Tutorial on Bandit

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Overview

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- After say pulling each arm once we are presented with an *exploration-exploitation* trade-off, that is whether to continue to pull the arm for which we have observed the highest estimated reward till now(exploitation) or to explore a new arm(exploration).
- If we become too greedy and always exploit we may miss the chance of actually finding the optimal arm and get stuck with a sub-optimal arm.

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- They form the linking pieces of a larger problem.
- They are easy to implement.

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- Selecting the best possible route for a message to pass through in a peer-to-peer network connection.

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- The distributions for each of the arms are fixed throughout the time horizon.

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- Cumulative Regret: The loss we suffer because of not pulling the optimal arm till the total number of timesteps T.

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$$R_T = r^* T - \sum_{i \in A} r_i T_i(T),$$

- The expected regret of an algorithm after T rounds can be written as

$$\mathbb{E}[R_T] = \sum_{i=1}^{K} \mathbb{E}[T_i(T)]\Delta_i,$$

- $\Delta_i = r^* - r_i$ denotes the gap between the means of the optimal arm and of the i-th arm.

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- Can we have a policy which achieves the minimum regret among all the possible environments available?
- This is called the worst case gap-independent regret or sometimes called the minimax regret.
- It is generally found by setting all the gaps to equal values of order $O\left(1/\sqrt{T}\right)$.

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  - **Bayesian Approach:** In this approach you start with a prior guess over the performance of each of the arms, then you pull an arm and based on the reward received we update our posterior guess on the performance of the arm.

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  - **Bayesian Approach:** In this approach you start with a prior guess over the performance of each of the arms, then you pull an arm and based on the reward received we update our posterior guess on the performance of the arm.
- We will be focusing on UCB based approaches in our work.

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  - **Mean and Variance based Estimation:** Here, at every timestep we choose an arm based on $\hat{r}_i$, $\hat{V}_i$ and its confidence interval c_i. Eg: UCB-Norma [Auer et al.(2002a)Auer, Cesa-Bianchi, and Fischer]l, UCB-V [Audibert et al.(2009)Audibert, Munos, and Szepesvári].

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  - **Divergence based methods:** Eg: KL-UCB [Garivier and Cappé(2011)], DMED [Honda and Takemura(2010)].

UCB1 Algorithm ([Auer et al.(2002a)Auer, Cesa-Bianchi, and Fischer])

Algorithm 1 UCB1

1: Pull each arm once
2: **for** $t = K + 1, ..., T$ **do**
3: Pull the arm such that $\max_{i \in A} \left\{ \hat{r}_i + \sqrt{\frac{2 \log t}{s_i}} \right\}$
4: $t := t + 1$
5: **end for**

- Maintain an upper confidence bound (c_i) for each of the arms
- This c_i will help in sufficiently exploring sub-optimal arms and then exploiting the optimal arm.
- The gap-independent regret bound of $O\left(\sqrt{KT \log T}\right)$ and gap-dependent bound of $O\left(\frac{K \log(T)}{\Delta}\right)$.

Minimax Optimal Strategy in the Stochastic Case ([Audibert and Bubeck(2009)])

Algorithm 2 MOSS

1: Pull each arm once
2: **for** $t = K+1, ..., T$ **do**
3: Pull the arm such that $\max_{i \in A} \left\{ \hat{r}_i + \sqrt{\frac{\max\{0, \log(\frac{T}{Ks_i})\}}{s_i}} \right\}$
4: $t := t + 1$
5: **end for**

- UCB1 suffers from a worst case regret of $O\left(\sqrt{KT \log T}\right)$.
- MOSS corrects this and gives us a gap-independent regret bound of $O\left(\sqrt{KT}\right)$ and gap-dependent bound of $O\left(\frac{K^2 \log(\frac{T\Delta^2}{K})}{\Delta}\right)$.

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- Pull all surviving arms equal number of times during a round.
- At the end of the round eliminate some arms based on some criteria.
- Reset parameters and proceed to next round.
- UCB-Imp achieves a gap-independent regret bound of $O\left(\sqrt{KT \log K}\right)$ and gap-dependent bound of $O\left(\frac{K \log(T\Delta^2)}{\Delta}\right)$.

UCB-Improved ([Auer and Ortner(2010)])

Algorithm 3 UCB-Improved

Input: Time horizon T

Initialization: Set $B_0 := A$ and $\tilde{\Delta}_0 := 1$.

for $m = 0, 1, .. \lfloor \frac{1}{2} \log_2 \frac{T}{e} \rfloor$ **do**

Pull each arm in B_m, $n_m = \left\lceil \frac{2 \log (T \tilde{\Delta}_m^2)}{\tilde{\Delta}_m} \right\rceil$ number of times.

Arm Elimination

For each $i \in B_m$, delete arm i from B_m if,

$$\bar{X}_i + \sqrt{\frac{\log (T \tilde{\Delta}_m^2)}{2 n_m}} < \max_{j \in B_m} \left\{ \bar{X}_j - \sqrt{\frac{\log (T \tilde{\Delta}_m^2)}{2 n_m}} \right\}$$

Set $\tilde{\Delta}_{m+1} := \frac{\tilde{\Delta}_m}{2}$, Set $B_{m+1} := B_m$

Stop if $|B_m| = 1$ and pull $i \in B_m$ till n is reached.

end for

Some technical details of UCB-Improved

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- As opposed to UCB1, MOSS and OCUCB, UCB-Improved has fixed confidence interval $c_m = \sqrt{\dfrac{\log(T\tilde{\Delta}_m^2)}{2n_m}}$ for all arms in a particular phase.

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- As opposed to UCB1, MOSS and OCUCB, UCB-Improved has fixed confidence interval $c_m = \sqrt{\dfrac{\log(T\tilde{\Delta}_m^2)}{2n_m}}$ for all arms in a particular phase.
- c_m ensures that whenever $\tilde{\Delta}_m < \dfrac{\Delta_i}{2}$ in the m-th round, the arm i gets eliminated.

Optimally confident UCB ([Lattimore(2015)])

Algorithm 4 MOSS

Input: K,T, α, ψ

Pull each arm once

for $t = K + 1, ..., T$ **do**

  Pull the arm such that $\max_{i \in A} \left\{ \hat{r}_i + \sqrt{\alpha \frac{\max\{0, \log(\frac{\psi T}{s_i})\}}{s_i}} \right\}$

  $t := t + 1$

end for

- UCB1 is too conservative in exploiting, MOSS is not conservative enough and tends to explore more often than required.
- OCUCB correctly balances this and achieves a gap-independent regret bound $O\left(\sqrt{KT}\right)$ and gap-dependent bound $O\left(\frac{K \log(T/H)}{\Delta}\right)$.

Comparison of UCB1, MOSS, OCUCB, UCB-Improved

Table: Cumulative Regret of Algorithms

Algorithm	Upper bound on Cumulative Regret
UCB1	$\min\left\{O\left(\frac{K\log T}{\Delta}\right), O\left(\sqrt{KT\log T}\right)\right\}$
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OCUCB	$\min\left\{O\left(\frac{K\log(T/H)}{\Delta}\right), O\left(\sqrt{KT}\right)\right\}$
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- The arm elimination condition of UCB-Imp is very conservative.
- When the gaps are small and uniform UCB-Imp performs very badly.
- There is a gap in theoretical guarantee and empirical performance.

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  - Can we bridge the theoretical versus empirical performance of UCB-Imp?
  - Can we use ideas from Clustering to achieve this?
  - Can we study the effect of Clustering in SMAB?
- The answer to all of this is ClusUCB.

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- Also eliminate clusters with all of its arms by comparing its performance against the globally best arm.
- Reset parameters and move to the next round.

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- At the end of the round eliminate arms inside each cluster by comparing its performance against the best arm in the cluster.
- Also eliminate clusters with all of its arms by comparing its performance against the globally best arm.
- Reset parameters and move to the next round.
- At a higher level ClusUCB behaves like p independently running UCB-Imp with the exploration parameters ρ_a, ρ_s and ψ helping in overcoming early exploration.

Approach of ClusUCB I

Input: Number of clusters p, time horizon T, exploration parameters ρ_a, ρ_s and ψ.

Initialization: Set $B_0 := A$, $S_0 = S$ and $\epsilon_0 := 1$.

Create a partition S_0 of the arms at random into p clusters of size up to $\ell = \left\lceil \frac{K}{p} \right\rceil$ each.

for $m = 0, 1, .. \left\lfloor \frac{1}{2} \log_2 \frac{7T}{K} \right\rfloor$ **do**

  Pull each arm in B_m so that the total number of times it has been pulled is $n_m = \left\lceil \frac{2 \log\left(\psi T \epsilon_m^2\right)}{\epsilon_m} \right\rceil$.

  Arm Elimination

    For each cluster $s_k \in S_m$, delete arm $i \in s_k$ from B_m if

$$\hat{r}_i + \sqrt{\frac{\rho_a \log\left(\psi T \epsilon_m^2\right)}{2 n_m}} < \max_{j \in s_k} \left\{ \hat{r}_j - \sqrt{\frac{\rho_a \log\left(\psi T \epsilon_m^2\right)}{2 n_m}} \right\}$$

Approach of ClusUCB II

Cluster Elimination

Delete cluster $s_k \in S_m$ and remove all arms $i \in s_k$ from B_m if

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Set $\epsilon_{m+1} := \frac{\epsilon_m}{2}$

Set $B_{m+1} := B_m$

Stop if $|B_m| = 1$ and pull $i \in B_m$ till T is reached.

end for

UCB1, MOSS, OCUCB, UCB-Imp, ClusUCB

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OCUCB	$\min\left\{O\left(\frac{K\log(T/H)}{\Delta}\right), O\left(\sqrt{KT}\right)\right\}$
UCB-Improved	$\min\left\{O\left(\frac{K\log(T\Delta^2)}{\Delta}\right), O\left(\sqrt{KT\log K}\right)\right\}$
ClusUCB	$\min\left\{O\left(\frac{K\log(T\Delta^2/\sqrt{\log K})}{\Delta}\right), O\left(\sqrt{KT\log K}\right)\right\}$

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- One simple solution is to pull the arm with the highest UCB at every timestep. This is called optimistic greedy sampling for UCB-Imp (see [Liu and Tsuruoka(2016)]).

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- We introduce this in Efficient ClusUCB or EClusUCB.

Input: Number of clusters p, time horizon T, exploration parameters ρ_a, ρ_s and ψ.
Initialization: Set $m := 0$, $B_0 := A$, $S_0 = S$, $\epsilon_0 := 1$,
$M = \left\lfloor \frac{1}{2} \log_2 \frac{7T}{K} \right\rfloor$, $n_0 = \left\lceil \frac{2 \log(\psi T \epsilon_0^2)}{\epsilon_0} \right\rceil$ and $N_0 = K n_0$.
Create a partition S_0 of the arms at random into p clusters of size up to $\ell = \left\lceil \frac{K}{p} \right\rceil$ each.
Pull each arm once
for $t = K + 1, .., T$ **do**

Pull arm $i \in B_m$ such that $\text{argmax}_{j \in B_m} \left\{ \hat{r}_j + \sqrt{\frac{\rho_s \log(\psi T \epsilon_m^2)}{2 z_j}} \right\}$,
where z_j is the number of times arm j has been pulled

$t := t + 1$

Arm Elimination

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if $t \geq N_m$ and $m \leq M$ **then**

Approach of ClusUCB III

$\epsilon_{m+1} := \frac{\epsilon_m}{2}$

$B_{m+1} := B_m$

$n_{m+1} := \left\lceil \frac{2\log(\psi T \epsilon_{m+1}^2)}{\epsilon_{m+1}} \right\rceil$

$N_{m+1} := t + |B_{m+1}| n_{m+1}$

$m := m + 1$

Stop if $|B_m| = 1$ and pull $i \in B_m$ till T is reached.

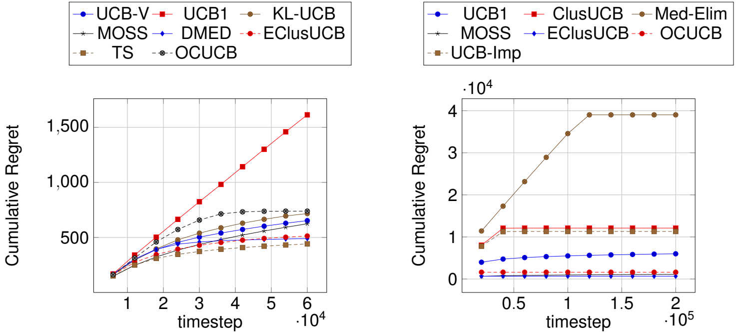
end if

end for

UCB1, MOSS, OCUCB, UCB-Imp, EClusUCB

Algorithm	Upper bound on Cumulative Regret
UCB1	$\min\left\{O\left(\frac{K\log T}{\Delta}\right), O\left(\sqrt{KT\log T}\right)\right\}$
MOSS	$\min\left\{O\left(\frac{K^2\log(T\Delta^2/K)}{\Delta}\right), O\left(\sqrt{KT}\right)\right\}$
OCUCB	$\min\left\{O\left(\frac{K\log(T/H)}{\Delta}\right), O\left(\sqrt{KT}\right)\right\}$
UCB-Improved	$\min\left\{O\left(\frac{K\log(T\Delta^2)}{\Delta}\right), O\left(\sqrt{KT\log K}\right)\right\}$
EClusUCB	$\min\left\{O\left(\frac{K\log(T\Delta^2/\sqrt{\log K})}{\Delta}\right), O\left(\sqrt{KT\log K}\right)\right\}$

Finally, experiment!!!



(a) Experiment 1: 20 Bernoulli-distributed arms with $r_{i_{i \neq *}} = 0.07$ and $r^* = 0.1$.

(b) Experiment 2: 100 Gaussian-distributed arms with $r_{i_{i \neq *:1-33}} = 0.1$, $r_{i_{i \neq *:34-99}} = 0.6$ and $r^*_{i=100} = 0.9$.

Figure: Cumulative regret for various bandit algorithms on two stochastic K-armed bandit environments.

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Thank You