Efficient-UCBV: An Almost Optimal Algorithm using Variance Estimates

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Overview

Stochastic Multi-Armed Bandit Problem (SMAB)

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- **Vital Assumption:** $D_i, \forall i \in \mathbb{A}$ are fixed throughout the time horizon denoted by T.

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  where $\Delta_i = r^* - r_i$ is the gap.
- $LB(R_T) \geq \Omega\left(\sqrt{KT}\right)$ and good learner should have atmost $UB(R_T) \leq O\left(\sqrt{KT \log T}\right)$.

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- **Algorithmic:** No round-based arm-elimination variance-aware algorithm exists which performs well empirically.
- **Theoretical:** No round-based arm-elimination variance-aware algorithm exists which reaches order-optimal regret bound of $O\left(\sqrt{KT}\right)$.

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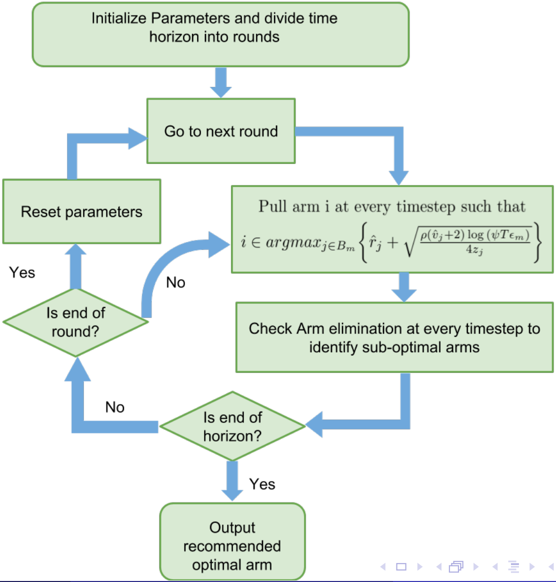
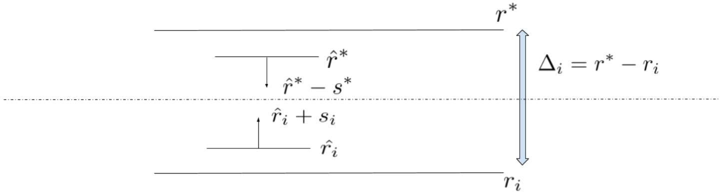
- Empirically, it outperforms all the state-of-the-art algorithms for the considered environments.

EUCBV Algorithm for SMAB



EUCBV Arm Elimination

Arm Elimination: $\hat{r}_i + s_i < \max_{j \in B_m} \{\hat{r}_j - s_j\}$ **Definition:** $s_i = \dfrac{\rho(\hat{v}_i + 2)\log(T\epsilon_m)}{2n_i}$



Expected Regret of EUCBV

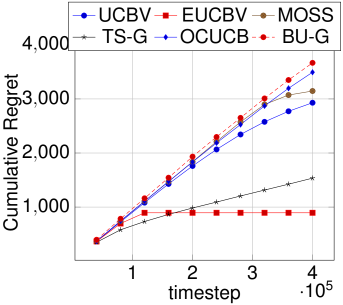
Corollary (***Gap-Independent Bound***)

The regret of EUCBV is upper bounded by the following gap-independent expression:

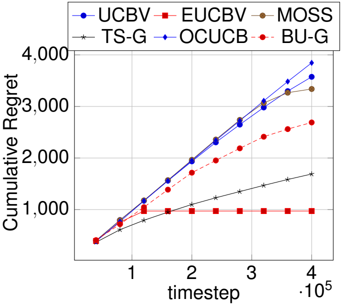
$$\mathbb{E}[R_T] \leq \frac{C_3 K^5}{T^{\frac{1}{4}}} + 80\sqrt{KT}.$$

Algorithm	GD Bound	GI Bound	Var
EUCBV	$O\left(\frac{K\sigma^2_{\max}\log(\frac{T\Delta^2}{K})}{\Delta}\right)$	$O\left(\sqrt{KT}\right)$	Yes
UCBV	$O\left(\frac{K\sigma^2_{\max}\log T}{\Delta}\right)$	$O\left(\sqrt{KT\log T}\right)$	Yes
MOSS	$O\left(\frac{K^2\log(T\Delta^2/K)}{\Delta}\right)$	$O\left(\sqrt{KT}\right)$	No
OCUCB	$O\left(\frac{K\log(T/H_i)}{\Delta}\right)$	$O\left(\sqrt{KT}\right)$	No

Experiments in SMAB



(a) Expt-1: Failure of TS



(b) Expt-2: 3 Group Variance

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- **Acknowledgement:** We thank Google India for granting us with a generous travel grant to present our work in AAAI 2018.

Thank You