MATHEMATICS 1B

MATH1231 ASSIGNMENT

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Question 1:

The function is defined by

Show that is linear.

By definition of a linear transformation, a transformation is **linear** if it preserves the conditions of **addition** and **scalar multiplication**.

**Addition:**

**Scalar Multiplication:**

The transformation defined has a domain and co-domain which are both known vector spaces. Now, we check if the transformation preserves the properties of addition and scalar multiplication.

1. **Addition**

Similarly,

Hence as , the transformation preserves the **addition** property.

1. **Scalar Multiplication**

Similarly,

Hence, as , the transformation preserves **scalar multiplication.**

Since the domain and co-domain of ( and ) are known vector spaces, and preserves the properties of addition and scalar multiplication, is a **linear** transformation.

Question 2:

Show that

is a subspace of

By the Subspace theorem, a subset of a *vector space* over a *field* , *under the same rules for addition and multiplication by scalars*, is a subspace of *if and only if*

1. The vector in also belongs to .
2. is closed under *vector addition*, and
3. is closed under *multiplication by scalars* from .

We are given , a set of vectors of the known vector space .

To prove that is a subspace of , we must prove the existence of zero vector, closure under vector addition, and closure under scalar multiplication.

1. **Existence of zero vector.**

The zero vector, in is given by

and,

Hence,

1. **Closure under vector addition.**

If  then

or

and

Now,

Equations (1), and (2) suggest

Hence , so is closed under vector addition.

1. **Closure under scalar multiplication.**

If , and then

or

We have,

or

Equation (1) suggests

Hence is closed under scalar multiplication.

**(I), (II) and (III)** imply that **S** is a vector subspace of , by the Subspace Theorem.

Question 3:

Let be the set of real polynomials of degree at most . Show that

Is a subspace of

We are given , a set of vectors of the known vector space . To prove that is a subspace, we must show that satisfies the conditions of the Subspace Theorem.

1. **Existence of zero polynomial.**

If then

As , we can write as

Now, equation (1) can be expressed as

The zero polynomial in is

Since , a known vector space, is defined at the zero polynomial, thus

Hence, the zero polynomial is in .

1. **Closure under addition.**

If then

and

Now,

Since and ,

Therefore, equation (1) implies and is a factor of .

Hence so is closed under addition.

1. **Closure under scalar multiplication.**

If and then

As , we can write as

Then,

Since is an arbitrary polynomial in , , and is still a factor of .

Hence , so is closed under scalar multiplication.

**(I), (II) and (III)** imply that **S** is a subspace of , by the Subspace Theorem.