Thresholding Bandits with Augmented UCB

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Overview

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- After say pulling each arm once, we are presented with an *exploration-exploitation* trade-off, that is whether to continue to pull the arm for which we have observed the highest estimated reward till now(exploitation) or to explore a new arm(exploration).
- If we become too greedy and always exploit, we may miss the chance of actually finding the optimal arm and get stuck with a sub-optimal arm.

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- The above goal has to be achieved within T timesteps of exploration/exploitation and this is termed as a fixed-budget problem.
- At the end of the given T timesteps the learner must recommend a set of arms which (according to it) are the arms having reward mean above τ.

Problem Definition

- We define the set $S_\tau = \{i \in \mathcal{A} : r_i \geq \tau\}$. Note that, S_τ is the set of all arms whose reward mean is greater than τ. Let S_τ^c denote the complement of S_τ, i.e., $S_\tau^c = \{i \in \mathcal{A} : r_i < \tau\}$.

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- Let $\hat{S}_\tau$ denote the recommendation of a learning algorithm after T time units of exploration, while $\hat{S}_\tau^c$ denotes its complement.
- The performance of the learning agent is measured by the accuracy with which it can classify the arms into S_τ and S_τ^c after time horizon T. Equivalently, the *loss* $\mathcal{L}(T)$ is defined as

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- The goal of the learning agent is to minimize the expected loss:

$$\mathbb{E}[\mathcal{L}(T)] = \mathbb{P}\big(\{S_\tau \cap \hat{S}_\tau^c \neq \emptyset\} \cup \{\hat{S}_\tau \cap S_\tau^c \neq \emptyset\}\big).$$

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- In anomaly detection and classification (see [Locatelli et al.(2016)Locatelli, Gutzeit, and Carpentier]).

Contribution

- We propose the Augmented UCB (AugUCB) [Mukherjee et al.(2017)Mukherjee, Naveen, Nandan, and Ravindran] algorithm for the fixed-budget TBP setting.

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- It is the first variance-based arm elimination algorithm for the considered TBP settings.
- It address an open problem discussed in [Auer and Ortner(2010)] of designing an algorithm that can eliminate arms based on variance estimates.
- It is the first algorithm on the larger pure exploration setting which uses empirical variance estimates along with arm elimination with a new problem complexity.

Previous Works

- The Anytime Parameter Free (APT) [Locatelli et al.(2016)Locatelli, Gutzeit, and Carpentier] algorithm was proposed for TBP setting in ICML 2016.

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- Theoretically they proved this algorithm to be almost optimal when only mean estimation is used as a metric of comparison.
- Empirically it outperformed other state-of-the-art algorithms which were modified to perform in the TBP setting.

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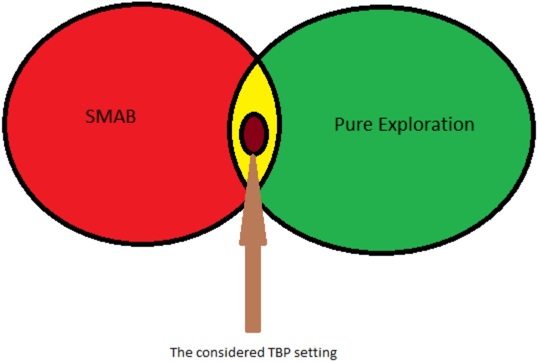
- Our considered TBP is a fixed budget pure exploration problem.
- Both APT and AugUCB reuses several ideas from Pure exploration problem.

Previous Works (Diagram)



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- Pull all surviving arms equal number of times during a round.
- At the end of the round eliminate some sub-optimal arms (as judged by learner) based on elimination criteria.
- Reset parameters and proceed to next round.

UCB-Improved ([Auer and Ortner(2010)])

Algorithm 1 UCB-Improved

Input: Time horizon T

Initialization: Set $B_0 := A$ and $\epsilon_0 := 1$.

for $m = 0, 1, .. \lfloor \frac{1}{2} \log_2 \frac{T}{e} \rfloor$ **do**

  Pull each arm in B_m, $n_m = \left\lceil \frac{2 \log(T\epsilon_m^2)}{\epsilon_m} \right\rceil$ number of times.

  Arm Elimination

    For each $i \in B_m$, delete arm i from B_m if,

$$\hat{r}_i + \sqrt{\frac{\log(T\epsilon_m^2)}{2n_m}} < \max_{j \in B_m} \left\{ \hat{r}_j - \sqrt{\frac{\log(T\epsilon_m^2)}{2n_m}} \right\}$$

  Set $\epsilon_{m+1} := \frac{\epsilon_m}{2}$, Set $B_{m+1} := B_m$

  Stop if $|B_m| = 1$ and pull $i \in B_m$ till n is reached.

end for

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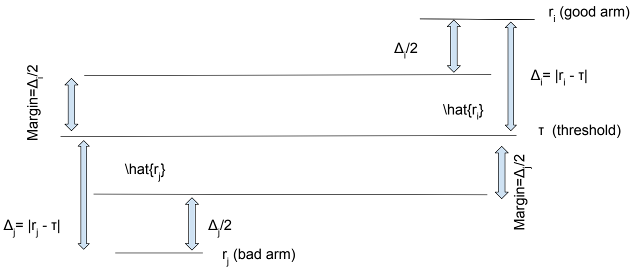
- UCB-Improved has fixed confidence interval $c_m = \sqrt{\dfrac{\log(T\epsilon_m^2)}{2n_m}}$ for all arms in a particular phase.
- c_m ensures that whenever $\epsilon_m < \dfrac{\Delta_i}{2}$ in the m-th round, the arm i gets eliminated.

AugUCB algorithm

$\Delta_i = |r_i - \tau|$. It is risky to eliminate while estimated mean of arm i ($\hat{r_i}$) is inside Margin. Confidence interval s_i will make sure arm i is not eliminated while inside Margin. So we have to bound $P\{\hat{r_i}\} <= r_i - 2s_i\}$. Till that time arm i will not be accepted as good arm. For arm j we have to bound $P\{\hat{r_j}\} >= r_j + 2s_j\}$ since till that time j will not be rejected as bad arm.



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- At the end of the phase we reset the parameters.
- Note that the length of the phase, the exploration parameters and the confidence interval term $s_i = \sqrt{\frac{\rho\psi_m(\hat{v}_i+1)\log(T\epsilon_m)}{4n_i}}$ are set through detailed theoretical analysis.

AugUCB algorithm I

Input: Time budget T; parameter ρ; threshold τ
Initialization: $B_0 = \mathcal{A}$; $m = 0$; $\epsilon_0 = 1$;

$$M = \left\lfloor \frac{1}{2} \log_2 \frac{T}{e} \right\rfloor; \quad \psi_0 = \frac{T\epsilon_0}{128\left(\log(\frac{3}{16}K\log K)\right)^2};$$

$$\ell_0 = \left\lceil \frac{2\psi_0 \log(T\epsilon_0)}{\epsilon_0} \right\rceil; \quad N_0 = K\ell_0$$

Pull each arm once

for $t = K+1, .., T$ **do**
  Pull arm $j \in \arg\min_{i \in B_m} \left\{ |\hat{r}_i - \tau| - 2s_i \right\}$
  for $i \in B_m$ **do**
    if $(\hat{r}_i + s_i < \tau - s_i)$ or $(\hat{r}_i - s_i > \tau + s_i)$ **then**
      $B_m \leftarrow B_m \backslash \{i\}$ (Arm deletion)
    end if

AugUCB algorithm II

end for

if $t \geq N_m$ and $m \leq M$ **then**

Reset Parameters

$\epsilon_{m+1} \leftarrow \frac{\epsilon_m}{2}$

$B_{m+1} \leftarrow B_m$

$\psi_{m+1} \leftarrow \frac{T\epsilon_{m+1}}{128(\log(\frac{3}{16}K\log K))^2}$

$\ell_{m+1} \leftarrow \left\lceil \frac{2\psi_{m+1}\log(T\epsilon_{m+1})}{\epsilon_{m+1}} \right\rceil$

$N_{m+1} \leftarrow t + |B_{m+1}|\ell_{m+1}$

$m \leftarrow m + 1$

end if

end for

Output: $\hat{S}_\tau = \{i : \hat{r}_i \geq \tau\}$.

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- The relationship between H_1 and H_2 can be derived as,

$$H_1 \leq \log(2K) H_2 \text{ and } H_1 \leq \log(K) H_{CSAR,2}.$$

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- For variance aware algorithm H_1^σ ([Gabillon et al.(2011)Gabillon, Ghavamzadeh, Lazaric, and Bubeck]) that incorporates reward variances into its expression as:

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- Note that H_1, H_2 and $H_{\sigma,1}, H_{\sigma,2}$ are not directly comparable to each other except in a special case when variances are very low we can say that $H_{\sigma,1} < H_1$.

Expected Loss

Theorem

For $K \geq 4$ and $\rho = 1/3$, the expected loss of the AugUCB algorithm is given by,

$$\mathbb{E}[\mathcal{L}(T)] \leq 2KT \exp\left(-\frac{T}{4096\log(K\log K)H_{\sigma,2}}\right).$$

Table: AugUCB vs. State of the art

Algorithm	Upper Bound on Expected Loss
AugUCB	$\exp\left(-\frac{T}{4096\log(K\log K)H_{\sigma,2}} + \log(2KT)\right)$
UCBEV	$\exp\left(-\frac{1}{512}\frac{T-2K}{H_{\sigma,1}} + \log(6KT)\right)$
APT	$\exp\left(-\frac{T}{64H_1} + 2\log((\log(T)+1)K)\right)$
CSAR	$\exp\left(-\frac{T-K}{72\log(K)H_{CSAR,2}} + 2\log(K)\right)$

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- In the second module,we define a favourable event that will yield an upper bound on the expected loss and use union bound and module-1 (on the arm elimination probability) to derive the result through a series of simplifications.

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- By access we mean that an oracle supplies them the H_1 or $H_{\sigma,1}$, they do not have access to individual means and variances.
- APT, AugUCB, CSAR, UA do not require access to H_1 or $H_{\sigma,1}$.
- UCBE, UCBEV, CSAR and UA come from the pure exploration lineage and are modified suitably to perform in TBP setting.

Experimental Setup

- This setup involves Gaussian reward distributions with $K = 100$, $T = 10000$ and $\tau = 0.5$ with the reward means set in two groups.

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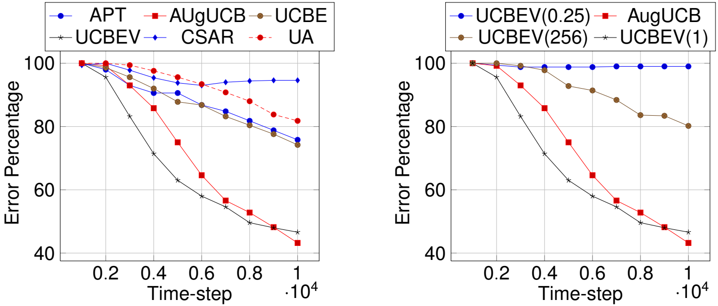
- The means of arms $i = 11 : 100$ are chosen same as $r_{11:100} = 0.4$.
- Variances are set as $\sigma^2_{1:5} = 0.3$ and $\sigma^2_{6:10} = 0.8$; $\sigma^2_{11:100}$ are independently and uniformly chosen in the interval $[0.2, 0.3]$.

Experimental Result



(a) Expt-1: Two Group Setting (Advance)

(b) Expt-2: Two Group Setting (Advance)

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Thank You