System Identification for Unmanned Marine Vehicles using Interval Analysis

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Abstract—**The aim in the present paper remains to describe a system identification method for unmanned marine vehicles using interval analysis in order to come up with estimation of parameter intervals instead of real values, such that the actual dynamics of the system shall remain confined within guaranteed bounds. The bounds are propagated through the system equations using such interval based parameters. The proposed method exploits guaranteed error bounds for the state variables as observed by different sensors. Unlike conventional Kalman estimators the adopted method rules out the requirement for determining co-variance matrices and approximated Jacobians. Interval based variables are used in constructing interval matrices. Principle of Least Squares is used in solving the system equation involving such non-punctual (interval) matrices. In this context a sophisticated interval matrix inversion technique is employed within the least squares framework in finally determining the parameter intervals.**

I. Introduction

Unmanned Marine Vehicles are systems capable of autonomous maneuvering and navigation targeted towards oceanography or respective sea operations. A thorough understanding of the system's dynamic behavior as governed by the various modalities of the system along with several uncertainties, is thus essential for successful control and guidance of the vehicle. This is often realized by open loop system identification, in order that the model parameters are fitted against data collected from actual operations of the vehicle under sophisticated configurations as for instance fixed thrust or motion profiles etc. The parameters are initialized with values known from some initial analysis or design. However, the accuracy of parameter identification is largely defined by the observations as collected during experiments with the vehicle, mainly accelerations, linear and angular velocities. The uncertainties associated with such data in the form of sensor noise as well as external disturbances, may significantly affect the parameters evaluated through identification. Conventional estimators like Kalman filters have been employed in order to estimate the parameters using system model equations, thereby taking into consideration the measurement noise associated with sensors. In the same line of thought the aim in the present paper remains to describe a sensor fusion approach using interval analysis in order to come up with parameter intervals instead of real values, such that the actual dynamics of the system shall remain confined within guaranteed bounds propagated through the system equations using such interval based parameters. The proposed method exploits guaranteed error bounds for the state variables as observed by different sensors. Unlike conventional Kalman estimators the adopted method rules out the requirement for determining co-variance matrices and approximated Jacobians. Interval based variables are used in constructing interval matrices. Principle of Least Squares is used in solving the system equation involving such non-punctual (interval) matrices. In this context a sophisticated interval matrix inversion technique is employed within the least squares framework in finally determining the parameter intervals. Conventional approaches are proved to be inapplicable for the present case. Hansen's interval hull technique for finding out an inverted interval matrix is used as the basis for the proposed approach.

The organization of the paper is as follows- section II summarizes a priori art in the field of system identifications carried out for unmanned marine vehicles. Conventional estimators like Kalman filters are discussed in this regard. Section III describes the problem associated with identifying model parameters. The general dynamics of such systems are formulated and a Least Square solution framework is established. Subsequently, section IV presents the methodology adopted towards identifying parameter intervals. A discussion regarding the problems encountered with available methods due to Krawczyk or Nirmala and Ganesan, is presented at the outset. Following the inconsistencies, a partitioned hull approach towards finding interval matrix inverse is described. Finally, experimental setup is detailed and field results are obtained. Parameter intervals are estimated using a data set collected during a single trial. Overestimated intervals are constricted heuristically. The constricted intervals are verified with another data set. Results are summarized accordingly.

II. Priori Art

A multi-layered connectionist neural network (N-N) model is adopted in the work of Hassan et al [1]. Two phase N-N has been designed with the first layer identifying single degrees of freedom parameters, and the subsequent identifying couped parameters. However, the work ignores typical stochastic errors associated with sensors from which data are collected during the trials. As a result, training of the neural network remains subject to stochastic disturbances associated with corresponding measurements.

Jay et al in his thesis work [2], adopted a least squares fit to side slip and turn rate data using maximum likelihood of batch processing, in order to estimate steering equation parameters. The identification process requires some controller

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to be designed initially. Subsequently, the identified parameters are utilized to improve upon the controllers thus formulated.

Mark et al adopted Nelder-Mead simplex method to optimize parameters for improved flight-control of an unmanned underwater vehicle [3]. Using some initial controller, the system is made to follow roughly a desired set-point. Subsequently, controllers are turned off and open-loop maneuvers are tested. Nelder Mead simplex method involves functional evaluation at vertices of a simplex and shrinking the simplex iteratively. The parameters thus optimized are finally utilized to improve upon the chosen control system.

Contrastingly, Nikola et al adopted a self-oscillations approach in order to identify the parameters for yaw motion of an unmanned marine vehicle [4]. Self-oscillations is the behavior created within a process if it is a closed loop one and contains a non-linear element. The work involves considering relay with hysteresis as an instrumented nonlinear element in order to generate self-oscillations within the systems behavior. Once the self-oscillations are established within the closed loop system, magnitude and frequency of the obtained self-oscillations are used as inputs to the non-linear element. Finally, from intersection between Nyquist frequency characteristics of the process and inverse negative describing function of the nonlinear element, unknown process parameters are identified. Although, the algorithm consumes lesser time than open-loop methods, it is subject to some initial closed-loop controlled framework.

Juan et al adopted parameter estimation approaches using EKF and maximum likelihood estimation, in order to compare performance measures in identifying model parameters for a nominal AUV [5]. Levenberg-Marquardt algorithm is further used to optimize the non-linear least square solution framework in combination with MLE method. MLE however, exhibits better convergence rate than EKF.

III. Problem Description

Unmanned Marine vehicles are usually not multi-body systems and are therefore modeled using the 6-DOF (degrees of freedom) rigid body dynamics [6]. The expressions for the corresponding system model are thus defined as follows:

$$M\dot{v} + C(v)v + D(v)v + g(\eta) = \tau + \tau_d \tag{1}$$

$$M = M_r + M_a \tag{2}$$

where the terms preserve their usual meaning with M including the rigid body and added mass values denoted as M_r, M_a respectively. C(v)v and D(v)v denote the Coriolis and damping forces respectively. Now, hydrodynamic mass is as important as the rigid body mass, which is because together they contribute to the inertial force required by the motion of the vehicle. On the other hand high velocities are faced with large drag forces since unmanned marine vehicles often have speeds ranging from $4-7$ Knots. Precisely therefore, the hydrodynamic mass and drag components are quite predominant in governing the dynamic behavior of such vehicles. In the scope of the present paper we shall however, contract the above equation in terms of state variables in order to portray a basic 2-DOF vehicle attributed with surge and steer motions, defined as follows:

$$F_x = M\dot{u} - Mvr + C_{u|u}u^2 \tag{3}$$

$$F_y = M\dot{v} + Mur + C_{v|v}v^2 \tag{4}$$

$$\tau = I_{zz}\dot{r} + Mx_G\dot{v} + Mx_Gur - My_G\dot{u} + My_Gvr \tag{5}$$

where I_{zz} is the moment of inertia about Z axis, x_G and y_G are the position differences between geometric center and center of gravity, u and v are surge and sway velocity components expressed in body-fixed frame of reference, and r remains the yaw rate. The above equations can be expressed further in a linear form as follows:

$$A.\theta = Y \tag{6}$$

Where $A = \begin{bmatrix} \dot{u} - vr & u^2 & 0 \\ \dot{v} + ur & v^2 & 0 \\ 0 & 0 & \dot{r} \end{bmatrix}$ is the design matrix, $\theta = \begin{bmatrix} M \\ C_{u|u} \\ I_{zz} \end{bmatrix}$ and $Y = \begin{bmatrix} F_x \\ F_y \\ \tau \end{bmatrix}$ being the parameter and force vectors respectively. It may be mentioned here that the coupled terms in (3) are ignored while formulating the design matrix A, wherein τ is defined in a reduced form as $I_{zz}.\dot{r}$. By referring to the value of M in (2),we can substitute for its value in (3) and (4). The modified θ and Y matrices are revised as follows:

$\theta = \begin{bmatrix} M_a \\ C_{u|u} \\ I_{zz} \end{bmatrix}$ and $Y = \begin{bmatrix} F_x - M_r\dot{u} - vr \\ F_y - M_r\dot{v} + ur \\ \tau \end{bmatrix}$ This is to be considered in particular because the rigid mass M_r can be known easily while the hydrodynamic added mass M_a happens to be the actual parameter to be identified.

However, the subsequent issue in solving (6) is that the design matrix involves variables which are observation dependent and is therefore subject to stochastic variations. The randomness in the design matrix is given birth by measurement noises associated with Inertial Navigation Systems eking out data like accelerations and angular rates as well as Velocity Dopplers providing relative velocity of the vehicle with respect to the bottom floor (if used in bottom locking mode). Talking about the other sensor, Doppler navigation for marine vehicles is vividly reported in [7], [8], with an extensive experimentation carried out with the HUGIN AUV as reported in [9]. On the other side as well, significant studies have gone into the field of realizing and mitigating stochastic disturbances associated with low cost MEMs based Inertial Sensors. The seminal work of David Allan in computing correlation co-efficients simply through a time averaged procedure has been subsequently put to error analysis of inertial sensors profoundly in the thesis work of Nassar [10], Park [11] and Naser et al [12]. The same method is adopted in order to compute critical noise coefficients associated with the measurements and is discussed in details in section IV of this paper. DVL noise margins are also dealt with in the same section. Coming to the present context of the paper, each of the variables viz. u, v, $\dot{u}$, $\dot{v}$ and r are finitely bounded by their respective accuracies (i.e. in terms of $3\sigma \pm \%fs$) defined in terms of noise margins and scale factor errors. This in turn affects the parameter vector to be identified. Hence, the design matrix in (4) needs to be modified as an interval based matrix instead of real-valued one. The interval based matrix denoted as $\tilde{A}$ is defined as

$\begin{bmatrix} \dot{\tilde{u}} - \tilde{v}\tilde{r} & \tilde{u}^2 & 0 \\ \dot{\tilde{v}} + \tilde{u}\tilde{r} & \tilde{v}^2 & 0 \\ 0 & 0 & \dot{\tilde{r}} \end{bmatrix}$, where the interval based variables are denoted as follows:

$\dot{\tilde{u}} = [\underline{\dot{u}}, \overline{\dot{u}}], \dot{\tilde{v}} = [\underline{\dot{v}}, \overline{\dot{v}}], \tilde{u} = [\underline{u}, \overline{u}], \tilde{v} = [\underline{v}, \overline{v}]$ and $\tilde{r} = [\underline{r}, \overline{r}]$.

It may be intuitively stated that the parameter vector also shall no longer remain a real-valued one and hence θ in (4) is replaced with an interval based denotation $\tilde{\theta}$ and hence (4) may be restated as follows:

$$\tilde{A}.\tilde{\theta} = Y \tag{7}$$

IV. Interval solution to Parameter Identification

In the context of this section, discussion on interval methods adopted for finding solution to (5) shall be presented. The section comprises few subsections in order to systematically establish the requirement and effectiveness of the proposed method. The section begins with a brief introduction to fundamental concepts of interval arithmetic and methods formulated to attack problems such as system of equations and computing inverse interval matrices. In the final subsection, problems faced in applying such methods for the present case are described, thereby defining the scope for the proposed method which is elaborated subsequently.

A. Interval Arithmetic and System of Equations

The interval analysis approach treats intervals as a new kind of numbers represented by pairs of endpoints. An interval [x] of R is a closed bounded set defined as follows.

$$[x] = [\underline{x}, \overline{x}] = \{x | x \in R, \underline{x} \le x \le \overline{x}\}$$

Where $\underline{x}$ and $\overline{x}$ are the (finite or infinite) inferior and superior interval endpoints, respectively. Interval analysis applies all standard set and arithmetic operations on intervals. Interval matrices are defined as matrices whose elements are non-punctual, i.e. interval valued. For example following is an example of an interval matrix:

$$Q = \begin{bmatrix} [2,4] & [-2,2] \\ [-1.5,4.5] & [-2,3.5] \end{bmatrix}$$

The interval matrix Q can be denoted as a hull of two boundary real-valued matrices [$\underline{Q}$,$\overline{Q}$], where $\underline{Q}$ and $\overline{Q}$ are defined as follows:

$$\underline{Q} = \begin{bmatrix} 2 & -2 \\ -1.5 & -2 \end{bmatrix} \text{ and } \overline{Q} = \begin{bmatrix} 4 & 2 \\ 4.5 & 3.5 \end{bmatrix}$$

A few basic terms need to be defined for working with interval matrices in this context. The norm of an interval matrix is defined as an extension of the maximum row sum norm of a real-valued matrix, i.e. $\|Q\| = max_i \sum_j Q_{ij}$. Mid-point and radius of an interval matrix are denoted as Q_c and Δ. The mid-point of an interval matrix is one which has for its elements the mid-vales of the elements of the interval matrix. Therefore, mid-point of an interval matrix is a real-valued one. For the given example, the parameters defined so far can be computed for the matrix Q in the example as follows:

$\|Q\| = max\{|[2,4]| + |[-2,2]|, |[-1.5,4.5]| + |[-2,3.5]|\} = max\{6,8\} = 8$

$$Q_c = \begin{bmatrix} 3 & 0 \\ 1.5 & 0.75 \end{bmatrix} \text{ and } \Delta_Q = \begin{bmatrix} 1 & 2 \\ 3 & 2.25 \end{bmatrix}$$

Coming back to our problem, finding solution to system of linear equations with interval matrices is a well studied problem. It started with Hansen [13], and was taken along by other works including those of Ning and Klearfott [14]. However, in the present context of this paper, two straightforward approaches have been studied - the one belonging to Krawczyk [15 Moore's book] and the other formulated in the work of Nirmala and Ganesan [16]. In the present sub-section we shall briefly describe Krawczyk's and Nirmala's methods, and the basic requirements for applying them.

1) Krawczyk's Method of solving Interval Based Linear System of Equations: Let us consider the equation $Ax = B$, such that we need a solution in vector x. In order to apply Krawczyk method we need to define a real-valued matrix E as follows:

$$E = I - YA \tag{8}$$

where I is a unit matrix and Y can be any matrix for which the norm value of E is much lesser than unity, i.e. $\|E\| < 1$. Usually Y is chosen to be the mid-point of the interval matrix A. Upon successful identification of an E matrix the following steps are conducted:

Step 1: If the matrix B is a real-valued matrix, then $x(0)$ as the first iteration is computed as follows.

$$x_i(0) = \frac{\|YB\|}{1 - \|E\|}[-1, 1] \tag{9}$$

If the matrix B is an interval matrix, then the unit interval is not applied to (9).

Step 2: The value of x is sequentially computed as follows:

$$x(k+1) = YB + Ex(k) \cap Ex(k) \tag{10}$$

The above step is carried out for refining the intervals as obtained in Step 1, thereby iteratively reducing the overestimation errors introduced in the previous steps. However, the key to applying Krawczyk's method remains in the identification of an E matrix as defined above.

B. Nirmala and Ganesan's method towards finding interval inverse

In the context of determining interval inverse in finding solutions to system of equations, Nirmala and Ganesan have proposed a new approach. Reconsidering equation $Ax = B$, solution in terms of the vector x may be obtained as follows:

Step 1: The determinant $|A|$ of the interval matrix A is computed.

Step 2: Compute are the co-determinants for interval matrices obtained by replacing each column of an $n \times n$ matrix A with the column of $n - ary$ vector B, as follows:

$$\left|A^{(1)}\right| = \begin{vmatrix} B_{11} & A_{12} & \cdots \\ B_{21} & A_{22} & \cdots \\ \vdots & \vdots & \vdots \\ B_{n1} & A_{n2} & \cdots \end{vmatrix} \quad \left|A^{(2)}\right| = \begin{vmatrix} A_{11} & B_{11} & \cdots \\ A_{21} & B_{21} & \cdots \\ \vdots & \vdots & \vdots \\ A_{n1} & B_{n1} & \cdots \end{vmatrix}$$

Step 3: Finally, elements of the x vector are obtained as ratios between corresponding co-determinants and the determinant of A. This may be denoted as follows:

$$x_1 = \frac{|A^{(1)}|}{|A|} x_2 = \frac{|A^{(2)}|}{|A|} \cdots x_n = \frac{|A^{(n)}|}{|A|},$$

Where $x = |x_1 \;\; x_2 \;\; \cdots \;\; x_n|^T$

However, the above-defined algorithm is constrained as $0 \notin |A|$. This is because if the said determinant interval contains a zero, then step 3 would involve a division by zero and hence could not be carried out and the algorithm is inapplicable.

By referring to the constraints of the above-mentioned approaches, it may be concluded that the chosen methods may not be applicable to a vast set of problems wherein the interval matrices are not conditioned properly. It is in this context that a partitioned hull method towards finding an inverse interval matrix is proposed and presented in the subsequent section.

C. Partitioned Hull method in finding out an inverse interval matrix

The proposed method is based upon Kuttler's proposition [17] which may be stated as follows:

For a regular interval matrix A, the inverse is defined as: $A^{-1} = [\underline{A}^{-1}, \overline{A}^{-1}]$.

The inverse is thus obtained as a hull of the inverses of corresponding end-point matrices obtained from the original interval matrix. The partitioned hull method may be described as partitioning the interval matrix into sub-matrices symmetrically on both sides of its end-points. This results in the formulation of a set of infinum matrices and supremum matrices. Subsequently, inverse for each sub-matrix is obtained. Finally the element-wise minimum of infinum matrix inverses is chosen for the infinum matrix going into the final inverse. Similarly, the supremum of inverse of the final inverse is chosen as element-wise maximum of supremum matrices. By referring to an equation of the form $Ax = B$, the proposed method can now be illustrated in the form of an algorithm as follows:

Step 1: Determine the radius matrix Δ for given interval matrix A, and element wise divide the radius by the number of partitions required, which may be denoted by N. This can also be expressed as $\Delta = N.\delta$, where δ is the granularity factor, such that lesser is the value of granularity, higher goes the significance of partitioning.

Step 2: Compute a set of unique matrices defined by the infinum and supremum from each interval element belonging to the original interval A. The set of matrices thus obtained may be formally defined as follows:

$$I = \{(A_c - \Delta), (A_c - \Delta + \delta)...(A_c - \Delta + [n-1]\delta)\}$$

$$S = \{(A_c + \Delta - [n-1]\delta), ..., (A_c + \Delta - \delta), (A_c + \Delta)\}$$

Where A_c, Δ are mid-point and radius of the matrix A such that Δ is an integral multiple of the granularity factor δ as defined above.

Step 3: Finally, the interval inverse for A is obtained as a hull matrix containing elements from both the sets of infinum and supremeum matrices in a manner which is portrayed as follows:

$$A^{-1} = [min_i(I_i^{-1}), max_i(S_i^{-1})] \tag{11}$$

where the minimum and maximum operations are defined in the context of element-wise comparisons between the matrices belonging to sets I and S. In matrix form infinum and supremum of the inverse matrix can be realized as follows:

$$min(I^{-1}) = \begin{bmatrix} min_j\{I(j)_{11}^{-1}\} & min_j\{I(j)_{12}^{-1}\} & \cdots \\ min_j\{I(j)_{21}^{-1}\} & min_j\{I(j)_{22}^{-1}\} & \cdots \\ \vdots & & \end{bmatrix} \tag{12}$$

$$max(I^{-1}) = \begin{bmatrix} max_j\{I(j)_{11}^{-1}\} & max_j\{I(j)_{12}^{-1}\} & \cdots \\ max_j\{I(j)_{21}^{-1}\} & max_j\{I(j)_{22}^{-1}\} & \cdots \\ \vdots & & \end{bmatrix} \tag{13}$$

However, with the proposed method as defined so far, we shall come back to the linear system of equations formulated in the context of the present paper (refer to section III). Recalling from equation (7) defined in section III, the solution in terms of the parameter vector can be obtained by applying the Principle of Least Squares. It may be proved that even with interval inputs at the design matrix and real-valued output vector, principle of Least Squares can be applied to obtain solution to the parameter vector. Interested readers are requested to refer to the proof given by Markov [18]. The above defined state model is linear in terms of model parameters and hence the solution to (4) for $\tilde{\theta}$ can be obtained from principle of Least Squares as follows: Hence the parameter vector may be obtained in the form of Interval based solution to linear systems. The methodology adopted in this regard is described in details in the subsequent section.

$$\tilde{\theta} = \{\tilde{A}^T \tilde{A}\}^{-1} \tilde{A}^T \tilde{Y} \tag{14}$$

The inverse of the product matrix $\tilde{A}^T \tilde{A}$ is computed using the proposed partitioned hull approach. The subsequent section deals with the practical implementation of the proposed solution method for the problem in context. It shall be observed that the obtained parametric interval matrix θ is broad. However heuristically, a sharper interval is progressively evolved.

V. Experiment and Discussion

For a performance based analysis of the proposed method, an experimental setup was prepared. The targeted platform remained in the form of a lab-scale 2-DOF unmanned surface vehicle. Equipped with low-cost MEMs Inertial Sensors and a Doppler sensor, the system was driven by a pair of Brushless DC (BLDC) thrusters responsible for the surge and steer motions, thereby imparting to it 2 Degrees of Freedom (DOF) and suitably defining the dynamics by equations (2)-(3). Specifications for the system are summarized in Table 1. The present section is once again divided into few sub-sections for a systematic discussion of the experimental setup and trial results. The section begins with a presentation on error analysis for the sensors being used in the platform. The bounds are defined from the densities of stochastic noise associated with the measurements from the sensors. The bounds are utilized in converting real valued measurements into corresponding intervals. Following the analysis of errors, an actuation model for computing force and torque is described. Finally the trial data are discussed and parameter intervals are obtained for a single data-set. The estimated interval parameters are then used in the dynamic equations as defined in Section III, in

TABLE I. Specifications for a Lab-Scale ASV

Dimensions (mm): LxBxH	1090 x 755 x 587
Weight (in air)	52.45 kg
Weight (in water)	60.94 kg
Energy Source	Li-polymers, 26Ah, 14.8V, Max 16V
Inertial Sensor	(MEMs) Mtx, Xsens make
Doppler Sensor	WHN 300, RDI make
Actuation	Tunnel Thrusters, MCT01, Seaeye make

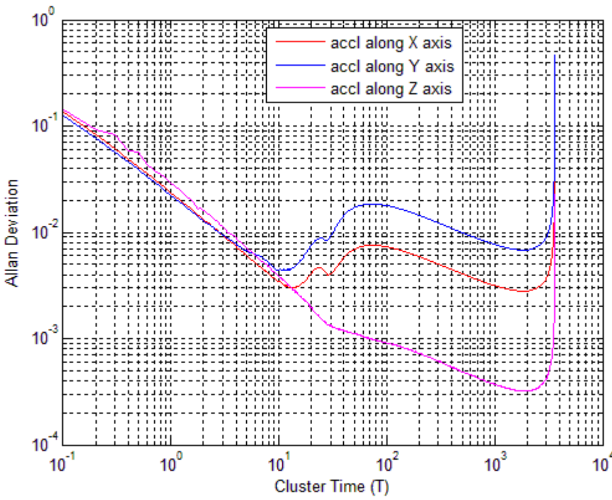

Fig. 1. Plot for Allan Deviations against acceleration measurements during static tests with Mtx INS

order that the external force and moment may be computed. The predicted force and moment are compared with the corresponding values observed during a different trial. Errors are computed against predicted intervals for different data, with the interval having minimum error being selected for the final solution of the parameter vector in (7).

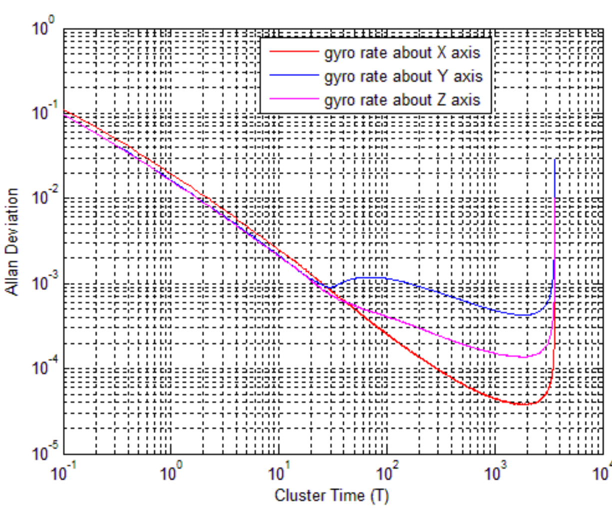

Fig. 2. Plot for Allan Deviations against angular rate measurements during static tests with Mtx INS

TABLE II. Noise coefficients for Acceleration and angular rate information coming from Mtx Inertial Sensor

Physical Quantity	Coefficient	X-axis	Y-axis	Z-axis
Acceleration	N (m/s/s)	0.0018	0.002	0.0028
	B (m/s2)	3.0x10-4 (670 secs)	2.8x10-4 (100 secs)	2.8x10-4 (600 secs)
Angular Rate	N (0/s)	6.0	9.0	6.0
	B (0/s)	0.007 (600 secs)	0.005 (500 secs)	0.012(270 secs) (270 secs)

Fig. 3. Image of the Test Vehicle - SWAN (Surface Water Autonomous Navigator)

A. Error Bounds for Inertial and Doppler sensors

MEMs based inertial sensors are cost effective measures used in localization problems of robotics. However, such devices are fairly accurate and associations with stochastic noise are significant. Systematic errors emanating from misalignment of sensitive axes as well as quantization noise, can be filtered out using proper calibration methods. However, stochastic errors originating from time-varying correlations, need to be analyzed in order that they may be properly modeled. In the present context of the sensor used, the most widely adopted Allan Variance analysis method is employed for realizing various noise coefficients. Data from the Mtx was recorded over an uninterrupted duration of 2 hours, and the stochastic variations in the observed errors were studied in order to compute the random walk and bias instability co-efficients. Angle / Velocity Random Walk is denoted by N. The error grows as $N \times \sqrt{sec}$, and is modeled as white noise. On the other hand, Bias Instability is denoted by B, and defined as a random variable variance of which grows over time in correlated manner. It is usually modeled as a first-ordered Gauss Markov noise variable. The random walk and bias instability coefficients are as provided in Table 2.

The cumulative effect of both the above-defined types of errors may be consolidated into a single white noise representative standard deviation σ_c as follows:

$$\sigma_c = \frac{N}{\sqrt{\delta t}} + \sqrt{\frac{\delta t}{t}}.B \tag{15}$$

Consequently, the intervals for accelerations and angular rates

TABLE III. THRUST VERSUS DEMAND CHARACTERISTICS

Demand (%)	Average Thrust (N)
15	4.654
20	6.297
25	8.204
30	10.407
35	12.325
40	14.463
45	17.347
50	19.526
55	24.024
60	33.221
65	40.725
70	54.319
75	59.243
80	72.768
85	78.924

can be drawn out as follows: $\dot{\tilde{u}} = [\dot{u} - 3\sigma_c, \dot{u} + 3\sigma_c]$.

Doppler Velocity Log (DVL) SONARs reportedly suffer from +/-2$\sim$3mm/sec accuracy levels thereby ending up with 0.4$\sim$0.6$\%$ scale factor error. This contributes to an along track error drift of 0.4$\%$ of traveled distance, or 28.8 m/hour for an AUV traveling at 2 m/s (4 knots). With the present information on the typical error bounds for DVL measurements, we define the following intervals:

$$u_m - acc \leq u_t(1 - sfe) \leq u_m + acc$$

where u_m, acc, u_t and sfe are the measured u, accuracy, true value for u and scale factor error respectively. Hence the body-fixed frame referenced velocity intervals $[u]$ and $[v]$ can be constructed as follows:

$$[u] = \left[\frac{(u - 0.002)}{0.996}, \frac{(u - 0.002)}{0.996}\right] \tag{16}$$

$$[v] = \left[\frac{(v - 0.002)}{0.996}, \frac{(v - 0.002)}{0.996}\right] \tag{17}$$

The intervals thus obtained are applied to corresponding velocity information as obtained from the recorded data sets. However, in order to construct the Y matrix in equation (7), a model is introduced in the following sub-section.

B. Force and Torque Model

The actuation used for the proposed test platform is in the form of a couple of tunnel thrusters run by Brushless DC (BLDC) motors. The thrusters are of Seaeye make, model number MCT01, and are specified with a maximum torque at 12.9 kgf consuming a maximal power of 300 Watts. Maneuvering with the thrusters is achieved by sending digital demands (desired percentage of the maximal thrust) to the embedded control drives. Thrust versus demand characteristics were established through Bollard Pull tests both in the forward and reverse directions. Table 3 summarizes the average thrusts obtained for different demand levels.

Finally, a thrust model is defined in terms of demand supplied to the drives through 3-order polynomial fitting as follows:

$$\tau_{model} = c_1.d^3 + c_2.d^2 + c_3.d + c_4 \tag{18}$$

where $c_1 = 0.0003 c_2 = -0.015 c_3 = 0.6008 and c_4 = -1.0902$.

TABLE IV. DESIGN PARAMETERS FOR THE VEHICLE

Parameter	Geometric Center as the Origin		
	X-axis	Y-axis	Z-axis
Center of Gravity	-1.06 mm	-2.95	-117.8
Center of Buoyancy	1.77 mm	0.5	18.9

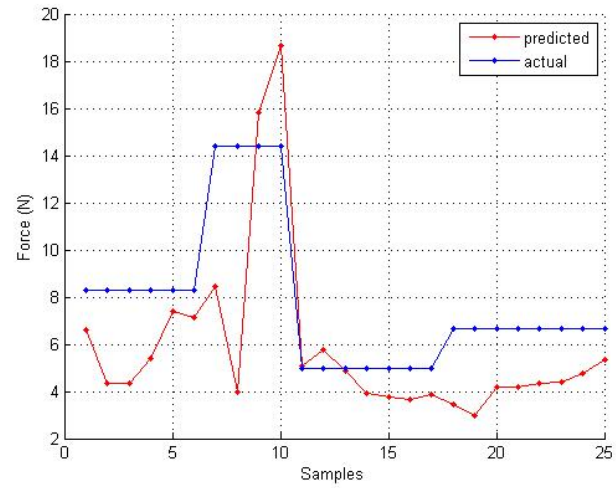

Fig. 4. Plot for mid-values of non-punctual predicted force against measured values

Consequently, the forces and torque are computed as follows:

$$F_{eff} = F_1 + F_2 \tag{19}$$

$$\tau_{eff} = F_1 \times l_1 + F_2 \times l_2 \tag{20}$$

where, F_1, F_2, l_1 and l_2 are respectively the forces delivered by the thrusters and their distances from the center of gravity of the vehicle. In this context it may be mentioned that the design for the vehicle has been done in a manner such that its center of gravity is close to the geometric center. Table 4 provides the design parameters for the vehicle.

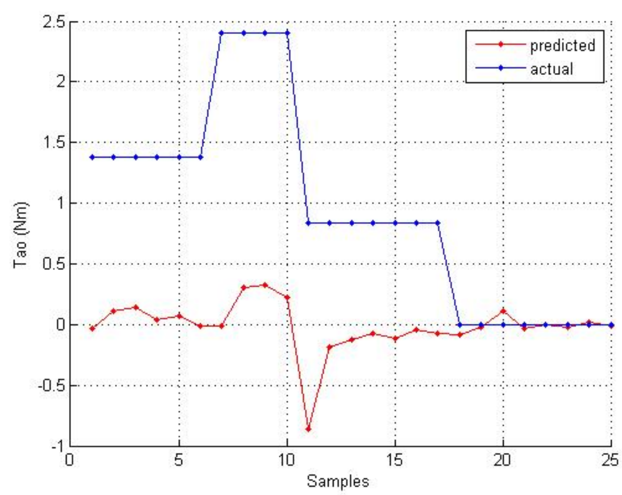

Fig. 5. Plot for mid-values of non-punctual predicted torque against measured values

TABLE V. ESTIMATED PARAMETER INTERVALS

Parameter	Interval
Added Mass	[-31.8686,8.1316]
Drag Parameter	[235.7778,275.7781]
Moment of Inertia	[-41.7606, 48.2396]

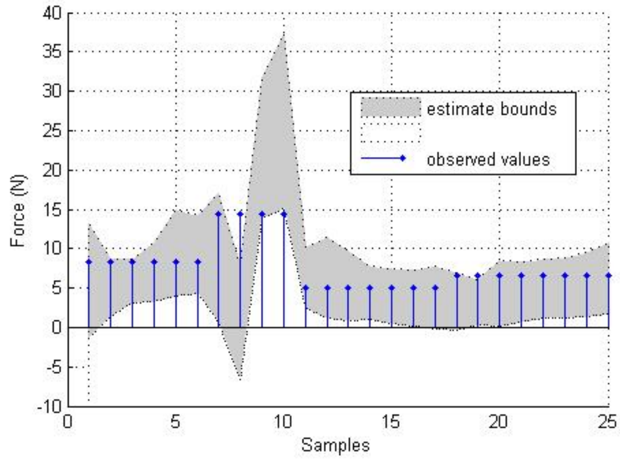

Fig. 6. Guaranteed bounds for estimated force against measured values

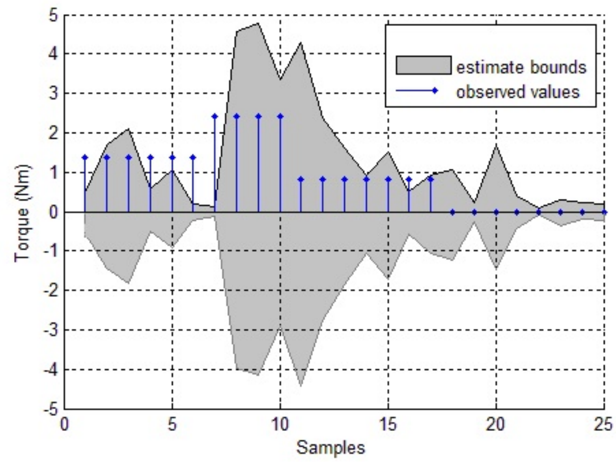

Fig. 7. Guaranteed bounds for estimated moment against measured values

C. Discussion on identified interval based parameters

Finally, non-punctual (interval based) parameters are computed by substituting the design matrix A obtained from interval bound data from sensors, as well as the Y matrix into equation (14). A couple of runs were conducted and data collected fro the respective sensors. The identification was carried out using one data set, while the parameter intervals were substituted into equations (3)-(5) by taking into consideration values from the second data set. In the course of identifying the parameters from the first data set using the partitioned hull method as described in section IV, error intervals were also computed (through direct comparison with the observed force and moment values). Heuristically, the interval with the minimum diameter (i.e. having minimum end-to-end absolute error), was selected. Force and moment intervals were thus computed from the typical model equations using the selected parameter intervals. However, for the purpose of comparing against observed values of external force and moment, mid-values of the intervals were considered, plots of which are shown in Fig 1 and 2 respectively. The errors for computed force and torque having mean of 1.955 N and 0.963 N-m with σ values of 2.699 N and 0.785 N-m respectively. The interval based parameters thus identified are listed in Table V.

The intervals for estimated force and torque values are illustrated in Fig 6 and 7. Evidently, the estimated parameter intervals can predict guaranteed bounds for the force and moment acting on the vehicle.

VI. CONCLUSION

It is presented in this paper as a system identification method using interval arithmetic as a basic means of handling uncertainties associated with sensors. Since, data coming from the sensors are converted into intervals instead of dealing with real valued numbers, therefore the matrices formed out of them are also interval based. The identification problem is expressed in a linear form and various interval inclined solution methods are discussed in light of the formulated problem. However the constraints of such methods are elucidated and a partitioned hull approach for computing interval inverse matrix is proposed. The new method is defined by partitioning the original interval matrix into a set of infinum and supremum matrices. Inverse for each such real valued matrix is computed. Finally through element wise minimum and maximum comparisons between all such matrices, the inverse interval matrix is computed. The method is exercised over data collected from test runs for a lab-scale unmanned surface vehicle. Interval based parameters thus identified are substituted into the model equations for the system, in order to estimate force and moment values. Performance of identification is analyzed from comparison of the values thus computed with observed values. The interval approach is seen to provide guaranteed bounds to the dynamic behavior of the system in the presence of measurement uncertainties. Future work may be conducted in reducing overestimates by the proposed interval method and the bounds even narrower.

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