

Thresholding Bandits with Augmented UCB

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CS15S300

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Overview

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- 2 Problem Definition
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- The rewards for each of the arms are identical and independent random variables drawn from distribution specific to the arm.
- The learner does not know the mean $r_i, \forall i \in A$ of the distribution or the variance σ_i^2 .

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- The more we pull arm i the closer \hat{r}_i gets to r_i .
- Due to the uncertainty in \hat{r}_i we have to carefully conduct exploration.

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- **Condition:** This has to be achieved within T timesteps of exploration and this is termed as a fixed-budget problem.

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- Let \hat{S}_τ denote the recommendation of a learning algorithm after T time units of exploration, while \hat{S}_τ^c denotes its complement.
- The goal of the learning agent is to minimize the expected loss:

$$\mathbb{E}[\mathcal{L}(T)] = \mathbb{P}\left(\underbrace{\{S_\tau \cap \hat{S}_\tau^c \neq \emptyset\}}_{\text{Rejected good arms}} \cup \underbrace{\{\hat{S}_\tau \cap S_\tau^c \neq \emptyset\}}_{\text{Accepted bad arms}} \right)$$

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- Higher the variance of reward distribution of the arms' \Rightarrow harder the problem.

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- In anomaly detection and classification (see Locatelli et al. (2016)).

- We propose the Augmented UCB (AugUCB) [Mukherjee et al. (2017)] algorithm for the fixed-budget TBP setting.

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- It is the first variance-based arm elimination algorithm for the considered TBP settings.
- It addresses an open problem discussed in Auer and Ortner (2010) of designing an algorithm that can eliminate arms based on variance estimates.
- We also define a new problem complexity which uses empirical variance estimates along with arm's mean for giving the theoretical bound.

The Upper Confidence Bound (UCB) Approach

- Since there is an initial uncertainty in the estimated mean (\hat{r}_i) introduce a confidence interval term c_i .

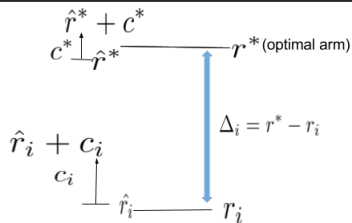
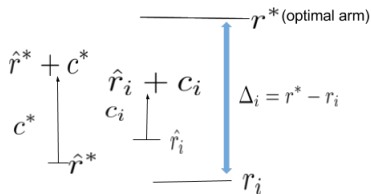
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- c_i ensures that the arm i is properly explored and is gradually reduced with time as one pulls the arm i more.

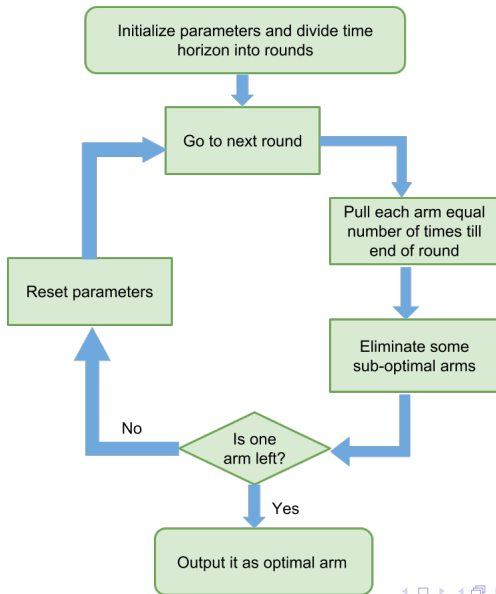
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- c_i represents the uncertainty about \hat{r}_i . Higher the c_i , higher is the uncertainty.
- c_i ensures that the arm i is properly explored and is gradually reduced with time as one pulls the arm i more.
- At every timestep pull arm $j \in \arg \max_{i \in A} \{\hat{r}_i + c_i\}$ and this will ensure that proper exploration is done.

The UCB Approach

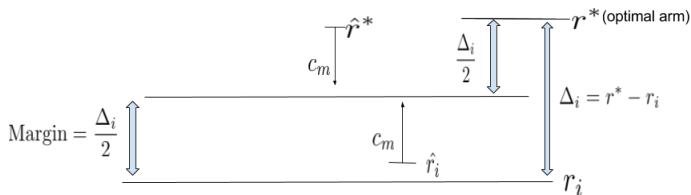


Approach of UCB-Improved (UCB-Imp)



Intuition of UCB-Improved (UCB-Imp)

Arm Elimination: $\hat{r}_i + c_m < \hat{r}_{max} - c_m$



APT Approach

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- This algorithm uses only mean estimation to find the S_τ .
- Theoretically they proved this algorithm to be almost optimal when only mean estimation is used as a metric of comparison.
- Empirically it outperformed other state-of-the-art algorithms which were modified to perform in the TBP setting.

Algorithm 1 APT

Input: Time horizon T , threshold τ , tolerance factor $\epsilon \geq 0$

Pull each arm once

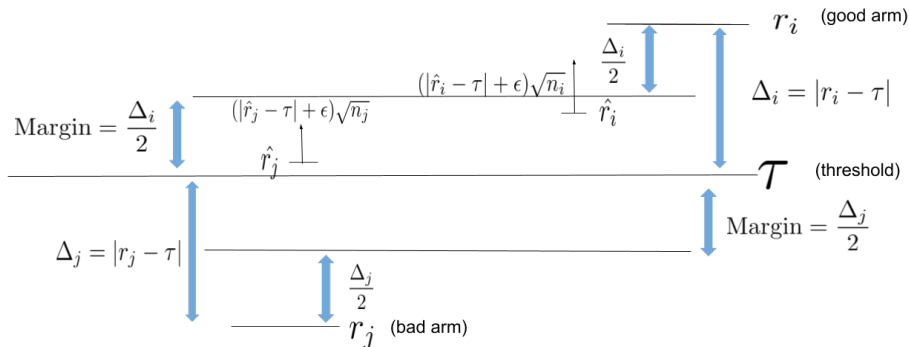
for $t = K + 1, \dots, T$ **do**

 Pull arm $j \in \arg \min_{i \in A} \left\{ (|\hat{r}_i - \tau| + \epsilon) \sqrt{n_i} \right\}$ and observe the reward for arm j .

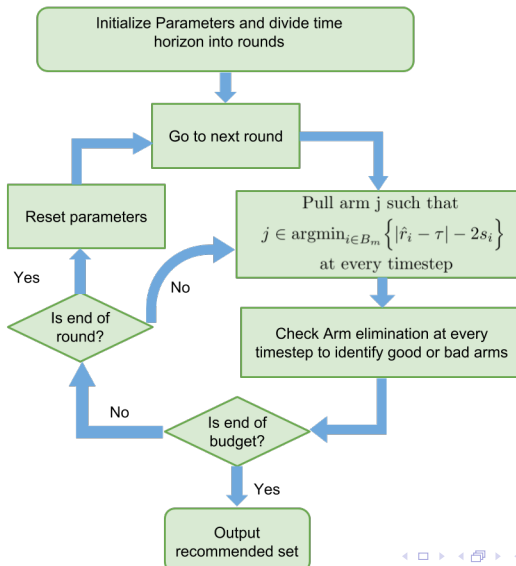
end for

Output: $\hat{S}_\tau = \{i : \hat{r}_i \geq \tau\}$.

Intuition of APT

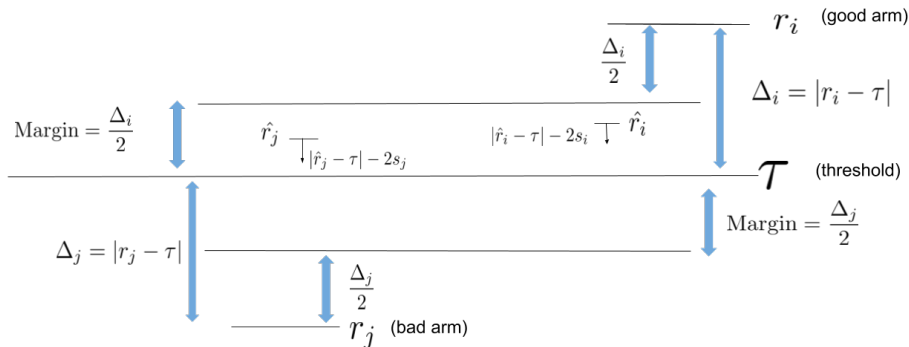


AugUCB algorithm



AugUCB algorithm (Intuition, Arm pulling)

- Like UCB-Imp, AugUCB also divides the time budget T into rounds.
- At every timestep we pull arm j s.t. $j \in \arg \min_{i \in B_m} \left\{ |\hat{r}_i - \tau| - 2s_i \right\}$ (like APT).



AugUCB algorithm (Intuition, Arm Elimination)

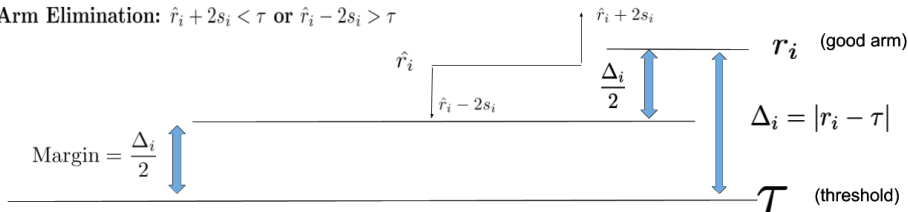
- We eliminate an arm when we are sure that \hat{r}_i is close to r_i with high probability and hence identify it as good or bad arm.
- It's risky to eliminate an arm when \hat{r}_i is inside *Margin*.
- Confidence interval s_i will make sure arm i is not eliminated while inside Margin with a high probability.

Arm Elimination: $\hat{r}_i + 2s_i < \tau$

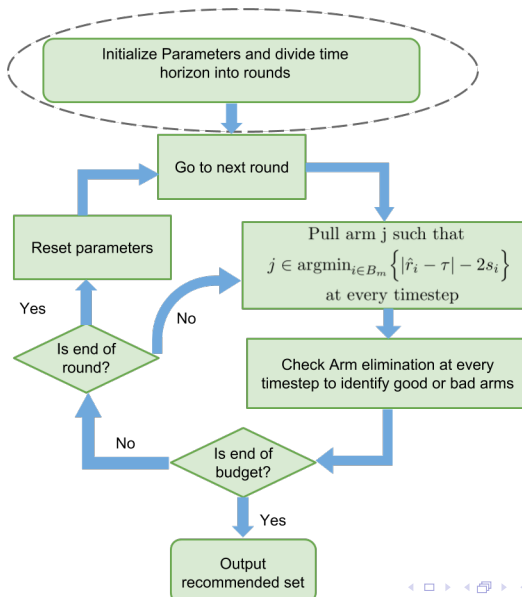
AugUCB algorithm (Intuition, Arm Elimination)

- Now we see that \hat{r}_i has moved close to its true estimate r_i .
- We eliminate i and re-allocate the remaining budget to pull arms close to the threshold

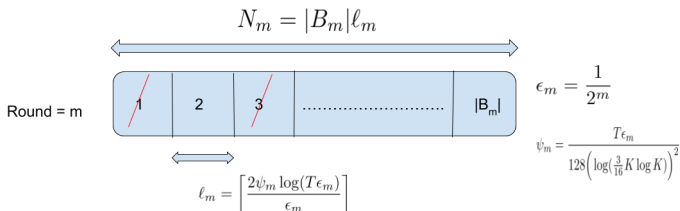
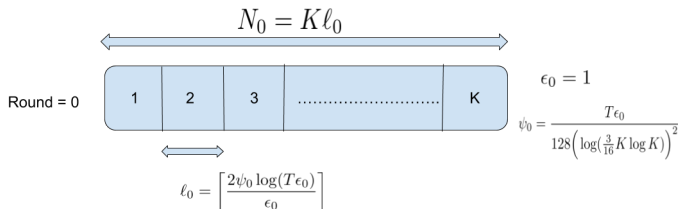
Arm Elimination: $\hat{r}_i + 2s_i < \tau$ or $\hat{r}_i - 2s_i > \tau$



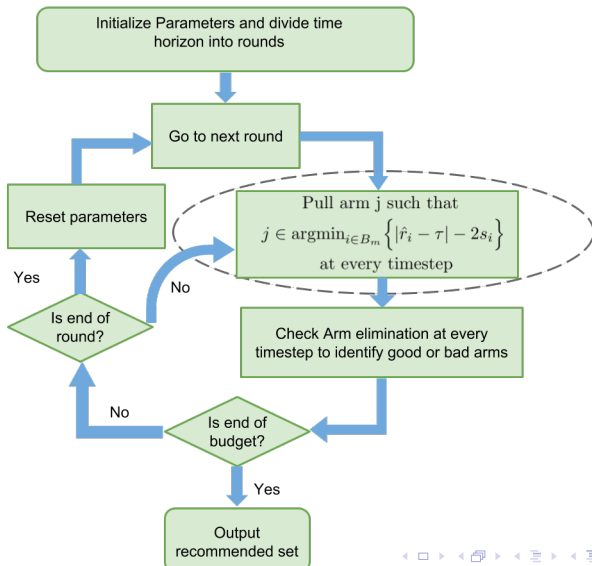
AugUCB parameter initialization



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AugUCB arm pull



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Arm pull

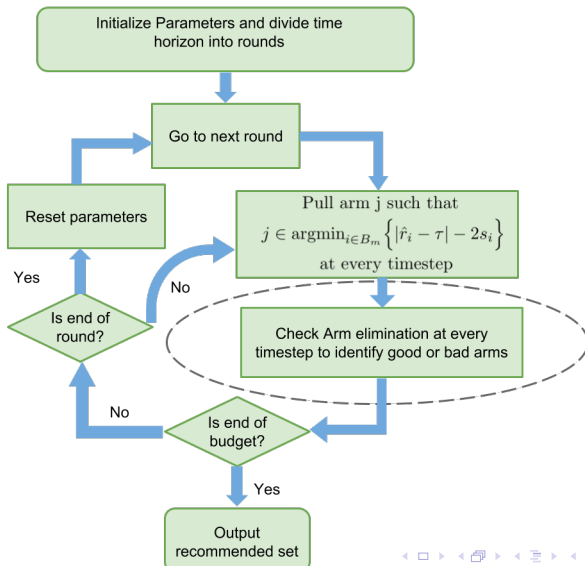
- We pull the arm that minimizes $j \in \arg \min_{i \in B_m} \left\{ |\hat{r}_i - \tau| - 2s_i \right\}$
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- s_i decreases with more n_i and ψ_m and ρ ensures that it decreases at a correct rate.
- Note that \hat{v}_i estimated variance in s_i makes the algorithm pull the arm which shows more variance.

AugUCB arm elimination



Arm elimination

- Arm elimination condition is checked at every timestep.

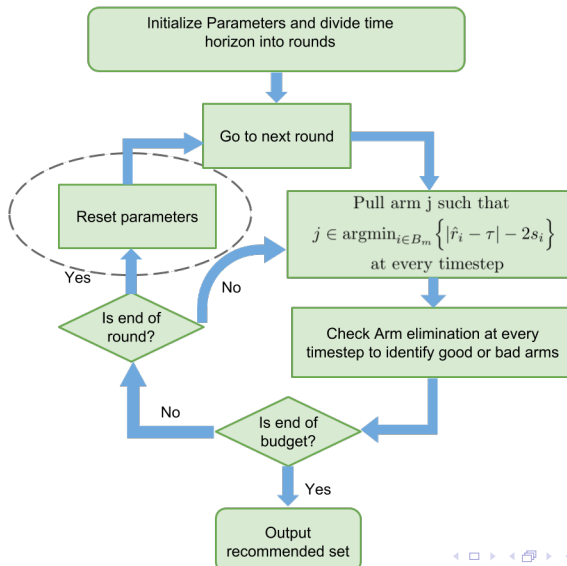
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- It eliminates the arms which have been identified as good or bad arms (with a high probability) and re-allocates the remaining budget for surviving arms.

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- Recalculate the length of next round on the number of surviving arms.

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- From Audibert and Bubeck (2010) the relationship between H_1 and H_2 can be derived as,

$$H_2 \leq H_1 \leq \log(2K)H_2$$

Problem Complexity

- For a variance aware algorithm we define $H_{\sigma,1}$ (as in Gabillon et al. (2011)) that incorporates reward variances into its expression as:

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- Finally, analogous to H_2 , we introduce $H_{\sigma,2}$, such that $H_{\sigma,2} = \max_{i \in \mathcal{A}} \frac{i}{\tilde{\Delta}_{(i)}^2}$, where $\tilde{\Delta}_i^2 = \frac{\Delta_i^2}{\sigma_i + \sqrt{\sigma_i^2 + (16/3)\Delta_i}}$, $(\tilde{\Delta}_{(i)})$ is an increasing ordering of $(\tilde{\Delta}_i)$.

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- Note that H_1 , H_2 and $H_{\sigma,1}$, $H_{\sigma,2}$ are not directly comparable to each other except in a special case when variances and gaps (Δ_i) are very low we can say that $H_{\sigma,1} < H_1$.

Expected Loss of AugUCB

Theorem

For $K \geq 4$ and $\rho = 1/3$, the expected loss of the AugUCB algorithm is given by,

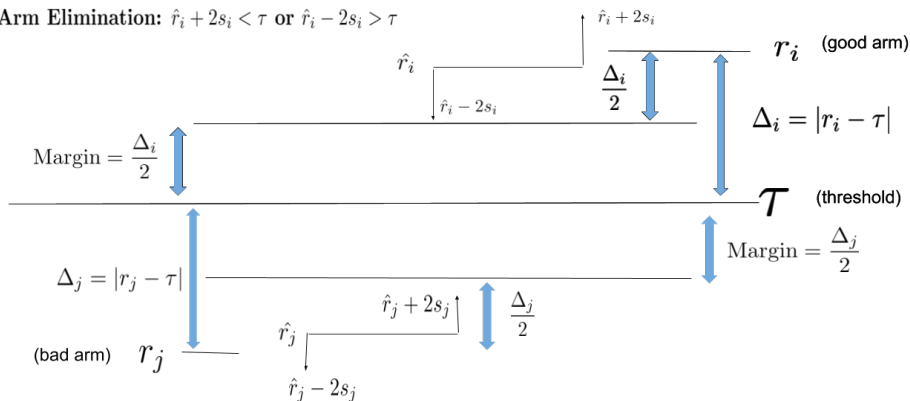
$$\mathbb{E}[\mathcal{L}(T)] \leq 2KT \exp \left(- \frac{T}{4096 \log(K \log K) H_{\sigma,2}} \right).$$

Table: AugUCB vs. State of the art

Algorithm	Upper Bound on Expected Loss	Oracle
AugUCB	$\exp \left(- \frac{T}{4096 \log(K \log K) H_{\sigma,2}} + \log(2KT) \right)$	No
UCBEV	$\exp \left(- \frac{1}{512} \frac{T-2K}{H_{\sigma,1}} + \log(6KT) \right)$	Yes
APT	$\exp \left(- \frac{T}{64H_1} + 2 \log((\log(T) + 1)K) \right)$	No
UCBE	$\exp \left(- \frac{T-K}{18H_1} - 2 \log(\log(T)K) \right)$	Yes

Sketch of the proof

Arm Elimination: $\hat{r}_i + 2s_i < \tau$ or $\hat{r}_i - 2s_i > \tau$



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- APT, AugUCB, UA do not require access to H_1 or $H_{\sigma,1}$.

Experimental Setup

- This setup involves Gaussian reward distributions with $K = 100$, $T = 10000$ and $\tau = 0.5$ with the reward means set in two groups.

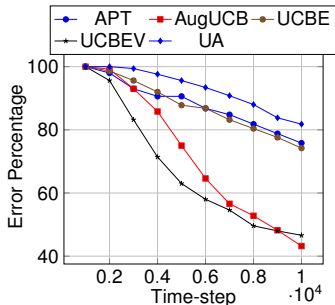
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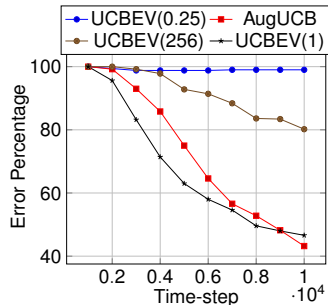
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- The first 10 arms partitioned into two groups; the respective means are $r_{1:5} = 0.45$, $r_{6:10} = 0.55$.
- The means of arms $i = 11 : 100$ are chosen same as $r_{11:100} = 0.4$.
- Variances are set as $\sigma_{1:5}^2 = 0.3$ and $\sigma_{6:10}^2 = 0.8$; $\sigma_{11:100}^2$ are independently and uniformly chosen in the interval $[0.2, 0.3]$.

Experimental Results

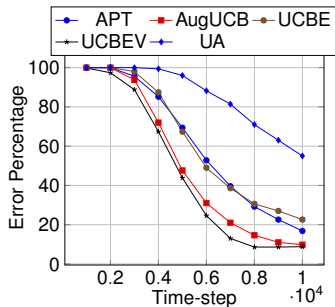


(a) Expt-1: Two Group Setting (Advance)

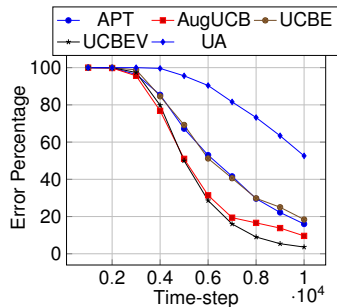


(b) Expt-2: Two Group Setting (Advance)

Experimental Results



(c) Expt-1: Arithmetic Progression (Gaussian)



(d) Expt-2: Geometric Progression (Gaussian)

Conclusion

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- A more detailed analysis of the non-uniform arm selection and parameter selection is also required.

Thank You