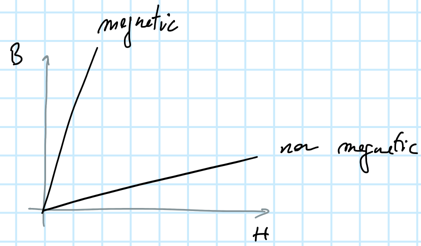
28/10 Bianchi

giovedì 3 dicembre 2020 12:57

MAGNETIC MATERIALS:



- B: flux density (induzione) [T]
- H: magnetic field [A/m]

$$B = \mu H \quad , \quad \mu : \text{permeability } [H/m]$$

$$\mu_0 = 4\pi \cdot 10^{-7} \; [H/m]$$

→ Saturation of the flux density when the atoms are all oriented they do not contribute anymore to the permeability $\mu \rightarrow \mu_0$

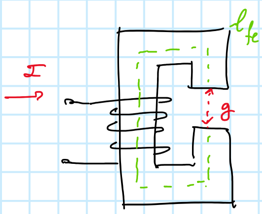
AMPERE'S LAW

$$\oint_l H \, dl = I$$

The integral of the magnetic field in a closed line gives the current

$$\downarrow$$

$$\Sigma H \Delta l = I$$



N turns → $H_{fe} \cdot l_{fe} + H_g \cdot g = N \cdot I$

$$\begin{cases} B_{fe} = \mu_{fe} H_{fe} \\ B_g = \mu_0 H_g \end{cases}$$

$$\frac{B_{fe}}{\mu_{fe}} l_{fe} + \frac{B_g}{\mu_0} l_g = NI \quad \circledast$$

GAUSS LAW

$$\oint B \, dS = 0 \longrightarrow B_{fe} \cdot S_{fe} = B_g \cdot S_g$$

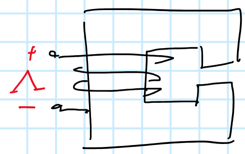
$$S_{fe} \cong S_g$$

$$\boxed{B_{fe} \simeq B_g}$$

Ⓐ $B\left(\frac{l_{fe}}{\mu_{fe}} + \frac{g}{\mu_0}\right) = NI$ $\qquad g \ll l_{fe}$, $\mu_0 \ggg \mu_{fe}$ $\longrightarrow$ $\boxed{B \simeq \frac{NI\mu_0}{g}}$

LOW AIRGAP TO INCREASE MAXIMUM FLUX

$\boxed{\Phi = B \cdot S \quad [Wb]}$ FLUX $\longrightarrow$ $\boxed{\Lambda = N \cdot \Phi}$ LINKAGE FLUX



Flux linked by the coil

RELUCTANCE

$$\mathcal{R} = \frac{NI}{\Phi} = \frac{MMF}{\Phi} \quad \left[\frac{A}{Wb}\right]$$

$$I \longrightarrow mmf \longrightarrow H = \frac{NI}{g} \longrightarrow B = \mu \cdot H \longrightarrow \Phi = B \cdot S \longrightarrow \Lambda = N\Phi$$

↓ (t)

$$i(t) \longrightarrow MMF = N \cdot i(t) \longrightarrow h(t) \longrightarrow b(t) \longrightarrow \varphi(t) \longrightarrow \lambda(t) \longrightarrow e(t)$$

electromotive force EMF [V]

FARADAY'S LAW

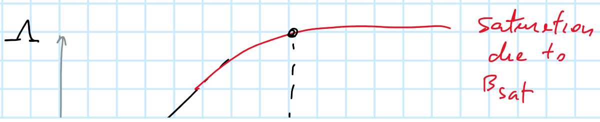
$$\boxed{e(t) = -\frac{d\lambda(t)}{dt}}$$

generator : minus sign
motor : plus sign

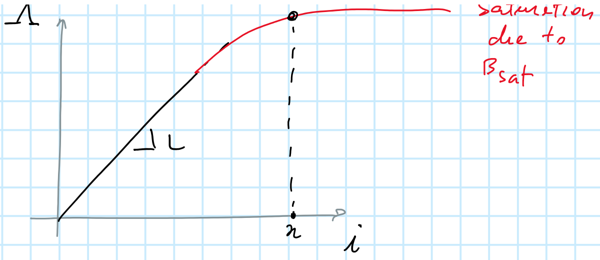
INDUCTANCE

$$\boxed{L = \frac{\Lambda}{I} \quad \left[\frac{V \cdot s}{A}\right] = [H]}$$

$$\Lambda = N\Phi = NBS = N\mu_0 HS$$
$$= N\mu_0 S \cdot \frac{NI}{g} = \underbrace{\frac{\mu_0 N^2 S}{g}}_{L} \cdot I$$



How to define the inductance



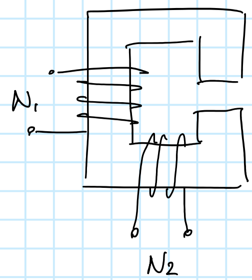
How to define the inductance in a curved point :

$$L_{APPARENT} = \frac{\Lambda(x)}{I(x)}$$

$$L_{DIFFERENTIAL} = \frac{\partial \Lambda}{\partial i} \quad \text{(INCREMENTAL)}$$

MUTUAL INDUCTANCE

$$M = L_M = \frac{\Lambda_A}{I_B}$$



$$M_{12} = \frac{\Lambda_2}{I_1} = \frac{\Lambda_1}{I_2}$$

MAGNETIC ENERGY

$$W_M = \frac{1}{2} B \cdot H \cdot \text{volume} \quad [J]$$

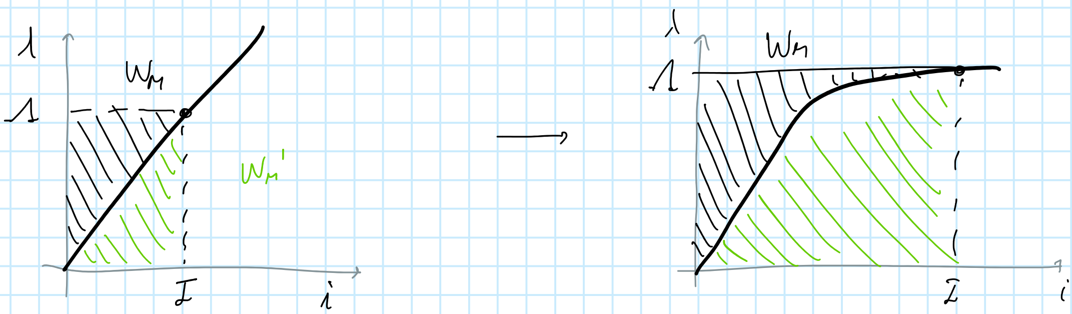
$$= \cancel{\frac{NI}{\delta}} \cdot \frac{\Lambda}{\cancel{N \cdot S}} \cdot \cancel{S \cdot \delta}\,\frac{1}{2}$$

$$= \frac{1}{2} \Lambda \cdot I$$

volume $= S \cdot \delta$

$H = \frac{NI}{\delta}$

$B = \frac{\Phi}{S} = \frac{\Lambda}{N \cdot S}$

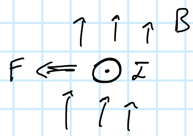


$$W_M = \int_0^{\Lambda} i \, d\lambda$$

W_M' MAGNETIC COENERGY

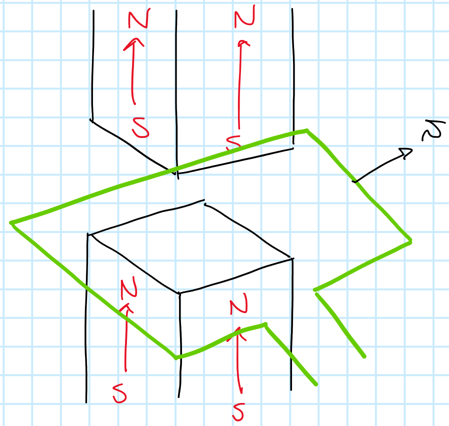
FORCE

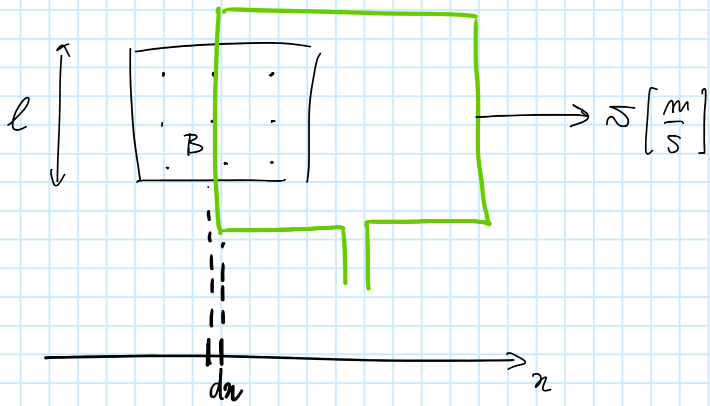
$$F_x = \frac{dW_M}{dx} \quad [N]$$



LORENTZ

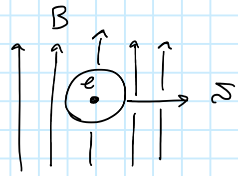
$$F = I \cdot B \cdot \ell$$

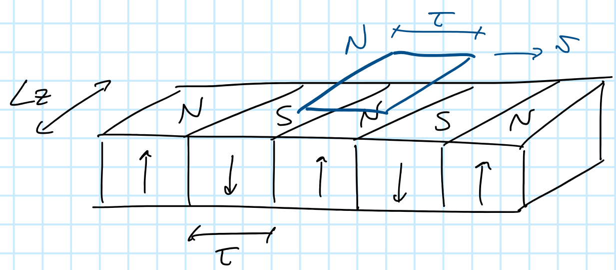


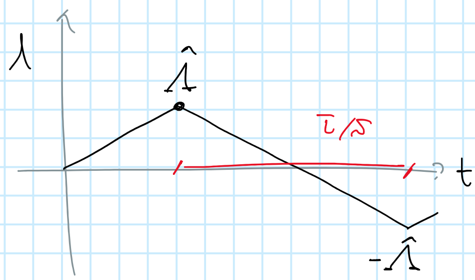


$$dt \longrightarrow \quad d\lambda = d\Phi = -B\ell\, dx$$

$$\boxed{e = -\frac{d\lambda}{dt} = vB\ell}$$



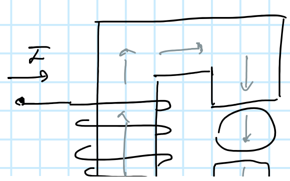


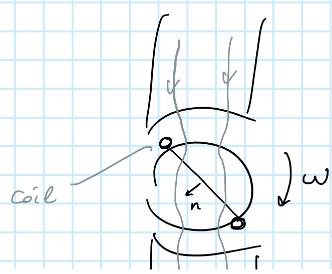


$$\hat{\Lambda} = NBS = NB\tau L_z$$

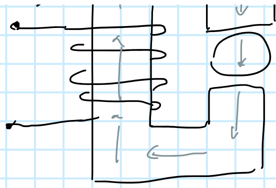
- $E_{tot} = \left|\frac{d\lambda}{dt}\right| = \frac{2\hat{\Lambda}}{\tau/v} = \frac{2v\hat{\Lambda}}{\tau} = 2vNBL_z$
- $E_{1turn} = \frac{E}{2N} = vBL_z$

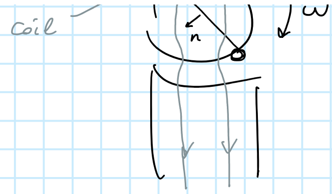
ROTATING COIL



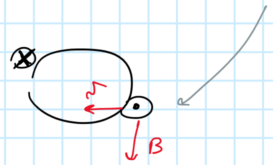


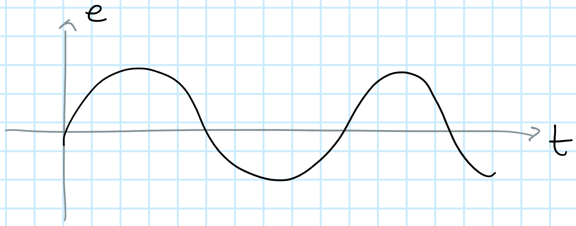
- $\lambda(t)$ decreases by rotation
- $e(t) = -\frac{d\lambda}{dt}$ is positive





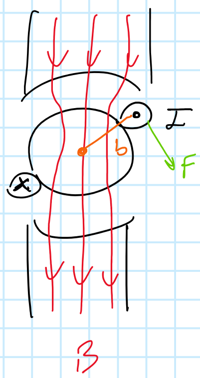
• $e(t) = -\frac{d\lambda}{dt}$ is positive



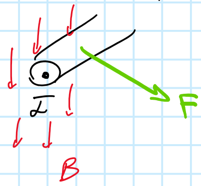


Sinusoidal
alternating
voltage

$\lambda(t) \longrightarrow e(t) \longrightarrow$ if closed circuit $\longrightarrow i(t)$



$(F = IBl)$



Torque = Force × braccio

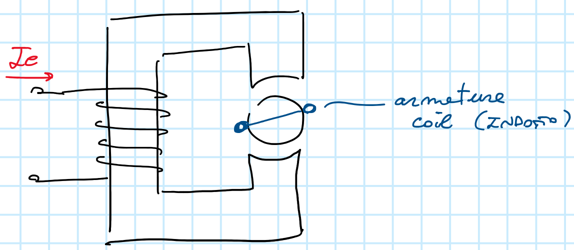
2 forces on 2 conductors

$\Rightarrow \boxed{T = 2F \cdot b}$

DC MOTOR

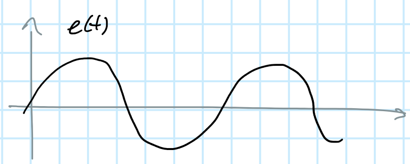
Ferromagnetic material $\longrightarrow$ High $B = \mu H$

Airgap is minimum $\longrightarrow$ just to avoid friction



If the rotor rotates at ω_m, emf e is induced in the coil :

$e(t) \propto \lambda(t), \omega_m$



$\boxed{E = K \Phi \omega_m}$

MOTOR EQUATIONS

$T = K \Phi I$ ... a: armature

EQUATIONS

$$\boxed{T_m = K \Phi I_a}$$

a: armature

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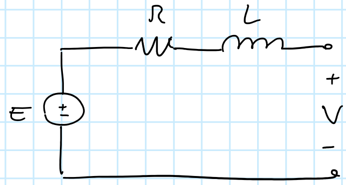
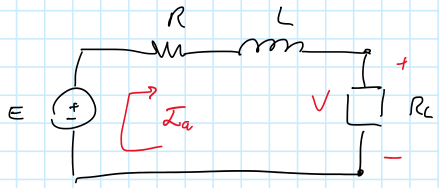
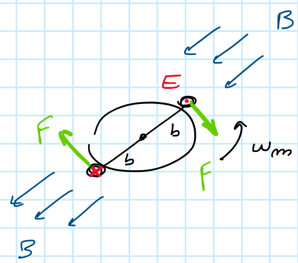
$E = K\Phi\omega_m$



$I = 0,\ V = E$

GENERATORE

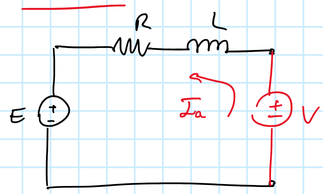




La corrente avrà la stessa direzione della fem

La forza di Lorentz si oppone al movimento rotatorio --> nasce una coppia elettromeccanica

MOTORE



$V > E$ → forzo la corrente dentro al motore vincendo la fem

$E = K\Phi\omega_m \longrightarrow T_{em} = K\Phi I_a$

BILANCIO POTENZE

$$P_e = P_m$$

$$E \cdot I_a = T_{em}\,\omega_m$$

$$(K\cdot\Phi\cdot\omega_m)\cdot I_a = (K\cdot\Phi\cdot I_a)\cdot\omega_m$$

KIRKHOFF

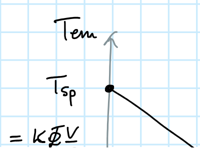
$$V = E + L\frac{dI_a}{dt} + RI_a$$

$$= E + RI_a$$

EQUAZIONI MOTORE DC

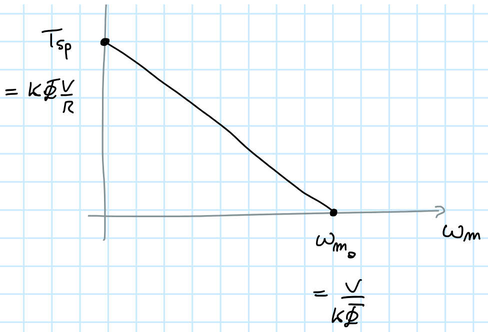
$$\boxed{\begin{aligned} E &= K\cdot\Phi\cdot\omega_m \\ T &= K\cdot\Phi\cdot I_a \\ V &= E + R\cdot I_a \end{aligned}}$$

CARATTERISTICA MECCANICA (A REGIME)



• B = cost (magnete permanente / I_e cost.)

$K\Phi$ = cost



• Φ = cost (magnete permanente / ec cost.)

$K\Phi$ = cost

• $V = V_N$ cost

$$T = K\Phi I = K\Phi\left(\frac{V-E}{R}\right) = K\Phi\left(\frac{V - K\Phi\omega_m}{R}\right) =$$

$$= K\Phi\frac{V}{R} - K^2\Phi^2\frac{\omega_m}{R}$$

EXERCISE 1

PM excitation

$R_a = 1\Omega$

$V_a = 100V$

$I_a = 15\,A$

$n = 3000$ rpm

T_{em} ?

Power ?

$$\boxed{\begin{aligned} E &= K\Phi\omega_m \\ T &= K\Phi I_a \\ V &= E + RI_a \end{aligned}}$$

$E = V - RI_a = 100 - 1\cdot 15 = 85\,V$

$\omega_m = \frac{2\pi}{60}3000 = 314$ rad/s

$K\Phi = \frac{E}{\omega_m} = 0{,}27$ Vs

• Torque $= K\Phi \cdot I_a = 0{,}27\cdot 15 = 4.06$ Nm

• Power :

– input power $\quad V\cdot I_a = 1500\,W$

– joule loss $\quad R\cdot I_a^2 = 15^2\,W = 225\,W$

– electromech. $\quad \left.\begin{aligned} T_{em}\cdot\omega_m &= E\cdot I_a \\ 4.06\cdot 314 &= 85\cdot 15 \end{aligned}\right\} = 1275\,W$

EXERCISE 2

PM DC motor

$V = 200V$

$I_a = 24.4A$

$R_a = 0.5\,\Omega$

$P_{out} = 4500W$

$n = 6000$ rpm

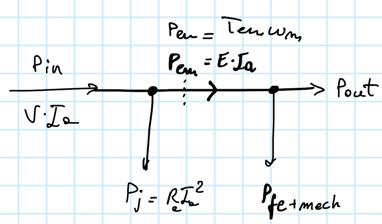
Torque ?

Joule loss ?

iron + mech losses ?

η ?

$K\Phi$?



$\omega_m = \frac{2\pi}{60} \cdot n = 628\ rad/s$

$T_{sh} = \frac{P_{out}}{\omega_m} = 7.16\ Nm$

$P_{in} = V_e \cdot I_e = 4880\ W$

$\eta = \frac{P_{out}}{P_{in}} = 92.2\ \%$

$P_J = R_e I_e^2 = 298 W$

$P_{fe+mech} = \underbrace{P_{in} - P_J}_{P_{em}} - P_{out} = 4880 - 298 - 4500 = 82\ W$

$T_m = \frac{P_{em}}{\omega_m} = 7.3\ Nm$

$K\Phi = \frac{T_m}{I_e} = 0.299\ Vs \longrightarrow E = K\Phi\,\omega_m = 187.8\ V$

EXERCISE 3

PM DC motor

$R_e = 0.2\ \Omega$

$P_N = 4000\ W$

$n_N = 3000\ rpm$

$V_N = 100 V$

$I_N = 45\ A$

T_N ?

η_N ?

$P_{fe+mech}$? $\propto \omega_m^2$

$② \rightarrow$

$T_L = 10\ Nm$

$n_L = 2000\ rpm$

V_e ?

$\omega_N = 314\ rad/s$

$T_N = \frac{P_N}{\omega_N} = 12.74\ Nm$

$P_{in_N} = V_N I_N = 4500 W$

$\eta_N = 88.9\ \%$

$P_J = R_e I_N^2 = 405\ W$

$P_{fe} = P_{in} - P_J - P_{out} = 95 W$

$T_{em_N} = \frac{P_{in} - P_J}{\omega_N} = 13 Nm$

$K\Phi = \frac{T_{em_N}}{I_N} = \frac{13}{45} = 0.29\ Vs$

$② \rightarrow$

$\omega_m = \frac{2\pi}{60} 2000 = 210\ rad/s$

$P_{fe}' = P_{fe} \left(\frac{\omega_m}{\omega_N}\right)^2 = 42 W$

$T_{fe} = \frac{P_{fe}}{\omega_t} = 0.2\ Nm$

$T_{em} = T_L + T_{fe} = 10.2\ Nm$

$I_e = \frac{T_{em}}{K\Phi} = 35.2\ A$

$V = E + R I_e = K\Phi\,\omega_m + R I_e = 0.29 \cdot 210 + 0.2 \cdot 35.2 = 66.9\ V$

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EXERCISE

DC motor independent excitation

$R_e = 1.5\,\Omega$

$V_e = 75\,V$

$\omega_{m0} = 400$ rad/s (at max flux)

- Requirement:

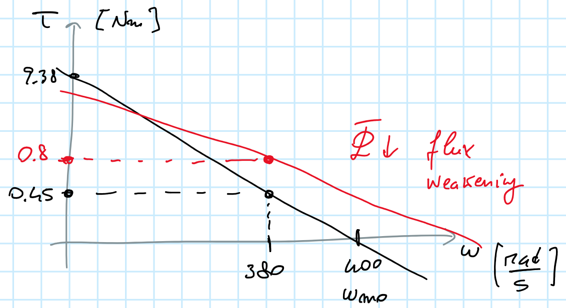
$T_L = 0.8\,Nm$

$\omega_m = 380$ rad/s

V_e ? I_e ? Power ?

$$\omega_{m0} = \frac{V_e}{K\Phi} \rightarrow K\Phi = \frac{75}{400} = 0.188\ Vs \quad \text{(NO LOAD)}$$

$$T_{sp} = K\Phi \frac{V_e}{R_e} = 9.38\ Nm \quad \text{(START UP)}$$



- at ω_m

$E = K\Phi\,\omega_m = 71.4\,V$

$I_e = \dfrac{V-E}{R_e} = 2.37\,A$

$T_{em} = \underline{K\Phi \cdot I_e} = 0.45\,Nm < T_L$

$$T_L = T_{em} = K\Phi \frac{V_e}{R_e} - (K\Phi)^2 \frac{\omega_m}{R_e}$$

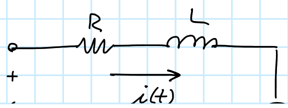
$$\downarrow$$

$K\Phi = \underline{\underline{0.179}} \vee 0.0176$

↓ low current

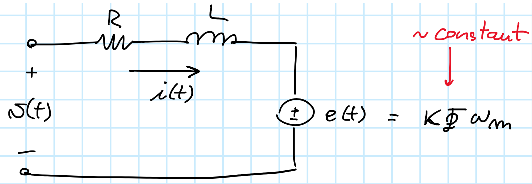
- $I_e = \dfrac{0.8}{0.179} = 4.47\,A$
- $V_e = E + R_e I_e =$
- $V_e I_e = E I_e + R_e I_e^2$

DYNAMICS



~ constant

- $e = K\Phi\,\omega_m$
- $T_m = K\Phi\, i$



$e(t) = K\Phi\,\omega_m$

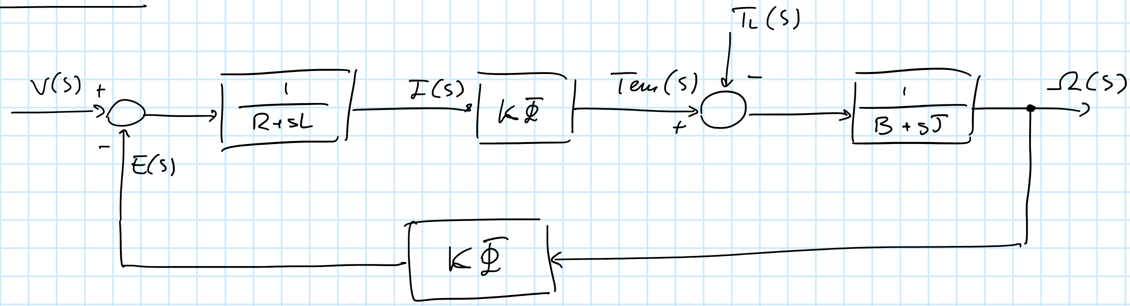
- $e = K\Phi\,\omega_m$
- $T_m = K\Phi\, i$
- $v = e + Ri + L\dfrac{di}{dt}$
- $\tau_{em} = \tau_L + \underbrace{B\,\omega_m}_{\text{Losses}} + \underbrace{J\dfrac{d\omega}{dt}}_{\text{Inertia}}$

LAPLACE

$$\begin{cases} V(s) = E(s) + R\,I(s) + sL\,I(s) \\ E(s) = K\Phi\,\Omega(s) \\ T_{em}(s) = K\Phi\, I(s) = T_L + B\,\Omega(s) + sJ\,\Omega(s) \end{cases}$$

$$\Omega(s) = \frac{T_{em}(s) - T_L(s)}{B + sJ}$$

BLOCK DIAGRAM



- Transfer function between Voltage and Speed :

$$\frac{\Omega_m(s)}{V(s)} = \frac{\dfrac{1}{R+sL}\cdot K\Phi \cdot \dfrac{1}{B+sJ}}{1 + \dfrac{1}{R+sL}\cdot (K\Phi)^2 \cdot \dfrac{1}{B+sJ}} = \frac{K\Phi}{(R+sL)(B+sJ) + (K\Phi)^2}$$

- B is negligible

$$\frac{\Omega_m(s)}{V(s)} = \frac{\dfrac{1}{K\Phi}}{s^2\dfrac{LJ}{(K\Phi)^2} + s\dfrac{RJ}{(K\Phi)^2} + 1}$$

At steady state $(s=0)$

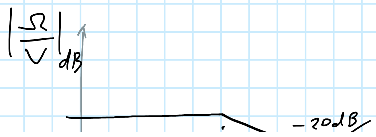
- $\Omega_{m_0} = \dfrac{V}{K\Phi}$ ✓

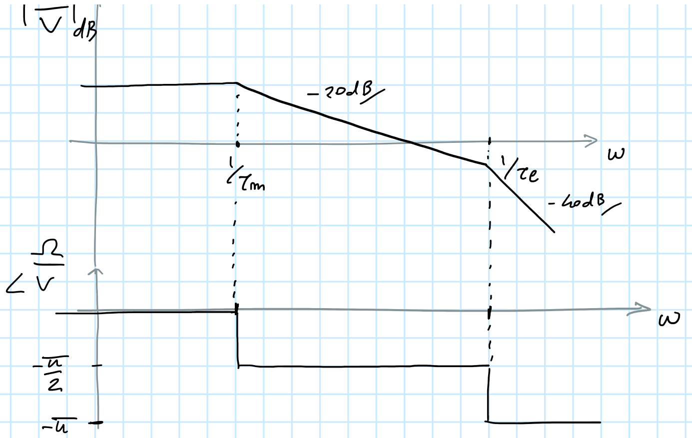
TIME CONSTANTS

$\tau_m = \dfrac{RJ}{(K\Phi)^2}$

$\tau_e = \dfrac{L}{R}$

$$\longrightarrow \quad \frac{\Omega_m}{V} = \frac{1}{1 + s\tau_m + s^2\tau_m\tau_e} \simeq \frac{1}{(s\tau_e + 1)(s\tau_m + 1)} \quad \text{if } \tau_m \gg \tau_e$$





$\Omega_m(s) \xrightarrow{t} ?$

$$\Omega_m(s) = \frac{A}{s} + \frac{B}{s+\frac{1}{\tau_1}} + \frac{C}{s+\frac{1}{\tau_2}}$$

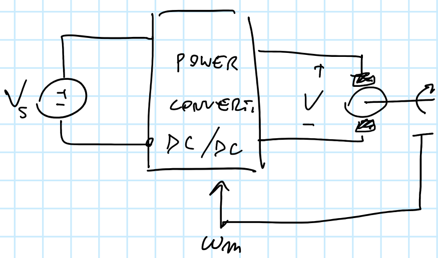
$$= \frac{A\left(s+\frac{1}{\tau_1}\right)\left(s+\frac{1}{\tau_2}\right) + B\left(s+\frac{1}{\tau_2}\right)s + C\left(s+\frac{1}{\tau_1}\right)s}{s\left(s+\frac{1}{\tau_1}\right)\left(s+\frac{1}{\tau_2}\right)}$$

$$\begin{cases} A = \lim_{s\to 0}\left(\Omega_m(s)\cdot s\right) \\ B = \lim_{s\to -\frac{1}{\tau_1}}\left[\Omega_m(s)\cdot\left(s+\frac{1}{\tau_1}\right)\right] \\ C = \lim_{s\to -\frac{1}{\tau_2}}\left[\Omega_m(s)\cdot\left(s+\frac{1}{\tau_2}\right)\right] \end{cases}$$

$$\boxed{\omega_m(t) = A + Be^{-t/\tau_1} + Ce^{-t/\tau_2}}$$

steady state after ~ 3 time constants

POWER CONVERTER

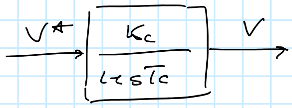


From before $\frac{\Omega(s)}{V(s)}$: (numeri dati)

$$V \rightarrow \boxed{\frac{s}{\left(1+\frac{s}{100}\right)\left(1+\frac{s}{2}\right)}} \rightarrow \Omega$$

Power converter: $V^* \rightarrow \boxed{K_c} \rightarrow V$

Power converter:

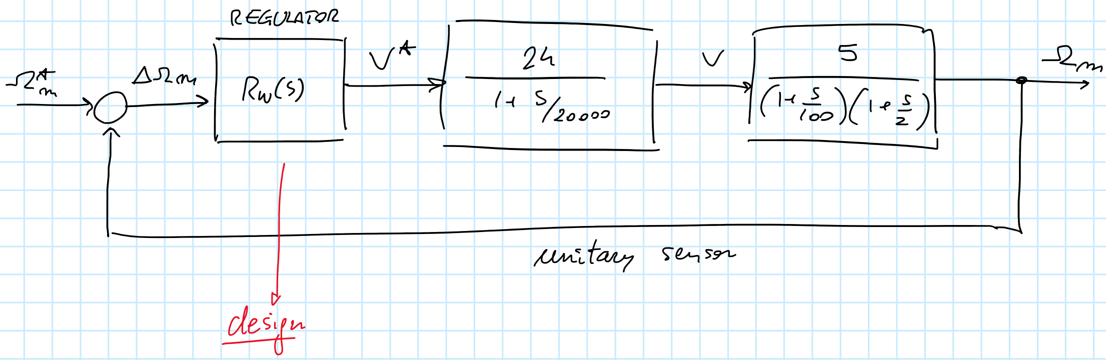


$K_c = 24$

$V_N = 120\,V$

$V_{REF} = V^* = 5\,V$

$f_{sw} = 10\,kHz \rightarrow T_{sw} = 0.1\,ms \rightarrow T_c = \frac{T_{sw}}{2}$



05/11 Bianchi

sabato 5 dicembre 2020 16:08

$\varphi_m = 60° \longrightarrow$ PROPORZIONALE

$$\angle G(j\omega_A) = -180 + \varphi_m = -120°$$

$$= -\operatorname{atan}\left(\frac{\omega_A}{0.5}\right) - \operatorname{atan}\left(\frac{\omega_A}{100}\right) - \cancel{\operatorname{atan}\left(\frac{\omega_A}{20000}\right)} = -120°$$

$$\Rightarrow \omega_A = 60 \text{ rad/s}$$

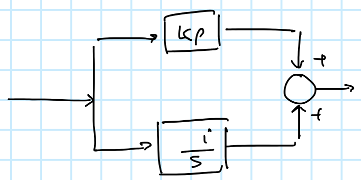
- K_p : $|G(j60)| = 1$

$$\left| K_p \cdot \frac{120}{\left(1+j\frac{60}{0.5}\right)\left(1+j\frac{60}{100}\right)\left(1+j\frac{60}{20k}\right)} \right| = 1$$

$$\Rightarrow K_p = \dots$$

$$\text{errore} = e(\infty) = \frac{1}{|G(0)|+1} \neq 0$$

PI



$$R_\omega(s) = K_p + \frac{K_i}{s} = \frac{sK_p + K_i}{s} = K_I \frac{\left(1+s\frac{K_p}{K_i}\right)}{s}$$

$$\frac{K_p}{K_i} = T_i \rightarrow \boxed{R_\omega(s) = K_I \cdot \frac{1+sT_i}{s}}$$

EXERCISE

MOTOR

$K\Phi = 0.1$ Vs

$R = 2.5\ \Omega$

$L = 10$ mH

$B \simeq 0$

$J = 10^{-3}$ kg m^2

POWER

$K_c = 8$

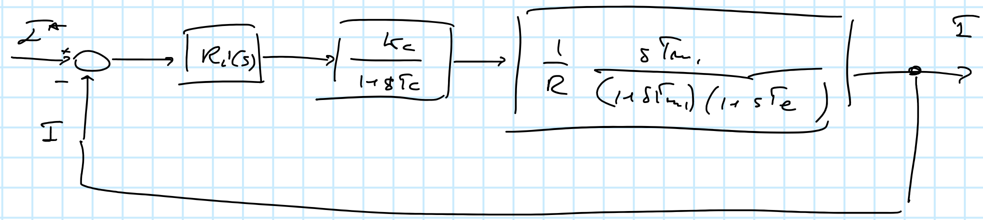
$f_{sw} = 2.5$ kHz

REQUIREMENT

Speed control

- $\omega_B \geq 40$ rad/s
- $\varphi_m \geq 60°$

Current control :



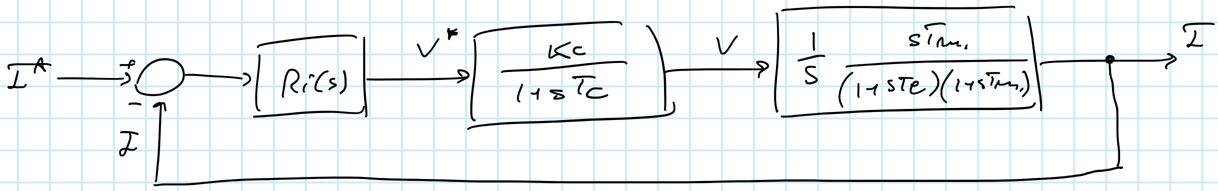
$$T_c = \frac{T_{sw}}{2} = \frac{1}{2}\,\frac{1}{2.5\,kHz} = 0.0002\ s$$

$$T_e = \frac{L}{R} = 0.004\ s$$

$$T_{m1} = \frac{RJ}{(k\bar{\Phi})^2} = 0.25\ s$$

06/11

lunedì 7 dicembre 2020 11:53

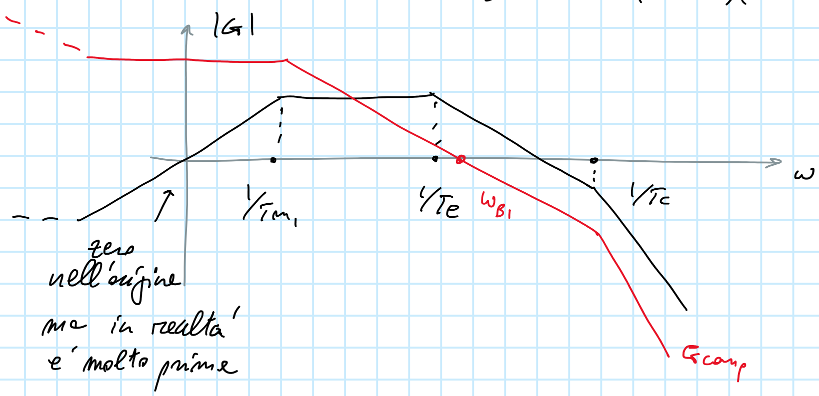


$T_e = 0.04\ s$

$T_{m1} = 0.25\ s$

$T_c = 0.0002\ s$

$$G(s) = R_i(s) \cdot \frac{K_c}{1+sT_c} \cdot \frac{1}{s} \frac{sT_{m1}}{(1+sT_e)(1+sT_{m1})}$$



Regolatore PI :

$$R_{pi}(s) = K_i \cdot \frac{1+sT_{pi}}{s}$$

$$T_{pi} = T_e$$

$$\omega_{Bi} = \frac{1}{10} T_c = 500\ rad/s$$

Impongo : $|G(j\omega_{Bi})| = 1$

$$\left| K_i \cdot \frac{1+j\omega_{Bi}T_{pi}}{j\omega_{Bi}} \cdot \frac{K_c}{1+j\omega_{Bi}T_c} \cdot \frac{1}{R} \frac{j\omega_{Bi}T_{m1}}{(1+j\omega_{Bi}T_e)(1+j\omega_{Bi}T_{m1})} \right| = 1$$

⋮

$$\boxed{K_i = 157}$$

$$\boxed{K_p = T_{pi} \cdot K_i = 0.628}$$

CON LUOGO DELLE RADICI :

$$G(s) = K_i \cdot \frac{\cancel{1+sT_{pi}}}{\cancel{s}} \cdot \frac{K_c}{1+sT_c} \cdot \frac{1}{R} \frac{\cancel{s}T_{m1}}{\cancel{(1+sT_e)}(1+sT_{m1})}$$

$$G(s) = k_i \cdot \frac{\cancel{(1+s\tau_{m_1})}}{\cancel{s}} \cdot \frac{k_c}{1+s\tau_c} \cdot \frac{1}{R}\,\frac{\cancel{s}\tau_{m_1}}{\cancel{(1+s\tau_c)}(1+s\tau_{m_1})}$$

$$= \underbrace{\frac{k_i k_c \tau_{m_1}}{R}}_{k'} \frac{1}{(1+s\tau_{m_1})(1+s\tau_c)}$$

$$= \frac{k'}{(1+s\tau_{m_1})(1+s\tau_c)}$$

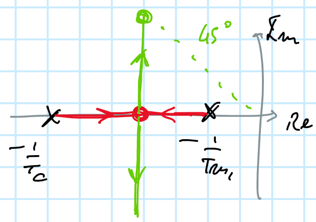
Closed loop :
$$\frac{G}{1+G} = \frac{k'}{(1+s\tau_{m_1})(1+s\tau_c)+k'}$$

$$s^2(\tau_{m_1}\tau_c) + s(\tau_{m_1}+\tau_c) + (1+k') = 0$$

$$s_{1,2} = \frac{-(\tau_{m_1}+\tau_c) \pm \sqrt{(\tau_{m_1}+\tau_c)^2 - 4(\tau_{m_1}\tau_c)(1+k')}}{2\tau_{m_1}\tau_c}$$

<u>Vario k'</u>:

- $k' = 0 \longrightarrow s_{12} = \begin{cases} -\frac{1}{\tau_{m_1}} \\ -\frac{1}{\tau_c} \end{cases}$



- $k'_\Delta \mid \Delta = 0 \longrightarrow (\tau_{m_1}+\tau_c)^2 - 4(\tau_{m_1}\tau_c)(1+k'_\Delta) = 0$

$$k'_\Delta = \frac{(\tau_{m_1}-\tau_c)^2}{4\tau_{m_1}\tau_c} - 1 \longrightarrow s_{12} = -\frac{\tau_{m_1}+\tau_c}{2\tau_{m_1}\tau_c}$$

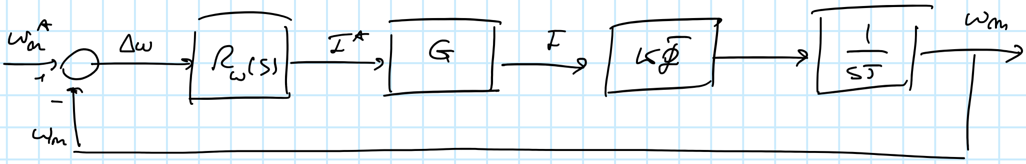
- $k' \mid \Delta < 0 \longrightarrow k' > k'_\Delta \qquad s_{12} = Re \pm j\,Im \qquad \xi = \cos 45° = 0.707$ ✓

con $Re = Im$

$$k' = 625 = \frac{k_i k_c \tau_{m_1}}{R}$$

$$\longrightarrow k_i = \frac{k'}{k_c \tau_{m_1}} \cdot R = \underline{\underline{781}}$$

$$k_p = \tau_{m_1} k_i = \underline{\underline{3.13}}$$



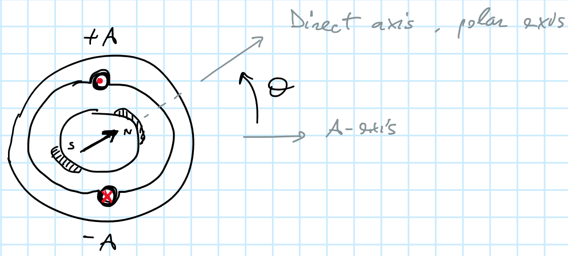
$$\omega_m = \frac{K_\rho K_A}{B} \Delta\omega = \omega^* - \Delta\omega \rightarrow \quad \Delta\omega = \frac{\omega^*}{1 + \frac{K_\rho K_A}{B}}$$

10/11 SPM

lunedì 7 dicembre 2020 16:14

MOTORE SINCRONO A MAGNETE PERMANENTE SUPERFICIALE

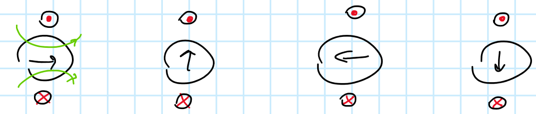
$\boxed{\text{SPM MOTORS}}$



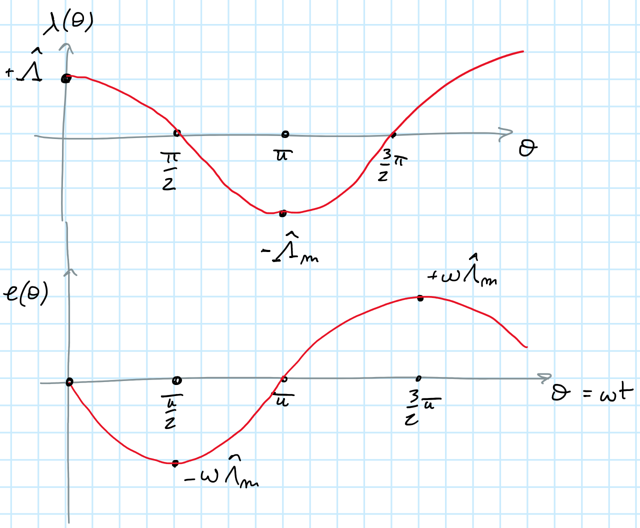
θ: angular position of the rotor axis

$\lambda(\theta)$ flux linkage is dependent to the angle

θ: 0° 90° 180° 270°



maximum of linkage | zero linkage | maximum negative | no linkage



$$\lambda(\theta) = \hat{\Lambda}_m \cos(\theta)$$

$$\omega = \frac{d\theta}{dt} \rightarrow \text{constant}$$

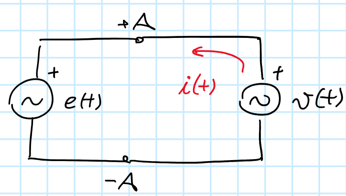
$$\theta = \omega t$$

$$\Rightarrow \boxed{e(t) =} \frac{d\lambda(t)}{dt} = \frac{d\lambda}{d\theta}\frac{d\theta}{dt}$$

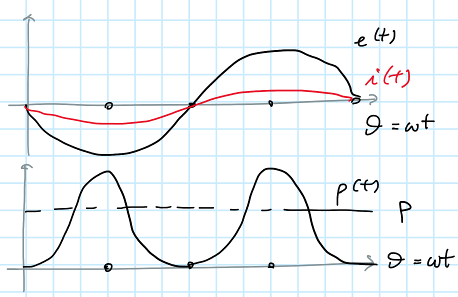
$$\boxed{= -\omega\hat{\Lambda}_m \sin(\theta)}$$

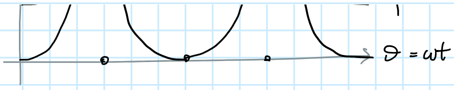
- $E = \omega \Lambda_m$ in DC: $E = K\Phi\omega_m$

EFFECT OF THE CURRENT



We want to impose a current which is in phase with the electromotive force $e(t)$





$i(\theta) = I \sin \theta$

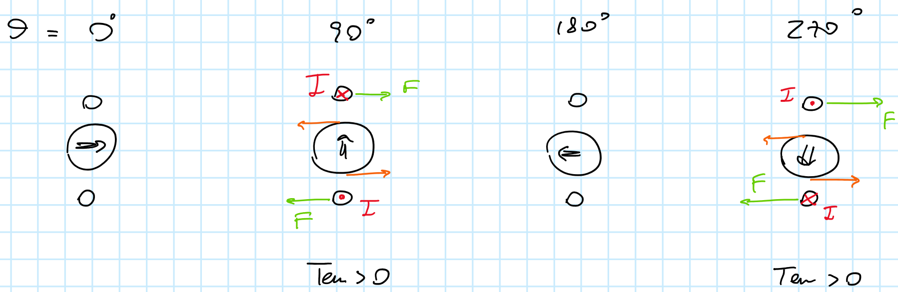
$p(\theta) = i(\theta)\, e(\theta) = \omega \Lambda_m \sin\vartheta \cdot I \sin\vartheta = \omega \Lambda_m I \cdot \sin^2\vartheta$ INSTANTANEOUS POWER

$\cos(\alpha+\beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$

$\sin\alpha \sin\beta = \frac{\cos(\alpha-\beta) - \cos(\alpha+\beta)}{2} \longrightarrow \sin^2\vartheta = \frac{1-\cos(2\vartheta)}{2}$

$\Rightarrow p(\theta) = \omega \Lambda_m I \left(\frac{1-\cos 2\vartheta}{2} \right)$ $P = \frac{\omega \Lambda_m I}{2}$ AVERAGE POWER

$T_{em} = \frac{P}{\omega_m} = \frac{1}{2} \Lambda_m I_m$ AVERAGE TORQUE



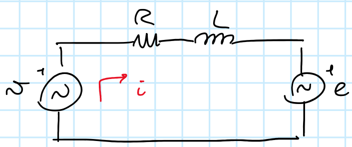
$F = IBL$

Force on the rotor

$T = F \cdot b$

b = radius of the rotor

VOLTAGE EQUATION



$v = e + Ri + L\frac{di}{dt}$

$e = -E \sin(\omega t)$

$i = -I \sin(\omega t)$

$a(t) = A \cos(\omega t + \alpha)$

$\downarrow$

$\bar{A} = |A| e^{j\alpha}$ $\longrightarrow$ $A_{RMS} = \frac{A_{PEAK}}{\sqrt{2}}$

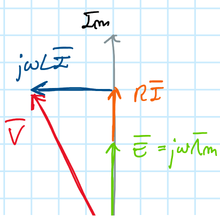
$\frac{da}{dt} \longrightarrow \bar{A} j\omega$

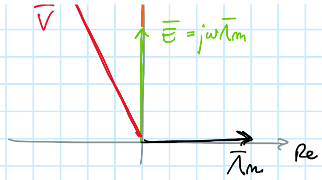
$\Rightarrow v = e + Ri + L\frac{di}{dt}$

$$\boxed{\bar{V} = \bar{E} + R\bar{I} + j\omega L \bar{I}}$$

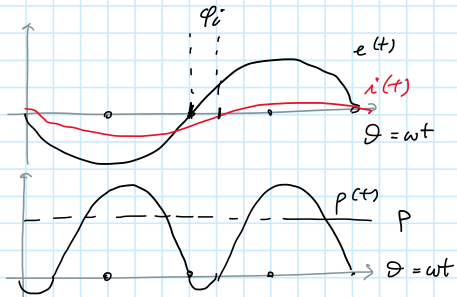
VECTOR FORM

$e = \frac{d\Lambda_m}{dt} \longrightarrow \bar{E} = j\omega \cdot \bar{\Lambda}_m$

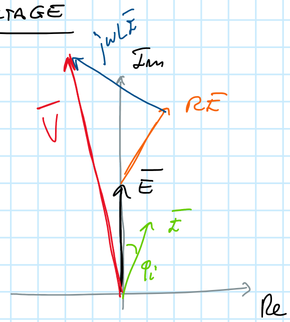




IF THE CURRENT IS NOT IN PHASE WITH THE VOLTAGE

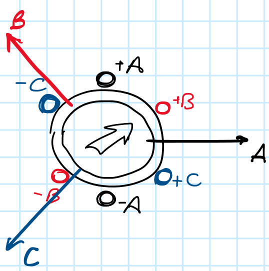


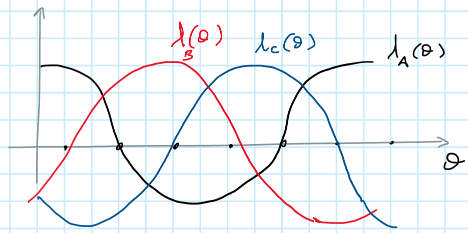
torque is going negative



Torque is not constant even if the current is in phase

How to mesure a constant torque ?

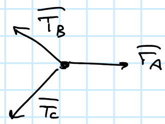




$$T_{em}(\vartheta) = T_{em_{AVG}} + \Delta T_{em}(\vartheta)$$

↪ ZERO

$$\boxed{T_{em} = \frac{3}{2}\Lambda_m I}$$



EXERCISE

SPM motor , steady state operation

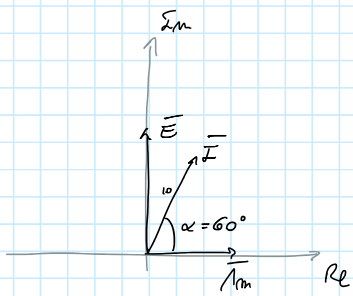
$\Lambda_m = 0.27$ Vs

$R = 0.5\,\Omega$

$L = 17.3$ mH

$\omega = 200$ rad/s

$\bar{I} = 10 \cdot e^{j60°}$ $\begin{cases} I = 10\,A \\ \alpha = 60° \end{cases}$



$$\bar{E} = j\omega \bar{\Lambda}_m = j\,53$$

$$\bar{I} = 5 + j\,8.66 \text{ A}$$

$$\bar{V} = \bar{E} + R\bar{I} + j\omega L\bar{I}$$

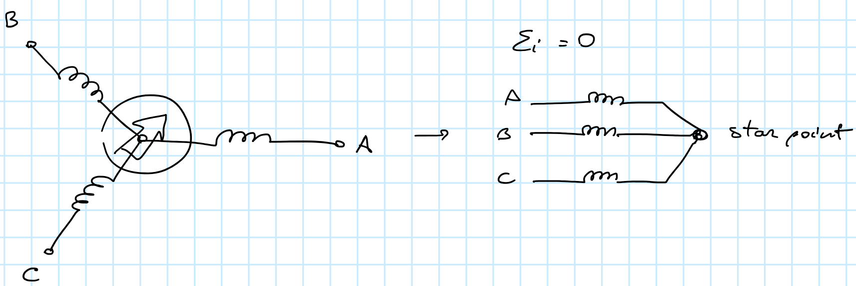
$$\stackrel{!}{=} \bar{E} + (R I_R - \omega L I_I) + j\,(R I_I + \omega L I_R)$$

$$V_R = 0.5 \cdot 5 - 200 \cdot 0.0173 \cdot 8.66 = -27.5 \text{ V}$$

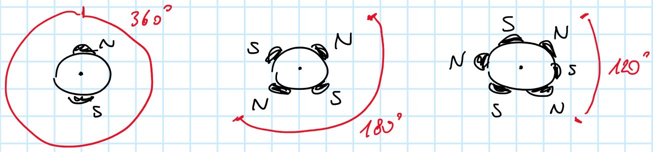
$$V_I = E + 0.5 \cdot 8.66 + 200 \cdot 0.0173 \cdot 5 = 74.6 \text{ V}$$

$$V = \sqrt{V_R^2 + V_I^2} = 80.45 \text{ V}$$

$$\alpha_V = \operatorname{atan} \frac{V_I}{V_R} = 110°$$



Number of poles:



$p = 1$ $\quad \vartheta = \vartheta_m$

$p = 2$ $\quad \vartheta = 2\vartheta_m$

$p = 3$ $\quad \vartheta = 3\vartheta_m$

p: number of pole pairs

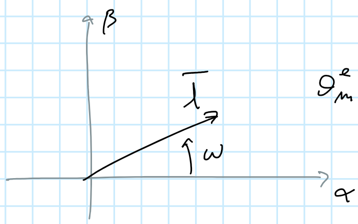
The mechanical angle is always 360°
→ it is a turn of the rotor

$$\boxed{\begin{aligned} \vartheta &= \vartheta_m^e = p\vartheta_m \\ \omega &= \omega_m^e = p\omega_m \end{aligned}}$$

SPACE VECTOR

$$\begin{cases} \lambda_{m_A} = \Lambda_m \cos \vartheta_m^e \\ \lambda_{m_B} = \Lambda_m \cos\left(\vartheta_m^e - \frac{2\pi}{3}\right) \\ \lambda_{m_C} = \Lambda_m \cos\left(\vartheta_m^e - \frac{4\pi}{3}\right) \end{cases} \longrightarrow \bar{\lambda}_{\alpha,\beta} = \Lambda_m e^{j\vartheta_m^e}$$

$$\frac{2}{3}\left[\lambda_A - \lambda_B e^{-j\frac{2}{3}\pi} + \lambda_C e^{j\frac{2}{3}\pi}\right] \cdot 8$$

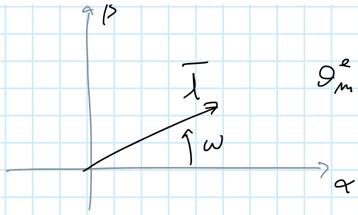


$$\vartheta_m^e = \omega t$$

$$\lambda_\alpha = \Lambda_m \cos \vartheta_m^e$$

$$\lambda_\beta = \Lambda_m \sin \vartheta_m^e$$

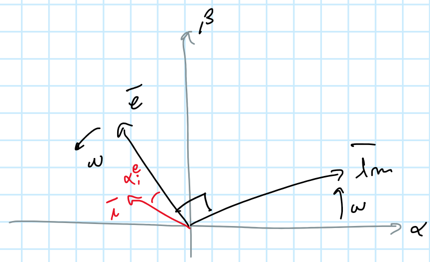
$$\lambda_{m_C} = \Lambda_m \cos\left(\vartheta_m^e - \frac{4}{3}\pi\right)$$



$$\vartheta_m^e = \omega t \qquad \lambda_\alpha = \Lambda_m \cos\vartheta_m^e \qquad \lambda_\beta = \Lambda_m \sin\vartheta_m^e$$

$$\begin{cases} e_A = \dfrac{d\lambda_A}{dt} = -\omega\Lambda_m \sin\vartheta_m^e \\ e_B = \dfrac{d\lambda_B}{dt} = -\omega\Lambda_m \sin\left(\vartheta_m^e - \frac{2}{3}\pi\right) \\ e_C = \dfrac{d\lambda_C}{dt} = -\omega\Lambda_m \sin\left(\vartheta_m^e - \frac{4}{3}\pi\right) \end{cases}$$

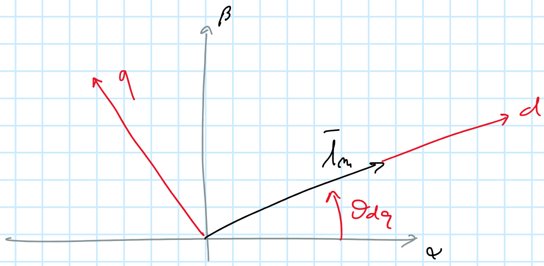
$$\bar{e}_{\alpha\beta} = \omega\Lambda_m e^{j\left(\vartheta_m^e + \frac{\pi}{2}\right)}$$



$$\begin{cases} i_A = I\cos\left(\vartheta_m^e + \alpha_i^e\right) \\ i_B = I\cos\left(\vartheta_m^e - \frac{2\pi}{3} + \alpha_i^e\right) \\ i_C = I\cos\left(\vartheta_m^e - \frac{4}{3}\pi + \alpha_i^e\right) \end{cases}$$

$$i_{\alpha\beta} = I e^{\left(j\vartheta_m^e + \alpha_i^e\right)}$$

- dq - reference system:



$$\bar{\lambda}_{\alpha\beta} \cdot e^{-j\vartheta_{dq}} = \bar{\lambda}_{dq} \qquad \left[\vartheta_{dq} = \vartheta_m^e\right]$$

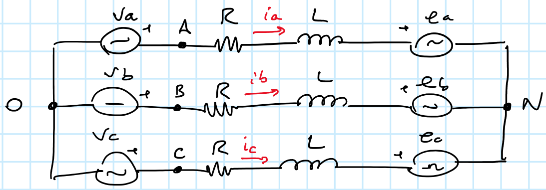
$$= \Lambda_m e^{j\vartheta_m^e} e^{-j\vartheta_{dq}} = \Lambda_m$$

- $\bar{e}_{dq} = \bar{e}_{\alpha\beta} e^{j\left(\vartheta_m^e + \frac{\pi}{2}\right)} e^{-j\vartheta_{dq}} = \bar{e}_{\alpha\beta} e^{j\frac{\pi}{2}} = \omega\Lambda_m e^{j\frac{\pi}{2}} = j\omega\Lambda_m$
- $\bar{\lambda}_{dq} = \Lambda_m$
- $\bar{i}_{dq} = \bar{i}_{\alpha\beta} e^{j\left(\vartheta_m^e + \alpha_i^e\right)} e^{-j\vartheta_{dq}} = I e^{j\alpha_i^e} = I\cos\alpha_i^e - jI\sin\alpha_i^e$

constant values in dq

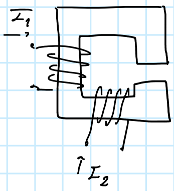
12/11

mercoledì 9 dicembre 2020 09:26



$$\begin{cases} v_{AN} = R i_a + L \dfrac{di_a}{dt} + e_a = v_a + v_{0N} \\ v_{BN} = R i_b + L \dfrac{di_b}{dt} + e_b = v_b + v_{0N} \\ v_{CN} = R i_c + L \dfrac{di_c}{dt} + e_c = v_c + v_{0N} \end{cases}$$

→ SPACE VECTOR : $R\bar{i} + L\dfrac{d\bar{i}}{dt} + \bar{e} = \bar{v} + \emptyset$



$\dfrac{\Lambda_1}{I_1} = L_1$ (self inductance)

$\dfrac{\Lambda_2}{I_1} = M_{12}$ (mutual inductance)

→

PHASE LINKAGE

$\lambda_a = L_a i_a + M_{ab} i_b + M_{ac} i_c$

- by geometry $M_{ab} = M_{ac} = -\dfrac{L_a}{2}$

$\lambda_a = L_a i_a + M(i_b + i_c)$

- $i_a + i_b + i_c = 0$

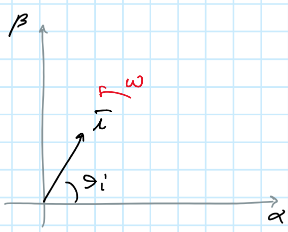
$$\boxed{\lambda_a = i_a (L_a - M) \simeq \frac{3}{2} L_a i_a}$$

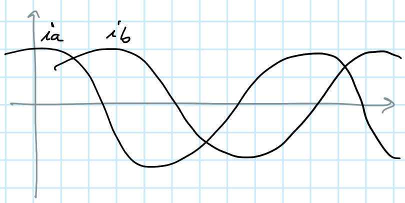
in $\alpha\beta$ reference : $\bar{v} = \bar{e} + R\bar{i} + L\dfrac{d\bar{i}}{dt}$

- $\bar{e} = \dfrac{d\bar{\lambda}_m}{dt}$
- $\dfrac{d}{dt}(L\bar{i}) = \dfrac{d(\bar{\lambda}_i)}{dt}$

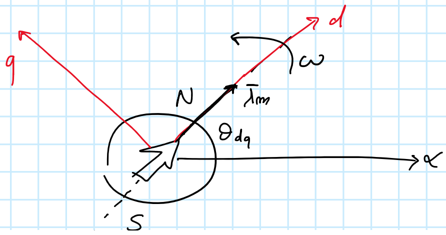
$$\boxed{\bar{v} = R\bar{i} + \frac{d}{dt}\bar{\lambda}}$$

$\bar{\lambda} = \bar{\lambda}_m + \bar{\lambda}_i$



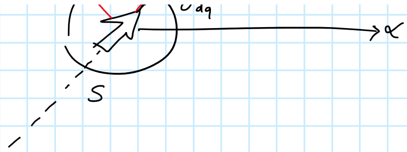


in dq reference :



synchronous reference frame

$\vartheta_{dq} = \theta_m$



$\alpha\beta$: $\quad \bar{\lambda}_{m\,\alpha\beta} = \Lambda_m e^{j\vartheta_m^e} = \Lambda_m \cos\vartheta_m^e + j\Lambda_m \sin\vartheta_m^e$

dq: $\quad \bar{\lambda}_{m\,dq} = \bar{\lambda}_{m\,\alpha\beta}\, e^{-j\vartheta_{dq}} = \Lambda_m e^{j\vartheta_m^e} e^{-j\vartheta_{dq}} = \begin{cases} \lambda_{md} = \lambda_{m\alpha}\cos\vartheta_{dq} + \lambda_{m\beta}\sin\vartheta_{dq} = \Lambda_m \\ \lambda_{mq} = 0 \end{cases}$

$\Rightarrow \boxed{\bar{\lambda}_{m\,dq} = \Lambda_m}$

EMF

$\alpha\beta$: $\quad \bar{e}_{\alpha\beta} = \dfrac{d\bar{\lambda}_{m\,\alpha\beta}}{dt} = \quad = \bar{e}_{dq}\cdot e^{+j\vartheta_{dq}} = \dfrac{d}{dt}\left(\bar{\lambda}_{m\,dq}\cdot e^{+j\vartheta_{dq}}\right)$

dq: $\quad \bar{e}_{dq} = \bar{e}_{\alpha\beta}\cdot e^{-j\vartheta_{dq}}$

$$\bar{e}_{dq}\cdot e^{j\vartheta_{dq}} = \frac{d}{dt}\left(\bar{\lambda}_{m\,dq}\right)\cdot e^{j\vartheta_{dq}} + \bar{\lambda}_{m\,dq}\cdot\frac{d}{dt}\left(e^{j\vartheta_{dq}}\right)$$

$$= 0 + \Lambda_m \frac{d}{dt} e^{j\omega t}$$

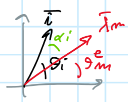
$$= j\omega\Lambda_m e^{j\vartheta_{dq}}$$

$\vartheta_{dq} = \omega t = \omega_m^e t$

$\circ\ \lambda_{m\,dq} = \Lambda_m = \text{const.}$

$\Rightarrow \boxed{\bar{e}_{dq} = j\omega\Lambda_m}$

CURRENT



$$\bar{i}_{\alpha\beta} = I\cdot e^{j\vartheta_i} = I\cdot e^{j(\vartheta_m^e + \alpha_i)}$$

$$\bar{i}_{dq} = \bar{i}_{\alpha\beta}\, e^{-j\vartheta_{dq}} = I\cdot e^{j(\vartheta_m^e + \alpha_i - \vartheta_{dq})} = I e^{j\alpha_i}$$

$$\boxed{\bar{i}_{dq} = I e^{j\alpha_i}}$$

$$\begin{cases} i_d = I\cos\alpha_i \\ i_q = I\sin\alpha_i \end{cases}$$

In steady state α_i is constant because ω_m is constant

CURRENT LINKAGE

$$\bar{\lambda}_{i\,\alpha\beta} = L\cdot\bar{i}_{\alpha\beta}$$

$$\frac{d}{dt}\bar{\lambda}_{i\,\alpha\beta} = \frac{d}{dt}\left(L\bar{i}_{\alpha\beta}\right) = L\frac{d\bar{i}_{\alpha\beta}}{dt} = L\frac{d}{dt}\left(\bar{i}_{dq}\cdot e^{+j\vartheta_{dq}}\right) = L\frac{d}{dt}\left(\bar{i}_{dq}\right)e^{j\vartheta_{dq}} + L\bar{i}_{dq}\frac{d}{dt}\left(e^{j\vartheta_{dq}}\right) =$$

$$= L\frac{d}{dt}\left(\bar{i}_{dq}\right)e^{j\vartheta_{dq}} + L\bar{i}_{dq}\, j\omega e^{j\vartheta_{dq}}$$

$\vartheta_{dq} = \omega_m^e\cdot t$

$$= L\,\frac{d}{dt}(\bar{i}_{dq})\,e^{j\vartheta_{dq}} + L\,\bar{i}_{dq}\,j\omega\,e^{j\vartheta_{dq}}$$

$$\vartheta_{dq} = \omega_m^e \cdot t$$

$$= \frac{d}{dt}\left(\bar{\lambda}_{i_{dq}}\right) e^{j\vartheta_{dq}} + \underbrace{\bar{\lambda}_{i_{dq}}\,j\omega\,e^{j\vartheta_{dq}}}_{\text{motional EMF}}$$

$$\bar{v}_{\alpha\beta} = R\,\bar{i}_{\alpha\beta} + L\,\frac{d\bar{i}_{\alpha\beta}}{dt} + \bar{e}_{\alpha\beta} \qquad \left(\bar{e}_{\alpha\beta} = j\omega\,\bar{\Lambda}_{m_{\alpha\beta}}\right)$$

$$\bar{v}_{dq} = R\,\bar{i}_{dq} + L\,\frac{d\bar{i}_{dq}}{dt} + j\omega\,\bar{\lambda}_{i_{dq}} + \bar{e}_{dq} \qquad \left(\bar{e}_{dq} = j\omega\,\bar{\Lambda}_{m_{dq}}\right)$$

$$= R\,\bar{i}_{dq} + L\,\frac{d\bar{i}_{dq}}{dt} + j\omega\,\bar{\lambda}_{dq}$$

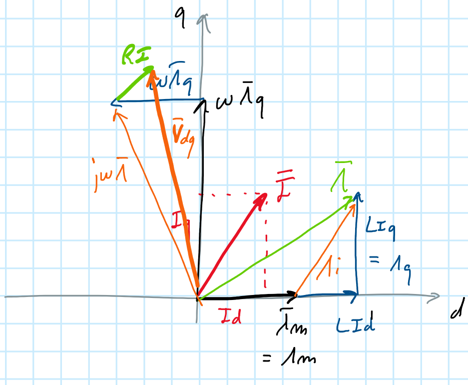
- $\lambda_{dq} = \lambda_{i_{dq}} + \lambda_{m_{dq}}$

$$\begin{cases} v_d = R i_d + L\dfrac{di_d}{dt} - \omega\lambda_q \\ v_q = R i_q + L\dfrac{di_q}{dt} + \omega\lambda_d \end{cases} \qquad \begin{cases} \lambda_d = \lambda_{i_d} + \lambda_{m_d} = \Lambda_m + L i_d \\ \lambda_q = \lambda_{i_q} + \lambda_{m_q} = 0 + L i_q \end{cases}$$

In steady state all the terms are constant:

$$V_{dq} = V_d + V_q$$

$$\begin{cases} V_d = R I_d - \omega\Lambda_q \\ V_q = R I_q + \omega\Lambda_d \end{cases} \qquad \begin{cases} \Lambda_d = \Lambda_m + L I_d \\ \Lambda_q = L I_q \end{cases}$$



POWER BALANCE

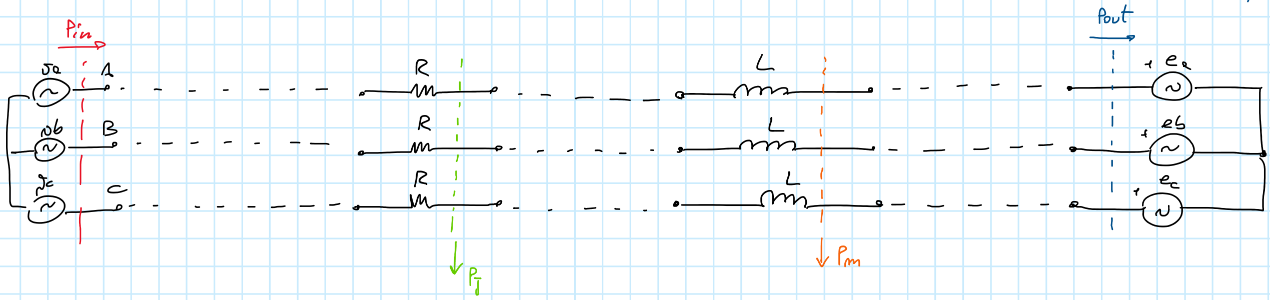
$$v_d \cdot i_d = R i_d^2 + i_d\,L\frac{di_d}{dt} - \omega L\,i_d\,i_q$$

$$v_q \cdot i_q = R i_q^2 + i_q\,L\frac{di_q}{dt} + \omega L\,i_d \cdot i_q + \omega\Lambda_m i_q$$

$\Sigma(\dots)\cdot\frac{3}{2}$ → 3 phase system
→ peak to RMS ($\underset{v}{\sqrt{2}}\cdot\underset{i}{\sqrt{2}}$)

$$\underbrace{\frac{3}{2}\left(v_d i_d + v_q i_q\right)}_{\text{Input power } P_{in}} = \underbrace{\frac{3}{2} R\left(i_d^2 + i_q^2\right)}_{\text{Joule losses}} + \frac{d}{dt}\left[\underbrace{\frac{3}{2}\left(\frac{1}{2}L i_d^2 + \frac{1}{2} L i_q^2\right)}_{\text{magnetic energy } W_m}\right] + \underbrace{\frac{3}{2}\omega\left[-\cancel{L i_d i_q} + \cancel{L i_d i_q} + \Lambda_m i_q\right]}_{\text{Power converted } P_{out} = P_{em}}$$

electromechanical power



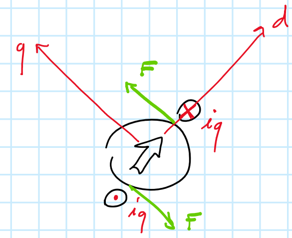
$P_{em} = T_{em} \cdot \omega_m$

$\omega = p \cdot \omega_m$

$$\boxed{T_{em} = \frac{3}{2} p \Lambda_m i_q}$$

TORQUE

The torque depends on the pole numbers, Λ_m and i_q



There is no need for i_d current as it only produces losses

EXERCISE

$K_T = 1.05 \quad Nm/A \quad$ [in RMS]

$$\left.\begin{aligned} T_{em} &= K_T \cdot I_{RMS} = K_T \cdot \frac{I}{\sqrt{2}} \\ &= \frac{3}{2} p \Lambda_m I \end{aligned}\right\} \quad \frac{3}{2} p \Lambda_m = \frac{K_T}{\sqrt{2}} \rightarrow \boxed{\Lambda_m = \frac{2 K_T}{3\sqrt{2} \cdot p}}$$

- Speed $\quad rpm \rightarrow rad/s \qquad \omega_m = rpm \cdot \frac{2\pi}{60} \rightarrow \omega = p \cdot \omega_m \ (= \omega_m^e)$
- Phase to phase – Line to line voltage (tensione concatenata) $\rightarrow V_{pp_{RMS}} = 400V$

$$V = \frac{V_{pp_{RMS}}}{\sqrt{3}} \cdot \sqrt{2} = 327 \ V$$

2.4

Synchronous SPM motor 3phase

data:	at no load:	with load: (current controlled) ↗ only i_q current
$2p = 2$	$V_{ll_{RMS}} = 38.48V$	$V_{pp_{RMS}} = 53.64V$

data:

$2p = 2$

$R = 2\,\Omega$

$L = 25\,mH$

$P_{fe+mech} \approx 0$

at no load:

$V_{ll\,rms} = 38.48\,V$

$\omega_{m_0} = 314.16\,\frac{rad}{s}$

with load: (current controlled)

$V_{ll\,rms} = 53.64\,V$

$I_{rms} = 4\,A$

determine : ω_m at load, torque at load

$$V_{NL} = 38.48 \cdot \frac{\sqrt{2}}{\sqrt{3}} = 31.42\ V$$

$$\omega = p \cdot \omega_{m_0} = 314.16\ \frac{rad}{s}$$

$$\Lambda_m = \frac{V}{\omega} = 0.1\ V \cdot s$$

$$V_{LOAD} = 53.64 \cdot \sqrt{\frac{2}{3}} =$$

$$I = \sqrt{2}\, I_{rms} = 5.66\,A$$

$$I_q = I$$

$$T_{em} = \frac{3}{2} p \Lambda_m I_q = \frac{3}{2} \cdot 0.1 \cdot 5.66 = 0.849\ Nm$$

At steady state :

$$V_{dq} = \begin{cases} V_d = R I_d - \omega \Lambda_q \\ V_q = R I_q + \omega \Lambda_d \end{cases} \longrightarrow V^2 = V_d^2 + V_q^2 = (R I_d - \omega \Lambda_q)^2 + (R I_q + \omega \Lambda_d)^2$$

$$\Lambda_{dq} = \begin{cases} \Lambda_q = L i_q \\ L_d = \Lambda_m + L i_d \end{cases}$$

$$\omega = 209\ \frac{rad}{s}$$

$$\omega_m = \frac{\omega}{p} \longrightarrow n = \frac{\omega_m \cdot 60}{2\pi} = \text{in rpm}$$

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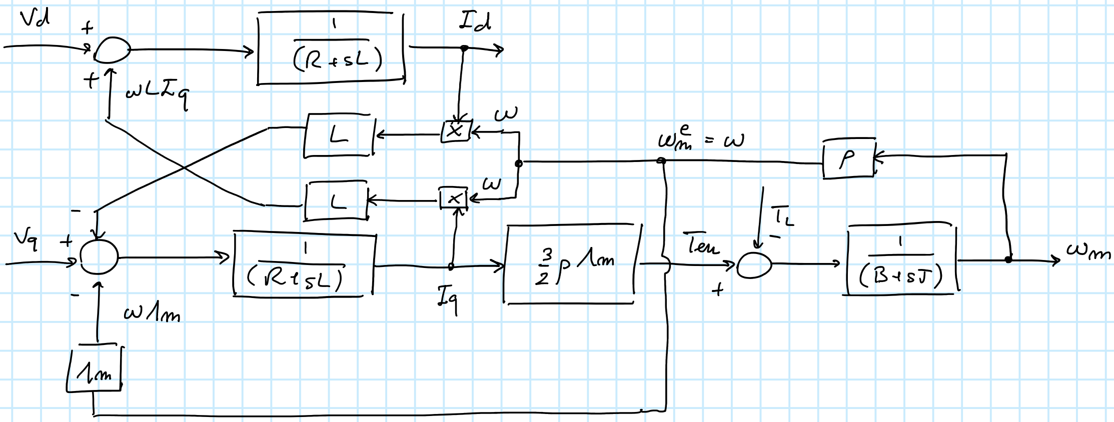
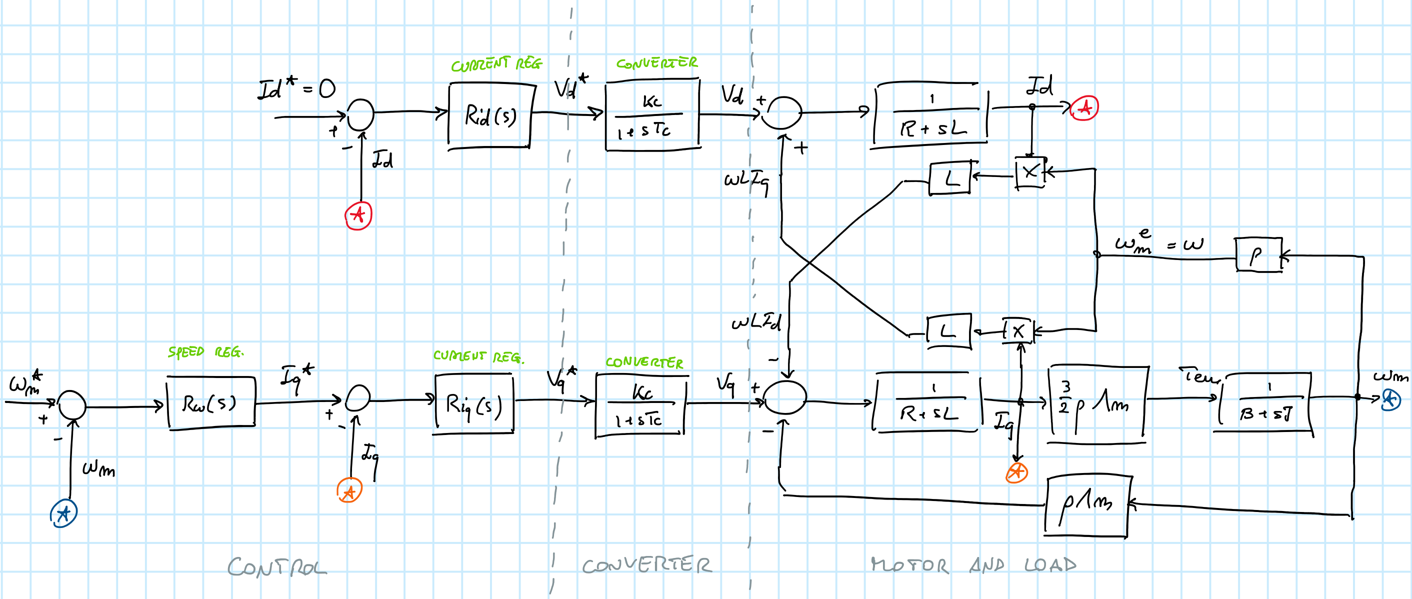
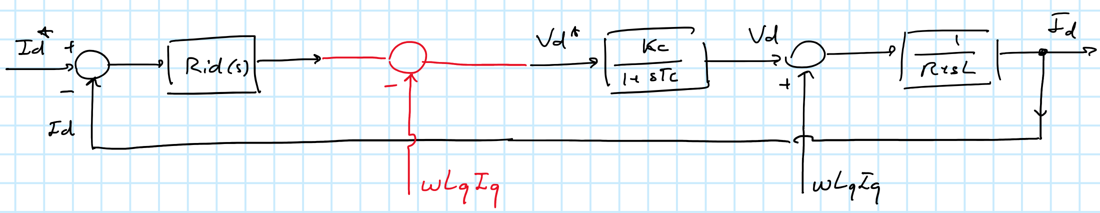
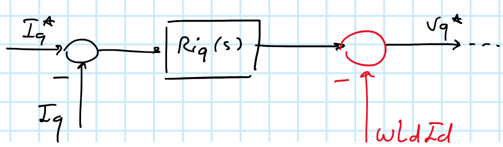
SPM BLOCK DIAGRAM

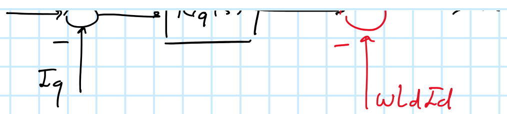




CURRENT LOOP, decoupling of the two axis : dq current decoupling







EXERCISE

Design control of SPM motor:

Source:

$V_{dc} = 400V$

$\Lambda_m = 0{,}25 Vs$

$R = 0.5\Omega$

$L = 10 mH$

$2p = 8$

$J_m = 0.02\ kg\,m^2$

inverter:

$f_{sw} = 10 kHz$

$K_c = 15$

current loop:

$B_W = 500 Hz$

$\varphi_m \geq 60°$

load:

$T_{load} \propto \omega_{load}$; $\dfrac{262 Nm}{400 rpm}$, $J_L = 48\ kg\,m^2$

$K_{gear} = \frac{1}{10}$

speed loop:

$B_W = 32 Hz$

$\varphi_m \geq 60°$

$V_{pp} = 400V \rightarrow V = \dfrac{V_{pp}}{\sqrt{3}} = 230V$

- gearbox : $\omega_L = K_g\,\omega_m$ $\qquad \omega_m \rightarrow \boxed{K_g} \rightarrow \omega_L$

  $T_m = K_g T_L$ $\qquad T_m \rightarrow \boxed{\frac{1}{K_g}} \rightarrow T_L$

- From data : $T_L \propto \omega_L = \dfrac{262}{100}\dfrac{Nm}{rpm}$

$$T_L = B_L\omega_L + J\frac{d}{dt}\omega_L \rightarrow B_L = \frac{262}{100\cdot\frac{2\pi}{60}} = 25\ Nm\,s$$

$$T_m = K_g T_L = K_g J_L \frac{d}{dt}\omega_L + K_g B_L\omega_L =$$

$$= \underbrace{K_g^2 J_L}_{J_L'} \frac{d}{dt}\omega_m + \underbrace{K_g^2 B_L}_{B}\omega_m$$

$J_L' = 0.1^2 \cdot 48 = 0.48\ kg\,m^2$

$B = 0.1^2 \cdot 25 = 0.25\ Nm\cdot s$

- Total inertia : $J = J_m + J_L' = 0.02 + 0.48 = 0.5\ kg\,m^2$

- $\tau_e = \dfrac{L}{R} = \dfrac{0.01}{0.5} = 0.2\ s$

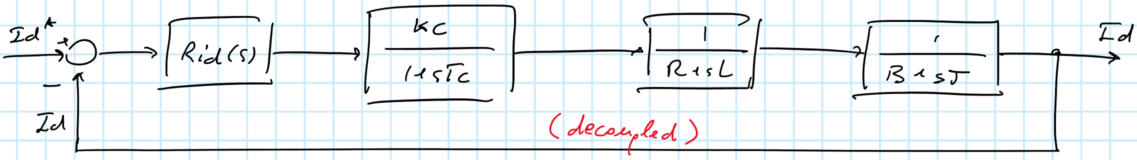
- $\tau_{m_1} = \dfrac{J}{B} = \dfrac{0.5}{0.25} = 2\ s$

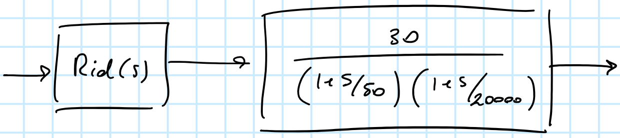
- $\tau_c = \dfrac{1}{2f_s} = \dfrac{1}{2\cdot 10000} = 50\mu s$

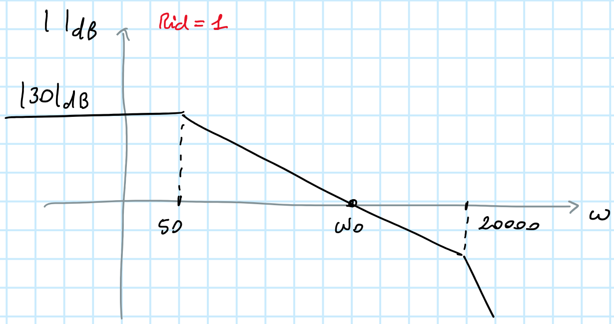
$$\boxed{\frac{1}{R+sL}} = \frac{1}{R}\,\frac{1}{1+sT_e} = \frac{2}{1+s/50}$$

$$\boxed{\frac{1}{B+sJ}} = \frac{1}{B}\,\frac{1}{1+sT_{m_1}} = \frac{4}{1+s/0.5} \quad \text{(NEGLECTED)}$$

$$\boxed{\frac{K_C}{1+sT_C}} = \frac{15}{1+s/20000}$$







$$\omega_0 \overset{?}{=}$$

$$\left| \frac{30}{\left(1+j\omega_0/50\right)\left(1+\frac{j\omega_0}{20000}\right)} \right| = 1$$

$$\simeq \frac{30}{\sqrt{1+\left(\frac{\omega_0}{50}\right)^2}} = 1 \longrightarrow \omega_0 \simeq 1500\ \frac{rad}{s}$$

- With a PI regulator: compensate the first pole + pole in the origin $\quad R_{PI} = K_i \cdot \frac{1+sT_i}{s}$

$f_{BW} = 500\ Hz \longrightarrow \omega_{BW} = 2\pi\, 500 = 3140\ \frac{rad}{s}$

$T_i = T_e = \frac{1}{50} = 0.02$

$$K_i \overset{?}{=} \quad \left| K_i \cdot \frac{(\cancel{1+sT_i})}{s} \cdot \frac{30}{(\cancel{1+sT_e})\,(1+sT_{m_1})} \right|_{s=j\omega_{BW}} = \quad \frac{K_i}{3140} \cdot \frac{30}{\sqrt{1+\left(\frac{3140}{20000}\right)^2}} = 1$$

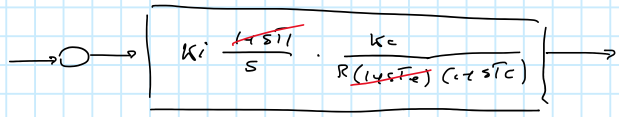
$$\boxed{K_i \simeq 106}$$

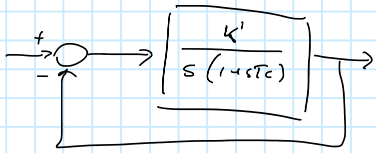
$K_p = K_i T_i = 2.16$

$$\varphi_m = 180 + \angle G(3140) = 180^\circ - 90^\circ - \operatorname{atan}\frac{3140}{20000} = 81^\circ \quad ✓$$

Using root locus:

$T_i = T_e$





$$\frac{G}{1+G} = \frac{K'}{s(1+sT_c)+K'}$$

$$s(1+sT_c) + K' = 0$$

$$s^2T_c + s + K' = 0$$

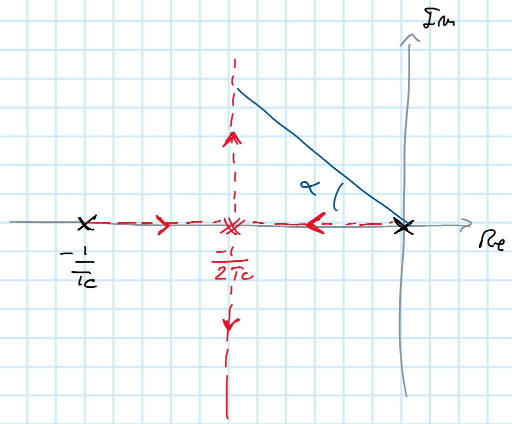
$$s = \frac{-1 \pm \sqrt{1-4T_cK'}}{2T_c}$$

- $K' = 0 \rightarrow s_{1,2} = \frac{-1 \pm 1}{2T_c} = 0,\ -\frac{1}{T_c}$

- $K'_\Delta \rightarrow 1 - 4T_cK'_\Delta = 0$

  $K'_\Delta = \frac{1}{4T_c} \rightarrow s_{1,2} = -\frac{1}{2T_c}$

- K' : $\Delta < 0 \rightarrow$ two imaginary solution



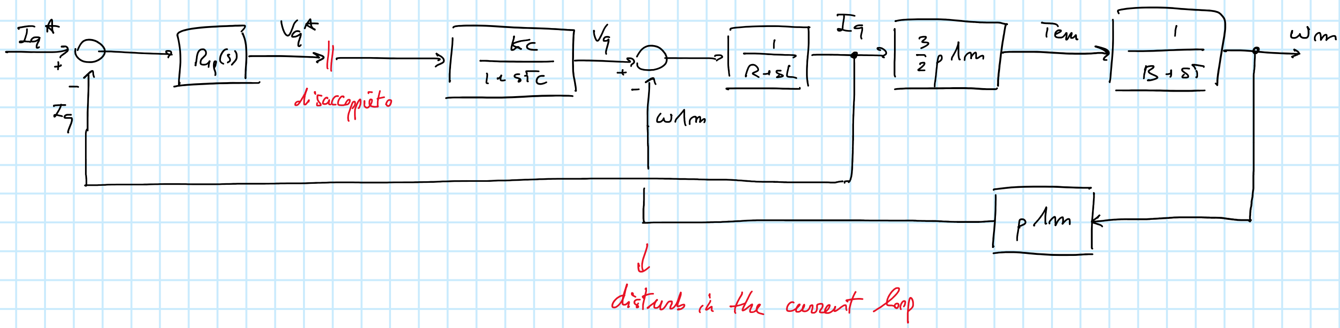
$\xi = \cos\alpha \rightarrow \alpha = 45^\circ$: $Re = Im$

Good solution for $\xi \geq 0.707$

$$\left|\frac{1}{2T_c}\right| = \left|\frac{1}{2T_c}\sqrt{1-4T_cK'}\right| \rightarrow K' = \frac{1}{2T_c} \rightarrow K_i = \frac{K'R}{K_c} = 333$$

$$K_p = 6.67$$

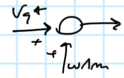
I_q SIDE



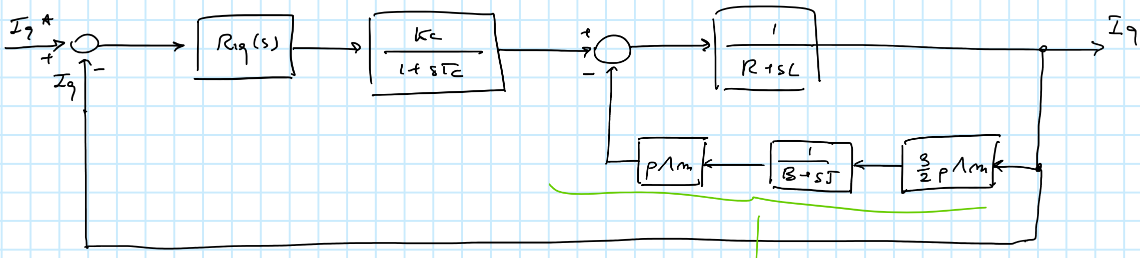
STRATEGIES

1) the speed has a dynamic slower than the current → neglect the $\omega \lambda_m$

2) compensate $p\lambda_m$ in the control loop in V_q^*

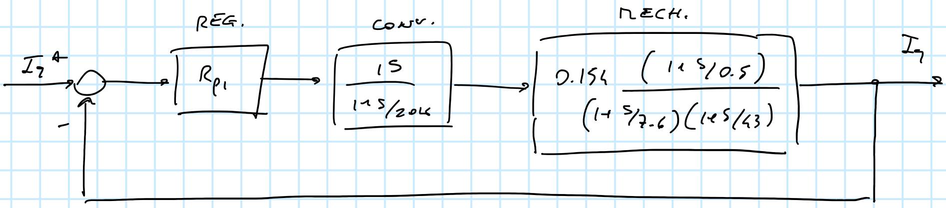


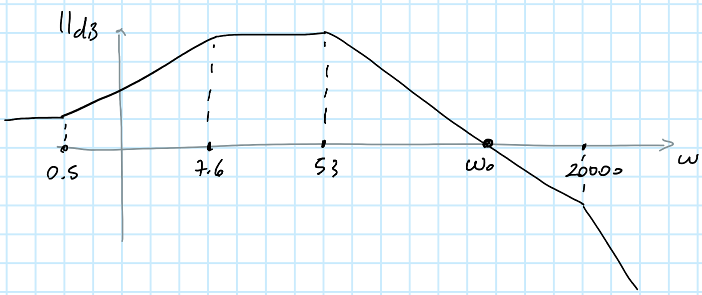
3) introduce the mechanical feedback



$$\frac{G}{1+GH} = \frac{\frac{1}{R}\,\frac{1}{1+sT_e}}{1+\frac{1}{R}\,\frac{1}{1+sT_e}\cdot\frac{3}{2}p\lambda_m\frac{1}{B}\,\frac{1}{1+sT_m}\cdot p\lambda_m} =$$

$$= \frac{B}{\frac{3}{2}(p\lambda_m)^2}\cdot\frac{1+sT_m}{\frac{RB}{\frac{3}{2}(p\lambda_m)^2}(1+sT_e)(1+sT_m)+1} = 0.154\cdot\frac{(1+s/0.5)}{(1+s/7.6)\,(1+s/43)}$$





$\omega_0 \simeq 1500$ rad/s

PI regulator : $k_i\,\frac{(1+sT_i)}{s}$, $T_i = \frac{1}{7.6}$

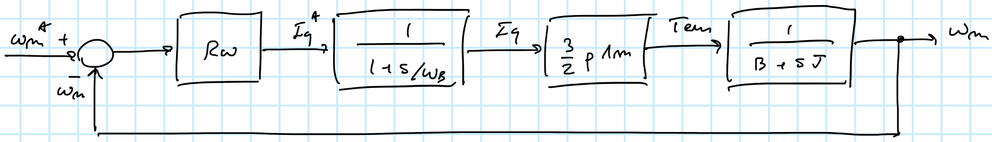
$$\left| k_i\,\frac{\cancel{(1+sT_i)}}{s}\cdot 2.31\,\frac{1+s/0.5}{\cancel{(1+s/7.6)}\,(1+s/43)}\right|_{s=j\omega_B=j3140} = 1$$

⋮

$k_i = 16$, $k_p = k_i\cdot T_i = 2.1$

$$\varphi_m = 180 - 90 - \text{atan}\left(\frac{3160}{0.5}\right) - \text{atan}\left(\frac{3160}{43}\right) - \text{atan}\left(\frac{3160}{20000}\right) = 82° \checkmark$$

SPEED REGULATOR :



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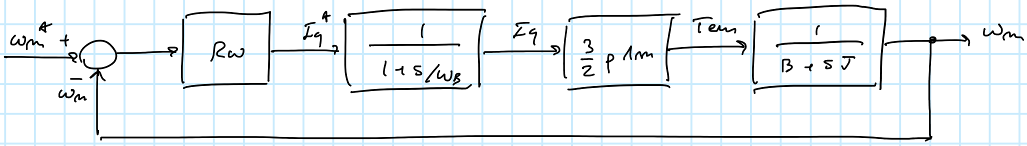
SPEED REGULATOR :



$$G_\omega(s) = R_\omega(s)\cdot \frac{3}{2}p\Lambda_m \frac{1}{B}\frac{1}{(1+sT_m)(1+s/\omega_B)} \overset{PI}{=} k_i \frac{\cancel{1+sT_i}}{s}\frac{6}{(\cancel{1+3/10\,s})(1+s/3140)}$$

$$= \frac{6k_i}{s}\frac{1}{(1+s/3140)}$$

- $\omega_B = 200$ rad/s

$|G_\omega(200)| = 1 \rightarrow k_i = 33.4$, $k_p = k_i T_i = 66.8$

$$\varphi_m = 180 - 90 - \mathrm{atan}\left(\frac{200}{3140}\right) = 86°$$

LIMITI DI FUNZIONAMENTO

$\bar{I} = I_d + jI_q$

$\bar{\Lambda} = \Lambda_d + j\Lambda_q \qquad \begin{cases}\Lambda_d = \Lambda_m + LI_d \\ \Lambda_q = LI_q\end{cases}$

$\bar{V} = V_d + jV_q \qquad \begin{cases} V_d = RI_d - \omega\Lambda_q \\ V_q = RI_q + \omega\Lambda_d\end{cases}$ At limit we can neglect $\overset{\simeq 0}{RI_d}$ and consider only the speed : $\begin{cases} V_d \simeq -\omega\Lambda_q \\ V_q \simeq +\omega\Lambda_d\end{cases}$

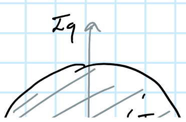
$T_{em} = \frac{3}{2}p\Lambda_m I_q$

limits : $I \le I_N$, $V \le V_N$

electrical source $\rightarrow$ Power electronics $\rightarrow$ motor

$\Rightarrow$ max T_{em} and ω_m ?

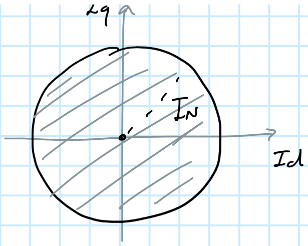
$I_d^2 + I_q^2 \le I_N^2$



$V_d^2 + V_q^2 \le V_N^2$

$(\omega\Lambda_q)^2 + (\omega\Lambda_d)^2 \le V_N^2$

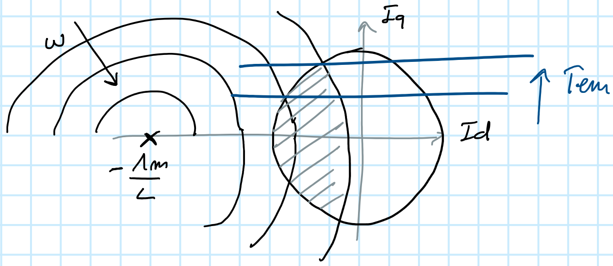
$(LI_q)^2 + (\Lambda_m + LI_d)^2 \le \left(\frac{V_N}{\omega}\right)^2$



$$(\omega\Lambda_q)^2 + (\omega\Lambda_d)^2 \le V_N^2$$

$$(L I_q)^2 + (\Lambda_m + L I_d)^2 \le \left(\frac{V_N}{\omega}\right)^2$$

$$I_q^2 + \left(\frac{\Lambda_m}{L} + I_d\right)^2 \le \left(\frac{V_N}{\omega L}\right)^2$$



Torque : $\frac{3}{2} p \Lambda_m I_q$ → max torque @ ω_m very low (zero) → no V_N imperative

<u>consider only I_N</u>

$$I_q = \max I = I_N$$

$$T_{em,max} = \frac{3}{2} p \Lambda_m I_N \qquad \begin{cases} I_q = I_N \\ I_d = 0 \end{cases} \rightarrow \begin{cases} \Lambda_d = \Lambda_m + L I_d = \Lambda_m \\ \Lambda_q = L I_q = L I_N \end{cases}$$

$$\begin{cases} V_d = -\omega L I_N \\ V_q = \omega \Lambda_m \end{cases}$$

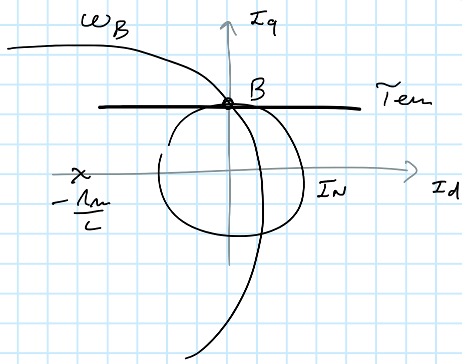
$$V^2 = \omega^2\left(\Lambda_m^2 + (L I_N)^2\right) = \omega^2 \Lambda^2$$

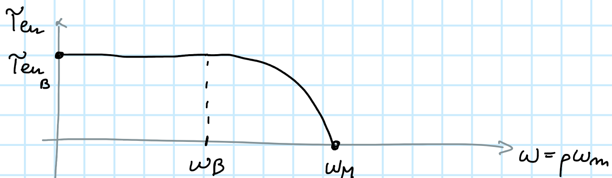
$(V = V_N)$

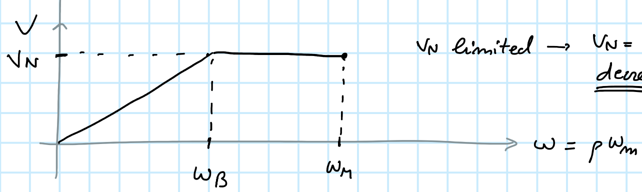
$$\boxed{\omega = \frac{V_N}{\Lambda} = \frac{V_N}{\sqrt{\Lambda_m^2 + (L I_N)^2}}}$$

BASE SPEED ω_B

$\boxed{\omega \text{ ELECTRICAL}}$

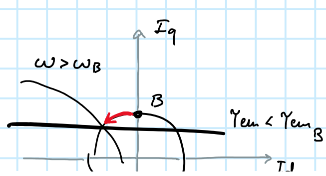


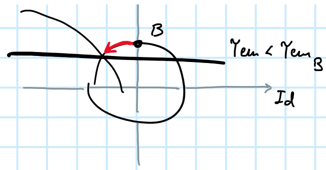




V_N limited → $V_N = \omega\Lambda$

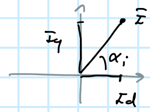
<u>decrease Λ</u>





$$V^2 = \omega^2\left(\Lambda_d^2 + \Lambda_q^2\right) = \omega^2\left[\left(\Lambda_m + L I_d\right)^2 + \left(L I_q\right)^2\right]$$

$$I_N^2 = I_d^2 + I_q^2 \qquad \begin{cases} I_d = I_N \cos\alpha_i \\ I_q = I_N \sin\alpha_i \end{cases}$$



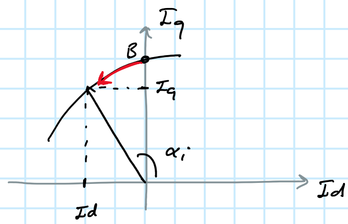
- imposing $\omega_m^e = \omega$

$$V_N^2 = \omega_m^{e\,2}\left[\left(\Lambda_m + L I_N \cos\alpha_i\right)^2 + \left(L I_N \sin\alpha_i\right)^2\right]$$

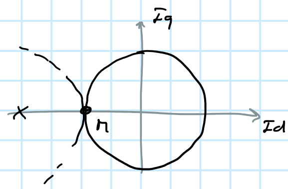
$$\frac{V_N^2}{\omega_m^{e\,2}} = \Lambda_m^2 + 2\Lambda_m L I_N \cos\alpha_i + \left(L I_N \cos\alpha_i\right)^2 + \left(L I_N \sin\alpha_i\right)^2 = \Lambda_m^2 + 2\Lambda_m I_N \cos\alpha_i + L^2 I_N^2$$

FLUX WEAKENING

$$\boxed{\cos\alpha_i = \frac{\left(\frac{V_N}{\omega_m^e}\right)^2 - \Lambda_m^2 - L^2 I_N^2}{2\Lambda_m L I_N} < 0}$$



maximum speed: $\boxed{\cos\alpha_i = 1}$



$$\begin{cases} I_q = 0 \\ I_d = -I_N \end{cases}$$

$\vdots$

$$\boxed{\omega_M = \frac{V_N}{\Lambda_m - L I_N}}$$

MAXIMUM SPEED

EXERCISE

$R \simeq 0$ $\qquad V_{pp} = 250V$ $\qquad V = \frac{\sqrt{2}}{\sqrt{3}} \cdot 250 = 204\,V$ (phase peak)

$L = 0.15\,mH$ $\qquad I_{rms} = 200A$ $\qquad I = 200 \cdot \sqrt{2} = 283\,A$

$2p = 8$

$K_E = 0.6 \frac{V_{rms\,pp}}{rad/s}$ $\rightarrow$ $V = \sqrt{\frac{2}{3}} \cdot K_E\,\omega_m = \Lambda_m \omega = \Lambda_m p\omega_m$

$$\Lambda_m = \sqrt{\frac{2}{3}}\,\frac{K_E}{p} = 0.122\,Vs$$

maximum speed at no load:

$I_d = 0$, $\quad \Lambda_d = \Lambda_m$, $\quad V_d = 0$

$I_q = 0$, $\quad \Lambda_q = 0$, $\quad V_q = \omega\Lambda_m$

$V_N = \omega_0 \Lambda_m \rightarrow \omega_0 = \frac{V_N}{\Lambda_m} = \frac{204}{0.122} = 1672\,\frac{rad}{s}$

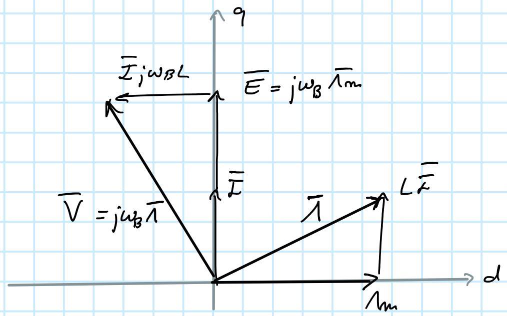
$$\omega_{m0} = \frac{\omega_0}{p} = 418 \ \frac{rad}{s} \qquad n_0 = \omega_{m0} \cdot \frac{60}{2\pi} = 4000 \ rpm$$

BASE POINT

$$\omega_B = \frac{V_N}{\sqrt{(\Lambda_m)^2 + (L I_N)^2}} = \frac{204}{\sqrt{(0.122)^2 + (215e\text{-}3 \cdot 283)^2}} = 1580 \ \frac{rad}{s}$$

$$\omega_{mB} = \frac{\omega_B}{p} = 395 \ \frac{rad}{s} \qquad n_B = 3780 \ rpm$$

$$T_{em_B} = \frac{3}{2} p \ \Lambda_m I_N = 207 \ Nm$$



MAXIMUM SPEED

$$\omega_M = \frac{V_N}{\Lambda_m - L I_N} = 2570 \ \frac{rad}{s} \rightarrow n_M = \frac{2570}{4} \cdot \frac{60}{2\pi} = 6125 \ rpm$$

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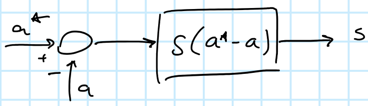
3 PHASE CURRENT CONTROL

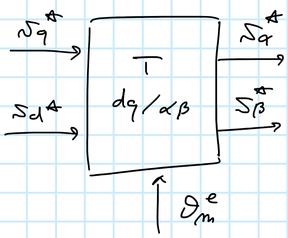
control → stationary ref frame $\alpha\beta$
control → synchronous ref frame dq

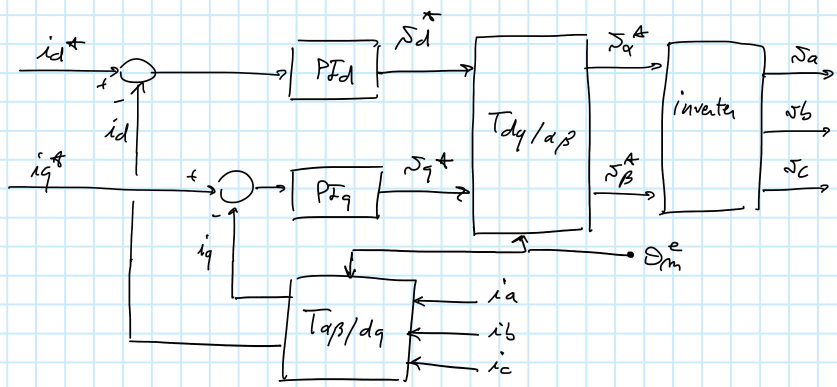
control → PWM voltage inverter
control → direct command of the switch



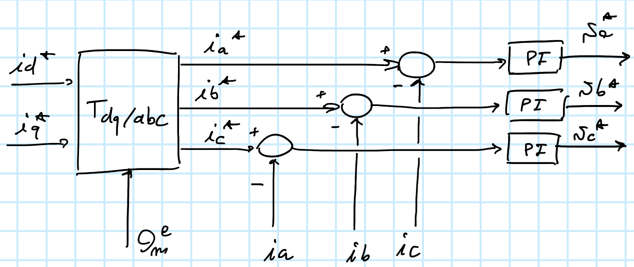
gate command to the leg

- The motor works in $\alpha\beta$ quantities





PI in abc



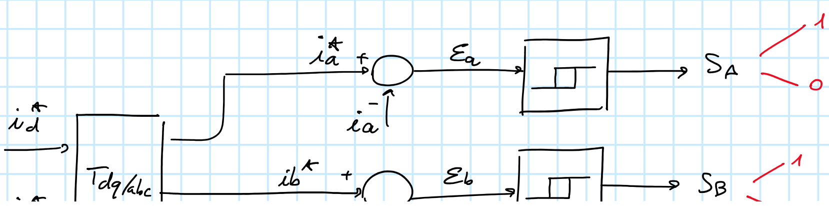
DRAWBACK

the PI receive a sinusoidal error

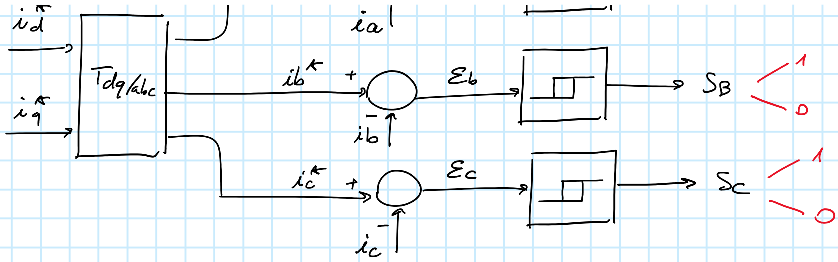
→ at steady state the error is $\neq 0$

same if we use $T_{dq/\alpha\beta}$ and use i_α^* i_β^*

DIRECT CONTROL HISTERESIS

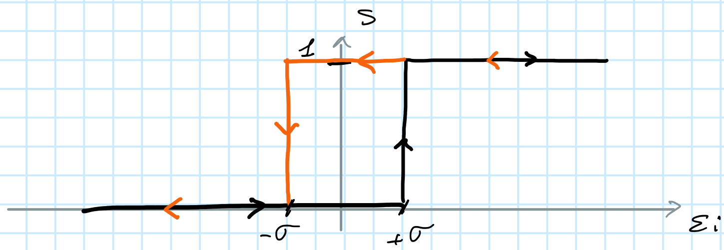


indipendent
hysteresis
control



hysteresis control

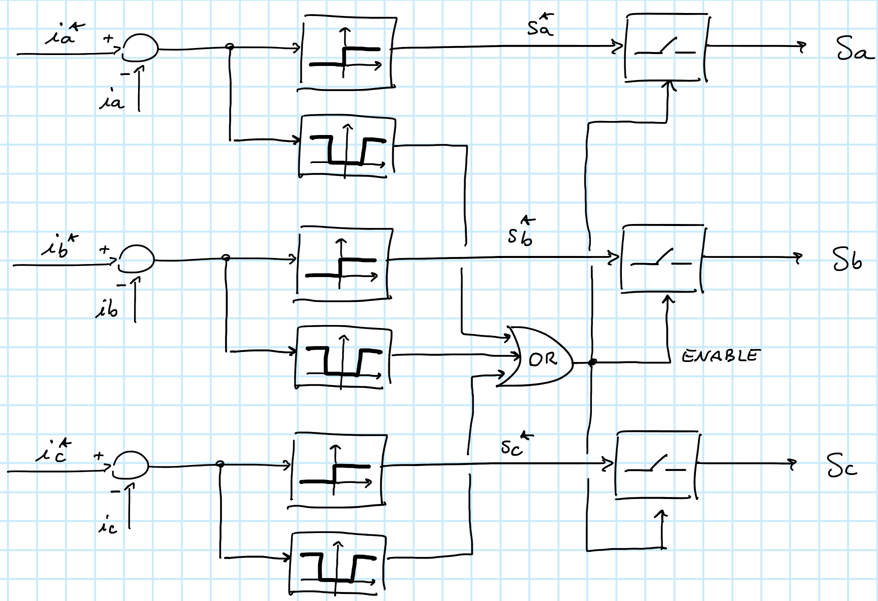
→ the error can be 2σ because every phase can give an error

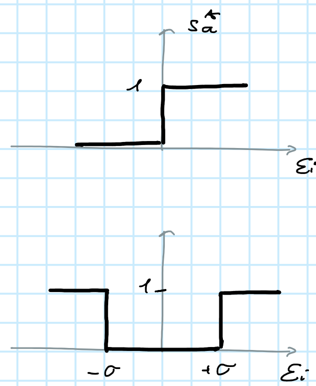


- $\varepsilon_i < 0$: $i_i > i_i^*$ → REDUCE = open switch
- $\varepsilon_i > 0$: $i_i < i_i^*$ → INCREASE = closed switch

- $\varepsilon \leq 2\sigma$
- f_{sw} depends on the current behaviour
  - ↳ high sw losses
  - ↳ distortion of voltage

DEPENDENT 3ph HYSTERESIS CONTROL





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SPM

$\bar{\lambda}_m$

$$\bar{e}_{\alpha\beta} = \frac{d\bar{\lambda}_m}{dt}$$

$$\bar{v}_{\alpha\beta} = R\bar{i}_{\alpha\beta} + L\frac{d\bar{i}_{\alpha\beta}}{dt} + \bar{e}_{\alpha\beta} = R\bar{i}_{\alpha\beta} + \frac{d\bar{\lambda}_{\alpha\beta}}{dt}$$

$$\bar{\lambda}_{\alpha\beta} = \bar{\lambda}_m + L\bar{i}_{\alpha\beta}$$

$\boxed{\text{in } dq}$ At steady state all the quantities are CONSTANTS

$$\bar{i}_{dq} = \bar{i}_{\alpha\beta} \cdot e^{-j\vartheta_m^e}$$

$$\bar{v}_{dq} = R\bar{i}_{dq} + L\frac{d\bar{i}_{dq}}{dt} + j\omega\bar{\lambda}_{dq}$$

$$v_d = Ri_d + \frac{d\lambda_d}{dt} - \omega\lambda_q$$

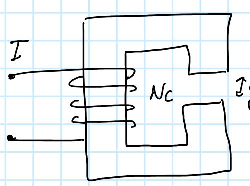
$$v_q = Ri_q + \frac{d\lambda_q}{dt} + \omega\lambda_d$$

$$T_{em} = \frac{3}{2}p\Lambda_m i_q$$

IPM MOTORS : same equations in $\alpha\beta$ and in dq

$$\begin{cases} \lambda_d = \Lambda_m + L_d i_d \\ \lambda_q = L_q i_q \end{cases}$$

the inductances are DIFFERENT



$$L = \frac{\Lambda}{I} = \frac{N_c\Phi}{I} \qquad \Phi \propto \frac{1}{g}$$

$L_d < L_q$ in the q axis the flux flows in the air gap

in the d axis the flux is inside the iron

$$\begin{cases} v_d = Ri_d + L_d\frac{di_d}{dt} - \omega L_q i_q \\ v_q = Ri_q + L_q\frac{di_q}{dt} + \omega L_d i_d + \omega\Lambda_m \end{cases}$$

$$\begin{cases} v_d i_d = R i_d^2 + i_d L_d \dfrac{di_d}{dt} - \omega L_q i_d i_q \\ v_q i_q = R i_q^2 + i_q L_q \dfrac{di_q}{dt} + \omega \Lambda_m i_q + \omega L_d i_d i_q \end{cases}$$

$L i \dfrac{di}{dt} = \dfrac{d}{dt}\left(\dfrac{1}{2} L i^2\right)$

$\downarrow +,\ \times \frac{3}{2}$ to get POWER

$$\frac{3}{2}\left(v_d i_d + v_q i_q\right) = \frac{3}{2} R\left(i_d^2 + i_q^2\right) + \frac{3}{2}\left[\frac{d}{dt}\left(\frac{1}{2} L_d i_d^2 + \frac{1}{2} L_q i_q^2\right)\right] + \frac{3}{2}\omega\left(\Lambda_m i_q + L_d i_q i_d - L_q i_d i_q\right)$$

P_{in} — Joule losses — Magnetic energy variation — P_m

$$P_m = \frac{3}{2}\omega\left[\Lambda_m i_q + (L_d - L_q) i_d i_q\right]$$

SALIENCY $\xi = \dfrac{L_q}{L_d} = 2 \div 8$

$$T_{em} = \frac{P_m}{\omega_m} = \frac{3}{2} p \left[\underbrace{\Lambda_m i_q}_{\text{PM component}} + \underbrace{(L_d - L_q) i_d i_q}_{\text{Reluctance Torque component}}\right]$$

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IPM MOTOR LIMIT OPERATING CONDITION

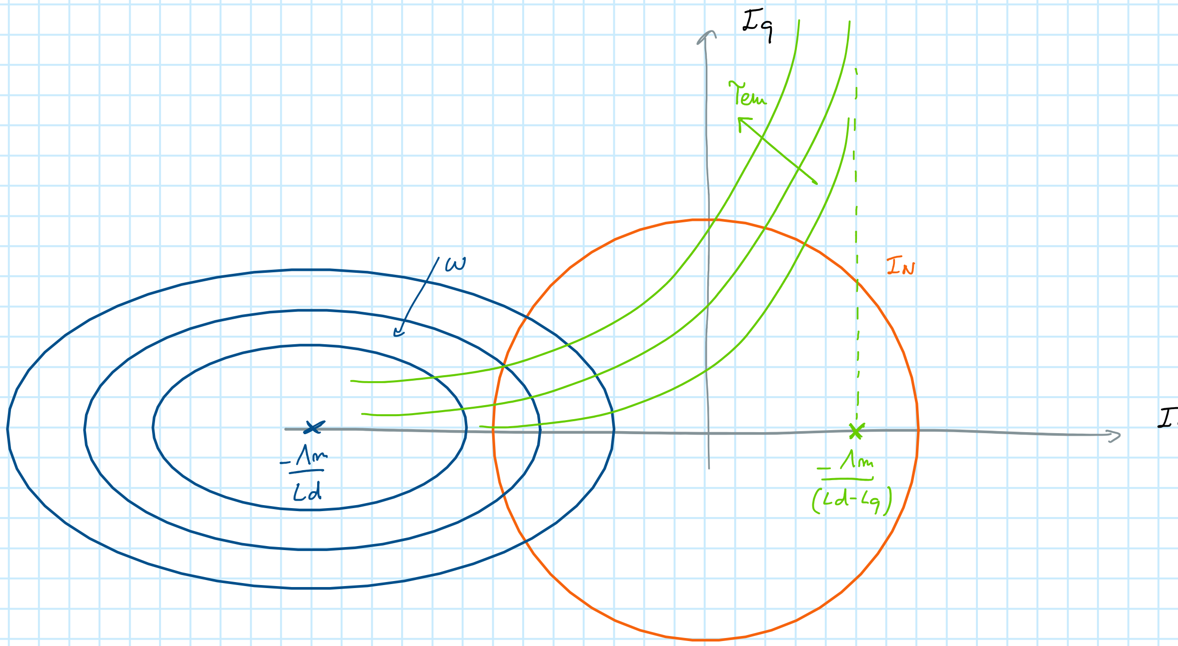
1. $I_d^2 + I_q^2 = I_N^2$

2. $V_d^2 + V_q^2 = V_N^2$

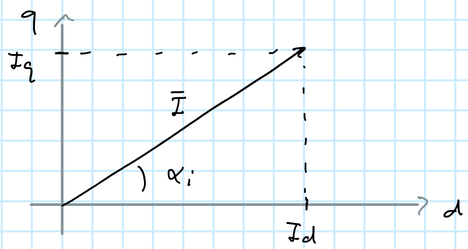
at high speed $R \simeq 0 \rightarrow (\omega \Lambda_q)^2 + (\omega \Lambda_d)^2 = V_N^2$

$$(L_q I_q)^2 + (\Lambda_m + L_d I_d)^2 = \left(\frac{V_N}{\omega}\right)^2$$

3. $$T_{em} = \frac{3}{2} p \left[\Lambda_m I_q + (L_d - L_q) I_q I_d \right] = \frac{3}{2} p \left[\Lambda_m + (L_d - L_q) I_d \right] \cdot I_q$$



$\boxed{\text{MTPA :}}$ Maximum Torque Per Ampere, at zero speed the Voltage is zero → no V_N limit



$$T_{em} = \frac{3}{2} p \left[\Lambda_m I_N \sin\alpha_i + (L_d - L_q)\, I_N^2 \sin\alpha_i \cos\alpha_i \right]$$

$$\frac{d T_{em}}{d\alpha_i} = 0 \rightarrow \Lambda_m \cancel{I_N} \cos\alpha_i + (L_d - L_q) I_N^{\cancel{2}} \left(-\underbrace{\sin^2\alpha_i}_{1-\cos^2\alpha_i} + \cos^2\alpha_i \right) = 0$$

$$\Lambda_m \cos\alpha_i + (L_d - L_q) I_N (2\cos^2\alpha_i - 1) = 0$$

$$\left[2 \cdot (L_d - L_q) I_N \right] \cos^2\alpha_i + \Lambda_m \cos\alpha_i + \left[-(L_d - L_q) I_N \right] = 0$$

MTPA $\longrightarrow$ $\cos\alpha_i = \dots$ solution in the II quadrant $\rightarrow$ $\alpha_i \geq 90°$

$\Rightarrow \boxed{\cos\alpha_i \leq 0}$

- Particular case ; synchronous reluctance motor ($\Lambda_m = 0$)

$$\cos\alpha_i = \frac{-\sqrt{8}}{4} = -\frac{\sqrt{2}}{2} \longrightarrow \alpha_i = 90 + 45° = 135°$$

⋮

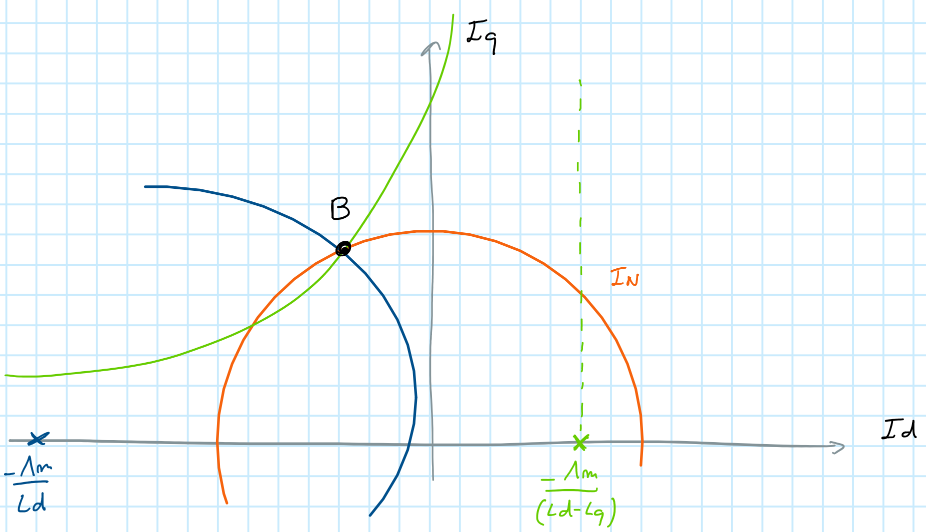
$$\begin{aligned} I_d &= I_N \cos\alpha_{i_{MTPA}} \\ I_q &= I_N \sin\alpha_{i_{MTPA}} \end{aligned} \longrightarrow \begin{aligned} V_d &= -\omega\Lambda_q = -\omega L I_q \\ V_d &= +\omega\Lambda_d = \omega(\Lambda_m + L_d I_d) \end{aligned}$$

$$V = \sqrt{V_d^2 + V_q^2} = \omega\sqrt{\Lambda_d^2 + \Lambda_q^2} = \omega\Lambda$$

Λ is fixed by the current

$$V \leq V_N \longrightarrow \boxed{\omega_B \leq \frac{V_N}{\Lambda}}$$

speed in BASE POINT



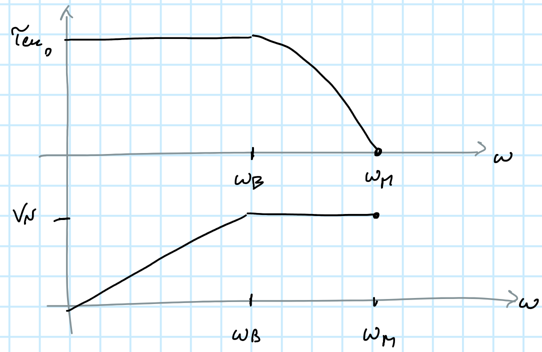
If $V = V_N$ is fixed , the ellipse collapses with increasing speed

→ Field weakening

→ Field weakening

$$V_N = \omega \Lambda$$

$$\Lambda = \begin{cases} \Lambda_q = L_q I_q & \longrightarrow \text{Lower } I_q \\ \Lambda_d = \Lambda_m + L_d I_d & \longrightarrow \text{more negative } I_d \end{cases} \Bigg\} \; \underline{\text{Torque DECREASES}}$$



maximum speed : $I_d = -I_N$, $I_q = 0$

$$\boxed{\omega_M = \frac{V_N}{\Lambda_m - L_d I_N}}$$

26/11 mtpa ipm

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EXERCISE

IPM motor

$\Lambda_m = 0.4\ Vs$

$L_d = 15.9\ mH$

$L_q = 21.2\ mH$

$2p = 4$

$R \simeq 0$

$V_N = 210\,V$
$I_N = 20\,A$ } peak value

Determine

- $T_{em\,max}$
- $\omega(T_{em,max}) = \omega_B$ (MTPA base point)
- ω_M maximum speed
- T_{em} at $\omega_m = 2200$ rpm

MTPA : only I_N limit

$$\cos\alpha_i = \frac{-\Lambda_m + \sqrt{\Lambda_m^2 + 8(L_d - L_q)^2 I_N^2}}{4(L_d - L_q) I_N} = -0.236 \longrightarrow \alpha_i = 103.6°$$

$$\begin{cases} I_d = I_N \cos\alpha_i = -4.72\ A \\ I_q = I_N \sin\alpha_i = 19.44\ A \end{cases} \qquad \begin{cases} \Lambda_d = \Lambda_m + L_d I_d = 0.325\ Vs \\ \Lambda_q = L_q I_q = 0.412\ Vs \end{cases}$$

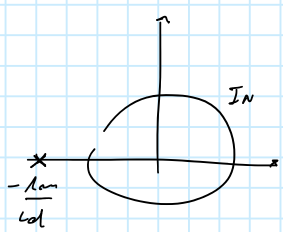
$$T_{em} = \frac{3}{2} p \left(\Lambda_d I_q - \Lambda_q I_d \right) = 24.78\ Nm$$

$$R \simeq 0 \rightarrow V = \omega\Lambda \longrightarrow \omega_B = \frac{V_N}{\Lambda_B} = \frac{210}{\sqrt{0.325^2 + 0.412^2}} = 400\ rad/s$$

$$\rightarrow \omega_{mB} = \frac{\omega_B}{p} \cdot \frac{60}{2\pi} = 1910\ rpm$$

maximum speed :

$$\frac{\Lambda_m}{L_d} = \frac{0.4}{0.015\,A} = 25.16\ A > I_N \rightarrow$$



$$\begin{cases} I_d = -I_N \\ I_q = 0 \end{cases}$$

$$\begin{cases} \Lambda_d = \Lambda_m - L_d I_N = \Lambda_M \\ \Lambda_q = 0 \end{cases} \longrightarrow \omega_M = \frac{V_N}{\Lambda_M} = 2560 \ \frac{rad}{s}$$

$$\omega_{m_M} = \frac{\omega_M}{p} \frac{60}{2\pi} = 12230 \ rpm$$

T_{em} at 2200 rpm :

$$\omega = \omega_m \cdot p \cdot \frac{2\pi}{60} = 460 \ \frac{rad}{s} = \frac{V_N}{\Lambda} \longrightarrow \Lambda = \frac{V_N}{\omega} = 0.456 \ Vs$$

$$\Lambda = \sqrt{\Lambda_d^2 + \Lambda_q^2} = \sqrt{(\Lambda_m + L_d I_N \cos\alpha_i)^2 + (L_q I_N \sin\alpha_i)^2}$$

$$\longrightarrow \cos\alpha_i = -0.455 \longrightarrow \alpha_i = 117^\circ$$

$$\begin{cases} I_d = -9.08 \ A \\ I_q = 17.82 \ A \end{cases}$$

$$\begin{cases} \Lambda_d = 0.256 \ Vs \\ \Lambda_q = 0.378 \ Vs \end{cases} \longrightarrow T_{em} = \frac{3}{2} p \left(\Lambda_d I_q + \Lambda_q I_d \right) = 24 \ Nm$$

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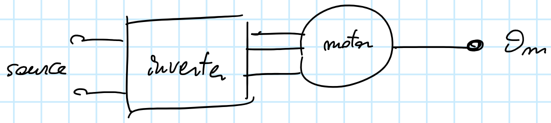
We want to remove the position sensor



RECOGNIZE THE INITIAL POSITION

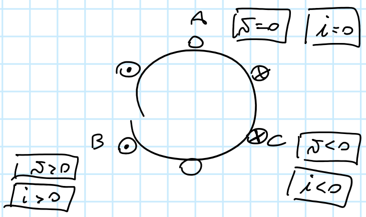
① SPM ROTOR:

$L = L_d = L_q$ isotropic rotor

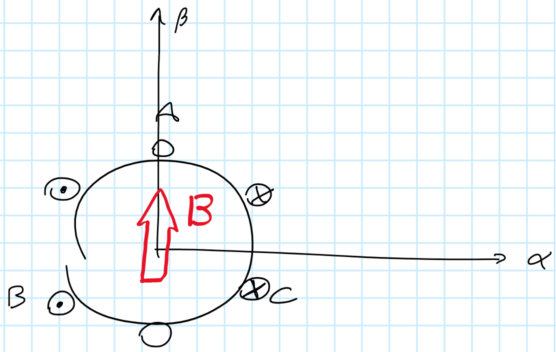
$T_{em} = \frac{3}{2} p \Lambda_m I_q$

- Assumption : $T_L = 0$

→ Voltages are imposed in the stator winding to have $\begin{cases} v_\alpha = 0 \\ v_\beta = V \end{cases}$



$v = Ri + L\frac{di}{dt}$, after transient $\begin{cases} i_\alpha = 0 \\ i_\beta = \frac{V}{R} \end{cases}$

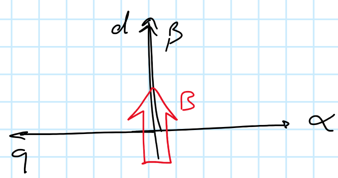


if $T_L = 0$, $T_{em} = 0$ → $\boxed{I_q = 0}$

$\begin{cases} I_q = I_\alpha = 0 \\ I_d = I_\beta \end{cases}$

$\boxed{\text{The } dq \text{ axis is aligned with the } \alpha\beta \text{ axis}}$

→ $\boxed{\vartheta_m^e = 90°}$



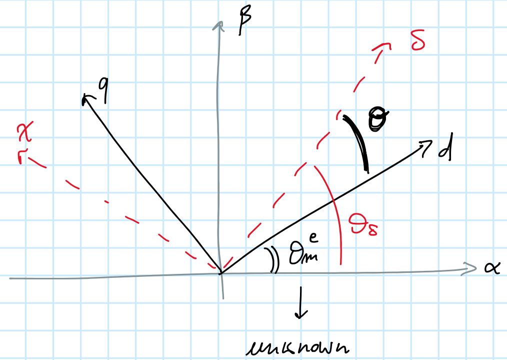
② IPM ROTOR:

$L_d \neq L_q$

standstill rotor : $\omega_m = 0$

rotor equations: $\begin{cases} v_d = R i_d + L_d \dfrac{di_d}{dt} \\ v_q = \quad '' \end{cases}$ $\qquad \begin{cases} \omega \lambda_q = 0 \\ \omega \lambda_d = 0 \end{cases}$

$$\bar{v}_{dq} = R\,\bar{i}_{dq} + [L_{dq}]\,\frac{d\,\bar{i}_{dq}}{dt}$$



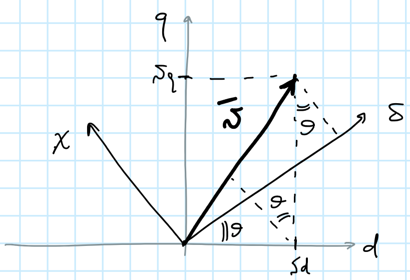
when $\vartheta = 0$ the δx is aligned with the dq reference frame

$\vartheta = \vartheta_\delta - \vartheta_m^e$

- We inject a signal on the δx frame to recognize the position of dq, that gives ϑ_m^e

$$\bar{v}_{\delta x} = \bar{v}_{dq} \cdot e^{-j\vartheta} = (v_d + j v_q)(\cos\vartheta - j\sin\vartheta) = (v_d\cos\vartheta + v_q\sin\vartheta) + j(v_q\cos\vartheta - v_d\sin\vartheta)$$

$$\begin{bmatrix} v_\delta \\ v_x \end{bmatrix} = \begin{bmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{bmatrix} \cdot \begin{bmatrix} v_d \\ v_q \end{bmatrix} \qquad \begin{bmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{bmatrix} = T$$



$$\bar{v}_{\delta x} = T\,\bar{v}_{dq}$$

$$\Rightarrow \bar{v}_{dq} = T^{-1}\,\bar{v}_{\delta x}$$

$$\Rightarrow T^{-1}\,\bar{v}_{\delta x} = R\,T^{-1}\,\bar{i}_{\delta x} + [L_{dq}] \cdot \frac{d}{dt}\left(T^{-1}\,\bar{i}_{\delta x}\right)$$

The rotor is still $\rightarrow \vartheta = \text{constant} \Rightarrow \dfrac{d}{dt} T^{-1} = 0$

$$\bar{v}_{\delta x} = R\,\bar{i}_{\delta x} + T\,[L_{dq}]\,T^{-1}\,\frac{d}{dt}\,\bar{i}_{\delta x}$$

- $[L_{\delta x}] = T\,[L_{dq}]\,T^{-1}$

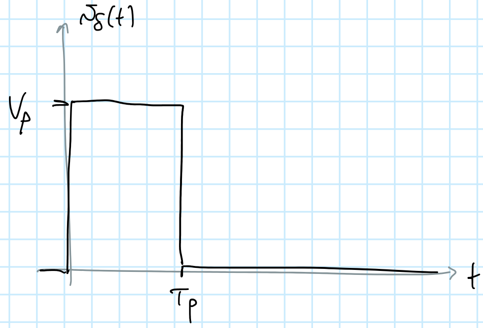
$$= \begin{bmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{bmatrix} \begin{bmatrix} L_d & 0 \\ 0 & L_q \end{bmatrix} \begin{bmatrix} \cos\vartheta & -\sin\vartheta \\ \sin\vartheta & \cos\vartheta \end{bmatrix} = \begin{bmatrix} L_d\cos^2\vartheta + L_q\sin^2\vartheta & \dfrac{L_q - L_d}{2}\sin 2\vartheta \\ \dfrac{L_q - L_d}{2}\sin 2\vartheta & L_d\sin^2\vartheta + L_q\cos^2\vartheta \end{bmatrix}$$

mutual interaction

between δx axis

- $[L_{\delta x}] = [L_{\delta x}(\theta)]$ is a function of the position θ

→ Apply a voltage pulse along the δ-axis:



T_p short $\longrightarrow \left| R\, \bar{i}_{\delta x} \right| < \left| [L_{\delta x}] \dfrac{d\,\bar{i}_{\delta x}}{dt} \right|$

the drop can be approximated to be only inductive ~ no change in the current

- **EXTRACTION OF THE CURRENT**

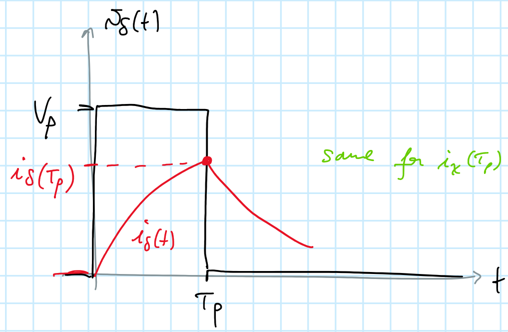
$$\frac{d\,\bar{i}_{\delta x}}{dt} = [L_{\delta x}]^{-1} \cdot \bar{v}_{\delta x} \qquad v_\delta = \begin{cases} V_d & \text{during } T_p \\ 0 & \text{elsewhere} \end{cases}, \quad v_x = 0$$

$$\bar{i}_{\delta x} = [L_{\delta x}]^{-1} \cdot \int_0^t \bar{v}_{\delta x}\, dt$$

$$\bar{i}_{\delta x}(T_p) = [L_{\delta x}]^{-1} \cdot \int_0^{T_p} \bar{v}_{\delta x}\, dt = [L_{\delta x}]^{-1} \cdot \begin{bmatrix} V_p \\ 0 \end{bmatrix} \cdot T_p$$

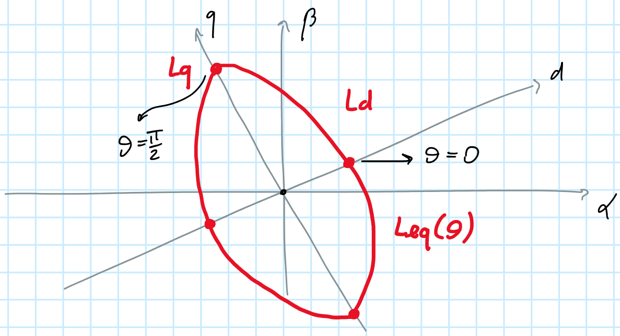
$$= \begin{bmatrix} \dfrac{\cos^2\theta}{L_d} + \dfrac{\sin^2\theta}{L_q} & \dfrac{\sin 2\theta}{2}\,\dfrac{L_d - L_q}{L_d L_q} \\ \dfrac{\sin 2\theta}{2}\,\dfrac{L_d - L_q}{L_d L_q} & \dfrac{\cos^2\theta}{L_q} + \dfrac{\sin^2\theta}{L_d} \end{bmatrix} \cdot \begin{bmatrix} V_p \\ 0 \end{bmatrix} \cdot T_p$$

$$\begin{cases} i_\delta(T_p) = \left(\dfrac{\cos^2\theta}{L_d} + \dfrac{\sin^2\theta}{L_q} \right) V_p T_p \\ i_x(T_p) = \left(\dfrac{\sin 2\theta}{2}\,\dfrac{L_d - L_q}{L_d L_q} \right) V_p T_p \end{cases}$$

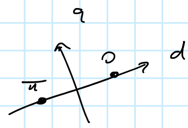


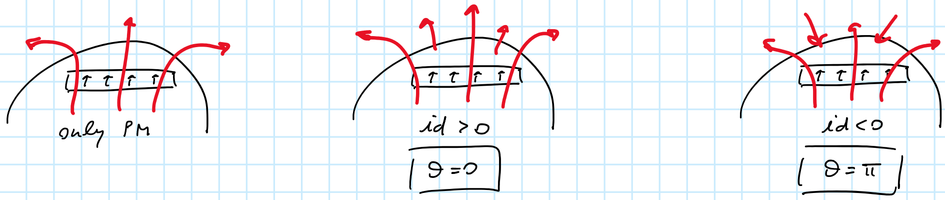
$$\left| \bar{i}_{\delta x}(T_p) \right| = \sqrt{i_\delta(T_p)^2 + i_x(T_p)^2} = V_p T_p \cdot \sqrt{\frac{\cos^2\theta}{L_d^2} + \frac{\sin^2\theta}{L_q^2}}$$

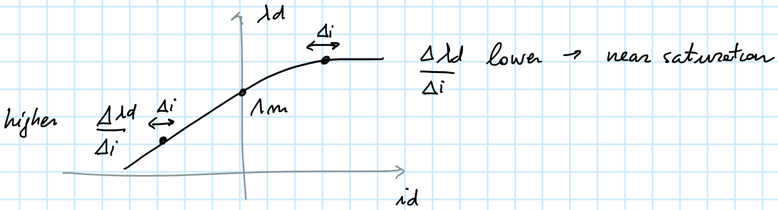
$$\boxed{\left| \bar{i}_{\delta x}(T_p) \right| = \frac{V_p T_p}{L_{eq}(\theta)}} \qquad \boxed{L_{eq}(\theta) = \frac{L_d L_q}{\sqrt{L_d^2 \sin^2\theta + L_q^2 \cos^2\theta}}}$$



HOW TO RECOGNIZE $\vartheta = 0$ or $\vartheta = \pi$



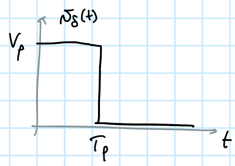




EXERCISE

$L_d = 100\,mH$

$L_q = 400\,mH$



$V_p = 60V$

$T_p = 10ms$

$\vartheta_\delta = 30°$

- $i_\delta, i_\chi \, (T_p)$?
- $|i_{\delta\chi}|$?
- $L_{eq}(\vartheta)$?

$$i_\delta(T_p) = \left(\frac{\cos^2 30°}{0.100} + \frac{\sin^2 30°}{0.400} \right) 60 \cdot 0.010 = 0.8125\,A$$

$$i_\chi(T_p) = \left(\frac{\sin 2 \cdot 30°}{2} \cdot \frac{0.100 - 0.400}{0.1 \cdot 0.4} \right) 60 \cdot 0.010 = -0.325A$$

$$|\bar{i}_{\delta\chi}(T_p)| = \sqrt{0.8125^2 + 0.325^2} = 0.875\,A$$

$$|\bar{i}_{s\alpha}(T_p)| = \sqrt{0.8125^2 + 0.325^2} = 0.875\,A$$

$$L_{eq}(30^\circ) = \frac{V_p T_p}{|i_{s\alpha}|} = \frac{10 \cdot 0.010}{0.875} = 114\ mH$$

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giovedì 17 dicembre 2020 08:51

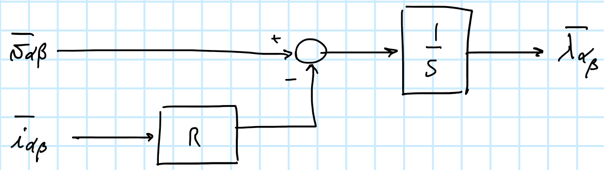
NO POSITION SENSOR

Recognize the position of the rotor from the flux linkage:

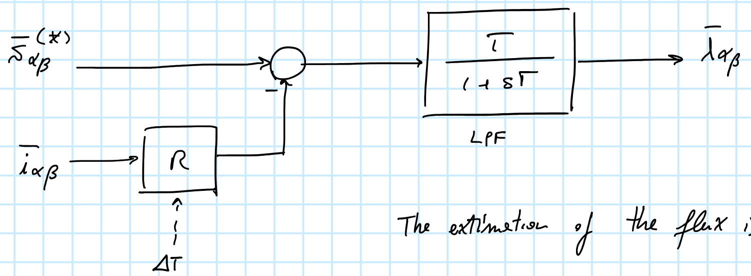
$$\bar{\lambda}_{\alpha\beta} = \int \left(\bar{v}_{\alpha\beta} - R\,\bar{i}_{\alpha\beta} \right) dt$$

Performance of the computation:

- accuracy from hall sensor
- switching ripple / noise → usually reconstructed from V_{DC} + state inverter
- drift of the integral due to DC offset
- don't know the initial condition $\lambda(0) = \lambda_m(0)$, $i(0) = 0$
- parameter accuracy: $R_{t20} = R_{20}(1 + \alpha \Delta T)$, $\alpha \Delta T = 40\%$

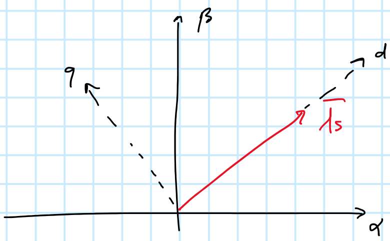


PRACTICAL IMPLEMENTATION



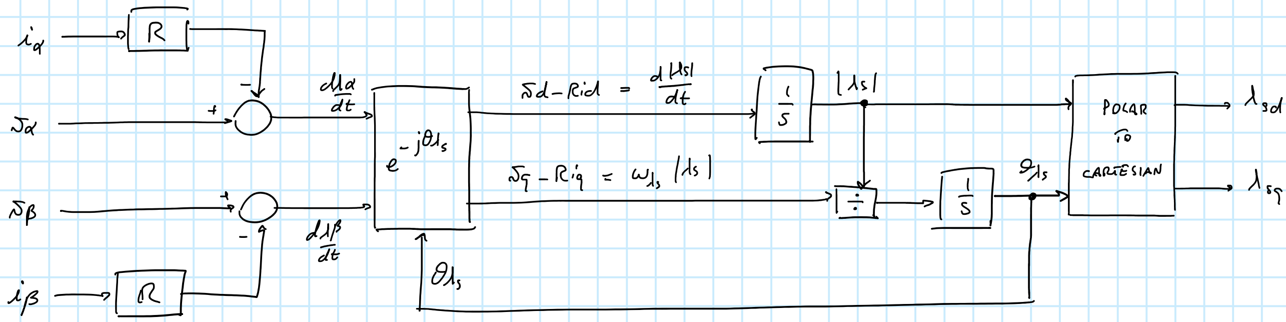
The estimation of the flux is accurate for $\omega > \frac{1}{\tau}$

STATOR LINKAGE



$$\bar{\lambda}_s = \begin{cases} \lambda_{sd} = |\lambda_s| \\ \lambda_{sq} = 0 \end{cases}$$

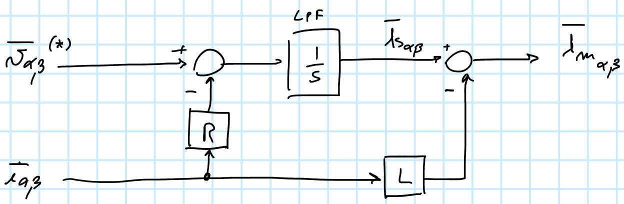
$$\bar{v}_{dq} = R\,\bar{i}_{dq} + \frac{d\bar{\lambda}_{dq}}{dt} + j\omega\bar{\lambda}_{dq} \rightarrow \begin{cases} v_d = R i_d + \frac{d|\lambda_s|}{dt} \\ v_q = R i_q + \omega_{\lambda_s} |\lambda_s| \end{cases}$$



SPM motor has permanent magnet

$\bar{\lambda}_s = \bar{\lambda}_m + L\bar{i}$

Recognized by voltage and current integration



- $\bar{\lambda}_m = \Lambda_m e^{j\vartheta_m^e} = \bar{\lambda}_s - L\bar{i}$

EXERCISE

steady state (sinusoidal)

$\Lambda_m = ?$

$R = 3.05\,\Omega$

$L = 17.6\,mH$

$\bar{v} = 100\, e^{j(\omega t + \frac{2}{3}\pi)}$

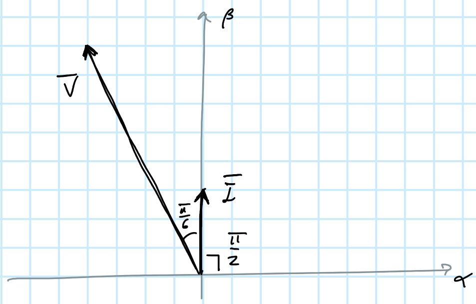
$\bar{i} = 12\, e^{j(\omega t + \frac{\pi}{2})}$

$\alpha\beta$ ref.

$\omega = 100\ rad/s$

$\bar{V} = 100\, e^{j\frac{2}{3}\pi} = 100\, e^{j(\frac{\pi}{2} + \frac{\pi}{6})}$

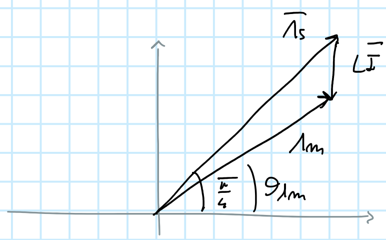
$\bar{I} = 12\, e^{j\frac{\pi}{2}}$



$j\omega\bar{\Lambda}_s = \bar{V} - R\bar{I} = 100\, e^{j(\frac{\pi}{2}+\frac{\pi}{6})} - 36.6\, e^{j\frac{\pi}{2}} = -50 + j50$

$\bar{\Lambda}_s = \frac{\bar{V} - R\bar{I}}{j\omega} = \frac{-50 + j50}{j100} = 0.5 + j0.5$

$\bar{\Lambda}_m = \bar{\Lambda}_s - L\bar{I} = 0.5 + j0.289$



$\vartheta_{\lambda_m} = \vartheta_m^e$

$\vartheta_{\lambda_m} = \operatorname{atan}\frac{0.289}{0.5} = \frac{\pi}{6}$

$|\Lambda_m| = \sqrt{0.5^2 + 0.289^2} = 0.578\ Vs$

$\bar{\Lambda}_m = |\Lambda_m|\, e^{j\vartheta_{\lambda_m}} = 0.578 \cdot e^{j(\omega t + \frac{\pi}{6})}$

$$\bar{\lambda}_m = |\lambda_m| \, e^{j \vartheta_{\lambda_m}} = 0.578 \cdot e^{j(\omega t + \frac{\pi}{6})}$$

EXTIMATION OF THE SPEED FROM λ_s

$$\omega_m^e = \frac{d \vartheta_{\lambda_s}}{dt} \qquad \vartheta_{\lambda_s} = \operatorname{atan}\left(\frac{\lambda_{s\beta}}{\lambda_{s\alpha}}\right) \qquad \bar{\lambda}_{s_{\alpha\beta}} = \lambda_{s\alpha} + j \lambda_{s\beta}$$

$$\frac{d}{dx} \operatorname{atan}(x) = \frac{1}{1+x^2}$$

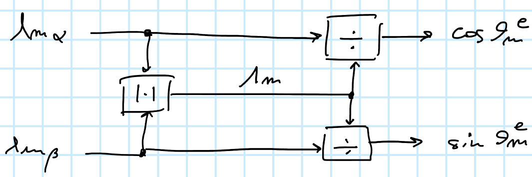
$$\frac{d}{dt} \operatorname{atan}(f(t)) = \frac{1}{1+f(t)^2} \cdot \frac{d}{dt} f(t)$$

$$\omega_m^e = \frac{1}{1 + \left(\frac{\lambda_{s\beta}}{\lambda_{s\alpha}}\right)^2} \cdot \frac{d}{dt}\left(\frac{\lambda_{s\beta}}{\lambda_{s\alpha}}\right) = \frac{\cancel{\lambda_{s\alpha}^2}}{\lambda_{s\alpha}^2 + \lambda_{s\beta}^2} \cdot \frac{\lambda_{s\beta}' \lambda_{s\alpha} - \lambda_{s\alpha}' \lambda_{s\beta}}{\cancel{\lambda_{s\alpha}^2}} = \frac{\lambda_{s\alpha} \lambda_{s\beta}' - \lambda_{s\beta} \lambda_{s\alpha}'}{\lambda_{s\alpha}^2 + \lambda_{s\beta}^2}$$

- in digital : $\frac{d}{dt} \lambda_{s\alpha} = \frac{\lambda_{s\alpha}[k] - \lambda_{s\alpha}[k-1]}{T_s}$

$$\Rightarrow \omega_m^e = \frac{\lambda_{s\alpha}[k-1]\, \lambda_{s\beta}[k] - \lambda_{s\alpha}[k]\, \lambda_{s\beta}[k-1]}{T_s \left(\lambda_{s\alpha}^2[k] + \lambda_{s\beta}^2[k]\right)}$$

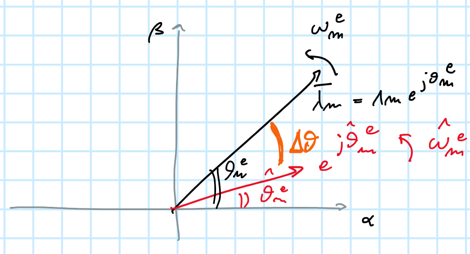
$\boxed{\hat{\vartheta}_m^e}$ MATH APPROACH :



$$\vartheta_m^e = \operatorname{atan}\left(\frac{\sin\vartheta}{\cos\vartheta}\right)$$

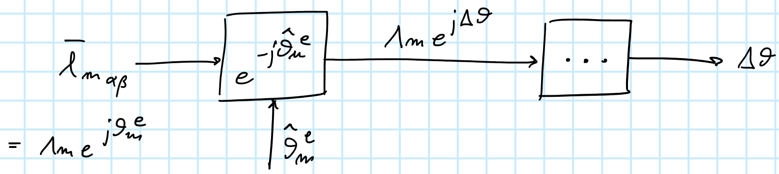
$$\omega_m^e = \frac{d \vartheta_m^e}{dt}$$

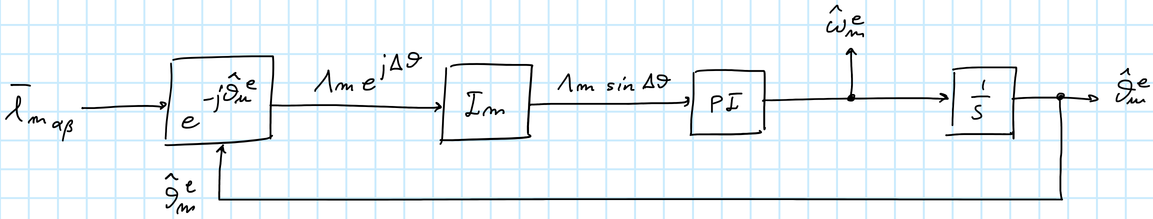
Implementation approach : PLL PHASE-LOCKED-LOOP



If $\Delta\vartheta > 0$, $\hat{\omega}_m^e$ must increase

If $\Delta\vartheta < 0$, $\hat{\omega}_m^e$ must decrease





Linearization: $\sin\Delta\vartheta \simeq \Delta\vartheta$

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SYSTEM DESCRIPTION IN STATE-SPACE

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases}$$

x: state

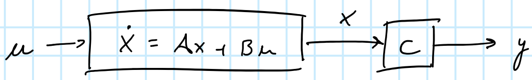
A: system matrix

B: input matrix

C: output matrix

u: input

y: output



EXAMPLE: DC motor

$$\begin{cases} v = Ri + L\dfrac{di}{dt} + k\Phi\omega_m \\ T_{em} = k\Phi i = b\omega_m + J\dfrac{d\omega_m}{dt} \end{cases} \longrightarrow \begin{cases} \dfrac{di}{dt} = \dfrac{1}{L}\left(v - Ri - k\Phi\omega_m\right) \\ \dfrac{d\omega_m}{dt} = \dfrac{1}{J}\left(k\Phi i - b\omega_m\right) \end{cases}$$

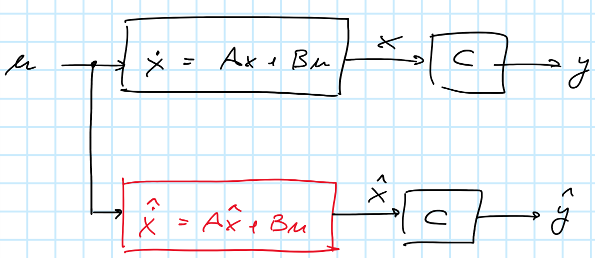
$$\begin{bmatrix} \dfrac{di}{dt} \\ \dfrac{d\omega_m}{dt} \end{bmatrix} = \begin{bmatrix} -\dfrac{R}{L} & -\dfrac{k\Phi}{L} \\ \dfrac{k\Phi}{J} & -\dfrac{b}{J} \end{bmatrix} \cdot \begin{bmatrix} i \\ \omega_m \end{bmatrix} + \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix} v$$

$$\dot{x} = A \cdot x + B \cdot u$$

$$\omega_m = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} i \\ \omega_m \end{bmatrix}$$

$$y = C \cdot x$$

OBSERVER



$\hat{x}$: estimated value of state variables

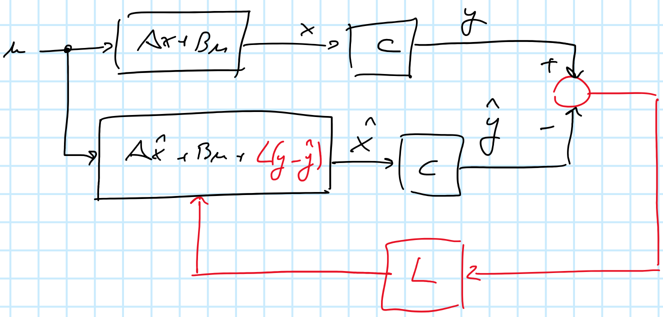
open loop observer :

→ no check of the estimation accuracy

$$x_{err} = x - \hat{x}$$

$$\dot{x}_{err} = \dot{x} - \dot{\hat{x}} = \left[Ax + \cancel{Bu}\right] - \left[A\hat{x} + \cancel{Bu}\right] = A x_{err}$$

CLOSED LOOP OBSERVER



$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y})$$

$$\dot{x}_{err} = \dot{x} - \dot{\hat{x}} = A(x-\hat{x}) - L(Cx - C\hat{x}) = (A - LC)\cdot x_{err}$$

L to have fast and stable convergence $\hat{x} \to x$

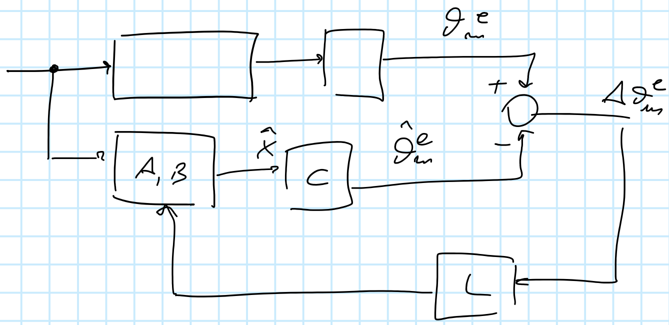
$$L = \begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{bmatrix}$$

$$\det\left[sI - (A - LC)\right] = 0$$

$\beta_1 \; \beta_2 \ldots \qquad (s-\beta_1)(s-\beta_2)\ldots = 0$

$$\Rightarrow \begin{bmatrix} k_1 \\ \vdots \\ k_n \end{bmatrix}$$

ROTOR POSITION OBSERVER OF 2nd ORDER



$$x = \begin{bmatrix} \vartheta_m^e \\ \omega_m^e \end{bmatrix} \qquad \begin{cases} \dot{\vartheta}_m^e = \omega_m^e \\ \dot{\omega}_m^e = p \dfrac{T_{em} - T_L}{J} = 0 \end{cases}$$

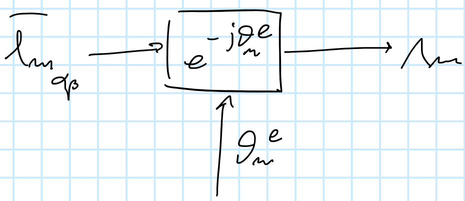
imposed $\rightarrow$ constant speed, no acceleration

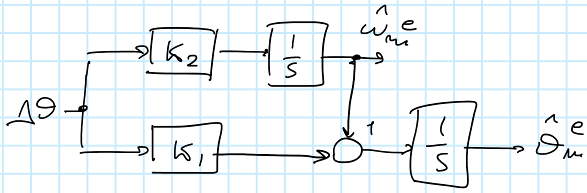
$$\underbrace{\begin{bmatrix} \dot{\hat{\vartheta}}_m^e \\ \dot{\hat{\omega}}_m^e \end{bmatrix}}_{} = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \hat{\vartheta}_m^e \\ \hat{\omega}_m^e \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{B} u + \underbrace{\begin{bmatrix} k_1 \\ k_2 \end{bmatrix}}_{L} \left(\vartheta_m^e - \hat{\vartheta}_m^e \right)$$

$$y = \hat{\vartheta}_m^e = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} \hat{\vartheta}_m^e \\ \hat{\omega}_m^e \end{bmatrix}}_{x}$$

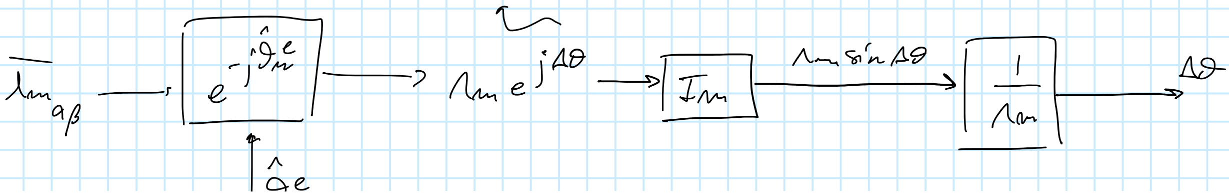
$$\begin{cases} \dot{\hat{\vartheta}}_m^e = \hat{\omega}_m^e + k_1 \left(\vartheta_m^e - \hat{\vartheta}_m^e \right) \\ \dot{\hat{\omega}}_m^e = k_2 \left(\vartheta_m^e - \hat{\vartheta}_m^e \right) \end{cases}$$

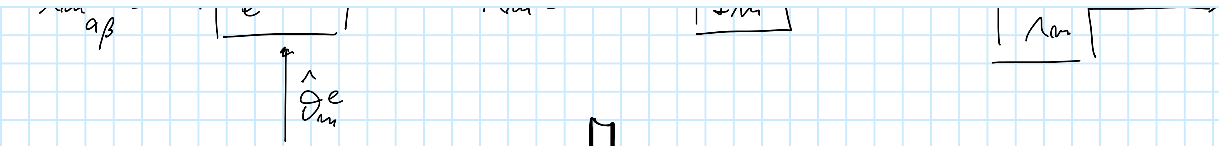
$$\bar{\lambda}_{m_{\alpha\beta}} = \Lambda_m e^{j\vartheta_m^e}$$

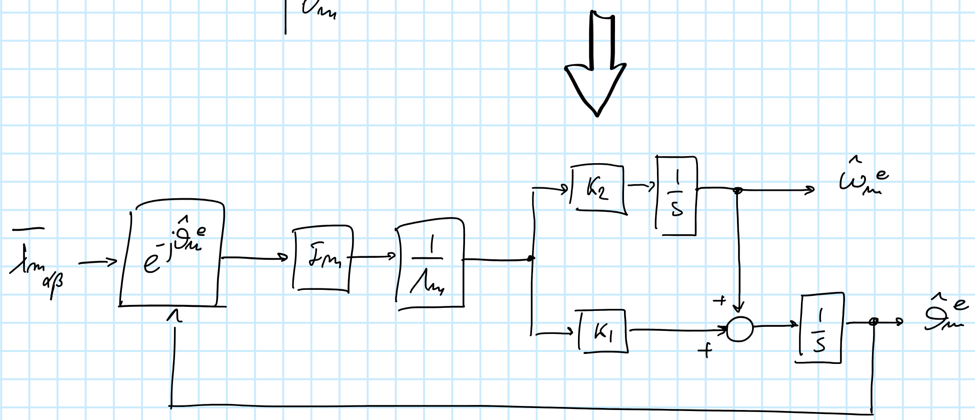




$$\Delta\vartheta = \vartheta_m^e - \hat{\vartheta}_m^e$$







K_2 integral gain
K_1 proportional gain
similar to PLL

DETERMINE K_1, K_2

$$\dot{x}_{err} = A x_{err} - LC x_{err} = (A - LC)\, x_{err}$$

$$\det \left[sI - (A - LC) \right] = 0$$

$$\det \left[\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} \right) \right] = 0$$

$$\det \left[\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} + \begin{bmatrix} K_1 & -1 \\ K_2 & 0 \end{bmatrix} \right] = 0$$

$$\det \begin{bmatrix} s + K_1 & -1 \\ K_2 & s \end{bmatrix} = 0$$

$$\Rightarrow s(s + K_1) + K_2 = 0 \quad \Rightarrow \quad \boxed{s^2 + K_1 s + K_2 = 0}$$

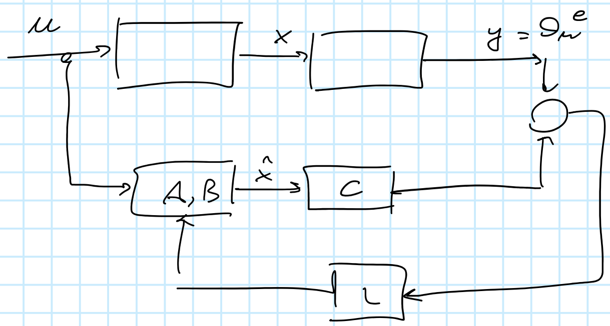
FIXED, I WANT A DOUBLE POLE

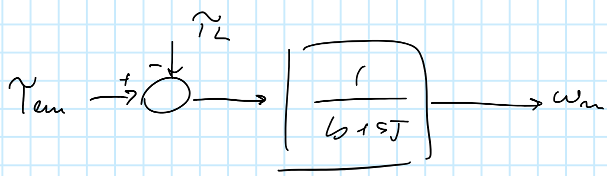
$$f = 10\,Hz, \quad \omega = 60 \, \frac{rad}{s}$$

$$p_1 = p_2 = -60 \longrightarrow (s + 60)^2 = \boxed{s^2 + 120 s + 3600}$$

$$\boxed{\begin{aligned} K_1 &= 120 \\ K_2 &= 3600 \end{aligned}}$$

3RD ORDER OBSERVER





- $\hat{\tau}_{em} = \frac{3}{2} p \Lambda_m \hat{i}_q$

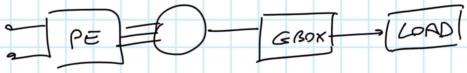
$$x = \begin{bmatrix} \vartheta_m \\ \omega_m \\ \tau_L \end{bmatrix}$$

$$\begin{cases} \dot{\vartheta}_m = \omega_m \\ \dot{\omega}_m = \dfrac{\tau_{em} - \tau_L}{J} - \dfrac{b}{J}\omega_m \\ \dot{\tau}_L = 0 \end{cases} \longrightarrow \begin{bmatrix} \dot{\vartheta}_m \\ \dot{\omega}_m \\ \dot{\tau}_L \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{b}{J} & -\frac{1}{J} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \vartheta_m \\ \omega_m \\ \tau_L \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{J} \\ 0 \end{bmatrix} \tau_{em}$$

$$\begin{bmatrix} \dot{\hat{\vartheta}}_m \\ \dot{\hat{\omega}}_m \\ \dot{\hat{\tau}}_L \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{b}{J} & -\frac{1}{J} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{\vartheta}_m \\ \hat{\omega}_m \\ \hat{\tau}_L \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{J} \\ 0 \end{bmatrix} \hat{\tau}_{em} + \begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix} \left(\vartheta_m^e - \hat{\vartheta}_m^e \right)$$

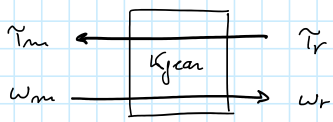
Gearbox

sabato 19 dicembre 2020 17:02



$T_{em} = k\bar{\phi} \cdot I_a$ (DC) $P_J = R_a I_a^2$

$T_{em} = \frac{3}{2} p \Lambda_m I$ (SPM) $P_J = \frac{3}{2} R_s I^2$



$K_{gear} < 1$

$$\begin{cases} T_r = \dfrac{T_m}{K_{gear}} \\ \omega_r = \omega_m \cdot K_{gear} \end{cases}$$

η_{gear} : $$\begin{cases} \omega_r = K_{gear}\,\omega_m \\ T_r = \eta_{gear} \cdot \dfrac{T_m}{K_{gear}} \end{cases}$$

gli ingranaggi girano alla stessa velocita'

la coppia e' ridotta a causa delle perdite

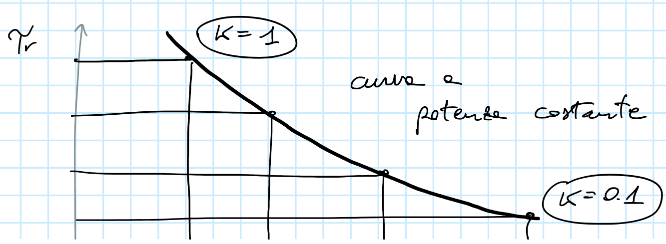
CARICO GENERICO (DINAMICO)

$$T_{carico} = T_r + J_r \frac{d\omega_r}{dt} \longrightarrow T_{motore} = T_m - J_m \frac{d\omega}{dt}$$

$$\left(T_m - J_m \frac{d\omega_m}{dt}\right)\omega_m = \left(T_r + J_r \frac{d\omega_r}{dt}\right)\omega_r$$

$$\boxed{T_m - T_r \cdot K_{gear} = \left(\frac{J_m}{K_{gear}} + K_{gear} J_r\right)\frac{d\omega_r}{dt}}$$

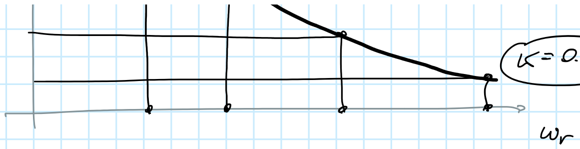
CARICO STATICO



curva a potenza costante

$T_r\,\omega_r = T_m\,\omega_m$

per il motore la relazione



per il motore la relazione è inversa, per K crescenti ω_m diminuisce

$$\boxed{\eta \cdot P_m > P_r}$$

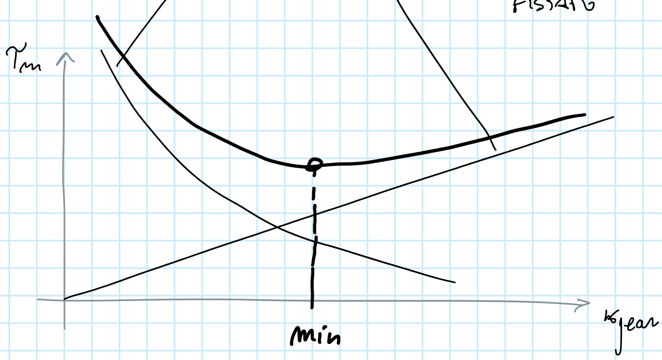
$$\boxed{\begin{aligned} \omega_r &= \omega_m K_{gear_{min}} \\ \tilde{T}_r &= \frac{T_m}{K_{gear_{max}}} \end{aligned}}$$

CARICO DINAMICO PURAMENTE INERZIALE

$$T_{carico} = \cancel{T_K} + J_r \frac{d\omega_r}{dt}$$

$$T_m = \left(\frac{J_m}{K_{gear}} + K_{gear} J_r \right) \frac{d\omega_r}{dt} \qquad \omega_m = \frac{\omega_r}{K_{gear}}$$

FISSATO



$$\frac{d}{dK}\left(\frac{J_m}{K} + K J_r \right) = 0$$

$$\boxed{K_{opt} = \sqrt{\frac{J_m}{J_r}}}$$

$$T_{m,min} = T_m(K_{opt})$$

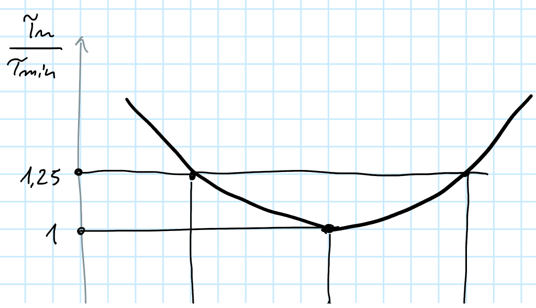
$$\boxed{T_{m,min} = 2\sqrt{J_m J_r}\, \frac{d\omega_r}{dt}}$$

$$\frac{J_m}{K_{opt}} = J_r K_{opt} \rightarrow \boxed{J_m = K_{opt}^2 J_r}$$

Inerzia del carico riportata al motore $= J_r'$

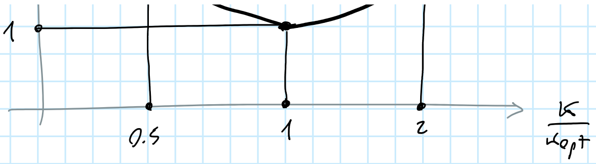
RIDUTTORE NON OTTIMO:

$$\boxed{\frac{\tilde{T}_m}{\tilde{T}_{m,min}} = \frac{1}{2}\left(\frac{K_{opt}}{K} + \frac{K}{K_{opt}} \right)}$$



cambia del 25% "sbagliando" del doppio il rapporto di K

→ varia poco



→ Varia poco

COPPIA DI PICCO

$$K = \frac{\tilde{T}_{pk} \pm \sqrt{\tilde{T}_{pk}^2 - 4 J_m J_r \dot{\omega}_{r\,max}}}{2 \dot{\omega}_{r\,max}} = \begin{cases} K_{min} \\ K_{max} \end{cases}$$

COPPIA NOMINALE

$$K = \frac{\tilde{T}_N \pm \sqrt{\tilde{T}_N^2 - 4 J_m J_r \dot{\omega}_{r\,rms}}}{2 \dot{\omega}_{r\,rms}} = \begin{cases} K_{min} \\ K_{max} \end{cases}$$

$$\boxed{K_{min} = \frac{\omega_{r\,max}}{\omega_{m_N}}}$$

Gearbox es

domenica 20 dicembre 2020 11:52

SPM MOTOR

$R \simeq 0$

$L = 0.01\ H$

$\Lambda_m = 0.2\ Vs$

$2p = 8$

$I_N = 10\ A$

$I_p = 20 A$

$V_N = 100 V$

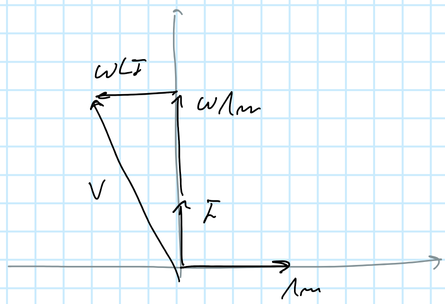
$\tau_r = 100\ Nm$

$\omega_r = 45\ rad/s$

k_{gear} ? $\eta_{gear} = 90\%$

$T_{em} = \frac{3}{2} p \Lambda_m I_q \longrightarrow \tau_N = 12\ Nm$

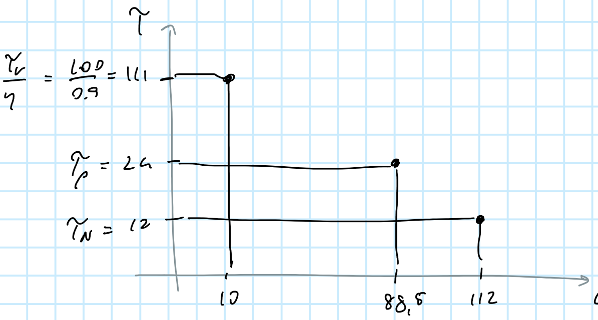
$\tau_p = 24\ Nm$



$$V_N^2 = (\omega \Lambda_m)^2 + (\omega L I)^2$$

$$\omega = \frac{V_N}{\sqrt{\Lambda_m^2 + (L I)^2}}$$

- $I_N = 10$, $\omega_N = 447\ rad/s \longrightarrow \omega_{m_N} = 112\ rad/s$
- $I_p = 20$, $\omega_p = 354\ rad/s \longrightarrow \omega_{m_p} = 88.5\ rad/s$



$P_N = 112 \cdot 12 = 1344\ W$

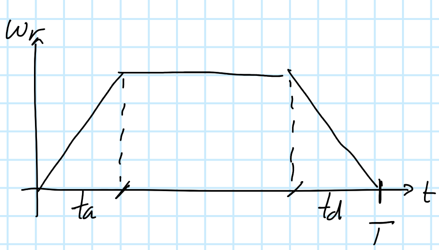
$P_r = 111 \cdot 10 = 1110\ W$

$\frac{\omega_r}{\omega_N} = \frac{10}{112} = 0.0893$

$\frac{\tau_N}{\tau_r} = \frac{12}{111} = 0.108$

$\longrightarrow \boxed{0.0893 \le k_{gear} \le 0.108}$

CARICO PURAMENTE INERZIALE



$J_r = 8 \cdot 10^{-3}\ kg m^2$

$\dot\omega_a = 500 rad/s^2$ $t_a = 400\ ms$

$\dot\omega_d = -400\ rad/s^2$ $T = 2s$

	1	2	3
T_p	5	2.5	1
T_N	2	1.3	0.5
ω_N	1000	1000	1600
J_m	$2e^{-3}$	0.5e-3	0.08e-3

Quale motore e' il più adeguato?

$\omega_{r\,max} = \dot{\omega}_A t_A = 200\ rad/s$

$t_D = 0.5s$

$\dot{\omega}_{r\,max} = \dot{\omega}_A$

$\dot{\omega}_{r\,RMS} = \sqrt{\frac{1}{2}\left(500^2 \cdot 0.4 + 400^2 \cdot 0.5\right)} = 300\ rad/s^2$

$T_A = \dot{\omega}_A J_r = 0.008 \cdot 500 = 4\ Nm$

$T_D = J_r \dot{\omega}_D = 3.2\ Nm$

$T_{r\,max} = T_A \quad , \quad T_{r\,RMS} = \sqrt{\frac{1}{2}\left(4^2 \cdot 0.4 + 3.2^2 \cdot 0.5\right)} = 2.4\ Nm = J_r \dot{\omega}_{rms}$

(1) $k_{opt} = \sqrt{\frac{J_m}{J_r}} = \sqrt{\frac{2}{8}} = 0.5$

$T_{p\,min} \geq 2\sqrt{J_r T_m}\ \dot{\omega}_{r\,max} = 4\ Nm$ ✓

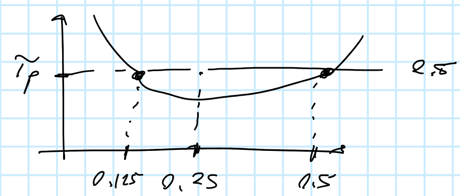
$T_N \geq 2\sqrt{J_r T_m}\ \dot{\omega}_{r\,RMS} = 2.4\ Nm$ ✗

(2) $k_{opt} = \sqrt{\frac{0.5}{8}} = 0.25$

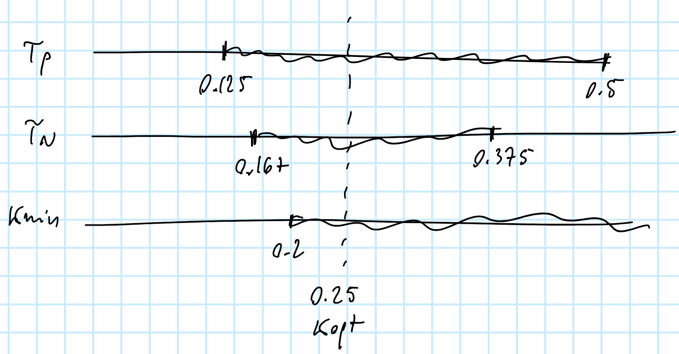
$T_{p\,min} \geq 2\sqrt{J_r T_m}\ \dot{\omega}_{r\,max} = 2\ Nm$ ✓

$T_N \geq 2\sqrt{J_r T_m}\ \dot{\omega}_{r\,RMS} = 1.2\ Nm$ ✓

- T_p : $k_{1,2} = \frac{1}{2}\ \frac{2.5 \pm \sqrt{2.5^2 - 4 \cdot 500^2 \cdot 8 \cdot 0.5 \cdot 10^{-6}}}{500 \cdot 0.008} = \begin{cases} 0.125 \\ 0.5 \end{cases}$



- T_N : $k_{1,2} = \frac{1}{2}\ \frac{1.3 \pm \sqrt{1.3^2 - 4 \cdot 300^2 \cdot 8.0.5 \cdot 10^{-6}}}{300 \cdot 0.008} = \begin{cases} 0.167 \\ 0.375 \end{cases}$
- $\omega_N = 1000\ rad/s \quad , \quad k_{min} = \frac{\omega_{r\,max}}{\omega_N} = \frac{200}{1000} = 0.2$



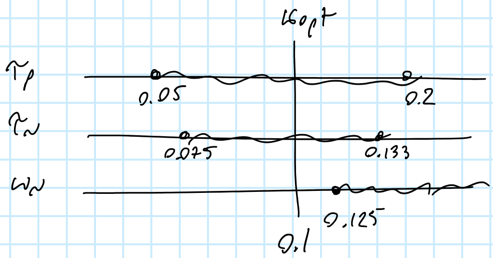
$\boxed{0.2 \leq k_{rean} \leq 0.375}$ per il motore (2)

$$\boxed{0.2 \le K_{gear} \le 0.375}$$ per il motore 2

3 $K_{opt} = 0.1$

$T_p = 9.8\ Nm$ ✓ $\quad T_N = 0.48\ Nm$ ✓ $\quad \omega_N = 2000$ ✗ , però OK fuori dal K_{opt}

$T_p : K_{1,2} < \begin{matrix} 0.05 \\ 0.2 \end{matrix}$ $\quad T_N : K_{1,2} = \begin{matrix} 0.075 \\ 0.133 \end{matrix}$ $\quad \omega_N : K_{min} = \dfrac{\omega_{max}}{\omega_N} = 0.125$



→ fuori dal K_{opt}

$$\boxed{0.125 \le K_{gear} \le 0.133}$$ motore 3

10/12

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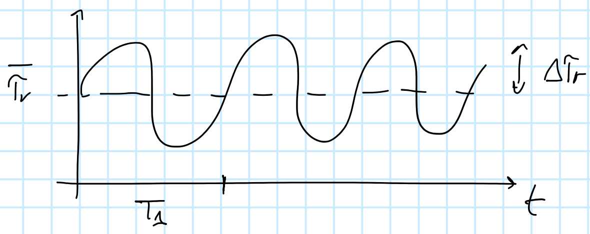
$T_r(t)$



$\overline{T_r} = 50\ Nm$

$\Delta T_r = 40\ Nm$

$T_1 = 2\ s$

$J_r = 0.138\ kgm^2$

$k_{gear} = \frac{1}{4}$

DC motor

$K_\omega = 0.4$

$\omega_{m0} = 200\ rad/s$

$J_m = 7 \cdot 10^{-3}\ kgm^2$

$T_m = K_\omega(\omega_{m0} - \omega_m)$

$\overline{T_r}' = k_{gear}\ \overline{T_r} = 12.5\ Nm$ coppia riportata al motore

$\Delta T_r' = k_{gear}\ \Delta T_r = 10\ Nm$

$J_r' = k_{gear}^2\ J_r = 0.0086\ kgm^2$

$J = J_m + J_r' = 15.625 \cdot 10^{-3}\ kgm^2$

$f_1 = 0.5\ Hz$

$T_r'(t) = \overline{T_r}' + \Delta T_r' \sin(2\pi f_1 t) = 12.5 + 10 \sin(\pi t)$

$$\begin{cases} T_m = K_\omega(\omega_{m0} - \omega_m) \\ T_m = \overline{T_r}'(t) + J\dot{\omega}_m \end{cases} \longrightarrow J\dot{\omega}_m + \overline{T_r}' + \Delta T_r' \sin(\pi t) = K_\omega(\omega_{m0} - \omega_m)$$

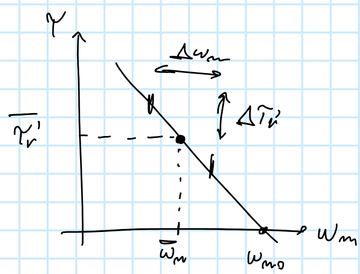
$\downarrow$

the speed is oscillating

$\omega_m = \overline{\omega_m} + \Delta\omega_m \Rightarrow \dot{\omega}_m = \Delta\dot{\omega}_m$

$$J\Delta\dot{\omega}_m + \underline{\overline{T_r}'} + \Delta T_r' \sin(\pi t) = \underline{K_\omega(\omega_{m0} - \overline{\omega_m})} - K_\omega \Delta\omega_m$$

constant part



$$\Delta\dot{\omega}_m + \frac{K_\omega}{J}\Delta\omega_m = -\frac{\Delta T_r'}{J}\sin(\pi t)$$

$\Rightarrow \Delta\omega_m(t) = A\sin(2\pi f_1 t) + B\cos(2\pi f_1 t) + \cancel{D e^{-t/\tau}}$ → we focus on steady state

$\Delta\dot{\omega}(t) = A 2\pi f_1 \cos(2\pi f_1 t) - B 2\pi f_1 \sin(2\pi f_1 t)$

$$\left[A\Omega_1\cos(\Omega_1 t) + B\Omega_1\sin(\Omega_1 t)\right] + \frac{K_\omega}{J}\left[A\sin(\Omega_1 t) + B\cos(\Omega_1 t)\right] = -\frac{\Delta T_r'}{J}\sin(\Omega_1 t)$$

$$\begin{cases} J A \Omega_A = -K_w B \\ -J B \Omega_A + \Delta \tilde{T}_r' = -K_w A \end{cases} \longrightarrow \Delta \omega_m = \sqrt{A^2 + B^2}$$

$$\boxed{\Delta \omega_m = \frac{\Delta \tilde{T}_r'}{\sqrt{J^2 \Omega_A^2 + K_w^2}}} = \underline{\underline{24,81 \text{ rad/s}}}$$

- In this case $J\Omega_A \simeq 0$

=> the oscillation is almost only dependent on the motor characteristics

The requirement is to reduce the oscillation to 1.5% of the average speed:

$$\Delta \omega_m \leq 1.5\% \, \bar{\omega}_m$$

$$\bar{T}_r' = K_w (\omega_{m0} - \bar{\omega}_m) \longrightarrow \bar{\omega}_m = \frac{-\bar{T}_r'}{K_w} + \omega_{m0} = 168,75 \text{ rad/s}$$

$$\Rightarrow \Delta \omega_m \leq 2.53 \text{ rad/s} \quad \underline{\text{REQUIREMENT}}$$

The only method to reduce the oscillations is to increase J by adding a flywheel

$$\Delta \omega_m = \frac{\Delta \tilde{T}_r'}{\sqrt{J^2 \Omega_A^2 + K_w^2}} \longrightarrow J^2 = \frac{\left(\frac{\Delta \tilde{T}_r'}{\Delta \omega_m}\right)^2 - K_w^2}{\Omega_A^2} = 1.567$$

$$J = 1.25 \text{ kgm}^2 = J_m + K_{gear}^2 (J_r + J_{fw})$$

added from before

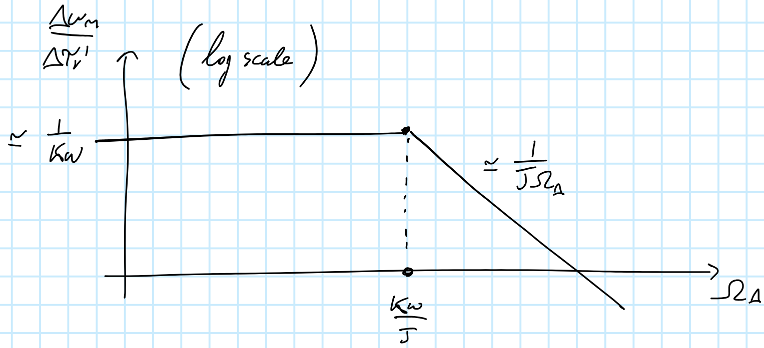
$$J_{fw} = \frac{J - J_m}{K_{gear}^2} - J_r = 19,75 \text{ kgm}^2$$

We cannot add the flywheel on the motor shaft → the flywheel should be on the shaft that presents load variations, which is the load itself

$$\frac{\Delta \omega_m}{\Delta \tilde{T}_r'} = \frac{1}{\sqrt{J^2 \Omega_A^2 + K_m^2}} = \begin{cases} \simeq \frac{1}{K} & \text{if } \Omega_A \ll \frac{K_w}{J} \quad (1) \\ \simeq \frac{1}{J \Omega_A} & \text{if } \Omega_A \gg \frac{K_w}{J} \quad (2) \end{cases}$$

(1) The motor limits the oscillations of the speed

(2) The inertia of the system limits the oscillations

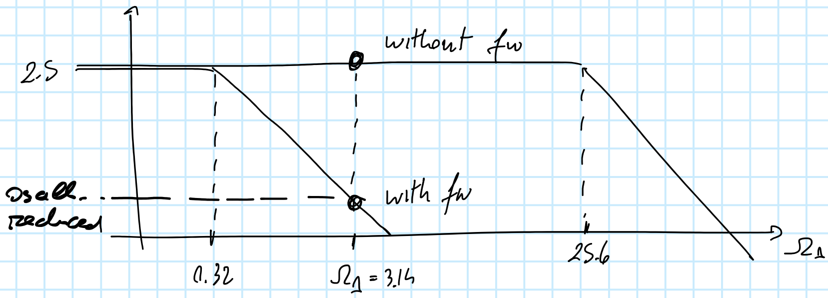


in the exercise:

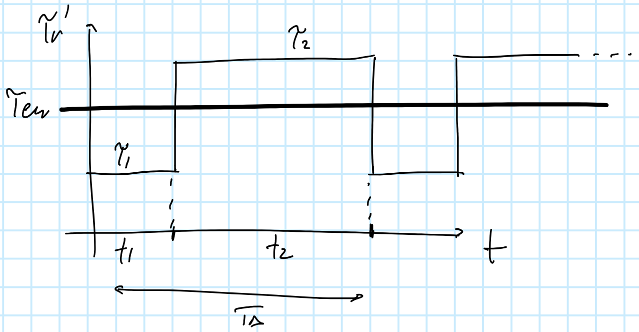
$\Omega_\Delta = 3.14 \ \frac{rad}{s}$

$\frac{1}{K\omega} = 2.5$

$\frac{K\omega}{J} = $ 25.6 (w/o fw) rad/s ; 0.32 (w/ fw) rad/s



The inertia has the same effect of a low pass filter



τ_1 in t_1

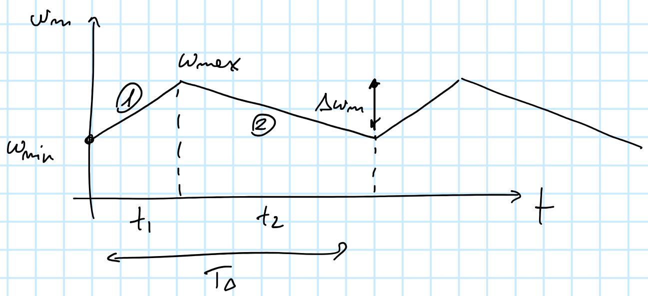
τ_2 in t_2

$t_1 + t_2 = T_\Delta$

τ_{em} not changing with load

$\tau_{em} \rightarrow$ constant (semplification)

[In reality the motor helps in reduce the oscillations by changing its torque]



$\tau_{em} = \tau_r' + J\,\dot{\omega}_m$

$\tau_r' = \tau_r \cdot \kappa_{gear}$

$J = J_m + J_r' + J_{fw}$

$\dot{\omega}_m = \frac{\tau_{em} - \tau_r'}{J} \rightarrow \omega_m = \frac{\tau_{em} - \tau_r'}{J} \int dt$

(1) : $\omega_{min} + \frac{\tau_{em} - \tau_1}{J} \cdot t_1 = \omega_{max}$

(2) : $\omega_{max} + \frac{\tau_{em} - \tau_2}{J} \cdot t_2 = \omega_{min}$

$\Delta\omega = \omega_{max} - \omega_{min} = \frac{\tau_{em} - \tau_1}{J} = \frac{-\tau_{em} + \tau_2}{J} t_2$

$\Longrightarrow \quad (T_{em} - T_1)\, t_1 = (-T_{em} + T_2)\, t_2$

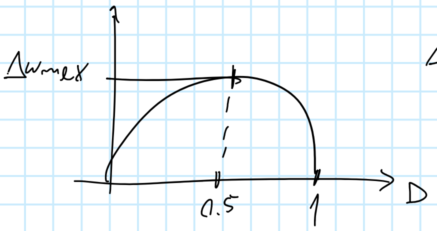
$$\boxed{T_{em} = \frac{T_1 t_1 + T_2 t_2}{t_1 + t_2}}$$ torque average required from the motor

$$\Delta\omega_m = \frac{T_{em} - T_1}{J}\, t_1 = \frac{T_1 t_1 + T_2 t_2 - T_1 T_\Delta}{J} \cdot \frac{t_1}{T_\Delta} =$$

$$\boxed{\Delta\omega_m = \frac{T_2 - T_1}{f_\Delta J} \cdot \frac{t_1 t_2}{T_\Delta^2}}$$

$$\begin{cases} \dfrac{t_1}{T_\Delta} = D \\ \dfrac{t_2}{T_\Delta} = 1 - D \end{cases} \longrightarrow D(1-D) \quad \text{(Duty cycle)}$$

similar to the oscillation of the current in a buck converter

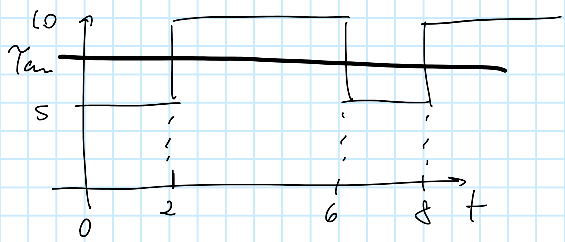


$$\Delta\omega_{max} = \frac{T_2 - T_1}{4 f_\Delta J} \quad \text{for } D = 0.5$$

$$\boxed{\Delta\omega_m = \frac{T_2 - T_1}{J} \cdot \frac{D(1-D)}{f_\Delta}}$$

$$J = \frac{\Delta T_r'}{\Delta\omega_m} = \frac{D(1-D)}{f_\Delta} \qquad J_{max} := \frac{\Delta T_r'}{\Delta\omega_m} \cdot \frac{1}{4 f_\Delta}$$

EXERCISE



$\Delta\omega_m \le 11.1$ rad/s

$T_1 = 5$ Nm, $t_1 = 2$s

$T_2 = 10$ Nm, $t_2 = 4$s

$T_\Delta = 6$s $\longrightarrow D = \frac{t_1}{T_\Delta} = \frac{2}{6} = \frac{1}{3}$

$f_\Delta = \frac{1}{6}$ Hz

$$T_{em} = D T_1 + (1-D) T_2 = \frac{1}{3} 5 + \frac{2}{3} \cdot 10 = \frac{25}{3} = 8.33 \text{ Nm}$$

$$\Delta T_r' = |5 - 10| = 5 \text{ Nm}$$

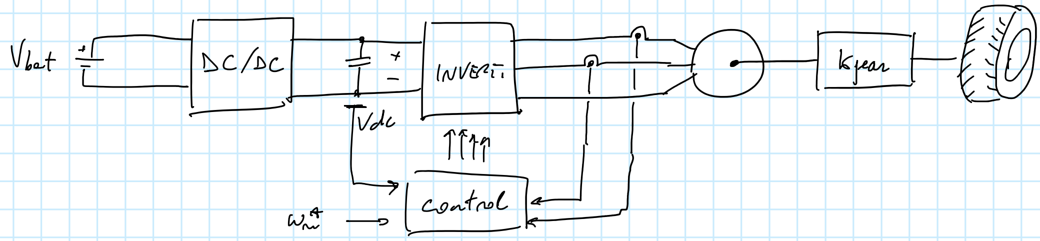
$$J = \frac{\Delta T_r'}{\Delta\omega_m} \cdot \frac{D(1-D)}{f_\Delta} = \frac{5}{11.1} \cdot \frac{\frac{1}{3}\left(\frac{2}{3}\right)}{\frac{1}{6}} = 0.6 \text{ kgm}^2$$

$$J_{max} = \frac{\Delta T_r'}{\Delta\omega_m} \cdot \frac{1}{4 f_\Delta} = 0.68 \text{ kgm}^2 \longrightarrow \underline{\text{worst case}}$$

Esempio

lunedì 21 dicembre 2020 15:03

Electric car : battery, PE, motor, gearbox



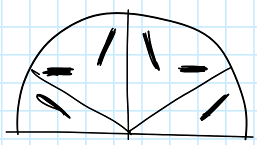
$V_{bat} = 96V$

$V_{dc} : 400V$

max speed = 150 km/h $\rightarrow$ 42 m/s

wheel diameter = 0.4 m $\longrightarrow$ $\omega_{wheel} = \frac{v}{r} = \frac{42}{0.2} = 210$ rad/s

MOTOR (IPM)



Vshape $2p = 8$ (4 in picture)

$\Lambda_m = 0.1 Vs$

$L_d = 0.32$ mH, $L_q = 0.64$ mH $\Big\}$ $\xi = 2$

$I_N = 150A$

$I_p = 300A$

$\omega_{m_{Lim}} = 2000$ rad/s

REQUIREMENTS

① high slope

$\tau_L = 400 Nm$

$\omega_L = 30$ rad/s

② high speed operation

$\tau_L = 120 Nm$

$\omega_L = 200$ rad/s

DCDC is a boost converter

$V_N = \frac{V_{dc}}{\sqrt{3}} = 230V$ V_{DC} = phase to phase peak voltage

Nominal current (MTPA)

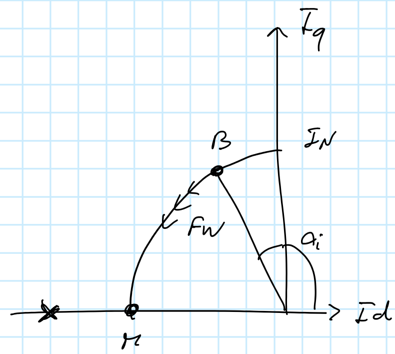
$$\cos\alpha_i = \frac{-\Lambda_m \pm \sqrt{\Lambda_m^2 + 8(L_d-L_q)^2 I_N^2}}{4(L_d-L_q)I_N} = -0.357$$

$I_{dN} = -53.6\,A$ $\qquad \Lambda_d = \Lambda_m + L_d I_{dN} = 0.0828\ Vs$

$I_{qN} = 140.1\,A$ $\qquad \Lambda_q = L_q I_{qN} = 0.0897\ Vs$

$\Lambda = 0.122\ Vs$

- $T_{em} = \frac{3}{2} p \left[\Lambda_d I_q - \Lambda_q I_d \right] = 98.45\ Nm$
- $\omega_B = \left.\frac{V_N}{\Lambda}\right|_{R \simeq 0} = \frac{230}{0.122} = 1884\ rad/s \longrightarrow \omega_{mB} = \frac{\omega_B}{p} = 471\ rad/s$
- $P_N = T_{em}\,\omega_{B_m} = 98.45 \times 471 = 46\,370\ W$



<u>Above nominal speed</u>

$\Lambda_m = 0.1$

$L_d I_N = 0.00032 \cdot 150 = 0.048$

$\Lambda_m > L_d I_N$ outside the circle

- maximum speed: $\omega_M = \frac{V_N}{\Lambda_m - L_d I_N} = 4423\ rad/s$ $\qquad \omega_{m_M} = \frac{\omega_M}{p} = 1106\ rad/s$

$\boxed{\text{Same consideration for the peak values}}$

$\cos\alpha_i = -0.493 \longrightarrow T_{em_p} = 230.7\ Nm$

$\alpha_i = 119.5°$ $\qquad \omega_{B_p} = 1313\ rad/s \longrightarrow \omega_{m_p} = 328\ rad/s$

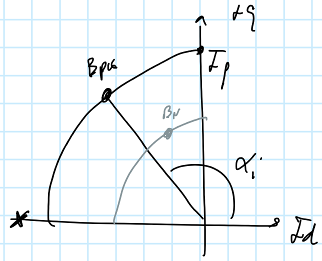
$I_d = -148\,A$ $\qquad P_{peak} = T_{em_p}\,\omega_{m_p} = 75750\ W$

$I_q = 261\,A$

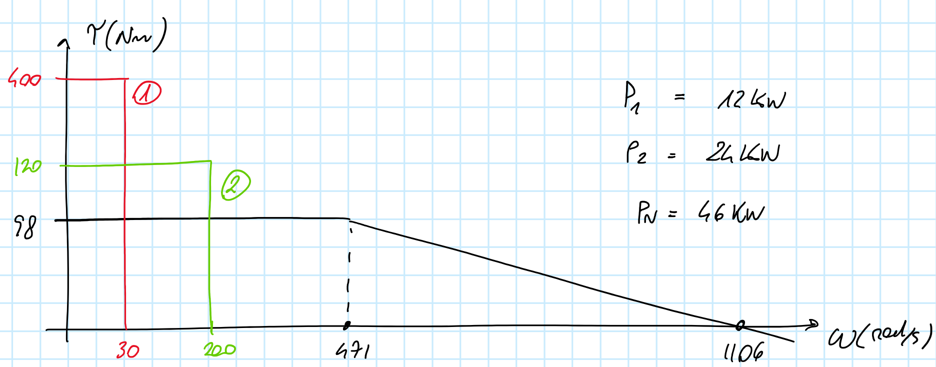
$\Lambda_d = 0.0526\ Vs$ $\qquad \Lambda_m > L_d I_N$

$\Lambda_q = 0.167\ Vs$ $\qquad$ outside as before

$\Lambda = 0.175$



$\omega_M = \ldots$



$P_1 = 12\ kW$

$P_2 = 24\ kW$

$P_N = 46\ kW$

$\eta_{gear} = 90\%$

gearbox (1)

$K'_{gear} = \frac{30}{471} = 0.064$

$K''_{gear} = \frac{88}{400} \simeq 0.22$

(2)

$K'_{gear} = \frac{200}{471} = 0.425$

$K''_{gear} = \frac{88}{120} = 0.735$

0.064 — 0.22 (1) ... 0.425 — 0.735 (2)

the second load could operate in flux weakening:

$K_{gear} = 0.2 \longrightarrow T_{em(2)} = K_{gear} \frac{T_L}{\eta_{gear}} = 0.2 \frac{120}{0.9} = 26.7\ Nm$

$\omega_{m(2)} = \frac{\omega_L}{K_{gear}} = \frac{200}{0.2} = 1000\ rad/s$

$T_{em} = \frac{3}{2} p \left[\Lambda_m I_N \sin\alpha_i + (L_d - L_q)\, I_N^2 \sin\alpha_i \cos\alpha_i \right] = 20 \sin\alpha_i - 43.2 \sin\alpha_i \cos\alpha_i \geq 26.7\ Nm$

$\Rightarrow$ iteration $\longrightarrow$ $\boxed{\alpha_i = 168°}$ $\boxed{T_{em} = 27.5\ Nm}$

$I_d =$

$I_q =$

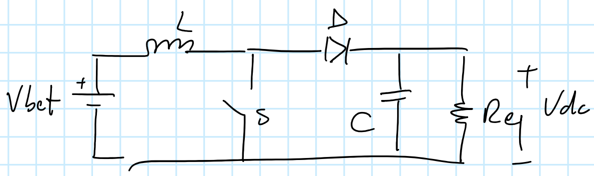
$\Lambda_d =$

$\Lambda_q =$

$\omega =$

$\omega_m = 1013\ rad/s$ $\boxed{OK}$

BOOST CONVERTER



$V_o = V_{dc}$

$P_o = P_{peak} = 75kW$

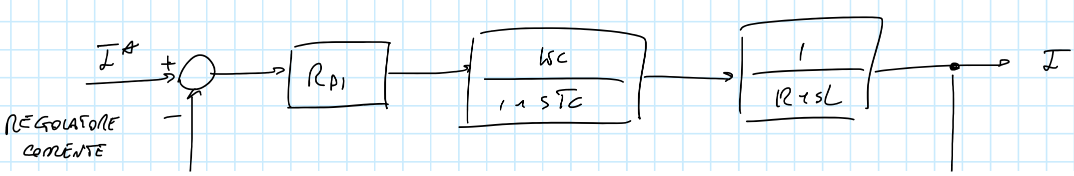
$I_o = \frac{P_o}{V_o} = 120\ A$

$R_{eq} = \frac{V_o}{I_o} = 2.11\ \Omega$

$M = \frac{V_{dc}}{V_{bat}}$

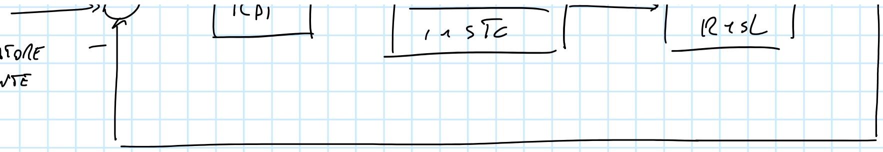
$M = \frac{1}{1-D}$

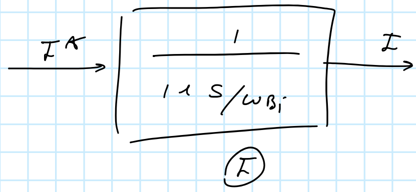
Control loop Regulator



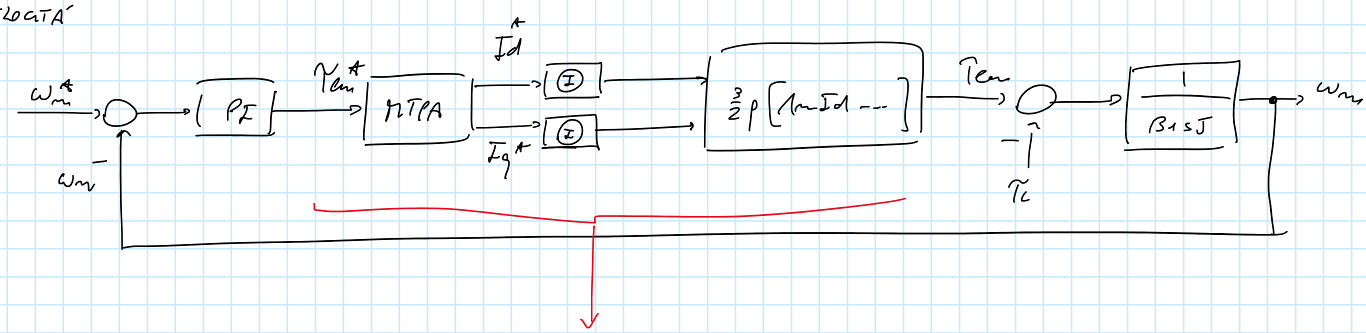
REGOLATORE CORRENTE

d/q

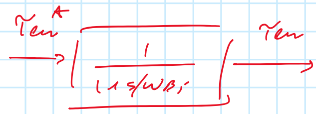




REGOLATORE VELOCITÀ

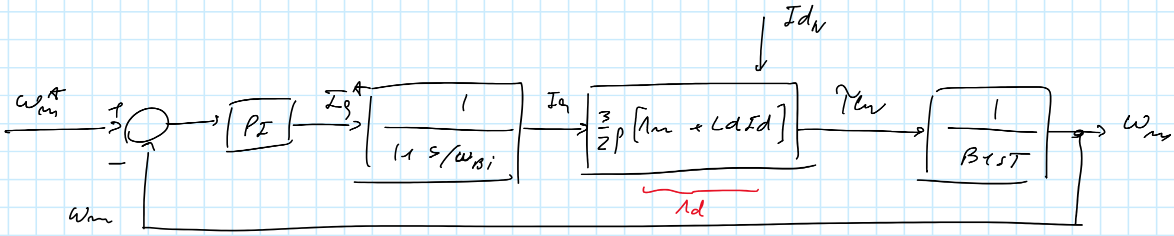


Soluzione 1 :



unico blocco con le dinamiche delle correnti

Soluzione 2 :



Davide Fogagnolo 1233994, Laurea Magistrale, progetto Arduino (drone ?) durante la prima sessione.

martedì 22 dicembre 2020 08:18

MOTOR : $L_d \neq L_q \Rightarrow$ IPM MOTOR

$2p = 8$ $\quad J_m = 0.035 \ kg m^2$ $\quad V_{DC} = 200V = V_{ph\text{-}ph,pk}$

$\Lambda_m = 0.085 \ Vs$ $\quad b = 10^{-3} \ Nms$ $\quad V_N = \frac{V_{DC}}{\sqrt{3}} = 115,5 \ V$

$L_d = 5 mH$ $\quad \omega_{m,lim} = 1500 \ rad/s$

$L_q = 20 mH$ $\quad I_N = 15 A$

$R \simeq 0 \ \Omega$ $\quad I_p = 30 A$

c) AC MOTOR

$$\cos\alpha_i = \frac{\Lambda_m \pm \sqrt{\Lambda_m^2 + 8(L_d - L_q)^2 I_N^2}}{4(L_d - L_q) I_N} = -0.6189$$

$\boxed{\text{NOMINAL}}$

$$\alpha_i = 128.24°$$

$$\begin{cases} I_{d_N} = I_N \cos\alpha_i = -9.28 \ A \\ I_{q_N} = I_N \sin\alpha_i = 11,78 \ A \end{cases} \qquad \begin{cases} \Lambda_{d_N} = \Lambda_m + L_d I_{d_N} = 0.0386 \ Vs \\ \Lambda_{q_N} = L_q I_{q_N} = 0.2356 \ Vs \end{cases}$$

$$\Lambda_N = \sqrt{\Lambda_{d_N}^2 + \Lambda_{q_N}^2} = 0.2388 \ Vs$$

$$T_{em_N} = \frac{3}{2} p \left[\Lambda_d I_{q_N} - \Lambda_q I_{d_N} \right] = 15,85 \ Nm$$

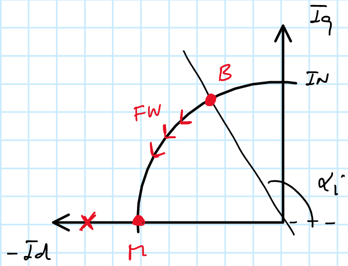
$$\omega_{B_N} = \frac{V_N}{\Lambda_N} = 483,6 \ rad/s \longrightarrow \omega_{m_{BN}} = \frac{\omega_{BN}}{p} = 120.9 \ rad/s$$

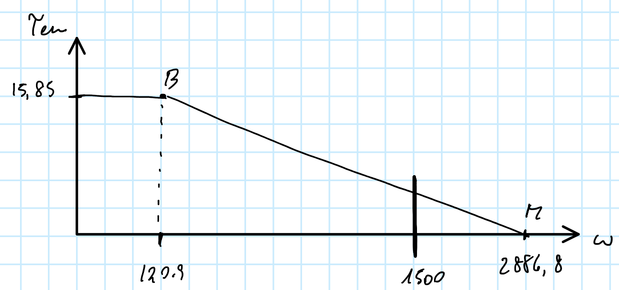
$$P_{m_N} = T_{em_N} \cdot \omega_{m_{BN}} = 1916 \ W$$

- Flux weakening : $\Lambda_m \overset{?}{>} L_d I_N$

$0.085 > 0.075$ ✓ outside the circle

$$\omega_{M_N} = \frac{V_N}{\Lambda_m - L_d I_N} = 11547 \ \frac{rad}{s} \longrightarrow \omega_{m_{MN}} = \frac{\omega_{MN}}{p} = 2886,8 \ rad/s$$





PEAK

$\cos\alpha_i = -0.6615 \rightarrow \alpha_i = 131.4°$

$\begin{cases} I_{dp} = -19.84 \text{ A} \\ I_{qp} = 22.5 \text{ A} \end{cases}$ $\begin{cases} \lambda_{dp} = -0.0142 \text{ Vs} \\ \lambda_{qp} = 0.45 \text{ Vs} \end{cases} \longrightarrow \lambda_p = 0.4502 \text{ Vs}$

$T_{em_p} = 51.65$ Nm

$\omega_{B_p} = 256.5$ rad/s $\rightarrow \omega_{m_{Bp}} = 64$ rad/s

$P_{m_p} = T_{em_p} \cdot \omega_{m_{Bp}} = 3312$ W

- flux weakening : $\lambda_m \overset{?}{>} L_d I_p$

  $0.085 > 0.15$ ✗ inside the circle

$\xi = \frac{L_q}{L_d} = 4$

$\boxed{\text{MTPV}} \longrightarrow |I_d| = \xi |I_q|$

$\tan\alpha_F = -\frac{1}{\xi} = -0.25 \longrightarrow \alpha_F = 166°$

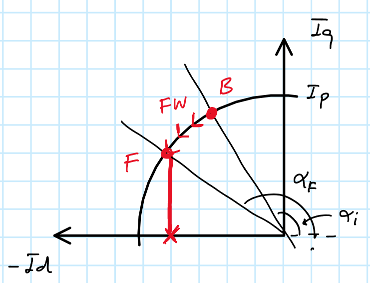
$\begin{cases} I_{dF} = I_p \cdot \cos\alpha_F = -22.10 \text{ A} \\ I_{qF} = I_p \cdot \sin\alpha_F = 7.27 \text{ A} \end{cases}$ $\begin{cases} \lambda_{dF} = \lambda_m + L_d I_{dF} = -0.06 \text{ Vs} \\ \lambda_{qF} = L_q I_{qF} = 0.14 \text{ Vs} \end{cases}$

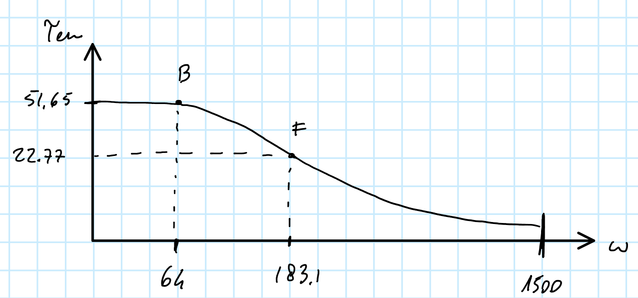
$\lambda_F = 0.1576$ Vs

$T_{em_F} = 22.77$ Nm

$\omega_F = \frac{V_N}{\lambda_F} = 732.6$ rad/s $\longrightarrow \omega_{m_F} = 183.1$ rad/s

$P_{em_F} = T_{em_F} \cdot \omega_{m_F} = 4170$ W





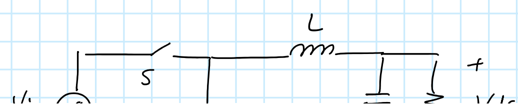
Ⓐ DC-DC converter

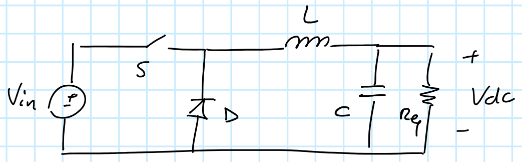
$V_{in} > V_{out} \rightarrow$ buck

$V_g = V_{in}$ $V_o = V_{dc}$

P_{em_F} = peak power

$I_o = \frac{P_o}{\quad} = 20.85$ A





$$I_o = \frac{P_o}{V_o} = 20.85\ A$$

$$R_{eq} = \frac{V_o}{I_o} = 9{,}6\ \Omega$$

$$M = D = \frac{V_o}{V_g} = 0.625$$

$$\Delta i_L = 2\%\ I_L = 2\%\ I_o = 417\ mA \Rightarrow L = \frac{V_g}{f_s \Delta i_L} D(1-D) = 30\ mH$$

$$\Delta V_{out} = 1\%\ V_o = 2\ V \Rightarrow C = \frac{\Delta i_L}{8 f_s \Delta V_o} = 4.34\ \mu F$$

$$I_{sw} = D\, I_L = 13.03 A \qquad I_{sw_{pk}} = I_o + \frac{\Delta i_L}{2} = 21.06\ A$$

$$I_D = (1-D)\, I_L = 7{,}817\ A \qquad I_{o_{pk}} = I_{sw_{pk}}$$

Maximum reverce voltage : $V_g = 320\ V$

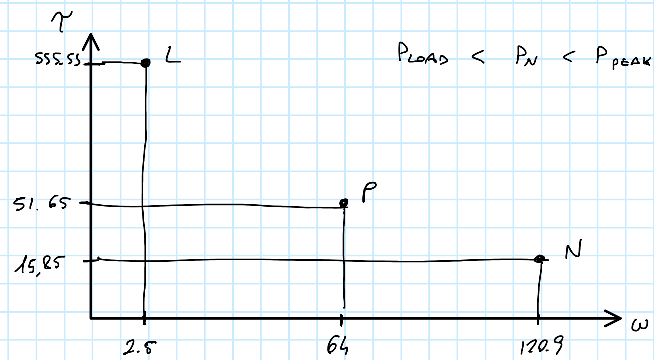
D STATIC LOAD

$\tau_L = 500\ Nm$ $\qquad \omega_{m_N} = 120.9\ rad/s$, $\tau_{em,N} = 15{,}85\ Nm$

$\omega_L = 2.5\ Nm$ $\qquad \omega_{m_p} = 64\ rad/s$, $\tau_{em,p} = 51{,}65\ Nm$

$\eta_{gear} = 90\%$

$$\tau_r = \frac{\tau_L}{\eta_{gear}} = 555.55\ Nm$$



$P_{LOAD} < P_N < P_{PEAK}$ ✓

$$K_{min} = \frac{\omega_r}{\omega_N} = 0.02067$$

$$K_{max} = \frac{\tau_N}{\tau_r} = 0.0285$$

$$K_{min} \le K_{gear} \le K_{max}$$

- $K_{gear} = \frac{1}{40} = 0.025$

$$\Rightarrow \quad \omega_m = \frac{\omega_r}{K_{gear}} = 100\ rad/s$$

$$\Rightarrow \quad \tau_m = K_{gear}\, \tau_r = 13.89\ Nm$$

$$\tau_{em} = \frac{3}{2} p \left[\Lambda_m I_N \sin\alpha_i + (L_d - L_q) I_N^2 \sin\alpha_i \cos\alpha_i \right]$$

$$= \frac{3}{2} \cdot 4 \left[0.085 \cdot 15 \sin\alpha_i + (0.005 - 0.02)\, 15^2 \sin\alpha_i \cos\alpha_i \right]$$

$$= 7.65 \sin\alpha_i - 20{,}25 \sin\alpha_i \cos\alpha_i = \boxed{13.89\ Nm}$$

α_i	130°	120°	110°	111°
T_{em}	15.83	15.33	13.69	13.91

$$\begin{cases} I_d = I_N \cos\alpha_i = -5.37\ A \\ I_q = I_N \sin\alpha_i = 14\ A \end{cases} \rightarrow I = 15 e^{j111°}$$

$$\begin{cases} \Lambda_d = \Lambda_m + L_d I_d = 0.05815\ Vs \\ \Lambda_q = L_q I_q = 0.280\ Vs \end{cases} \rightarrow \Lambda = 0.286\ Vs$$

$$V = \omega_m \Lambda = 28.6\ V$$

E DYNAMIC LOAD

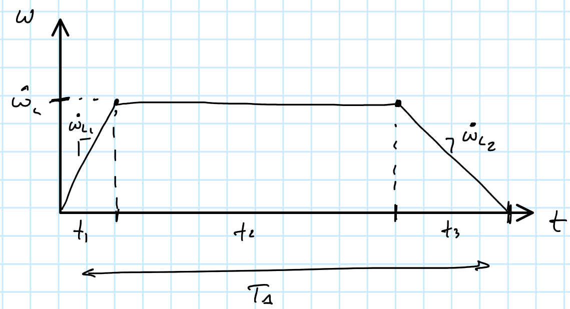
$J_L = 0.45\ kgm^2$

$\dot{\omega}_{L_1} = 150\ rad/s^2$, $t_1 = 1s$

$\hat{\omega}_L$ const , $t_2 = 9s$

$\dot{\omega}_{L_2} = ?$, $t_3 = 2s$

$T_a = 1+9+2 = 12s$



$\hat{\omega}_L = \dot{\omega}_{L_1} \cdot t_1 = 150\ rad/s$

$\hat{\omega}_L - t_2 \dot{\omega}_{L_2} = 0$

$\hookrightarrow \dot{\omega}_{L_2} = \dfrac{\hat{\omega}_L}{t_2} = 75\ rad/s^2$

- $K_{opt} = \sqrt{\dfrac{J_m}{J_L}} = 0.2789$

$\dot{\omega}_{L\,rms} = \sqrt{\dfrac{1}{T_a}\left(\dot{\omega}_{L_1}^2 t_1 + \dot{\omega}_{L_2}^2 t_2\right)} = 53\ rad/s^2$

$\dot{\omega}_{L\,max} = \dot{\omega}_{L_1} = 150\ rad/s^2$

$\left(T_p = 51,65\ Nm, \quad T_N = 15,85\ Nm \right)$

- $K_{p_{12}} = \dfrac{T_p \pm \sqrt{T_p^2 - 4 J_m J_L \dot{\omega}_{L\,max}}}{2 \dot{\omega}_{L\,max}} = \begin{cases} 0.344 \\ 0.0003 \end{cases}$

- $K_{N_{12}} = \dfrac{T_N \pm \sqrt{T_N^2 - 4 J_m J_L \dot{\omega}_{L\,rms}}}{} = 0.298$

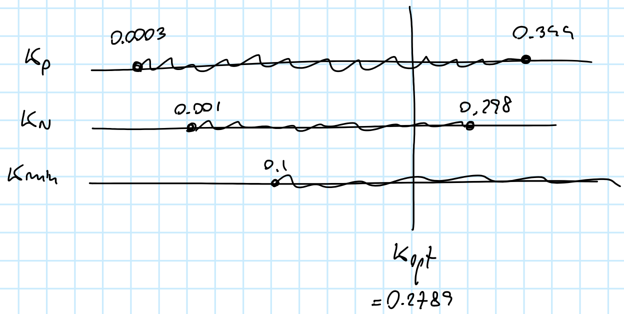
• $K_{N_{1,2}} = \dfrac{T_N \pm \sqrt{T_N^2 - 4 J_m T_L \dot{\omega}_{L_{RMS}}}}{2 \cdot \dot{\omega}_{L_{RMS}}} = \begin{cases} 0.298 \\ 0.001 \end{cases}$

• $K_{min} = \dfrac{\omega_{L\,max}}{\omega_{max}} = \dfrac{150}{1500} = 0.1$

$\omega_{max} \geq \dfrac{\omega_{L\,max}}{K_{opt}}$ ✓

$T_p \geq 2\sqrt{J_L J_m}\;\dot{\omega}_{L\,max}$ ✓

$T_N \geq 2\sqrt{J_L J_m}\;\dot{\omega}_{L\,RMS}$ ✓



$\Rightarrow \boxed{0.1 \leq K_{gear} \leq 0.298}$

$T_{L_{RMS}} = J_L \cdot \dot{\omega}_{L_{RMS}} = 23.85$ Nm

$\rightarrow T_{em_{RMS}} = \dfrac{T_{L\,RMS} \cdot K_{gear}}{\eta_{gear}}$

I can use the optimal K_{gear} since it is inside the range:

• $T_{em_{RMS}}(K_{opt}) = \dfrac{T_{L\,RMS}}{\eta_{gear}} K_{opt} = 7.39$ Nm

F) Design of the regulators

CURRENT LOOP

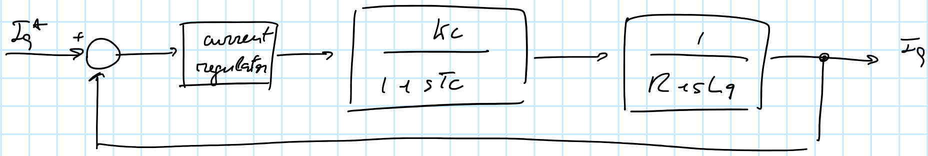
$K_c = 20$

$T_c = 0.5$ ms

$K_{fb} = 1$

$\omega_B = 250$ rad/s

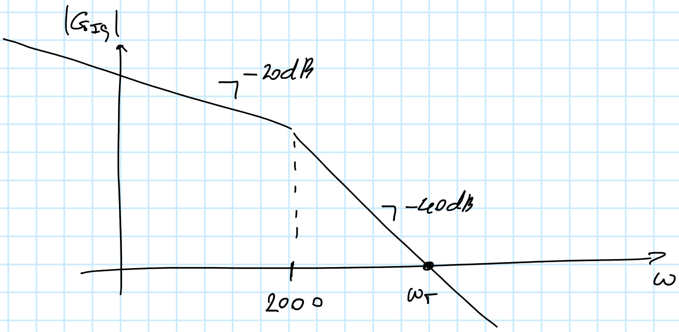
$\varphi_m = 60°$



• Since $R \simeq 0$: $\dfrac{1}{R + sL} \simeq \dfrac{1}{sL}$ → No need for a PI regulator → only proportional

There is already a pole in the origin for zero static error of the loop

$$\frac{K_C}{1+sT_C}\cdot\frac{1}{sL_q} = \frac{20}{1+s\cdot 0{,}0005}\cdot\frac{1}{s\cdot 0.02} = \frac{1000}{s\left(1+\frac{s}{2000}\right)}$$

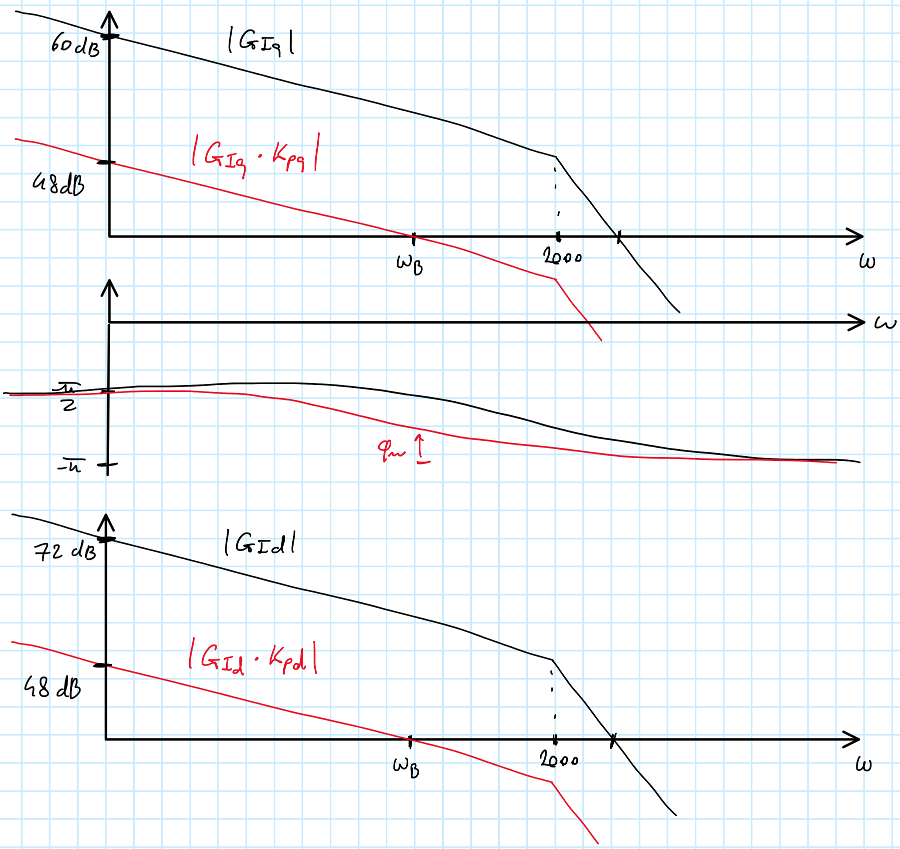


$$\varphi_m = 180 + \angle G_{Iq} = 180 - 90 - \arctan\frac{250}{2000} = 82.875° > 60° \checkmark$$

- $\left|G_{Iq}(\omega_B)\right| = 1$

$$\Rightarrow \left| K_{pq}\cdot\frac{K_C}{1+sT_C}\cdot\frac{1}{sL_q}\right|_{s=j\omega_B} = 1 \longrightarrow \boxed{K_{pq} = 0.2519}$$

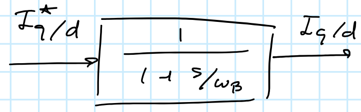
- for I_d: $\left| K_{pd}\cdot\frac{K_C}{1+sT_C}\cdot\frac{1}{sL_d}\right| = 1 \longrightarrow \boxed{K_{pd} = 0.063}$



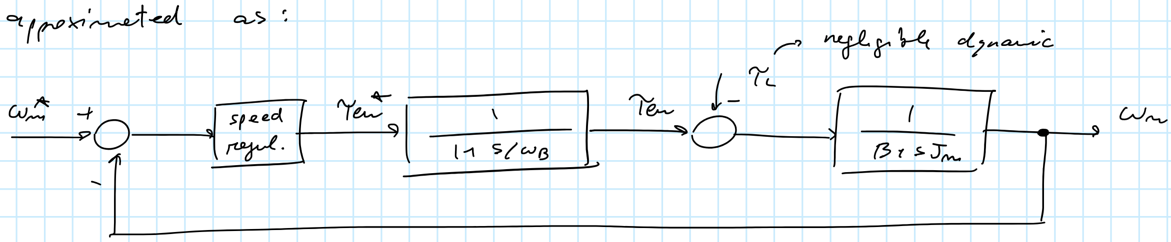
phase valid for both

SPEED LOOP CONTROL

The whole current loop can be approximated as



Since the torque has the same dynamic of the current, the speed loop can be approximated as:



In this case we need a PI regulator to have a pole in the origin.

$$G_{\omega_m} = \frac{1}{1+\frac{s}{\omega_B}} \cdot \frac{1}{B+sJ_m} = \frac{1/B}{\left(1+\frac{s}{\omega_B}\right)\left(1+s\frac{J}{B}\right)} =$$

$$= \frac{1000}{\left(1+\frac{s}{250}\right)\left(1+s\cdot 35\right)}$$

$\varphi_m = 180 - \angle G_{\omega_m} = 85.5^\circ$ ✓ → no need to compensate but just a pole in the origin

- PI regulator : compensate the first pole

$$G_{PI} = K_i \cdot \frac{1+sT_i}{s}$$

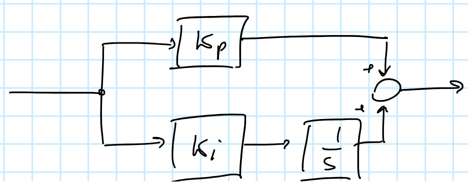
$$\boxed{T_i = \frac{J}{B} = 35}$$

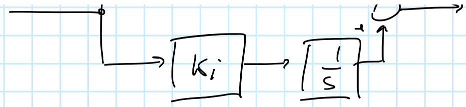
$\varphi_m' = 180 - \angle G_{\omega_m} \cdot G_{PI} = 180 - 90 - \text{atan}\left(\frac{20}{250}\right) = 85.4^\circ$ ✓

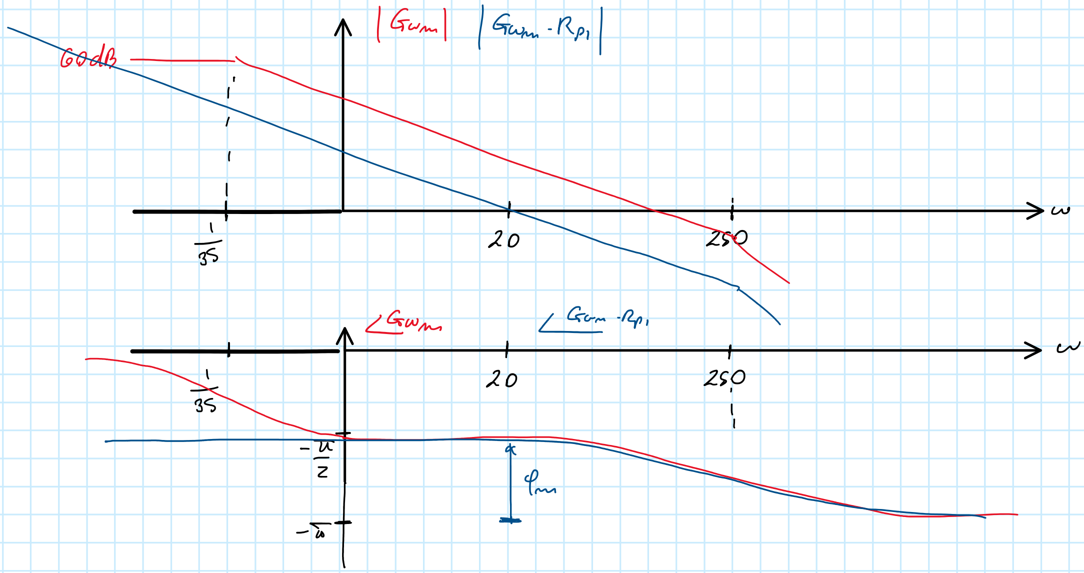
K_i : $\left| K_i \cdot \frac{\cancel{1+sT_i}}{s} \cdot \frac{\frac{1}{B}}{\left(1+\frac{s}{\omega_B}\right)\left(\cancel{1+s\frac{J}{B}}\right)} \right| = 1$

$$\boxed{\begin{aligned} &K_i = 0.02 \\ &K_p = K_i T_i = 0.02 \cdot 35 = 0.7 \end{aligned}}$$

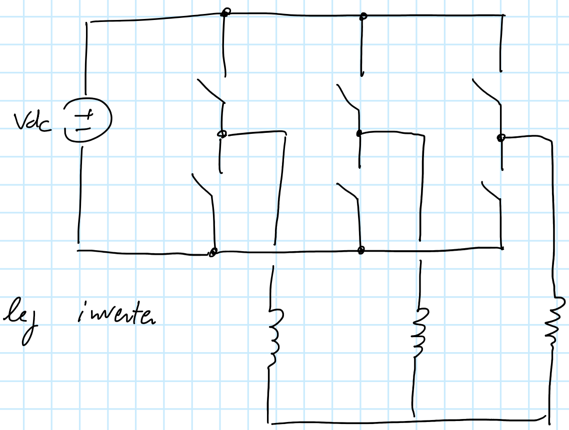
R_{PI}







(B) Inverter design



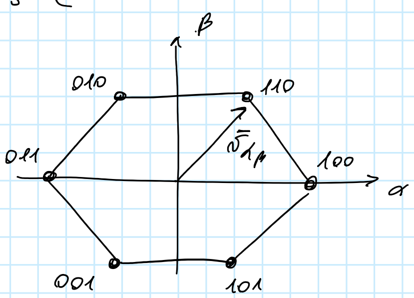
three leg inverter

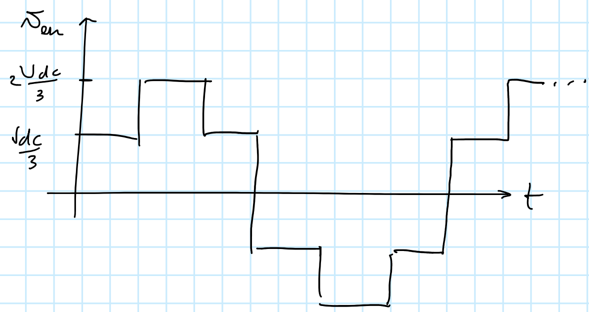
peak current $= I_{o\,buck} + \Delta i_{c\,buck}$

$= 21.06$ A

maximum voltage $= V_{dc} = 200$ V

$$\bar{v}_{abc} = \frac{2}{3}\left[v_a + v_b e^{j\frac{2}{3}\pi} + v_c e^{-j\frac{2}{3}\pi}\right] \Rightarrow \begin{bmatrix} v_\alpha \\ v_\beta \\ v_0 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\ 0 & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}$$

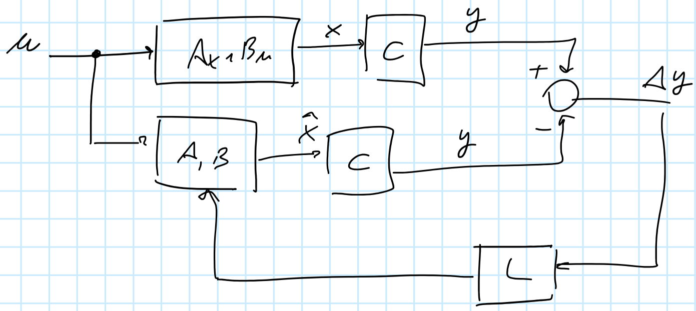




every state has a binary code

G) Sensorless control

I can implement a second order observer to get both position and speed. This technique requires the motor to run at a speed sufficiently high to avoid errors in the measurements.



$$x = \begin{bmatrix} \vartheta_m^e \\ \omega_m^e \end{bmatrix} \qquad \begin{cases} \dot{\vartheta}_m^e = \omega_m^e \\ \dot{\omega}_m^e = p \dfrac{T_{em} - T_L}{J} = 0 \end{cases} \rightarrow \text{assume constant speed}$$

$$\begin{bmatrix} \dot{\hat{\vartheta}}_m^e \\ \dot{\hat{\omega}}_m^e \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{\vartheta}_m^e \\ \hat{\omega}_m^e \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u + \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} \left(\vartheta_m^e - \hat{\vartheta}_m^e \right)$$

$$\dot{\hat{x}} = A \hat{x} + B u + L \Delta\theta$$

$$y = \hat{\vartheta}_m^e = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} \hat{\vartheta}_m^e \\ \hat{\omega}_m^e \end{bmatrix}}_{x}$$

$$\begin{cases} \dot{x} = Ax + Bu \\ \dot{\hat{x}} = A\hat{x} + Bu + LC(x - \hat{x}) \end{cases}$$

$$\dot{x}_{err} = Ax_{err} - LC x_{err} = (A - LC) x_{err}$$

$$\det \left[sI - (A - LC) \right] = 0$$

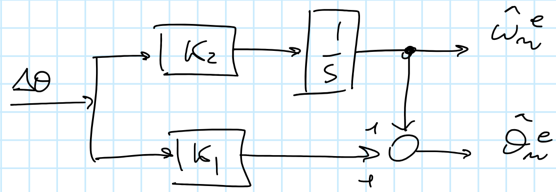
$$\Rightarrow s^2 + k_1 s + k_2 = 0$$

From specification ω_B speed $= 20$ rad/s

I want two coincident poles :

$$(s+20)^2 = s^2 + 40s + 400$$

$$\boxed{\begin{array}{l} k_1 = 40 \\ k_2 = 400 \end{array}}$$



- $\bar{\lambda}_{m_{\alpha\beta}} = \Lambda_m e^{j\vartheta_m^e}$