Semester 2 – 2020 Batch

MA1024 – Methods of Mathematics

Section: Probability and Statistics

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1. Introduction to statistical experiments and outcomes

In Statistics we mainly have observational or experimental data.

- A Statistical Experiment is a process which is designed and conducted to acquire necessary experimental data under a controlled or an uncontrolled background.

  Rolling a die is an example for an experiment with an uncontrolled background where as observing the growth of a plant under a treatment exemplifies an experiment with an controlled background.

- The experiment can have more than one possible outcome and each possible discrete outcome or the scale of continuous outcome can be specified in advance.

Discrete Outcomes	**Continuous Outcomes**
Number of Heads resulted in 5 flips of a coin S={0,1,2,3,4,5}	The excess amount of water/day in a power station in m^3.
Number of motor vehicles crashed in an accident S={0,1,2,3,4,5, more than 5}	Duration for recurrence of a cancer in weeks.

- The outcome of the experiment purely depends on chance.

- A sample survey, a census or a cohort study will be carried out usually to gather observational data.

❖ *In a sample survey a **sample** of elements from a target population is selected to conduct the **survey.***

❖ *A **census** acquires information about the members of a given **population of interest.***

❖ *A cohort is a group of subjects who share a particular event together during a particular time span.*

- Gathered data will be effectively used to perform statistical analysis to make decisions in scientific research.

- A **sample space** is a set of elements that represents all possible outcomes of a statistical experiment.

  When you randomly pick a card out of 52 cards in a deck, the sample space is the 52 cards with a distinct sign and a number/letter combination in the deck.

- A **sample point** is an element of a sample space.

  Eg: The card with 9-Hearts

- An **event** is a subset of a sample space - one or more sample points.

  Eg: Aces

Types of events

Two events are **mutually exclusive** if they have no sample points in common.

Consider the events A={Heart-J, Spade-Q, Heart-K, Spade-A}, B={Club-A, Diamond-A} and C={Hearts-A, Clubs-A}

Events A and B are mutually exclusive, and Events A and C are mutually exclusive; since they have no points in common.

Events B and C have common sample points, so they are not mutually exclusive.

- Two events are **independent** when the occurrence of one does not affect the probability of the occurrence of the other.

  Suppose you pick a card from one deck and pick another card from a different deck.

  Then each picking of a card is an independent event since neither card which is picked will effect the picking of the other.

Counting Rules

The solution to many statistical experiments involves being able to count the number of points in a sample space.

- **Rule 1:** If an experiment consists of a sequence of k steps in which there are n_1 possible results for the first step, n_2 possible results for the second step, and so on, then the total number of experimental outcomes is given by $(n_1)(n_2) \ldots (n_k)$.

Eg: A code with a capital English letter followed by a digit.

26*10

Example

A businessman has 4 shirts and 7 ties. How many different shirt-tie outfits can he create?

Solution: For each outfit, he can choose one of four shirts and one of seven ties. Therefore, the business man can create (4)(7) = 28 different shirt-tie outfits.

- A helpful graphical representation of a multiple-step experiment is a **Tree Diagram**. A tree diagram is a graphical representation that helps in visualizing a multiple step experiment.

In general, n objects can be arranged in $n(n - 1)(n - 2) \ldots (3)(2)(1)$ ways. This product is represented by the symbol $n!$, which is called **n factorial**. (By convention, $0! = 1$.)

A **permutation** is an arrangement of all or part of a set of objects, *with* regard to the order of the arrangement.

Example: The letters A, B & C can be arranged a number of different ways ABC, ACB, BAC, BCA, CBA and CAB. Each of these arrangements is a permutation.

The number of permutations of *n* objects taken *r* at a time is denoted by ^nP_r.

Rule 2: The number of permutations of *n* objects taken *r* at a time is

$$^nP_r = n(n-1)(n-2)\ldots(n-r+1)$$

$$^nP_r = \frac{n!}{(n-r)!}$$

A **combination** is a selection of all or part of a set of objects, *regardless of* the order in which they were selected.

The number of combinations of *n* objects taken *r* at a time is denoted by ${}^nC_r$.

Rule 3: The number of Combinations of *n* objects taken *r* at a time is

$$ {}^nC_r = \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!} = \frac{n!}{r!(n-r)!} = \frac{{}^nP_r}{r!} $$

Rule 4: Suppose that a multi-set M contains n items of k different types, such that there are exactly r_1 items of type 1, r_2 items of type 2, and so on. Then the number of distinct permutations of the items in M is

$$\frac{n!}{r_1!\,r_2!\ldots r_3!}$$

Ex: Five beads, one of each of the colours red, green, yellow, blue, and pink, are threaded on a circular wire. In how many different orders can the beads occur round the circle?