M Section CM II Quiz
Partial Derivatives and Applications

Department of Mathematics
January 18, 2026

Instructions

- Time allowed: 50 minutes
- Answer all questions
- Show all your work clearly
- Justify your answers where necessary
- Calculators are not allowed
- Total marks: 70 (10 marks each)

Problem 1: Chain Rule Application (10 marks)

If $z = f\left(\frac{y}{x}\right) + \sqrt{x^2 + y^2}$, show that:

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \sqrt{x^2 + y^2}$$

Problem 2: Mixed Partial Derivatives (10 marks)

If $u = f(r, s, t)$ and $r = x/y$, $s = y/z$, $t = z/x$, prove that:

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$$

Problem 3: Coordinate Transformation (10 marks)

Given that $u(x, y, z) = f(x^2 + y^2 + z^2)$ where:

$$x = r\cos\theta\cos\phi, \quad y = r\cos\theta\sin\phi, \quad z = r\sin\theta$$

Find $\frac{\partial u}{\partial \theta}$ and $\frac{\partial u}{\partial \phi}$.

Problem 4: Variable Transformation (10 marks)

Let $z = f(x, y)$, where $x = u^2 - v^2$ and $y = 2uv$. Find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ in terms of partial derivatives of f.

Problem 5: Second Order Chain Rule (10 marks)

If $u = f(x, y)$, $x = e^s \cos t$, $y = e^s \sin t$, show that:

$$\frac{\partial^2 u}{\partial s^2} + \frac{\partial^2 u}{\partial t^2} = e^{2s}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

Problem 6: Linear Approximation Application (10 marks)

A rectangular metal plate has length $L = 50.0$ cm and width $W = 30.0$ cm. The coefficient of linear expansion is $\alpha = 1.2 \times 10^{-5}$ per °C for both dimensions.

The area of the plate is $A = LW$. If the temperature increases by $\Delta T = 10$°C:

(a) Use linear approximation to estimate the change in area ΔA.

(b) Calculate the exact change in area using $L' = L(1+\alpha\Delta T)$ and $W' = W(1+\alpha\Delta T)$.

(c) Find the percentage error in your linear approximation and explain why it's small.

Problem 7: Error Propagation Analysis (10 marks)

The period T of a simple pendulum is given by $T = 2\pi\sqrt{\frac{L}{g}}$, where L is the length and g is acceleration due to gravity.

Given: $L = 1.00$ m with possible error $\Delta L = \pm 0.01$ m, and $g = 9.80$ m/s^2 with possible error $\Delta g = \pm 0.05$ m/s^2.

(a) Use linear approximation to estimate the maximum possible error in T.

(b) Which variable (L or g) contributes more to the error in T? Justify mathematically.

(c) If we want to reduce the error in T by 50%, by what factor should we reduce the error in the more significant variable?

Name: _____ **Student ID:** _____

Date: _____ **Section:** _____
