

## 3D-Transformation

### Curve Modeling

#### Bezier Spline Curves

- Bezier splines have a number of properties that make them highly useful and convenient for curve.
- Bezier curve can be fitted to any number of control points.
- The number of control point to be approximated and their relative position determine the degree of Bezier polynomial.
- Bezier curve can be specified with boundary conditions.
- control points using blending fn, characterizing matrix, or boundary conditions.

\*

#### \* Bezier Curve Equation \*

\* Suppose we are giving  $n+1$  control-point positions  $\uparrow$   $n$  number of control position + 1.

$P_K = (x_K, y_K, z_K)$  with  $K$  varying from 0 to  $n$ .

→  $P_0, P_1, P_2, \dots, P_n$  (every point of position  $P_K$ ,  $P_K$  expressed as  $x_K, y_K, z_K$ )

\* These co-ordinate points can be blended to produce the following position vector  $P(u)$ , which describes the path of an approximating Bezier polynomial fn between  $P_0$  and  $P_n$ .

\* All these points they will be blended or put together, they will be combined together to pr...

$$* P(u) = \sum_{K=0}^n P_K BE_{K,n}(u), \quad 0 \leq u \leq 1 \quad \text{--- (1)}$$

$P(u) \Rightarrow$  Describe the path of the polynomial fn betw the point  $P_0$  and  $P_n$ .  $P(u)$  express that the summation of  $K$  is equal to 0 to  $n$  ( $K=0 \dots n$ )

$P(K)$  and then we have  $BE_{K,n}(u)$ , where  $u$  is varying between 0 and 1.



3E4 → this called as the ~~Bernstein~~ <sup>Bezier</sup> polynomial, or this is called the blended  $f^n$ . The BEZ blending  $f^n$  is also called as the BEZ of  $k, n$  of  $(u)$ , where  $u$  is varying from  $(0, 1)$

The Bezier blending  $f^n$   $BEZ_{k,n}(u)$  are the Bernstein Polynomials/ $b_{i,n}(t) = \frac{n!}{i!(n-i)!} t^i (1-t)^{n-i}$   $(k=0 \text{ The value of } (0-1))$   
 $BEZ_{k,n}(u) = \frac{n!}{k!(n-k)!} u^k (1-u)^{n-k}$  — (2)  $n = \text{number of the control point}$

where, parameters  $C(n,k)$  are the binomial co-efficients

$$* \text{ } nCk / C(n,k) = \frac{n!}{k!(n-k)!} \quad \text{--- (3) } \frac{n!}{i!(n-i)!}$$

Equation 1 represents a set of three parametric equations for the <sup>three</sup> individual curve co-ordinates

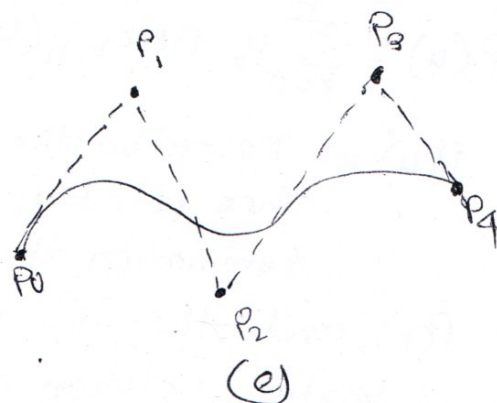
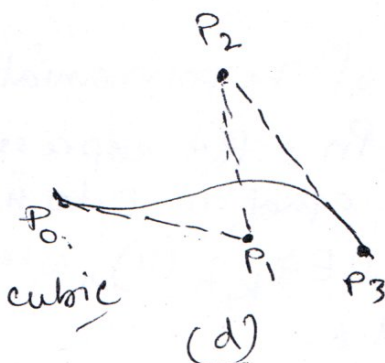
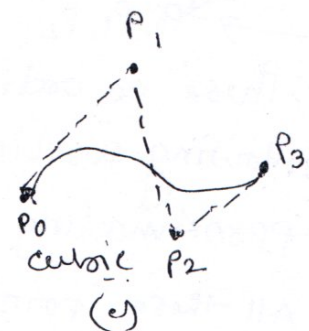
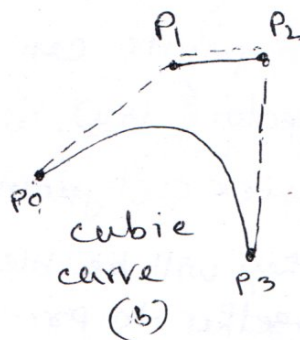
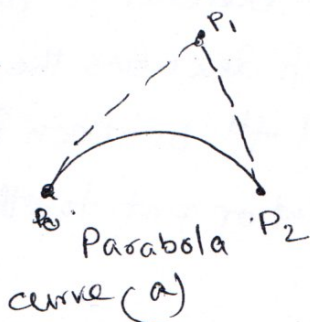
$$x(u) = \sum_{k=0}^n x_k BEZ_{k,n}(u)$$

$$y(u) = \sum_{k=0}^n y_k BEZ_{k,n}(u)$$

$$z(u) = \sum_{k=0}^n z_k BEZ_{k,n}(u)$$

$\left\{ \begin{array}{l} P(u) \text{ is nothing but} \\ \text{Bezier Polynomial } f^n \\ \text{which is express the} \\ \text{--- (4) term of} \end{array} \right.$

Spline representation



\* Two dimensional Bezier curve generated with three, 4, 5 control point \*



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\* Figure appearance of some Bezier curves for various selection of control points in the  $xy$  plane ( $z=0$ )

\* Suppose we have control point at same co-ordinate points position, these set of control-point produce a Bezier curve, that is only a single point.

a)  $P_1$  it is pulling the curve up ~~to~~<sup>towards</sup> direction.  $P_0$  is starting point  $P_2$  is the ending point.  $P_1$  is another point which is lying to  $P_1$  and  $P_2$ . That's why curve is oriented towards to  $P_1$ .

b) How  $P_0$  has control point?  $P_3$  is a last control point in between  $P_0$  and  $P_3$  and  $P_1, P_2$ .  $P_1$  and  $P_2$  their controlling the curve direction. so the curve has been drag towards  $P_1$  and  $P_2$ . It is more towards because the  $P_1$  is closer here.  $P_2$  also having control...

c) It has four control points but we can see that after control point  $P_0$  there having  $P_1$  and having  $P_2$  and  $P_3$ .  $P_0 \rightarrow$  towards to  $P_1$  and towards  $P_2 \dots P_3$ .

d)  $P_0, P_1, P_2, P_3$  (curve is similarly oriented)

e)

lets take 4 control point, where  $n=3$

$P_0, P_1, P_2, P_3$

$$B(t) = P_0 b_{0,3}(t) + P_1 b_{1,3}(t) + P_2 b_{2,3}(t) + P_3 b_{3,3}(t) \quad \boxed{0 \leq t \leq 1}$$

$$\begin{aligned} * b_{0,3}(t) &= 3c_0 t^0 (1-t)^3 \\ &= (1-t)^3, \quad 3c_0 = \frac{3!}{0!(3-0)!} \quad \checkmark \end{aligned}$$

Same way

$$B_{1,3}(t) = 3t(1-t)^2$$

$$B_{2,3}(t) = 3t^2(1-t)$$

$$B_{3,3}(t) = t^3$$

$$x_0(t) = x_0(1-t)^3 + x_{1,3}t(1-t)^2 + x_{2,3}t^2(1-t) + x_{3,3}t^3$$

$$y_0(t) = y_0(1-t)^3 + y_{1,3}t(1-t)^2 + y_{2,3}t^2(1-t) + y_{3,3}t^3$$



# Clipping

\* what we understand clipping?

→ we have clipping the part, means we are cutting the part, so, whatever the unnecessary part it is that we have to removed. that we call it as a clipping.

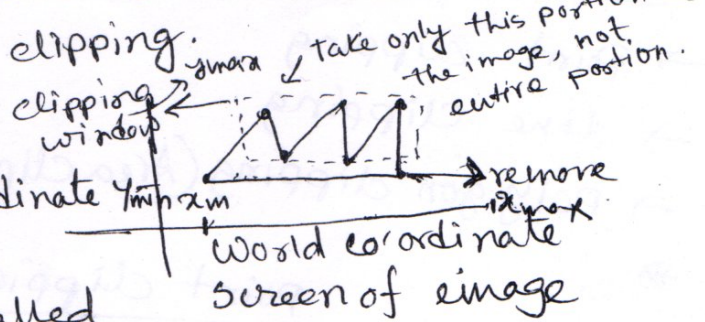
→ let us take, a image.

→ window is given with a co-ordinate  $y_{min}$   $x_{min}$

$x$  - minimum,  $y$  - minimum.

$x$  - maximum,  $y$  - maximum is called

clipping window.



→ One window is given with co-ordinates  $(x_{min}, y_{min})$  &  $(x_{max}, y_{max})$  is called clipping window.

→ Only this portion of image has to be displayed on.

→ So, we want to remove, portion is called clipping window.

→ Why maximum, and minimum?

→ Means the portion of the clipping window we are assigning with  $x$  minimum and  $x$  - maximum values. and the  $y_{min}$  and  $y_{max}$  values.

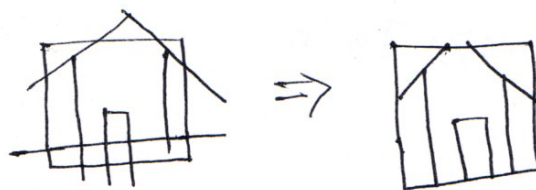
→ we have to display the object which is coming inside clipping window and destroy the part that is outside the window has to be clipped.

## Applications of clipping

→ clipping will extract part what we ~~design~~ desire.

\* It will extract part we ~~design~~ desire

\* To identify the visible and invisible area in 3D object





- \* For creating object using solid modeling
- \* For drawing operations.
- \* For deleting, copying, moving part of an objects.

## Types of clipping

- Point clipping
- Line clipping
- Polygon clipping (Area clipping)



### point clipping

- It is used to determining whether the point is inside the window or not.
- If point is coming inside the clipping window, we have display it; otherwise no need to display it

\* Entire the window, having a point which is  $(x, y)$

\* We need to check whether the point is inside the window or not we don't know.

\* we need to check condition  $\Rightarrow$  whether its inside the

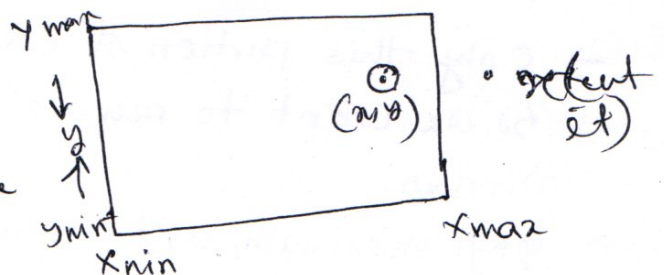
using window or not

1.  ~~$x \leq x_{max}$~~   $x \leq x_{max}$  ( $x$  should be less than or equal to  $x_{max}$ )

2.  $x \geq x_{min}$

3.  $y \leq y_{max}$

4.  $y \geq y_{min}$





## Line clipping

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\* Line clipping is same as point clipping

\* The line appear outside the clipping window that line has to be discarded

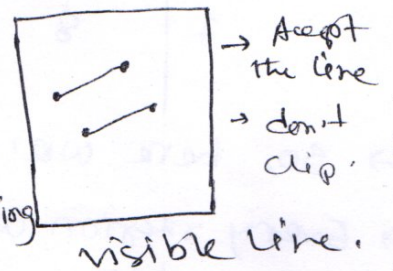
\* Line that is present inside that has to be accepted.

\* So, we need a intersecting point.

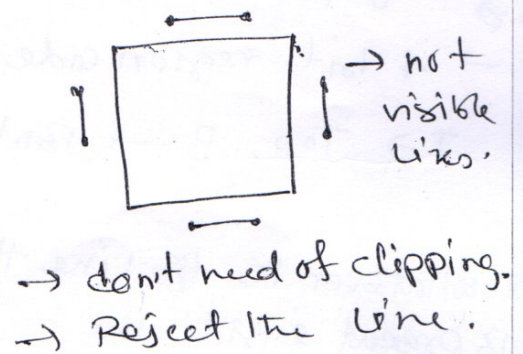
→ The part of the line inside the window is kept & the part of the line appearing outside is removed - the line clipping.

\* if line and both the endpoints or present inside the clipping window

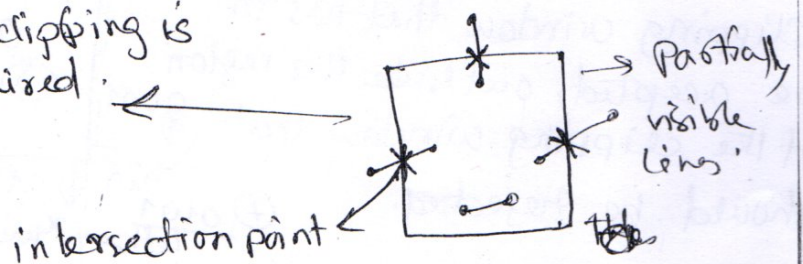
\* both points are present inside the clipping window / visible lines



→ If the line / both the endpoints are present outside the window, then we call not visible lines.

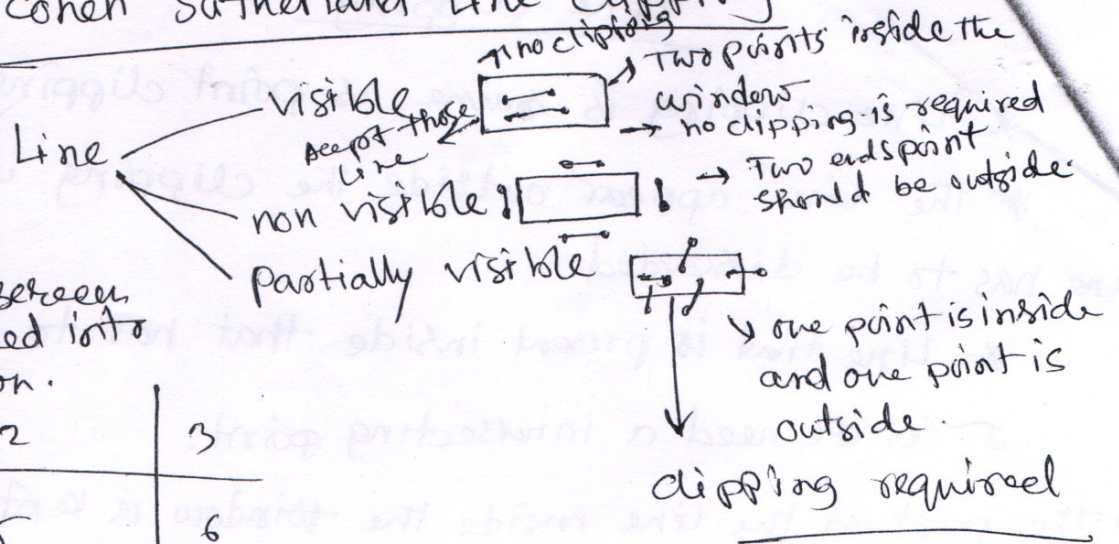


Here, clipping is required.





Cohen Sutherland Line Clipping



→ The complete screen is completely divided into total line region.

region 1	2	3
4	5	6
7	8	9

- So, here we've divided the screen into 9 regions.
- Every region we have to divided in into code [code is 4 bit]

→ 4 bit region code represents - TBR1

T  $\rightarrow$  Top, B  $\rightarrow$  Bottom, R  $\rightarrow$  Right, L  $\rightarrow$  Left.

\* whatever the ~~the~~ line that is present inside the clipping window that has to be accepted, outside the region of the clipping window that should be rejected.

Diagram illustrating a clipping window region. The window is defined by coordinates (0,0), (1,0), (1,1), and (0,1). A line segment is shown passing through the window. The line is divided into segments labeled 1 through 7. Segment 1 is outside the window (y > 1). Segment 2 is on the top edge (y=1). Segment 3 is inside the window (0 < y < 1). Segment 4 is on the right edge (x=1). Segment 5 is inside the window (0 < x < 1). Segment 6 is on the bottom edge (y=0). Segment 7 is outside the window (y < 0). Annotations include: 'clipping window region' with a downward arrow, 'right 4 bit (x,y), two coord' with a downward arrow, and 'two end points' with an arrow pointing to the line segment. Bit representations for coordinates are shown: (0,1) as 1001, (1,1) as 1010, (1,0) as 0100, and (0,0) as 0000. The line segment is labeled 'TBR L' at both ends.

→ we already divided complete screen into 4 region, the centre area having the code 0000. [we said each code is a 4 bit]

→  $x_i$  should be  $z_{\min}$  and  $z_{\max}$ .

→  $y$  should be  $y_{\min}$  and  $y_{\max}$ .



## Condition — 4 bit form - TBRL

T:  $y > y_{max}$  →  $y$  is coordinate that has to be present in — between  $y_{min}$ ,  $y_{max}$ .

if  $y$  is greater than  $y_{max}$  is crossing the window <sup>as</sup>

B:  $y < y_{min}$  → below the window

R:  $x > x_{max}$  → crossing the window.

L:  $x < x_{min}$  → left the window.

lets take, top ⇒ crossing the outside the window —

Top - 1001

1 1 1 1

Top has to said 1, if above the window.

Bottom

Right

Left [because is the  $y_{max}$  in left side]

← This vision code in 4 bit.