Thresholding Bandits with Augmented UCB

Subhojyoti Mukherjee

IIT Madras

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Overview

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- After (say) pulling each arm once, we are presented with an *exploration-exploitation* trade-off, that is whether to continue to pull the arm for which we have observed the highest estimated reward till now(exploitation) or to explore a new arm(exploration).
- If we become too greedy and always exploit, we may miss the chance of actually finding the optimal arm and get stuck with a sub-optimal arm.

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- The distributions for each of the arms are fixed throughout the time horizon denoted by T.

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- At the end of the given T timesteps the learner must recommend a set of arms which (according to it) are the arms having reward mean above τ.

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- Let $\hat{S}_\tau$ denote the recommendation of a learning algorithm after T time units of exploration, while $\hat{S}_\tau^c$ denotes its complement.
- The performance of the learning agent is measured by the accuracy with which it can classify the arms into S_τ and S_τ^c after time horizon T. Equivalently, the *loss* $\mathcal{L}(T)$ is defined as

$$\mathcal{L}(T) = \mathbb{I}\big(\{S_\tau \cap \hat{S}_\tau^c \neq \emptyset\} \cup \{\hat{S}_\tau \cap S_\tau^c \neq \emptyset\}\big).$$

- The goal of the learning agent is to minimize the expected loss:

$$\mathbb{E}[\mathcal{L}(T)] = \mathbb{P}\big(\{S_\tau \cap \hat{S}_\tau^c \neq \emptyset\} \cup \{\hat{S}_\tau \cap S_\tau^c \neq \emptyset\}\big).$$

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- Selecting a small set of best workers (out of a very large pool of workers) whose productivity is above a threshold.
- In anomaly detection and classification (see Locatelli et al. (2016)).

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- It is the first variance-based arm elimination algorithm for the considered TBP settings.
- It address an open problem discussed in [Auer and Ortner(2010)] of designing an algorithm that can eliminate arms based on variance estimates.
- It is the first algorithm on the larger pure exploration setting which uses empirical variance estimates along with arm elimination with a new problem complexity.

Previous Works

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- Theoretically they proved this algorithm to be almost optimal when only mean estimation is used as a metric of comparison.
- Empirically it outperformed other state-of-the-art algorithms which were modified to perform in the TBP setting.

APT Algorithm

Algorithm 1 APT

Input: Time horizon T, threshold τ, tolerance factor $\epsilon \geq 0$
Pull each arm once

for $t = K + 1, .., T$ **do**
Pull arm $j \in \arg\min_{i \in A} \left\{ (|\hat{r}_i - \tau| + \epsilon)\sqrt{n_i} \right\}$ and observe the reward for arm j.
end for
Output: $\hat{S}_\tau = \{i : \hat{r}_i \geq \tau\}$.

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- Our considered TBP is a fixed budget pure exploration problem.
- Both APT and AugUCB reuses several ideas from Pure exploration problem.

Previous Works (Diagram)

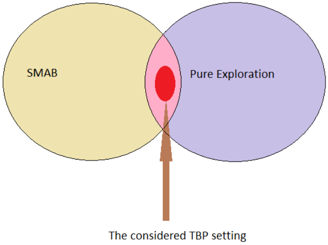

Figure: TBP place within SMAB and Pure exploration

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- Pull all surviving arms equal number of times during a round.
- At the end of the round eliminate some sub-optimal arms (as judged by learner) based on elimination criteria.
- Reset parameters and proceed to next round.

Algorithm 2 UCB-Improved

Input: Time horizon T
Initialization: Set $B_0 := A$ and $\epsilon_0 := 1$.
for $m = 0, 1, .. \lfloor \frac{1}{2} \log_2 \frac{T}{e} \rfloor$ **do**
    Pull each arm in B_m, $n_m = \left\lceil \frac{2 \log(T\epsilon_m^2)}{\epsilon_m} \right\rceil$ number of times.
    Arm Elimination
        For each $i \in B_m$, delete arm i from B_m if,

$$\hat{r}_i + \sqrt{\frac{\log(T\epsilon_m^2)}{2n_m}} < \max_{j \in B_m} \left\{ \hat{r}_j - \sqrt{\frac{\log(T\epsilon_m^2)}{2n_m}} \right\}$$

    Set $\epsilon_{m+1} := \frac{\epsilon_m}{2}$, Set $B_{m+1} := B_m$
    Stop if $|B_m| = 1$ and pull $i \in B_m$ till T is reached.
end for

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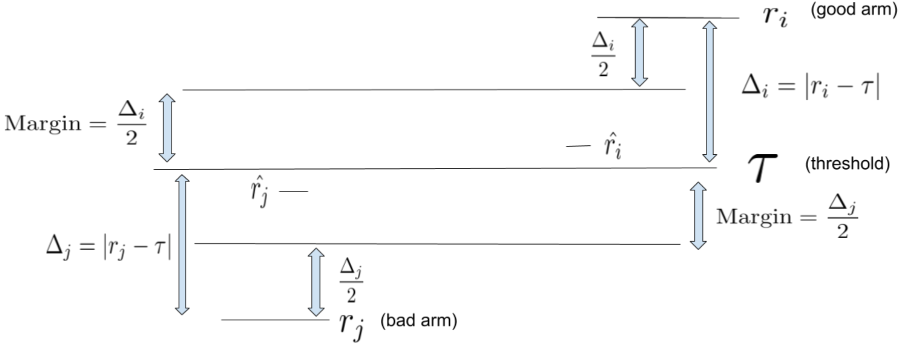
- UCB-Improved has fixed confidence interval $c_m = \sqrt{\dfrac{\log(T\epsilon_m^2)}{2n_m}}$ for all arms in a particular round.
- c_m ensures that whenever $\epsilon_m < \dfrac{\Delta_i}{2}$ in the m-th round, the arm i gets eliminated.

AugUCB algorithm (Intuition)

- We define $\Delta_i = |r_i - \tau|$.
- It is risky to eliminate arm i while $\hat{r}_i$ is inside *Margin*.
- Confidence interval s_i will make sure arm i is not eliminated while inside Margin with a high probability.



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- At every timestep now we run the arm elimination check to eliminate sub-optimal arms.
- At the end of the phase we reset the parameters.
- Note that the length of the phase, the exploration parameters and the confidence interval term $s_i = \sqrt{\frac{\rho\psi_m(\hat{v}_i + 1)\log(T\epsilon_m)}{4n_i}}$ are set through detailed theoretical analysis.

AugUCB algorithm I

Input: Time budget T; parameter ρ; threshold τ
Initialization: $B_0 = \mathcal{A}$; $m = 0$; $\epsilon_0 = 1$;

$$M = \left\lfloor \frac{1}{2} \log_2 \frac{T}{e} \right\rfloor; \quad \psi_0 = \frac{T\epsilon_0}{128\Big(\log(\frac{3}{16}K\log K)\Big)^2};$$

$$\ell_0 = \left\lceil \frac{2\psi_0 \log(T\epsilon_0)}{\epsilon_0} \right\rceil; \quad N_0 = K\ell_0$$

Pull each arm once

for $t = K+1, .., T$ **do**
  Pull arm $j \in \arg\min_{i\in B_m} \Big\{ |\hat{r}_i - \tau| - 2s_i \Big\}$
  for $i \in B_m$ **do**
    if $(\hat{r}_i + s_i < \tau - s_i)$ or $(\hat{r}_i - s_i > \tau + s_i)$ **then**
      $B_m \leftarrow B_m \backslash \{i\}$ (Arm deletion)
    end if

AugUCB algorithm II

end for

if $t \geq N_m$ and $m \leq M$ **then**

Reset Parameters

$\epsilon_{m+1} \leftarrow \frac{\epsilon_m}{2}$
$B_{m+1} \leftarrow B_m$
$\psi_{m+1} \leftarrow \frac{T\epsilon_{m+1}}{128(\log(\frac{3}{16}K\log K))^2}$
$\ell_{m+1} \leftarrow \left\lceil \frac{2\psi_{m+1}\log(T\epsilon_{m+1})}{\epsilon_{m+1}} \right\rceil$
$N_{m+1} \leftarrow t + |B_{m+1}|\ell_{m+1}$
$m \leftarrow m+1$

end if

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- The relationship between H_1 and H_2 can be derived as,

$$H_2 \leq H_1 \leq \log(2K) H_2$$

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- For a variance aware algorithm we define $H_{\sigma,1}$ (as in Gabillon et al. (2011)) that incorporates reward variances into its expression as:

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- Finally, analogous to H_2, we introduce $H_{\sigma,2}$, such that $H_{\sigma,2} = \max_{i \in \mathcal{A}} \frac{i}{\tilde{\Delta}_{(i)}^2}$, where $\tilde{\Delta}_i^2 = \frac{\Delta_i^2}{\sigma_i + \sqrt{\sigma_i^2 + (16/3)\Delta_i}}$, $(\tilde{\Delta}_{(i)})$ is an increasing ordering of $(\tilde{\Delta}_i)$.

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$$H_{\sigma,2} \leq H_{\sigma,1} \leq \log(2K) H_{\sigma,2}.$$

- Note that H_1, H_2 and $H_{\sigma,1}, H_{\sigma,2}$ are not directly comparable to each other except in a special case when variances are very low we can say that $H_{\sigma,1} < H_1$.

Expected Loss of AugUCB

Theorem

For $K \geq 4$ and $\rho = 1/3$, the expected loss of the AugUCB algorithm is given by,

$$\mathbb{E}[\mathcal{L}(T)] \leq 2KT \exp\left(-\frac{T}{4096 \log(K \log K) H_{\sigma,2}}\right).$$

Table: AugUCB vs. State of the art

Algorithm	Upper Bound on Expected Loss
AugUCB	$\exp\left(-\frac{T}{4096 \log(K \log K) H_{\sigma,2}} + \log(2KT)\right)$
UCBEV	$\exp\left(-\frac{1}{512} \frac{T-2K}{H_{\sigma,1}} + \log(6KT)\right)$
APT	$\exp\left(-\frac{T}{64 H_1} + 2\log((\log(T)+1)K)\right)$
CSAR	$\exp\left(-\frac{T-K}{72 \log(K) H_{CSAR,2}} + 2\log(K)\right)$

Concentration Bounds

- Let $X_1, ..., X_n$ be random variables with common support $[0, 1]$ and such that $E[X_t|X_1, ..., X_{t-1}] = r_i$. Let $\hat{r}_i = \frac{X_1+, ..., +X_n}{n}$. Then for all $c \geq 0$,

$$\mathbb{P}\{\hat{r}_i \geq r_i + c\} \leq \exp\left(-2c^2 n\right)$$
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- Along with the above information if we know that $\text{Var}[X_t|X_1, ..., X_{t-1}] = \sigma_i^2$ then Bernstein inequality gives us,

$$\mathbb{P}\{\hat{r}_i \geq r_i + c\} \leq \exp\left(-\frac{c^2 n}{2\sigma_i^2 + \frac{2c}{3}}\right)$$
$$\mathbb{P}\{\hat{r}_i \leq r_i - c\} \leq \exp\left(-\frac{c^2 n}{2\sigma_i^2 + \frac{2c}{3}}\right)$$

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- We bound the arm-elimination probability by Bernstein inequality (as in Audibert et al. (2009)) rather that the Chernoff-Hoeffding bounds (used in UCB-Imp).
- In the second module,we define a favourable event that will yield an upper bound on the expected loss and use union bound and module-1 (on the arm elimination probability) to derive the result through a series of simplifications.

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- APT, AugUCB, CSAR, UA do not require access to H_1 or $H_{\sigma,1}$.

Finally, experiment!!!

- We experiment with APT, AugUCB, UCBE, UCBEV, CSAR, UA.
- Note that UCBE and UCBEV require access to H_1 and $H_{\sigma,1}$ as input and hence not implementable in real life.
- By access we mean that an oracle supplies them the H_1 or $H_{\sigma,1}$. They do not have access to individual means and variances.
- APT, AugUCB, CSAR, UA do not require access to H_1 or $H_{\sigma,1}$.
- UCBE, UCBEV, CSAR and UA come from the pure exploration lineage and are modified suitably to perform in TBP setting.

Experimental Setup

- This setup involves Gaussian reward distributions with $K = 100$, $T = 10000$ and $\tau = 0.5$ with the reward means set in two groups.

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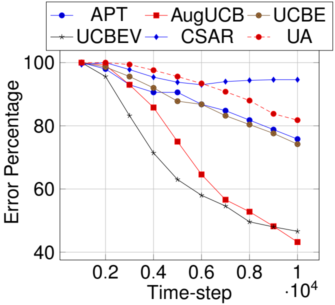
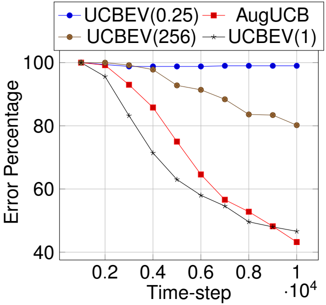
- The first 10 arms partitioned into two groups; the respective means are $r_{1:5} = 0.45$, $r_{6:10} = 0.55$.
- The means of arms $i = 11 : 100$ are chosen same as $r_{11:100} = 0.4$.
- Variances are set as $\sigma^2_{1:5} = 0.3$ and $\sigma^2_{6:10} = 0.8$; $\sigma^2_{11:100}$ are independently and uniformly chosen in the interval $[0.2, 0.3]$.

Experimental Result



(a) Expt-1: Two Group Setting (Advance)



(b) Expt-2: Two Group Setting (Advance)

Conclusion

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- Further studies are required to establish a lower bound on the expected loss of AugUCB.
- A more detailed analysis of the non-uniform arm selection and parameter selection is also required.

Future direction in MS

- We have proposed a very similar algorithm as in AugUCB for the SMAB problem with more detailed study of non-uniform arm selection and parameter selection.

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- Termed as Efficient UCBV, we are preparing for submission for AAAI 2018.
- I am going to INRIA, SequeL Lab, Lille, France for a three month internship starting from 1st September under Dr. Odalric Maillard.

Any Questions?

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Thank You