Coding Assignment
AE323 Optimization Techniques in Engineering

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This is a report summarizing the results obtained by the application of Real Coded Genetic Algorithim for the minimization of several benchmark unimodal and multimodal functions.

Unimodal Functions

Bohachevsky Unimodal Function

The Bohachevsky Unimodal function is shown in Eqn. 1.

$$f(x,y) = x^2 + 2y^2 - 0.3cos(3\pi x) - 0.4cos(4\pi y) + 0.7 \tag{1}$$

The function is evaluated in the region $x, y \in [-100, 100]$.
The global minima occurs at (0,0) with f = 0. The GA algorithim was run 5 times and the results are displayed in the Tab. I.

TABLE I: Results obtained for the Bohachevsky Unimodal function.

Function value	Point
3.4575E-06	(-0.00021785,0.0002876)
3.07264E-05	(-0.000255,-0.00094193)
0.000335	(-0.00482511,-0.00026019)
0.004489	(0.01466948,0.00648435)
0.001238	(0.0091049,0.00124047)

Rotated Hyper Ellipsoidal Function

The Rotated Hyper Ellipsoidal function is shown in Eqn. 2.

$$f(\mathbf{x}) = \sum_{i=1}^{d} \sum_{j=1}^{i} x_j^2 \tag{2}$$

The function is evaluated in the region $x_i \in [-65.536, 65.536]$.
The global minima occurs at (0,. . . ,0) with f = 0. The GA algorithim was run 5 times and the results are displayed in the Tab. II.

TABLE II: Results obtained for the Rotated Hyper Ellipsoidal function.

Function value	Point
1.482155E-06	(0.000705,-0.0006973)
6.680540E-06	(-0.0018216,-0.00020874)
1.40123E-05	(-0.0022259,-0.0020255)
4.20125E-06	(-0.000257, -0.0020170)
1.4047411E-06	(0.0003359, 0.001085)

Powell Sum Function

The Powell Sum function is shown in Eqn. 3.

$$f(\mathbf{x}) = \sum_{i=1}^{d} |x_i|^{i+1} \tag{3}$$

The function is evaluated in the region $x_i \in [-1, 1]$.
The global minima occurs at (0,. . . ,0) with f = 0. The GA algorithim was run 5 times and the results are displayed in the Tab. III.

TABLE III: Results obtained for the Powell Sum function.

Function value	Point
0.0008074	(-0.000167, -0.0252882)
2.6557245E-05	(-1.85529E-05, 0.002829)
0.0004259	(-0.000305, -0.010988)
0.000137	(0.000136, 0.001228)
0.000136	(4.39469E-06, 0.01148)

Dixon Price Function

The Dixon Price function is shown in Eqn. 4.

$$f(\mathbf{x}) = (x_1 - 1)^2 + \sum_{i=2}^{d} i(2x_i^2 - x_{i-1})^2 \tag{4}$$

The function is evaluated in the region $x_i \in [-10, 10]$.
The global minima occurs at $x_i = 2^{-\frac{2^i-2}{2^i}}$ with f = 0. The GA algorithim was run 5 times and the results are displayed in the Tab. IV.

TABLE IV: Results obtained for the Dixon Price function.

Function value	Point
2.638940E-05	(0.9991, 0.70501)
8.46782E-06	(0.998425, -0.7056834)
2.106204E-05	(0.99565, -0.7050428)
2.010103E-05	(0.995750, -0.705096)
7.49619E-07	(0.999325, 0.706676)

Brown Function

The Brown function is shown in Eqn. 5.

$$f(\mathbf{x}) = \sum_{i=1}^{d-1} (x_i^2)^{(x_{i+1}^2+1)} + (x_{i+1}^2)^{(x_i^2+1)} \tag{5}$$

The function is evaluated in the region $x_i \in [-1, 4]$.
The global minima occurs at (0,. . . , 0) with f = 0. The GA algorithim was run 5 times and the results are displayed in the Tab. V.

TABLE V: Results obtained for the Brown function.

Function value	Point
9.76832E-08	(0.000302,-7.734096E-05)
2.59299E-06	(0.000910,-0.00137)
1.12574E-07	(5.584413E-05,0.0003308)
1.62486E-06	(-0.000207, -0.001257)
6.11415E-07	(0.000698,0.00035)

Multimodal Functions

Ackely's Function

The Ackely's function is shown in Eqn. 6.

$$f(\mathbf{x}) = -aexp\left(-b\sqrt{\frac{1}{d}\sum_{i=1}^{d}x_i^2}\right) - exp\left(\frac{1}{d}\sum_{i=1}^{d}cos(cx_i)\right) + a + exp(1) \tag{6}$$

The values of a, b and c are generally taken to be $a = 20$, $b = 0.2$ and $c = 2\pi$. The function is evaluated in the region $x_i \in [-32.768, 32.768]$.
The global minima occurs at (0,. . . , 0) with f = 0. The GA algorithim was run 5 times and the results are displayed in the Tab. VI.

TABLE VI: Results obtained for the Ackely's function.

Function value	Point
0.003938	(0.000185,-0.00136)
0.011421	(0.003894,-4.62902E-05)
0.0066468	(-0.001894,0.001305)
0.015415	(-0.00456,-0.002483)
0.0009774	(-0.000104,0.0003281)

Rosenbrock's Function

The Rosenbrock's function is shown in Eqn. 7.

$$f(\mathbf{x}) = \sum_{i=1}^{n-1}[b(x_{i+1} - x_i^2)^2 + (a - x_i)^2] \tag{7}$$

The values of a and b are generally taken to be $a = 100$ and $b = 1$. The function is evaluated in the region $x_i \in [-5, 10]$.
The global minima occurs at (1,. . . , 1) with f = 0. The GA algorithim was run 5 times and the results are displayed in the Tab. VII.

TABLE VII: Results obtained for the Rosenbrock's function.

Function value	Point
0.0001943	(1.01212,1.0250769)
0.0002855	(0.99112,0.98089)
2.675727E-05	(1.00438,1.00905)
0.0028021	(0.992767,0.98034)
0.001109	(0.9680,0.937997)

Levy's Function

The Levy's function is shown in Eqn. 8.

$$f(\mathbf{x}) = sin^2(\pi w_1) + (w_d - 1)^2[1 + sin^2(2\pi w_d)] + \sum_{i=1}^{d-1}(w_1 - 1)^2[1 + 10sin^2(\pi w_i + 1)] \tag{8}$$

Here $w_i = 1 + \frac{x_i - 1}{4}$. The function is evaluated in the region $x_i \in [-10, 10]$.
The global minima occurs at (1,. . . , 1) with f = 0. The GA algorithim was run 5 times and the results are displayed in the Tab. VIII.

TABLE VIII: Results obtained for the Levy's function.

Function value	Point
0.001358	(0.89466,-0.28374)
3.42484	(-1.53805,3.71271)
4.65123	(-1.6167,-0.19894)
8.76861	(-2.403670,3.13758)
6.580024	(-1.8798, 1.815841)

Rastrigin's Function

The Rastrigin's function is shown in Eqn. 9.

$$f(\mathbf{x}) = Ad + \sum_{i=1}^{d}[x_i^2 - Acos(2\pi x_i)] \tag{9}$$

The value of A is generally taken to be $A = 10$. The function is evaluated in the region $x_i \in [-5.12, 5.12]$.
The global minima occurs at (0,. . . , 0) with f = 0. The GA algorithim was run 5 times and the results are displayed in the Tab. IX.

TABLE IX: Results obtained for the Rastrigin's function.

Function value	Point
10.00000001	(-3.10901,-9.67684E-06)
10.000000002	(-4.10264,-1.16545E-06)
10.000000006	(6.6862,5.753642E-06)
10.00000000007	(-1.86995,-6.00623E-07)
10.000000001	(4.00154,-2.485189E-06)

Bohachevsky Multimodal Function

The Bohachevsky Multimodal function is shown in Eqn. 10.

$$f(x, y) = x^2 + 2y^2 - 0.3cos(3\pi x)cos(4\pi y) + 0.3 \tag{10}$$

The function is evaluated in the region $x, y \in [-100, 100]$.
The global minima occurs at (0,0) with f = 0. The GA algorithim was run 5 times and the results are displayed in the Tab. X.

TABLE X: Results obtained for the Bohachevsky Multimodal function.

Function value	Point
0.000651	(0.000771,0.00500)
0.003075	(0.00797,-0.009194)
0.005719	(-0.01792,-0.00667)
0.00080694	(0.002839,-0.0051898)
0.0053730	(-0.017288,-0.0065880)

1. CONCLUSION

From the results we can see that for Unimodal functions the results for all the 5 functions are obtained upto 3 decimal places accuartely in atleast two of the 5 runs conducted.
For the multimodal functions apart from Levy's and Rastrigin's functions, other functions reach the global minima in atleast one run out of the 5 runs atleast upto 1 decimal place.
The Rastrigin and Levy functions have a lot of local minima and hence the GA did not converge to the global minima.

[1] https://www.sheffield.ac.uk/polopoly_fs/1.60188!/file/manual.pdf
[2] http://www.iitg.ac.in/rkbc/presentation/Genetic%20Algorithms.pdf
[3] http://benchmarkfcns.xyz/benchmarkfcns/bohachevskyn2fcn.html
[4] https://www.sfu.ca/~ssurjano/dixonpr.html
[5] http://benchmarkfcns.xyz/benchmarkfcns/brownfcn.html
[6] http://benchmarkfcns.xyz/benchmarkfcns/rosenbrockfcn.html
[7] https://www.sfu.ca/~ssurjano/levy.html
[8] https://en.wikipedia.org/wiki/Rastrigin_function
[9] http://benchmarkfcns.xyz/benchmarkfcns/bohachevskyn1fcn.html
[10] https://www.iitk.ac.in/kangal/resources.shtml
[11] https://www.sfu.ca/~ssurjano/rothyp.html#:~:text=The%20Rotated%20Hyper%2DEllipsoid%20function,shows%20its%20two%2Ddimensional%20form.
[12] http://benchmarkfcns.xyz/benchmarkfcns/powellsumfcn.html#:~:text=The%20function%20can%20be%20defined,%3D%201%20%2C%20%E2%80%A6%20%2C%20n%20.
[13] https://www.sfu.ca/~ssurjano/ackley.html#:~:text=The%20Ackley%20function%20is%20widely%20used%20for%20testing%20optimization%20algorithms.&text=The%20function%20poses%20a%20risk,%3D%200.2%20and%20c%20%3D%202%CF%80.