MATHEMATICS 1B

MATH1231 ASSIGNMENT

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Question 1:

The function is defined by

Show that is linear.

By definition of a linear transformation, a transformation is **linear** if it preserves the conditions of **addition** and **scalar multiplication**.

**Addition:**

**Scalar Multiplication:**

The transformation defined has a domain and co-domain which are both known vector spaces. Now, we check if the transformation preserves the properties of addition and scalar multiplication.

1. **Addition**

Similarly,

Hence as , the transformation preserves the **addition** property.

1. **Scalar Multiplication**

Similarly,

Hence, as , the transformation preserves **scalar multiplication.**

Since the domain and co-domain of ( and ) are known vector spaces, and preserves the properties of addition and scalar multiplication, is a **linear** transformation.

Question 2:

Show that

is a subspace of

By the Subspace theorem, a subset of a *vector space* over a *field* , *under the same rules for addition and multiplication by scalars*, is a subspace of *if and only if*

1. The vector in also belongs to .
2. is closed under *vector addition*, and
3. is closed under *multiplication by scalars* from .

We are given , a set of vectors of the known vector space .

To prove that is a subspace of , we must prove the existence of zero vector, closure under vector addition, and closure under scalar multiplication.

1. **Existence of zero vector.**

The zero vector, in is given by

and

Hence,

1. **Closure under vector addition.**

If  then

or

and

Now,

Equations (1), and (2) suggest

Hence , so is closed under vector addition.

1. **Closure under scalar multiplication.**

If , and then

or

We have,

or

Equation (1) suggests

Hence is closed under scalar multiplication.

**(I), (II) and (III)** imply that **S** is a vector subspace of , by the Subspace Theorem.

Question 3:

Let be the set of real polynomials of degree at most . Show that

Is a subspace of

We are given , a set of vectors of the known vector space . To prove that is a subspace, we must show that satisfies the conditions of the Subspace Theorem.

1. **Existence of zero polynomial.**

If then

As , we can write as

Now, equation (1) can be expressed as

The zero polynomial in is

Since , a known vector space, is defined at the zero polynomial, thus

Hence, the zero polynomial of is in .

1. **Closure under addition.**

If then

and

Now,

Since and ,

Therefore, equation (1) implies and is a factor of .

Hence so is closed under addition.

1. **Closure under scalar multiplication.**

If and then

As , we can write as

Then,

Since is an arbitrary polynomial in , , and is still a factor of .

Hence , so is closed under scalar multiplication.

**(I), (II) and (III)** imply that **S** is a subspace of , by the Subspace Theorem.

Question 4:

The air in a   kitchen is initially clean, but when Paul burns his toast while making breakfast, smoke is mixed with the room's air at a rate of . An air conditioning system exchanges the mixture of air and smoke with clean air at a rate of  .  Assume that the pollutants are mixed uniformly throughout the room and that burnt toast is taken outside after . Let   be the amount of smoke in in the room at time   after the toast first began to burn.

1. Find a differential equation obeyed by
2. Find for by solving the differential equation in (a) with an appropriate initial condition.
3. What is the level of pollution in per after .
4. How long does it take for the level of pollution to fall to per after the toast is taken outside?
5. Find a differential equation obeyed by

If Smoke at time , then the rate of smoke entering the room is given by the ordinary differential equation

is the rate at which smoke is produced by the toaster, which is given to be ,

is the rate at which smoke exits the room, due to the air conditioning. The rate at which air is exchanged is given to be . Thus, the amount of **smoke** exiting the room is

Hence the differential equation, obeyed by , according to equation (1) becomes,

1. Find for by solving the differential equation (2) with an appropriate initial condition.

We have the differential equation (2)

Rearranging the terms, we get

We can see that the equation is a linear differential equation of the form

The integrating factor can be found by

The solution for the differential equation will then be

For our differential equation,

The solution for the equation will be

Taking the constants out of the integral

Expanding the bracket

The kitchen was initially clean implies that at , the amount of smoke in the room . In the differential equation, that would be expressed as

Rearranging this equation, we obtain

Thus, equation (3) becomes

1. What is the level of pollution in per after ?

After , , thus the equation becomes

Thus, the pollution will be the amount of smoke in the kitchen per of the kitchen,

1. How long does it take for the level of pollution to fall to per after the toast is taken outside?

It is given that the toaster has been taken out of the room, so the rate of smoke entering the room is now . The differential equation (2) now becomes

We can see that this is a separable differential equation, which can be rearranged as

Integrating both sides, we get

Exponentiating both sides, we get

or

We know that the toaster was removed from the room after , thus when , or after the toaster was removed, the amount of smoke in the room was , this is inferred from (c).

Thus, we now have an initial condition that .

When ,

Thus,

Equation (5) then becomes

Rearranging, we obtain

The pollution in the room at some time can be calculated by where is the volume of the room, and is the amount of smoke in the room.

Thus, for the pollution to drop to ,

Equation (5) then becomes

Therefore, it takes for the pollution to drop to after the toast has been taken out of the kitchen.