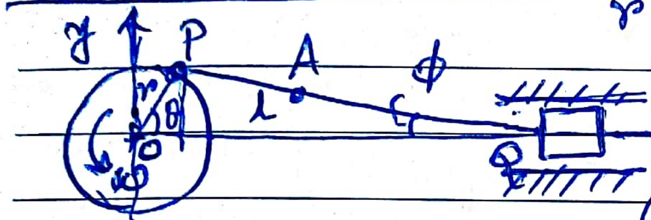
Slider Crank mechanism: ω = Angular velo. of crank
r = Crank radius



Assume, $l >> r$.
To find:–
(1) Displacement–time rel^n for motion of piston
(2) Max^m Velo. & Acc^n of piston
(3) Motion of point A.

Here, $\theta = \omega t$

$$x = r\cos\theta + l\cos\phi \quad ---(1)$$

from ΔOPQ:– $\frac{\sin\phi}{r} = \frac{\sin\theta}{l}$ or, $\sin\phi = \frac{r}{l}\sin\theta$.

or, $\cos\phi = \sqrt{1 - \frac{r^2}{l^2}\sin^2\theta}$

So, $x = r\cos\theta + l\left[1 - \frac{r^2}{l^2}\sin^2\theta\right]^{\frac{1}{2}}$

As $l >> r$, $\frac{r}{l} \approx 0$.

$$x = r\cos\theta + l = r\cos\omega t + l.$$

$$v = \dot{x} = -r\omega\sin\omega t$$

$$a = \ddot{x} = \dot{v} = -r\omega^2\cos\omega t$$

Velocity will be max^m when $\theta = \omega t = 90^\circ$

Acceleration will be max^m when $\theta = \omega t = 0^\circ$

Let point A is p distance away from point P

$$x_A = r\cos\theta + p\cos\phi$$
$$= r\cos\omega t + p\left(1 - \frac{r^2}{l^2}\sin^2\omega t\right)^{\frac{1}{2}}$$

$$y_A = (l-p)\sin\phi = (l-p)\frac{r}{l}.\sin\theta$$
$$= \left(\frac{l-p}{l}\right)r\sin\omega t$$

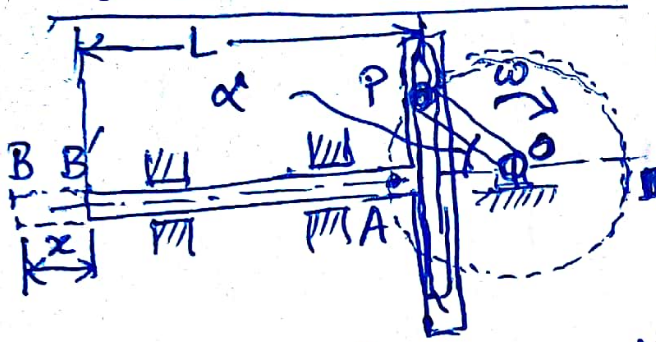
At $l = r$,

$$x_A = (r+p)\cos\omega t$$
$$y_A = (r-p)\sin\omega t$$

$$\frac{x_A^2}{(r+p)^2} + \frac{y_A^2}{(r-p)^2} = \cos^2\omega t + \sin^2\omega t = 1$$

This is the eq^n of ellipse.

Scotch Yoke Mechanism:



ω = Angular velocity of crank (OP)

Find displacement, velocity and Acc^n of Sliding Bar AB.

- At horizontal position, point P and point A become same ie, at $\alpha = 0^\circ$

- At other position, some displacement $BB' = x$

Now, $x = OB - OB' = OB - OP\cos\alpha - AB'$

$$x = OB - OP\cos\alpha - L \quad \text{---(1)}$$

At $\alpha = 0^\circ$, $\quad OB = L + OA = L + OP$

From eq^n (1) $\quad x = OB - OP\cos\alpha - L$

$$= (L + OP) - OP\cos\alpha - L$$

$$= OP(1 - \cos\alpha)$$

Here, length of crank OP can be taken as radius R of rotating circle.

$$x = R(1 - \cos\alpha) = R(1 - \cos\omega t)$$

$$v = \dot{x} = R\omega\sin\omega t$$

$$a = \dot{v} = \ddot{x} = R\omega^2\cos\omega t.$$

Prob A stuntman likes to cross the ditch as shown in figure. Find the max^m velocity reqd at point P. Determine the direction & magnitude of velocity of the stuntman just at the instant of clearing ditch.

Let, V = Velo. at point P

ie, $Vt = 7$ and $\frac{1}{2}gt^2 = 4$

or, $\frac{1}{2}g\left(\frac{7}{V}\right)^2 = 4$

or, $V = 7.75$ m/s $= V_x$ and $V_y = 0$

At clearing the ditch,

$$V_y = \sqrt{2g\times4} = \sqrt{2\times9.81\times4} = 8.858 \text{ m}$$

At point Q,

$$V = \sqrt{V_x^2 + V_y^2} = \sqrt{7.75^2 + 8.858^2} = 11.77 \text{ m/s}$$

and $\alpha = \tan^{-1}\left(\frac{V_y}{V_x}\right) = 48.8^\circ$

Prob Show that curvature of plane curve at any point can be expressed as.

$$\frac{1}{r} = \frac{\dot{x}\ddot{y} - \ddot{x}\dot{y}}{\left(\dot{x}^2 + \dot{y}^2\right)^{3/2}}$$

We know,

$$\frac{1}{r} = \frac{\frac{d^2y}{dx^2}}{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}} \qquad \text{—— (1)}$$

$$\frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx} = \frac{(dy/dt)}{(dx/dt)} = \frac{\dot{y}}{\dot{x}}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right)\cdot\frac{dt}{dx} = \frac{d}{dt}\left(\frac{\dot{y}}{\dot{x}}\right)\frac{1}{\dot{x}}$$

$$= \left(\frac{\ddot{y}\dot{x} - \dot{y}\ddot{x}}{\dot{x}^2}\right) \times \frac{1}{\dot{x}}$$

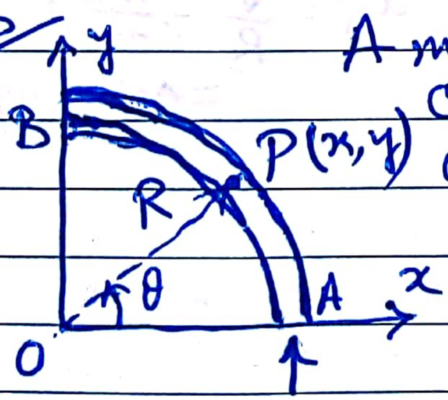
$$= \frac{\dot{x}\ddot{y} - \ddot{x}\dot{y}}{\dot{x}^3}$$

Substituting in eq^n (1)

$$\frac{1}{r} = \frac{\dot{x}\ddot{y} - \ddot{x}\dot{y}}{\dot{x}^3} \times \frac{1}{\left(1 + \frac{\dot{y}^2}{\dot{x}^2}\right)^{3/2}}$$

$$= \frac{\dot{x}\ddot{y} - \ddot{x}\dot{y}}{\left(\dot{x}^2 + \dot{y}^2\right)^{3/2}}$$

Prob



A motorcycle enters a path AB curved in a quadrant of a circle of radius 230 m. When it enters at a point A, it was moving in 40 Km/hr speed and maintains throughout the path. Determine the components of velocity and acceleration of the motorcycle along x and y directions after 17sec of passing through A. Express the velocity and acceleration in terms of n-t co-ordinate system.

Linear velocity of motorcycle $V = 40$ Km/hr.

$$= \frac{40 \times 1000}{3600}$$

$$= 11.11 \text{ m/s}.$$

Angular velocity

$$\omega = \frac{V}{R} = \frac{11.11}{230} = 0.0483 \text{ rad/s}.$$

Let's assume after 't' sec the motorcycle reaches at point 'P' subtending angle $\theta = \omega t$

$$x = R\cos\theta = R\cos\omega t$$

$$y = R\sin\theta = R\sin\omega t$$

$\dot{x} = -R\omega\sin\omega t$ $\qquad$ $\dot{y} = R\omega\cos\omega t$

$\ddot{x} = -R\omega^2\cos\omega t$ $\qquad$ $\ddot{y} = -R\omega^2\sin\omega t$

Putting $R = 230$, $\omega = 0.0483$ rad/s, $t = 17$ sec we get.

$\dot{x} = -0.1592$ m/s $\qquad$ $\dot{y} = 11.1078$ m/s.

$\ddot{x} = -0.5365$ m/s^2 $\qquad$ $\ddot{y} = -7.689 \times 10^{-3}$ m/s^2

In n-t co-ordinate system

$V_t = v = 11.11$ m/s $\quad|\quad a_t = \frac{dv}{dt} = 0$

$V_n = 0.$ $\quad|\quad a_n = \frac{V^2}{R} = \frac{11.11^2}{230}$

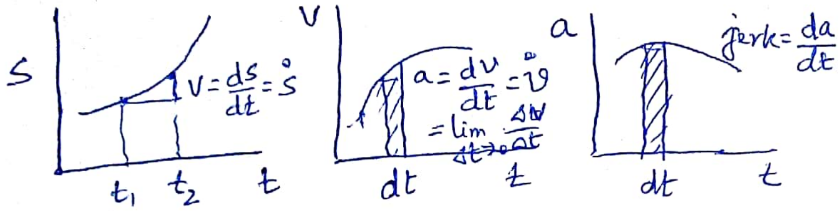
$$= 0.536 \text{ m/s}^2$$

$\left(\frac{d}{dt}\right)\uparrow$ $\qquad$ $\downarrow \int dt$

Pounce (P)
flounce (C)
jounce (S)
jerk (j)
Acc^n (a)
velo (v)
displ. (x)
absement (A)
absity (B)
abselération (C)
abserk (D)

Power (P)
workdone (W)
Actergy (X)

Rectilinear Motion:



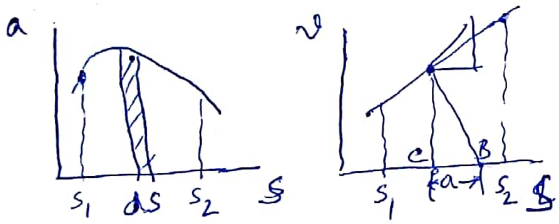
$$v = \frac{ds}{dt} = \dot{s}$$

$$a = \frac{dv}{dt} = \dot{v} = \ddot{s}$$

$$= \frac{d^2s}{dt^2} = \frac{dv}{dt} = \frac{dv}{ds}\cdot\frac{ds}{dt} = v\frac{dv}{ds}$$

$$v\,dv = a\,ds$$

$$\dot{s}\,d\dot{s} = \ddot{s}\,ds$$



$$\boxed{\frac{ds}{dt} = v}$$

$$ds = v\,dt$$

$$\int_{s_1}^{s_2} ds = \int_{t_1}^{t_2} v\,dt$$

$(s_2 - s_1)$ = Area Under v-t curve

$$\boxed{\frac{dv}{dt} = a}$$

$$dv = a\,dt$$

$$\int_{v_1}^{v_2} dv = \int_{t_1}^{t_2} a\,dt$$

$(v_2 - v_1)$ = Area Under a-t curve

$$\boxed{v\,dv = a\,ds}$$

$$\int_{v_1}^{v_2} v\,dv = \int_{s_1}^{s_2} a\,ds$$

$\frac{1}{2}(v_2^2 - v_1^2)$ = Area Under a-s curve

Case I : Constant Acc^n. (a = Const)

$$\boxed{\frac{dv}{dt} = a}$$

$$dv = a\,dt$$

$$\int_{v_0}^{v} dv = a\int_0^t dt$$

$$v - v_0 = at$$

$$\boxed{v = v_0 + at}$$

$$\boxed{v\,dv = a\,ds}$$

$$\int_{v_0}^{v} v\,dv = \int_{s_0}^{s} a\,ds$$

$$\frac{1}{2}(v^2 - v_0^2) = a(s - s_0)$$

$$\boxed{v^2 = v_0^2 + 2a(s - s_0)}$$

$$\boxed{\frac{ds}{dt} = v_0 + at}$$

$$ds = (v_0 + at)\,dt$$

$$\int_{s_0}^{s} ds = \int_0^t (v_0 + at)\,dt$$

$$(s - s_0) = v_0t + \frac{1}{2}at^2$$

$$\boxed{s = s_0 + v_0t + \frac{1}{2}at^2}$$

Case II : a = f(t)

$$\boxed{f(t) = \frac{dv}{dt}}$$

$$dv = f(t)\cdot dt$$

$$\int_{v_0}^{v} dv = \int_0^t f(t)\,dt$$

$$v - v_0 = \int_0^t f(t)\,dt$$

$$\boxed{v = v_0 + \int_0^t f(t)\,dt}$$

$$\boxed{\frac{ds}{dt} = v}$$

$$ds = v\,dt$$

$$\int_{s_0}^{s} ds = \int_0^t v\,dt$$

$$s - s_0 = \int_0^t v\,dt$$

$$\boxed{s = s_0 + \int_0^t v\,dt}$$

$$\boxed{\begin{aligned} s &= ut + \tfrac{1}{2}ft^2 \\ v &= u + ft \\ v^2 &= u^2 + 2fs \end{aligned}}$$

[f = g when gravitational acc^n is considered)

Case III : a = f(v)

$$f(v) = \frac{dv}{dt}$$

$$dt = \frac{dv}{f(v)}$$

$$\boxed{\int_0^t dt = \int_{v_0}^{v} \frac{dv}{f(v)} = t}$$

$$\boxed{v\,dv = a\,ds}$$

$$v\,dv = f(v)\cdot ds$$

$$\frac{v\,dv}{f(v)} = ds$$

$$\int_{v_0}^{v} \frac{v\,dv}{f(v)} = \int_{s_0}^{s} ds$$

$$s - s_0 = \int_{v_0}^{v} \frac{v\,dv}{f(v)}$$

$$\boxed{s = s_0 + \int_{v_0}^{v} \frac{v\,dv}{f(v)}}$$

Case IV : a = f(s)

$$\boxed{v\,dv = a\,ds}$$

$$v\,dv = f(s)\,ds$$

$$\int_{v_0}^{v} v\,dv = \int_{s_0}^{s} f(s)\,ds$$

$$\frac{1}{2}(v^2 - v_0^2) = \int_{s_0}^{s} f(s)\,d$$

$$\boxed{v^2 = v_0^2 + 2\int_{s_0}^{s} f(s)\,ds}$$

If: $v = g(s)$

$$\frac{ds}{dt} = v = g(s)$$

$$\frac{ds}{g(s)} = dt$$

$$\int_{s_0}^{s} \frac{ds}{g(s)} = \int_0^t dt$$

$$t = \int_{s_0}^{s} \frac{ds}{g(s)}$$

Example - SHM

$$x = a\cos\omega t$$

$$\dot{x} = -a\omega\sin\omega t = v$$

$$\ddot{x} = -a\omega^2\cos\omega t = f$$

$$= -\omega^2 x$$

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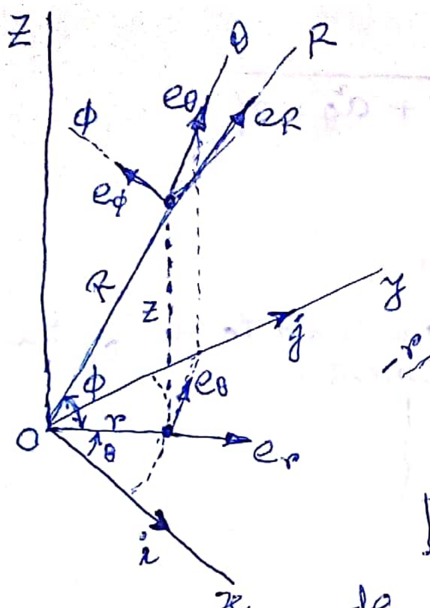
Space Curvilinear Motion: (a) Rectangular Co-ordinate System $(x-y-z)$

$$\vec{R} = x\hat{i} + y\hat{j} + z\hat{k}$$ [R= Position vector]

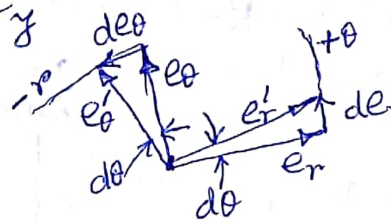
$$V = \dot{R} = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}$$

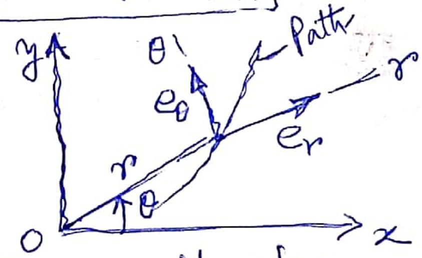
$$a = \dot{V} = \ddot{R} = \ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k}$$



(b) Cylindrical Co-ordinate System $(r-\theta-z)$

$$\boxed{R = r\hat{e}_r + z\hat{k}}$$





$$\boxed{r = r e_r}$$

Here, e_r & e_θ are unit vectors

$$de_r = e_\theta \cdot d\theta$$
$$\frac{de_r}{d\theta} = e_\theta .$$
$$\frac{de_r}{dt} = \left(\frac{d\theta}{dt}\right) \cdot e_\theta$$
$$\dot{e}_r = \dot{\theta} e_\theta .$$

$$de_\theta = -e_r d\theta$$
$$\frac{de_\theta}{d\theta} = -e_r$$
$$\frac{de_\theta}{dt} = -\left(\frac{d\theta}{dt}\right) e_r$$
$$\dot{e}_\theta = -\dot{\theta} e_r .$$

$$V = \dot{r} = \dot{r} e_r + r\dot{e}_r$$

$$\boxed{V = \dot{r} e_r + r\dot{\theta} e_\theta} \quad [\because \dot{e}_r = \dot{\theta} e_\theta] \quad V = V_r\hat{e}_r + V_\theta \cdot \hat{e}_\theta$$

Where, $\left.\begin{array}{l} V_r = \dot{r} \\ V_\theta = r\dot{\theta} \end{array}\right] - V = \sqrt{V_r^2 + V_\theta^2}$

$$a = \dot{V} = (\ddot{r} e_r + \dot{r}\dot{e}_r) + (\dot{r}\dot{\theta} e_\theta + r\ddot{\theta} e_\theta + r\dot{\theta}\dot{e}_\theta)$$

Putting $\dot{e}_r = \dot{\theta} e_\theta$ and $\dot{e}_\theta = -\dot{\theta} e_r$, we get:

$$\boxed{a = (\ddot{r} - r\dot{\theta}^2) e_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta}) e_\theta} \quad \text{ie, } a = a_r\hat{e}_r + a_\theta\hat{e}_\theta$$

where; $\left.\begin{array}{l} a_r = \ddot{r} - r\dot{\theta}^2 \\ a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} \end{array}\right] - a = \sqrt{a_r^2 + a_\theta^2}$

So,
$$R = r e_r + z k$$
$$V = \dot{R} = \dot{r} e_r + r\dot{\theta} e_\theta + \dot{z} k$$

where, $\left.\begin{array}{l} V_r = \dot{r} \\ V_\theta = r\dot{\theta} \\ V_z = \dot{z} \end{array}\right] - V = \sqrt{V_r^2 + V_\theta^2 + V_z^2}$

$$a = \ddot{R} = \dot{V} = (\ddot{r} - r\dot{\theta}^2)e_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})e_\theta + \ddot{z}K$$

$$\left.\begin{aligned} a_r &= \ddot{r} - r\dot{\theta}^2 \\ a_\theta &= r\ddot{\theta} + 2\dot{r}\dot{\theta} \\ a_z &= \ddot{z} \end{aligned}\right] \; a = \sqrt{a_r^2 + a_\theta^2 + a_z^2}$$

(c) <u>Spherical Co-ordinate System</u> $(R-\theta-\phi)$:

When a radial distance and two angles are used to specify the position of a particle as in radar measurement.

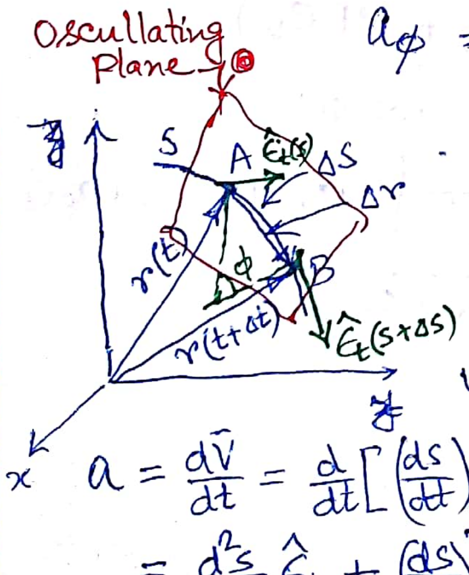
Here, $\boxed{V = v_R\hat{e}_R + v_\theta\hat{e}_\theta + v_\phi\hat{e}_\phi}$

Where, e_R, e_θ and e_ϕ are unit vectors

$$v_R = \dot{R}$$
$$v_\theta = R\dot{\theta}\cos\phi$$
$$v_\phi = R\dot{\phi}$$

Similarly, $\boxed{a = a_R\hat{e}_R + a_\theta\hat{e}_\theta + a_\phi\hat{e}_\phi}$

where,
$$a_R = \ddot{R} - R\dot{\phi}^2 - R\dot{\theta}^2\cos\phi$$
$$a_\theta = \frac{\cos\phi}{R}\frac{d}{dt}(R^2\dot{\theta}) - 2R\dot{\theta}\dot{\phi}\sin\phi$$
$$a_\phi = \frac{1}{R}\frac{d}{dt}(R^2\dot{\phi}) + R\dot{\theta}^2\sin\phi\cos\phi$$



$$V = \frac{d\bar{r}}{dt} = \underset{\Delta t \to 0}{Lt}\frac{r(t+\Delta t) - r(t)}{\Delta t} = \underset{\Delta t \to 0}{Lt}\frac{\Delta r}{\Delta t}$$

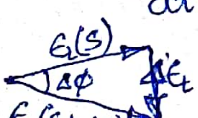
$$= \underset{\Delta t \to 0}{Lt}\left[\left(\frac{\Delta r}{\Delta s}\right)\left(\frac{\Delta s}{\Delta t}\right)\right] = \left(\frac{ds}{dt}\right)\hat{\epsilon}_t$$

Where $\hat{\epsilon}_t$ = Unit vector with dir^n along tangent; $\frac{ds}{dt}$ = Speed

$$a = \frac{d\bar{v}}{dt} = \frac{d}{dt}\left[\left(\frac{ds}{dt}\right)\hat{\epsilon}_t\right] = \frac{d^2s}{dt^2}\hat{\epsilon}_t + \left(\frac{ds}{dt}\right)\left(\frac{d\hat{\epsilon}_t}{dt}\right) = \frac{d^2s}{dt^2}\hat{\epsilon}_t + \left(\frac{ds}{dt}\right)\left(\frac{ds}{dt}\right)\left(\frac{d\hat{\epsilon}_t}{ds}\right)$$

$$= \frac{d^2s}{dt^2}\hat{\epsilon}_t + \left(\frac{ds}{dt}\right)^2\left(\frac{d\hat{\epsilon}_t}{ds}\right) \cdots\cdots a = \left(\frac{d^2s}{dt^2}\right)\hat{\epsilon}_t + \left(\frac{ds}{dt}\right)^2\frac{1}{R}\hat{\epsilon}_n$$

$R = \frac{[1+y_1^2]^{3/2}}{y_2}$ = Radius of curvature



$$\frac{d\epsilon_t}{ds} = \underset{\Delta s \to 0}{Lt}\frac{\epsilon_t(s+\Delta s) - \epsilon_t(s)}{\Delta s} = \underset{\Delta s \to 0}{Lt}\frac{\Delta\epsilon_t}{\Delta S} = \underset{\Delta s \to 0}{Lt}\frac{\Delta S/R\,\epsilon_n}{\Delta S} = \frac{\epsilon_n}{R}$$

Now $\Delta\epsilon_t \simeq |\epsilon_t|\Delta\phi = \Delta\phi = \frac{\Delta S}{R}$; $\Delta\phi = \frac{\Delta S}{R}$

Hence, $\epsilon_t = \frac{\Delta S}{R}\epsilon_n$

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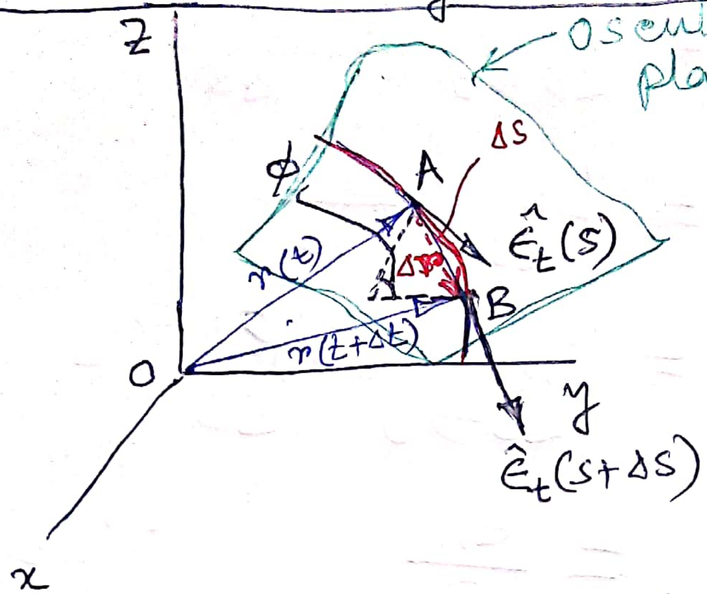
Normal and Tangential Reference System:



If the velocity & accl^n in curvillinear motion along normal and tangential dir^n are needed to express, consider two subsequent points A & B with radius of curvature as 'ρ'

Let, the unit vectors along the tangential & normal dir^n are $\hat{\epsilon}_t$ and $\hat{\epsilon}_n$, respectively.

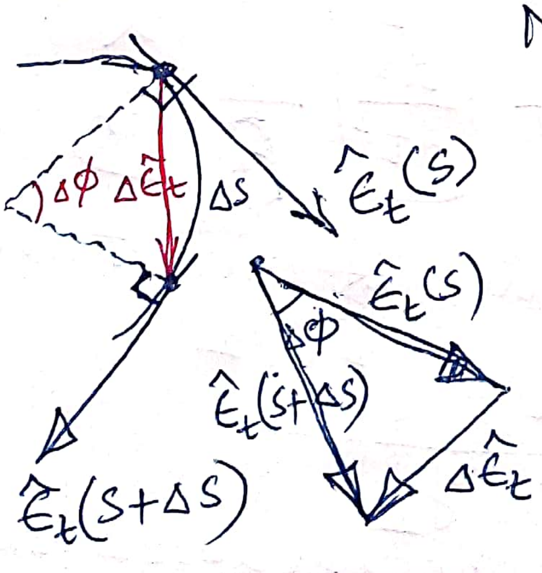
Now, the instanteneous velocity can be expressed as,

$$V = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t} = \frac{dr}{dt} = \frac{dr}{ds}\left(\frac{ds}{dt}\right)$$

$$\lim_{dt \to 0} \frac{r(t+\Delta t) - r(t)}{\Delta t}$$

$$= \underbrace{\left(\frac{ds}{dt}\right)}_{\text{speed}} \cdot \hat{\epsilon}_t$$

In this case, the normal component of velocity is zero ie, $\hat{\epsilon}_n = 0$



Now, the instanteneous acceleration

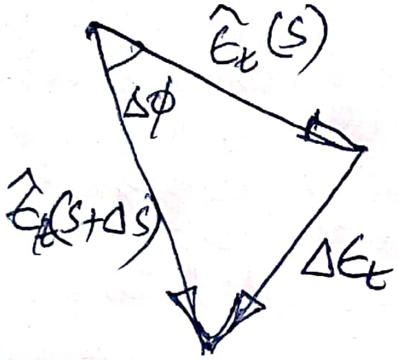
$$a = \frac{dV}{dt} = \frac{d}{dt}\left[\left(\frac{ds}{dt}\right)\hat{\epsilon}_t\right]$$

$$= \frac{d^2 s}{dt^2} \cdot \hat{\epsilon}_t + \left(\frac{ds}{dt}\right)\left(\frac{d\hat{\epsilon}_t}{dt}\right)$$

$$= \frac{d^2 s}{dt^2} \cdot \hat{\epsilon}_t + \left(\frac{ds}{dt}\right)\left(\frac{ds}{dt}\right)\left(\frac{d\hat{\epsilon}_t}{ds}\right)$$

$$= \left(\frac{d^2 s}{dt^2}\right)\hat{\epsilon}_t + \left(\frac{ds}{dt}\right)^2 \cdot \left(\frac{d\hat{\epsilon}_t}{ds}\right)$$

Now,



$$\frac{d\epsilon_t}{ds} = \lim_{\Delta s \to 0} \frac{\epsilon_t(s+\Delta s) - \epsilon_t(s)}{\Delta s}$$

$$= \lim_{\Delta s \to 0} \frac{\Delta \epsilon_t}{\Delta s}$$

$$= \lim_{\Delta s \to 0} \frac{(\Delta s/\rho)\, \epsilon_n}{\Delta s}$$

[Putting the value of eq^n (A)]

$$= \frac{\epsilon_n}{\rho}$$

Now,

$$\Delta \epsilon_t \simeq |\epsilon_t| \Delta\phi$$

$$= \Delta\phi$$

$$= \frac{\Delta s}{\rho}$$

Hence, $$\hat{\epsilon}_t = \frac{\Delta s}{\rho} \cdot \hat{\epsilon}_n \qquad \text{(A)}$$

Thus,

$$a = \left(\frac{d^2 s}{dt^2}\right) \cdot \hat{\epsilon}_t + \left(\frac{ds}{dt}\right)^2 \cdot \left(\frac{d\epsilon_t}{ds}\right)$$

$$= \underbrace{\left(\frac{d^2 s}{dt^2}\right) \hat{\epsilon}_t}_{a_t} + \underbrace{\left(\frac{d^2 s}{dt^2}\right) \frac{1}{\rho} \cdot \hat{\epsilon}_n}_{a_n}$$

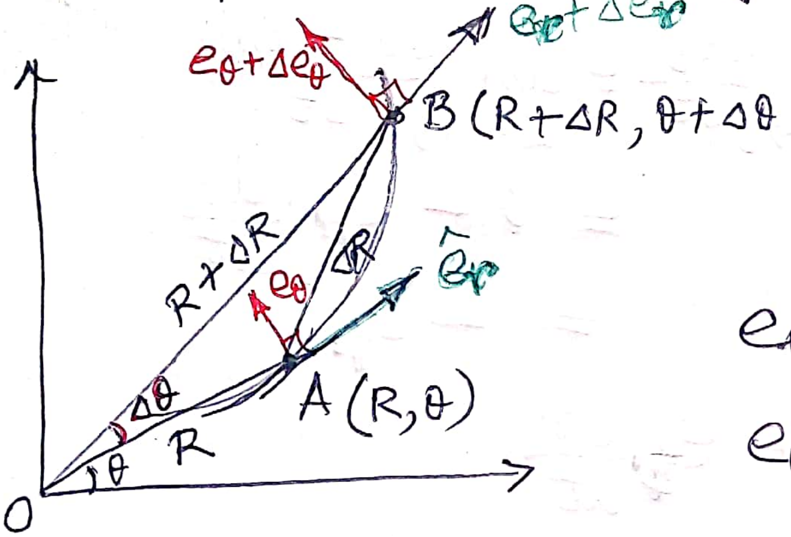
Where, ρ = Radius of curvature

$$= \frac{(1+y_1^2)^{3/2}}{y_2}$$

a_n and a_t are the normal acceleration and Tangential acceleration components, respectively.

Velocity and Acceleration in Polar reference System :

In polar coordinate system, initial position of a particle at A (R, θ) and after time (Δt) it reaches to point B $(R+\Delta R, \theta+\Delta\theta)$



$e_t \rightarrow$ Unit vector at radial dir^n

$e_\theta \rightarrow$ Unit vector at θ-dir^n.

From geometry, we can write

$$\boxed{e_r = \Delta\theta \, e_\theta}. \quad \text{and} \quad \boxed{\Delta e_\theta = \Delta\theta(-e_r)}$$

The position vector of points A and B are.

At point A,

$$R = (r e_r)$$

At point B,

$$R + \Delta R = (r + \Delta r)(e_r + \Delta e_r)$$

$$\text{or, } R + \Delta R = r e_r + r \Delta e_r + \Delta r \, e_r + \cancel{\Delta r \Delta e_t}$$

(as very small)

$$\text{or, } R + \Delta R = (r e_r) + r \Delta e_r + e_t \Delta r$$

$$\text{or, } \cancel{R} + \Delta R = \cancel{R} + r \Delta e_r + e_r \Delta r$$

$$\text{or, } \boxed{\Delta R = r \Delta e_r + e_r \Delta r}$$

The instanteneous velocity

$$V = \lim_{\Delta t \to 0} \frac{\Delta R}{\Delta t} = \lim_{\Delta t \to 0} \frac{r \Delta e_r + e_r \Delta r}{\Delta t} = r \frac{de_r}{dt} + e_r \frac{dr}{dt}$$

Now,

$$\dot{e}_r = \frac{de_r}{dt} = \lim_{\Delta t \to 0} \frac{\Delta e_r}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta \theta\, e_\theta}{\Delta t} = e_\theta \frac{d\theta}{dt}$$

$$= e_\theta \cdot \dot{\theta}$$

$$\dot{e}_\theta = \frac{de_\theta}{dt} = \lim_{\Delta t \to 0} \frac{\Delta e_\theta}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta \theta(-e_r)}{\Delta t}$$

$$= \cancel{\theta} - e_r \cdot \frac{d\theta}{dt}$$

$$= -e_r \cdot \dot{\theta}$$

Finally,

velocity

$$V = r\frac{de_r}{dt} + e_r \frac{dr}{dt}$$

$$= \underbrace{r\dot{\theta}e_\theta}_{v_\theta} \cdot + \underbrace{\dot{r}\, e_r}_{v_r}$$

Acceleration

$$a = \frac{dV}{dt} = \frac{d}{dt}(r\dot{\theta}e_\theta + \dot{r}e_r)$$

$$= \dot{r}e_\theta\dot{\theta} + r(\dot{e}_\theta)\dot{\theta} + re_\theta \cdot \ddot{\theta} + \ddot{r}e_r + \dot{r}\dot{e}_r$$

$$= \dot{r}e_\theta\dot{\theta} + r\dot{\theta}(-e_r\dot{\theta}) + re_\theta \cdot \ddot{\theta} + \ddot{r}e_r + \dot{r}(e_\theta\dot{\theta})$$

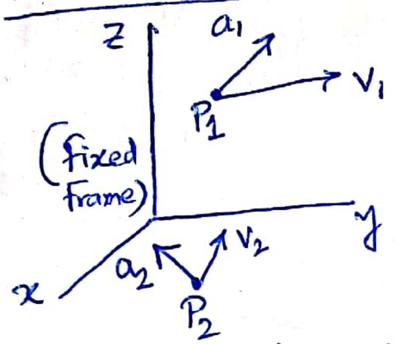
$$= \underbrace{(\ddot{r} - r\dot{\theta}^2)}_{a_r} e_r + \underbrace{(r\ddot{\theta} + 2\dot{r}\dot{\theta})}_{a_\theta} e_\theta$$

Case 1: When rotation is in constant angular speed, Here, $\dot{\theta}$ = constant then, $\ddot{\theta} = 0$

Case 2: When rotation is a plane circular motion, Here, r = constant, ie, $\dot{r} = \ddot{r} = 0$ then $V = V_\theta = (r\dot{\theta})e_\theta$. and $a = a_r + a_\theta$ where, $a_r = -r\dot{\theta}^2 e_r$. and $a_\theta = r\ddot{\theta} e_\theta$.

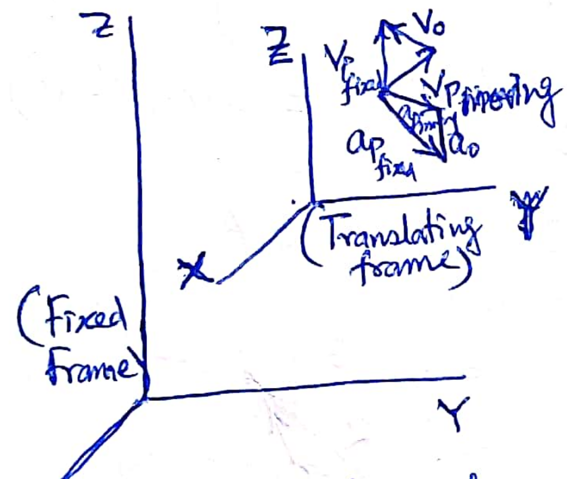
Motion referred to moving frames of reference:

(a) Relative motion of two points:



$$\boxed{\begin{aligned} V_{12} &= V_1 - V_2 \\ a_{12} &= a_1 - a_2 \end{aligned}} \quad \text{and} \quad \boxed{\begin{aligned} V_{21} &= V_2 - V_1 = -V_{12} \\ a_{21} &= a_2 - a_1 = -a_{12} \end{aligned}}$$

(b) Translation of moving frame:

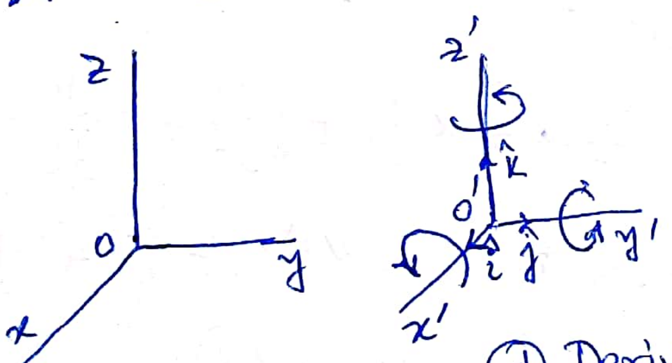


V_o = Velocity of moving frame
a_o = Accl^n of " "

$$\boxed{\begin{aligned} V_{P_{fixed}} &= V_{P_{moving}} + V_o \\ a_{P_{fixed}} &= a_{P_{moving}} + a_o \end{aligned}}$$

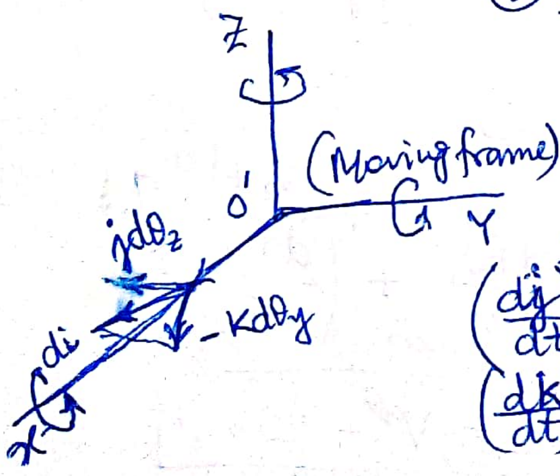
(c) Rotation of a moving frame:

Let, $d\theta_x, d\theta_y, d\theta_z \rightarrow$ Infinitesimal rotation of moving frame



$$\omega = \omega_x \hat{i} + \omega_y \hat{j} + \omega_z \hat{k}$$
$$= \frac{d\theta_x}{dt}\hat{i} + \frac{d\theta_y}{dt}\hat{j} + \frac{d\theta_z}{dt}\hat{k}$$

(d) Derivatives of moving unit vector:

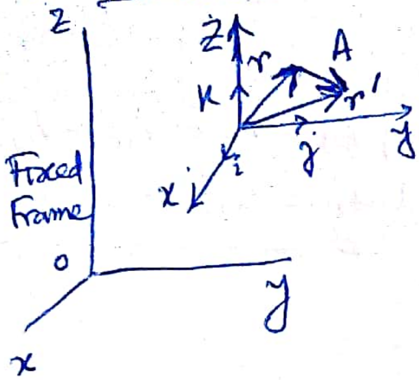


$$\left(\frac{di}{dt}\right)_f = j\frac{d\theta_z}{dt} - k\frac{d\theta_y}{dt}$$
$$= j\omega_z - k\omega_y$$
$$= \omega \times i$$

Similarly,

$$\left(\frac{dj}{dt}\right)_f = \omega \times j$$
$$\left(\frac{dk}{dt}\right)_f = \omega \times k$$

(e) Derivative of Constant vector in moving frame:



$$r = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\left(\frac{dr}{dt}\right)_{fixed} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k} + x\left(\frac{di}{dt}\right)_{fixed} + y\left(\frac{dj}{dt}\right)_{fixed} + z\left(\frac{dk}{dt}\right)_{fixed}$$

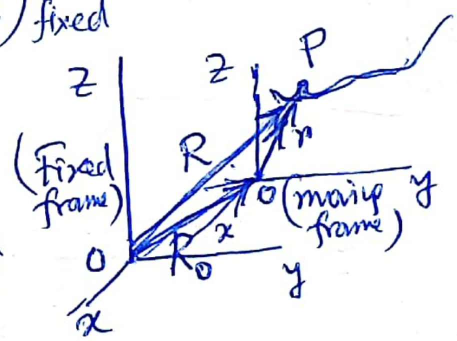
$$\left(\frac{dr}{dt}\right)_{moving} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k} = 0$$

$$\left(\frac{dr}{dt}\right)_{fixed} = x(\omega \times i) + y(\omega \times \hat{j}) + z(\omega \times k)$$
$$= \omega \times (xi + yj + zk) = \omega \times r$$

Similarly, $\left(\frac{dr'}{dt}\right)_{fixed} = \omega \times r'$ $\quad\therefore \left\{\frac{d}{dt}(r' - r)\right\}_{fixed} = \omega \times (r' - r)$

$$\left(\frac{dA}{dt}\right)_{fixed} = \omega \times A.$$

(f) Derivative of position vector for Different reference:



$$r = x\hat{i} + y\hat{j} + \hat{k}z$$
$$R = R_0 + r$$

$$\left(\frac{dr}{dt}\right)_{moving} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{z}$$

$$\left(\frac{dr}{dt}\right)_{fixed} = \frac{dx}{dt}i + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k} + x\frac{di}{dt} + y\frac{dj}{dt} + z\frac{dk}{dt}$$
$$= \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k} + x(\omega \times \hat{i}) + y(\omega \times \hat{j}) + z(\omega \times k)$$
$$= \left(\frac{dr}{dt}\right)_{moving} + \omega \times (xi + yj + zk)$$
$$= \left(\frac{dr}{dt}\right)_{moving} + \omega \times r$$

(g) Velocity of a point:

$$V_{P_{fixed}} = \left(\frac{dR}{dt}\right)_{fixed}$$

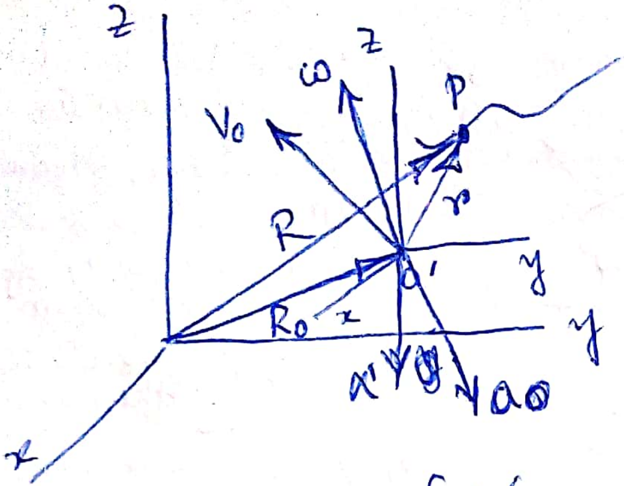
$$V_{P_{moving}} = \left(\frac{dr}{dt}\right)_{moving}$$

$$V_{P_{fixed}} = V_0 + \left[\left(\frac{dr}{dt}\right)_{moving} + \omega \times r\right]$$

$$\left(\frac{dR}{dt}\right)_{fixed} = \left(\frac{dR_0}{dt}\right)_{fixed} + \left(\frac{dr}{dt}\right)_{fixed}$$

$$\boxed{V_{P_{fixed}} = V_{P_{moving}} + V_0 + \omega \times r}$$

(h) Acceleration of a point :



$$a_{P_{fixed}} = \left[\frac{d}{dt}\left(V_{P_{fixed}}\right)\right]_{fixed}$$

$$a_{P_{moving}} = \left[\frac{d}{dt}\left(V_{P_{moving}}\right)\right]_{moving}$$

$$= \left[\frac{d^2 r}{dt^2}\right]_{moving}$$

$$a_{P_{fixed}} = \left[\frac{d}{dt}\left(V_{P_{fixed}}\right)\right]_{fixed}$$

$$= a_0 + \left[\frac{d}{dt}\left(V_{P_{moving}}\right)\right]_{fixed} + \left[\frac{d}{dt}\left(\omega \times r\right)\right]_{fixed}$$

$$\left[\frac{d}{dt}\left(V_{P_{moving}}\right)\right]_{fixed} = \left[\frac{d}{dt}\left(V_{P_{moving}}\right)\right]_{moving} + \omega \times V_{P_{moving}}$$

$$= a_{P_{moving}} + \omega \times V_{P_{moving}}$$

$$\left[\frac{d}{dt}\left(\omega \times r\right)\right]_{fixed} = \omega \times \left(\frac{dr}{dt}\right)_{fixed} + \left(\frac{d\omega}{dt}\right)_{fixed} \times r$$

$$= \omega \times \left[\left(\frac{dr}{dt}\right)_{moving} + \omega \times r\right] + \alpha \times r$$

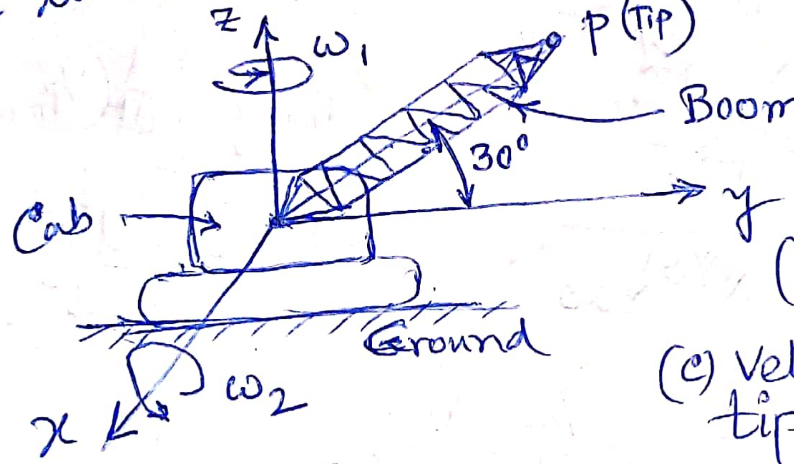
$$= \omega \times \left[V_{P_{moving}} + \omega \times r\right] + \alpha \times r$$

$$= \omega \times V_{P_{moving}} + \omega \times (\omega \times r) + \alpha \times r$$

Acceleration of a point P w.r.t fixed frame of reference

$$a_{P_{fixed}} = \underbrace{a_0}_{\text{Translational acc}^n} + \underbrace{a_{P_{moving}}}_{\text{Acc of pt 'P' w.r.t moving frame of reference}} + \underbrace{(\alpha \times r)}_{\text{acc}^n\text{ due to moving frame accelerating with }\alpha} + \underbrace{\omega \times (\omega \times r)}_{\text{normal component of acc}^n\text{ (centripetal acc}^n)} + \underbrace{2\omega \times V_{P_{moving}}}_{\text{Corriolis component of acc}^n}$$

Problem A crane rotates about a vertical axis with a constant angular velocity of 0.4 rad/s. While the boom is being raised with a constant angular velocity of 0.5 rad/s relative to the cab as shown in figure. If the length of boom is 10m, determine



(a) Angular Velocity of the boom

(b) Angular acc^n of the boom

(c) Velocity of the tip of the boom.

(d) Acce^n of the tip of the boom

Hints: $r = 10(\cos 30\hat{j} + \sin 30\hat{k})$
$= 0.865\hat{j} + 0.5\hat{k}$

$\omega_{boom} = \omega_1 + \omega_2$
$= 0.4\hat{k} + 0.5\hat{i}$ rad/s

$\omega_1 \rightarrow$ of Cab
$\omega_2 \rightarrow$ of boom w.r.t Cab

$$\alpha_{boom} = \left(\frac{d\omega_{boom}}{dt}\right)_f = \left(\frac{d\omega_1}{dt}\right)_f + \left(\frac{d\omega_2}{dt}\right)_f$$

Now, $\left(\frac{d\omega_1}{dt}\right)_f = 0$ as cab rotates at constant angular velocity

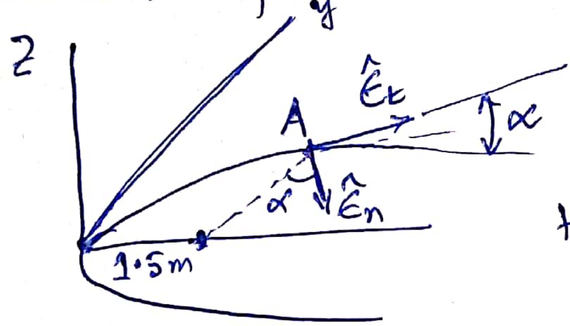
$$\alpha_{boom} = \left(\frac{d\omega_2}{dt}\right)_f = \omega_1 \times \omega_2 = 0.4\hat{k} \times 0.5\hat{i}$$
$$= 0.2\hat{j} \text{ rad/s}^2$$

Velocity of Tip (P) $(V_P) = \omega \times r$
$= (0.4\hat{k} + 0.5\hat{i}) \times (0.865\hat{j} + 0.5\hat{k})$
$= -3.465\hat{i} - 2.5\hat{j} + 4.33\hat{k}$ m/s

Acceleration of Tip (P)

$$a_P = \alpha_{boom} \times r + \omega_{boom} \times (\omega_{boom} \times r)$$
$$= 0.2\hat{j} \times (0.866\hat{j} + 5\hat{k}) + (0.4\hat{k} + 0.5\hat{i}) \times [(0.4\hat{k} + 0.5\hat{i}) \times (0.866\hat{j} + 5\hat{k})]$$
$$= 2\hat{i} - 3.55\hat{j} - 1.25\hat{k} \text{ m/s}^2$$

Problem A particle is moving in the xy plane along a parabolic path as $y = 1.22\sqrt{x}$. At position A $(x = 1.5\,m)$, the particle has a speed of 3 m/sec and rate of change of speed 3 m/s² along the path. What is the acceleration vector of the particle at this position?



Now,

$$\epsilon_t = \cos\alpha\,\hat{i} + \sin\alpha\,\hat{j}$$

Here, $\tan\alpha = \frac{dy}{dx} = \frac{d}{dx}(1.22\sqrt{x})$

$$= \frac{0.610}{\sqrt{x}}$$

At $x = 1.5\,m$,

$$\tan\alpha = \frac{0.610}{\sqrt{1.5}} = \frac{1}{2}$$

$$\text{or, } \alpha = 26.5^\circ$$

Hence,

$$\hat{\epsilon}_t = 0.895\hat{i} + 0.446\hat{j} \quad \text{--- (1)}$$

And from diagram, $\hat{\epsilon}_n = \sin\alpha\,\hat{i} - \cos\alpha\,\hat{j}$

$$\text{or, } \hat{\epsilon}_n = 0.446\hat{i} - 0.895\hat{j} \quad \text{--- (2)}$$

Now,

$$y = 1.22\sqrt{x}$$

$$y_1 = \frac{dy}{dx} = 0.61\,x^{-\frac{1}{2}}$$

$$y_2 = \frac{d^2y}{dx^2} = -0.305\,x^{-3/2}$$

$$\therefore R\Big|_{x=1.5m} = \frac{(1+y_1^2)^{3/2}}{y_2} = 8.4\,m$$

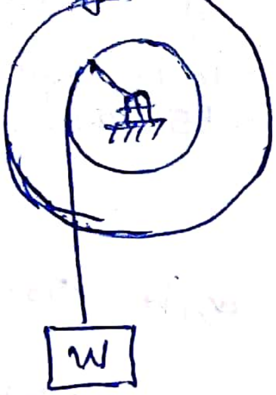
We know,

$$\vec{a} = \left(\frac{d^2s}{dt^2}\right)\hat{\epsilon}_t + \left(\frac{ds}{dt}\right)^2 \cdot \frac{\hat{\epsilon}_n}{R}$$

$$= 3(0.895\hat{i} + 0.446\hat{j}) + \frac{3^2}{8.4}(0.446\hat{i} - 0.895\hat{j})$$

$$\bar{a} = 3.16\hat{i} + 0.379\hat{j}\ \ m/s^2$$

Problem — A string is wound around a pulley of 0.28 m dia. One of its ends is fixed to the pulley while the other end is fixed to a weight 'W' as shown in figure. This weight moves a distance of 7.7 m starting from rest in 4.3 sec.



Determine the angular speed of the pulley. Also determine the total length ~~of~~ traversed by the weight to make the pulley rotate by 425 rpm.

$$s = ut + \frac{1}{2}at^2$$

$$7.7 = 0 \times t + \frac{1}{2} a \times 4.3^2$$

$$a = 0.833 \text{ m/s}^2$$

Angular accel^n of pulley, $\alpha = \frac{a}{R} = \frac{0.833}{\left(\frac{0.28}{2}\right)} = 5.95 \text{ rad/s}^2$

$$\omega = \omega_0 + \alpha t$$

$$= 0 + 5.95 \times 4.3$$

$$= 25.585 \text{ rad/s}$$

Final speed of pulley, $\omega_{final} = \frac{2\pi \times 425}{60} =$

Time reqd to reach the final speed, $t = \frac{\omega_{final} - \omega_0}{\alpha} = \frac{\frac{2\pi \times 425}{60} - 0}{5.95}$

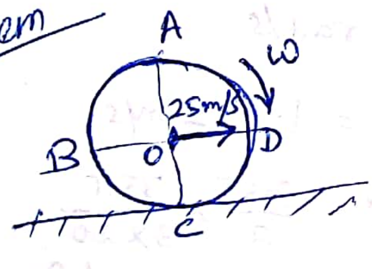
$= 7.48$ sec.

Total length traversed by the weight to make pulley rotate by 425 rpm

$$S = ut + \frac{1}{2}at^2$$

$$= 0 \times t + \frac{1}{2} \times 0.833 \times 7.48^2 = 23.303 \text{ m}$$

Problem

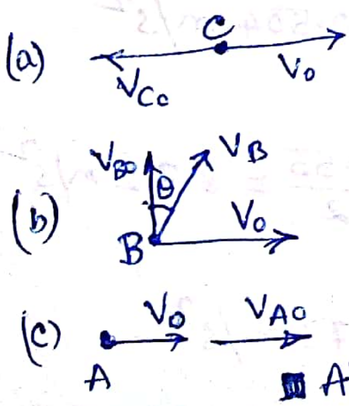


A roller of radius 2.5 m rolls without slipping along a horizontal plane. The centre O has uniform velocity 25 m/s. Determine the velocity of the points A and B of the roller.

■ At no slip cond^n ($V_c = 0$)

$V_c = V_{co} + V_o = -r\omega + V_o = 0$ or, $V_o = r\omega$

$$\omega = \frac{V}{r} = \frac{25}{2.5} = 10 \text{ rad/s}$$



■ $V_{BO} = V_{AO} = V_{DO} = \ldots\ldots = r\omega$

At point B, $V_B = V_{Bo} + V_o$

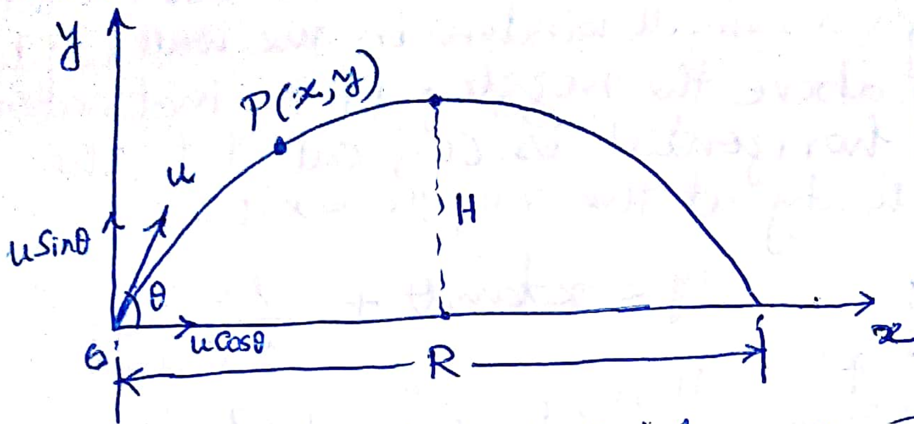
$V_B = \sqrt{V_{Bo}^2 + V_o^2} = \sqrt{(r\omega)^2 + V_o^2} = \sqrt{(10 \times 2.5)^2 + 25^2}$

$= 35.35$ m/s

$\theta = \tan^{-1}\left(\frac{25}{25}\right) = 45°$

■ At pt A, $V_A = V_{Ao} + V_o = 2 \times 25 = 50$ m/s.

Projectile motion :



$$x = u\cos\theta \times t \quad \text{------------- (I)}$$

$$y = u\sin\theta \times t - \frac{1}{2}gt^2 \quad \text{------------- (II)}$$

$$\text{or, } y = (u\sin\theta)\cdot\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$$

$$\boxed{y = x\tan\theta - \frac{1}{2}g\cdot\frac{x^2}{u^2\cos^2\theta}} \quad \text{------ (A)}$$

To find the time of flight :

For this, put $y=0$ in eq^n (II)

$$\boxed{t = \frac{2u\sin\theta}{g}}$$

To find the max^m height reached :

$$v^2 = u^2 - 2gH$$

$$0 = (u\sin\theta)^2 - 2gH$$

$$\text{or, } \boxed{H = \frac{u^2\sin^2\theta}{2g}}$$

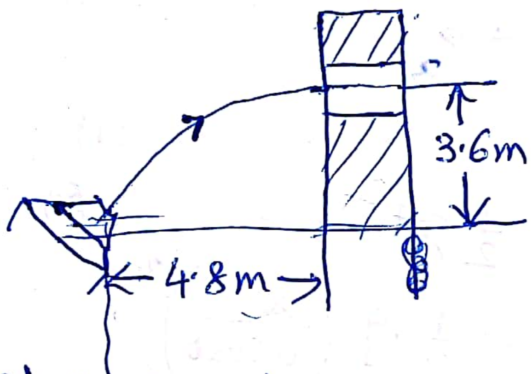
To find the max^m horizontal range :-

$$R = (u\cos\theta) \times \text{time of flight}$$

$$= u\cos\theta \times \frac{2u\sin\theta}{g}$$

$$\boxed{R = \frac{u^2\sin 2\theta}{g}}$$

Problem A fireman holding a nozzle at a horizontal distance of 4.8 m from a vertical wall, wishes to send a jet of water through a small window in the wall located 3.6 m vertically above the nozzle. If the inclination of the jet with horizontal is 60°, calculate the required jet velocity at the nozzle exit.



$$y = x\tan\theta + \frac{gx^2}{2u^2\cos^2\theta}$$

Here, $\theta = 60^\circ$, $y = 3.6$ m.
$x = 4.8$ m

Putting these values :-

$u = 9.78$ m/s.

Problem A particle is projected at such an angle with the horizontal that the horizontal range is 4 times the greatest height attained by the particle. Find the angle of projection.

Here, $R = 4 H_{max}$

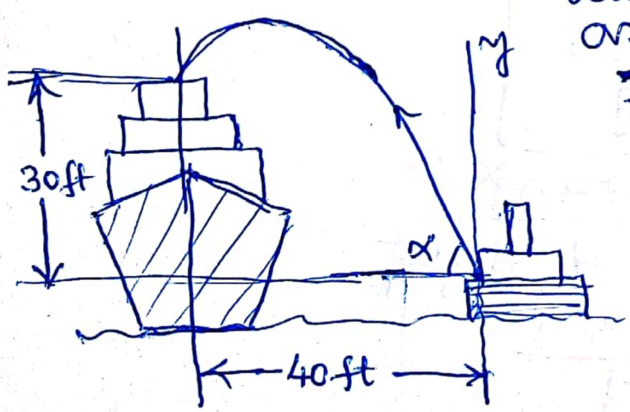
or, $$\frac{u^2\sin 2\theta}{g} = 4 \cdot \frac{u^2\sin^2\theta}{2g}$$

or, $2\sin\theta\cos\theta = 2\sin^2\theta$.

or, $\tan\theta = 1$ or, $\theta = 45^\circ$

Problem

The engine room of a freighter is on fire. A fire-fighter tugboat has drawn alongside and is directing a stream of water to enter the stack of the freighter as shown in figure.

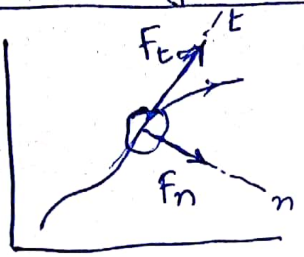


If the initial speed of jet is 70 ft/sec, calculate the angle of projection of jet.

[Ans :- $\alpha = 81.37^\circ$]

Particle Dynamics:

$$F = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$



$$F_x = ma_x \qquad F_y = ma_y \qquad F_z = ma_z$$

$$F_t = ma_t = m\dot{s} = mr\ddot{\theta} = m\alpha$$

$$F_n = ma_n = m\frac{v^2}{r} = m\dot{\theta}^2 r = m\omega^2 r$$

$$s = ut \pm \frac{1}{2}ft^2 \qquad v^2 = u^2 + 2fs \qquad v = u + ft$$

$$\theta = \omega t + \frac{1}{2}\alpha t^2 \qquad \omega^2 = \omega_0^2 + 2\alpha\theta \qquad \omega = \omega_0 + \alpha t$$

For cylindrical Co-ordinate Systm:

$$F = F_r\hat{e}_r + F_\theta\hat{e}_\theta + F_z\hat{e}_z$$

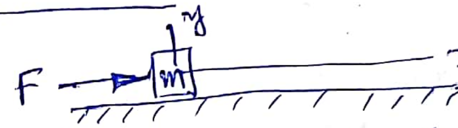
$$F_r = m(\ddot{r} - r\dot{\theta}^2)$$

$$F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

$$F_z = m\ddot{z}$$

(1) Force is a function of Time or Constant:

$$a = \frac{d^2x}{dt^2}$$



$$F(t) = m\vec{a} = m\frac{d^2x}{dt^2} = m\frac{dV}{dt}$$

Example – Tractive force of automobile w.r.t time

$$d\left(\frac{dx}{dt}\right) = F(t)\,dt$$

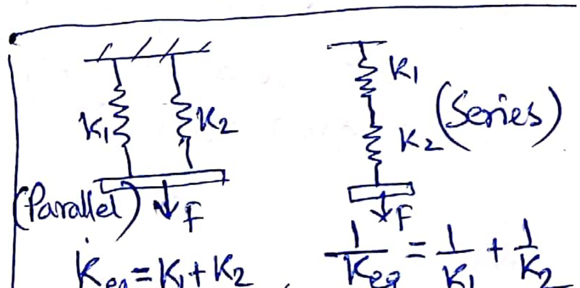
Integrating:–

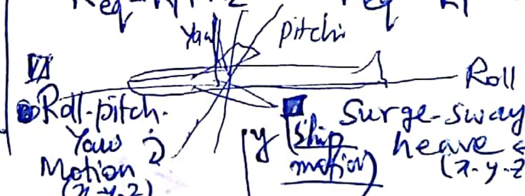
or, $$V = \int F(t)\,dt + C_1$$

Integrating –

or, $$x = \iint F(t)\,dt + C_1 t + C_2$$

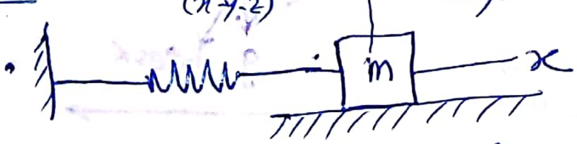
Apply B.C. $t = 0,\ V = V_0$; $t = 0,\ x = x_0$ (Say)





(2) Force is a function of position:

$$m\frac{dV}{dt} = F(x)$$



$$m\frac{dV}{dt} = m\left(\frac{dV}{dx}\right)\left(\frac{dx}{dt}\right) = F(x)$$

Example - Restoring force of spring-mass System

$$F_{spring} = -\int k(x)\,dx$$

or, $$mV\frac{dV}{dx} = F(x)$$

Integrating.–

$$m\frac{V^2}{2} = \int F(x)\,dx + C_1$$

or $$V = \left[\frac{2}{m}\int F(x)\,dx + C_1\right]^{\frac{1}{2}}$$

$$\frac{dx}{dt} = \left[\frac{2}{m}\int F(x)\,dx + C_1\right]^{\frac{1}{2}}$$

$$dt = \frac{dx}{\left[\frac{2}{m}\int F(x)\,dx + C_1\right]^{\frac{1}{2}}}$$

or, $$t = \int \frac{dx}{\left[\frac{2}{m}\int F(x)\,dx + C_1\right]^{\frac{1}{2}}} + C_2$$

Now, Apply B.C.

③ Force is a function of speed: $F(V) = m \cdot \frac{dV}{dt}$.

$$\frac{dV}{dt} = \frac{F(V)}{m}$$

$$\frac{dV}{F(V)} = \frac{1}{m} dt$$

$$\int \frac{dV}{F(V)} = \frac{1}{m} \cdot t + C_1$$

Here, $V = H(t, C_1)$ or, $\frac{dx}{dt} = H(t, C_1)$.

Where, H is a function of t & C_1

$$x = \int H(t, C_1)\, dt + C_2$$

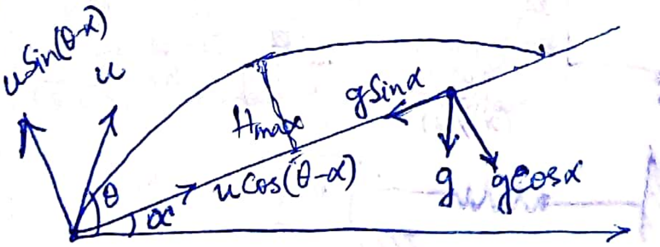
Apply B.C. to find the values of C_1 and C_2.

Lagrange-D'Alembert's Principle: The sum of differences between the forces acting on the system and the time derivative of momentum of the system along a virtual displacement consistent with the constraints of the system is zero.

$$\sum (F_i - m_i a_i)\, \delta r_i = 0 \quad \text{or,} \quad \sum (F_i - m_i a_i) = 0$$

ie, $F - ma = 0$.

Projectile motion on Inclined plane:



(i) $u\cos(\theta-\alpha)$ along inclined plane

(ii) $u\sin(\theta-\alpha)$ along ⊥ to inclined plane

$$s = ut + \frac{1}{2}at^2$$

$$0 = u\sin(\theta-\alpha) \cdot t - \frac{1}{2}(g\cos\alpha)\, t^2$$

$$\boxed{t = \frac{2u\sin(\theta-\alpha)}{g\cos\alpha}} \quad \text{(Time of flight)} \quad ⓐ$$

~~Max^m height reached w.r.t inclined plane~~

$$v^2 = u^2 - 2gH$$

$$0 = u^2\sin^2(\theta-\alpha) - 2(g\cos\alpha) \cdot H_{max} \quad \text{or,} \quad \boxed{H_{max} = \frac{u^2\sin^2(\theta-\alpha)}{2g\cos\alpha}}$$

Max^m horizontal Range

$$S = ut - \frac{1}{2}gt^2$$

$$R = u\cos(\theta-\alpha) \cdot \frac{2u\sin(\theta-\alpha)}{g\cos\alpha} - \frac{1}{2}(g\sin\alpha)\left[\frac{2u\sin(\theta-\alpha)}{g\cos\alpha}\right]^2$$

$$R = \frac{2u^2\sin(\theta-\alpha)}{g\cos^2\alpha}\cos\theta = \frac{u^2}{g\cos^2\alpha}\left[\sin(2\theta-\alpha) - \sin\alpha\right]$$

For projectile down inclined plane $+ \to -$

For max^m Range $\theta = 45 + \frac{\alpha}{2}$

$$\boxed{R_{max} = \frac{u^2}{g\cos^2\alpha}(1-\sin\alpha)}$$

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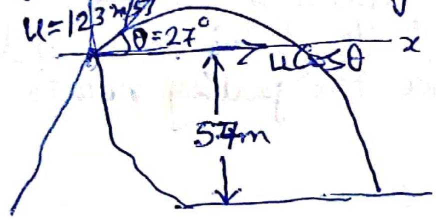
Problem A gun is fired from hill top at 27° upward at the speed of 123 m/s. The enemy is 57 m below (vertically). Determine the max^m rise of bullet from horizontal, velocity and time to hit the enemy.



$u_x = u\cos\theta = 123\cos 27° = 109.6$ m/s

$u_y = u\sin\theta = 123\sin 27° = 55.84$ m/s.

At max^n height

$v_y^2 = u_y^2 - 2gH$

$0 = 55.84^2 - 2gH$ $\Rightarrow$ $\boxed{H = 158.97 \text{ m}}$

At the instant of hitting the enemy, if the vertical component of velocity

$u_{hit}^2 = 55.84^2 + 2\times 9.8\times 158.97$

$\boxed{u_{hit} = 78.96 \text{ m/s}}$

At this instant,

final velocity $= \sqrt{109.6^2 + 78.96^2}$

$= 135.08$ m/s.

If the time span required to hit the enemy

$v = u + gt$

$78.96 = 55.84 + 9.81\times t$

$t = 2.358$ sec.

Problem A flywheel of 550 mm dia is brought uniformly from rest up to a speed of 350 rpm in 20 sec. Find the velocity and acceleration of a point on it's rim 3 sec after starting from rest.

Initial angular speed $\omega_0 = 0$

Speed at 20sec time, $\omega = \dfrac{2\pi\times 350}{60} = \dfrac{35\pi}{3}$ rad/s.

$\alpha = \dfrac{\omega - \omega_0}{t} = \dfrac{35\pi}{3\times 20}$ rad/s^2

Now, at t=3 sec.

$\omega_t = \omega_0 + \alpha t = 0 + \dfrac{35\pi}{3\times 20}\times 3 = \dfrac{7\pi}{4}$ rad/s.

- Velocity of the point; $v = r\omega_t = \dfrac{0.55}{2}\times\dfrac{7\pi}{4} = 1.512$ m/s.

- Accl^n — a) Tangential accl^n $(a_t) = r\alpha = \dfrac{0.55}{2}\times\dfrac{35\pi}{3\times 20}$

$= 0.504$ m/s^2

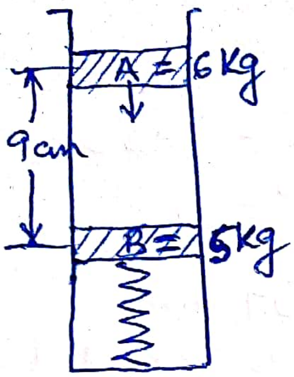
b) Normal accl^n $(a_n) = \omega_t^2 r$

$= \left(\dfrac{7\pi}{4}\right)^2\times\dfrac{0.55}{2} = 8.312$ m/s^2

Total accl^n $(a) = \sqrt{a_t^2 + a_n^2}$

$= \sqrt{0.504^2 + 8.312^2} = 8.3273$ m/s^2

Problem.. Block A of wt 6 kg drops on block B of weight 5 kg from a height of 9 cm as shown in figure. Considering the spring constant as $k = 1.85$ N/mm, determine the amount of compression of spring over and above the static deformation due to block B.



Sol^n Velocity of block A just before impact with block B,

$$u_A = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 0.09}$$
$$= 1.328 \text{ m/s}.$$

After impact, block A & block B both will move down at a common velocity (say V).

Using principle of conservation of momentum,

$$m_A u_A + m_B \cancel{u_B}^{0} = (m_A + m_B) V$$

or, $6 \times 1.328 + 0 = (6+5) V$

or, $V = 0.7247$ m/s.

Static deformation of spring due to block B,

$$\delta_{st} = \frac{W_B}{k} = \frac{m_B g}{K} = \frac{5 \times 9.81}{1.85} = 0.0265 \text{ m}$$

If due to impact, the spring gets compressed by an amount of δ, total compression

$$\Delta = \delta_{st} + \delta = (0.0265 + \delta) \text{ m}$$

Just after impact, the total energy stored in the system

$$E_1 = \frac{1}{2} k \delta_{st}^2 + \frac{1}{2}(m_A + m_B) V^2$$
$$= \frac{1}{2} \times 1.85 \times 0.0265^2 + \frac{1}{2}(6+5) \times 0.7247^2$$
$$= 3.538 \text{ N-m}$$

When both the blocks come to rest after compressing the spring by δ, total energy stored in the system.

$$E_2 = \frac{1}{2} K \Delta^2 - (m_A + m_B) \times g \times \delta$$
$$= \frac{1}{2} \times 1.85 \times 10^{3} \Delta^2 - (6+5) \times 9.81 \times \delta$$
$$= 925 \Delta^2 - 107.87 \delta$$

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from principle of conservation of energy

$$E_2 = E_1$$

$$925\Delta^2 - 107.878\delta = \cancel{35}\,3.538$$

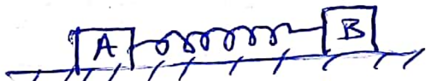
or, $$925(\delta + 0.0265)^2 - 107.878\delta = 3.538$$

or, $$\delta^2 - 0.0636\delta - 3.1226\times 10^{-3} = 0$$

or, $$\delta = 0.09609 \text{ m} \approx 96.09 \text{ mm}$$

Problem A spring of unstretched length 0.2 m is connected to blocks A and B having weights of 3 kg and 2 kg, respectively. If ~~to~~ initially, the spring is compressed by 5 cm and the system is released from rest, when the spring comes again in its normal length? Take spring const. k = 2 N/mm

From principle of conservation of momentum,



$$m_A u_A + m_B u_B = 0$$ (as initial ~~velo~~ momentum of blocks is zero)

$$3u_A + 2u_B = 0 \quad \text{or,} \quad \boxed{u_A = -\tfrac{2}{3}u_B}$$

If the spring expands by ~~an~~ amount 's' from its compressed position to come back to its original length, the compressive force, $F = k(0.05 - s) = 2\times 10^3(0.05 - s)$

For displacement of 'ds',

$$dW = F \times ds$$

$$W = \int_0^{0.05} dW = \int_0^{0.05} 2\times 10^3 (0.05 - s)\, ds.$$

$$= 2\times 10^3\left[0.05s - \frac{s^2}{2}\right]_0^{0.05}$$

$$= 2.5 \text{ N-m}$$

Initial kinetic energy of the block are zero.

Final kinetic energy $= \frac{1}{2}m_A u_A^2 + \frac{1}{2}m_B u_B^2$

$$= \frac{1}{2}\times 3\times\left(-\frac{2}{3}u_B\right)^2 + \frac{1}{2}\times 2 . u_B^2$$

$$= \frac{5}{3}u_B^2$$

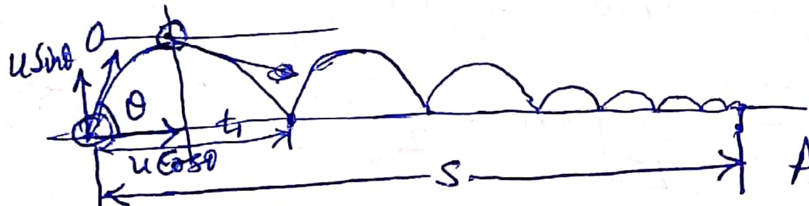
From work-energy principle,

$$\frac{5}{3}u_B^2 = 2.5$$

or, $u_B = 1.224$ m/s $\therefore u_A = -\frac{2}{3}u_B = -0.816$ m/s.

Problem A glass ball of mass 24 gm is thrown from a horizontal floor at an angle of 'θ' at velocity u. When the ball stop bouncing after number of impacts, the horizontal distance travelled is 's'. If 'e' is symbolised as the co-efficient of restitution, prove that,

$$S = \frac{u^2}{g}\frac{\sin 2\theta}{(1-e)}$$



$(u\sin\theta) \rightarrow$ for bouncing effect acting vertically

After 1st impact,
velocity $= eu\sin\theta$.

After 2nd impact
velocity $= e^2 u\sin\theta$

- - - and so on

Time interval:

$$t_1 = \frac{2u\sin\theta}{g}$$

$$t_2 = \frac{2eu\sin\theta}{g}$$

... — — and so on.

$$\text{Total time } (T) = t_1 + t_2 + t_3 + \cdots$$

$$= \frac{2u\sin\theta}{g} + \frac{2eu\sin\theta}{g} + \frac{2e^2u\sin\theta}{g} + \frac{2e^3u\sin\theta}{g} + \cdots$$

$$= \frac{2u\sin\theta}{g}(1 + e + e^2 + e^3 + \cdots)$$

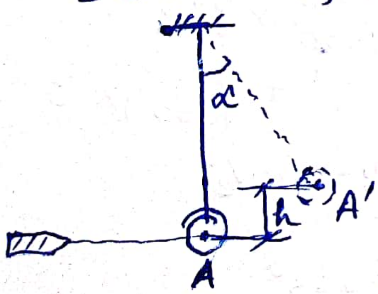
$$= \frac{2u\sin\theta}{g} \times \frac{1}{(1-e)}$$ [from Bionomial expansion formulation]

Total distance travelled

$$(S) = (u\cos\theta) \times \text{Total time } (T)$$

$$= u\cos\theta \times \frac{2u\sin\theta}{g} \times \frac{1}{(1-e)} = \frac{u^2\sin 2\theta}{g(1-e)}$$

Problem A 0.303 bullet weighing 5.678 gm hits a pendulum of bob 707 gm suspended by an inextensible string of length 1.1 m. If the velocity of strike is 97.3 m/s, determine the max^m angle of swing (α) when the bullet gets embedded within the bob and also when the bullet escapes the bob at a velocity of 17 m/s. If the bullet rebounds the bob at a velocity of 24.5 m/s, what will be the angle of swing? [$\alpha_1 = 11.26^\circ$, $\alpha_2 = 10.213^\circ$]



(a) When the bullet gets embedded, applying principle of conservation of momentum,

$$\left(\frac{5.678}{1000}\right)V + \left(\frac{707}{1000}\right)V = \left(\frac{5.678}{1000}\right)\times 97.3 + \left(\frac{707}{1000}\right)\times 0$$

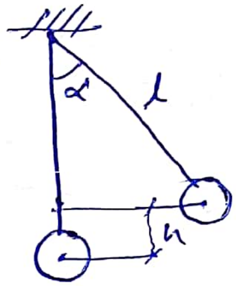
$V = 0.7752$ m/s. (Combined velocity)

Let, the system gets displaced from position A to A' raising a height 'h'
Now, applying principle of conservation of energy.

$$\left(\frac{5.678}{1000} + \frac{707}{1000}\right)\times g\times h = \frac{1}{2}\left(\frac{5.678}{1000} + \frac{707}{1000}\right)V^2$$

or, $h = \frac{V^2}{2g}$.

or, $h = 0.0306$ m.



Angle of swing (α)

$$\cos\alpha = \frac{l-h}{l} = \frac{1.1 - 0.0306}{1.1}.$$

$\alpha = 13.546°$

(b) If escape velocity of bullet $v_1 = 17$ m/s and velocity of bob is 'V_1' then using principle's of conservation of momentum,

$$\left(\frac{5.678}{1000}\right)\times 17 + \left(\frac{707}{1000}\right)\times V_1 = \left(\frac{5.678}{1000}\right)\times 97.3 + \left(\frac{707}{1000}\right)\times 0$$

or; $V_1 = 0.6449$ m/s.

Using principle's of conservation of energy, $mgh_1 = \frac{1}{2}mV_1^2$

$$\left(\frac{707}{1000}\right)\times g\times h_1 = \frac{1}{2}\left(\frac{707}{1000}\right)\times 0.6449^2$$

or, $h_1 = \frac{V_1^2}{2g} = 0.0212$ m

So, Angle of swing $(\alpha_1) = \cos^{-1}\left(\frac{1.1 - 0.0212}{1.1}\right) = 11.267°$

(c) When the bullet rebounds at $v_2 = 24.5$ m/s, from principles of conservation of momentum,

$$\left(\frac{5.678}{1000}\right)\times 24.5 + \left(\frac{707}{1000}\right)V_2 = \left(\frac{5.678}{1000}\right)\times 97.3 + \left(\frac{707}{1000}\right)\times 0$$

$V_2 = 0.5847$ m/s.

Using principle of conservation of energy, $mgh_2 = \frac{1}{2}mV_2^2$

$$\left(\frac{707}{1000}\right)\times g\cdot h_2 = \frac{1}{2}\left(\frac{707}{1000}\right)\times 0.5847^2$$

or, $h_2 = 0.01743$ m

$\therefore$ Angle of Swing $(\alpha_3) = \cos^{-1}[(1.1 - 0.01743)/1.1] = 10.213°$

Impulse & Momentum

From Newton's 2nd law of motion,

$$F = \frac{d}{dt}(mv)$$

$$\underbrace{\int_{t_1}^{t_2} F\,dt}_{\text{Impulse}} = \int_{v_1}^{v_2} d(mv) = m\int_{v_1}^{v_2} dv = m(v_2 - v_1) = m\,\Delta v$$

$$\underbrace{mv_1}_{\text{Initial momentum}} + \underbrace{\int_{t_1}^{t_2} F\,dt}_{\text{Impulse}} = \underbrace{mv_2}_{\text{Final momentum}}$$

$$\frac{d}{dt}(I_z\omega) = M_z$$ (Gyroscope rotating about axis of symmetry/spin maintains constant angular velocity of spin)

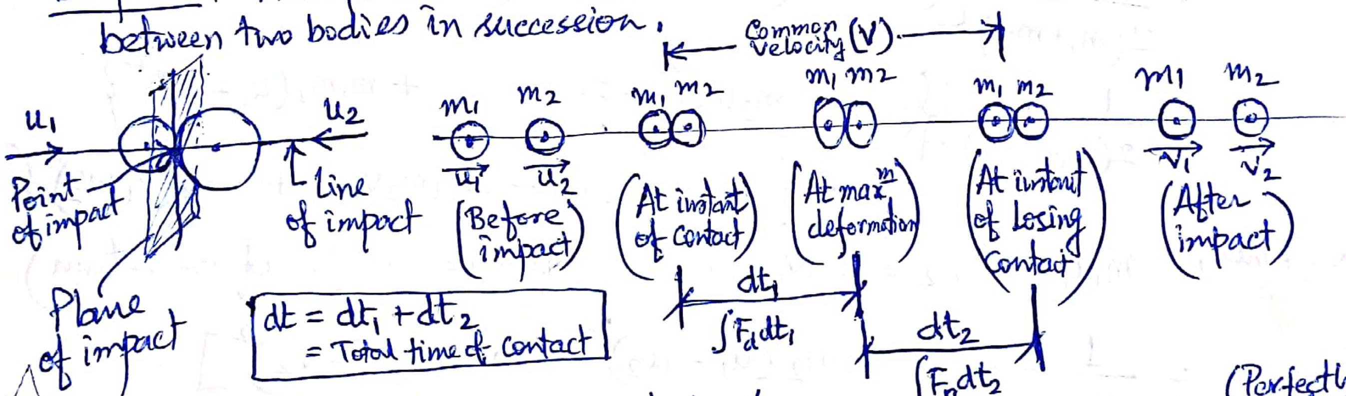
Example of impulse

a) Bouncing of ball
b) Ramming of pile driver
c) Hitting a baseball by bat

$$\sum_{i=1}^{n}(m_i v_{1i})_{x/y/z} + \sum_{i=1}^{n}\int_{t_1}^{t_2} F_i\,dt_{x/y/z} = \sum_{i=1}^{n}(m_i v_{2i})_{x/y/z}$$

Principle of conservation of linear momentum: $\sum_{i=1}^{n}(m_i v_i)_{initial} = \sum_{i=1}^{n}(m_i v_i)_{final}$

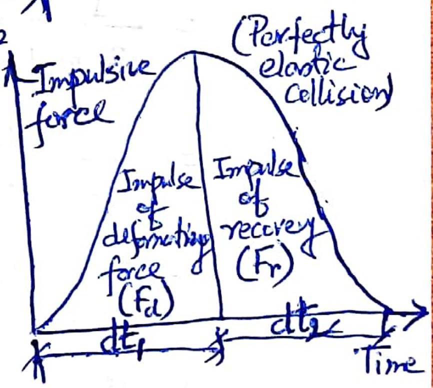
Impact: Abrupt short duration encounter between two bodies in succession.



$$\boxed{dt = dt_1 + dt_2 = \text{Total time of contact}}$$

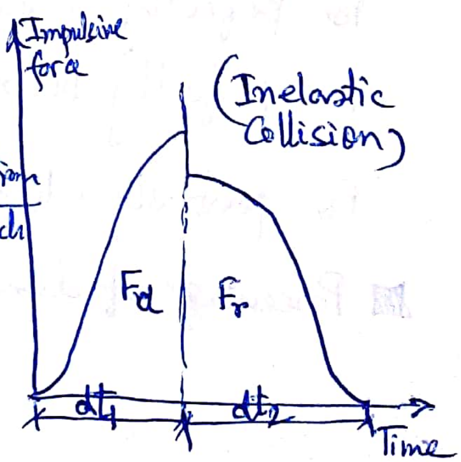
Coeff of restitution (COR)(e): Ratio of magnitude of impulse of force of recovery to impulse of force of deformation.
(in other word, relative velocity after impact to before impact)

$$e = \frac{\int F_r dt_2}{\int F_d dt_1}$$



$$e = \frac{m_1V - m_1v_1}{m_1u_1 - m_1V} = \frac{V - v_1}{u_1 - V} \quad \text{(for Body A)} \text{ — (I)}$$

$$e = \frac{m_2v_2 - m_2V}{m_2V - m_2u_2} = \frac{v_2 - V}{V - u_2} \quad \text{(for body B)} \text{ — (II)}$$



From eq^n (I) & (II) applying concept of identity

$$e = \frac{(V - v_1) + (v_2 - V)}{(u_1 - V) + (V - u_2)} = \frac{v_2 - v_1}{u_1 - u_2} = \frac{\text{Velo. of Separation}}{\text{Velo of approach}}$$

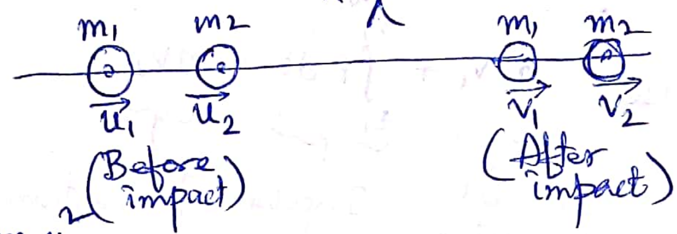
- If $e = 1 \Rightarrow$ Perfectly elastic collision
- If $e = 0 \Rightarrow$ Perfectly plastic collision

Impact types :- (a) Depending on direction of velocity
(i) Central (Normal), (ii) Eccentric (Oblique)

~~(b) Depending line of~~

Example of impact :- Collision bet^n atoms, Collision bet^n two billiard ball. ~~by golf club, Firing shot gun~~

Inelastic collision :



$$KE_{Initial} = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$$

$$KE_{final} = \frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2$$

Loss of KE during inelastic collision,

$$\Delta KE = KE_{Initial} - KE_{final} = \left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) - \left(\frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2\right)$$

$$= \frac{1}{2(m_1+m_2)}\left[(m_1+m_2)(m_1u_1^2+m_2u_2^2) - (m_1+m_2)(m_1V_1^2+m_2V_2^2)\right]$$

$$= \frac{1}{2(m_1+m_2)}\left[\left\{\cancel{(m_1u_1+m_2u_2)^2} - 2m_1m_2u_1u_2 + m_1m_2(u_1+u_2)^2\right\} - \left\{\cancel{(m_1V_1+m_2V_2)^2} - 2m_1m_2V_1V_2 + m_1m_2(V_1+V_2)^2\right\}\right]$$

We know, $m_1u_1 + m_2u_2 = m_1V_1 + m_2V_2$ (from Conservation of momentum)

$$= \frac{1}{2(m_1+m_2)}\left[m_1m_2(u_1-u_2)^2 - m_1m_2(V_1-V_2)^2\right]$$

$$= \frac{m_1m_2}{2(m_1+m_2)}\left[(u_1-u_2)^2 - e^2(u_1-u_2)^2\right] \qquad \left[e = \frac{V_2-V_1}{u_1-u_2}\right]$$

$$= \frac{m_1m_2(u_1-u_2)^2}{2(m_1+m_2)}(1-e^2)$$

For perfectly elastic collision, $e = 1$. ie, $\Delta KE = 0$.

For perfectly plastic collision, $e = 0$ ie, $\Delta KE = \frac{m_1m_2(u_1-u_2)^2}{2(m_1+m_2)}$

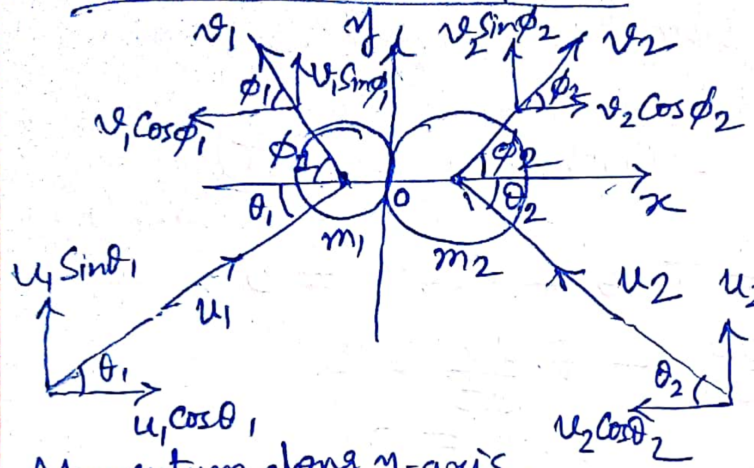
For partially elastic collision, $0 < e < 1$

Percentage of dissipation of energy $= \frac{\Delta KE}{KE_{Initial}} \times 100\%$

Oblique Central Impact:

θ_1, θ_2 are the angles which initial velocities u_1, u_2 makes with x-axis.

$\phi_1, \phi_2 \rightarrow$ — v_1, v_2 make with y-axis.



Momentum along y-axis,

$$m_1 u_1 \sin\theta_1 = m_1 v_1 \sin\phi_1 \quad \text{or,} \quad u_1 \sin\theta_1 = v_1 \sin\phi_1 \text{ --- (i)}$$

$$m_2 u_2 \sin\theta_2 = m_2 v_2 \sin\phi_2 \quad \text{or,} \quad u_2 \sin\theta_2 = v_2 \sin\phi_2 \text{ --- (ii)}$$

From conservation of momentum,

$$m_1 u_1 \cos\theta_1 + m_2 u_2 \cos\theta_2 = m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2 \text{ --- (iii)}$$

From Newton's law of Collision of elastic body valid for oblique impacts.

$$\text{Co-eff of restitution } (e) = \frac{v_2 \cos\phi_2 - v_1 \cos\phi_1}{u_1 \cos\theta_1 - u_2 \cos\theta_2} \text{ --- (iv)}$$

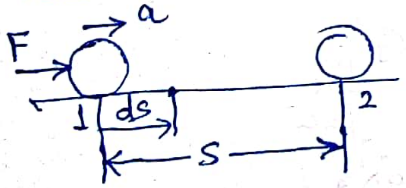
For perfectly elastic collision, $e = 1$

$$\text{or,} \quad v_2 \cos\phi_2 - v_1 \cos\phi_1 = u_1 \cos\theta_1 - u_2 \cos\theta_2 \text{ --- (iva)}$$

Using eq^n (i) to (iva), solving eq^n we can get four unknowns such as, v_1, v_2, ϕ_1 and ϕ_2, considering u_1, u_2, θ_1 & θ_2 are known.

Work-Energy Principle:

If a body is subjected to force 'F' to travel elementary 'ds' distance in time 'dt' then Workdone by the force,



$$dW = F \cdot ds$$

From Newton's 2nd law of motion, $F = m \cdot a$

where, $a = \frac{dv}{dt} = \frac{dv}{ds} \cdot \frac{ds}{dt} = v\frac{dv}{ds}$

$$\therefore dW = (m \cdot a) \cdot ds = m \cdot v\frac{dv}{ds} \cdot ds = mv \cdot dv$$

Integrating both sides:-

$$W_{1-2} = \int_1^2 dW = m\int_1^2 v\,dv = \frac{1}{2}m(v_2^2 - v_1^2) = KE_2 - KE_1$$

i.e, Workdone on the object equals to change in K.E. of the object. This is called Work-energy principle.

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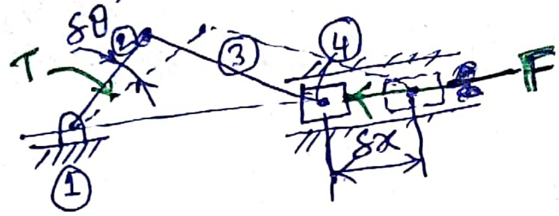
Principles of Virtual Work:

The sum of workdone during a virtual small displacement from the equilibrium is equal to zero.



Total Workdone
= Workdone by Torque (T)
+ Workdone by force (F)

$$W = T.\delta\theta + F.\delta x = 0$$

or, $$T.\frac{d\theta}{dt} + F.\frac{dx}{dt} = 0.$$

or, $$T.\omega + F.v = 0.$$

or, $$\boxed{T = -\frac{F}{\omega}.v}$$

D'Alembert's Principle:

Inertia forces and Couples + External forces and torques on a body together provide the static equilibrium.

Mathematically,

$$\boxed{\begin{aligned}\sum F + F_{inertia} &= 0\\ \sum T + C_{inertia} &= 0.\end{aligned}}$$

where,
$F_{inertia} = -m f_g.$
$C_{inertia} = -I_g.\alpha.$
$\sum F = F_1 + F_2 + F_3 - - + F_n$ (external forces)
$\sum T = T_1 + T_2 + T_3 + \cdot\cdot T_n$ (external torques)

For static equilibrium,

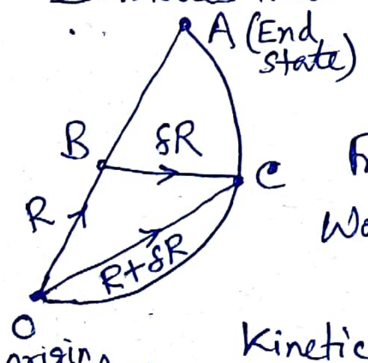
$F_{inertia} = 0$ and $C_{inertia} = 0.$

then, $$\boxed{\begin{aligned}\sum F &= 0\\ \sum T &= 0\end{aligned}}$$

D'Alembert's principle states:— The system of forces acting on a body in motion is in dynamic equilibrium with the inertia forces & inertia couples of the body.

Hamilton's Principle:

It states that — Time integral of the difference between the kinetic and potential energies of dynamical system is stationary.



From Newton's 2nd law, $F = m\frac{d^2R}{dt^2}$

Workdone during virtual displacement 'δR',

$$\delta W = F \times \delta R = m\frac{d^2R}{dt^2}\delta R \quad \text{--- (i)}$$

Kinetic energy, $KE = \frac{1}{2}m\left(\frac{dR}{dt}\right)^2$ or, $\delta KE = m\left(\frac{dR}{dt}\right).\delta\left(\frac{dR}{dt}\right)$

$$= m\frac{dR}{dt}\frac{d}{dt}(\delta R) \quad \text{(ii)}$$

Adding eq^n (i) & (ii)

$$\delta W + \delta KE = m\frac{d}{dt}\left(\frac{dR}{dt}.\delta R\right)$$

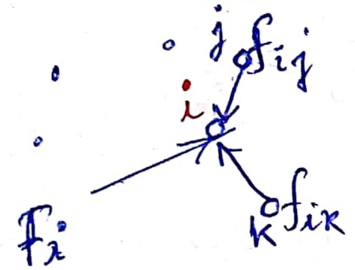
$$\int(\delta W + \delta KE)dt = \left(m\frac{dR}{dt}.\delta R\right]_{t_0}^{t_1}$$

Since, $\delta R = 0$ at t_0 & t_1

$$\boxed{\int(KE - PE)dt = 0}$$

where, $\delta W = -PE$

General motion of a System of Particle :



F_i = Applied force

f_{ij} = Interactive force between particles

from Newton's 2nd law,

$$m_i \frac{d^2 r_i}{dt^2} = F_i + \sum_{\substack{j=1 \\ (i \neq j)}}^{n} f_{ij}$$

$$\sum_{i=1}^{n} m_i \frac{d^2 r_i}{dt^2} = \sum_{i=1}^{n} F_i + \sum_{i=1}^{n}\sum_{j=1}^{n} f_{ij} \qquad \text{Since, } f_{ij} = -f_{ji}$$

$$F = \sum_{i=1}^{n} m_i \frac{d^2 r_i}{dt^2} = \frac{d^2}{dt^2}\sum_{i=1}^{n} m_i r_i$$

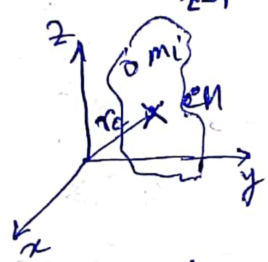
First moment vector $= \sum m_i r_i$

$$r_c \sum_{i=1}^{n} m_i = \sum_{i=1}^{n} m_i r_i$$

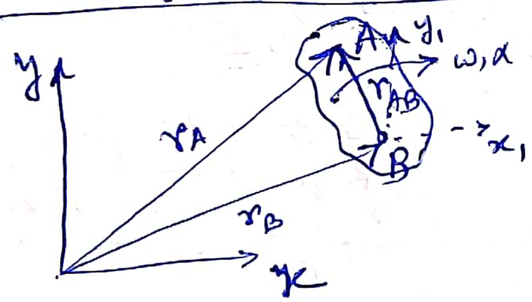
$$\boxed{r_c = \frac{\sum m_i r_i}{\sum m_i} = \frac{\sum m_i r_i}{M}}$$

$$F = \frac{d^2}{dt^2}(M r_c)$$

$$= M\frac{d^2 r_c}{dt^2}$$



Governing kinetic eq^n of Rigid body :



$$r_A = r_B + r_{AB}$$

$$v_A = v_B + v_{AB}$$

where, $v_{AB} = \dfrac{dr_{AB}}{dt} = \omega \times r_{AB}$

$$v_A = \underbrace{v_B}_{\text{Translation component}} + \underbrace{\omega \times r_{AB}}_{\text{Rotational component}}$$

Now, $a_A = a_B + a_{AB}$

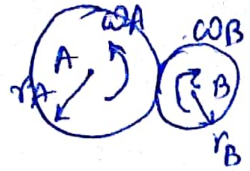
$$a_{AB} = \frac{d}{dt}(\omega \times r_{AB})$$

$$= \frac{d\omega}{dt} \times r_{AB} + \omega \times \frac{dr_{AB}}{dt}$$

$$= \alpha \times r_{AB} + \omega \times (\omega \times r_{AB})$$

$$a_A = \underbrace{a_B}_{\text{translation part}} + \underbrace{\alpha \times r_{AB} + \omega \times (\omega \times r_{AB})}_{\text{Rotational part}} = a_B + (a_{AB})_t + (a_{AB})_n$$

$$\begin{cases} (a_{AB})_t = \alpha \times r_{AB} \\ (a_{AB})_n = \omega^2 \times r_{AB} \end{cases}$$

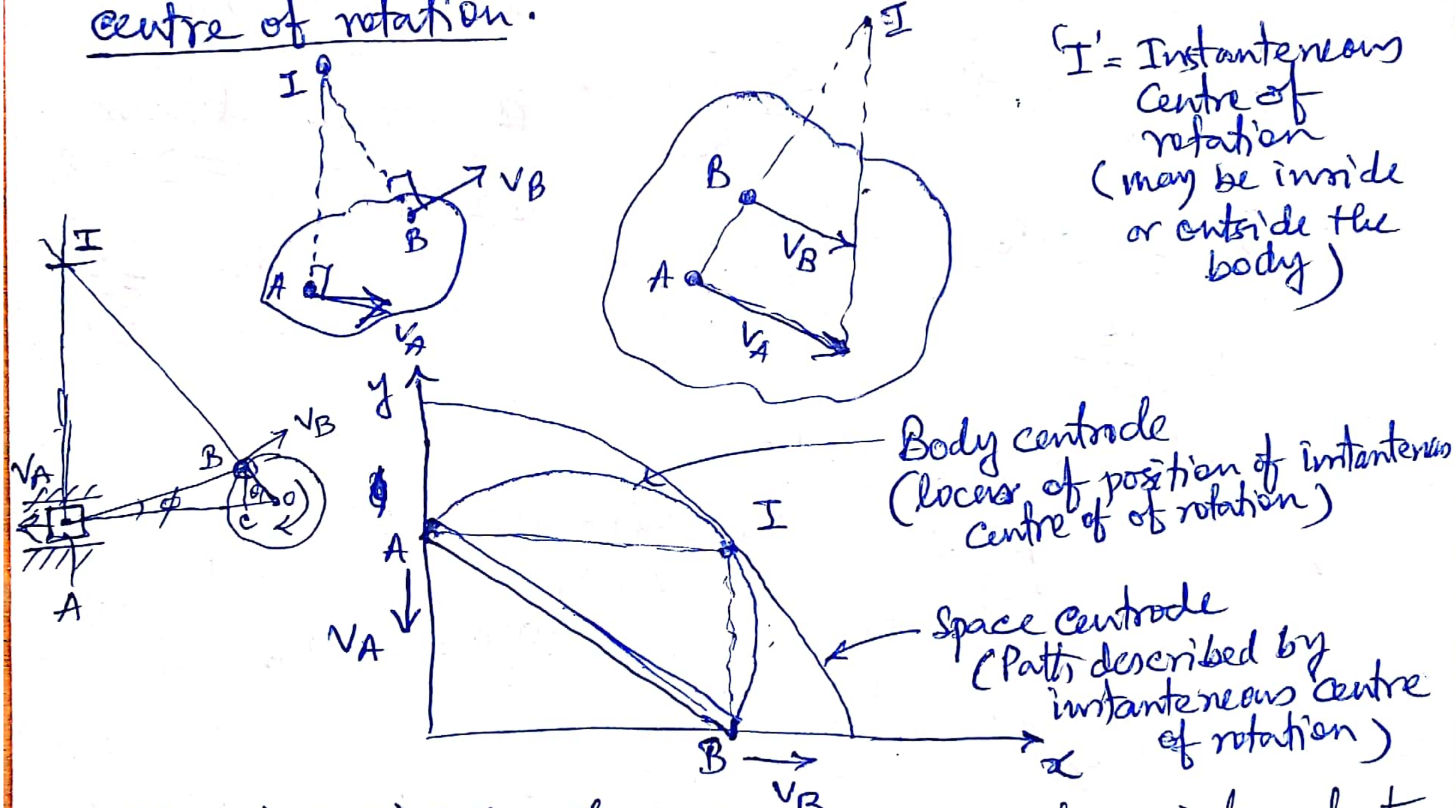


$$V_A = V_B$$
$$r_A \omega_A = r_B \omega_B$$
$$\frac{\omega_A}{\omega_B} = \frac{r_B}{r_A}.$$

$$r_A \alpha_A = - r_B \alpha_B$$
$$\frac{\alpha_A}{\alpha_B} = \frac{-r_B}{r_A}.$$

Instanteneous Centre of Rotation :

At a given instant, the velocities of various particles of a body could be expressed as the result of a pure rotation whose axis is perpendicular to the plane. This axis intersects the plane at a point called instanteneous centre of rotation.



The axis passing through the instanteneous centre point and at right angles to the plane of motion is called instanteneous axis of rotation. The instanteneous centre changes it's position at every moment and it's locus is called centrode. The surface generated by the instanteneous axis is called axode.

Important points on instanteneous centre :–

(a) It is the point about which the body appears to rotate
(b) It is not a fixed point but changes from one instant to another as the body rotates.
(c) The velocity at an instanteneous centre is zero.
(d) Instanteneous centre of a body may be inside or outside the body
(e) The dir^n of velocity at a point on a body is normal to the line joining the point and the instanteneous centre.
(f) I.C. lies at infinite distance when velocities at two points are equal, parallel & in same dir^n.

Corriolis' law :

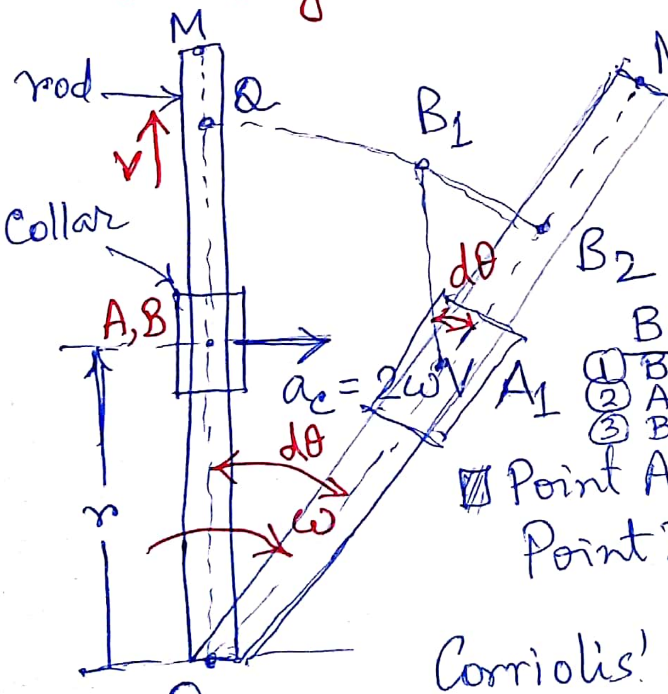
If a point moves along a path that has a rotational motion, the absolute acceleration of the point is given by the vector sum of three acclerations :—

(a) acceleration of the coincident point relative to which the point under consideration is moving

(b) acceleration of the point relative to the coincident point and —

(c) corriolis component of acceleration.

Mathematically, $\underbrace{\vec{a}_A}_{\text{Absolute acc}^n\text{ of particle at point A}} = \underbrace{\vec{a}_B}_{\text{Acc}^n\text{ of point B of moving frame '}\omega\text{' coinciding with A}} + \underbrace{\vec{a}_{A\omega}}_{\text{Acc}^n\text{ of 'A' relative to moving frame '}\omega\text{'}} + \underbrace{\vec{a}_c}_{\text{Corriolis' acc}^n}$



Consider a collar slides at const velocity along the rod OM towards point 'Q'. Simulteneously the rod rotates at const angular velocity 'ω' about 'O'. When the collar moves on the rod to point 'Q' in time 'dt', the rod rotates through an angle 'dθ'. So, finally $A \rightarrow A_1$, $B \rightarrow B_2$

① B to A_1 due to rotation of rod.
② A_1 to B_1 due to outward velocity of collar on the rod.
③ B_1 to B_2 due to corriolis component of acc^n and

☒ Point A is on the rod

Point B is on the collar.

Corriolis' component acts in the direction of relative velocity vector for the two coincident point (A and B) rotated by 90° in the direction of angular velocity of rod and it is expressed as, $\boxed{\vec{a}_c = 2\omega V_{ba}}$

Corriolis force $= m\vec{a}_c$ where m = mass of the collar

The concept of Coriolis acceleration is very useful in the study of long range projectiles and the bodies whose motions are appreciably affected by the rotation of earth.

Chasle's theorem

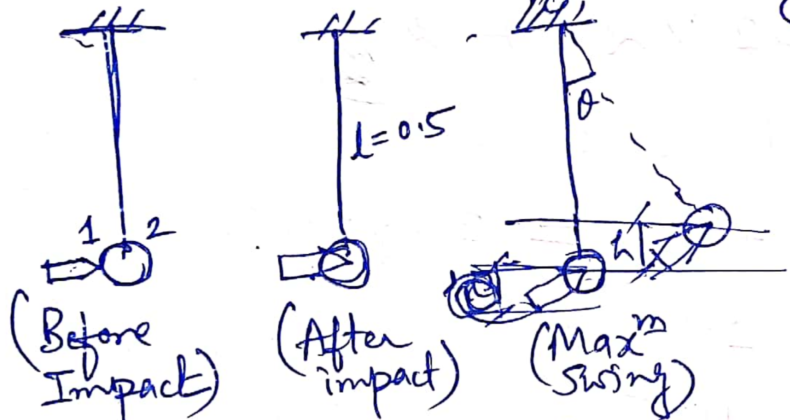
It states that any general displacement of a rigid body can be represented by a combination of translatory motion and rotational motion.

Problem A bullet of mass 20 g moving with a velocity of 100 m/s. hits a 2 kg bob of a simple pendulum horizontally as shown in figure. Determine the max^m angle through which the pendulum string 0·5 m long may swing if:-

(a) the bullet get embedded in the bob.

(b) the bullet escapes from other end at 20 m/s.

(c) the bullet is rebounded from the surface of the bob at 20 m/s.



Given,

$V_1 = 100$ m/s.
$m_1 = 0{\cdot}02$ kg.
$m_1V_1 = 2$ kg-m/s. } (for bullet)

$V_2 = 0$
$m_2 = 2$ kg
$m_2V_2 = 0$. } (for bob)

~~Now, as per conservation of momentum,~~

So, $m_1V_1 + m_2V_2 = 2$ kg m/s → (Initial momentum)

Case (a): Just after impact, the bullet and bob becomes united to travel at a velocity of V' m/s and their momentum $= (2+0{\cdot}02)V' = 2{\cdot}02V'$ kg-m/s.

Equating initial and final momentum,

$$2{\cdot}02V' = 2 \quad \text{or,} \quad V' = 0{\cdot}99 \text{ m/s}.$$

After the impact, the bob must rise by a distance h according to principle of energy conservation,

$$\frac{1}{2}mV'^2 = mgh$$

$$\text{or,} \quad h = \frac{V'^2}{2g} = 0{\cdot}05 \text{ m}.$$

$$\therefore \quad \theta = \cos^{-1}\left(\frac{0{\cdot}5 - 0{\cdot}05}{0{\cdot}5}\right) = 25{\cdot}85^\circ$$

Case (b): If the bullet escaped from other end of bob at a velocity of 20 m/s, the final momentum would be given by, $= (0{\cdot}02 \times 20) + 2 \times V_2'$

Equating with initial momentum, we get,

$$0{\cdot}02 \times 20 + 2V_2' = 2 \text{ kg-m/s} \quad \text{or,} \quad V_2' = 0{\cdot}8 \text{ m/s}.$$

So, $h = \frac{V_2'^2}{2g} = \frac{0.8^2}{2\times 9.81} = 0.0326\ m.$

$$\theta = \cos^{-1}\left(\frac{l-h}{l}\right) = \cos^{-1}\left(\frac{0.5-0.0326}{0.5}\right) = 20.8^0$$

Case (e): If instead the bullet rebounded at 20 m/s, the final momentum would be given by,

$$= -0.02 \times 20 + 2V_3'$$

Equating with initial momentum,

$$(-0.02\times 20) + 2V_3' = 2\ kg\text{-}m/s.$$

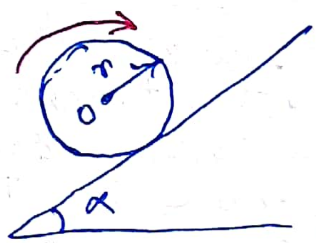
or, $V_3' = 1.2\ m/s.$

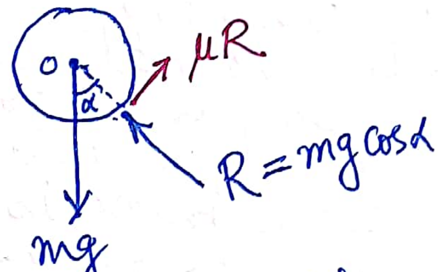
$$\therefore\ h = \frac{V_3'^2}{2g} = \frac{1.2^2}{2\times 9.81} = 0.0734\ m$$

$$\theta = \cos^{-1}\left(\frac{l-h}{l}\right) = \cos^{-1}\left(\frac{0.5-0.0734}{0.5}\right)$$

$$= 31.4^0.$$

Problem A uniform circular cylinder of mass 'm' and radius 'r' is given an initial angular velocity 'ω_0' and no initial translational velocity. It is placed in contact with the plane inclined at 'α' to the horizontal and it moves in upward direction. If 'μ' = co-eff of friction ~~bet~~ for sliding between cylinder and the plane, find the distance the cylinder moves before sliding stops. Assume, $\mu > \tan\alpha$





Using angular momentum eqn to determine the time 't' required to decrease the velocity from 'ω_0' to 'ω'

$$-\mu R r t = I(\omega-\omega_0) = \frac{mr^2}{2}(\omega-\omega_0)$$

$$\text{or, } -\mu(\cancel{m}g\cos\alpha).\cancel{r}.t = \frac{\cancel{m}r^{\cancel{2}}}{2}(\omega-\omega_0)$$

$$\text{or, } t = \frac{r(\omega_0-\omega)}{2\mu g\cos\alpha} \quad ---- (1)$$

The velocity of the centre 'O' parallel to the plane is obtained by applying linear momentum equation,

$$(\mu mg\cos\alpha - mg\sin\alpha)\times t = m(v-0) \quad --(2)$$

(with positive dirn up the plane)

From eqn (1) & (2)

$$\boxed{\begin{aligned}&\cancel{m}g(\mu\cos\alpha-\sin\alpha)\times t = \cancel{m}\omega r\\ &\text{or, } \cancel{g}(\mu\cos\alpha-\sin\alpha)\times\frac{\cancel{r}(\omega_0-\omega)}{2\mu\cancel{g}\cos\alpha} = \omega\cancel{r}\end{aligned}}$$

Note: At the instant sliding stops and pure rolling begins.
$v = r\omega$.

$$\omega = \frac{\mu\cos\alpha-\sin\alpha}{3\mu\cos\alpha-\sin\alpha}\cdot\omega_0 \quad\text{and}\quad t = \frac{r\omega_0}{g(3\mu\cos\alpha-\sin\alpha)}$$

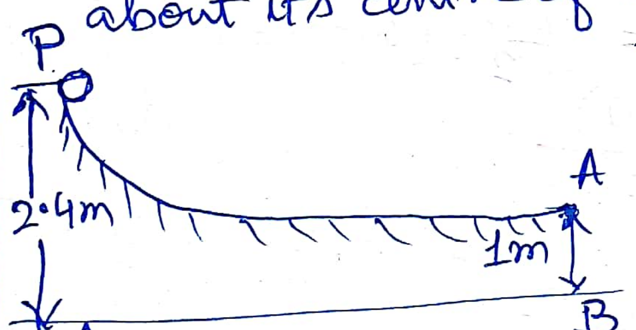
The forces acting on cylinder remain constant during sliding and hence, acceleration is constant.

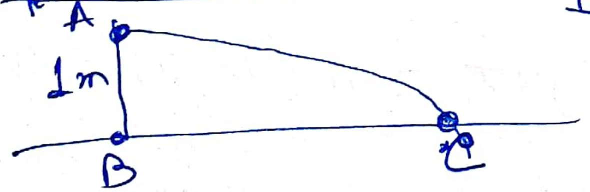
The distance travelled by sliding

$$d = \frac{1}{2} v t = \frac{1}{2} \omega r . t$$

$$= \frac{r^2 \omega_0^2 (\mu \cos\alpha - \sin\alpha)}{2g(3\mu\cos\alpha - \sin\alpha)^2}$$

Problem — A small sphere rolls down without slipping from the top of a track in a vertical plane. The track has an elevated section and a horizontal part as shown in Figure. The horizontal part is 1 m above the ground level and the top of the track is 2·4 m above the ground. Find the distance on the ground with respect to point 'B' (which is vertically below the end of the track as shown) where the sphere lands. During ~~its~~ flight projectile, does the sphere continue to rotate about it's centre of mass?





At Point P,

$KE_1 = 0$

$PE_1 = 2{\cdot}4\, mg$.

At point A,

$$KE_2 = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$$

$$= \frac{1}{2} m v^2 + \frac{1}{2}\left(\frac{2}{5} m R^2\right)\frac{v^2}{R^2}$$

$$= \frac{1}{2} m v^2 + \frac{1}{5} m v^2$$

$$= \frac{7}{10} m v^2$$

$PE_2 = 1 \cdot mg$.

By conservation of energy

$$0 + 2{\cdot}4\, mg = \frac{7}{10} m v^2 + 1 \times mg$$

$$1{\cdot}4\, mg = 0{\cdot}7\, m v^2$$

or, $V = \sqrt{2g}$

$$z = -\frac{g x^2}{2 V_0^2}$$

or $x = \sqrt{2 \times 2g \times 1/g}$

$= 2$ m.

After leaving 'A', the sphere would continue to rotate at the same rotational speed in absence of any external moment acting on it.

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Problem The rectilinear motion of a particle is defined by the time-displacement eqn: $x = x_0(2e^{-kt} - e^{-2kt})$, in which x_0 is the initial displacement, e is natural logarithmic base. Find the maxm velocity of the particle.

$$x = x_0(2e^{-kt} - e^{-2kt})$$

$$\dot{x} = 2x_0(-k)e^{-kt} - x_0(-2k)e^{-2kt}$$

$$= -2kx_0\left[e^{-kt} - e^{-2kt}\right]$$

For attaining maxm velocity.

$$\ddot{x} = 0$$

$$e^{-kt} - e^{-2kt} = 0$$

$$\text{or} \quad e^{kt} = 2$$

$$\text{or,} \quad \boxed{t = \frac{1}{k}\ln 2}$$

$$\therefore \text{Max}^m \text{ velocity} = \dot{x}|_{max} = \dot{x}|_{t=\frac{1}{k}\ln 2} = -\frac{kx_0}{2}.$$

Problem An automobile starting from rest increases its speed from '0' to 'v' with constant acceleration 'α', runs at this speed for a time and finally comes to rest with constant acceleration 'β'. If the total distance travelled is 'S', find the total time (t) required.

While moving with acceln 'α', time reqd $(t_1) = \frac{v-0}{\alpha} = \frac{v}{\alpha}$

While moving with deceln 'β', time reqd $(t_3) = \frac{v-0}{\beta} = \frac{v}{\beta}$

Distance travelled during 't_1', $S_1 = \frac{1}{2}\alpha t_1^2 = \frac{1}{2}(\alpha t_1).t_1$

$$= \frac{1}{2}vt_1$$

Distance travelled during 't_3', $S_3 = \frac{1}{2}\beta t_3^2 = \frac{1}{2}(\beta t_3).t_3$

$$= \frac{1}{2}vt_3$$

Distance travelled during 't_2', $S_2 = vt_2$

Now, $S = S_1 + S_2 + S_3 = \frac{1}{2}vt_1 + vt_2 + \frac{1}{2}vt_3$

or, $vt_2 = s - \frac{1}{2}v(t_1+t_3)$

or, $t_2 = \frac{s}{v} - \frac{1}{2}(t_1+t_3)$.

Total time of travel,

$$t = t_1 + t_2 + t_3$$
$$= \frac{v}{\alpha} + \left[\frac{s}{v} - \frac{1}{2}(t_1+t_3)\right] + \frac{v}{\beta}.$$
$$= \frac{v}{\alpha} + \left[\frac{s}{v} - \frac{1}{2}\frac{v}{\alpha} - \frac{1}{2}\frac{v}{\beta}\right] + \frac{v}{\beta}.$$
$$= \frac{s}{v} + \frac{v}{2}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right).$$

If, $\alpha = \beta$ then

$$t = \frac{s}{v} + \frac{v}{\alpha}$$

Problem The acceleration of a particle along a straight line is given by the eq^n; $a = 4 - t^2/9$. If the particle starts with zero velocity from a position $x = 0$, find:-
(i) it's velocity after 6 sec. (ii) distance travelled in 6 sec.

Here, $a = 4 - \frac{t^2}{9} = \frac{dv}{dt}$.

or, $dv = \left(4 - \frac{t^2}{9}\right)dt$.

Integrating:—

$$\int_0^v dv = \int_0^t \left(4 - \frac{t^2}{9}\right)dt$$
$$\boxed{v = 4t - \frac{t^3}{27}}$$

$$\frac{dx}{dt} = \left(4t - \frac{t^3}{27}\right)$$
$$dx = \left(4t - \frac{t^3}{27}\right)dt$$

Integrating:—

$$\int_0^x dx = \int_0^t \left(4t - \frac{t^3}{27}\right)dt$$
$$\boxed{x = 2t^2 - \frac{t^4}{108}}$$

At $t = 6$ sec,

$v = 16$ m/s and $x = 60$ m.

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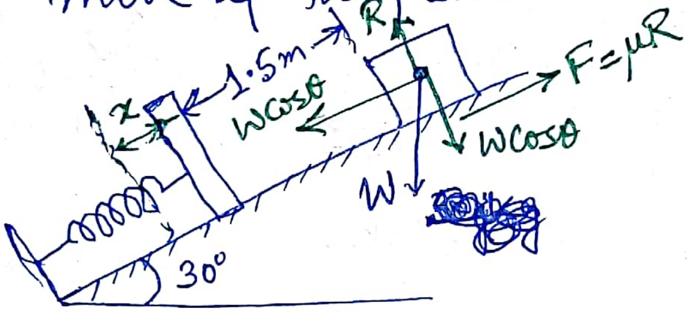
Problem A block of 50 kg mass is released from rest and slides down a 30° inclined plane as shown. After sliding 1.5 m down the plane, the block hits spring of spring constant (K) = 25 N/mm. If the co-eff of friction between the block and the plane is 0.2, make calculations for maximum deflection of the spring and the maxm velocity of the block. Also find the distance the block will move up the plane due to rebound.



Net force on the block when it starts moving downward

$= W\sin\theta - \mu R$

$= mg\sin\theta - \mu W\cos\theta$

$= mg\sin\theta - \mu mg\cos\theta$

$= mg(\sin\theta - \mu\cos\theta)$

$= 160.29$ N.

Let, x is the deformation of the spring.

$\therefore$ Distance moved by the block $= (1.5 + x)$

$\therefore$ Workdone by the block (W_B) = Net force on block × distance moved by the block

$= 160.29 \times (1.5 + x)$

Now, Workdone by the spring (W_s) $= -\frac{1}{2}Kx^2 = -\frac{1}{2}\times(25\times10^3)\times x^2$

$= -12.5\times10^3 x^2$

$\therefore$ Total Workdone $= W_B + W_s$

$= 160.29\times(1.5+x) - 12.5\times10^3 x^2$

The block starts from rest and finally comes to rest when there is maxm deflection of spring. obviously the change in K.E. is zero.

Applying Work-energy principle,

Workdone = change in kinetic energy = 0

$160.29\times(1.5+x) - 12.5\times10^3 x^2 = 0$

or, $12.5x^2 - 0.16x - 0.24 = 0$

or, $x = 0.145$ m or -0.132 m

Hence acceptable $x = 0.145$ m

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(i) According to Newton's Second law, $F = ma$

$$160.29 = 50a$$

$$\text{or, } a = 3.21\ m/s^2$$

From kinematic relations,

$$v^2 - \cancel{u^2} = 2as$$

$$v^2 = 2 \times a \times s = 2 \times 3.21 \times 1.5$$

$$v \simeq 3.10\ m/s \text{ (Max}^m \text{ velo. of block)}$$

(ii) When the block moves up the inclined plane due to rebound, both the components of weight of the block along the plane and the frictional forces act in the same direction.

$$\therefore \text{ force on the block} = W\sin\theta + \mu mg\cos\theta$$

$$= mg\sin\theta + \mu mg\cos\theta$$

$$= 330.21\ N$$

If 's' be the distance which is moved up by the block due to rebound,

Workdone by forces acting on the block

$$= 330.21 \times s$$

This workdone equals to energy stored in spring

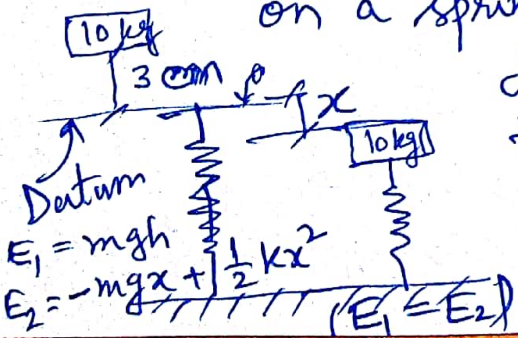
$$\therefore \frac{1}{2}kx^2 = 330.21 \times s$$

$$\text{or } \frac{1}{2} \times (25 \times 10^3) \times (0.145)^2 = 330.21 \times s$$

$$\text{or, } s = 0.796\ m$$

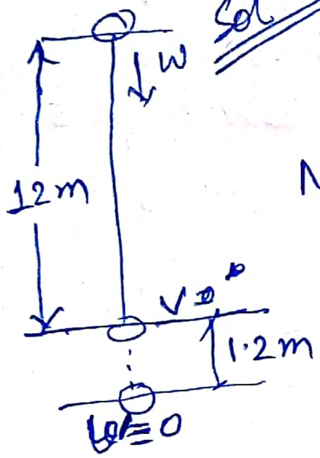
Ex Problem

A body of mass 10 kg is made to fall 3 cm height on a spring of stiffness 120 N/cm. Find the displacement of spring using the concept that total energy of mass-spring system remains constant. [Ans $x = 3.178$ cm]



Problem A body of weight 100 N falls from a height of 12 m on a sand bed. It is estimated that the body penetrates 1.2 m into the sand before coming to rest. Determine the average thrust exerted by the sand on the body.

Soln Velocity after falling 12 m height



$$v = \sqrt{2gh} = 15.34 \text{ m/s}.$$

Now, $v'^2 = v^2 + 2as$.

$0 = 15.34^2 + 2 \times a \times 1.2$

or, $a = -98.05 \text{ m/s}^2$

Let, F_r = Avg. resistance exerted by sand on the body, in upward direction

$\therefore$ Net force acting on body in upward direction

$F_n = F_r - W$ [As, $F_n = ma$]

or, $\frac{100}{9.81} \times 98.05 = F_r - 100$

$F_r = 1100$ N

or,

Problem A train weighing 4000 kN has a frictional resistance of 5 N/kN of weight. Determine the steady pull which the locomotive must exert if the speed of the train is to be increased from 30 km/hr to 60 km/hr within a period of 1.5 min.

Solution Initial velocity $(u) = 30 \text{ km/hr} = 8.33 \text{ m/s}$.

Final ,, $(v) = 60 \text{ km/hr} = 16.66 \text{ m/s}$.

$v = u + at$

or, $a = \frac{v-u}{t} = \frac{16.66 - 8.33}{1.5 \times 60} = 0.092 \text{ m/s}^2$

Frictional resistance (F_f) = 5 N per kN of train weight
$= 5 \times 4000 = 20{,}000$ N.

Net force (F) = Tractive force (F_t) or steady pull − Frictional resistance (F_f)

or, $\frac{4000 \times 10^3}{9.81} \times 0.092 = F_t - 20{,}000$ [where $F = ma$]

$F_t = 57.512$ kN.

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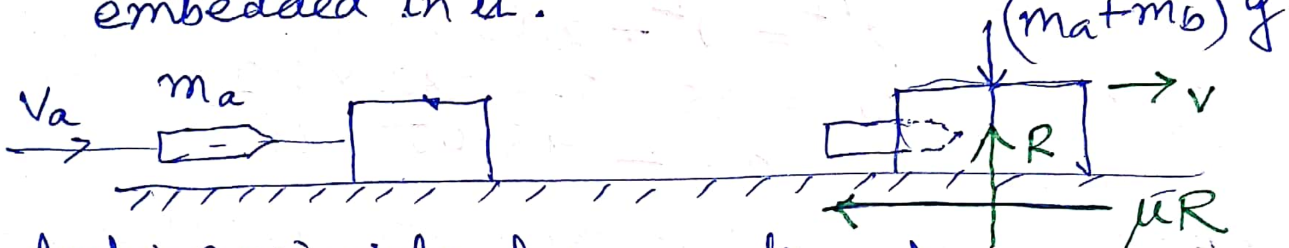
Problem: A wooden block of weight 40 N rests on a rough horizontal plane having friction co-eff $\mu = 0.35$. The block is stuck by a bullet travelling horizontally with a velocity of 750 m/s and weighing 0.25 N. Workout the distance by which the block is displaced from its initial position considering that the bullet after striking the block gets embedded in it.



Applying principles of conservation of momentum,

$$m_a V_a + m_b V_b = (m_a + m_b) V.$$

$$\left(\frac{0.25}{9.81}\right) \times 750 + \left(\frac{40}{9.81}\right) \times 0 = \left(\frac{0.25+40}{9.81}\right) \times V.$$

or $V = 4.657$ m/s.

Force of friction $(F_f) = \mu R = \mu (m_a + m_b) g$

$$= 0.35 \left[\frac{0.25+40}{9.81}\right] \times 9.81$$

$$= 14.087 \text{ N}.$$

Applying Work-energy Correlation,

Workdone to overcome the frictional force = kinetic energy lost by the block with bullet embedded

or, $F_f \times S = \frac{1}{2}(m_a + m_b) \times V^2$

or, $14.087 \times S = \frac{1}{2} \times \left(\frac{0.25+40}{9.81}\right) \times (4.657)^2$

or, $S = 3.097$ m.

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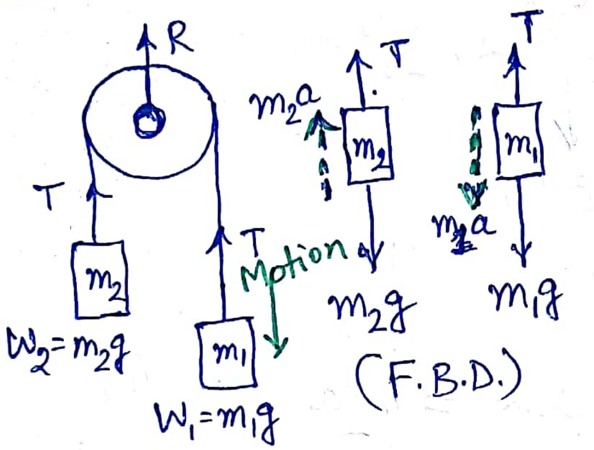
Motion of Connected Bodies:

Motion of two bodies connected each other when are subjected to some external forces, both bodies would be moving with same velocity and acceleration.

Case-1: Two bodies connected by a string passing over smooth pulley



For mass m_1:

$$m_1 g - T = m_1 a \quad \text{.....(i)}$$

For mass m_2:

$$T - m_2 g = m_2 a \quad \text{.....(ii)}$$

Adding eq^n (i) & (ii) we get:-

$$m_1 g - m_2 g = (m_1 + m_2) a.$$

or, $$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g = \left(\frac{W_1 - W_2}{W_1 + W_2}\right) g$$

Putting the value of 'a' in eq^n (i)

$$\left[m_1 = \frac{W_1}{g} \ \& \ m_2 = \frac{W_2}{g}\right]$$

$$m_1 g - T = m_1 a = m_1 \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g$$

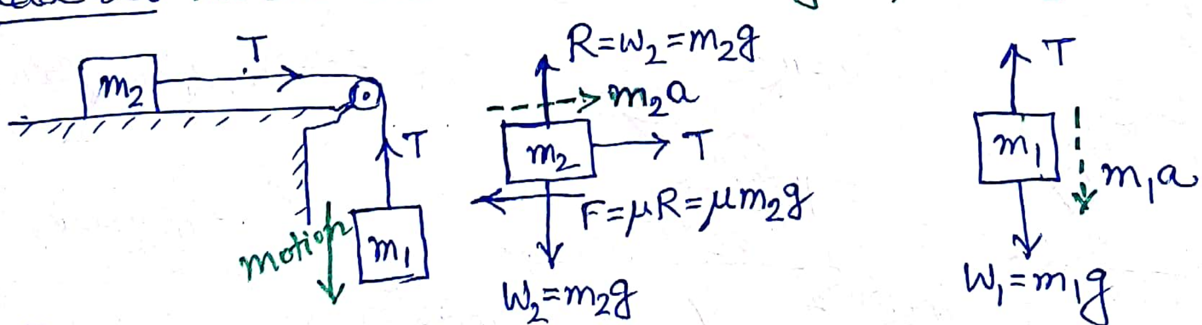
or, $$T = m_1 g \left[1 - \frac{m_1 - m_2}{m_1 + m_2}\right] = \left(\frac{2 m_1 m_2}{m_1 + m_2}\right) g = \left(\frac{2 W_1 W_2}{W_1 + W_2}\right)$$

Pressure on the pulley

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} = \sqrt{T^2 + T^2 + 2.T.T.\cos 0^\circ} = \sqrt{4T^2} = 2T.$$

$$\therefore R = 2T = \frac{4 W_1 W_2}{W_1 + W_2}$$

Case-2: Two bodies connected at the edge of a horizontal surface



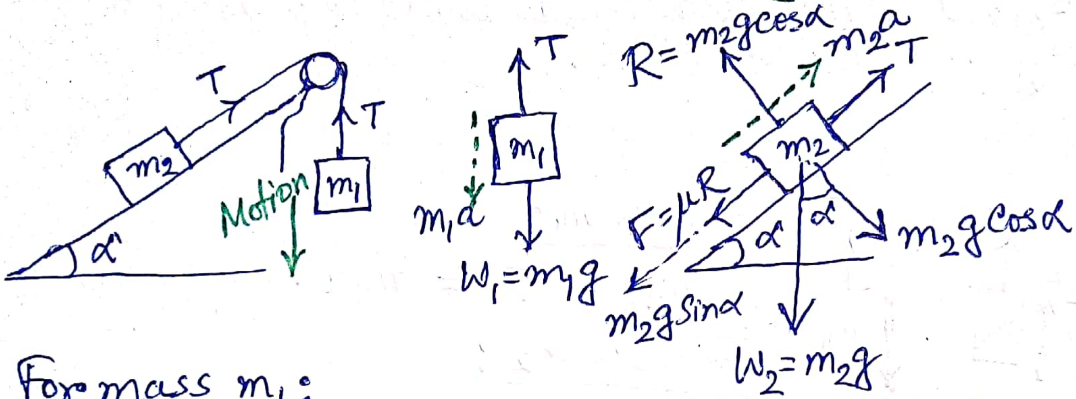
For mass m_1: $$T - \mu m_2 g = m_2 a \quad \text{....(i)}$$

For mass m_2: $$m_1 g - T = m_1 a \quad \text{....(ii)}$$

Solving eq^n (i) & (ii) we get,

$$a = \frac{m_1 g - \mu m_2 g}{m_1 + m_2} = \frac{W_1 - \mu W_2}{W_1 + W_2} \quad \text{and} \quad T = \frac{m_1 m_2 (1 + \mu)}{m_1 + m_2} . g = \frac{W_1 W_2 (1 + \mu)}{W_1 + W_2}$$

Case-3 : Two bodies connected by a string one hangs free and the other on rough inclined plane



For mass m_1:

$$m_1 g - T = m_1 a \quad \text{------ (i)}$$

For mass m_2:

$$T - m_2 g \sin\alpha - \mu m_2 g \cos\alpha = m_2 a \quad \text{------ (ii)}$$

Solving eq^n (i) & (ii), we get,

$$a = \frac{m_1 g - m_2 g \sin\alpha - \mu m_2 g \cos\alpha}{m_1 + m_2} = \left(\frac{W_1 - W_2 \sin\alpha - \mu W_2 \cos\alpha}{W_1 + W_2}\right) \times g$$

$$T = \frac{m_1 m_2 (1 + \sin\alpha + \mu \cos\alpha) g}{m_1 + m_2} = \frac{W_1 W_2 (1 + \sin\alpha + \mu \cos\alpha)}{W_1 + W_2}$$

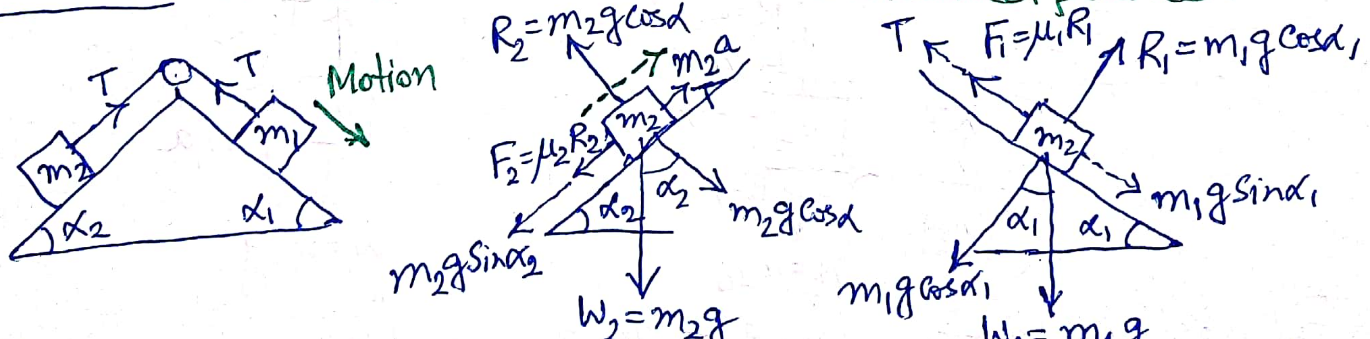
☑ If, $\mu = 0$, $a = \left(\frac{W_1 - W_2 \sin\alpha}{W_1 + W_2}\right) \times g$ and $T = \frac{W_1 W_2 (1 + \sin\alpha)}{W_1 + W_2}$

Pressure on Pulley

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} = \sqrt{T^2 + T^2 + 2.T.T.\cos(90^\circ - \alpha)} = \sqrt{2T^2(1 + \sin\alpha)}$$

$$= T\sqrt{2(1 + \sin\alpha)}$$

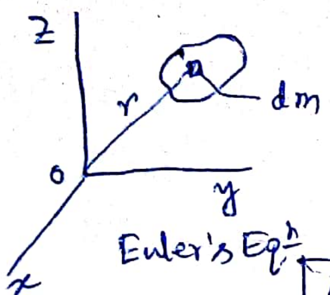
Case-4 : Two bodies connected over inclined planes.



For mass m_1: $m_1 g \sin\alpha_1 - T - \mu_1 m_1 g \cos\alpha_1 = m_1 a$ ------ (i)

For mass m_2: $T - m_2 g \sin\alpha_2 - \mu_2 m_2 g \cos\alpha_2 = m_2 a$ --- (ii)

Momentum and Angular momentum



Momentum of elementary mass $= dm \times v = dm \times \omega r$

Moment of momentum of " " $=$ momentum $\times$ radius

$= dm \times \omega r \times r$

For entire mass, moment of momentum $= \int dm\,\omega r^2$

$= \omega \int dm\, r^2$

$= \omega I_c$ [for circular section, $I = \frac{\pi}{64} d^4$]

Euler's Eqn: $\boxed{M_c = I_c \alpha}$

Impulse-momentum:

From Newton's 2nd law, $F = ma = m \frac{dV}{dt}$

$$\int_{t_1}^{t_2} F\,dt = m \int_{V_1}^{V_2} dV = m(V_2 - V_1)$$

From Euler's eqn; $M_c = I_c \alpha = I_c \cdot \frac{d\omega}{dt}$

$$\int_{t_1}^{t_2} M_c\,dt = I_c \int_{\omega_1}^{\omega_2} d\omega = I_c(\omega_2 - \omega_1) .$$

Linear momentum Conservation principle: $m_1 V_1 + m_2 V_2 = m_1 V_1' + m_2 V_2'$

Angular " " " : $I_a \omega_1 + I_b \omega_2 = I_a \omega_1' + I_b \omega_2'$

Work-Energy formulation:

Linear: $dW = F \cdot dr$

$$W = \int dW = \int_{r_1}^{r_2} F \cdot dr$$

Angular: $dW = M\,d\theta$

$$W = \int dW = \int_{\theta_1}^{\theta_2} M \cdot d\theta .$$

where, $M =$ Moment

$\boxed{\oint F \cdot dr = 0}$ (for Conservative force)

$PE = mg \int_{h_1}^{h_2} dh$ and $KE = \frac{1}{2} m v^2 + \frac{1}{2} I_c \omega^2$

$$W_{Linear} = \int_{r_1}^{r_2} F \cdot dr = \frac{1}{2} m (V_2^2 - V_1^2) = KE_{2_{Lin}} - KE_{1_{Lin}} .$$

$$W_{Angular} = \int_{\theta_1}^{\theta_2} M\,d\theta = \frac{1}{2} I_c(\omega_2^2 - \omega_1^2) = KE_{2_{Angu}} - KE_{1_{Angular}}$$

$$W_{Total} = W_{Linear} + W_{Angular} = \int_{r_1}^{r_2} F \cdot dr + \int_{\theta_1}^{\theta_2} M_c\,d\theta$$

$$= \frac{1}{2} m (V_2^2 - V_1^2) + \frac{1}{2} I_c(\omega_2^2 - \omega_1^2)$$

$$= \left[\frac{1}{2} m V_2^2 + \frac{1}{2} I_c \omega_2^2\right] - \left[\frac{1}{2} m V_1^2 + \frac{1}{2} I_c \omega_1^2\right]$$

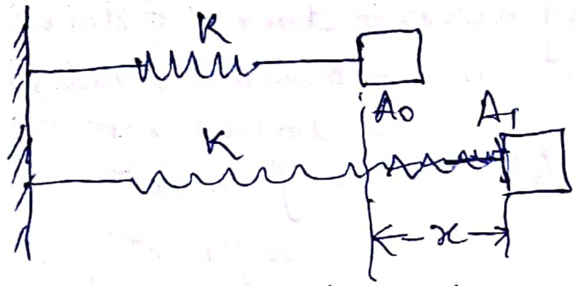
$$= KE_2 - KE_1$$

$$(PE_1 - PE_2)_{Total} = (KE_2 - KE_1)_{Total}$$

$\therefore (KE_1 + PE_1)_T = (KE_2 + PE_2)_T$

(Principle of conservation of energy)

Workdone by Spring:



Spring force $\propto$ displacement

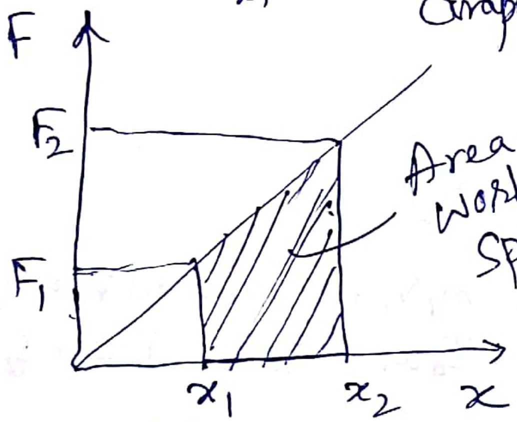
$F \propto x$

$\therefore F = -Kx$ K = Spring stiffness in N/m

Elementary workdone by spring $= F\,dx = -Kx\,dx$

$$W = -\int Kx\,dx = -\frac{1}{2}Kx^2$$

$$W = \int_{x_1}^{x_2} Kx\,dx = -\frac{1}{2}(x_2^2 - x_1^2) = \frac{1}{2}(x_1^2 - x_2^2)$$



Area represents workdone by Spring force

Graphically, Again, $W = \frac{1}{2} \times (F_1 + F_2) \times (x_2 - x_1)$

= Area of trapezium

$= \frac{1}{2}\{(-Kx_1) + (-Kx_2)\} \times (x_2 - x_1)$

$= -\frac{1}{2}K(x_2 + x_1)(x_2 - x_1)$

$= -\frac{1}{2}K(x_2^2 - x_1^2)$

$= \frac{1}{2}K(x_1^2 - x_2^2)$

Workdone by the spring force is positive when $x_2 < x_1$

ie, when the spring is returning to its original position

Kinetics of Particle:

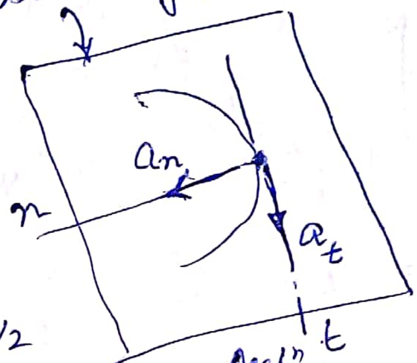
$$\boxed{\Sigma F = ma}$$

$$\Sigma F_x = ma_x$$

$$\Sigma F_y = ma_y$$

$$\Sigma F_z = ma_z$$

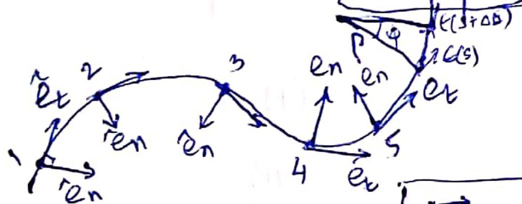
motion only occurs in Osculating plane



$$\boxed{a_n = \frac{v^2}{\rho}}$$ where, $\rho = \dfrac{(1+y_1^2)^{3/2}}{y_2}$

Note; Accl^n or Force along bi-normal dir^n is zero

a_b or $F_b = 0$ $(a_t \times a_n)$



$e_t \rightarrow$ tangential component of unit vector (Dir^n of velocity or motion) (Example-headlight of car)

$$\boxed{v = v\hat{e}_t}$$

$$\boxed{\vec{a} = a_t\hat{e}_t + a_n\hat{e}_n}$$

$a_t\hat{e}_t$: Changes speed (By brake or accelerator)

$a_n\hat{e}_n$: Changes dir^n (By steering wheel)

$e_n \rightarrow$ normal component of unit vector, along radius of curvature. (Dir^n of turning)

$$\bar{a} = \frac{d\bar{v}}{dt} = \frac{d}{dt}(v\hat{e}_t)$$

$$= v\left(\frac{de_t}{dt}\right) + \frac{dv}{dt}\hat{e}_t$$

Now, $\dfrac{de_t}{dt} = \left(\dfrac{d\hat{e}_t}{d\theta}\right)\left(\dfrac{d\theta}{dt}\right) = \hat{e}_n \cdot \omega$

$$= v\omega\hat{e}_n + \frac{dv}{dt}\hat{e}_t$$

Now, $v = \dfrac{ds}{dt} = \rho\dfrac{d\theta}{dt} = \rho\omega$

$$= \rho\cdot\omega\cdot\omega\cdot\hat{e}_n + \frac{dv}{dt}\cdot\hat{e}_t$$

$$= \rho\omega^2\hat{e}_n + \frac{dv}{dt}\hat{e}_t$$

$$= \frac{(\rho\omega)^2}{\rho}\cdot\hat{e}_n + \left(\frac{dv}{dt}\right)\cdot\hat{e}_t$$

$$= \frac{v^2}{\rho}\cdot\hat{e}_n + \left(\frac{dv}{dt}\right)\cdot e_t$$

$$= \underbrace{\dot{v}\hat{e}_t}_{a_t \text{ or tangential accl}^n} + \underbrace{\left(\frac{v^2}{\rho}\right)\hat{e}_n}_{\text{Centripetal accl}^n (a_n)}$$

Straight line motion:

$$\rho \rightarrow \infty, \quad a_n = \frac{v^2}{\rho} = 0, \quad a = a_t = \dot{v}$$

Constant speed along the path

$$a_t = \dot{v} = 0, \quad a = a_n = \frac{v^2}{\rho}$$

Constant tangential accl^n.

$a_t = (a_t)_c$

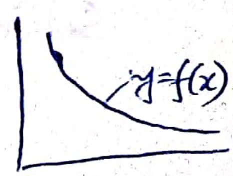
$$v = (v_0) + (a_t)_c t$$

$$s - s_0 = (v_0)t + \frac{1}{2}(a_t)_c t^2$$

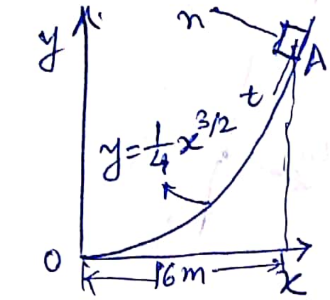
$$v^2 - v_0^2 = 2(a_t)_c(s - s_0)$$

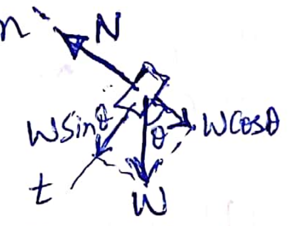
Path: $y = f(x)$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left(\frac{d^2y}{dx^2}\right)}$$



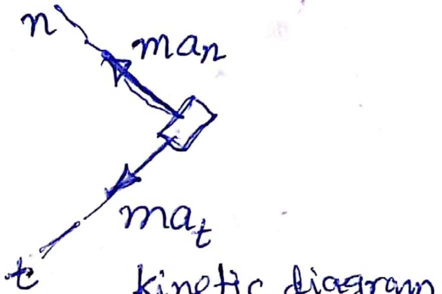
Problem: An 10 kg crate travels along a smooth slope as shown. If at the point 'A', its speed is 20 m/s, determine the normal force exerted by the slope to the crate. Also what is it's acceleration?





Free-body diagram

$\equiv$



Kinetic diagram

By force balancing,

$$\Sigma F_n = N - W\cos\theta = m a_n \quad \text{------ (I)}$$

$$\Sigma F_t = W\sin\theta = m a_t \quad \text{------ (II)}$$

Now, $y = \frac{1}{4}x^{3/2}$ or, $\frac{dy}{dx} = \text{Slope} = \frac{3}{8}x^{\frac{1}{2}}$

At $x = 16\,m$, slope $= \frac{3}{8} \times 16^{\frac{1}{2}} = 1.5 = \tan\theta$

or, $\theta = \tan^{-1}(1.5) = 56.3^\circ$

We know, $a_n = \frac{v^2}{\rho}$ Here, $v = 20\,m/s$.

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left(\frac{d^2y}{dx^2}\right)}, \quad \text{Now, } \frac{d^2y}{dx^2} = \frac{3 \times 1 x^{-1/2}}{8 \times 2} = \frac{3}{16}x^{-1/2}$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=16} = \frac{3}{64}$$

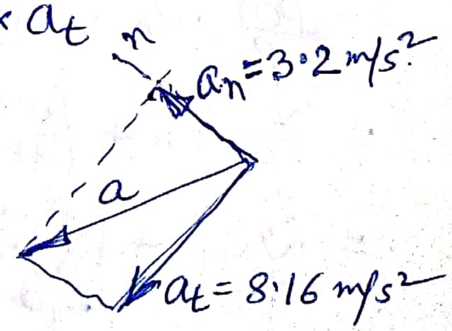
$\rho = 125\,m$

So, $a_n = \frac{v^2}{\rho} = \frac{20^2}{125} = 3.2\ m/s^2$

From eqn (I) & (II).

$$N - 10 \times 9.81 \cos 56.3^\circ = 10 \times 3.2$$

$$10 \times 9.81 \sin 56.3^\circ = 10 \times a_t$$



Solving the above eqn we get

$$a_t = 8.16\ m/s^2$$

$$\underline{N = 86.4\,N}$$

$$a = \sqrt{a_n^2 + a_t^2} = \underline{8.77\ m/s^2}$$