

\* Divide and Conquer: It is an Algorithm of dividing a bigger problem into two equal subproblems & then two subproblems into 2-2 each sub-subproblems. As Merge sort & Quick sort are great examples of it.

### ① Quick sort

ex: I/P: 

0	1	2	3	4	5	6
8	3	4	1	20	50	30

#### Quick sort Algorithm:

- ↳ Pick one element & place it at right position. [Partition Logic]
- ↳ Then everything else leave it to Recursion.

#### \* Partition Logic:

- ① Choose any pivot element. it could be starting one, or ending one or any random element from an array.
- ② Now place that element at its right position.
- ③ Elements left to the pivot element should be smaller than pivot element & elements right to the pivot element should be greater than pivot element.

ex: 

0	1	2	3	4	5	6
8	3	4	1	20	50	30

↓  
pivot element

0	1	2	3	4	5	6
1	3	4	8	20	50	30

↑ smaller than pivot element
↑ greater than pivot element

↪ pivot element at its right position



0	1	2	3	4	5	6
8	1	3	4	20	50	30

Partition logic: ① choose pivot element  
i.e. = 8.

② pivot at right place.

count.

8 > 1 Yes count = 1

8 > 3 Yes

8 > 4 Yes

8 > 20 No  
8 > 50 No  
8 > 30 No  
swap(arr[pivotIndex], arr[start + count])

Here we have added start as we can use just count because for every recursive call the value of start varies as it doesn't begin from 0. The array divided into sub parts so start value become different.

\* Partition Logic:

① choose pivotIndex & pivotElement. i.e. is start.

② Place pivot Element at its right position.

i.e. arr[i] < arr[pivotElement]

0	1	2	3	4	5	6
8	1	3	4	20	50	30

count ++;

i = indexes

count = 5

8 > 20

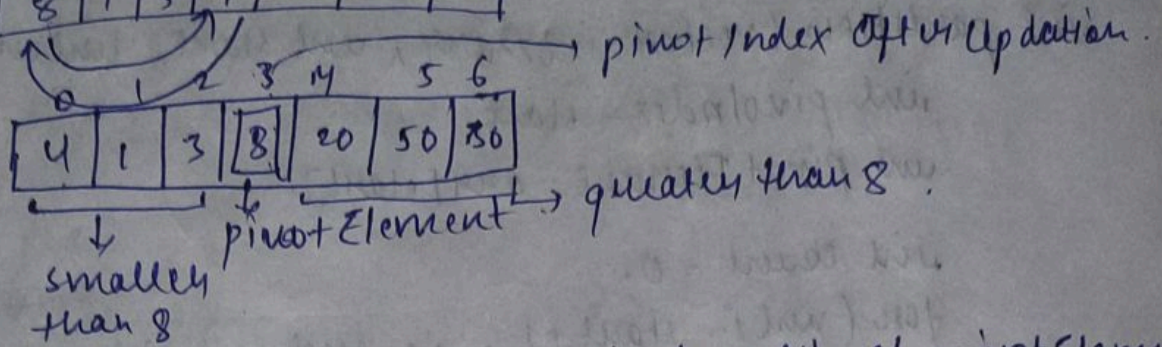
8 will be placed at 3 index & it is 4th element of an array.

③ swap(arr[pivotIndex], arr[start + count]).

swap(arr[0], arr[3]).



0	1	2	3	4	5	6
8	1	3	4	20	50	30



④ Recursive calls to sort left & right part of pivotElement

Left → 

0	1	2
4	1	3

 start = 0, end = 2

Right → 

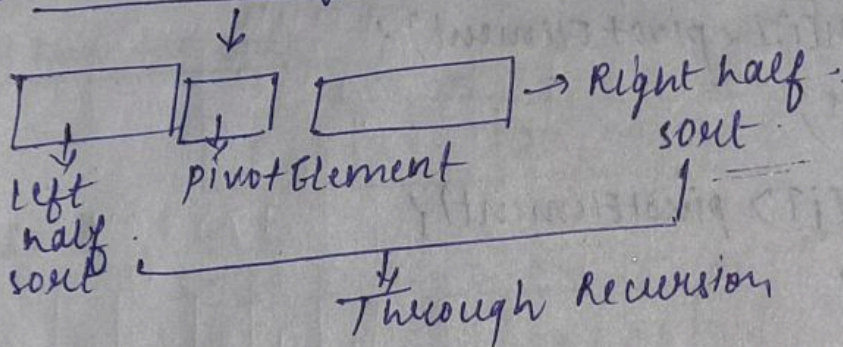
4	5	6
20	50	30

 start = 4, end = 6

## Quick sort

① Partition logic

② Recursive logic



## \* Partition logic

① ~~int~~ int pivotIndex = start

② if (arr[i] < arr[pivotIndex]) {  
    count++;  
} → right place of pivot element

③ swap(arr[pivotIndex], arr[start + count]); → swap

④ 

left	PE	right
------	----	-------

  
left < PE      PE > right

⑤ return pivotIndex



## Quick Sort Code

```
int partition (vector<int> &arr, int start, int end) {  
    int pivotIndex = start;  
    int pivotElement = arr[start];  
    int count = 0;  
    for (int i = start + 1; i <= end; i++) {  
        if (arr[i] < pivotElement) {  
            count++;  
        }  
    }  
    swap (arr[pivotIndex], arr[start + count]);  
    pivotIndex = start + count;  
    int i = start;  
    int j = end;  
    while (i < pivotIndex && j > pivotIndex) {  
        while (arr[i] < pivotElement) {  
            i++;  
        }  
        while (arr[j] > pivotElement) {  
            j--;  
        }  
        if (i < pivotIndex && j > pivotIndex) {  
            swap (arr[i], arr[j]);  
        }  
    }  
    return pivotIndex;  
}  
void quickSort (vector<int> &arr, int start, int end) {  
    if (start >= end) {  
        return;  
    }  
}
```



```

int pivotIndex = partition(arr, start, end);
quickSort(arr, start, pivotIndex-1);
quickSort(arr, pivotIndex+1, end);

```

```

int main() {
    int num;
    cin >> num;
    vector<int> arr(num);
    for (int i=0; i<arr.size(); i++) {
        cin >> arr[i];
    }
    int start=0, end=arr.size()-1;
    quickSort(arr, start, end);
    for (int i=0; i<arr.size(); i++) {
        cout << arr[i] << " ";
    }
}

```

Dry run the code

0	1	2	3	4	5	6
8	7	3	4	20	50	30

$s=0, e=6$

$s \rightarrow 0$	1	2	3	4	5	6 $\leftarrow e$
8	7	3	4	20	50	30

$s \neq e$

index = partition(arr, 0, 6)

pivotIndex = 0

$\therefore \text{ele} = \text{arr}[0] = 8$

$\text{arr}[1] = 7 < 8 \checkmark$  count = 1

$= 3 < 8 \checkmark$

$4 < 8 \checkmark$

$20 < 8 \times$

swap(arr[0], arr[0+3])

$s \rightarrow 0$   $6 \leftarrow e$

0	1	2	3	4	5	6
8	7	3	4	20	50	30

swap (arrow from index 0 to index 3)



0	1	2	3	4	5	6
4	1	3	8	20	50	30

↓  
pivot element

PI = 3  
PI = 3

$i = 0$   
 $i = 6$   
 $3 < 8$   
 $4 < 8$   
 $1 < 8$   
 $3 < 8$   
 $20 > 8$   
 $50 > 8$   
 $30 > 8$

PI = 3

quicksort(arr, 0, 2)

0	1	2
4	1	3

PI = 0 PE = 4

arr[1]  $1 < 4$  ✓ count = 2  
arr[2]  $3 < 4$  ✓

swap(arr[0], arr[0+2])

0	1	2
4	1	3

0	1	2
3	1	4

PI = 3 + count  
= 0 + 2 = 2

$i = 0, j = 2$

$0 < 2$  ✓

PI = 2

quicksort(arr, 0, 1)

0	1
3	1

PI = 0

PE = 3

$i = 1$   
arr[1]  $1 < 3$  ✓ count = 1

swap(arr[0], arr[0+1])

0	1
3	1

0	1
1	3

PI = 3 + count  
= 0 + 1 = 1

$i = 0, j = 1$

$0 < 1$  ✓

Original Array

0	1	2	3	4	5	6
3	1	4	8	20	50	30

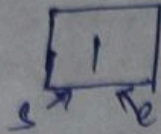
Original Array

0	1	2	3	4	5	6
3	1	4	8	20	50	30

P.B.R



quicksort(arr, 0, 0)

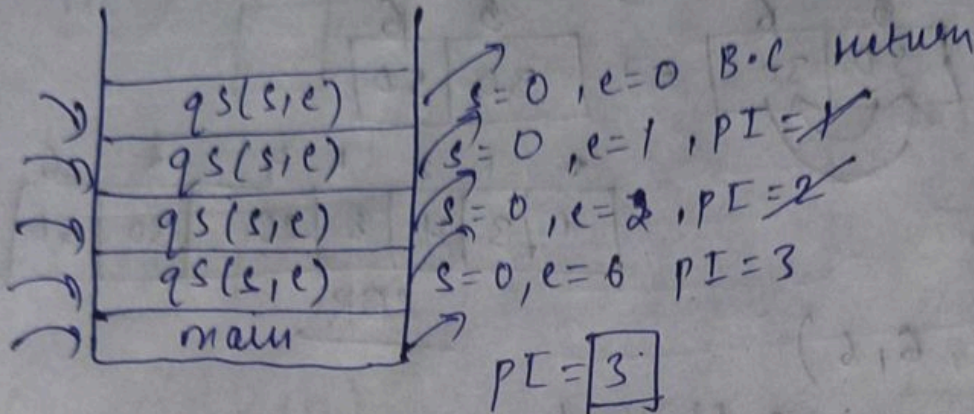


return  
Base case

0	1	2	3	4	5	6	
1	3	4	8	20	50	30	0.A

P.B.R

Recursive call stack



quicksort(arr, 4, 6)

4	5	6
20	50	30

$s = 4, e = 6$

$PI = 4, PE = 20$

$i = 5$  arr[5] ~~50~~  $50 < 20$   $\times$  count = 0  
arr[6]

swap(arr[4], arr[5 + count])

arr[4], arr[4 + 0]

4	5	6
20	50	30

$PI = 4, PE = 20$

$i = 4$  &  $j = 6$

$i < 4$  &  $6 > 4$   $\times$

$PI = 4$

quicksort(arr, 5, 6)

0	1	2	3	4	5	6	
1	3	4	8	20	50	30	0.A

PBR



quicksort(arr, 5, 6)

5	6
50	30

$s=5, e=6$

$PI=5$   
 $PE=50$

$i = 5+1 = 6$

$arr[6] < PE$  count = 1

$30 < 50$

swap(arr[5], arr[5+1])  $arr[s+count]$   
 $5+1=6$

arr(5), arr(6)

5	6
50	30

→

5	6
30	50

$PI = 6$

0	1	2	3	4	5	6	
1	3	4	8	20	30	50	OR

PBR

quicksort(arr, 6, 6)

50
----

$s=6, e=6$

single Element  
B.C

return

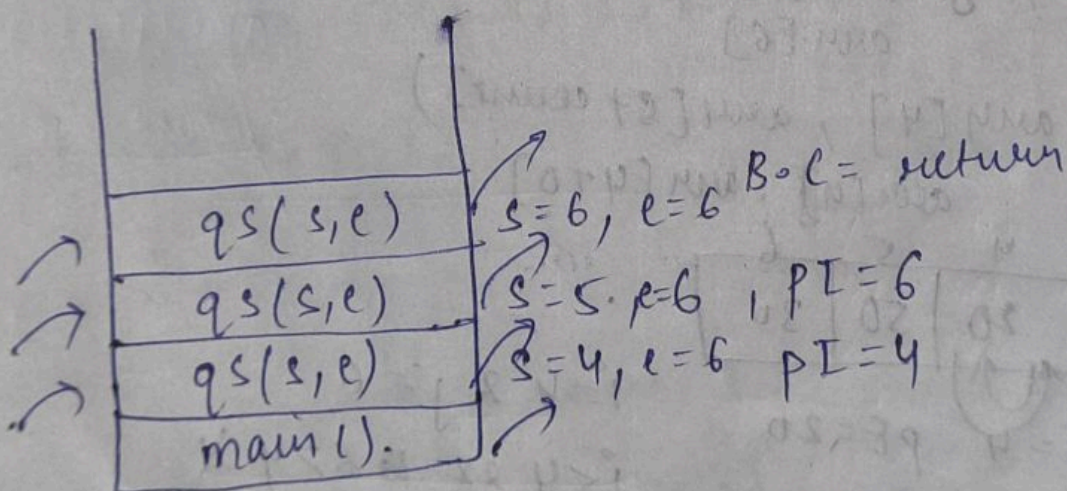
~~OR~~

0	1	2	3	4	5	6	
1	3	4	8	20	30	50	<u>O.R</u>

Pass By Reference

~~Call~~

Recursive Call Stack



1	3	4	8	20	30	50
---	---	---	---	----	----	----

ORIGINAL ARRAY

PRINT THE ARRAY  
IN MAIN FUNCTION

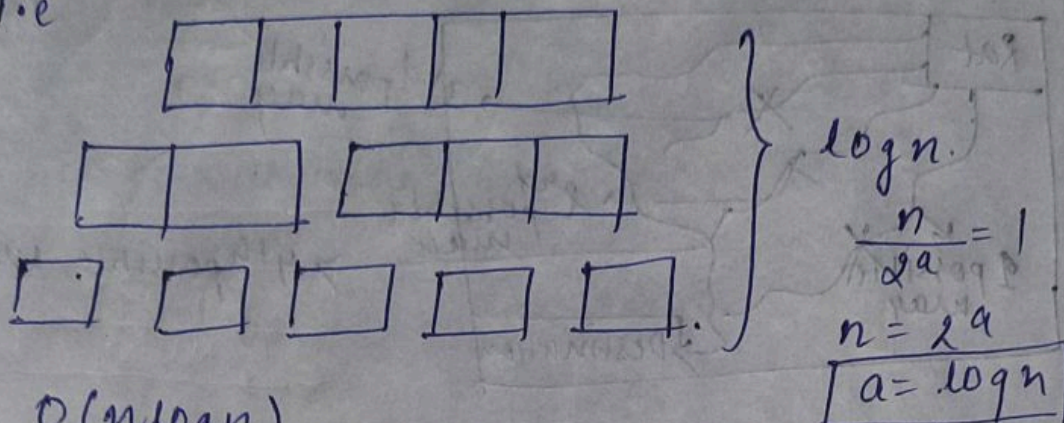


## \* Time Complexity:

0	1	2	3	4	5	6	7
1	5	5	6	8	20	30	60

As partition logic ~~the~~ the T.C will be  $O(2n) \rightarrow O(n)$

As the array divided into ~~part~~ subarray  
i.e



T.C =  $O(n \log n)$   
for the best case

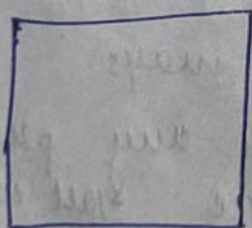
T.C for worst case i.e.  
when array is reverse  
order.  
T.C =  $O(n^2)$

Here equals to 1 until the array become single element as single element is sorted.  
order.

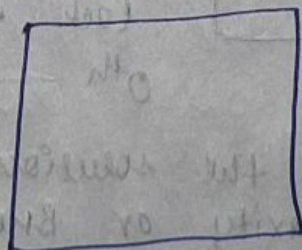
## \* Space Complexity: $O(\log n)$

## \* Backtracking: It is specific form of Recursion.

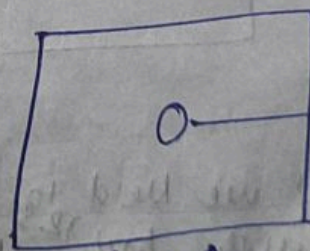
- It will check all possible solution.
- Once one solution is checked, it never goes back.



Glass 1



Glass 2



Glass 3

gold coin

To find gold coin is in which glass.

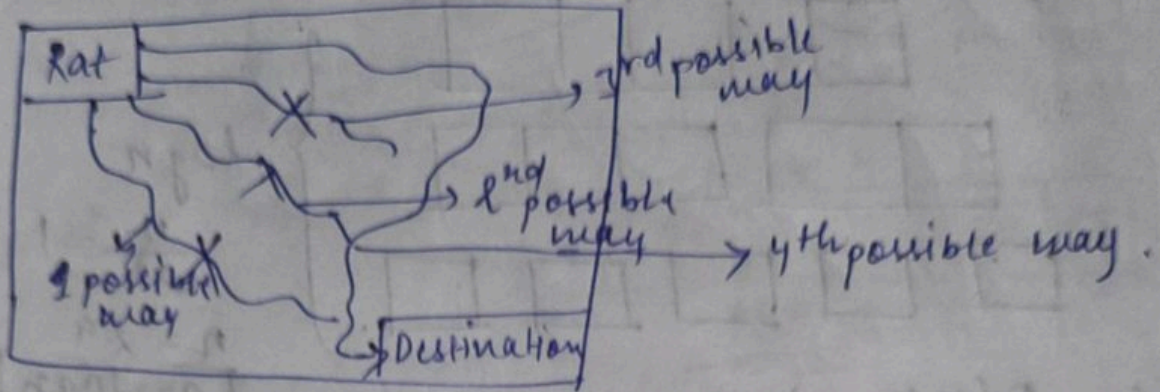


~~It will check glass 1 / if found return else check in~~

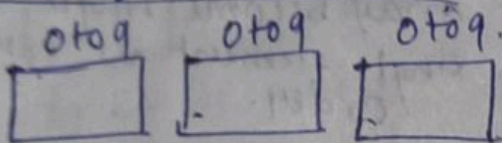
It will check glass 1 if found return else check in glass 2. found return else it won't go back to glass 1. then go to glass 3 found return.

### \* Famous Question

Rat in Maze:



### \* Password find in lock

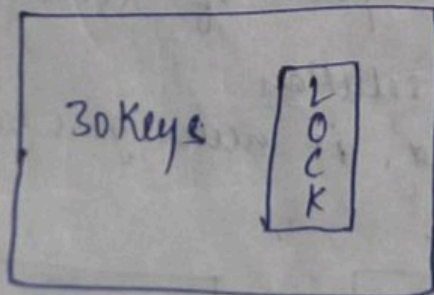


In each block you can put number from 0 to 9

→ so you start from 000 till 999 possible values you can check.

DESI TARIKA

### \* Escape Room:



You have Lock & have 30 Keys. • You have to open the lock.

→ so we can check for each key to open the lock means 0th → 30th ways.

### \* NOTE:

If we need to find the solution with bad complexity we use BACKTRACKING.

or Brute force you can



\* When do we use Backtracking?  
 → we use when we don't have optimum solution of any problem when you can only go for brute force then we will use Backtracking.

\* Permutation of string:

Input string "abc"

All permutation

"abc, bac, bca, cab, cba, acb"

string = "xy"

Permutation = "xy, yx"

string = "abcd"

abcd	baed	cabd	dabc
abdc	badc	cadb	clab
debd	bcad	cbad	dbac
acdb	bcdc	cbdc	dbca
adcb	bdae	cdab	clcab
adbc	bdca	cdba	dcbe

It can't be done with inc/exc ~~proway~~

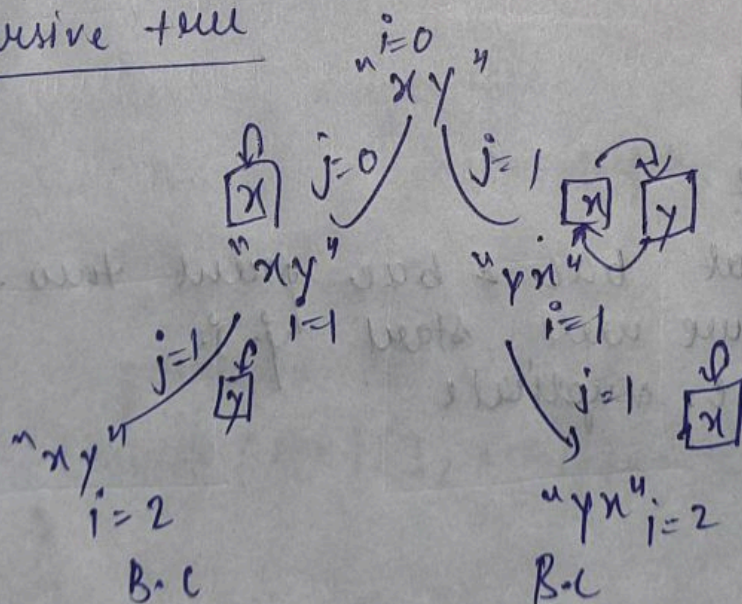
string str = "abc"

abc, bac, bca, cab, cba, acb

□ □ □

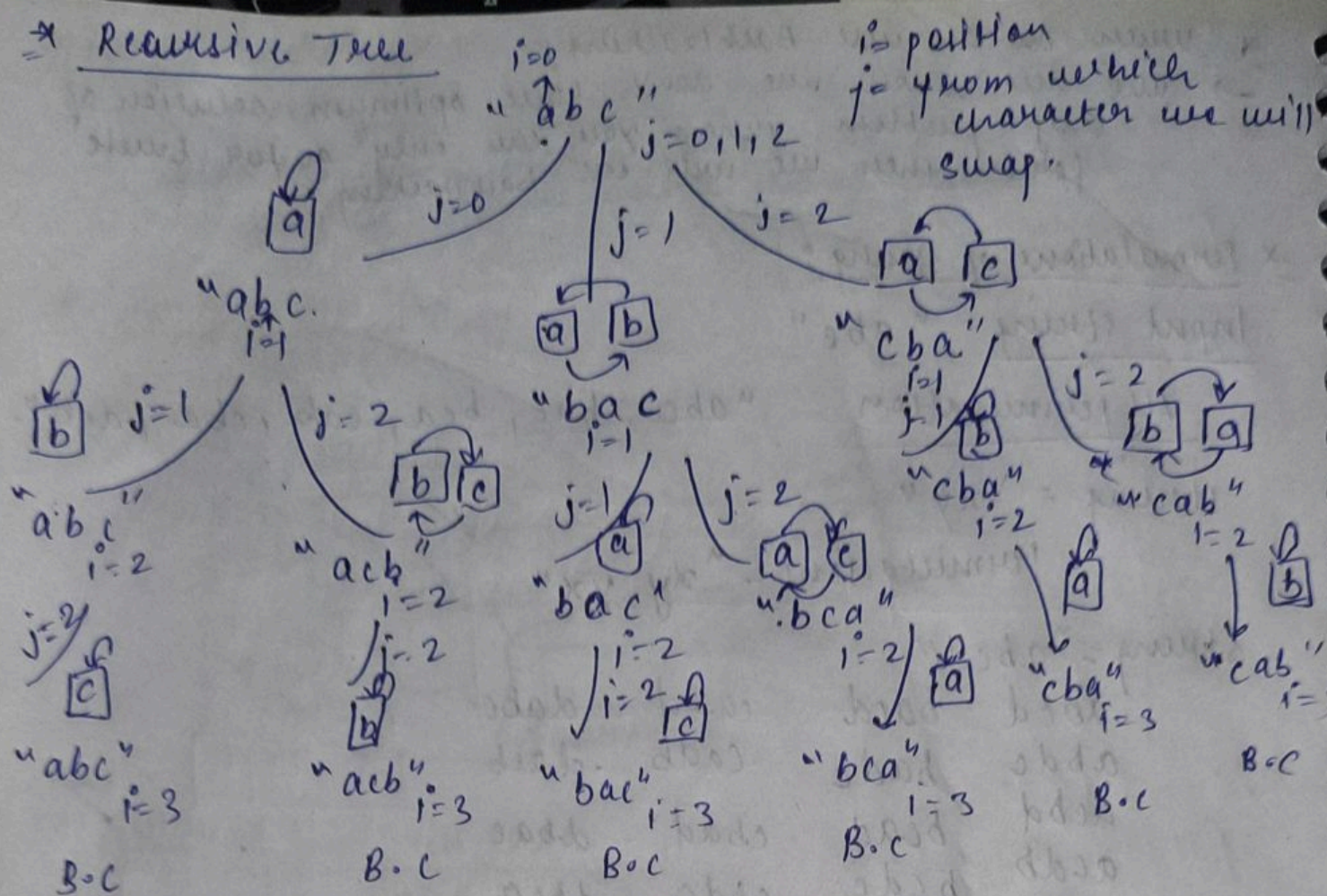
In each block each character comes.

\* Recursive tree

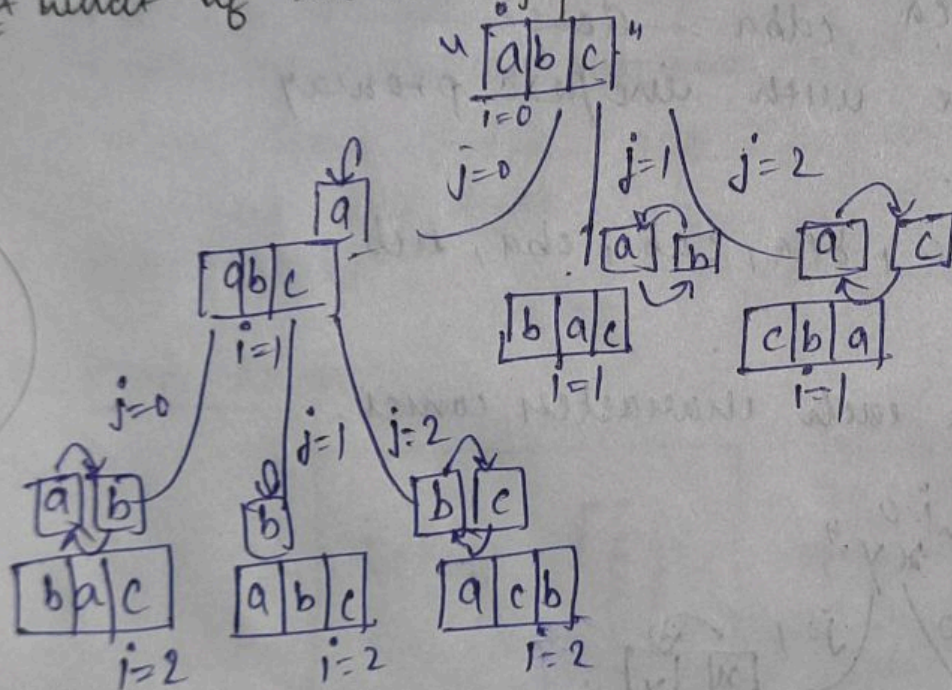




# \* Recursive Tree



\* what - if we start  $j$  from 0 ~~with~~ for each case



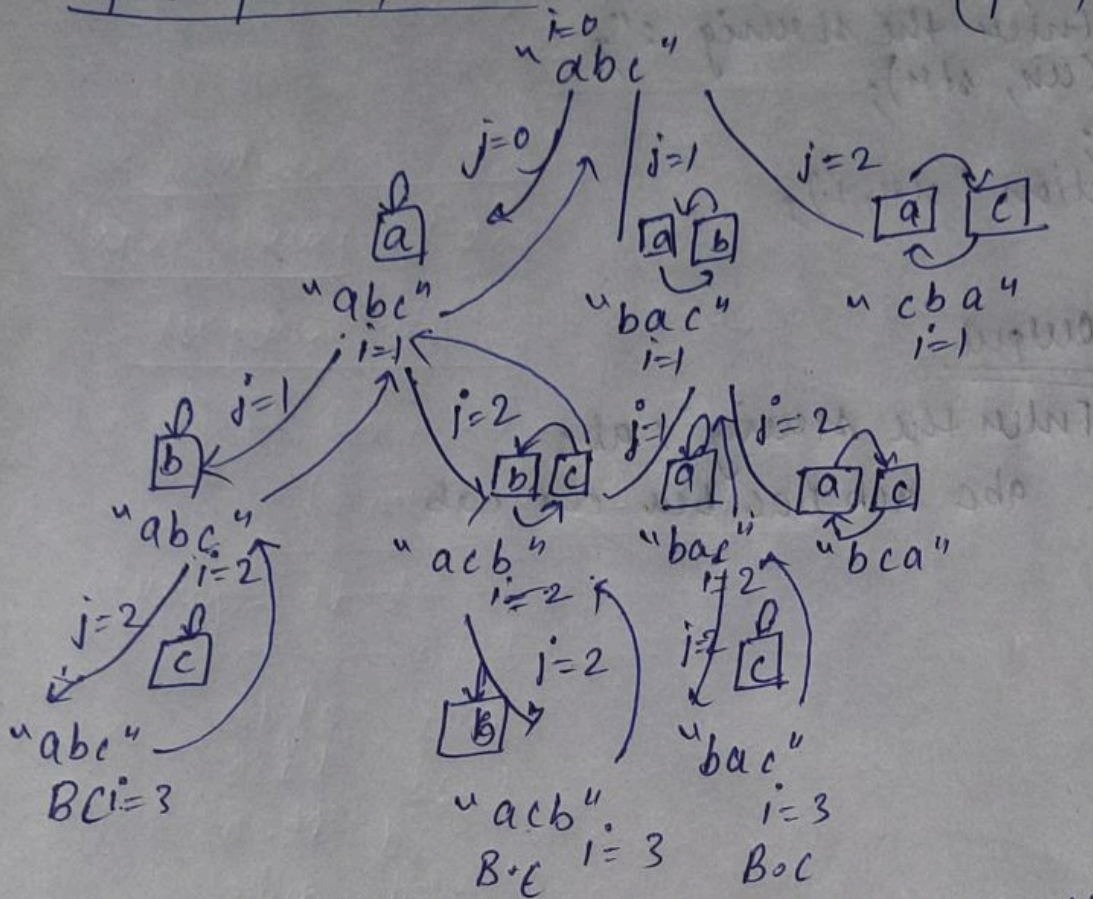
we can see that bac & bac print two times so we will start  $j=i$  to not to create duplicate.



\* we need to pass the vector by reference:

if we pass by value

( $j = i$ ;  $j < str.length()$ ;  $j++$ ;



In pass by value you will find that there will create the duplicate strings

void permutations (string str, int i) {  
 // base case.

if ( $i \geq str.length()$ ) {

cout << str << " ";

return;

// processing single case

for (int  $j = i$ ;  $j < str.length$ ;  $j++$ ) {

~~// swap~~

swap ( $str[i]$ ,  $str[j]$ );

permutation (str,  $i+1$ );

swap ( $str[i]$ ,  $str[j]$ ); // backtracking.



4. 

```
int main () {  
    string str;  
    cout << "Enter the string :";  
    getline (cin, str);  
    int i = 0;  
    permutation (str, i);  
}
```

### Output

Enter the string : abc  
abc acb bac bca eba cab