

3D-Transformation

Curve Modeling

Bezier Spline Curves

- Bezier splines have a number of properties that make them highly useful and convenient for curve.
- Bezier curve can be fitted to any number of control points.
- The number of control point to be approximated and their relative position determine the degree of Bezier polynomial.
- Bezier curve can be specified with boundary conditions.
- control points using blending f^n , characterizing matrix, or boundary conditions.

*

* Bezier Curve Equation *

* Suppose we are giving $\frac{1}{n}$ number of control position + 1.

$P_k = (x_k, y_k, z_k)$ with k varying from 0 to n .

→ $P_0, P_1, P_2, \dots, P_n$ (every point of position P_k , P_k expressed as x_k, y_k, z_k)

* These co-ordinate points can be blended to produce the following position vector $P(u)$, which describes the path of an approximating Bezier polynomial f^n between P_0 and P_n .

* All these points they will be blended or put together, they will be combined together to P_0, \dots

$$* P(u) = \sum_{k=0}^n P_k BE_{k,n}(u), \quad 0 < u < 1 \quad \text{--- (1)}$$

$P(u) \Rightarrow$ Describe the path of the polynomial f^n between the point P_0 and P_n . $P(u)$ express that the summation of k is equal to 0 to n ($k=0 \dots n$)

$P(k)$ and then we have $BE_{k,n}(u)$, where u is varying between 0 and 1.

3. BEZ → this called as the ~~Binomial~~ ^{Bezier} polynomials or this is called the blended f^n . The BEZ blending f^n is also called as the BEZ of k, n of (u) , where u is varying from $(0, 1)$

The Bezier blending f^n $BEZ_{k,n}(u)$ are the Bernstein Polynomials/ $b_{i,n}(t) = \frac{n!}{i!(n-i)!} t^i (1-t)^{n-i}$ $(k=0 \text{ The value of } (0-1))$
 $BEZ_{k,n}(u) = C(n,k) u^k (1-u)^{n-k}$ — (2) $n = \text{number of the control points}$

where, parameters $C(n,k)$ are the binomial coefficients

$$C(n,k) = \frac{n!}{k!(n-k)!} \quad \text{--- (3)}$$

Equation 1 represents a set of three parametric equations for the ^{three} individual curve co-ordinates

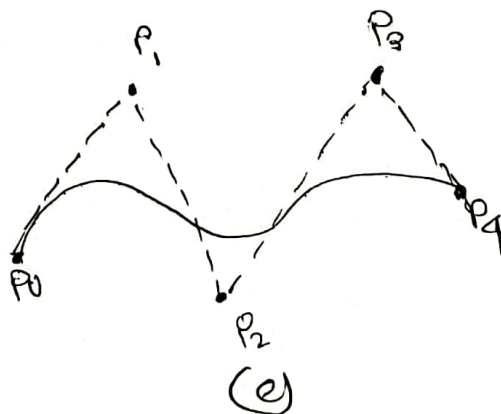
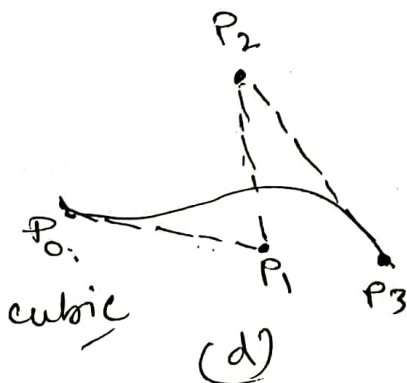
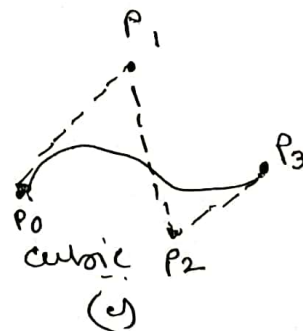
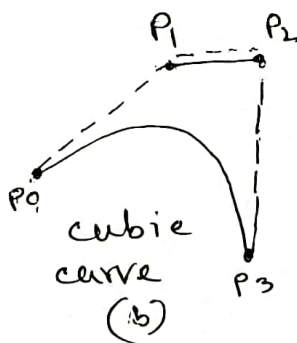
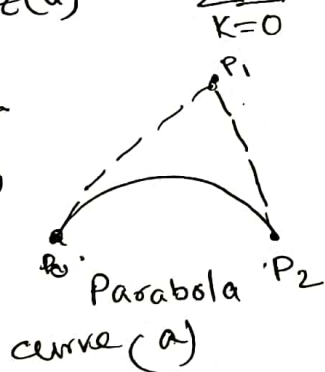
$$x(u) = \sum_{k=0}^n x_k BEZ_{k,n}(u)$$

$$y(u) = \sum_{k=0}^n y_k BEZ_{k,n}(u)$$

$$z(u) = \sum_{k=0}^n z_k BEZ_{k,n}(u)$$

$P(u)$ is nothing but Bezier Polynomial f^n which is express the — (4) term of

Spline presentation



* Two dimensional Bezier curve generated with three, 4, 5 control points.

Figure appearance of some Bezier curves for various selection of control points in the xy plane ($z=0$)

* Suppose we have control point at same co-ordinate points position, these set of control-point produce a Bezier curve, that is only a single point.

a) P_1 it is pulling the curve up ^{towards} ~~to~~ direction. P_0 is starting point P_2 is the ending point. P_1 is another point which is lying to P_1 and P_2 . That's why curve is oriented towards to P_1 .

b) How P_0 has control point? P_3 is a last control point in between P_0 and P_3 and P_1, P_2 . P_1 and P_2 their controlling the curve direction. so the curve has been drag towards P_1 and P_2 . It is more towards because the P_1 is closer here. P_2 also having control . . .

c) It has four control points but we can see that after control point P_0 there having P_1 and having P_2 and P_3 . $P_0 \rightarrow$ towards to P_1 and towards $P_2 \dots P_3$.

d) P_0, P_1, P_2, P_3 (curve is similarly oriented)

e)

lets take 4 control point, where $n=3$

P_0, P_1, P_2, P_3

$$B(t) = P_0 b_{0,3}(t) + P_1 b_{1,3}(t) + P_2 b_{2,3}(t) + P_3 b_{3,3}(t)$$

$$0 < t < 1$$

$$\begin{aligned} * b_{0,3}(t) &= 3c_0 t^0 (1-t)^3 \\ &= (1-t)^3 \end{aligned} \quad , \quad 3c_0 = \frac{3!}{0!(3-0)!} \quad \checkmark$$

Same way

$$B_{1,3}(t) = 3t(1-t)^2$$

$$B_{2,3}(t) = 3t^2(1-t)$$

$$B_{3,3}(t) = t^3$$

$$x_0(t) = x_0(1-t)^3 + x_{1,3}t(1-t)^2 + x_{2,3}t^2(1-t) + x_3t^3$$

$$y_0(t) = y_0(1-t)^3 + y_{1,3}t(1-t)^2 + y_{2,3}t^2(1-t) + y_3t^3$$

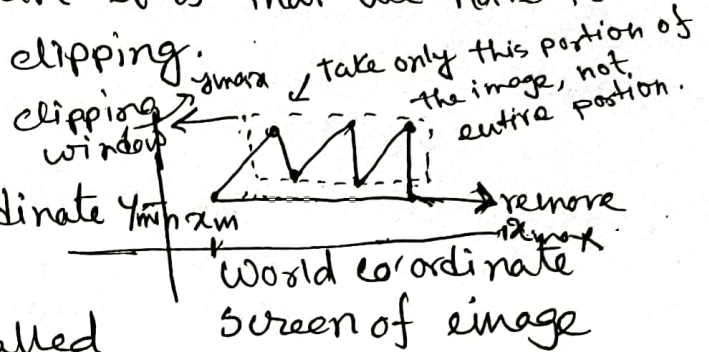
Clipping

* what we understand clipping?

→ we have clipping the part, means we are cutting the part, so, whatever the unnecessary part it is that we have to removed. that we call it as a clipping.

→ let us take, a image.

→ window is given with a co-ordinate x_{min} , y_{min} , x_{max} , y_{max} is called clipping window.



→ One window is given with co-ordinates (x_{min}, y_{min}) & (x_{max}, y_{max}) is called clipping window.

→ Only this portion of image has to be displayed on.
→ So, we want to remove, portion is called clipping window.

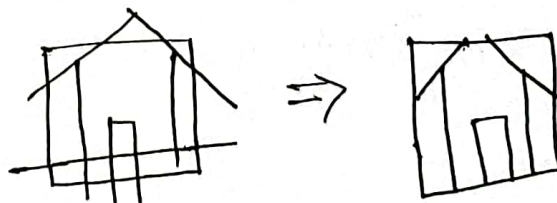
→ Why maximum, and minimum?

→ Means the portion of the clipping window we are assigning with x_{min} and x_{max} values, and the y_{min} and y_{max} values.

→ we have to display the object which is coming inside clipping window and destroy the part that is outside the window has to be clipped.

Applications of clipping

→ clipping will extract part what we ~~design~~ desire.



* It will extract part we ~~design~~ desire

* To identify the visible and invisible area in 3D object

- * For creating object using solid modeling
- * For drawing operations.
- * For deleting, copying, moving part of an objects.

Types of clipping

- Point clipping
- Line clipping
- Polygon clipping (Area clipping)

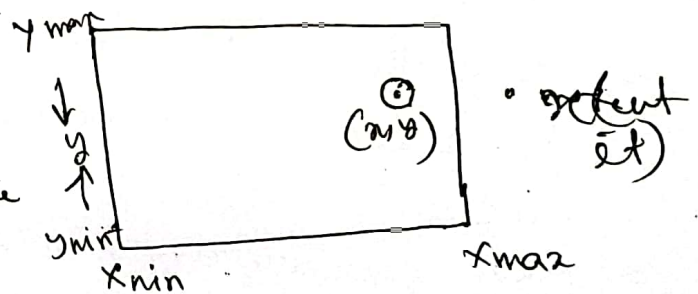
→

point clipping

- It is used to determining whether the point is inside the window or not.
- If point is coming inside the clipping window, we have to display it; otherwise no need to display it.

* Entire the window, having a point which is (x, y)

* We need to check whether the point is inside the window or not we don't know.



* we need to check condition \Rightarrow whether its inside the

clipping window or not

1. ~~$x \leq x_{max}$~~ $x \leq x_{max}$ (x should be less than or equal to x_{max})
2. $x \geq x_{min}$
3. $y \leq y_{max}$
4. $y \geq y_{min}$

* Line clipping is same as point clipping

* The line appear outside the clipping window that line has to be discarded

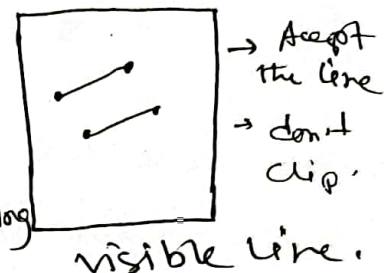
* Line that is present inside that has to be accepted.

* So, we need a intersecting point.

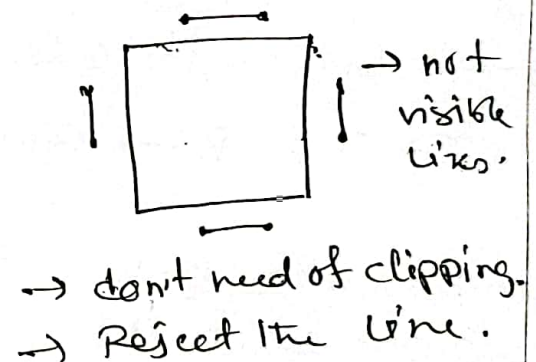
→ The part of the line inside the window is kept & the part of the line appearing outside is removed - ~~the~~ line clipping.

* if line and both the endpoints or present inside the clipping window

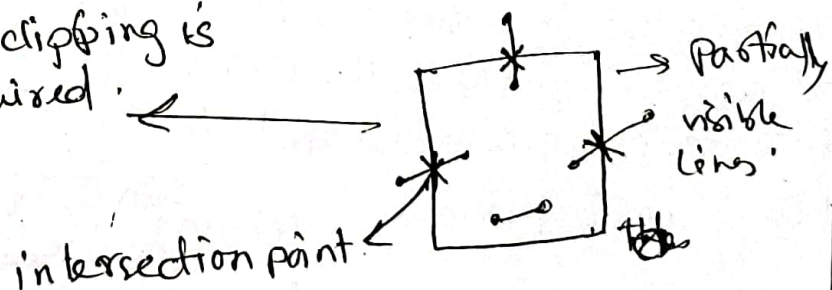
* both points are present inside the clipping window / visible lines



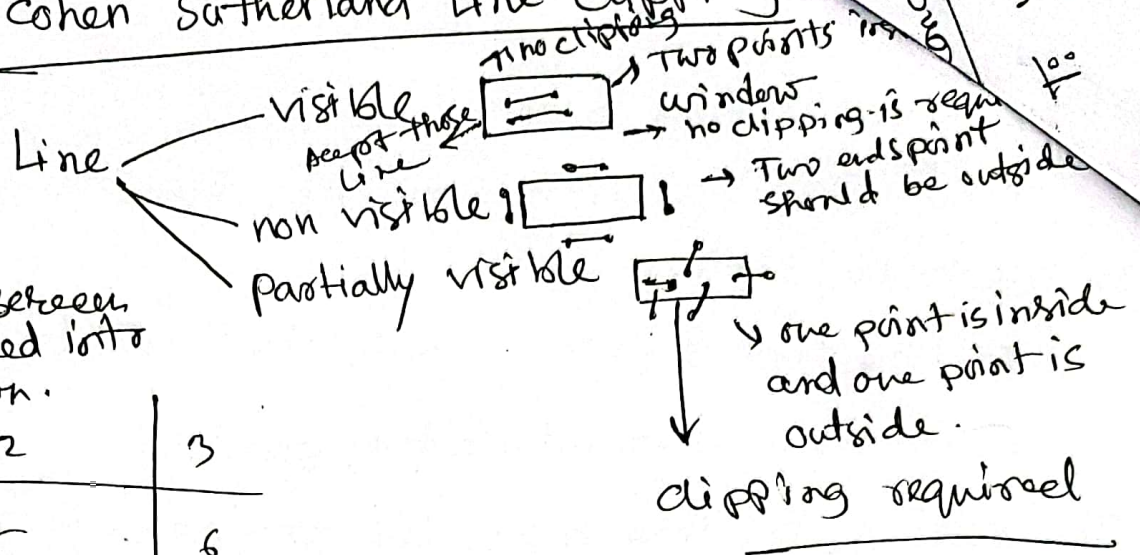
→ If the line / both the endpoints are present outside the window, then we call not visible lines.



Here, clipping is required.



Cohen Sutherland Line Clipping



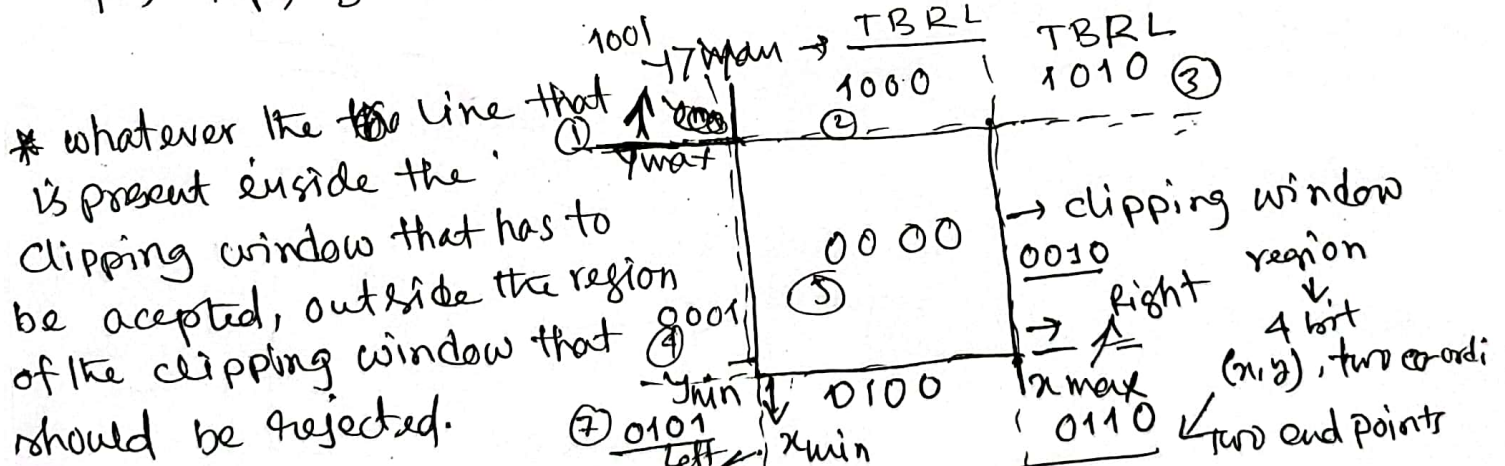
→ The complete screen is completely divided into 9 regions.

region 1	2	3
4	5	6
7	8	9

→ So, here we've divided the screen into 9 regions.
 → Every region we have to divide it into code [code is 4 bit]

→ 4 bit region code represents - TBRL

T → Top, B → Bottom, R → Right, L → Left.



→ we already divided complete screen into 9 regions, the centre area having the code 0000. [we said each code is a 4 bit]

→ x, should be x_{min} and x_{max} .

→ y should be y_{min} and y_{max} .

Condition — 4 bit form - TBRL

T: $y > y_{max}$ → y is coordinate that has to be present in between y_{min} , y_{max} .

if y is greater than y_{max} is crossing the window as

B: $y < y_{min}$ → below the window

R: $x > x_{max}$ → crossing the window.

L: $x < x_{min}$ → left the window.

lets take, top ⇒ crossing the outside the window —

Top - 1001

1

Top has to said 1, if above the window.

Bottom

Right

Left

[because is the y_{max} in left side]

This vision code in 4 bit.