



# hw5

November 29, 2020

## 1 Computer Vision

## 2 Jacobs University Bremen

## 3 Fall 2020

## 4 Homework 5

*This notebook includes both coding and written questions. Please hand in this notebook file with all the outputs and your answers to the written questions.*

This assignment covers K-Means and HAC methods for clustering and image segmentation.

```
[2]: # Setup
from time import time
import numpy as np
import matplotlib.pyplot as plt
from matplotlib import rc
from skimage import io

from __future__ import print_function

%matplotlib inline
plt.rcParams['figure.figsize'] = (15.0, 12.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'

# for auto-reloading external modules
%load_ext autoreload
%autoreload 2
```

The autoreload extension is already loaded. To reload it, use:

```
%reload_ext autoreload
```

### 4.1 Introduction

In this assignment, you will use clustering algorithms to segment images. You will then use these segmentations to identify foreground and background objects.

Your assignment will involve the following subtasks: - **Clustering algorithms:** Implement K-Means clustering and Hierarchical Agglomerative Clustering. - **Pixel-level features:** Implement a feature vector that combines color and position information and implement feature normalization. - **Quantitative Evaluation:** Evaluate segmentation algorithms with a variety of parameter settings by comparing your computed segmentations against a dataset of ground-truth segmentations.

## 4.2 1 Clustering Algorithms (40 points)

```
[15]: # Generate random data points for clustering

# Set seed for consistency
np.random.seed(0)

# Cluster 1
mean1 = [-1, 0]
cov1 = [[0.1, 0], [0, 0.1]]
X1 = np.random.multivariate_normal(mean1, cov1, 100)

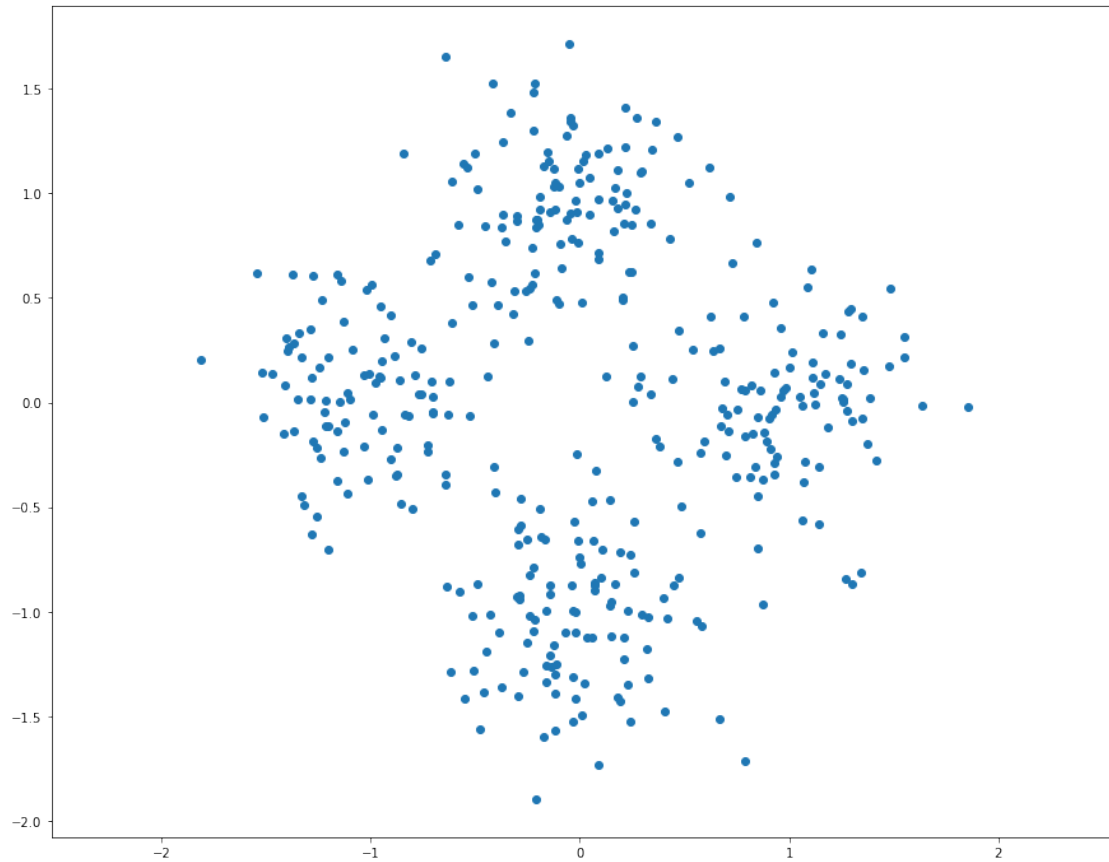
# Cluster 2
mean2 = [0, 1]
cov2 = [[0.1, 0], [0, 0.1]]
X2 = np.random.multivariate_normal(mean2, cov2, 100)

# Cluster 3
mean3 = [1, 0]
cov3 = [[0.1, 0], [0, 0.1]]
X3 = np.random.multivariate_normal(mean3, cov3, 100)

# Cluster 4
mean4 = [0, -1]
cov4 = [[0.1, 0], [0, 0.1]]
X4 = np.random.multivariate_normal(mean4, cov4, 100)

# Merge two sets of data points
X = np.concatenate((X1, X2, X3, X4))

# Plot data points
plt.scatter(X[:, 0], X[:, 1])
plt.axis('equal')
plt.show()
```



#### 4.2.1 1.1 K-Means Clustering (20 points)

As discussed in class, K-Means is one of the most popular clustering algorithms. We have provided skeleton code for K-Means clustering in the file `segmentation.py`. Your first task is to finish implementing `kmeans` in `segmentation.py`. This version uses nested for loops to assign points to the closest centroid and compute a new mean for each cluster.

```
[46]: from segmentation import kmeans

np.random.seed(0)
start = time()
assignments = kmeans(X, 4)
end = time()

kmeans_runtime = end - start

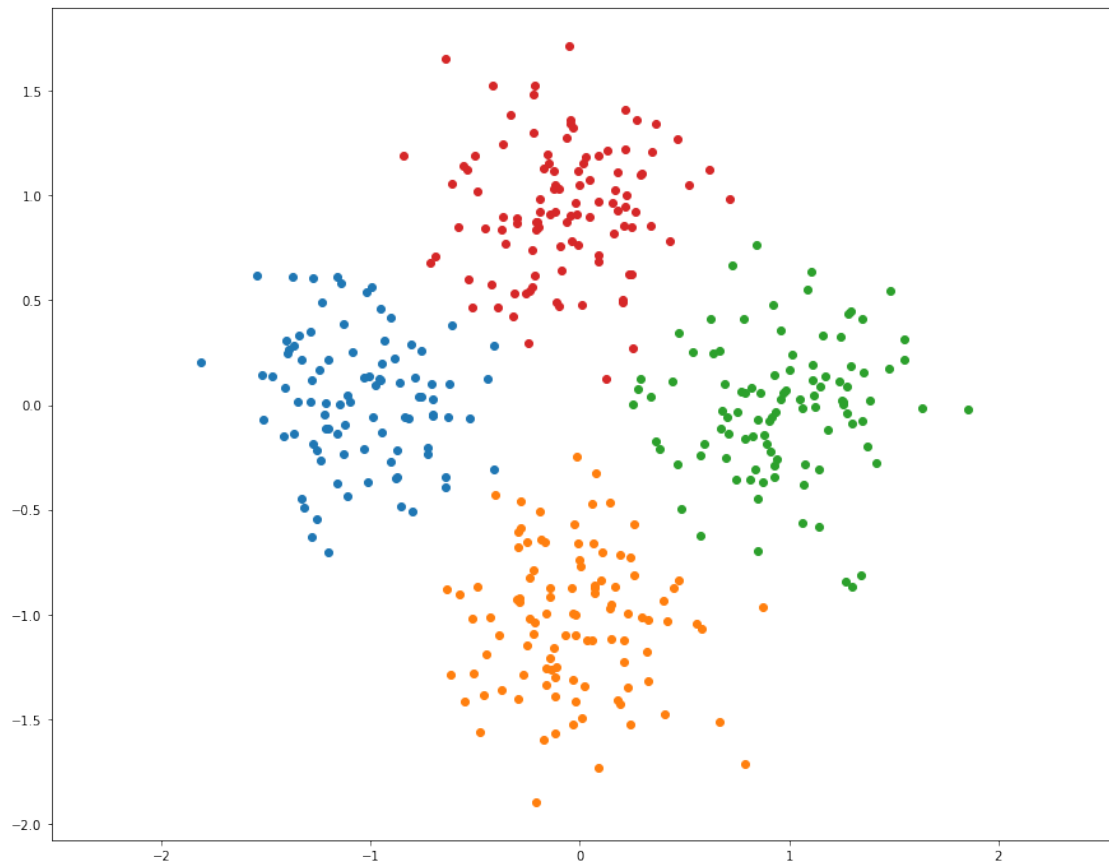
print("kmeans running time: %f seconds." % kmeans_runtime)

for i in range(4):
    cluster_i = X[assignments==i]
```

```
plt.scatter(cluster_i[:, 0], cluster_i[:, 1])

plt.axis('equal')
plt.show()
```

kmeans running time: 0.088948 seconds.



We can use numpy functions and broadcasting to make K-Means faster. Implement `kmeans_fast` in `segmentation.py`. This should run at least 10 times faster than the previous implementation.

```
[47]: from segmentation import kmeans_fast

np.random.seed(0)
start = time()
assignments = kmeans_fast(X, 4)
end = time()

kmeans_fast_runtime = end - start
print("kmeans running time: %f seconds." % kmeans_fast_runtime)
print("%f times faster!" % (kmeans_runtime / kmeans_fast_runtime))
```

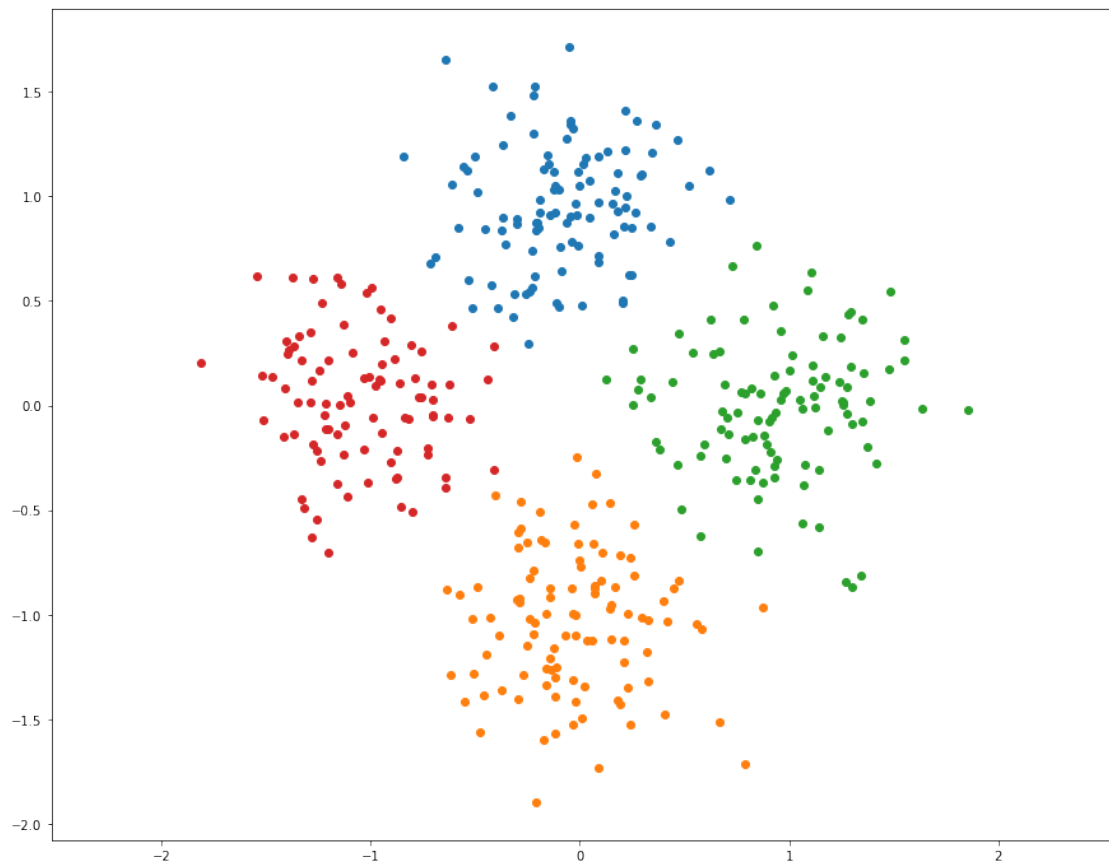
```

for i in range(4):
    cluster_i = X[assignments==i]
    plt.scatter(cluster_i[:, 0], cluster_i[:, 1])

plt.axis('equal')
plt.show()

```

kmeans running time: 0.003999 seconds.  
 22.242712 times faster!



## 5 1.2 K-Means Convergence (10 points)

Implementations of the K-Means algorithm will often have the parameter `num_iters` to define the maximum number of iterations the algorithm should run for. Consider that we opt to not include this upper bound on the number of iterations, and that we define the termination criterion of the algorithm to be when the cost  $L$  stops changing.

Recall that  $L$  is defined as the sum of squared distance between all points  $x$  and their nearest cluster center  $c$ :

$$L = \sum_{i \in \text{clusters}} \sum_{x \in \text{cluster}_i} (x - c_i)^2$$

Show that for any set of points  $D$  and any number of clusters  $k$ , the K-Means algorithm will terminate in a finite number of iterations.

**Your answer here:**

### 5.0.1 1.2 Hierarchical Agglomerative Clustering (10 points)

Another simple clustering algorithm is Hierarchical Agglomerative Clustering, which is sometimes abbreviated as HAC. In this algorithm, each point is initially assigned to its own cluster. Then cluster pairs are merged until we are left with the desired number of predetermined clusters (see Algorithm 1).

Implement `hiererachical_clustering` in `segmentation.py`.

attachment: algo1.png

```
[62]: from segmentation import hierarchical_clustering

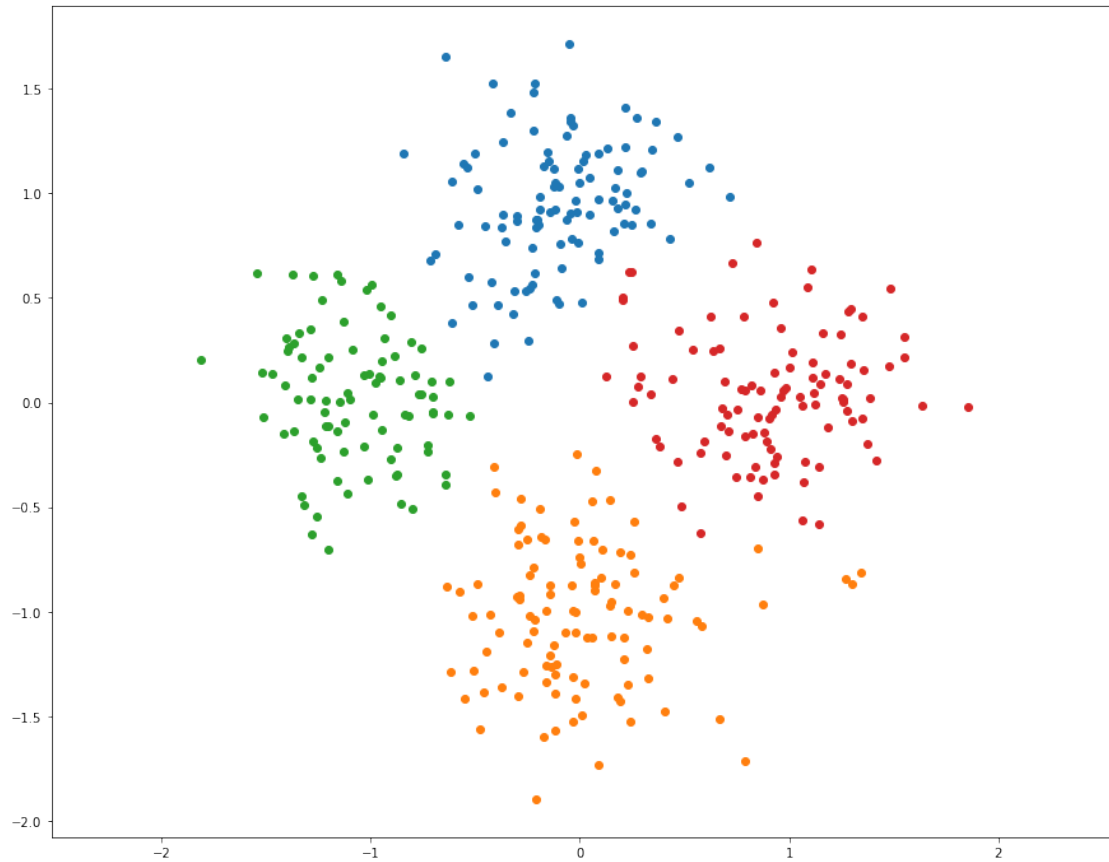
start = time()
assignments = hierarchical_clustering(X, 4)
end = time()

print("hierarchical_clustering running time: %f seconds." % (end - start))

for i in range(4):
    cluster_i = X[assignments==i]
    plt.scatter(cluster_i[:, 0], cluster_i[:, 1])

plt.axis('equal')
plt.show()
```

hierarchical\_clustering running time: 1.307263 seconds.



## 5.1 2 Pixel-Level Features (30 points)

Before we can use a clustering algorithm to segment an image, we must compute some *feature vector* for each pixel. The feature vector for each pixel should encode the qualities that we care about in a good segmentation. More concretely, for a pair of pixels  $p_i$  and  $p_j$  with corresponding feature vectors  $f_i$  and  $f_j$ , the distance between  $f_i$  and  $f_j$  should be small if we believe that  $p_i$  and  $p_j$  should be placed in the same segment and large otherwise.

```
[154]: # Load and display image
img = io.imread('train.jpg')
H, W, C = img.shape

plt.imshow(img)
plt.axis('off')
plt.show()
```





### 5.1.1 2.1 Color Features (15 points)

One of the simplest possible feature vectors for a pixel is simply the vector of colors for that pixel. Implement `color_features` in `segmentation.py`. Output should look like the following:

attachment:color\_features.png

```
[155]: from segmentation import color_features
np.random.seed(0)

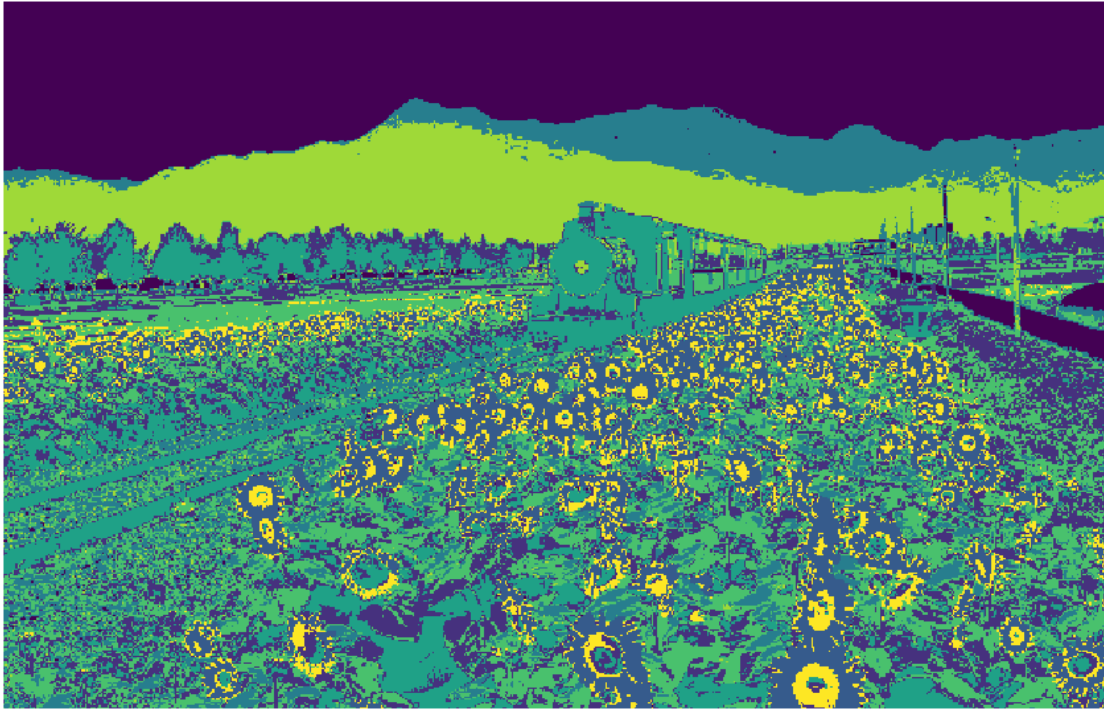
features = color_features(img)

# Sanity checks
assert features.shape == (H * W, C),\
    "Incorrect shape! Check your implementation."

assert features.dtype == np.float,\
    "dtype of color_features should be float."

assignments = kmeans_fast(features, 8)
segments = assignments.reshape((H, W))
```

```
# Display segmentation
plt.imshow(segments, cmap='viridis')
plt.axis('off')
plt.show()
```



In the cell below, we visualize each segment as the mean color of pixels in the segment.

```
[156]: from utils import visualize_mean_color_image
visualize_mean_color_image(img, segments)
```



### 5.1.2 2.2 Color and Position Features (15 points)

Another simple feature vector for a pixel is to concatenate its color and position within the image. In other words, for a pixel of color  $(r, g, b)$  located at position  $(x, y)$  in the image, its feature vector would be  $(r, g, b, x, y)$ . However, the color and position features may have drastically different ranges; for example each color channel of an image may be in the range  $[0, 1)$ , while the position of each pixel may have a much wider range. Uneven scaling between different features in the feature vector may cause clustering algorithms to behave poorly.

One way to correct for uneven scaling between different features is to apply some sort of normalization to the feature vector. One of the simplest types of normalization is to force each feature to have zero mean and unit variance.

Implement `color_position_features` in `segmentation.py`.

attachment:color\_position\_features.png

Output segmentation should look like the following:

```
[157]: visualize_mean_color_image(img, segments)
```



### 5.1.3 Extra Credit: Implement Your Own Feature

For this programming assignment we have asked you to implement a very simple feature transform for each pixel. While it is not required, you should feel free to experiment with other feature transforms. Could your final segmentations be improved by adding gradients, edges, SIFT descriptors, or other information to your feature vectors? Could a different type of normalization give better results?

Implement your feature extractor `my_features` in `segmentation.py`

Depending on the creativity of your approach and the quality of your writeup, implementing extra feature vectors can be worth extra credit (up to 1 point).

```
[158]: from segmentation import color_position_features
np.random.seed(0)

features = color_position_features(img)
# Sanity checks
assert features.shape == (H * W, C + 2),\
    "Incorrect shape! Check your implementation."

assert features.dtype == np.float,\
    "dtype of color_features should be float."

assignments = kmeans_fast(features, 8)
```

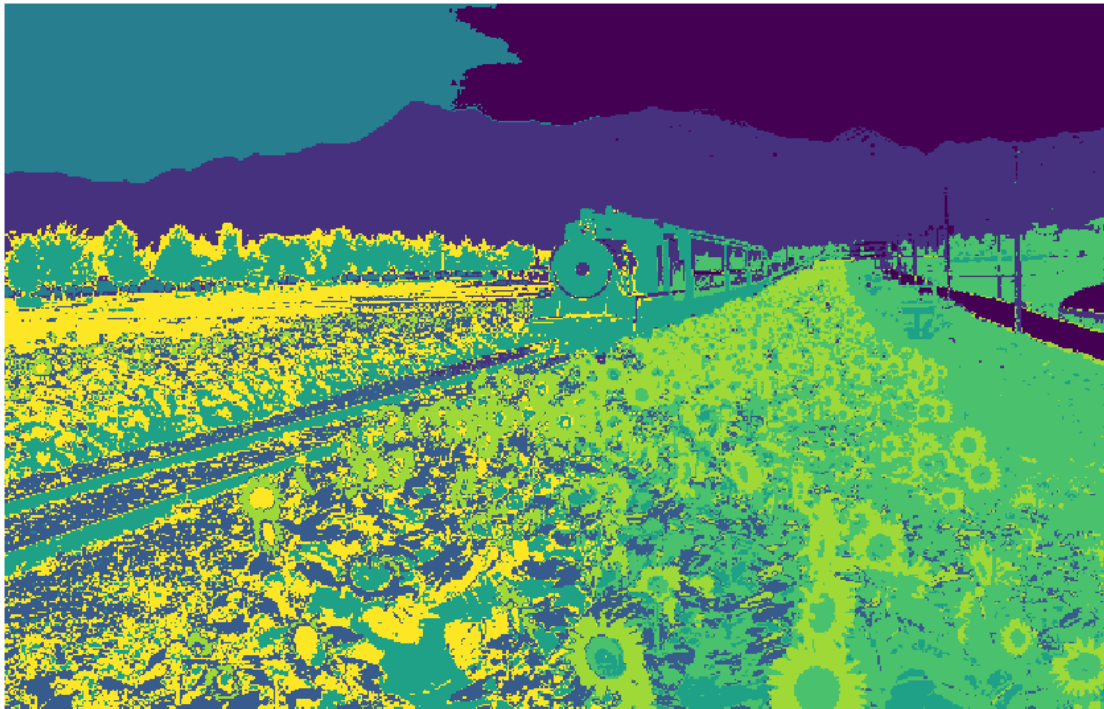


```

segments = assignments.reshape((H, W))

# Display segmentation
plt.imshow(segments, cmap='viridis')
plt.axis('off')
plt.show()

```



**Describe your approach: (YOUR APPROACH)**

```

[ ]: from segmentation import my_features

# Feel free to experiment with different images
# and varying number of segments
img = io.imread('train.jpg')
num_segments = 8

H, W, C = img.shape

# Extract pixel-level features
features = my_features(img)

# Run clustering algorithm
assignments = kmeans_fast(features, num_segments)

```

```

segments = assignments.reshape((H, W))

# Display segmentation
plt.imshow(segments, cmap='viridis')
plt.axis('off')
plt.show()

```

## 5.2 3 Quantitative Evaluation (30 points)

Looking at images is a good way to get an idea for how well an algorithm is working, but the best way to evaluate an algorithm is to have some quantitative measure of its performance.

For this project we have supplied a small dataset of cat images and ground truth segmentations of these images into foreground (cats) and background (everything else). We will quantitatively evaluate different segmentation methods (features and clustering methods) on this dataset.

We can cast the segmentation task into a binary classification problem, where we need to classify each pixel in an image into either foreground (positive) or background (negative). Given the ground-truth labels, the accuracy of a segmentation is  $(TP + TN) / (P + N)$ .

Implement `compute_accuracy` in `segmentation.py`.

```

[150]: from segmentation import compute_accuracy

mask_gt = np.zeros((100, 100))
mask = np.zeros((100, 100))

# Test compute_accuracy function
mask_gt[20:50, 30:60] = 1
mask[30:50, 30:60] = 1

accuracy = compute_accuracy(mask_gt, mask)

print('Accuracy: %0.2f' % (accuracy))
if accuracy != 0.97:
    print('Check your implementation!')

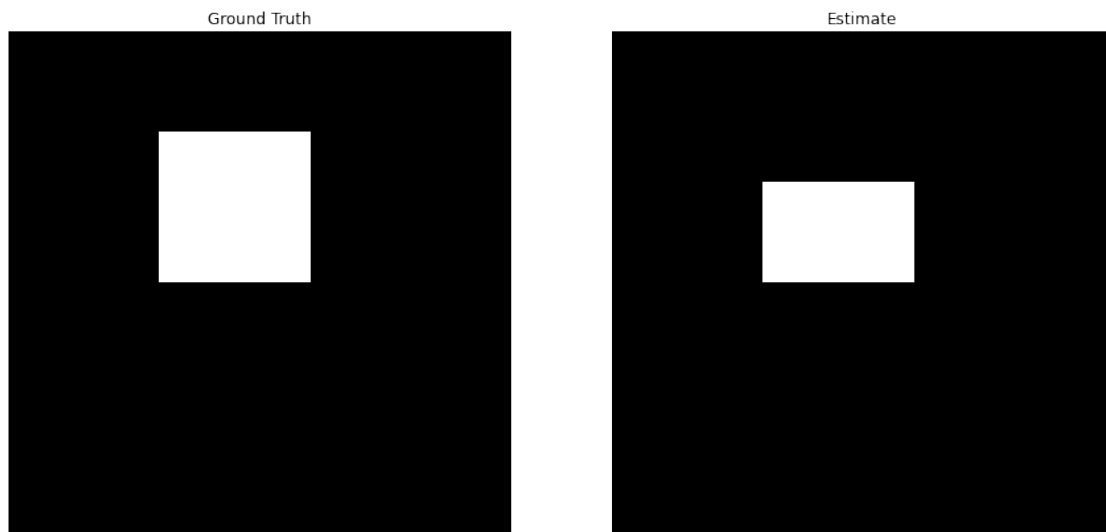
plt.subplot(121)
plt.imshow(mask_gt)
plt.title('Ground Truth')
plt.axis('off')

plt.subplot(122)
plt.imshow(mask)
plt.title('Estimate')
plt.axis('off')

plt.show()

```

Accuracy: 0.97



You can use the script below to evaluate a segmentation method's ability to separate foreground from background on the entire provided dataset. Use this script as a starting point to evaluate a variety of segmentation parameters.

```
[151]: from utils import load_dataset, compute_segmentation
from segmentation import evaluate_segmentation

# Load a small segmentation dataset
imgs, gt_masks = load_dataset('./data')

# Set the parameters for segmentation.
num_segments = 3
clustering_fn = kmeans_fast
feature_fn = color_position_features
scale = 0.5

mean_accuracy = 0.0

segmentations = []

for i, (img, gt_mask) in enumerate(zip(imgs, gt_masks)):
    # Compute a segmentation for this image
    segments = compute_segmentation(img, num_segments,
                                    clustering_fn=clustering_fn,
                                    feature_fn=feature_fn,
                                    scale=scale)

    segmentations.append(segments)
```

```

# Evaluate segmentation
accuracy = evaluate_segmentation(gt_mask, segments)

print('Accuracy for image %d: %0.4f' %(i, accuracy))
mean_accuracy += accuracy

mean_accuracy = mean_accuracy / len(imgs)
print('Mean accuracy: %0.4f' % mean_accuracy)

```

```

Accuracy for image 0: 0.8158
Accuracy for image 1: 0.9245
Accuracy for image 2: 0.9831
Accuracy for image 3: 0.8008
Accuracy for image 4: 0.6997
Accuracy for image 5: 0.6627
Accuracy for image 6: 0.6390
Accuracy for image 7: 0.5687
Accuracy for image 8: 0.8981
Accuracy for image 9: 0.9576
Accuracy for image 10: 0.8855
Accuracy for image 11: 0.8206
Accuracy for image 12: 0.7226
Accuracy for image 13: 0.6558
Accuracy for image 14: 0.7571
Accuracy for image 15: 0.6196
Mean accuracy: 0.7757

```

```

[152]: # Visualize segmentation results

N = len(imgs)
plt.figure(figsize=(15,60))
for i in range(N):

    plt.subplot(N, 3, (i * 3) + 1)
    plt.imshow(imgs[i])
    plt.axis('off')

    plt.subplot(N, 3, (i * 3) + 2)
    plt.imshow(gt_masks[i])
    plt.axis('off')

    plt.subplot(N, 3, (i * 3) + 3)
    plt.imshow(segmentations[i], cmap='viridis')
    plt.axis('off')

plt.show()

```





Include a detailed evaluation of the effect of varying segmentation parameters (feature transform, clustering method, number of clusters, resize) on the mean accuracy of foreground-background segmentations on the provided dataset. You should test a minimum of 10 combinations of parameters. To present your results, add rows to the table below (you may delete the first row).

Feature Transform	Clustering Method	Number of segments	Scale	Mean Accuracy
Color	K-Means	3	0.5	0.7775
Color and Position	K-Means	3	0.5	0.7757

Observe your results carefully and try to answer the following question: 1. Based on your quantitative experiments, how do each of the segmentation parameters affect the quality of the final foreground-background segmentation? 2. Are some images simply more difficult to segment correctly than others? If so, what are the qualities of these images that cause the segmentation algorithms to perform poorly? 3. Also feel free to point out or discuss any other interesting observations that you made.

Write your analysis in the cell below.

**Your answer here:**

While we could not test out hierarchical agglomerative clustering, since the algorithm doesn't scale up well with regard to large number of data points, we could test the kmeans algorithm and we varied the feature space to only color and a combination of color and position. The accuracy obtained on using either of the feature spaces is very similar.

Some images are difficult to segment because the colors of the foreground and the background are very similar and the distance between the feature vector representing these points are very less, thus making them the part of the same cluster, whereas the expectation is that they would be parts of different clusters.

Whenever we encounter a scenario as mentioned above, it is imperative to incorporate the position of the pixel into the feature vector, as the distance between the corresponding vectors in the feature space of the pixel is not solely determined by their color, thus reducing the chances that pixels belonging to the background and the foreground will be put under the same cluster. But even this approach will fail if the foreground and background share a border and have the same color composition.