# Support Vector Machine

Prof. Dr. Christina Bauer

christina.bauer@th-deg.de

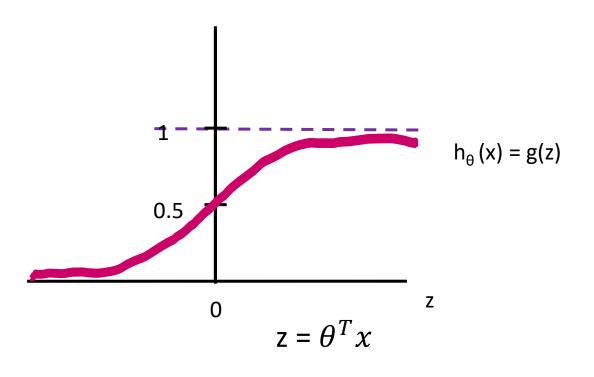
Faculty of Computer Science

#### ALTERNATIVE VIEW OF LOGISTIC REGRESSION

$$h_{\theta}(x) = \frac{1}{1 + e^{-\theta^T x}}$$

If y = 1, we want  $h_{\theta}(x) \sim 1 \rightarrow \theta^T x \gg 0$ 

If y = 0, we want  $h_{\theta}(x) \sim 0 \rightarrow \theta^T x \ll 0$ 



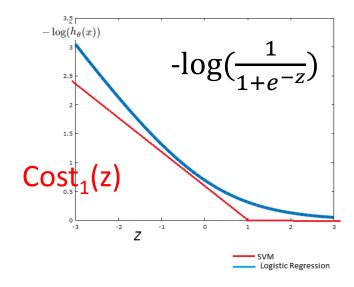
#### ALTERNATIVE VIEW OF LOGISTIC REGRESSION

#### Cost of one example:

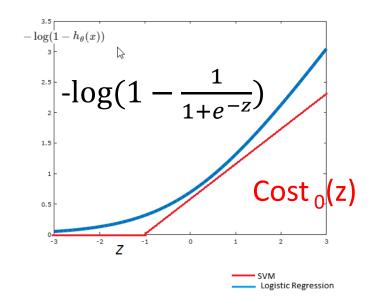
$$-(y \log(h_{\theta}(x)) + (1-y)\log(1-h_{\theta}(x)))$$

= 
$$-y \log(\frac{1}{1+e^{-\theta^T x}}) - (1-y) \log(1 - \frac{1}{1+e^{-\theta^T x}})$$

If 
$$y = 1 \rightarrow [z = \theta^T x] \gg 0$$



If 
$$y = 0 \rightarrow [z = \theta^T x] \ll 0$$



#### SUPPORT VECTOR MACHINE

#### **Cost function Logistic Regression**

$$\min_{\theta} \frac{1}{m} \left[ \sum_{i=1}^{m} y^{(i)} \left( -\log(h_{\theta}(x^{(i)})) + (1-y^{(i)}) \left( -\log(1-h_{\theta}(x^{(i)})) \right) \right] + \frac{\lambda}{2m} \sum_{j=1}^{n} \theta_{j}^{2} \cos_{t_{0}(\theta^{T}x)} \cos_{t_{0}(\theta^{T}x)} \right]$$

#### **Support Vector Machine**

$$\min_{\theta} \frac{1}{m} \left[ \sum_{i=1}^{m} y^{(i)} \operatorname{cost}_{1}(\theta^{T} x) + (1 - y^{(i)}) \operatorname{cost}_{0}(\theta^{T} x) \right] + \frac{\lambda}{2m} \sum_{j=1}^{n} \theta_{j}^{2} \xrightarrow{\text{C instead of } Z} \operatorname{cinstead of } Z$$

$$\min_{\theta} C \sum_{i=1}^{m} \left[ y^{(i)} \operatorname{cost}_{1}(\theta^{T} x) + (1 - y^{(i)}) \operatorname{cost}_{0}(\theta^{T} x) \right] + \frac{1}{2} \sum_{j=1}^{n} \theta_{j}^{2}$$

# QUESTION



Consider the following minimization problems:

1. 
$$\min_{\theta} \frac{1}{m} \left[ \sum_{i=1}^{m} y^{(i)} \cot_{1}(\theta^{T}x) (1-y^{(i)}) \cot_{0}(\theta^{T}x) \right] + \frac{\lambda}{2m} \sum_{j=1}^{n} \theta_{j}^{2}$$
  
2.  $\min_{\theta} C \sum_{i=1}^{m} \left[ y^{(i)} \cot_{1}(\theta^{T}x) (1-y^{(i)}) \cot_{0}(\theta^{T}x) \right] + \frac{1}{2} \sum_{j=1}^{n} \theta_{j}^{2}$ 

2. 
$$\min_{\theta} C \sum_{i=1}^{m} [y^{(i)} \operatorname{cost}_{1}(\theta^{T} x) (1-y^{(i)}) \operatorname{cost}_{0}(\theta^{T} x)] + \frac{1}{2} \sum_{j=1}^{m} \theta_{j}^{2}$$

These two optimization problems will give the same value of  $\theta$  (i.e., the same value of  $\theta$  gives the optimal solution to both problems) if:

A:  $C=\lambda$ 

B:  $C=-\lambda$ 

C:  $C=1/\lambda$ 

D:  $C=2/\lambda$ 

# QUESTION



Consider the following minimization problems:

1. 
$$\min_{\theta} \frac{1}{m} \left[ \sum_{i=1}^{m} y^{(i)} \cot_{1}(\theta^{T}x) (1-y^{(i)}) \cot_{0}(\theta^{T}x) \right] + \frac{\lambda}{2m} \sum_{j=1}^{n} \theta_{j}^{2}$$
  
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2. 
$$\min_{\theta} C \sum_{i=1}^{m} [y^{(i)} \operatorname{cost}_{1}(\theta^{T} x) (1-y^{(i)}) \operatorname{cost}_{0}(\theta^{T} x)] + \frac{1}{2} \sum_{j=1}^{m} \theta_{j}^{2}$$

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B; 
$$C=-\lambda$$

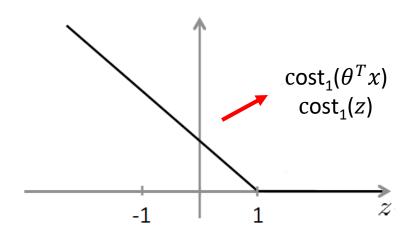
#### **SVM Hypothesis**

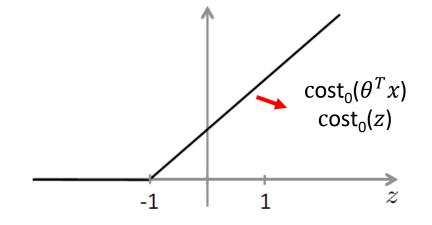
$$\min_{\theta} C \sum_{i=1}^{m} [y^{(i)} \cot_{1}(\theta^{T} x) (1 - y^{(i)}) \cot_{0}(\theta^{T} x)] + \frac{1}{2} \sum_{j=1}^{n} \theta_{j}^{2}$$

$$h_{\theta}(x) - \begin{cases} 1 & \text{if } \theta^T x \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

### SVM – LARGE MARGIN CLASSIFIERS

$$\min_{\theta} C \sum_{i=1}^{m} [y^{(i)} \cot_{1}(\theta^{T} x) (1 - y^{(i)}) \cot_{1}(\theta^{T} x)] + \frac{1}{2} \sum_{j=1}^{n} \theta_{j}^{2}$$





If y = 1, we want 
$$\theta^T x \ge 1$$
 (not just  $\ge 0$ )

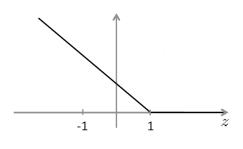
If y = 0, we want 
$$\theta^T x \le -1$$
 (not just < 0)

### SVM — DECISION BOUNDARY

$$\min_{\theta} C \sum_{i=1}^{m} [\underline{y}^{(i)} \cot_{1}(\theta^{T} x) (1 - \underline{y}^{(i)}) \cot_{1}(\theta^{T} x)] + \frac{1}{2} \sum_{j=1}^{n} \theta_{j}^{2}$$

(With C being a large number  $\rightarrow$  first term of function should be near 0)

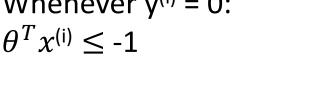
Whenever  $y^{(i)} = 1$ :  $\theta^T x^{(i)} \ge 1$  (not just  $\ge 0$ )

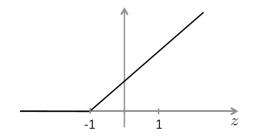


Optimization problem:

$$\min_{\theta} \frac{\mathsf{C*0}}{\mathsf{C}} + \frac{1}{2} \sum_{\mathsf{j}=1}^{n} \theta_{\mathsf{j}}^{2}$$

Whenever  $y^{(i)} = 0$ :

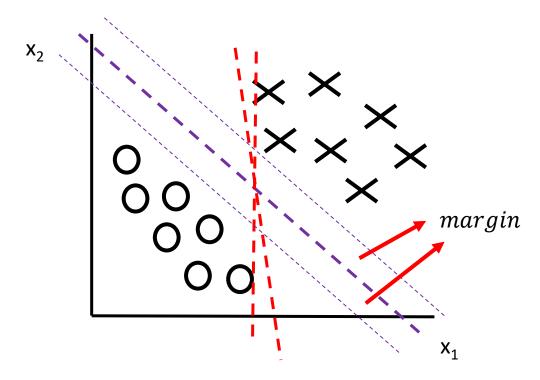




s.t.  

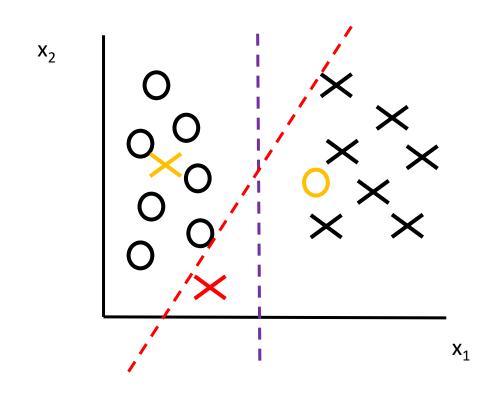
$$\theta^T x^{(i)} \ge 1 \text{ if } y^{(i)} = 1$$
  
 $\theta^T x^{(i)} \le -1 \text{ if } y^{(i)} = 0$ 

# SVM — DECISION BOUNDARY: LINEARLY SEPARABLE



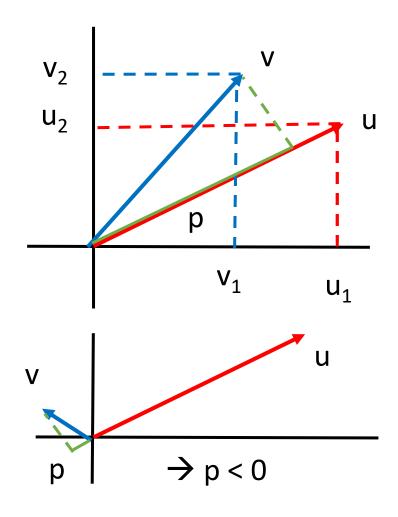
→ Large margin classifier

# SVM — Large Margin classifier in the presence of Outliers



- → C very large → decision boundary will change to red one (good idea?)
- → C not too large → decision boundary will stay at the purple line
- → If C is not too large SVM is more robust to outliers (red point) and if your data is not linearly separable (orange points)

### **VECTOR INNER PRODUCTS**



$$\mathbf{u} = \begin{bmatrix} \mathbf{u}_1 \\ \mathbf{u}_2 \end{bmatrix} \qquad \mathbf{v} = \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{bmatrix}$$

 $\|\mathbf{u}\| = \text{length of vector } \mathbf{u} = \sqrt{u_1^2 + u_2^2} \in \mathbb{R}$ p = length of projection of v onto  $\mathbf{u} \in \mathbb{R}$ 

inner product:  $\mathbf{u}^{\mathsf{T}}\mathbf{v} = \mathbf{v}^{\mathsf{T}}\mathbf{u}$ 

$$u^{T}v = p * ||u||$$
  
 $u^{T}v = u_{1}*v_{1} + u_{2}*v_{2}$ 

# SVM DECISION BOUNDARY

#### Optimization problem:

$$\min_{\theta} \frac{1}{2} \sum_{j=1}^{n} \theta_{j}^{2}$$

$$\Rightarrow \text{Simplification: } \theta_{0} = 0; \text{ n} = 2$$

$$\Rightarrow \frac{1}{2} (\theta_{1}^{2} + \theta_{2}^{2}) \Rightarrow \frac{1}{2} (\sqrt{\theta_{1}^{2} + \theta_{2}^{2}})^{2}$$

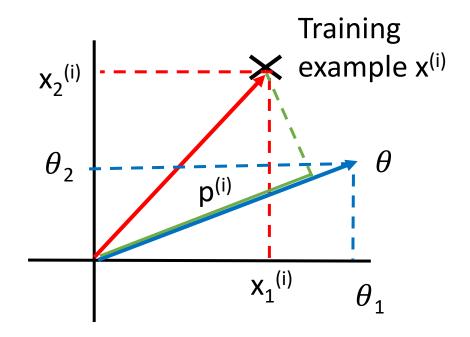
$$\Rightarrow \sqrt[2]{\theta_{1}^{2} + \theta_{2}^{2}} = ||\theta|| \text{ (length of the vector theta)}$$

$$\Rightarrow \frac{1}{2} ||\theta||^{2}$$
s.t.
$$\theta^{T} x^{(i)} \ge 1 \text{ if } y^{(i)} = 1$$

$$\theta^{T} x^{(i)} \le -1 \text{ if } y^{(i)} = 0$$

$$\Rightarrow \theta^{T} x^{(i)} \Rightarrow \text{ inner product}$$

$$\Rightarrow p^{(i)} * ||\theta|| = \theta_{1} x_{1}^{(i)} + \theta_{2} x_{2}^{(i)}$$



# SVM DECISION BOUNDARY

Optimization problem (C is very large):

$$\min_{\theta} \frac{1}{2} \sum_{j=1}^{n} \theta_j^2 = \frac{1}{2} ||\theta||^2$$

→ Minimize the length of the vector theta

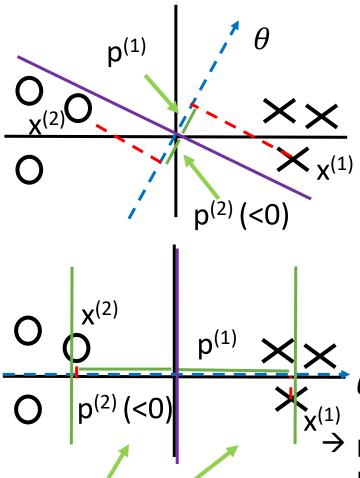
s.t.

$$|p^{(i)}| \|\theta\| \ge 1$$
 if  $y^{(i)} = 1$ 

$$p^{(i)*} ||\theta|| \le -1 \text{ if } y^{(i)} = 0$$

$$\rightarrow \theta^T \chi^{(i)} = p^{(i)*} ||\theta||$$

- $\rightarrow$ p<sup>(i)</sup> is the projection of  $x^{(i)}$  onto the vector  $\theta$
- ⇒Simplification:  $\theta_0$  = 0 (Decision Boundary passes through (0;0))



→ SVM hypothesis → large margin

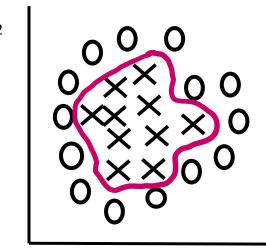
- → The upper decision boundary will not be chosen
- ⇒  $p^{(1)} * \|\theta\| \ge 1$ ; if  $p^{(1)}$  is small  $\|\theta\|$  has to be large
- ⇒  $p^{(2)} * ||\theta|| \le -1$ ; if  $p^{(2)}$  is small negative number  $||\theta||$  has to be large
- ⇒  $\min_{\theta} \frac{1}{2} ||\theta||^2$  ⇒ the norm of  $\theta$  should be small

 $p^{(1)} * ||\theta|| \ge 1 \rightarrow p^{(1)}$  is bigger now  $||\theta||$  can be smaller (equivalent for  $p^{(2)}$ )

#### Non-Linear Decision Boundary

• Predict e.g. y = 1 if  $\theta_0 + \theta_1 \underline{x_1} + \theta_2 \underline{x_2} + \theta_3 \underline{x_1} \underline{x_2} + \theta_4 \underline{x_1}^2 + \theta_4 \underline{x_2}^2 + \theta_5 \underline{x_2}^3 + ... >= 0$ 

$$h_{\theta}(x) = \begin{cases} 1 & \text{if } \theta^T x \ge 0 \\ 0 & \text{otherwise} \end{cases}$$



- → Predict e.g. y = 1 if  $\theta_0 + \theta_1 f_1 + \theta_2 f_2 + \theta_3 f_3 + \theta_4 f_4 + \theta_4 f_5 + \theta_5 f_6 + ... >= 0$
- Is there a different/better choice of the features f<sub>1</sub>,f<sub>2</sub>,f<sub>3</sub>,...?

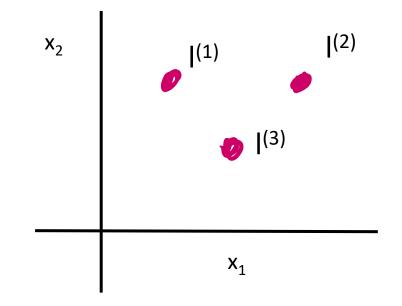
#### KERNEL

Given x, compute new features depending on proximity to landmarks  $I^{(1)},I^{(2)},I^{(3)}$ 

$$f_{1} = similarity(x, I^{(1)}) = e^{-\frac{\left|\left|x-l^{(1)}\right|\right|^{2}}{2\sigma^{2}}}$$

$$f_{2} = similarity(x, I^{(2)}) = e^{-\frac{\left|\left|x-l^{(2)}\right|\right|^{2}}{2\sigma^{2}}}$$

$$f_{3} = similarity(x, I^{(3)}) = e^{-\frac{\left|\left|x-l^{(3)}\right|\right|^{2}}{2\sigma^{2}}}$$



Similarity = kernel (here: Gaussian kernel)  $\rightarrow$  k(x, l)

# KERNELS AND SIMILARITY

$$f_1 = \text{similarity}(\mathbf{x}, \, \mathsf{I}^{(1)}) = \mathrm{e}^{-\frac{\left|\left|x-l^{(1)}\right|\right|^2}{2\sigma^2}} \, (\text{where } \|\mathbf{x}-\mathsf{I}\|^2 = \sum_{j=1}^n (x_j - lj)^2)$$

$$\text{If } \mathbf{x} \sim \mathsf{I}^{(1)}:$$

$$f_1 \sim \mathrm{e}^{-\frac{0^2}{2\sigma^2} \sim 1}$$

If x is far from  $I^{(1)}$ :

$$f_1 \sim e^{-\frac{large\ number^2}{2\sigma^2}} \sim 0$$

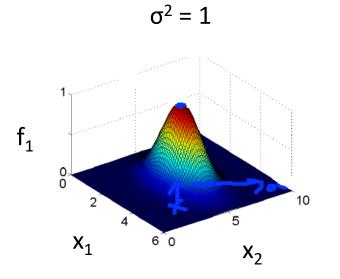
$$I^{(1)} \rightarrow f_1$$

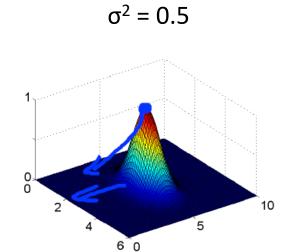
$$I^{(2)} \rightarrow f_2$$

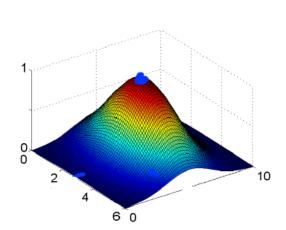
$$I^{(3)} \rightarrow f_3$$

# **KERNELS**

•  $\sigma^2$  defines how narrow the gaussian kernels are





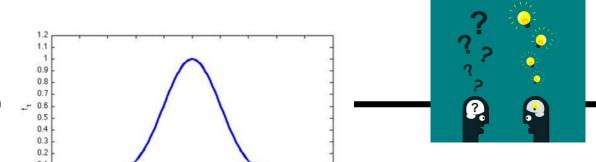


 $\sigma^2 = 3$ 

Example with 
$$I^{(1)} = ((3);(5))$$
 and  $f_1 = e^{-\frac{\|x - l^{(1)}\|^2}{2\sigma^2}}$ 

Plots by Andrew Ng

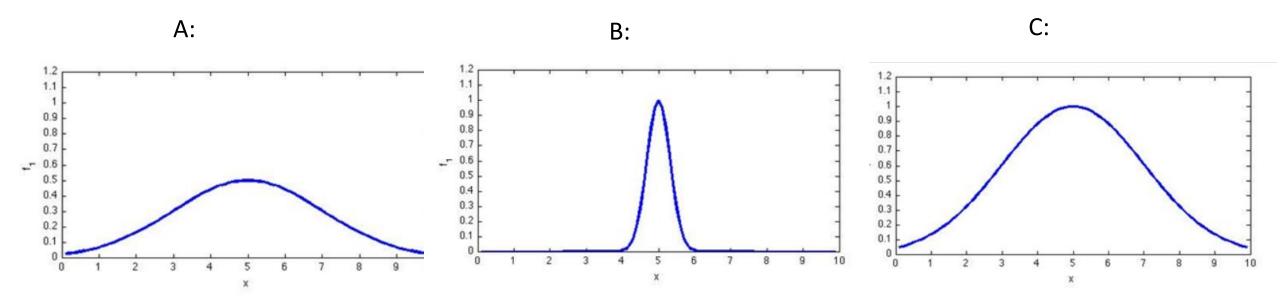
# QUESTION



Consider a 1-D example with one feature  $x_1$ . Suppose  $I^{(1)}=5$ .

This is a plot of 
$$f_1 = e^{-\frac{\left|\left|x-l^{(1)}\right|\right|^2}{2\sigma^2}}$$
 when  $\sigma^2 = 1$ .

Suppose we now change  $\sigma^2$  = 4. Which of the following is a plot of  $f_1$  with the new value of  $\sigma^2$ ?



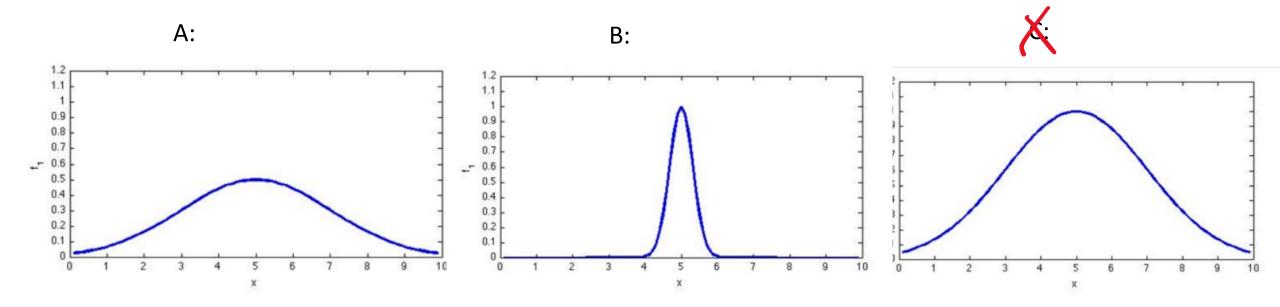
# QUESTION

1.2 1.1 1 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2

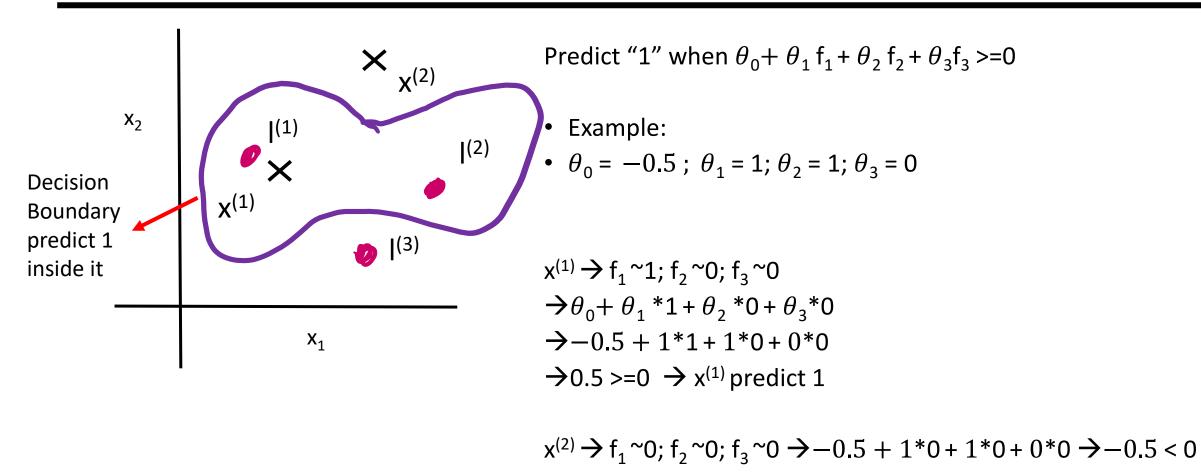
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### **KERNELS**

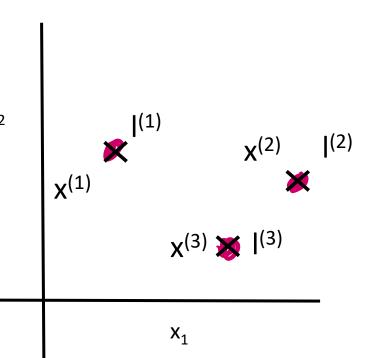


#### CHOOSING THE LANDMARKS

• Given x:

• 
$$f_1$$
= similarity(x,  $I^{(1)}$ ) =  $e^{-\frac{||x-l^{(1)}||^2}{2\sigma^2}}$ 

- Predict "1" when  $\theta_0$ +  $\theta_1$  f<sub>1</sub>+  $\theta_2$  f<sub>2</sub>+  $\theta_3$ f<sub>3</sub> >=0
- Where to get  $I^{(1)}, I^{(2)}, I^{(3)}$ ?
- Idea: set  $I^{(1)},...,I^{(m)}$  to  $x^{(1)},...,x^{(m)}$



#### SVM WITH KERNELS

- Given  $(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), ..., (x^{(m)}, y^{(m)})$
- Choose  $I^{(1)} = x^{(1)}, I^{(2)} = x^{(2)}, ..., I^{(m)} = x^{(m)}$
- Given example x:
  - $f_1 = similarity(x, I^{(1)})$  (where  $I^{(1)} = x^{(1)}$ )
  - $f_2$ = similarity(x,  $I^{(2)}$ )
- For training example (x<sup>(i)</sup>,y<sup>(i)</sup>)

$$x^{(i)} \rightarrow$$

$$f_1^{(i)} = similarity(x^{(i)}, I^{(1)})$$

$$f_2^{(i)} = similarity(x^{(i)}, I^{(2)})$$

•••

$$f_m^{(i)} = similarity(x^{(i)}, I^{(m)})$$

→ Somewhere in this list: 
$$f_i^{(i)} = similarity(x^{(i)}, I^{(i)})$$
 (where  $I^{(i)} = x^{(i)}$ )

$$\rightarrow$$
 With Gaussian kernel:  $e^{-\frac{0}{2\sigma^2}}=1$ 

$$f = \begin{bmatrix} f_0 \\ f_1 \\ \vdots \\ f_m \end{bmatrix}$$

$$f_0 = 1$$

$$\mathbf{x}^{(i)} = \mathbf{f}^{(i)} \qquad \begin{vmatrix} \mathbf{f_0}^{(i)} \\ \mathbf{f_1}^{(i)} \\ \vdots \\ \mathbf{f_m}^{(i)} \end{vmatrix} \qquad \mathbf{f_0}^{(i)} = \mathbf{1}$$

#### SVM WITH KERNELS

- Hypothesis: Given x, compute features  $f \in \mathbb{R}^{m+1}$  (m = training set size and number of landmarks)
  - Predict "1" if  $\theta^T f >= 0$  (with  $\theta \in \mathbb{R}^{m+1}$ )
- Training:
- Before:

$$\min_{\theta} C \sum_{i=1}^{m} [y^{(i)} \cos t_1(\theta^T x) (1-y^{(i)}) \cos t_0(\theta^T x)] + \frac{1}{2} \sum_{j=1}^{n} \theta_j^2$$

• Now:

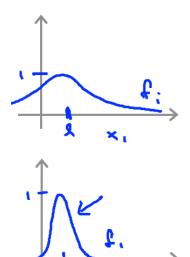
$$\min_{\theta} C \sum_{i=1}^{m} [y^{(i)} \cos t_1(\theta^T f^{(i)}) (1 - y^{(i)}) \cos t_1(\theta^T f^{(i)})] + \frac{1}{2} \sum_{j=1}^{m} \theta_j^2$$

Note:  $\sum_{j=1}^{m} \theta_{j}^{2} = \theta^{T}\theta$ ; often implemented as  $\theta^{T}M\theta$  (M depends on the kernel)

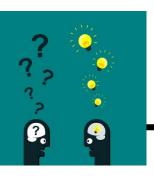
### **SVM** PARAMETERS

• C = 
$$(\frac{1}{\lambda})$$
:

- Large C: lower bias, high variance (think of small  $\lambda$ )
- Small C: higher bias, low variance (think of large  $\lambda$ )
- $\sigma^2$ 
  - Large  $\sigma^2$ : Features  $f_i$  vary more smoothly
  - → higher bias, lower variance
  - Small  $\sigma^2$ : Features  $f_i$  vary less smoothly
  - > lower bias, higher variance



# QUESTION



Suppose you train an SVM and find it overfits your training data. Which of these would be a reasonable next step? Check all that apply.

A: Increase C

B: Decrease C

C: Increase  $\sigma^2$ 

D: Decrease  $\sigma^2$ 

# QUESTION



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🗴 Decrease C

 $\chi$ : Increase  $\sigma^2$ 

D: Decrease  $\sigma^2$ 

#### Using A SVM

- Use a SVM software package to solve for the parameters  $\theta$
- Specify:
  - Parameter C
  - Kernel
- E. g. no kernel ("linear kernel")
  - Predict "y = 1" if  $\theta^T x >= 0$
  - Linear Decision Boundary
  - Applied if n is large and m is small (risk of overfitting)
- Gaussian kernel  $f_i = e^{\frac{\|x-l^{(i)}\|^2}{2\sigma^2}}$ , where  $l^{(i)} = x^{(i)}$ 
  - Choose  $\sigma^2$
  - Applied if n is small and m is large

#### KERNEL FUNCTION EXAMPLE

Function f = kernel (x1,x2)  

$$fi = \exp(-\frac{||x_1-x_2||^2}{2\sigma^2})$$
return

Note: Do perform feature scaling before using the Gaussian kernel

$$||x - l||^2 \rightarrow (x_1 - l_1)^2 + (x_2 - l_2)^2 + ... + (x_n - l_n)^2$$

→ If one x is very large it will "dominate" the value

#### OTHER KERNEL

- Note: Not all similarity functions make valid kernels.
- → Need to satisfy technical condition called "Mercer's Theorem" to make sure SVM packages' optimizations run correctly and do not diverge

- Other valid kernels:
  - Polynomial kernel:  $(x^Tl+constant)^{degree}$  e.g.  $(x^Tl)^3$ ,  $(x^Tl+1)^3$ ,  $(x^Tl+5)^4$
  - String kernel, chi-square kernel, histogram intersection kernel,...

# QUESTION



Suppose you are trying to decide among a few different choices of kernels and are also choosing parameters such as C,  $\sigma^2$ , etc. How should you make the choice?

A: Choose whatever performs best on the training data.

B: Choose whatever performs best on the cross-validation data.

C: Choose whatever performs best on the test data.

D: Choose whatever gives the largest SVM margin.

# QUESTION



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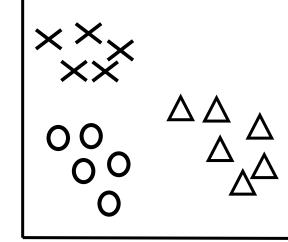
D: Choose whatever gives the largest SVM margin.

### Multiclass Classification

 Many SVM packages already have built-in multi-class classification functionality

• Otherwise, use one-vs-all method (Train K SVMs, one to distinguish y= i from the rest, for i = 1,2,..,K), get  $\theta^{(1)}$ ,  $\theta^{(2)}$ ,.., $\theta^{(K)}$ , pick class i with largest  $(\theta^i)^T x$ 

• Y ∈ {1,2,..,K}



# LOGISTIC REGRESSION VS. SVM - RECOMMENDATION

- If n is large (relative to m) (e. g. text classification)
  - Use logistic regression, or SVM without a kernel (the "linear kernel")
- If n is small and m is intermediate
  - Use SVM with a Gaussian Kernel
- If n is small and m is large
  - Create/add more features, then use logistic regression or SVM without a kernel.
- Note: a neural network is likely to work well for any of these situations, but may be slower to train.

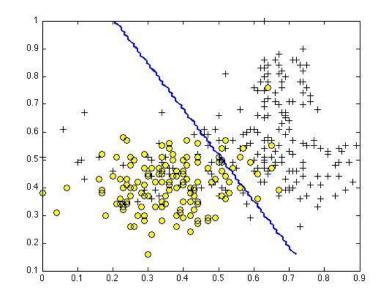
- Suppose you have trained an SVM classifier with a Gaussian kernel, and it learned the following decision boundary on the training set:
- You suspect that the SVM is underfitting your dataset. Should you try increasing or decreasing C? Increasing or decreasing  $\sigma^2$ ?

A: It would be reasonable to try increasing C. It would also be reasonable to try increasing  $\sigma^2$ .

B: It would be reasonable to try decreasing C. It would also be reasonable to try increasing  $\sigma^2$ .

C: It would be reasonable to try decreasing C. It would also be reasonable to try decreasing  $\sigma^2$ .

D: It would be reasonable to try increasing C. It would also be reasonable to try decreasing  $\sigma^2$ .



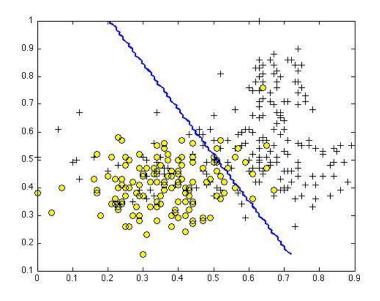
- Suppose you have trained an SVM classifier with a Gaussian kernel, and it learned the following decision boundary on the training set:
- You suspect that the SVM is underfitting your dataset. Should you try increasing or decreasing C? Increasing or decreasing  $\sigma^2$ ?

A: It would be reasonable to try increasing C. It would also be reasonable to try increasing  $\sigma^2$ .

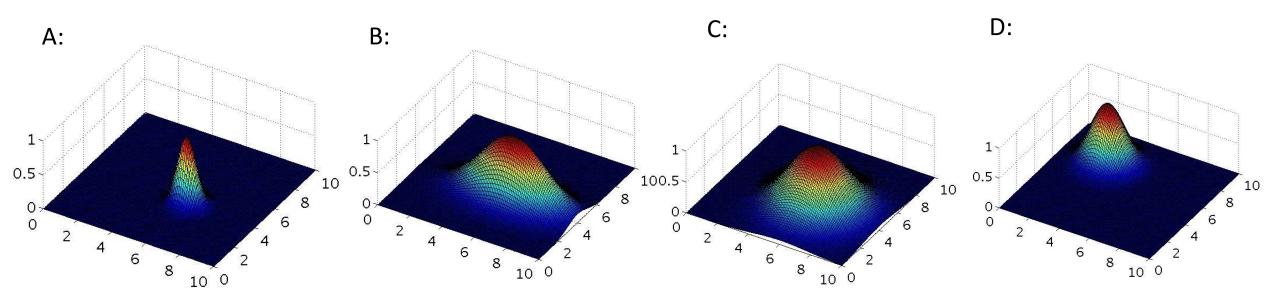
B: It would be reasonable to try decreasing C. It would also be reasonable to try increasing  $\sigma^2$ .

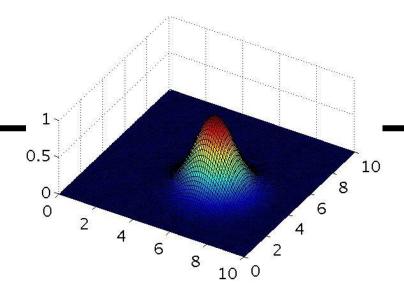
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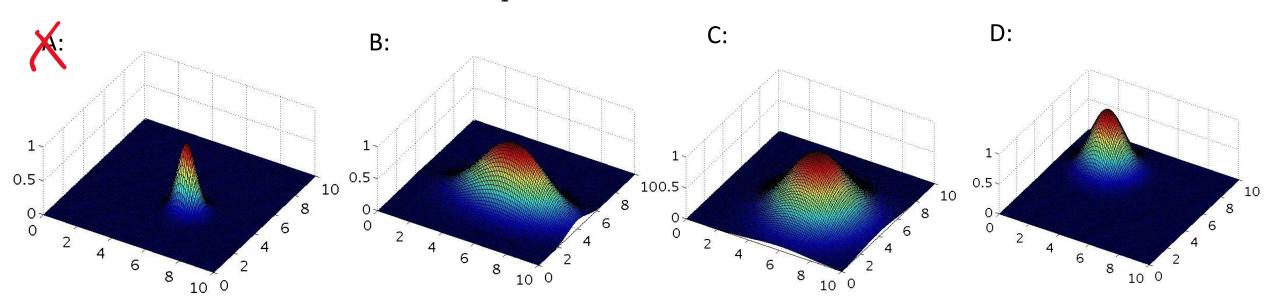


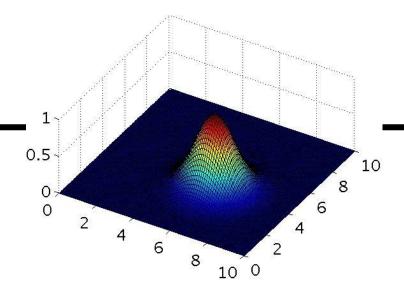
- The formula for the Gaussian kernel is given by  $f_1$ = similarity(x,  $I^{(1)}$ ) =  $e^{-\frac{||x-l^{(1)}||^2}{2\sigma^2}}$
- The figure shows a plot of  $f_1$ = similarity(x,  $I^{(1)}$ ) with  $\sigma^2$  = 1
- Which of the following is a plot of  $f_1$  when  $\sigma^2$  =0.25?

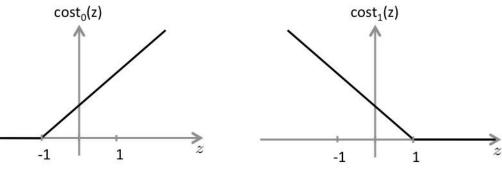




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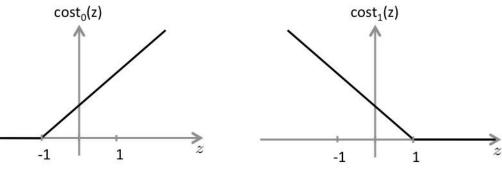
- The SVM solves  $\min_{\theta} C \sum_{i=1}^{m} [y^{(i)} \cos t_1(\theta^T x) (1-y^{(i)}) \cos t_0(\theta^T x)] + \frac{1}{2} \sum_{j=1}^{n} \theta_j^2$
- The first term in the objective is:  $C \sum_{i=1}^{m} [y^{(i)} \cos t_1(\theta^T x) (1-y^{(i)}) \cos t_0(\theta^T x)]$
- This first term will be zero if two of the following four conditions hold true.
   Which are the two conditions that would guarantee that this term equals zero?

A: For every example with  $y^{(i)}=1$ , we have that  $\theta^T x^{(i)} \ge 1$ .

B: For every example with  $y^{(i)}=0$ , we have that  $\theta^T x^{(i)} \le -1$ .

C: For every example with  $y^{(i)}=1$ , we have that  $\theta^T x^{(i)} \ge 0$ .

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Suppose you have a dataset with n = 10 features and m = 5000 examples.

After training your logistic regression classifier with gradient descent, you find that it has underfit the training set and does not achieve the desired performance on the training or cross validation sets.

Which of the following might be promising steps to take? Check all that apply.

A: Use an SVM with a linear kernel, without introducing new features.

B: Increase the regularization parameter  $\lambda$ .

C: Use an SVM with a Gaussian Kernel.

D: Create/add new polynomial features.

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Which of the following statements are true? Check all that apply.

A: It is important to perform feature normalization before using the Gaussian kernel.

B: If the data are linearly separable, an SVM using a linear kernel will return the same parameters  $\theta$  regardless of the chosen value of C (i.e., the resulting value of  $\theta$  does not depend on C).

C: The maximum value of the Gaussian kernel (i.e.,  $sim(x, l^{(1)})$  is 1.

D: Suppose you are using SVMs to do multi-class classification and would like to use the one-vs-all approach. If you have K different classes, you will train K - 1 different SVMs.

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