8E and 8F: Finding the Probability P(Y==1|X)

8E: Implementing Decision Function of SVM RBF Kernel

After we train a kernel SVM model, we will be getting support vectors and their corresponsing coefficients α_i
Check the documentation for better understanding of these attributes:

https://scikit-learn.org/stable/modules/generated/sklearn.svm.SVC.html (https://scikit-learn.org/stable/modules/generated/sklearn.svm.SVC.html)

Attributes:

support_ : *array-like, shape = [n_SV]*
Indices of support vectors.

support_vectors_ : *array-like, shape = [n_SV, n_features]*
Support vectors.

n_support_ : *array-like, dtype=int32, shape = [n_class]*
Number of support vectors for each class.

dual_coef_ : *array, shape = [n_class-1, n_SV]*
Coefficients of the support vector in the decision function. For multiclass, coefficient for all 1-vs-1 classifiers. The layout of the coefficients in the multiclass case is somewhat non-trivial. See the section about multi-class classification in the SVM section of the User Guide for details.

coef_ : *array, shape = [n_class * (n_class-1) / 2, n_features]*
Weights assigned to the features (coefficients in the primal problem). This is only available in the case of a linear kernel.

coef_ is a readonly property derived from `dual_coef_` and `support_vectors_`.

intercept_ : *array, shape = [n_class * (n_class-1) / 2]*
Constants in decision function.

fit_status_ : *int*
0 if correctly fitted, 1 otherwise (will raise warning)

probA_ : *array, shape = [n_class * (n_class-1) / 2]*

probB_ : *array, shape = [n_class * (n_class-1) / 2]*
If probability=True, the parameters learned in Platt scaling to produce probability estimates from decision values. If probability=False, an empty array. Platt scaling uses the logistic function `1 / (1 + exp(decision_value * probA_ + probB_))` where `probA_` and `probB_` are learned from the dataset [R20c70293ef72-2]. For more information on the multiclass case and training procedure see section 8 of [R20c70293ef72-1].

As a part of this assignment you will be implementing the `decision_function()` of kernel SVM, here decision_function() means based on the value return by `decision_function()` model will classify the data point either as positive or negative

Ex 1: In logistic regression After traning the models with the optimal weights w we get, we will find the value $\frac{1}{1+\exp(-(wx+b))}$, if this value comes out to be < 0.5 we will mark it as negative class, else its positive class

Ex 2: In Linear SVM After traning the models with the optimal weights w we get, we will find the value of $sign(wx + b)$, if this value comes out to be -ve we will mark it as negative class, else its positive class.

Similarly in Kernel SVM After traning the models with the coefficients α_i we get, we will find the value of $sign(\sum_{i=1}^{n}(y_i\alpha_i K(x_i, x_q)) + intercept)$, here $K(x_i, x_q)$ is the RBF kernel. If this value comes out to be -ve we will mark x_q as negative class, else its positive class.

RBF kernel is defined as: $K(x_i, x_q)$ = $exp(-\gamma||x_i - x_q||^2)$

For better understanding check this link: https://scikit-learn.org/stable/modules/svm.html#svm-mathematical-formulation (https://scikit-learn.org/stable/modules/svm.html#svm-mathematical-formulation)

Task E

1. Split the data into X_{train}(60), X_{cv}(20), X_{test}(20)
2. Train $SVC(gamma = 0.001, C = 100.)$ on the (X_{train}, y_{train})
3. Get the decision boundry values f_{cv} on the X_{cv} data i.e. f_{cv} = `decision_function(`X_{cv}`)` you need to implement this decision_function()

In [1]:

```
import numpy as np
import pandas as pd
from sklearn.datasets import make_classification
import numpy as np
from sklearn.svm import SVC
```

In [2]:

```
X, y = make_classification(n_samples=5000, n_features=5, n_redundant=2,
                           n_classes=2, weights=[0.7], class_sep=0.7, random_state=15)
```

Splitting the data into Train ,CV and Test sets:

In [3]:

```
from sklearn.model_selection import train_test_split
X_train,X_test,y_train,y_test= train_test_split(X,y,test_size=0.2,stratify=y, random_state=15)
X_train,X_cv,y_train,y_cv = train_test_split(X_train,y_train,test_size=0.25,stratify=y_train, random_state=42)
print(X_train.shape,y_train.shape)
print(X_cv.shape,y_cv.shape)
print(X_test.shape,y_test.shape)
```

```
(3000, 5) (3000,)
(1000, 5) (1000,)
(1000, 5) (1000,)
```

Implementing SVC and finding the decision boundary:

In [4]:

```
model=SVC(gamma=0.001,C=100,kernel='rbf',random_state=15)
model.fit(X_train,y_train)
```

Out[4]:

```
SVC(C=100, gamma=0.001, random_state=15)
```

In [5]:

```
f_cv=model.decision_function(X_cv)
print(f_cv)
```

```
[ 1.80695751e+00  1.74878058e+00 -2.79214542e+00 -1.97298263e+00
  5.15429922e-01 -1.66912828e+00  2.70427940e+00 -3.57264780e+00
  1.74853092e+00  1.69490108e+00  1.84770514e+00 -1.76684121e+00
  1.27521004e+00 -2.90892641e+00 -7.14224742e-01 -4.14731562e+00
  4.67040993e-01  1.38753847e+00 -1.60363554e+00 -2.97884988e+00
 -3.67451776e+00 -8.89530860e-01 -2.86929846e+00 -9.16129354e-01
  1.41778659e+00  1.72776123e+00 -2.08462515e+00 -2.17691643e+00
  1.72860604e+00  2.21212969e+00 -3.10565637e+00 -2.14039812e+00
 -3.86256365e+00 -3.67452455e+00  8.46438676e-01 -8.45657034e-01
  9.09252205e-01  1.46856799e+00  1.64431643e+00  9.57008630e-01
  1.63135573e+00  1.34595199e+00 -2.17672082e+00 -1.44959809e+00
 -2.55185733e+00 -3.06075772e+00 -3.92572914e+00 -9.83550442e-01
  1.47684491e+00 -2.01481131e+00 -1.35710848e+00  2.22817539e+00
 -1.38113935e+00 -3.23984732e+00 -4.07098022e+00 -2.38985160e+00
 -2.82074476e+00  2.33558494e+00 -2.80342975e+00 -3.29997660e+00
 -7.03276367e-01 -2.60356599e+00  1.42222945e+00 -2.53408632e+00
 -1.98105718e+00 -5.33508275e-01 -2.70513355e+00 -2.09316696e+00
 -1.18606318e+00 -4.01425241e+00 -3.23715011e+00 -8.62010950e-01
 -3.01135166e+00 -8.61628227e-02 -3.41693889e+00 -2.37969523e-01
 -1.16568221e+00 -2.52849767e+00 -2.75132750e+00 -3.17855550e+00
 -3.09405251e+00 -3.07924505e+00 -3.69292416e+00 -2.15130435e+00
  1.35161606e+00 -1.04316879e+00  1.59928055e-01 -2.19184414e+00
  3.52625979e-01 -2.75796428e+00  1.88694294e+00 -2.83182344e+00
 -2.44719362e+00 -1.15766164e+00 -1.98387285e+00  7.40806589e-01
 -5.47368051e-01 -3.45842681e+00 -1.75856942e+00  2.95547317e+00
 -2.47805777e+00  6.50577837e-01  2.55194543e-01 -4.19139208e+00
 -2.58036654e+00  3.81034830e+00 -1.88197828e-01 -1.55203681e+00
 -3.85996975e+00 -2.18007760e+00 -3.23137733e+00 -2.31488967e+00
  1.16160889e+00 -3.72589957e-01 -8.37305041e-01  1.61314807e+00
 -3.10927396e+00  1.59097033e+00 -1.85823503e-01  2.02624468e+00
```

```
-2.56200104e+00 -1.98619184e+00 -2.90086662e+00 -2.08450564e+00
-2.83332230e+00 -2.35514782e+00 -2.35412585e+00  3.97479134e-01
 1.48705498e+00 -1.47760559e+00 -2.79443160e+00  3.32343606e+00
-2.86623256e+00 -2.32081211e+00 -1.90424561e+00 -2.10781364e-01
-3.90985358e+00 -4.59477213e+00  1.99647803e+00 -1.71836037e+00
 1.52943327e-02 -1.11959511e+00  1.47863991e+00  2.27521980e+00
-2.12797162e+00  1.59461867e+00 -2.31327622e+00  1.96223238e+00
-5.28339297e-01 -4.47816884e-01  1.05578724e+00 -2.78045681e+00
-2.73214940e+00 -7.16138943e-02 -2.90565159e+00 -2.50443046e+00
-2.95981921e+00 -2.79781479e+00 -1.83576833e+00 -3.19310912e+00
 1.82464350e+00 -4.52054191e+00 -8.33424486e-01 -1.90934401e+00
-3.67087683e+00 -1.45465408e+00 -2.56071904e+00  6.87377163e-01
 1.92831992e+00 -1.73663105e+00 -1.49779120e+00  1.66945041e+00
-1.60849958e+00 -4.01619885e+00  2.11606156e+00 -1.88072155e+00
-2.39937127e+00  1.46538200e+00 -2.91266332e+00 -2.82632194e+00
-2.82427583e+00 -4.00697542e+00 -1.58308455e+00 -1.50238040e+00
-3.91455376e+00  2.67052002e+00 -3.83383048e-01 -2.49786348e-01
-2.01322092e+00 -3.15834828e+00  2.54473306e+00  1.95643725e+00
-2.98068399e+00 -1.66542727e+00 -3.48761775e+00  2.74319689e-01
-3.65458212e+00 -2.21007292e+00  1.89121470e+00 -3.61497483e-01
-1.77264306e+00 -2.72170791e+00 -2.46214979e+00 -2.90133625e+00
-1.05095431e+00 -2.43275058e+00 -2.07902700e+00 -2.57332239e+00
 9.03304324e-01  5.74194897e-01 -3.02721672e+00  1.54109116e+00
-1.84042644e+00 -5.74352962e-01 -2.66367959e+00  8.16237416e-01
-2.54755864e+00 -2.09170729e+00  1.26943213e+00 -2.97639307e+00
 9.39834168e-01 -1.80718224e-01 -1.46808742e+00  7.69891248e-01
-2.37750010e-01  1.00635558e+00 -1.66009545e+00  5.67136692e-01
-7.93313366e-01 -2.86802835e+00  1.14640495e+00  8.56847630e-01
-2.62017410e+00  2.59562881e-01  1.73574091e+00 -1.32647546e+00
-3.32222247e+00  1.31475782e+00 -3.16135645e+00 -2.50920161e+00
 1.73172951e+00  1.85795258e+00 -4.75951905e-01 -3.98035171e+00
-2.46195015e+00 -2.36405354e+00  9.08920324e-01 -2.59929534e+00
-4.68964923e-01  3.18143482e+00  1.63939266e+00 -2.14540129e+00
 1.89322383e+00 -3.26242111e+00 -5.00544964e-02  2.49621472e+00
-8.48177045e-01 -3.84268998e+00 -2.90397860e+00 -3.33692303e+00
-2.37085623e+00 -2.40483925e+00 -1.88403255e+00 -3.15395542e+00
-1.94627245e+00 -1.20884803e+00 -1.81517472e+00  2.65491088e+00
 8.71209708e-01 -2.80535798e+00 -2.40014112e+00 -1.07065999e+00
-4.61270778e-01 -3.70941377e+00  2.32160036e+00  2.22841317e+00
-3.63711436e+00 -1.98129873e+00 -2.91131084e+00 -1.65991329e+00
-7.43954430e-01  4.80534181e-01 -3.22642544e+00 -2.10548228e+00
 1.35996662e+00 -1.98188356e+00 -1.22677118e+00  1.57569631e+00
-3.27510373e+00 -6.07467799e-01  2.28937043e+00 -2.67621107e+00
-1.80710820e+00  2.39550410e+00  1.38230631e+00  1.44659073e+00
-2.72616905e+00 -3.01267074e+00 -1.84721157e-01 -1.59669799e+00
-2.56753047e+00 -2.31781303e+00 -2.92803938e+00  1.19516265e+00
-2.45082212e+00 -3.78831516e-01 -2.53593188e+00 -1.54738726e+00
 1.99697042e+00 -2.98917134e+00 -3.38888438e+00 -3.04871305e+00
-2.13232274e+00  2.09371784e+00 -2.74383720e+00 -1.93564337e+00
 1.47053982e+00  2.13390996e+00 -3.27951723e+00 -2.28463528e+00
-2.99590754e+00 -2.77573757e+00 -3.64216859e+00  1.65496029e-01
-2.61222313e+00  2.55088865e+00 -1.28797783e+00 -2.08023166e+00
 3.50348056e+00 -4.12166908e+00 -2.35478196e+00 -5.90985587e-01
-3.93889437e+00 -3.00560584e+00  2.95047373e+00 -1.59370727e+00
-1.71865956e+00 -2.93736007e+00  2.03224795e+00  1.90030892e+00
-3.60032581e+00 -2.43596502e+00 -3.31270459e+00  2.23967084e+00
-1.56753330e+00 -3.08349875e+00 -2.43314782e+00 -1.77798579e+00
-3.47458731e-02 -2.00508183e+00 -3.00179367e+00 -2.89574913e+00
-5.77428588e-01  1.07507444e+00 -2.17629405e+00 -3.42542327e-01
-5.81841990e-01 -2.68094172e+00 -2.05908648e+00 -3.12725836e+00
 1.77717685e+00 -2.19986119e+00  1.52058775e+00 -2.26421456e+00
 2.65710194e-01  1.64006424e+00 -3.36035206e+00 -1.02299517e-01
 3.26328157e-01 -2.48051958e+00 -8.68341622e-02 -1.95343482e+00
-3.20787385e+00 -2.99275566e+00 -4.45610450e-01 -2.98547867e+00
 1.03777368e+00 -2.23433448e+00 -7.43326221e-01 -2.14289351e+00
-2.79770749e+00  2.70785678e-01 -2.68864564e+00  1.57562908e+00
-2.72591515e+00 -3.29132253e+00 -2.59463819e+00 -3.42866075e+00
-2.35198352e+00 -2.23418751e+00  1.68175294e+00 -6.62918338e-01
-2.04918438e+00  2.05948717e+00  1.66222172e+00 -2.29060172e+00
-2.45306380e+00 -9.80761501e-01 -1.99693536e+00  1.72895412e+00
 2.06541859e+00 -1.28594129e+00 -3.37239024e+00 -4.22756418e+00
-6.88675927e-01 -4.28483264e+00  2.27665319e+00  8.63727183e-01
-1.17091996e+00  2.15782033e+00 -3.05895221e+00  1.83136494e+00
-1.10346520e+00 -2.25901522e+00 -3.08443897e+00 -1.65473550e+00
 1.51518802e+00  1.96252638e+00 -3.95609162e+00 -2.99508598e+00
```

```
-4.38313542e-01 -1.31280926e+00 -2.18861797e+00 -4.27758049e+00
-2.61080641e+00  1.13854900e-01 -3.09120284e+00 -2.87075145e+00
 1.98091500e+00 -2.14785389e+00 -3.65480899e+00 -1.74486267e+00
-3.55208745e+00 -2.85026702e+00 -8.82628327e-01 -1.83057984e+00
-2.70861880e+00  2.16575521e+00 -3.10901439e+00 -1.19624665e+00
-1.32950950e+00 -6.08246518e-02 -2.44507867e+00  1.66837720e+00
-2.58183040e+00 -2.49931676e+00 -1.88438662e+00 -2.02774652e+00
 1.71756054e+00 -6.61005955e-01 -3.75526800e-01 -1.28049995e+00
 7.91887539e-01  2.30500533e+00  5.70830325e-01 -1.33596429e+00
-2.68449728e+00  1.66290028e+00 -4.05607446e+00 -8.82914889e-01
-1.60580466e+00 -5.29847519e-01  1.73476745e+00  4.07702360e-01
-3.17181495e+00 -1.07887771e+00  1.99206862e+00 -7.75884234e-01
-2.00697085e+00 -3.62708836e+00 -2.13745401e+00 -1.67684796e+00
-3.38707120e+00 -1.85349650e-01 -1.83123745e+00  1.32696361e+00
 1.74859125e+00  2.21083428e+00  2.05228161e+00  1.48025033e+00
 2.11934517e+00 -3.10556462e+00 -2.59426867e+00  1.78608880e+00
-4.51163007e+00  1.19107427e+00 -3.18411131e+00 -3.38178254e+00
-3.70086376e+00  1.55507641e+00  2.14480947e+00 -3.11862374e+00
-2.61890394e+00 -2.40604208e+00  1.90590447e+00 -2.31467674e+00
-4.17197212e+00  1.67897470e+00 -1.56065634e-01  2.55302666e+00
-1.73979400e+00  3.48892680e-01  1.77385143e+00 -3.96056969e+00
-2.16807635e+00  1.20496211e+00 -1.15219577e+00 -2.01392140e+00
-2.44430394e+00  5.19477468e-01  1.30821073e+00 -3.23801970e+00
-1.63291902e-01 -4.75696500e+00  1.36124303e+00 -2.77021470e+00
-8.12778231e-01 -3.08247138e+00 -3.60270802e+00  1.66353457e+00
 2.63551619e+00 -4.26777970e+00  3.23762010e+00  1.83404227e+00
-2.85589065e+00 -2.36579874e+00  1.00298660e+00  9.29298059e-01
 2.16477000e+00 -6.52482072e-01  1.52486193e+00 -2.72639299e+00
-2.83842205e+00 -3.84184160e+00  9.83969726e-01 -4.83040307e-04
-2.07021700e+00  1.60860521e+00  1.38841990e+00  1.08202163e+00
-4.32892878e-01 -1.10092831e+00  2.38369766e+00  4.66570835e-01
 1.42893884e+00 -2.30456965e+00  8.60150966e-01 -2.85024379e+00
-4.25005399e+00  3.22000158e-01 -2.48069464e+00 -2.85134255e+00
-4.61982519e-01 -7.97160149e-01 -3.09997839e+00 -1.08743111e+00
 1.84684936e+00 -4.75387148e-01  1.61715312e+00 -3.10519114e+00
-1.74773606e+00 -3.38672653e+00 -3.85162602e+00 -2.41005950e+00
 1.75359718e+00 -2.95297593e+00 -3.00524604e+00  1.82981604e+00
-1.80071845e+00  3.74081621e-01  1.51407850e+00 -1.49967939e+00
 1.12640151e+00  1.92409292e-01 -2.92227970e+00 -1.47040148e+00
-2.45341842e+00 -1.42238828e+00 -8.72684415e-01 -3.88945818e+00
-1.04730058e+00  1.44775312e+00 -2.21572880e+00 -2.37326962e+00
-3.27560050e+00  6.47831910e-01  2.74316035e+00 -1.96526673e+00
-2.61430229e+00 -1.74162557e+00  3.06476412e+00  2.47769130e+00
-2.49596971e+00 -3.67394664e-01  1.72575752e+00 -1.93891065e+00
-2.62463332e+00 -3.13519468e+00 -2.86467915e+00 -1.71799849e+00
-2.89693868e+00 -3.69205391e+00 -1.26580419e+00 -4.90431993e-02
 7.97344970e-01  1.86445268e+00 -3.53086342e+00 -1.94875771e+00
-3.03020337e+00 -1.21053151e+00 -2.42602050e+00 -3.48061200e+00
-9.71310572e-01 -1.72444625e+00  1.84584074e+00 -6.92944902e-01
 2.00155633e+00 -8.56784553e-01 -4.41830714e+00 -4.35131416e+00
-3.79976229e+00 -1.49695885e+00 -4.97231853e-01 -4.01191735e+00
 1.84939310e+00 -3.91319616e-01 -6.54356754e-01 -5.50940014e-01
-3.04605122e+00 -2.68921679e+00 -2.74381163e+00  9.60732080e-01
-2.60582870e+00  1.67500513e+00 -3.95580949e+00 -7.21936304e-02
-2.31254708e+00  2.01912384e+00 -4.20220232e+00 -4.17518222e+00
-5.76676512e-02 -2.65099228e+00 -1.43403907e+00 -3.76710677e+00
 2.25459460e+00 -3.76466165e+00  1.54123759e+00  1.98924452e+00
-3.05872696e+00 -2.42069339e+00 -6.36702087e-01  1.95880792e+00
-1.08680901e+00 -1.36008705e+00 -2.46129741e+00 -4.65696430e+00
-3.05361081e+00 -2.45720962e+00 -2.37441593e+00 -1.68564248e+00
-3.34996121e+00 -2.02517476e+00 -3.38998533e-01  7.44240573e-01
-3.10414683e-01  1.64509528e+00 -2.18510100e+00 -3.23873171e+00
-6.20153336e-01  1.37630842e+00  1.84911741e+00 -3.32319529e+00
-3.37210448e+00 -3.95779727e+00 -2.48070880e+00 -2.54480178e+00
 1.92681122e+00 -2.47227287e+00 -4.54979072e-01 -2.32171325e+00
-2.62822162e+00  2.09422876e+00 -2.44730710e+00 -1.92883050e+00
-2.69270825e+00  1.94900553e+00 -1.39132637e+00  1.84888760e+00
 2.44775276e+00 -4.04295429e+00 -1.02040774e+00 -1.91315833e+00
-2.49693493e+00 -2.67676006e+00  3.19870834e+00 -2.12953913e+00
-2.04528154e+00 -3.25607324e+00 -3.13190132e+00 -2.08988902e+00
-2.88658788e+00 -2.40179162e+00 -3.31006984e+00 -2.34489064e+00
-2.96637176e+00 -3.20360960e+00 -4.04588716e+00 -1.29252008e+00
 1.27762193e+00 -1.10996406e+00 -2.28599439e+00 -1.74256019e+00
-3.22614962e+00  1.79682045e+00  1.54673855e+00 -2.80482025e+00
-1.32699309e+00 -2.56071649e+00 -6.86701216e-01 -2.65713567e+00
```

```
 -3.72588557e+00 -1.44892443e+00 -4.10999524e+00 -3.46671509e+00
  1.78476206e+00 -1.38543413e+00  2.40575715e+00 -1.66162658e+00
 -2.07164517e+00  1.78575176e+00 -3.10309606e+00 -1.96555903e+00
 -3.34718526e+00 -1.95125910e+00 -3.52921576e+00  1.13215285e+00
 -3.04664067e+00 -1.23855320e+00 -2.55784284e+00  2.41525846e+00
 -1.51580089e+00 -3.30817902e+00 -2.90889562e+00  9.72226500e-01
 -1.72145144e-01 -3.40941168e+00 -7.78266313e-01 -2.41333706e+00
  8.82819900e-01  7.41124335e-01 -3.24194820e+00  1.10646588e+00
  2.43521650e+00 -3.35045532e+00 -3.91221722e+00 -2.22276286e+00
 -3.51671677e+00 -3.14933463e+00 -8.22258621e-01 -2.28865005e+00
 -2.42390445e+00  1.79193216e+00  9.73286981e-02  8.03610196e-02
 -3.30085125e+00 -1.06641964e+00  5.88436968e-01 -2.75256886e+00
  4.98187538e-01  2.38273588e+00 -6.91682936e-01  1.83970793e+00
 -9.05701609e-01 -3.46625240e+00 -4.02321484e-01  1.78578074e+00
 -3.25978423e+00 -2.81872714e+00 -9.46512643e-01 -1.27623866e+00
 -3.44082541e+00 -2.66086385e-01  9.97361824e-02 -2.61908593e+00
 -3.19521891e-01 -1.22642009e+00 -1.77724435e-01 -4.90566389e+00
 -3.55183820e+00  3.72858901e+00  1.80519381e+00 -3.83580875e+00
 -2.57410013e+00  1.68384516e+00 -3.19983249e+00 -1.25852179e+00
 -2.48604726e+00 -1.32150979e+00  3.20613452e-01  2.14187431e+00
 -6.10643312e-01  1.17349944e+00  1.48837717e+00 -2.48848565e+00
 -2.91121161e+00  5.72515819e-01  7.82268490e-01 -2.28271440e+00
  1.35763066e+00 -1.64305359e+00 -2.16045463e+00 -2.38336626e+00
 -8.93619076e-01 -1.84877906e+00 -2.22833647e+00 -3.61555076e+00
 -1.55315823e+00 -4.78589280e+00 -3.56775155e+00  1.33561055e+00
 -9.13793818e-01  4.96498365e-01  2.43586081e+00 -1.44276817e+00
 -3.52877942e+00  3.34007036e+00  2.21313825e+00 -3.10824957e+00
 -3.83290735e+00  1.83569007e+00  1.70587918e+00 -2.77572943e+00
 -7.63089433e-01  1.85185192e+00 -2.40576715e+00 -2.88324189e+00
 -3.92993891e+00  1.17694102e+00  1.86708156e+00 -3.04813609e-01
 -2.71775056e+00 -2.99584828e+00  4.11515309e-01  1.34924484e-01
  2.58773754e-01 -1.86646713e+00 -1.44120427e+00 -2.50027155e+00
 -7.81363631e-01 -2.79217728e+00  1.78662522e+00  1.74748266e+00
  1.65661575e+00 -1.84732032e+00 -2.56503347e+00 -3.54750239e+00
 -1.51603679e+00 -4.20839569e+00 -2.42565431e+00 -2.54143792e+00
  7.66925311e-01 -2.39462759e+00  3.12074155e+00  1.77099714e+00
 -8.39551926e-01  1.64333994e+00 -6.55886672e-01 -2.47957629e+00
 -1.84846359e+00 -3.28301205e+00 -2.83883321e+00 -2.55349856e-02
 -4.06938971e+00 -3.80319304e+00  1.66354724e+00 -1.71499457e+00
 -2.17089641e+00  1.87223592e+00  1.31095174e+00 -1.57857952e+00
  2.04237853e+00  1.92914859e+00  1.88554744e+00  1.49642308e+00
 -3.06070098e+00  3.75344409e-02  1.90214814e+00  1.88582200e+00
 -2.97982095e+00 -7.56299548e-01 -2.35670010e+00 -1.59428634e-02
 -2.67557040e+00 -2.96158573e+00 -1.84634701e+00 -3.27448068e+00
 -3.47907354e+00  1.70339542e+00 -2.86614883e+00  3.02102499e+00
  3.41542788e-01 -6.58876412e-01  1.95511623e+00 -2.97809103e+00
 -2.18419486e+00 -2.46493839e+00 -2.21617769e+00 -2.61491989e+00
 -3.33164726e+00  1.45348095e+00  7.37492697e-01 -2.82915944e+00
 -1.53311590e+00 -2.84607838e+00 -2.61721066e+00  1.95257405e+00
  1.80896800e+00  1.79098431e+00  2.02926963e+00 -1.41543702e+00
 -3.72527775e+00 -1.50871363e+00  1.97327807e+00 -3.24398411e+00
 -2.47661040e+00 -4.35912657e-01  2.18123251e+00 -2.26900180e+00
 -3.20001564e+00 -2.87496556e+00 -4.67708399e+00 -1.64169163e+00
 -3.11721120e+00 -2.00213387e-01  1.69548706e+00 -1.55420872e+00
 -3.49368836e-01 -4.10033574e+00 -1.03407476e+00 -1.86014981e-01
 -8.56241668e-01 -2.88690954e+00  2.30255804e+00 -1.92365372e+00
 -2.59614060e+00 -2.91316966e+00 -2.58455009e+00 -2.07081901e+00
 -2.25266258e+00 -3.37148497e+00  2.32443268e-02 -2.82932671e+00
 -8.35765619e-01 -2.93356300e+00  1.75645090e+00 -3.70777205e+00
 -6.99109257e-02 -3.76597041e+00 -1.02215643e+00 -2.23706648e+00
 -2.50668983e+00 -7.11609215e-01  1.27902797e+00  1.63913181e+00
 -2.43268929e+00 -4.06788525e+00  2.02726377e+00 -2.95050934e+00
 -3.06143935e+00 -1.50629848e+00 -1.98394237e+00 -1.35128150e+00
 -2.52628409e+00 -3.38617478e+00  2.17599546e+00  1.56731126e+00
 -3.18694519e+00 -2.19986701e+00 -3.59317886e+00 -2.40373973e+00
 -2.67274345e+00 -3.41444738e+00 -1.72379521e+00  5.16997667e-01
  1.89098687e+00 -1.80324619e+00  1.32391233e+00 -1.06262998e+00
 -2.83482745e+00  6.29620859e-01 -4.00640112e+00 -3.14761329e+00
  2.02867886e+00 -3.99739550e+00 -1.32595110e+00 -1.60190261e+00
 -1.81864356e-02 -3.30534690e+00 -2.69560577e+00 -3.02408799e+00]
```

Pseudo code

clf = SVC(gamma=0.001, C=100.)
clf.fit(Xtrain, ytrain)

def decision_function(Xcv, ...): #use appropriate parameters
for a data point x_q in Xcv:
#write code to implement $(\sum_{i=1}^{\text{all the support vectors}}(y_i\alpha_iK(x_i, x_q)) + intercept)$, here the values y_i, α_i, and $intercept$ can be obtained from the trained model
return *# the decision_function output for all the data points in the Xcv*

fcv = decision_function(Xcv, ...) *# based on your requirement you can pass any other parameters*

Note: Make sure the values you get as fcv, should be equal to outputs of clf.decision_function(Xcv)

In [6]:

```
# you can write your code here
def decision_function(X_cv):
    alpha=model.dual_coef_[0]
    decision_boundary=[]
    for Xq in X_cv:
        sum1 = model.intercept_[0]
        for i,sup_vec in enumerate(model.support_vectors_):
            norm = np.linalg.norm(sup_vec - Xq)**2
            rbf = np.exp(-0.001*norm)
            sum1 += (alpha[i]*rbf)
        decision_boundary.append(sum1)
    return np.array(decision_boundary)
```

In [7]:

```
f_cv=decision_function(X_cv)
f_cv
```

Out[7]:

```
array([ 1.80695751e+00,  1.74878058e+00, -2.79214542e+00, -1.97298263e+00,
        5.15429922e-01, -1.66912828e+00,  2.70427940e+00, -3.57264780e+00,
        1.74853092e+00,  1.69490108e+00,  1.84770514e+00, -1.76684121e+00,
        1.27521004e+00, -2.90892641e+00, -7.14224742e-01, -4.14731562e+00,
        4.67040993e-01,  1.38753847e+00, -1.60363554e+00, -2.97884988e+00,
       -3.67451776e+00, -8.89530860e-01, -2.86929846e+00, -9.16129354e-01,
        1.41778659e+00,  1.72776123e+00, -2.08462515e+00, -2.17691643e+00,
        1.72860604e+00,  2.21212969e+00, -3.10565637e+00, -2.14039812e+00,
       -3.86256365e+00, -3.67452455e+00,  8.46438676e-01, -8.45657034e-01,
        9.09252205e-01,  1.46856799e+00,  1.64431643e+00,  9.57008630e-01,
        1.63135573e+00,  1.34595199e+00, -2.17672082e+00, -1.44959809e+00,
       -2.55185733e+00, -3.06075772e+00, -3.92572914e+00, -9.83550442e-01,
        1.47684491e+00, -2.01481131e+00, -1.35710848e+00,  2.22817539e+00,
       -1.38113935e+00, -3.23984732e+00, -4.07098022e+00, -2.38985160e+00,
       -2.82074476e+00,  2.33558494e+00, -2.80342975e+00, -3.29997660e+00,
       -7.03276367e-01, -2.60356599e+00,  1.42222945e+00, -2.53408632e+00,
       -1.98105718e+00, -5.33508275e-01, -2.70513355e+00, -2.09316696e+00,
       -1.18606318e+00, -4.01425241e+00, -3.23715011e+00, -8.62010950e-01,
       -3.01135166e+00, -8.61628227e-02, -3.41693889e+00, -2.37969523e-01,
       -1.16568221e+00, -2.52849767e+00, -2.75132750e+00, -3.17855550e+00,
       -3.09405251e+00, -3.07924505e+00, -3.69292416e+00, -2.15130435e+00,
        1.35161606e+00, -1.04316879e+00,  1.59928055e-01, -2.19184414e+00,
        3.52625979e-01, -2.75796428e+00,  1.88694294e+00, -2.83182344e+00,
       -2.44719362e+00, -1.15766164e+00, -1.98387285e+00,  7.40806589e-01,
       -5.47368051e-01, -3.45842681e+00, -1.75856942e+00,  2.95547317e+00,
       -2.47805777e+00,  6.50577837e-01,  2.55194543e-01, -4.19139208e+00,
       -2.58036654e+00,  3.81034830e+00, -1.88197828e-01, -1.55203681e+00,
       -3.85996975e+00, -2.18007760e+00, -3.23137733e+00, -2.31488967e+00,
        1.16160889e+00, -3.72589957e-01, -8.37305041e-01,  1.61314807e+00,
       -3.10927396e+00,  1.59097033e+00, -1.85823503e-01,  2.02624468e+00,
       -2.56200104e+00, -1.98619184e+00, -2.90086662e+00, -2.08450564e+00,
       -2.83332230e+00, -2.35514782e+00, -2.35412585e+00,  3.97479134e-01,
        1.48705498e+00, -1.47760559e+00, -2.79443160e+00,  3.32343606e+00,
       -2.86623256e+00, -2.32081211e+00, -1.90424561e+00, -2.10781364e-01,
       -3.90985358e+00, -4.59477213e+00,  1.99647803e+00, -1.71836037e+00,
        1.52943327e-02, -1.11959511e+00,  1.47863991e+00,  2.27521980e+00,
       -2.12797162e+00,  1.59461867e+00, -2.31327622e+00,  1.96223238e+00,
```

```
-5.28339297e-01, -4.47816884e-01,  1.05578724e+00, -2.78045681e+00,

-2.73214940e+00, -7.16138943e-02, -2.90565159e+00, -2.50443046e+00,
-2.95981921e+00, -2.79781479e+00, -1.83576833e+00, -3.19310912e+00,
 1.82464350e+00, -4.52054191e+00, -8.33424486e-01, -1.90934401e+00,
-3.67087683e+00, -1.45465408e+00, -2.56071904e+00,  6.87377163e-01,
 1.92831992e+00, -1.73663105e+00, -1.49779120e+00,  1.66945041e+00,
-1.60849958e+00, -4.01619885e+00,  2.11606156e+00, -1.88072155e+00,
-2.39937127e+00,  1.46538200e+00, -2.91266332e+00, -2.82632194e+00,
-2.82427583e+00, -4.00697542e+00, -1.58308455e+00, -1.50238040e+00,
-3.91455376e+00,  2.67052002e+00, -3.83383048e-01, -2.49786348e-01,
-2.01322092e+00, -3.15834828e+00,  2.54473306e+00,  1.95643725e+00,
-2.98068399e+00, -1.66542727e+00, -3.48761775e+00,  2.74319689e-01,
-3.65458212e+00, -2.21007292e+00,  1.89121470e+00, -3.61497483e-01,
-1.77264306e+00, -2.72170791e+00, -2.46214979e+00, -2.90133625e+00,
-1.05095431e+00, -2.43275058e+00, -2.07902700e+00, -2.57332239e+00,
 9.03304324e-01,  5.74194897e-01, -3.02721672e+00,  1.54109116e+00,
-1.84042644e+00, -5.74352962e-01, -2.66367959e+00,  8.16237416e-01,
-2.54755864e+00, -2.09170729e+00,  1.26943213e+00, -2.97639307e+00,
 9.39834168e-01, -1.80718224e-01, -1.46808742e+00,  7.69891248e-01,
-2.37750010e-01,  1.00635558e+00, -1.66009545e+00,  5.67136692e-01,
-7.93313366e-01, -2.86802835e+00,  1.14640495e+00,  8.56847630e-01,
-2.62017410e+00,  2.59562881e-01,  1.73574091e+00, -1.32647546e+00,
-3.32222247e+00,  1.31475782e+00, -3.16135645e+00, -2.50920161e+00,
 1.73172951e+00,  1.85795258e+00, -4.75951905e-01, -3.98035171e+00,
-2.46195015e+00, -2.36405354e+00,  9.08920324e-01, -2.59929534e+00,
-4.68964923e-01,  3.18143482e+00,  1.63939266e+00, -2.14540129e+00,
 1.89322383e+00, -3.26242111e+00, -5.00544964e-02,  2.49621472e+00,
-8.48177045e-01, -3.84268998e+00, -2.90397860e+00, -3.33692303e+00,
-2.37085623e+00, -2.40483925e+00, -1.88403255e+00, -3.15395542e+00,
-1.94627245e+00, -1.20884803e+00, -1.81517472e+00,  2.65491088e+00,
 8.71209708e-01, -2.80535798e+00, -2.40014112e+00, -1.07065999e+00,
-4.61270778e-01, -3.70941377e+00,  2.32160036e+00,  2.22841317e+00,
-3.63711436e+00, -1.98129873e+00, -2.91131084e+00, -1.65991329e+00,
-7.43954430e-01,  4.80534181e-01, -3.22642544e+00, -2.10548228e+00,
 1.35996662e+00, -1.98188356e+00, -1.22677118e+00,  1.57569631e+00,
-3.27510373e+00, -6.07467799e-01,  2.28937043e+00, -2.67621107e+00,
-1.80710820e+00,  2.39550410e+00,  1.38230631e+00,  1.44659073e+00,
-2.72616905e+00, -3.01267074e+00, -1.84721157e-01, -1.59669799e+00,
-2.56753047e+00, -2.31781303e+00, -2.92803938e+00,  1.19516265e+00,
-2.45082212e+00, -3.78831516e-01, -2.53593188e+00, -1.54738726e+00,
 1.99697042e+00, -2.98917134e+00, -3.38888438e+00, -3.04871305e+00,
-2.13232274e+00,  2.09371784e+00, -2.74383720e+00, -1.93564337e+00,
 1.47053982e+00,  2.13390996e+00, -3.27951723e+00, -2.28463528e+00,
-2.99590754e+00, -2.77573757e+00, -3.64216859e+00,  1.65496029e-01,
-2.61222313e+00,  2.55088865e+00, -1.28797783e+00, -2.08023166e+00,
 3.50348056e+00, -4.12166908e+00, -2.35478196e+00, -5.90985587e-01,
-3.93889437e+00, -3.00560584e+00,  2.95047373e+00, -1.59370727e+00,
-1.71865956e+00, -2.93736007e+00,  2.03224795e+00,  1.90030892e+00,
-3.60032581e+00, -2.43596502e+00, -3.31270459e+00,  2.23967084e+00,
-1.56753330e+00, -3.08349875e+00, -2.43314782e+00, -1.77798579e+00,
-3.47458731e-02, -2.00508183e+00, -3.00179367e+00, -2.89574913e+00,
-5.77428588e-01,  1.07507444e+00, -2.17629405e+00, -3.42542327e-01,
-5.81841990e-01, -2.68094172e+00, -2.05908648e+00, -3.12725836e+00,
 1.77717685e+00, -2.19986119e+00,  1.52058775e+00, -2.26421456e+00,
 2.65710194e-01,  1.64006424e+00, -3.36035206e+00, -1.02299517e-01,
 3.26328157e-01, -2.48051958e+00, -8.68341622e-02, -1.95343482e+00,
-3.20787385e+00, -2.99275566e+00, -4.45610450e-01, -2.98547867e+00,
 1.03777368e+00, -2.23433448e+00, -7.43326221e-01, -2.14289351e+00,
-2.79770749e+00,  2.70785678e-01, -2.68864564e+00,  1.57562908e+00,
-2.72591515e+00, -3.29132253e+00, -2.59463819e+00, -3.42866075e+00,
-2.35198352e+00, -2.23418751e+00,  1.68175294e+00, -6.62918338e-01,
-2.04918438e+00,  2.05948717e+00,  1.66222172e+00, -2.29060172e+00,
-2.45306380e+00, -9.80761501e-01, -1.99693536e+00,  1.72895412e+00,
 2.06541859e+00, -1.28594129e+00, -3.37239024e+00, -4.22756418e+00,
-6.88675927e-01, -4.28483264e+00,  2.27665319e+00,  8.63727183e-01,
-1.17091996e+00,  2.15782033e+00, -3.05895221e+00,  1.83136494e+00,
-1.10346520e+00, -2.25901522e+00, -3.08443897e+00, -1.65473550e+00,
 1.51518802e+00,  1.96252638e+00, -3.95609162e+00, -2.99508598e+00,
-4.38313542e-01, -1.31280926e+00, -2.18861797e+00, -4.27758049e+00,
-2.61080641e+00,  1.13854900e-01, -3.09120284e+00, -2.87075145e+00,
 1.98091500e+00, -2.14785389e+00, -3.65480899e+00, -1.74486267e+00,
-3.55208745e+00, -2.85026702e+00, -8.82628327e-01, -1.83057984e+00,
-2.70861880e+00,  2.16575521e+00, -3.10901439e+00, -1.19624665e+00,
-1.32950950e+00, -6.08246518e-02, -2.44507867e+00,  1.66837720e+00,
-2.58183040e+00, -2.49931676e+00, -1.88438662e+00, -2.02774652e+00,
```

```
 1.71756054e+00, -6.61005955e-01, -3.75526800e-01, -1.28049995e+00,
 7.91887539e-01,  2.30500533e+00,  5.70830325e-01, -1.33596429e+00,
-2.68449728e+00,  1.66290028e+00, -4.05607446e+00, -8.82914889e-01,
-1.60580466e+00, -5.29847519e-01,  1.73476745e+00,  4.07702360e-01,
-3.17181495e+00, -1.07887771e+00,  1.99206862e+00, -7.75884234e-01,
-2.00697085e+00, -3.62708836e+00, -2.13745401e+00, -1.67684796e+00,
-3.38707120e+00, -1.85349650e-01, -1.83123745e+00,  1.32696361e+00,
 1.74859125e+00,  2.21083428e+00,  2.05228161e+00,  1.48025033e+00,
 2.11934517e+00, -3.10556462e+00, -2.59426867e+00,  1.78608880e+00,
-4.51163007e+00,  1.19107427e+00, -3.18411131e+00, -3.38178254e+00,
-3.70086376e+00,  1.55507641e+00,  2.14480947e+00, -3.11862374e+00,
-2.61890394e+00, -2.40604208e+00,  1.90590447e+00, -2.31467674e+00,
-4.17197212e+00,  1.67897470e+00, -1.56065634e-01,  2.55302666e+00,
-1.73979400e+00,  3.48892680e-01,  1.77385143e+00, -3.96056969e+00,
-2.16807635e+00,  1.20496211e+00, -1.15219577e+00, -2.01392140e+00,
-2.44430394e+00,  5.19477468e-01,  1.30821073e+00, -3.23801970e+00,
-1.63291902e-01, -4.75696500e+00,  1.36124303e+00, -2.77021470e+00,
-8.12778231e-01, -3.08247138e+00, -3.60270802e+00,  1.66353457e+00,
 2.63551619e+00, -4.26777970e+00,  3.23762010e+00,  1.83404227e+00,
-2.85589065e+00, -2.36579874e+00,  1.00298660e+00,  9.29298059e-01,
 2.16477000e+00, -6.52482072e-01,  1.52486193e+00, -2.72639299e+00,
-2.83842205e+00, -3.84184160e+00,  9.83969726e-01, -4.83040312e-04,
-2.07021700e+00,  1.60860521e+00,  1.38841990e+00,  1.08202163e+00,
-4.32892878e-01, -1.10092831e+00,  2.38369766e+00,  4.66570835e-01,
 1.42893884e+00, -2.30456965e+00,  8.60150966e-01, -2.85024379e+00,
-4.25005399e+00,  3.22000158e-01, -2.48069464e+00, -2.85134255e+00,
-4.61982519e-01, -7.97160149e-01, -3.09997839e+00, -1.08743111e+00,
 1.84684936e+00, -4.75387148e-01,  1.61715312e+00, -3.10519114e+00,
-1.74773606e+00, -3.38672653e+00, -3.85162602e+00, -2.41005950e+00,
 1.75359718e+00, -2.95297593e+00, -3.00524604e+00,  1.82981604e+00,
-1.80071845e+00,  3.74081621e-01,  1.51407850e+00, -1.49967939e+00,
 1.12640151e+00,  1.92409292e-01, -2.92227970e+00, -1.47040148e+00,
-2.45341842e+00, -1.42238828e+00, -8.72684415e-01, -3.88945818e+00,
-1.04730058e+00,  1.44775312e+00, -2.21572880e+00, -2.37326962e+00,
-3.27560050e+00,  6.47831910e-01,  2.74316035e+00, -1.96526673e+00,
-2.61430229e+00, -1.74162557e+00,  3.06476412e+00,  2.47769130e+00,
-2.49596971e+00, -3.67394664e-01,  1.72575752e+00, -1.93891065e+00,
-2.62463332e+00, -3.13519468e+00, -2.86467915e+00, -1.71799849e+00,
-2.89693868e+00, -3.69205391e+00, -1.26580419e+00, -4.90431993e-02,
 7.97344970e-01,  1.86445268e+00, -3.53086342e+00, -1.94875771e+00,
-3.03020337e+00, -1.21053151e+00, -2.42602050e+00, -3.48061200e+00,
-9.71310572e-01, -1.72444625e+00,  1.84584074e+00, -6.92944902e-01,
 2.00155633e+00, -8.56784553e-01, -4.41830714e+00, -4.35131416e+00,
-3.79976229e+00, -1.49695885e+00, -4.97231853e-01, -4.01191735e+00,
 1.84939310e+00, -3.91319616e-01, -6.54356754e-01, -5.50940014e-01,
-3.04605122e+00, -2.68921679e+00, -2.74381163e+00,  9.60732080e-01,
-2.60582870e+00,  1.67500513e+00, -3.95580949e+00, -7.21936304e-02,
-2.31254708e+00,  2.01912384e+00, -4.20220232e+00, -4.17518222e+00,
-5.76676512e-02, -2.65099228e+00, -1.43403907e+00, -3.76710677e+00,
 2.25459460e+00, -3.76466165e+00,  1.54123759e+00,  1.98924452e+00,
-3.05872696e+00, -2.42069339e+00, -6.36702087e-01,  1.95880792e+00,
-1.08680901e+00, -1.36008705e+00, -2.46129741e+00, -4.65696430e+00,
-3.05361081e+00, -2.45720962e+00, -2.37441593e+00, -1.68564248e+00,
-3.34996121e+00, -2.02517476e+00, -3.38998533e-01,  7.44240573e-01,
-3.10414683e-01,  1.64509528e+00, -2.18510100e+00, -3.23873171e+00,
-6.20153336e-01,  1.37630842e+00,  1.84911741e+00, -3.32319529e+00,
-3.37210448e+00, -3.95779727e+00, -2.48070880e+00, -2.54480178e+00,
 1.92681122e+00, -2.47227287e+00, -4.54979072e-01, -2.32171325e+00,
-2.62822162e+00,  2.09422876e+00, -2.44730710e+00, -1.92883050e+00,
-2.69270825e+00,  1.94900553e+00, -1.39132637e+00,  1.84888760e+00,
 2.44775276e+00, -4.04295429e+00, -1.02040774e+00, -1.91315833e+00,
-2.49693493e+00, -2.67676006e+00,  3.19870834e+00, -2.12953913e+00,
-2.04528154e+00, -3.25607324e+00, -3.13190132e+00, -2.08988902e+00,
-2.88658788e+00, -2.40179162e+00, -3.31006984e+00, -2.34489064e+00,
-2.96637176e+00, -3.20360960e+00, -4.04588716e+00, -1.29252008e+00,
 1.27762193e+00, -1.10996406e+00, -2.28599439e+00, -1.74256019e+00,
-3.22614962e+00,  1.79682045e+00,  1.54673855e+00, -2.80482025e+00,
-1.32699309e+00, -2.56071649e+00, -6.86701216e-01, -2.65713567e+00,
-3.72588557e+00, -1.44892443e+00, -4.10999524e+00, -3.46671509e+00,
 1.78476206e+00, -1.38543413e+00,  2.40575715e+00, -1.66162658e+00,
-2.07164517e+00,  1.78575176e+00, -3.10309606e+00, -1.96555903e+00,
-3.34718526e+00, -1.95125910e+00, -3.52921576e+00,  1.13215285e+00,
-3.04664067e+00, -1.23855320e+00, -2.55784284e+00,  2.41525846e+00,
-1.51580089e+00, -3.30817902e+00, -2.90889562e+00,  9.72226500e-01,
-1.72145144e-01, -3.40941168e+00, -7.78266313e-01, -2.41333706e+00,
```

```
 8.82819900e-01,  7.41124335e-01, -3.24194820e+00,  1.10646588e+00,
 2.43521650e+00, -3.35045532e+00, -3.91221722e+00, -2.22276286e+00,
-3.51671677e+00, -3.14933463e+00, -8.22258621e-01, -2.28865005e+00,
-2.42390445e+00,  1.79193216e+00,  9.73286981e-02,  8.03610196e-02,
-3.30085125e+00, -1.06641964e+00,  5.88436968e-01, -2.75256886e+00,
 4.98187538e-01,  2.38273588e+00, -6.91682936e-01,  1.83970793e+00,
-9.05701609e-01, -3.46625240e+00, -4.02321484e-01,  1.78578074e+00,
-3.25978423e+00, -2.81872714e+00, -9.46512643e-01, -1.27623866e+00,
-3.44082541e+00, -2.66086385e-01,  9.97361824e-02, -2.61908593e+00,
-3.19521891e-01, -1.22642009e+00, -1.77724435e-01, -4.90566389e+00,
-3.55183820e+00,  3.72858901e+00,  1.80519381e+00, -3.83580875e+00,
-2.57410013e+00,  1.68384516e+00, -3.19983249e+00, -1.25852179e+00,
-2.48604726e+00, -1.32150979e+00,  3.20613452e-01,  2.14187431e+00,
-6.10643312e-01,  1.17349944e+00,  1.48837717e+00, -2.48848565e+00,
-2.91121161e+00,  5.72515819e-01,  7.82268490e-01, -2.28271440e+00,
 1.35763066e+00, -1.64305359e+00, -2.16045463e+00, -2.38336626e+00,
-8.93619076e-01, -1.84877906e+00, -2.22833647e+00, -3.61555076e+00,
-1.55315823e+00, -4.78589280e+00, -3.56775155e+00,  1.33561055e+00,
-9.13793818e-01,  4.96498365e-01,  2.43586081e+00, -1.44276817e+00,
-3.52877942e+00,  3.34007036e+00,  2.21313825e+00, -3.10824957e+00,
-3.83290735e+00,  1.83569007e+00,  1.70587918e+00, -2.77572943e+00,
-7.63089433e-01,  1.85185192e+00, -2.40576715e+00, -2.88324189e+00,
-3.92993891e+00,  1.17694102e+00,  1.86708156e+00, -3.04813609e-01,
-2.71775056e+00, -2.99584828e+00,  4.11515309e-01,  1.34924484e-01,
 2.58773754e-01, -1.86646713e+00, -1.44120427e+00, -2.50027155e+00,
-7.81363631e-01, -2.79217728e+00,  1.78662522e+00,  1.74748266e+00,
 1.65661575e+00, -1.84732032e+00, -2.56503347e+00, -3.54750239e+00,
-1.51603679e+00, -4.20839569e+00, -2.42565431e+00, -2.54143792e+00,
 7.66925311e-01, -2.39462759e+00,  3.12074155e+00,  1.77099714e+00,
-8.39551926e-01,  1.64333994e+00, -6.55886672e-01, -2.47957629e+00,
-1.84846359e+00, -3.28301205e+00, -2.83883321e+00, -2.55349856e-02,
-4.06938971e+00, -3.80319304e+00,  1.66354724e+00, -1.71499457e+00,
-2.17089641e+00,  1.87223592e+00,  1.31095174e+00, -1.57857952e+00,
 2.04237853e+00,  1.92914859e+00,  1.88554744e+00,  1.49642308e+00,
-3.06070098e+00,  3.75344409e-02,  1.90214814e+00,  1.88582200e+00,
-2.97982095e+00, -7.56299548e-01, -2.35670010e+00, -1.59428634e-02,
-2.67557040e+00, -2.96158573e+00, -1.84634701e+00, -3.27448068e+00,
-3.47907354e+00,  1.70339542e+00, -2.86614883e+00,  3.02102499e+00,
 3.41542788e-01, -6.58876412e-01,  1.95511623e+00, -2.97809103e+00,
-2.18419486e+00, -2.46493839e+00, -2.21617769e+00, -2.61491989e+00,
-3.33164726e+00,  1.45348095e+00,  7.37492697e-01, -2.82915944e+00,
-1.53311590e+00, -2.84607838e+00, -2.61721066e+00,  1.95257405e+00,
 1.80896800e+00,  1.79098431e+00,  2.02926963e+00, -1.41543702e+00,
-3.72527775e+00, -1.50871363e+00,  1.97327807e+00, -3.24398411e+00,
-2.47661040e+00, -4.35912657e-01,  2.18123251e+00, -2.26900180e+00,
-3.20001564e+00, -2.87496556e+00, -4.67708399e+00, -1.64169163e+00,
-3.11721120e+00, -2.00213387e-01,  1.69548706e+00, -1.55420872e+00,
-3.49368836e-01, -4.10033574e+00, -1.03407476e+00, -1.86014981e-01,
-8.56241668e-01, -2.88690954e+00,  2.30255804e+00, -1.92365372e+00,
-2.59614060e+00, -2.91316966e+00, -2.58455009e+00, -2.07081901e+00,
-2.25266258e+00, -3.37148497e+00,  2.32443268e-02, -2.82932671e+00,
-8.35765619e-01, -2.93356300e+00,  1.75645090e+00, -3.70777205e+00,
-6.99109257e-02, -3.76597041e+00, -1.02215643e+00, -2.23706648e+00,
-2.50668983e+00, -7.11609215e-01,  1.27902797e+00,  1.63913181e+00,
-2.43268929e+00, -4.06788525e+00,  2.02726377e+00, -2.95050934e+00,
-3.06143935e+00, -1.50629848e+00, -1.98394237e+00, -1.35128150e+00,
-2.52628409e+00, -3.38617478e+00,  2.17599546e+00,  1.56731126e+00,
-3.18694519e+00, -2.19986701e+00, -3.59317886e+00, -2.40373973e+00,
-2.67274345e+00, -3.41444738e+00, -1.72379521e+00,  5.16997667e-01,
 1.89098687e+00, -1.80324619e+00,  1.32391233e+00, -1.06262998e+00,
-2.83482745e+00,  6.29620859e-01, -4.00640112e+00, -3.14761329e+00,
 2.02867886e+00, -3.99739550e+00, -1.32595110e+00, -1.60190261e+00,
-1.81864356e-02, -3.30534690e+00, -2.69560577e+00, -3.02408799e+00])
```

8F: Implementing Platt Scaling to find P(Y==1|X)

Check this PDF (https://drive.google.com/open?id=133odBinMOIVb_rh_GQxxsyMRyW-Zts7a)

Let the output of a learning method be $f(x)$. To get calibrated probabilities, pass the output through a sigmoid:

$$P(y = 1|f) = \frac{1}{1 + exp(Af + B)} \quad (1)$$

where the parameters A and B are fitted using maximum likelihood estimation from a fitting training set (f_i, y_i). Gradient descent is used to find A and B such that they are the solution to:

$$\underset{A,B}{argmin}\{-\sum_i y_i log(p_i) + (1 - y_i) log(1 - p_i)\}, \quad (2)$$

where

$$p_i = \frac{1}{1 + exp(Af_i + B)} \quad (3)$$

Two questions arise: where does the sigmoid train set come from? and how to avoid overfitting to this training set?

If we use the same data set that was used to train the model we want to calibrate, we introduce unwanted bias. For example, if the model learns to discriminate the train set perfectly and orders all the negative examples before the positive examples, then the sigmoid transformation will output just a 0,1 function. So we need to use an independent calibration set in order to get good posterior probabilities. This, however, is not a draw back, since the same set can be used for model and parameter selection.

To avoid overfitting to the sigmoid train set, an out-of-sample model is used. If there are N_+ positive examples and N_- negative examples in the train set, for each training example Platt Calibration uses target values y_+ and y_- (instead of 1 and 0, respectively), where

$$y_+ = \frac{N_+ + 1}{N_+ + 2};\ y_- = \frac{1}{N_- + 2} \quad (4)$$

For a more detailed treatment, and a justification of these particular target values see (Platt, 1999).

TASK F

1. Apply SGD algorithm with (f_{cv}, y_{cv}) and find the weight W intercept b `Note: here our data is of one dimensional so we will have a one dimensional weight vector i.e W.shape (1,)`

Note1: Don't forget to change the values of y_{cv} as mentioned in the above image. you will calculate y+, y- based on data points in train data

Note2: the Sklearn's SGD algorithm doesn't support the real valued outputs, you need to use the code that was done in the `'Logistic Regression with SGD and L2'` Assignment after modifying loss function, and use same parameters that used in that assignment.

```
def log_loss(w, b, X, Y):
    N = len(X)
    sum_log = 0
    for i in range(N):
        sum_log += Y[i]*np.log10(sig(w, X[i], b)) + (1-Y[i])*np.log10(1-sig(w, X[i], b))
    return -1*sum_log/N
```

if Y[i] is 1, it will be replaced with y+ value else it will replaced with y- value

1. For a given data point from X_{test}, $P(Y=1|X) = \frac{1}{1+exp(-(W*f_{test}+b))}$ where f_{test} = `decision_function(`X_{test}`)`, W and b will be learned as metioned in the above step

Note: in the above algorithm, the steps 2, 4 might need hyper parameter tuning, To reduce the complexity of the assignment we are excluding the hyerparameter tuning part, but intrested students can try that

If any one wants to try other calibration algorithm istonic regression also please check these tutorials

1. http://fa.bianp.net/blog/tag/scikit-learn.html#fn:1 (http://fa.bianp.net/blog/tag/scikit-learn.html#fn:1)
2. https://drive.google.com/open?id=1MzmA7QaP58RDzocB0RBmRiWfl7Co_VJ7 (https://drive.google.com/open?id=1MzmA7QaP58RDzocB0RBmRiWfl7Co_VJ7)
3. https://drive.google.com/open?id=133odBinMOIVb_rh_GQxxsyMRyW-Zts7a (https://drive.google.com/open?id=133odBinMOIVb_rh_GQxxsyMRyW-Zts7a)
4. https://stat.fandom.com/wiki/Isotonic_regression#Pool_Adjacent_Violators_Algorithm (https://stat.fandom.com/wiki/Isotonic_regression#Pool_Adjacent_Violators_Algorithm)

In [8]:

```
# Storing all the positive examples and negative examples in separate lists
N_p=[]
N_n=[]
for i in y_train:
    if i==1:
        N_p.append(i)
    else:
        N_n.append(i)
print(len(N_p))
print(len(N_n))
```

```
908
2092
```

In [9]:

```
N_p = len(N_p)
N_n = len(N_n)
```

In [10]:

```
# Since Platt calibration uses y+ and y- ,hence we calculate y+ and y-
y_pos= (N_p+1)/(N_p+2)
print(y_pos)
y_neg= 1/(N_n+2)
print(y_neg)
```

```
0.9989010989010989
0.0004775549188156638
```

In [11]:

```python
# replace the values of 1,0 in y_cv dataset with y+ and y-
y_cv_new=[]
for i in y_cv:
    if i==1:
        y_cv_new.append(y_pos)
    elif i!=1:
        y_cv_new.append(y_neg)
y_cv_new
```

Out[11]:

```
[0.9989010989010989,
 0.9989010989010989,
 0.0004775549188156638,
 0.0004775549188156638,
 0.9989010989010989,
 0.0004775549188156638,
 0.9989010989010989,
 0.0004775549188156638,
 0.9989010989010989,
 0.9989010989010989,
 0.9989010989010989,
 0.0004775549188156638,
 0.9989010989010989,
 0.0004775549188156638,
 0.9989010989010989,
 0.0004775549188156638,
 0.0004775549188156638,
 0.9989010989010989,
 0.0004775549188156638,
 0.0004775549188156638,
 0.0004775549188156638,
 0.0004775549188156638,
 0.0004775549188156638,
 0.9989010989010989,
 0.9989010989010989,
 0.9989010989010989,
 0.0004775549188156638,
 0.0004775549188156638,
 0.9989010989010989,
 0.9989010989010989,
 0.0004775549188156638,
 0.0004775549188156638,
 0.9989010989010989,
 0.0004775549188156638,
 0.0004775549188156638,
 0.0004775549188156638,
 0.0004775549188156638,
 0.9989010989010989,
 0.9989010989010989,
 0.9989010989010989,
 0.9989010989010989,
 0.9989010989010989,
 0.0004775549188156638,
 0.0004775549188156638,
 0.0004775549188156638,
 0.0004775549188156638,
 0.0004775549188156638,
 0.9989010989010989,
 0.9989010989010989,
 0.0004775549188156638,
 0.0004775549188156638,
 0.9989010989010989,
 0.0004775549188156638,
 0.0004775549188156638,
 0.0004775549188156638,
 0.0004775549188156638,
 0.0004775549188156638,
 0.9989010989010989,
 0.0004775549188156638,
 0.0004775549188156638,
 0.9989010989010989,
 0.0004775549188156638,
```

```
    0.9989010989010989,
    0.0004775549188156638,
    0.0004775549188156638,
    0.0004775549188156638,
    0.0004775549188156638,
    0.0004775549188156638,
    0.0004775549188156638,
    0.0004775549188156638,
    0.0004775549188156638,
    0.0004775549188156638,
    0.0004775549188156638,
    0.0004775549188156638,
    0.0004775549188156638,
    0.9989010989010989,
    0.0004775549188156638,
    0.0004775549188156638,
    0.0004775549188156638,
    0.0004775549188156638,
    0.0004775549188156638,
    0.0004775549188156638,
    0.0004775549188156638,
    0.0004775549188156638,
    0.9989010989010989,
    0.0004775549188156638,
    0.9989010989010989,
    0.0004775549188156638,
    0.0004775549188156638,
    0.0004775549188156638,
    0.9989010989010989,
    0.0004775549188156638,
    0.0004775549188156638,
    0.0004775549188156638,
    0.0004775549188156638,
    0.9989010989010989,
    0.0004775549188156638,
    0.0004775549188156638,
    0.0004775549188156638,
    0.9989010989010989,
    0.0004775549188156638,
    0.9989010989010989,
    0.9989010989010989,
    0.0004775549188156638,
    0.0004775549188156638,
    0.9989010989010989,
    0.0004775549188156638,
    0.0004775549188156638,
    0.0004775549188156638,
    0.0004775549188156638,
    0.0004775549188156638,
    0.0004775549188156638,
    0.0004775549188156638,
    0.0004775549188156638,
    0.9989010989010989,
    0.9989010989010989,
    0.0004775549188156638,
    0.9989010989010989,
    0.0004775549188156638,
    0.9989010989010989,
    0.0004775549188156638,
    0.0004775549188156638,
    0.0004775549188156638,
    0.0004775549188156638,
    0.0004775549188156638,
    0.0004775549188156638,
    0.0004775549188156638,
    0.0004775549188156638,
    0.9989010989010989,
    0.0004775549188156638,
    0.0004775549188156638,
    0.9989010989010989,
    0.0004775549188156638,
    0.0004775549188156638,
    0.0004775549188156638,
    0.9989010989010989,
    0.0004775549188156638,
```

```
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
```

```
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
```

0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,

0.9989010989010989,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,

```
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.9989010989010989,
0.9989010989010989,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
```

0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,

```
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
```

0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,

0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,

```
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
```

0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.9989010989010989,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.0004775549188156638,
0.9989010989010989,

```
 0.9989010989010989,
 0.9989010989010989,
 0.0004775549188156638,
 0.0004775549188156638,
 0.9989010989010989,
 0.0004775549188156638,
 0.0004775549188156638,
 0.0004775549188156638,
 0.0004775549188156638,
 0.9989010989010989,
 0.0004775549188156638,
 0.0004775549188156638,
 0.9989010989010989,
 0.9989010989010989,
 0.0004775549188156638,
 0.0004775549188156638,
 0.0004775549188156638,
 0.0004775549188156638,
 0.0004775549188156638,
 0.0004775549188156638,
 0.0004775549188156638,
 0.0004775549188156638,
 0.9989010989010989,
 0.0004775549188156638,
 0.9989010989010989,
 0.0004775549188156638,
 0.0004775549188156638,
 0.9989010989010989,
 0.0004775549188156638,
 0.0004775549188156638,
 0.9989010989010989,
 0.0004775549188156638,
 0.0004775549188156638,
 0.0004775549188156638,
 0.9989010989010989,
 0.0004775549188156638,
 0.0004775549188156638,
 0.0004775549188156638]
```

Applying Logistic Regression with SGD and L2 regularization on the newly created dataset:

In [12]:

```
from tqdm import tqdm
import math
from math import log

def initialize_weights(dim):
    dim = X_train[0]
    w = np.zeros((1,))
    b = 0

    return w,b

dim = X_train[0]
w,b = initialize_weights(dim)


def sigmoid(z):

    return 1. / (1 + np. exp(-z))

def logloss(y_true,y_pred):

    loss = -sum(map(lambda y_true, y_pred: y_true*np.log10(y_pred) + (1-y_true)*np.log10(1-y_pred), y_true, y_pred))/len(y
_true)

    return loss

def gradient_dw(x,y,w,b,alpha,N):

    dw = x * (y - sigmoid(np.dot(w, x) + b)) - (alpha / N) * w

    return dw

def gradient_db(x,y,w,b):

    db = y - sigmoid(np.dot(w, x) + b)

    return db

log_loss=[]
epoch_list = []
def train(X_train,y_train):

    w,b = initialize_weights(X_train[0])
    eta0 = 0.0001

    for e in tqdm(range(epochs)):
        y_tr_pred=[]
        y_ts_pred=[]
        for i in range(len(X_train)):

            w = w + (eta0 * gradient_dw(X_train[i], y_train[i], w, b, alpha, N))
            b = b + (eta0 * gradient_db(X_train[i], y_train[i], w, b))

        for k in range(len(X_train)):
            z = np.dot(w,X_train[k])+b
            s = sigmoid(z)
            y_tr_pred.append(s)
        l_tr = logloss(y_train,y_tr_pred)
        log_loss.append(l_tr)


        epoch_list.append(e)

    return w,b
```

In [13]:

```
alpha=0.0001
eta0=0.0001
N=len(f_cv)
epochs=50
w,b=train(f_cv,y_cv_new)
```

```
100%|██████████████████████████████████████████| 50/50 [00:03<00:00, 15.39it/s]
```

In [14]:

```
print('weights:',w)
```

```
weights: [1.17043809]
```

In [15]:

```
print('Intercept:',b)
```

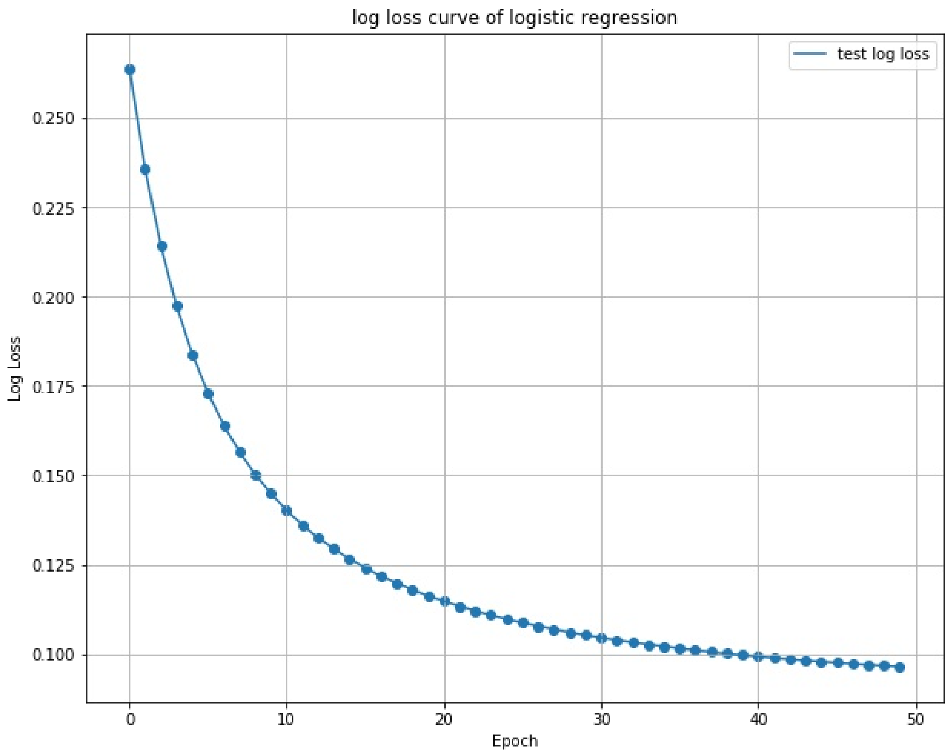
```
Intercept: [-0.16813985]
```

In [17]:

```
import matplotlib.pyplot as plt
plt.figure(figsize=(10,8))
plt.grid()
plt.plot(epoch_list,log_loss,label='test log loss')
plt.scatter(epoch_list,log_loss)
plt.xlabel('Epoch')
plt.ylabel('Log Loss')
plt.title('log loss curve of logistic regression')
plt.legend()
```

Out[17]:

```
<matplotlib.legend.Legend at 0x4943fac8>
```



Probability of Y==1| X :

In [18]:

```
f_test=model.decision_function(X_test)

P = 1/(1 + np.exp(-((w*f_test)+b)))
P
```

Out[18]:

```
array([0.61373282, 0.083824  , 0.01026194, 0.0201278 , 0.04300734,
       0.13706624, 0.10753921, 0.04128707, 0.82334818, 0.8796553 ,
```

```
       0.0879715 , 0.67130637, 0.02507694, 0.54946646, 0.10150171,
       0.05403434, 0.02919601, 0.97026782, 0.32575782, 0.05527751,
       0.14294209, 0.61305754, 0.81932576, 0.01106932, 0.10336141,
       0.75960892, 0.82176401, 0.05428534, 0.89327014, 0.77588384,
       0.10843147, 0.20112698, 0.07699283, 0.00850466, 0.93553815,
       0.00119108, 0.64234382, 0.92265084, 0.12503242, 0.87561775,
       0.02077811, 0.00343432, 0.03339706, 0.00710464, 0.84622414,
       0.01406702, 0.01840955, 0.86314   , 0.53493003, 0.00753556,
       0.02067913, 0.17947669, 0.00534215, 0.10594477, 0.17322029,
       0.04563505, 0.64505748, 0.05236154, 0.0998101 , 0.87926039,
       0.02477943, 0.07123137, 0.05760188, 0.02711193, 0.73650827,
       0.06118594, 0.04442075, 0.26051224, 0.0114734 , 0.05294436,
       0.80644912, 0.01737472, 0.07051795, 0.4543958 , 0.88691692,
       0.03407941, 0.13807607, 0.27608445, 0.08303254, 0.08384852,
       0.12088486, 0.63159357, 0.8778529 , 0.01581789, 0.08122304,
       0.02866465, 0.03083985, 0.13407051, 0.15832889, 0.04431747,
       0.0808517 , 0.40121324, 0.02414473, 0.02764881, 0.03640519,
       0.01115743, 0.53753883, 0.03485408, 0.15241976, 0.02000844,
       0.02788798, 0.0989188 , 0.88497934, 0.01391922, 0.08254807,
       0.10004618, 0.0125541 , 0.33201439, 0.05296177, 0.0125431 ,
       0.56832696, 0.134627  , 0.12251585, 0.71706274, 0.0544485 ,
       0.86549836, 0.07699578, 0.23165994, 0.83236292, 0.73833701,
       0.06036836, 0.01575679, 0.09157605, 0.10715812, 0.02513898,
       0.05108064, 0.00991795, 0.20756258, 0.10599642, 0.02866594,
       0.02969751, 0.67745436, 0.2735089 , 0.00961107, 0.02814355,
       0.01466827, 0.00191579, 0.0499972 , 0.11422128, 0.01921364,
       0.04011552, 0.87728423, 0.11463631, 0.64945732, 0.04120028,
       0.02843118, 0.01654341, 0.10185312, 0.0558587 , 0.029685  ,
       0.23753317, 0.88162944, 0.05128147, 0.00834696, 0.02199902,
       0.01332548, 0.860653  , 0.75177754, 0.80301344, 0.60161531,
       0.07650332, 0.01552989, 0.74069157, 0.08481979, 0.26472119,
       0.43679971, 0.92152037, 0.02903636, 0.02138787, 0.06531383,
       0.01445049, 0.84798667, 0.05408657, 0.04356021, 0.91582503,
       0.00482474, 0.58108621, 0.30567491, 0.04180707, 0.05503049,
       0.06198885, 0.01072638, 0.01297531, 0.10141923, 0.12004177,
       0.80882946, 0.98847331, 0.05688249, 0.0199649 , 0.77223372,
       0.07009877, 0.00924914, 0.02084746, 0.02960841, 0.93970436,
       0.56679241, 0.10443423, 0.05418532, 0.90553174, 0.32912778,
       0.03749585, 0.06035523, 0.00401258, 0.03059063, 0.02905995,
       0.14252933, 0.82304471, 0.01190739, 0.70003913, 0.03407246,
       0.81820366, 0.01019326, 0.02832216, 0.74464483, 0.02452275,
       0.06840074, 0.07438575, 0.05305618, 0.92265487, 0.26749043,
       0.85624311, 0.02025066, 0.55683613, 0.01499396, 0.0390698 ,
       0.75407694, 0.06712991, 0.74029066, 0.03508445, 0.03871936,
       0.19133582, 0.07801272, 0.49945949, 0.04666193, 0.02242184,
       0.0346491 , 0.01658619, 0.84294421, 0.02478641, 0.06322396,
       0.08475079, 0.83700083, 0.13834165, 0.01213889, 0.26041669,
       0.0131428 , 0.94370776, 0.96845704, 0.03234559, 0.08703134,
       0.92317499, 0.31557754, 0.54822151, 0.70028335, 0.02675177,
       0.14431257, 0.87481063, 0.04696766, 0.03518698, 0.00365944,
       0.16614671, 0.85070114, 0.98804381, 0.84585648, 0.00896683,
       0.00659515, 0.83311833, 0.00979529, 0.05087085, 0.05995735,
       0.0133944 , 0.64980002, 0.46422538, 0.93960286, 0.11223736,
       0.04889473, 0.00736508, 0.05490151, 0.01023728, 0.03839259,
       0.85736982, 0.02118203, 0.07765021, 0.86694917, 0.85227692,
       0.0487298 , 0.02680833, 0.90269909, 0.09057846, 0.01296659,
       0.01855171, 0.10018107, 0.1810688 , 0.02890154, 0.04053999,
       0.0407893 , 0.01285482, 0.29821505, 0.0221204 , 0.01819367,
       0.00657151, 0.00930537, 0.67321516, 0.46870978, 0.19223783,
       0.15867441, 0.04111818, 0.81560288, 0.58400007, 0.15019388,
       0.05258054, 0.11472131, 0.04803082, 0.08192604, 0.78305703,
       0.11916573, 0.93377004, 0.93588197, 0.30648825, 0.03770596,
       0.01216878, 0.00289025, 0.29865265, 0.81122478, 0.82294193,
       0.03609172, 0.76990559, 0.20015436, 0.01173903, 0.866921  ,
       0.09475674, 0.11752244, 0.03643039, 0.15154611, 0.72352742,
       0.85731396, 0.02707563, 0.85080043, 0.14922967, 0.00892463,
       0.02859878, 0.45476632, 0.43780064, 0.67787032, 0.91041897,
       0.05411749, 0.88385784, 0.10237982, 0.09832439, 0.02909878,
       0.75343337, 0.87477209, 0.57682147, 0.86207282, 0.94282859,
       0.04130761, 0.62891361, 0.10839812, 0.02744953, 0.04840612,
       0.83518862, 0.2908494 , 0.02970524, 0.47854392, 0.00663878,
       0.80986457, 0.57638528, 0.05586451, 0.70924779, 0.07672175,
       0.02114067, 0.02552807, 0.83073511, 0.33805721, 0.0134964 ,
       0.03911599, 0.20492844, 0.67544356, 0.04583182, 0.03606474,
       0.66356895, 0.69121342, 0.12909581, 0.75367927, 0.01852683,
```

0.06285423, 0.01167119, 0.92689521, 0.0139629 , 0.00721262,
0.03126108, 0.86724507, 0.4081962 , 0.01397131, 0.02408944,
0.16378469, 0.59106453, 0.83389371, 0.85216898, 0.02530465,
0.39975515, 0.02257892, 0.00178086, 0.22302994, 0.01181875,
0.71789074, 0.85451546, 0.81075354, 0.73411174, 0.01653544,
0.80367653, 0.81617066, 0.84364733, 0.09663394, 0.8085376 ,
0.57385121, 0.03729567, 0.10418809, 0.00978792, 0.03220767,
0.10039486, 0.10291093, 0.01988914, 0.07281163, 0.03362814,
0.01772904, 0.8434643 , 0.02428647, 0.86899844, 0.20861993,
0.66125212, 0.01514557, 0.03783981, 0.00795076, 0.02627944,
0.07098217, 0.01791241, 0.97583952, 0.72687032, 0.83130798,
0.30638584, 0.03546572, 0.01596176, 0.33769507, 0.11119377,
0.88543569, 0.67799695, 0.67228252, 0.0204658 , 0.88401515,
0.76986273, 0.51726878, 0.03326926, 0.67362269, 0.02203753,
0.91595835, 0.05653314, 0.03426701, 0.42119527, 0.07220642,
0.0230303 , 0.01682254, 0.44279546, 0.01156586, 0.03663627,
0.05097282, 0.00466443, 0.93237426, 0.74770092, 0.05466941,
0.04406869, 0.06834311, 0.00614723, 0.00347855, 0.08008172,
0.0063033 , 0.1497064 , 0.85468863, 0.36195956, 0.02652642,
0.00711046, 0.4185044 , 0.09273562, 0.02105216, 0.05043185,
0.05953126, 0.02420644, 0.04569261, 0.08601645, 0.98750229,
0.01385769, 0.00396819, 0.90670034, 0.00664491, 0.12331912,
0.86602128, 0.04374819, 0.03255365, 0.39986243, 0.19836494,
0.19383353, 0.45369412, 0.01414319, 0.77059708, 0.74713133,
0.20504584, 0.03316355, 0.88997941, 0.02036535, 0.78654392,
0.11085869, 0.03785381, 0.68860955, 0.20449039, 0.02031442,
0.17692822, 0.06828783, 0.01330811, 0.01919646, 0.87729324,
0.06447299, 0.0095701 , 0.05365441, 0.01337146, 0.080314 ,
0.03661812, 0.88486611, 0.45146491, 0.05104532, 0.01174406,
0.33441839, 0.02550573, 0.01762931, 0.03754855, 0.02782439,
0.01652264, 0.7034246 , 0.7841343 , 0.33963125, 0.31890607,
0.01211796, 0.05247843, 0.05874354, 0.80843464, 0.01243688,
0.66795389, 0.75704442, 0.77246853, 0.02052004, 0.04798232,
0.03164089, 0.02608322, 0.02247239, 0.01513467, 0.11418257,
0.0177113 , 0.06897555, 0.78601805, 0.03375413, 0.09091685,
0.14356049, 0.0036162 , 0.83909819, 0.01330556, 0.12761497,
0.02362731, 0.04858779, 0.82828481, 0.70574066, 0.20029953,
0.0728964 , 0.78456044, 0.11927392, 0.03087993, 0.53407079,
0.04286172, 0.51660593, 0.41224812, 0.04372684, 0.02670464,
0.18665628, 0.81168614, 0.00172947, 0.8432934 , 0.83425558,
0.36767526, 0.01036509, 0.0205045 , 0.11746995, 0.82306636,
0.01689519, 0.01243904, 0.01687553, 0.05785784, 0.93361607,
0.02642336, 0.02073317, 0.01281702, 0.15410424, 0.04769531,
0.23134194, 0.71197539, 0.0155303 , 0.84662498, 0.70561969,
0.85304527, 0.0758185 , 0.10458363, 0.10864419, 0.09209348,
0.20217518, 0.10623649, 0.15856275, 0.82721511, 0.06502615,
0.85942989, 0.04931247, 0.07025873, 0.09171489, 0.06636917,
0.0029788 , 0.00325608, 0.57688999, 0.1287959 , 0.21093858,
0.05577954, 0.70837665, 0.87665746, 0.65824236, 0.84070665,
0.09387643, 0.77431385, 0.82684174, 0.02599378, 0.0898662 ,
0.04627016, 0.00424807, 0.90840218, 0.63626964, 0.78660027,
0.01051739, 0.00229318, 0.04400091, 0.14890417, 0.02517256,
0.1094699 , 0.03912701, 0.09973343, 0.46210841, 0.04492259,
0.80699281, 0.06045612, 0.37689322, 0.05751905, 0.07462271,
0.18722667, 0.90362074, 0.09561408, 0.05320753, 0.03659101,
0.02293996, 0.1932805 , 0.0129314 , 0.04593972, 0.85741421,
0.09140014, 0.96740834, 0.62199857, 0.01266597, 0.91840898,
0.04154792, 0.03555273, 0.87439994, 0.01290608, 0.13750944,
0.02501941, 0.07771332, 0.01504425, 0.0304645 , 0.52265784,
0.00863445, 0.06781499, 0.77281844, 0.00221849, 0.00827765,
0.80190649, 0.80944334, 0.06415884, 0.02645596, 0.21795644,
0.01495632, 0.0143398 , 0.08261772, 0.84847043, 0.01594606,
0.62200472, 0.92681854, 0.3250246 , 0.85605404, 0.88447398,
0.0352783 , 0.33024266, 0.22768107, 0.02605956, 0.04465235,
0.07473111, 0.11319537, 0.02148094, 0.01927137, 0.91206831,
0.83115201, 0.0357506 , 0.01921307, 0.07325103, 0.45562221,
0.8354526 , 0.02050377, 0.60196355, 0.87888541, 0.05104411,
0.04299342, 0.32450039, 0.02216236, 0.03493077, 0.23078301,
0.73412721, 0.02023306, 0.02160694, 0.83075983, 0.06224409,
0.05891814, 0.01765666, 0.01340112, 0.03418851, 0.07867391,
0.058959 , 0.88492332, 0.91977284, 0.02079051, 0.48825868,
0.0451734 , 0.29968051, 0.00675173, 0.02263191, 0.82461973,
0.82643098, 0.06201779, 0.87613803, 0.75853389, 0.81185077,
0.12630753, 0.02278363, 0.10368122, 0.06762058, 0.73428101,
0.04932924, 0.04840009, 0.02046129, 0.12658931, 0.10099032,

```
0.88921451, 0.85719468, 0.87611521, 0.78823237, 0.00763621,
0.73936062, 0.50546144, 0.02007759, 0.68146109, 0.93496044,
0.76533966, 0.8718447 , 0.90199562, 0.90994301, 0.1342743 ,
0.47379867, 0.82086297, 0.10001263, 0.15316418, 0.00992458,
0.02697032, 0.0549621 , 0.47910931, 0.45215745, 0.02111779,
0.00863603, 0.08772819, 0.01378905, 0.0188054 , 0.05845037,
0.04001085, 0.01680849, 0.04819549, 0.7097239 , 0.10341547,
0.03147589, 0.06198132, 0.53030709, 0.77089033, 0.14849879,
0.28752797, 0.0360229 , 0.95850964, 0.02398944, 0.84110048,
0.08356906, 0.95851885, 0.03319909, 0.84384297, 0.61536361,
0.12211973, 0.88190512, 0.31078588, 0.93237191, 0.84468378,
0.86309198, 0.75442153, 0.02388059, 0.22117644, 0.04092229,
0.00604315, 0.72491895, 0.12642453, 0.07795605, 0.83887248,
0.15277248, 0.00144553, 0.89324148, 0.02332781, 0.0283224 ,
0.89933158, 0.11203178, 0.06755565, 0.69449787, 0.01735254,
0.81443955, 0.7600202 , 0.01864715, 0.01627533, 0.25459574,
0.84861561, 0.08868367, 0.75713953, 0.00653839, 0.0298798 ,
0.05423473, 0.53207935, 0.1037727 , 0.05268306, 0.34249949,
0.34171263, 0.09497561, 0.02173253, 0.86859983, 0.56160234,
0.07602486, 0.0304131 , 0.01204996, 0.20031693, 0.86613236,
0.54461924, 0.8602421 , 0.4093606 , 0.87182689, 0.52013618,
0.2960586 , 0.02921523, 0.1448751 , 0.84616979, 0.906945  ,
0.00290239, 0.11483234, 0.0767439 , 0.87542867, 0.918837  ,
0.84379824, 0.02604111, 0.90763042, 0.0327671 , 0.07337643,
0.65862612, 0.21588607, 0.09484564, 0.01038398, 0.0126433 ,
0.00794013, 0.05261474, 0.02240101, 0.94331757, 0.25571085,
0.06578367, 0.17850644, 0.00438951, 0.1994732 , 0.00844204,
0.09793874, 0.03271056, 0.02668556, 0.04854069, 0.77984288,
0.0720821 , 0.40494818, 0.60037158, 0.87396108, 0.89761323,
0.56591465, 0.78824917, 0.01621127, 0.05180765, 0.30559625,
0.04906263, 0.03922173, 0.01323849, 0.02471078, 0.01431633,
0.04832537, 0.00859741, 0.59077604, 0.01862037, 0.04691981,
0.06195326, 0.22272608, 0.05698632, 0.17164748, 0.09733976,
0.87026834, 0.01070659, 0.22205214, 0.12730312, 0.0199461 ,
0.81002658, 0.00571775, 0.76561253, 0.82340909, 0.78033911,
0.90140069, 0.07352872, 0.41088533, 0.84422918, 0.68680786,
0.06162065, 0.0098279 , 0.25353939, 0.01125576, 0.0386589 ,
0.89789359, 0.03612722, 0.03366581, 0.05480104, 0.01644568,
0.08722516, 0.06515379, 0.89104675, 0.94289524, 0.84917116,
0.14365642, 0.08659595, 0.12114735, 0.00701947, 0.10526736,
0.88119097, 0.51682502, 0.09908067, 0.25551129, 0.00843885,
0.74061278, 0.11030239, 0.02168157, 0.02602986, 0.00540139,
0.53933208, 0.07565749, 0.02314103, 0.05009531, 0.2957548 ,
0.07159197, 0.10210613, 0.00467212, 0.07112235, 0.85468476,
0.01154794, 0.00376768, 0.01301161, 0.01511886, 0.16252913,
0.04934219, 0.00860594, 0.07745496, 0.84722987, 0.08974733,
0.05948149, 0.86184531, 0.22186297, 0.03270814, 0.0570508 ,
0.85710252, 0.0592955 , 0.1099722 , 0.06402711, 0.15748488])
```