Art - 6.1 Rectilinear Translation

ENGINEERING MECHANICS (ME-101) Sl. No. :2018/ 6200

NATIONAL INSTITUTE OF TECHNOLOGY SILCHAR

DYNAMICS

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Prob-2 The velocity time relationship is $\dot{x} = \frac{1}{2} c \cdot t^2$ where $c = 8 \frac{ft}{sec^3}$

So, $\frac{dx}{dt} = \frac{1}{2} \times 8 \times t^2 = 4 \cdot t^2$

Integrating once, $x = \frac{4t^3}{3} + C$ Now, since the initial displacement is zero, $C = 0$

So, $x = \frac{4 \cdot t^3}{3}$

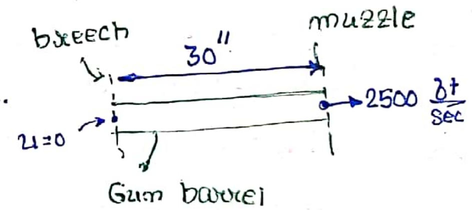
when $t = 3$ sec., $x = \frac{4 \times (3)^3}{3} = \boxed{36 ft}$

Prob-5 we use two formulas: $V = u + f \cdot t$ or, $10 = f \cdot t$ ....(1)

and $S = u \cdot t + \frac{1}{2} f \cdot t^2$ or, $25 = \frac{1}{2} f \cdot t^2$ ...(2)

From these two equations, we obtain $\boxed{f = 2 \frac{ft}{sec^2}}$

Prob-6. The bullet attains a velocity of 2500 ft/sec in time 't', starting from rest.



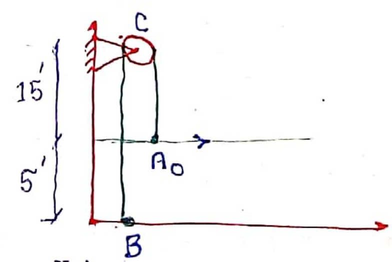
Thus we have two equations:

$v = f \cdot t$ or $2500 \times 12 = f \cdot t$ .....(1)

and $S = \frac{1}{2} f \cdot t^2$ or, $f \cdot t^2 = 60$ ...(2)

From these two equations, we obtain. $\boxed{t = 0.002 \text{ Sec.}}$

Prob-4 Total length of the rope $= 2 \times 15 + 5 = 35'$



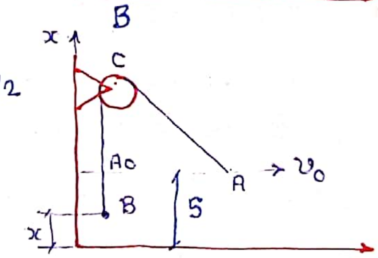
For the second case,

Length of rope $= AC + CA_0 + 5 - x$

$= AC + 20 - x$.

Since, the total length remains constant,

$AC + 20 - x = 35$ or $AC = x + 15$



Now, $AC = \left[AA_0^2 + CA_0^2\right]^{1/2}$

$= \left[v_0^2 t^2 + h^2\right]^{1/2}$

So, $\frac{dx}{dt} = \frac{d(AC)}{dt} =$

or, $\frac{dx}{dt} = \frac{d}{dt}\left[h^2 + v_0^2 t^2\right]^{1/2}$

or, $\frac{dx}{dt} = \frac{v_0^2 \cdot t}{\left[h^2 + v_0^2 \cdot t^2\right]^{1/2}}$ → This is the velocity of point B.

Now, when point B will reach the pulley at C, $AC = 35'$

So, $AA_0^2 + (CA_0)^2 = AC^2$ So, $AA_0 = 35^2 - 15^2 = 1{,}000$

or $v_0 \cdot t = (1000)^{1/2}$ or. $t = \frac{(1000)^{1/2}}{10} = \boxed{3.16 \text{ Sec.}}$

Prob-10 The displacement-time equation is, $x = x_0(2\cdot e^{-K\cdot t} - e^{-2Kt})$

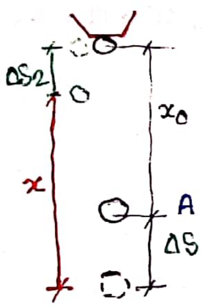
So, $\frac{dx}{dt} = x_0\left[-2K\cdot e^{-K\cdot t} + 2K\cdot e^{-2Kt}\right]$ .....(1)

Another integration yields, $\frac{d^2x}{dt^2} = x_0\left[2K^2\cdot e^{-K\cdot t} - 4K^2\cdot e^{-2Kt}\right]$ ——(2)

For maximum velocity, we set $\frac{d^2x}{dt^2} = 0$ and obtain $\boxed{t = \frac{1}{2}\ln K}$

The maximum velocity is obtained by substituting $t = \frac{1}{2}\ln K$ in eqn. (1), to obtain $\left(\frac{dx}{dt}\right)_{max} = \boxed{-\frac{K\cdot x_0}{2}}$

Prob-8



Time interval between two drops $= \frac{1}{m}$ sec.

For the first drop, $x_0 = \frac{1}{2}\cdot g\cdot t^2 = \frac{g}{2m^2}$

The velocity of first drop at $A = \sqrt{2g\cdot\frac{g}{2m^2}} = \frac{g}{m}$

So, $\Delta S_1 = \frac{g\cdot t}{m} + \frac{1}{2}g\cdot t^2$

For the second drop, $\Delta S_2 = \frac{1}{2}gt^2$

From the figure, $x = x_0 + \Delta S_1 - \Delta S_2$

or, $\boxed{x = \frac{g}{2m^2} + \frac{g\cdot t}{m}}$

Prob-13 let t_1, t_2 and t_3 be the different intervals. S_1, S_2 and S_3 are the corresponding distances.

For the first phase: $v = a_1\cdot t_1$ or $t_1 = \frac{v}{a_1}$ and $S_1 = \frac{1}{2}a_1\cdot t_1^2 = \frac{v^2}{2a_1}$

For the second phase, $S_2 = v\cdot t_2$ or $t_2 = \frac{S_2}{v}$

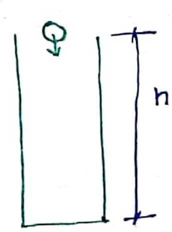
For the third phase, $S_3 = \frac{v^2}{2a_2}$ and $t_3 = \frac{v}{a_2}$

Now, $t = t_1 + t_2 + t_3 = \frac{v}{a_1} + \frac{S_2}{v} + \frac{v}{a_2} = v\cdot\left(\frac{1}{a_1} + \frac{1}{a_2}\right) + \frac{S_2}{v}$

or, $t = v\cdot\left(\frac{1}{a_1} + \frac{1}{a_2}\right) + \frac{1}{v}\cdot(S - S_1 - S_3)$

Substituting the values of S_1 and S_2, we obtain $\boxed{t = \frac{S}{v} + \frac{v}{2}\left(\frac{1}{a_1} + \frac{1}{a_2}\right)}$

Prob-9 Downward Journey: $h = \frac{1}{2}g\cdot t_1^2$ ....(1)



upward travel by sound wave: $h = 1120\cdot t_2$ ....(2)

Equating these two, $1120\cdot t_2 = \frac{32\cdot 2}{2}\cdot t_1^2$ or, $t_1^2 = 69.56\, t_2$

Now, $t_1 + t_2 = 6.5$ or $t_1 + \frac{t_1^2}{69.56} = 6.5$

Solving this quadratic equation, we obtain $t_1 = 5.98$ Secs.

So, using eqn. (1), $h = \frac{1}{2}\times 32.2\times(5.98)^2 = \boxed{577\text{ ft.}}$

Ax1- 6.3 :

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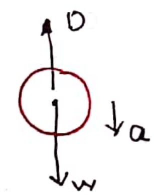
Prob: 6

According to the problem, $D = K \cdot V^2$

and when $V = 1$ ft/sec, $D = f$ $\therefore K = f$

Thus, $D = f \cdot V^2$



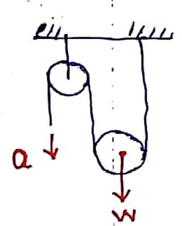
The equation for downward motion is,

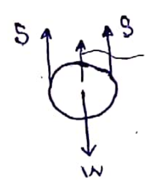
$$W - D = \frac{W}{g} \cdot a.$$

when uniform velocity will be attained, the acclm. will be zero.

$\therefore W = D = f \cdot V^2$ or, $\boxed{V = \sqrt{\frac{W}{f}}}$

Prob-7





$\frac{W}{g} \cdot \frac{a}{2}$

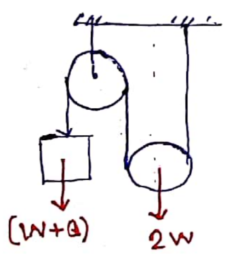
Force balance yields,

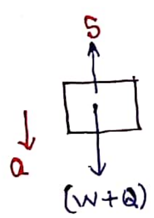
$$2S - W = \frac{W}{g} \cdot \frac{a}{2}$$

or, $S = \frac{1}{2} \cdot W \left[1 + \frac{a}{2g}\right]$

or, $S = \frac{1000}{2}\left[1 + \frac{6}{2 \times 32.2}\right] = \boxed{546.6 \text{ lb}}$

Prob- 8





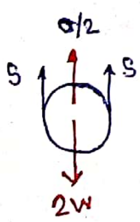
Force balance gives

$$W + Q - S = \frac{W+Q}{g} \cdot a$$

or, $S = (W+Q) - \frac{(W+Q) \cdot a}{g}$

or, $S = (W+Q)\left(1 - \frac{a}{g}\right)$

or $S = (W+Q) \times 0.9$



Force balance gives,

$$2S - 2W = \frac{2W}{g} \cdot \frac{a}{2}$$

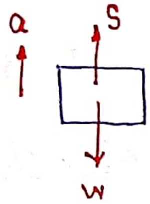
or, $S - W = \frac{W \cdot a}{2g} = 0.05W$. or $S = 1.05W$.

Substituting the value of S

$1.05W = (W+Q) \times 0.9$ or $W + Q = 1.166 W$.

or, $\boxed{Q = \frac{W}{6}}$

Prob-9



Equation of motion is. $\frac{W}{g}a = S - W$ or, $S = W\left(1+\frac{a}{g}\right)$.

Now let us consider the rim.

For every rotation (full) of the rim, the effective radius increases by 0.25" i.e the thickness of the chord.

So, rate of change of linear velocity of the chord

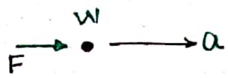
$$= \frac{V_2 - V_1}{\Delta t} = \frac{\omega.(R_2 - R_1)}{\Delta t} = \frac{\omega \times 0.25}{0.5} = \frac{2\pi \times 2 \times 0.25}{0.5}$$

one rev = 0.5 sec.

$$= 6.28 \frac{\text{inch}}{\text{sec}^2} = 0.523 \frac{\text{ft}}{\text{sec}^2} = a$$

On substitution, $S = W\left(1 + \frac{0.523}{32.2}\right) = \boxed{1.016\,W}$

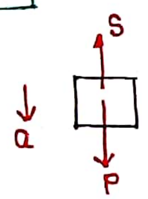
Art. 6.4 Motion of a particle acted upon by a constant force



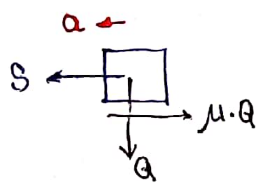
The equation of motion is, $F = \frac{W}{g}.a$ or $a = \frac{d^2x}{dt^2} = \frac{F.g}{W}$

Since, 'F' is constant, 'a' is also a constant. This equation can then easily be integrated to determine different quantities as displacement and velocity as a function of time.

Prob-5
P-271



Equation of motion for P is $P - S = \frac{P}{g}.a$ ....(1)



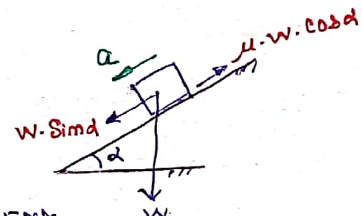
Equation of motion for Q is $S - \mu.Q = \frac{Q}{g}.a$ ....(2)

Adding (1) & (2) we obtain

$P - \mu.Q = \frac{P+Q}{g}.a$ or, $a = \frac{P-\mu Q}{P+Q}.g$

Substituting the values, $\boxed{a = \frac{3g}{11}}$

Prob-8 Equation of motion of the block



$\frac{W}{g}a = W.\sin\alpha - \mu.W.\cos\alpha$ ....(1)

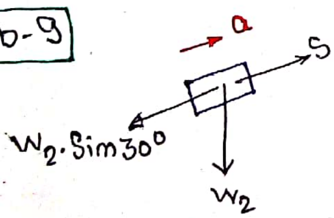
At α=30°, motion impends. So, $W.\sin\alpha = \mu.W.\cos\alpha$.

or, $\mu = \tan\alpha = \tan 30^\circ = 0.577$

when α = 45°, we have from eqn-(1),

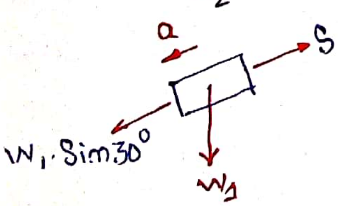
$\frac{a}{g} = \sin 45^\circ - 0.577 \times \cos 45^\circ = 0.3$ or, $\boxed{a = 0.3g}$

Prob-9



Equation of motion for W2,

$\frac{W_2}{g}.a = S - W_2.\sin 30^\circ$ ......(1)



Equation of motion for W1,

$\frac{W_1}{g}.a = W_1.\sin 30^\circ - S$ ...(2)

Adding (1) and (2), $\frac{W_1+W_2}{g}.a = (W_1 - W_2).\sin 30^\circ$

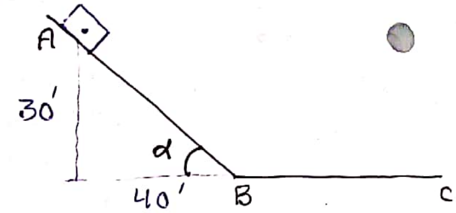
$\therefore a = g/6$

Considering the linear motion of W2,

$100 = \frac{1}{2}.a.t^2$ or, $t^2 = \frac{200}{a}$ or $\boxed{t = 6.1 \text{ sec.}}$

Prob-10 From the Figure, $\tan\alpha = \frac{30}{40}$ $\therefore \alpha = 36.87^\circ$ $AB = \sqrt{30^2+40^2} = 50'$

$\mu = 0.3$ (data)



Equation of motion for block A,

$$\frac{W}{g}\cdot a = W\cdot\sin\alpha - \mu\cdot W\cdot\cos\alpha$$

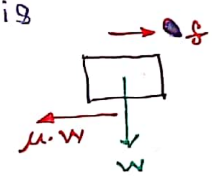
or, $a = 0.36\,g$.

The velocity at B, $V_B^2 = V_A^2 + 2\cdot a\cdot AB$

Since the block was initially at rest, $V_A = 0$.

So, $V_B = \sqrt{2\times 0.36g\times 50} = 34.04\ \frac{m}{sec}$.

For the motion along BC, the equation of motion is

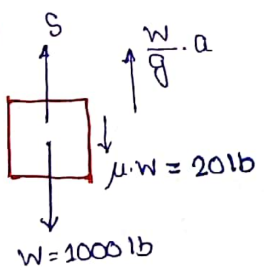


$$\frac{W}{g}\cdot f = \mu\cdot W \qquad \text{So, } f = \mu\cdot g = 0.3g$$

Now, $V_C^2 = V_B^2 - 2f\cdot BC$

Since, $V_C = 0$, we have $BC = \frac{V_B^2}{2f} = \frac{(34.04)^2}{2\times 0.3\times g} = \boxed{60'}$

Prob-3



The equation of motion is

$$\frac{W}{g}a = S - \mu\cdot W - W$$

The motion of the elevator is divided into two parts.

1> Uniform motion

For this case, $a = 0$ and $S = W(1+\mu) = 1020\,lb$

2> In the second case, power is switched off and consequently, the elevator decelerates.

So, $\frac{W}{g}a = -W - \mu\cdot W = -1020.$ Or, $a = -32.84\ \frac{ft}{sec^2}$.

Now, $V^2 = u^2 + 2a\cdot x$. where 'x' is the distance travelled before the elevator stops.

$\therefore 0^2 = (12)^2 + 2\times(-32.84)\cdot x$ or, $\boxed{x = 2.19\ ft.}$

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Asr: 6.5: Force as a function of time

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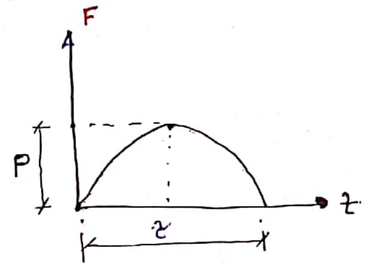
Prob. 4 The variation of force with respect to time is described by a parabola.



let the equation of the parabola is described by

$$F = a\cdot t^2 + b\cdot t + c$$

Now, the constants a, b and c are to be determined from the following:

when $t=0,\ F=0$ so, $c=0$

when $t=\tau,\ F=0$ so, $a\tau^2 + b\cdot\tau = 0$ or, $a\cdot\tau + b = 0$ ....(1)

when $t=\frac{\tau}{2}$ $F=P$ so, $P = a\cdot\frac{\tau^2}{4} + b\cdot\frac{\tau}{2}$ ...(2)

Replacing b, $P = a\cdot\frac{\tau^2}{4} - \frac{a\cdot\tau^2}{2} = -\frac{a\cdot\tau^2}{4}$ or, $a = -\frac{4P}{\tau^2}$

and $b = \frac{4P}{\tau}$

So, the equation of the curve

$$F = -\frac{4P}{\tau^2}\cdot t^2 + \frac{4P}{\tau}\cdot t$$

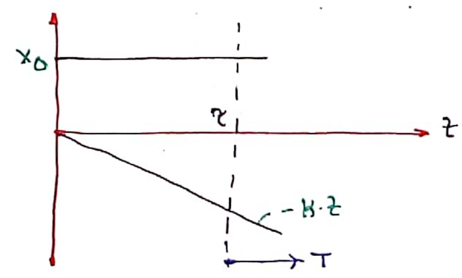
Now, $m\cdot\frac{d^2x}{dt^2} = -\frac{4P}{\tau^2}\cdot t^2 + \frac{4P}{\tau}\cdot t$

Assuming that at $t=0$, $x = \dot{x} = 0$, successive integration yields

$$m\cdot x(t) = -\frac{P\cdot t^4}{3\tau^2} + \frac{2P\cdot t^3}{3\tau}$$

So, the displacement at $t=\tau$ is, $\boxed{x(\tau) = \frac{P\cdot\tau^2}{3m}}$

Prob-8 Given $x_0 = 12\,lb$ and $K = 2\frac{lb}{sec}$



The governing eqn. of motion

$$m\cdot\frac{d^2x}{dt^2} = x_0 - K\cdot t$$

when $t=\tau$, $x_0 = K\cdot\tau$ So, $\tau = \frac{12}{2} = 6$ sec.

At this point of time, the acceleration in the positive x-direction becomes zero. Since the force become negative, the forward motion of the particle is not possible. So, the particle begins it motion back to the origin. let T be the new time variable such that $t = T+\tau$. Replacing x by $-x$ in the governing eqn., it assumes the form $-m\cdot\frac{d^2x}{dT^2} = x_0 - K\cdot T$ or, $m\cdot\frac{d^2x}{dT^2} = K\cdot T - x_0$

or, $m\cdot\frac{dx}{dT} = \frac{K\cdot T^2}{2} - x_0 T = 0$ So, $T = \frac{2x_0}{K} = \frac{2\times 12}{2} = 12$ sec. $\therefore t = 12+6 = \boxed{18\text{ sec.}}$

Prob-2 The governing eqn. is $m\cdot\frac{d^2x}{dt^2} = K\cdot t$.

Integrating once, $m\cdot\frac{dx}{dt} = K\cdot\frac{t^2}{2} + a$. Now, at $t=0$, $\frac{dx}{dt}=0$ $\therefore a=0$.

Integrating once more, $m\cdot x = \frac{Kt^3}{6} + b$

at $t=0$, $x=0$; So, $b=0$.

Thus $\frac{dx}{dt} = \frac{K\cdot t^2}{2m}$ and $x = \frac{Kt^3}{6m}$

So, $\frac{x}{\dot{x}} = \frac{Kt^3}{6m} \times \frac{2m}{K\cdot t^2} = \boxed{\frac{t}{3}}$

Prob-6 The equation of motion is, $m\cdot\frac{d^2x}{dt^2} = X_0\cdot \sin\omega t$ or, $\frac{d^2x}{dt^2} = A\cdot \sin\omega t$ where $A = \frac{X_0}{m}$

where ω is the angular speed $= \frac{2\pi}{T}$ and T = time period.

Since, $\omega = 8$ we have $T = \frac{2\pi}{8} = \frac{\pi}{4}$.

Integrating the governing equation once, $\left.\frac{dx}{dt}\right|_0^t = \left[-\frac{A}{\omega}\cdot\cos\omega t + a\right]$

At $t=0$, $\frac{dx}{dt}=0$ $\therefore a = \frac{A}{\omega}$ Thus $\frac{dx}{dt} = \frac{A}{\omega} - \frac{A}{\omega}\cdot\cos\omega t$

Integrating once again,

$x = \frac{A\cdot t}{\omega} - \frac{A}{\omega^2}\cdot\sin\omega t + b$ Now, at $t=0$, $x=0$. $\therefore b=0$.

So, $x = \frac{A\cdot t}{\omega} - \frac{A\cdot\sin\omega t}{\omega^2}$

Now, according to the problem, for a complete force cycle (i.e, $t=T$), $x=10$.

Thus, $10 = \frac{A\cdot T}{\omega}$ or $\frac{X_0}{m}\cdot\frac{T}{\omega} = 10$ or, $X_0 = \frac{10\times8\times3.14}{32.2\times\pi/4} = \boxed{9.93\text{lb}}$

Art 6.6 Force proportional to displacement.

Prob-1 According to the problem $W = 10\,lb$; $K = 5 \frac{lb}{inch}$

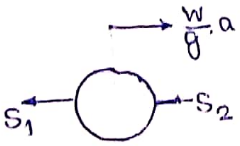
So, $\delta = \frac{W}{K} = \frac{10}{5} = 2$ inch

Hence, $T = 2\pi\sqrt{\frac{\delta}{g}} = 2\pi\sqrt{\frac{2}{32.2\times12}} = \boxed{0.452\ s}$

Prob-2 Since a 10 lb tension produces an elongation of 1", $K = \frac{10}{1} = 10 \frac{lb}{inch}$

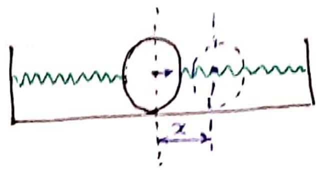
Frequency $= \frac{1}{2\pi}\sqrt{\frac{K\cdot g}{W}} = \frac{1}{2\pi}\sqrt{\frac{10\times32.2\times12}{1}} = \boxed{9.9\ sec^{-1}}$

Prob-6



The equation of motion

$\frac{W}{g}\cdot a = -S_1 - S_2 = -x - x = -2x.$



or, $\frac{d^2x}{dt^2} = -\frac{2\cdot g}{W}\cdot x = -m^2\cdot x$ where $m^2 = \frac{2\cdot g}{W} = \frac{2\times32.2\times12}{1}$

or, $m = 27.8$

Now, $m = \frac{2\pi}{T}$ $\therefore T = \frac{2\pi}{27.8} = \boxed{0.226\ Sec}$ Now $x_0 = 1''$

The soln. to the eqn. of motion is, $x = x_0\cdot \cos m\cdot t$

So, $x = \cos mt$ $\therefore \frac{dx}{dt} = -m\cdot \sin mt$

when $x = 0$, $mt = \frac{\pi}{2}$ At the origin, $\left.\frac{dx}{dt}\right|_{t=\frac{\pi}{2m}} = -m\cdot\sin\left(m\cdot\frac{\pi}{2m}\right) = \boxed{-27.8\ inch/sec}$

$\therefore t = \frac{\pi}{2m}$

Prob-8 let the weight of the pan be X lb.

So, according to the problem, $\tau_0 = 2\pi\sqrt{\frac{X}{K\cdot g}}$ ...(1) $\tau_1 = 2\pi\sqrt{\frac{P+X}{K\cdot g}}$ ...(2)

and $\tau_2 = 2\pi\sqrt{\frac{W+X}{K\cdot g}}$ ...(3)

Thus $\frac{\tau_0^2}{4\pi^2} = \frac{X}{K\cdot g}$; $\frac{\tau_1^2}{4\pi^2} = \frac{P+X}{K\cdot g}$ and $\frac{\tau_2^2}{4\pi^2} = \frac{W+X}{K\cdot g}$

Replacing $\frac{X}{K\cdot g}$ from the second and third equations, we obtain

$\frac{P}{K\cdot g} = \frac{\tau_1^2 - \tau_0^2}{4\pi^2}$ and $\frac{W}{K\cdot g} = \frac{\tau_2^2 - \tau_0^2}{4\pi^2}$

Thus $\boxed{W = \frac{\tau_2^2 - \tau_0^2}{\tau_1^2 - \tau_0^2}\cdot P}$

Prob-9 The arrangement is same as the above problem.

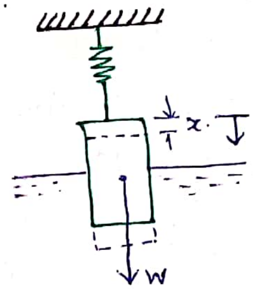
So, $\frac{P+X}{K\cdot g} = \frac{\tau_1^2}{4\pi^2}$ and $\frac{Q+X}{K\cdot g} = \frac{\tau_2^2}{4\pi^2}$

on subtraction, we obtain $\frac{P-Q}{K\cdot g} = \frac{\tau_1^2 - \tau_2^2}{4\pi^2}$

So, $\boxed{K = \frac{4\pi^2(P-Q)}{g(\tau_1^2 - \tau_2^2)}}$

Prob-12 Increase in tension of the spring $= -K \cdot x$.

Increase in buoyancy force $= -\rho_\omega \cdot A \cdot x \cdot g$.



Total unbalanced force in x-direction is $= -x(K + \rho_\omega \cdot A \cdot g)$

The differential equation of motion of the cylinder is $\frac{W}{g} \cdot \frac{d^2x}{dt^2} = -(K + \rho_\omega \cdot A \cdot g) \cdot x$.

So, $\omega^2 = \frac{g(K + \rho_\omega \cdot A g)}{W} = \frac{(32.2 \times 12) \times \left(1 + \frac{18 \times 62.4}{(12)^3}\right)}{10}$

$\therefore \omega = 7.98 \quad \therefore T = \frac{2\pi}{\omega} = \frac{2\pi}{7.98} = \boxed{0.786}$ sec

Note: here $\rho_\omega g = \frac{62.4}{(12)^3} \frac{lb}{inch^3}$

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Ass- 6.7

D'Alembert's principle

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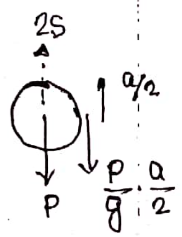
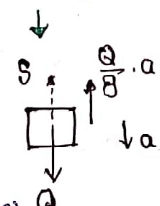
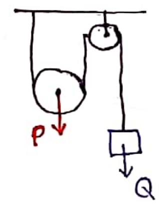
Prob-1 FBD of P → $2S = P + \frac{P}{g}\cdot\frac{a}{2}$ ....(1)



FBD of Q





→ $S = Q - \frac{Q}{g}\cdot a$ .. (2)

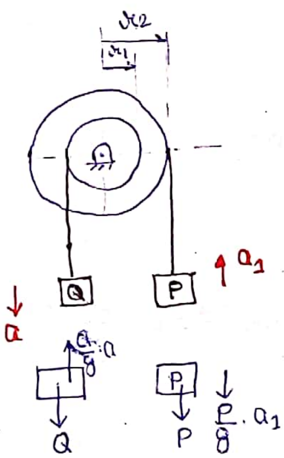
Eliminating S between (1) & (2),

$$P + \frac{P}{g}\cdot\frac{a}{2} = 2Q - \frac{2Q}{g}\cdot a$$

Substituting the values of P and Q, we obtain $\boxed{a = 8.05\ ft/sec^2}$

Prob-4 We apply the principle of virtual work so that we need to consider only the active forces.

Let δx_1 be the downward displacement for Q and δx_2 be the corresponding upward displacement for P.



Thus, $(Q - \frac{Q}{g}a)\cdot\delta x_1 = (P + \frac{P}{g}\cdot a_1)\cdot\delta x_2$

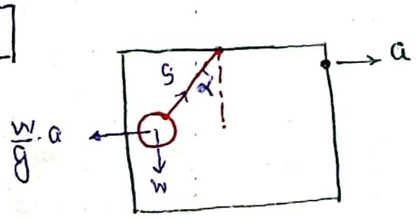
Now, $\delta x_1 = r_1\cdot\delta\theta$ and $\delta x_2 = r_2\cdot\delta\theta$.

Thus, we have $r_1\cdot(Q - \frac{Q}{g}\cdot a) = r_2\cdot(P + \frac{P}{g}\cdot a_1)$

Now, $a = r_1\cdot\ddot{\theta}$ and $a_1 = r_2\cdot\ddot{\theta}$ Thus, $\frac{a_1}{a} = \frac{r_2}{r_1} = 2$

On substitution, $a_1 = -\frac{2g}{11}$ → This is negative. It means that P moves down instead of up.

Prob-5



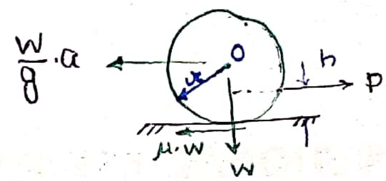
Resolving the forces,

$\frac{W}{g}\cdot a = S\cdot\sin\alpha$ and $W = S\cdot\cos\alpha$.

Replacing S between the two equations,

we obtain $\boxed{a = g\cdot\tan\alpha}$

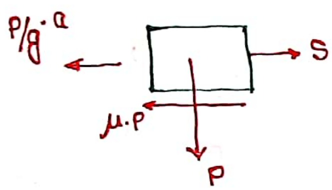
Prob-7 Taking moment of the forces about O, we obtain



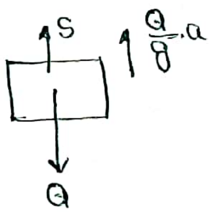
$\mu.W.x = P(x-h)$

or, $\boxed{h = x.\left(1 - \frac{\mu.W}{P}\right)}$

Prob-9 let us consider the FBD of each block



$\Rightarrow \quad \frac{P.a}{g} + \mu.P = S \quad \cdots\cdots (1)$



$\Rightarrow \quad S + \frac{Q}{g}.a = Q \quad \cdots\cdots (2)$

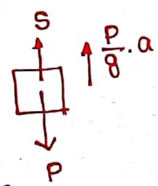
Equating (1) & (2), $\frac{P.a}{g} + \mu.P = Q - \frac{Q.a}{g}$

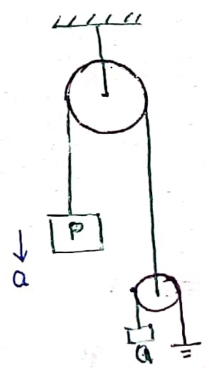
or, $a = \frac{Q - \mu.P}{P+Q}.g = \boxed{3.58 \frac{ft}{sec^2}}$.

Substituting the value of a in eqn. (2),

$S = Q\left[1 - \frac{a}{g}\right] = 6 \times \left(1 - \frac{3.58}{32.2}\right) = \boxed{5.33 \text{ lb}}$

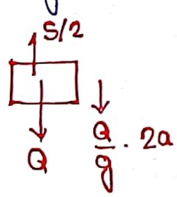
Prob-11 let P be falling.





So, $S = P - \frac{P.a}{g} \cdots (1)$

The FBD of Q $\Rightarrow$



$\Rightarrow \frac{S}{2} = Q + \frac{Q}{g}.2a \cdots (2)$

Equating (1) & (2),

$P - \frac{P.a}{g} = 2Q + \frac{2Q}{g} \times 2a$ or, $P - 2Q = \frac{a}{g}(P + 4Q)$

Since, P = Q, we have $a = -\frac{g}{5}$. → This is the acclm. of P.

The acclm. of Q is $= 2a = \boxed{-\frac{2g}{5}}$

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Prob Set- 7.1

Curvilinear Translation

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Important Points

Tangential acceleration $= \frac{dv}{dt} = a_t$

Normal acceleration $= \frac{v^2}{\rho} = a_n$ $\rho \to$ Radius of curvature

* when the path is a straight line, $\rho \to \infty$ So, $a_n = 0$
* In a curved path, if speed remains constant, $a_t = 0$ and we have $a_n \neq 0$.

Prob-2 After 60 sec, the locomotive attains a speed of 15 mph $(= 22 \frac{ft}{sec})$.

So, it moves with a change in velocity of 22 ft/sec in 60 sec.

$$\therefore a_t = \frac{22}{60} = \boxed{0.366 \frac{ft}{sec^2}}$$

Since acceleration remains const, the speed at 30 sec

$$= 0.366 \times 30 = 10.98 \frac{ft}{sec}$$

$$a_N = \frac{v^2}{\rho} = \frac{(10.98)^2}{2000} = \boxed{0.06 \frac{ft}{sec^2}}$$

Prob-7 The path travelled is described by $S = c \cdot t^2$

$$\therefore v = \frac{dS}{dt} = 2c \cdot t \quad \text{and} \quad a_t = \frac{dv}{dt} = \boxed{2c}$$

$$\text{and } a_n = \frac{v^2}{r} = \boxed{\frac{4c^2 t^2}{r}}$$

Prob-8 Since the particle travels with constant speed, there will be no tangential acceleration i.e, $a_t = 0$

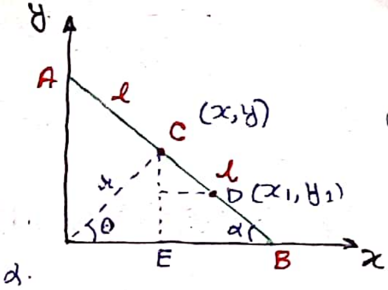
The trajectory is a parabola described by $y = K \cdot x^2$; So, $\frac{dy}{dx} = 2Kx$; $\frac{d^2y}{dx^2} = 2K$.

$$\text{So, } \rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} \quad \text{or, } \rho = \frac{\left[1 + 4K^2x^2\right]^{3/2}}{2K}$$

$$\text{on substitution } a_n = \frac{v^2}{\rho} \quad \text{or, } a_n = \frac{2Kv^2}{\left[1 + 4K^2x^2\right]^{3/2}}$$

This acceleration will be maximum when the denominator is minimum i.e, at $x = 0$. Thus, $\boxed{a_n = 2Kv^2}$

Prob-4 $CB = AC = l$ and $CD = b$



The co-ordinate of the point C is (x, y)

Now $x = r \cdot \cos\theta$ and $y = r \cdot \sin\theta$

The co-ordinate of the point D is (x_1, y_1)

Thus, $x_1 = x + b \cdot \cos\alpha$ and $y_1 = y - b \cdot \sin\alpha$.

From the geometry, $CE = r \cdot \sin\theta = l \cdot \sin\alpha$.

Now, $y_1^2 = (y - b \cdot \sin\alpha)^2 = (r \cdot \sin\theta - b \cdot \sin\alpha)^2$

or, $y_1^2 = (l \cdot \sin\alpha - b \cdot \sin\alpha)^2$ or, $\dfrac{y_1^2}{(l-b)^2} = \sin^2\alpha$ ....(1)

Similarly $x_1^2 = (x + b \cdot \cos\alpha)^2 = (r \cdot \cos\theta + b \cdot \cos\alpha)^2$

Now, $\sin\theta = \dfrac{l}{r} \cdot \sin\alpha$ $\therefore \cos\theta = \sqrt{1 - \dfrac{l^2}{r^2} \cdot \sin^2\alpha}$

when $l = r$ we get $\cos\theta = \cos\alpha$.

So, $x_1^2 = (r \cdot \cos\alpha + b \cdot \cos\alpha)^2$ or, $\dfrac{x_1^2}{(l+b)^2} = \cos^2\alpha$ ...(2)

Adding ① and ②, we obtain

$$\boxed{\dfrac{x_1^2}{(l+b)^2} + \dfrac{y_1^2}{(l-b)^2} = 1}$$

Asst-8.3

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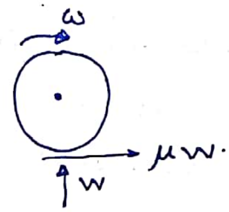
Prob-1



For the problem, $\omega_f^2 = \omega_i^2 + 2\ddot{\theta}\cdot\theta$

Now, $\omega_f = 0$ and $\omega_i = \omega$.

Since, it is a problem of deceleration.

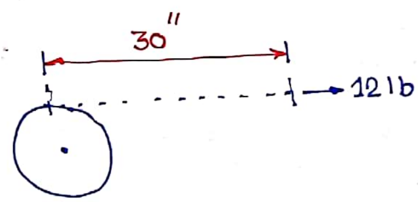
we have $\theta = \dfrac{\omega^2}{2\ddot{\theta}}$ ......(1)

For the rotor, under condition of dynamic equilibrium,

$M = I\cdot\ddot{\theta}$ or, $r\cdot\mu w = \dfrac{w}{g}\cdot\dfrac{r^2}{2}\cdot\ddot{\theta}$ or, $\ddot{\theta} = \dfrac{2\mu g}{r}$ ....(2)

Substituting (2) into (1), $\boxed{\theta = \dfrac{\omega^2\cdot r}{4\mu g}}$

Prob-2



For the rotor, applied torque

$M = 12\times\dfrac{0.5}{2} = 3$ lb-in

Setting $M = I\cdot\ddot{\theta}$ we have $\ddot{\theta} = \dfrac{M}{I} = \dfrac{3}{0.05} = 60\ \dfrac{rad}{sec^2}$

Now, let N be the number of revolutions.

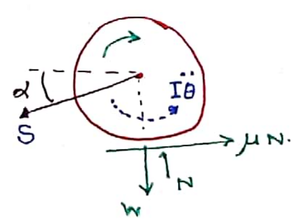
So, $2\pi r\cdot N = 30$ or, $N = \dfrac{30}{2\pi\times 0.25} = 19.1$ rev.

corresponding angular displacement $= 19.1\times 2\pi = 120$ rad.

Now applying the formula, $\omega_f^2 = \omega_i^2 + 2\ddot{\theta}\cdot\theta$.

we have. $\omega_f^2 = 2\times 60\times 120$ $\therefore \boxed{\omega_f = 120 \text{ rad/sec.}}$

Prob-4. let us consider the FBD of the rotor.



Resolving the forces, we have

$S.\cos\alpha = \mu N$ ...... (1)

and $N = W + S.\sin\alpha$ ....(2)

Eliminating S between (1) & (2), we obtain

$N = \dfrac{W}{1-\mu\cdot\tan\alpha} = \dfrac{W}{1-0.25\times\tan 15^0} = 1.071\ W$

So, $M = \mu\cdot N\cdot r = 0.25\times 1.071\ W\times 1 = 0.268 W$ lb-ft.

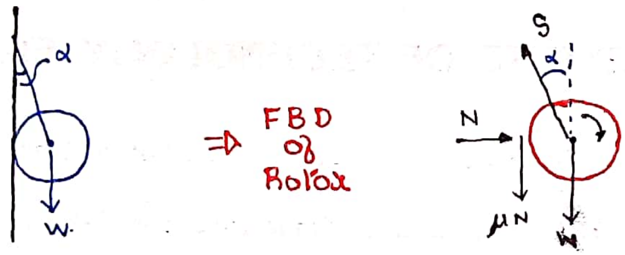
Making a dynamic balance, $M + I\ddot{\theta} = 0$ or, $\ddot{\theta} = -\dfrac{0.268\ W}{\frac{W}{g}\times(1)^2/2} = -17.25\ \dfrac{rad}{sec^2}$

Applying the formula, $\omega_f = \omega_i + \ddot{\theta}\cdot t$

Noting that $\omega_f = 0$ and $\omega_i = 20\pi \frac{rad}{sec}$

we have, $t = \frac{20\pi}{17.25} = \boxed{3.64 \text{ Sec.}}$

Prob-5



Resolving the forces horizontally & vertically, we obtain

$N = S.\sin\alpha$ .... (1) and $S.\cos\alpha = W + \mu.N$ ..... (2)

Removing S between (1) & (2), we have $N = \frac{W}{\cot\alpha - \mu} = 0.287\,W.$

Applying the condition of dynamic equilibrium, $M + I.\ddot{\theta} = 0$

Here, $M = \mu.N.r = 0.25 \times 0.287\,W \times 1 = 0.0717\,W.$ lb-ft.

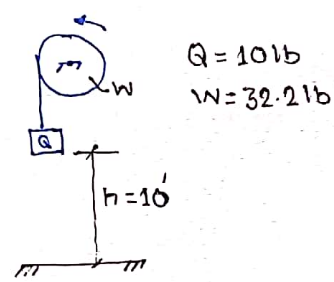
and $I = \frac{W}{g}.\frac{r^2}{2} = \frac{W.}{32.2} \times \frac{(1)^2}{2} = 0.016\,W.$

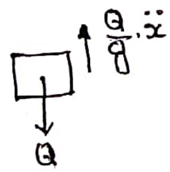
So, $\ddot{\theta} = -\frac{M}{I} = -\frac{0.0717\,W.}{0.016\,W} = -4.617 \frac{rad}{sec^2}$

Applying the formula, $\omega_f = \omega_i + \ddot{\theta}.t$

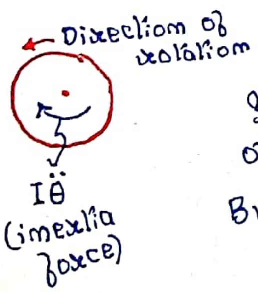
$\therefore t = \frac{20\pi}{4.617} = \boxed{13.6 \text{ Sec}}$

Prob-6 Such problem can be solved by applying the principle of virtual work.





For the falling weight Q, the work done by only the active forces $= (Q - \frac{Q}{g}.\ddot{x}).\delta x$



For the rotor, the moment in the direction of rotation $= -I.\ddot{\theta}$ the direction

If '$\delta\theta$' be the angular displacement in the direction of rotation, the work done by inertia component $= (-I\ddot{\theta}).\delta\theta$.

By principle of virtual work, we have

$(Q - \frac{Q}{g}.\ddot{x}).\delta x = I.\ddot{\theta}.\delta\theta.$

Now, we employ the following relationships: $\delta x = r.\delta\theta$ and $\ddot{x} = r.\ddot{\theta}$

on substitution and arrangements, we have $\ddot{x} = 12.33$ ft/sec^2

Now, for the falling body Q, we have, $V^2 = u^2 + 2\ddot{x} \times h$ or, $V = [2 \times 12.33 \times 10]^{1/2}$

or, $\boxed{V = 15.7 \frac{ft.}{sec}}$

The falling time can be calculated from $V = u + \ddot{x}.t$ or, $t = \frac{15.7}{12.33} = \boxed{1.273 \text{ Sec.}}$

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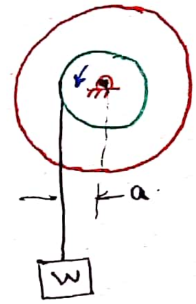
Prob-7 Moment of inertia of the rotor and shaft $= \frac{500}{g} \times \left(\frac{10}{12}\right)^2 = 10.78$ lb-sec^2-ft.



like Prob-6, we apply the principle of virtual work.
Thus work done by active forces of W in the downward direction $= \left(W - \frac{W}{g}\cdot\ddot{x}\right)\delta x$

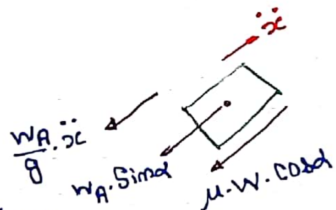
work done on the rotor and shaft $= (-I\cdot\ddot{\theta})\cdot\delta\theta$.

Now, they are to be equated to give $\left(W - \frac{W}{g}\cdot\ddot{x}\right)\delta x = I\cdot\ddot{\theta}\ \delta\theta$

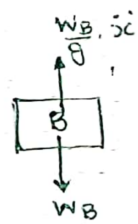
The additional relationships are : $\delta x = a\cdot\delta\theta$ and $\ddot{x} = a\cdot\ddot{\theta}$

on substitution, we obtain, $\boxed{\ddot{x} = \frac{g}{21}}$

Prob-10 let us consider Block A first.
let δx be the displacement in the upward direction.

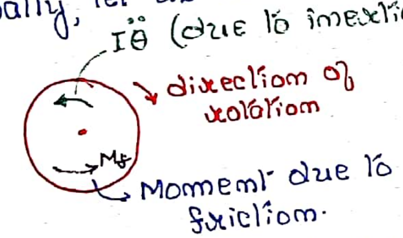


So, work done on block A by the active forces $= -\left[\frac{W_A}{g}.\ddot{x} + W_A\cdot\sin\alpha + \mu.W_A\cos\alpha\right]\cdot\delta x$.



work done by block B $= \left(W_B - \frac{W_B}{g}.\ddot{x}\right).\delta x$.

Finally, let us consider the pulley.



work done on the pulley by moments in the direction of rotation $= -(M_f + I\cdot\ddot{\theta})\cdot\delta\theta$.

So, according to the principle of virtual work, we have the following

$$\left(W_B - \frac{W_B}{g}.\ddot{x}\right)\cdot\delta x = \left[\frac{W_A}{g}.\ddot{x} + W_A\cdot\sin\alpha + \mu.W_A\cdot\cos\alpha\right]\cdot\delta x + (M_f + I\cdot\ddot{\theta})\delta\theta.$$

Noting that r = radius of the pulley $= 1'$, $\delta\theta = \delta x$ and $\ddot{\theta} = \ddot{x}$

on substitution, $W_B - W_A(\sin\alpha + \mu\cdot\cos\alpha) - M_f = \ddot{x}\left[\frac{W_B}{g} + \frac{W_A}{g} + I\right]$

Now, $I = \frac{W_{pulley}}{g} \times \frac{r^2}{2} = \frac{W_{Pulley}}{2g} = \frac{322}{2\times 32.2} = 5$ lb-sec^2-ft.

on substitution, $\boxed{\ddot{x} = 2.43 \text{ ft/sec}^2}$

For determining tension in the string we consider dynamic equilibrium of block B. Accordingly, $S = W_B - \frac{W_B}{g}.\ddot{x} = 644 \times \left(1 - \frac{2.43}{32.2}\right) = \boxed{595 \text{ lb.}}$