Rubik's cubes and permutation group theory

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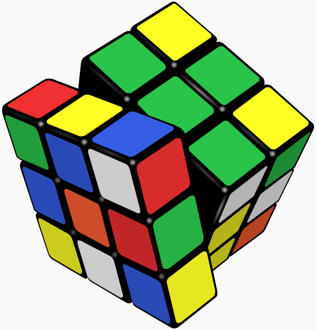
Honours presentation



Contents

Questions about Rubik's cube

- How can we represent *move sequences* and *states* of a cube?
- How can we tell *how many* states a Rubik's cube can take?
- If we repeat a move, do we eventually *get back to the start*?
- If a Rubik's cube is *restickered*, is it *solvable*?
- How can we use maths to *solve* a Rubik's cube?

One answer: using permutations and *computational group theory*!

(J. A. Paulos, Innumeracy)

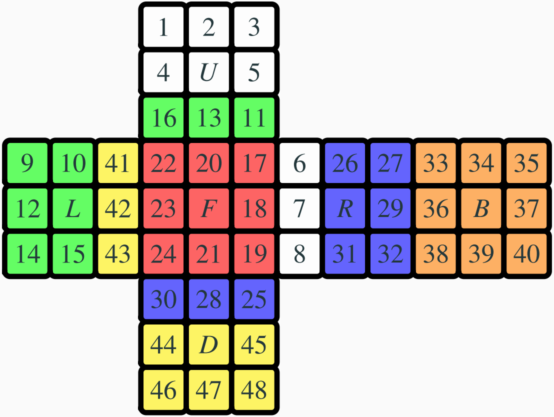
> *Ideal Toy Company stated on the package of the original Rubik cube that there were more than three billion possible states the cube could attain. It's analogous to McDonald's proudly announcing that they've sold more than 120 hamburgers.*

Some basic group theory

The Rubik's group

Representing the cube and its moves

From *solved state* 1, consider F which rotates front face clockwise:



Under F: $[17, 18, 19, 20, 21, 22, 23, 24] \mapsto [22, 20, 17, 23, 18, 24, 21, 19]$ (red), etc. So

$$F = (17, 22, 24, 19)(18, 20, 23, 21)(6, 16, 43, 25)(7, 13, 42, 28)(8, 11, 41, 30) \in \mathrm{Sym}(48).$$

Analysing the Rubik's group

Concluding remarks

References and resources

- Analyzing Rubik's cube with GAP: `https://www.gap-system.org/Doc/Examples/rubik.html`
- J.A. Paulos — *Innumeracy* (book)
- Holt — *Handbook of Computational Group Theory* (textbook)
- Dixon and Mortimer — *Permutation Groups* (textbook)
- Orders of elements in Rubik's group (1260 largest, 13 smallest without, 11 rarest, 60 most common, median 67.3, 73 options): `https://www.jaapsch.net/puzzles/cubic3.htm#p34`
- Thistlethwaite's 52 move algorithm (using group theory): `https://www.jaapsch.net/puzzles/thistle.htm`