Exercise 3
A - Frame Crane Preliminary Analysis
AE332 - Modelling and Analysis Lab

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This is a report based on the preliminary analysis done to decide upon the A-Frame Crane. Two models and several variations for the Lifting mechanism are presented and their merits and demerits are discussed.

PROBLEM DESCRIPTION

In the previous analysis of the Dragon Capsule, it was concluded that, a single rope to lift the Capsule was insuffient, not due to the fact that the Capsule is too heavy for any rope, but due to the fact that the capsule would have oscillations which can damage both the capsule and the Crane lifting it. Hence additional ropes would have to be employed to limit and kill the oscillations of the capsule. The tension in the rope lifting the capsule is found to be about 25,000N. Now we have to design a crane which would hoist the capsule out of water and place it on the deck, by means of a pulley fixed at its top. The motion of the crane can be controlled using a hydraulic piston-cylinder actuator. We consider the time taken for the lifting operation to be about a minute. For modelling purposes since the crane is symmetrical, we model only half of it.

SOME BASIC CONSIDERATIONS

Positioning of a Piston-Cylinder setup with respect to a Link

Consider a link OA pivoted at O as shown in Fig. 1. The link is supported through a piston cylinder attached to OA at B and pivoted on the ground at C. Let $OB = l_f$, $OA = L$ and $OC = D$. Assuming the system to be in equillibrium we can write Eqns. 1 and 2.

$$Nsin\theta + Fcos\phi = R \tag{1}$$

$$LNcos\theta - l_f Fsin\phi = \tau \tag{2}$$

Here R and τ are the reaction force and moment provided by the pivot at point O.
Lower the reaction forces and moments, less is the stress on the joint at point O. Thus for R to be lower we would want ϕ to be an obtuse angle i.e. $\phi > 90°$. For τ to be lower we would want l_f to be as high as possible.
For a system such as the one shown in Fig 1, based on the above mentioned observations we can conclude that:

1. Preferably OC must be short.

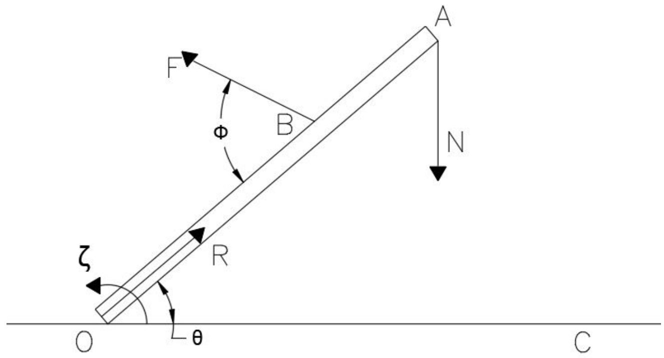

FIG. 1: A Simple Pivoted Link.

2. Preferably OB must be long.

Length Constraints for the Crane

The height of the crane required can be estimated as follows:

- The Diameter of the Capsule is $4m$.
- A cylinder (floating in water) must be present between the capsule and the deck of the ship to avoid any collision of the capsule and the ship. Its diameter can be assumed to be half of that of the capsule i.e. $2m$.
- The crane can't be placed at the end of the deck and hence needs certain clearance from the edge of the deck. The clearance can assumed to be about $1 - 2m$.

Based on the above mentioned conditions we would require a crane of height of about $4 + 2 + 2 = 8m$.

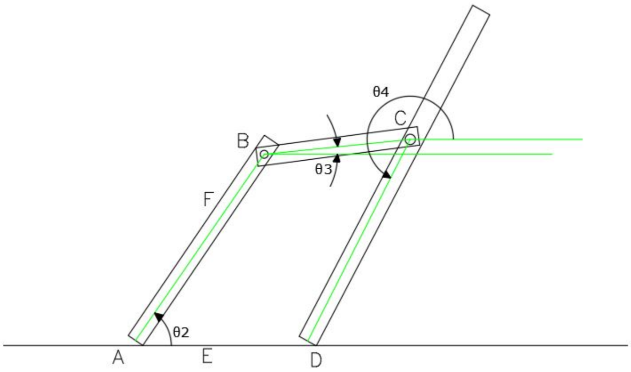

FIG. 2: Four Bar mechanism for the Crane.

MODEL 1

This is similar to the setup shown in Fig. 1. The actuators would pull the crane down after which the capsule would be attached and then the actuators would push the crane so that the capsule is lifted from the ocean.

Considerations in Model 1

The main consideration in this model is that since the actuator mechanism must be attached, it would require some more space and this will lead to a crane of higher length. This increase in length would cost us in terms of the strength of the joint at pivot O, the life of the crane as this can deform the crane. To compensate this more stronger materials would be required, increasing the cost.

MODEL 2

The fact that using the crane as in Model 1 would require more clearance space and would lead to consequences discussed above, gives way for another model for the crane where the actuation mechanism is attached to AB whereas the crane which lifts the capsule is part of CD.

Here ABCD form a Four-Bar Mechanism as shown in Fig. 2. F is the point of application of force by the piston cylinder mechanism. AB,BC,CD,DA,AF are known link lengths. EF is connected through a piston cylinder arrangement. The link AB is actuated through piston cylinder mechanism EF at a *constant angular velocity.*

For the fourbar there can be three possibilities, where when AB is perpendicular to the ground:

1. DC is also perpendicular to the ground thus making $AD = BC$.
2. DC leans towards the sea thus $AD < BC$.
3. DC leans away from the sea thus $AD > BC$.

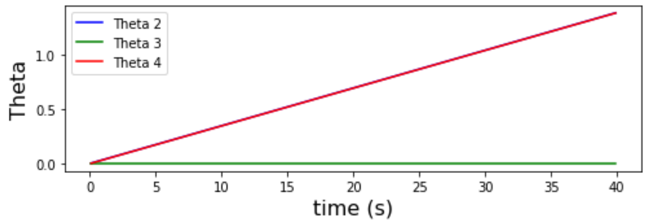

FIG. 3: Variation of angles of the links with time for case 1.

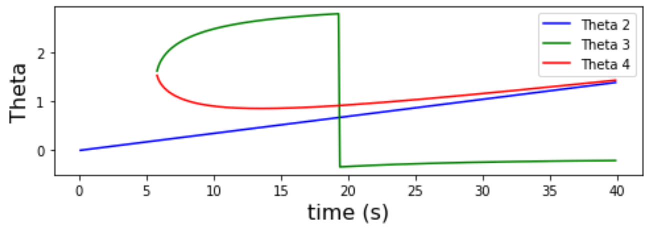

FIG. 4: Variation of angles of the links with time for case 2.

Since an approximate of the height obtained is about $8m$, and following the discussions in the initial sections we can consider that the lengths of the links of the fourbar can be around $5 - 6m$. In each of these cases, variations in the lengths AB and CD are also possible. All the possible cases are examined below and the results are discussed subsequently.

Case 1

Here we have considered $AB = BC = CD = DA = 5m$, $AE = 2m$ and $AF = 3m$. The plot of variation of angles of the links with time is shown in Fig. 3. The minimum and maximum length of EF are $1m$ and $3.3m$ respectively and occur at $10°$ and $90°$ respectively and this variation is linear.

Case 2

Here we have considered $AB = 5m$, $CD = 6m$ $BC = AD = 5m$, $AE = 2m$ and $AF = 3m$. The plot of variation of angles of the links with time is shown in Fig. 4. Minimum crane angle achievable is only $52.8°$.

Case 3

Here we have considered $AB = 6m$, $CD = 5m$ $BC = AD = 5m$, $AE = 2m$ and $AF = 4m$. The plot of variation of angles of the links with time is shown in Fig. 5. The crane(CD) achieves an angle of $10°$ when θ_2 is $28°$ and the length of EF is $2.4m$ at this instant.

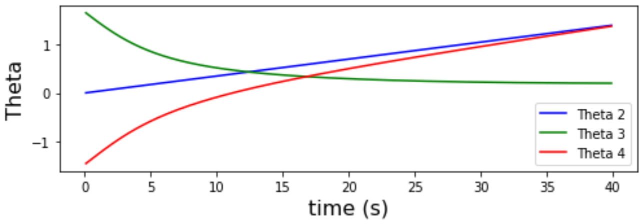

FIG. 5: Variation of angles of the links with time for case 3.

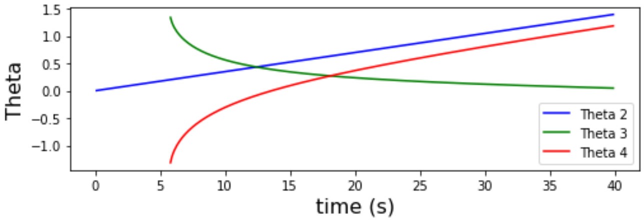

FIG. 6: Variation of angles of the links with time for case 4.

Case 4

Here we have considered $AB = 5m$, $CD = 5m$ $BC = 6m$ $AD = 5m$, $AE = 2m$ and $AF = 3m$. The plot of variation of angles of the links with time is shown in Fig. 6. The crane(CD) achieves an angle of 10° when θ_2 is 32° and the length of EF is $1.7m$ at this instant.

Case 5

Here we have considered $AB = 5m$, $CD = 5m$ $BC = 5m$ $AD = 6m$, $AE = 2m$ and $AF = 3m$. The plot of variation of angles of the links with time is shown in Fig. 7. The minimum angle that can be achieved by the crane here is 49°.

Case 6

Here we have considered $AB = 7m$, $CD = 5m$ $BC = 5m$ $AD = 6m$, $AE = 2m$ and $AF = 3m$. The plot of variation of angles of the links with time is shown in Fig. 8. The crane achieves 10° when θ_2 is about 25°.

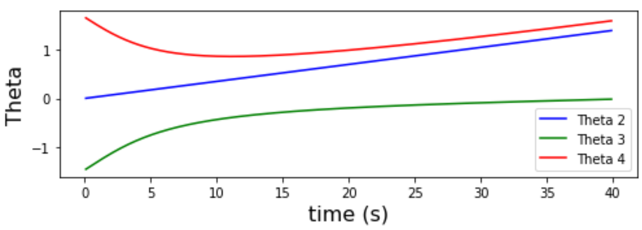

FIG. 7: Variation of angles of the links with time for case 5.

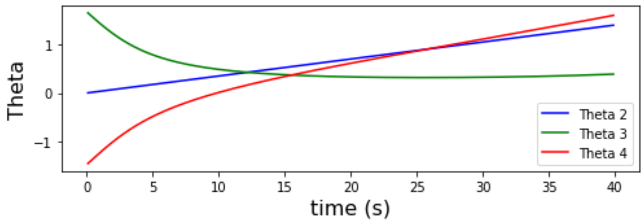

FIG. 8: Variation of angles of the links with time for case 6.

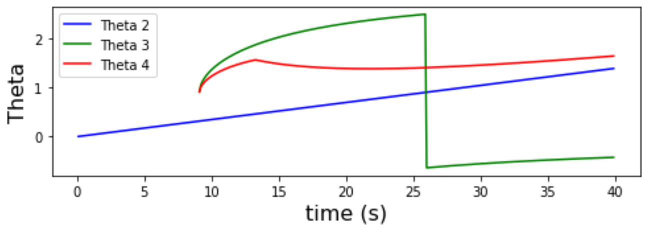

FIG. 9: Variation of angles of the links with time for case 7.

Case 7

Here we have considered $AB = 5m$, $CD = 7m$ $BC = 5m$ $AD = 6m$, $AE = 2m$ and $AF = 3m$. The plot of variation of angles of the links with time is shown in Fig. 9. The minimum angle that can be achieved by the crane here is 80°.

Case 8

Here we have considered $AB = 7$, $CD = 5$ $BC = 6$ $AD = 5m$, $AE = 2m$ and $AF = 3m$. The plot of variation of angles of the links with time is shown in Fig. 10. The minimum length of the EF is $1.82m$. θ_2 is about 37° when the crane is horizontal.

Case 9

Here we have considered $AB = 5m$, $CD = 7m$ $BC = 6m$ $AD = 5m$, $AE = 2m$ and $AF = 3m$. The plot of variation of angles of the links with time is shown in Fig. 11. The minimum angle that can be achieved by the

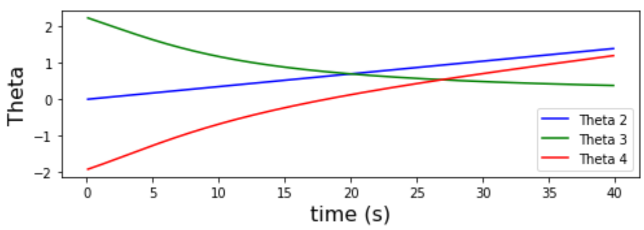

FIG. 10: Variation of angles of the links with time for case 8.

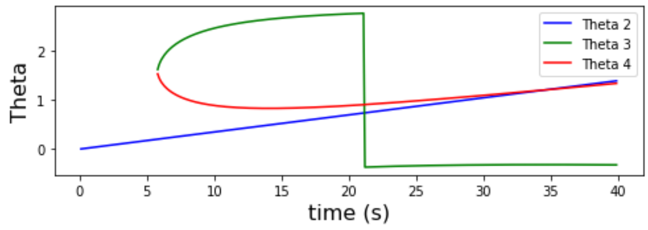

FIG. 11: Variation of angles of the links with time for case 9.

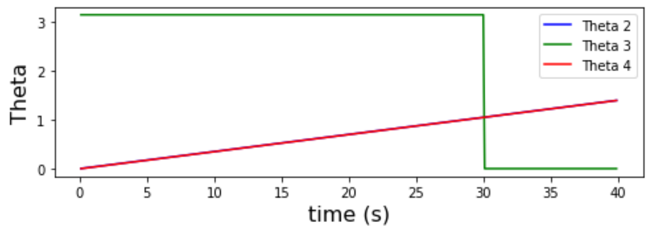

FIG. 12: Variation of angles of the links with time for case 10.

crane here is 51.5°.

Case 10

Here we have considered $AB = CD = 6m$ $BC = AD = 3m$, $AE = 2m$ and $AF = 3m$. The plot of variation of angles of the links with time is shown in Fig. 12. The minimum and maximum length of EF are $1m$ and $3.3m$ respectively and occur at 10° and 90° respectively and this variation is linear.

All the cases are summarized in Tab. I and the animations for different cases can be found here.[1, 2]

TABLE I: Model 2 - Summary

Case	Remarks
Case 1	Minimum and maximum length of piston-cylinder is $1m$ and $3.3m$
Case 2	Minimum crane angle achievable is only 52.8°
Case 3	Crane achieves an angle of 10° when θ_2 is 28°. Length of EF is $2.4m$ at this instant.
Case 4	The crane achieves an angle of 10° when θ_2 is 32°. Length of EF is $1.7m$ at this instant.
Case 5	Minimum crane angle achievable is only 49°.
Case 6	The crane achieves 10° when θ_2 is about 25°.
Case 7	Minimum crane angle achievable is only 80°.
Case 8	θ_2 is about 37° when the crane is horizontal. The minimum length of the EF is $1.82m$.
Case 9	Minimum crane angle achievable is only 51.5°.
Case 10	Minimum and maximum length of piston-cylinder is $1m$ and $3.3m$

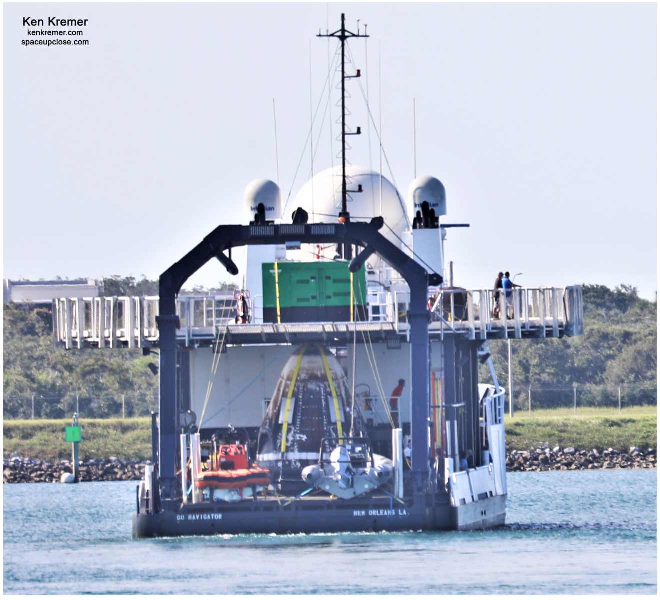

FIG. 13: A-Frame Crane on Go-Navigator

DISCUSSIONS

From the above results we can see that Cases 2,5,7 and 9 are not of our interest as the crane cant reach 10° for θ_2 varying between 0° and 90°.
In Cases 1 and 10, where the fourbars form parallelograms the crane follows θ_2.
In Cases 3,4,6 and 8, for the crane to vary from 10° - 90°, θ_2 also need not vary from 10°-90° as in cases 1 and 10.

DETERMINATION OF APPROXIMATE LENGTHS OF THE A-FRAME

Using the Fig. 13, with the knwoledge that the width of the ship is $11m$ from Exercise 0. We can approximate that the height of the crane is about $8.56m$.[3]
Subsequently using Fig. 14 we can approximate the value of AB to be about $5.65m$ using the height of the crane obtained earlier.[4]

CONCLUSION

For all the cases which are feasible for our purpose, we have $AB \geq CD$ along with no specific relation between BC and DA. But $AB \geq CD$ along with $BC \geq AD$ result in the least angular motion for AB that is required to provide the crane 10° - 90° motion. Also even though due to this argument, case 8 seems to be better than other cases, the dynamics of the system need to be analysed for all the feasible cases as the reaction forces in the links

FIG. 14: A-Frame Crane on Go-Navigator

are important parameters too for design purpose. This would be done as Exercise 4.

APPENDIX

The equations for Position Velocity and Acceleration analysis are presented below where the lengths AB,BC,CD,DA,AF are known. θ_2, $\dot{\theta}_2$ and $\ddot{\theta}_2$ are known. The unknowns are θ_3, θ_4, θ_f and EF.

Position Analysis:

$$ABcos(\theta_2) + BCcos(\theta_3) + CDcos(\theta_4) = AD$$

$$ABsin(\theta_2) + BCsin(\theta_3) + CDsin(\theta_4) = 0$$

$$AFcos(\theta_2) + EFcos(\theta_f) = AE$$

$$AFsin(\theta_2) - EFsin(\theta_f) = 0$$

Where $\theta_f = \angle AEF$

Velocity Analysis:

1.

$$\begin{bmatrix} AB\dot{\theta}_2 sin(\theta_2) \\ -AB\dot{\theta}_2 cos(\theta_2) \end{bmatrix} = \begin{bmatrix} BCcos(\theta_3) & CDsin(\theta_4) \\ BCcos(\theta_3) & CDcos(\theta_4) \end{bmatrix} \begin{bmatrix} \dot{\theta}_3 \\ \dot{\theta}_4 \end{bmatrix}$$

2.

$$\begin{bmatrix} AF\dot{\theta}_2 sin(\theta_2) \\ AF\dot{\theta}_2 cos(\theta_2) \end{bmatrix} = \begin{bmatrix} cos(\theta_f) & -AFsin(\theta_f) \\ sin(\theta_f) & AFcos(\theta_f) \end{bmatrix} \begin{bmatrix} \dot{EF} \\ \dot{\theta}_f \end{bmatrix}$$

Acceleration Analysis:

1.

$$\begin{bmatrix} -(CDcos(\theta_4)\dot{theta_4}^2) - (BCcos(\theta_3)\dot{\theta}_3^2) \\ -(ABcos(\theta_2)\dot{\theta}_2^2) - (ABsin(\theta_2)\ddot{\theta}_2) \\ \\ (CDsin(\theta_4)\dot{\theta}_4^2) + (BCsin(\theta_3)\dot{\theta}_3^2) \\ +(ABsin(\theta_2)\dot{\theta}_2^2) - (ABcos(\theta_2)\ddot{\theta}_2) \end{bmatrix} =$$

$$\begin{bmatrix} BCsin(\theta_3) & CDsin(\theta_4) \\ BCcos(\theta_3) & CDcos(\theta_4) \end{bmatrix} \begin{bmatrix} \ddot{\theta}_3 \\ \ddot{\theta}_4 \end{bmatrix}$$

2.

$$\begin{bmatrix} AF(\ddot{\theta}_2 sin(\theta_2) + \dot{\theta}_2^2 cos(\theta_2)) \\ +2\dot{EF}\dot{\theta}_f sin(\theta_f) + EF\dot{\theta}_f^2 cos(\theta_f) \\ \\ AF(\ddot{\theta}_2 cos(\theta_2) - \dot{\theta}_2^2 sin(\theta_2)) \\ -2\dot{EF}\dot{\theta}_f cos(\theta_f) + EF\dot{\theta}_f^2 sin(\theta_f) \end{bmatrix} =$$

$$\begin{bmatrix} cos(\theta_f) & -AFsin(\theta_f) \\ sin(\theta_f) & AFcos(\theta_f) \end{bmatrix} \begin{bmatrix} \ddot{EF} \\ \ddot{\theta}_f \end{bmatrix}$$

[1] https://mechanicalexpressions.com/explore/kinematics/4-bar-linkage.html; Accessed on 07 October 2020 7:45 pm.

[2] https://pythonforundergradengineers.com/piston-motion-with-python-matplotlib.html; Accessed on 10 October 2020 11:21 am.

[3] https://www.spaceupclose.com/2020/08/nasa-targets-net-october-23-for-1st-operational-\spacex-crew-dragon-launch-from-kennedy/; Accessed on 08 October 2020 9:42 pm.

[4] https://en.wikipedia.org/wiki/GO_Navigator#/media/File:SpaceX_Demo-2_Pre-Landing_(NHQ202008020003).jpg; Accessed on 08 October 2020 9:58 pm.