MEEN 672	**Fall Semester 2021**

An Introduction to the Finite Element Method

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ASSIGNMENT No. 7

(Tests the understanding of the material from Chapters 9 and 10)

Date: Oct. 21, 2021 **Due: Oct. 29, 2021**

Problem 1: Consider the steady-state heat transfer (or other phenomenon with a single unknown function u) in a square region shown in Figure 1. The governing equation and boundary conditions are given as follows:

$$-\frac{\partial}{\partial x}\left(k\frac{\partial u}{\partial x}\right)-\frac{\partial}{\partial y}\left(k\frac{\partial u}{\partial y}\right)=f_0 \tag{1.1}$$

$$u(0,y)=y^2\ ,\ u(x,0)=x^2\ ,\ u(1,y)=1-y\ ,\ u(x,1)=1-x \tag{1.2}$$

Assuming $k=1$ and $f_0=2$, determine the unknown nodal value of u using the finite element mesh shown in Figure 1.

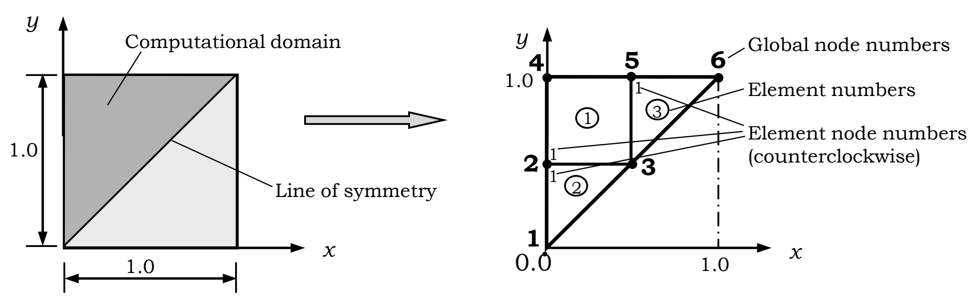

Figure 1: Heat transfer in a 2D region

Problem 2: Consider a **single-degree freedom** problem governed by the differential equation

$$-\frac{\partial}{\partial x}\left(k\frac{\partial u}{\partial x}\right)-\frac{\partial}{\partial y}\left(k\frac{\partial u}{\partial y}\right)=f_0 \tag{2.1}$$

over the domain shown in Figure 2.

(a) Write the finite element equation associated with global node 1 in terms of *element* coefficients $k_{ij}^{(e)}$ and $F_i^{(e)}$ (include only the non-zero contributions).

(b) Compute the contributions of the flux q_0 to F_I (global source vector) for $I = 1$ and $I = 4$.

(c) Compute the explicit (i.e., numerical) contribution of the boundary condition $q_n + 5u = 0$ to both K_{1J} and F_1.

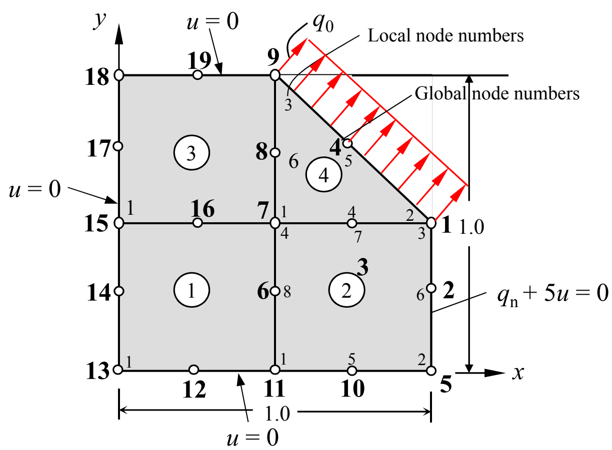

Figure 2: A single-degree of freedom problem in a 2D region

Problem 3: The nodal values of a linear triangular element Ω^e in the finite-element analysis of the field problem

$$-\nabla^2 u = f_0 \tag{3.1}$$

are $u_1^e = U_{10} = 389.79$, $u_2^e = U_{12} = 337.19$, and $u_3^e = U_{11} = 395.08$.

(a) Find the gradient of the solution,

$$\boldsymbol{\nabla} u \equiv \frac{\partial u}{\partial x}\hat{\mathbf{e}}_x + \frac{\partial u}{\partial y}\hat{\mathbf{e}}_y \tag{3.2}$$

in the element domain Ω^e (see Figure 3).

(b) Determine the places where the $u = 392$ isoline intersects the boundary of the element domain Ω^e shown in Figure 3.

(c) Find the contribution of a point source Q_0 located at $(x = 3.75, y = 1.25)$ to the nodes of the element.

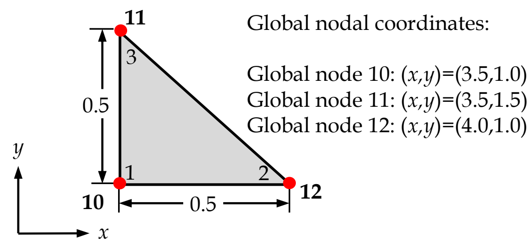

Figure 3: A linear triangular element with nodal locations

Note: The next three problems, **Problems 4–6**, must be solved using computer program **FEM2D**. You must check the results of the problems analyzed for their accuracy by whatever means you have (e.g., exact and numerical solutions from known sources and hand-calculations). You may present the results in tabular and/or graphical form (i.e., you postprocess the results). Submit the output (which includes an echo of the input data) along with the edited results (to limit the amount of output you submit) and a brief discussion of the problem data (how the input data is generated, including the governing differential equation) and the results obtained.

Problem 4: Solve **Problem 9.18** on page 579 (see Figure 4) with 4×8 mesh of linear (a) triangular elements (add diagonals oriented towards the right) and (b) rectangular elements (i.e., two separate analyses), and compare the results for $u(0.5, y)$ vs. y in graphical form with the analytical solution given on page 579:

$$u(x, y) = \frac{\sin \pi x \, \sinh \pi y}{\sinh \pi} \tag{4.1}$$

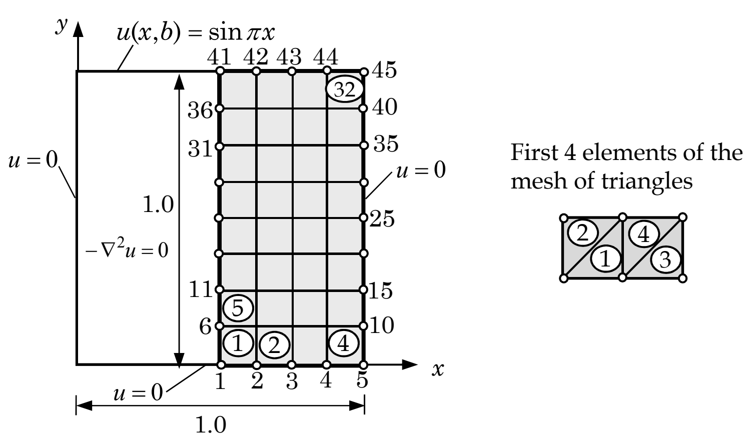

Figure 4: Laplace equation in a square domain

Problem 5: Solve the eigenvalue problem of **Example 9.5.3** (see Figure 5) on page 570 using 8×8 mesh of linear (a) triangular elements and (b) rectangular elements (i.e., two separate analyses) in the **whole domain** and compare with the analytical solution given in the book (see page 570 and Table 9.5.4 on page 571).

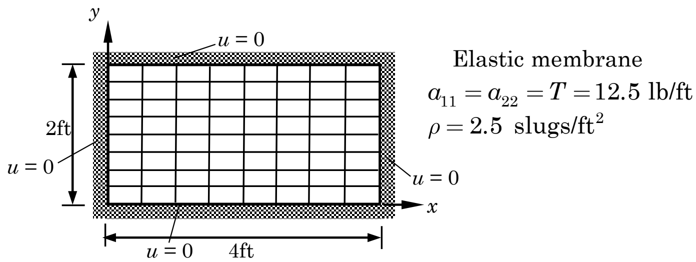

Figure 5: Natural vibrations of a rectangular membrane

Problem 6: Solve the time-dependent problem of **Example 9.5.2** (see Figure 6) and duplicate the results presented in graphical form in Figures 9.5.2 and 9.5.3 on page 568 of the text book.

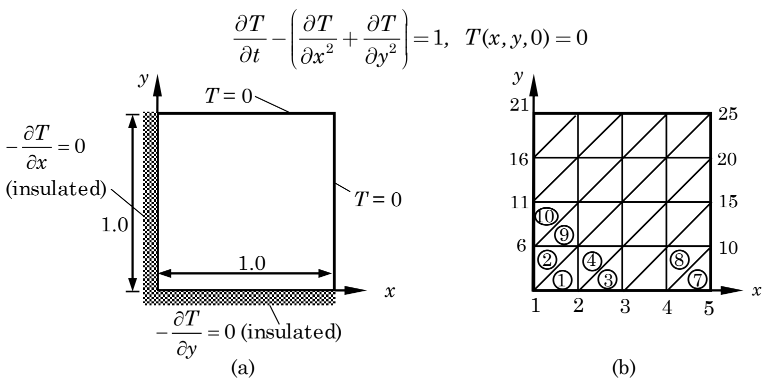

Figure 6: Transient analysis of a heat transfer problem

REMINDER: Test 2 is scheduled for **November 4, 2021**, and it will have questions from the material covered after Test 1 (i.e., Chapters 7 and 9; no computer program related questions will be asked). **Assignments 6 and 7 are most useful for Test 2.**