

Thresholding Bandits with Augmented UCB

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Overview

- 1 Problem Definition
- 2 Contribution
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- 4 Theoretical Analysis
- 5 Experiments
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Problem Definition of TBP

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- **Primary aim:** Identify *all* the arms whose expected mean of the reward distribution (r_i) is above a particular threshold τ given as input.
- **Condition:** This has to be achieved within T timesteps of exploration and this is termed as a fixed-budget problem.

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- Let \hat{S}_τ denote the recommendation of a learning algorithm after T time units of exploration, while \hat{S}_τ^c denotes its complement.
- The goal of the learning agent is to minimize the **expected** loss at the end of budget T :

$$\mathbb{E}[\mathcal{L}(T)] = \mathbb{P}\left(\underbrace{\{S_\tau \cap \hat{S}_\tau^c \neq \emptyset\}}_{\text{Rejected good arms}} \cup \underbrace{\{\hat{S}_\tau \cap S_\tau^c \neq \emptyset\}}_{\text{Accepted bad arms}} \right)$$

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- Lesser the budget \Rightarrow Harder the problem.
- Higher the variance of an arm's reward distribution \Rightarrow Harder the problem.

- We propose the **Augmented UCB (AugUCB)** algorithm for the fixed-budget TBP setting.

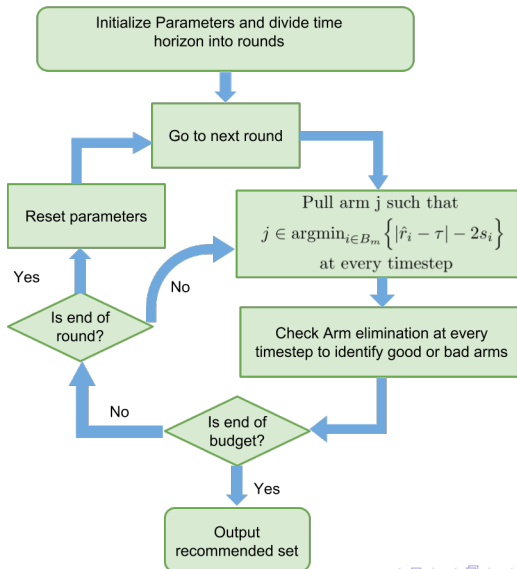
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- It **addresses an open problem** discussed in Auer and Ortner (2010) of designing an algorithm that can eliminate arms based on variance estimates.

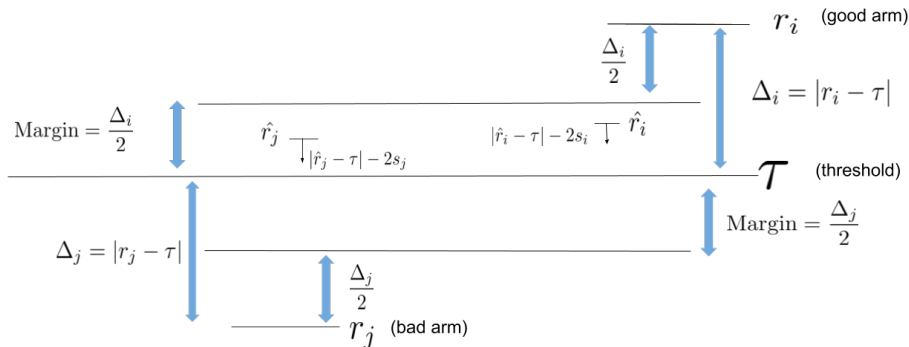
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- It is the **first variance-based arm elimination algorithm** for the considered TBP settings.
- It **addresses an open problem** discussed in Auer and Ortner (2010) of designing an algorithm that can eliminate arms based on variance estimates.
- We also define a **new problem complexity** which uses empirical variance estimates along with arm's mean for giving the theoretical bound.

AugUCB algorithm



AugUCB algorithm (Intuition, Arm pulling)

- Like UCB-Imp, AugUCB also divides the time budget T into rounds.
- At every timestep we pull arm j s.t. $j \in \arg \min_{i \in B_m} \left\{ |\hat{r}_i - \tau| - 2s_i \right\}$ (like APT).



- We pull the arm that minimizes $j \in \arg \min_{i \in B_m} \left\{ |\hat{r}_i - \tau| - 2s_i \right\}$

Arm pull

- We pull the arm that minimizes $j \in \arg \min_{i \in B_m} \left\{ |\hat{r}_i - \tau| - 2s_i \right\}$
- We define the confidence interval $s_i = \sqrt{\frac{\rho \psi_m(\hat{v}_i + 1) \log(T \epsilon_m)}{4n_i}}$.

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- s_i decreases with more n_i and ψ_m and ρ ensures that it decreases at a correct rate.

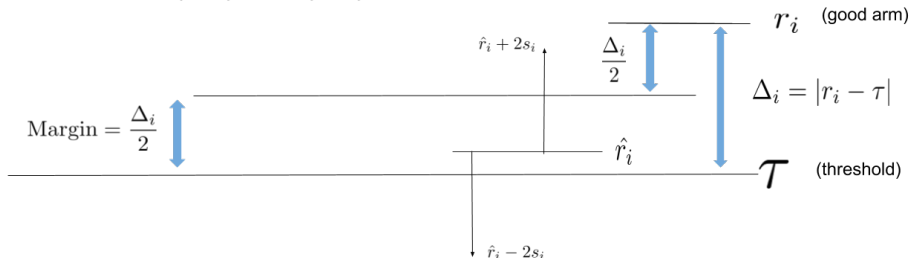
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- s_i decreases with more n_i and ψ_m and ρ ensures that it decreases at a correct rate.
- Note that \hat{v}_i estimated variance in s_i makes the algorithm pull the arm which shows more variance.

AugUCB algorithm (Intuition, Arm Elimination)

- We eliminate an arm when we are sure that \hat{r}_i is close to r_i with high probability and hence identify it as good or bad arm.
- It's risky to eliminate an arm when \hat{r}_i is inside *Margin*.
- Confidence interval s_i will make sure arm i is not eliminated while inside Margin with a high probability.

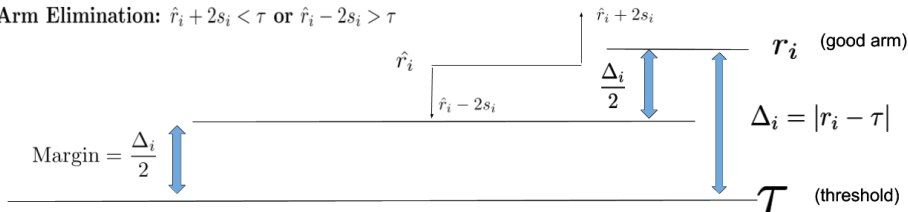
Arm Elimination: $\hat{r}_i + 2s_i < \tau$ or $\hat{r}_i - 2s_i > \tau$



AugUCB algorithm (Intuition, Arm Elimination)

- Now we see that \hat{r}_i has moved close to its true estimate r_i .
- We eliminate i and re-allocate the remaining budget to pull arms close to the threshold

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Expected Loss of AugUCB

Theorem

For $K \geq 4$ and $\rho = 1/3$, the expected loss of the AugUCB algorithm is given by,

$$\mathbb{E}[\mathcal{L}(T)] \leq 2KT \exp \left(- \frac{T}{4096 \log(K \log K) H_{\sigma,2}} \right).$$

Table: AugUCB vs. State of the art

Algorithm	Upper Bound on Expected Loss	Oracle
AugUCB	$\exp \left(- \frac{T}{4096 \log(K \log K) H_{\sigma,2}} + \log(2KT) \right)$	No
UCBEV	$\exp \left(- \frac{1}{512} \frac{T-2K}{H_{\sigma,1}} + \log(6KT) \right)$	Yes
APT	$\exp \left(- \frac{T}{64H_1} + 2 \log((\log(T) + 1)K) \right)$	No
UCBE	$\exp \left(- \frac{T-K}{18H_1} - 2 \log(\log(T)K) \right)$	Yes

- We compare with APT, AugUCB, UCB_E, UCB_{EV}, UA.

Experiments

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- Note that UCBE and UCBEV require access to H_1 and $H_{\sigma,1}$ as input and hence not implementable in real life.
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- APT, AugUCB, UA do not require access to H_1 or $H_{\sigma,1}$.

Experimental Setup

- This setup involves Gaussian reward distributions with $K = 100$, $T = 10000$ and $\tau = 0.5$ with the reward means set in two groups.

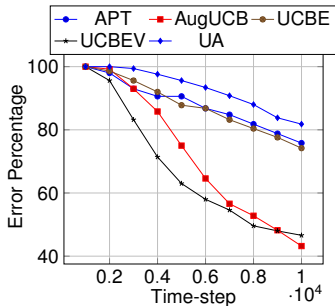
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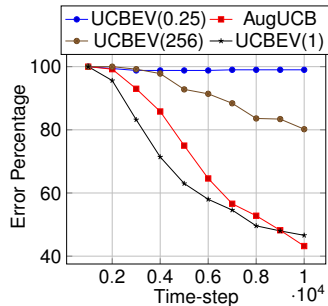
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- The first 10 arms partitioned into two groups; the respective means are $r_{1:5} = 0.45$, $r_{6:10} = 0.55$.
- The means of arms $i = 11 : 100$ are chosen same as $r_{11:100} = 0.4$.
- Variances are set as $\sigma_{1:5}^2 = 0.3$ and $\sigma_{6:10}^2 = 0.8$; $\sigma_{11:100}^2$ are independently and uniformly chosen in the interval $[0.2, 0.3]$.

Experimental Results

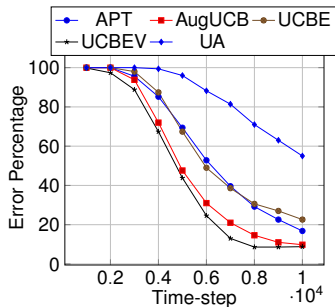


(a) Expt-1: Two Group Setting (Advance)

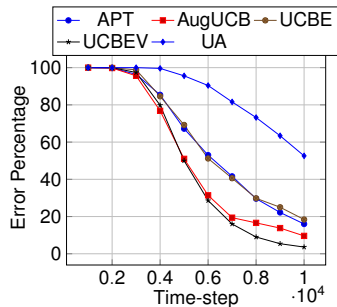


(b) Expt-2: Two Group Setting (Advance)

Experimental Results



(c) Expt-1: Arithmetic Progression (Gaussian)



(d) Expt-2: Geometric Progression (Gaussian)

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- Further studies are required to establish a lower bound on the expected loss of AugUCB.
- A more detailed analysis of the non-uniform arm selection and parameter selection is also required.

Thank You