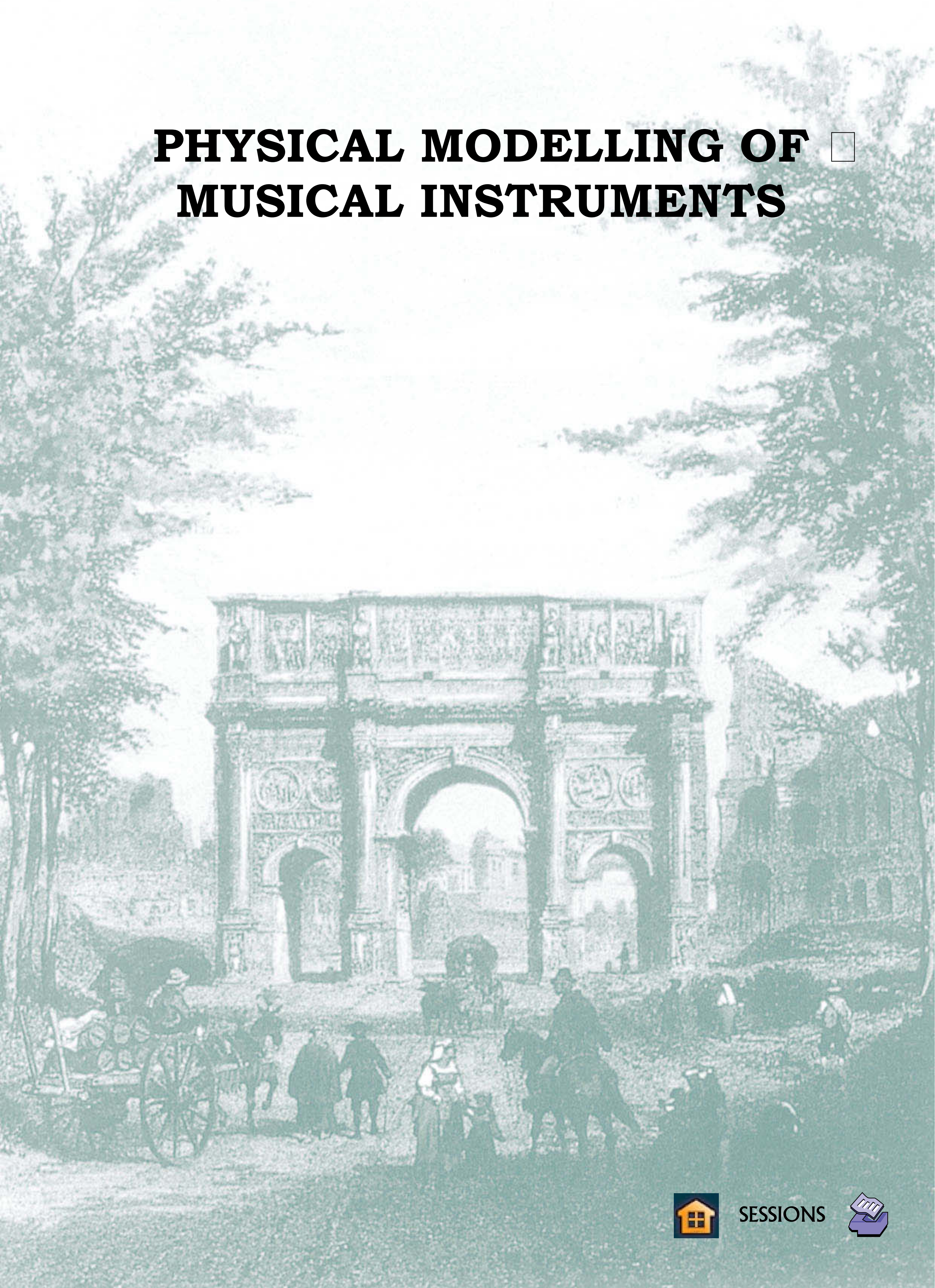
PHYSICAL MODELLING OF MUSICAL INSTRUMENTS



Tactics for efficient and realistic sound synthesis of the piano

D. Rocchesso[a], G. Borin[a,b]

[a]*Dipartimento di Informatica, Università di Verona, Italy.* `http://www.sci.univr.it/~rocchess`
[b]*Centro di Sonologia Computazionale dell'Università di Padova, Italy*

Real-time synthesis of piano tones by physical modelling requires special tricks and approximations to overcome problems of computability, complexity, parametric control. A handful of tactics have been deployed to implement a complete piano model that runs in real time with full polyphony on current general purpose processors.

INTRODUCTION

As part of a collaboration between the University of Padova and Generalmusic, we demonstrated the first polyphonic real-time piano model in 1997 [1]. Such collaboration has produced a number of byproducts that have been used to enhance sampling-based digital pianos with pieces of physical models (i.e., the pedal and damper effects). Recently, electro-mechanical piano models developed after the research of G. Borin hit the market.

Most of the constraints that affected our modelling efforts came from the need of having a complete model running in real-time on inexpensive hardware. This limitation forced us to develop special techniques and new research lines. Some of these contributions are listed in the following sections. The section titles refer to the specific parts of the piano that required special tactics to be tackled with physical modelling.

THE HAMMER

Classical models of piano hammer are based on the parallel connection of a mass and a nonlinear spring which accounts for the felt compression characteristics [2]. Several problems arise when translating this class of models into a discrete-time implementation. Besides the problems of retaining the possibility to do the "hammer voicing" by manual adjustment of k and α, a more fundamental difficulty comes from the noncomputable loops that emerge when discretizing the nonlinear hammer-string system. We easily come up with an implicit system relating the force $f(n)$ and the string velocity $\dot{y}(n)$. This implicit relationship can be made explicit either by iterative search of a solution or by assuming that $f(n) \approx f(n-1)$. Both solutions are not acceptable for our task: the first because the running time increases and cannot be assumed constant; the second because it may introduce instabilities when the sample rate is not very high and high-pitched notes are played. We devised a method [3] that rearranges the equations in such a way that instantaneous dependencies across the nonlinearity are dropped. As it is shown in fig. 1, the insertion of a fictitious delay element has severe consequences on the simulation of high-pitched notes at reasonable sample rates. Our implementation avoids artificial instabilities and reproduces a reliable force signal, thus producing a much more natural sound.

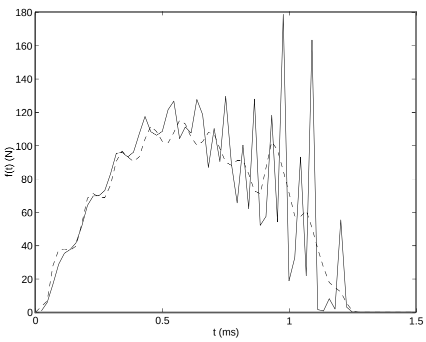


FIGURE 1. Time evolution of the hammer force for a C6 with $F_s = 44100Hz$, $v_h = 6.8m/s$ (*fff*), $m_h = 0.0066Kg$, $\alpha = 3.0$, $k = 200 \cdot 10^9 N/m^3$. Delay-free loop resolution with (solid line) and without (dashed line) a fictitious delay element.

THE STRING

Piano strings exhibit frequency-dependent losses and dispersion, which have to be simulated in order to attain realistic sounds. Following the tradition of digital waveguides, we lump losses and dispersions in the whole string, and we simulate them by lowpass and allpass filters, respectively.

The problem of simulating string dispersion is the most demanding in terms of computations. We adapted to our needs the design method proposed by Lang and Laakso [4], which is fast and provides a weighted least-squares phase error approximation. With this method [5] it is possible to set a frequency-dependent weight, in such a way that the partials in low frequency are more accurately put on their exact (inharmonic) positions. Figure 2 illustrates the approximation of the theoretical (in-

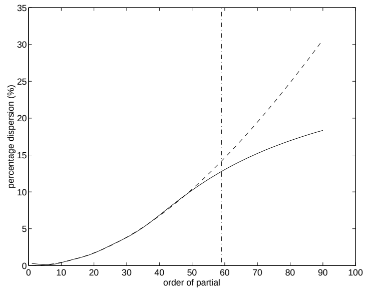


FIGURE 2. Synthesis of dispersive filters: C2. 3 sixth-order filters. Dashed line: theoretical distribution of partials. Solid line: approximation by allpass filters.

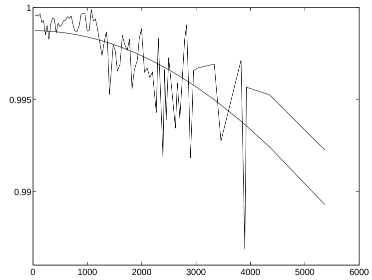


FIGURE 4. Attenuation per cycle for the partials of a C1 string. Approximation by a length-3 FIR filter is overlapped.

harmonic) distribution of partials obtained by using 3 sixth-order filters for note C2.

Since the range of correct phase approximation (bounded by a vertical line in fig. 2) can be extended at the expense of increasing the filter order, we became interested in understanding how accurate is the perception of inharmonicity in piano tones, so that such range can be limited to the bandwidth of perceived inharmonicity. We ran some subjective tests with expert listeners [6], and the results are summarized in fig. 3.

As far as the string losses are concerned, the profile of the attenuation per loop cycle that one gets by measurement of actual piano tones is very complicated (see fig. 4). Moreover, such measurement can not be precise due to the amplitude modulations that many partials are subject to. Indeed, it does not make much sense to model the detailed distribution of peaks by string loop filter, as much of the variability in decay time can be attributed to the soundboard. Therefore, we only follow the general lowpass trend of the attenuation profile and we simulate it by means of a low order filter.

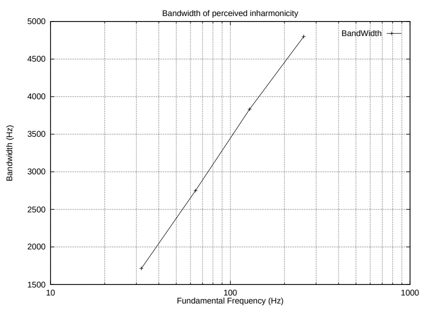


FIGURE 3. Bandwidth of perceived inharmonicity for piano tones.

THE SOUNDBOARD

The soundboard has a strong influence on piano sounds, not only because it radiates and filters the velocity waves that reach the bridge. Its role is essential in determining the time-domain modulations of the partials. In fact, the bridge/soundboard system connects the strings together and acts as a distributed driving-point impedance for string terminations. We developed finite-difference models to study the properties of soundboards [7], but we found computationally advantageous to lump the soundboard impedance at one point in the real-time model. Such lumped driving-point impedance has been simulated by a feedback delay network [8], whose poles are designed by proper tuning of the delay lines.

REFERENCES

1. G. Borin, D. Rocchesso and F. Scalcon, Proc. Int. Comp. Mus. Conf., 350-353 (1997).
2. H. Suzuki, J. Acoust. Soc. Am., **82**, 1145–1151 (1987).
3. G. Borin, G. De Poli and D. Rocchesso, IEEE Trans. on Speech and Audio Proc., **8**, 597–605 (2000).
4. M. Lang and T.I. Laakso, IEEE Trans. on Circuit and Syst., **41**, 40–48, (1994).
5. D. Rocchesso and F. Scalcon, Proc. Nordic Acoust. Meeting, 407–414, 1996.
6. D. Rocchesso and F. Scalcon, Trans. on Speech and Audio Proc., **7**, 597–601 (1999).
7. B. Bazzi and D. Rocchesso, Proc. XIII Colloq. Mus. Inf., 2000.
8. D. Rocchesso and J.O. Smith, IEEE Trans. on Speech and Audio Proc., **5**, 51–63 (1997).

Trumpet and trumpeter: physical modeling for sound synthesis

Christophe Vergez[a,b], Xavier Rodet[a]

[a] *Analysis/Synthesis Team, IRCAM, 1 Pl. I. Stravinsky, 75004 Paris, France*
[b] *Musical Acoustics Team, IRCAM, 1 Pl. I. Stravinsky, 75004 Paris, France*

The goal of this paper is to present a physical model of trumpet aimed at sound synthesis. This work is a part of the PhD thesis [4].

TYPICAL BRASS MODEL

A simple physical model of trumpet-like instruments has first been developed. It is derived from the typical single mass lips model, nonlinearly coupled with a linear model for the bore (cf. [1]).

The lips model includes a single parallelepipedic mass m attached to a spring k and a damper r. The acoustic pressure field inside the bore is decomposed into outgoing and incoming travelling waves p_o and p_i.

The bore of the trumpet is modelled by its time-domain reflection function $r'(t)$, derived from complex input impedances measured in an anechoic room ([5]).

The air flow is supposed laminar in the mouth and in the lips channel. An air jet formed after the lips, is supposed to dissipate by turbulence all its kinetic energy in the cup of the mouthpiece without pressure recovery ([3]). Therefore, nonlinear coupling between the lips and the bore is represented by the Bernoulli equation linking volume flow between the lips $u(t)$, lips aperture $x(t)$ and pressure difference between the mouth and the mouthpiece $(p_m - p(t))$ (air velocity in the mouth is neglected).

Finally, this basic physical model is described by the following system of equations for positive $x(t)$:

$$(1)\left\{\begin{array}{l} m\ddot{x}(t) + r\dot{x}(t) + kx(t) = \sum F_{\text{aero-acoustics}} \\ p_i(t) = (r' * p_o)(t) \\ u(t) = l\,x(t)\,\mathrm{sgn}(p_m - p(t))\sqrt{\frac{2}{\rho}|p_m - p(t)|} \\ p(t) = p_o(t) + p_i(t) \\ u(t) = \frac{1}{Z_c}(p_o(t) - p_i(t)) \end{array}\right.$$

where l is the length of the mass in the transversal dimension, $Z_c = \frac{\rho c}{A_{cup}}$ is the characteristic impedance at the entry of the mouthpiece (ρ is the air density, c the sound velocity and A_{cup} is the cross section area of the mouthpiece entry).

When lips are closed ($x(t) \leq 0$) the lower lip is taken into account by additional stiffness and damping coefficients $3k$ and $4r$. The volume flow is set to zero.

MOST SIGNIFICANT IMPROVEMENTS

We now focus on the most important improvements brought to the basic model presented above.

Lips Oscillation

A one dimensional model for the lips is a crude approximation. In fact, it has been shown in [6] that this approximation leads to a discontinuous volume flow derivative at lips closure. To cope with this problem, the length of the mass l is made dependent on x for small openings. This accounts for the fact that lips begin to close from the corners . Practically, l is replaced by $l\Theta_1(x)$ with $\Theta_1(x) = \tanh(\frac{b}{l}x)$. Then the air flow partial derivative becomes continous and a significant part of disturbing high frequencies in the sound is eliminated.

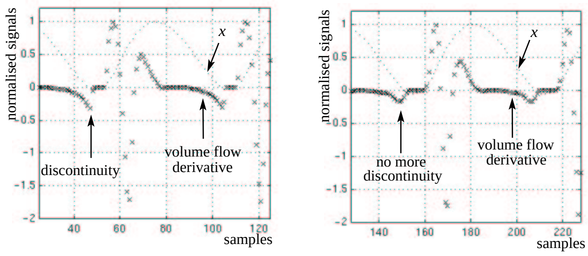


FIGURE 1. Effect of a varying lips length $l\tanh(\frac{6}{l}x)$ on the volume flow derivative at exact lips closure.

Air Flow Between The Lips

Just after the opening of the mass, a rapid alternance (apparently non realistic) of a strongly positive and negative volume flow has been observed. This occurs since the air flow model ignores viscothermal losses and inertia.

The hypothesis of a Poiseuille flow under the mass has then been studied (inertial effects are neglected). Considering the 2D-geometry of the model and the coupling between the mouth and the bore, the analytical expression for u is given (after some calculus) by equation (2).

$$u = \left[-u_1 + \sqrt{u_1^2 + 4|p_m - p_h|\frac{\rho}{2A_{lip}^2}}\right] \Big/ \frac{\rho\xi}{A_{lip}^2} \qquad (2)$$

with $u_1 = \left(\frac{3\eta e}{\left(\frac{x}{2}\right)^2 A_{lip}} + Z_c\right)$ where $A_{lip} = lx$, η is air dynamic viscosity and e is the width of the mass.

This complex equation increases the computational cost. However, viscous effects are only significant for small apertures. An approximation of equation (2) for small x leads to:

$$u \sim_{x=0} \frac{l}{12\eta e}(p_m - p_h)x^3 \qquad (3)$$

However, for large x, the Bernoulli equation is a good approximation since viscosity can be neglected. Therefore, we propose to switch smoothly between a Poiseuille flow (small openings) and a viscousless flow (large openings):

$$u = \Theta_2 \times \frac{-1}{2\,Z_c}\xi A(x)\left[A(x) - \sqrt{A^2(x) + 4|p_m - p_h|}\right] \qquad (4)$$

with $\Theta_2 = \tanh\left(\alpha_1 x^{\alpha_2}(p_m - p_h)^{0.5}\right)$, and $\alpha_1 = \frac{1}{12\eta e}\sqrt{\frac{\rho}{2}}$ $(= O(10^5))$, and $\alpha_2 = 2$. Practically, it is interesting to let the player modify α_1 and α_2 around these values. This allows a finer control on the timbre of the sound. For example, figure 2 shows the influence of α_1 (α_2 being set to 3). The smaller α_1, the softer the sound.

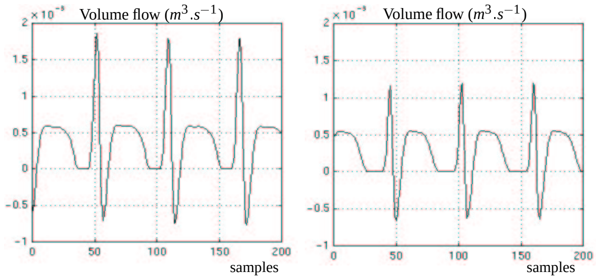


FIGURE 2. Volume flow calculated according through equation (4) with $\alpha_2 = 3$. From left to right $\alpha_1 = 10^6$, $\alpha_1 = 2.10^5$.

Acoustic Wave Propagation

It is now known that nonlinear propagation effects should be taken into account for precise simulation at high sonic levels ([2]). Indeed they are mainly responsible for the characteristic brassy sounds obtained when a trombone or a brass instrument is played fortissimo.

A new algorithm to simulate waveform distortion due to nonlinear propagation of an air pressure wave has been developed. This algorithm is derived from the simple-wave differential equation (Burgers equation without dispersion or dissipation), which assumes that the solution is always a C^1 function. However, a physically-based supplementary constraint included in our computational model, allows us to simulate shock-waves.

The extension of this method to the nonlinear propagation of a wave in a resonator fabricated by cylindrical tubes has then been studied and applied to the real-time model in [7] (Fig.3). Moreover, a *hybrid* linear/nonlinear formulation has been tested to include visco-thermal losses and a more accurate geometrical description: the inner dynamics is simulated using a measured reflection function while the sound at the bell is calculated through the nonlinear propagation algorithm. This solution is both perceptively convincing and less time consuming.

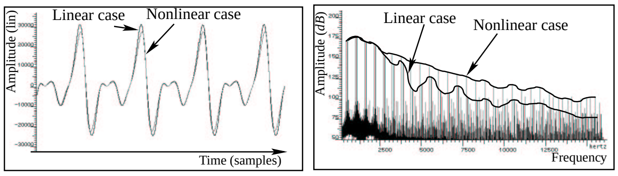


FIGURE 3. Acoustic pressure at the bell of the trumpet model: difference between linear and nonlinear propagation.

EXPERIMENTAL VALIDATION

Validation of the global behavior of the model is first made by ear (the real-time model will be demonstrated at the conference). However, to check the influence of precise parameters, an artificial mouth with water-filled latex lips has been conceived. Comparisons between simulations and experiments can be found in the PhD thesis [4]. They confirm that a simple physical model can reproduce most of the behavior of the real instrument.

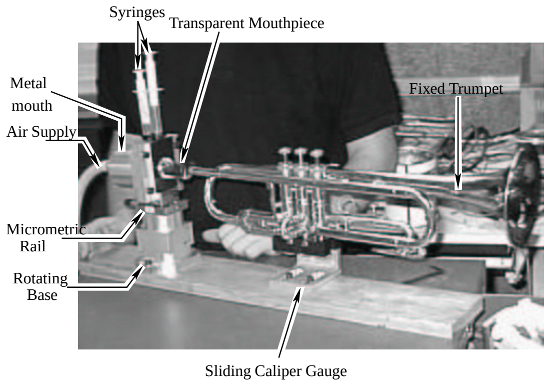


FIGURE 4. Photograph of an artificial mouth device (from [5]).

REFERENCES

1. Elliott and Bowsher. Regeneration in Brass Wind Instruments. *Journal of Sound and Vibration*, 83(2):181–217, 1982.
2. J. Gilbert and J. F. Petiot. Nonlinéarités dans les instruments de type cuivre: résultats expérimentaux. In *Actes de Colloque du quatrième CFA, Marseille*, 1996.
3. A. Hirschberg. *Mechanics of Musical Instruments*, chapter Aero-acoustics of wind instruments. Springer Verlag, 1995.
4. C. Vergez. *Trompette et trompettiste: un système dynamique non linéaire analysé modélisé et simulé dans un contexte musical.* PhD thesis, Université Paris 6, January 2000.
5. C. Vergez and X. Rodet. Model of the trumpet functioning: real time simulation and experiments with an artificial mouth. In *Proceedings ISMA'97*, Edinburgh, August 1997.
6. C. Vergez and X. Rodet. Air flow related improvements for basic physical models of brass instruments. In *Proceedings of ICMC'2000*, Berlin, German, August 2000.
7. C. Vergez and X. Rodet. New algorithm for nonlinear propagation of a sound wave. application to a physical model of a trumpet. *Journal of Signal Processing*, 4(1):79–87, January 2000. Special issue on nonlinear signal processing.

A computer system for acoustical input admittance measurements of violins giving clues for the prediction of sound characteristics.

W. Kausel

Institut für Wiener Klangstil, University of Music and Performing Arts, Vienna, Austria

The new, portable violin instrument analysis system (VIAS) offers a variety of different graphical representations of the measured resonance spectrum of an instrument, which are related to its sound characteristics. For that purpose, the violin's bridge is set vibrating by an electro-mechanical transmitter, applying a sinusoidal sound pressure signal close to the edge of the bridge. With a laser-optical sensor the actual displacement of the contact point is measured as a function of frequency. Phase information is used to separate the active component contributing to the radiated sound energy from the idle component. Pictorial representations of the resonance spectrum are generated, assuming stimulation of the measured resonator by the saw-tooth like signal of a bowed string. The synthetic sound spectra are plotted against the chromatic scale and are made audible using the computer's soundcard. Results have been correlated with transmission curves taken in the reverberant chamber. Eliminating the measured resonance profile from a recording of a violin played by an expert player, a 'violin-neutral' recording is obtained and the resonance profile of a different instrument can be superimposed. This way, resonator characteristics can be compared objectively and luthiers can get valuable hints during the manufacturing process.

INTRODUCTION

Very often it is not possible to move precious violins away from their storage location, whether this is now a private instrument collection or a museum. Attempts to get objective measures related to the sound of a historical instrument therefore must not rely on any studio-like or laboratory environment such as an an-echoic chamber. This rules out methods like transmission function measurements using far-field microphones with bridge stimulation or using sound field stimulation in combination with sensors glued to the bridge. On the other hand luthiers are most interested to learn as much as possible about sound and resonance characteristics of great violins, especially those who would like to make their own instruments sound like a Stradivarius, Guarneri or Armati [1].

It was the aim of this work to develop some portable tool, by means of which accurate resonance profiles of string instruments can be measured without having to move the instrument away from its place. The result was a laser-optical velocity (actually displacement) sensor in conjunction with an electromechanical sound pressure source, both built into a compact measurement head which gently touches the edge of the instrument's bridge during the measurement.

The acoustical input admittance at the bridge edge measured as a complex function of frequency represents the resonance profile of the instrument, because the bridge is set vibrating just the same way as it would be by bowing a string.

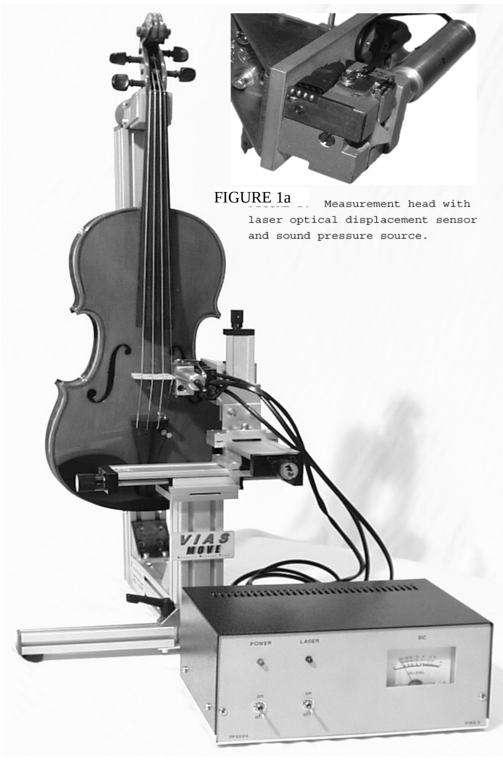


FIGURE 1. VIAS system & measurement head

MEASUREMENT HEAD

The measurement head consists of a magnetic circuit driven by a strong permanent magnet out of a rare earth compound, a spring mounted piece of epoxy plate with a thin copper trace carrying the signal current, a 3mW red laser diode with a convex lens with a focal length of 6mm and an optical detector circuit based on a photo resistor. The laser beam is partially masked by the vibrating plate which therefore modulates the amount of light impinging on the photo detector. If stray light is minimized and the rectangular laser beam adjusted properly then there is a linear dependency between displacement of the protrusive probe and the detector signal. AC-currents of about 1A are sufficient even to drive a double bass bridge.

The protrusive probe allows admittance measurements on other instruments, too, finished and unfinished ones, and not limited to the bridge. It has successfully been applied to guitars, percussion instruments, and even violin bows.

ANALYSIS

The measurement system consisting of admittance head and amplifier subsystem is connected to the sound card of a PC. The VIAS software generates a sine wave sweep between two corner frequencies with adjustable duration. A logarithmic sweep with a length of 120sec is sufficiently slow to excite even subtle resonances of violins.

The response signal is split into a DC component, which is an indication for the contact pressure, and an AC component, which is synchronously demodulated with the stimulus sweep and plotted over frequency. A 2^{nd} order high pass filter helps to avoid overloading the sound card input when slow mechanical vibrations are strongly modulating the contact pressure. Its loss is compensated for by the plot module.

By evaluating recorded phase information effective power (radiated sound + thermal losses) and idle power can be separated. The real part of the resonance profile has been found to clearly correlate with a transmission function measured in the reverberation chamber, when the Helmholtz (air-) resonances are emphasized according to their smaller thermal losses and when some extra resonances are added, which can be found by measuring bridge admittance vertically to the top plate [2].

This justifies the use of input admittance curves as a means for sound estimation. Knowing the Fourier decomposition of a saw-tooth signal:

$$f(t) = \sum_{i=1}^{\infty} (-1)^{(i+1)} \frac{\sin(i\omega t)}{i}, \qquad (1)$$

which is the kind of signal excited when a string is bowed, then all its harmonic components can be filtered according to a corrected resonance profile.

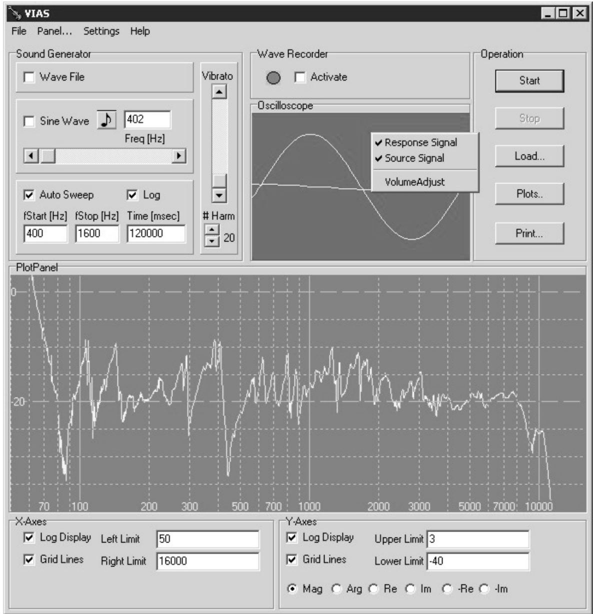


FIGURE 2. VIAS control panel with violin curve.

If this is done for each note within the playing range of the instrument an intensity sound plot can be created. The bottom row shows the intensity of the fundamental, the upper rows the relative levels of 2^{nd}, 3^{rd}, 4^{th}... harmonics, plotted over the played note. This virtual sound can be synthesized by VIAS to listen to it.

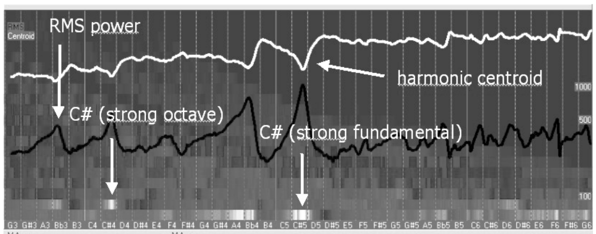


FIGURE 3. VIAS estimated sound prediction.

REFERENCES

1. M. Schleske., "On making 'tonal copies' of a violin" in *Journal of the CAS*, **Vol.3/Nr.2/Ser.II**, Montclair/NJ: Catgut Ac.Soc., 1996, pp.18-28.

2. S. Zopf and A. Brown., *Input admittance measurements and their relation to the sound characteristics of violins and violas*, Proceedings ISMA, Peruggia, 2001.

Measuring and Data Processing of a Plucked Violin

I. Bláhová

Department of Physics, Faculty of Mechanical Engineering, Czech Technical University, Technická 4, 166 07 Praha, Czech Republic

"The violin is surely one of the most astounding and complicated acoustical devices ever created by and for the human nervous system", wrote C. M. Hutchins in the introduction of her book [1]. Many researchers have tried to characterise the sound quality of violins by objective quality parameters. An experimental method described here is very similar to the technique of violin playing termed "pizzicato". A violin is placed horizontally on two supports with its string down. Small mass hangs from its string in a standard playing position. After releasing, the transient response is picked up by a Brüel&Kjær 4374 accelerometer placed on the back plate opposite to the bridge. For the first set of measurement's a measuring card ADSP2115 was used for recording signals; for the second set of measurement's a Brüel&Kjær 2825 PULSE. A number of tests proved the reproducibility of excitation end measurement. Several violins of different quality were measured. Data were processed in a system MATLAB. STFT was used for calculation of frequency spectrum. The obtained frequency spectra are dependent on time. Characteristics which could be important for a tonal quality of a violin have also been noted.

INTRODUCTION

MUDr. Honěk is a violinmaker who has tried to determine objective characteristics of a violin's sound. In his research for the simplest method, he measured the duration and the intensity of a sound of plucked violins strings. An accelerometer and a sound level meter were used for measuring, and the data obtained were processed. Three coefficients were determined and are presented as characteristic of the sound quality of an instrument [2]. His idea found application in a construction of a professional experimental apparatus, but needs further elaboration for the more complex matter of signal processing [3].

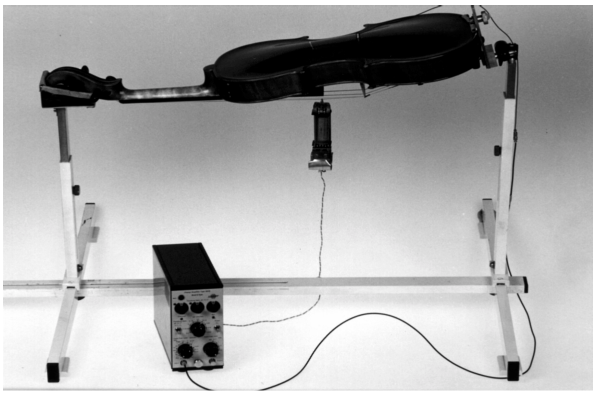


FIGURE 1. The experimental apparatus (the Brüel&Kjær 4374 accelerometer is placed on the back plate, the box in the left down corner is the Brüel&Kjær amplifier, the plucking mass is hanging down).

THE EXPERIMENTAL METHOD

The principal feature of the experimental method described is very similar to the manner of violin playing named "pizzicato". A violin is placed horizontally on two supports with its string down [Figure 1]. A small mass hangs from its string in a standard playing position. After its release, the transient response is picked up by a Brüel&Kjær 4374 accelerometer, which is placed on the back plate. For the first set of measurement a measuring card ADSP2115 was used for recording signals with a sampling frequency 44,1 kHz for the second set of measurement a Brüel&Kjær 2825 PULSE was used with a sampling frequency 65,536 kHz. Data are processed in the system MATLAB.

Initial Tests

Initial tests have already been performed, revealing clearly that a very small change in the position of an accelerometer or in position of an excitation point could not greatly change a violin response. In short, the measuring method must be reproducible. A number of signals under the same conditions were recorded and correlation coefficients were calculated; the programme for calculation of a maximum correlation coefficient had been created previously.
In the following table, for example, correlation coefficients can be seen for pairs of all couples of five signals. These signals were registered under the same condition, but a violin and an accelerometer were removed and then replaced. String **a** was plucked.

Table 1. Correlation Coefficients.

Signal	1	2	3	4
2	0,98	0,96	0,87	0,98
3		0,94	0,88	0,95
4			0,76	0,96
5				0,83

These coefficients are very numerically close to one, meaning that picked signals are very similar. It follows that the way of plucking of a violin and the way of registering a response is repeatable.

Experimental Violins

After a number of initial tests, eight different quality violins were measured. The best and also the oldest violin was the one was signed “Carl Ludwig Bachmann, Berlin 1760”. Figure 2 shows the transient response of a string **a** of this violin.

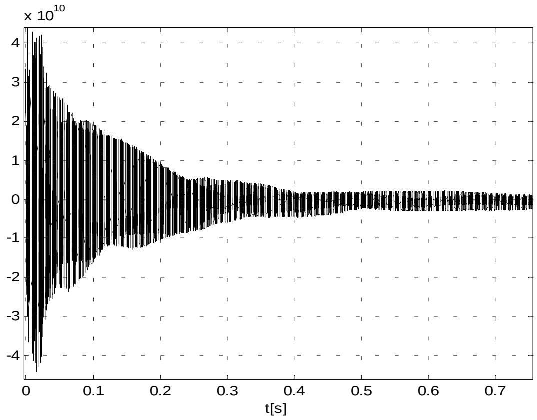


FIGURE 2. The response of a string **a** of the violin “Carl Ludwig Bachmann, Berlin 1760”.

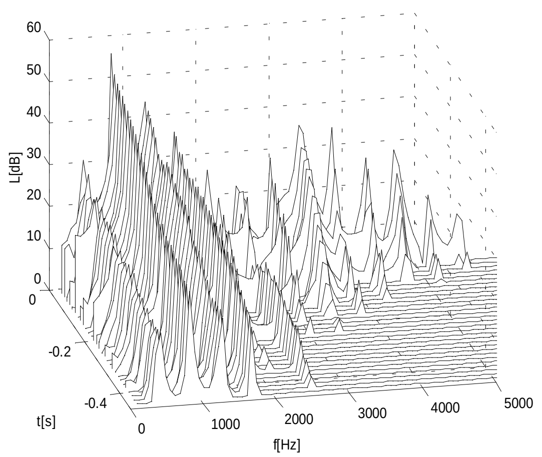


FIGURE 3. Spectrum of the response of a string **a** of the violin “Carl Ludwig Bachmann, Berlin 1760”.

RESULTS

STFT was used for calculation of frequency spectrum [Figure 3]. The obtained frequency spectra depend on time. Certain characteristics of possible importance for the tonal quality of a violin are found: for example, the dependence of overtones on time is quite interesting; overtones can reach more than one local maximum. Another interesting effect is the level of intensity of the second, third and fourth harmonics which is higher then the level of intensity of the fundamental.

CONCLUSIONS

An experimental method for violin testing was prepared. Initial tests proved the reproducibility of excitation end measurement. Eight different quality violins were measured. A summary of all results of this experiment, and a corresponding physical interpretation are in preparation. Another method for data processing (the Discrete Wavelet Transformation) will be attemped.

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REFERENCES

1. C. M. Hutchins, *Research in Violin Acoustics, 1975-1993*, ASA, Woodbury 1997.

1. A. Honěk, *Evaluation of Tonal Qualities,* STRAD, pp. 597-595 (1973).

2. I. Bláhová, *Measuring and Data Processing of a Plucked Violin*, Fine Mechanics and Optics 10/2000, pp. 282-284 (2000).

4. T. D. Rossing, N. H. Fletcher, *The Physics of Musical Instruments*, Springer, New York 1991.

Phenomenological Models for Musical Instruments

A. Barjau[a] and V. Gibiat[b]

[a]*Department of Mechanical Engineering, Universitat Politècnica de Catalunya, Barcelona, Spain*
[b]*Laboratoire Ondes et Acoustique, Université de Paris 7 Denis Diderot, Paris, France*

We propose a general philosophy to build simplified models for the three families of self-sustained musical instruments (woodwinds, flutes and strings). These models are represented mathematically by a delayed differential equation and work with just one variable. Though very simple, they allow to reproduce many different behaviours (periodic, quasiperiodic, chaotic) corresponding to real instrument possibilities.

INTRODUCTION

When modeling musical instruments, there are two clearly different philosophies. One consists of a very realistic description of the system, and this leads to a high number of equations with a very high number of variables and parameters [1,2,3]. These models have obviously no analytic solution and call for a numerical resolution which in general is very time and power consuming. Moreover the estimation of parameters is extremely delicate. The global result is that the model manipulation is as hard as learning to play the real instrument. A totally different philosophy is that provided by the phenomenological modeling (already used in different domains of science, as biology and fluid dynamics). The main idea is to build a differential model based upon the main physical phenomena implied in the studied process. It is possible to reduce then the number of variables (sometimes one -the output of the system- is enough) and parameters.

GENERAL DESCRIPTION OF SELF-SUSTAINED INSTRUMENTS

The general structure of a self-sustained musical instrument is a continuous system (air column or string) coupled to an energy source (the player) and sometimes to a secondary resonant system (string family). The most outstanding difference between these instruments and those based on free oscillations is the essential nonlinearity transforming the constant energy flow into an oscillating one. As a first approach, the nonlinearity can be localized at one single point (the excitation point), and the continuous system can be considered as a linear one. The coupling between these two elements leads to different oscillating regimes.

Nonlinear element

Several treatments of the nonlinear element can be found in literature. They go from a single-degree-of-freedom (DOF) system to a discrete system with just a few DOFs. In some cases, its behaviour is governed by a set of ordinary differential equations while in others they are just described through an algebraic equation. However, in all cases the final output is a variable $g(t)$ which is the input of the linear element.
We have chosen the simplified description through an algebraic equation relating $g(t)$ to the variable $f(t)$ which will be taken as response of the continuous system at the excitation point. The variables $g(t)$ and $f(t)$ are, respectively, the air flow and the air pressure for woodwinds, the air flow and the jet deflection for flutes, and the friction force and the transverse velocity for strings. The nonlinear characteristics $g(t) = F_{nonlin}\left[f(t)\right]$ have been taken to be the same as those presented in [4].

Linear element

The linear element is usually considered as a 1D continuous system (and so with an infinite number of DOFs). The continuous essence can be represented mathematically through three different kinds of equations governing the variable $f(x,t)$: partial differential equations, convolution integrals or delayed differential equations. If partial derivatives are used, the infinite DOFs are included in the x dependence of the $f(x,t)$. If convolution or delayed differential equations are used, the $f(x,t)$ is only calculated at one point of the system, $f(x_0,t) \equiv f(t)$, but the knowledge of the past history of $f(t)$ is required in order to perform the integrals. Strictly speaking, this implies an infinite amount of information, which corresponds well to the existence of an infinite number of DOF.

DELAYED DIFFERENTIAL EQUATION

The delayed differential equation for the three families of instruments can be built in a very intuitive way from a physical rationale. We have done it in three steps which correspond to three different states of the system.

Free response of an infinite 1D system

If no ends are considered, the propagation phenomenom along the system is responsible for an exctinction of any initial condition introduced at the excitation point. This disparition is not a sudden one, and it can be represented through a relaxation equation,

$$\frac{df(t)}{dt} = -\alpha f(t) \qquad (1)$$

Forced response of an infinite 1D system

The excitation $g(t)$ (the output of the nonlinear element) has to be added to the previous equation,

$$\frac{df(t)}{dt} = -\alpha f(t) + \beta g(t) \qquad (2)$$

Forced response of a finite 1D system

The existence of end conditions is responsible for a feed-back coming from them. As a first approach, we have considered the ends effect through just a reflection coefficient R. For the general case of a system where the excitation takes place at an arbitrary point P, this feed-back reinjects the input variable $g(t-2\tau_1)$, $g(t-2\tau_2)$, $g(t-2T)$(where $2\tau_1$, $2\tau_2$ are the time intervals needed for a round trip from point P to each end, and $T = \tau_1 + \tau_2$) during the time interval$[0,2T]$,

$$\begin{aligned}\frac{df(t)}{dt} &= -\alpha f(t) + \beta g(t) + \beta x_1 R_1 g(t-2\tau_1) + \\ &+ \beta x_2 R_2 g(t-2\tau_2) + \beta(x_1+x_2) R_1 R_2 g(t-2T)\end{aligned} \qquad (3)$$

The coefficients x_i represent the amount of $g(t)$ propagating towards each end.
For longer intervals, more and more delayed inputs would have to be considered:

$$\begin{aligned}\frac{df(t)}{dt} &= -\alpha f(t) + \beta g(t) + \\ &+ \beta \sum_{i=1}^{2} x_i R_i \left[\sum_{n=0}^{\infty} (R_1 R_2)^n g(t - 2\tau_i - 2nT) \right] + \\ &+ \beta (x_1 + x_2) \sum_{n=1}^{\infty} (R_1 R_2)^n g(t - 2nT)\end{aligned} \qquad (4)$$

A more compact formulation can be obtained if we rewrite eq. (3) at $(t-2T)$, isolate the sumations and substitute into (4):

$$\begin{aligned}\frac{df(t)}{dt} + \alpha f(t) - R\left[\frac{df}{dt} + \alpha f\right]_{t-2T} &= \beta g(t) + \\ + \beta \left[(x-1) R g(t-2T) + \sum_{i=1}^{2} x_i R_i g(t-2\tau_i) \right]\end{aligned} \qquad (5)$$

where $x = x_1 + x_2$. If the excitation point is placed at one end (as in the case of woodwinds), the equation simplifies because $x_2 = 0,\ \tau_1 = T$:

$$\begin{aligned}\frac{df(t)}{dt} + \alpha f(t) - R\left[\frac{df}{dt} + \alpha f\right]_{t-2T} &= \\ = \beta\left[g(t) + R g(t-2T)\right]\end{aligned} \qquad (6)$$

RESULTS

Many numerical simulations have been done for woodwinds and strings. As an example, Figure 1 shows the transverse velocity of the bowing point for a bowing pressure of 1 Pa and a bowing speed of 0.5 m/s. The bowing point is located at a distance from the bridge equal to 1/7th of the string length. Results for cylindrical and conical woodwinds can be found in [3,4].

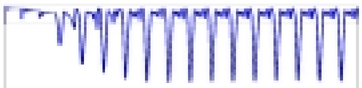

FIGURE 1. Velocity at the bowing point

REFERENCES

1. Schumacher, R.T. “Self sustained oscillations of the clarinet: an integral equation approach”, Acustica **40**, 298-309 (1978)

2. McIntyre, M.; Schumacher, R.; Woodhouse, J. “On the oscillations of musical instruments”, J. Acoust. Soc. Am. **74**(5), 1325-1345 (1983)

3. Barjau, A., Gibiat, V., Grand, N. "Study of woodwind-like systems through non linear differential equations. Part I: simple geometry", J. Acoust. Soc. Am.**102**(5),3023-3031 (1997)

4. Barjau, A., Gibiat, V. "Study of woodwind-like systems through non linear differential equations. Part II: real geometry", J. Acoust. Soc. Am. **102**(5),3032-3037 (1997)

Recent Developments in Woodwind Instrument Physical Modeling

G. P. Scavone[1], S. Lakatos[2]

[1] *CCRMA, Department of Music, Stanford University, Stanford, CA 94305, USA*
[2] *Department of Psychology, Washington State University, Vancouver, WA 98686, USA*

This paper briefly reviews several recent developments in physical modeling of woodwind musical instruments. In particular, issues with regard to real-time synthesis of toneholes, the single-reed excitation mechanism, and conical air columns are discussed. A programming environment appropriate for real-time physical modeling synthesis is presented, as well as issues of control demanded by recent increases in model complexity. Psychoacoustic studies have begun to exploit the parametric flexibility of physical models to study complex auditory perception. One recent study addressing auditory learning and memory is described.

MODEL DEVELOPMENTS

This paper addresses time-domain models of woodwind musical instruments which can be used for real-time sound synthesis. In particular, digital waveguide techniques are employed to efficiently model wave propagation within the instrument air column.

Toneholes

Keefe (1981) presents a rigorous study of woodwind instrument tonehole acoustics and provides frequency-domain results calibrated in part through experimental measurements. Scavone (1997) and Smith and Scavone (1997) translate these results for efficient implementation in time-domain digital waveguide models. Keefe's approach provides two distinct models for the open and closed tonehole states. Scavone and Cook (1998) present a single tonehole model capable of dynamic state changes from fully open through fully closed which shows good agreement with the Keefe model. An alternate approach using wave digital filter techniques resolves a limitation on the minimum tonehole height inherent in the earlier model (van Walstijn and Scavone, 2000).

The Single-Reed Excitation

The reed mechanism of woodwind instruments is traditionally modeled as a second-order oscillator and a nonlinear volume flow characteristic. For clarinet-like systems, the reed behavior is dominated by stiffness. Under this assumption, it is common to neglect the reed mass to produce a simplified, memory-less model. Recent work has concentrated on efficient numerical techniques to solve the simultaneous reed/bore and nonlinear flow equations (Borin *et al.*, 2000; Avanzini, 2000).

Conical Waveguide Issues

Ayers *et al.* (1985) provides a detailed study of conical air column acoustics. While a complete cone can support harmonically-aligned partials, the resonances of a truncated and stopped conic frustrum are "warped" in proportion to the length of the truncated section. When an appropriately designed digital waveguide structure is used to model a truncated cone, the resulting inharmonicity of the air column can complicate the production of a stable "regime of oscillation". Several approaches have been investigated to yield stable "conical" air column behavior.

REAL-TIME SYNTHESIS

Digital waveguide techniques have been used to implement real-time woodwind instrument synthesis models on computer host processors since the mid-1990s (and on special purpose digital signal processing hardware since the late-1980s). Continuing advances in desktop computing power are allowing ever greater model complexity. The digital waveguide technique computes the air column reflection function in real time (as opposed to the use of a fixed reflection function stored in memory). This allows smooth modification of the air column parameters, such as the opening and closing of toneholes or muting of a brass instrument bell. A cross-platform synthesis environment has been written in the C++ programming language to aid in the prototyping and testing of the models discussed above.

The Synthesis ToolKit (STK) in C++

The Synthesis ToolKit (Cook and Scavone, 1999) provides an object-oriented, C++ framework for the programming of audio signal processing algorithms. Specific design goals have included cross-platform functionality, ease of use, real-time synthesis and control, and user extensibility. STK provides "unit generator" classes for a variety of filter and synthesis algorithms, as well as input/output functionality for internet streaming, realtime computer audio hardware, and .wav, .snd, .aif, and .mat (Matlab MAT-file) formatted files. The ToolKit currently runs with realtime support (audio and MIDI) on Linux, SGI (Irix), and Windows computer platforms. Generic, non-realtime support has been tested under NeXTStep, but should work with any standard C++ compiler.

Realtime Control

One advantage of physical models is parametric control of instrument features. The complex parameter space which often results, however, can prove to be nearly as difficult to master as that of real musical instruments. This has stimulated research and development in human-computer interface technologies, a rapidly growing field of study. While commercially available MIDI wind controllers provide a more appropriate interface to woodwind instrument models, these devices remain limited in their functionality, in part because of limitations in commercial synthesizers. Extensions have been proposed and implemented to address the control of dynamic tonehole models as discussed above (Scavone and Cook, 1998).

PSYCHOACOUSTIC STUDIES

The parametric flexibility of physical models offers new opportunities for the study of complex auditory perception. Recent experiments were conducted to test listeners' ability to attend selectively to the properties of a physical model comprising collisions between multiple independent sound-producing objects (Lakatos *et al.*, 2000). Percussion instrument sounds were synthesized using physically informed sonic modeling (PhISM) techniques (Cook, 1997). Results showed that listeners are able to correlate some common physical properties across different target and cue object types.

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REFERENCES

Avanzini, F. (2000). On the use of weighted sample methods in digitizing the clarinet equations. In ICMC (2000), pp. 46–49.

Ayers, D. R., Eliason, L. J., and Mahgerefteh, D. (1985). The conical bore in musical acoustics. *Am. J. Phys.*, 53(6):528–537.

Borin, G., De Poli, G., and Rocchesso, D. (2000). Elimination of delay-free loops in discrete-time models of nonlinear acoustic systems. *IEEE Transactions on Speech and Audio Processing*, 8(5):597–605.

Cook, P. R. (1997). Physically informed sonic modeling (phism): Synthesis of percussive sounds. *Computer Music J.*, 21(3):38–49.

Cook, P. R. and Scavone, G. P. (1999). The Synthesis ToolKit (STK). In *Proc. 1999 Int. Computer Music Conf.*, pp. 164–166, Beijing, China. Computer Music Association.

ICMC (2000). *Proc. 2000 Int. Computer Music Conf.*, Berlin, Germany. Computer Music Association.

Keefe, D. H. (1981). *Woodwind Tone-hole Acoustics and the Spectrum Transformation Function.* Ph.D. thesis, Case Western Reserve University.

Lakatos, S., Cook, P. R., and Scavone, G. P. (2000). Selective attention to the parameters of a physically informed sonic model. *J. Acoust. Soc. Am.*, 107(5):L31–L36.

Scavone, G. P. (1997). *An Acoustic Analysis of Single-Reed Woodwind Instruments with an Emphasis on Design and Performance Issues and Digital Waveguide Modeling Techniques*. Ph.D. thesis, Music Dept., Stanford University.

Scavone, G. P. and Cook, P. R. (1998). Real-time computer modeling of woodwind instruments. In *Proc. Int. Symp. on Musical Acoustics (ISMA-98), Leavenworth, WA*, pp. 197–202.

Smith, J. O. and Scavone, G. P. (1997). The one-filter Keefe clarinet tonehole. In *Proc. IEEE Workshop on Applied Signal Processing to Audio and Acoustics*, New York. IEEE Press.

van Walstijn, M. and Scavone, G. P. (2000). The wave digital tonehole model. In ICMC (2000), pp. 465–468.

Sound Synthesis of Plucked String Instruments Using a Commuted Waveguide Model

V. Välimäki[a], C. Erkut[a], and M. Laurson[b]

[a]Laboratory of Acoustics and Audio Signal Processing, Helsinki Univ. of Tech., Espoo, Finland
[b]Centre for Music and Technology, Sibelius Academy, Helsinki, Finland

A model-based synthesis method for the synthesis of plucked string instruments is reviewed. This technique is based on commuted waveguide synthesis, in which a sophisticated yet efficient model for the vibrating string is used. The body response, pluck excitation, and any extra sounds are incorporated as samples. This methodology is applicable to a broad family of plucked string instruments, such as guitars and lutes. We discuss the model structure, parameter estimation from recordings, and control issues. A new contribution is a discussion on the use of high-order digital filters within the feedback loop of the string models. At low fundamental frequencies, the decay characteristics of tones may be complicated, and the traditional first-order loop filter may not be sufficiently flexible for their reproduction. Based on our experiments on the model-based synthesis of other string instruments, we briefly survey some points of modification needed to calibrate the synthesis system for different string instruments.

SYNTHESIZER STRUCTURE

Figure 1 describes the commuted waveguide synthesis system for a single plucked string [1]. It consists of two sample databases and two string models, $S_h(z)$ and $S_v(z)$, which have been coupled to implement the two polarizations of vibration, and two low-order digital filters (Timbre control and Plucking-point filter) to process the samples used as the input for the string model. The two string models have been coupled also to the other strings (five in the case of the classical acoustic guitar) through a sympathetic coupling matrix, which enables leakage of energy to all strings from any one that is currently sounding.

The string models are based on the filter structure proposed by Jaffe and Smith [2]. It is a feedback loop where the loop delay implemented with a delay line and an interpolation filter controls the pitch of the tone, and where a lowpass loop filter controls the decay rate that depends on frequency. We have been using a one-pole IIR filter as the loop filter, because it is easy to design and has been sufficient for good sound quality [1].

The excitation signal database contains processed samples taken from the attack part of recorded plucked string tones. The spectrum of these signals has been whitened to cancel the harmonics, but the aim has been to leave the noises related to plucking of the string and resonances of the body intact. In addition, we have created a database of special performance effects, such as rubbing and scraping of the string and different types of knockings on the guitar body, which have turned out to be essential in synthesizing modern guitar repertoire. These samples can be mixed with the synthetic signal at the output, as shown in Fig. 1. A more detailed description of our current synthesizer will be published soon elsewhere [1].

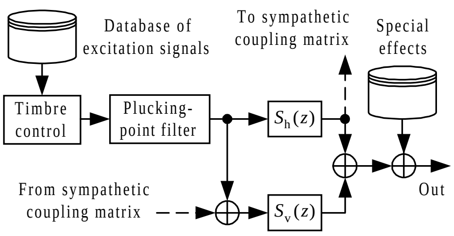

Figure 1. Plucked string synthesis model.

CONTROL SCHEME

We use a music notation software called Expressive Notation Package (ENP) for controlling the synthesizer. The use of notation in expressive synthesis control is motivated by the lack of adequate real-time controllers, familiarity with music notation, and precision of control.

The user enters in ENP the musical material in standard notation. The system requires no textual input. The user can also add both standard and non-standard expressions that allow to specify instrument specific playing styles with great precision. Expressions can be applied to a single note (such as string number, pluck position, vibrato, or dynamics) or to a group of notes (e.g., left-hand slurs or finger-pedals). Groups can overlap and they may contain other objects, such as breakpoint functions. Macro expressions generate additional note events, such as tremolo, trills, portamento, and rasgueado. ENP allows fine-tuning of timing with the help of graphical tempo functions. Besides tempo functions, ENP supports user definable performance rules which allow modification of score information.

The calculation of the control information for the synthesizer is executed in two main steps. In the first one, the note information provided by the input score is modified by the tempo functions and ENP performance rules. In the second step, all notes of the input score are scheduled. While the scheduler is running, each note sends a special method to its instrument which in turn starts other scheduled methods which produce the final control data. These methods are responsible for creating discrete control data (such as excitation information) or continuous data (gain of the loop filter, filter coefficients, or other low-level data).

HIGH-ORDER LOOP FILTERS

The one-pole loop filter guarantees a smooth magnitude roll-off in accordance with a model of viscous drag of air. However, other physical mechanisms, such as dry friction and the string-body coupling complicate this rather simplistic model. A one-pole filter cannot accurately account for these loss mechanisms. Therefore, a higher-order filter would provide a better simulation, especially at low frequencies where the dry friction of wound strings is most effective and most prominent modes of the instrument body alter the decay times [3]. However, the design of higher order filters is not straightforward, since there is a nonlinear relation between the decay times of the synthetic tone and the magnitude response of the filter. Moreover, no technique guarantees the stability.

A transformation technique has been used in loop filter design for piano tones [5]. Based on matching the decay times between the analyzed and synthetic tones, this technique ensures the stability. Since the decay characteristics of all the tones of a string are dictated solely by corresponding string and its termination, this technique may be adopted into the design of loop filters of plucked string instruments as follows.

The decay time $\tau_{m,k}$ of the kth harmonic is obtained by pitch-synchronous short-time Fourier transform and linear regression for each fret m. The decay rates $\sigma_{m,k} = 1/\tau_{m,k}$ are obtained as a function of frequency. Figure 2(a) shows the extracted decay rates of the sixth guitar string. A 6th-order polynomial has been fitted in the mean square sense to the data using polynomial regression (solid curve in Fig. 2(a)). A weighting function may be used [4]. This polynomial decay rate function can be used as a prototype, and the filter magnitude responses for any tone of the string can be obtained by

$$G_{\mathrm{m}}(f) = e^{-\sigma(f)/f_0} \qquad (1)$$

Figure 2(b) compares the magnitude responses of the target filter, a one-pole filter, and a 4th-order pole-zero filter that yields a good match.

DIFFERENT INSTRUMENTS

In addition to the classical acoustic guitar [1], our recent work has concentrated on natural-sounding synthesis of the clavichord, the renaissance lute, the ud, the tanbur, and the kantele. Each instrument has its own special characteristics which must be accounted for by modifying

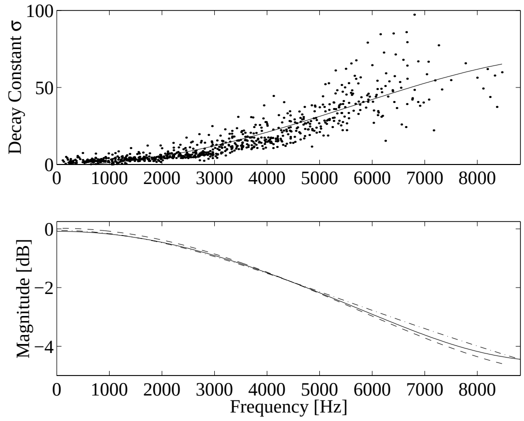

Figure 2. (a) Extracted decay rates for harmonics of guitar tones played on the 6th string (all frets) and a polynomial fit (solid line). (b) Magnitude responses of target filter (dashed line), one-pole filter (dash-dot line), and 4th-order IIR filter (solid line) for fret #7 (f_0 = 123.3 Hz). The sampling rate is 44100 Hz.

the synthesis model. Naturally, excitation samples and loop-filter parameters must be extracted for each instrument. Furthermore, in the case of the clavichord, for example, an additional sample database is needed that contains the impulse responses of the soundbox obtained by hitting the bridge with an impulse hammer at various points. One of these samples is selected according to the key and is added to the output signal. This method allows a freedom to adjust the amount of soundbox reverberation in the synthetic sound. The tanbur synthesis model [5] requires the use of a nonlinear version of the string model [6].

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REFERENCES

1. M. Laurson, C. Erkut, V. Välimäki, and M. Kuuskankare, *Computer Music J.* **25**(3) (2001).
2. D. A. Jaffe and J. O. Smith, *Computer Music J.* **7**(2), 76–87 (1983).
3. C. Valette, "The Mechanics of Vibrating Strings," in *Mechanics of Musical Instruments*, edited by A. Hirschberg, J. Kergomard, and G. Weinreich, Springer, 1995, pp. 116–183.
4. B. Bank, *Physics-Based Sound Synthesis of the Piano*. Report no. 54, Helsinki Univ. of Tech., Lab. of Acoustics and Audio Signal Processing, Espoo, Finland, 2000.
5. C. Erkut and V. Välimäki, "Model-Based Sound Synthesis of Tanbur, a Turkish Long-Necked Lute," in *Proc. ICASSP'2000*, IEEE, Istanbul, Turkey, 2000, vol. 2, 769–772.
6. T. Tolonen, V. Välimäki, and M. Karjalainen, *IEEE Trans. Speech and Audio Process.* **8**(3), 300–310 (2000).

Influence of wall curvature on the resonance behavior of glass bowls

A. A. Piacsek, K. K. Roberts

Department of Physics, Central Washington University, Ellensburg, WA, USA

Measurements were made of the lowest resonance frequency of a variety of glass vessels to test how well theoretical predictions based on a simplified vessel shape can be applied toward more complex shapes. The types of vessels studied include standard wine glasses (slightly bulbous), flutes, brandy snifters (bulbous), martini glasses (conical), and tumblers (cylindrical, with flat bottom).

Background

Thin-walled bowls are commonly employed as percussion instruments, typically in the form of bells. The acoustics and vibrational behavior of bells has been well described by Rayleigh, Rossing, and others [1, 2, 3]. Bells are made in a variety of shapes, most of which were established empirically before modern vibrational analysis. A complete analysis of vibrating bells or bowls is quite complex; few bells are designed from scratch using theory alone.

Bells are generally made of metal, but glass drinking vessels are also known for their musical qualities. A rigorous analysis of the vibrational behavior of wine glasses was published in 1983 by French [4]. By modeling glasses as cylindrical vessels with a flat bottom and an effective height that depended on the variation of wall thickness, French derived the following expression for the frequency of the hum mode (the mode excited by rubbing a finger around the rim of the glass):

$$\nu_{2,0} = \frac{1}{2\pi}\left(\frac{3Y}{5\rho}\right)^{1/2}\frac{T}{R^2}\left[1+\frac{4}{3}\left(\frac{R}{H}\right)^4\right]^{1/2} \qquad (1)$$

in which Y is Young's modulus, ρ is the density of the glass (assumed homogeneous), T is the glass thickness near the rim, and R is the radius at the rim. The term that depends on the extrinsic dimensions of the glass we shall denote $\gamma(T,R,H)$.

An important simplification in the above theory is that a constant radius is assumed for the vessel bowl. It was our goal to perform a simple experiment that would allow us to quantify the influence of wall curvature on the resonance frequency of glass vessels and, by extension, of other bowl shaped obects. Our hypothesis was that the brandy glasses, having a pronounced wall curvature that increases their stiffness relative to a cylindrical shell wall, would have correspondingly higher frequencies, all other factors being identical.

Experiment

A total of thirty-one differently shaped drinking glasses were obtained, all of which were relatively inexpensive and are assumed to be made of comparable glass. A subset of these were measured for density; a value of 2.4 g/cm^3 was obtained for each. The 31 glasses were divided into four categories corresponding to shape and function, as depicted in Fig. 1.

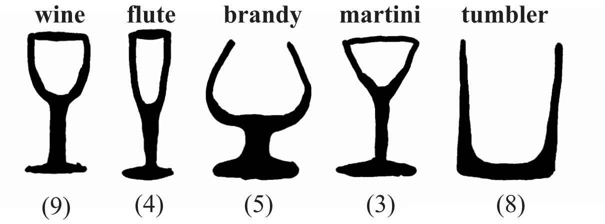

FIGURE 1. Silhouettes of vessel shapes used in the experiment. Figure in parentheses indicates number of each type studied.

The diameter was measured at the rim and the thickness just below the rim. An approximate value for the effective height of the glass, H^*, was measured by spanning the glass wall with thumb and forefinger, sliding down until the thickness was perceived to be twice that at the rim. The distance from this point to the rim was then recorded. This criterion was chosen more or less arbitrarily, with the assumption that below this point, where the glass becomes ever thicker, the the vibration amplitude is negligible.

An alternate method of measuring the effective height was performed, whereby water was slowly added to the glass while it was being tapped; pouring stopped when there was a barely perceptible change in pitch. The distance from rim to water level was recorded as H^*. The rationale behind this method is that if fluid loading up to a certain height does not perceptibly alter the pitch, then the submerged portion of the glass can be considered below the effective fixed boundary of the glass wall.

After determining the effective height via the water method, an effective volume enclosed by the vibrating

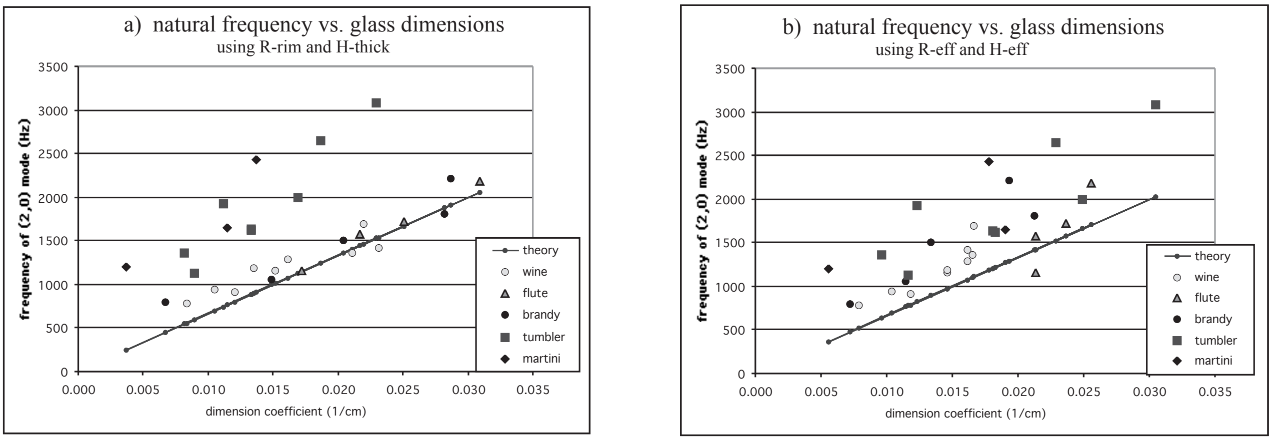

FIGURE 2. Measured natural frequency of all glass vessels plotted against the parameter $\gamma(R,T,H)$; each vessel type is coded with a different symbol. Prediction from Eq. 1 is shown for comparison.

shell could be measured. From this volume and height, we computed an effective radius corresponding to a cylinder of equivalent volume.

Some of the tumblers have a significant wall thickness gradient, which is accounted for by averaging the thickness at the rim and at a point near the effective bottom.

The natural frequencies of each glass were measured by recording the sound of it being tapped, then computing the FFT. The first peak corresponded to the (2,0) mode; this was confirmed by observing the spectrum of the tone produced by rubbing a finger around the rim.

Results

The measured frequency of each glass is plotted against the parameter $\gamma = \frac{T}{R^2}\left[1+\frac{4}{3}\left(\frac{R}{H}\right)^4\right]^{1/2}$ from Eq. 1. Figure 2 shows two versions of this plot: in a), γ is computed using the radius at the rim and the height measured by the thickness method; in b), γ is computed using the effective radius and height determined via the water method. In both plots, a theoretical frequency is calculated using a value of 2.4 g/cm^3 for the density of glass and $7 \cdot 10^{11}$ dyne/cm^2 for the Young's modulus.

Two rather surprising results are evident: 1) that most vessel types have natural frequencies that agree quite well with the French model that assumes a cylindrical shape, and 2) those shapes that agree best with the theory are not cylinders. Although the tumblers best fit the simplified model of a cylindrical shell, the predicted frequencies for these glasses are too low by a factor of two or more.

In plot b) the largest effect is seen with the brandy glasses, which have the most dramatic variation of radius with height. The position of the brandy glass frequencies in plot b) is shifted upward, confirming our hypothesis that these glasses would exhibit a higher natural frequency than that of corresponding cylindrical glasses (indicated by the theory points), as well as those of standard wine and flute glasses; however, the tumblers and martini glasses do not conform to expectations.

One possible explanation for the lack of agreement between predicted and measured frequencies of tumblers is that the effective height for these glasses is much smaller (by a factor of two) than that supposed.

Conclusion

Comparison of measured natural frequencies of variously shaped glass vessels with the theory derived for cylindrical vessels indicates that wall curvature plays a small role in determining the frequency of the lowest (2,0) mode, although bulbous (brandy) glasses show a measurably higher frequency. Tumblers do not behave as predicted; an explanation may be found by precisely determining the value of effective height via measurements of vibration amplitude.

REFERENCES

1. Rayleigh (J. W. Strutt), *The Theory of Sound*, reprinted by Dover, New York, 1945, Vol. 1, 383-432.
2. T. D. Rossing, *American Scientist* **72**, 440-447 (1984).
3. T. D. Rossing and R. Perrin,*Applied Acoustics* **20**, 41-70 (1987).
4. A. P. French, *Am. J. Phys.*, **51**, No. 8, 688-694 (1983).

Vibrations and Sound Quality of C*utaway* guitars

P. Ricciardi

D.I.T.E.C.,Università di Genova, Via all'Opera Pia 15A, 16145 Genova, Italy

Among all the different characteristics of guitars, such as various height of bindings, number of strings, shape and dimension of the sound-hole, configuration of braces, the *cutaway* guitars present peculiar and relevant acoustical not yet fully analyzed. Every school of luthery produce singular shapes of the *cutaway* guitar, characterized by different approach to music which can often lead to the variance between a rich and poor quality of the musical instrument.

In this paper a cutaway guitar has been compared with a classical one. The acoustical and vibrational properties of the two instruments have been studied.

Differences in sound quality and vibration behavior have been pointed out, and compared with subjective evaluation given by performers.

THEORY

Considering the sound of a string instrument that reaches the ear, the main part of the acoustic propagation is coming from the sound chest. The conversion of mechanical energy, coming from an excitation point and going toward a receiving point, into a different form of energy (mainly heat) is called damping.

The damping phenomenon has been analyzed starting from the basic equation of elasticity (Hooke law) . The relation between stress and strain should be related to the "strain history", superposed linear [1, 2, 3, 4], i.e.:

$$\sigma(t) = D_1 \varepsilon(t) - \int_0^{\infty} \varepsilon(t - \Delta t) \cdot \varphi(\Delta t) \cdot d(\Delta t) \qquad (1)$$

with the positions of sinusoidal time-variations and the stress-strain, relation (1) becomes:

$$\sigma(t) = D_1 \tilde{\varepsilon} \cos(\omega t) - \frac{D_2}{\tau} \tilde{\varepsilon} \int_0^{\infty} \cos\omega(t - \Delta t) \cdot e^{-\Delta t/\tau} d(\Delta t) \qquad (2)$$

The equation (2) underlines the phase-shift between stress and strain. In order to simplify this relation, it is possible to rewrite the complex modulus of elasticity [3]:

$$D = D' + jD'' = D'(1 + j\eta) \qquad (3)$$

and to define the parameter *loss factor* as:

$$\eta = \frac{D''}{D'} \qquad (4)$$

From an energetic point of view, a different expression for the parameter can be obtained. The loss factor can be described as the ratio between the quantity of energy that is lost and the whole reversible mechanical energy [5]:

$$\eta = \frac{E_l}{2\pi E_R} \qquad (5)$$

Considering the energy converted into heat in the time *t*:

$$E_l(t) = \int_0^{t} \left(E_{R0}(t) - E_l(t)\right) \eta\omega \, dt \qquad (6)$$

and differentiating eq. (6) with respect to time, the following equation can be determined as:

$$E_l(t) = E_{R0}\left(1 - e^{-\eta\omega t}\right) \quad \Rightarrow \quad E_{RT_{60}} = E_{R0} e^{-\eta\omega T_{60}} \qquad (7)$$

that leads to the equation:

$$\eta = \frac{\ln(10^6)}{2\pi f \cdot T_{60}} \qquad (8)$$

Considering the last equation (8), the experimental measurements have been carried out by getting the *structural reverberation time* in 26 points of the sound board, and calculating the corresponding loss factor in one-third octave band, for the two different guitars

EXPERIMENTAL MEASUREMENTS

Vibrational measurements in two guitars (namely Yamaha LD-10E and LW-5C) were performed. A miniaturized impedance-head hammer (B&K 8202) and accelerometers B & K 4398) were used. The output signal was directly stored in a PC by meaning of SPDIF connectors. During the post-processing analysis, the *structural RT* was obtained and filtered for each frequency. Finally, Loss Factor was calculated. The following pictures represent the different results obtained at 1 kHz.

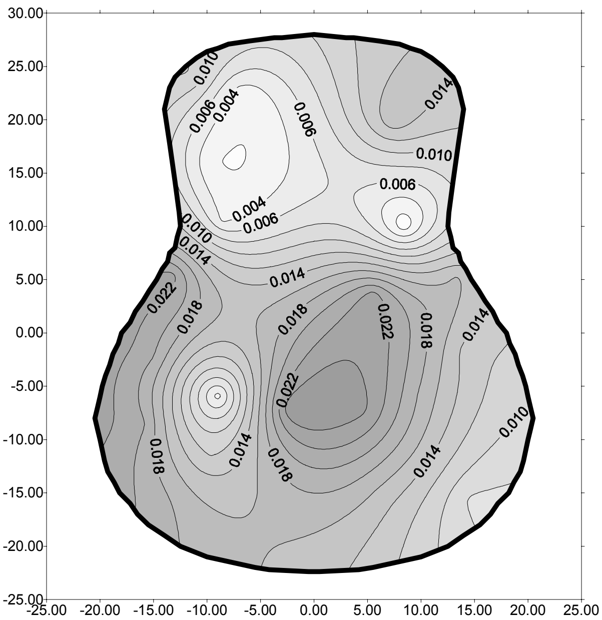

FIGURE 1. Loss Factor measured for classical guitars (Yamaha LD-10E) at 1kHz

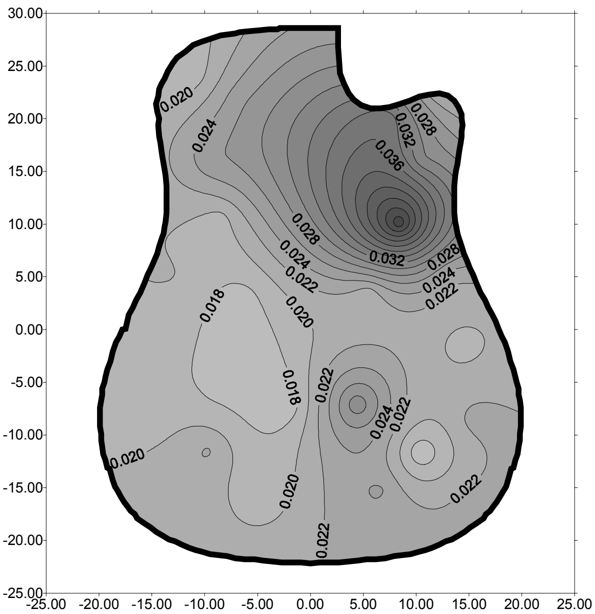

FIGURE 2. Loss Factor measured for cutaway guitars (Yamaha LW-5C) at 1kHz.

DISCUSSION

The force propagation in the soundboard appeared very different for the two guitars. The magnitude of loss factor decreased for the classical guitar, whilst the *cutaway* one had higher values of damping. The phenomenon is accentuated in the top of the body, especially in the right side. Also the distribution of damping is quite different between the two instruments. While the normal guitar was more “reverberant” in the upper zone of the soundboard, the *cutaway* guitar “sounded” more in the lower area.

CONCLUSIONS

Experimental measurements put in evidence the acoustic differences, among the two guitars especially in the timber, due to higher values of damping in the cutaway guitars.

These variances pointed out the necessity of further experimental researches, such as acoustical radiation and modal analysis, as suggested by Suzuki [3]. In this way a more specific correlation between sound radiation and vibrational behavior of the sound-chest could be underlined.

Furthermore, the applied experimental techniques could also be employed for a virtual reconstruction of the sound quality in these musical instruments [6].

REFERENCES

1. L. Cremer, M. Heckl, *Structure-Borne Sound*, Springer-Verlag, New York

2 H. N. Fletcher, T. D. Rossing: *The Physics of Musical Instruments*, Springer-Verlag, 1991 New York.

3 H. Suzuki: *Vibration and sound radiation of a piano soundboard*, Journal of Acoustical Society of America, 80, 1986

4. D.J. Ewins: *Modal Testing: theory and Practice*, Research Studies Press, 1984, London

5. L. Tronchin, A. Cocchi: *On the toning of cello: the effect on damping in the sound board* Proc of ISMA International Symposium on Musical Acoustics, Leavenworth, USA, 1998.

6. A. Farina, L. Tronchin: *On the“virtual” reconstruction of sound quality of trumpets*, Acustica United with Acta Acustica, 86(4), 737-745, 2000