Tutorial on Bandit

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Overview

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- After say pulling each arm once we are presented with an *exploration-exploitation* trade-off, that is whether to continue to pull the arm for which we have observed the highest estimated reward till now(exploitation) or to explore a new arm(exploration).
- If we become too greedy and always exploit we may miss the chance of actually finding the optimal arm and get stuck with a sub-optimal arm.

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- They are easy to implement.

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- Selecting the best possible route for a message to pass through in a peer-to-peer network connection.

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- The learner has to find the optimal arm the mean of whose distribution is denoted by μ^* such that $\mu^* > \mu_i, \forall i \in A$.
- The distributions for each of the arms are fixed throughout the time horizon.

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$$R_n = \mu^* n - \sum_{i \in A} \mu_i T_i(n),$$

- The expected regret of an algorithm after n rounds can be written as

$$\mathbb{E}[R_n] = \sum_{i=1}^{K} \mathbb{E}[T_i(n)] \Delta_i,$$

- $\Delta_i = \mu^* - \mu_i$ denotes the gap between the means of the optimal arm and of the i-th arm.

UCB 1 Algorithm

Algorithm 1 UCB1

1: Pull each arm once
2: **for** $t = K + 1, ..., n$ **do**
3: Pull the arm such that $\max_{i \in A} \left\{ \bar{X}_i + \sqrt{\frac{2 \log t}{s_i}} \right\}$
4: **end for**

[Auer et al.(2002a)Auer, Cesa-Bianchi, and Fischer]

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- Let $X_1, ..., X_n$ be random variables with common range [0, 1] and such that $E[X_t | X_1, ..., X_{t-1}] = \mu$. Let $\bar{X}_n = \frac{X_1 +, ..., +X_n}{n}$. Then for all $a \geq 0$,

$$\mathbb{P}\{\bar{X}_n \geq \mu + a\} \leq e^{-2a^2 n}$$
$$\mathbb{P}\{\bar{X}_n \leq \mu - a\} \leq e^{-2a^2 n}$$

UCB1 Theorem on Regret Bound

Theorem

For all $K > 1$, if policy UCB1 is run on K arms having arbitrary reward distributions $P_1, ..., P_K$ with support in $[0, 1]$, then its expected regret after any number n plays is at most,

$$\mathbb{E}[R_n] \leq \sum_{i \in A} \frac{8 \log n}{\Delta_i} + \sum_{i \in A} \Delta_i \left(1 + \frac{\pi^2}{3}\right)$$

where $\mu_1, ..., \mu_K$ are the expected values of $P_1, ..., P_K$.

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- But, this is nothing but the probability that how many times after the ℓ pulls the UCB of $*$ is less than the UCB of i which will have the highest UCB among all arms in A to be selected,

$$T_i(n) \leq \ell + \sum_{t=1}^{\infty} \sum_{s=1}^{t-1} \sum_{s_i=\ell}^{t-1} \{\bar{X}_s^* + c_{t,s} \leq \bar{X}_{i,s_i} + c_{t,s_i}\}$$

- The main argument lies in this,

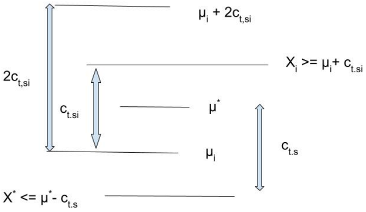

Figure 1

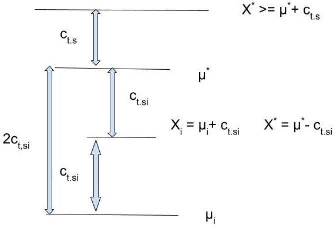

Figure 2

-

$$\bar{X}_s^* \leq \mu^* - c_{t,s}; \bar{X}_{i,s_i} \geq \mu_i + c_{t,s_i}; \mu^* < \mu_i + 2c_{t,s_i}$$

UCB1 Proof

- Now, we get the value of confidence interval $c_{t,s_i} = \sqrt{\frac{2 \ln t}{s_i}}$ by plugging its value in the below equations,
- $\mathbb{P}\{\bar{X}^*_s \leq \mu^* - c_{t,s}\} \leq \exp\left(-2(\sqrt{\frac{2 \ln t}{s}})^2 s\right) \leq e^{-4 \log t} \leq t^{-4}$

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- $\mathbb{P}\{\bar{X}_{i,s_i} \geq \mu + c_{t,s_i}\} \leq \exp\left(-2(\sqrt{\frac{2\ln t}{s_i}})^2 s_i\right) \leq e^{-4\log t} \leq t^{-4}$

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- $\mathbb{P}\{\bar{X}_{i,s_i} \geq \mu + c_{t,s_i}\} \leq \exp\left(-2(\sqrt{\frac{2\ln t}{s_i}})^2 s_i\right) \leq e^{-4\log t} \leq t^{-4}$
- And by plugging $\ell = s_i = \left\lceil \frac{8\log n}{\Delta_i^2} \right\rceil$ in,

$$\mu^* - \mu_i - 2c_{t,s_i} = \mu^* - \mu_i - 2\sqrt{\frac{2\log t}{s_i}} \geq \mu^* - \mu_i - \Delta_i = 0$$

we get $\mu^* - \mu_i - 2c_{t,s_i} \geq 0$. So for any pulls greater than ℓ, μ^* will surely be atleast $2c_{t,s_i}$ more than μ_i and one of the rest two events will occur with high probability.

UCB1 Proof

- Summing everything up, any sub-optimal arm i will get pulled atleast ℓ times and then the two events $\bar{X}_s^* \leq \mu^* - c_{t,s}$ and $\bar{X}_{i,s_i} \geq \mu + c_{t,s_i}$ will occur with atmost t^{-4} probability.

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$$\mathbb{E}[T_i(n)] \leq \left\lceil \frac{8 \log n}{\Delta_i^2} \right\rceil + \sum_{t=1}^{\infty} \sum_{s=1}^{t-1} \sum_{s_i=\ell}^{t-1} 2t^{-4}$$

$$\leq \frac{8 \log n}{\Delta_i^2} + 1 + \frac{\pi^2}{3}, \text{by Bazel's equation}$$

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- $$\begin{aligned}\mathbb{E}[T_i(n)] &\leq \left\lceil \frac{8 \log n}{\Delta_i^2} \right\rceil + \sum_{t=1}^{\infty} \sum_{s=1}^{t-1} \sum_{s_i=\ell}^{t-1} 2t^{-4} \\ &\leq \frac{8 \log n}{\Delta_i^2} + 1 + \frac{\pi^2}{3}, \text{by Bazel's equation}\end{aligned}$$
- So finally the cumulative regret is,

$$\mathbb{E}[R_n] \leq \sum_{i \in A} \mathbb{E}[T_i(n)]\Delta_i \leq \sum_{i \in A} \frac{8 \log n}{\Delta_i} + \sum_{i \in A} \Delta_i \left(1 + \frac{\pi^2}{3}\right)$$

Looking beyond Cumulative regret

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- We are interested in finding an arm such that it's ϵ close to the optimal arm and we can guarantee this with $1 - \delta$ probability.
- Note, that ϵ and δ are given as input and the main aim is to *minimize the number of pulls of an arm i so that it is ϵ close to the optimal arm with $1 - \delta$ probability*. This is called Sample complexity.

A Naive Algorithm

Algorithm 2 Naive Algorithm

Input: $\epsilon > 0$, $\delta > 0$
Output: An arm
for each arm $i \in A$ **do**
    Sample it for $\ell = \frac{4}{\epsilon^2} \log \frac{2K}{\delta}$
    Let $\bar{X}_i$ be the average reward of arm i
end for
Output $argmax_{i \in A}\{\bar{X}_i\}$

[Even-Dar et al.(2006)Even-Dar, Mannor, and Mansour]

Sample Complexity of Naive Algorithm

Theorem

The sample complexity of Naive Algorithm for a set of arms K is given by,

$$O\left(\frac{K}{\epsilon^2}\log\left(\frac{K}{\delta}\right)\right)$$

[Even-Dar et al.(2006)Even-Dar, Mannor, and Mansour]

Naive Algorithm Sample Complexity Proof

- We want to bound the probability of the event $\bar{X}_i > \bar{X}^*$. The goal is to find the minimum number of pulls required for an arm i so that $\mu^* - \mu_i < \epsilon$.

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- So, we only need to bound the probability of,

$$\mathbb{P}\{\bar{X}_i > \bar{X}^*\} \leq \mathbb{P}\left\{\bar{X}_i > \mu_i + \frac{\epsilon}{2}\right\} + \mathbb{P}\left\{\bar{X}^* < \mu^* - \frac{\epsilon}{2}\right\}$$
$$\leq 2\exp\left(-2\left(\frac{\epsilon}{2}\right)^2\ell\right) \leq 2\exp\left(-2\frac{\epsilon^2}{4}.\frac{4}{\epsilon^2}\log\frac{2K}{\delta}\right) \leq \frac{\delta}{K}$$

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- Summing over all the $K - 1$ arms (arms excluding $*$) we get, $\dfrac{(K-1)\delta}{K} < \delta$

Intuition about Median Elimination

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- One simple way to modify Naive Algorithm is to divide the time horizon into phases.
- In each phase pull all the surviving arms equal number of times.
- After that eliminate half the surviving arms with a high guarantee that they are surely not ϵ-optimal arms.

Median Elimination

Algorithm 3 Median Elimination

Input: $\epsilon > 0$, $\delta > 0$
Output: An arm
Set $S_1 = A$, $\epsilon_1 = \epsilon/4$, $\delta_1 = \delta/2$ and $\ell = 1$
for Repeat till $|S_\ell| = 1$ **do**
    Sample every arm in S_ℓ for $\frac{4}{\epsilon_\ell^2} \log(\frac{3}{\delta_\ell})$ times and let $\bar{X}_i$ denote the average estimated payoff of i.
    Find median m_ℓ of all surviving arms based on their $\bar{X}_i, \forall i \in S_\ell$
    Eliminate all arms from S_ℓ such that $\bar{X}_i < m_\ell$ and create $S_{\ell+1}$.
    Reset Parameters: $\epsilon_{\ell+1} = \frac{3}{4}\epsilon$; $\delta_{\ell+1} = \frac{1}{2}\delta$; $\ell = \ell + 1$
end for

[Even-Dar et al.(2006)Even-Dar, Mannor, and Mansour]

Comparison of Median Elimination, Naive Algorithm

Table: Sample Complexity of Median Elimination, Naive Algorithm

Algorithm	Upper bound on Sample Complexity
Naive	$O\left(\frac{K}{\epsilon^2}\log\left(\frac{K}{\delta}\right)\right)$
ME	$O\left(\frac{K}{\epsilon^2}\log\left(\frac{1}{\delta}\right)\right)$

- So clearly Naive algorithm uses more samples than Median Elimination to give us the same (ϵ, δ) guarantee

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- We have seen that Median Elimination algorithm which is an AE algorithm, is more powerful than Naive Algorithm.
- Can we have such algorithm for minimizing Cumulative Regret?

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- At the end of the phase eliminate some arms based on some criteria.
- Reset parameters and proceed to next phase.

Enter UCB-Improved

Algorithm 4 UCB-Improved

Input: Time horizon n

Initialization: Set $B_0 := A$ and $\tilde{\Delta}_0 := 1$.

for $m = 0, 1, ..\lfloor \frac{1}{2} \log_2 \frac{n}{e} \rfloor$ **do**

  Pull each arm in B_m, $n_m = \left\lceil \frac{2 \log (n \tilde{\Delta}_m^2)}{\epsilon_m} \right\rceil$ number of times.

  Arm Elimination

    For each $i \in B_m$, delete arm i from B_m if,

$$\bar{X}_i + \sqrt{\frac{\log (n \tilde{\Delta}_m^2)}{2 n_m}} < \max_{j \in B_m} \left\{ \bar{X}_j - \sqrt{\frac{\log (n \tilde{\Delta}_m^2)}{2 n_m}} \right\}$$

  Set $\tilde{\Delta}_{m+1} := \frac{\tilde{\Delta}_m}{2}$, Set $B_{m+1} := B_m$

  Stop if $|B_m| = 1$ and pull $i \in B_m$ till n is reached.

end for

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- As opposed to UCB1, UCB-Improved has fixed confidence interval $c_m = \sqrt{\dfrac{\log(n\tilde{\Delta}_m^2)}{2n_m}}$ for all arms in a particular phase.
- c_m ensures that whenever $\tilde{\Delta}_m < \dfrac{\Delta_i}{2}$ in the m-th round, the arm i gets eliminated.

Comparison of UCB-Improved,ME and UCB1

Table: Cumulative Regret of UCB-Improved, UCB1

Algorithm	Upper bound on Cumulative Regret
UCB1	$O\left(\frac{K \log n}{\Delta}\right)$
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- So, UCB-Improved is more powerful than UCB1 theoretically.

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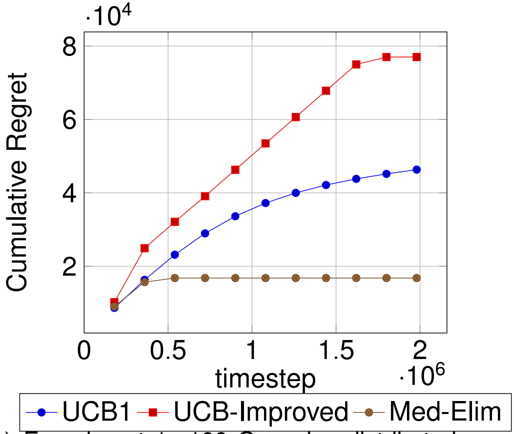
- Empirically, UCB-Improved beats UCB1 when K is very large and gaps ($\Delta_i, \forall i \in A$) are very small.

Finally, an experiment!!!



(a) Experiment 1: 100 Gaussian-distributed arms with $\mu_{i_{i \neq *:1-33}} = 0.01$, $\mu_{i_{i \neq *:34-99}} = 0.06$, $r^*_{i=100} = 0.1$ and $\sigma^2_{i:1-100} = 0.3$. For ME, $\epsilon = 0.03, \delta = 0.1$

Figure: Experiment with bandit

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- Pure exploration Bandits : In this setup the sole consideration is to conduct as much exploration as possible within a limited number of pulls and then suggest the best arm(s). Used in Clinical trials.

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Thank You