EN1014 Electronic Engineering

Review of Number Systems

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Representation of Numbers

- A number represented in the base (or radix) r has the form

$$\boxed{(a_n a_{n-1} \cdots a_0 . a_{-1} a_{-2} \cdots a_{-m})_r .}$$

- The coefficients a_i, $i = n, n-1, \ldots, -m$ takes
  - the integers from 0 to $r - 1$, if $r \leq 10$
  - the integers from 0 to 9 and letters from the English alphabet, if $r > 10$.

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  - the integers from 0 to $r-1$, if $r \leq 10$
  - the integers from 0 to 9 and letters from the English alphabet, if $r > 10$.
- If $r \neq 10$, the equivalent base-10 (decimal) number can be obtained from

$$a_n r^n + a_{n-1} r^{n-1} + \cdots + a_1 r + a_0 + a_{-1} r^{-1} + \cdots + a_{-m} r^{-m}.$$

- In this module, we are especially interested in numbers represented in base-2 (binary), base-10 and base-16 (hexadecimal).

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- The conversion of a decimal integer to a number in base r is done by dividing the number and all successive quotients by r and accumulating the remainders.

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  - Convert $(41)_{10}$ to binary and octal (base-8)
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- Examples:
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- The conversion of a decimal fraction to a number in base r is done by multiplying the number and all fraction parts by r and accumulating the integers.

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- The conversion of a decimal fraction to a number in base r is done by multiplying the number and all fraction parts by r and accumulating the integers.
- Examples:
  - Convert $(0.6875)_{10}$ to binary and octal
  - Convert $(0.515)_{10}$ to hexadecimal.

Number Base Conversion *cont'd*

- From previous examples:
  - $(41.6875)_{10} = (101001.1011)_2 = (51.54)_8$
  - $(15225.515)_{10} = (3B79.83D70A\ldots)_{16}$.

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- Example:
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- The conversion from hexadecimal to binary is done by reversing the preceding procedure.
- Example:
  - Convert $(A2.24)_{16}$ and $(57.C)_{16}$ to binary.

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- Example:
  - Convert $(A2.24)_{16}$ and $(57.C)_{16}$ to binary.
- The procedure for the conversion between binary to octal and octal to binary is similar.
- Hexadecimal (or sometimes octal) number system is used to represent binary numbers compactly.

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- For examples,
  - 2's complement and 1's complement for binary
  - 10's complement 9's complement for decimal.

Complements of Numbers *cont'd*

- Given a number N in base r having n digits, the $(r-1)$'s complement of N is defined as

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  - n 9's for decimal.
- In finding 1's complement, just change 0's to 1's and 1's to 0's.

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- Examples:
  - Find 9's complement of $(546700)_{10}$ and $(012398)_{10}$
  - Find 1's complement of $(1011000)_2$ and $(0101101)_2$.

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- The r's complement of N can be obtained by adding 1 to the $(r-1)$'s complement of N.

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- The r's complement of N can be obtained by adding 1 to the $(r-1)$'s complement of N.
- Examples:
  - Find 10's complement of $(546700)_{10}$ and $(012398)_{10}$
  - Find 2's complement of $(1011000)_2$ and $(0101101)_2$.

Complements of Numbers *cont'd*

- If the original number N contains a radix point,
  - the point should be removed temporarily in order to form the r's or $(r-1)$'s complement.
  - Then, the radix point is restored to the complemented number in the same relative position.

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- The subtraction of two n-digit unsigned numbers M and N, i.e. $M - N$, in base r is done as follows:
  - Add M to the r's complement of N, which gives $M - N + r^n$.
  - If $M \geq N$, discard the carry produced by r^n.
  - If $M < N$, there is no carry, and the sum is equal to $r^n - (N - M)$ (r's complement of $(N - M)$).

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  - If $M \geq N$, discard the carry produced by r^n.
  - If $M < N$, there is no carry, and the sum is equal to $r^n - (N - M)$ (r's complement of $(N - M)$).
- In the case $M < N$, take the r's complement of the sum and place a negative sign in front in order to obtain the answer in the conventional form.
- Examples:
  - Find $(72532)_{10} - (3250)_{10}$ and $(3250)_{10} - (72532)_{10}$
  - Find $(1010100)_2 - (1000011)_2$ and $(1000011)_2 - (1010100)_2$.

Complements of Numbers *cont'd*

- The subtraction can also be performed by using $(r-1)$'s complement. However, if $M \geq N$, in addition to discarding the carry, 1 must be added to the sum to get the correct answer (called end-around carry).

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- The subtraction can also be performed by using $(r-1)$'s complement. However, if $M \geq N$, in addition to discarding the carry, 1 must be added to the sum to get the correct answer (called end-around carry).
- Home work:
  - Find $(72532)_{10} - (3250)_{10}$ and $(3250)_{10} - (72532)_{10}$ using 9's complement
  - Find $(1010100)_2 - (1000011)_2$ and $(1000011)_2 - (1010100)_2$ using 1's complement.

Signed Binary Numbers

- Non-negative numbers can be represented as unsigned numbers. However, a sign (or notation) is necessary to represent negative numbers.
- In computer arithmetic with signed numbers, the sign of the number is indicated with a bit placed in the leftmost position of the number.

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- The convention is to make the sign bit 0 for positive and 1 for negative.
- There are two main representations for signed numbers.
  - signed-magnitude representation
  - signed-complement representation.
- In the signed-magnitude representation, a number is negated by changing its sign.
- In the signed-complement representation, a number is negated by taking its complement.

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- Example:
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- With n bits,
  - $-(2^{n-1}-1)$ to $(2^{n-1}-1)$ can be represented with the signed-magnitude representation (with two zeros)
  - $-(2^{n-1}-1)$ to $(2^{n-1}-1)$ can be represented with the signed-1's-complement representation (with two zeros)
  - -2^{n-1} to $(2^{n-1}-1)$ can be represented with the signed-2's-complement representation (with one zero).

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  - -2^{n-1} to $(2^{n-1}-1)$ can be represented with the signed-2's-complement representation (with one zero).
- In computer arithmetic, the signed-2's-complement representation is mostly used.
- The signed-1's-complement representation may be used in logical operations because the change of 0's to 1's and 1's to 0's is equivalent to a logical complement operation.

Signed Binary Numbers *cont'd*

- Home work:
  - Obtain -7 to 7 in the signed-magnitude and the signed-1's-complement, and -8 to 7 in the signed-2's-complement representations with 4 bits.

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  - Obtain -7 to 7 in the signed-magnitude and the signed-1's-complement, and -8 to 7 in the signed-2's-complement representations with 4 bits.
  - In the signed-2's-complement representation with 4 bits, find
    - (a) $5_{10} + 2_{10}$ (b) $6_{10} + 4_{10}$ (c) $2_{10} - 6_{10}$ (d) $-3_{10} - 4_{10}$ (e) $-4_{10} - 7_{10}$ (f) $-3_{10} - (-5)_{10}$.
    - Note that a carry out of the sign-bit position needs to be discarded in the addition and the subtraction in the signed-2's-complement representation. Also, overflow may occur in some additions and subtractions.

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- Home work:
  - Read about the binary-coded decimal (BCD) code and the gray code.