Exercise 4
A - Frame Crane Dynamic Analysis
AE332 - Modelling and Analysis Lab

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This is a report based on the dynamic analysis of the A-Frame Crane. Dynamics of models considered acceptable in Exercise 3 are presented and their merits and demerits discussed.

PROBLEM DESCRIPTION

In Exerise 3 we found that certain combinations of lengths for the links in the fourbar are in accordance to our requirement. For the retrieval mechanism we need the crane to move with constant angular velocity and the time limit for the process is about a minute or so. Thus the task now is to find how this can be done through a piston cylinder arrangement actuating the crane. Hence we perform inverse dynamic analysis of the system to determine the kind of actuation required. We will do this for all the feasible cases for the various link lengths of the fourbar mechanism.

FOURBAR MODEL

The fourbar crane model is as shown in the Fig. 1. Here N is the tension in the rope which carries the Crew module.
EF is connected through a piston cylinder, a schematic of which is shown in Fig. 2. F_p is the force due to the pressure in the fluid present inside the cylinder. This is a external force applied to the crane to hoist the crew module out of the ocean.
From Exercise 3 we know the kind of motion we expect from the crane. Now we find the actuating force F_p which can produce such a motion.

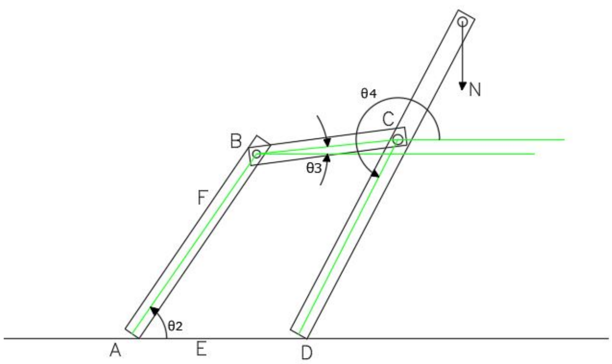

FIG. 1: A-Frame Crane.

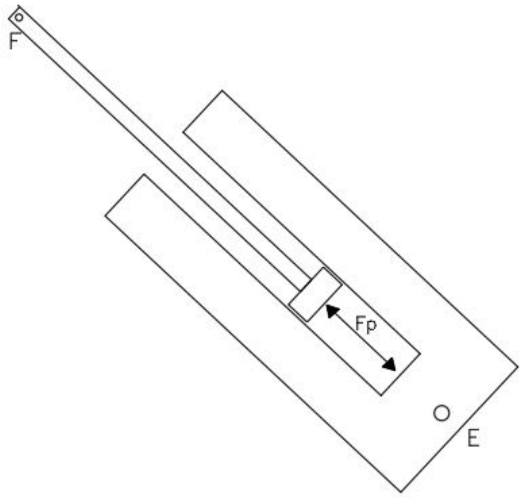

FIG. 2: Schematic of the Piston Cylinder Actuator.

PROBLEM VARIABLES

Link Lengths

In exercise 3, considering physical and practical constraints we had arrived at an estimate for the link length $AB = 6m$. Also certain variations in the lengths provided certain advantages. In total 6 combinations of link lengths are to be analysed which are listed in Tab. I.

TABLE I: Various Link Lengths - Summary

	AB (in m)	BC (in m)	CD (in m)	DA (in m)
Case 1	5	5	5	5
Case 2	6	5	5	5
Case 3	5	6	5	5
Case 4	7	5	5	6
Case 5	7	5	6	5
Case 6	6	3	6	3

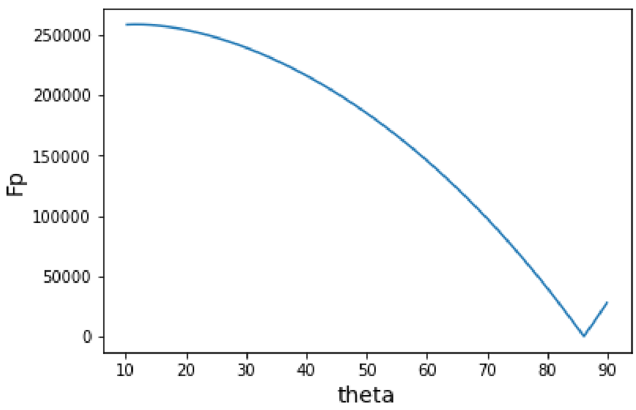

FIG. 3: F_p vs θ plot for Case 1 with AE = 4m

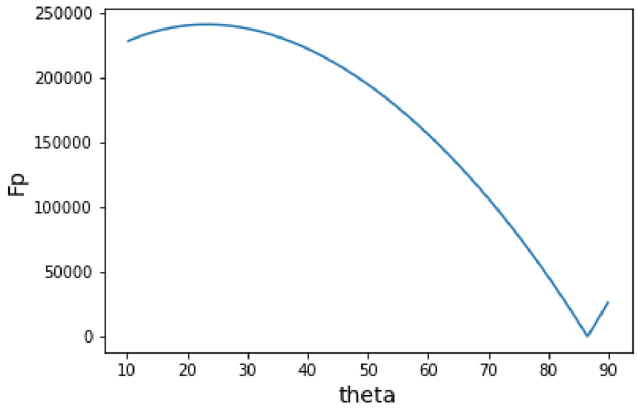

FIG. 4: F_p vs θ plot for Case 2 with AE = 4m

Masses of Links

The material of the link is considered to be steel which has a density of $8,050kg/m^3$. The link is assumed to be rectangular and width and depth of the links are considered to be $0.5m$ each. Hence the mass of a link of length L can be calculated as :

$$M = 8050 \times 0.3 \times 0.3L$$

Moments of Inertia of Links

The moments of Inertia of the links is found using the formula [1]:

$$I = \frac{1}{12}M(0.3^2 + L^2) + M\frac{L^2}{4}$$

RESULTS AND DISCUSSION

The plots of F_p versus the angle of the crane for different cases are shown in Figs. 3, 4, 5, 6 and 7.

For case 3 the cure for F_p is non differentiable and not monotonous. In these cases the order of the force is 10^5N and the curves are almost monotonous. The lowest value of F_p is for the Case 6 where the force is about $2.2 \times 10^5 N$. This force corresponds to a pressure of about 4000psi for a piston diameter of $80mm$. Even lesser pressures are achievable for other combinations of link length but those variations are not continuous and monotonous. This value for pressure is just an approximate and it can be influenced by other factors such as the mass of the links. For the purpose of calculations we have used steel here but other materials can be used.[2, 3]

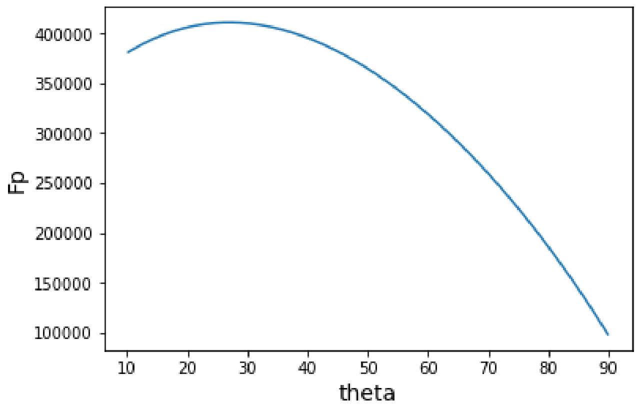

FIG. 5: F_p vs θ plot for Case 4 with AE = 4m

CONCLUSION

From the analysis done so far, Case 6 requires a maximum F_p of about $2.2 \times 10^5 N$.
Even though this case requires the least value of F_p among the cases discussed there is further scope for improvement which can be achieved through better material selection which provides lighter links. But any material can't be chosen as it must also be able to withstand the reaction forces, this aspect would be more clear when the static mechanical analysis of the A-Frame is performed. Also for the analysis we have not considered the side support ropes which can improve the F_p value.

APPENDIX

The Equations used for the dynamic analysis are provided below:

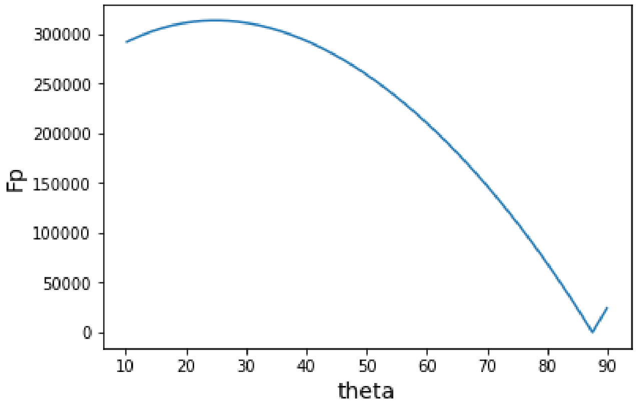

FIG. 6: F_p vs θ plot for Case 5 with AE = 4m

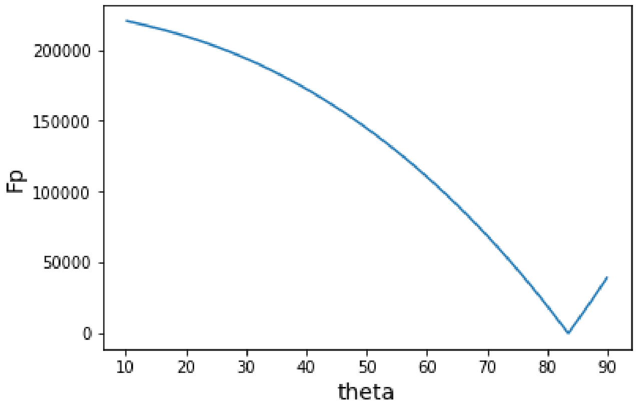

FIG. 7: F_p vs θ plot for Case 6 with AE = 4m

Equations of motion for link AB

The free body diagram of Link AB (Link 2) is shown in Fig. 8.

$$F_x + R_3 + R_1 = m_2\ddot{x}_2$$

$$F_y + R_4 + R_2 - m_2 g = m_2\ddot{y}_2$$

$$R_1(y_2 - y_A) + R_4(x_B - x_2) - R_3(y_B - y_3)$$

$$+F_y(x_F - x_2) - R_2(x_2 - x_A) - F_x(y_F - y_2) = I_2\ddot{\theta}_2$$

Equations of motion for link BC

The free body diagram of Link BC (Link 3) is shown in Fig. 9.

$$R_5 - R_3 = m_3\ddot{x}_3$$

$$R_6 - R_4 - m_3 g = m_3\ddot{y}_3$$

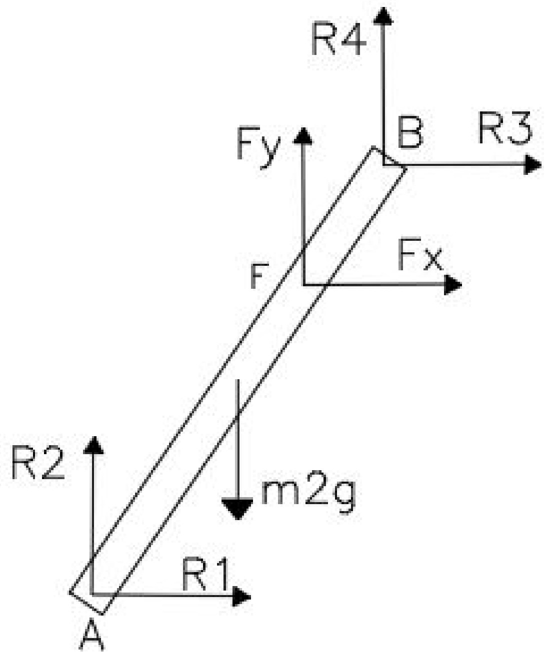

FIG. 8: Free body diagram for Link 2

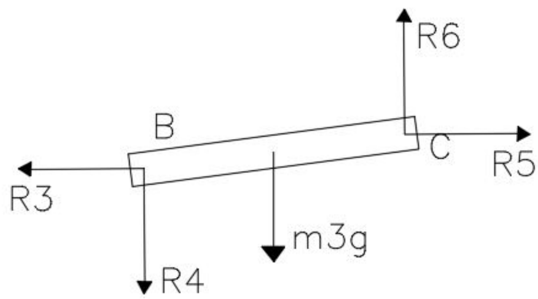

FIG. 9: Free body diagram for Link 3

$$R_4(x_3 - x_B) - R_5(y_C - y_3)$$

$$-R_6(x_C - x_3) - R_3(y_3 - y_B) = I_3\ddot{\theta}_3$$

Equations of motion for link CD

The free body diagram of Link CD (Link 4) is shown in Fig. 10.

$$R_7 - R_5 = m_4\ddot{x}_4$$

$$R_8 - R_6 - m_4 g - N = m_4\ddot{y}_4$$

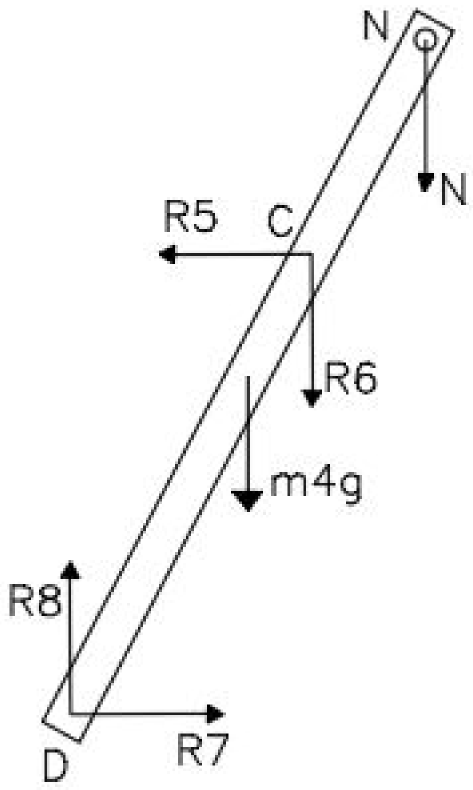

FIG. 10: Free body diagram for Link 4

$$R_5(y_C - y_4) - R_8(x_4 - x_D) + R_7(y_4 - y_D)$$

$$-R_6(x_C - x_4) - N(x_N - x_4) = I_4\ddot{\theta}_4$$

Equations of motion for the piston

The free body diagram for the piston (Link 5) is shown in Fig. 11.

$$R_{12}sin\theta_f - R_9cos\theta_f - F_x = m_5\ddot{x_5}$$

$$R_{12}cos\theta_f + R_9sin\theta_f - F_y - m_5g = m_5\ddot{y_5}$$

$$F_x(y_F - y_5) + F_y(x_5 - x_F) + R_{12}(l_1 - \delta)$$

$$-M_1 = I_5\ddot{\theta_f}$$

Equations of motion for the cylinder

The free body diagram for the Cylinder (Link 6) is shown in Fig. 12.

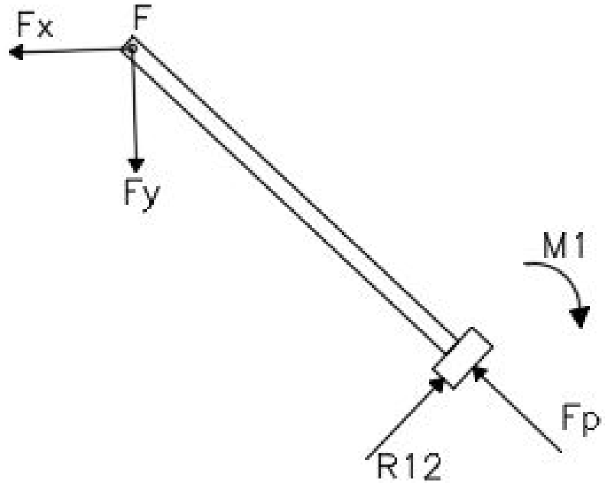

FIG. 11: Free body diagram for Link 5

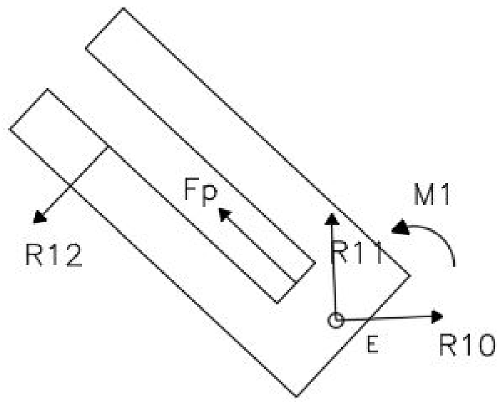

FIG. 12: Free body diagram for Link 6

$$-R_{12}sin\theta_f + R_9cos\theta_f + R_{10} = m_6\ddot{x_6}$$

$$-R_{12}cos\theta_f - R_9sin\theta_f + R_{11} - m_6g = m_6\ddot{y_6}$$

$$R_{10}(y_6 - y_E) + R_{11}(x_E - x_6) + R_{12}(l + l_2) + M_1 = I_6\ddot{\theta_f}$$

Acceleration Equations

In the subsequent equations the subsript 2,3,4,5,6 refer to the center of mass of the link.

Link AB

$$\ddot{x_2} = -r_2(\dot{\theta_2}^2 cos\theta_2 + \ddot{\theta}_2 sin\theta_2)$$

$$\ddot{y_2} = r_2(-\dot{\theta_2}^2 sin\theta_2 + \ddot{\theta}_2 cos\theta_2)$$

Link BC

$$\ddot{x_3} = -r_2(\dot{\theta_2}^2 cos\theta_2 + \ddot{\theta}_2 sin\theta_2) - r_3(\dot{\theta_3}^2 cos\theta_3 + \ddot{\theta}_3 sin\theta_3)$$

$$\ddot{y_3} = r_2(-\dot{\theta_2}^2 sin\theta_2 + \ddot{\theta}_2 cos\theta_2) + r_3(-\dot{\theta_3}^2 sin\theta_3 + \ddot{\theta}_3 cos\theta_3)$$

Link CD

$$\ddot{x_4} = -r_4(\dot{\theta_4}^2 cos\theta_4 + \ddot{\theta}_4 sin\theta_4)$$

$$\ddot{y_4} = r_4(-\dot{\theta_4}^2 sin\theta_4 + \ddot{\theta}_4 cos\theta_4)$$

Piston

$$\ddot{x_5} = -l_f(\dot{\theta_2}^2 cos\theta_2 + \ddot{\theta}_2 sin\theta_2) - r_5(\dot{\theta_f}^2 cos\theta_f + \ddot{\theta}_f sin\theta_f)$$

$$\ddot{y_5} = l_f(-\dot{\theta_2}^2 sin\theta_2 + \ddot{\theta}_2 cos\theta_2) - r_5(-\dot{\theta_f}^2 sin\theta_f + \ddot{\theta}_f cos\theta_f)$$

Cylinder

$$\ddot{x_6} = r_6(\dot{\theta_f}^2 cos\theta_f + \ddot{\theta}_f sin\theta_f)$$

$$\ddot{y_4} = r_6(-\dot{\theta_f}^2 sin\theta_f + \ddot{\theta}_f cos\theta_f)$$

[1] https://en.wikipedia.org/wiki/List_of_moments_of_inertia; Accessed on 17 October 2020 10:09 am.

[2] https://www.roemheld-gruppe.de/fileadmin/user_upload/downloads/technische_informationen/Wissenswertes_Hydraulikzylinder_en_0212.pdf; Accessed on 16 October 2020 8:46 pm.

[3] https://www.parker.com/Literature/Industrial%20Cylinder/cylinder/cat/english/HY08-1114-4-2H-3H-Heavy-Duty-Cyl.pdf; Accessed on 16 October 2020 8:33 pm.