equations to **linear state-space equations**.
  - Formulating the problem as ***Linear MPC.***
  - Implementing *MPC and mp-MPC* algorithms on the system and comparing **runtime operation**.

Runtime operation reduced by 83.52%.

  - Designing of ***Bilevel Control Structures*** and solving them using multi-parametric bilevel programming algorithm.
  - Performing closed-loop validation.

Linear Model Predictive Control on TCLab

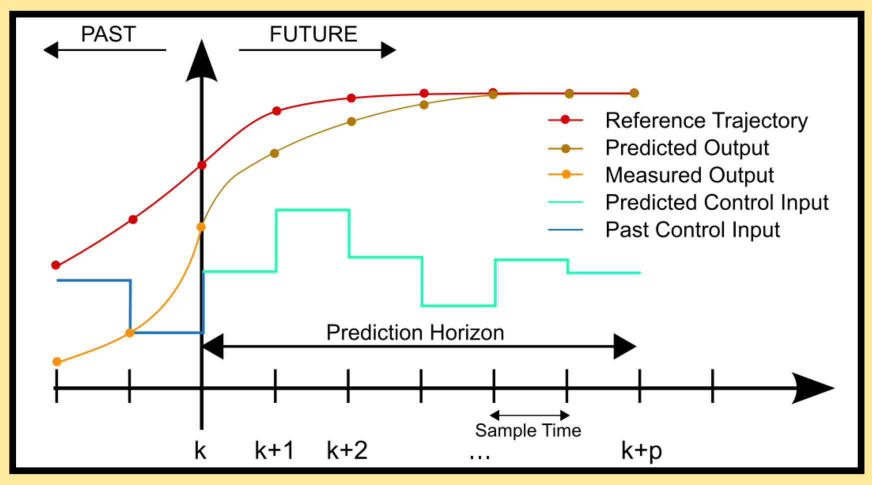
Objective function

$$\min_{u_1,u_2} \Sigma\left(y_k - y_k^R\right)^T Q_k\left(y_k - y_k^R\right) + \Sigma\left(u_k - u_k^R\right)^T R_k\left(u_k - u_k^R\right) + \Sigma \Delta u_k^T R1_k(\Delta u_k)$$

s.t.

$$\left.\begin{aligned} x_{k+1} &= Ax_k + Bu_K \\ y_k &= Dx_k + Eu_k \end{aligned}\right\} \text{System Model}$$

$$\left.\begin{aligned} u_{min} &\le u_k \le u_{max} \\ y_{min} &\le y_k \le y_{max} \end{aligned}\right\} \text{Constraints}$$



MPC solves an optimization problem at each k^{th} time step.

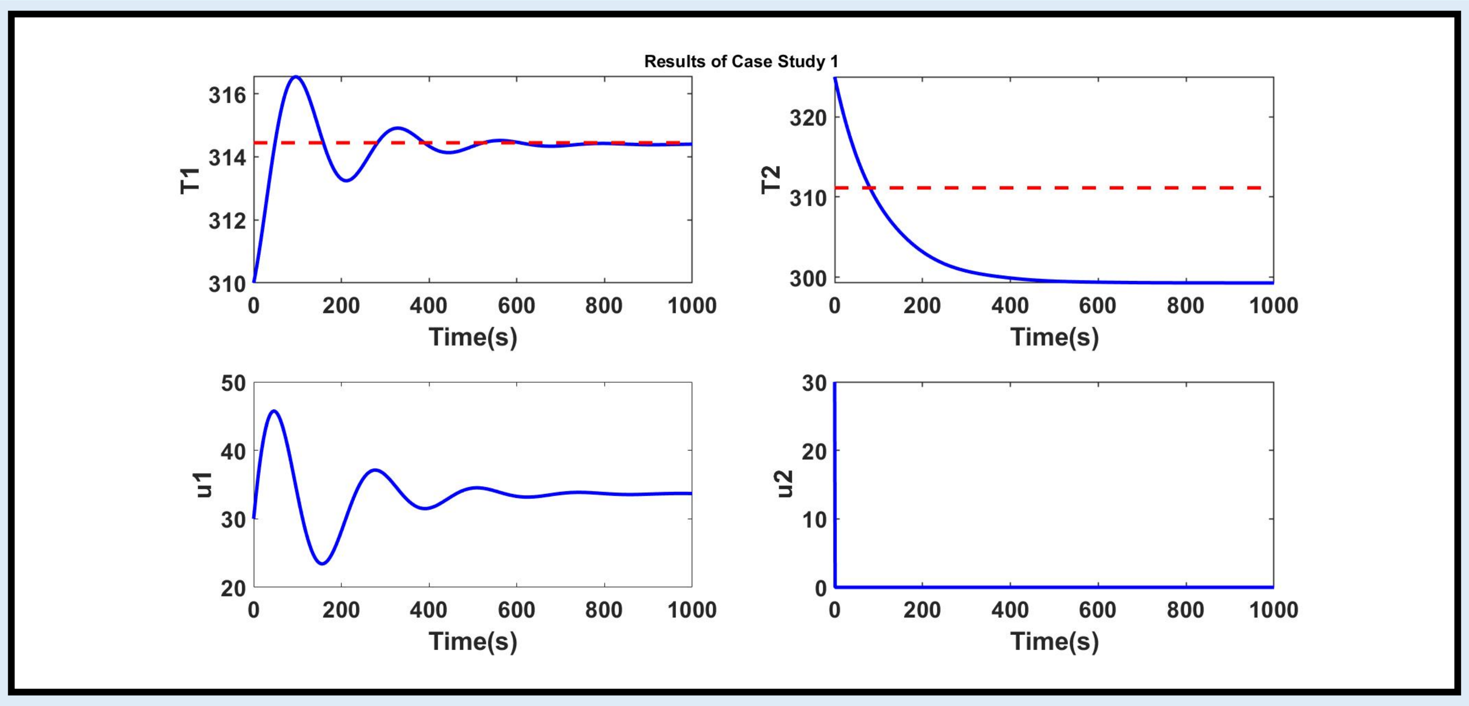
Comparison based on runtime operation.

No.	*Type of MPC*	*Total runtime (s)*
1	Online MPC	5.351
2	mp-MPC	0.912

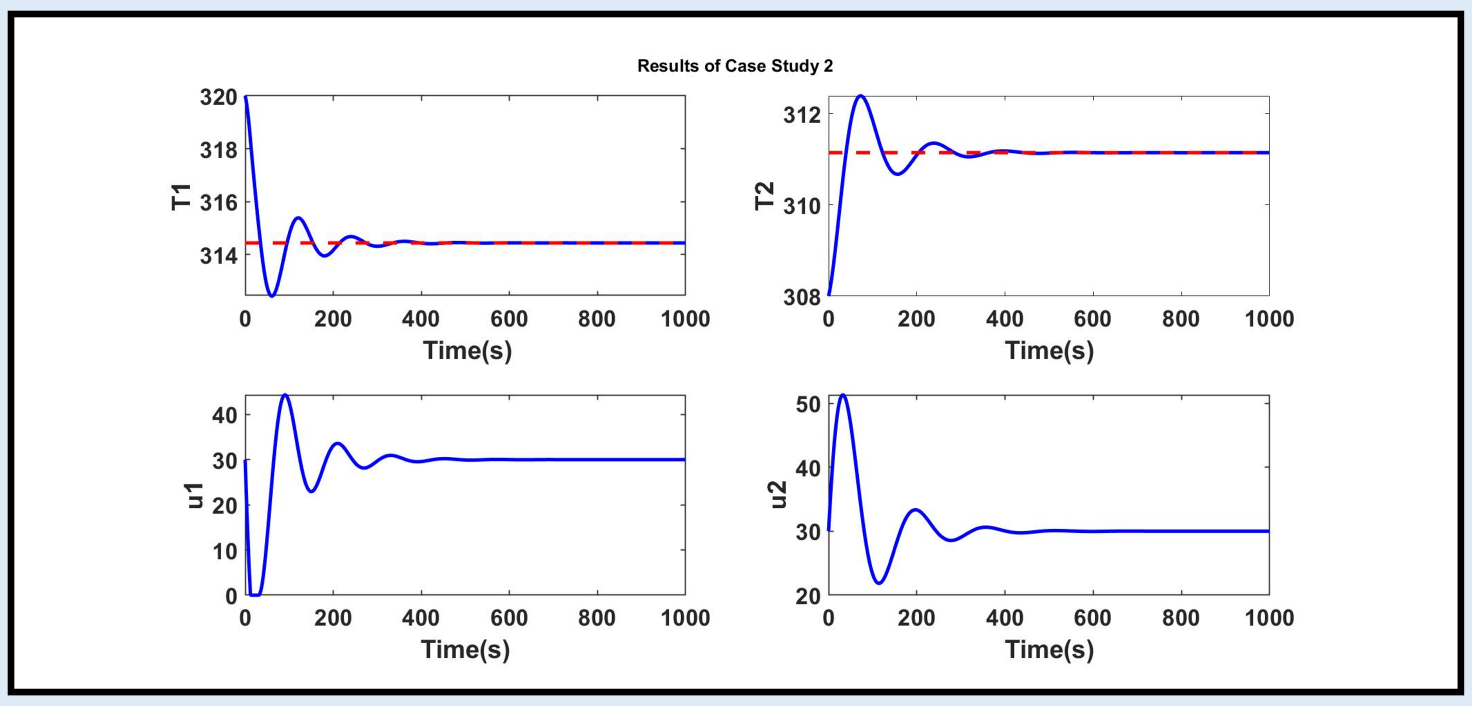
4. Results

Bilevel MPC Control Structures

$$\min_{u_2} u_1 + u_2$$
$$\min_{u_1} \left(T_1 - T_1^R\right)^T Q_1\left(T_1 - T_1^R\right) + \left(u_1 - u_1^R\right)^T R_1\left(u_1 - u_1^R\right) + \Delta u_1^T R1_1 \Delta u_1$$



$$\min_{u_1} \left(T_1 - T_1^R\right)^T Q_1\left(T_1 - T_1^R\right) + \left(u_1 - u_1^R\right)^T R_1\left(u_1 - u_1^R\right) + \Delta u_1^T R1_1 \Delta u_1$$
$$\min_{u_2} \left(T_2 - T_2^R\right)^T Q_2\left(T_2 - T_2^R\right) + \left(u_2 - u_2^R\right)^T R_2\left(u_2 - u_2^R\right) + \Delta u_2^T R1_2 \Delta u_2$$



5. Conclusion

- Bilevel mp-MPC framework applied to TCLab manage to follow the temperature set-point while optimizing an upper-level objective function.