1 Theory

This chapter starts with a short overview of Intensity Interferometry, followed by an explanation of how Intensity Interferometry is done with Cherenkov Imaging Telescopes. The second part includes a description of Generative Adversarial Networks and the specific network architecture used for this work.

1.1 Intensity Interferometry

Intensity Interferometry (II) was first carried out by R. Hanbury Brown with the Narrabri Stellar Intensity Interferometer, the first optical experiment that resolved the diameter of bright stars at visible wavelengths [1, 2].

In a simple case, an intensity interferometer consists of two light collectors. This can be two optical telescopes, for example, MAGIC-I and MAGIC-II, which are pointed to a star and measure each an intensity $I_1(t), I_2(t)$ [3, 4]. The signals from both detectors are cross-correlated and measured over time. The second-order coherence function is given by:

$$g^{(2)} = \frac{\langle I_1(t) \cdot I_2(t+\tau) \rangle}{\langle I_1(t) \rangle \cdot \langle I_2(t) \rangle} \tag{1}$$

Where τ is the time delay between both telescopes. If the light source is chaotic and randomly polarized, we have:

$$g^{(2)} = 1 + \frac{\Delta f}{\Delta \nu} \cdot |V_{12}| \tag{2}$$

Here, Δf is the electronic bandwidth, $\Delta \nu$ is the optical bandwidth and we have $\Delta f \ll \Delta \nu$. V_{12} is the complex visibility, the Fourier transform of the source brightness distribution [3]. The phase information of V_{12} can not be measured, hence it is lost.

The correlation is often expressed in terms of the normalized contrast, given by:

$$c = \frac{\langle (I_1(t) - \langle I_1 \rangle) \cdot (I_2(t+\tau) - \langle I_2 \rangle) \rangle}{\langle I_1(t) \rangle \cdot \langle I_2(t) \rangle} = g^{(2)} - 1 \tag{3}$$

1.2 Intensity Interferometry with MAGIC

The two MAGIC telescopes are two Imaging Atmospheric Cherenkov Telescopes (IACTs) located at the Roque de los Muchachos Observatory in La Palma, Spain. The main physics channel of MAGIC is the study of Very

High Energy (VHE) gamma rays ($\geq$ 30 GeV) originating from particle showers in the atmosphere. Both telescopes have an array of mirrors with a total diameter of 17m, which focuses the light onto an array of photomultiplier tubes (PMTs) [5]. For Intensity Interferometry, the light of a stellar source is focused on a single PMT, which has a filter in front. The purpose of this filter is to efficiently transmit light around 432nm, which protects the PMTs from excessive light. This is necessary, as II observations are mainly done during full moon nights, where it is too bright for VHE γ-ray observations. The optical signal of the PMTs is transmitted with optical fibers, converted to an electrical signal and digitized [3].
The measurable observable is the Pearson's correlation coefficient:

$$\rho(\tau) = \frac{\langle (I_1(t) - \langle I_1 \rangle) \cdot (I_2(t+\tau) - \langle I_2 \rangle) \rangle}{\sqrt{\langle (I_1(t) - \langle I_1 \rangle)^2 \rangle} \cdot \sqrt{\langle (I_2(t) - \langle I_2 \rangle)^2 \rangle}} \tag{4}$$

Since I_1 and I_2 are proportional to the direct current of the PMTs (DC_i), the normalized contrast can be calculated as in equation 5. One must also correct for a non-constant gain of the PMTs, as well as the moon. The latter can be done by measuring the background light with an additional PMT with no mirrors focused on it [3].

$$c \propto \frac{\rho}{\sqrt{(DC_1 \cdot DC_2)}} \tag{5}$$

It must be noted that the calculation of Pearson's correlation coefficient is non-trivial: MAGIC makes use of the Convolution Theorem for discrete Fourier transforms, because it is computationally more efficient.

The significance of the signal can be expressed as the signal-to-noise ratio S/N, which depends on a lot of factors, but most importantly it is proportional to the complex visibility, which itself depends on the distance between the telescopes, shown in equation (6) [3].

$$S/N = A \cdot \alpha \cdot q \cdot n \cdot |V_{12}|^2(d) \cdot \sqrt{b_v} \cdot F^{-1} \cdot \sqrt{\frac{T}{2}} \cdot \sigma \tag{6}$$

Here, A is the total mirror area, α the quantum efficiency of the PMTs, q the quantum efficiency of the optics, n the differential photon flux from the source, and b_v the cross-correlation bandwidth. The noise of the PMTs is accounted for with F, T denotes the observation time, and σ is the normalized spectral distribution of the light (including filters) [3]. While most of the parameters can be optimized with hardware, mainly higher observation time will lead to better signal to noise ratios.

1.3 Baseline considerations

Since the complex visibility depends on the distance between the telescopes, also called the baseline d, one can measure the angular size of the star using equation (7). The idea is to measure with different baselines, such that the size of the star can be fitted. This has been done for several examples [3, 6].

$$V_{12}(d) = \frac{c(d)}{c(0)} \tag{7}$$

If the source is at the zenith, that is perpendicular to the line of sight, the coordinates in the Fourier plane (u, v) are given by:

$$(u, v) = \frac{1}{\lambda}(d_N, d_E) \tag{8}$$

Where B_N and B_E are the baseline coordinates expressed in north and east coordinates. Since not all sources are at the zenith, and the telescopes are stationary, the source moves across the sky during the observation. The baseline rotation is then given by equation (9), which traces an ellipse for every pair of telescopes. Furthermore, the different altitudes d_A of the telescopes must be considered [4, 7].

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = R_x(\delta) \cdot R_y(h) \cdot R_x(-l) \begin{pmatrix} d_N \\ d_E \\ d_A \end{pmatrix} \tag{9}$$

Here, δ is the declination and h is the hour angle of the stellar source, and l is the latitude of the telescopes. The three matrices R_i correspond to the fundamental representation of the SO(3) group [7].

Since every pair of telescopes traces an ellipse in the Fourier plane, the total number of ellipses scales as:

$$\mathcal{N} = \frac{N_T \cdot (N_T - 1)}{2} \tag{10}$$

Hence Intensity Interferometry benefits especially well from a large number of telescopes, as it is planned with the Cherenkov Telescope Array (CTA).

1.4 Generative Adversarial Networks

Generative Adversarial Networks (GANs) were invented by Ian Goodfellow in 2014. The concept is rather simple, as it consists of two competing networks. The first network, called the generator, creates new images based on an input image. For consistency, the images generated by the generator will be denoted as output images. The second network, the discriminator, tries to distinguish between the output image and the real image, also called the ground truth [8]. As a consequence of training both networks alternately, the output images eventually become indistinguishable from the real images. This is nothing than a two-player min-max game, a famous problem in game theory. Goodfellow initially proposed the problem as:

$$\min_G \max_D V(D,G) = \mathbb{E}_{x \sim p_{data}(x)} [\log D(x)] + \mathbb{E}_{z \sim p_z(z)} [\log (1 - D(G(z)))] \tag{11}$$

Here, $V(D,G)$ is the value function of the min-max game. The goal is now to learn the distribution of the generator p_g over the data x. We have input noise variables $p_z(z)$, as well as the two perceptrons $G(z;\theta_g)$ and $D(x;\theta_d)$ with parameters θ_i. G represents a differential function that maps from z to the data space x, while $D(x)$ represents the probability that x is from the real data [8].
The problem can be reformulated as:

$$\max_D V(G,D) = \mathbb{E}_{x \sim p_{data}} [\log D^*_G(x)] + \mathbb{E}_{x \sim p_g} [\log (1 - D^*_G(x))] \tag{12}$$

Where D^*_G denotes the optimum of the discriminator for a given fixed generator, shown in equation (13). It can now be shown that the global optimum of equation (12) is reached if and only if $p_g = p_{data}$. Furthermore, if both G and D are allowed to reach their optima, p_g converges to p_{data}. A more detailed discussion of the problem, including proofs can be found in [8].

$$D^*_G(x) = \frac{p_{data}(x)}{p_{data}(x) + p_g(g)} \tag{13}$$

GANs can be extended to a conditional model [9]. In this case, both generator and discriminator receive extra information y, such that the value function of the conditional GAN can be expressed as:

$$V(D,G) = \mathbb{E}_{x \sim p_{data}(x)} [\log D(x|y)] + \mathbb{E}_{z \sim p_z(z)} [\log (1 - D(G(z|y))] \tag{14}$$

Furthermore, [10] observed that combining the conditional GAN given in equation 14 with the traditional L1-loss improves results, as the generator tries to be close to the ground truth. Hence the function, that is minimized is:

$$L_{tot} = \arg \min_{G} \max_{D} V(D,G) + \lambda \cdot L_1(G) \tag{15}$$

Where $L_1(g)$ is given by equation (16), and $\lambda = 100$.

$$\mathbb{E}_{x,y,z} \left[\|y - G(x,z)\|_1 \right] \tag{16}$$

This type of network is shown to be very robust, as it has been applied to several problems. Examples include creating colored images from grayscale images, creating images of facades based on architectural labels, changing from day to night in different pictures, and predicting a map based on satellite data. A longer list of applications is given in [10].

1.4.1 Generator

The generator is a common U-Net convolutional network, described in [11]. It consists of a contracting part, reducing the image size, and an expansive part, where the image is enlarged. The contracting part consists of repeated application of alternating convolutional layers and rectified linear unit (ReLU) layers. The expansion consists of repeated convolution, batch normalization, and ReLU layers. The first contracting step also includes a dropout layer [10]

1.4.2 Discriminator

The discriminator, named PatchGAN, does not work globally but rather classifies single patches as real or fake. The overall result is then estimated based on the average. The model architecture is as follows: zero padding, convolution, batch-norm, ReLu, zero-padding, and convolution [10].

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