Efficient-UCBV: An Almost Optimal Algorithm using Variance Estimates

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Feb 6, 2018

Overview

Stochastic Multi-Armed Bandit Problem (SMAB)

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- The rewards for each of the arms are i.i.d random variables drawn from distribution specific to the arm which are fixed throughout the time horizon denoted by T.
- The learner does not know the mean $r_i, \forall i \in \mathbb{A}$ of the distribution or the variance σ_i^2.

Problem Definition of SMAB

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- **Condition:** This has to be achieved within a finite T timesteps.
- The expected regret of an algorithm after T timesteps is give by,

$$\mathbb{E}[R_T] = \sum_{i=1}^{K} \mathbb{E}[z_i(T)]\Delta_i,$$

where $\Delta_i = r^* - r_i$ is the gap.

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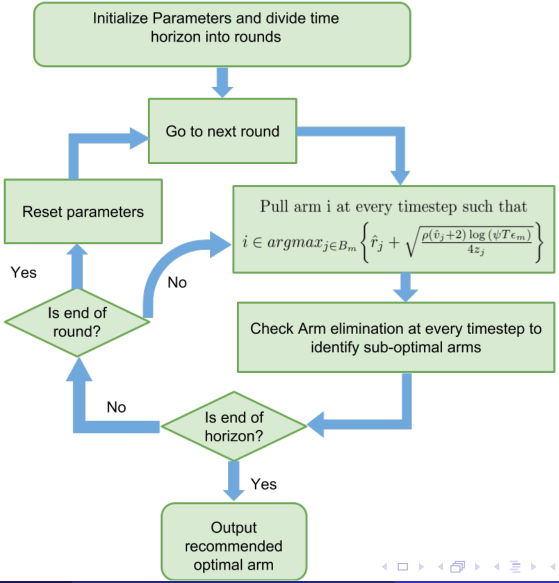
- Theoretically it achieves an order-optimal regret bound, the first for an arm elimination algorithm in SMAB setting.
- Empirically, it outperforms all the state-of-the-art algorithms for the considered environments.

EUCBV Algorithm for SMAB



Expected Regret of EUCBV

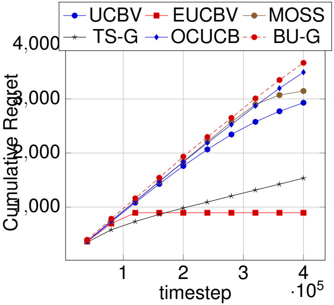
Corollary (*Gap-Independent Bound*)

The regret of EUCBV is upper bounded by the following gap-independent expression:

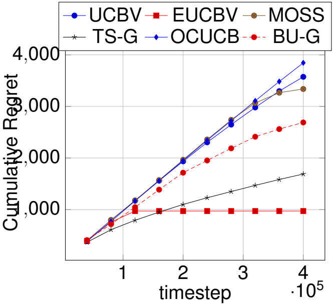
$$\mathbb{E}[R_T] \leq \frac{C_3 K^5}{T^{\frac{1}{4}}} + 80\sqrt{KT}.$$

Algorithm	GD Bound	GI Bound	Var
EUCBV	$O\left(\frac{K\sigma_{\max}^2 \log(\frac{T\Delta^2}{K})}{\Delta}\right)$	$O\left(\sqrt{KT}\right)$	Yes
UCBV	$O\left(\frac{K\sigma_{\max}^2 \log T}{\Delta}\right)$	$O\left(\sqrt{KT \log T}\right)$	Yes
MOSS	$O\left(\frac{K^2 \log(T\Delta^2/K)}{\Delta}\right)$	$O\left(\sqrt{KT}\right)$	No
OCUCB	$O\left(\frac{K \log(T/H_i)}{\Delta}\right)$	$O\left(\sqrt{KT}\right)$	No

Experiments in SMAB



(a) Expt-3: Failure of TS



(b) Expt-4: 3 Group Variance

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- Theoretically, EUCBV achieves an order-optimal regret guarantees, but further studies are required to reduce the constants.
- A more detailed analysis of the non-uniform arm selection and parameter selection is also required for EUCBV.

Thank You