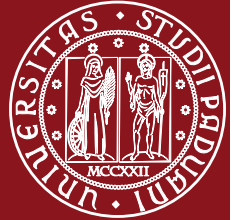




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Power Electronics – a.a. 2019-2020



Summary of Steady-State Equations of Basic dc-dc Converters

Luca Corradini

Power Electronics Group

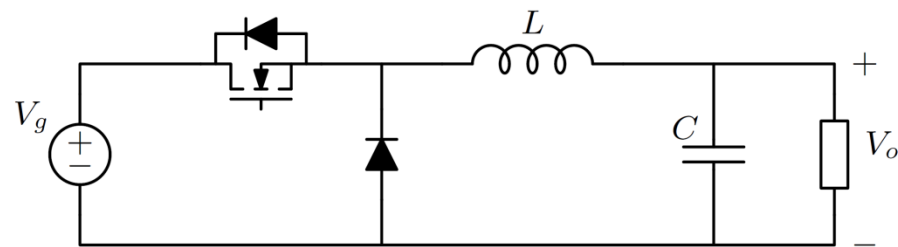
<http://pelgroup.dei.unipd.it>

University of Padova

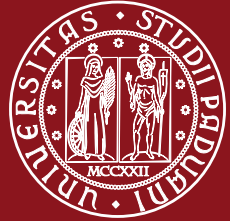
Department of Information Engineering – DEI



Buck Converter



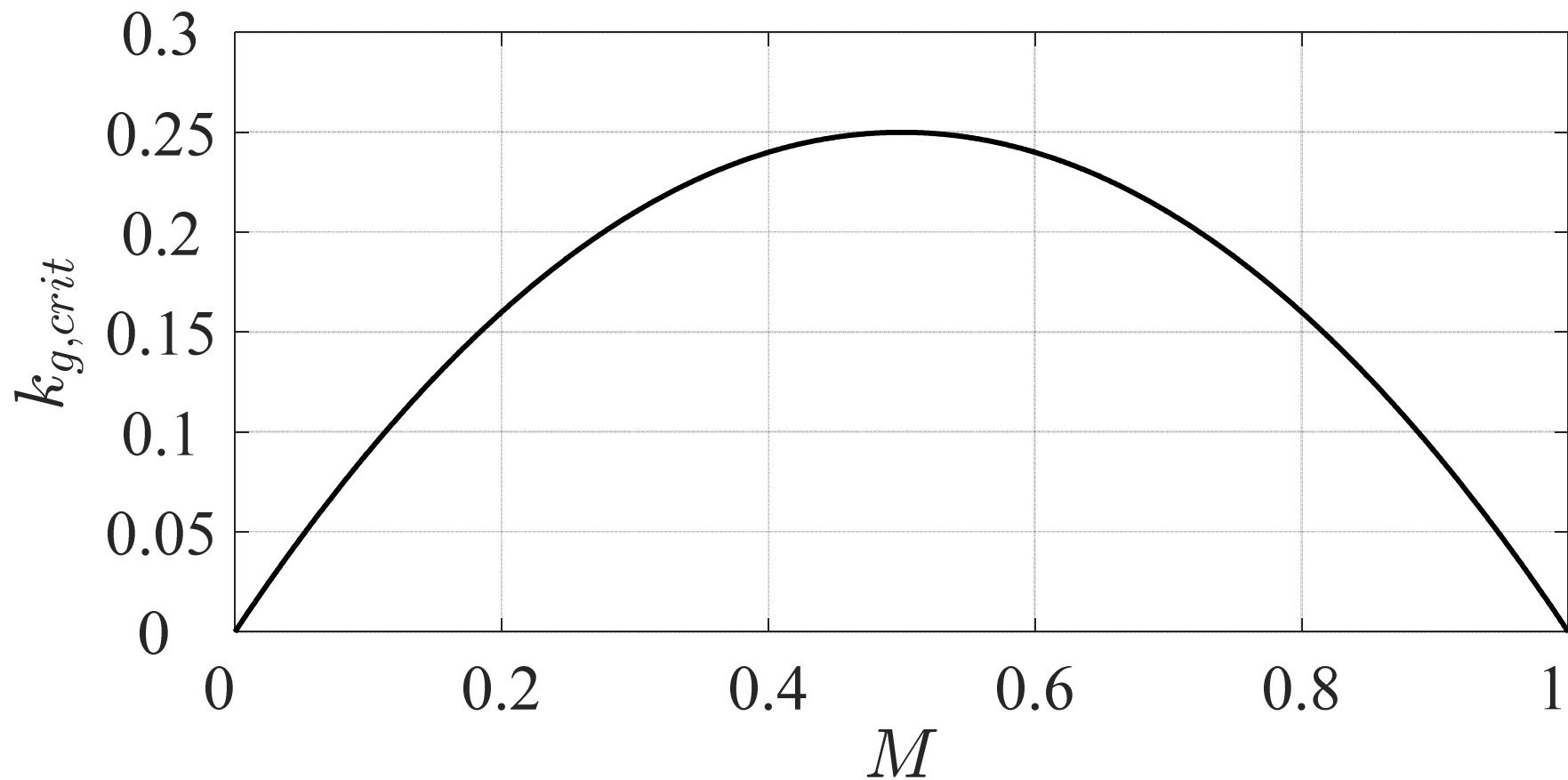
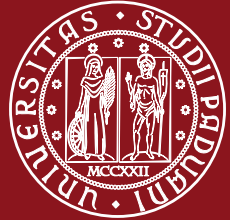
Buck converter with constant current load: voltage conversion ratio $M(D, k_g)$



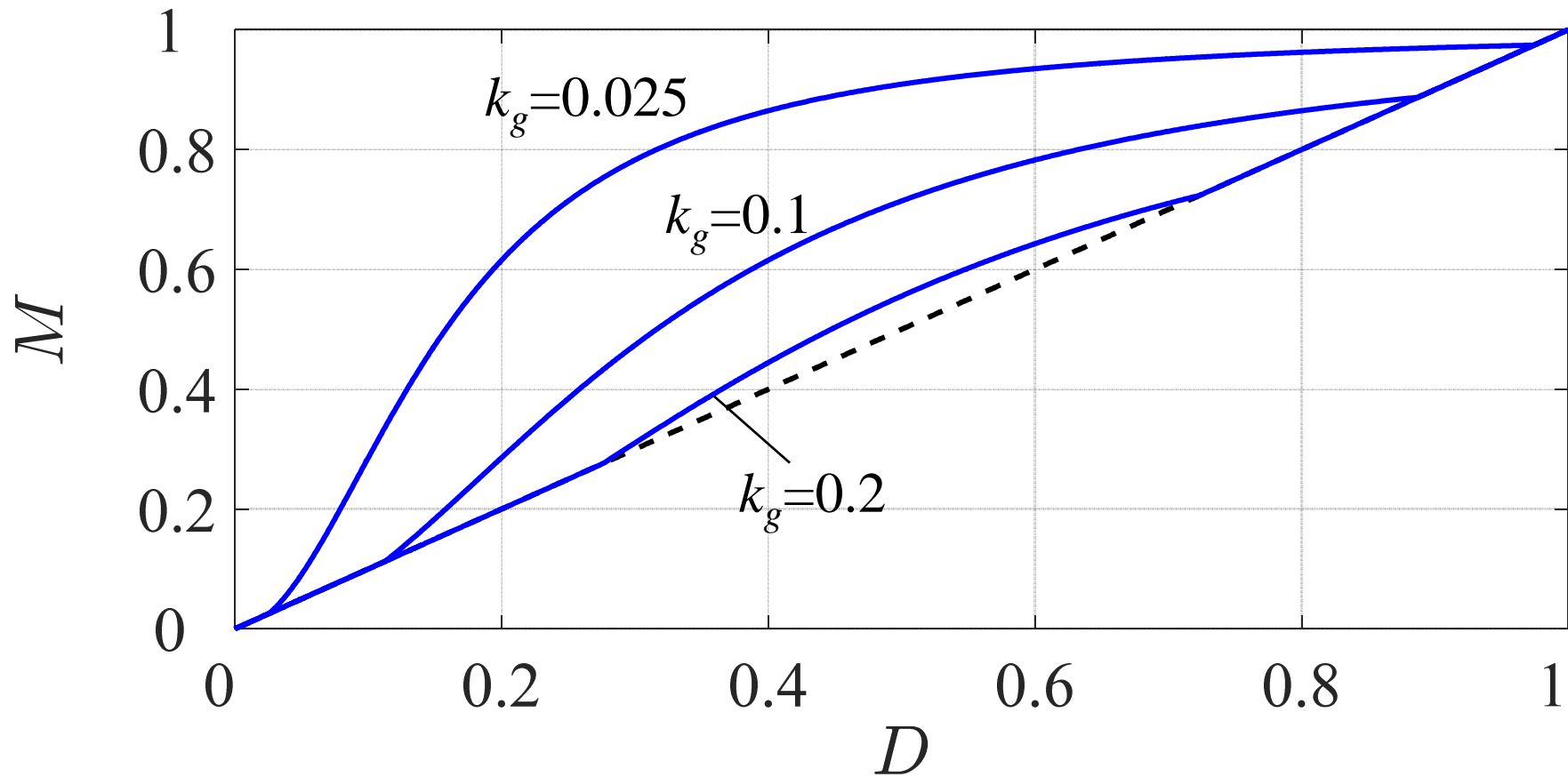
$$M(D, k_g) = \begin{cases} D, & k_g > k_{g, crit} \quad (\text{CCM}) \\ \frac{1}{1 + \frac{k_g}{D^2}}, & k_g < k_{g, crit} \quad (\text{DCM}) \end{cases}$$

$$k_g \triangleq \frac{2f_s L I_o}{V_g}, \quad k_{g, crit}(D) \triangleq D(1 - D) \quad \text{or} \quad k_{g, crit}(M) \triangleq M(1 - M)$$

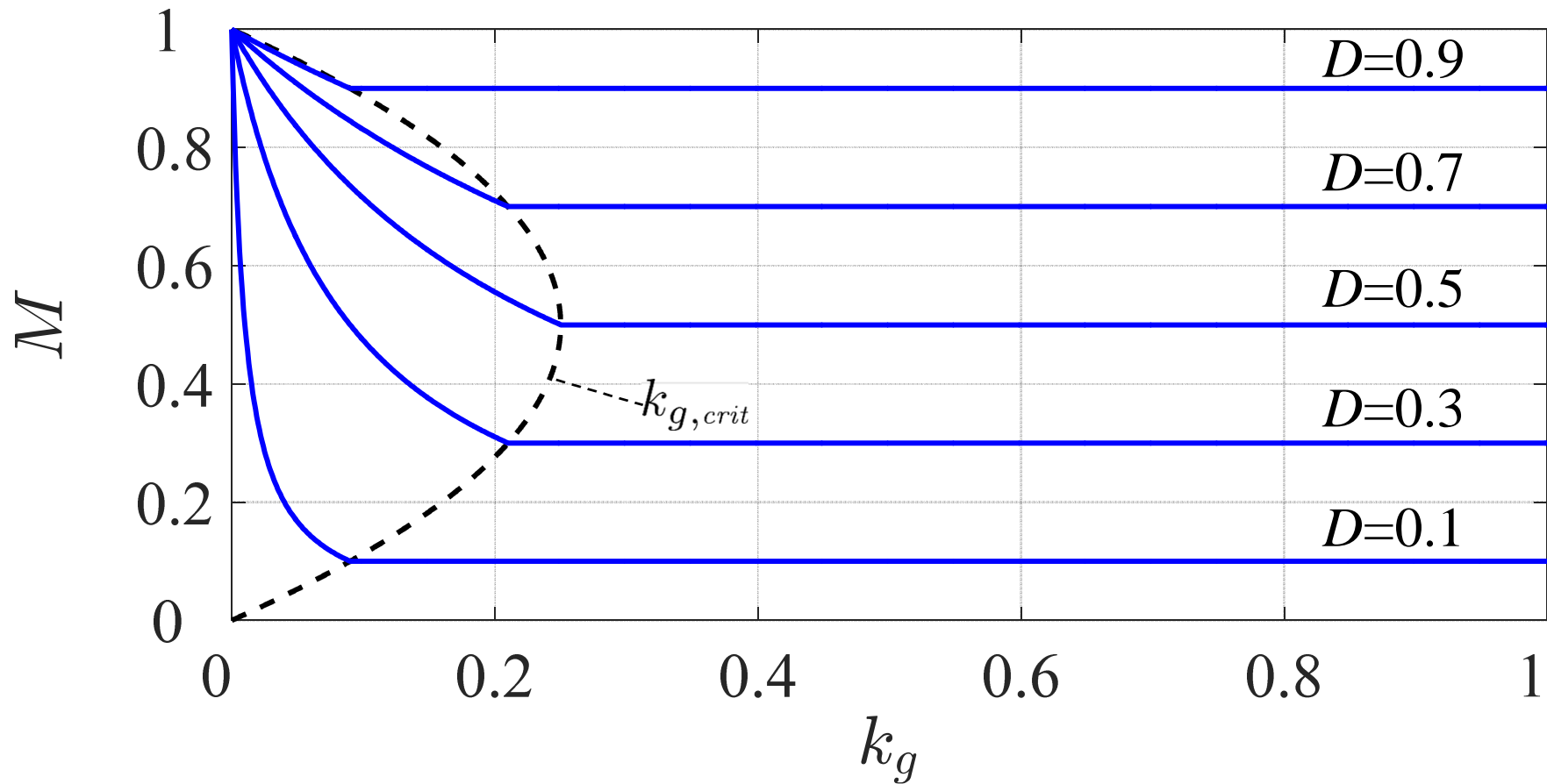
Buck converter with constant current load: critical k_g vs. M



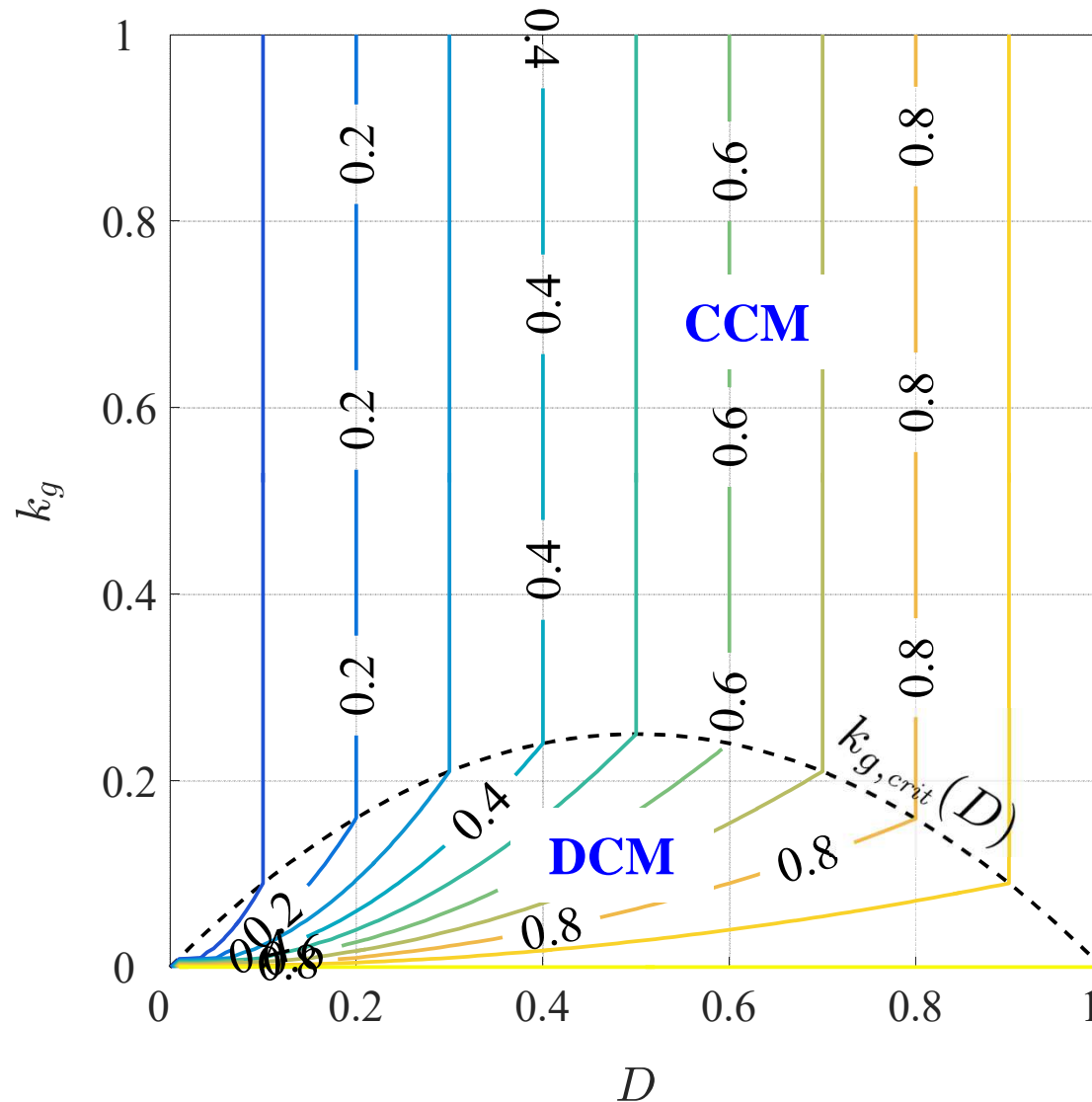
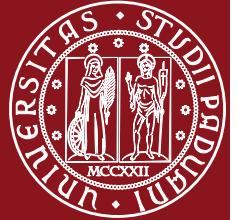
Buck converter with constant current load: control characteristics



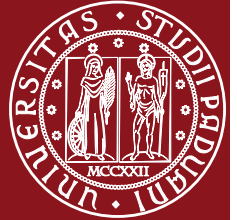
Buck converter with constant current load: output characteristics



Buck converter with constant current load: voltage conversion ratio $M(D, k_g)$



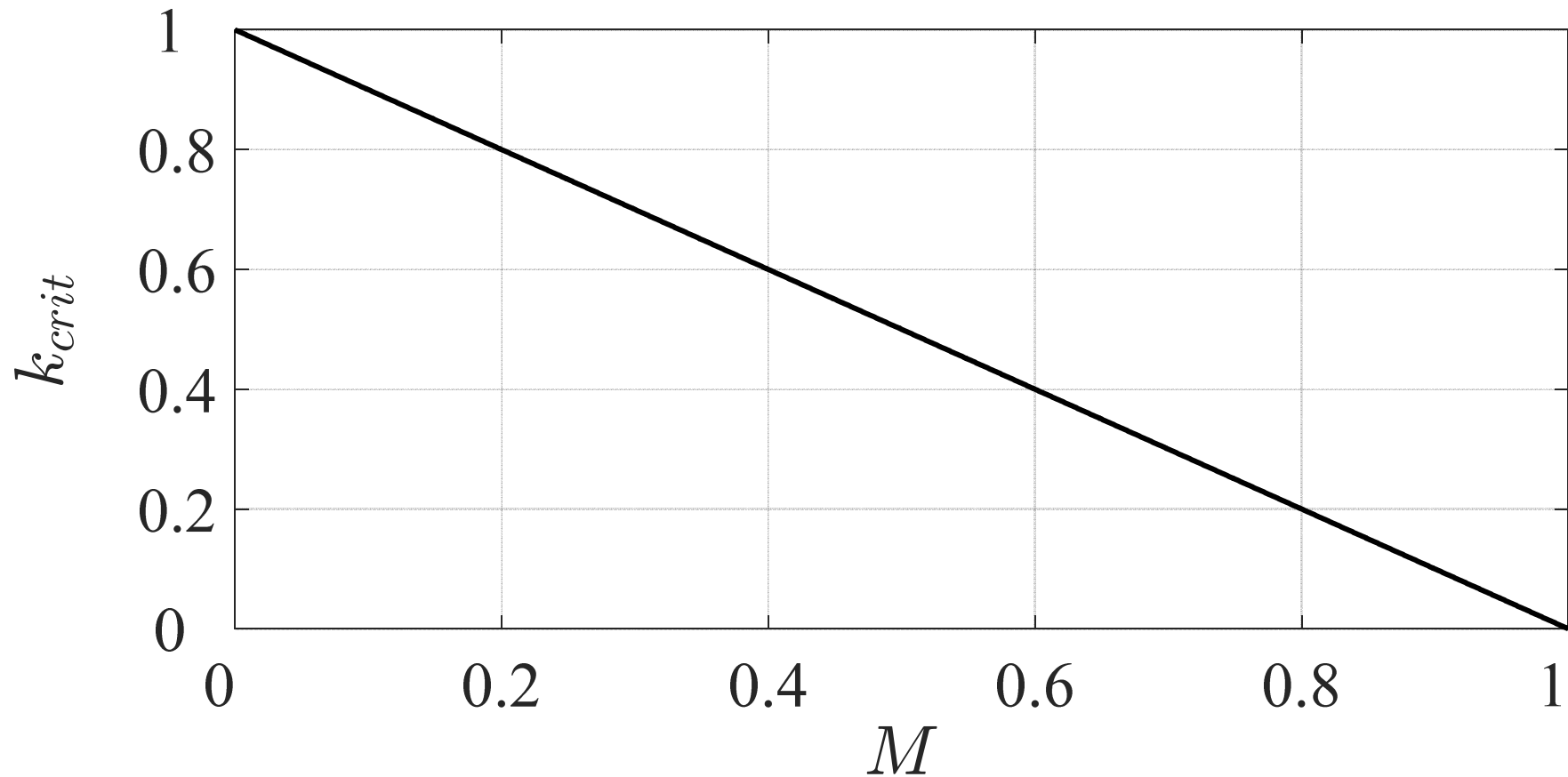
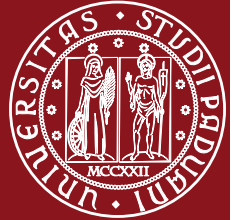
Buck converter with resistive load: voltage conversion ratio $M(D,k)$



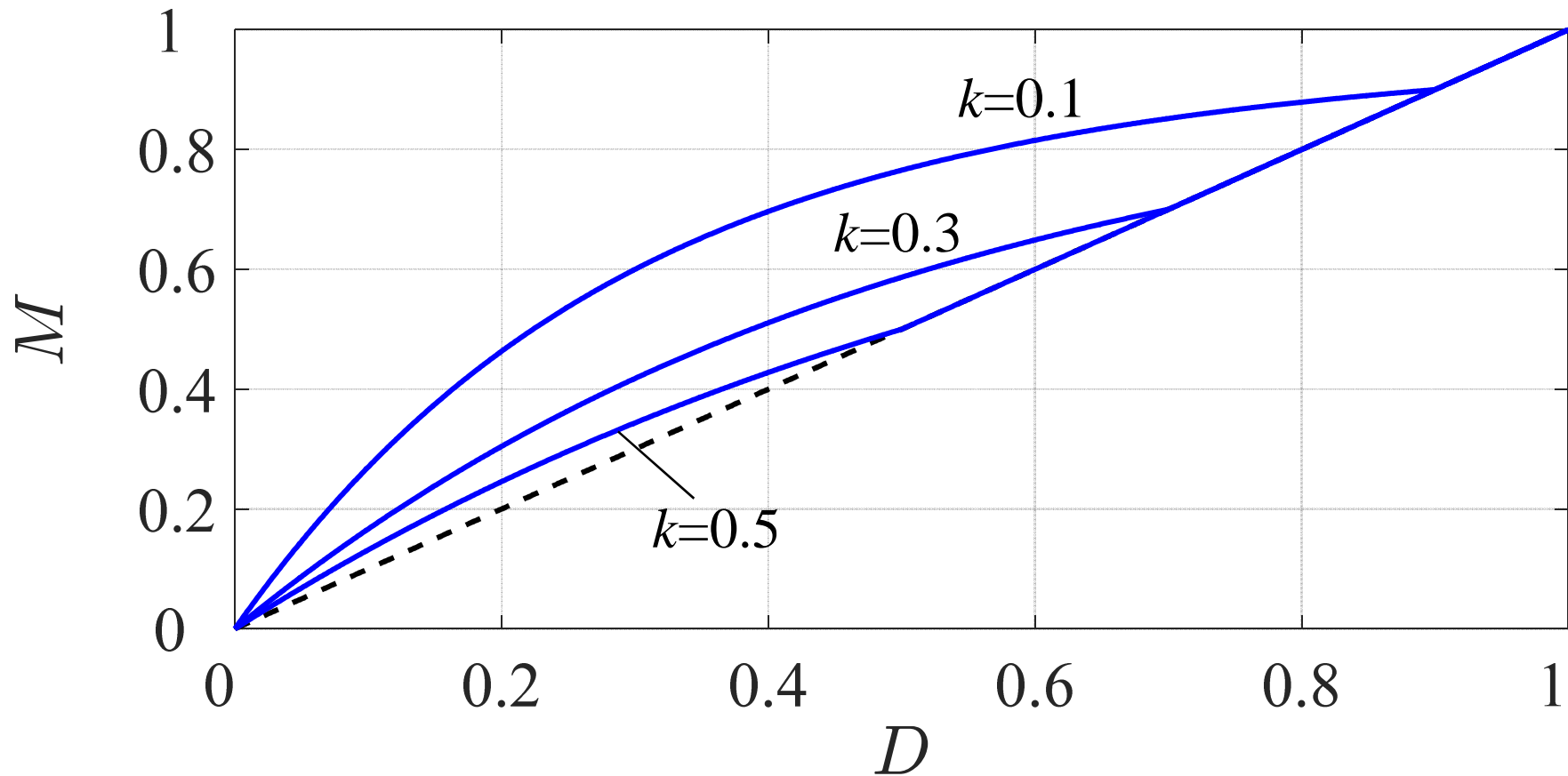
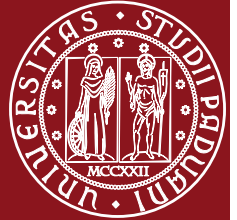
$$M(D, k) = \begin{cases} D, & k > k_{crit} \quad (\text{CCM}) \\ \frac{2}{1 + \sqrt{1 + \frac{4k}{D^2}}}, & k < k_{crit} \quad (\text{DCM}) \end{cases}$$

$$k \triangleq \frac{2f_s L I_o}{V_o}, \quad k_{crit}(D) \triangleq 1 - D \quad \text{or} \quad k_{crit}(M) \triangleq 1 - M$$

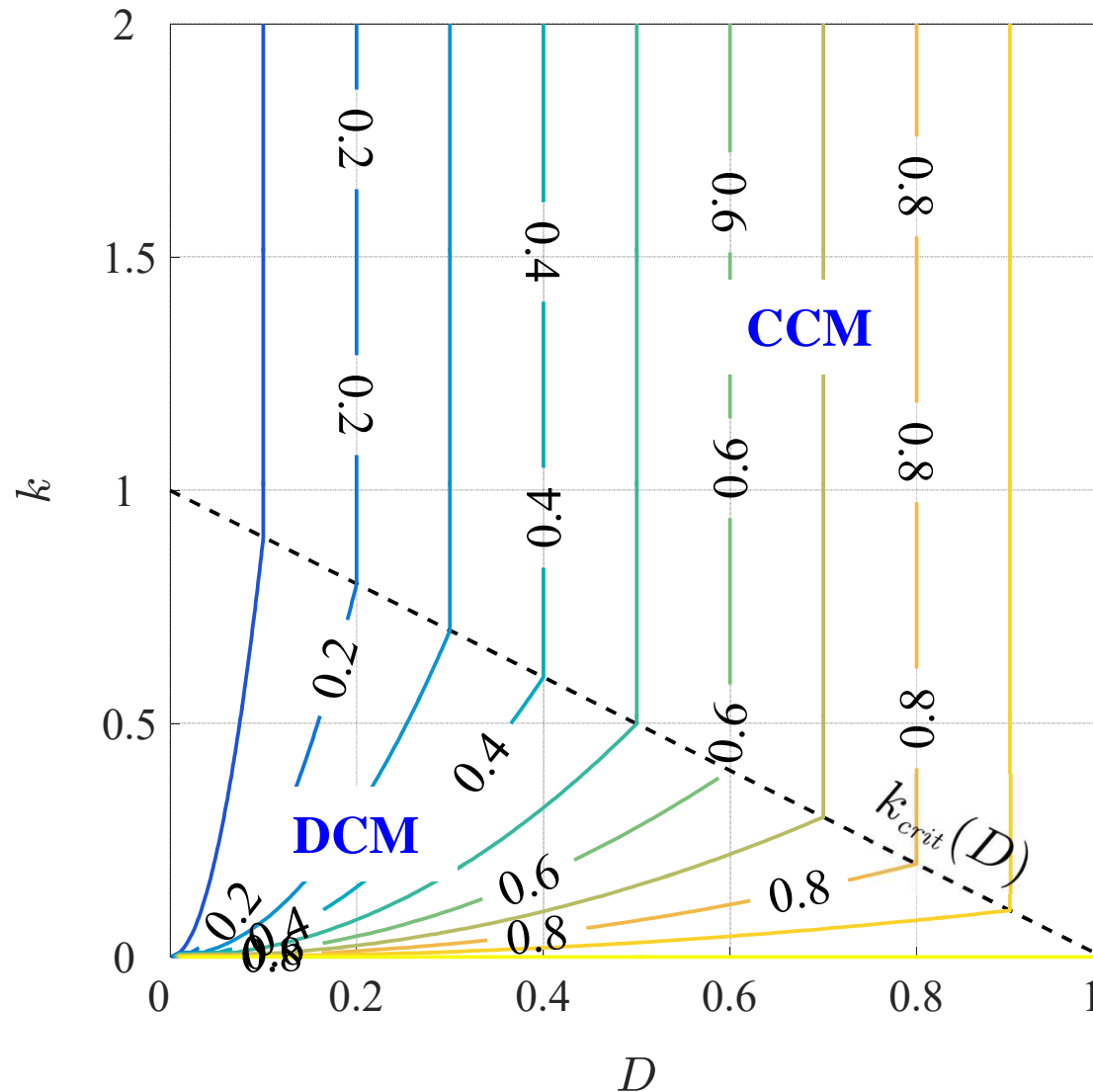
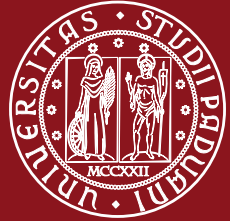
Buck converter with resistive load: critical k vs. M



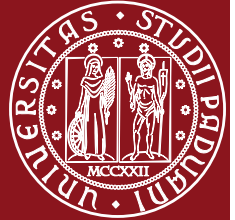
Buck converter with resistive load: control characteristics



Buck converter with resistive load: voltage conversion ratio $M(D,k)$

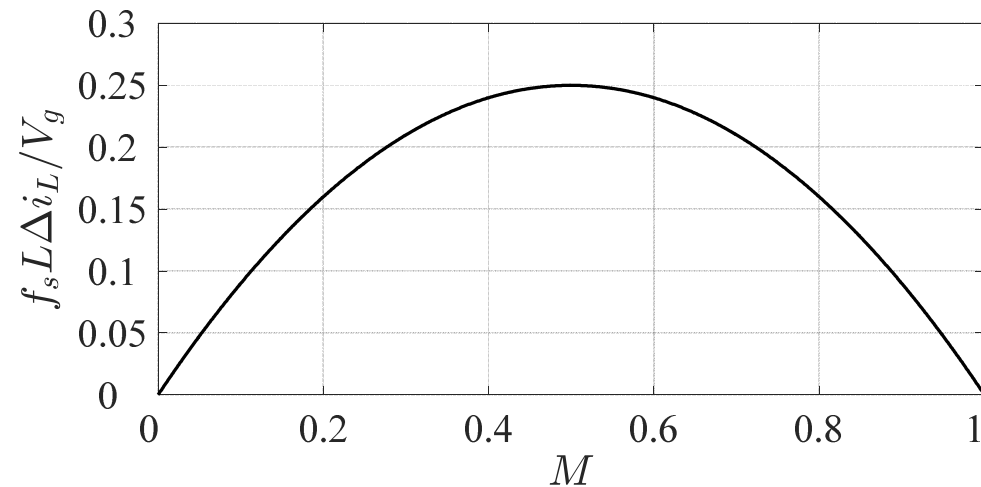


Buck converter: inductor current ripple in CCM



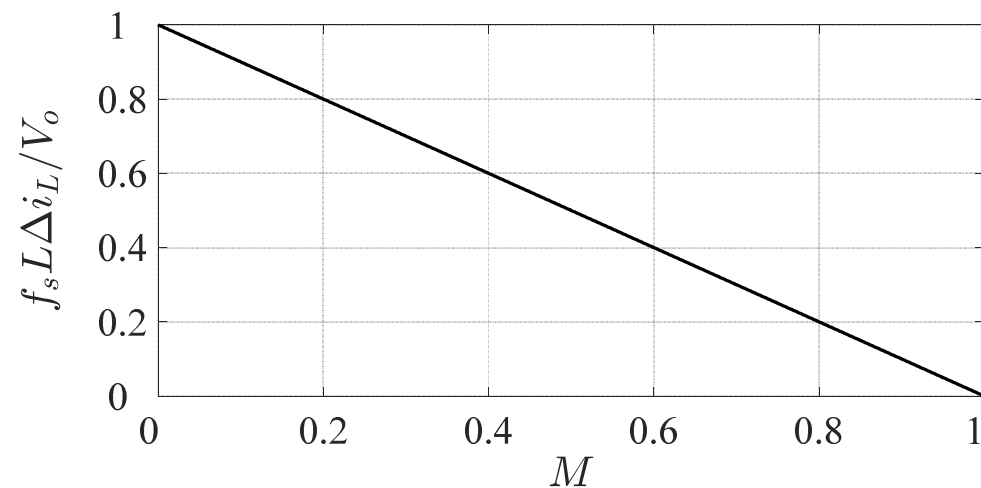
- Constant V_g :

$$\begin{aligned}
 \Delta i_L &= \frac{V_g}{f_s L} D(1 - D) \\
 &= \frac{V_g}{f_s L} M(1 - M)
 \end{aligned}$$

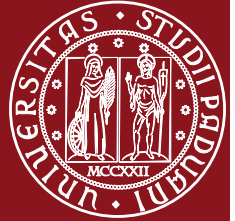


- Constant V_o :

$$\begin{aligned}
 \Delta i_L &= \frac{V_o}{f_s L} (1 - D) \\
 &= \frac{V_o}{f_s L} (1 - M)
 \end{aligned}$$

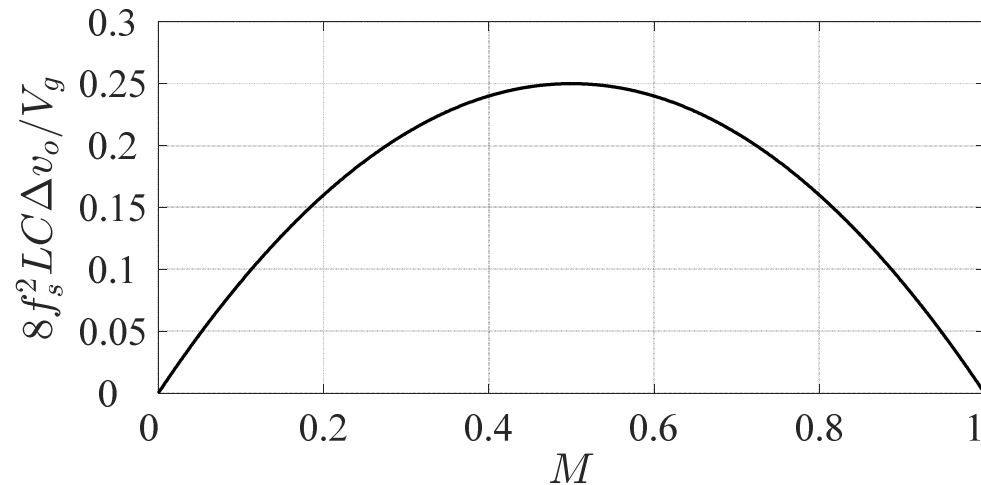


Buck converter: output voltage ripple in CCM



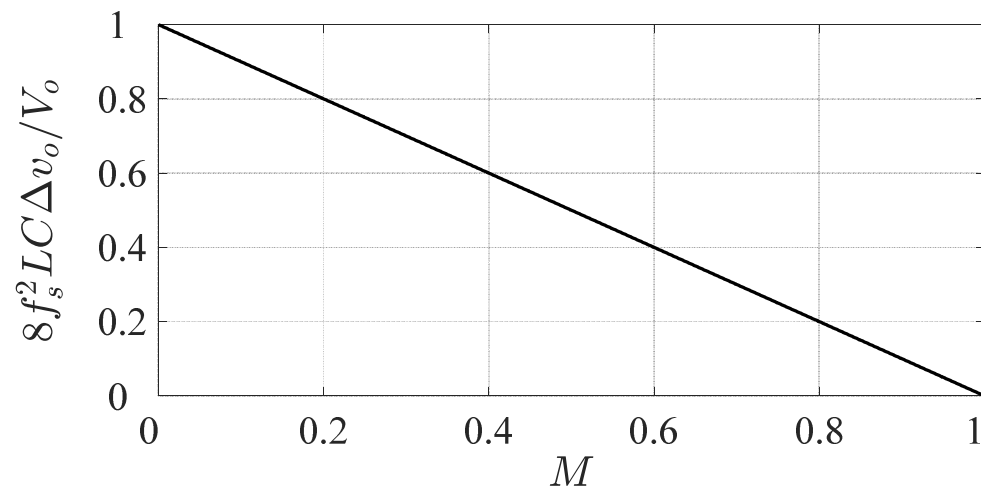
- Constant V_g :

$$\begin{aligned}
 \Delta v_o &= \frac{\Delta i_L}{8f_s C} \\
 &= \frac{V_g}{8f_s^2 LC} M(1 - M)
 \end{aligned}$$



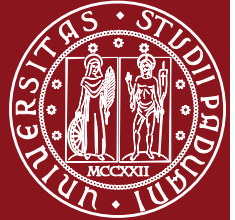
- Constant V_o :

$$\begin{aligned}
 \Delta v_o &= \frac{\Delta i_L}{8f_s C} \\
 &= \frac{V_o}{8f_s^2 LC} (1 - M)
 \end{aligned}$$

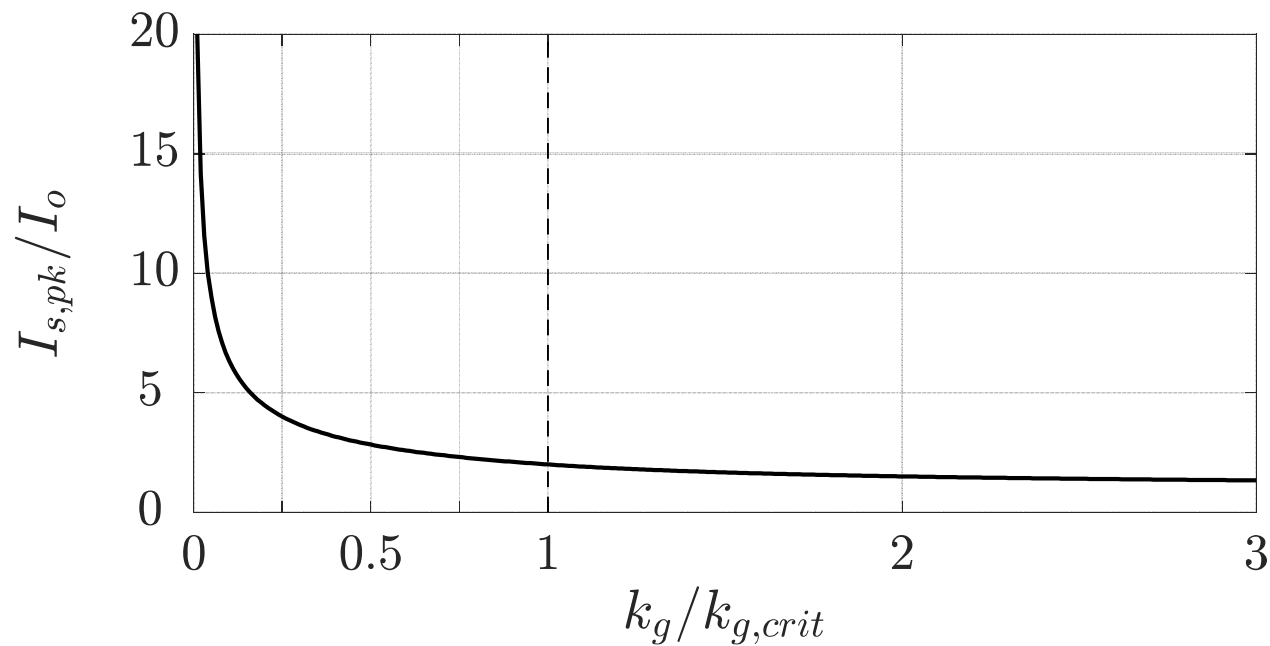




Buck converter: current stress

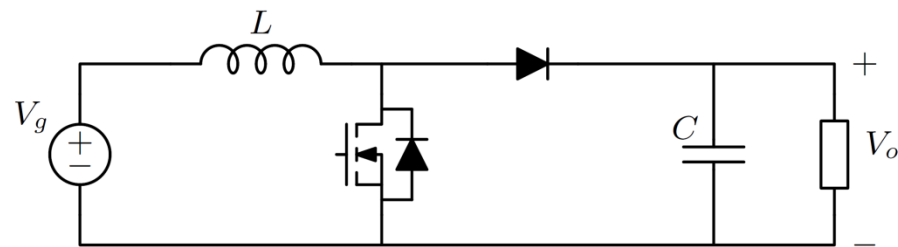


$$\frac{I_{s,pk}}{I_o} = \begin{cases} 1 + \frac{1}{k_g/k_{g,crit}} & \text{(CCM)} \\ \frac{2}{\sqrt{k_g/k_{g,crit}}} & \text{(DCM)} \end{cases}$$





Boost Converter



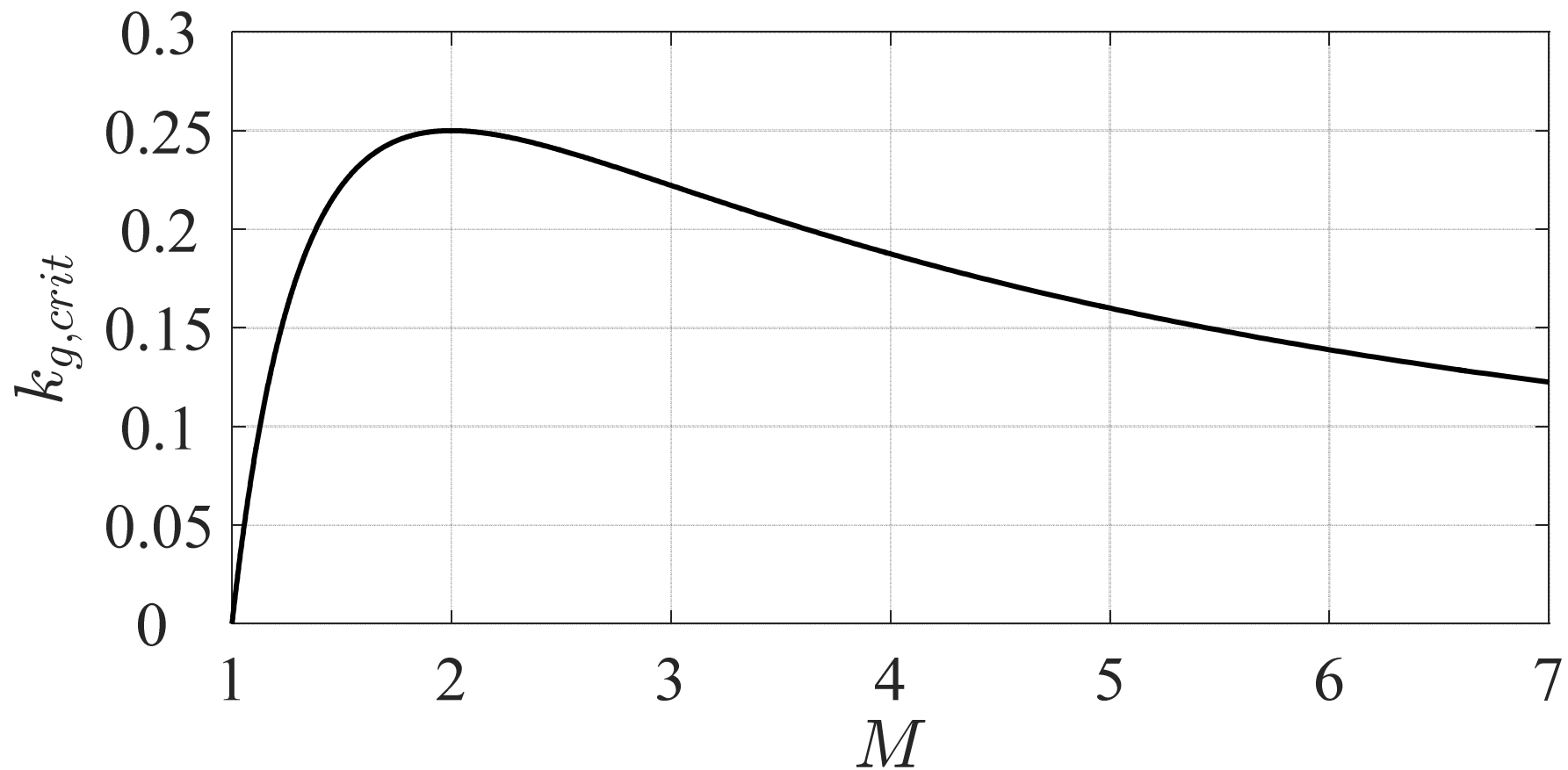
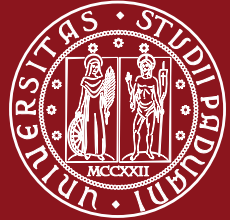
Boost converter with constant current load: voltage conversion ratio $M(D, k_g)$



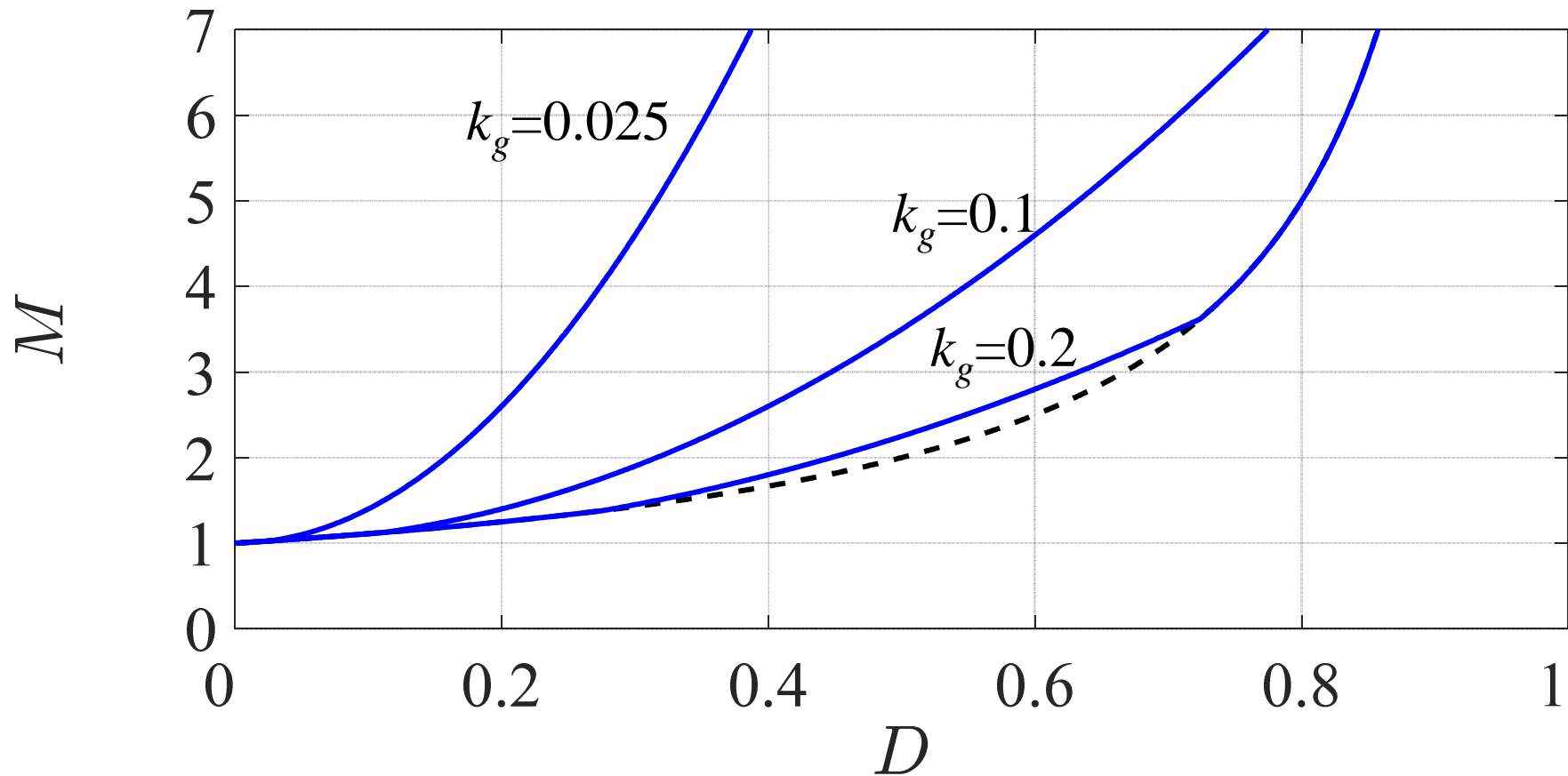
$$M(D, k_g) = \begin{cases} \frac{1}{1-D}, & k_g > k_{g,crit} \quad (\text{CCM}) \\ 1 + \frac{D^2}{k_g}, & k_g < k_{g,crit} \quad (\text{DCM}) \end{cases}$$

$$k_g \triangleq \frac{2f_s L I_o}{V_g}, \quad k_{g,crit}(D) \triangleq D(1-D) \quad \text{or} \quad k_{g,crit}(M) \triangleq \frac{M-1}{M^2}$$

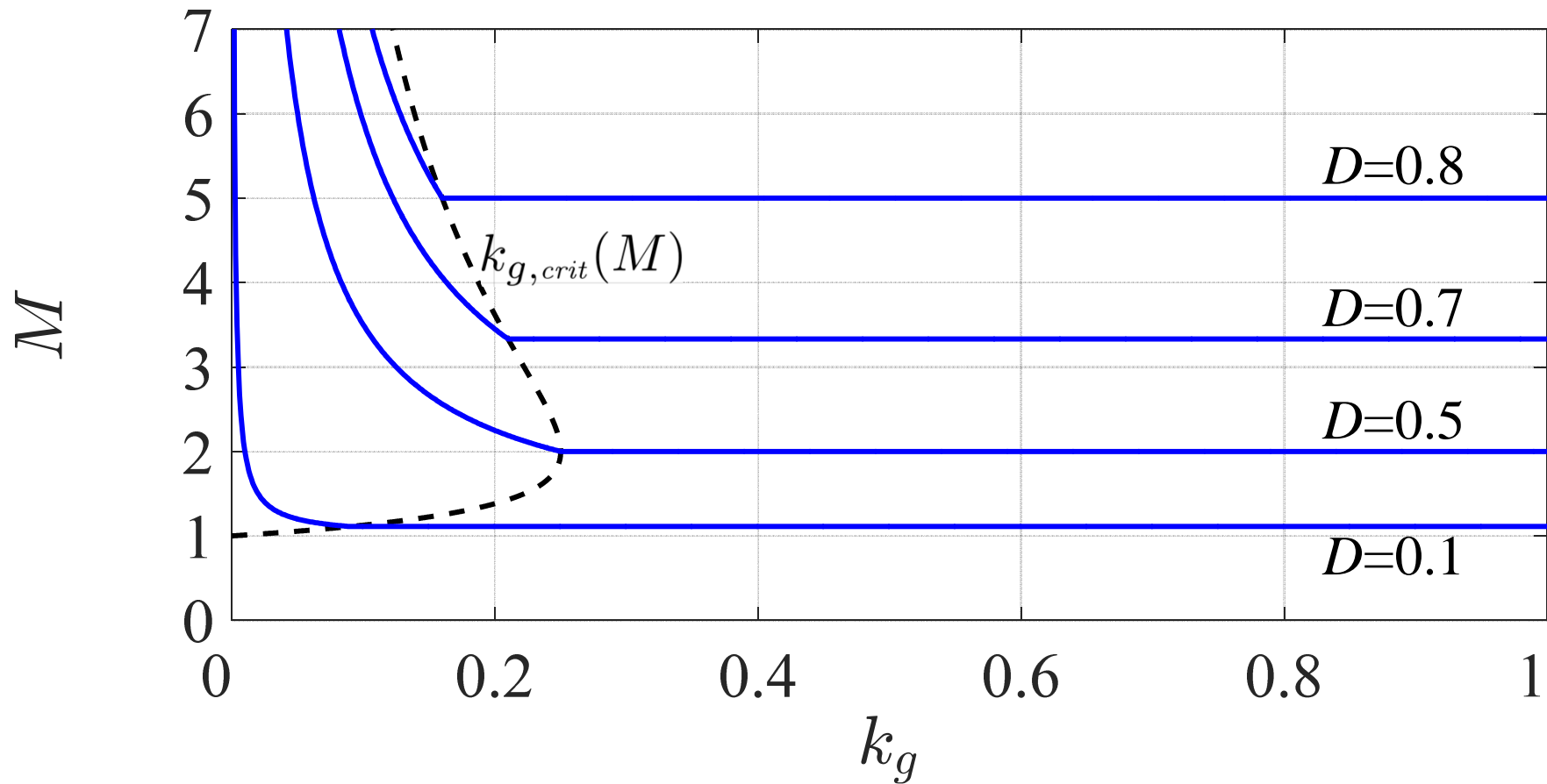
Boost converter with constant current load: critical k_g vs. M



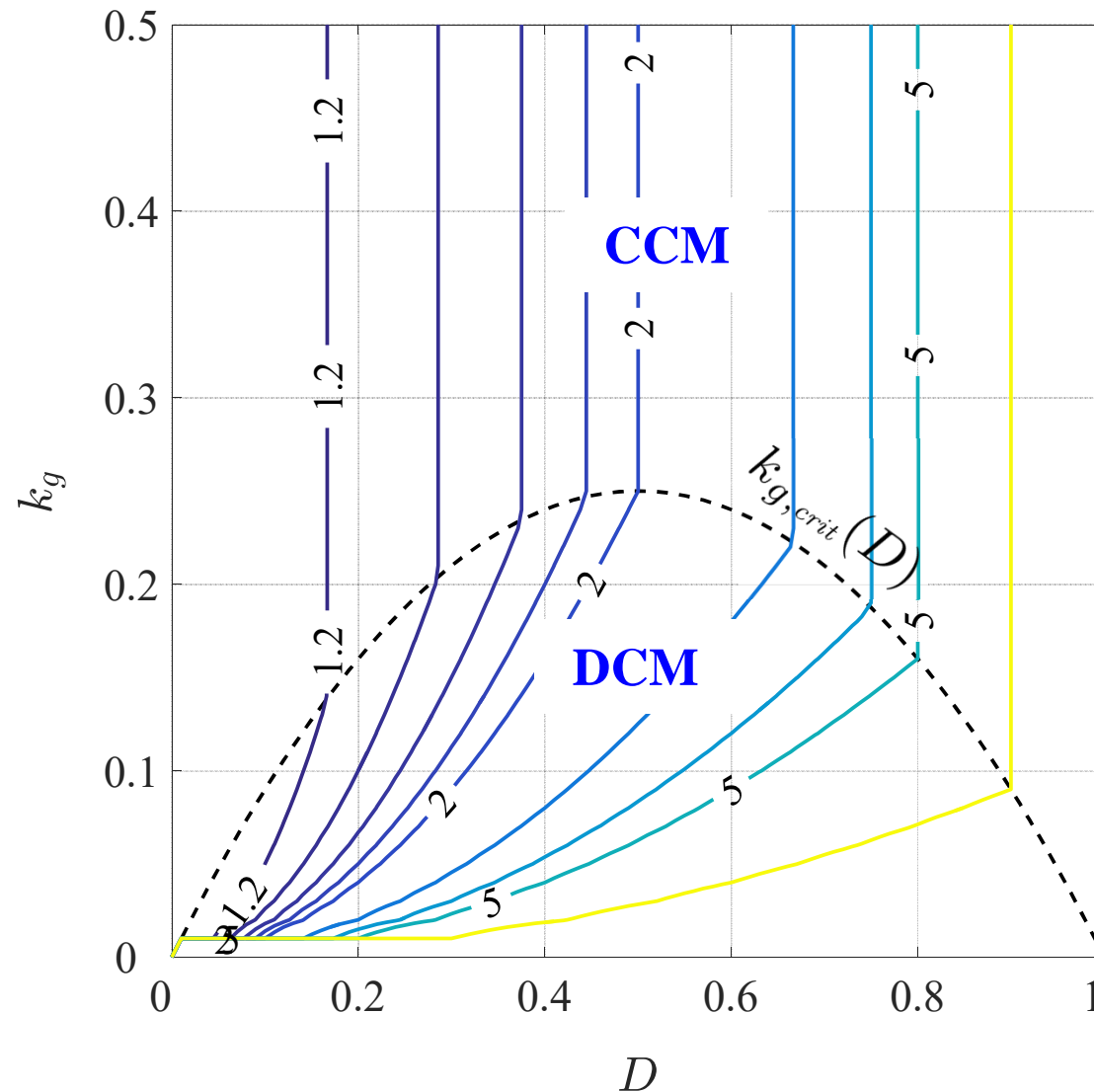
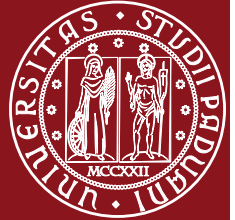
Boost converter with constant current load: control characteristics



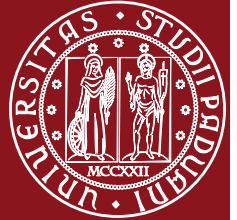
Boost converter with constant current load: output characteristics



Boost converter with constant current load: voltage conversion ratio $M(D, k_g)$



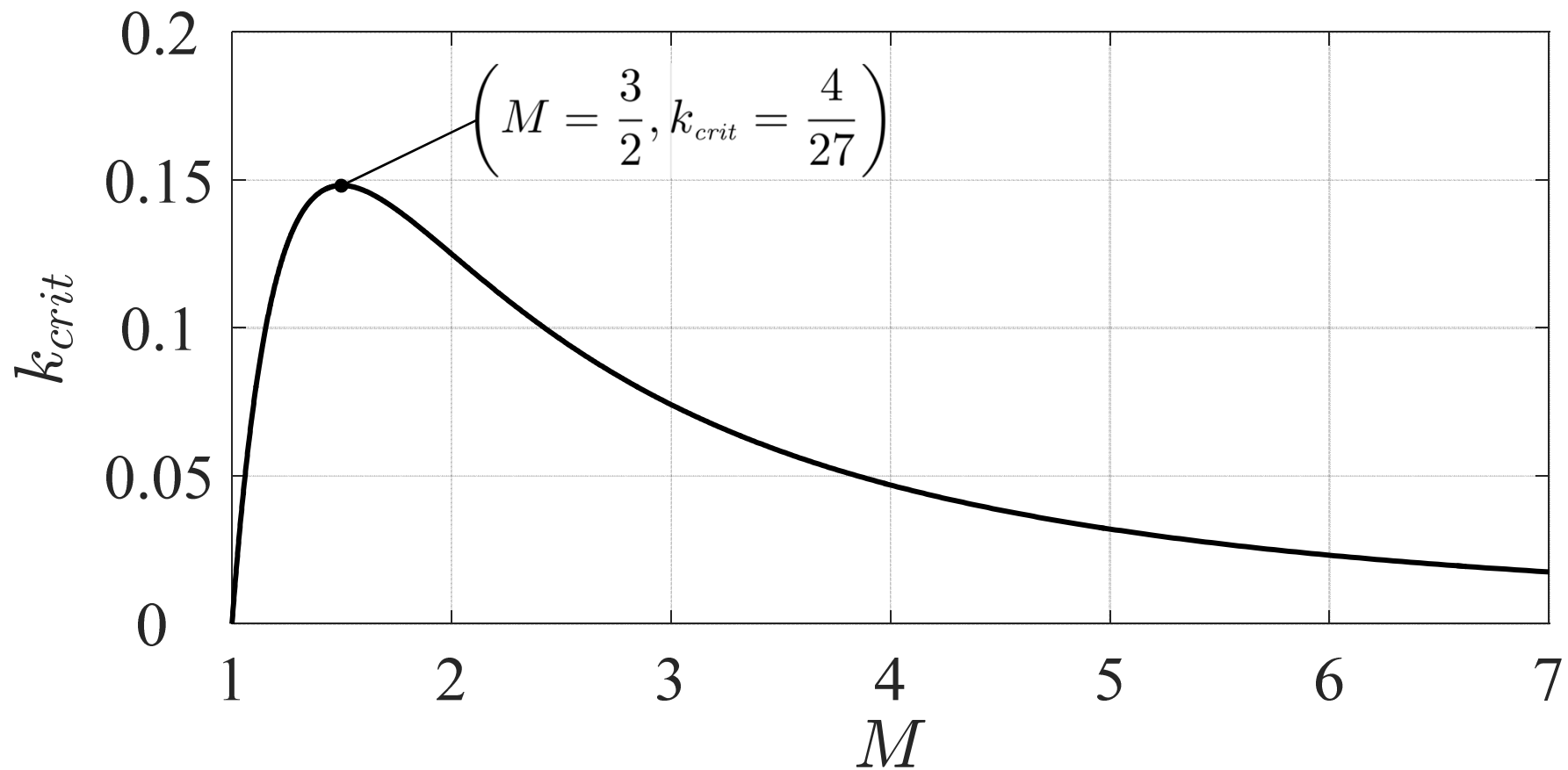
Boost converter with resistive load: voltage conversion ratio $M(D,k)$



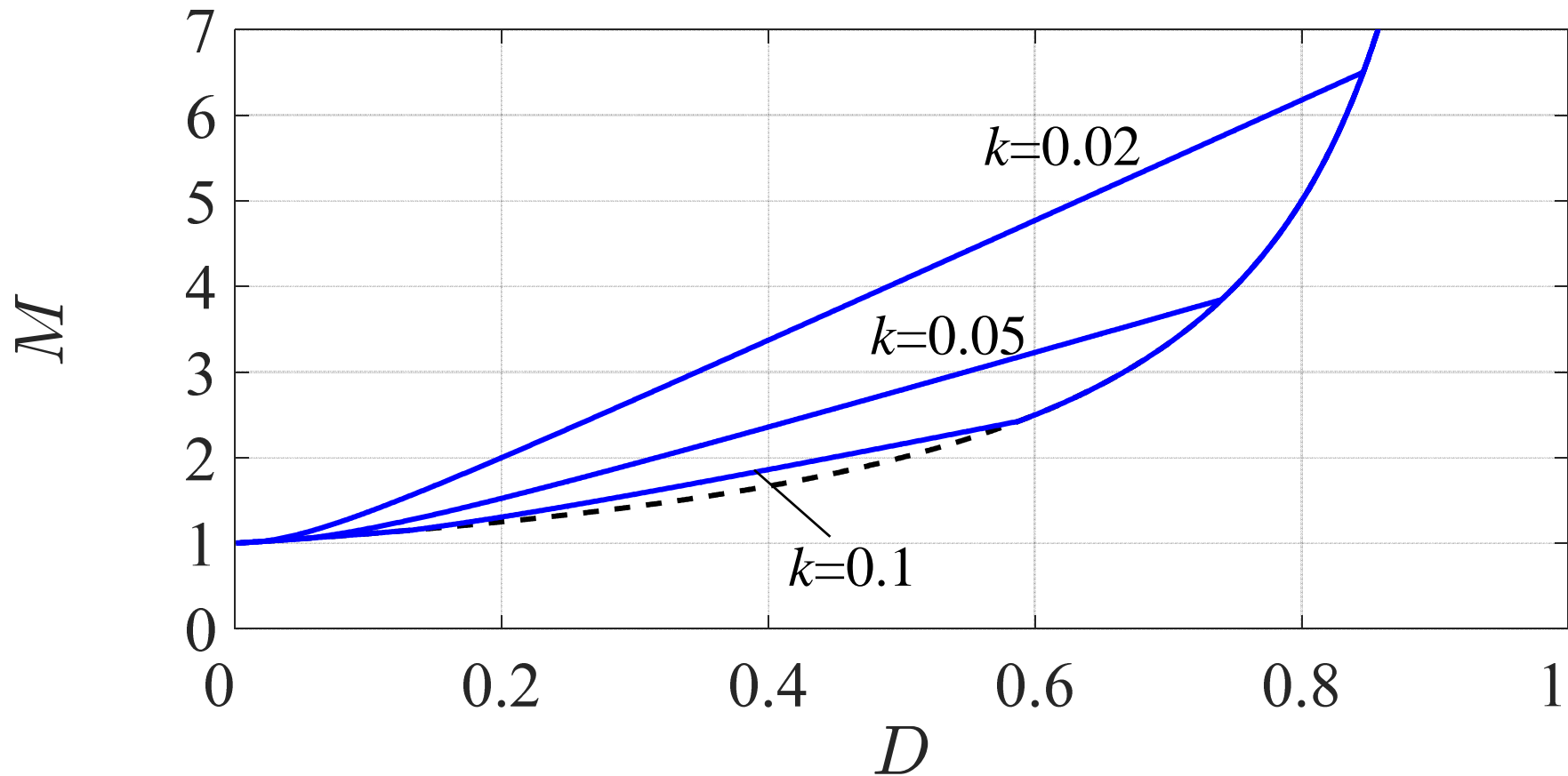
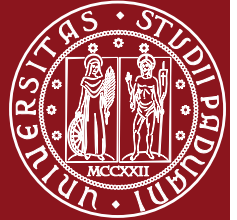
$$M(D, k) = \begin{cases} \frac{1}{1-D}, & k > k_{crit} \quad (\text{CCM}) \\ \frac{1 + \sqrt{1 + \frac{4D^2}{k}}}{2}, & k < k_{crit} \quad (\text{DCM}) \end{cases}$$

$$k \triangleq \frac{2f_s L I_o}{V_o}, \quad k_{crit}(D) \triangleq D(1-D)^2 \quad \text{or} \quad k_{crit}(M) \triangleq \frac{M-1}{M^3}$$

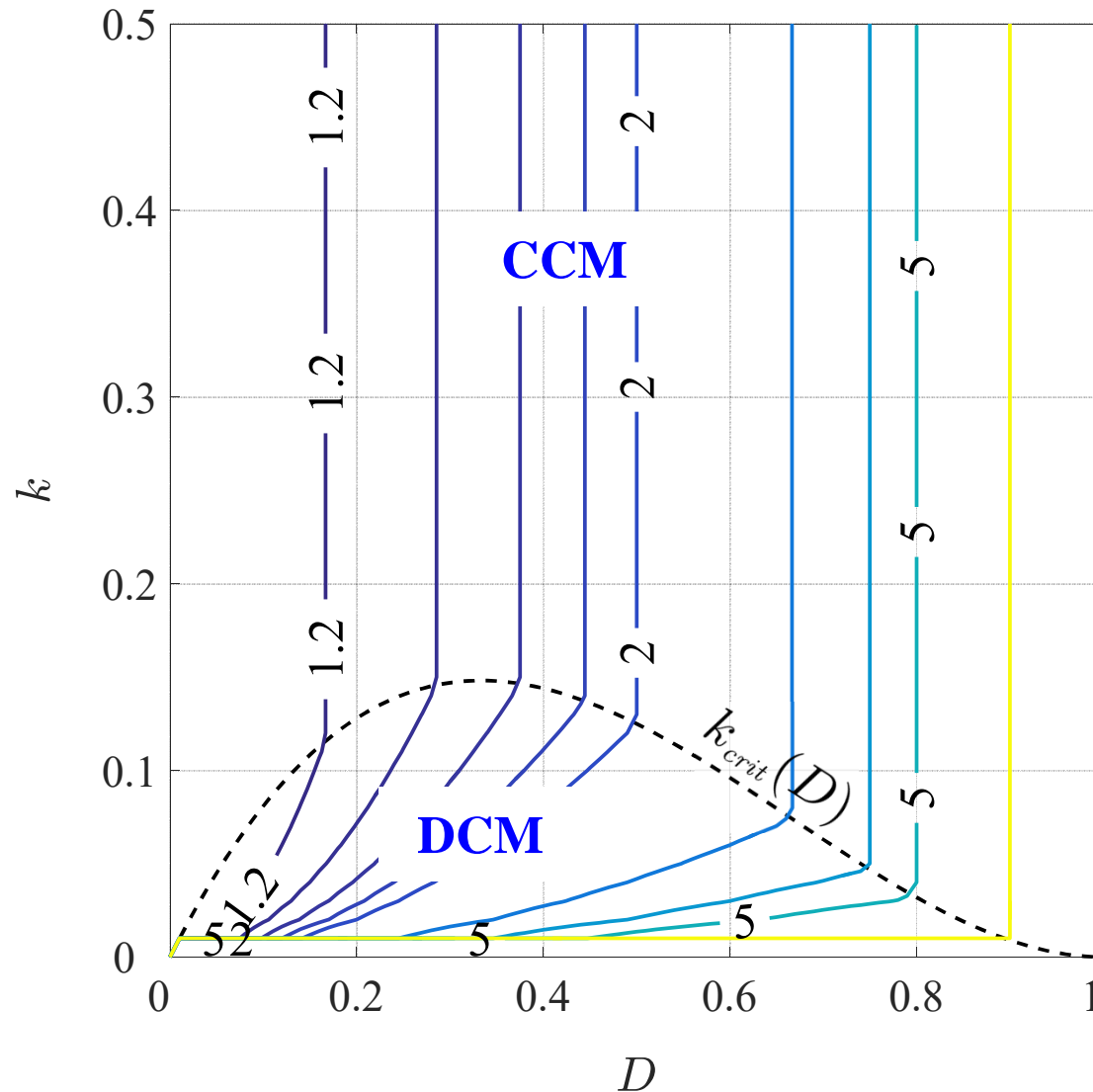
Boost converter with resistive load: critical k vs. M



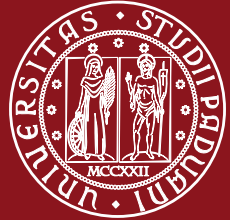
Boost converter with resistive load: control characteristics



Boost converter with resistive load: voltage conversion ratio $M(D,k)$

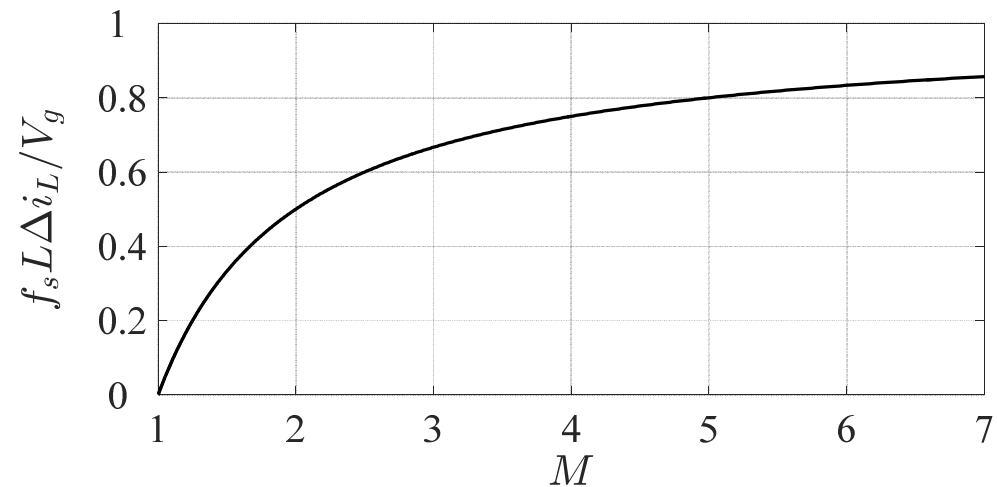


Boost converter: inductor current ripple in CCM



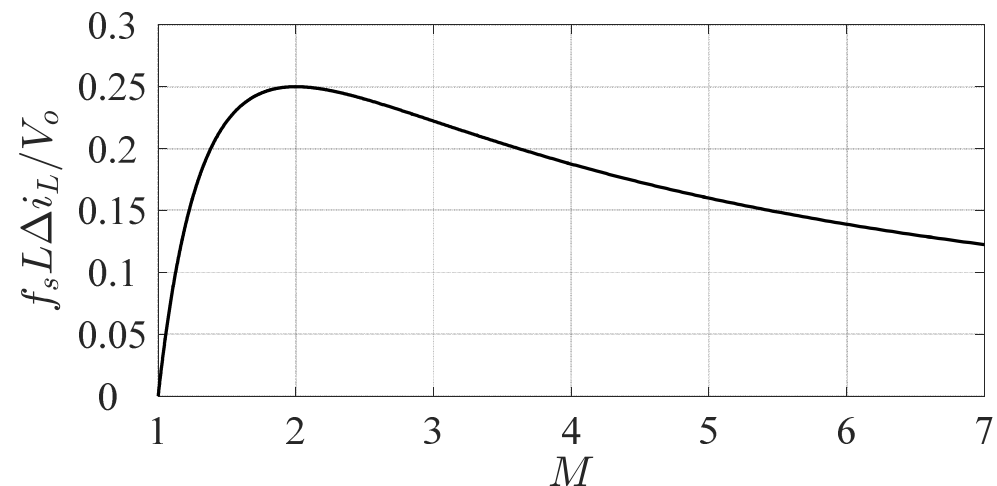
- Constant V_g :

$$\begin{aligned}
 \Delta i_L &= \frac{V_g}{f_s L} D \\
 &= \frac{V_g}{f_s L} \frac{M - 1}{M}
 \end{aligned}$$

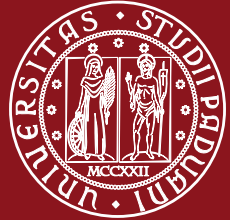


- Constant V_o :

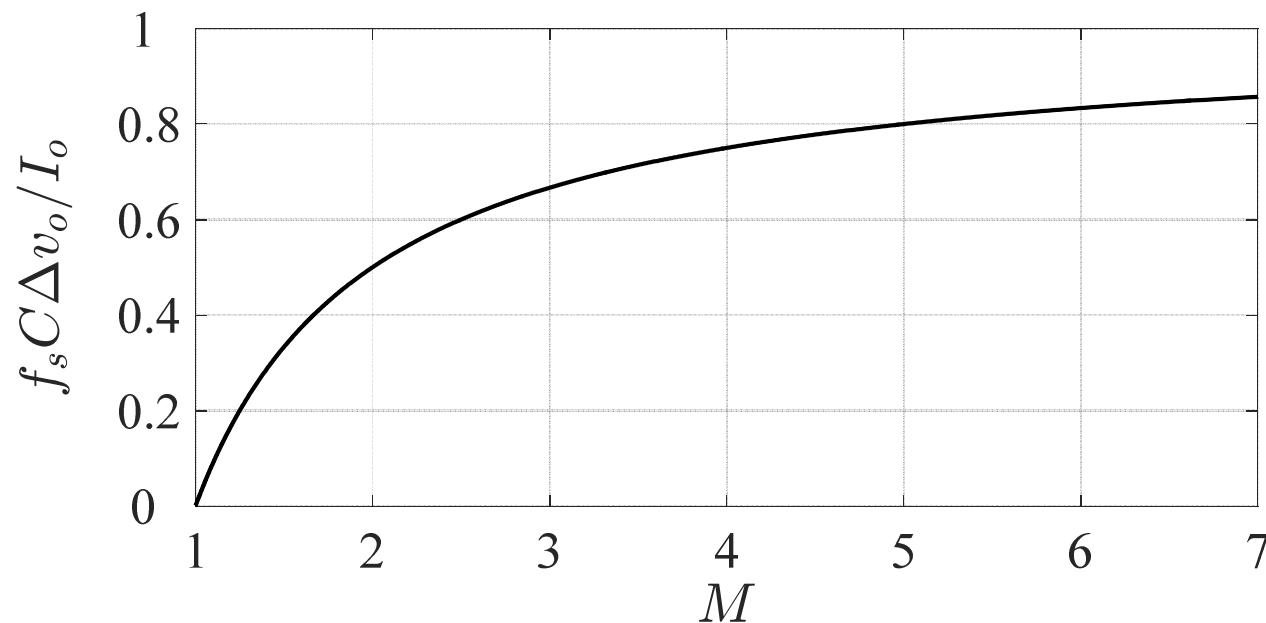
$$\begin{aligned}
 \Delta i_L &= \frac{V_o}{f_s L} D(1 - D) \\
 &= \frac{V_o}{f_s L} \frac{M - 1}{M^2}
 \end{aligned}$$



Boost converter: output voltage ripple in CCM

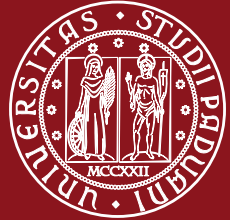


$$\Delta v_o = \frac{DI_o}{f_s C} = \frac{I_o}{f_s C} \frac{M - 1}{M}$$

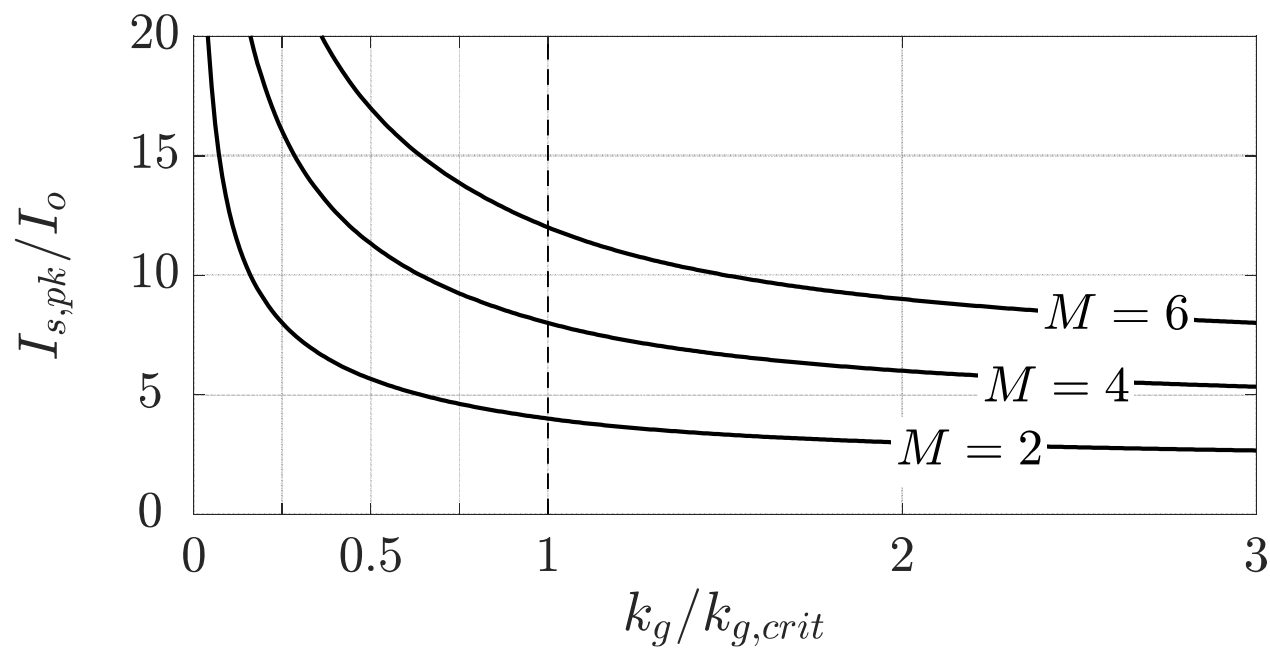




Boost converter: current stress

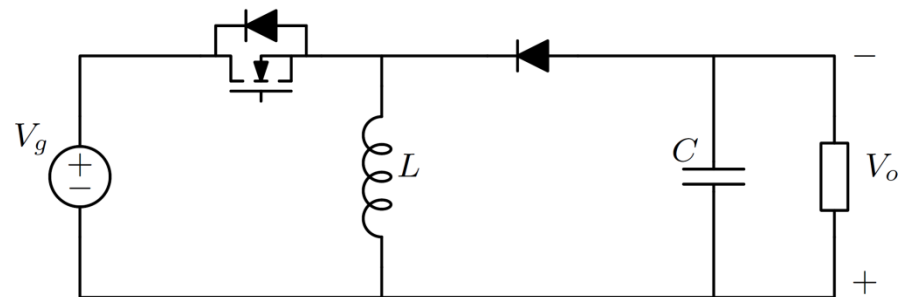


$$\frac{I_{s,pk}}{I_o} = \begin{cases} M \left(1 + \frac{1}{k_g/k_{g,crit}} \right) & \text{(CCM)} \\ \frac{2M}{\sqrt{k_g/k_{g,crit}}} & \text{(DCM)} \end{cases}$$

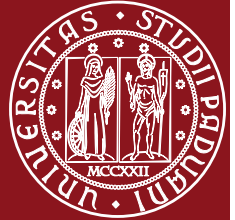




Buck-Boost Converter



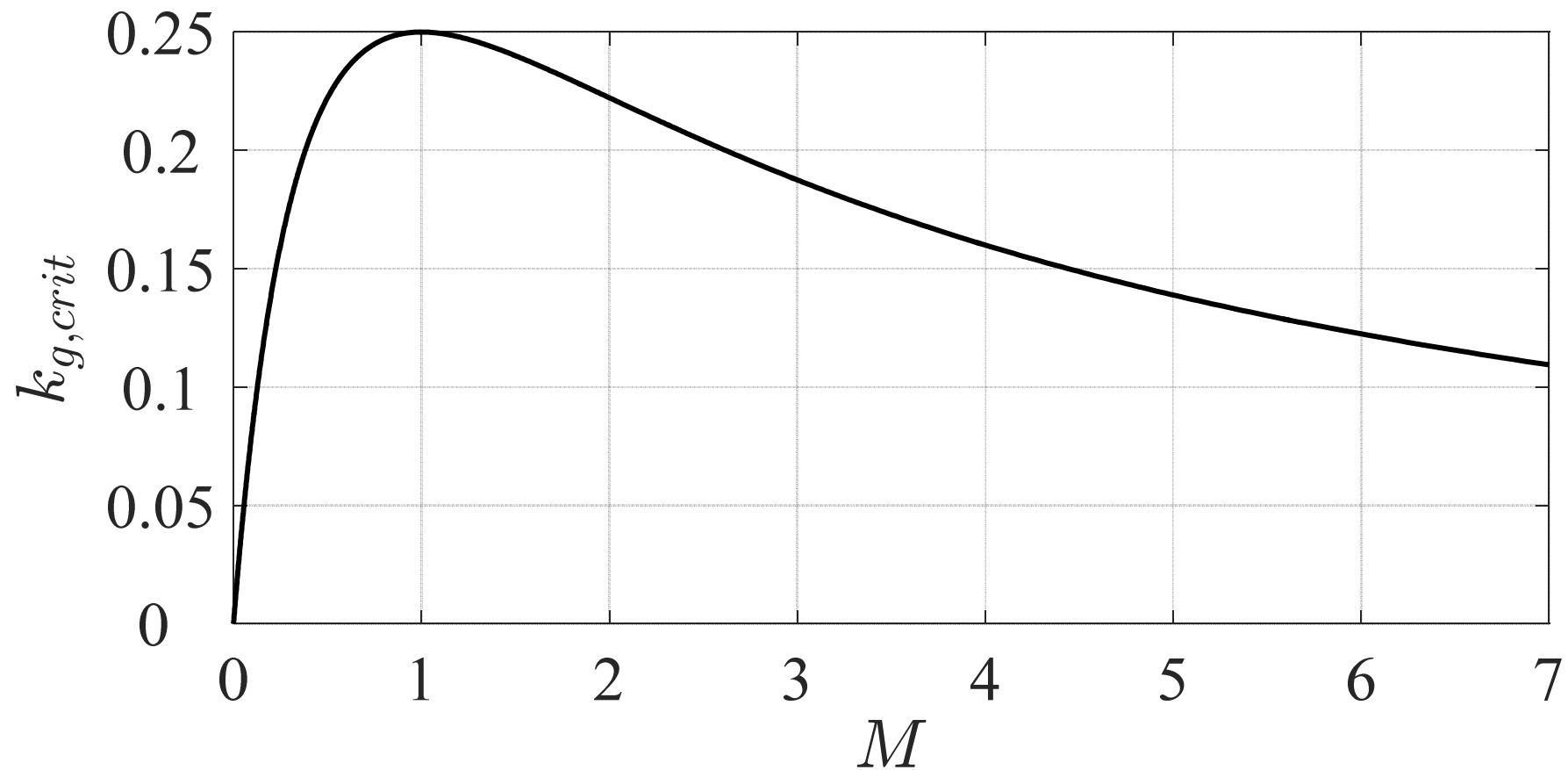
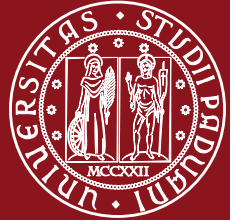
Buck-Boost converter with constant current load: voltage conversion ratio $M(D, k_g)$



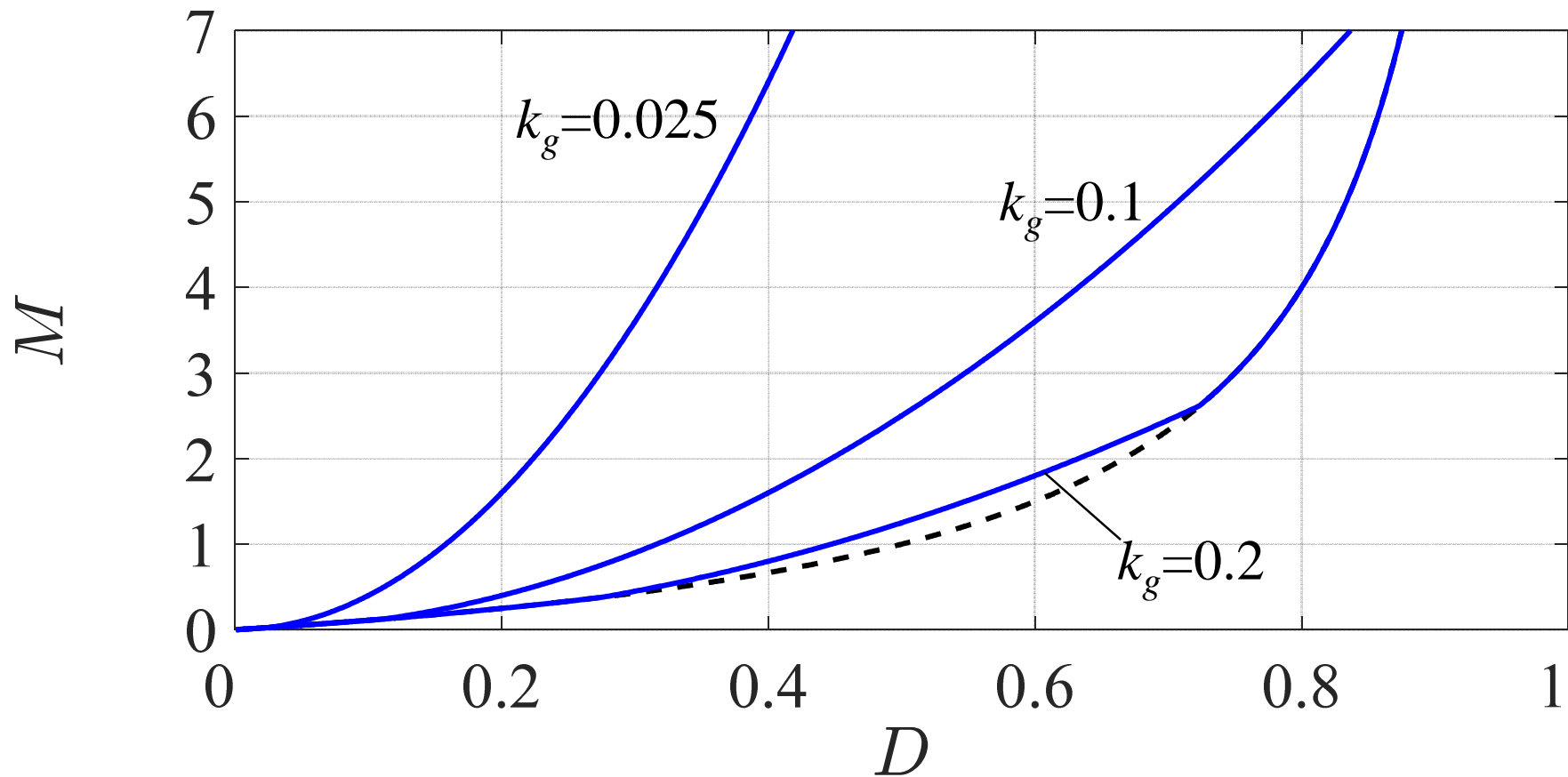
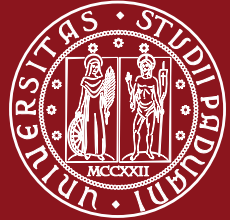
$$M(D, k_g) = \begin{cases} \frac{D}{1-D}, & k_g > k_{g,crit} \quad (\text{CCM}) \\ \frac{D^2}{k_g}, & k_g < k_{g,crit} \quad (\text{DCM}) \end{cases}$$

$$k_g \triangleq \frac{2f_s L I_o}{V_g}, \quad k_{g,crit}(D) \triangleq D(1-D) \quad \text{or} \quad k_{g,crit}(M) \triangleq \frac{M}{(1+M)^2}$$

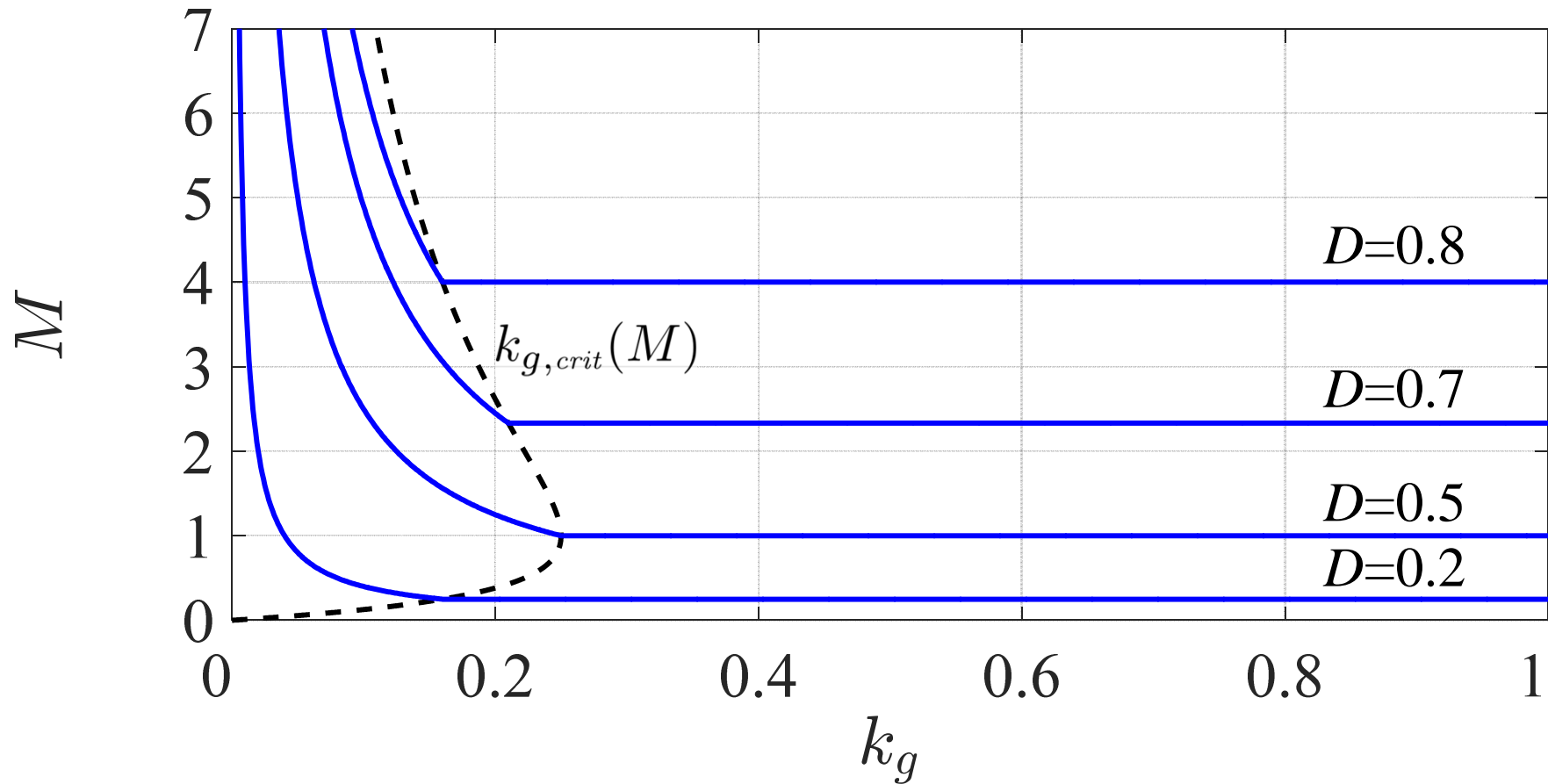
Buck-Boost converter with constant current load: critical k_g vs. M



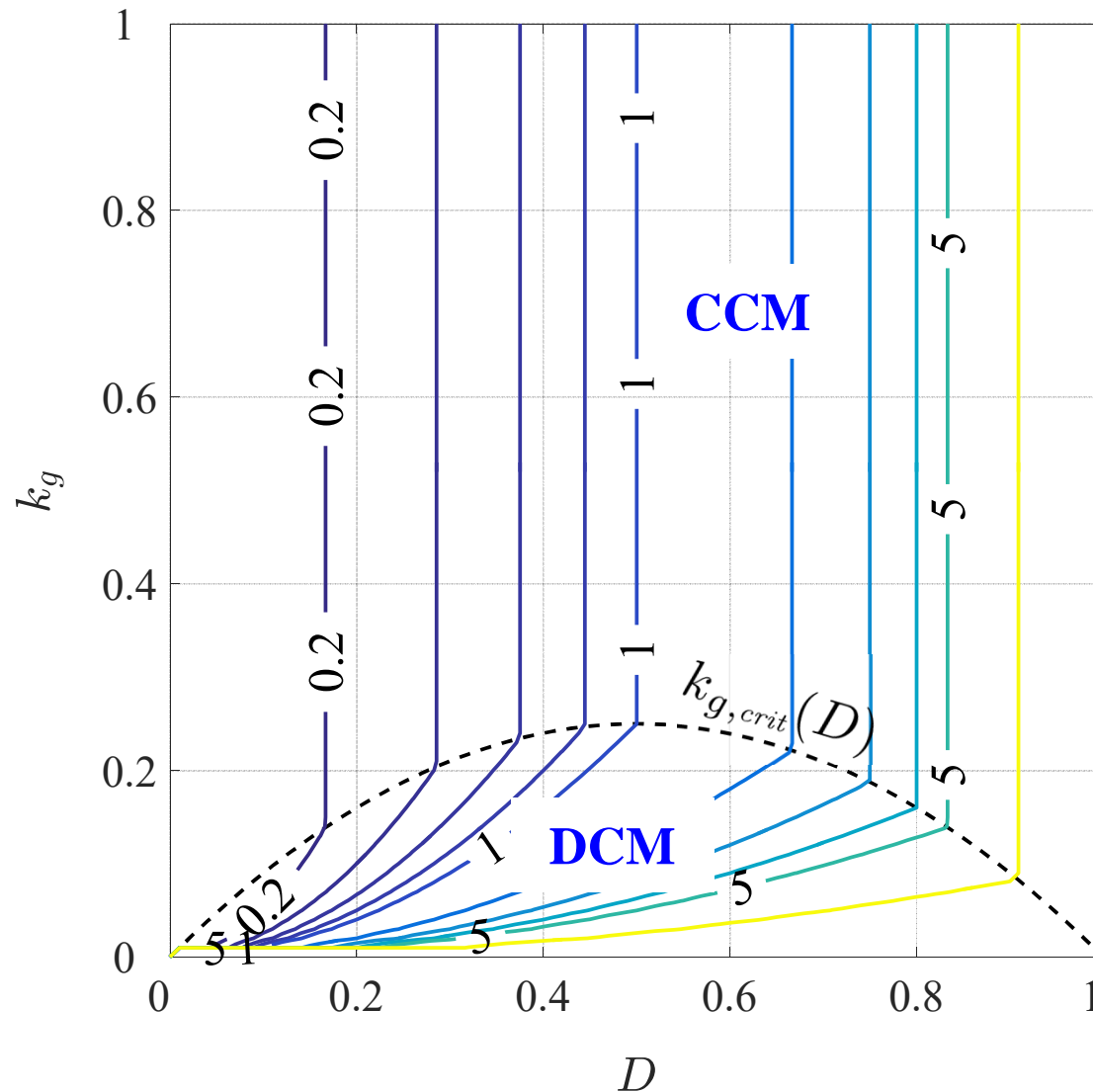
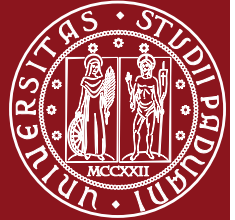
Buck-Boost converter with constant current load: control characteristics



Buck-Boost converter with constant current load: output characteristics



Buck-Boost converter with constant current load: voltage conversion ratio $M(D, k_g)$



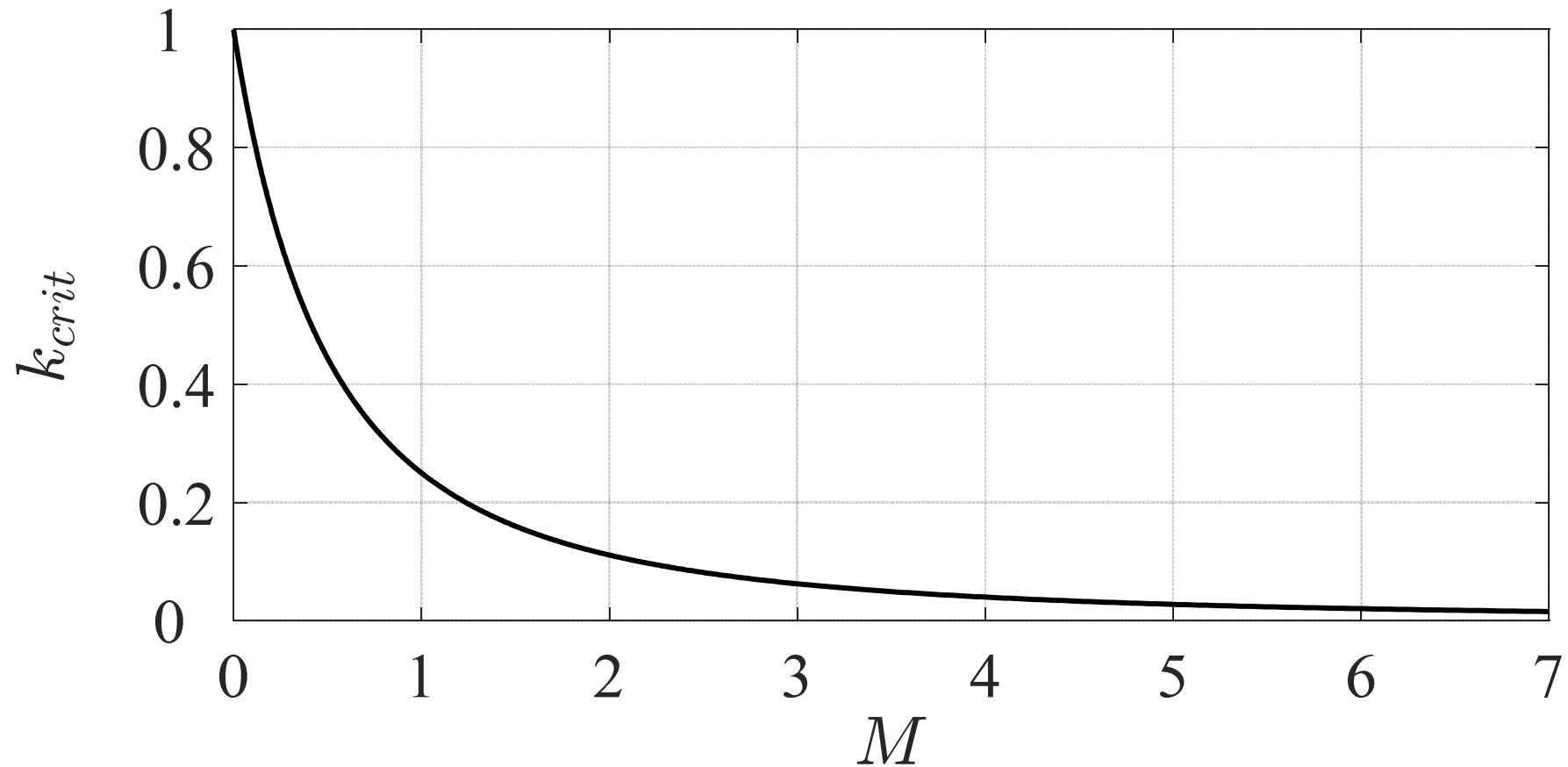
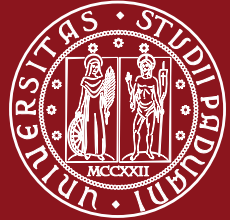
Buck-Boost converter with resistive load: voltage conversion ratio $M(D,k)$



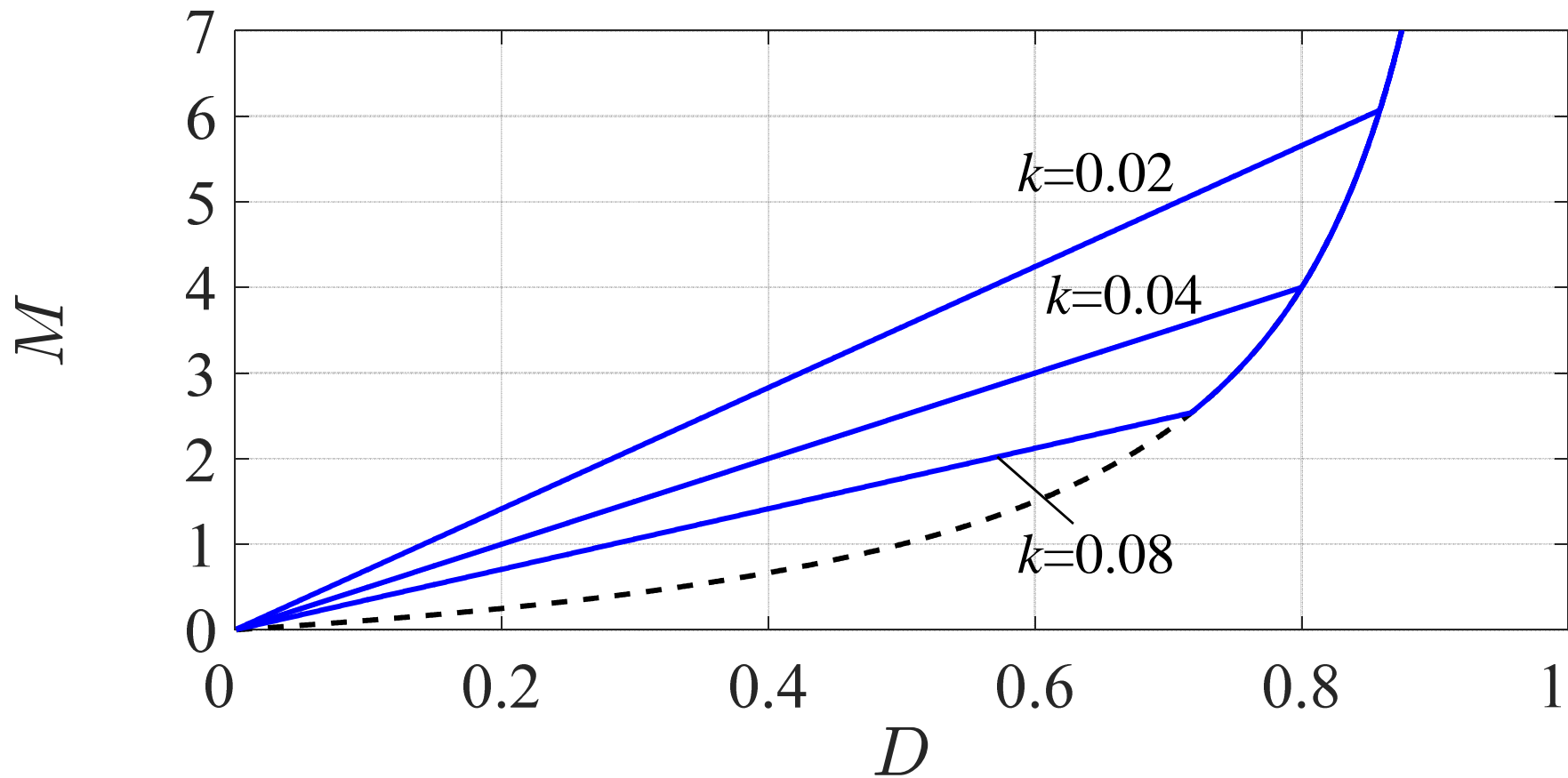
$$M(D, k) = \begin{cases} \frac{D}{1-D}, & k > k_{crit} \quad (\text{CCM}) \\ \frac{D}{\sqrt{k}}, & k < k_{crit} \quad (\text{DCM}) \end{cases}$$

$$k \triangleq \frac{2f_s L I_o}{V_o}, \quad k_{crit}(D) \triangleq (1-D)^2 \quad \text{or} \quad k_{crit}(M) \triangleq \frac{1}{(1+M)^2}$$

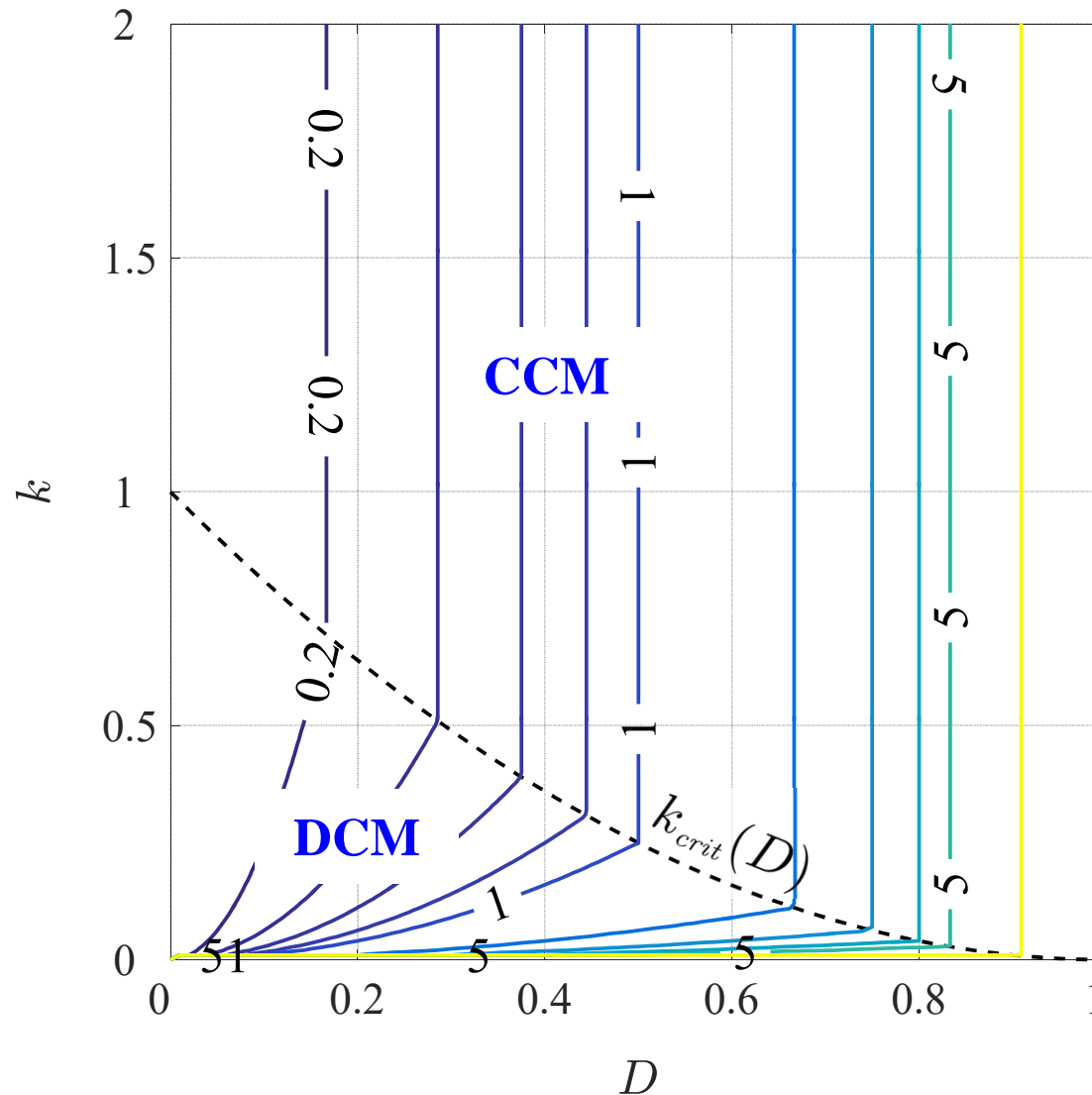
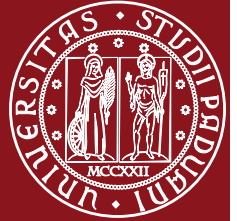
Buck-Boost converter with resistive load: critical k vs. M



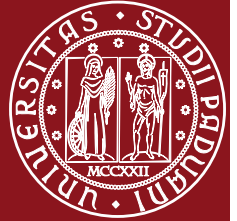
Buck-Boost converter with resistive load: control characteristics



Buck-Boost converter with resistive load: voltage conversion ratio $M(D,k)$

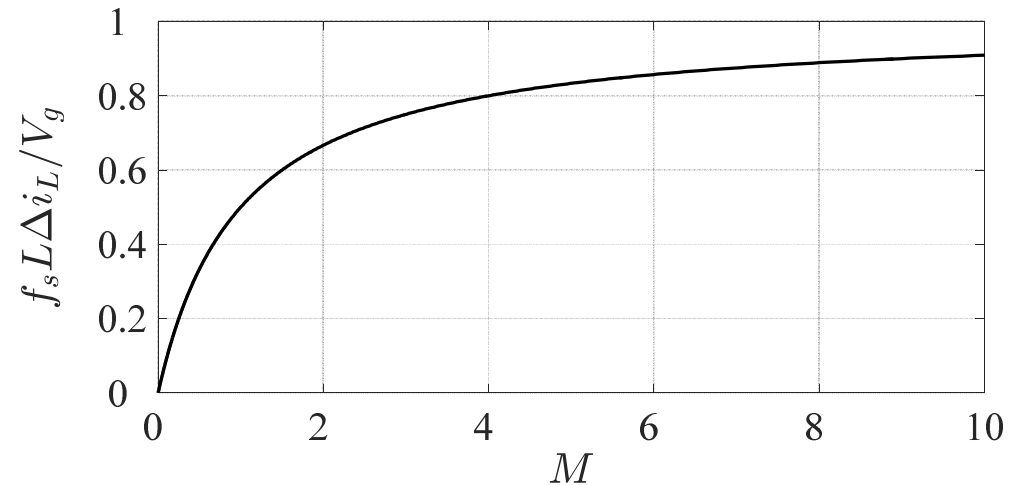


Buck-Boost converter: inductor current ripple in CCM



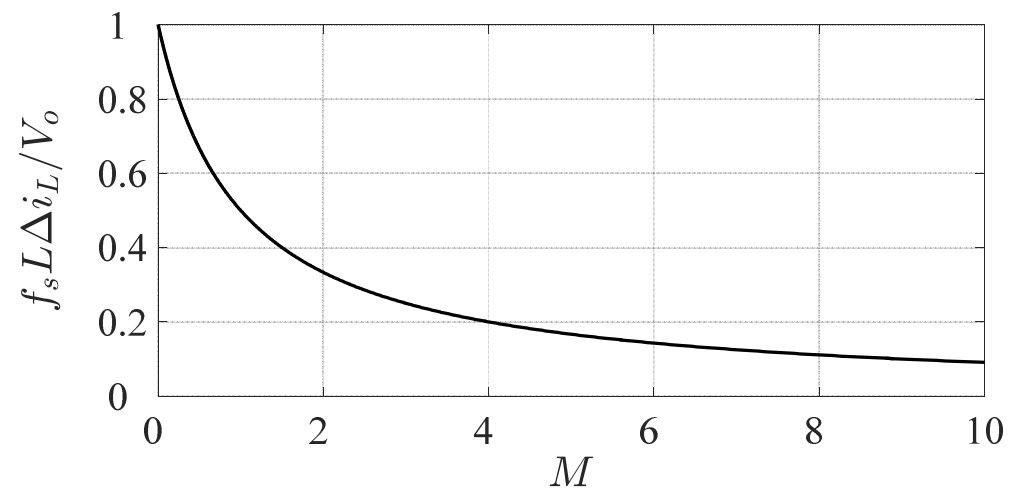
- Constant V_g :

$$\begin{aligned}
 \Delta i_L &= \frac{V_g}{f_s L} D \\
 &= \frac{V_g}{f_s L} \frac{M}{1 + M}
 \end{aligned}$$



- Constant V_o :

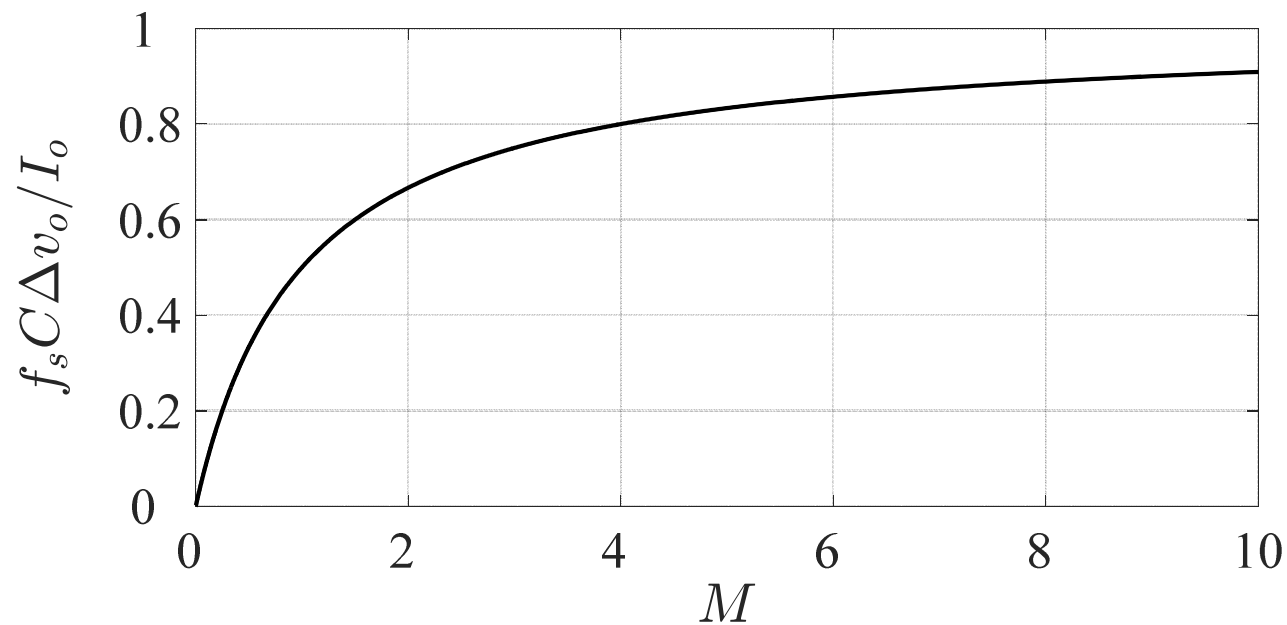
$$\begin{aligned}
 \Delta i_L &= \frac{V_o}{f_s L} (1 - D) \\
 &= \frac{V_o}{f_s L} \frac{1}{1 + M}
 \end{aligned}$$



Buck-Boost converter: output voltage ripple in CCM



$$\Delta v_o = \frac{DI_o}{f_s C} = \frac{I_o}{f_s C} \frac{M}{1+M}$$



Buck-Boost converter: current stress

$$\frac{I_{s,pk}}{I_o} = \begin{cases} (1 + M) \left(1 + \frac{1}{k_g/k_{g,crit}} \right) & \text{(CCM)} \\ \frac{2(1 + M)}{\sqrt{k_g/k_{g,crit}}} & \text{(DCM)} \end{cases}$$

