

## Problem 16.1

Since we are using double hashing, the final hash function is.

$$h(k, i) = (h_1(k) + i h_2(k)) \bmod 5.$$

Now, inserting 3

$$\begin{aligned} h(3, 0) &= (h_1(3) + 0 h_2(3)) \bmod 5. \\ &= h_1(3) \bmod 5 \\ &= (3 \bmod 5) \bmod 5 \\ &= 3 \bmod 5 \\ &= 3. \end{aligned}$$

So, the hash table is.

| Index | value. |
|-------|--------|
| 0     |        |
| 1     |        |
| 2     |        |
| 3     | 3.     |
| 4.    |        |

Inserting 10.

$$\begin{aligned}
 h(10, 0) &= (h_1(10) + 0 \times h_2(10)) \bmod 5. \\
 &= (h_1(10)) \bmod 5. \\
 &= (10 \bmod 5) \bmod 5 \\
 &= 0 \bmod 5 \\
 &= 0
 \end{aligned}$$

So, the hash table is.

| Index | value. |
|-------|--------|
| 0     | 10     |
| 1     |        |
| 2     |        |
| 3     | 3.     |
| 4.    |        |

Inserting 2.

$$\begin{aligned}
 h(2, 0) &= (h_1(2) + 0 \times h_2(2)) \bmod 5 \\
 &= h_1(2) \bmod 5 \\
 &= (2 \bmod 5) \bmod 5 \\
 &= 2 \bmod 5 \\
 &= 2
 \end{aligned}$$

So, the hash table is

| Index | value. |
|-------|--------|
| 0     | 10     |
| 1     |        |
| 2     | 2      |
| 3     | 3.     |
| 4.    |        |

Now, inserting 4.

$$\begin{aligned}
 h(u, 0) &= (h_1(u) + 0 \times h_2(u)) \bmod 5 \\
 &= (4 \bmod 5 + 0) \bmod 5 \\
 &= (4 \bmod 5) \bmod 5 \\
 &= 4 \bmod 5 \\
 &= 4.
 \end{aligned}$$

So, the hash table is

| Index | value. |
|-------|--------|
| 0     | 10     |
| 1     |        |
| 2     | 2      |
| 3     | 3.     |
| 4.    | 4      |

Ans (10.2.)

Ans a)

we have the following time table of activities arranged according to their finishing times

| Activity number | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|-----------------|---|---|---|---|---|---|---|---|---|---|----|----|----|----|----|
| 1               | X | X | X | X |   |   |   |   |   |   |    |    |    |    |    |
| 2               |   |   |   | X | X |   |   |   |   |   |    |    |    |    |    |
| 3               |   |   |   | X | X | X | X | X |   |   |    |    |    |    |    |
| 4               |   |   |   |   | X | X | X | X |   |   |    |    |    |    |    |
| 5               |   |   |   |   |   |   | X | X | X |   |    |    |    |    |    |
| 6               |   |   | X | X | X | X | X | X | X |   |    |    |    |    |    |
| 7               |   |   |   |   |   |   |   |   | X | X | X  | X  |    |    |    |
| 8               |   |   |   |   |   |   |   | X | X | X | X  | X  | X  |    |    |
| 9               |   |   |   |   |   |   |   |   |   |   |    | X  | X  |    |    |
| 10              |   |   |   |   |   |   |   |   |   |   |    |    | X  | X  | X  |
| 11              |   |   |   |   |   |   |   | X | X | X | X  | X  | X  | X  | X  |

So, we know that the optimal solution of the activity selector problem is.

|                                   |   |   |   |   |   |   |   |   |   |   |    |    |    |    |    |
|-----------------------------------|---|---|---|---|---|---|---|---|---|---|----|----|----|----|----|
| Time slot                         | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| Activity that fills the time slot | 1 | 1 | 1 | 1 | 4 | 4 | 4 | 4 | 7 | 7 | 7  | 7  | 10 | 10 | 10 |

Now, sorting the activities according to their duration in ascending order we get:

|                 |   |   |   |   |   |   |   |   |   |   |    |    |    |    |    |
|-----------------|---|---|---|---|---|---|---|---|---|---|----|----|----|----|----|
| Activity number | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 2               |   |   |   | X | X |   |   |   |   |   |    |    |    |    |    |
| 9               |   |   |   |   |   |   |   |   |   |   |    | X  | X  |    |    |
| 5               |   |   |   |   |   |   | X | X | X |   |    |    |    |    |    |
| 10              |   |   |   |   |   |   |   |   |   |   |    |    | X  | X  | X  |
| 1               | X | X | X | X |   |   |   |   |   |   |    |    |    |    |    |
| 4               |   |   |   |   | X | X | X | X |   |   |    |    |    |    |    |
| 7               |   |   |   |   |   |   |   |   | X | X | X  | X  |    |    |    |
| 3               |   |   |   | X | X | X | X | X |   |   |    |    |    |    |    |
| 8               |   |   |   |   |   |   |   | X | X | X | X  | X  | X  |    |    |
| 6               |   |   | X | X | X | X | X | X | X |   |    |    |    |    |    |
| 11              |   |   |   |   |   |   |   | X | X | X | X  | X  | X  | X  | X  |

Now, the greedy solution generated by the above permutation of activities is

| Time slot                         |  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|-----------------------------------|--|---|---|---|---|---|---|---|---|---|---|----|----|----|----|----|
| Activity that fills the time slot |  |   |   |   | 2 | 2 |   | 5 | 5 | 5 |   |    | 9  | 9  |    |    |

The above solution just consists of 3 mutually compatible activities and that is lower than the four activities in the optimal solution.

Due to this counterexample, we can conclude that the greedy approach for the activity selector problem when the activities are sorted in order of their duration is not always effective.