MATH 1081 – Discrete Mathematics

Assignment 1 (draft)

**Q1. Consider the following sets:**

1. **Show that is a proper subset of .**
2. **Show that is a proper subset of .**
3. **Show that there is no containment relation between and .**
4. We begin by assuming that . This means that we can write

, for some .

We will prove that , that is, we will show that there exists some integer , such that

5, where

Hence, we have proven that for every element , can be represented as , and thus . This means that

For to be a proper subset of , but , or . There must be some such that .

We know as , and , but for

thus .

Hence .

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1. Containment relations are

:

Assume .

Then, for some element , must also be in . Then, by definition of the sets,

, and

, for

We know as ,

But as

Thus, there exists an integer that is in but not in .

This contradicts our assumption that

Therefore .

or :

Assume .

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Thus, there exists an integer that is in but not in .

This contradicts our assumption that

Therefore .

:

Since we have proven that and , .

Thus, since , , and , there is no containment relation between the sets and .

**Q2. A relation  is defined on  by**

**if and only if for some non-negative integer .**

**Prove that is a partial order.**

We know from the definition of a partial order that for a relation to be a partial order, the relation must be reflexive, antisymmetric and transitive. We will prove these properties for the relation .

**Reflexive:**

We can see that

and since 0 is a non-negative number, .

Thus, or relation is reflexive.

**Antisymmetric:**

We know that if ,

and if ,

for non-negative integers and .

From ,

thus, so , but we know from the definition of the relation that and are non-negative, thus .

From , , and so .

Hence, we have proven that if and then , so the relation is said to be antisymmetric.

**Transitive:**

We know that if ,

and if ,

,

for non-negative integers and .

From ,

and since and are non-negative integers, is also a non-negative integer, thus we can say or that if and , we have proven that . Hence the relation is transitive.

Since relation is reflexive, antisymmetric, and transitive, is a partial order.

**Q3. Prove is irrational.**

Let us assume that is rational.

The definition of a rational number implies that can be expressed as a ratio of integers and , where , and , or

Since is a multiple of 7, and therefore must also be a multiple of 7, or and thus may be written as , where is an arbitrary integer.

now becomes

This implies that , and since , and therefore, . Thus, we know that and . This contradicts our initial assumption that the as is a common divisor of and and . Since the , cannot be expressed as a rational number.

Hence, must be irrational.