Normalizing flows in tf_utils

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Planar Flows:

$$f(z) = z + h(w^\top z + b)$$
$$\frac{df(z)}{dz} = someexpression$$

Cholesky Product Flows:

$$f(z) = \text{vec}(L(z)L(z)^\top)$$

$$L(z) = \begin{bmatrix} e^{z_1} & 0 & ... & 0 & \\ z_2 & e^{z_3} & & & \\ z_4 & z_5 & e^{z_6} & & \\ ... & & & & \end{bmatrix}$$

$$\text{diag}(z) = \begin{bmatrix} z_1 \\ z_3 \\ z_6 \\ .. \end{bmatrix}, \text{diag}(L(z)) = \begin{bmatrix} e^{z_1} \\ e^{z_3} \\ e^{z_6} \\ .. \end{bmatrix}$$

$$vec(M) = \begin{bmatrix} M_{11} \\ M_{21} \\ M_{22} \\ M_{31} \\ ... \\ M_{DD} \end{bmatrix}$$

$$\left|\frac{dL(z)}{dz}\right| = \prod_{d=1}^{D} \text{diag}(L(z))_d = \prod_{d=1}^{D} e^{\text{diag}(z)_d}$$

$$\log(\left|\frac{dL(z)}{dz})\right| = \log(\prod_{d=1}^{D} e^{\text{diag}(z)_d}) = \sum_{d=1}^{D} \text{diag}(z)_d$$

$$\Sigma(z) = L(z)L(z)^\top$$

$$\left|\frac{d\text{vec}(\Sigma(z))}{dL(z)}\right| = 2^D \prod_{d=1}^{D} L_{dd}^{D-d+1}$$

$$\log(\left|\frac{d\text{vec}(\Sigma(z))}{dL(z)}\right|) = D\log(2) \sum_{d=1}^{D} (D-d+1)\log(\text{diag}(L(z))_d)$$

$$= D\log(2) \sum_{d=1}^{D} (D-d+1)\text{diag}(z)_d$$

$$\log(\left|\frac{df(z)}{dz}\right|) = \log(\left|\frac{d\text{vec}(\Sigma(z))}{dL(z)}\frac{dL(z)}{dz}\right|) = \log(\left|\frac{d\text{vec}(\Sigma(z))}{dL(z)}\right|\left|\frac{dL(z)}{dz}\right|) = \log(\left|\frac{d\text{vec}(\Sigma(z))}{dL(z)}\right|) + \log(\left|\frac{dL(z)}{dz}\right|)$$