Thresholding Bandits with Augmented UCB

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August 20, 2017

Overview

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- **Condition:** This has to be achieved within **T** timesteps of exploration and this is termed as a fixed-budget problem.

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- Let $\hat{S}_\tau$ denote the recommendation of a learning algorithm after T time units of exploration, while $\hat{S}_\tau^c$ denotes its complement.
- The goal of the learning agent is to minimize the **expected** loss at the end of budget T:

$$\mathbb{E}[\mathcal{L}(T)] = \mathbb{P}\Big(\underbrace{\{S_\tau \cap \hat{S}_\tau^c \neq \emptyset\}}_{\textbf{Rejected good arms}} \cup \underbrace{\{\hat{S}_\tau \cap S_\tau^c \neq \emptyset\}}_{\textbf{Accepted bad arms}} \Big)$$

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- Lesser the budget $\Rightarrow$ Harder the problem.
- Higher the variance of reward distribution of the arms' $\Rightarrow$ Harder the problem.

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- It is the **first variance-based arm elimination algorithm** for the considered TBP settings.
- It **addresses an open problem** discussed in Auer and Ortner (2010) of designing an algorithm that can eliminate arms based on variance estimates.
- We also define a **new problem complexity** which uses empirical variance estimates along with arm's mean for giving the theoretical bound.

APT Approach

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- This algorithm uses only mean estimation to find the S_τ.
- Theoretically they proved this algorithm to be almost optimal when only mean estimation is used as a metric of comparison.
- Empirically it outperformed other state-of-the-art algorithms which were modified to perform in the TBP setting.

APT Algorithm

Algorithm 1 APT

Input: Time horizon T, threshold τ, tolerance factor $\epsilon \geq 0$

Pull each arm once

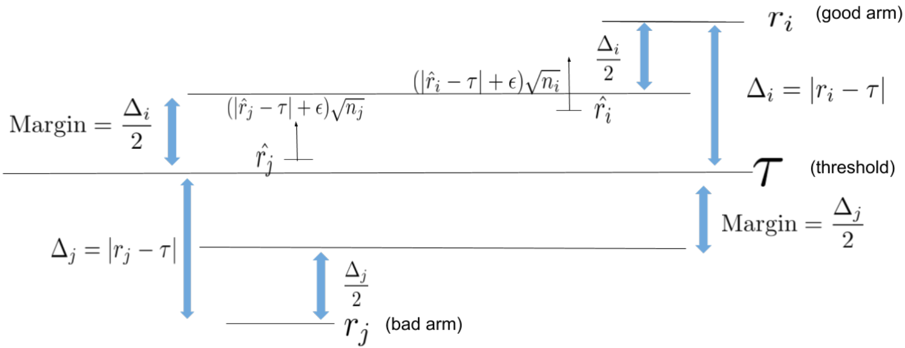
for $t = K + 1, .., T$ **do**

Pull arm $j \in \arg\min_{i \in A} \left\{ (|\hat{r}_i - \tau| + \epsilon) \sqrt{n_i} \right\}$ and observe the reward for arm j.

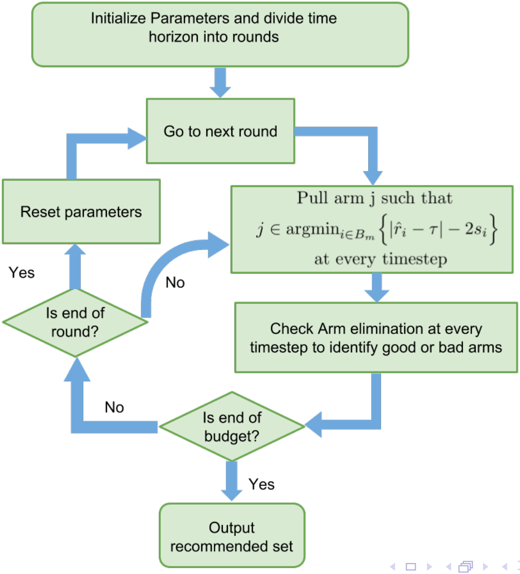
end for

Output: $\hat{S}_\tau = \{i : \hat{r}_i \geq \tau\}$.

Intuition of APT

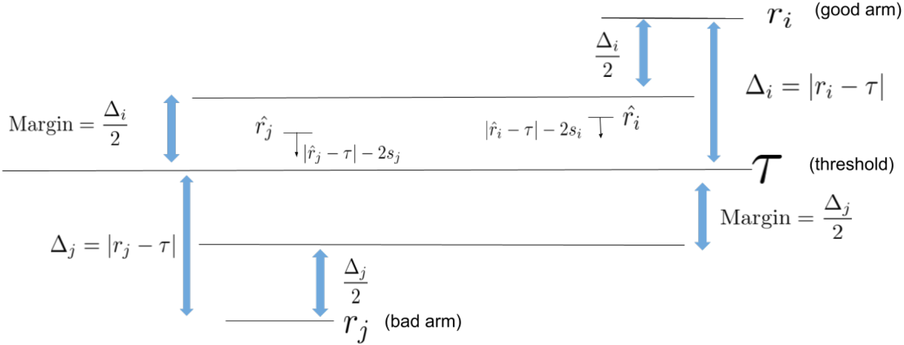


AugUCB algorithm



AugUCB algorithm (Intuition, Arm pulling)

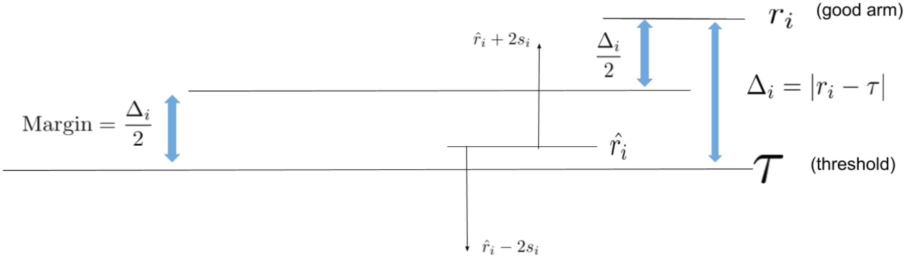
- Like UCB-Imp, AugUCB also divides the time budget T into rounds.
- At every timestep we pull arm j s.t. $j \in \arg\min_{i \in B_m} \left\{ |\hat{r}_i - \tau| - 2s_i \right\}$ (like APT).



AugUCB algorithm (Intuition, Arm Elimination)

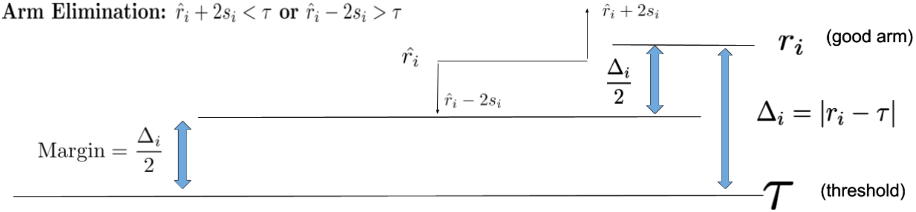
- We eliminate an arm when we are sure that $\hat{r}_i$ is close to r_i with high probability and hence identify it as good or bad arm.
- It's risky to eliminate an arm when $\hat{r}_i$ is inside *Margin*.
- Confidence interval s_i will make sure arm i is not eliminated while inside Margin with a high probability.

Arm Elimination: $\hat{r}_i + 2s_i < \tau$ **or** $\hat{r}_i - 2s_i > \tau$

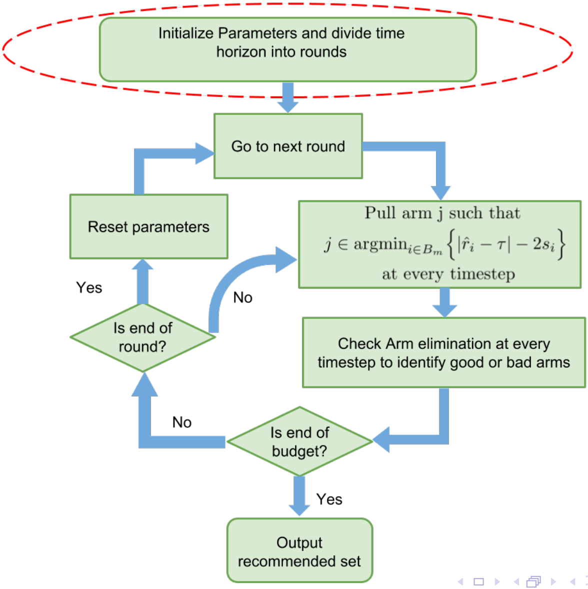


AugUCB algorithm (Intuition, Arm Elimination)

- Now we see that $\hat{r}_i$ has moved close to its true estimate r_i.
- We eliminate i and re-allocate the remaining budget to pull arms close to the threshold

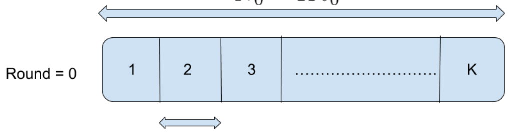


AugUCB parameter initialization



Parameter initialization

$$N_0 = K\ell_0$$

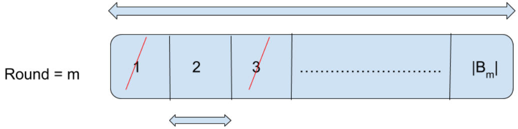


$$\epsilon_0 = 1$$

$$\psi_0 = \frac{T\epsilon_0}{128\left(\log(\frac{3}{16}K\log K)\right)^2}$$

$$\ell_0 = \left\lceil \frac{2\psi_0 \log(T\epsilon_0)}{\epsilon_0} \right\rceil$$

$$N_m = |B_m|\ell_m$$



$$\epsilon_m = \frac{1}{2^m}$$

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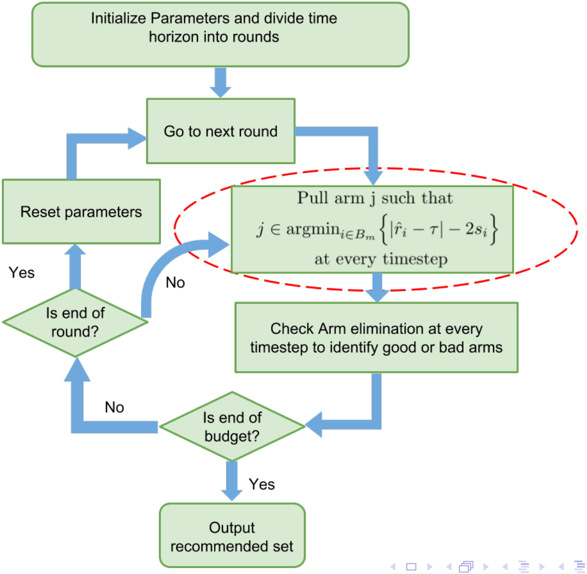
AugUCB arm pull



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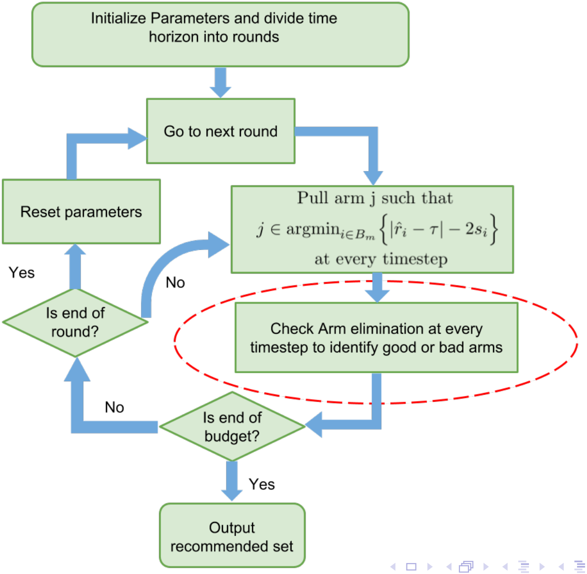
- s_i decreases with more n_i and ψ_m and ρ ensures that it decreases at a correct rate.
- Note that $\hat{v}_i$ estimated variance in s_i makes the algorithm pull the arm which shows more variance.

AugUCB arm elimination



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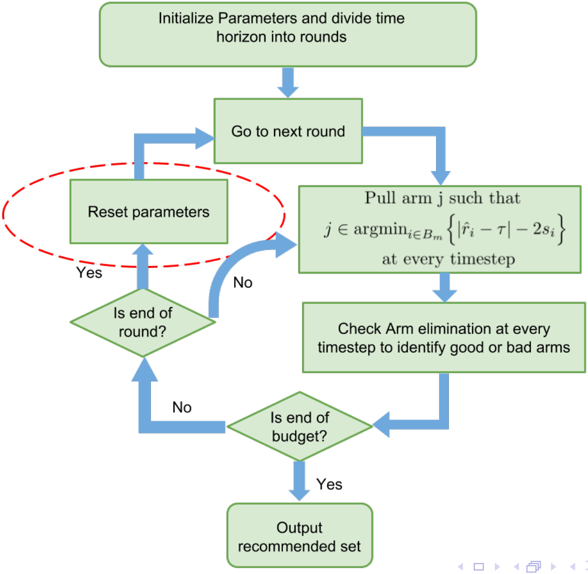
- It identifies the arm whose estimates lies close to their true mean and thus help in identifying the good or bad arms.
- It eliminates the arms which have been identified as good or bad arms (with a high probability) and re-allocates the remaining budget for surviving arms.

AugUCB parameter reset



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- Recalculate the length of next round on the number of surviving arms.

Expected Loss of AugUCB

Theorem

For $K \geq 4$ and $\rho = 1/3$, the expected loss of the AugUCB algorithm is given by,

$$\mathbb{E}[\mathcal{L}(T)] \leq 2KT \exp\left(-\frac{T}{4096\log(K\log K)H_{\sigma,2}}\right).$$

Table: AugUCB vs. State of the art

Algorithm	Upper Bound on Expected Loss	Oracle
AugUCB	$\exp\left(-\frac{T}{4096\log(K\log K)H_{\sigma,2}} + \log(2KT)\right)$	No
UCBEV	$\exp\left(-\frac{1}{512}\frac{T-2K}{H_{\sigma,1}} + \log(6KT)\right)$	Yes
APT	$\exp\left(-\frac{T}{64H_1} + 2\log((\log(T)+1)K)\right)$	No
UCBE	$\exp\left(-\frac{T-K}{18H_1} - 2\log(\log(T)K\right)$	Yes

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- By access we mean that an oracle supplies them the H_1 or $H_{\sigma,1}$. They do not have access to individual means and variances.
- APT, AugUCB, UA do not require access to H_1 or $H_{\sigma,1}$.

Experimental Setup

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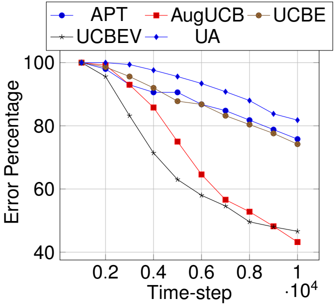
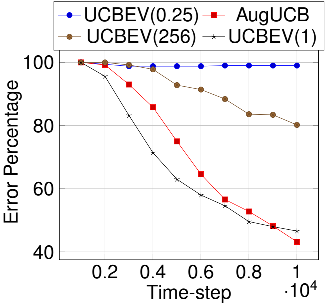
- The first 10 arms partitioned into two groups; the respective means are $r_{1:5} = 0.45$, $r_{6:10} = 0.55$.
- The means of arms $i = 11 : 100$ are chosen same as $r_{11:100} = 0.4$.
- Variances are set as $\sigma^2_{1:5} = 0.3$ and $\sigma^2_{6:10} = 0.8$; $\sigma^2_{11:100}$ are independently and uniformly chosen in the interval $[0.2, 0.3]$.

Experimental Results

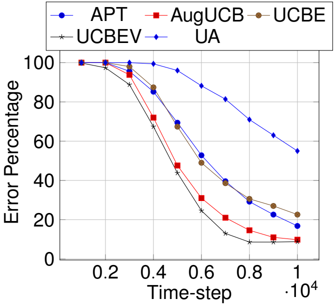


(a) Expt-1: Two Group Setting (Advance)

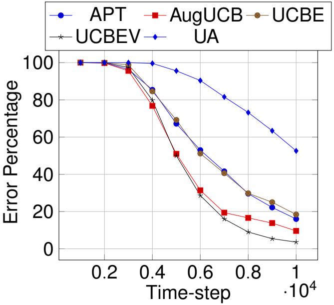


(b) Expt-2: Two Group Setting (Advance)

Experimental Results



(c) Expt-1: Arithmetic Progression (Gaussian)



(d) Expt-2: Geometric Progression (Gaussian)

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- Further studies are required to establish a lower bound on the expected loss of AugUCB.
- A more detailed analysis of the non-uniform arm selection and parameter selection is also required.

Thank You