Computation of 2D Equillibrium Flow over an Expansion Profile

A Report submitted

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in

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by

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(SC18B029 & SC18B030)

pursued in

Department of Aerospace Engineering

Indian Institute of Space Science and Technology

to

INDIAN INSTITUTE OF SPACE SCIENCE AND TECHNOLOGY

THIRUVANANTHAPURAM

May 2021

CERTIFICATE

This is to certify that the thesis titled '**Computation of 2D Equillibrium Flow over an Expansion Profile**', submitted by **Siddharth J Gantawar & Subrahmanya V Bhide**, to the Indian Institute of Space Science and Technology, Thiruvananthapuram, in partial fullfilment for the award of the degree of **Bacholer of Technology** in **Aerospace Engineering**, is a bonafide record of the research work done by **them** under my supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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Place: IIST, Thiruvanathapuram
May 2021

Declaration

We declare that this thesis titled '**Computation of 2D Equillibrium Flow over an Expansion Profile**' submitted in partial fulfillment of the Degree of **Bachelor of Technology in Aerospace Engineering** is a record of original work carried out by me under the supervision of **Dr Satheesh K** , and has not formed the basis for the award of any degree, diploma, associateship, fellowship or other titles in this or any other Institution or University of higher learning. In keeping with the ethical practice in reporting scientific information, due acknowledgments have been made wherever the findings of others have been cited.

Place: IIST, Thiruvanathapuram
May 2021

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Acknowledgements

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Abstract

This project involves generation of a code for the MOC computation of 2D equillibrium flow over an expansion proflie. Tanehill curve fits for equillibrium air have been used in the process of obatining the various properties like Pressure, Temperature, Density . . . etc. The variations of different properties with the angle of turn/deviation of the flow have been plotted.

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Chapter 1

INTRODUCTION & PROBLEM DESCRIPTION

A supersonic expansion fan, technically known as Prandtl–Meyer expansion fan, a two-dimensional simple wave, is a centered expansion process that occurs when a supersonic flow turns around a convex corner. The fan consists of an infinite number of Mach waves, diverging from a sharp corner. When a flow turns around a smooth and circular corner, these waves can be extended backwards to meet at a point.

Across the expansion fan, the flow accelerates (velocity increases) and the Mach number increases, while the static pressure, temperature and density decrease. Since the process is isentropic, the stagnation properties (e.g. the total pressure and total temperature) remain constant across the fan.

Here, the only difference is that the initial conditions are high temperature conditions. But

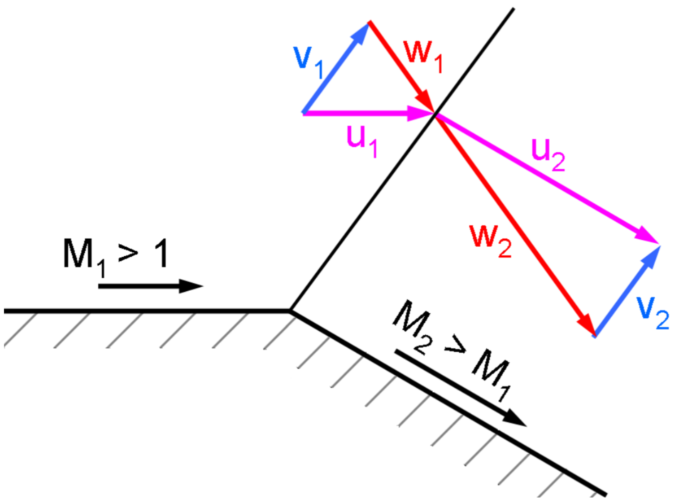

Fig. 1.1 Expansion fan across a profile.

because of the high temperature conditions, a lot of involved calculations are needed. The state relations and equations can't be directly used, as the molecules and atoms show quite different behaviour at high temperatures.

1. For temperatures less than 600 K, air can be assumed to be calorifically perfect gas, but after 600 K, a part of any energy supplied to the molecules is used to increase the vibrational energy of the molecules i.e. the vibrational modes also become active.

2. For temperature range of 600 K to 2500 K, air now behaves as a thermally perfect gas; as now the vibrational energy starts getting prominent.

3. As more higher temperatures are provided, bonds will start to break and again a major change in properties will be exhibited, then the gas will no longer be thermally perfect too.

4. As higher temperatures are provided, ionization will occur and again a major change in properties will occur.

Relations have be derived while invoking statistical and quantum mechanics approaches and these have been curve fitted into complex polynomials which have been explicitly mentioned and tabulated in the Tannehill tables.
In here, we have tried to find pressure, temperature, density and other such parameters in each $d\theta$ arc.

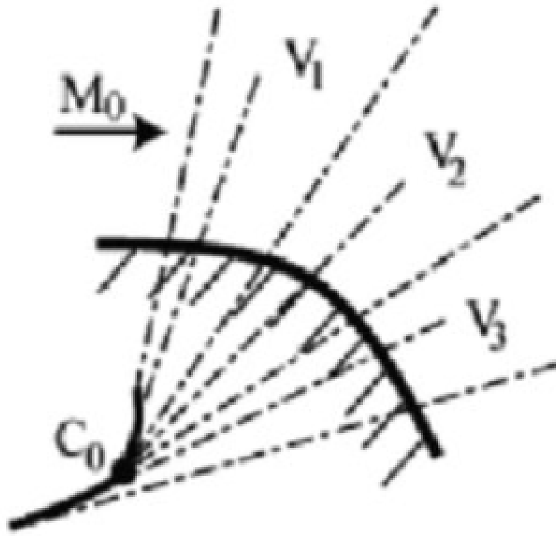

Fig. 1.2 Expansion fans across a smooth profile.

Chapter 2

SOLUTION METHODOLOGY

For each $d\theta$ arc, a particular flow velocity, velocity of sound, pressure and other thermodynamic parameters are associated with it.

Initial conditions of the flow that are known is: Inlet Pressure, Density, and Mach number.

2.0.1 Obtaining all initial parameters

The inlet internal energy (e) was calculated implicitly from the Tannehill table for $p = p(e, \rho)$ (since here we know inlet pressure p and inlet density ρ).
For this type of flows, we know that entropy is constant. So inlet entropy was constant throughout the flow; and was calculated from Tannehill Table of $s = s(e, \rho)$.
Inlet speed of sound (a) was obtained from the Tannehill table of $a = a(p, s)$; while inlet velocity (v) was obtained from the relation:

$$v = Ma \tag{2.1}$$

With this information, inlet enthalpy h was obtained using the Tannehill Table of $h = h(p, \rho)$; and hence Total Enthalpy (h_0) was obtained from the relation:

$$h_0 = h + v^2/2 \tag{2.2}$$

Also, for a Prandtl Meyer flow, we know:

$$d\theta = (\sqrt{\left(\frac{v}{a}\right)^2 - 1})dv/v \tag{2.3}$$

For a given θ, $d\theta$ was calculated as:

$$d\theta = \theta / N \tag{2.4}$$

where N is the number of divisions into which we divide the domain.

2.0.2 For First $d\theta$ section

For the very first $d\theta$ section, dv was obtained from Eqn 2.3, using v and a from the initial conditions. With this dv, velocity was obtained as $v + dv$ for the first section.
Using Eqn 2.1, h_1 for this section was obtained, since h_0 (already found before, and is constant) and v (for this section) are known.
An array of pressure points was generated as follows:

$$p_i = i \times p_{initial} / \bar{N} \tag{2.5}$$

$\bar{N}$ is the resolution used for the search calculations.
With this array of p_i, an array of e_i was obtained from the Tannehill table of $e = e(p, s)$ (s is taken from the initial parameters).
Now with the array of e_i, an array of ρ_i was obtained from impicit relation using the Tannehill table of $s = s(e, \rho)$.
Now, h_i array was calculated from the Tannehill tables of $h = h(p, \rho)$, using the arrays of p_i and ρ_i.
Now, since we already had h_1, we can pin pointedly get that particular h_k which matches with h_1. With the found out h_k, we now also have ρ_k and e_k.
This ρ_k, e_k and h_k is the density, internal energy and enthalpy of this section. a for this section can be found out using the Tannehill table of $a = a(p, s)$ for $p = p_k$ and s from initial conditions.

2.0.3 For Following $d\theta$ sections

Using the above algorithm for each section of $d\theta$, each parameter associated with that $d\theta$ arc can be found out.

Chapter 3

RESULTS

RESULTS & OBSERVATIONS

We consider an initial horizontal flow at a pressure of 4atm, a density of $1.1kg/m^3$ and with Mach number M = 1.8. The temperature of this flow is 1203K. The search calculations require more computation time and as the resolution of our search increases the time required also increases manifold. The results for the initial conditions are solved for an angle of 40° and plotted. All the codes were run on a Personal Laptop and hence there were restrictions in the resolutions that could be chosen.
Figures 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, show the variation of Pressure, Density, Velocity, Mach Number, Speed of Sound, Mach Angle and Temperature with θ respectively.

1. We know that for a supersonic flow across an expansion profile, pressure, density, temperature decreases. The flow accelerates and the Mach number increases.

2. All the values plotted are in their respective SI units.

3. The calculations are performed for a $d\theta$ of 0.4° .

4. The variations of different properties obtained for the flow through the procedure described in the previous section, show behaviour which is in accordance to the expected behaviour.

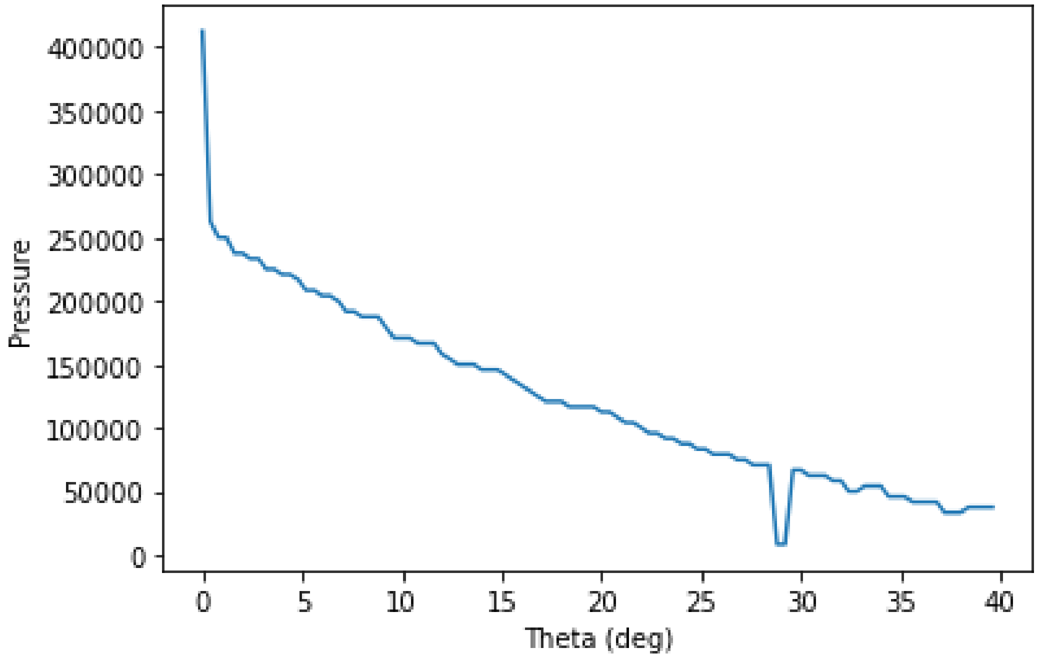

Fig. 3.1 Variation of Pressure with θ.

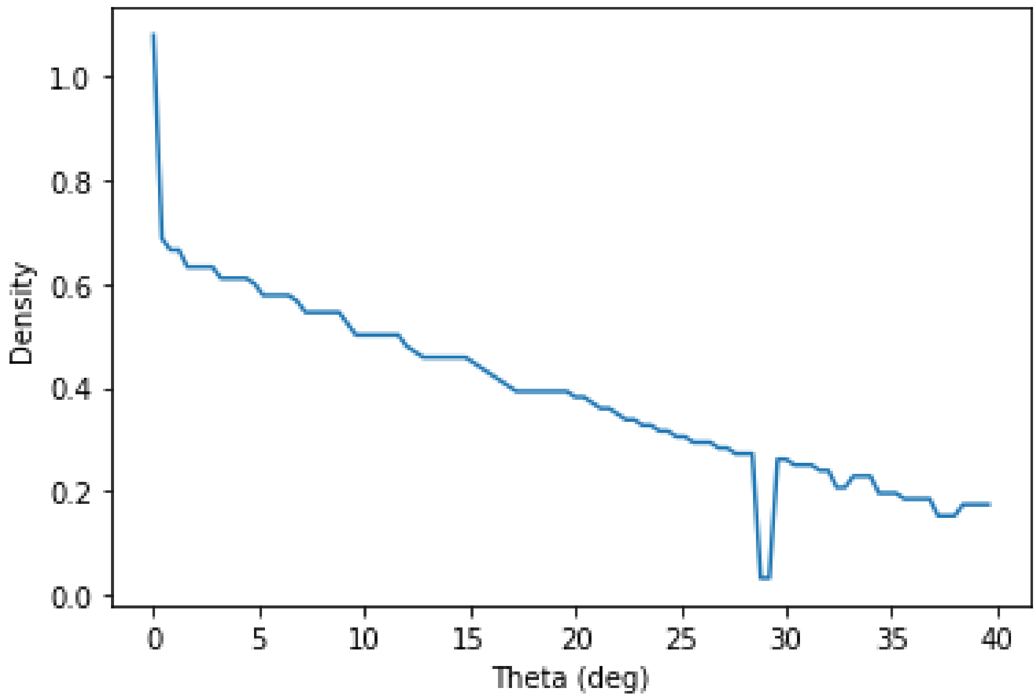

Fig. 3.2 Variation of Density with θ.

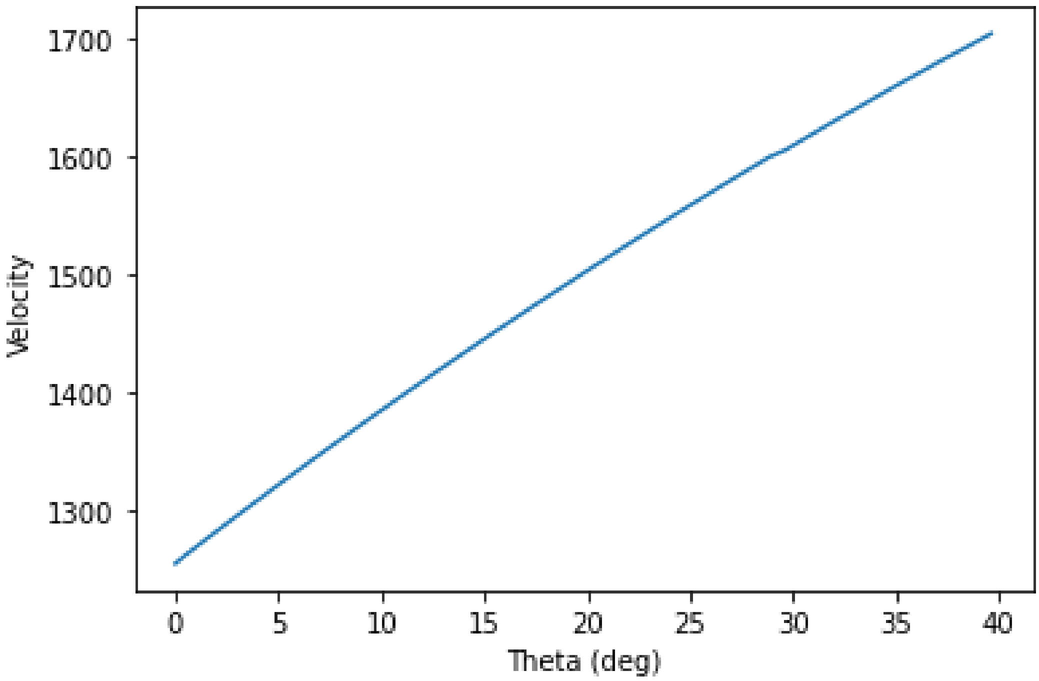

Fig. 3.3 Variation of Velocity with θ.

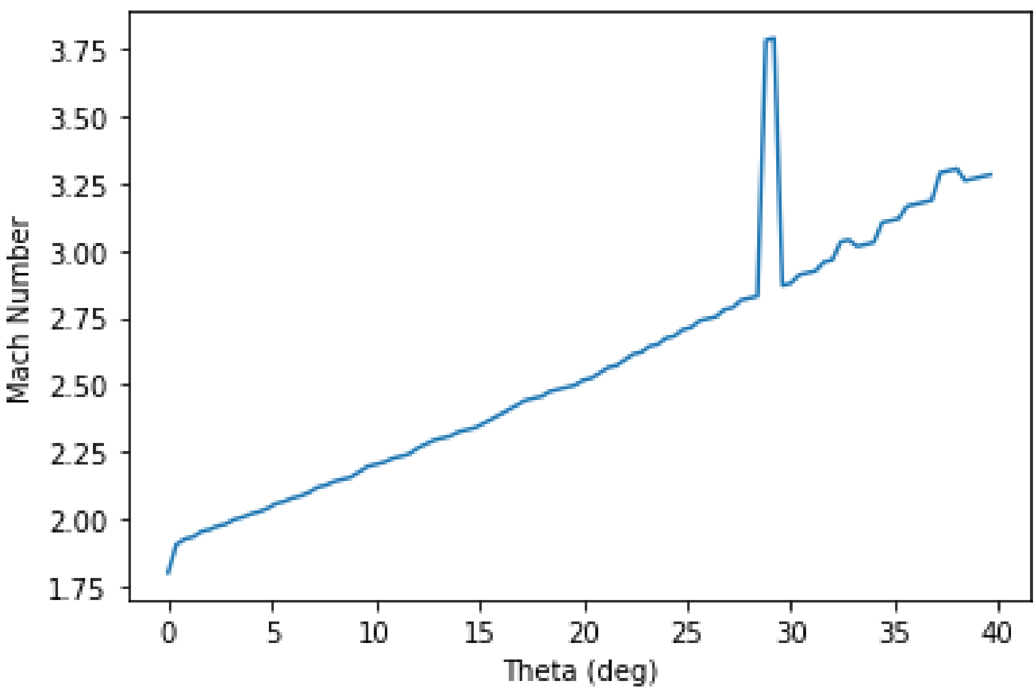

Fig. 3.4 Variation of Mach Number with θ.

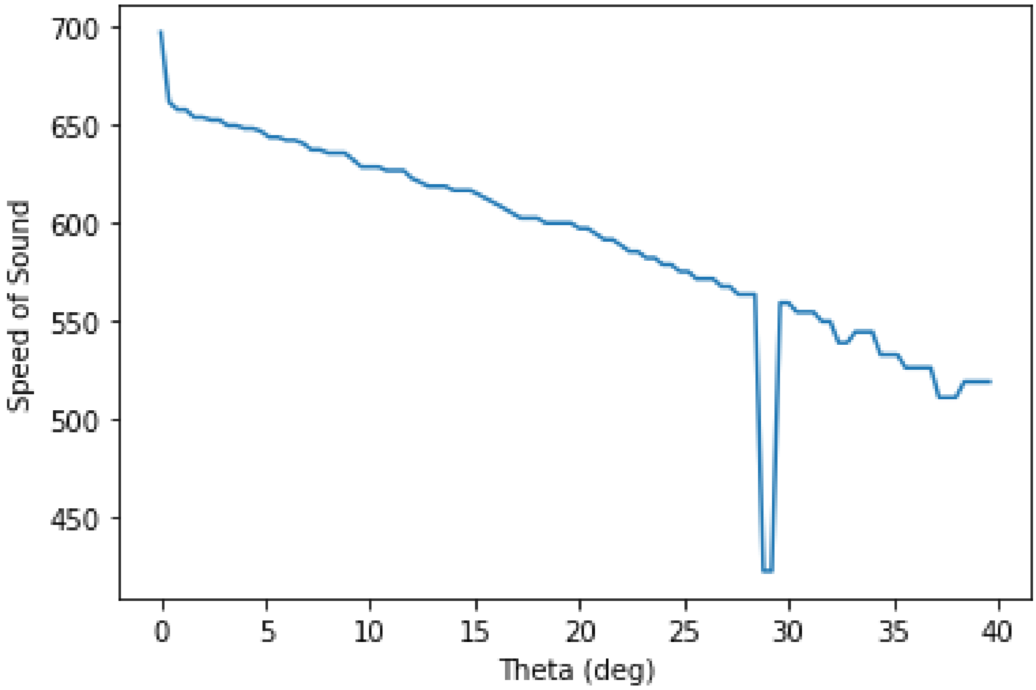

Fig. 3.5 Variation of Speed of Sound with θ.

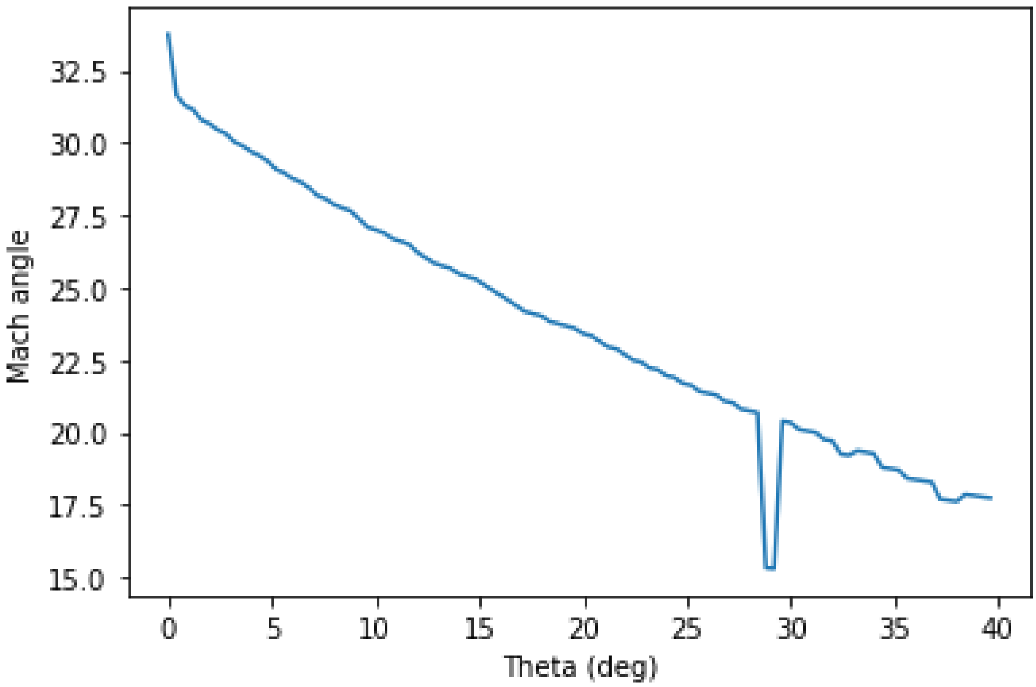

Fig. 3.6 Variation of Mach Angle with θ.

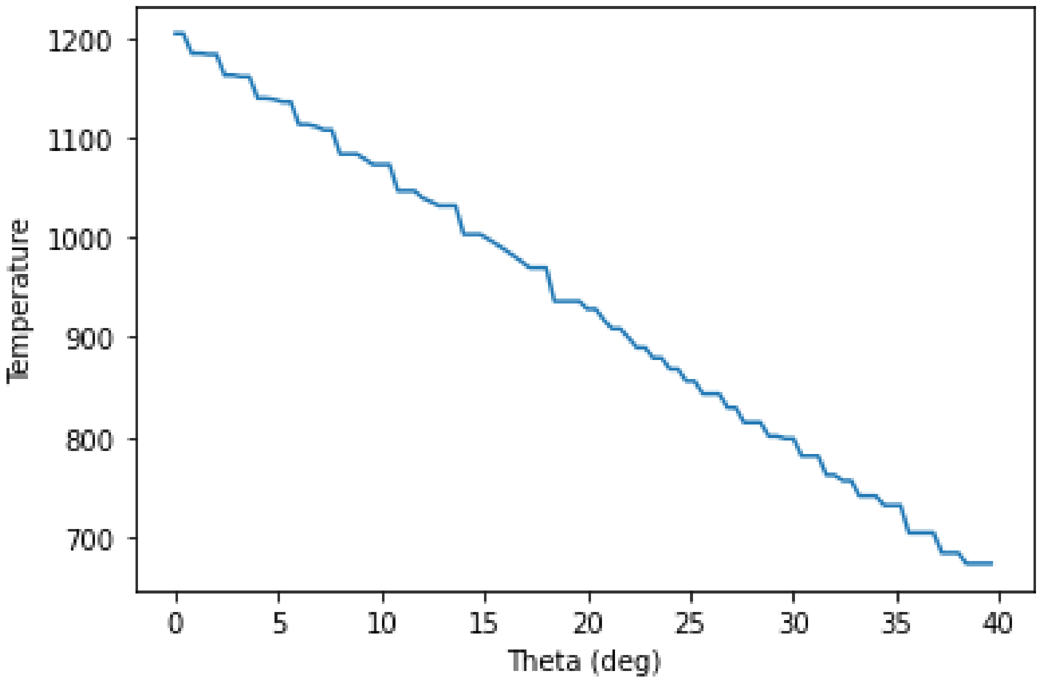

Fig. 3.7 Variation of Temperature with θ.

5. We can observe that the Mach number increases to about 3.25 from the initial value of 1.8 when the flow is rotated through an angle of 40°.

6. However we can observe that properties have sudden rise and fall at certain places and also the variation is not smooth but, highly discontinuous and not unidirectional.

7. This is a consequence of the low resolution of the solution method followed to obtain the results.

The plot of the Mach waves for a $d\theta$ of 4° is shown in Fig. 3.8.

The above calculations were run for 100 steps for the search algorithim ($\bar{N} = 100$). The calculations were performed again for 500 search steps. This increased the time required for calculations by a factor of about 10.

Figure 3.9, 3.10 and 3.11 show the variation of Pressure, Density and Temperature with θ for an improved search resolution of 500 steps respectively.

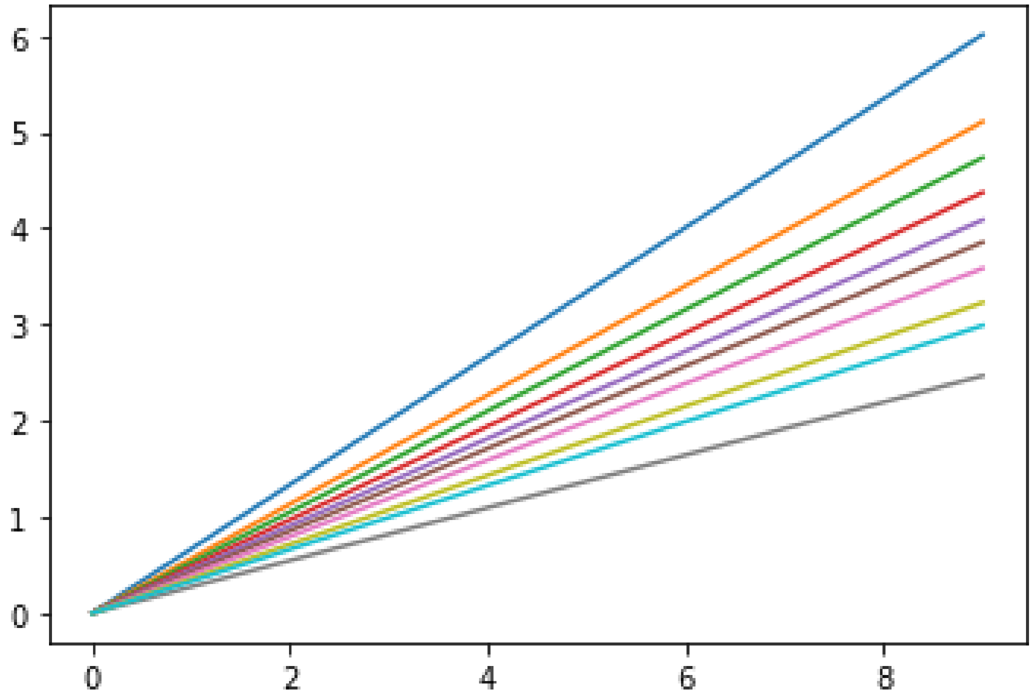

Fig. 3.8 Plot of Mach Waves.

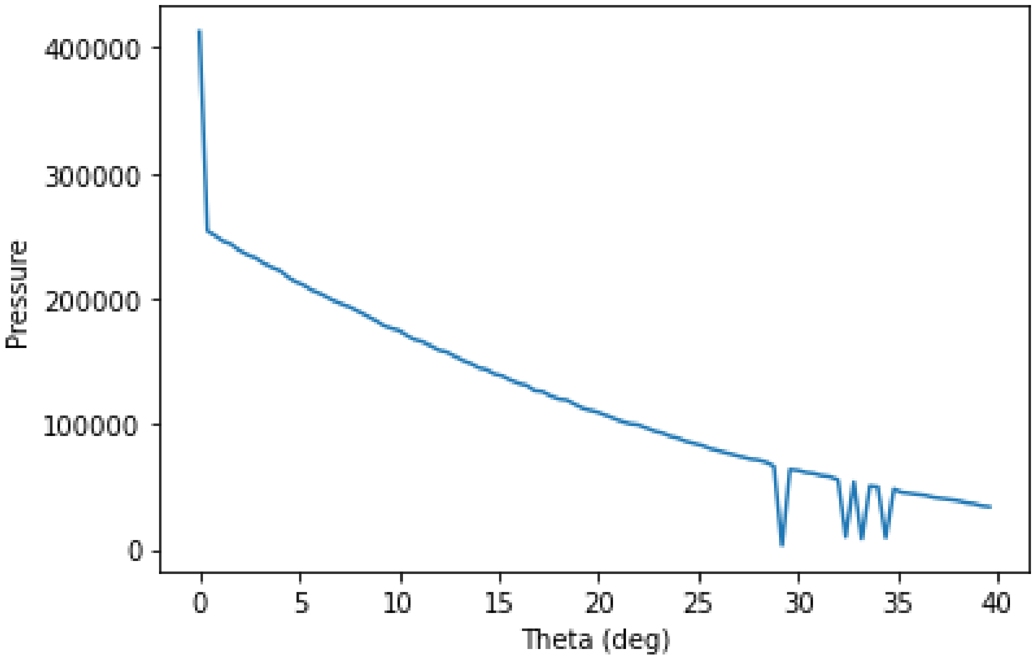

Fig. 3.9 Variation of Pressure with θ.

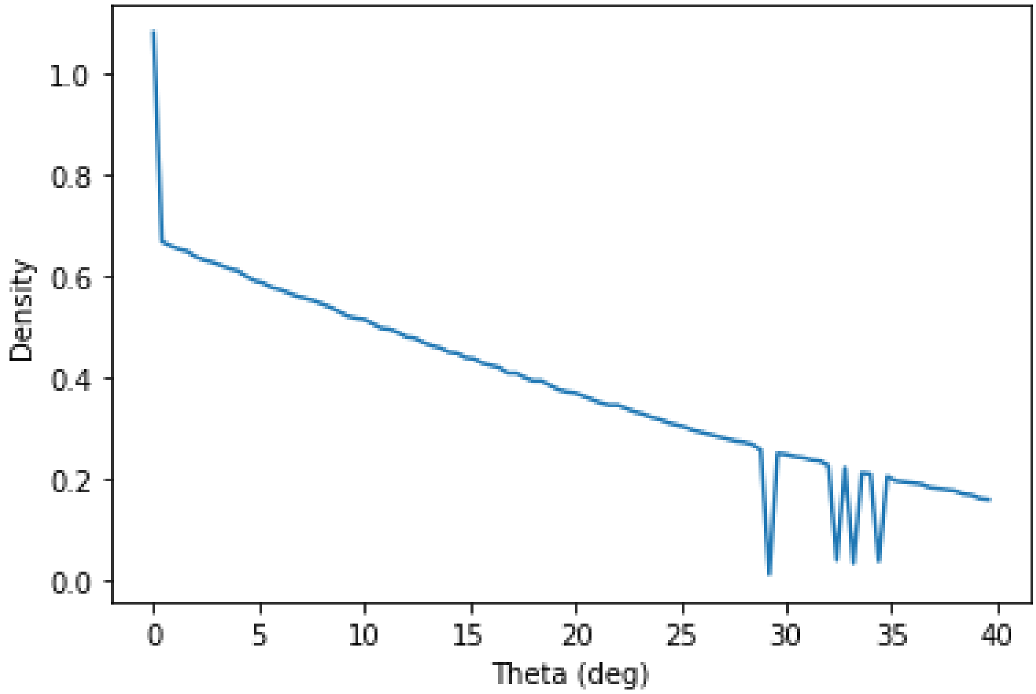

Fig. 3.10 Variation of Density with θ.

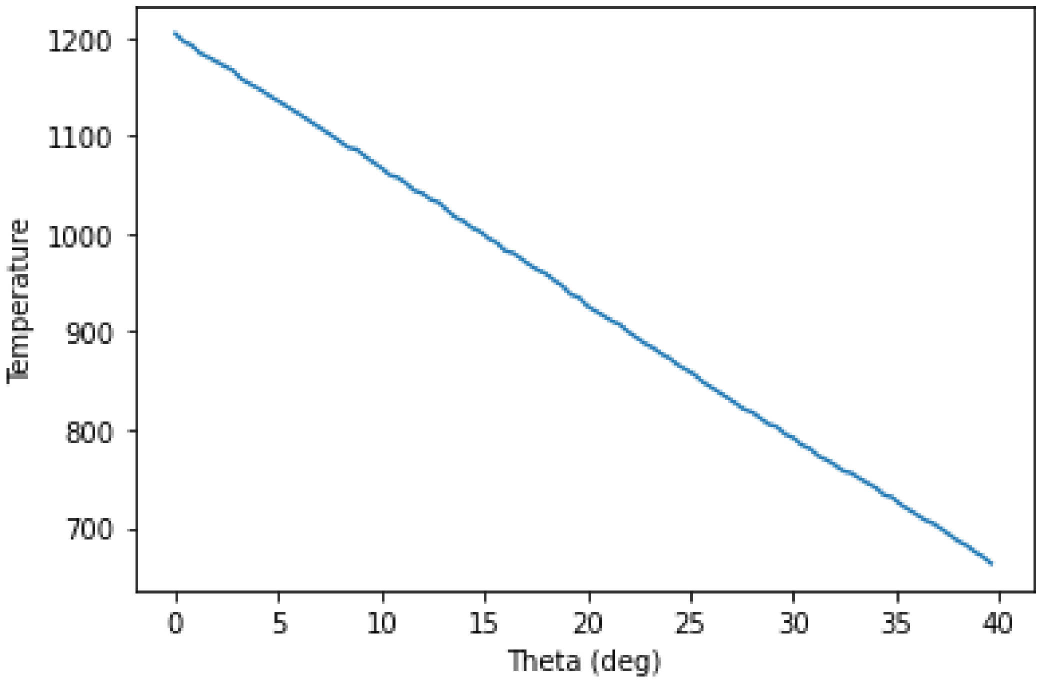

Fig. 3.11 Variation of Temperature with θ.

DISCUSSIONS & CONCLUSION

The calculations and the solution procedure was carried out for 100 divisions of the total θ, i.e. $d\theta = \frac{\theta}{100}$. The resolution for the search methods was also restricted to 500 steps. Thus as a consequence of this the obtained results are not very accurate even though they follow the general trend and behaviour. A least square fit can be considered at an initial level, but higher resolutions are required to solve for more accurate values of the parameteres. Also the search method used for the parameters Pressure and Density are simple magnitude based search algorithims. Methods such as the Golden sections method or other gradient based methods can also be used for the search process which would improve the solution quality but would also require higher computation time and capabilities. Also it is observed that there is a sudden drop or rise in the values for almost all parameters except for Velocity and Temeprature. We suspect that this is a consequence of the resolution of the solution methodology. These variations still persist even though the resolution was increased to 500 steps in the search process. Thus maybe a different approach to the probelm can solve this issue. Also in the Prandtl Meyer flow expression relating $d\theta$ and dv, the assumption was made that $d\theta = \Delta\theta$ and $dv = \Delta v$ for the calculations. This also would have affected the quality of the solution obtained.

PYTHON CODES DEVELOPED FOR THE PROJECT

The python codes developed for the project can be found here.

References

1. Simplified Curve Fits for th Thermodynamic Properties of Equillibrium Air
   https://ntrs.nasa.gov/api/citations/19870016876/downloads/19870016876.pdf
   NASA Reference Publication 1181
   S Srinivasan, J C Tanehill and K J Weilmunester
   Aug 1987

2. https://en.wikipedia.org/wiki/Prandtl%E2%80%93Meyer_expansion_fan

3. Introduction to Physical Gas Dynamics
   W G Vincenti, C H Kruger Jr
   Krieger Publishing Company, Malabar Florida
   1965