Online sequential Learning using Bandits

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Overview

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- If we become too greedy and always exploit, we may miss the chance of actually finding the optimal arm and get stuck with a sub-optimal arm.

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- Selecting the best possible route for a message to pass through in a peer-to-peer network connection.

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- Bandits allows us to study this behavior in a more formal way giving us strict guarantees regarding the performance of our algorithm.
- They are easy to implement.

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- The distributions for each of the arms are fixed throughout the time horizon.

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$$R_T = r^* T - \sum_{i \in A} r_i T_i(T),$$

- The expected regret of an algorithm after T rounds can be written as

$$\mathbb{E}[R_T] = \sum_{i=1}^{K} \mathbb{E}[T_i(T)]\Delta_i,$$

- $\Delta_i = r^* - r_i$ denotes the gap between the means of the optimal arm and of the i-th arm.

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- It is generally found by setting all the gaps to equal values of $\min_{i \in A} \Delta_i = \Delta = \sqrt{\dfrac{f(K)}{T}}$.
- Also we will define the hardness parameter H as $H = \sum_{i=1}^{K} \dfrac{1}{\Delta_i^2}$

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  - **Mean and Variance based Estimation:** Here, at every timestep we choose an arm based on $\hat{r}_i$, $\hat{V}_i$ and its confidence interval c_i. Eg: UCB-Normal [**?**]l, UCB-V [**?**].

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- We will be focusing on Mean based estimation.

UCB1 Algorithm ([?])

Algorithm 1 UCB1

1: Pull each arm once
2: **for** $t = K + 1, ..., T$ **do**
3: Pull the arm such that $\max_{i \in A} \left\{ \hat{r}_i + \sqrt{\frac{2 \log t}{s_i}} \right\}$
4: $t := t + 1$
5: **end for**

- Maintain an upper confidence bound (c_i) for each of the arms
- This c_i will help in sufficiently exploring sub-optimal arms and then exploiting the optimal arm.
- The gap-independent regret bound of $O\left(\sqrt{KT \log T}\right)$ and gap-dependent bound of $O\left(\frac{K \log(T)}{\Delta}\right)$.

Algorithm 2 MOSS

1: Pull each arm once
2: **for** $t = K+1, ..., T$ **do**
3: Pull the arm such that $\max_{i \in A} \left\{ \hat{r}_i + \sqrt{\frac{\max\{0, \log(\frac{T}{Ks_i})\}}{s_i}} \right\}$
4: $t := t+1$
5: **end for**

- UCB1 suffers from a worst case regret of $O\left(\sqrt{KT \log T}\right)$.
- MOSS corrects this and gives us a gap-independent regret bound of $O\left(\sqrt{KT}\right)$ and gap-dependent bound of $O\left(\frac{K^2 \log(\frac{T\Delta^2}{K})}{\Delta}\right)$.

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- Pull all surviving arms equal number of times during a round.
- At the end of the round eliminate some arms based on some criteria.
- Reset parameters and proceed to next round.
- UCB-Imp achieves a gap-independent regret bound of $O\left(\sqrt{KT\log K}\right)$ and gap-dependent bound of $O\left(\frac{K\log(T\Delta^2)}{\Delta}\right)$.

UCB-Improved ([?])

Algorithm 3 UCB-Improved

Input: Time horizon T

Initialization: Set $B_0 := A$ and $\tilde{\Delta}_0 := 1$.

for $m = 0, 1, ..\lfloor \frac{1}{2} \log_2 \frac{T}{e} \rfloor$ **do**

Pull each arm in B_m, $n_m = \left\lceil \frac{2 \log(T\tilde{\Delta}_m^2)}{\tilde{\Delta}_m} \right\rceil$ number of times.

Arm Elimination

For each $i \in B_m$, delete arm i from B_m if,

$$\bar{X}_i + \sqrt{\frac{\log(T\tilde{\Delta}_m^2)}{2n_m}} < \max_{j \in B_m} \left\{ \bar{X}_j - \sqrt{\frac{\log(T\tilde{\Delta}_m^2)}{2n_m}} \right\}$$

Set $\tilde{\Delta}_{m+1} := \frac{\tilde{\Delta}_m}{2}$, Set $B_{m+1} := B_m$

Stop if $|B_m| = 1$ and pull $i \in B_m$ till n is reached.

end for

Some technical details of UCB-Improved

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- As opposed to UCB1, MOSS and OCUCB, UCB-Improved has fixed confidence interval $c_m = \sqrt{\dfrac{\log(T\tilde{\Delta}_m^2)}{2n_m}}$ for all arms in a particular phase.

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- As opposed to UCB1, MOSS and OCUCB, UCB-Improved has fixed confidence interval $c_m = \sqrt{\dfrac{\log(T\tilde{\Delta}_m^2)}{2n_m}}$ for all arms in a particular phase.
- c_m ensures that whenever $\tilde{\Delta}_m < \dfrac{\Delta_i}{2}$ in the m-th round, the arm i gets eliminated.

Optimally confident UCB ([?])

Algorithm 4 MOSS

1: **Input:** K,T, α, ψ
2: Pull each arm once
3: **for** $t = K + 1, ..., T$ **do**
4: Pull the arm such that $\max_{i \in A} \left\{ \hat{r}_i + \sqrt{\alpha \frac{\max\{0, \log(\frac{\psi T}{s_i})\}}{s_i}} \right\}$
5: $t := t + 1$
6: **end for**

- UCB1 is too conservative in exploiting, MOSS is not conservative enough and tends to explore more often than required.
- OCUCB correctly balances this and achieves a gap-independent regret bound $O\left(\sqrt{KT}\right)$ and gap-dependent bound $O\left(\frac{K \log(T/H)}{\Delta}\right)$.

Comparison of UCB1, MOSS, OCUCB, UCB-Improved

Table: Cumulative Regret of Algorithms

Algorithm	Upper bound on Cumulative Regret
UCB1	$\min\left\{O\left(\frac{K\log T}{\Delta}\right), O\left(\sqrt{KT\log T}\right)\right\}$
MOSS	$\min\left\{O\left(\frac{K^2\log(T\Delta^2/K)}{\Delta}\right), O\left(\sqrt{KT}\right)\right\}$
UCB-Improved	$\min\left\{O\left(\frac{K\log(T\Delta^2)}{\Delta}\right), O\left(\sqrt{KT\log K}\right)\right\}$
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- UCB-Improved conducts too much early exploration.
- The arm elimination condition of UCB-Imp is very conservative.
- When the gaps are small and uniform UCB-Imp performs very badly.
- There is a gap in theoretical guarantee and empirical performance.

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  - Can we bridge the theoretical versus empirical performance of UCB-Improved?
  - Can we use ideas from Clustering to achieve this?
  - Can we study the effect of Clustering in SMAB?
- The answer to all of this is ClusUCB.

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- Also eliminate clusters with all of its arms by comparing its performance against the globally best arm.
- Reset parameters and move to the next round.

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- At the end of the round eliminate arms inside each cluster by comparing its performance against the best arm in the cluster.
- Also eliminate clusters with all of its arms by comparing its performance against the globally best arm.
- Reset parameters and move to the next round.
- At a higher level ClusUCB behaves like *p* independently running UCB-Imp with the exploration parameters ρ_a, ρ_s and ψ helping in overcoming early exploration.

ClusUCB Algorithm I

1: **Input:** Number of clusters p, time horizon T, exploration parameters ρ_a, ρ_s and ψ.

2: **Initialization:** Set $B_0 := A$, $S_0 = S$ and $\epsilon_0 := 1$.

3: Create a partition S_0 of the arms at random into p clusters of size up to $\ell = \left\lceil \frac{K}{p} \right\rceil$ each.

4: **for** $m = 0, 1, .. \left\lfloor \frac{1}{2} \log_2 \frac{14T}{K} \right\rfloor$ **do**

5: Pull each arm in B_m so that the total number of times it has been pulled is $n_m = \left\lceil \frac{2\log\left(\psi T \epsilon_m^2\right)}{\epsilon_m} \right\rceil$.

6: ***Arm Elimination***

7: For each cluster $s_k \in S_m$, delete arm $i \in s_k$ from B_m if

$$\hat{r}_i + \sqrt{\frac{\rho_a \log\left(\psi T \epsilon_m^2\right)}{2n_m}} < \max_{j \in s_k} \left\{ \hat{r}_j - \sqrt{\frac{\rho_a \log\left(\psi T \epsilon_m^2\right)}{2n_m}} \right\}$$

ClusUCB Algorithm II

Cluster Elimination

Delete cluster $s_k \in S_m$ and remove all arms $i \in s_k$ from B_m if

$$\max_{i \in s_k} \left\{ \hat{r}_i + \sqrt{\frac{\rho_s \log(\psi T \epsilon_m^2)}{2n_m}} \right\} < \max_{j \in B_m} \left\{ \hat{r}_j - \sqrt{\frac{\rho_s \log(\psi T \epsilon_m^2)}{2n_m}} \right\}.$$

Set $\epsilon_{m+1} := \frac{\epsilon_m}{2}$

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UCB1, MOSS, OCUCB, UCB-Imp, ClusUCB

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UCB-Improved	$\min\left\{O\left(\frac{K\log(T\Delta^2)}{\Delta}\right), O\left(\sqrt{KT\log K}\right)\right\}$
ClusUCB	$\min\left\{O\left(\frac{K\log(T\Delta^2/\sqrt{\log K})}{\Delta}\right), O\left(\sqrt{KT\log K}\right)\right\}$
OCUCB	$\min\left\{O\left(\frac{K\log(T/H)}{\Delta}\right), O\left(\sqrt{KT}\right)\right\}$

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- One simple solution is to pull the arm with the highest UCB at every timestep. This is called optimistic greedy sampling for UCB-Imp (see [**?**]).

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- We introduce this in Efficient ClusUCB or EClusUCB.

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- Divide the horizon into rounds and for each round allocate $|B_m|n_m$ pulls, where $n_m := \left\lceil \frac{2\log(\psi T\epsilon_m^2)}{\epsilon_m} \right\rceil$.
- Within a round pull an arm that has the maximum UCB among all arms.
- The arm and cluster elimination is done in same way.

EClusUCB Algorithm I

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Initialization: Set $m := 0$, $B_0 := A$, $S_0 = S$, $\epsilon_0 := 1$,
$M = \left\lfloor \frac{1}{2}\log_2 \frac{14T}{K} \right\rfloor$, $n_0 = \left\lceil \frac{2\log(\psi T\epsilon_0^2)}{\epsilon_0} \right\rceil$ and $N_0 = Kn_0$.
Create a partition S_0 of the arms at random into p clusters of size up to $\ell = \left\lceil \frac{K}{p} \right\rceil$ each.
Pull each arm once
for $t = K+1, .., T$ **do**

Pull arm $i \in B_m$ such that $\operatorname{argmax}_{j \in B_m} \left\{ \hat{r}_j + \sqrt{\frac{\rho_s \log(\psi T\epsilon_m^2)}{2z_j}} \right\}$,
where z_j is the number of times arm j has been pulled

$t := t+1$

Arm Elimination

For each cluster $s_k \in S_m$, delete arm $i \in s_k$ from B_m if

$$\hat{r}_i + \sqrt{\frac{\rho_a \log(\psi T \epsilon_m^2)}{2z_i}} < \max_{j \in s_k} \left\{ \hat{r}_j - \sqrt{\frac{\rho_a \log(\psi T \epsilon_m^2)}{2z_j}} \right\}$$

Cluster Elimination

Delete cluster $s_k \in S_m$ and remove all arms $i \in s_k$ from B_m if

$$\max_{i \in s_k} \left\{ \hat{r}_i + \sqrt{\frac{\rho_s \log(\psi T \epsilon_m^2)}{2z_i}} \right\} < \max_{j \in B_m} \left\{ \hat{r}_j - \sqrt{\frac{\rho_s \log(\psi T \epsilon_m^2)}{2z_j}} \right\}.$$

if $t \geq N_m$ and $m \leq M$ **then**

$\epsilon_{m+1} := \frac{\epsilon_m}{2}$

$B_{m+1} := B_m$

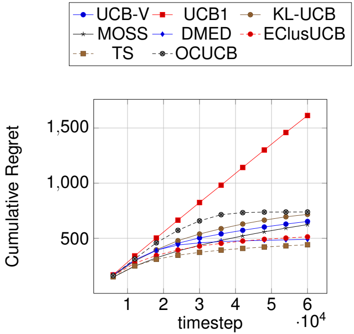
$n_{m+1} := \left\lceil \frac{2 \log(\psi T \epsilon_{m+1}^2)}{\epsilon_{m+1}} \right\rceil$

$N_{m+1} := t + |B_{m+1}| n_{m+1}$
$m := m + 1$
Stop if $|B_m| = 1$ and pull $i \in B_m$ till T is reached.
end if
end for

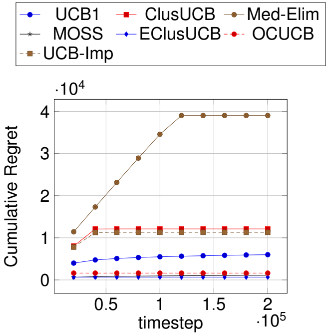
UCB1, MOSS, OCUCB, UCB-Imp, EClusUCB

Algorithm	Upper bound on Cumulative Regret
UCB1	$\min\left\{O\left(\frac{K\log T}{\Delta}\right), O\left(\sqrt{KT\log T}\right)\right\}$
MOSS	$\min\left\{O\left(\frac{K^2\log(T\Delta^2/K)}{\Delta}\right), O\left(\sqrt{KT}\right)\right\}$
UCB-Improved	$\min\left\{O\left(\frac{K\log(T\Delta^2)}{\Delta}\right), O\left(\sqrt{KT\log K}\right)\right\}$
EClusUCB	$\min\left\{O\left(\frac{K\log(T\Delta^2/\sqrt{\log K})}{\Delta}\right), O\left(\sqrt{KT\log K}\right)\right\}$
OCUCB	$\min\left\{O\left(\frac{K\log(T/H)}{\Delta}\right), O\left(\sqrt{KT}\right)\right\}$

Finally, experiment!!!



(a) Experiment 1: 20 Bernoulli-distributed arms with $r_{i_{i\neq *}} = 0.07$ and $r^* = 0.1$.



(b) Experiment 2: 100 Gaussian-distributed arms with $r_{i_{i\neq *:1-33}} = 0.1$, $r_{i_{i\neq *:34-99}} = 0.6$ and $r^*_{i=100} = 0.9$.

Figure: Cumulative regret for various bandit algorithms on two stochastic K-armed bandit environments.

Conclusions

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References I

Thank You