Tutorial on Bandit

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Overview

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- After say pulling each arm once we are presented with an *exploration-exploitation* problem, that is whether to continue to pull the arm for which we have observed the highest estimated reward till now(exploitation) or to explore a new arm(exploration).
- If we become too greedy and always exploit we may miss the chance of actually finding the optimal arm and get stuck with a sub-optimal arm.

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- They form the linking pieces of a larger problem.
- They are easy to implement.

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- Selecting the best possible route for a message to pass through in a peer-to-peer network connection.

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- The distributions for each of the arms are fixed throughout the time horizon.

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- $\Delta_i = r^* - r_i$ denotes the gap between the means of the optimal arm and of the i-th arm.

UCB 1 Algorithm

Algorithm 1 UCB1

1: Pull each arm once
2: **for** $t = K + 1, ..., T$ **do**
3: Pull the arm such that $\max_{i \in A} \left\{ \hat{\mu} + \sqrt{\frac{2 \log t}{n_i}} \right\}$
4: **end for**

[Auer et al.(2002a)Auer, Cesa-Bianchi, and Fischer]

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- Let $X_1, ..., X_n$ be random variables with common range [0, 1] and such that $E[X_t | X_1, ..., X_{t-1}] = \mu$. Let $\bar{S}_n = \frac{X_1 +, ..., +X_n}{n}$. Then for all $a \geq 0$,

$$\mathbb{P}\{\bar{S}_n \geq \mu + a\} \leq e^{-2a^2 n}$$
$$\mathbb{P}\{\bar{S}_n \leq \mu - a\} \leq e^{-2a^2 n}$$

UCB1 Theorem on Regret Bound

Theorem

For all $K > 1$, if policy UCB1 is run on K arms having arbitrary reward distributions $P_1, ..., P_K$ with support in $[0, 1]$, then its expected regret after any number n plays is at most,

$$\mathbb{E}[R_n] \leq \sum_{i \in A} \frac{8 \log n}{\Delta_i} + \sum_{i \in A} \Delta_i \left(1 + \frac{\pi^2}{3}\right)$$

where $\mu_1, ..., \mu_K$ are the expected values of $P_1, ..., P_K$.

UCB1 Proof

- The main goal is to bound the number of pulls ($T_i(n)$) of the sub-optimal arm i till the n-th timestep.

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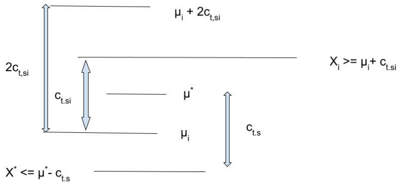
- But, this is nothing but the probability that how many times after the ℓ pulls the UCB of $*$ is less than the UCB of i which will have the highest UCB among all arms in A to be selected,

$$T_i(n) \le \ell + \sum_{t=1}^{\infty} \sum_{s=1}^{t-1} \sum_{s_i=\ell}^{t-1} \{\bar{X}_s^* + c_{t,s} \le \bar{X}_{i,s_i} + c_{t,s_i}\}$$

UCB1 Proof

- The main argument lies in this,



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$$\bar{X}_s^* \leq \mu^* - c_{t,s}$$
$$\bar{X}_{i,s_i} \geq \mu + c_{t,s_i}$$
$$\mu^* < \mu_i + 2c_{t,s_i}$$

UCB1 Proof

- Now, we get the value of confidence interval $c_{t,s_i} = \sqrt{\frac{2\ln t}{s_i}}$ by plugging its value in the below equations,
- $\mathbb{P}\{\bar{X}_s^* \leq \mu^* - c_{t,s}\} \leq \exp\left(-2(\sqrt{\frac{2\ln t}{s}})^2 s\right) \leq e^{-4\log t} \leq t^{-4}$

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- $\mathbb{P}\{\bar{X}_{i,s_i} \ge \mu + c_{t,s_i}\} \le \exp\left(-2(\sqrt{\frac{2\ln t}{s_i}})^2 s_i\right) \le e^{-4\log t} \le t^{-4}$

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- $\mathbb{P}\{\bar{X}_{i,s_i} \ge \mu + c_{t,s_i}\} \le \exp\left(-2(\sqrt{\frac{2\ln t}{s_i}})^2 s_i\right) \le e^{-4\log t} \le t^{-4}$
- And by plugging $\ell = \left\lceil \frac{8\log n}{\Delta_i^2} \right\rceil$ in,

$$\mu^* - \mu_i - 2c_{t,s_i} = \mu^* - \mu_i - 2\sqrt{\frac{2\log t}{s_i}} \ge \mu^* - \mu_i - \Delta_i = 0$$

we get $\mu^* - \mu_i - 2c_{t,s_i} \ge 0$. So for any pulls greater than ℓ, μ^* will surely be atleast $2c_{t,s_i}$ more than μ_i and one of the rest two events will occur with high probability.

UCB1 Proof

- Summing everything up, any sub-optimal arm i will get pulled atleast ℓ times and then the two events $\bar{X}^*_s \leq \mu^* - c_{t,s}$ and $\bar{X}_{i,s_i} \geq \mu + c_{t,s_i}$ will occur with atmost t^{-4} probability.

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- $$\begin{aligned}\mathbb{E}[T_i(n)] &\leq \left\lceil \frac{8 \log n}{\Delta_i^2} \right\rceil + \sum_{t=1}^{\infty} \sum_{s=1}^{t-1} \sum_{s_i=\ell}^{t-1} 2t^{-4} \\ &\leq \frac{8 \log n}{\Delta_i^2} + 1 + \frac{\pi^2}{3}, \text{by Bazel's equation}\end{aligned}$$

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- $$\begin{aligned}\mathbb{E}[T_i(n)] &\leq \left\lceil \frac{8 \log n}{\Delta_i^2} \right\rceil + \sum_{t=1}^{\infty} \sum_{s=1}^{t-1} \sum_{s_i=\ell}^{t-1} 2t^{-4} \\ &\leq \frac{8 \log n}{\Delta_i^2} + 1 + \frac{\pi^2}{3}, \text{by Bazel's equation}\end{aligned}$$
- So finally the cumulative regret is,

$$\mathbb{E}[R_n] \leq \sum_{i \in A} \mathbb{E}[T_i(n)]\Delta_i \leq \frac{8 \log n}{\Delta_i} + \Delta_i \left(1 + \frac{\pi^2}{3}\right)$$

Looking beyond Cumulative regret

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- We are interested in finding an arm such that it's ϵ close to the optimal arm and we can guarantee this with $1 - \delta$ probability.
- Note, that ϵ and δ are given as input and the main aim is to *minimize the number of pulls of an arm i so that it is ϵ close to the optimal arm with $1 - \delta$ probability*. This is called Sample complexity.

A Naive Algorithm

Algorithm 2 Naive Algorithm

1: Input: $\epsilon > 0$, $\delta > 0$
2: Output: An arm
3: **for** each arm $i \in A$ **do**
4: Sample it for $\ell = \dfrac{4}{\epsilon^2} \log \dfrac{2K}{\delta}$
5: Let $\bar{X}_i$ be the average reward of arm i
6: **end for**
7: Output $argmax_{i \in A}\{\bar{X}_i\}$

[Even-Dar et al.(2006)Even-Dar, Mannor, and Mansour]

Sample Complexity of Naive Algorithm

Theorem

The sample complexity of Naive Algorithm for a set of arms K is given by,

$$O\left(\frac{K}{\epsilon^2}\log\left(\frac{K}{\delta}\right)\right)$$

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- So, we only need to bound the probability of,

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$$\leq 2\exp\left(-2\left(\frac{\epsilon}{2}\right)^2 \ell\right) \leq 2\exp\left(-2\frac{\epsilon^2}{4}.\frac{4}{\epsilon^2}\log\frac{2K}{\delta}\right) \leq \frac{\delta}{K}$$

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- Summing over all the $K-1$ arms (arms excluding $*$) we get, $\frac{(K-1)\delta}{K} < \delta$

Intuition about Median Elimination

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- One simple way to modify Naive Algorithm is to divide the time horizon into phases.
- In each phase pull all the arms equal number of times.
- After that eliminate half the arms with a high guarantee that they are surely not ϵ-optimal arms.

Median Elimination

Algorithm 3 Median Elimination

Input: $\epsilon > 0$, $\delta > 0$
Output: An arm
Set $S_1 = A$, $\epsilon_1 = \epsilon/4$, $\delta_1 = \delta/2$ and $\ell = 1$
for Repeat till $|S_\ell| = 1$ **do**
    Sample every arm in S_ℓ for $\frac{4}{\epsilon_\ell^2}\log(\frac{3}{\delta_\ell})$ times and let $\bar{X}_i$ denote the average estimated payoff of i.
    Find median m_ℓ of all the surviving arms based on their $\bar{X}_i, \forall i \in A$
    Eliminate all arms from S_ℓ such that $\bar{X}_i < m_\ell$ and create $S_{\ell+1}$.
    Reset Parameters: $\epsilon_{\ell+1} = \frac{3}{4}\epsilon$; $\delta_{\ell+1} = \frac{1}{2}\delta$; $\ell = \ell + 1$
end for

[Even-Dar et al.(2006)Even-Dar, Mannor, and Mansour]

Comparison of Median Elimination, Naive Algorithm

Table: Sample Complexity of Median Elimination, Naive Algorithm

Algorithm	Upper bound on Sample Complexity
Naive	$O\left(\frac{K}{\epsilon^2} \log\left(\frac{K}{\delta}\right)\right)$
ME	$O\left(\frac{K}{\epsilon^2} \log big(\frac{1}{\delta})\right)$

- So clearly Naive algorithm uses more samples than Median Elimination to give us the same (ϵ, δ) guarantee

Arm Elimination Algorithm

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- Arm Elimination (AE) algorithms proceeds in a phase-wise manner whereby they pull all the surviving arms equal number of times in each phase and then at the end of the phase proceeds to eliminate one or more arms from its active set.
- We have seen that Median Elimination algorithm which is an AE algorithm, is more powerful than Naive Algorithm.
- Can we have such algorithm for minimizing Cumulative Regret?

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- Reset parameters and proceed to next phase.

Enter UCB-Improved

Algorithm 4 UCB-Improved

Input: Time horizon n

Initialization: Set $B_0 := A$ and $\epsilon_0 := 1$.

for $m = 0, 1, .. \lfloor \frac{1}{2} \log_2 \frac{n}{e} \rfloor$ **do**

Pull each arm in B_m so that the total number of times it has been pulled is $n_m = \left\lceil \frac{2 \log (n\epsilon_m^2)}{\epsilon_m} \right\rceil$.

Arm Elimination

For each $i \in B_m$, delete arm i from B_m if,

$$\bar{X}_i + \sqrt{\frac{\log (n\epsilon_m^2)}{2n_m}} < \max_{j \in B_m} \left\{ \bar{X}_j - \sqrt{\frac{\log (n\epsilon_m^2)}{2n_m}} \right\}$$

Reset Parameters

Set $\epsilon_{m+1} := \frac{\epsilon_m}{2}$, Set $B_{m+1} := B_m$

Comparison of UCB-Improved,ME and UCB1

Table: Cumulative Regret of UCB-Improved, UCB1

Algorithm	Upper bound on Cumulative Regret
UCB1	$O\left(\frac{K \log n}{\Delta}\right)$
UCB-Improved	$O\left(\frac{K \log(n\Delta^2)}{\Delta}\right)$

- So, UCB-Improved is more powerful than UCB1 theoretically.

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- UCB-Improved is more powerful than ME, because ME will always take $\log_2 K$ number of phases to come up with the best arm (since it eliminates half the number of arms after every phase) whereas UCB-Improved eliminates arbitrary number of arms in any phase.

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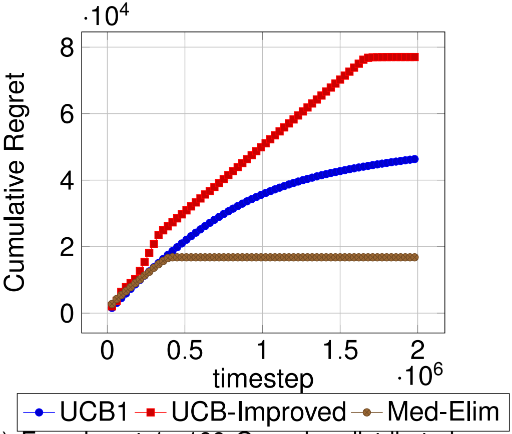
- UCB-Improved is more powerful than ME, because ME will always take $\log_2 K$ number of phases to come up with the best arm (since it eliminates half the number of arms after every phase) whereas UCB-Improved eliminates arbitrary number of arms in any phase.
- Empirically, UCB-Improved beats UCB1 when K is very large and gaps ($\Delta_i, \forall i \in A$) are very small.

Finally, an experiment!!!



(a) Experiment 1: 100 Gaussian-distributed arms with $\mu_{i_{i \neq *:1-33}} = 0.01$, $\mu_{i_{i \neq *:34-99}} = 0.06$, $r^*_{i=100} = 0.1$ and $\sigma^2_{i:1-100} = 0.3$

Figure: Experiment with bandit

Some Other Bandits and Applications

- Adversarial Bandits : Used in Investment in Stock Markets
- Contextual Bandits : Used in online Advertisement/news article selection
- Budgeted Bandits : Used in Clinical trials
- Distributed Bandits : Used in packet routing through a network

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Thank You