

Online sequential Learning using Bandits

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Introduction

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- After say pulling each arm once, we are presented with an *exploration-exploitation* trade-off, that is whether to continue to pull the arm for which we have observed the highest estimated reward till now (exploitation) or to explore a new arm (exploration).
- If we become too greedy and always exploit, we may miss the chance of actually finding the optimal arm and get stuck with a sub-optimal arm.

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- Selecting the best possible route for a message to pass through in a peer-to-peer network connection.

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- Bandits allows us to study this behavior in a more formal way giving us strict guarantees regarding the performance of our algorithm.
- They are easy to implement.

Stochastic Multi-Armed Bandit Problem

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- The distributions for each of the arms are fixed throughout the time horizon.

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$$R_T = r^*T - \sum_{i \in A} r_i T_i(T),$$

- The expected regret of an algorithm after T rounds can be written as

$$\mathbb{E}[R_T] = \sum_{i=1}^K \mathbb{E}[T_i(T)] \Delta_i,$$

- $\Delta_i = r^* - r_i$ denotes the gap between the means of the optimal arm and of the i -th arm.

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- Also we will define the hardness parameter H as $H = \sum_{i=1}^K \frac{1}{\Delta_i^2}$

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 - **Mean-based estimation:** In this approach at every timestep we choose an arm based on \hat{r}_i and its confidence interval c_i . Eg: UCB1 [Auer et al.(2002a)Auer, Cesa-Bianchi, and Fischer], MOSS [Audibert and Bubeck(2009)], UCB-Improved [Auer and Ortner(2010)]

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 - **Mean and Variance based Estimation:** Here, at every timestep we choose an arm based on \hat{r}_i , \hat{V}_i and its confidence interval c_i . Eg: UCB-Normal [Auer et al.(2002a)Auer, Cesa-Bianchi, and Fischer], UCB-V [Audibert et al.(2009)Audibert, Munos, and Szepesvári].

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- We will be focusing on Mean based estimation.

UCB1 Algorithm

([Auer et al.(2002a)Auer, Cesa-Bianchi, and Fischer])

Algorithm 1 UCB1

```
1: Pull each arm once
2: for  $t = K + 1, \dots, T$  do
3:   Pull the arm such that  $\max_{i \in A} \left\{ \hat{r}_i + \sqrt{\frac{2 \log t}{s_i}} \right\}$ 
4:    $t := t + 1$ 
5: end for
```

- Maintain an upper confidence bound (c_i) for each of the arms
- This c_i will help in sufficiently exploring sub-optimal arms and then exploiting the optimal arm.
- The gap-independent regret bound of $O\left(\sqrt{KT \log T}\right)$ and gap-dependent bound of $O\left(\frac{K \log(T)}{\Delta}\right)$.

Minimax Optimal Strategy in the Stochastic Case ([Audibert and Bubeck(2009)])

Algorithm 2 MOSS

1: Pull each arm once

2: **for** $t = K + 1, \dots, T$ **do**

3: Pull the arm such that $\max_{i \in A} \left\{ \hat{r}_i + \sqrt{\frac{\max\{0, \log(\frac{T}{Ks_i})\}}{s_i}} \right\}$

4: $t := t + 1$

5: **end for**

- UCB1 suffers from a worst case regret of $O\left(\sqrt{KT \log T}\right)$.
- MOSS corrects this and gives us a gap-independent regret bound of $O\left(\sqrt{KT}\right)$ and gap-dependent bound of $O\left(\frac{K^2 \log(\frac{T\Delta^2}{K})}{\Delta}\right)$.

Approach of UCB-Improved

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- Pull all surviving arms equal number of times during a round.
- At the end of the round eliminate some arms based on some criteria.
- Reset parameters and proceed to next round.
- UCB-Imp achieves a gap-independent regret bound of $O\left(\sqrt{KT \log K}\right)$ and gap-dependent bound of $O\left(\frac{K \log(T \Delta^2)}{\Delta}\right)$.

UCB-Improved ([Auer and Ortner(2010)])

Algorithm 3 UCB-Improved

- 1: **Input:** Time horizon T
- 2: **Initialization:** Set $B_0 := A$ and $\tilde{\Delta}_0 := 1$.
- 3: **for** $m = 0, 1, \dots, \lfloor \frac{1}{2} \log_2 \frac{T}{e} \rfloor$ **do**
- 4: Pull each arm in B_m , $n_m = \left\lceil \frac{2 \log (T \tilde{\Delta}_m^2)}{\tilde{\Delta}_m} \right\rceil$ number of times.
- 5: **Arm Elimination**
- 6: For each $i \in B_m$, delete arm i from B_m if,

$$\bar{X}_i + \sqrt{\frac{\log (T \tilde{\Delta}_m^2)}{2n_m}} < \max_{j \in B_m} \left\{ \bar{X}_j - \sqrt{\frac{\log (T \tilde{\Delta}_m^2)}{2n_m}} \right\}$$

- 7: Set $\tilde{\Delta}_{m+1} := \frac{\tilde{\Delta}_m}{2}$, Set $B_{m+1} := B_m$
- 8: Stop if $|B_m| = 1$ and pull $i \in B_m$ till n is reached.
- 9: **end for**

Some technical details of UCB-Improved

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- As opposed to UCB1, MOSS and OCUCB, UCB-Improved has fixed confidence interval $c_m = \sqrt{\frac{\log(T \tilde{\Delta}_m^2)}{2n_m}}$ for all arms in a particular phase.

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- As opposed to UCB1, MOSS and OCUCB, UCB-Improved has fixed confidence interval $c_m = \sqrt{\frac{\log(T \tilde{\Delta}_m^2)}{2n_m}}$ for all arms in a particular phase.
- c_m ensures that whenever $\tilde{\Delta}_m < \frac{\Delta_i}{2}$ in the m -th round, the arm i gets eliminated.

Algorithm 4 MOSS

- 1: **Input:** K, T, α, ψ
 - 2: Pull each arm once
 - 3: **for** $t = K + 1, \dots, T$ **do**
 - 4: Pull the arm such that $\max_{i \in A} \left\{ \hat{r}_i + \sqrt{\alpha \frac{\max\{0, \log(\frac{\psi T}{s_i})\}}{s_i}} \right\}$
 - 5: $t := t + 1$
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- UCB1 is too conservative in exploiting, MOSS is not conservative enough and tends to explore more often than required.
- OCUCB correctly balances this and achieves a gap-independent regret bound $O(\sqrt{KT})$ and gap-dependent bound $O\left(\frac{K \log(T/H)}{\Delta}\right)$.

Comparison of UCB1, MOSS, OCUCB, UCB-Improved

Table: Cumulative Regret of Algorithms

Algorithm	Upper bound on Cumulative Regret
UCB1	$\min \left\{ O \left(\frac{K \log T}{\Delta} \right), O \left(\sqrt{KT \log T} \right) \right\}$
MOSS	$\min \left\{ O \left(\frac{K^2 \log(T \Delta^2 / K)}{\Delta} \right), O \left(\sqrt{KT} \right) \right\}$
UCB-Improved	$\min \left\{ O \left(\frac{K \log(T \Delta^2)}{\Delta} \right), O \left(\sqrt{KT \log K} \right) \right\}$
OCUCB	$\min \left\{ O \left(\frac{K \log(T/H)}{\Delta} \right), O \left(\sqrt{KT} \right) \right\}$

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- The arm elimination condition of UCB-Imp is very conservative.
- When the gaps are small and uniform UCB-Imp performs very badly.
- There is a gap in theoretical guarantee and empirical performance.

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 - Can we study the effect of Clustering in SMAB?
- The answer to all of this is ClusUCB.

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- Also eliminate clusters with all of its arms by comparing its performance against the globally best arm.
- Reset parameters and move to the next round.

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- At the end of the round eliminate arms inside each cluster by comparing its performance against the best arm in the cluster.
- Also eliminate clusters with all of its arms by comparing its performance against the globally best arm.
- Reset parameters and move to the next round.
- At a higher level ClusUCB behaves like p independently running UCB-Imp with the exploration parameters ρ_a, ρ_s and ψ helping in overcoming early exploration.

ClusUCB Algorithm I

- 1: **Input:** Number of clusters p , time horizon T , exploration parameters ρ_a , ρ_s and ψ .
- 2: **Initialization:** Set $B_0 := A$, $S_0 = S$ and $\epsilon_0 := 1$.
- 3: Create a partition S_0 of the arms at random into p clusters of size up to $\ell = \left\lceil \frac{K}{p} \right\rceil$ each.
- 4: **for** $m = 0, 1, \dots, \left\lfloor \frac{1}{2} \log_2 \frac{14T}{K} \right\rfloor$ **do**
- 5: Pull each arm in B_m so that the total number of times it has been pulled is $n_m = \left\lceil \frac{2 \log(\psi T \epsilon_m^2)}{\epsilon_m} \right\rceil$.
- 6: **Arm Elimination**
- 7: For each cluster $s_k \in S_m$, delete arm $i \in s_k$ from B_m if

$$\hat{r}_i + \sqrt{\frac{\rho_a \log(\psi T \epsilon_m^2)}{2n_m}} < \max_{j \in s_k} \left\{ \hat{r}_j - \sqrt{\frac{\rho_a \log(\psi T \epsilon_m^2)}{2n_m}} \right\}$$

ClusUCB Algorithm II

8: **Cluster Elimination**

9: Delete cluster $s_k \in S_m$ and remove all arms $i \in s_k$ from B_m if

$$\max_{i \in s_k} \left\{ \hat{r}_i + \sqrt{\frac{\rho_s \log(\psi T \epsilon_m^2)}{2n_m}} \right\} < \max_{j \in B_m} \left\{ \hat{r}_j - \sqrt{\frac{\rho_s \log(\psi T \epsilon_m^2)}{2n_m}} \right\}.$$

10: Set $\epsilon_{m+1} := \frac{\epsilon_m}{2}$

11: Set $B_{m+1} := B_m$

12: Stop if $|B_m| = 1$ and pull $i \in B_m$ till T is reached.

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UCB1, MOSS, OCUCB, UCB-Imp, ClusUCB

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UCB-Improved	$\min \left\{ O \left(\frac{K \log(T \Delta^2)}{\Delta} \right), O \left(\sqrt{KT \log K} \right) \right\}$
ClusUCB	$\min \left\{ O \left(\frac{K \log(T \Delta^2 / \sqrt{\log K})}{\Delta} \right), O \left(\sqrt{KT \log K} \right) \right\}$
OCUCB	$\min \left\{ O \left(\frac{K \log(T/H)}{\Delta} \right), O \left(\sqrt{KT} \right) \right\}$

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- One simple solution is to pull the arm with the highest UCB at every timestep. This is called optimistic greedy sampling for UCB-Imp (see [Liu and Tsuruoka(2016)]).

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- We introduce this in Efficient ClusUCB or EClusUCB.

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- Divide the horizon into rounds and for each round allocate $|B_m|n_m$ pulls, where $n_m := \left\lceil \frac{2 \log(\psi T \epsilon_m^2)}{\epsilon_m} \right\rceil$.
- Within a round pull an arm that has the maximum UCB among all arms.

Technical Details of EClusUCB

- EClusUCB has three exploration parameters ρ_a , ρ_s and ψ .
- It also creates p fixed clusters (given as an input) and randomly assign arms into each of them such that each cluster contains equal number of arms.
- Divide the horizon into rounds and for each round allocate $|B_m|n_m$ pulls, where $n_m := \left\lceil \frac{2 \log(\psi T \epsilon_m^2)}{\epsilon_m} \right\rceil$.
- Within a round pull an arm that has the maximum UCB among all arms.
- The arm and cluster elimination is done in same way.

EClusUCB Algorithm I

Input: Number of clusters p , time horizon T , exploration parameters ρ_a, ρ_s and ψ .

Initialization: Set $m := 0$, $B_0 := A$, $S_0 = S$, $\epsilon_0 := 1$,

$$M = \lfloor \frac{1}{2} \log_2 \frac{14T}{K} \rfloor, n_0 = \left\lceil \frac{2 \log(\psi T \epsilon_0^2)}{\epsilon_0} \right\rceil \text{ and } N_0 = Kn_0.$$

Create a partition S_0 of the arms at random into p clusters of size up to $\ell = \left\lceil \frac{K}{p} \right\rceil$ each.

Pull each arm once

for $t = K + 1, \dots, T$ **do**

$$\text{Pull arm } i \in B_m \text{ such that } \operatorname{argmax}_{j \in B_m} \left\{ \hat{r}_j + \sqrt{\frac{\rho_s \log(\psi T \epsilon_m^2)}{2z_j}} \right\},$$

where z_j is the number of times arm j has been pulled

$$t := t + 1$$

Arm Elimination

EClusUCB Algorithm II

For each cluster $s_k \in S_m$, delete arm $i \in s_k$ from B_m if

$$\hat{r}_i + \sqrt{\frac{\rho_a \log(\psi T \epsilon_m^2)}{2z_i}} < \max_{j \in s_k} \left\{ \hat{r}_j - \sqrt{\frac{\rho_a \log(\psi T \epsilon_m^2)}{2z_j}} \right\}$$

Cluster Elimination

Delete cluster $s_k \in S_m$ and remove all arms $i \in s_k$ from B_m if

$$\max_{i \in s_k} \left\{ \hat{r}_i + \sqrt{\frac{\rho_s \log(\psi T \epsilon_m^2)}{2z_i}} \right\} < \max_{j \in B_m} \left\{ \hat{r}_j - \sqrt{\frac{\rho_s \log(\psi T \epsilon_m^2)}{2z_j}} \right\}.$$

if $t \geq N_m$ and $m \leq M$ then

$$\epsilon_{m+1} := \frac{\epsilon_m}{2}$$

$$B_{m+1} := B_m$$

$$n_{m+1} := \left\lceil \frac{2 \log(\psi T \epsilon_{m+1}^2)}{\epsilon_{m+1}} \right\rceil$$

EClusUCB Algorithm III

$$N_{m+1} := t + |B_{m+1}|n_{m+1}$$

$$m := m + 1$$

Stop if $|B_m| = 1$ and pull $i \in B_m$ till T is reached.

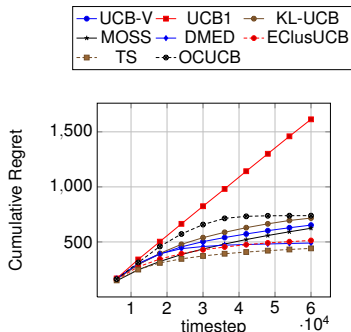
end if

end for

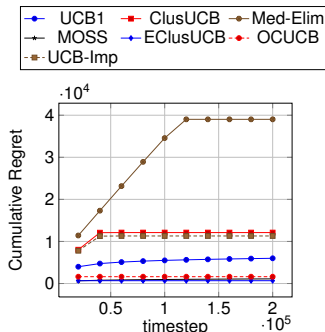
UCB1, MOSS, OCUCB, UCB-Imp, EClusUCB

Algorithm	Upper bound on Cumulative Regret
UCB1	$\min \left\{ O \left(\frac{K \log T}{\Delta} \right), O \left(\sqrt{KT \log T} \right) \right\}$
MOSS	$\min \left\{ O \left(\frac{K^2 \log(T \Delta^2 / K)}{\Delta} \right), O \left(\sqrt{KT} \right) \right\}$
UCB-Improved	$\min \left\{ O \left(\frac{K \log(T \Delta^2)}{\Delta} \right), O \left(\sqrt{KT \log K} \right) \right\}$
EClusUCB	$\min \left\{ O \left(\frac{K \log(T \Delta^2 / \sqrt{\log K})}{\Delta} \right), O \left(\sqrt{KT \log K} \right) \right\}$
OCUCB	$\min \left\{ O \left(\frac{K \log(T/H)}{\Delta} \right), O \left(\sqrt{KT} \right) \right\}$

Finally, experiment!!!



(a) Experiment 1: 20 Bernoulli-distributed arms with $r_{i \neq *} = 0.07$ and $r^* = 0.1$.



(b) Experiment 2: 100 Gaussian-distributed arms with $r_{i \neq *:1-33} = 0.1$, $r_{i \neq *:34-99} = 0.6$ and $r_{i=100}^* = 0.9$.

Figure: Cumulative regret for various bandit algorithms on two stochastic K-armed bandit environments.

Conclusions

- We introduced the ClusUCB and EClusUCB algorithms which achieves a better gap-dependent regret bound than UCB1, UCB-Imp and Moss.

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Thank You