# Thresholding Bandits with Augmented UCB

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# Overview

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- **Primary aim:** Identify *all* the arms whose expected mean of the reward distribution ($r_i$) is above a particular threshold $\tau$ given as input.
- **Condition:** This has to be achieved within $T$ timesteps of exploration and this is termed as a fixed-budget problem.

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- Let $\hat{S}_\tau$ denote the recommendation of a learning algorithm after $T$ time units of exploration, while $\hat{S}_\tau^c$ denotes its complement.
- The goal of the learning agent is to minimize the **expected** loss at the end of budget $T$:

$$\mathbb{E}[\mathcal{L}(T)] = \mathbb{P}\Big( \underbrace{\{S_\tau \cap \hat{S}_\tau^c \neq \emptyset\}}_{\textbf{Rejected good arms}} \cup \underbrace{\{\hat{S}_\tau \cap S_\tau^c \neq \emptyset\}}_{\textbf{Accepted bad arms}} \Big)$$

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- Lesser the budget $\Rightarrow$ Harder the problem.
- Higher the variance of an arm's reward distribution $\Rightarrow$ Harder the problem.

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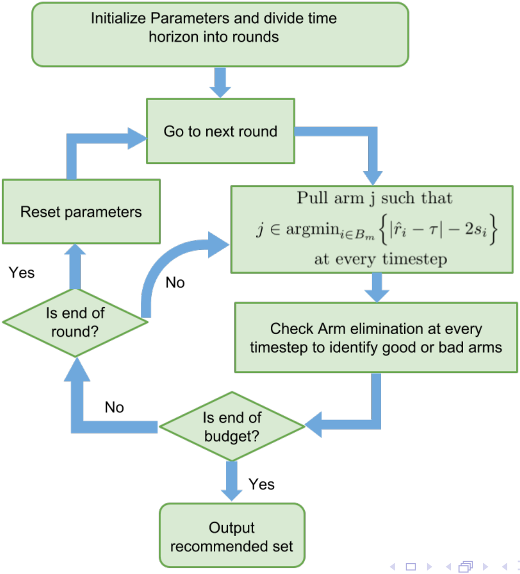
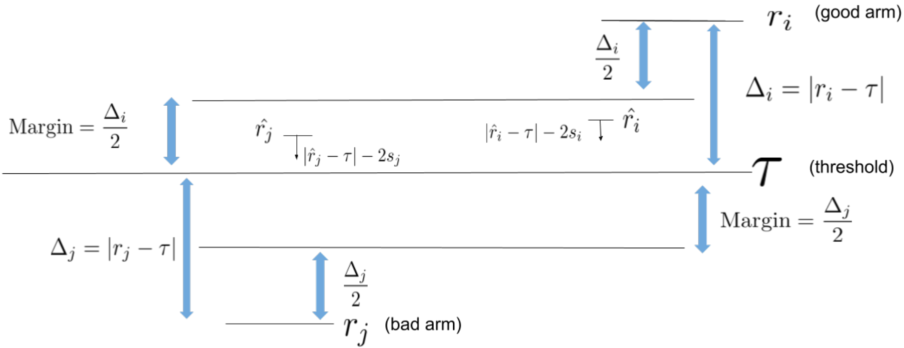
- It is the **first variance-based arm elimination algorithm** for the considered TBP settings.
- It **addresses an open problem** discussed in Auer and Ortner (2010) of designing an algorithm that can eliminate arms based on variance estimates.
- We also define a **new problem complexity** which uses empirical variance estimates along with arm's mean for giving the theoretical bound.

# AugUCB algorithm



# AugUCB algorithm (Intuition, Arm pulling)

- Like UCB-Imp, AugUCB also divides the time budget $T$ into rounds.
- At every timestep we pull arm j s.t. $j \in \arg\min_{i \in B_m} \left\{ |\hat{r}_i - \tau| - 2s_i \right\}$ (like APT).



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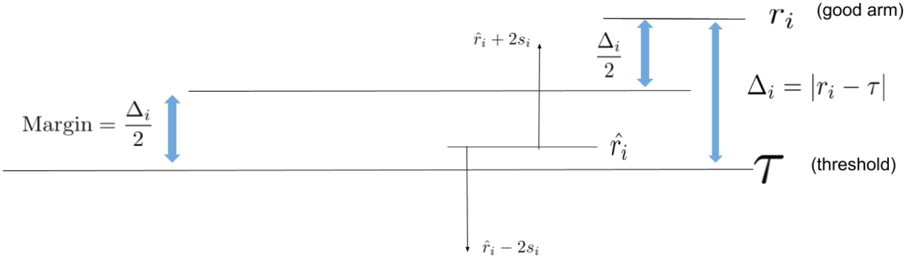
- $s_i$ decreases with more $n_i$ and $\psi_m$ and $\rho$ ensures that it decreases at a correct rate.
- Note that $\hat{v}_i$ estimated variance in $s_i$ makes the algorithm pull the arm which shows more variance.

# AugUCB algorithm (Intuition, Arm Elimination)

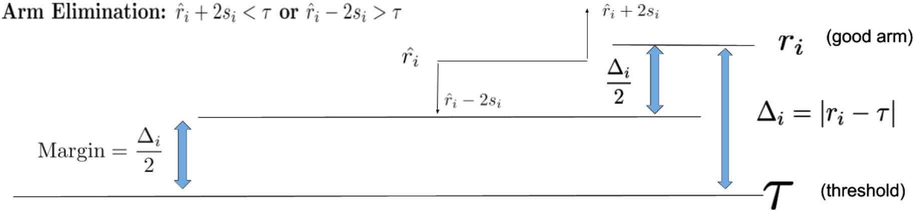
- We eliminate an arm when we are sure that $\hat{r}_i$ is close to $r_i$ with high probability and hence identify it as good or bad arm.
- It's risky to eliminate an arm when $\hat{r}_i$ is inside *Margin*.
- Confidence interval $s_i$ will make sure arm $i$ is not eliminated while inside Margin with a high probability.

**Arm Elimination:** $\hat{r}_i + 2s_i < \tau$ **or** $\hat{r}_i - 2s_i > \tau$



# AugUCB algorithm (Intuition, Arm Elimination)

- Now we see that $\hat{r}_i$ has moved close to its true estimate $r_i$.
- We eliminate $i$ and re-allocate the remaining budget to pull arms close to the threshold



# Expected Loss of AugUCB

## Theorem

*For $K \geq 4$ and $\rho = 1/3$, the expected loss of the AugUCB algorithm is given by,*

$$\mathbb{E}[\mathcal{L}(T)] \leq 2KT \exp\left(-\frac{T}{4096 \log(K \log K) H_{\sigma,2}}\right).$$

Table: AugUCB vs. State of the art

| Algorithm | Upper Bound on Expected Loss | Oracle |
|---|---|---|
| AugUCB | $\exp\left(-\frac{T}{4096 \log(K \log K) H_{\sigma,2}} + \log(2KT)\right)$ | No |
| UCBEV | $\exp\left(-\frac{1}{512}\frac{T-2K}{H_{\sigma,1}} + \log(6KT)\right)$ | Yes |
| APT | $\exp\left(-\frac{T}{64H_1} + 2\log((\log(T)+1)K)\right)$ | No |
| UCBE | $\exp\left(-\frac{T-K}{18H_1} - 2\log(\log(T)K\right)$ | Yes |

- We compare with APT, AugUCB, UCBE, UCBEV, UA.

# Experiments

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- Note that UCBE and UCBEV require access to $H_1$ and $H_{\sigma,1}$ as input and hence not implementable in real life.
- By access we mean that an oracle supplies them the $H_1$ or $H_{\sigma,1}$. They do not have access to individual means and variances.

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- APT, AugUCB, UA do not require access to $H_1$ or $H_{\sigma,1}$.

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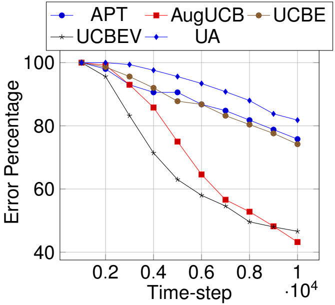
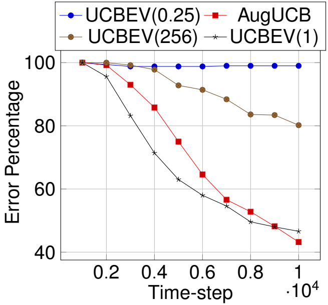
- The first 10 arms partitioned into two groups; the respective means are $r_{1:5} = 0.45$, $r_{6:10} = 0.55$.
- The means of arms $i = 11 : 100$ are chosen same as $r_{11:100} = 0.4$.
- Variances are set as $\sigma^2_{1:5} = 0.3$ and $\sigma^2_{6:10} = 0.8$; $\sigma^2_{11:100}$ are independently and uniformly chosen in the interval $[0.2, 0.3]$.

# Experimental Results




(a) Expt-1: Two Group Setting (Advance)




(b) Expt-2: Two Group Setting (Advance)

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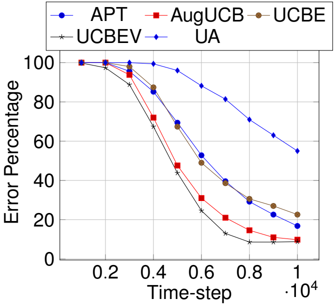
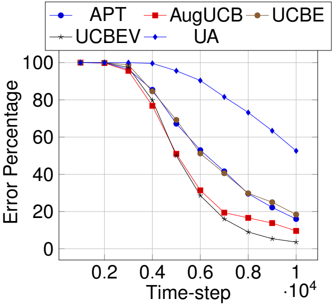



(c) Expt-1: Arithmetic Progression (Gaussian)




(d) Expt-2: Geometric Progression (Gaussian)

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- Further studies are required to establish a lower bound on the expected loss of AugUCB.
- A more detailed analysis of the non-uniform arm selection and parameter selection is also required.

# Thank You