MEEN 672 **Fall Semester 2021**

An Introduction to the Finite Element Method

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ASSIGNMENT No. 6

(Tests the understanding of material from Chapters 7 and 8 on eigenvalue and transient analysis)

Date: Oct. 13, 2021 **Due: Oct. 21, 2021**

Problem 1: Determine the first two longitudinal frequencies of a rod (with Young's modulus E, area of cross section A, and length L) that is fixed at $x = 0$ and supported axially at the other end (at $x = L$) by a linear elastic spring (with spring constant k), as shown in Fig. 7.3.4 on page 368 of the book. The governing differential equation is

$$-EA\frac{\partial^2 u}{\partial x^2} + \rho A\frac{\partial^2 u}{\partial t^2} = 0 \quad \text{for} \quad 0 < x < L \tag{1.1}$$

$$u(0) = 0, \quad \left(EA\frac{du}{dx} + ku\right)\bigg|_{x=L} = 0 \tag{1.2}$$

Use (a) two linear elements, and (b) one quadratic element to determine the frequencies and mode shapes. Formulate the eigenvalue problems to determine the eigenvalues (square of the frequency). Then compute the numerical values of the frequencies using the following data: $L = 1$, $EA = 1$, $\rho A = 1$ and $EA/k = 0.01$ (it amounts to a normalization of the variables).

Problem 2: The natural vibration of a beam (according to the Euler–bernoulli beam theory) under axial compressive load N^0 is governed by the differential equation

$$EI\frac{d^4 w}{dx^4} + N^0\frac{d^2 w}{dx^2} = \rho A\omega^2\, w$$

where ω denotes frequency of natural vibration, EI is the flexural stiffness, and ρA is the mass per unit length of the beam. (a) Determine the fundamental (i.e., smallest) natural frequency ω of a cantilever beam (i.e., fixed at one end and free at the other end) of length L **when the axial compressive load is zero,** $N_0 = 0$; use one beam finite element. (b) Determine the critical buckling load N^0 of a non-vibrating beam (i.e., $\omega = 0$) using one beam finite element.

Problem 3: The equations governing the motion of a beam according to the Timoshenko beam theory can be reduced to the single equation

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 w}{\partial x^2}\right) + \rho A\frac{\partial^2 w}{\partial t^2} - \frac{\partial}{\partial x}\left[\rho I\left(1 + \frac{E}{K_s G}\right)\frac{\partial^3 w}{\partial x \partial t^2}\right] = 0$$

where E is Young's modulus, G is the shear modulus, ρ is the mass per unit length, A is the area of cross section, and I is the moment of inertia. (a) Formulate the eigenvalue problem associated with the equation and develop its finite element model for the determination of natural frequencies, and (b) formulate the fully discretized problem for the determination of the transient response [use Eqs. (7.4.37a) and (7.4.37b) to write your final answer as $\hat{\mathbf{K}}\boldsymbol{\Delta}^{s+1} = \hat{\mathbf{F}}^{s,s+1}$].

Note: The next two problems, **Problems 4 and 5**, must be solved using computer program **FEM1D**. You must check the results of the problems analyzed for their accuracy by whatever means you have (e.g., exact and numerical solutions from known sources and hand-calculations). You may present the results in tabular and/or graphical form (i.e., you postprocess the results). Submit the output (which includes an echo of the input data) along with the edited results (to limit the amount of output you submit) and a brief discussion of the problem data (how the input data is generated, including the governing differential equation) and the results obtained.

Problem 4: (**Problem 1** above) Determine the first two longitudinal frequencies of a rod (with Young's modulus E, area of cross section A, and length L) that is fixed at $x = 0$ and supported axially at the other end (at $x = L$) by a linear elastic spring (with spring constant k), as shown in Figure 1.

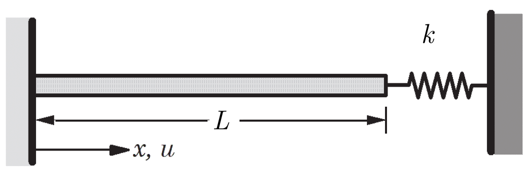

Figure 1: Axial vibrations of a bar.

The governing differential equation and boundary conditions are

$$-EA\frac{\partial^2 u}{\partial x^2} + \rho A\frac{\partial^2 u}{\partial t^2} = 0 \quad \text{for } 0 < x < L$$

$$u(0) = 0, \qquad \left(EA\frac{du}{dx} + ku\right)\Big|_{x=L} = 0$$

Use (a) four linear elements, and (b) two quadratic elements to determine the frequencies and mode shapes. Plot the first two modes shapes. Take $L = 1$, $EA = 1$, $\rho A = 1$ and $EA/k = 0.01$ (it amounts to a normalization of the variables).

Problem 5: Consider the axial motion of an elastic bar, governed by

$$EA\frac{\partial^2 u}{\partial x^2} = \rho A\frac{\partial^2 u}{\partial t^2} \quad \text{for } 0 < x < L$$

with the following data: length of bar $L = 500$ mm, cross-sectional area $A = 1$ mm^2, modulus of elasticity $E = 20,000$ N/mm^2, density $\rho = 0.008$ kg/mm^3, $\alpha = 0.5$, and $\gamma = 0.5$. The boundary conditions are

$$u(0,t) = 0, \quad EA\frac{\partial u}{\partial x}(L,t) = 1 \text{ N}$$

Assume zero initial conditions. Using 20 linear elements and $\Delta t = 0.002$ s, determine the axial displacement and plot the displacement as a function of position along the bar for $t = 0.8$ s. Use Newtons (N) and meters (m) in your input ($EA = 20 \times 10^3$ N and $\rho A = 8$ kg/m).