Canadian Math Challengers Society

Exam Archive

2005 to 2019

2005 Regional and Provincial Questions

2006 Regional and Provincial Questions
2006 Regional and Provincial Answers

2007 Regional and Provincial Questions
2007 Regional and Provincial Answers

2008 Regional and Provincial Questions
2008 Regional and Provincial Answers

2009 Regional and Provincial Questions
2009 Regional and Provincial Answers

2010 Regional and Provincial Questions
2010 Regional and Provincial Answers

2011 Regional and Provincial Questions
2011 Regional and Provincial Answers

2012 Regional and Provincial Questions
2012 Regional and Provincial Answers

2013 Regional and Provincial Questions
2013 Regional and Provincial Answers

2014 Regional and Provincial Questions   face off missing
2014 Regional and Provincial Answers   face off missing

2015 Regional and Provincial Questions
2015 Regional and Provincial Answers

2015 Regional and Provincial Questions
2016 Regional and Provincial Answers

2017 Regional and Provincial Questions

2018 Regional Questions

2108 Provincial Questions
2019 Regional Questions

Page 1: Problem Solving

1. A class of 30 students took a test, and the class average was 70. The five students who failed had marks of 20, 25, 25, 30, and 40. What was the average mark among the students who didn't fail? Give the answer correct to 1 decimal place.
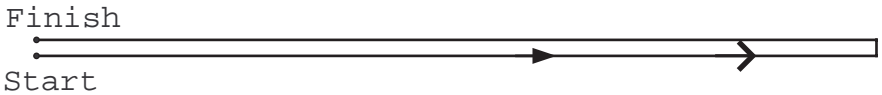
   1. _____

2. Beta has twice as many pennies as Alpha. Gamma has three times as many pennies as Beta. Between the three of them, they have fewer than 80 pennies. What is the largest possible number of pennies that they could have between them?

   2. _____

3. Xaviera jogs at a steady rate of 5 minutes per kilometre. Yolande jogs at 7 minutes per kilometre. They start at the same time on an out and back run that consists of 15 km due east followed by 15 km due west. How many kilometres from the finish line is Xaviera when they pass next to each other? Give the answer correct to 1 decimal place.

   3. _____

   ```
   Finish
   •————————————————————————————————————————>
   Start
   ```

4. It so happens that 7700625 is a perfect square, that is, the square of a whole number, and that 7706176 is the next perfect square. What is the first perfect square greater than 7706176?

   4. _____

Page 2: Combinatorics

5. In a game, you toss two standard dice, a silver one and a gold one. Your *score* is the number showing on the silver one, plus twice the number showing on the gold one. What is the probability that your score is 8? Express your answer as a fraction in lowest terms.

5. [ ]

6. There are 5 people in a family, all of different heights. We want to line them up for a picture, with the tallest person in the middle, and so that as we go from left to right, the heights of the people increase and then decrease. How many ways are there to do this?

6. [ ]

7. Let A, B, and C be the measures, in degrees, of the angles of a triangle, where $A \leq B \leq C$. How many possibilities are there for the ordered triple (A, B, C), given that each of A, B, and C is a multiple of 15?
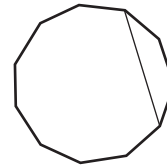
7. [ ]

8. How many 4-digit numbers are there which obey all three of the following rules: (i) no digit other than 1, 2, 3, or 4 is to be used; (ii) a digit may occur more than once; (iii) as you read the number from left to right, digits never decrease (so 1134 is OK, but 3314 is not)?
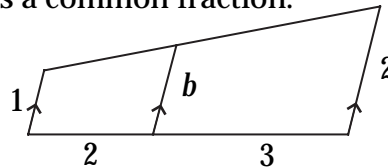
8. [ ]

Page 3: Geometry

9. A regular polygon has 11 sides. How many diagonals does it have? A *diagonal* is a line segment that joins two corners but is not a side. One diagonal is shown in the picture.
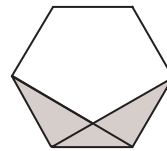
9.

10. Lines that look parallel in the picture *are* parallel. Express b, the length of the "middle" line segment, as a common fraction.
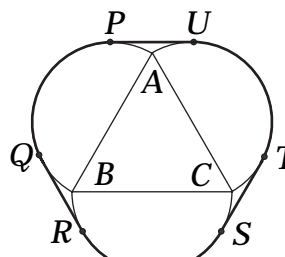
10.

11. The figure below is a regular hexagon with area 1. Express the area of the shaded region as a common fraction.

11.

12. Each side of triangle ABC has length 2. Three semicircles are constructed outside ABC with the three sides AB, BC, CA as diameters. A thread $PQRSTUP$ is tied tightly around the resulting "three-leaf clover" shape so that PQ, RS, TU are arcs of the semicircles with diameters AB, BC, CA respectively and QR, ST, UP are line segments. Find the area enclosed by the thread. Give your answer in terms of π.

12.

1. What is the largest number that is less than 100 and has exactly 3 whole number factors?

1.

2. Triangle ABC has a right angle at A. The two legs AB and AC have lengths 9 centimetres and 40 centimetres. What is the length, in centimetres, of the hypotenuse BC?

2.

3. Given that $\dfrac{1}{2x} + \dfrac{3}{2} = \dfrac{3}{4x} + \dfrac{5}{4}$, what is the value of x?

3.

4. If a computer is worth a certain amount, it is worth 40% less a year later. Alphonse's computer is now worth \$2000. How many dollars will it be worth 3 years from now?

4.

5. Suppose that a and b are real numbers such that $a^b = 5$. What is the value of $a^{3b} - 6$?

5.

6. The cost of sending a parcel is \$4.00 for the first kilogram, and \$0.60 for each additional kg. A certain parcel weighs a whole number of kg, and costs \$40.00 to send. How many kg does the parcel weigh?

6.

7. A rectangular field is 50% longer than it is wide. The perimeter of the field is 300 metres. What is the area of the field, in square metres?

7.

8. Alphonse sold a house to Beth for \$300,000. She sold it immediately to Gamal at a 10% profit. Then Gamal sold the house back to Alphonse at a 10% loss. What was Alphonse's overall profit (in dollars)?

8.

9. A string of length 120 centimetres is cut into three parts whose lengths are proportional to 4, 5, and 6. What is the length, in centimetres, of the longest part?

9.

10. To get some money, Tom decided to sell his CDs. After he had sold six-sevenths of his collection for \$8.00 per CD, he had three CDs left that he couldn't sell. How much money did Tom get (in dollars)?

10.

11. The ratio of x to y is $\dfrac{3}{4}$, and the ratio of x to z is $\dfrac{5}{6}$. What is the ratio of y to z? Express your answer as a common fraction.

11.

12. When an integer n is divided by 12, the remainder is 7. What is the remainder when $7n$ is divided by 12?

12.

13. The two legs of a right-angled triangle have length 20 and 100. To the nearest integer, what is the length of the hypotenuse?

13.

14. If a car travels at 70 kilometres per hour, how many metres does it travel in 18 seconds?

14.

Page 3

15. There are 12 tickets (numbered 1 to 12) in a hat. Alfonso takes two tickets, chosen at random. What is the probability that the sum of the numbers on Alfonso's two tickets is odd? Express your answer as a common fraction.

15.

16. The number of cubic millimetres in a cubic kilometre is 10^n. What is n?

16.

17. Line segments PA, AB, BC, CD, DE, and EF have length 1, and angles PAB, PBC, PCD, PDE, PEF are right angles. Find the length of PF.
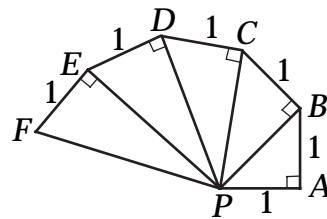
17.

18. The sum of two positive whole numbers is 144. If the larger of the two numbers is divided by the smaller, the quotient is 3 and the remainder is 12. What is the smaller of the two numbers?

18.

19. Suppose that you play the following game: you toss a fair nickel, dime, and quarter at the same time. If you get at least one "head," stop (game over). If you don't, you toss the coins again. If you get at least one head, stop. Otherwise, go on ....

When you toss for the last time, what is the probability that all three coins show heads? Express your answer as a common fraction.

19.

20. If we start adding the consecutive positive integers like this, $1 + 2 + 3 + 4 + 5 \cdots$, and we stop adding when the next number would put our sum over 1000, what sum do we get?

20.

Page 4

21. There were three candidates for mayor of Mathville, Alpha, Beta, and Gamma. Gamma came in last with 5000 votes. Alpha was 1000 votes ahead of Beta, who was 1500 votes ahead of Gamma. How many people voted?

21.

22. Let $N = 2^{22}$. What is the second digit from the left in the decimal expansion of N? (If instead we had $N = 3^8$, then the answer would be 5, since $3^8 = 6561$.)

22.

23. Alphonse lost all his marbles. Some were blue, some were white, and the rest were red. All but 99 were blue, all but 85 were white, and all but 70 were red. How many marbles did Alphonse lose?

23.

24. How many three-digit numbers have exactly one 9 in their decimal expansion?

24.

25. Beth is one-fifth of the way through her cross-country race. After she runs a further three-quarters of a kilometre, she will be one-quarter of the way through the race. Over how many kilometres is the whole race?

25.

26. A gambler started off with 1 dollar. She placed a series of 1 dollar bets, winning a dollar or losing a dollar each time. After a total of 9 bets, the gambler was broke. In how many orders could this have happened? If you have no money you can't bet.

26.

1. The product of three consecutive positive integers is equal to 4080. What is the sum of the three integers?

1.

2. In the picture below, the two smaller circles are equal in size, go through the center of the larger circle, and are tangent to each other and to the larger circle. If the area of each smaller circle is 17 square units, what is the area, in square units, of the shaded region?
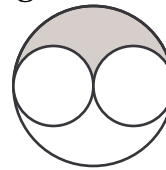
2.

3. How many integers n are there such that $1 \leq n \leq 64$ and n^n is a perfect square?

3.

4. For any whole number n, the number $n!$ is defined by

$$n! = (1)(2)(3)(4) \cdots (n-1)(n).$$

For example $4! = (1)(2)(3)(4) = 24$.

What is the the remainder when $2! + 3! + 4! + \cdots + 89! + 90!$ is divided by 90?

4.

5. How many zeros are there in the decimal representation of

$$1000000^{1000000}$$

(one million to the power one million)?

5.

6. What is the remainder when 222,222,222 is divided by 99?

6.

7. How many ways are there to express 105 as a sum of two or more consecutive positive whole numbers? One of the ways is

$$105 = 34 + 35 + 36.$$

7.

8. How many ordered pairs (x, y) are there such that x and y are integers, with $x \leq y$ and

$$(3x - 16)(3y - 16) = 256?$$

Note that the integers are 0, 1, -1, 2, -2, 3, -3, and so on.

8.

9. How many ways are there to express 22 as a sum of six (not necessarily different) positive odd integers? (The order of summation does not matter, so for example the expression $1 + 1 + 1 + 3 + 3 + 13$ is to be considered the "same" as $1 + 3 + 1 + 13 + 1 + 3$.)

9.

10. One of the factors of 10^{20} is chosen at random. What is the probability that this factor is actually a factor of 10^{10}? Express your answer as a common fraction.

10.

STAGE 4 QUESTIONS, 2005 REGIONAL

**1.** A zoo has some ostriches and some tigers. Altogether they have 34 eyes and 48 legs. How many tigers are there? (Ostriches have 2 eyes and 2 legs, tigers 2 eyes and 4 legs.)

2. How many integers n satisfy the inequality $(n-6)(2n+7) < 0$?

3. What is the surface area, in square centimetres, of the outside of a 50 cm by 50 cm by 40 cm closed cardboard box?

4. When the number N is divided by 8, the quotient is 17 and the remainder is 5. What is N?

5. The rain started at 9:20 am, and continued at a steady rate all day. By 12:00 noon, 4 mm of rain had fallen. How many mm of rain fell from 7:20 am to 10:00 pm?

6. If $x * y = x^2 + 4xy + y^2$, what is $45 * 5$?

7. Express $\dfrac{2^{100}}{2^{101} - 2^{100} + 2^{99}}$ as a fraction in lowest terms.

8. How many real numbers satisfy the equation
$$(x)(x^2 - 1)(x^3 - 2)(x^4 - 3) = 0?$$

9. The *harmonic mean* of a and b is $\dfrac{1}{\dfrac{1}{2}\left(\dfrac{1}{a} + \dfrac{1}{b}\right)}$. What is the harmonic mean of 40 and 60?

10. What is the sum of the integers from 1 to 50 that are *not* divisible by 5?

11. Square $\mathcal{A}$ has area 192 square centimetres, which is 75% of the area of square $\mathcal{B}$. How many centimetres are in the side of square $\mathcal{B}$?

12. How many ordered pairs (x, y) are there such that x and y are integers and $xy = 64$?

13. Joan completes a 26.2 mile race in 2.5 hours. What is her average speed in miles per hour? Give the answer correct to 2 decimal places.

14. A box of 50 cookies is divided between A, B, C, and D. Together, A and D get 24 cookies; A, B, and C together get 40 cookies. How many cookies does A get?

15. What is the product of 20.5 and 30.5, correct to 2 decimal places?

16. In how many ways can 6 (different) CDs be given to Adam, Beth, and Charlie so each gets 2 CDs?

17. What is the smallest prime that is the sum of 3 distinct primes?

18. If $\dfrac{x+y}{x+2y} = \dfrac{7}{11}$, what is $\dfrac{x}{y}$?

19. A survey of 200 students showed that 150 like cupcakes, 120 like muffins, and 20 like neither. How many of the students like both cupcakes and muffins?

20. Two integers add up to 99. What is the largest possible value of their product?

21. Simplify: $\dfrac{5^2 - 1}{3^2 - 1} \cdot \dfrac{9^2 - 1}{7^2 - 1} \cdot \dfrac{13^2 - 1}{11^2 - 1}$

22. What is the least positive integer that is divisible by all of 1^2, 2^2, 3^2, 4^2, 5^2, and 6^2?

23. The average of two common fractions is $1/3$. If one fraction is $1/5$, what is the other fraction?

24. What is the largest prime factor of 2772?

25. The area of a triangle with vertices $(-1, 0)$, $(a, 0)$, and $(0, a)$ is 36. If $a > -1$, what is a?

26. What is the smallest prime number that is greater than 199?

27. The areas of squares A and B in the picture are 25 and 36 square units respectively. How many square units are in the combined area of the two shaded rectangles?
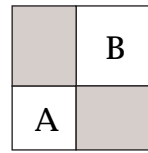
28. How many pairs (x, y) of positive integers are there such that $x + 2y = 50$?

29. Each of two dice has the numbers 1, 3, 5, 7, 9, and 11 on its faces instead of the usual 1, 2, ..., 6. If you toss the two dice, what is the probability of getting a sum of 10?

30. What is the greatest common factor of 144 and 264?

31. Let $\mathcal{L}$ consist of all points in the coordinate plane that have integer coordinates. What is the sum of the x-coordinates of all points in $\mathcal{L}$ that are at distance 5 from the point $(10, 0)$.

32. What is the largest integer that is less than 2005 and is divisible by 21?

33. Two different numbers are chosen at random from 1, 2, 3, ..., 10. What is the probability that one of the numbers is twice the other? Express the answer as a fraction in lowest terms.

34. The point P is on the line segment that joins $(2, 8)$ and $(3, 27)$, and halfway between them. What is the positive difference of the coordinates of P?

35. The side lengths of a triangle are 10 units, 13 units, and 13 units. How many square units are in the area of the triangle?

36. What is the largest prime factor of $24^4 - 24^2$?

37. What is $99^2 - 1$?

38. Each side of a rectangle is divided into 3 equal parts. Some of the division points are joined as in the picture. If the rectangle has area 108 square metres, how many square metres are in the shaded region?
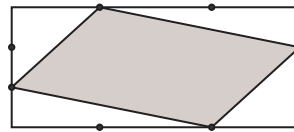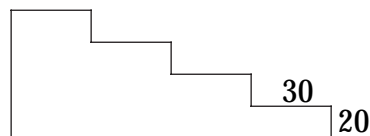

39. What is 112.5% of 56?

40. What is the least integer whose cube is greater than 10,000?

41. What is the greatest integer n such that $3^n < 2^{n+3}$?

42. How many integers n satisfy the inequality $|n| < 2005$?

43. How many square centimetres are in the area of the "staircase" figure below, given that each "stair" is 30 cm wide and 20 cm high?

30

20

44. What is the least perfect square that is greater than 7^4?

45. The sum of the integers from 1 to 100 is 5050. What is the sum of the integers from 201 to 299?

46. Beth left the campground at 8:45 in the morning, and arrived home at 11:05 the same morning. The distance from campground to home is 189 km. What was Beth's average speed, in km per hour?

47. What is the units digit of $1^2 + 2^2 + 3^2 + \cdots + 30^2$?

48. Simplify: $\left(10 + \dfrac{3}{10}\right)^2 - \left(10 - \dfrac{3}{10}\right)^2$.

49. Let ℓ be the line that goes through the points $(0, 2)$ and $(3, 11)$. What is the *x*-coordinate of the point where the line ℓ meets the *x*-axis?

50. If $16y + 5 = 9y - 1$, what is the value of $28y$?

51. Amy has a total of \$16.00 in nickels, dimes, and quarters. If Amy has an equal number of coins of each kind, what is the total number of coins that Amy has?

52. A movie ended at 11:00 pm and ran for 108 minutes. At what time did the movie start?

53. Solve for *x*: $\dfrac{1}{15} - \dfrac{1}{18} = \dfrac{1}{x}$.

54. A 48 centimetre-long piece of wire is cut into two parts, one twice as long as the other. Each part is bent to form a square. What is the sum, in square centimetres, of the areas of the two squares?

55. What is the remainder when 200520052005 is divided by 9?

56. The altitude to the hypotenuse of a right-angled triangle divides the hypotenuse into segments of length 3 and 12 units. How many square units are in the area of the triangle?

$$3 \qquad\qquad 12$$
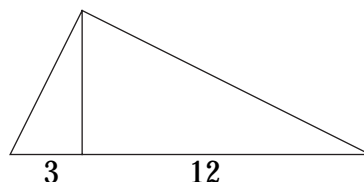
57. Solve for *x*: $|x + 6| = |x - 66|$.

58. Alan bought three textbooks, and spent a total of $210. The calculus book cost $5 more than the economics book, which cost $5 more than the psychology book. How many dollars did the calculus book cost?

59. How many whole numbers between 1 and 100 (inclusive) are divisible by neither 2 nor 5?

60. What is the greatest integer n such that 99! is divisible by 99^n?

61. Alphonse spent one-third of his money, and then lost one third of what remained. After that, he had $20 left. How many dollars did he start with?

62. You toss two fair standard dice, a red one and a blue one. What is the probability that the number on the red one is bigger than the number on the blue one? Express your answer as a fraction in lowest terms.

63. Six is five percent of four percent of what number?

64. Given that a, b, c, d, and e are five consecutive integers with $b + c + d = 135$, what is $a + b + c + d + e$?

65. How many triangles (including triangles of all sizes) are in the picture below?
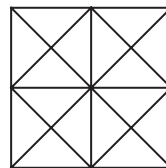
66. If $f(x) = x^2 - 2x + 2$, what is $f(f(f(3)))$?

67. A litre of regular gas costs $0.70. A litre of premium costs $0.80. Richie spent $36 to fill up with premium. How much could he have saved by filling up with regular? Give your answer in dollars and cents.

68. A circle passes through the points $(0, 0)$, $(10, 0)$, and $(0, 10)$. Find the area of the circle. Express your answer in terms of π.

69. There are 120 five-digit numbers that can be formed using each of the digits 1, 2, 3, 4, and 5 once. How many of these are divisible by 4?

70. Three distinct circles and one line are drawn in the plane. What is the largest possible number of points where two or more of these figures meet?

71. A fair coin is flipped 4 times in a row. What is the probability that we get 3 or more consecutive heads during these 4 tosses? Express your answer as a fraction in lowest terms.

72. Express x as a common fraction, given that $1 - \dfrac{1}{1 - \dfrac{1}{x}} = 7$

73. Simplify: $1 - 2 + 3 - 4 + 5 - 6 + \cdots + 99 - 100$.

74. A movie started at 7:47 pm and ended at 9:37 pm. At what time was the movie exactly halfway through?

75. How many perfect squares are there among the positive integers that are less than 10000 and are multiples of 99?

76. What is the 2005th term of the arithmetic sequence 1, 6, 11, 16, …?

77. How many more square metres are there in the area of a circle with diameter 20 than in the area of a circle with diameter 10? Express your answer in terms of π.

78. Two fair dice are tossed. What is the probability that the product of the numbers showing is a prime number? Express your answer as a fraction in lowest terms.

79. What is the leftmost digit in the decimal representation of $9^{100} + 10^{100}$?

80. Express $\dfrac{8! - 7!}{8! + 7!}$ as a fraction in lowest terms.

81. Given that 5 is a solution of the equation $x^2 + ax + 5a = 2005$, what is the value of a?

Answers, Stage 1

1. 78.4

2. 72

3. 12.5

4. 7711729

5. $\dfrac{1}{12}$

6. 6

7. 12

8. 35

9. 44

10. $\dfrac{7}{5}$

11. $\dfrac{5}{18}$

12. $3 + \dfrac{\sqrt{3}}{4} + \pi$

Answers, Stage 2

1. 49

2. 41

3. 1

4. 432

5. 119

6. 61

7. 5400

8. 3000

9. 48

10. 144

11. $\dfrac{10}{9}$

12. 1

13. 102

14. 350

15. $\dfrac{6}{11}$

16. 18

17. $\sqrt{6}$ or $6^{1/2}$

18. 33

19. $\dfrac{1}{7}$

20. 990

21. 19000

22. 1

23. 127

24. 225

25. 15

26. 14

Answers, Stage 3

1. 48

2. 17

3. 36

4. 62

5. 6,000,000

6. 90

7. 7

8. 5

9. 20

10. $\dfrac{121}{441}$

Answers, Stage 4

1. 7
2. 9
3. 13000
4. 141
5. 19
6. 2950
7. $\frac{2}{3}$
8. 6
9. 48
10. 1000
11. 16
12. 14
13. 10.48
14. 14
15. 625.25
16. 90
17. 19
18. $\frac{3}{4}$
19. 90
20. 2450
21. 7

22. 3600
23. $\frac{7}{15}$
24. 11
25. 8
26. 211
27. 60
28. 24
29. $\frac{5}{36}$
30. 24
31. 120
32. 1995
33. $\frac{1}{9}$
34. 15
35. 60
36. 23
37. 9800
38. 60
39. 63
40. 22
41. 5
42. 4009

43. 6000
44. 2500
45. 25050
46. 81
47. 5
48. 12
49. $-\frac{2}{3}$
50. -24
51. 120
52. 9:12 (pm)
53. 90
54. 80
55. 3
56. 45
57. 30
58. 75
59. 40
60. 9
61. 45
62. $\frac{5}{12}$
63. 3000

64. 225
65. 44
66. 257
67. 4.50
68. 50π
69. 24
70. 12
71. $\frac{3}{16}$
72. $\frac{6}{7}$
73. -50
74. 8:42 (pm)
75. 3
76. 10021
77. 75π
78. $\frac{1}{6}$
79. 1
80. $\frac{7}{9}$
81. 198

From Stage 1

1. In a math contest, each contestant's test uses 12 sheets of paper, of which 9 are printed on one side only, and 3 are printed on both sides. Each sheet of paper costs 2 cents. Photocopying costs an additional 5 cents per printed side. How much does it cost to produce enough tests for 250 contestants? Please give the answer in dollars. An answer like 123.00 or 123.45 is of the right shape.

2. In a math contest, every student wrote two tests. One-half of the students wrote Tests 1 and 2, one-quarter wrote Tests 2 and 3, one-sixth wrote Tests 3 and 4, and the remaining 11 students wrote Tests 4 and 5. How many students wrote Test 2?

3. The contract of the CEO of a corporation specifies that she will be paid a bonus of 20% of the net profit, *after* that bonus is deducted. The gross profit, *before* the bonus was deducted, was \$2,850,000. What is the CEO's bonus, in dollars?

4. When the positive rational numbers x and y are expressed as common fractions in lowest terms, the denominators are 126 and 180 respectively. What is the least possible denominator when $x + y$ is expressed as a common fraction?

From Stage 2

1. How many different rectangles are there whose sides are integers and whose perimeter is 98?

2. A fair coin is flipped 5 times. What is the probability that it lands heads exactly once?

3. Alfonso bought 10 identical shirts and 10 identical sweaters, paying a total of \$600. A shirt costs 8 dollars less than a sweater. How much did each shirt cost?

4. What is the volume, in cubic cm, of the solid below? The solid is a box that has a hole with square cross-section drilled all the way from the front to the back. All distances in the diagram are in cm.
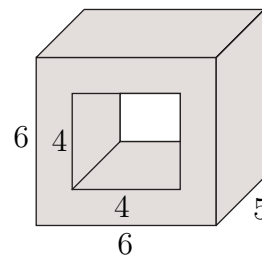
5. A box of 34 cookies is divided between A, B, C, and D. Together, A and D get 14 cookies; A, B, and C together get 28 cookies; and C and D together get 15 cookies. How many cookies does A get?

6. 66 is 120% of what number?

7. A baker sells cookies in either small boxes (6 cookies to a box) or large boxes (12 cookies to a box). At the end of the day the baker has sold 30 boxes of cookies, 210 cookies altogether. How many small boxes did the baker sell?

From Stage 3

1. What is $98 + 198 + 298 + 398 + \cdots + 998$?

2. Runner A runs at a steady 18 km per hour, while B runs at 16 km per hour. In a race, A crosses the finish line 10 minutes ahead of B. How many km was the race?

3. The hypotenuse AB of right-angled triangle ABC has length $\sqrt{13}$ metres. The sum of the lengths of the other two sides is $\sqrt{15}$ metres. How many square metres are in the area of the triangle? Express your answer as a common fraction.

From Stage 4

1. Simplify: $1 - 2 + 3 - 4 + 5 - 6 + \cdots + 99 - 100$.

2. The diagram represents a regular hexagon with perimeter 90 cm. What is the number of cm in the diagonal AB?

3. What whole number is closest to the square root of 2005?

4. Express $7\frac{1}{7}\%$ as a common fraction in lowest terms.

5. There were three candidates for Student Council president. Alicia got twice as many votes as Beti, who got twice as many votes as Chris. Altogether, 350 votes were cast. How many votes did Alicia get?

6. What is the value of $\dfrac{25.3}{0.11}$?

7. Alan went to the casino. Each time he bet, he won 1 dollar or lost 1 dollar. By the end of the evening, he had won 10 times and had a net loss of 10 dollars. How many times did Alan bet?

8. Name the smallest positive even integer which is *not* a factor of 7!.

9. If $\dfrac{1}{2x+1} = \dfrac{2}{3}$, what is $\dfrac{1}{2x+2}$?

10. How many ways are there to make change for a ten dollar bill using only pennies and/or dimes?

11. The school basketball team played 28 games. It never won more than 4 games in a row. What is the largest number of games it could have won?

12. Let $\mathcal{C}$ and $\mathcal{D}$ be two different circles. What is the largest number of common tangents that $\mathcal{C}$ and $\mathcal{D}$ can have?

13. A square is divided into 3 congruent rectangles as in the diagram. If the perimeter of any of the 3 rectangles is 12 units, what is the perimeter of the square?
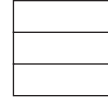
14. If $\dfrac{2}{xy} = 7$, what is $2xy$?

15. How many positive integers are factors of 1024?

16. How many three-digit numbers are there such that the three digits are not all the same?

17. Beth has 3 pennies, 4 dimes, and 5 quarters. If she loses 3 of her coins, how many different amounts of money can she lose?

18. In a certain community, 15% of households own no car, 60% own 1 cars, 20% own 2 cars, and 5% own 3 cars. How many cars are there for every 100 households?

19. The large circle has area 100 square units. The small circles are all equal, and their centers lie along two diameters of the large circle. What is the number of square units in the shaded area?
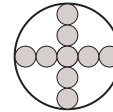
20. What is the sum of the positive integers that divide 64?

From Stage 1

1. Alphonse and Beth play the following game. Gamal, a neutral third party, flips one fair coin repeatedly. He continues until sometime or other during the tossing either (i) the sequence "HH" (two heads in a row) appears, in which case Alphonse wins or (ii) the sequence "TH" (tail followed by a head) appears, in which case Beth wins. What is the probability that Beth wins? Express your answer as a common fraction.

2. There are three balls in a box, numbered 1, 2, and 3. You remove a ball from the box at random, record its number, and put it back in the box. You do this a total of three times. What is the probability that the sum of the three numbers you recorded is 6? Express your answer as a common fraction.

3. The diagram shows a network of one-way streets. How many ways are there to get from A to B if you can only travel in the direction of the arrows?
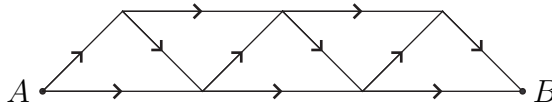
4. Alpha has four white socks and four black socks. She randomly divides these eight socks into four pairs, paying no attention to colour. What is the probability that each pair consists of 2 socks that match in colour? Express your answer as a common fraction.

From Stage 2

1. Today it is Saturday. What day of the week will it be 1000 days from now? Your answer should be one of Sunday, Monday, . . . , Saturday.

2. A piggy bank contains only dimes and quarters. There are three times as many dimes as there are quarters, and the total amount in the piggy bank is $8.25. How many coins are in the piggy bank?

3. What is the sum of the integers between -100 to 103, including -100 and 103?

4. The first act of a three act play began at 7:30 pm. The third act ended at 10:35 pm. All three acts were of the same length, and there were two 10 minute intermissions, one between Act I and Act II, and one between Act II and Act III. At what time did the first intermission begin?

5. In the diagram, grid lines (dashed) are 1 unit apart. How many square units are in the shaded area?
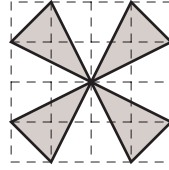
6. A grocer buys oranges at 6 for a dollar, and sells them at 5 for two dollars. How many oranges must she sell in order to make a profit of 70 dollars?

7. Beth has a total of $6.80 in nickels, dimes, and quarters. She has the same number of coins of each kind. How many coins does Beth have?

From Stage 3

1. Alan went to the casino. On his first bet, he doubled the money he had. On the second, he lost 32 dollars. On the third bet, he doubled the money he had. On the fourth, he lost 32 dollars. He made a total of 10 bets, alternately doubling the money he had and losing 32 dollars. After the 10 bets, Alan was left without money. How many dollars did Alan start out with?

2. We want to divide the hexagon of the diagram below into triangles by drawing 3 diagonals that do not meet inside the hexagon. How many ways are there to do this?
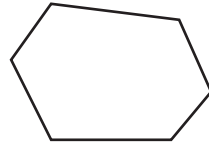
3. The first two terms of the *Fibonacci sequence* are 1 and 1. After that, each term is the sum of the previous two. So the Fibonacci sequence goes

$$1, \quad 1, \quad 2, \quad 3, \quad 5, \quad 8, \quad 13, \quad 21, \quad 34,$$

and so on. How many of the first 100 terms of the Fibonacci sequence are multiples of 4?

From Stage 4

1. Solve for x: $\dfrac{x}{2} - \dfrac{x}{3} + \dfrac{x}{4} = 10$.

2. What is the sum of the roots of the equation $(x - 10)^2 = 16$?

3. If Alan gets paid $8 per hour, he needs to work 24 days to earn enough for a month's food and rent. How many days does it take if he gets paid $6 per hour?

4. The surface area of a cube is 24 square cm. What is the sum, in cm, of the edge lengths of the cube?

5. The four corners of a 16 by 10 rectangle are folded inward as in the diagram. What is the perimeter of the resulting hexagon? Give the answer in simplest radical form.
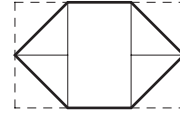
6. Express $\dfrac{8! - 7!}{8! + 7!}$ as a fraction in lowest terms.

7. Given that 5 is a solution of the equation $x^2 + ax + 5a = 2005$, what is the value of a?

8. How many integers are there between $\sqrt{11}$ and $\sqrt{111}$?

9. Fifty gold coins are divided among pirates A, B, and C. Pirate A gets more coins than B, who gets more than C. What is the largest number of coins C can get?

10. The title MATH CHALLENGERS must be centered on a line that contains 80 spaces. If each letter takes up one space, and the space between the two words takes up one space, how many spaces must be put before the M?

11. Two diagonally opposite vertices of a square are at $(0,0)$ and $(5,1)$. How many square units are in the area of the square?
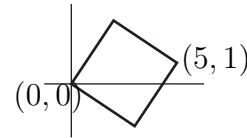
12. What is the value of $\dfrac{x + y}{x - y}$ if $x = \dfrac{5}{6}$ and $y = \dfrac{4}{5}$?

13. What is the value of $\dfrac{8!}{4!4!}$?

14. When N is divided by 40, the quotient is 39 and the remainder is 39. What is N?

15. One-sixth of the hospital salary budget goes to the manager, and the remaining five-sixths goes to the 100 workers. If each worker earns $240 a week, how many dollars a week does the manager earn?

16. What is the smallest positive integer that is a solution of $32x - 16 > 1600$?

17. Duper Store sells evaporated milk for $1 a can for the first 3 cans, and for the regular price of $1.50 a can for every can over 3 cans. If you buy 10 cans of milk, how many percent do you save over the regular price?

18. How many square units are in the area of the trapezoid with vertices at $(0,0)$, $(8,0)$, $(5,3)$, and $(1,3)$?
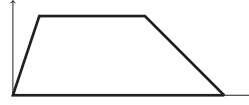
19. What is the smallest prime number that leaves a remainder of 24 when it is divided by 25?

20. The speed of light is $300\,000$ km per second, and the Sun is 150 million km from the Earth. How many seconds does it take for light to travel from the Sun to the Earth?
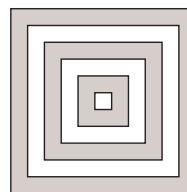
List 3 of Questions from 2006
Provincial Competition

From Bull's Eye Stage

1. Assume that the Earth is a sphere of radius 6400 km, and that Mont Blanc, the highest mountain in the Alps, is 4.8 km high. If we make a scale model of the Earth of radius 0.20 metres, how many millimetres high should Mont Blanc be on the model? Give the answer as a decimal, correct to 2 decimal places.

   1. _____ mm

2. The first term of an arithmetic sequence is 1 and the last term is 4. The sum of all the terms is 30. What is the second term? Express your answer as a common fraction.

   2. _____

3. A tall cylindrical cooking pot has a 12 cm inner base radius, and has some water in it; the depth of the water is 5 cm. A tall heavy closed cylindrical can is placed in the pot, with one of the flat sides down. The base radius of the can is 4 cm. How many cm deep is the water in the pot now? Give the answer correct to 3 decimal places.
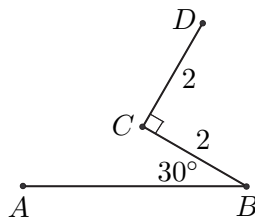
   3. _____ cm

4. What is the shaded area in the diagram below? The six squares have sides of length 1, 3, 5, 7, 9, and 11 units.

   4. _____ units^2

From Blitz Stage

1. Alfie spends one-third of his allowance on books and two-thirds on (healthy) snacks. Suppose the price of books goes down by 8% and the price of snacks goes up by 10%. What percent increase in allowance should Alfie get so that he can keep on buying as many books and as many snacks as before prices changed?

   1. _____ %

2. In the country of Decima, instead of dividing the usual clock into 12 hours, they divide it into 10 equal parts. What time does an ordinary Canadian clock show when a Decima clock shows 8.00? Give your answer in the usual hours : minutes format.

   2. _____

3. Together, A and B own 64 DVDs (that is, the number of DVDs owned by A plus the number of DVDs owned by B is 64). Together, B and C own 81 DVDs. And together, C and D own 100 DVDs. How many DVDs do A and D own together?

   3. _____ DVDs

4. Using the letters a, b, c, d, and e, we can form 625 four-letter "words." Suppose we list these words in alphabetical order. The first six words are aaaa, aaab, aaac, aaad, aaae, and aaba. What is the 235-th word in the list?

4. _____

5. What is the smallest positive integer n such that $10n + 1$ is a power of 7?

5. _____

6. The integers from 1 to 24 are written on index cards, one number to each card. Alicia picks a card at random. Let x be the probability that the number on her card is divisible both by 2 and by 3, and let y be the probability that the number is divisible by 2 or by 3 (or both). What is $\dfrac{x}{y}$? Express your answer as a common fraction.

6. _____

7. In the figure below, $\angle ABC$ has measure $30°$, $\angle BCD$ is a right angle, and $BC = CD = 2$. What is the (perpendicular) distance from D to the line AB? Give the answer in simplest radical form.

7. _____ units

8. A bowl contains 600 slips of paper. Each slip has one of the six letters A, B, C, D, E, or Q written on it, and there are 100 of each kind. How many slips must you grab in order to be sure that among the slips you grab there is at least 1 A, or at least 2 B's, or at least 3 C's, or at least 4 D's, or at least 5 E's?

8. _____ slips

9. Xavier has 5 friends, A, B, C, D, and E. He wants to have dinner with 1 or more of these. Unfortunately, A and B dislike each other and cannot both be invited to the same dinner. In how many ways can Xavier select the people he will have dinner with?

9. _____ ways

From Co-op Stage

1. The integers from 1 to 5 are written on index cards, one number to each card. The cards are placed in a box. Alan removes two randomly chosen cards from the box. He then calculates the product of the numbers on the two cards. What is the average value (mean) of the result that he gets? Express your answer as a common fraction.

1. _____

2. Let $N = 1 \cdot 3 \cdot 5 \cdot 7 \cdot 9 \cdot 11 \cdots 163$. What is the largest positive integer n such that 3^n is a factor of N?

2. _____

3. A group of robbers stole a quantity of thin silk, and decided to share it equally. If each robber received 6 $p'i$ of silk, there would be 6 $p'i$ left over. If each robber got 7 $p'i$, then 7 $p'i$ more silk would be needed than they stole. How many $p'i$ of silk did each robber actually get? Express your answer as a common fraction.

3. _____ $p'i$

4. In the figure below, $ABCD$ is a parallelogram, DP is perpendicular to AB, and DQ is perpendicular to BC. Given that the lengths of DP, PB, BQ, and QD are 4, 7, 1, and 8 centimetres respectively, how many square centimetres are in the area of parallelogram $ABCD$? Express your answer as a common fraction.
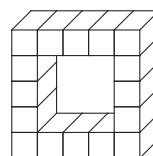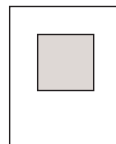
4. _____ cm^2

From Face-off Stage

1. How many even numbers are there between -101 and 101?

1. _____

2. What is the value of $2007^2 - 2005^2$?

2. _____

3. On July 1, 2006, the sun rises at 5:11 AM and sets at 9:21 PM. At what time on July 1, 2006 is it exactly halfway between sunrise and sunset?

3. _____

4. The sum of 11 consecutive integers is 110. What is the largest of these 11 integers?

4. _____

5. A car travels 5 kilometres in 4 minutes. At this speed, how many seconds does it take to travel 1 kilometre?

5. _____

6. The figure below represents a square grid of 9 points in which every point is at unit distance from its nearest horizontal or vertical neighbours. How many lines are there that contain 2 or more points of the grid?

7. Express
$$\frac{4.8 \times 10^{18}}{1.2 \times 10^{20}}$$
as a common fraction.

7. _____

8. Alan started with a number x. He added 10 to it, multiplied the result by 10, then subtracted 10, ending up with 200. What is the value of x?

8. _____

9. Express $\sqrt{\dfrac{1}{25} + \dfrac{1}{144}}$ as a common fraction.

9. _____

10. The figure below was constructed by cementing together sixteen 1 cm by 1 cm by 1 cm cubes. What is the surface area of the figure, in cm^2?

10. _____

11. A rectangular poster is $40\,$cm wide. There is a $20\,$cm by $20\,$cm square picture on the poster. The picture takes up one-fifth of the area of the poster. How many cm are in the height of the poster?

11. _____

12. The average of five numbers is 80. The average of the first three of these numbers is 70. What is the average of the last two of the numbers?

12. _____

13. Alphonse rolls three standard dice once. What is the probability that the sum of the numbers rolled is equal to 4? Express your answer as a common fraction.

13. _____

14. What is the largest digit k such that the five-digit number that has decimal representation $88k88$ is a multiple of 12?

14. _____

15. If $\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{2}{3}$ and $x + y = 8$, what is the value of xy?

15. _____

16. Suppose that for all n,

$$f(n+2) = f(n) + f(n+1).$$

Given that $f(3) = 8$ and $f(4) = 5$, what is the value of $f(1)$?

16. _____

17. What is the number of units in the perimeter of a rhombus whose diagonals have lengths 3 and 4 units?

17. _____

18. How many of the perfect squares between 1^2 and 100^2 have decimal representation with the units digit equal to 4?

18. _____

19. Four points A, B, C, D are on the same line as in the picture below. If

$$\frac{AB}{BC} = \frac{1}{2} \quad \text{and} \quad \frac{BC}{CD} = \frac{4}{5},$$

what is the value of $\overline{\dfrac{AB}{BD}}$? Express your answer as a common fraction.

19. _____

20. What is the 40-th number in the sequence 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, 6, …?
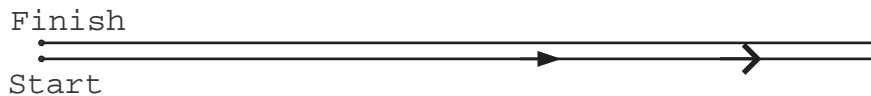
20. _____

Page 1: Problem Solving

1. A class of 30 students took a test, and the class average was 70. The five students who failed had marks of 20, 25, 25, 30, and 40. What was the average mark among the students who didn't fail? Give the answer correct to 1 decimal place.

   1. [   ]

2. Beta has twice as many pennies as Alpha. Gamma has three times as many pennies as Beta. Between the three of them, they have fewer than 80 pennies. What is the largest possible number of pennies that they could have between them?

   2. [   ]

3. Xaviera jogs at a steady rate of 5 minutes per kilometre. Yolande jogs at 7 minutes per kilometre. They start at the same time on an out and back run that consists of 15 km due east followed by 15 km due west. How many kilometres from the finish line is Xaviera when they pass next to each other? Give the answer correct to 1 decimal place.

   3. [   ]

   ```
   Finish
   •————————————————————————————————————→
   Start
   ```

4. It so happens that 7700625 is a perfect square, that is, the square of a whole number, and that 7706176 is the next perfect square. What is the first perfect square greater than 7706176?

   4. [   ]

Canadian Math Challengers Society

Page 2: Combinatorics

5. In a game, you toss two standard dice, a silver one and a gold one. Your *score* is the number showing on the silver one, plus twice the number showing on the gold one. What is the probability that your score is 8? Express your answer as a fraction in lowest terms.

5.

6. There are 5 people in a family, all of different heights. We want to line them up for a picture, with the tallest person in the middle, and so that as we go from left to right, the heights of the people increase and then decrease. How many ways are there to do this?

6.

7. Let *A*, *B*, and *C* be the measures, in degrees, of the angles of a triangle, where $A \leq B \leq C$. How many possibilities are there for the ordered triple (A, B, C), given that each of *A*, *B*, and *C* is a multiple of 15?

7.

8. How many 4-digit numbers are there which obey all three of the following rules: (i) no digit other than 1, 2, 3, or 4 is to be used; (ii) a digit may occur more than once; (iii) as you read the number from left to right, digits never decrease (so 1134 is OK, but 3314 is not)?

8.

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Page 3: Geometry

9. A regular polygon has 11 sides. How many diagonals does it have? A *diagonal* is a line segment that joins two corners but is not a side. One diagonal is shown in the picture.
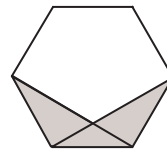
9.

10. Lines that look parallel in the picture *are* parallel. Express b, the length of the "middle" line segment, as a common fraction.
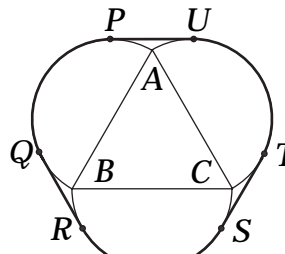
10.

11. The figure below is a regular hexagon with area 1. Express the area of the shaded region as a common fraction.

11.

12. Each side of triangle *ABC* has length 2. Three semicircles are constructed outside *ABC* with the three sides *AB*, *BC*, *CA* as diameters. A thread *PQRSTUP* is tied tightly around the resulting "three-leaf clover" shape so that *PQ*, *RS*, *TU* are arcs of the semicircles with diameters *AB*, *BC*, *CA* respectively and *QR*, *ST*, *UP* are line segments. Find the area enclosed by the thread. Give your answer in terms of π.

12.

1. Three concentric circles have radii 5, 12, and 13. What is the length of the shortest line segment that contains one point on each circle?

1. _____ units

2. What is $\dfrac{20^6}{40^4}$?

2. _____

3. Solve for x: $\sqrt{x} + \sqrt{x} = \sqrt{15}$.

3. _____

4. Alphonse went to the bank to get change for a $20 bill. The change consisted of equal numbers of nickels, dimes, and quarters. How many coins did Alphonse get altogether?

4. _____ coins

5. How many points are there, both of whose coordinates are integers, on the line segment that joins the point $(0,0)$ to the point $(15,10)$? Include both end points in your count.

5. _____ points

6. Alfie took 5 math tests, in which possible marks ranged from 0 to 100. His average on the 5 tests was exactly 88.8. What is the lowest mark he could possibly have received on any one of the tests?

6. _____

7. What is

$$1 + 2 - 3 - 4 + 5 + 6 - 7 - 8 + \cdots - 2003 - 2004 + 2005 + 2006?$$

7. _____

8. Suppose that $x \circ y = \dfrac{x}{y}$. Express $1 \circ (2 \circ (3 \circ (4 \circ 5)))$ as a fraction in lowest terms.

8. _____

9. What is the square of 11111?

9. _____

10. A disabled person's annual pension is 10,000 dollars. How many dollars per year will her pension be after three consecutive 10% cuts?

10. _____ dollars

11. What is the least positive integer N such that $45N$ is a perfect cube?

11. _____

12. In a triangle that contains an obtuse angle, the lengths of the sides, in increasing order, are the integers 6, 8, and n. What is the smallest possible value of n?

12. _____

13. How many squares can be found in the figure below? The figure has been constructed using fifteen 1×1 squares.

13. _____ squares
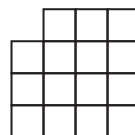
14. Abigail has some nickels, dimes, and quarters. The ratio of the number of dimes to the number of nickels is the same as the ratio of the number of quarters to the number of dimes. If Abigail has 12 dimes, and more quarters than nickels, what is the largest number of nickels that Abigail could have?

14. _____ nickels

15. For every box of cereal that I buy at the regular price, I get an additional box for 1 penny. My grocery cart contained 12 boxes of cereal, for which I paid a total of $17.88 dollars. What is the regular price (in dollars) of a box of cereal?

15. _____ dollars

16. Odin and Eve play the following game. A fair die is tossed. If the result is an odd number, Odin gets 1 point; if the result is an even number, Eve gets 1 point. The first person to get 10 points wins the game. Right now Odin has 7 points, and Eve has 9. What is the probability that Eve wins the game? Express the answer as a common fraction.

16. _____

17. What is the product of the solutions of the equation

$$\sqrt{4 - \frac{1}{x}} = 4 - \frac{1}{x}?$$

17. _____

18. The figure below is bounded by four arcs. Each is one-quarter of the boundary of a circle with radius 5. What is the number of square units in the area of the figure?

18. _____ units^2
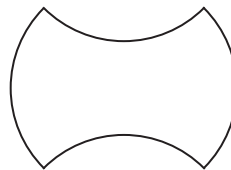
19. A train usually takes 13 hours to get from A to B. If the train's average speed is 5 km per hour less than usual, the trip takes an hour longer. What is the distance, in km, from A to B?

19. _____ km

20. A grasshopper is hopping on the number line below. If it is on a number which is not a multiple of 7, it hops to the right by 2. If it is on a multiple of 7, it hops to the left by 1.

The grasshopper starts on the number 1. On what number is it after 100 hops?

20. _____

21. A UBC graduate left two-fifths of her estate to daughter #1, two-fifths of the rest to daughter #2, and two-fifths of the rest to daughter #3. What remained was given to UBC. What fraction of the estate was given to UBC?

21. _____

22. At a meeting, there are 32 people who are not blonde, 35 who are not brown-haired, and 38 who are not black-haired. What is the largest number of people that could be at the meeting? Note that maybe there are red-haired people at the meeting.

22. _____ people

23. A student has 3 physics books, 3 chemistry books, and 3 biology books. She arranges them on a bookshelf so that books on the same subject are together. In how many ways can she do this?

23. _____ ways

24. In the following diagram, $\triangle ABC$ and $\triangle DEF$ are equilateral, and $\triangle DEF$ is inscribed in $\triangle ABC$, with ED perpendicular to AB. Given that $\triangle ABC$ has area 1, what is the area of $\triangle DEF$?

24. _____ units^2
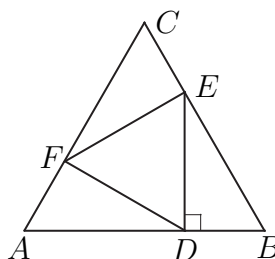
25. What is the smallest positive fraction x such that $\dfrac{x}{y}$ is an integer for both $y = \dfrac{8}{21}$ and $y = \dfrac{6}{35}$?

25. _____

26. What fraction of $\triangle ABC$ is shaded?

26. _____
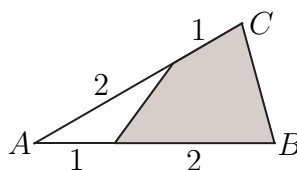
Co-op Round, Page 1: Team answers must be on the *coloured* page. Answers on a white page will not be graded.

1. In Vancouver taxis, the meter starts at $2.44. For that you get to ride a distance of at most 71 metres. Each additional 71 metres or fraction thereof costs 10 cents. You are also charged 10 cents for every 14 seconds or fraction thereof that the taxi is not moving.

   Alan took a taxi ride in Vancouver. The distance covered was 7 kilometres, and the taxi was not moving for a total of 6 minutes and 59 seconds. How many dollars did the taxi ride cost?

1. _____ dollars

2. At the beginning, the red bowl contains 1 litre of water, and the blue bowl is empty. Then one-half of the water in the red bowl is poured into the blue bowl. Then one-third of the water in the blue bowl is poured into the red bowl. Then one-quarter of the water in the red bowl is poured into the blue bowl. Finally, one-fifth of the water in the blue bowl is poured into the red bowl.

   At the end, how many litres of water are in the red bowl?

2. _____ litres

3. For any number x, $\lfloor x \rfloor$ is the greatest integer which is less than or equal to x. For example, $\lfloor 2.45 \rfloor = 2$.

   Find the positive number x such that the product of x and $\lfloor x \rfloor$ is equal to $\dfrac{64}{7}$.

3. _____

4. Starting at a point P_0 on a circle, we mark off successive arcs of 75 degrees counterclockwise around the circle. Let P_1, P_2, ... be the points that we reach in turn. What is the smallest positive integer n such that P_n coincides with P_0?

4. _____
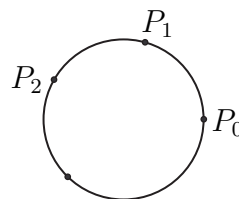
Co-op Round, Page 2: Team answers must be on the *coloured* page. Answers on a white page will not be graded.

5. Let x be the repeating decimal $1.\overline{6}$, that is, $1.6666\ldots$. Express x^2 as a repeating decimal.

5. _____

6. In the figure below, the shaded circle is inscribed in the circular sector OPQ. Given that $\angle QOP$ has measure 60 degrees, and that the shaded circle has area 1, what is the area of the circular sector?

6. _____ units^2
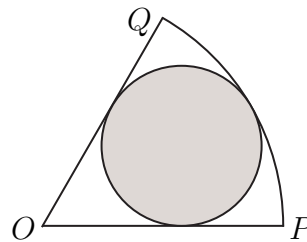
7. How many ways are there to express 15 as the sum of three positive integers? Note that for example $3 + 9 + 3$ is to be viewed as 'the same' as $9 + 3 + 3$.

7. _____ ways

8. How many right-angled triangles can be formed whose vertices are 3 of the 9 points below? The 9 points have coordinates (a, b), where a and b range over the integers from 0 to 2.
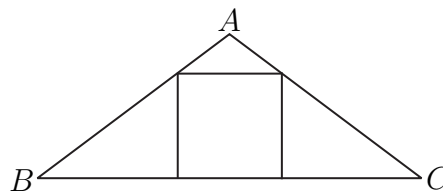
8. _____ triangles

$\cdot \quad \cdot \quad \cdot$

$\cdot \quad \cdot \quad \cdot$

$\cdot \quad \cdot \quad \cdot$

9. Express the following sum as a common fraction:

9. _____

$$\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \cdots + \frac{1}{17 \cdot 19} + \frac{1}{19 \cdot 21}.$$

10. In $\triangle ABC$ below, $AB = AC = 10$ and $BC = 16$. A square is inscribed in $\triangle ABC$, with one side along the line BC. What is the number of units in a side of the square? Express your answer as a common fraction.

10. _____ units

_____

Regional 2006 Face-off Questions

1. What number is 50 less than 50% of 50?

2. If $\dfrac{6}{x} = \dfrac{1}{5}$, what is $6x$?

3. An ant starts at $(0,0)$ and travels in a straight line towards $(19, -3)$. One-quarter of the way to $(19, -3)$, it takes a nap. What is the sum of the coordinates of the point where it naps?

4. Mall-Wart's normal profit on a dress is 200%. At the end of the season it holds a 50% off sale. What is Mall-Wart's percent profit on a dress when it is on sale?

5. A house has 5 basement windows and 3 doors to the outside. In how many different ways can a burglar enter the house through a basement window and leave through a door?

6. A circle is 1 metre in diameter. What is the number of square metres in the area of the circle? Express your answer in terms of π.

7. The price of 120 grams (net weight) of canned tuna is $1.32. What is the cost, in dollars, per 100 grams of tuna?

8. Alice and Bob play a game in which ties are impossible. The probability that Bob wins is three-quarters of the probability that Alice wins. What is the probability that Alice wins?

9. The diameter of a circle is increased by 10%. By how many percent does the circumference increase?

10. If $x + 6y = 9$ and $x - 6y = -2$, what is the value of y?

11. Alan went to Mall-Wart and bought 2 pants and 3 shirts for a total of $100. Each pant cost $11.50 more than each shirt. How many dollars did each shirt cost?

12. How many different primes divide 50! but do not divide 25!?

13. The figure below consists of 11 points: the 10 vertices of a regular decagon and the center of the decagon. How many triangles have one vertex at the center of the decagon, and the other two at vertices of the decagon?
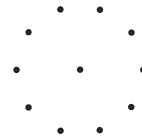
14. What is the units digit in the decimal expansion of $\dfrac{99!}{95!}$?

15. What is $\dfrac{3}{4}$ divided by $\dfrac{5}{6}$?

16. Alicia and Beti together drank a 750 ml bottle of wine. Alicia drank 50% more wine than Beti. How many ml of wine did Alicia drink?

17. The greatest common factor of 4807 and 4853 is not equal to 1. What is it?

18. The gas tank of Alicia's car has capacity 57 litres, and was one-third full when she went to the gas station. She filled up the rest of the tank. If gas cost \$1.02 a litre, how many dollars did the fill-up cost?

19. If n is a perfect square, how many possible values are there for the units digit in the decimal expansion of n?

20. Each of the 10 small squares in the figure below has area 1 square unit. What is the number of square units in the area of the shaded triangle?
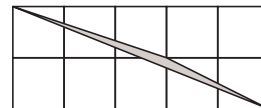
21. Alan puts \$12 under his mattress every month. How many years will it take for these savings to build up to \$3600?

22. If x is 60% of y, what percent is $3x$ of $2y$?

23. A trapezoidal region is bounded by the x-axis, the y-axis, and the lines $x + y = 4$ and $x + y = 10$. What is the number of square units in the area of the region?
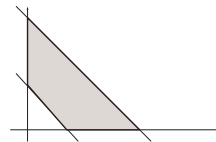

24. What is the positive difference between 2 and the square of $\dfrac{17}{12}$? Express your answer as a common fraction.

25. Four fair coins are tossed. What is the probability we get more than one head?

26. A high school sprinter ran 100 metres in 12 seconds. What was the sprinter's average speed in kilometres per hour?

27. A tub of Alicia's favourite ice cream costs \$4.80. What is the largest number of tubs she can buy if she has \$500?

28. How many squares have two of their vertices among the vertices of the equilateral triangle ABC?
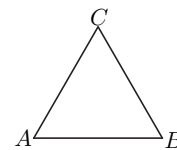
**29.** Suppose that $a_1 = 2$ and $a_{n+1} = 2a_n - 1$ for all $n \geq 1$. What is the value of a_7?

30. A cat sleeps for 20 hours a day. It spends 60% of its waking hours doing nothing, 15% grooming itself, and 15% eating. The rest of the time it scratches the furniture. How many minutes a day does it scratch the furniture?

31. Three cubical dice each have the number 1 marked on three of the faces, and -1 on the other three. The three dice are tossed. What is the probability that the sum of the numbers on the 'top' faces is 1?

32. A 1 km jog uses up 70 calories. A small Dairy Queen chocolate malt has 525 calories. How many km must one jog to use up the calories in a small chocolate malt? Give the answer to the nearest tenth of a km.

3

33. What is the value of $(0.4)^2 - (0.4)^3$?

34. The NHL once had 6 teams. Each year, every team played 50 regular season games. How many regular season NHL games were played each year?

35. Imelda has 365 different pairs of shoes. If she picks two of the shoes at random, what is the probability that the shoes match?

36. Alan has 5 pennies and 10 dimes. In how many ways can he distribute these coins between his left pocket and his right pocket? Either pocket could end up empty. The pennies are identical, as are the dimes.

37. The square of the sum of two numbers is 100. The square of their difference is 99. What is the product of the two numbers?

38. There are 120 ways to arrange the letters of the word 'ANGLE' in a row. Suppose these 120 arrangements are listed alphabetically, from AEGLN to NLGEA. What is the 60-th arrangement in the list?

39. What is $0.\overline{540}$ (that is, $0.540540\ldots$)? Give the answer as a fraction in lowest terms.

40. A sequence has the property that for any three consecutive terms, the third term is equal to the sum of the previous two. If the 100-th term of the sequence is 1, and the 99-th term is 3, what is the 96-th term?

41. An operation $\star$ is defined by $x \star y = x^2 - y^2$. What is the value of $(4 \star 5) \star 6$?

42. Six fair coins are tossed. What is the probability that the positive difference between the number of heads and the number of tails is not equal to 1?

43. Of the students in Mr. Alpher's Math 9 class, 62.5% are in grade 9, 25% are in grade 8, and the remaining 6 students are in grade 10. How many students are in Mr. Alpher's Math 9 class?

44. What is the sum of the solutions of the equation
$3(x-4)^2 + 5 = 6$?

45. What is the remainder when $2003 + 2004 + 2005$ is divided by 2006?

46. Alan says to Beth: "I only have 65 dollars. Give me 10 dollars. If you do, you will still have twice as much money as I will have." How many dollars does Beth have?

4

47. An integer from 1 to 1000 (inclusive) is chosen at random. What is the probability that its cube root is greater than 5?

48. Let A have coordinates $(3,0)$ and let B have coordinates $(0,5)$. What is the number of square units in a square that has the segment AB as a diagonal?

49. If one-half of one-third of one-quarter of a number is 120, what is one-third of one-quarter of one-fifth of the number?

50. What is the sum of all integer values of x such that
$(x+1)^2 \leq 25$?

51. Given that n is an integer and $15n$ is a multiple of 21 and $21n$ is a multiple of 15, what is the largest integer that *must* be a factor of n?

52. What is the sum of all positive integer values of n such that the least common multiple of 12 and n is 24?

53. What is the value of $\dfrac{15^2 - 9^2}{13^2 - 5^2}$?

54. Alan has 32 red and 32 blue $1 \times 1 \times 1$ cubes. He glues all of these together to make a $4 \times 4 \times 4$ cube. What is the least possible number of red 1×1 faces on the outside of the $4 \times 4 \times 4$ cube?
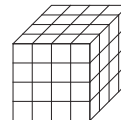
55. If $\dfrac{2x+1}{x} = \dfrac{5}{6}$, what is the value of x?

56. If 75% of a number is 60, what is 85% of that number?

57. What is the least common multiple of 18, 24, 32, and 36?

58. What time is it 1500 seconds after 1:20?

**59.** What is the positive difference between 90% of 125,000 and 90% of 25,000?

5

60. The sides of a square are divided into 5 equal parts to create the pattern below. What is the ratio of the area of the shaded part of the square to the area of the whole square? Express the answer as a common fraction.
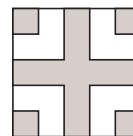
61. What is the product of the odd integers from -5 to 5, inclusive?

62. What is the positive difference between $\dfrac{5}{6}$ and its reciprocal?

63. In the triangular glass prism below, the sides of the end triangles are 3, 4, and 5 cm. The prism has height 10 cm. What is the number of square cm in the total surface area of the prism?
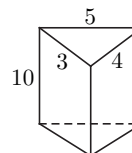
64. What is the sum of the positive integers that are less than 12 and not a multiple of 3?

65. Let $f(x) = x^2 + x + 41$. What is $f(40) - f(-40)$?

66. What is $\dfrac{10!7!4!}{9!6!3!}$?

67. What is the product of the first five positive odd integers?

68. A cubic centimetre of gold has mass 19.32 grams. How many kilograms are in the mass of a 1 metre by 1 metre by 1 metre cube of gold?

69. On July 1, 2006, the sun rises at 5:11 AM and the sun sets at 9:21 PM. At what time on July 1, 2006 is it exactly halfway between sunrise and sunset?

70. Alicia's restaurant meal cost \$125. She decided to leave a 12% tip. What total number of dollars did she pay (bill plus tip)?

List 1 of Questions from 2006
# Provincial Competition

From Bull's Eye Stage

1. A math contest consists of two stages. The Bull's-eye Stage has 12 questions, worth 2 points each, and the Blitz Stage has 26 questions, worth 1 point each. Alicia answered 28 of the 38 questions correctly, and got 35 points. How many Blitz Stage questions did she answer correctly?

   1. _____ questions

2. If the price of eggs was increased by N cents per dozen, where N is a positive integer, it would cost $10N$ more cents to buy $N + 1$ eggs than it would cost if their price was reduced by N cents per dozen. What is the value of N?

   2. _____

3. Suppose that $a_1 = 1/2$, $a_2 = (2/3)(a_1 + 1)$, $a_3 = (3/4)(a_2 + 1)$, $a_4 = (4/5)(a_3 + 1)$, and so on. What is the value of a_{13}? Express your answer as a common fraction.

   3. _____

4. Triangle ABC below is isosceles and right-angled at A. A circle with center A and radius 1 unit intersects the hypotenuse BC at points P and Q, where $BP = PQ = QC$. What is the area of $\triangle ABC$? Express your answer as a common fraction.

   4. _____ units^2
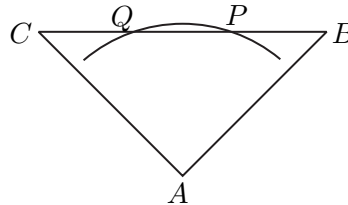
From Blitz Stage

1. The population of a country increased by 25%, and food production increased by 20%. By how many percent did food production per person decrease?

   1. _____ %

2. If $x^{64} = 64$, what is x^{32}?

   2. _____

3. Two equilateral triangles, each with area equal to 1, are placed so as to form a six-pointed star, with a regular hexagon as their part in common. What is the area of the star? Express your answer as a common fraction.

   3. _____ units^2
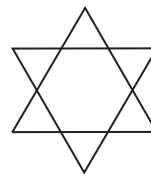
4. What is the largest integer that always divides the difference between the squares of any two odd multiples of 3?

   4. _____

5. In an election, everyone voted for Alan or for Beth. After 60% of the votes had been counted, Alan was leading 60% to 40%. What percentage of the rest of the vote must Beth get in order to end up tied with Alan?

5. _____ %

6. How many positive integers less than 2006 are divisible by 20 or by 25 but not by both?

6. _____ integers

7. Suppose that $f(n+2) = f(n) + 2f(n+1)$ for all n. Given that $f(2) = 9$ and $f(3) = 25$, what is the value of $f(1)$?

7. _____

8. A stadium is divided into two sections, Section A and Section B. Section A has 2000 less than three-quarters of the total number of seats in the stadium. Section B has 1000 less than one-third of the total number of seats in the stadium. What is the total number of seats in the stadium?

8. _____ seats

9. The number in each square is the sum of the numbers in the two squares immediately "above" it. So for example the number in the leftmost square of the second row must be 6. Given the information in the diagram below, what is the value of x?

9. _____

| 1 | 5 | x | 5 | 1 |

_____

0
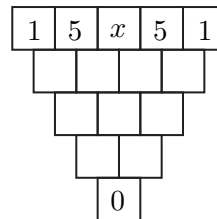
From Co-op Stage

1. Call an integer *unlucky* if the sum of its decimal digits is equal to 13. How many unlucky integers are there between 1 and 888?

1. _____

2. How many digits in total are there in the decimal representation of $(5^5)^5$?

2. _____ digits

3. The figure below is a 5 by 5 grid of points. Each point is 1 cm from its nearest horizontal and vertical neighbours. How many ways are there to choose two distinct grid points that are an integer number of cm from each other? Note that choosing P and Q is to be considered the same as choosing Q and P.

3. _____ ways

From Face-off Stage

1. If $5x + 6y + 2 = \dfrac{5}{7}$, what is the value of $10x + 12y + 3$?

1. _____

2. Let $\lfloor x \rfloor$ denote the greatest integer which is less than or equal to x. What is the value of
$$\left\lfloor \frac{22}{7} \right\rfloor - \left\lfloor -\frac{22}{7} \right\rfloor ?$$

2. _____

3. A turntable makes 45 revolutions per minute. Through how many degrees does the turntable rotate per second?

3. _____

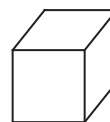4. What is the product of the first five positive odd integers?

4. _____

5. There are pheasants and rabbits in a barn, 78 animals altogether. Pheasants have 2 feet and rabbits have 4. If I remove half of the rabbits, how many feet remain in the barn?

5. _____

6. The sum of all the edge lengths of a cube is $36\,\text{cm}$. What is the surface area of the cube, in cm^2?

6. _____

7. If Alicia loses one and a half pounds a week, how many weeks will it take her to go from 154 pounds to 130 pounds?

7. _____

8. If $3x - 1 = \dfrac{1}{4}$, express x as a common fraction.

8. _____

9. A cubic centimetre of gold has mass 19.32 grams. How many kilograms are in the mass of a 1 metre by 1 metre by 1 metre cube of gold?

9. _____

10. Alicia's restaurant meal cost \$125. She decided to leave a 12% tip. What total number of dollars did she pay (bill plus tip)?

10. _____

11. Simplify:
$$(2^{10} + 2^{10} + 2^{10} + 2^{10})^{1/3} .$$

11. _____

12. How many positive factors of $2^4 \times 5^4$ are perfect squares?

12. _____

13. Alice can paint a room in one day, and Bob can paint a room in two days. Working together, how many days will it take for them to paint 15 rooms?

13. _____

14. What is the value of
$$(\sqrt{10} + \sqrt{5})^2 + (\sqrt{10} - \sqrt{5})^2 ?$$

14. _____

15. In an election, Alfie got 65% of the votes and Beth got the rest. If Alfie got 120 more votes than Beth, how many people voted?

15. _____

16. The sum of the positive integers from 1 to n (inclusive) is greater than 275. What is the smallest possible value of n?

16. _____

17. Alphonse and Beti walk side by side. Alphonse averages 120 steps per minute and each of his steps is 90 cm long. Beti's steps are 75 cm long. How many steps does Beti average per minute?

17. _____

18. In the right-angled triangle below, the hypotenuse has length 17 units, and one of the legs has length 15 units. How many units^2 are in the area of the triangle?

18. _____
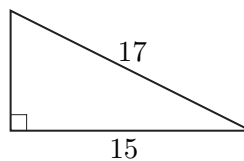
19. Let A be the sum of the 25 integers from 0 to 24, and let B be the sum of the 25 integers from 76 to 100. What is the value of $A + B$?

19. _____

20. Suppose that $4^{x+3} = 8^{x-3}$. What is the value of x?

20. _____

List 2 of Questions from 2006
Provincial Competition

From Bull's Eye Stage

1. Wolf can eat a lamb in 1/4 of an hour. Bear can eat a lamb in 1/5 of an hour. How many hours will it take for Wolf and Bear, dining together at their usual speed, to eat a lamb? Express your answer as a common fraction.

   1. _____ hours

2. There are six balls in an urn, with the numbers 1, 2, 3, 4, 5, and 6 written on them. You reach into the urn and simultaneously remove two randomly chosen balls. What is the probability that the sum of the numbers on these two balls is equal to 6? Express your answer as a common fraction.

   2. _____

3. A small produce store has two cashiers. In how many different ways can 5 customers line up to pay? A sample lineup is given below. Note that all the customers could line up in front of one cashier.

   3. _____ ways

   D   C
   A   B
         E
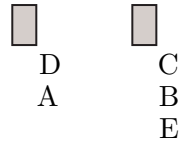
4. The figure below is a regular hexagon. Each of the six sides has length 1 unit. What is the sum of the lengths of all the diagonals? Express your answer in simplest radical form. Note that there are three diagonals through each corner.

   4. _____ units
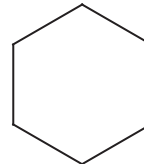
From Blitz Stage

1. What is the smallest prime whose square does not divide 40! ?

   1. _____

2. Richie has 3 dimes, 3 quarters, and 3 loonies. How many different non-zero amounts of money can he make by using one or more of these 9 coins?

   2. _____ amounts

3. Sixty percent of sixty percent of a number is 666. What is the number?

   3. _____

4. If the shaded region below has area 150 cm^2, what is the value of x? Give your answer in simplest radical form.

   4. _____ cm

   x 3
   3   3
   x x
      x 3
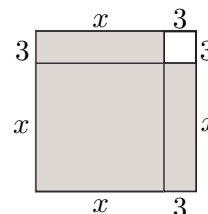
5. What is the smallest positive value of $6-\dfrac{2006}{n}$, as n ranges over the positive integers? Express your answer as a common fraction.

5. _____

6. A group of 5 students splits into two study groups. (A study group has at least two students.) In how many ways can this be done?

6. _____ ways

7. In how many ways can 100 be expressed as the sum of two (positive) primes? (The decompositions $100 = 3+97$ and $100 = 97+3$ are considered the same.)

7. _____ ways

8. The first term of a sequence is 1. If x is any term in the sequence, then the next term is the reciprocal of $1 + x$. (So the second term is $1/2$.) What is the product of the first 6 terms of the sequence? Express your answer as a common fraction.

8. _____

From Co-op Stage

1. Calculators are packaged in boxes of 12 or boxes of 25. (Boxes must be full.) What is the least number of boxes required to package 2006 calculators?

1. _____ boxes

2. Two triangles with sides 3 cm, 4 cm, and 5 cm are placed on top of each other so that the right angles coincide but the triangles do not. What is the number of cm^2 in the region of overlap of the two triangles? Express your answer as a common fraction.

2. _____ cm^2
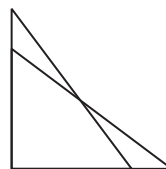
3. Let N be the smallest positive integer such that (i) 24 divides N and (ii) N has exactly 24 positive divisors. What is the value of N? Note that for any positive integer k, both 1 and k are divisors of k.

3. _____

From Face-off Stage

1. The length of a rectangle is four times the width. If the perimeter of the rectangle is 5 metres, how many square metres are in the area of the rectangle?

1. _____

2. When N is divided by 11, the quotient is 12 and the remainder is 7. What is the value of N?

2. _____

3. A cup of flour has 400 calories. A cup of lard has 1700 calories. A pie crust is made using two cups of flour and one cup of lard. How many percent of the calories in the pie crust come from lard?

3. _____

4. Express
$$\frac{1+2+3+4+5+6}{1+2+3+4+5+6+7}$$
as a fraction in lowest terms.

4. _____

5. What is the value of $\dfrac{27.3}{0.13}$?

5. _____

6. What is the sum of the positive integers that divide 24?

6. _____

7. A GMC Yukon on average uses 21.3 litres of gas to travel 100 km. A Toyota Prius uses 4.1 litres of gas to travel 100 km. Beti drives 10,000 km a year. How many dollars in gas costs would she save in a year if she drove a Prius instead of her Yukon? Assume that gas costs \$1.00 per litre.

7. _____

8. What is the largest integer x such that $\dfrac{60}{12-x}$ is an integer?

8. _____

9. The two equal sides of the isosceles triangle below each have length 3 cm. The third side has length 2 cm. What is the area of the triangle, in square cm?

9. _____
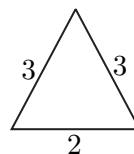
10. The product of 12 positive integers is equal to 12. What is the smallest possible sum of the 12 integers?

10. _____

11. Alice and Bob select, independently and at random, a positive integer that divides 16. What is the probability that they select the same number? Express your answer as a common fraction.

11. _____

12. What common fraction is halfway between $\dfrac{3}{4}$ and $\dfrac{4}{3}$ on the number line?

12. _____

13. If 4 distinct circles are drawn, what is the largest possible number of points that lie on more than one circle?

13. _____

14. What is the sum of the prime factors of $2^8 - 1$?

14. _____

15. Sixty people are arranged in 3 rows. If the back row has three more people than the middle row, and the middle row has three more people than the front row, how many people are in the front row?

15. _____

16. The line segment that joins the points $(-4, 2)$ and $(3, 0)$ is drawn. What is the y-coordinate of the point where this line segment meets the y-axis? Express your answer as a common fraction.

16. _____
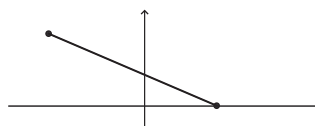
17. A circle has radius 7 units. How many units are in the circumference of the circle? Give the answer rounded to the nearest integer.

17. _____

18. Let $f(x) = x^2 + x + 41$. What is the value of $f(40) - f(-40)$?

18. _____

19. If $4! \times 4! \times N = 8!$, what is the value of N?

19. _____

20. Aleph started with \$2700. He gave one-third of his money to Beth and four-fifths of the rest to Gimel. How many dollars does Aleph have left?
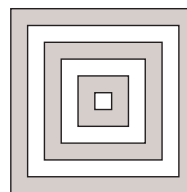
20. _____

List 3 of Questions from 2006
Provincial Competition

From Bull's Eye Stage

1. Assume that the Earth is a sphere of radius 6400 km, and that Mont Blanc, the highest mountain in the Alps, is 4.8 km high. If we make a scale model of the Earth of radius 0.20 metres, how many millimetres high should Mont Blanc be on the model? Give the answer as a decimal, correct to 2 decimal places.

   1. _____ mm

2. The first term of an arithmetic sequence is 1 and the last term is 4. The sum of all the terms is 30. What is the second term? Express your answer as a common fraction.

   2. _____

3. A tall cylindrical cooking pot has a 12 cm inner base radius, and has some water in it; the depth of the water is 5 cm. A tall heavy closed cylindrical can is placed in the pot, with one of the flat sides down. The base radius of the can is 4 cm. How many cm deep is the water in the pot now? Give the answer correct to 3 decimal places.

   3. _____ cm

4. What is the shaded area in the diagram below? The six squares have sides of length 1, 3, 5, 7, 9, and 11 units.

   4. _____ units^2

From Blitz Stage

1. Alfie spends one-third of his allowance on books and two-thirds on (healthy) snacks. Suppose the price of books goes down by 8% and the price of snacks goes up by 10%. What percent increase in allowance should Alfie get so that he can keep on buying as many books and as many snacks as before prices changed?

   1. _____ %

2. In the country of Decima, instead of dividing the usual clock into 12 hours, they divide it into 10 equal parts. What time does an ordinary Canadian clock show when a Decima clock shows 8.00? Give your answer in the usual hours : minutes format.

   2. _____

3. Together, A and B own 64 DVDs (that is, the number of DVDs owned by A plus the number of DVDs owned by B is 64). Together, B and C own 81 DVDs. And together, C and D own 100 DVDs. How many DVDs do A and D own together?

   3. _____ DVDs

4. Using the letters a, b, c, d, and e, we can form 625 four-letter "words." Suppose we list these words in alphabetical order. The first six words are aaaa, aaab, aaac, aaad, aaae, and aaba. What is the 235-th word in the list?

4. _____

5. What is the smallest positive integer n such that $10n + 1$ is a power of 7?

5. _____

6. The integers from 1 to 24 are written on index cards, one number to each card. Alicia picks a card at random. Let x be the probability that the number on her card is divisible both by 2 and by 3, and let y be the probability that the number is divisible by 2 or by 3 (or both). What is $\dfrac{x}{y}$? Express your answer as a common fraction.

6. _____

7. In the figure below, $\angle ABC$ has measure $30°$, $\angle BCD$ is a right angle, and $BC = CD = 2$. What is the (perpendicular) distance from D to the line AB? Give the answer in simplest radical form.

7. _____ units

8. A bowl contains 600 slips of paper. Each slip has one of the six letters A, B, C, D, E, or Q written on it, and there are 100 of each kind. How many slips must you grab in order to be sure that among the slips you grab there is at least 1 A, or at least 2 B's, or at least 3 C's, or at least 4 D's, or at least 5 E's?

8. _____ slips

9. Xavier has 5 friends, A, B, C, D, and E. He wants to have dinner with 1 or more of these. Unfortunately, A and B dislike each other and cannot both be invited to the same dinner. In how many ways can Xavier select the people he will have dinner with?

9. _____ ways

From Co-op Stage

1. The integers from 1 to 5 are written on index cards, one number to each card. The cards are placed in a box. Alan removes two randomly chosen cards from the box. He then calculates the product of the numbers on the two cards. What is the average value (mean) of the result that he gets? Express your answer as a common fraction.

1. _____

2. Let $N = 1 \cdot 3 \cdot 5 \cdot 7 \cdot 9 \cdot 11 \cdots 163$. What is the largest positive integer n such that 3^n is a factor of N?

2. _____

3. A group of robbers stole a quantity of thin silk, and decided to share it equally. If each robber received 6 p'i of silk, there would be 6 p'i left over. If each robber got 7 p'i, then 7 p'i more silk would be needed than they stole. How many p'i of silk did each robber actually get? Express your answer as a common fraction.

3. _____ p'i

4. In the figure below, $ABCD$ is a parallelogram, DP is perpendicular to AB, and DQ is perpendicular to BC. Given that the lengths of DP, PB, BQ, and QD are 4, 7, 1, and 8 centimetres respectively, how many square centimetres are in the area of parallelogram $ABCD$? Express your answer as a common fraction.
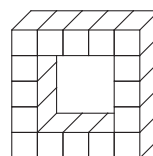
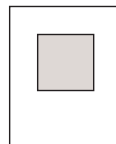4. _____ cm^2

From Face-off Stage

1. How many even numbers are there between -101 and 101?

1. _____

2. What is the value of $2007^2 - 2005^2$?

2. _____

3. On July 1, 2006, the sun rises at 5:11 AM and sets at 9:21 PM. At what time on July 1, 2006 is it exactly halfway between sunrise and sunset?

3. _____

4. The sum of 11 consecutive integers is 110. What is the largest of these 11 integers?

4. _____

5. A car travels 5 kilometres in 4 minutes. At this speed, how many seconds does it take to travel 1 kilometre?

5. _____

6. The figure below represents a square grid of 9 points in which every point is at unit distance from its nearest horizontal or vertical neighbours. How many lines are there that contain 2 or more points of the grid?

6. _____

7. Express
$$\frac{4.8 \times 10^{18}}{1.2 \times 10^{20}}$$
as a common fraction.

7. _____

8. Alan started with a number x. He added 10 to it, multiplied the result by 10, then subtracted 10, ending up with 200. What is the value of x?

8. _____

9. Express $\sqrt{\dfrac{1}{25} + \dfrac{1}{144}}$ as a common fraction.

9. _____

10. The figure below was constructed by cementing together sixteen 1 cm by 1 cm by 1 cm cubes. What is the surface area of the figure, in cm^2?

10. _____

11. A rectangular poster is $40\,\text{cm}$ wide. There is a $20\,\text{cm}$ by $20\,\text{cm}$ square picture on the poster. The picture takes up one-fifth of the area of the poster. How many cm are in the height of the poster?

11. _____

12. The average of five numbers is 80. The average of the first three of these numbers is 70. What is the average of the last two of the numbers?

12. _____

13. Alphonse rolls three standard dice once. What is the probability that the sum of the numbers rolled is equal to 4? Express your answer as a common fraction.

13. _____

14. What is the largest digit k such that the five-digit number that has decimal representation $88k88$ is a multiple of 12?

14. _____

15. If $\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{2}{3}$ and $x + y = 8$, what is the value of xy?

15. _____

16. Suppose that for all n,

$$f(n+2) = f(n) + f(n+1).$$

Given that $f(3) = 8$ and $f(4) = 5$, what is the value of $f(1)$?

16. _____

17. What is the number of units in the perimeter of a rhombus whose diagonals have lengths 3 and 4 units?

17. _____

18. How many of the perfect squares between 1^2 and 100^2 have decimal representation with the units digit equal to 4?

18. _____

19. Four points A, B, C, D are on the same line as in the picture below. If

$$\frac{AB}{BC} = \frac{1}{2} \quad \text{and} \quad \frac{BC}{CD} = \frac{4}{5},$$

what is the value of $\dfrac{AB}{BD}$? Express your answer as a common fraction.

19. _____

20. What is the 40-th number in the sequence 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, 6, ...?

20. _____

Answers, Bull's-eye Stage

1. 20

2. 40

3. 12:48

4. 36

5. $\frac{2}{5}$

6. 25

7. $\frac{1}{52}$

8. 8

9. 15

10. $\frac{104}{3}$

11. 110π

12. $\frac{94}{17}$

Answers, Blitz Stage

1. 8

2. 25

3. $\frac{15}{4}$

4. 150

5. 6

6. 44

7. 2007

8. $\frac{15}{8}$

9. 123454321

10. 7290

11. 75

12. 11

13. 26

14. 9

15. 2.97

16. $\frac{7}{8}$

17. $\frac{1}{12}$

18. 50

19. 910

20. 141

21. $\frac{27}{125}$

22. 52

23. 1296

24. $\frac{1}{3}$

25. $\frac{24}{7}$

26. $\frac{7}{9}$

Answers, Co-op Stage

1. 15.24

2. $\frac{3}{5}$

3. $\frac{64}{21}$

4. 24

5. $2.\overline{7}$ or 2.7777...

6. $\frac{3}{2}$

7. 19

8. 44

9. $\frac{10}{21}$

10. $\frac{48}{11}$

2007 Canadian Math Challengers Regional Questions

Bull's-eye, Page 1: Combinatorics and Numbers

1. What is the smallest positive integer which is *not* a factor of 25!?

1. _____

2. Richie has one penny, two nickels, and three dimes. How many different amounts of money can he make using one or more of these six coins?

2. _____ amounts

3. The picture below shows 7 stools arranged in a row. Initially they were all unoccupied. Alfie chose a stool at random and sat down. Then Beti chose an empty stool at random and sat down. What is the probability that Alfie and Beti chose stools that are next to each other? Express your answer as a common fraction.

3. _____

4. Suppose that we write $(2 \times 2007)^2$ as $a_1 \times a_2 \times \cdots \times a_n$, where a_1, a_2, ..., a_n are prime numbers, not necessarily distinct. What is the value of $a_1 + a_2 + \cdots + a_n$? Note that 1 is not a prime.

4. _____

5. Alicia's heart beats 50 times per minute when she is asleep, and 70 times per minute when she is awake, except that it beats 90 times per minute when she is doing mathematics—which she never does while she sleeps. In the last 24 hours, Alicia slept for 8 hours, and did mathematics for 8 hours. What was Alicia's average number of heart beats per minute over the last 24 hours?

5. _____ beats/min

6. Every week, Alfie works 20 hours at Burper Queen for $6.50 per hour, and 20 hours at Florida Fried Fat for $6.00 per hour. How many weeks of work will Alfie need in order to pay his $4500 university fees?

6. _____ weeks

7. Alphonse and Beth drove on the freeway, in separate cars, from Miniburg to Microville. They left Miniburg at the same time. Alphonse drove at a steady 90 km per hour, and Beth drove at a steady 80 km per hour. Alphonse arrived in Microville 10 minutes before Beth did. What is the freeway distance from Miniburg to Microville?

7. _____ km

8. Alicia is driving at 60 miles per hour. How many feet does she drive per second? (There are 5280 feet in 1 mile.)

8. _____ feet/sec

9. In the diagram below, which is not drawn to scale, the shaded rectangle has width 14 cm and height 12 cm, and is contained in trapezoid $ABCD$. Given that $AB = 13$ cm, $BC = 14$ cm, and $CD = 15$ cm, what is the perimeter of trapezoid $ABCD$?
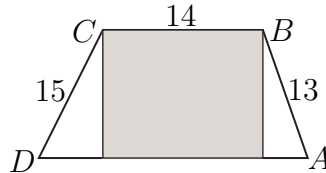
9. _____ cm

10. A spruce plank which is 18 cm wide, 100 cm long, and 2 cm thick weighs 1.62 kg. What is the weight, in kg, of a spruce plank which is 12 cm wide, 200 cm long, and 2 cm thick? Give your answer as a decimal, to the nearest hundredth of a kg.

10. _____ kg

11. In the figure below, $ABCD$ is a square, and P is a point inside the square such that the distances from P to C, to D, and to AB are all equal. What fraction of the area of the square is shaded?
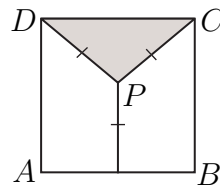
11. _____

12. The figure below is a quarter-circle with center O and radius 12. Point M bisects the line segment OP. Point N is on the quarter-circle, with $\angle NOP = 30°$. What is the area of the shaded region? Express your answer in terms of π
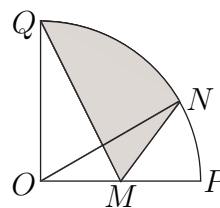
12. _____ units^2

1. In the figure below, the shaded region is a rectangle with base 1 cm and height 3 cm. An outward-facing square has been erected on each side of the rectangle. What is the area of the entire figure (including both the shaded part and the unshaded part)?
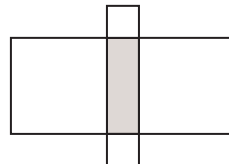
1. _____ cm^2

2. A fair coin is flipped 6 times. What is the probability of getting 6 heads in a row? Express your answer as a common fraction.

2. _____

3. What is three-quarters of one-third of two-sevenths? Express your answer as a common fraction.

3. _____

4. What is the value of $\dfrac{10^6}{20^3}$?

4. _____

5. What is the (positive) value of $\sqrt{x^3 + y^2}$ when $x = 6$ and $y = 3$?

5. _____

6. Today is Saturday. What day of the week was it 1000 days ago?

6. _____

7. A special breed of cow produces 25% more milk per year than a regular cow, but it needs to eat 20% more grain. A farmer replaces her regular cows with special breed cows. How many percent less grain do the special breed cows eat per litre of milk produced?

7. _____ %

8. The figure below is a regular hexagon. Each side has length 2 cm. What is the area of the hexagon? Express the answer in simplest radical form.

8. _____ cm^2
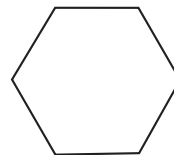
9. At the instant that it is midnight in Vancouver, it is 4:30 am in St. John's, Newfoundland. An airplane left Vancouver at 10:00 am (Vancouver time) and flew directly to St. John's. The total flight time was 7 hours and 50 minutes. What time was it in St. John's when the plane arrived? Express your answer in the usual hours:minutes format, as in 6:25.

9. _____ pm

10. A house and a garage are built on a rectangular lot that is 33 feet by 120 feet. The house occupies 1500 square feet of the lot, the garage occupies an additional 360 square feet, and the rest of the lot is lawn. What is the area of the lawn, in square feet?

10. _____ feet^2

11. Beti has 40% more money than Alfie. Between them they have $1800. How many dollars does Alfie have?

11. _____ dollars

12. On a farm that has genetically engineered animals, chickens have 1 head and 3 feet, and rabbits have 1 head and 5 feet. These are the only animals on the farm. Altogether, the animals on the farm have 90 heads and 310 feet. How many chickens are on the farm?

12. _____ chickens

13. Richie went to the bank to cash a $2000 cheque. He got an equal number of 10 dollar bills, 20 dollar bills, and 50 dollar bills. How many bills did Richie get altogether?

13. _____ bills

14. How many points are there, both of whose coordinates are integers, on the boundary of the rectangle whose corners are $(-20, 5)$, $(20, 5)$, $(20, 35)$, and $(-20, 35)$?

14. _____ points

15. Suppose that

$$a + b = 12, \quad b + c = 13, \quad c + d = 14, \quad \text{and} \quad a + 2d = 14.$$

What is the value of a?

15. _____

16. Let x and y be real numbers such that $x^{10} = 11$ and $y^{20} = 100$. What is the value of $x^{20} \times y^{10}$?

16. _____

17. A multiple choice quiz has 12 easy questions, worth 5 marks each, and 5 harder questions, worth 8 marks each. On any question, the only possible mark is 0 or full marks. Alfie's mark was 54. What is the *total* number of questions that Alfie answered correctly?

17. _____ questions

18. An integer N is chosen at random from the integers in the interval from 1 to 30, inclusive. What is the probability that N is a factor of 30? Express your answer as a common fraction.

18. _____

19. Alicia went on a 40 km bicycle ride. The first 20 km segment took 1 hour. On the last 20 km she rode 4 km per hour faster than on the first 20 km. How many minutes did the whole ride take?

19. _____ minutes

20. The picture below represents a quadrilateral with two opposite right angles. If three of the sides have lengths as shown in the picture, what is the area of the quadrilateral?

20. _____ units^2
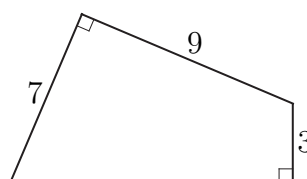
21. Suppose that $x = 1 - \dfrac{1}{2} + \dfrac{1}{4} - \dfrac{1}{8} + \dfrac{1}{16} - \dfrac{1}{32} + \dfrac{1}{64} - \dfrac{1}{128}$. What is the value of $128x$?

21. _____

22. The product of 5 consecutive positive integers is divisible by 1120. What is the smallest possible value of the sum of the 5 consecutive integers?

22. _____

23. Twelve white unit cubes are assembled to make a $3 \times 2 \times 2$ rectangular prism. The outside surface of this prism is painted blue. Then the prism is disassembled into the original 12 unit cubes, and one of these cubes chosen at random is tossed like a die. What is the probability that the cube lands with the "up" face painted blue?

23. _____
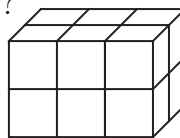
24. A math contest has 38 questions, of which the first 26 are worth 1 point each, and the last 12 are worth 2 points each. (So the maximum possible mark for the entire contest is 50.) There is no partial credit on any question. A certain number N of students participated. Their average score on the first 26 questions was 20 (out of 26), and their average final score was 34 (out of 50). The total combined number of questions that were answered correctly was 621. What is the value of N?

24. _____ students

25. In the kite below, $AB = AD = 1$ cm and $CB = CD = 3$ cm. Angles ABC and ADC are right angles. Express the area of triangle ABD as a common fraction.

25. _____ cm^2
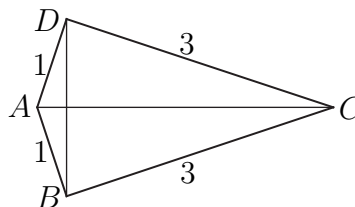
26. In a certain type of game, B beats A 60% of the time, C beats B 60% of the time, and A always beats C. Player A played against B, and then the winner played against C. Given that C lost that game, what is the probability that C played against A? Write your answer as a common fraction.

26. _____

Canadian Math Challengers Society

Co-op, Page 1: Team answers must be on the *coloured* page.
   Answers on a white page will not be graded.

1. The positive difference between two perfect squares is 60. What is the largest possible sum of the two perfect squares?

1. _____

2. How many digits are there in the decimal representation of

$$10 \times 10^2 \times 10^3 \times 10^4 \times \cdots \times 10^{2006} \times 10^{2007}?$$

2. _____ digits

3. When plotted in the standard rectangular coordinate system, $\triangle ABC$ has vertices $A(12, 0)$, $B(0, 10)$, and $C(2, 1)$. How many square units are in the area of $\triangle ABC$?

3. _____ units^2

4. Call an integer n "triply odd" if when you divide n by 3, you get an odd integer whose digits add up to an odd multiple of 3. How many triply odd integers are there between 1 and 300?

4. _____ integers

5. The figure below is a 4 by 4 grid of points. Each point is 1 cm from its nearest horizontal and vertical neighbours. Two of these 16 points are chosen at random. What is the probability that they do not lie in the same horizontal row? Express your answer as a common fraction.

5. _____

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.   .   .   .
.   .   .   .
.   .   .   .
.   .   .   .
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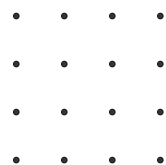
6. How many ordered pairs (a, b) are there such that a and b are integers and $a^2 b = -1024$?

6. _____ pairs

7. An election was held for a joint grade 8/9 rep. Only grade 8 and grade 9 students could vote, and everyone voted for one of Alicia and Beti. Of the students who voted for Alicia, three-quarters were in grade 8. (Note that this does *not* mean that three-quarters of grade 8 students voted for Alicia.) Of the students who voted for Beti, four-fifths were in grade 9. An equal number of grade 8 and grade 9 students voted. What fraction of all the students voting voted for Alicia?

7. _____

8. Let p_1, p_2, p_3, p_4, ... be all the primes, listed in increasing order. So $p_1 = 2$, $p_2 = 3$, $p_3 = 5$, and so on. Let N be the smallest integer such that

$$p_1 + p_2 + p_3 + \cdots + p_N > 2007.$$

What is the value of p_N? Note that we want p_N, not N.

8. _____

9. How many ordered 5-tuples (p, q, r, s, t) are there such that: (i) the sequence p, q, r, s, t is an *arithmetic sequence*; (ii) p, q, r, s, and t are positive integers; (iii) $p < q$; and (iv) $p + q + r + s + t = 1000$?

9. _____ sequences

10. Let $\triangle ABC$ be right-angled at A. A semicircle with radius 3 units has center O on the line AB, passes through A, and is tangent to the line segment BC. If AC has length 5 units, what is the length of AB? Express your answer as a common fraction. (The diagram is not drawn to scale.)
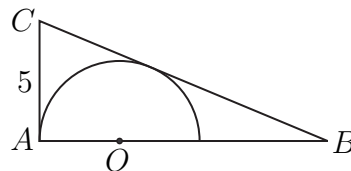
10. _____ units

Regional 2007 Face-off Questions

1. Alfie walked 0.6 km in 12 minutes. At this rate, how many metres can Alfie walk in 30 minutes?

1.

2. Express $\dfrac{1}{2} + \dfrac{1}{4} + \dfrac{1}{6}$ as a common fraction.

2.

3. Two standard dice are tossed. What is the probability that the product of the two numbers obtained is 5? Express the answer as a common fraction.

3.

**4.** In the diagram below, $BC = BD = DA$ and $\angle BCA = 50°$. What is the degree measure of $\angle ABC$?

4.
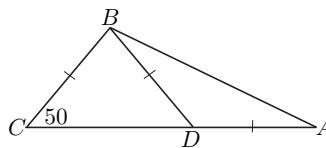
5. How many positive integers less than 2007 are divisible by both 30 and 100?

5.

6. The sum of three different positive integers is equal to 10. What is the largest possible value of the sum of their squares?

6.

7. You are told that 1 is halfway between x and 17. What is the value of x?

7.

8. The triangle below is right-angled. The hypotenuse has length 17 cm and one leg has length 8 cm. What is the number of cm^2 in the area of the triangle?

8.
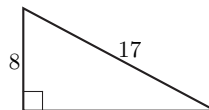
9. Let x be the 2007-th term of the geometric sequence 3, 6, 12, ..., and let y be the 2004-th term of the same sequence. What is the value of x/y?

9.

10. Fifty percent of 50% of a certain number is 50. What is the number?

10.

11. Suppose that N and $N + 17$ are both perfect squares. What is the value of N?

11.

12. Two prime numbers p and q add up to 70. What is the smallest possible positive value of $p - q$?

12.

13. The figure $ABCD$ below is a rhombus, with the vertices A, B, C, and D going counterclockwise. Vertex A has coordinates $(0,0)$, vertex B is on the positive x-axis, and vertex D has coordinates $(5, 12)$. What is the area of the rhombus?

13.
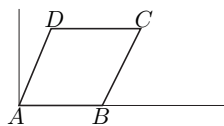
14. A circle has radius $\dfrac{2007}{\pi}$. What is the circumference of the circle?

14.

15. Suppose that $\dfrac{x}{a} = \dfrac{a}{5}$ and $a = 20$. What is the value of x?

15.

16. A basketball team won 20% more games than it lost. The team played 88 games. How many games did it win? (Ties are impossible in basketball.)

16.

17. What is the smallest positive integer n such that n is a multiple of 5, $n + 1$ is a multiple of 3, and $n + 2$ is a multiple of 2?

17.

18. The product of two consecutive odd integers is 255. What is the sum of the two odd integers?

18.

19. What is the sum of all the positive factors of 62?

19.

20. One foot is equal to 12 inches. Alan is jogging at 600 feet per minute. What is his speed in inches per second?
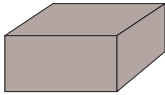
20.

21. The sum of two numbers is 100 and one of the numbers is 17. What is the positive difference between the two numbers?

21.

2

22. One corner of a square is at $(1, 2)$. The diagonally opposite corner is at $(7, 8)$. How many square units are in the area of the square?

22.

23. Express $\dfrac{10! - 9! - 8!}{10! + 9! + 8!}$ as a common fraction.

23.

24. What is the value of 2007^2?

24.

25. If $3x - 2 = \dfrac{1}{7}$, what is the value of x?

25.

26. An equilateral triangle has side 4. What integer is closest to the area of the triangle?

26.

27. Simplify: $\sqrt{\dfrac{1}{16} + \dfrac{3}{50}}$.

27.

28. Suppose that x and y are positive integers and $x^2 - y^2 = 64$. What is the smallest possible value of $x + y$?

28.

29. The surface area of a cube is 216 cm^2. What is the number of cm^3 in the volume of the cube?

29.

30. What is the value of $\sqrt{1^3 + 2^3 + 3^3 + 4^3 + 5^3}$?

30.

31. What is the remainder when 2^{2007} is divided by 9?

31.

32. Alphonse's hourly wage is 15% less than Beth's. Gamal's hourly wage is 2% greater than Beth's. How many percent is Gamal's wage greater than Alphonse's?

32.

33. The perimeter of the regular hexagon $ABCDEF$ is 192 cm. How many cm are in the perimeter of the trapezoid $ABCF$?

33.

3
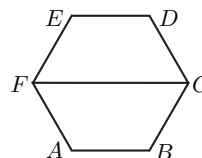
34. What is the remainder when 1111 is divided by 7 ?   34.

35. What is the value of   35.

$$1 + 2 - 3 + 4 + 5 - 6 + 7 + 8 - 9 + \cdots$$
$$+ 70 + 71 - 72 + 73 + 74 - 75 ?$$

36. The sum of the squares of two positive integers is equal to 73. What is the sum of the two integers ?   36.

37. The opera consists of 2 acts of equal length with an intermission in between. The length of the opera (including the 30 minute intermission) is exactly 3 hours. What is the ratio of the length of the intermission to the length of the first act? Express as a common fraction.   37.

38. What is the smallest positive integer n such that $5^n > 4^{n+1}$?   38.

39. What is the value of $(\sqrt{4} + \sqrt{2})^2 + (\sqrt{4} - \sqrt{2})^2$?   39.

40. Solve for x: $(4^5)^6 = (2^x)^3$.   40.

## Bull's-eye, Page 1: Problem Solving

1. Dina runs 5 kilometres in 20 minutes. At this speed, how many seconds does it take her to run 100 metres?

   1. _____ seconds

2. This year, UBC has 14000 seats of classroom space, which is exactly enough to accommodate current student enrollment. An additional 4% of classroom space seats are under construction and will be available next year. UBC plans to admit 10% more students next year than this year. How many more seats of classroom space (beyond those available now or under construction) must be provided to accommodate next year's planned enrollment?

   2. _____ seats

3. The mass of 1000 cubic mm of a certain metal is 9 grams. A box with (inner) dimensions 4 cm by 3 cm by 2 cm is filled with the metal. What is the mass, in kilograms, of the metal in the box? Give your answer as a decimal, to 3 decimal places.

   3. _____ kg

4. Alicia to Beti: "If you gave me one-half of your money, I would have $100." Beti to Gamal: "If you gave me one-third of your money, I would have $100." Gamal to Alicia: 'If you gave me one-quarter of your money, I would have $100." How many dollars do Alicia, Beti, and Gamal have between them?

   4. _____ dollars

Bull's-eye, Page 2: Combinatorics and Numbers

5. If 5 distinct circles are drawn, what is the largest possible number of points of intersection of the circles?

5. _____ points

6. On any standard die, the sum of the numbers on opposite faces is equal to 7. If two standard dice are tossed, what is the probability that the sum of the numbers on the 10 visible faces is equal to 31? Express your answer as a common fraction.

6. _____

7. What is the smallest positive integer n such that $11n + 1$ is a power of 2?

7. _____

8. In a school cafeteria, there are 2 different soups, 3 different main courses, and 3 different desserts. You are allowed to take at most one soup, at most one main course, and up to three desserts (but you cannot have two or more servings of the *same* dessert). How many different meals could you have? Include in your count the "meal" in which you eat nothing.
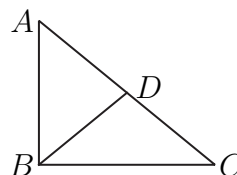
8. _____ meals

9. In the picture below (which is not drawn to scale) the large trapezoid has been cut into two trapezoids by a vertical line. Given that dimensions are as shown in the diagram, what is the value of x?

9. _____

60 x 20
60 40
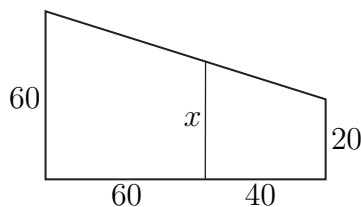
10. In the figure below, $\triangle ABC$ is right-angled at B, $AB = 12$ cm and $BC = 14$ cm. The point D bisects the hypotenuse AC. What is the area of $\triangle ABD$?

10. _____ cm^2

A D B C

11. In the figure below, $ABCD$ is a rectangle whose length AB is 6 cm and whose width BC is 4 cm. A circle of radius 2 cm is drawn, with its center at the center of the rectangle. Point P on AB is such that DP is tangent to the circle. What is the length of DP (in cm)? Express your answer as a common fraction.

11. _____ cm

D C A P B
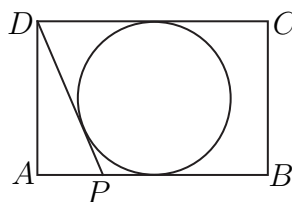
12. A triangle has vertices $A(0,0)$, $B(13,0)$, and $C(5,7)$. The triangle is inscribed in a circle. What are the coordinates of the center of the circle? Express your answer in the form (x, y), where x and y are common fractions.
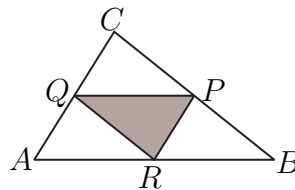
12. _____

Blitz, Page 1

1. You toss two fair dice. What is the probability of getting a "double," that is, two 1's or two 2's or two 3's or two 4's or two 5's or two 6's? Express your answer as a common fraction.

    1. _____

2. Triangle ABC has area 100 cm^2. Point P is the midpoint of BC, Q is the midpoint of CA, and R is the midpoint of AB. What is the area of the shaded triangle PQR?

    2. _____ cm^2

3. Three numbers form an arithmetic sequence. The smallest of the three numbers is -10 and the largest is 22. What is the sum of the three numbers?

    3. _____

4. At a club picnic, 30 people ate 1 hot dog each, 40 people ate 2 hot dogs each, and everyone else ate 3 hot dogs each. Altogether, 200 hot dogs were eaten. How many people ate 3 hot dogs?

    4. _____ people

5. What is the sum of the cubes of the solutions of the equation

    $$x^2 + x - 2 = 0 \,?$$

    5. _____

6. The figure below is made up of a number of small squares, each of which has sides of length 2 cm. What is the total area of the figure?

    6. _____ cm^2
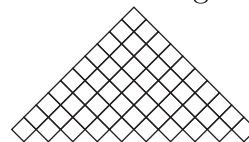
7. Alicia and Beti ran for Student Council president. Alicia got 55% of the votes, and Beti got the rest. Alicia got 80 more votes than Beti. How many votes did Alicia get?

    7. _____ votes

8. Four people of different heights are lined up in a row, from tallest down to shortest. Two people who are next to each other swap positions. Then, again two neighbours swap positions, and so on. What is the least total number of swaps that gets the people lined up from shortest to tallest?
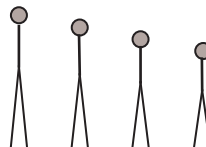
8. _____ swaps

9. Alfred walks for an hour every day. If he increases the daily distance walked by 20%, and decreases his average speed by 20%, how many *extra* minutes will he walk each day?

9. _____ minutes

10. The picture below shows four circles with the same center, with two of the "rings" between them coloured gray. The four circles have radius 5, 7, 9, and 11 units respectively. What is the ratio of the area of the inner gray ring to the area of the outer gray ring? Express your answer as a common fraction.
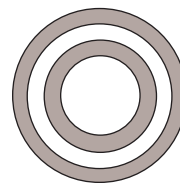
10. _____

11. Let $Q = 2 \times 3 \times 5 \times 7$. How many primes are there among the numbers $Q+2$, $Q+3$, $Q+4$, $Q+5$, $Q+6$, $Q+7$, $Q+8$, $Q+9$, and $Q+10$?

11. _____ primes

12. Alper rolls 3 standard dice once. What is the probability that the sum of the numbers rolled is 5 or more? Express your answer as a common fraction.

12. _____

13. Let (a, b) be the coordinates of the center of the circle that passes through $(1, 0)$, $(0, 1)$, and $(5, 5)$. What is the value of $a + b$? Express your answer as a common fraction.

13. _____

14. In the figure below, the cross (shaded) is made up of five congruent squares. The cross is inscribed in a big square, with the diagonals of the big square parallel to the arms of the cross. What is the ratio of the area of the cross to the area of the big square? Express your answer as a common fraction.
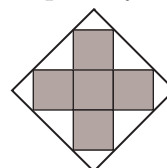
14. _____

15. If x is the temperature measured in degrees Celsius ($^\circ$C), then the temperature y measured in degrees Fahrenheit is given by the formula $y = (9/5)x + 32$. If the temperature at the North Pole is -40 degrees Fahrenheit, what is it in $^\circ$C?

15. _____ $^\circ$C

16. In the figure below, rectangle $PQRS$ has been divided into 7 squares as shown. If the smallest squares in the figure (there are 2 of them) each have side length equal to 1 unit, what is the length of PQ?

16. _____ units
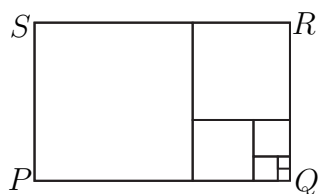
17. The figure below is a 3 by 3 grid of points. Each point is 1 cm from its nearest horizontal and vertical neighbours. There are 10 rectangles whose corners are points of the grid. What is the average area of these 10 rectangles? Give the answer as a decimal, to 1 decimal place.

17. _____ cm^2

18. Let $N = 2^{17} \times 5^{10}$. What is the total number of digits in the decimal representation of N?

18. _____ digits

19. One corner of a square is at the origin $(0, 0)$. The diagonally opposite corner is at $(7, 1)$. How many square units are in the area of the square?

19. _____ units^2

20. A computer prints the squares of all the integers from 1 to 99 inclusive. What is the total number of digits that the computer prints?

20. _____ digits

21. Suppose that

$$ab = 6, \qquad bc = 8, \qquad cd = 10, \qquad \text{and} \qquad de = 12.$$

What is the value of $\dfrac{a}{e}$? Express your answer as a common fraction.

21. _____

22. The wheels of a car travelling down Main Street make 7.5 revolutions per second. The diameter of each wheel is $2/\pi$ metres. What is the speed of the car in kilometres per hour?

22. _____ km/hr

23. Triangle ABC is right-angled at C, with $AC = 3$ and $BC = 4$. The large shaded square $PQRS$ is inscribed in $\triangle ABC$, with P and Q on AB. The small shaded square is inscribed in $\triangle SRC$, with one side along SR. What is the ratio of the side of the small shaded square to the side of the large shaded square? Express your answer as a common fraction.

23. _____
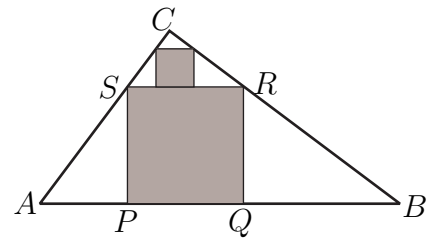
24. If x is any real number, then $\lfloor x \rfloor$ denotes the greatest integer which is less than or equal to x. For example, $\lfloor 2.33 \rfloor = 2$ and $\lfloor 6 \rfloor = 6$. For what integer n is

$$\lfloor \sqrt{1} \rfloor + \lfloor \sqrt{2} \rfloor + \lfloor \sqrt{3} \rfloor + \lfloor \sqrt{4} \rfloor + \lfloor \sqrt{5} \rfloor + \cdots + \lfloor \sqrt{n-1} \rfloor + \lfloor \sqrt{n} \rfloor$$
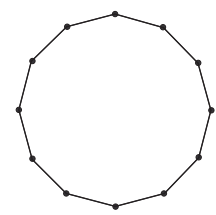
equal to 50?

24. _____

25. In the figure below, the square $DEFG$ is inscribed in $\triangle ABC$, with D and E on AB, F on BC and G on CA. Given that $DE = FC = GC = 1$ unit, what is the area of $\triangle ABC$? Express your answer in the form $\frac{a\sqrt{b}+c}{d}$, where a, b, c, and d are integers.

25. _____ units^2
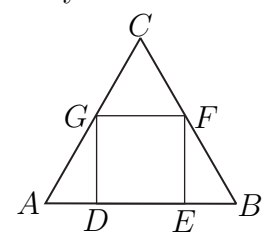
26. Three (distinct) vertices of the regular 12-gon below are chosen at random. What is the probability that no two of these vertices are adjacent to each other? Express your answer as a common fraction.

26. _____

1. How many of the perfect squares between 1 and 10000 have decimal representation with the units digit equal to 4?

1. _____

2. Let N be the result of multiplying 5^{32} by 160^5. How many zeros does the decimal expansion of N end with?

2. _____ zeros

3. In how many ways can a penny, a nickel, a dime, and a quarter be split between Alfie, Beth, and Gimel if each gets at least one coin?

3. _____ ways

4. The picture below (not drawn to scale) shows a rectangle $ABCD$, and a semi-circle with AB as diameter. The semi-circle meets side CD of the rectangle at P and Q. If the distance from A to P is 4 units, and the distance from B to P is 3 units, what is the distance from P to Q? Express your answer as a common fraction.
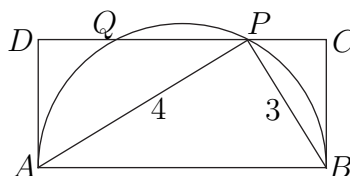
4. _____ units

5. A large bottle A contains 300 ml of a solution which is 4% acetic acid and the rest water. Bottle B has 300 ml of a solution which is 12.5% acetic acid and the rest water. How many ml of solution should you transfer from B to A so that after thorough mixing A will contain a solution which is 5% acetic acid?

5. _____ ml

6. An integer n is called *square-free* if 1 is the only perfect square that divides n. (The first few square-free integers are 1, 2, 3, 5, 6, 7, 10, 11, and 13.) What is the 100th square-free positive integer?

6. _____

7. The figure $ABCDEF$ below is a regular hexagon, and point P lies on side AB, with $AP = 3$ cm and $PB = 1$ cm. Line PE meets AD at Q. What is the ratio of the area of quadrilateral $AQEF$ to the area of hexagon $ABCDEF$? Express your answer as a common fraction.
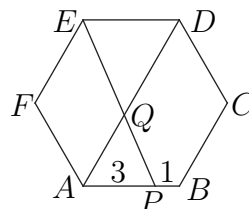
7. _____

8. There are integers a and b such that $(1 + \sqrt{2})^{16} = a + b\sqrt{2}$. What is the value of a?

8. _____

9. In the picture below, an equilateral triangle is inscribed in the large circle, and the smaller circle is inscribed in the equilateral triangle. Let X and Y be two of the points at which the smaller circle is tangent to the equilateral triangle. Suppose the line through X and Y meets the larger circle at A and B. What is the value of $\frac{AB}{XY}$?

9. _____
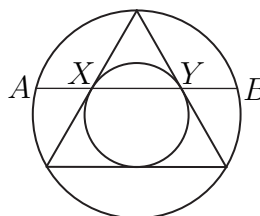
10. Halfy, Perfect, and Thirdy are mathematicians who participate in a target shooting competition to win a gold coin. When they shoot at the target, Halfy hits it $1/2$ of the time, Perfect hits it all the time, and Thirdy only hits it $1/3$ of the time. They have a total of 5 bullets.

10. _____

Halfy shoots first. If he hits, he eliminates one of the other two from the competition (at his choice, so as to maximize his chance of winning the gold). Next goes Perfect (if he was not eliminated already). He hits the target, and eliminates any of the other competitors still left (in a way that maximizes his chance of winning the gold). Then Thirdy gets a turn (if he was not eliminated already), and so on, until two of the three are eliminated *or* the 5 bullets are all gone. If two of the three are eliminated, the last remaining person wins the gold. If they run out of bullets before a winner is declared, no one wins the gold. What is the probability that no one wins the gold? Express your answer as a common fraction.

Math Challengers Regional 2007
Answers, Blitz Stage

1. 23

2. $\dfrac{1}{64}$

3. $\dfrac{1}{14}$

4. 125

5. 15

6. Sunday

7. 4

8. $6\sqrt{3}$

9. 10:20

10. 2100

11. 750

12. 70

13. 75

14. 140

15. 12

16. 1210

17. 9

18. $\dfrac{4}{15}$

19. 110

20. 48

21. 85

22. 30

23. $\dfrac{4}{9}$

24. 23

25. $\dfrac{3}{10}$

26. $\dfrac{5}{8}$

Math Challengers Regional 2007
Answers, Bull's-eye Stage

1. 29

2. 17

3. $\dfrac{2}{7}$

4. 462

5. 70

6. 18

7. 120

8. 88

9. 70

10. 2.16

11. $\dfrac{3}{16}$

12. $24\pi - 18$

Math Challengers Regional 2007
Answers, Co-op Stage

1. 452

2. 2015029

3. 44

4. 9

5. $\dfrac{4}{5}$

6. 12

7. $\dfrac{6}{11}$

8. 139

9. 99

10. $\dfrac{75}{8}$

Math Challengers Provincial 2007
Answers, Blitz Stage

1. $\frac{1}{6}$

2. 25

3. 18

4. 30

5. -7

6. 220

7. 440

8. 6

9. 30

10. $\frac{3}{5}$

11. 0

12. $\frac{53}{54}$

13. $\frac{49}{9}$

14. $\frac{5}{8}$

15. -40

16. 21

17. 1.8

18. 13

19. 25

20. 353

21. $\frac{5}{8}$

22. 54

23. $\frac{12}{37}$

24. 19

25. $\frac{7\sqrt{3}+12}{12}$

26. $\frac{28}{55}$

Math Challengers Provincial 2007
Answers, Bull's-eye Stage

1. 24

2. 840

3. 0.216

4. 220

5. 20

6. $\frac{1}{18}$

7. 93

8. 96

9. 36

10. 42

11. $\frac{13}{3}$

12. $\left(\frac{13}{2}, \frac{9}{14}\right)$

Math Challengers Provincial 2007
Answers, Co-op Stage

1. 20

2. 25

3. 36

4. $\frac{7}{5}$

5. 40

6. 163

7. $\frac{13}{42}$

8. 665857

9. $\sqrt{5}$

10. $\frac{1}{18}$

Math Challengers Provincial 2007
Answers, Blitz Stage

1. $\frac{1}{6}$

2. 25

3. 18

4. 30

5. -7

6. 220

7. 440

8. 6

9. 30

10. $\frac{3}{5}$

11. 0

12. $\frac{53}{54}$

13. $\frac{49}{9}$

14. $\frac{5}{8}$

15. -40

16. 21

17. 1.8

18. 13

19. 25

20. 353

21. $\frac{5}{8}$

22. 54

23. $\frac{12}{37}$

24. 19

25. $\frac{7\sqrt{3}+12}{12}$

26. $\frac{28}{55}$

Math Challengers Provincial 2007
Answers, Bull's-eye Stage

1. 24

2. 840

3. 0.216

4. 220

5. 20

6. $\frac{1}{18}$

7. 93

8. 96

9. 36

10. 42

11. $\frac{13}{3}$

12. $\left(\frac{13}{2}, \frac{9}{14}\right)$

Math Challengers Provincial 2007
Answers, Co-op Stage

1. 20

2. 25

3. 36

4. $\frac{7}{5}$

5. 40

6. 163

7. $\frac{13}{42}$

8. 665857

9. $\sqrt{5}$

10. $\frac{1}{18}$

Blitz, Page 1

1. What is 20% of 30% of 400?

1. _____

2. You flip 2 fair coins, and you win if you get 2 heads or 2 tails. What is the probability that you win? Express your answer as a common fraction.
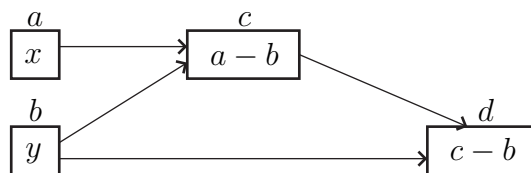
2. _____

3. What is the sum of all the primes between 10 and 20?

3. _____

4. Suppose that $\dfrac{x-2}{x+4} = \dfrac{1}{3}$. What is the value of x?

4. _____

5. The area of a circle is $\dfrac{49}{\pi}$ cm². What is the circumference of the circle?

5. _____ cm

_____ ———————$\left(\dfrac{49}{\pi}\right)$

6. In the flow chart, the value of y is 10 and the output value in d is 10. What is the value of x?

6. _____

a $\boxed{x}$ → c $\boxed{a-b}$
b $\boxed{y}$ → d $\boxed{c-b}$

7. Alfie had $10 to spend. He bought 3 chocolate bars and 8 cookies. The chocolate bars were priced at $1.60 each, tax included. What is the highest possible price of a cookie? Give your answer in cents.

7. _____ cents

8. Dan is riding his bicycle at 10 m/sec. What is his speed in km/hr?

8. _____ km/hr

9. What is the smallest positive integer N such that $2008 + N$ contains only odd digits in its decimal representation?

9. _____

10. The first five terms of an infinite arithmetic sequence are 4, 11, 18, 25, and 32. What is the value of the 16-th term of the sequence?

10. _____

_____

11. In the diagram below, $AR = PR$ and $PQ = BQ$. Also, $\angle ARP$ has measure 128 degrees, and $\angle PQB$ has measure 88 degrees. What is the degree measure of $\angle ACB$?

11. _____ degrees
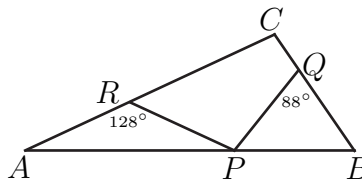
12. North American roulette wheels have red, black, and green pockets. The red pockets are labelled 1, 3, 5, ..., 35. The black pockets are labelled 2, 4, 6, ..., 36. The green pockets are labelled 0 and 00. When the wheel is spun, the ball is equally likely to land in any one of the pockets. What is the probability that the ball lands in a red pocket? Give your answer as a common fraction.

12. _____

13. Find the sum of all the real numbers x such that

$$|x - 99| + |x - 100| = 10.$$

13. _____

14. Alphonse and Beti have $41 between them, Beti and Gamay have $50 between them, and Gamay and Alphonse have $59 between them. How much money does Gamay have?

14. _____ dollars

15. At a Maritimes hockey tournament, each of the 4 Maritimes provinces has 3 teams, and each team plays one game against every team which is *not* from the same province. (So for example the 3 Nova Scotia teams do not play against each other.) What is the total number of games played in the tournament?

15. _____ games

16. A circle has area 300π units^2. An equilateral triangle is inscribed in the circle. What is the perimeter of this equilateral triangle?

16. _____ units

17. Xavier and Yolande, working in different places, put together a combined total of 480 lunch packages for a math competition. Yolande worked twice as fast as Xavier, and for 10% more time than Xavier. How many of the packages did Yolande put together?

17. _____ packages

18. The (internal) radius of the base of cylindrical cooking pan A is 8 cm, and the internal radius of the base of cylindrical cooking pan B is 12 cm. Pan A contains water which is 2 cm deep, and pan B is empty. If the water from pan A is poured into pan B, how deep will the water in pan B be? Express your answer as a common fraction.

18. _____ cm

19. In basketball, one can score points in three ways: a free throw basket (1 point), a 2 point basket (2 points), and a 3 point basket (3 points). In a tournament, Alicia scored a total of 160 points. She got 34 free throw baskets, and got three times as many 2 point baskets as 3 point baskets. What is the total number of baskets (free throw, 2 point, and 3 point) that she got in the tournament?

19. _____ baskets

20. What is the area of the convex quadrilateral with vertices $(0,0)$, $(6,0)$, $(2,5)$ and $(0,4)$?

20. _____ units^2

21. A generous teacher gives a 10 question true/false quiz in which you get 2 marks for every right answer, 0 marks for every wrong answer, and 1 mark for every question that you skip (10 is a pass). In how many different ways can you get a mark of 18? You could for example have the wrong answer on the fifth question, with the rest right. Or you could get the right answer on the first 8 questions, and skip the last two.

21. _____ ways

22. A right pyramid has a square base, and all edges of the pyramid are equal. If the volume of the pyramid is 144 cm^3, what is the area of the base of the pyramid?

22. _____ cm^2

23. Define the sequence a_1, a_2, a_3, and so on by $a_1 = 3$ and

$$a_{n+1} = \frac{a_n - 1}{a_n + 1}$$

for all $n \geq 1$. What is the value of a_{2008}?
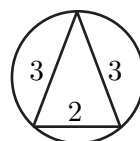
23. _____

24. The figure $ABCDE$ is a regular pentagon, and the point P in the interior of the pentagon is chosen so that $\triangle ABP$ is equilateral. What is the degree measure of $\angle BPC$?

24. _____ degrees
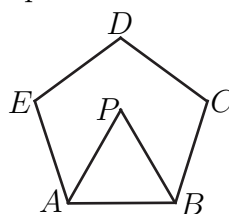
25. In the multiplication problem below, the letters G, M, A, T, and H represent *different* digits. What is the value of $G + M + A + T + H$?

$$\begin{array}{r} 2\,0\,0\,8 \\ \times\quad\ \ H\,T \\ \hline G\,M\,A\,T\,H \end{array}$$

25. _____

26. A triangle with sides 3, 3, and 2 is inscribed in a circle. For what number x is the area of the circle equal to πx? Express x as a common fraction.

26. _____

1. A distance of 1 cm on a map represents a distance of 5 km on the ground. A lake has area 200 km^2. How many cm^2 should the image of the lake occupy on the map?

1. _____ cm^2

2. A cup of coffee costs \$3.00, and a muffin costs \$4.00. This week, Sabrina spent exactly \$29.00 on coffee and muffins. What is the sum of the smallest number of muffins and the largest number of muffins that Sabrina could have bought this week?

2. _____ muffins

3. Alicia's Toyota Camry uses 11.5 litres of gasoline to travel 100 km in the city, and 7.5 litres to travel 100 km on the highway. Eighty percent of the distance Alicia travels per year is in the city, and 20% is on the highway. What is the average number of litres Alicia uses to travel 100 km? Give the answer correct to 1 decimal place.

3. _____ litres

4. Alicia, Beti, and Gamay ran for Student Council president. Alicia won with 45% of the votes, Beti got 40%, and Gamay got 15%. If 20 people had switched their vote from Gamay to Beti, then Beti would have ended up with 1 more vote than Alicia. How many people voted?

4. _____ people

Canadian Math Challengers Society

Bull's-eye, Page 2: Combinatorics and Numbers

5. Alicia has 6 pairs of shoes, identical except for colour: 3 of the pairs (6 shoes) are brown, 2 pairs are red, and 1 pair is green. Alicia is completely colour blind, so she picks a left shoe and a right shoe at random. What is the probability that the two shoes are of the same colour? Express your answer as a common fraction.

5. _____

6. The positive integer N is a multiple of 8. When N is divided by 25, the remainder is 9. What is the smallest possible value of N?

6. _____

7. The positive integers a, b, and c satisfy the equation

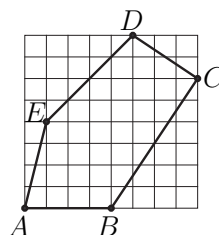$$\frac{4}{5} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$$

What is the largest possible value of $a + b + c$?

7. _____

8. What is the smallest prime that divides 2047?

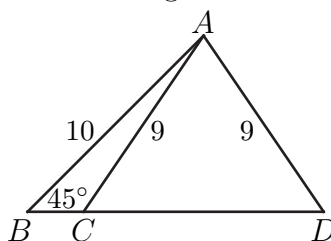8. _____

Bull's-eye, Page 3: Geometry

9. Each small square in the diagram below has area 1 unit^2. The diagram consists of 64 squares. What is the area enclosed by the 5-sided figure $ABCDE$? The points A, B, C, D, and E are all grid points.
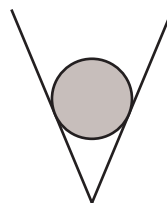
9. _____ units^2

10. In the figure below, C is on the line segment BD, and $\angle ABC$ has measure $45°$. Also, we have $AB = 10$, and $AC = AD = 9$. What is the length of CD? Express your answer in the form $\sqrt{N}$, where N is an integer. For example, an answer of $\sqrt{160}$ is of the right form.
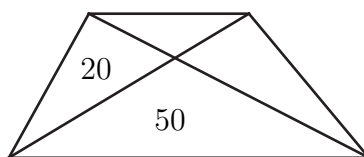
10. _____ units

11. A spherical ball of radius 2 is dropped into a cup. The cup is a right-circular cone, with the radius of the top equal to 5 and the height equal to 12. When the ball reaches as low as it can, how far is the bottom of the ball from the vertex of the cone? Express your answer as a common fraction.

11. _____ units

12. In the trapezoid below, lines that look parallel are parallel. The trapezoid is divided into four regions by its diagonals. The areas of two of these regions are 50 cm^2 and 20 cm^2, as indicated. What is the area of the entire trapezoid?

12. _____ cm^2

Co-op, Page 1: Team answers must be on the *coloured* page.
Answers on a white page will not be graded.

1. How many five-digit positive integers have two 2's and three 3's in their decimal representation?

1. _____

2. There are four different positive integers a, b, c, and d such that

$$a^3 + b^3 = c^3 + d^3 = 1729.$$

What is the value of $a + b + c + d$?

2. _____

3. What is the largest number of $20 \times 20 \times 10$ wooden boxes that can be put in a $50 \times 50 \times 40$ box? All measurements are in centimetres. Boxes cannot be cut, or changed in shape or size.

3. _____ boxes

4. Some positive integers can be expressed as the sum of two perfect squares. For example, $1 = 0^2 + 1^2$, $2 = 1^2 + 1^2$, $4 = 0^2 + 2^2$, $5 = 1^2 + 2^2$. But, for example, 3, 6, and 7 cannot be expressed as the sum of two perfect squares. How many of the integers from 1 to 25 *cannot* be expressed as the sum of two perfect squares?

4. _____

Co-op, Page 2: Team answers must be on the *coloured* page.
Answers on a white page will not be graded.

5. In the 3×3 grid below, every point is at a unit distance from its nearest horizontal or vertical neighbours. Call a line segment *good* if the endpoints of the line segment are grid points, and the line segment contains no grid point other than its endpoints. (Two good line segments have been drawn in the diagram.) How many different good line segments are there?

5. _____ segments
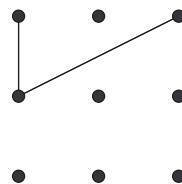
6. The picture below (on the left) shows a 4×4 "checker board." How many ways are there to place a white piece and a black piece on two different squares of this board, so that the two pieces are *not* on squares that are next to each other. (Two squares are considered next to each other if they have a side or a vertex in common.) The three pictures below (on the right) show three of the allowed ways to place the white piece and the black piece.

6. _____ ways
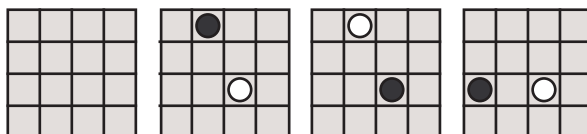
7. Let N be a positive integer. Define $f(N)$ to be the total number of occurrences of primes in the prime factorization of N. For example, $f(60) = 4$, since 60 is the product of the primes 2, 2, 3, and 5. How many positive integers $N \leq 125$ are there such that $f(N) = 4$?

7. _____

8. In the 4×4 grid below, every point is at a unit distance from its nearest horizontal or vertical neighbours. How many ways are there to choose a collection of 4 points of the grid so that these 4 points form the corners of a square?

8. _____ ways

9. Imagine listing, in increasing order, the positive integers which are neither perfect squares nor perfect cubes, nor perfect fourth powers, nor perfect fifth powers, and so on. The first few numbers in the list are 2, 3, 5, 6, 7, and 10. What is the 250-th number in the list?

9. _____

10. The figure below is a quarter-circle with center O and radius 4. The point M is on the curved part of the quarter-circle, and the length of the arc PM is equal to the length of the arc MQ. The point N bisects the line segment OP. What is the area of the shaded region? Express your answer in terms of π.

10. _____ units^2
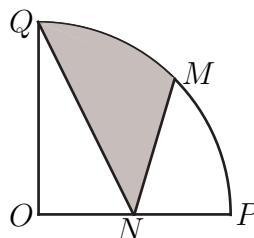
1. What is the value of 2^{13} ?

....

2. The perimeter of a rectangle is 18 cm. The length of the rectangle is twice the width. What is the number of cm^2 in the area of the rectangle?

....

3. Twenty percent of 20% of a certain number is 20. What is the number?

4. The measures of the angles of a triangle are in the ratio $2:3:4$. What is the degree measure of the largest angle?

....

5. What is the (positive) value of $\sqrt{x^3 + y^3}$ when $x = 8$ and $y = 4$?

....

6. Simplify: $\dfrac{35}{0.014}$

7. Beti has 25% more loonies than Alfie. Between them they have 99 loonies. How many loonies does Alfie have?

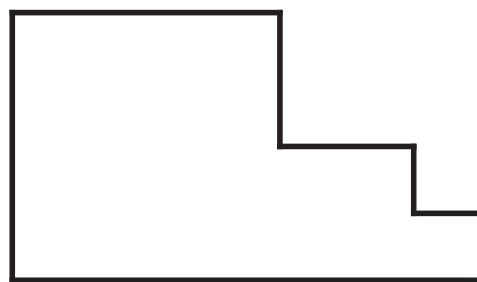...

8. The area of a circle is less than 300π cm^2. If the radius of the circle is an integer number of cm, what is the largest possible number of cm in the radius of the circle ?

....

9. At Mall-Wart$^{\text{TM}}$, 4 pants and 6 shirts cost $\$288$. How much do 10 pants and 15 shirts cost?

10. The figure below is made by sliding together a 4×4 square, a 2×2 square, and a 1×1 square. What is the perimeter of the figure?

....

11. What is the value of $4^3 + 5^3 + 6^3$?

....

12. What is the 40-th number in the sequence $1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5,$ $5, 5, 5, 5, 6, \ldots$?

13. A triangle with sides 6, 8, and 10 is inscribed in a circle. What is the area of the circle? Express your answer in terms of π.

⋯⋯⋯⋯⋯⋯⋯⋯⋯⋯⋯⋯⋯⋯⋯⋯⋯⋯⋯⋯⋯

14. A drawer contains 4 white socks and 4 black socks. Alicia removes 2 socks from the drawer, chosen at random. What is the probability that the socks are of the same colour? Express your answer as a common fraction.

⋯⋯⋯⋯⋯⋯⋯⋯⋯⋯⋯⋯⋯⋯⋯⋯⋯⋯⋯⋯⋯

15. A prime number p is called a *Sophie Germain* prime if $2p + 1$ is also prime. What is the smallest Sophie Germain prime that is bigger than 30?

16. At $3\!:\!00$ o'clock the angle between the hour hand and the minute hand of a clock is $90°$. What is the angle between the two hands at $3\!:\!10$?

. . . .

17. Peter can pick 3 pecks of peppers in 4 hours. Petra can pick 4 pecks of peppers in 3 hours. Working together, how many pecks of peppers can Peter and Petra pick in 12 hours?

. . . .

18. What is the sum of the two positive integers x such that $x^2 + 40$ is a perfect square?

19. A mother who is 180 cm tall is standing in the sunshine beside her 120 cm tall child. If the child casts a 70 cm shadow, what is the number of cm in the shadow cast by the mother?

....

20. Three circles have, respectively, radius 4, 6, and 12 units. What is the radius of the circle whose area is the sum of the areas of the three circles?
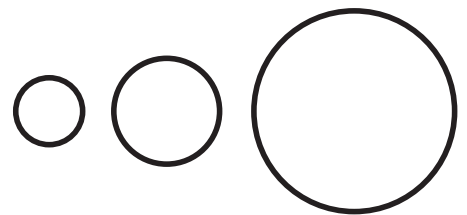
....

21. Two fair dice are tossed. What is the probability that the positive difference between the numbers obtained is 1? Express your answer as a common fraction.

22. Four numbers form an arithmetic sequence. The smallest of the four numbers is 5 and the largest is 15. What is the sum of the four numbers?

....

23. Let $D(x, y) = x^2 - y^2$. What is the value of $D(125, 25)$?

....

24. A square is inscribed in an isosceles right-angled triangle. Two vertices of the square are on the hypotenuse of the triangle. What is the ratio of the area of the square to the area of the triangle? Express your answer as a common fraction.
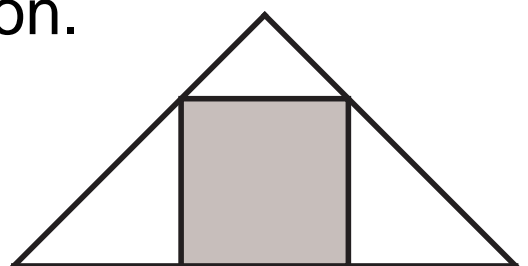
25. The length of rectangle $\mathcal{R}$ is 30% more than the side of square $\mathcal{S}$, and the width of $\mathcal{R}$ is 30% less than the side of $\mathcal{S}$. The area of $\mathcal{R}$ is how many percent less than the area of $\mathcal{S}$?
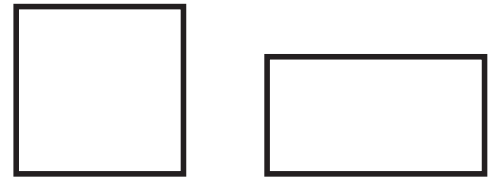
26. What is the sum of the positive integers that divide 81?

27. What is the smallest positive integer n such that $n-4$, $n-2$, $n+2$, and $n+4$ are all prime?

28. How many integers $k \geq 8$ are there such that there is a triangle with sides 8, 8, and k?

....

29. There are two different pairs (a, b) and (c, d) such that

$$\frac{a!}{b!} = \frac{c!}{d!} = 30.$$

What is the value of $a + c$?

....

30. How many millilitres of milk must you mix with 180 ml of black coffee so that the mixture is 20% milk?

31. What is the least common multiple of the first five positive odd integers?

....

32. What is the smallest integer which is greater than 2008 and is divisible by both 5 and 9?

....

33. Alfie and Beth each pick a number from 1 to 10 (inclusive), independently and at random (they could pick the same number). What is the probability that the sum of their two numbers is equal to 16? Express your answer as a common fraction.

34. What is the largest number n less than 100 such that $n - 1$ and $n + 1$ are both prime?

....

35. One can of paint is required to paint all the faces of a big cube. The big cube is cut up into 64 equal little cubes. How many cans of paint are required to paint all the faces of the 64 little cubes?

....

36. Suppose that $4^{x+3} = 8^{x-3}$. What is the value of x?

2008 Canadian Math Challengers Provincial Questions

Blitz, Page 1
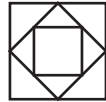
1. Let $x = (1.1)^2 + (0.1)^2$. Express x as a decimal.

   1. _____

2. The volume of a cone is 9 cm^3. What is the volume of a cylinder which has the same base and the same height as the cone?

   2. _____ cm^3

3. Suppose that $x > 0$ and $\dfrac{10}{x} + \dfrac{40}{x} = x$. What is the *integer* which is nearest to x?

   3. _____

4. The price of A is \$2.60. The price of B is 25% of the price of A. The price of C is 16/13 times the price of B. What is the price of C? Express your answer in cents.

   4. _____ cents

5. Suppose that $1 + 2 + 3 + 4 + N + 6 + 7 + 8 + 9 = 20$. What is the value of N?

   5. _____

6. The figure below consists of three squares with the same centre. The middle-sized square is inscribed diagonally in the large square, and the small square is inscribed diagonally in the middle-sized square. The diagonal of the smallest square has length 5 cm. What is the area of the largest square?

   6. _____ cm^2

7. Two fair dice are tossed. What is the probability that the sum is 8? Express your answer as a common fraction.

   7. _____

8. Let $\mathcal{S}$ be the 6-element set $\{A, B, C, D, E, F\}$. How many of the subsets of $\mathcal{S}$ have exactly 2 elements?

8. _____

9. On a trip, Alicia drove at 50 km per hour for 30 minutes, then at 80 km/hr for 1 hour, and then at 100 km/hr for 1 hour. What was her average speed for the trip, in km/hr?

9. _____ km/hr

10. What is the area of the convex quadrilateral whose vertices are at $(0,0)$, $(0,2)$, $(10,4)$, and $(10,0)$?

10. _____ units^2

11. What is the smallest integer which is greater than 300 and has exactly four different prime factors?

11. _____

12. Alicia chooses at random a multiple of 6 between 1 and 121. Beth chooses at random a multiple of 15 between 1 and 121. What is the probability that they choose the same number? Express your answer as a common fraction.

12. _____

13. A small cup of coffee costs \$1.50, a medium cup costs \$2.00, and a large cup costs \$2.50. Last month, Sabrina spent a total of \$100.00 on cups of coffee. Of the cups of coffee she bought, 20 were small, 20 were medium, and the rest were large. What is the total number of cups of coffee that Sabrina bought last month?

13. _____ cups

14. Four equal squares are removed from the corners of square $ABCD$, leaving the shaded cross below. The arms of the cross have width 5 units. If the area of the cross is 375 units^2, what is the area of the square $ABCD$?

14. _____ units^2
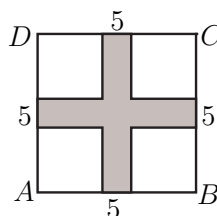
15. What is the sum of the solutions of the equation $|1+x|+|1-x| = 4$?

15. _____

16. The figure below is constructed from ~~1 by 1 squares.~~ What is the total number of squares (of all sizes) in the figure?

16. _____ squares
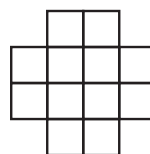
17. How many positive integers divide 120 but do not divide 24? Note that for any positive integer n, 1 divides n and n divides n.

17. _____

18. How many degrees are there in the acute angle between the hour hand and the minute hand of an ordinary clock at $12\!:\!12$ PM?

18. _____ degrees

19. The picture below is a *top view* of a big solid cube that has been put together from 125 little cubes, each of side 1 cm. The centre little cube of each face of the big cube is removed. (The little cube removed from the top face has been marked with an x.) What is the total surface area of the solid thus created?


19. _____ cm^2
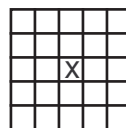
20. How many digits are there in the decimal representation of 5^{40}?

20. _____ digits

21. The lines in the figure below represent the streets of a village. The shaded region is a park with no road through it. In how many different ways can Alicia drive from home (H) to school (S), if she can never be driving South (downward) or West (leftward)?

21. _____ ways
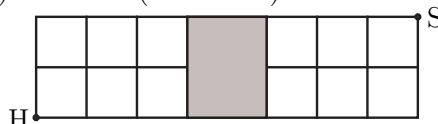
22. Alphonse, Beth, and their mother Gamay were all born in January. In Februry 2008, their ages were all prime numbers: 11, 13, and 43. What is the first calendar year after 2008 that their ages in February will all be prime?

_____

22. _____

23. The picture shows a triangle ABC, with $AB = 4$, $AC = 1$, and a right angle at A. So $\triangle ABC$ has area 2. How many points P are there, in the plane of $\triangle ABC$, such that (i) $\triangle ABP$ has area 2, and (ii) $\triangle ABP$ has a right angle somewhere, not necessarily at A. Include in your count the point C of the picture.

23. _____ points
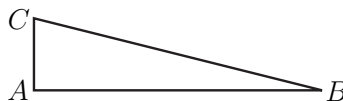
24. What is the smallest possible positive value of $9 - \dfrac{2008}{N}$, if N is a positive integer? Express your answer as a common fraction.

24. _____

25. The two large shaded circles each have radius 1, and the distance between their centres is 5/2. The large circles and the small circle are tangent to the same line, and are on the same side of that line. The small circle lies between the two large circles and is tangent to them. What is the radius of the small circle? Express your answer as a common fraction.

25. _____ units
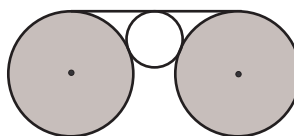
26. You have a Magic Money Machine (MMM). Whenever you put in a penny, the MMM keeps the penny, but spits out either 5 or 8 pennies. So if you have only 1 penny, and use the MMM twice, you may end up with 9, 12, or 15 pennies. What is the largest number of pennies that it is *impossible* to end up with, if you have only 1 penny and are allowed to use the MMM as many times as you want?

26. _____ pennies

1. A distance of 1 cm on a map represents a distance of 5 km on the ground. A lake has area 200 km^2. How many cm^2 should the image of the lake occupy on the map?

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$$\frac{4}{5} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$$

What is the largest possible value of $a + b + c$?

7. _____

8. What is the smallest prime that divides 2047?

8. _____

Bull's-eye, Page 3: Geometry

9. Each small square in the diagram below has area 1 unit^2. The diagram consists of 64 squares. What is the area enclosed by the 5-sided figure $ABCDE$? The points A, B, C, D, and E are all grid points.
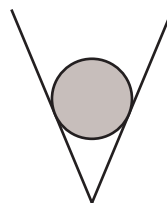
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10. _____ units

11. A spherical ball of radius 2 is dropped into a cup. The cup is a right-circular cone, with the radius of the top equal to 5 and the height equal to 12. When the ball reaches as low as it can, how far is the bottom of the ball from the vertex of the cone? Express your answer as a common fraction.

11. _____ units

12. In the trapezoid below, lines that look parallel are parallel. The trapezoid is divided into four regions by its diagonals. The areas of two of these regions are 50 cm^2 and 20 cm^2, as indicated. What is the area of the entire trapezoid?

12. _____ cm^2

Co-op, Page 1: Team answers must be on the *coloured* page.
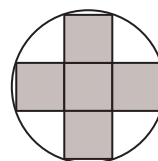   Answers on a white page will not be graded.

1. Line segment AB is 3 cm long. Line ℓ is parallel to AB and 1 cm from AB. In how many ways can we choose a point P on line ℓ so that $\triangle ABP$ is isosceles?

1. _____ ways

ℓ

A B

2. There are exactly two integers n between 10 and 99 such that the last two digits of n^2 are (in order) the same as the two digits of n. What is the sum of these two integers? (Note that 11 is not such an integer, because $11^2 = 121$.)

2. _____

3. How many 3-letter "words" can be made using letters chosen from the letters in "CANADA"? For example, "ADN" is an acceptable word, as are "AAA" and "DAA". But "NCC" is not acceptable, since it has two C's while "CANADA" has only one.

3. _____ words

4. In the diagram below, the cross (shaded) is constructed using 5 identical squares. The cross is inscribed in a circle of radius 6. What is the area of the cross?

4. _____ units^2

5. How many integers between 100 and 999 divide 96^2?

5. _____ integers

6. How many integers between 100 and 1000 have no 1's and no 2's in their decimal representation?
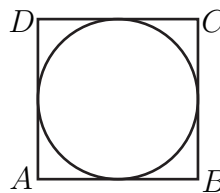
6. _____ integers

7. The diagram below shows a $3 \times 4 \times 7$ rectangular box. What is the length of the shortest path on the surface of the box from point X to point Y? Give your answer in the form $\sqrt{N}$, where N is an integer. For example, an answer of $\sqrt{172}$ is of the right form.

7. _____ units
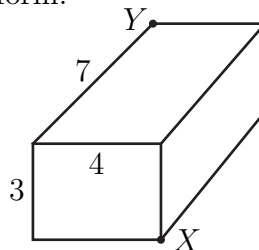
Co-op, Page 3: Team answers must be on the *coloured* page.
Answers on a white page will not be graded.

8. In the diagram below, a circle of radius greater than 9 cm is inscribed in the square $ABCD$. A point P on the circle is 8 cm from side AB of the square, and 9 cm from side AD. What is the radius of the circle?

8. _____ cm

9. The positive integer n will be called *fat* if n is divisible by every positive integer which is less than or equal to $\sqrt{n}$. How many fat positive integers are there?

9. _____

10. Define $A(n,i)$ by $A(n,i) = n^i - n^{i-1}$. Find the sum of all values of $A(n,i)$, as n ranges over the 10 integers from 1 to 10, and i independently ranges over the 3 integers from 1 to 3.

10. _____

PROVINCIAL 2008 FACE-OFF QUESTIONS
ACTUALLY USED

1. The first term of a geometric sequence is 0.3, and the second term is 0.6. What is the sixth term of the sequence? Express your answer as a decimal, to one decimal place.

2. What is the height of a cone which has the same base and the same volume as a cylinder of height $15 \, \text{cm}$?

3. What is the value of $11 + 22 + 33 + 44 + 55 + 66 + 77 + 88 + 99$?

4. Alicia bought two 2-inch binders and four 1-inch binders. The 2-inch binders cost $5.50 each, and the 1-inch binders cost $3.25 each. What was Alicia's average cost, in dollars, per binder?

5. If $3x + 5y = 345$ and $5x + 3y = 543$, what is the value of $x + y$?

6. What is the sum of all the positive integers that are perfect cubes and that are each less than 100?

7. An elephant can eat 20 bags of grain in 2 hours. At this rate, how many bags of grain can the elephant eat in 24 minutes?

8. In the right-angled triangle below, the hypotenuse has length 17 units, and one of the legs has length 15 units. How many units^2 are in the area of the triangle?
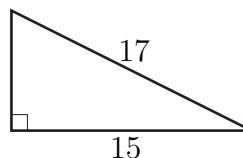
9. In an election, Alfie got 60% of the votes and Beth got the rest of the votes. If Alfie got 120 more votes than Beth, how many people voted?

10. What is the sum of the (decimal) digits of $10^6 - 8$?

11. The sum of all the positive integers from 1 to n (inclusive) is greater than 100. What is the smallest possible value of n?

12. How many positive factors of $2^6 \times 5^6$ are perfect squares?

13. The numbers 1, 2, 3, 4, and 5 are placed in the squares of the cross below, one to a square. Let H be the sum of the numbers in the horizontal line of squares, and let V be the sum of the numbers in the vertical line of squares. What is the smallest possible value of $H + V$?
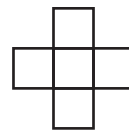
14. If $x \star y = x^2 + 3xy + y^2$, what is the value of $3 \star 7$?

15. A hen lays an egg every 26 hours. How many eggs will she lay in 13 weeks?

16. Alfie's back yard is a rectangle 10 metres wide and 20 metres long. He is tired of mowing the lawn. How many cubic metres of concrete does Alfie need in order to cover the back yard with an 8 cm thick layer of concrete?

17. The sum of two different positive integers is equal to 14. What is the smallest possible value of the sum of their squares?

18. The volume of a spherical ball is 288π cubic cm. What is the number of cm in the radius of the ball?

19. The Delta Blues basketball team played 50 games, and lost 24 more games than it won. (There are no ties in basketball.) How many games did the Blues win?

20. The table below shows daily rainfall, in mm, during one dismal November week.

day	Su	M	Tu	W	Th	F	Sa
mm	12	8	15	16	10	13	17

What was the average daily rainfall (in mm) that week?

21. What number is halfway between 1492 and 2008?

22. In old British coinage, a 1 shilling coin was worth 12 pence, and a 1 pound coin was worth 20 shillings. What was the value, in pence, of 1 pound, 5 shillings, and 3 pence?

23. Fruit "R" Us sells mangos at 5 for 4 dollars. SellHigh sells mangos at 4 for 5 dollars. How many more dollars does it take to buy 20 mangos at SellHigh than at Fruit "R" Us?

24. What is the smallest positive integer that does not divide 840?

25. Express $1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1}}}$ as a common fraction.

26. The fifteen numbers 1, 2, 3, ..., 14, 15 can be divided into three groups so that the sum S of the numbers in any group is the same. What is that sum S?

27. Alicia picks at random an integer from 1 to 6 (inclusive), with all choices equally likely. Beth independently picks at random an integer from 1 to 6. What is the probability that Alicia's number is *greater* than Beth's? Express your answer as a common fraction.

Math Challengers Regional 2008
Answers, Blitz Stage

1. 24

2. $\frac{1}{2}$

3. 60

4. 5

5. 14

6. 30

7. 65

8. 36

9. 1103

10. 109

11. 108

12. $\frac{9}{19}$

13. 199

14. 34

15. 54

16. 90

17. 330

18. $\frac{8}{9}$

19. 90

20. 19

21. 55

22. 72

23. -2

24. 66

25. 30

26. $\frac{81}{32}$

Math Challengers Regional 2008
Answers, Bull's-eye Stage

1. 1600

2. 576

3. $\frac{1}{81}$

4. 65

5. 8

6. 10

7. 130

8. 32

9. $\frac{9}{16}$

10. 28

11. 168

12. $\frac{31}{112}$

Math Challengers Regional 2008
Answers, Co-op Stage

1. 10   5. 28   8. 20

2. 32   6. 156   9. 273

3. 24   7. 13   10. $2\pi + 2\sqrt{2} - 4$

4. 12

Math Challengers Provincial 2008
Answers, Blitz Stage

1. 1.22

2. 27

3. 7

4. 80

5. −20

6. 50

7. $\dfrac{5}{36}$

8. 15

9. 82

10. 30

11. 330

12. $\dfrac{1}{40}$

13. 52

14. 1600

15. 0

16. 17

17. 8

18. 66

19. 174

20. 28

21. 20

22. 2026

23. 8

24. $\dfrac{1}{28}$

25. $\dfrac{25}{64}$

26. 18

Math Challengers Provincial 2008
Answers, Bull's-eye Stage

1. 8

2. 7

3. 10.7

4. 380

5. $\dfrac{7}{18}$

6. 184

7. 26

8. 23

9. 35

10. $\sqrt{124}$

11. $\dfrac{16}{5}$

12. 98

Math Challengers Provincial 2008
Answers, Co-op Stage

1. 5

2. 101

3. 34

4. 72

5. 9

6. 448

7. $\sqrt{98}$

8. 29

9. 8

10. 3015

2009 Canadian Math Challengers Regional Questions

Blitz, Page 1

1. Alicia tosses 3 fair coins. What is the probability that she gets at least 1 head? Express your answer as a common fraction.

1. _____

2. It took Anita 2.25 hours to walk 12.5 km. At this rate, how many hours will it take for her to walk a total of 50 km?

2. _____ hours

3. What is the value of $0.15 + 0.2 + 0.25 + 0.3 + 0.35$? Express your answer as a decimal.

3. _____

4. The points A, B, and C in the figure below are 3 consecutive vertices of a regular pentagon. What is the degree measure of angle ABC?
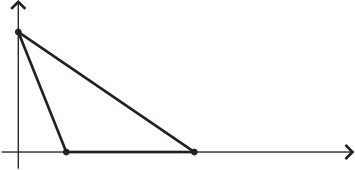
4. _____ degrees
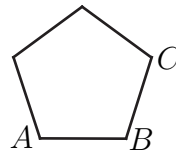
5. After getting all his cavities filled by the dentist, Andy celebrated by buying candies which were on sale at 32 per dollar. He spent $3.50 on the candies. How many candies did he buy?

5. _____ candies

6. Simplify: $\dfrac{4 + 5 + 6 + 7}{1 + \frac{1}{2} + \frac{1}{3}}$.

6. _____

7. What is the total (outer) surface area of a closed box with sides 1, 2, and 3?
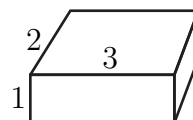
7. _____ units^2

8. Let $x = 1.23 + 4.56 + 7.89$. What is the integer nearest to x?

8. _____

9. Express $\dfrac{(99!)(101!)}{(98!)(102!)}$ as a common fraction.

9. _____

10. Alfie is running at a speed of 5 metres per second. What is his speed in kilometres per hour?

10. _____ km/hr

11. At Big Box Secondary School, each teacher teaches 4 classes per day, each student takes 5 classes per day, each class has 35 students, and there are 150 teachers. How many students are there at the school?

11. _____ students

12. The vertices of a triangle in the coordinate plane are at $(1, 0)$, $(x, 0)$, and $(0, x - 1)$. Given that $x > 1$, and that the triangle has area 18, what is the value of x? (The picture is not drawn to scale.)

12. _____

13. In a "mixed" chess tournament, every person played 5 games, 3 of them against a male player, and 2 against a female player. Altogether, there were 300 games. How many males played in the tournament?

13. _____ males

14. Suppose that $|x + 1| + |x - 1| \geq 10$ and $2x + |x - 1| \geq 6$. What is the smallest possible value of x?

14. _____

15. King Charles the Short-Sighted rode all the way around the boundary of his kingdom, which is a circle with diameter 100 kilometres. While he rode along, he could see only 20 metres in any direction. What is the total area (both inside and outside his kingdom) of the region that he saw? Assume the world is flat. Express your answer in terms of π.

15. _____ km^2

16. What is the greatest prime factor of $13^{12} + 13^{13} + 13^{14}$?

16. _____

17. We have 4 identical rectangular tables, each of which has short side equal to a, and long side equal to b. When the tables are pushed together short sides to short sides (left part of picture), the resulting long table has perimeter which is 2 times the perimeter when the tables are pushed together long sides to long sides (right part of picture). What is the ratio of a to b? Express your answer as a common fraction.

17. _____

18. The lines below represent the streets of a small town. All blocks in the town are squares. Alphonse is standing at the street intersection in the middle of town indicated by the small black circle. How many intersections can he end up at by walking for 3 or fewer blocks? Don't forget about the intersection he starts at, which he can reach by walking for 0 blocks, or 1 block north and 1 block south.
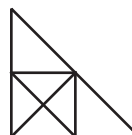
18. _____
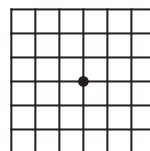
19. A car uses 12 litres of gas to travel 100 km. If gas costs \$1.50 a litre, how much does it cost to buy the gas to travel 12 km? Give your answer in dollars, to the nearest cent.

19. _____ dollars

20. Let $f(x,y,z) = \dfrac{xyz}{x+y+z}$. What is the value of $f(3\sqrt{7}, 2\sqrt{7}, \sqrt{7})$?

20. _____

21. How many triangles are there altogether in the figure below?

21. _____ triangles

22. What is the largest integer n such that $4n+1$ is a multiple of $n+4$?

22. _____

23. In how many different ways can 6 people (A, B, C, D, E, and F) be lined up in a row so that A and B are next to each other, C and D are next to each other, and E and F are *not* next to each other?
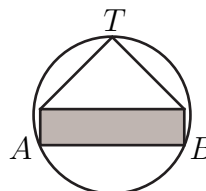
23. _____ ways

24. What the largest integer n such that $\dfrac{1440}{n^2 - 1}$ is an integer?

24. _____

25. The shaded rectangle below has base AB of length 4, and has height 1. An isosceles triangle is erected with base the side opposite to AB. The triangle is right-angled at T. A circle is drawn passing through A, B, and T. What is the radius of the circle? Express your answer as a common fraction.

25. _____ units

26. There is an integer a such that $a^{11} = 177917621779460413$. (Note that a^{11} is an 18-digit number.) What is the value of a?

26. _____

Bull's-eye, Page 1: Problem Solving

1. The average of x and $2y$ is 55. The average of x and $2z$ is 59. What is the average of x, y, and z?

1. _____

2. Alphonse works part-time. He earns \$6 per hour on weekdays (Monday to Friday) and \$8 per hour on weekends. This week he worked for a total of 32 hours, and earned \$220. How many weekday hours did Alphonse work?

2. _____ hours

3. Alicia weighed 168 pounds when she went on a diet, and her body was 30% fat by weight. On her diet, she lost fat only. After a year of dieting, her body was 20% fat by weight. How many pounds did she weigh then?

3. _____ pounds

4. Let a and b be real numbers such that

$$a + \frac{1}{b} = 9 \qquad \text{and} \qquad b + \frac{1}{a} = 10.$$

What is the value of $ab + \frac{1}{ab}$?

4. _____

Bull's-eye, Page 2: Combinatorics and Numbers

5. Two fair dice are tossed. What is the probability that the *product* of the two numbers so obtained is divisible by 3? Express your answer as a common fraction.

5. _____

6. Find the positive difference between the largest and the smallest of the fractions

$$\frac{3}{4}, \qquad \frac{4}{5}, \qquad \frac{11}{15}, \qquad \frac{23}{30}, \qquad \frac{47}{60}.$$

Express your answer as a common fraction.

6. _____

7. A combination for a simple bicycle lock is a sequence (a, b, c), where a, b, and c can be any of 0, 1, 2, 3, ~~4, 5, 6, or 7, but~~ *adjacent* numbers in the combination are different. For example $(5, 0, 7)$ is a legitimate combination, as is $(5, 0, 5)$, but $(2, 4, 4)$ is not allowed. How many combinations (combos) are possible, altogether?

7. _____ combos
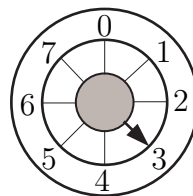
8. Suppose that x and y are *positive* integers such that $xy = x+y+2009$. What is the smallest possible value of $x + y$?

8. _____

9. The measures of the four interior angles of a 4-sided convex polygon form an arithmetic progression. The smallest angle has degree measure $33°$. What is the degree measure of the second smallest angle?

9. _____ degrees
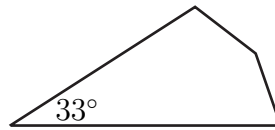
10. The pinwheel below is made by putting together four right-angled triangles whose legs are $\sqrt{7} - 1$ cm and $\sqrt{7} + 1$ cm in length. What is the number of cm in the perimeter of the pinwheel?

10. _____ cm
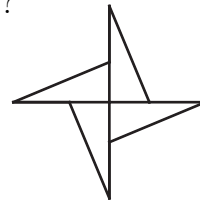
11. The vertices of the shaded triangle below have coordinates $(0, 1)$, $(1, 0)$, and (k, k) where k is positive. The triangle has area $\frac{8}{9}$ units^2. What is the value of k, as a common fraction?
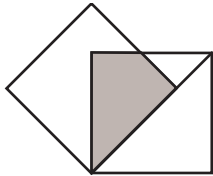
11. _____
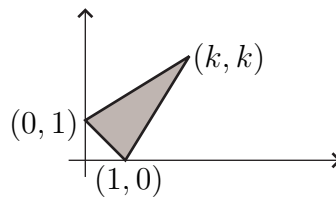
12. In the picture below, which is not drawn to scale, $\triangle ABC$ is right-angled at C. The two legs AC and BC have length 40 and 60. The shaded region consists of all points *inside* $\triangle ABC$ which are at a distance less than or equal to 6 from one (or both) of the two legs of $\triangle ABC$. What is the area of the shaded region?

12. _____ units^2
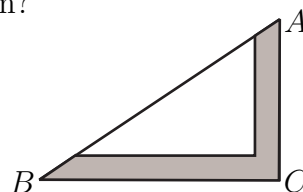
1. What is the probability that a randomly selected positive divisor of 2^{100} is a divisor of 2^{50}? Note that for any positive integer N, both 1 and N are divisors of N. Express your answer as a common fraction.

1. _____

2. How many of the integers in the interval from 1 to 40 have exactly 4 (positive) divisors?

2. _____ integers

3. Two 1×1 squares partially overlap as shown. What is the area of the region of overlap? Express your answer in simplest radical form.

3. _____ units^2

4. How many integers between 100 and 999 are there whose "hundreds" digit is equal to the sum of the other two digits?

4. _____ numbers

Co-op, Page 2: Team answers must be on the *coloured* page.
Answers on a white page will not be graded.

5. How many positive integers are there none of whose digits is 0 and the sum of whose digits is 5?

5. _____ integers

6. In the year k, where $1 \le k \le 4$, the village of Ratland had $P(k)$ people, $D(k)$ dogs, $C(k)$ cats, and $R(k)$ rats. Suppose that:

(a) For $1 \le k \le 4$, $P(k) = D(k) + C(k)$ (the number of people is equal to the number of dogs plus the number of cats);

(b) For $1 \le k \le 3$, $P(k+1) = P(k) + 10$, $D(k+1) = D(k) + 15$, and $R(k+1) = R(k) - 100\,C(k) + 10000$;

(c) $R(1) = 10000$, $P(2) = 100$, and $D(4) = 75$.

What was the value of $R(4)$, the rat population in year 4?

6. _____ rats

7. When Mr. Lucky starts betting, he has 3 dollars. On any bet, he wins with probability $1/3$ and loses with probability $2/3$. If he wins a bet, the total amount of money he has triples. If he loses a bet, he loses $2/3$ of the total amount of money he has. Mr. Lucky's objective is to walk away with 27 dollars in his pocket, and he can keep playing as long as he has at least 3 dollars. What is the probability that he reaches his objective? Express your answer as a common fraction.

7. _____

8. The numbers in the set $\{1, 2, 3, 4, 5, 6, 7\}$ are divided into two groups. Let a be the product of the numbers in one group, and let b be the product of the numbers in the other group. What is the smallest possible value of $a + b$?

8. _____

9. Three semicircles have diameters on the same line. The points A, B, C, D, E, and F are on the line, in that order. The leftmost semicircle has diameter $BC = 4$, the rightmost semicircle has diameter $DE = 2$. The middle semicircle has diameter $CD > 4$. A common tangent line to the leftmost two semicircles meets the line of diameters at A, and a tangent line to the rightmost two semicircles meets the line of diameters at F. Given that $AB = 6$, what is the length of EF? Express your answer as a common fraction.

9. _____
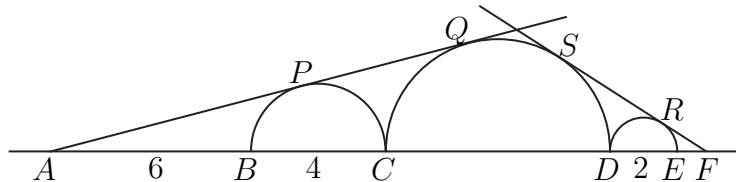
10. A 2×2 sheet of paper is folded once to make a 2×1 rectangle. The rectangle is then folded once to make a 1×1 square package. Finally, this package is cut all the way through along the two diagonals. How many separate pieces has the original sheet of paper been cut into?
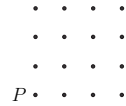
10. _____ pieces
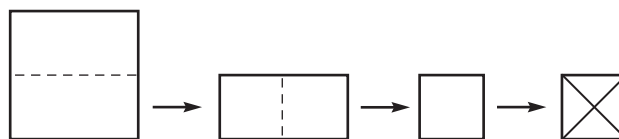
Regional 2009 Face-off Questions
Actually Used

1. Simplify: $\dfrac{66}{0.12}$

1. 550

2. In the 4×4 grid below, every grid point is 1 unit from its nearest horizontal or vertical neighbours. How many different lines pass through the grid point P at the lower left-hand corner of the grid, and through at least one other grid point?

2. 9 (lines)

```
                        . . . .
                        . . . .
                        . . . .
        _____ P . . . .
```

3. Alicia has \$1000 in her wallet. Half of the money is in \$20 bills, and the other half is in \$50 bills. How many bills does she have in her wallet?
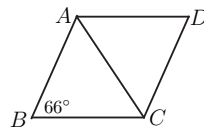
3. 35 (bills)

4. What is the mean, to 2 decimal places, of the following list of 8 numbers?

$$2, \quad 2, \quad 2, \quad 2, \quad 2, \quad 2, \quad 3, \quad 3$$

4. 2.25

5. Quadrilateral $ABCD$ is a rhombus, and the degree measure of $\angle ABC$ is 66°. What is the degree measure of $\angle DAC$?

5. 57 (degrees)

6. Two standard dice are tossed. What is the probability that the product of the two numbers obtained is even? Express your answer as a common fraction.

6. $\frac{3}{4}$

7. What is the sum of the solutions of the equation $(x-3)(x^2-4) = 0$?

7. 3

8. TicketScalper® is selling single tickets to the concert for \$95 each, and booklets of 5 tickets for \$449. Five friends decide to get a booklet and share the cost equally. How much money does *each* of them save over the single ticket price? Give the answer in dollars, to the nearest cent.

8. 5.20 (dollars)

9. If $x + 2y + 3 = \dfrac{3}{5}$, what is the value of $2x + 4y + 5$?

9. $\frac{1}{5}$

10. What is the product of 10.5 and 30.5? Give your answer as a decimal correct to 2 decimal places.

10. 320.25

11. In a class of 50 students, there are 20 boys and 30 girls. Each boy was asked to write down the number of boys in the class (including himself). Each girl was asked to write down the number of girls in the class, including herself. They all wrote down the correct numbers. What is the average (mean) of all the numbers they wrote down?

11. 26

12. The product of 3 consecutive positive integers, divided by their sum, is equal to 40. Of the 3 consecutive integers, which integer is in the middle?

12. 11

13. What is the number of cm in the height of a cone which has the same base and the same volume as a cylinder of height 15 cm?

13. 45 (cm)

14. We have 64 little cubes, each of which is $1 \times 1 \times 1$. They completely fill a lidless $4 \times 4 \times 4$ cubical box. How many of the little cubes touch a side or the bottom of the big box?

14. 52 (cubes)

15. Alicia started studying at 6:50 PM, and stopped studying 3 hours and 20 minutes later. At what time did she stop studying?

15. 10:10 or equivalent

16. A drawer contains 9 blue marbles, 7 white marbles, and 5 red marbles. What is the smallest number of marbles that we must take out to make sure that among the marbles we take out all 3 colours are represented?
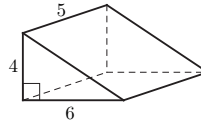
16. 17 (marbles)

17. The product of 3 consecutive even integers is equal to 960. What is the sum of these 3 even integers?

17. 30

18. A wedge of cheese has dimensions (in cm) shown below. (Technically, the wedge is a right triangular prism. The triangular faces of the prism are right triangles with legs of length 4 cm and 6 cm.) What is the volume, in cm^3, of the wedge of cheese?

18. 60 (cm^3)

19. There are 14 yellow, 15 black, and 16 red jelly beans in a bowl, and no others. If you pick a jelly bean at random from the bowl, what is the probability it is not black? Express your answer as a common fraction.

19. $\frac{2}{3}$

**20.** What is the remainder when 2009 is divided by 18?

20. 11

21. A mother weighs 14 times as much as either one of her two identical twin babies. Altogether, the mother and the two babies weigh 72 kg. How many kg does the mother weigh?

21. 63 (kg)

22. The two legs of a right angled triangle have lengths 1 and 2.4. What is the length of the hypotenuse? Express your answer as a decimal, correct to 1 decimal place.

22. 2.6 (units)

23. A full *small* (cylindrical) tomato paste can contains 156 ml of tomato paste. A *large* can has twice the radius and twice the height of a small can. How many ml of tomato paste does a full large can contain?

23. 1248 (ml)

24. Let N be the five-digit number whose digits, from left to right, are 7, d, 8, d, and 5. If N is a multiple of 9, what is the value of d?

24. 8

25. There were 4 candidates for Mayor. Alicia got 50% of the votes, Beti got 30% of the votes, Gamay got 16%, and Delbert got the remaining 100 votes. How many people voted for Alicia?

25. 1250 (people)

26. There are 2 different numbers each of which is twice as far from 0 as it is from 6. What is ~~the sum of these~~ 2 numbers?

26. 16
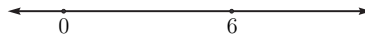
27. One cubic centimetre of red cedar weighs 0.38 grams. What is the weight, in kilograms, of 1 cubic metre of red cedar?

27. 380 (kg)

28. What is the smallest positive integer which is divisible by 30 and in whose decimal representation no digits occur other than 0 or 1?

28. 1110

29. The area of an equilateral triangle is $9\sqrt{3}$ cm². What is the number of cm in a side of the triangle?

29. 6 (cm)

30. Alicia has 1 five-dollar bill and 5 twenty-dollar bills. How many different sums of money can she make using 1 or more of these bills?

30. 11 (sums)

31. What is the sum of the x-coordinate and the y-coordinate of the point where the line $y = 2x$ meets the line $y = 7x - 30$?

31. 18

32. Define the function $f(x, y, z)$ by

$$f(x, y, z) = \frac{xy + yz + zx}{x^2 + y^2 + z^2}.$$

What is the value of $f(7, 7, 7)$?

32. 1

2009 Canadian Math Challengers Provincial Questions

Blitz, Page 1

1. Simplify: $\dfrac{\frac{1}{2} + \frac{3}{4} + \frac{5}{6}}{\frac{5}{12}}$

   1. _____

2. Approximately how many seconds are there in two-sevenths of a minute? Round your answer to the *nearest* second.

   2. _____ seconds

3. Alphonse has equal numbers of nickels, dimes, quarters, and loonies. If the total value of these coins is less than $20.00, what is the maximum possible total value of these coins? Give the answer in dollars, to the nearest cent.

   3. _____ dollars

4. Three friends (call them A, B, and C) go to a movie theatre. There are 5 consecutive empty seats in the front row. All other seats are occupied. How many ways are there to seat the friends, if they need to occupy consecutive seats? Two ways are shown in the picture below.

 | | | B | C | A | | | A | C | B | |

   4. _____ ways

5. A rectangle has area 1000 units^2. A new rectangle is constructed by increasing the length of the original rectangle by 10%, and decreasing the width by 10%. What is the area of the new rectangle?

   5. _____ units^2

6. What is the value of $2^5 - 2^4 + 2^3 - 2^2 + 2^1 - 1$?

   6. _____

7. A cubical die has its faces labelled with the numbers 1, 3, 5, 7, 9, and 11 instead of the usual 1 to 6. If two such dice are tossed, what is the probability that the sum of the numbers on the two "up" faces is 6? Express your answer as a common fraction.

   7. _____

8. Let $a_0 = 0$, and for any $n \geq 1$, let $a_n = n^2 - a_{n-1}$. What is the value of a_3?

8. _____

9. What is the integer closest to $\sqrt{2009}$?

9. _____

10. What is the smallest positive integer n such that 100 divides $n!$?

10. _____

11. Suppose that x and y are positive integers such that $400x + 9y = 2009$. What is the largest possible value of y?

11. _____

12. A box of 6 doughnuts costs \$3, and a box of 13 doughnuts costs \$6. What is the least number of dollars Alphonse needs to spend in order to buy *exactly* 175 doughnuts?

12. _____ dollars

13. After playing 20 games into the season, the Burnaby Bruisers had won 6 games and lost 14. After playing these 20 games, they fired the water boy. Over the rest of the season the Bruisers lost only 7 games and won the rest. Over the entire season, they won exactly two-thirds of the games they played. How many games did they play during the entire season?

13. _____ games

14. The figure below is a half-circle with centre O. Given that $PA = 13$ and $AQ = 3$, what is the ~~length of OC?~~ Express your answer as a common fraction.

14. _____ units
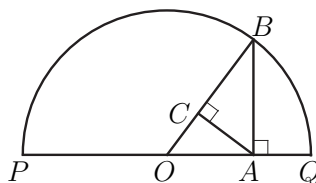
15. The mean and the median of a collection of 5 different positive integers are both equal to 20. What is the largest possible integer in the collection?

15. _____

16. The rectangle $ABCD$ is divided into 10 squares as in the picture below. If the side of one of the smallest squares (say the one at the corner D) is 3 units, how many units are in the base AB?

16. _____ units
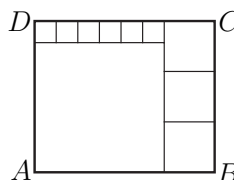
17. Squares are erected on the two legs of a right-angled triangle. These squares have areas 36 and 132 as shown. A semicircle (shaded) is drawn with the hypotenuse as diameter. What is the area of the semicircle? Give your answer in terms of π.

17. _____ units^2
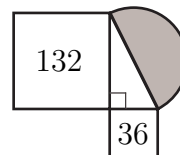
18. If $|x| + |2x + 3| \leq 60$, what is the largest possible value of $|x|$?

18. _____

19. In quadrilateral $ABCD$, $AB = AC = BD$, and $BC = CD = DA$. What is the degree measure of $\angle ABC$?

19. _____ degrees
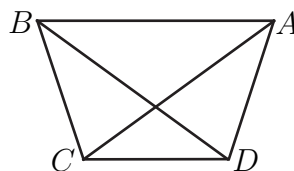
20. We are given 5 points, which have coordinates $(0,0)$, $(1,0)$, $(2,0)$, $(0,1)$, and $(0,2)$. How many ways are there to choose 3 of these points so that the 3 chosen points are the vertices of a triangle? Note that for example choosing $(2,0)$, $(0,1)$, and $(0,2)$ is the same as choosing $(0,1)$, $(2,0)$, and $(0,2)$.

20. _____ ways
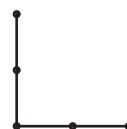
21. A large bottle contains 4 litres of a solution which is 5% acetic acid (and the rest water). How much of a solution which is 20% acetic acid should we add to the bottle to obtain a solution which is 7% acetic acid? Give your answer as a common fraction, in litres.

21. _____ litres

22. What is the area of the triangular region enclosed by the 3 lines that have equations $x - y = 0$, $x + y = 2$, and $x = 10$?

22. _____ units^2

23. Let $N = 4^8 \times 5^9$. How many digits are there in the decimal representation of N?

23. _____ digits

24. The rectangle on the left represents a 4×3 sheet of stamps, 12 stamps altogether. How many different ways are there to choose a set of 3 stamps which are *connected*? Connection must be through shared edges: a shared vertex is not good enough. The three pictures on the right show three different ways of doing the job.

24. _____ ways
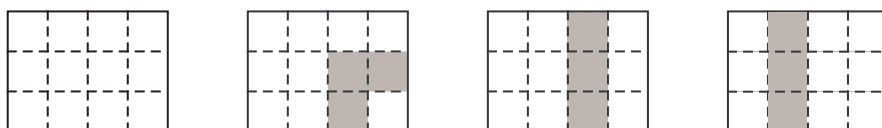
25. Five (5) cards with the number 1 written on them, and four (4) cards with the number 2 written on them, are placed in a box. You randomly select 3 of these 9 cards. What is the probability that the sum of the numbers written on the 3 selected cards is odd? Express your answer as a common fraction.

25. _____

26. The vertices of a trapezoid are $(0,0)$, $(10,0)$, $(10 + m, 4)$, and $(0, 4)$. The line $y = x/m$ divides the trapezoid into two polygons of equal area. What is the value of m? Express the answer as a common fraction.

26. _____

$(0,0)$
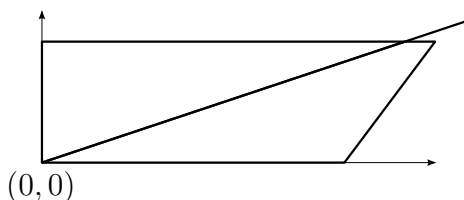
1. A store sells only bicycles (2 wheels each) and tricycles (3 wheels each). The store has exactly as many bicycles as tricycles. Given that the bicyles and tricycles in the store have a combined total of 330 wheels, how many tricycles are in the store?

1. _____ tricycles

2. SellHigh$^{\text{TM}}$ bought apples from a farmer, at 12 apples for \$1. SellHigh then sold all the apples in its Vancouver store at 2 apples for \$1. SellHigh's total profit on the apples was \$3000. How many dollars did SellHigh pay the farmer for the apples?

2. _____ dollars

3. A box-shaped pool is 25 metres long, 12 metres wide, uniformly 1 metre deep, and full of water. Water is leaking from the pool at 1000 cubic centimetres per minute. How many minutes will it take for the water level in the pool to go down by 1 centimetre?

3. _____ minutes

4. There are two candles, one short and thick, the other tall and thin. They burn at different rates. The short thick candle can burn for 120 minutes. Both candles were lit at the same time, and after 30 minutes they were both the same height. After 30 additional minutes, the (originally) tall candle was half the height of the (originally) short candle. What is the total expected burn time, in minutes, of the (originally) tall candle?

4. _____ minutes

5. What is the sum of all the positive integers that divide 60? (Note that 1 and 60 divide 60.)

5. _____

6. Four people (A, B, C, and D) line up in a row at random. What is the probability that A and B are next to each other but C and D are not next to each other? Express your answer as a common fraction.

6. _____

7. The sum of four *different* positive integers is equal to 300. If S is the smallest of the four positive integers, and B is the biggest, what is the smallest possible value of $S + B$?

7. _____

8. Twenty (20) people come to a party. We know that 11 of the people are friends with everyone else who came to the party. Also, the other 9 people each have exactly 13 friends at the party. (Assume that if A is a friend of B, then B is a friend of A. Assume also that A is never a friend of A.)

   Each person shakes hands with each of his/her friends. What is total number of handshakes?

8. _____ handshakes

Bull's-eye, Page 3: Geometry

9. How many lines of symmetry does a regular hexagon have?

9. _____ lines

_____

10. In the diagram below, $AX = 10$, $XB = 5$, $AY = 4$, and $YC = 8$. What is the ratio of the area of $\triangle AXY$ to the area of $\triangle ABC$? Express your answer as a common fraction.

10. _____
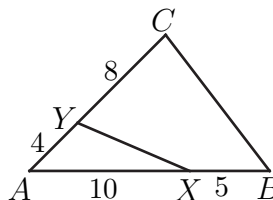
11. The *slant* height of a cone is 41, and the ordinary height (distance from the vertex to the centre of the base) is 40. If the volume of the cone is $N\pi$, what is the value of N?

11. _____

12. A ball of ice of radius 5 cm is placed in a tall empty cylindrical glass with the same radius. When the ice melts, every 11 cm^3 of ice turns into 10 cm^3 of liquid. When all the ice has melted, what is the height, in cm, of liquid in the glass? Express the answer as a common fraction.

12. _____ cm

Co-op, Page 1: Team answers must be on the *coloured* page.
Answers on a white page will not be graded.

1. Let N be the smallest positive integer such that each of N and $N+1$ has exactly 4 positive divisors. What is the value of N?

1. _____

2. Let a be the number of divisors of 6!, and let b be the number of divisors of 7!. What is the value of $\dfrac{a}{b}$? Express your answer as a common fraction.

2. _____

3. A cube is inscribed in a sphere of radius 3 cm. (So the cube is *inside* the sphere, and the all the corners of the cube touch the boundary of the sphere.) What is the surface area of the cube?

3. _____ cm^2

4. The black circles labelled A, B, C, D, and E represent cities, and the straight lines are highways between them. We want to travel from city to city in such a way that we travel on every road exactly once. (We may go through a city more than once.) It turns out that every path that works begins in a certain city X, and ends in a certain city Y, or vice-versa. What are these two cities? For example, if the path must begin at C and end at D, or vice-versa, your answer should be CD (or if you like, DC).
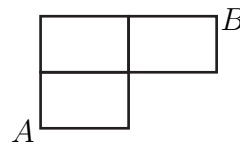
4. _____
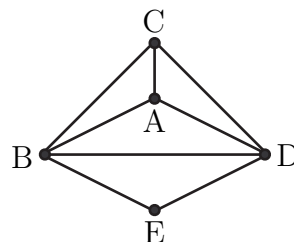
_____

Co-op, Page 2: Team answers must be on the *coloured* page.
Answers on a white page will not be graded.

5. What is the largest integer which is less than $\sqrt[3]{4100} \times \sqrt[4]{4100}$?

5. _____

6. Triangle ABC has $AB = 9$, $AC = 8$, and $BC = 4$. Line segment AC is extended to D in such a way that $\angle CBD = \angle CAB$. What is the length of the line segment CD? Express your answer as a common fraction.

6. _____ units
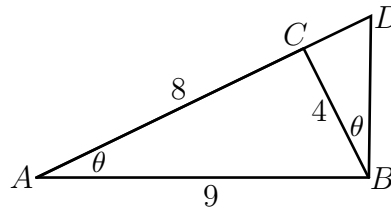
7. The lines in the diagram below represent the streets of a village. How many ways are there to drive from A to B, using village streets, if one is not allowed to travel along any block (segment) more than once? (One can pass through an *intersection* more than once.)
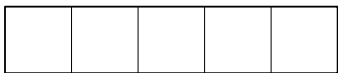
7. _____ ways

8. For any positive integer n, let $S(n)$ be the sum of the (decimal) digits of n. For example, $S(8) = 8$ and $S(47) = 11$. How many two-digit numbers n are there such that $S(S(n)) = 5$?

8. _____ numbers

9. Consider the angle between the hour hand and the minute hand of a watch. There are times when the angle between these hands is exactly 180 degrees (example: 6:00 o'clock). Find the sum of all these times, in the period from 1:00 pm to 4:00 pm the same day. Give the answer as a common fraction, in hours.

9. _____ hours

10. A class of 12 students is currently divided into 4 working groups of 3 students each, namely {A,B,C}, {D,E,F}, {G,H,I}, and {J,K,L}.

   Suppose that you want to regroup these 12 students into 4 groups of 3 students each, so that no 2 students who are currently in the same group will end up in the same group. In how many ways can this be done? An example of a valid way is {A,E,H}, {B,F,J}, {C,G,K}, and {D,I,L}. An example of an invalid way of regrouping is {A,E,F}, {B,G,J}, {C,H,K}, and {D,I,L}, because E and F are currently in the same group, so must end up in different groups.

10. _____ ways

Canadian Math Challengers Society

1. Simplify: $\dfrac{2^{-2} + 5^{-2}}{10^{-2}}$

1. 29

2. The rectangle below has perimeter 3 cm, and is divided into 5 congruent squares. What is the number of cm^2 in the area of the rectangle? Express your answer as a common fraction.

2. $\dfrac{5}{16}$ (cm^2)

3. Suppose that $n^2 - 4 = 50(n - 2)$ and n is not equal to 2. What is the value of n?

3. 48 (cm^2)

4. Simplify: $\dfrac{16.8}{0.014}$

4. 1200

5. Alicia walked 1000 metres in 12 minutes. At this rate, how many metres can Alicia walk in 27 minutes?

5. 2250 (metres)

6. We have 7 Scrabble$^{\text{TM}}$ tiles. They have the letters B, U, R, N, A, B, and Y written on them, one letter per tile. How many 2-letter "words" can be formed using 2 of these tiles? For our purposes a word is any string of 2 letters—it does not have to be a word of English.

6. 31 (words)

7. If $f(x) = x^2 - 6x$, what is the value of $f(f(1))$?

7. 55

8. In the first quarter of the football game, Alphonse punted the ball 3 times. The average length of his 3 punts was 35 yards. His longest punt was 41 yards. What was the average length, in yards, of his other 2 punts?

8. 32 (yards)

9. Given that $\dfrac{1}{2x+1} = \dfrac{7}{15}$, what is the value of $\dfrac{1}{2x-1}$?

9. 7

10. What is the sum of the x-coordinate and the y-coordinate of the point where the line $y = 2x$ meets the line $y = 7x - 30$?

10. 18

11. The sum of 4 consecutive integers is 182. What is the largest of the 4 integers?

11. 47

12. Alphonse wrote a 50 question true/false test. He never answered more than 5 questions in a row correctly. What is the largest number of questions he could have answered correctly?

12. 42 (questions)

13. A $3 \times 3 \times 3$ cube has been assembled using twenty-seven $1 \times 1 \times 1$ cubes. Then the three $1 \times 1 \times 1$ cubes shaded in the diagram are removed. What is the surface area of the remaining solid?

13. 60 (units^2)
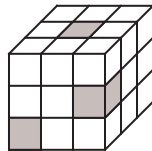

14. What is the sum of the roots of the equation $(x - 12)^2 = 100$?

14. 24

15. A survey of 500 students showed that 300 have an MP3 player, 250 have a cell phone, and 200 have both. How many of the students have neither an MP3 player nor a cell phone?

15. 150 (students)

16. What non-zero value of x satisfies the equation $(3x)^3 = (2x)^2$? Express the answer as a common fraction.

16. $\dfrac{4}{27}$

17. Alicia is driving at 100 km per hour. At this speed, her SUV uses 12 litres of gas for every 100 km driven. Alicia's gas tank contains 7 litres of gas now. For how many minutes can she drive before running out of gas?

17. 35 (minutes)

18. Organic potatoes come in 2 kg bags and in 3 kg bags. A 2 kg bag costs $2.99, and a 3 kg bag costs $3.99. Alphonse wants to buy 6 kg of organic potatoes. How much money will he save by buying them in 3 kg bags over buying them in 2 kg bags? Give your answer in cents.

18. 99 (cents)

19. Two adjacent vertices of a square have coordinates $(7, 1)$ and $(4, 14)$. What is the number of units^2 in the area of the square?

19. 178 (units^2)

20. Alphonse bought $10,000$ shares of SureThing Gold for 50 cents a share. Later, he sold one-half of these shares for 65 cents a share. Then SureThing crashed, and Alphonse sold his remaining shares for 5 cents a share. How many dollars did lose on SureThing Gold?

20. 1500 (dollars)

21. What is the largest integer that is less than 2009 and is a multiple of 45?

21. 1980

22. Ninety-nine (99) people are arranged in 3 rows. If the back row has 7 more people than the middle row, and the middle row has 7 more people than the front row, how many people are in the front row?

22. 26 (people)

23. A certain number N of people were interviewed about their knowledge of eminent Americans. Of the N people interviewed, 400 had heard of Britney Spears, 300 had heard of Snoop Dogg, and 50 had heard of neither. What is the smallest possible value of N?

23. 450

24. The outer circle has area 72 cm². The inner small circles are congruent to each other, and their centres lie along two perpendicular diameters of the outer circle. Circles that appear tangent to each other *are* tangent to each other. What is the number of cm² in the shaded region?

24. 40 (cm²)
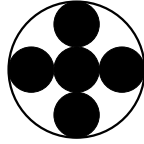
25. General admission tickets at a high school play were $10 each, and student tickets were $5 each. Equal numbers of general admission and student tickets were sold. Revenue from ticket sales was $3000. What was the total number of tickets sold?

25. 400 (tickets)

26. The point M is halfway between the points with coordinates $(1, 2)$ and $(8, 9)$. What is the sum of the x-coordinate and the y-coordinate of M?

26. 10
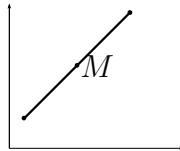
27. A 10 cent coin (dime) weighs 1.75 grams. What is the value, in dollars, of 3.5 kilograms of dimes?

27. 200 (dollars)

28. What is the smallest prime number that is one less than a multiple of 13?

28. 103

29. What is the sum of all the positive factors of 81?

29. 121

30. Suppose that you put numbers in the three empty boxes below, one number in each, so that the numbers in any three consecutive boxes add up to 200. (Note that two boxes already contain a number.) What number goes into the leftmost box?

30. 68

		44		88

31. Evaluate $\dfrac{8!}{5!3!}$

31. 56

32. Alicia and Beti walk side by side at the same speed. Alicia averages 120 steps per minute and each of her steps is 90 cm long. Beti's steps are 75 cm long. How many steps does Beti average per minute?

32. 144 (steps)

Math Challengers Regional 2009
Answers, Blitz Stage

1. $\dfrac{7}{8}$

2. 9

3. 1.25

4. 108

5. 112

6. 12

7. 22

8. 14

9. $\dfrac{33}{34}$

10. 18

11. 4200

12. 7

13. 72

14. 5

15. 4π

16. 61

17. $\dfrac{2}{7}$

18. 25

19. 2.16

20. 7

21. 13

22. 11

23. 48

24. 19

25. $\dfrac{13}{6}$

26. 37

Math Challengers Regional 2009
Answers, Bull's-eye Stage

1. 38

2. 18

3. 147

4. 88

5. $\dfrac{5}{9}$

6. $\dfrac{1}{15}$

7. 392

8. 99

9. 71

10. 24

11. $\dfrac{25}{18}$

12. 525

Math Challengers Regional 2009
Answers, Co-op Stage

1. $\dfrac{51}{101}$

2. 14

3. $\sqrt{2} - 1$ or $-1 + \sqrt{2}$

4. 54

5. 16

6. 23500

7. $\dfrac{1}{7}$

8. 142

9. $\dfrac{6}{7}$

10. 12

Math Challengers Provincial 2009
Answers, Blitz Stage

1. 5

2. 17

3. 19.60

4. 18

5. 990

6. 21

7. $\dfrac{1}{12}$

8. 6

9. 45

10. 10

11. 1

12. 81

13. 63

14. $\dfrac{25}{8}$

15. 56

16. 25

17. 21π

18. 21

19. 72

20. 8

21. $\dfrac{8}{13}$

22. 81

23. 12

24. 34

25. $\dfrac{10}{21}$

26. $\dfrac{20}{7}$

Math Challengers Provincial 2009
Answers, Bull's-eye Stage

1. 66

2. 600

3. 3000

4. 75

5. 168

6. $\dfrac{1}{6}$

7. 102

8. 163

9. 6

10. $\dfrac{2}{9}$

11. 1080

12. $\dfrac{200}{33}$

Math Challengers Provincial 2009
Answers, Co-op Stage

1. 14

2. $\dfrac{1}{2}$

3. 72

4. AC (or CA)

5. 128

6. 9

7. $\dfrac{128}{65}$

8. 10

9. $\dfrac{90}{11}$

10. 1296

2010 Canadian Math Challengers Regional Questions

1. Evaluate $7 + 8 + 9 + 10 + 11 + 12 + 13$.

   1. _____

2. The perimeter of a rectangle is 40 cm. One side of the rectangle is 7 cm. What is the area of the rectangle?

   2. _____ cm^2

3. Joshua answered 22 of the 55 Math Challengers practice questions correctly. How many percent of the practice questions did Joshua answer correctly?

   3. _____ percent

4. Three teams participated in a hockey tournament, and every team played every other team twice. How many games were played in the tournament?

   4. _____ games

5. The mean and the median of 5 different positive integers are both equal to 3. What is the largest of the 5 integers?

   5. _____

6. In the diagram below, ~~ABCD is a pa~~rallelogram, and the diagonals AC and BD meet at E. Given that angle BAC is 20 degrees, and angle CDB is 50 degrees, how many degrees are in angle AEB?

   6. _____ degrees
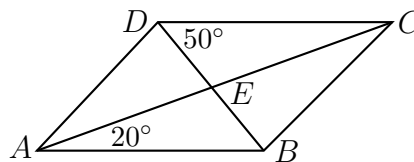
7. Evaluate $\dfrac{1 \times 4 \times 9 \times 16 \times 25}{(1 \times 2 \times 3 \times 4 \times 5)(1 \times 3 \times 5)}$.

   7. _____

8. Call a prime p *lonely* if neither $p-2$ nor $p+2$ is prime. What is the smallest lonely *odd* prime?

8. _____

9. Twenty percent of a certain positive number x multiplied by 40% of the same number is equal to 18. What is the value of x?

9. _____

10. You are told that a box contains a total of 6 coins, including at least one penny, at least one nickel, and at least one dime. Given this information, what is the smallest conceivable sum of money you can make using exactly 5 of these coins?

10. _____ cents

11. It took 25 minutes to drive from Alicia's home to the math contest at SFU. If the distance travelled was 20 km, what was the average speed of the drive in km/hr?

11. _____ km/hr

12. Two standard dice are tossed. What sum has the highest probability?

12. _____

13. Evaluate $\dfrac{2^3}{3^{-2}}$.

13. _____

14. What is the perimeter of an equilateral triangle whose heights are all equal to $\sqrt{3}$ units?

14. _____ units
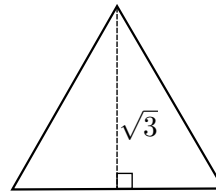
15. The hospital where Alicia works was privatized. The number of hours Alicia works per week went *up* by 20%, and her total weekly earnings went *down* by 13%. By how many percent did Alicia's hourly wage decrease? Give the answer as a decimal, to the nearest tenth of a percent. (An answer like 19.2 percent is of the right shape.)

15. _____ percent

16. Let $x = \sqrt{1} + \sqrt{2} + \sqrt{3} + \sqrt{4}$. What is the integer *nearest* to x?

16. _____

17. The interior of cooking pot A is a cylinder with base radius 8 cm and height 10 cm. The interior of cooking pot B is a cylinder with base radius 16 cm and height 40 cm. Pot A is filled with water and the contents are poured into pot B. After this has been done *twice*, how many cm deep is the water in pot B?

17. _____ cm

18. Two cubical dice each have the numbers 1, 2, 4, 8, 16, and 32 written on their faces, one number on each face. The two dice are rolled. What is the probability that the *sum* of the numbers on top of the two dice is odd? Write the answer as a common fraction.

18. _____

19. In the picture below, $\triangle ABC$ is right-angled at C. The leg CA has length 22 cm, and the leg CB has length 10 cm. Point P inside the triangle is at distance 5 cm from each of CA and CB. What is the area of $\triangle PAB$?

19. _____ cm^2
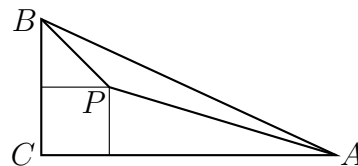
20. Let $N = 2 \times 4 \times 6 \times 8 \times \cdots \times 48 \times 50$. How many consecutive zeros does the decimal representation of N end with?

20. _____ zeros

21. A box contains 88 balls, with the numbers 1 to 88 written on them, one number per ball. How many balls must you remove from the box to be *certain* that among the numbers removed, there are at least two whose difference is divisible by 8?

21. _____ balls

22. An 800 metre long train travelling forward at 20 metres per second went through a tunnel. The *front* of the train emerged from the tunnel 50 seconds after the *rear* of the train entered the tunnel. What is the length of the tunnel in metres?

22. _____ metres

23. Twenty percent of the people who like chocolate like hot pepper. Ninety percent of the people who like hot pepper like chocolate. All people like one or the other or both. What fraction of the people like both?

23. _____

24. For how many positive integer values of n is $3n-1$ a factor of 2^{2010}?

24. _____ values

25. The lines in the picture below show the streets of a village. (Note the diagonal street.) A path from A to B is *efficient* if as you travel it you get ever closer to B. How many efficient paths are there from A to B?

25. _____ paths
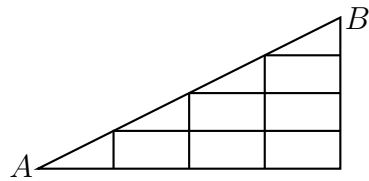
26. A desert outpost has its water supply stored in a cistern. Each person at the outpost uses water at the same constant rate. In addition, water evaporates from the cistern at a constant rate. There is enough water in the cistern to supply 40 people for 45 days, or 50 people for 40 days. For how many days can the cistern supply 60 people?

26. _____ days

Bull's-eye, Page 1: Problem Solving

1. A car travels 6.5 kilometres in 5 minutes. At this speed, how many km does it travel in an hour?

   1. _____ km

2. A carton of apple juice costs 60 cents. A carton of mango juice costs 1 dollar. How many different ways can Alicia spend a total of *exactly* $34.60 on cartons of apple juice and/or mango juice?

   2. _____ ways

3. A restaurant bought 1800 dollars' worth of wine at $9 a bottle and 1800 dollars' worth of wine at $15 a bottle. What was the restaurant's average cost per bottle? Give the answer in dollars, as a decimal, to the nearest cent. (An answer of 12.34 is of the right shape.)

   3. _____ dollars

4. Assume that weights of coins are as follows: 1 cent coin (penny), 3 grams; 5 cent coin (nickel), 5 grams; 10 cent coin (dime) 2 grams; 25 cent coin (quarter), 9 grams; 1 dollar coin (loonie), 13 grams; 2 dollar coin (toonie), 17 grams. Dan holds at least one of each of the above coins, with a total weight of exactly 220 grams. What is the maximum possible total value of his coins? Give the answer in dollars, to the nearest cent.

   4. _____ dollars

5. Four fair coins are tossed. What is the probability that there are exactly 2 heads? Express the answer as a common fraction.

5. _____

6. Alicia wrote down all the numbers from 40 to 600. How many times did she write the digit 5?

6. _____ times

7. Two standard dice are rolled, a white one and a black one. Let W be the number showing on the white one and B the number showing on the black one. What is the probability that $W < 2B$? Express the answer as a common fraction.

7. _____

8. What is the remainder when

$$1^4 + 3^4 + 5^4 + 7^4 + 9^4 + 11^4 + 13^4 + 15^4 + 17^4 + 19^4$$

is divided by 10?

8. _____

9. The volume of a cone is 484π cubic cm, and the height of the cone is 12 cm. What is the radius (in cm) of the base of the cone?

9. _____ cm

10. The radius of the base of a cylinder is increased by 50% and the height is decreased by 50%. By how many percent does the volume of the cylinder increase? Give the answer as a decimal to the nearest tenth.

10. _____ percent

11. A poster is 40 cm wide. There are two pictures on the poster. Each picture is 24 cm wide and 20 cm high. Together the pictures take up four-ninths of the area of the poster. What is the height (in cm) of the poster?

11. _____ cm
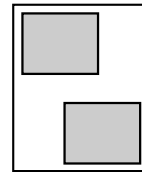
12. In older TVs, the ratio of screen width to screen height is 4:3. In newer flat-screen TVs, the ratio of screen width to screen height is 16:9. *Both* pictures below show a 32-inch 4:3 screen TV. (Here 32-inch means that the *diagonal* measures 32 inches.)

A newer TV show has been shot for 16:9 TVs. When it is viewed on a 4:3 TV, equal grey bands are produced at the top and bottom, so that the rest of the picture is in the correct 16:9 ratio. What is the vertical height in inches of *one* of these grey bands? Express the answer as a decimal, to the nearest tenth of an inch.
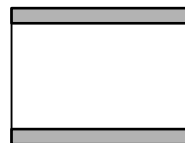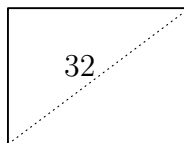
12. _____ inch

32

1. Let $x * y = \dfrac{x}{x + y}$ for $x \neq -y$. If $x * y = 9$, what is the value of $y * x$?

1. _____

2. How many integers between 0 and 999 are there that are divisible by 6 and whose rightmost decimal digit is equal to 6?

2. _____ integers

3. One corner of a square is at the origin $(0, 0)$. The diagonally opposite corner is at $(1, 11)$. What is the area of the square?

3. _____

4. The *Factoria*$^{\text{TM}}$ balance scale comes with one 1 gram weight, two 2 gram weights, three 6 gram weights, four 24 gram weights, five 120 gram weights, and six 720 gram weights. What is the total number of weights one must put in one pan of the scale so as to to balance a 2 kilogram bag of flour in the other pan?
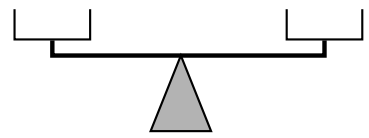
4. _____ weights

5. The trapezoid $ABCD$ has AB parallel to DC and $BC = AD$. Given that $AC = 14$ and $\angle CAB = 45°$, what is the area of trapezoid $ABCD$?

5. _____
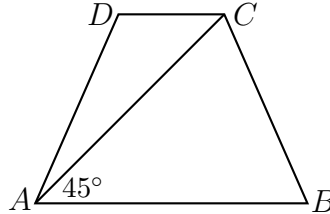
6. What is the least positive integer n such that n is a multiple of 6 and neither $n - 1$ nor $n + 1$ is prime?

6. _____

7. Four runners compete in a 100 meter race. How many possible orders of finish are there, if ties are allowed? For example, Alicia might win, with Beth, Cecil, and Deedee in a three-way tie for second. Or else Alicia and Cecil could tie for first, with Beth and Deedee tied for third. Or else Beth and Cecil could tie for first, with Alicia and Deedee tied for third. Or else there could be a four-way tie. There are other possibilities, including many with no tie at all.

7. _____ orders

8. There were 9 candidates for the 5 town council positions, and each
of the 1000 voters voted for 5 people. Each candidate got a different
total number of votes, and Epsilon came in fifth. What is the smallest
number of votes that Epsilon could have received?

8. _____ votes

9. Alan is at the origin O and starts walking. He walks 1 unit up, 1 unit
to the left, 2 units down, 2 units to the right, 3 units up, 3 units to
the left—he is now at the point with coordinates $(-2, 2)$ after walking
a total of 12 units. Then he walks 4 units down, 4 units to the right,
5 units up, and so on. Let the coordinates of the point where he ends
up after walking a total of 444 units be (a, b). What is the value of
$a + b$?

9. _____
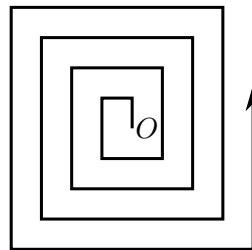
10. A 5-letter "word" that uses all the letters of the word RAINY is
called *deranged* if none of the letters occurs in the same position as
in RAINY. Thus IRAYN is deranged, while YANRI is not deranged
(since A occurs in the same position as in RAINY). Note also that
ARINY and RAINY are not deranged. How many deranged words
are there?

10. _____ words

Regional 2010 Face-off
First 40 Questions and Answers

1. What is 150% of 2010?

1. 3015

2. What is the value of $\dfrac{4^4}{8^2}$?

2. 4

3. What is the largest prime that divides $5! + 1$?

3. 11

4. What is the smallest positive integer which is simultaneously a multiple of 20, 25, and 30?

4. 300

5. If $6x + 10 = 101$, what is the value of $12x + 10$?

5. 192

6. What is the value of $\dfrac{3^5 - 3^3}{2^3}$?

6. 27

7. An accurate 12-hour clock shows that it is exactly 12:00. What time shows on the clock 600 seconds later? Express your answer in the usual hours:minutes format.

7. 12:10

8. Evaluate 15% of the reciprocal of 0.005.

8. 30

9. In the picture below, the smaller square has side 7 and the larger square has side 15. What is the area of $\triangle ABC$?

9. 60
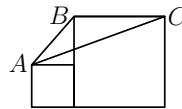
10. How many integers between 1 and 100 are divisible by 6?

10. 16

11. How many ordered pairs (a, b) are there such that a and b are integers (not necessarily positive) and $|a| + |b| \leq 1$?

11. 5

12. Alicia sold her condo for 20% more than she paid for it, and made a gross profit of $90,000. For how many dollars did she sell her condo?

12. $540,000$ (dollars)

13. How many four-letter "words" are there which use only the letters A and/or B, and in which there are no consecutive occurrences of B? (For example, AAAA and ABAB qualify, but ABBA does not.)

13. 8

14. There is an integer N such that $N^3 = 6859$. What is the value of N?

14. 19

15. The points M and N are the midpoints of two adjacent sides of a 1×1 square. What is the area of the shaded triangle? Give the answer as a common fraction.

15. $\dfrac{3}{8}$
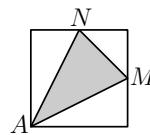
16. Evaluate $(6 \times 12 \times 24)^{1/3}$.

16. 12

17. Express $\left(1 - \dfrac{1}{3}\right) + \left(\dfrac{1}{9} - \dfrac{1}{27}\right)$ as a common fraction.

17. $\dfrac{20}{27}$

18. On Monday, 8 of the 10 students in the math class took a test. Their mean score was 60. On Tuesday, the remaining 2 students took the test. Their mean score was 95 (they had seen the questions). What was the mean class score on the test?

18. 67

19. Three standard dice are tossed. What is the probability that the sum is less than 3?

19. 0

20. A lidless $3 \times 3 \times 3$ box is completely filled with $1 \times 1 \times 1$ cubes. How many of the $1 \times 1 \times 1$ cubes touch a side or the bottom of the box?

20. 25

21. What is the measure, in degrees, of the acute angle between the hour hand and the minute hand of a clock at 3:30?

21. 75 (degrees)

22. The sum of 5 consecutive odd numbers is 55. What is the largest of the 5 numbers?

22. 15

23. The surface area of a cube is $3/2$ square metres. What is the volume of the cube, in cubic metres? Express the answer as a common fraction.

23. $\dfrac{1}{8}$ (square metres)

24. What is the value of $\dfrac{(4!)(6!)}{5!}$?

24. 144

25. In $\triangle ABC$, the angle at A is $84°$. The bisectors of the angles at B and C meet at P. <u>How many degrees</u> are in the measure of $\angle BPC$?

25. 132 (degrees)

3
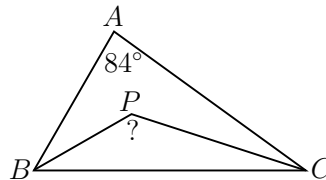
26. A list of 10 numbers has a mean of 50. One number is added to the list, and the mean of the new list of 11 numbers is 49. What number was added to the list?

26. 39

27. The 8 points below represent the vertices of a regular octagon. These vertices are alternately painted red and blue. How many lines are there that contain a red point and a blue point?

27. 16
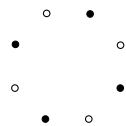
28. Simplify $\dfrac{2^{-3} + 3^{-3}}{6^{-3}}$.

28. 35

**29.** Suppose that $\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{1}{z}$. If $y = 10$ and $z = 8$, what is the value of x?

29. 40

30. What is the value of $2010^2 - 2009^2$?

30. 4019

31. Suppose that a and b are integers and $2^a - 2^b = 16$. What is the value of $a + b$?

31. 9

32. A line passes through the points $(-1, 2)$, $(2, -1)$, and $(x, -5)$. What is the value of x?

32. 6

4

33. A penny is placed on one of the squares of the 6×6 "chessboard" below, and then a dime is placed on a randomly chosen square different from the square occupied by the penny. What is the probability that the dime is neither in the same row nor in the same column as the penny? Express your answer as a common fraction.
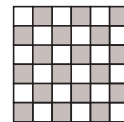
33. $\dfrac{5}{7}$

34. Simplify:
$$(2^{10} + 2^{10} + 2^{10} + 2^{10})^{1/3}$$

34. 16

35. How many positive integers between 1 and 30 (inclusive) are divisible by 3 or by 5 or by both?

35. 14

36. Alfie wrote four tests, each out of 100. His average over the four tests is 76. His average on the last two tests is 10 more than his average on the first two tests. What is his average on the first two tests?

36. 71

37. Express $\dfrac{\sqrt{8} - \sqrt{2}}{\sqrt{8} + \sqrt{2}}$ as a common fraction.

37. $\dfrac{1}{3}$

38. Alicia has 5 dollars less than Beti, and Cecille has as much money as Alicia and Beti have between them. Altogether, the three people have a total of 270 dollars. How many dollars does Alicia have?

38. 65 (dollars)

39. Evaluate $(1 \times 1!) + (2 \times 2!) + (3 \times 3!) + (4 \times 4!)$.

39. 119

40. Suppose that $f(n+2) = f(n) + f(n+1)$ for every positive integer n. Given that $f(1) = 1$ and $f(3) = 48$, what is the value of $f(2)$?

40. 47

2010 Canadian Math Challengers Provincial Questions

Blitz, Page 1

1. The sides of a cube are increased by 100%. By how many percent does the volume of the cube increase?

   1. _____ percent

2. How many primes are there between 90 and 100?

   2. _____

3. Approximately how many metres are there in 1/11 of a kilometre? Round your answer to the *nearest* metre.

   3. _____ metres

4. What is 40% of the number which is 25% greater than 144?

   4. _____

5. What is the smallest integer that is bigger than the reciprocal of 0.032?

   5. _____

6. What is the smallest sum of money (in cents) that you *cannot* pay using five or fewer standard Canadian coins?

   6. _____ cents

7. Find the sum, in degrees, of the internal angles of a convex heptagon (a convex 7-sided polygon).

   7. _____ degrees
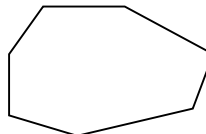
8. Express $\dfrac{1}{4} + \dfrac{2}{5} + \dfrac{3}{6}$ as a common fraction.

8. _____

9. Alana's necklace broke. She found one-third of the beads on the floor and one-quarter on the couch. One-sixth of the beads remained strung on the necklace. The rest of the beads (15 beads) were never found. How many beads were originally on the necklace?

9. _____ beads

10. A 385 ml can of orange juice concentrate was mixed in a jug with four 385 ml cans of water. Alan drank 385 ml of the mixture, and replaced it with 385 ml of water. After that, how many percent of the contents of the jug were orange juice concentrate?

10. _____ percent

11. What is the number of integer solutions of $-5 < \dfrac{n}{3} < 5$?

11. _____

12. A trucker stopped for diesel when her tank was one-eighth full. She bought 80 litres of diesel for \$90. She then noticed that her tank was only three-quarters full, so she returned to the station and filled the tank completely. How many dollars did she pay for that last one-quarter of a tank?

12. _____ dollars

13. Alex has three boxes of marbles. In the first box, 30% of the marbles are blue. There are twice as many marbles in the second box as in the first box, and 25% are blue. There are twice as many marbles in the third box as in the second box, and 20% are blue. If Alex loses the third box, how many percent of his blue marbles does he lose?

13. _____ percent

14. Three standard dice are tossed. How many different possible *sums* are there?

14. _____ sums

15. Suppose that

$$(x + 1) + (x + 2) + (x + 3) + \cdots + (x + 99) + (x + 100) = 12000.$$

What is the value of

$$(x + 101) + (x + 102) + (x + 103) + \cdots + (x + 199) + (x + 200)?$$

15. _____

16. For any real number x, let $\lfloor x \rfloor$ be the greatest integer which is less than or equal to x. For example, $\lfloor 3.65 \rfloor = \lfloor 3 \rfloor = 3$. What is the value of

$$\lfloor \sqrt[3]{1} \rfloor + \lfloor \sqrt[3]{2} \rfloor + \lfloor \sqrt[3]{3} \rfloor + \cdots + \lfloor \sqrt[3]{25} \rfloor + \lfloor \sqrt[3]{26} \rfloor?$$

16. _____

17. A car and a truck got on the freeway at the same place and time. After they had gone for 40 minutes in the same direction, the car was 6 kilometres ahead of the truck. The car's average speed was 100 km/hr. What was the truck's average speed, in km/hr?

17. _____ km/hr

18. What is the area of the square inscribed in a semicircle of radius 1? Express the answer as a common fraction.
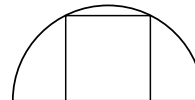
18. _____

19. Alphonse owes Beth 20 cents. In how many different ways can he pay the debt using pennies and/or nickels and/or dimes? Note that for example 5 pennies, 1 nickel, and 1 dime is the same way as 1 dime, 5 pennies, and 1 nickel.

19. _____ ways

20. What is the sum of the first 2010 terms of the sequence

$$0, \ 1, \ 2, \ 3, \ 0, \ 1, \ 2, \ 3, \ 0, \ 1, \ 2, \ 3, \ 0, \ 1, \ 2, \ldots?$$

20. _____

21. A sequence a, b, c, d, e of five numbers has the following properties: (i) $a = 1$; (ii) $e = 98$; (iii) every number after the second is the sum of the previous two numbers. What is the value of b?

21. _____

22. What is the smallest positive integer N such that the first four digits of N are 2, 0, 1, 0 (in that order) and such that N is divisible by 45? (Your answer should look like $2010\ldots$.)

22. _____

23. Two integers are *relatively prime* if 1 is the largest integer that divides both of them. For example, 21 and 47 are relatively prime. Alan and Beth each pick independently and at random an integer from 1 to 6 (inclusive). What is the probability that the numbers they pick are relatively prime? Express the answer as a common fraction.

23. _____

24. How many squares of all sizes are on a standard 8×8 chessboard?

24. _____ squares
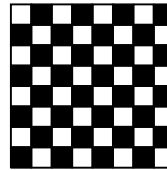
25. In the picture below, $\triangle ABC$ is right-angled at A, P lies on AB, Q lies on BC, R lies on CA, and $APQR$ is a square. The length of AB is 24 and the length of AC is 5. What is the length of AP? Write the answer as a common fraction.

25. _____
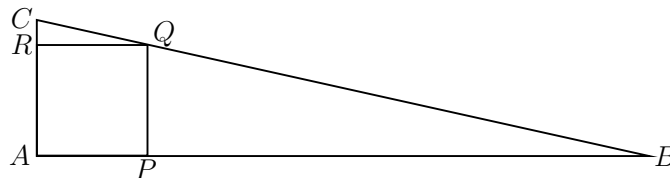
26. You play the "Lucky 4" game as follows. You roll a fair standard die and receive in pennies the number you rolled. You keep doing this until either you have accumulated exactly 4 pennies, in which case you win, and the game is over, or your last roll pushes you over 4 pennies, in which case you lose. What is the probability that you win? Express the answer as a common fraction.

26. _____

Bull's-eye, Page 1: Problem Solving

1. Some students went on a school ski trip by car, 3 per car, and the rest went by van, 5 per van. In total, 140 students went, using 40 vehicles. How many students went by car?

1. _____ students

2. When Beth goes from her home up to Mount Baker, her car uses on average 13 litres of gas for every 100 kilometres. On the way back from Mount Baker (using the same route), her car averages 11 litres of gas for every 100 km. The round trip uses 33 litres of gas. How many km in total are there in the round trip?

2. _____ km

3. Alfred wrote eight math tests, each graded out of 100. His average after six tests wasn't very good, but his seventh test raised his average by 2 marks, and his eighth test raised his average by another 2 marks. How many more marks did he get on the eighth test than on the seventh?

3. _____ marks

4. The average of nine positive integers is 18. The integers are not necessarily all different. What is the largest possible value of the median of the nine integers?

4. _____

Bull's-eye, Page 2: Numbers and Combinatorics

5. Alice picks an integer from 1 to 10 (inclusive) at random and Bob picks an integer from 11 to 20 (inclusive) at random. What is the probability that the product of Alice's number and Bob's number is a multiple of 3? Express the answer as a common fraction.

5. _____

6. What is the smallest number that can be written as the sum of two unequal primes in two different ways? Note that for example $5 = 2+3$ and $5 = 3 + 2$ should not be regarded as different ways.

6. _____

7. What is the remainder when 8! is divided by 256?

7. _____

8. Let $a = 888888$ and let $b = 999999$. What is the sum of the (decimal) digits of $a \times b$?

8. _____

Bull's-eye, Page 3: Geometry

9. A semicircle has the same area as a circle of radius 1. What is the radius of the semicircle? Express the answer in simplest radical form.

9. _____

10. What is the volume, in cm^3, of a right circular cone of height 8 cm whose base is a circle of radius $\dfrac{6}{\sqrt{\pi}}$?

10. _____ cm^3

11. The diagram shows a circle, and two tangent lines PA and PB. The points A, B, and Q are on the circle, and Q is on the line segment that joins the centre O of the circle to P. Suppose that the measure of $\angle APB$ is $34°$. What is the degree measure of $\angle AQB$?
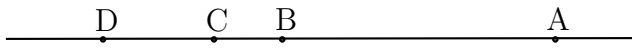
11. _____ degrees
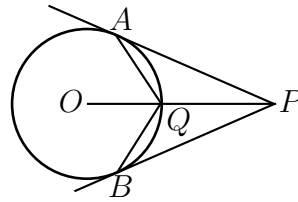
12. Consider all ordered pairs (A, B), where A and B are distinct vertices of a regular hexagon whose edges have length 5. What is the average value of the distance between A and B? Express the answer in simplest radical form.
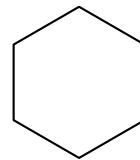
12. _____ units

1. The volume of a cylinder is 144π cubic inches. The radius of the base of the cylinder is a whole number of inches. The height of the cylinder is a whole number of inches. How many different such cylinders are there?

1. _____ cylinders

2. A, B, C, and D are running a marathon along a straight road. As usual, A is in front, B is next, C is behind B, and D is behind C. At this instant, A is 1 km ahead of C, B is 4 times as far from A as she is from C, and D is also 4 times as far from A as she is from C. What is the distance, in km, between B and D? Express the answer as a common fraction.

2. _____ km

$$\underline{\qquad \underset{\bullet}{\text{D}} \qquad \underset{\bullet}{\text{C}} \quad \underset{\bullet}{\text{B}} \qquad\qquad \underset{\bullet}{\text{A}} \qquad}$$

3. What is the value of $9515 \times 1595 - 9595 \times 1515$?

3. _____

4. What is the number of "digits" used when 5040 is written in base 2 (binary) notation? (For example, when 4 is written in base 2, the number of "digits" is 3.)

4. _____ "digits"

Co-op, Page 2: Team answers must be on the *coloured* page.
Answers on a white page will not be graded.

5. The 8 points below are the vertices of a regular octagon. How many right-angled triangles are there whose 3 vertices are chosen from these 8 points?

5. _____
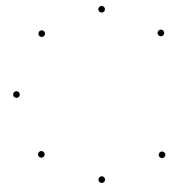
6. In the "Prime Challenger" competition, there are 10 questions. Each team gets an automatic n^2 points on the n-th question, whether or not the team answers the question correctly. Thus on Question 1, it gets an automatic 1 point, on Question 2 an automatic 4 points, and so on. In addition, each team gets 1 point for every correct answer. What is the highest prime score that a team can get?

6. _____

7. In the picture, ABC is a triangle, and R is a point on the line segment AB. The point P is on BC extended, with AP parallel to RC. Similarly, Q lies on AC extended, with BQ parallel to RC. Given that $AP = 5$ and $BQ = 6$, what is the length of the line segment RC? Express the answer as a common fraction.

7. _____ units
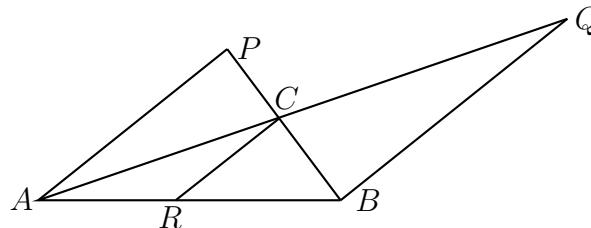
Co-op, Page 3: Team answers must be on the *coloured* page.
Answers on a white page will not be graded.

8. What is the smallest positive integer n such that the leftmost digit   8. _____
   in the decimal representation of 2^n is equal to 7? Hint: $2^{10} = 1024$.

9. There is a group of 7 women and m men arranged around a circular   9. _____
   table so that the number of people whose right-hand neighbour is of
   the same sex is the same as the number of people whose right-hand
   neighbour is of the opposite sex. What is the largest possible value
   of m?

10. Let $A(n) = \dfrac{n(3n-1)}{2}$. What is the smallest integer $n > 1$ such that   10. _____
    $A(n)$ is a perfect square?

Provincial 2010 Face-off
Questions and Answers

1. What is the value of $\dfrac{\dfrac{1}{2} - \dfrac{1}{5}}{\dfrac{1}{10}}$?

1. Answer: 3

2. How many perfect squares are there between 11 and 111?

2. Answer: 7 (perfect squares)

3. It is late afternoon, and a 1.2 metre tall child casts a 3 metre shadow. The child is standing next to an upright telephone pole, which casts a 40 metre shadow. What is the height, in metres, of the telephone pole?

3. Answer: 16 (metres)

4. What is the integer nearest to $(8.5)^2$?

4. Answer: 72

5. If $5x - 4x + 3x - 2x + x = 180$, what is the value of x?

5. Answer: 60

6. Today, sunrise in Burnaby is at 7 : 00 AM, and sunset is at 7 : 36 PM. At what time is it exactly halfway between sunrise and sunset? Express the answer in hour:minute format. A spoken answer like "twelve forty-two" is of the right shape.

6. Answer: 1 : 18 (PM)

7. The diagonals of a rhombus have lengths 16 and 12. What is the perimeter of the rhombus?
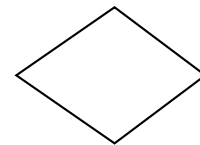
7. Answer: 40 (units)

8. What common fraction is halfway between $\dfrac{1}{3}$ and 3?

8. Answer: $\dfrac{5}{3}$

9. How many positive integers less than 61 are divisible by 2 or 3 (or both)?

9. Answer: 40

10. One dozen scarlet splendour roses cost \$98. At the same price per rose, what the cost in dollars of 2.5 dozen scarlet splendour roses?
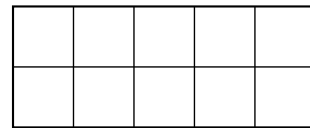
10. Answer: 245 (dollars)

11. If $x^{2010} = 9$, what is the value of x^{3015}?

11. Answer: 27

12. Among the three-letter "words" that only use the letters A and/or B and/or C, how many have exactly one C?

12. Answer: 12 ("words")

13. The rectangle below has perimeter 70 cm, and is split into 10 congruent squares as shown. What is the area, in cm^2, of the full rectangle?

13. Answer: 250 (cm^2)

14. Evaluate $\dfrac{7.2}{0.018}$

14. Answer: 400

15. When the radius of a circle is increased by 50%, the new circle has circumference 99. What is the circumference of the old circle?

15. Answer: 66 (units)

16. What is the value of the sum

$$1000 - 999 + 998 - 997 + \cdots + 4 - 3 + 2 - 1?$$

16. Answer: 500

17. A very long test has 108 questions, numbered 1 to 108. The test is 9 pages long, and each page has 12 questions. What is the number of the fifth question on the fifth page?

17. Answer: 53

18. Alphonse's auto repair bill is $3000, of which $1200 is for parts and the rest for labour. If labour is billed at $45 per hour, how many hours of labour were billed?

18. Answer: 40 (hours)

**19.** What is the largest prime which is less than 190?

19. Answer: 181

20. Let $\mu(a, b)$ be the mean of the numbers a and b. What is the value of $\mu(\mu(64, 32), 16)$?

20. Answer: 32

21. The sum of 4 consecutive integers is 90. What is the smallest of the 4 integers?

21. Answer: 21

22. In the 3×3 grid below, every grid point is 1 unit from its nearest horizontal or vertical neighbours. How many lines pass through *exactly* two grid points?
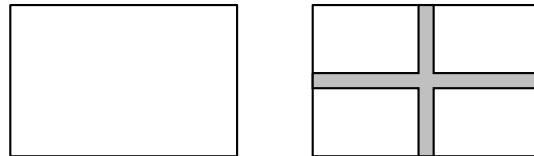
. . .

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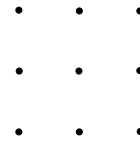
22. Answer: 12 (lines)

23. Evaluate $\dfrac{2^{10} - 1}{2^5 - 1}$.

23. Answer: 33

24. Alicia had a rectangular 20 feet by 30 feet garden (left-hand picture). She decided to make a 2 foot wide path in the garden as in the right-hand picture. How many percent of the area of the original garden is lost to the path?

24. Answer: 16 (percent)

25. Evaluate $\dfrac{7! - 6! - 5!}{6! - 5!}$.

25. Answer: 7

26. The sum of two consecutive primes is divisible by 2 but not by 4. What is the smallest possible value of this sum?

26. Answer: 18

27. Of the 30 marbles in a bag, 10 are red, 10 are green, and the remaining 10 are white. Two marbles are removed from the bag. What is the probability that these two marbles are of different colours? Express the answer as a common fraction.

27. Answer: $\dfrac{20}{29}$

28. How many minutes are there in two and a half days?

28. Answer: 3600 (minutes)

29. The average class size in the year 2000 was 27. Now the average class size is 30. By how many percent has the average class size increased from the year 2000 to now? Round the answer to the nearest integer. Thus an answer like 17 is of the right shape.
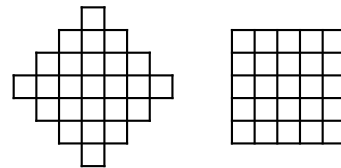
29. Answer: 11 (percent)

30. If $9^2 \times 27^3 = 3^n$, what is the value of n?

30. Answer: 13

31. The sum of three consecutive even integers is 30. What is the product of the three integers?

31. Answer: 960

32. Each figure below is made up using twenty-five 1×1 squares. What is the positive difference between the perimeters of the two figures?

32. Answer: 8 (units)

33. If $x^2 = \dfrac{9}{256}$, what is the value of $|x|$? Express the answer as a common fraction.

33. Answer: $\dfrac{3}{16}$

34. Last week, Alan read every second page of his 200 page textbook, starting with page 1. This week, Alan read every third page of the textbook, again starting with page 1. How many pages did Alan read twice?

34. Answer: 34 (pages)

35. A *rod* is 5.5 yards, and a *furlong* is 220 yards. How many rods are there in 6 furlongs?

35. Answer: 240 (rods)

36. Alicia and Beth differ in weight by 20 pounds. Beth and Gamal differ in weight by 30 pounds. And Gamal and Delbert differ in weight by 6 pounds. What is the least possible weight difference between Alicia and Delbert?

36. Answer: 4 (pounds)

37. Given that the least common multiple of the numbers 8, 10, and n is 80, what is the smallest possible positive value of n?

37. Answer: 16

38. Alphonse has three-sevenths as much money as Beti, and between them they have 400 dollars. How many dollars does Beti have?

38. Answer: 280 (dollars)

39. The mean of 20, 21, x, and $2x$ is 35. What is the value of x?

39. Answer: 33

40. The operation $\circledast$ is defined by the rule

$$x \circledast y = x^2 - 2xy + y^2.$$

What is the value of $6 \circledast (-4)$?

40. Answer: 100

41. The three cans of cola that Alphonse drinks every day together supply 20% of Alphonse's daily caloric requirements, which are 2100 calories. How many calories are in a can of cola?

41. Answer: 140 (calories)

42. Given that $x^2 = 0.2$, what is the value of x^{-4}?

42. Answer: 25

Math Challengers Regional 2010
Answers, Blitz Stage

1. 70

2. 91

3. 40

4. 6

5. 5

6. 110

7. 8

8. 23

9. 15

10. 9

11. 48

12. 7

13. 72

14. 6

15. 27.5

16. 6

17. 5

18. $\dfrac{5}{18}$

19. 30

20. 6

21. 9

22. 1800

23. $\dfrac{9}{46}$

24. 1005

25. 42

26. 36

Math Challengers Regional 2010
Answers, Bull's-eye Stage

1. 78

2. 12

3. 11.25

4. 22.52

5. $\dfrac{3}{8}$

6. 216

7. $\dfrac{3}{4}$

8. 8

9. 11

10. 12.5

11. 54

12. 2.4

Math Challengers Regional 2010
Answers, Co-op Stage

1. −8

2. 34

3. 61

4. 11

5. 98

6. 120

7. 75

8. 204

9. 18

10. 44

Math Challengers Provincial 2010
Answers, Blitz Stage

1. 700

2. 1

3. 91

4. 72

5. 32

6. 19

7. 900

8. $\dfrac{23}{20}$

9. 60

10. 16

11. 29

12. 36

13. 50

14. 16

15. 22000

16. 45

17. 91

18. $\dfrac{4}{5}$

19. 9

20. 3013

21. 32

22. 201015

23. $\dfrac{23}{36}$

24. 204

25. $\dfrac{120}{29}$

26. $\dfrac{343}{1296}$

Math Challengers Provincial 2010
Answers, Bull's-eye Stage

1. 90

2. 275

3. 4

4. 31

5. $\dfrac{51}{100}$

6. 16

7. 128

8. 54

9. $\sqrt{2}$

10. 96

11. 107

12. $4 + 2\sqrt{3}$

Math Challengers Provincial 2010
Answers, Co-op Stage

1. 6

2. $\dfrac{8}{15}$

3. $640,000$

4. 13

5. 24

6. 389

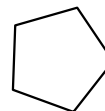7. $\dfrac{30}{11}$

8. 46

9. 21

10. 81

2011 Canadian Math Challengers Regional Questions

Blitz, Page 1

1. Eighty percent of eighty percent of a number is 144. What is the number?

   1. _____

2. How many diagonals does a regular pentagon have?

   2. _____ diagonals

3. A tiny test consists of 3 multiple choice questions. There are 4 choices of answer for each question, exactly one of which is correct. Alan chooses at random one of the 4 answers to each question. What is the probability that Alan answers all 3 questions correctly? Express the answer as a common fraction.

   3. _____

4. A cookie costs $1.20. Dina buys 7 cookies, and hands the cashier a 10 dollar bill. How much money should Dina get back in change? Express the answer in dollars, to the nearest cent. (An answer like 2.70 has the right shape, while 2.7 does not.)

   4. _____ dollars

5. What is the whole number which is nearest to $\sqrt{2011}$?

   5. _____

6. A cup of flour has 400 Calories. A cup of lard has 1700 Calories. A pie crust is made using two cups of flour, one cup of lard, and nothing else. How many percent of the Calories in the pie crust come from the lard?

   6. _____ percent

7. The height of a pyramid is increased by 30%. The base remains unchanged. By how many percent does the volume of the pyramid increase?

   7. _____ percent

8. The sum of the decimal digits of the whole number n is 33. What is the smallest possible value of n?

8. _____

9. Suppose that $f(n+2) = f(n) + f(n+1)$ for every whole number n. Given that $f(1) = 1$ and $f(3) = 48$, what is the value of $f(4)$?

9. _____

10. Alicia tosses 2 fair coins, and then Beti tosses 2 fair coins. What is the probability that they each get the same number of heads? Express the answer as a common fraction.

10. _____

11. Express as a common fraction:

11. _____

$$\frac{101 + 103 + 105 + \cdots + 197 + 199}{201 + 203 + 205 + \cdots + 297 + 299}.$$

12. A rectangle has area 1 square unit. Its width is nine-sixteenths of its length. How many units are in the perimeter of the rectangle? Express the answer as a common fraction.

12. _____ units

13. What is the smallest positive integer n such that the leftmost decimal digit of 3^n is 6?

13. _____

14. The first term of an arithmetic sequence is 1. The tenth term is 400. What is the fourth term of the sequence?

14. _____

15. What is the smallest possible value of $\left|x - \dfrac{1}{7}\right| + \left|x - \dfrac{1}{6}\right|$ as x travels over the real numbers? Express the answer as a common fraction.

15. _____

16. At a Math Challengers event, 240 people ate *one or more* slices of pizza, 140 ate *two or more* slices, 40 ate three slices, and no one ate more than three. How many slices of pizza were eaten?

16. _____ slices

17. A gambler tosses a fair coin 4 times. She wins if during the tossing she gets 3 or more heads in a row or 3 or more tails in a row. What is the probability that the gambler wins? Write the answer as a common fraction.

17. _____

18. On Thursday, Alphonse began to read a 360 page book. On Friday, he read twice as many pages of the book as he read Thursday. On Saturday, he read twice as many pages as he read Friday. On Sunday, he read twice as many pages as he read Saturday, and finished the book. How many pages of the book did Alphonse read on Thursday?

18. _____ pages

19. A letter E was made by pasting onto a sheet of paper three rectangular 2 cm by 10 cm strips of cardboard next to a rectangular 2 cm by 20 cm strip of cardboard, as in the picture below. No cutting was done. What is the perimeter, in cm, of this letter E?

19. _____ cm
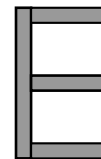
20. Define the number N by

$$N = 1223334444555556666667777777888888889999999999.$$

So the decimal expansion of N is 1 followed by two 2's followed by three 3's followed by four 4's, and so on, until it ends with nine 9's. Alan writes the number N over and over again. What is the 1000-th digit that Alan writes?

20. _____

21. The big rectangle is divided into 9 rectangles by lines parallel to the sides. If the areas of some of the little rectangles are as shown below, what is the value of x? Write the answer as a common fraction.

21. _____ units^2

3	4	
4		5
	5	x

22. Let r be the number of 8-letter "words" that can be formed using all the letters of the word "RICHMOND". Let v be the number of 9-letter "words" that can be formed using all the letters of the word "VANCOUVER". Express the ratio r/v as a common fraction.
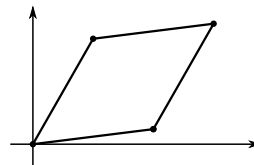
22. _____

23. What is the smallest positive integer whose square is divisible by every integer from 1 to 10?

23. _____

24. Three consecutive vertices of a rhombus have coordinates $(4, 7)$, $(0, 0)$, and $(8, 1)$ respectively. What is the area of the rhombus?
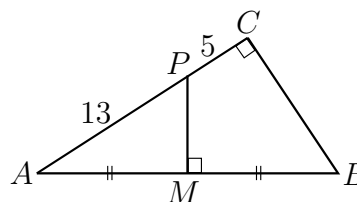
24. _____ units^2

25. What is the smallest prime number which is larger than 293?

25. _____

26. Triangle ABC is right-angled at C, and $AC > BC$. The perpendicular bisector of the hypotenuse AB meets the hypotenuse at M and meets AC at P. Given that $AP = 13$ and $PC = 5$, what is the ratio of the area of $\triangle APM$ to the area of $\triangle ABC$? Express the answer as a common fraction.

26. _____

1. Alan says to Beth: "Give me $100, and I shall become twice as rich as you." Beth replies: "Give me $10, and I shall become six times as rich as you." How many dollars does Alan have?

1. _____ dollars

2. A merchant carrying rice passes through three customs posts. At the first post, he has to give up one-third of his rice. At the second post, he has to give up one-fifth of what remains, and at the third post, one-seventh of what remains. He ends up with 5 measures of rice. How many measures of rice did he start out with? Express the answer as a common fraction.

2. _____ measures

3. Painter A can paint a room in 3.3 days, B can paint the room in 5.5 days, and C can paint the room in 6.6 days. They all worked together painting the room for 1 day. Then B and C got fired. How many (additional) days will it take for A to finish the job? Express the answer as a decimal, to the nearest tenth.

3. _____ days

4. The plane was full when it left Vancouver. In Kelowna, half the people got off and 28 got on. In Calgary, half the people got off, 40 got on, and the plane was full again. How many people were on the plane when it left Vancouver?

4. _____ people

5. How many six-letter "words" can be made using three A's and three B's if no two A's can be next to each other?

5. _____ words

6. The first number in a sequence is 2, and the second number is 3. Each new number in the sequence is obtained by dividing the previous number by the one before that. (Thus the third number is $\frac{3}{2}$.) What is the 100-th number in the sequence? Express the answer as a common fraction.

6. _____

7. What is the reciprocal of the repeating decimal $0.\overline{027}$?

7. _____

8. A 4×4 square is divided into sixteen 1×1 squares as shown. Two different squares are chosen at random from these sixteen squares. What is the probability that the two chosen squares have *exactly one* vertex in common? Express the answer as a common fraction.

8. _____
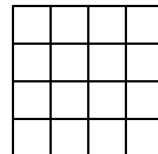
9. Two crystal pyramids each have a 2×2 square base. The two bases are cemented together to make a new crystal. Let V be the number of vertices of the new crystal, E the number of edges, and F the number of faces. What is the value of $V - E + F$?

9. _____

10. The vertices of a triangle are at $(0,0)$, $(48,1)$, and $(50,2)$. How many units^2 are in the area of the triangle?

10. _____ units^2

11. A circle of radius 3 meets a circle of radius 4 at points P and Q. The tangent lines at P to the two circles are perpendicular to each other. What is the length of the line segment PQ? Express the answer as a decimal, to the nearest tenth.

11. _____ units
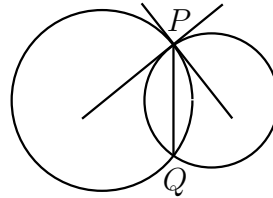
12. Triangle ABC is right-angled, with hypotenuse $AB = \sqrt{1001}$. Point D is on side CA, with $CB = CD$ and $DA = 11$. What is the area of triangle ABC?

12. _____ units^2
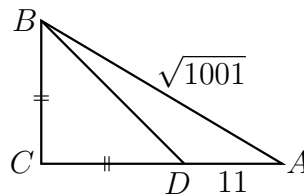
Co-op, Page 1: Team answers must be on the *coloured* page. Answers on a white page will not be graded.

1. How many of the multiples of 10 from 10 to 10^6 (inclusive) are perfect squares?

1. _____

2. The line with equation $y = 2x + 4$ meets the line with equation $y = -(5 + 6x)$ at the point with coordinates (a, b). What is the value of $a + b$? Express the answer as a common fraction.

2. _____

3. Two cards are dealt from a well-shuffled standard 52-card deck. What is the probability the cards are both of the same suit (that is, that both are spades, or both are hearts, or both are diamonds, or both are clubs)? Express the answer as a common fraction.

3. _____

4. What is the smallest possible value of $x + y$, given that x, y, and z are positive integers, z is odd, and $x^2 + y^2 = z^3$?

4. _____

Co-op, Page 2: Team answers must be on the *coloured* page.
Answers on a white page will not be graded.

5. A small island country in the middle of the ocean has the shape of a triangle with sides 10, 12, and 18 *kilometres.* The country has exclusive economic jurisdiction over all parts of the ocean that are within 200 *nautical miles* of its shores. (A nautical mile is 1.852 kilometres.) What is the area, in km^2, of the part of the ocean over which the country has exclusive economic jurisdiction? Give the answer to the nearest 1000 km^2 (thus your answer should have three 0's at the end). Assume the earth is flat.

5. _____ km^2

6. A positive integer is called *square-free* if it is not divisible by any perfect square greater than 1. For example, 1, 2, and 6 are square-free, while 4 and 18 are not. Two fair standard dice are tossed. What is the probability that the product of the two numbers obtained is square-free? Express the answer as a common fraction.

6. _____

7. Call an integer *lucky* if its decimal representation has two or more consecutive 8's. For example, 1881 is lucky, as is 8882, but 8087, 1289, and 4321 are not lucky. How many of the integers in the interval from 1000 to 9999 are lucky?

7. _____ integers

8. A right circular cone has base radius 3 cm and height 4 cm. What is the number of cm^2 in the total surface area (including the base) of the cone? Express the answer as a decimal, rounded to the nearest tenth of a cm^2. Note that π is approximately equal to 3.14159.

8. _____ cm^2

9. You have a large number of 10 cent coins, 25 cent coins, and 1 dollar coins, and no other coins. In how many ways can you make change for a 10 dollar bill?

9. _____ ways

10. What is the smallest possible value of $\left|\sqrt{2} - a/b\right|$, where a and b are positive integers and $b \leq 20$? Express the answer in scientific notation, correct to 4 significant digits. For example, an answer like 4.567×10^{-3} is of the right shape.

10. _____

REGIONAL 2011 FACE-OFF
QUESTIONS AND ANSWERS

1. How many integers between 1 and 100 are divisible by 7?
1. Answer: 14 (integers)

2. Express $\dfrac{\dfrac{1}{3} - \dfrac{1}{12}}{\dfrac{1}{3} + \dfrac{1}{12}}$ as a common fraction.

2. Answer: $\dfrac{3}{5}$

3. It takes 450 seconds for 4 teenagers to eat 3 pizzas. If all teenagers always eat at the same constant rate, how many seconds does it take 3 teenagers to eat 4 pizzas?
3. Answer: 800 (seconds)

4. What is the smallest prime number which is larger than 139?
4. Answer: 149

5. If $3x + 10 = 49$, what is the value of $6x + 10$?
5. Answer: 88

6. In a bag of jelly beans, $\frac{1}{5}$ are white, $\frac{1}{5}$ are yellow, $\frac{1}{4}$ are green, $\frac{1}{4}$ are red, and the remaining 12 are black. How many red jelly beans are in the bag?
6. Answer: 30 (red ones)

7. The temperature F in degrees Fahrenheit is related to the temperature C in degrees Celsius by the equation $F = \frac{9}{5}C + 32$. If the temperature of an oven in degrees Fahrenheit is 320, what is the temperature of the oven in degrees Celsius?
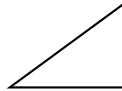**7. Answer:** 160 (degrees Celsius)

8. How many ordered pairs (a, b) of positive integers are there such that $a + b \le 5$?

8. Answer: 10 (ordered pairs)

9. The measures of the acute angles of a right triangle are in the ratio 2:3. What is the degree measure of the smallest angle of the triangle?

9. Answer: 36 (degrees)

10. The first few terms of a sequence are

$$1, 2, 3, 4, 5, 6, 7, 8, 9, 1, 2, 3, 4, 5, 6, 7, 8, 9, 1,$$

and the obvious pattern continues forever. What is the product of the 66-th term and the 67-th term of the sequence?

10. Answer: 12

11. Two fair dice are tossed. What is the probability that the sum is 6? Express the answer as a common fraction.
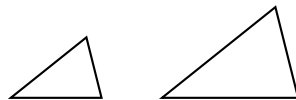
11. Answer: $\dfrac{5}{36}$

12. The number $1\frac{6}{7}$ is how many percent of $2\frac{6}{7}$?

12. Answer: 65 (percent)

13. Two similar triangles have area 4 square units and 9 square units respectively. If the perimeter of the smaller triangle is 10 units, how many units are in the perimeter of the larger triangle?

13. Answer: 15 (units)

14. On a planet far far away, a year is 500 days and a week is 7 days, with the usual English names. New Year's day this year was on a Sunday. On what day of the week is New Year's day next year?
14. Answer: Wednesday

15. An answering machine can hold up to 10 minutes of messages. Any message uses up at least 12 seconds. What is the largest number of messages that the machine can hold?
15. Answer: 50 (messages)

16. When the bowl of mixed nuts was set out, it was (by weight) 45% peanuts, 25% almonds, 20% cashews, and 10% hazelnuts. Alphonse picked out all the almonds and ate them. What percent (by weight) of the nuts in the bowl are now peanuts?
16. Answer: 60 (percent)

17. In a school election, every one of the 2011 students voted for exactly one of A or B. If A got 1100 votes, how many more votes than B did A get?
17. Answer: 189 (more votes)

18. Let $H(x,y) = \dfrac{2xy}{x+y}$. What is the value of $H(7,42)$?
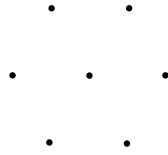**18. Answer:** 12

19. American Thanksgiving is on the fourth Thursday in November. What day of the month is the last possible day for American Thanksgiving? Your answer should be a number between 1 and 30.
19. Answer: 28

20. Two-thirds of a group of students are dark-haired, one-quarter are blonde, and the remaining 30 students are red-haired. How many of the students in the group are *not* red-haired?
20. Answer: 330 (students)

21. The figure below consists of 7 points, namely the 6 vertices of a regular hexagon and the centre of that hexagon. How many triangles have one vertex at the centre of the hexagon, and the other two vertices at vertices of the hexagon?

21. Answer: 12 (triangles)

**22.** What is the largest prime that divides 9999?
22. Answer: 101

23. A pack of 6 cans of cola costs \$4.50, plus 12% tax, plus a 5 cent deposit per can. How much does one have to pay for a pack of 6 cans? Give the answer in dollars, to the nearest cent.
23. Answer: 5.34 (dollars)

24. How many whole numbers are there between $\sqrt{88}$ and $\sqrt{888}$?
24. Answer: 20 (whole numbers)

25. What common fraction of the area of the trapezoid is shaded?
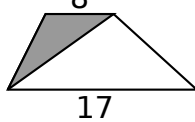
25. Answer: $\dfrac{8}{25}$

26. X has 60% as much money as Y. Between them they have \$1000. How many dollars does X have?
26. Answer: 375 (dollars)

27. Given that $(x - y)^2 = 39$ and $xy = 40$, what is the value of $(x + y)^2$?
27. Answer: 199

28. What is the smallest positive integer which is divisible by 12 and whose decimal representation contains no digits other than 0's and 1's?

28. Answer: 11100

29. Given that $\sqrt{1 + \sqrt{4 + x}} = 3$, what is the value of x?

29. Answer: 60

30. What is the area, in square cm, of the isosceles triangle that has sides 2, 4, and 4 cm? Express the answer in simplest radical form.

30. Answer: $\sqrt{15}$ (square cm)

31. The lengths of the three sides of a triangle are in the ratio 4:5:6. The triangle has perimeter 180 cm. How many cm are in the length of the longest side of the triangle?

31. Answer: 72 (cm)

32. Four fair coins are tossed. What is the probability of getting at least one head and at least one tail? Express the answer as a common fraction.

32. Answer: $\dfrac{7}{8}$

33. What is the smallest positive integer that cannot be expressed as the difference between two prime numbers?

33. Answer: 7

34. The top 100 rock and roll songs of all time are played one after the other. Each song lasts 3 minutes, and there is a 1 minute commercial break between songs. From the start of the first song to the end of the last song takes 6 hours plus how many minutes?

34. Answer: 39 (minutes)

35. A basketball team has 7 wins and 13 losses. To reach the playoffs it must win at least 50% of the regular-season games that it plays. What is the largest number of the remaining 35 regular season games that the team can lose and still reach the playoffs?

35. Answer: 14 (games)

36. If $4^{(4^4)} = 2^{(2^y)}$, what is the value of y?

36. Answer: 9

37. The first term of an arithmetic sequence is 1, and the sum of the first 4 terms of the sequence is 100. What is the sum of the first 5 terms of the sequence?

37. Answer: 165

38. What is the smallest whole number N such that $75N$ is a perfect cube?

38. Answer: 45

39. How many positive integers between 1 and 60 (inclusive) are divisible by 3 or by 5 or by both?

39. Answer: 28 (integers)

40. What is the largest perfect square which is less than 3^8?

40. Answer: 6400

41. What is the smallest perfect square which is in the arithmetic sequence 1, 6, 11, and also in the arithmetic sequence 7, 13, 19, ?

41. Answer: 121

42. How many integer values of n are there such that $|(6n-77)(n+30)|$ is a prime?

42. Answer: 3 (integer values)

43. How many 3-digit positive integers are there all of whose digits are odd?

43. Answer: 125

44. Alphonse took the same algebra test a total of 3 times. Each time he retook the test, the number of questions he answered correctly increased by 50%. If on the last test he got 36 of the 70 questions right, how many questions did he get right the first time he took the test?

44. Answer: 16 (questions)

45. What is the value of $\dfrac{\sqrt{b^2 - 4ac} - b}{2a}$ when $a = 1$, $b = 3$, and $c = -4$?

45. Answer: 1

2011 Canadian Math Challengers Provincial Questions

1. One-third of 105 is the same as seven-sixths of what number?
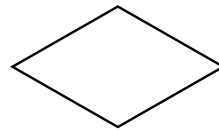
1. _____

2. A rectangle has length 16 and width 12. What is the radius of the circle that passes through the four vertices of the rectangle?

2. _____ units
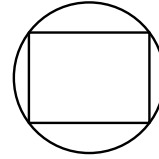
3. Which whole number is closest to 125% of 25?

3. _____

4. Express $1 - \dfrac{1}{2} + \dfrac{1}{3} - \dfrac{1}{4}$ as a fraction in lowest terms.

4. _____

5. What is the area of a rhombus with diagonals of length 4 and 7?

5. _____ units^2

6. Simplify $\dfrac{1}{1 + \frac{2}{3}} - \dfrac{1}{1 + \frac{3}{2}}$

6. _____

7. Let $F(x) = x^4 + 2x^3 + 3x^2 + 2x + 1$. What is the value of $F(2) - F(-2)$?

7. _____

8. Express $\dfrac{1}{2+\dfrac{1}{2+\dfrac{1}{2}}}$ as a common fraction.

8. _____

9. How many of the numbers from 1 to 100 can be written in the form $2^a 3^b$ where a and b are *positive* integers?

9. _____

10. One cell starts to divide at time $t = 0$. Any cell divides into 2 cells every minute, and the culture plate is full at $t = 20$ minutes. If we start with 4 cells starting to divide at $t = 0$ instead, how many minutes does it take until the plate is full?

10. _____ minutes

11. In $\triangle ABC$, the angle at A is $84°$. The bisectors of the angles at B and C meet at P. How many degrees are in the measure of $\angle BPC$?

11. _____ degrees
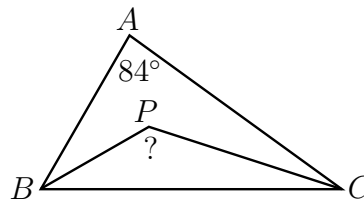
12. What is the greatest possible value of $p + q$ if $pq < 100$ and p and q are odd primes?

12. _____

13. A gambler went to the casino with 1 dollar. She made a series of six 1-dollar bets, winning or losing 1 dollar each time. She ended up with 1 dollar. In how many different orders (of winning/losing) could this have happened? Note that if you have no money you cannot bet.

13. _____ orders

14. A box contains 3 black beads, 3 blue beads, 6 red beads, and 8 yellow beads. If you are blindfolded, how many beads must you take out in order to be sure of taking out at least 2 beads of each colour?

14. _____ beads

15. What is the value of $17^3 - 17^2 \cdot 16 - 17 \cdot 16^2 + 16^3$?

15. _____

16. A line passes through the points $(-1, 10)$, $(10, -1)$, and $(x, -10)$. What is the value of x?

16. _____

17. For how many positive integers n is the positive difference between $\sqrt{n}$ and 9 less than 1?

17. _____

18. Vinegar is poured into a conical cup of height 3 inches until the vinegar is 2 inches deep at its deepest point (please see the left-hand diagram). Then olive oil is poured into the cup until the cup is full (right-hand diagram). After the oil has been poured in, what common fraction of the cup's contents is oil?

18. _____
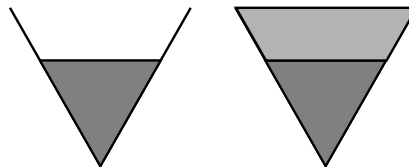
19. Alfred wrote down all the numbers from 111 to 999. How many times did he write the digit 0?

19. _____ times

20. An equilateral triangle with each side 6 cm is inscribed in a circle. If the area of the circle is $k\pi$ cm^2, what is the value of k?

20. _____
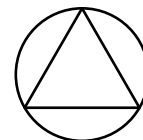
21. The large square is made up of nine 2×2 squares. Two circles whose center is the center of the large square pass through vertices of the small squares, as shown. What is the ratio of the area of the smaller circle to the area of the larger circle? Express the answer as a common fraction.

21. _____
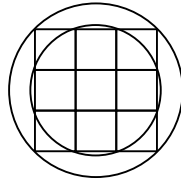
22. Given that $\left(x^2 + \dfrac{1}{x^2} \right)^2 = 100$, what is the value of $\left(x^2 - \dfrac{1}{x^2} \right)^2$?

22. _____

23. What is the smallest sum of money that can't be made up using 10 or fewer coins? Allowed coins are 1 cent, 5 cents, 10 cents, 25 cents, 1 dollar, and 2 dollars. Give the answer as a decimal, in dollars (an answer of 4.00 or 4.56 has the right shape).

23. _____ dollars

24. It so happens that you can put the numbers 1, 2, 3, 4, 5, 6, 8, 9, 10, and 12 (every number from 1 to 12 except 7 and 11) in the circles below, one to each circle, so that the sums of the numbers in any 4 circles whose centres lie on the same line are all equal. What must each of these sums be?

24. _____
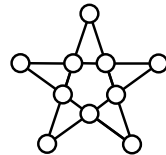
25. The larger square in the diagram has side 5 and the smaller square has side 3. What is the area of the shaded triangle? Express the answer as a common fraction.

25. _____ units^2
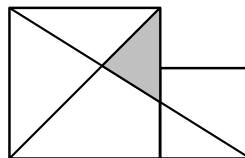
26. The number 130 has 8 positive factors, namely 1, 2, 5, 10, 13, 26, 65, and 130. How many positive integers *smaller* than 130 also have 8 positive factors?

26. _____ integers

Canadian Math Challengers Society

1. Alan answered 60 questions on a quiz. He had 40% more right answers than wrong answers. How many questions did Alan get right?

1. _____ questions

2. The time in St. John's, Newfoundland, is 4.5 hours ahead of the time in Vancouver. A cargo plane left St. John's at 4:00 AM and landed in Vancouver at 9:45 AM the same day. The plane left Vancouver for St. John's 45 minutes later. The flight back to St. John's was shorter by 105 minutes than the flight to Vancouver. The plane arrived at St. John's in the evening. At what time did it arrive? Express the answer in Hours:Minutes format, using a 12-hour clock. Thus 10:40 has the right shape, and 22:40 does not.

2. _____ PM

3. Dan decided to distribute $97 between his 5 kids, A, B, C, D, and E, giving each an integer number of dollars. He gave 1/2 (rounding up to the next $) to A, 1/4 (rounding up to the next $) to B, 1/5 (rounding up to the next $) to C, 1/40 (rounding up to the next $) to D, and the balance to E. How many dollars did E receive?

3. _____ dollars

4. We can put digits in the 5 squares below, one digit in each square, in a way that makes the statement below true. What 2-digit positive integer should be in the leftmost two squares?

4. _____

$\square\square$ percent of $\square\square\square$ is 777

5. How many ordered triples (x, y, z) of positive integers are there such that $x < y < z$ and $x + y + z = 12$?

5. _____ triples

6. Let $N = 1! + 2! + 4! + 8! + 16! + 32!$. What is the remainder when N^2 is divided by 16?

6. _____

7. How many ordered triples (a, b, c) are there such that a, b, and c are positive integers and $(a^b)^c = 64$? One such triple is $(64, 1, 1)$.

7. _____ triples

8. Five students will work on problem-solving in groups. Any group can consist of 1 to 5 students and each student must belong to exactly one group. In how many ways can the 5 students be divided into groups?

8. _____ ways

9. In the diagram, $PQ = SQ = SR$ and the degree measure of angle SPQ is 33°. What is the degree measure of angle QSR?
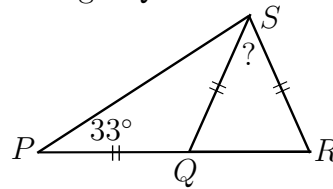
9. _____ degrees

10. Seven cubes are stacked one on top of the other. The volume of any cube is 10% more than the volume of the cube just above it. By how many percent is the side length of the bottom cube greater than the side length of the top cube?

10. _____ percent

11. The horizontal line segment at the bottom is the hypotenuse of two congruent right triangles. The legs of these triangles have lengths 1 and 2. What is the area of the shaded region? Express the answer as a common fraction.
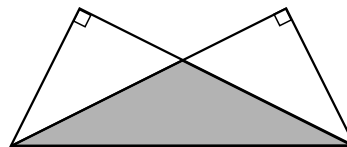
11. _____ units^2

12. The base of a right circular cylinder has diameter 6, and the height of the cylinder is 8. The cylinder is enclosed in a sphere which is just large enough to contain the cylinder. What common fraction of the volume of the sphere is taken up by the cylinder?

12. _____

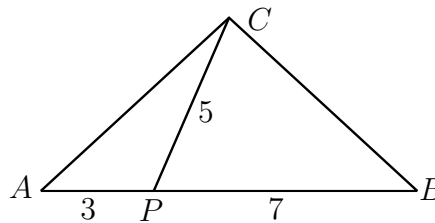Co-op, Page 1: Team answers must be on the *coloured* page.
Answers on a white page will not be graded.

1. Call an integer n *good* if the product of the positive integers that divide n is n^2. How many good integers are there in the interval from 1 to 27 (inclusive)? Please note that 1 is good, while 2 and 4 are not.

1. _____

2. Triangle ABC is isosceles, with $CA = CB$. Point P is on AB. Given that $AP = 3$, $BP = 7$, and $CP = 5$, what is the area of $\triangle ABC$? Express the answer as a decimal, rounded to the nearest tenth of a unit^2.

2. _____ units^2

3. A positive integer is called a *palindrome* if it doesn't change when the order of the digits is reversed (examples: 444 and 464). How many three-digit positive integers n are there such that n and $2n$ are both palindromes?

3. _____ integers

4. How many ordered pairs (a, b) are there such that a and b are integers (not necessarily positive) and $|a| + |b| \leq 3$?

4. _____ pairs

Co-op, Page 2: Team answers must be on the *coloured* page.
Answers on a white page will not be graded.

5. An equilateral triangle with area 1 cm² is inscribed in a circle. What
   is the number of cm² in the area of the circle? Express the answer
   as a decimal, rounded to the nearest one-hundredth of a cm². Note
   that π is approximately 3.14159.
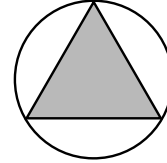
5. _____ cm²

6. What is the probability that the sum of two distinct randomly chosen
   positive factors of 420 is odd? Express the answer as a common
   fraction. Note that 1 and 420 are factors of 420.

6. _____

7. A hexagonal island is pictured below. The small triangles are all
   equilateral with side 1 km. The lines are the roads of the island. In
   how many ways can Alicia start at the center of the island and walk
   to the sea along island roads, walking a total distance of 3 km?
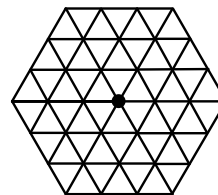
7. _____ ways

8. Boiling water, at 100 degrees Celsius, is to be cooled to below 5 degrees. The water cools by 10 degrees in the first minute (so it is 90 degrees at time $t = 1$ minute). The water cools by 10×0.9 degrees in the second minute, by $10 \times (0.9)^2$ degrees in the third minute, by $10 \times (0.9)^3$ degrees in the fourth minute, and so on. At what *integer* number of minutes will the water first be below 5 degrees?

8. _____ minutes

9. The rectangle below has length 10 cm and width 7 cm. An X-shaped figure (shaded) is drawn, with dimensions as shown. What is the area of the shaded figure, in cm^2? Express the answer as a common fraction.

9. _____ cm^2
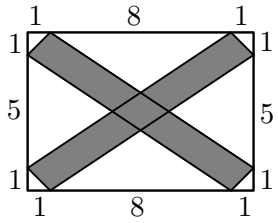
10. Starting at 6:00 AM, and then every 4 minutes, trains leave the end-stations A and G of the AG Train route and go all the way to the other end-stations G and A. All trains stop for 1 minute at stations B, C, D, E, and F. Distances between stations, in km, are shown on the diagram. Note that the distance from A to G is 38 km. The average train speed between consecutive stations is 1 km per minute.

Luciano got on a train that left Station A at exactly 9:00 AM, and got off a train at Station A before 11:00 AM the same day. On his trip he visited Station G. At any of the stations, Luciano had the choice to stay on the train, or get off the train and travel on the next train going in the opposite direction. Luciano maximized the distance he travelled. How many km did he travel in total?

10. _____ km
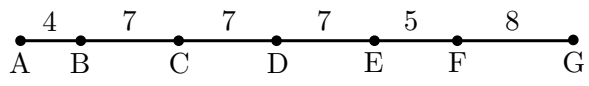
Provincial 2011 Face-off
Questions and Answers

1. What is the value of $\dfrac{(4!)(5!)}{6!}$?

1. Answer: 4

2. Evaluate $(2011)^2$.

2. Answer: 4,044,121 (4 million 44 thousand 121)

3. Three congruent squares are placed side by side to make a rectangle. If the perimeter of the rectangle is 40 cm, what is the number of cm^2 in the area of the rectangle?

3. Answer: 75 (cm^2)

4. A 10.5 fluid ounce can of tomato paste costs \$1.68. What is the cost, in cents, per fluid ounce?

4. Answer: 16 (cents)

5. What is the smallest integer which is a power of 4 and is larger than 2011?

5. Answer: 4096

6. Alice and Bob independently choose an integer from 1 to 10 at random. What is the probability that they choose *different* integers? Express the answer as a common fraction.

6. Answer: $\dfrac{9}{10}$

7. What is the smallest whole number that has 6 whole number divisors?
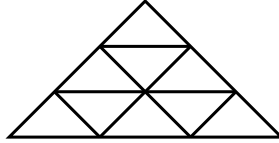
7. Answer: 12

8. Evaluate $5^5 - 55^2$.

8. Answer: 100

9. Two fair dice are tossed. What is the probability that the sum is *less* than 7? Express the answer as a common fraction.

9. Answer: $\dfrac{5}{12}$

10. How many triangles (including all sizes) are in the picture below?

10. Answer: 13 (triangles)

11. In how many ways can you make change for a quarter using pennies and/or nickels and/or dimes?

11. Answer: 12 (ways)

12. The surface area of a sphere is 36π cm^2. What is the number of cm^3 in the volume of the sphere? Express the answer in terms of π.

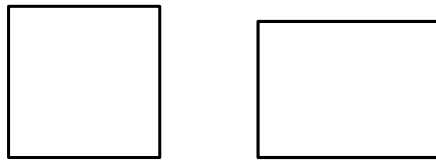**12. Answer:** 36π (cm^3)

13. Given that $(x + y)^2 = 200$, $x^2 = 65$, and $y^2 = 35$, what is the value of xy?

13. Answer: 50

14. Evaluate $(23 \times 8) + (8 \times 14) + (13 \times 8)$.

14. Answer: 400

15. Two opposite sides of a 10×10 square are each increased by 20%, and the other two sides are each decreased by 10%. By how many percent is the perimeter of the resulting rectangle greater than the perimeter of the original square?

15. Answer: 5 (percent)

16. What is the largest prime factor of the sum of the two smallest 3-digit primes?

16. Answer: 17

17. A movie has a running time of 100 minutes. The first showing starts at 7:00 PM, and there is a 20 minute interval between the end of the first showing and the beginning of the second showing. At what time does the second showing end? Give the answer in the usual Hours:Minutes format.

17. Answer: 10:40 (PM)

18. Three standard dice are tossed. How many possible sums are there?

18. Answer: 16 (sums)

19. How many integers x satisfy the equation

$$x^2(x^2 - 1)(x^2 - 2)(x^2 - 3)(x^2 - 4) = 0?$$

19. Answer: 5 (integers)

20. The width of the Atlantic Ocean is increasing at 2.5 centimetres per year. At this rate, how many years will it take for the width of the Atlantic Ocean to increase by 1 kilometre?

20. Answer: 40,000 (years)

21. Two fair dice are tossed. What is the probability that one of the dice shows a 5 and the other shows a number which is less than 5? Express the answer as a common fraction.

21. Answer: $\dfrac{2}{9}$

22. What is the sum of all the prime factors of 222?

22. Answer: 42

23. In the grid below, each of the 6 points is at distance 1 from its nearest horizontal and vertical neighbours. How many lines are there that pass through 2 or more points in the grid?

$$\cdot \qquad \cdot \qquad \cdot$$

$$\cdot \qquad \cdot \qquad \cdot$$
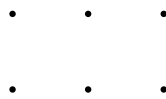
23. Answer: 11 (lines)

24. You buy an MP3 player whose price is $80, plus 12% HST. If you hand the cashier five $20 bills, how much should you get back in change? Give the answer in dollars, to the nearest cent. Thus an answer like 12.49 or 12.50 is of the right shape.

24. Answer: 10.40 (or 10 dollars and 40 cents, or ten forty)

25. Alicia has $1 coins, $2 coins, or a mixture, but no other coins. She has a total of 33 coins, worth a total of $52. How many $1 coins does she have?
25. Answer: 14 (1 dollar coins)

26. A string which is 80 cm long is cut into four pieces whose lengths are in the ratio 1:3:5:7. What is the number of cm in the length of the longest piece?
26. Answer: 35 (cm)

27. One (US) pound is 16 ounces. Given that chicken A weighs 3 pounds, 15 ounces, while chicken B weighs 5 pounds, 1 ounce, what is the ratio of the weight of A to the weight of B? Express the answer as a common fraction.
27. Answer: $\dfrac{7}{9}$

28. Of the people at the junior basketball game, 40% paid the full $10 ticket price, 40% got in at half-price, and the remaining 100 people paid nothing. What was the total income, in dollars, from ticket sales?
28. Answer: 3000 (dollars)

29. How many subsets of the set $\{1, 2, 3, 4\}$ contain at least one even number? (The whole set is a subset of itself.)
29. Answer: 12 (subsets)

30. The Executive Committee consists of 2 men and 3 women. In how many ways can these 5 people be seated in a row so that the 2 men are next to each other?

$$\bigcirc \quad \bigcirc \quad \bigcirc \quad \bigcirc \quad \bigcirc$$

30. Answer: 48 (ways)

31. Ball-point pens can be bought individually at 40 cents each, or at $4 per package of 12 pens. You need to buy 36 ball-point pens. How much do you save by buying the pens in packages of 12 instead of individually? Give the answer in dollars, to the nearest cent.
31. Answer: 2.40 (or "two-forty" or "two dollars and forty cents")

4

Math Challengers Regional 2011
Answers, Blitz Stage

1. 225

2. 5

3. $\dfrac{1}{64}$

4. 1.60

5. 45

6. 68

7. 30

8. 6999

9. 95

10. $\dfrac{3}{8}$

11. $\dfrac{3}{5}$

12. $\dfrac{25}{6}$

13. 8

14. 134

15. $\dfrac{1}{42}$

16. 420

17. $\dfrac{3}{8}$

18. 24

19. 104

20. 4

21. $\dfrac{75}{16}$

22. $\dfrac{2}{9}$

23. 420

24. 52

25. 307

26. $\dfrac{13}{36}$

Math Challengers Regional 2011
Answers, Bull's-eye Stage

1. 40

2. $\dfrac{175}{16}$

3. 1.2

4. 72

5. 4

6. $\dfrac{1}{2}$

7. 37

8. $\dfrac{3}{20}$

9. 2

10. 23

11. 4.8

12. 220

Math Challengers Regional 2011
Answers, Co-op Stage

1. 100

2. $\dfrac{5}{8}$

3. $\dfrac{4}{17}$

4. 13

5. 446000

6. $\dfrac{17}{36}$

7. 261

8. 75.4

9. 121

10. 2.449×10^{-3}

Math Challengers Provincial 2011
Answers, Blitz Stage

1. 30

2. 10

3. 31

4. $\dfrac{7}{12}$

5. 14

6. $\dfrac{1}{5}$

7. 40

8. $\dfrac{5}{12}$

9. 9

10. 18

11. 132

12. 34

13. 5

14. 19

15. 33

16. 19

17. 35

18. $\dfrac{19}{27}$

19. 168

20. 12

21. $\dfrac{5}{9}$

22. 96

23. 3.94

24. 24

25. $\dfrac{625}{208}$

26. 16

Math Challengers Provincial 2011
Answers, Bull's-eye Stage

1. 35

2. 11:30

3. 0

4. 84

5. 7

6. 9

7. 9

8. 52

9. 48

10. 21

11. $\dfrac{5}{8}$

12. $\dfrac{54}{125}$

Math Challengers Provincial 2011
Answers, Co-op Stage

1. 10

2. 22.9

3. 20

4. 25

5. 2.42

6. $\dfrac{32}{69}$

7. 42

8. 29

9. $\dfrac{335}{12}$

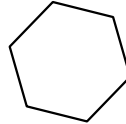10. 102

2012 Canadian Math Challengers Regional Questions

Blitz, Page 1

1. What is the value of 15% of 40?

1. _____

2. How many diagonals does a regular hexagon have? (A diagonal is any line segment that joins two vertices and is *not* an edge.)

2. _____ diagonals

3. For a school fundraiser, Dave sells chocolate bars. He wants to raise at least $100. He sells each chocolate bar for $1.25. What is the minimum number of bars he needs to sell?

3. _____ bars

4. Let $F(x) = x^2 + 1$. What is the value of $F(F(1))$?

4. _____

5. Round the following sum to the nearest whole number:

$$\frac{2}{1} + \frac{3}{2} + \frac{4}{3} + \frac{5}{4} + \frac{6}{5}.$$

5. _____

6. Four fair coins are tossed. What is the probability of getting an even number of heads? Express the answer as a common fraction.

6. _____

7. If you run at a speed of 12 km/hour for 20 minutes, and then at a speed of 10 km/hour for 45 minutes, what is the total distance, in km, that you ran? Express the answer as a decimal, correct to 1 decimal place.

7. _____ km

8. In a certain parallelogram, the degree measure of one of the internal angles is five-sevenths of the degree measure of another internal angle. How many degrees are in the measure of one of the smaller internal angles of the parallelogram?

8. _____ degrees

9. Simplify: $\dfrac{\sqrt{9 + \dfrac{1}{7}}}{\sqrt{\dfrac{1}{7}}}$

9. _____

10. What is the largest prime factor of 2012?

10. _____

11. Four hundred and forty students took part in a Math Challengers contest. Each student competed in the grade 8 category or the grade 9 category, but not in both. If 198 students were in the grade 9 category, how many percent of the students competed in the grade 8 category?

11. _____ percent

12. Each of the two figures below is made up using twenty-five 1×1 squares. What is the positive difference between the perimeters of the two figures?

12. _____ units
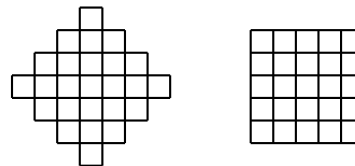
13. Express
$$\frac{1 + 3 + 3^2}{1 + 3 + 3^2 + 3^3 + 3^4 + 3^5}$$
as a common fraction.

13. _____

14. Alicia has 5 dollars less than Beti, and Cecille has as much money as Alicia and Beti have between them. Altogether, the three people have a total of 270 dollars. How many dollars does Alicia have?

14. _____ dollars

15. What is the value of the mean of the numbers in the following list? Express the answer as a common fraction.

15. _____

$$1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, 6, 6, 6, 6, 6, 6$$

16. Alan left home to go to the mall. He spent \$2.50 for the Skytrain trip. Then he spent six-sevenths of the money he had left on t-shirts. Then he spent two-thirds of the money he had left after buying the t-shirts on a snack. After that, all he had left was \$2.50 for the Skytrain trip home. How many dollars did Alan have immediately before he left home to go to the mall?

16. _____ dollars

17. There are 12 jelly beans on a tray, 6 blue and 6 yellow. Alphonse eats 2 of the jelly beans, chosen at random. What is the probability that these 2 jelly beans are of *different* colours? Express the answer as a common fraction.

17. _____

18. In the picture below, the smaller square has side 7 and the larger square has side 15. What is the area of $\triangle ABC$?

18. _____ units^2
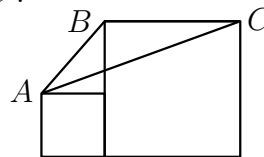
19. Call the integer n *special* if the sum of the decimal digits of n is 12. How many of the *perfect squares* from 1 to 400 are special?

19. _____ squares

20. Express $\dfrac{\sqrt{75} - \sqrt{3}}{\sqrt{75} + \sqrt{3}}$ as a common fraction.

20. _____

21. A convex hexagon has one internal right angle. The other five internal angles are all equal to each other. How many degrees are in the measure of one of these five angles?

21. _____ degrees
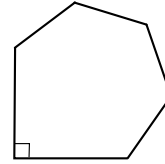
22. How many integers between 1 and 1000 are divisible by 3 but not by 9?

22. _____ integers

23. If $x^{2012} = 9$, what is the value of x^{3018}?

23. _____

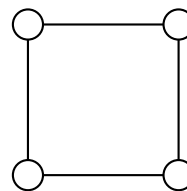24. What is the smallest positive integer n such that

$$1 + 2 + 3 + \cdots + (n-1) + n$$

is a multiple of 100?

24. _____

25. How many ways are there to colour the four circles below, using no colours other than blue, red, or yellow (only one colour to each circle), so that any two circles directly joined by a line are of different colours?

25. _____ ways

26. A half-circle has centre at the origin $O(0,0)$. Points $M(\sqrt{75}, 5)$ and $N(-5, \sqrt{75})$ are on the half-circle as shown. What is the area of the shaded region, correct to the nearest integer?

26. _____ units^2
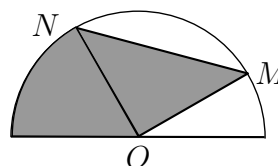
Bull's-eye, Page 1: Problem Solving

1. An exam consists of multiple choice questions, worth 2 marks each, and "short answer" questions, worth 4 marks each. The total number of questions on the exam is 39, and the maximum mark obtainable is 100. How many multiple choice questions are on the exam?

   1. _____ questions

2. A large group had a banquet. The food bill was $1000. In addition to that, the group had to pay the 12% HST. The group decided to leave a tip of 15% of the combined cost of food and HST. What was the amount, in dollars, of the tip?

   2. _____ dollars

3. Alicia has exactly $9.99 in standard Canadian coins. (The standard Canadian coins are 1 cent, 5 cents, 10 cents, 25 cents, $1, and $2). What is the smallest number of coins that Alicia could have?

   3. _____ coins

4. The lines below represent the streets of a town. Each side of every small triangle has length 1, and the additional segment has length 1. You want to walk from point A to point B, and travel at least once along each street of the town. What is the shortest possible length of the path?
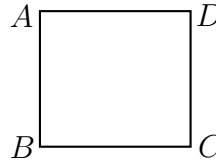
   4. _____ units
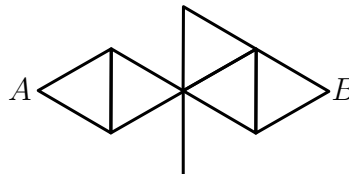
5. What is the smallest positive integer which can be expressed as the sum of 6 consecutive multiples of 6, some of which could be 0 or negative?

5. _____

6. A box contains 9 marbles, of which 4 are white, 3 are black, and 2 are blue. Dina removes 2 randomly chosen marbles from the box, and places them on a table. What is the probability that the 2 marbles are of the same colour? Express the answer as a common fraction.

6. _____

7. Call the integer n *good* if $n > 0$ and 2 divides n, 3 divides $n+1$, 4 divides $n+2$, and 5 divides $n+3$. The smallest good integer is 2. What is the next good integer?

7. _____

8. Let M be a three-digit number, with all digits different from 0 (so 234 and 665 are of the right kind, but 302 is not). Let N be a three-digit number obtained by changing the order of the digits of M. Given that $M + N = 949$, what is the largest possible value of N?

8. _____

9. The area of rectangle $ABCD$ is $\dfrac{5}{8}$ units^2. Side AB has length $\dfrac{3}{4}$ units. What is the length of side BC? Express the answer as a common fraction.
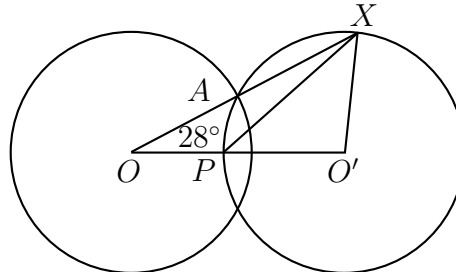
9. _____ units

10. Three gold spheres have, respectively, diameter 9 cm, 12 cm, and 15 cm. They are melted down and made into a single gold sphere. What is the diameter of that sphere?

10. _____ cm

11. The two circles below have the same radius, and have centres O and O'. The line segment joining O and O' meets the circle with centre O' at the point P. Point A is one of the intersection points of the circles, and the line OA meets the circle with centre O' at a second point X. Given that $\angle AOP$ is 28 degrees, how many degrees are in $\angle XPO'$?

11. _____ degrees

12. Triangle ABC is isosceles, with $CA = CB$. Point P is on AB. Given that $AP = 3$, $PB = 7$, and $CP = 5$, what is the area of $\triangle ABC$? Express the answer in simplest radical form.

12. _____ units^2
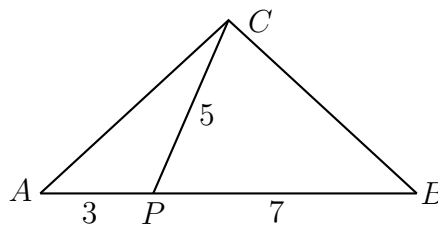
1. What is the value of

$$(1 \times 1!) + (2 \times 2!) + (3 \times 3!) + (4 \times 4!) + (5 \times 5!)?$$

1. _____

2. A large group had a banquet. The total bill was for $1000.15. This amount was for the food *plus* the HST: the 12% HST was included in the $1000.15 bill. The group decided to leave in addition a tip of 15% on the cost of the food only, not the HST. What was the total amount, in dollars, paid by the group, including the tip? Express the answer as a decimal, to the nearest cent..

2. _____ dollars

3. The figure $ABCD$ is a rectangle. The length of BM is $\frac{2}{3}$ times the length of side BC, and the length of CN is $\frac{4}{5}$ times the length of side CD. What is the ratio of the area of the shaded triangle to the area of rectangle $ABCD$? Express the answer as a common fraction.

3. _____
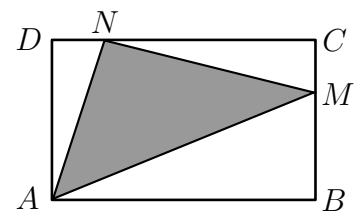
4. The function $f(x)$ satisfies the equation

$$f(a + b) = \frac{f(a) + f(b)}{1 - f(a)f(b)}$$

If $f(a) = \dfrac{1}{2}$ and $f(b) = \dfrac{1}{3}$, what is the value of $f(a + b)$?

4. _____

5. The number 999,999,999,999,999 is multiplied by 999. How many 9's are there in the decimal representation of the product?

5. _____ 9's

6. What is the smallest integer n such that $n(n+2012)(n-2013) > 0$?

6. _____

7. The energy released by an earthquake is measured using the Richter Scale. An earthquake of magnitude 9 releases 10 times as much energy as an earthquake of magnitude 8. In the same way, a quake of magnitude 8.2 releases 10 times as much energy as a quake of magnitude 7.2. What is the ratio of the energy released by a magnitude 8.5 earthquake to the energy released by a magnitude 8.0 earthquake? Express the answer as decimal, correct to 2 decimal places.

7. _____

8. Ten people, Alan and Beti and 8 others, are divided at random into two groups, one with 4 people and the other with 6 people. What is the probability that Alan and Beti end up in the same group? Express the answer as a common fraction.

8. _____

9. The five circles in the picture have the same centre, and their radii are 2.5, 3.5, 4.5, 5.5, and 6.5. How many percent of the area of the largest circle is shaded? Give the answer rounded to the nearest 1 percent.

9. _____ percent
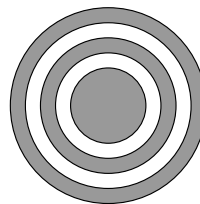
10. A poll of hockey fans reveals that if the price per ticket is $100, then 1 million people will attend over the entire season. If the ticket price is increased by a certain percentage, the attendance changes as in the graph. If management charges the price that will maximize its sales revenue, but definitely at least $100 and no more than $130 per ticket, what will be the total sales, in dollars?

10. _____ dollars
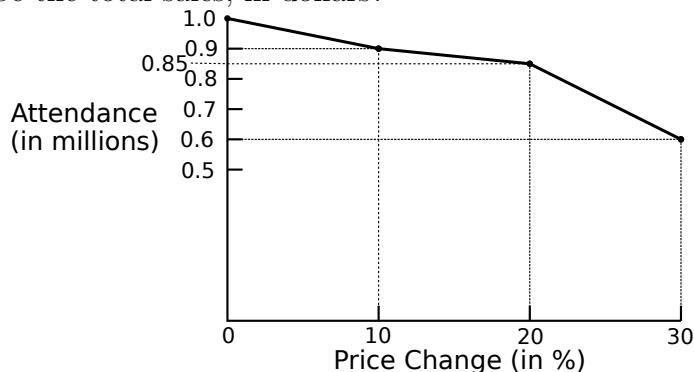
Co-op, Page 3: Team answers must be on the *coloured* page.
Answers on a white page will not be graded.

11. Suppose that a, b, c, and d are non-negative integers such that

    $$a + 8b + 64c + 512d = 2012.$$

   What is the smallest possible value of $a + b + c + d$?

    11. _____

12. The game of *Canadian checkers* is played on a 12×12 square board divided into 144 unit squares. A circle of radius 6 is drawn on a Canadian checkerboard, with centre the centre of the board. How many of the 144 unit squares have *all* their vertices on or inside the circle? (The unit squares are not shown in the picture.)

    12. _____ squares
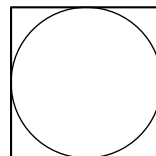
13. What is the sum of the positive divisors of the sum of the positive divisors of 200? Note that 1 and n are divisors of n.

    13. _____

14. Note that $1 + 2 + 3 + \cdots + 7 + 8 = 36$, and 36 is a perfect square. What is the smallest perfect square greater than 36 which is the sum of the first n positive integers for some n?

    14. _____

15. Triangle ABC has $AB = 10$ and $AC = 14$. The three heights AR, BQ, and CP are drawn and meet at O. The distance AP is equal to 6. Let $OQ = x$, and draw the circle with centre O and radius x. What is the area of the circle? Express the answer as a decimal, correct to 1 decimal place.

    15. _____ units^2
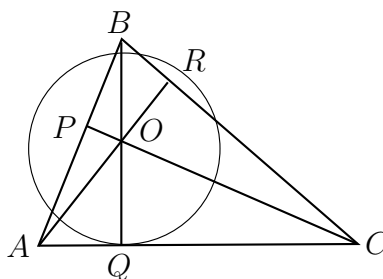
Regional 2012 Face-off
Questions and Answers

1. Evaluate $44 + 144 + 244$.
1. Answer: 432

2. What is the sum of all the primes that are smaller than 15?
2. Answer: 41

3. A rectangular box has volume 196000 cm^2 and a square base. If the height of the box is 40 cm, what is the number of cm in one side of the base?
3. Answer: 70 (cm)

4. What is the value of the sum

$$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15?$$

(Each term is 2 more than the previous one.)
4. Answer: 64

5. What is value of the smallest integer which is the sum of four distinct 2-digit primes?
5. Answer: 60

6. If $5x + y = 81$ and $5x - y = 69$, what is the value of x?
6. Answer: 15

7. The hypotenuse of a right-angled triangle has length 17, and one of the legs has length 8. What is the perimeter of the triangle?
7. Answer: 40

8. What is 250% of 2012?
8. Answer: 5030

9. An accurate 12-hour clock shows that it is exactly $12:00$. What time shows on the clock 2012 minutes later? An answer like $4:17$ or "four seventeen" is of the right shape.
9. Answer: $9:32$

10. What is the value of $\dfrac{4^8}{8^4}$?
10. Answer: 16

11. The triangle below is isosceles. Each of the two smaller angles is one-eighth of the largest angle. How many degrees are in the measure of the largest angle?

11. Answer: 144 (degrees)

12. A high school runner ran 3000 metres in 9 minutes. What was the runner's average speed in kilometres per hour?
12. Answer: 20 (km/hr)

13. What is the value of $(31 \times 41) - (31 + 41)$?
13. Answer: 1199

14. What is the area of the triangle whose vertices have coordinates $(-2, 0)$, $(2, 0)$, and $(7, 11)$?
14. Answer: 22

15. Evaluate $(23 \times 8) + (8 \times 14) + (13 \times 8)$?
15. Answer: 400

16. Ten years from now, the sum of the ages of the 30 people in the class will be 725 years. What will the sum of their ages be 5 years from now?
16. Answer: 575 (years)

17. Express $\dfrac{1 + 2 + 3 + 4 + 5}{1 \times 2 \times 3 \times 4 \times 5}$ as a common fraction. .
17. Answer: $\dfrac{1}{8}$

18. The product of two numbers is 210, and the sum of the two numbers is 29. What is the positive difference between the two numbers? .
18. Answer: 1

19. Evaluate $5^5 - 55^2$.
19. Answer: 100

20. What is the smallest positive integer which is divisible by all of 20, 24, 25, and 30?
20. Answer: 600

21. Express $\dfrac{4}{3} - \left(1 + \dfrac{1}{4} + \dfrac{1}{16}\right)$ as a common fraction.

21. Answer: $\dfrac{1}{48}$

22. Alicia drives 1 kilometre in 32 seconds. At this rate, how many km does she drive in 10 seconds? Express the answer as a decimal, *rounded* to the nearest hundredth of a km.

22. Answer: 0.31 (km)

23. For what value of x is $1^3 - 2^3 + 3^3 - x = 0$?

23. Answer: 20

24. Alicia sold her condo for 20% more than she paid for it, and made a gross profit of 90,000 dollars. For how many dollars did she sell her condo?

24. Answer: 540,000 (dollars)

25. What is the value of 95×95??

25. Answer: 9025 (years)

26. The sides of a rectangle are integers, and the perimeter of the rectangle is 42. What is the largest possible area of the rectangle?

26. Answer: 110 (years)

27. Three fair dice are tossed. What is the probability that the sum of the numbers obtained is equal to 4? Express the answer as a common fraction.

27. Answer: $\dfrac{1}{72}$

28. What is the product of the greatest common factor and the least common multiple of 15 and 48?

28. Answer: 720

29. How many integers n are there such that $\dfrac{11}{n} > \dfrac{9}{7}$.

29. Answer: 8 (values)

30. What is the smallest number of times that you must throw a fair coin to have probability of 80% or more of getting at least one head?

30. Answer: 3 (times)

31. What is the value of $\dfrac{(6!)(7!)}{8!}$?

31. Answer: 90

32. How many two-digit prime numbers are there that use two distinct digits chosen from $\{1, 2, 3, 4, 5, 6\}$?

32. Answer: 7 (primes)

33. What is the mean of the first 10 terms of the arithmetic sequence 1, 11, 21, 31, . . . ?

33. Answer: 46

**2012 Canadian Math Challengers Provincial Questions**

1. Express the reciprocal of 0.55 as a common fraction.

    1. _____

2. What is the smallest integer larger than $\sqrt{2012}$?

    2. _____

3. Each edge of a regular hexagon has length $\dfrac{4}{\sqrt{\pi}}$. The hexagon is inscribed in a circle. What is the area of the circle, in square units?

    3. _____ units^2
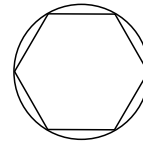
4. Alicia bought 45 litres of gasoline for \$54. If the price of gasoline goes up by 25%, how many litres of gasoline can Alicia buy for \$54?

    4. _____ litres

5. Simplify: $\left(1 + \dfrac{1}{4}\right)\left(1 + \dfrac{1}{5}\right)\left(1 + \dfrac{1}{6}\right)\left(1 + \dfrac{1}{7}\right)$

    5. _____

6. The area of equilateral triangle ABC is nine times the area of equilateral triangle APQ. What is the ratio of the perimeter of the trapezoid $PBCQ$ to the perimeter of $\triangle ABC$? Express the answer as a common fraction.

    6. _____
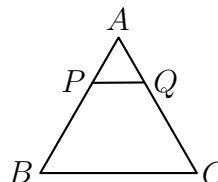
7. Let $x \circledast y = x^2 - 2y^2$. What is the value of $3 \circledast (2 \circledast 1)$?

    7. _____

8. Suppose that a and b are integers and $2^a - 2^b = 16$. What is the value of $a + b$?

8. _____

9. A prism has 12 edges. How many faces does it have? Recall that a prism is a polyhedron for which there is a face of the polyhedron such that when the polyhedron is placed on the floor with that face down, then all horizontal cross-sections are the same.

9. _____ faces

10. Ali has 50% more money than Beth, who has 50% more money than Cecil. All together, they have \$950. How many dollars does Ali have?

10. _____ dollars

11. Simplify: $\sqrt{\sqrt{8}\sqrt{16}\sqrt{32}}$

11. _____

12. At the university, 30% of the students have a car, and 80% of the students who don't have a car have a bike. How many percent of the students have neither a car nor a bike?

12. _____ percent

13. What is the area of the triangle whose sides are 17, 17, and 16?

13. _____ units^2
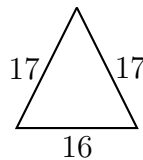
14. What is the sum of the first 2012 terms of the following arithmetic sequence?
$$-1005, -1004, -1003, -1002, \ldots$$

14. _____

15. Evaluate: $\dfrac{10!\,7!\,4!}{9!\,6!\,3!}$

15. _____

16. The median of a list of 11 positive integers (not necessarily distinct) is 20 and their mean is 25. What is the largest possible integer in the list?

16. _____

17. Rectangle $ABCD$ has base 20. A semicircle is drawn that has the base AB as a diameter. This semicircle meets side CD in the points P and Q, where $DP = CQ = 7$ and $PQ = 6$. What is the height of the rectangle (that is, what is the length of line segment BC)? Express the answer in simplest radical form.

17. _____ units
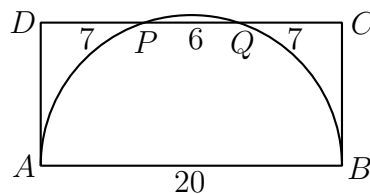
18. Six 5 dollar bills are placed in a row. Then every second bill is replaced by a 10 dollar bill. Then every third bill is replaced by a 20 dollar bill. After all the replacements are done, how many dollars in total are there in the row?

18. _____ dollars

19. A combined total of 2012 students participated in the last 8 Provincial Math Challengers competitions. The yearly participation numbers form an arithmetic sequence with a yearly increment of 3. What was the largest number of yearly participants during this period?

19. _____ students

20. In how many ways can 5 identical loonies be split between Aleph, Beth, and Gimel so that each of them gets at least 1 loonie? Only the totals that each person gets matter. For example, "Aleph is given 1 loonie, then Beth is given 1, then Alan is given 1, then Beth is given 1, then Gimel is given 1" is the same as "Beth is given 2, then Gimel is given 1, then Aleph is given 2."

20. _____ ways

Blitz, Page 4

21. Let $x = 2^{2012} + 3^{2012}$. What is the units digit of x?

21. _____

22. The integers i, j and k are even, and the integers l, m, and n are odd. Suppose that $0 < i < j < k < l < m < n$ and $\frac{i}{j} < \frac{k}{l} < \frac{m}{n}$. What is the smallest possible value of n?

22. _____

23. What is the smallest positive integer N such that N times 5! is a perfect cube?

23. _____

24. In the circle below, chord AB has length 22, and chord CD has length 16. Chord CD is twice as far from the centre of the circle as chord AB. What is the *square* of the radius of the circle?

24. _____ units^2
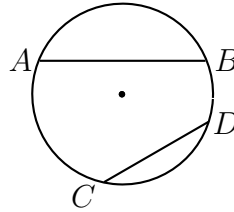
25. You toss 2 dice and record the sum. Then you do it again. What is the probability that the recorded sums are the same? Express the answer as a common fraction.

25. _____

26. A triangle has sides 3, 5, and 7. What is the *square* of its smallest height? Express the answer as a common fraction.

26. _____ units^2

Bull's-eye, Page 1: Problem Solving

1. Alfie gave B one-half of the loonies Alfie had, and then 7 more. Alfie then gave C one-half of the loonies he had left, and then 7 more. After that, Alfie had no loonies left. How many loonies did Alfie start out with?

1. _____ loonies

2. At the "Home Sweet Home" senior facility, the average age of the male residents is 70 years, the average age of the female residents is 75 years, and the average age of all residents is 73.5 years. What is the ratio of male residents to female residents of Home Sweet Home? Express the answer as a common fraction.

2. _____

3. Dean and Dina each run exactly 600 m. They start at the same time and finish at the same time. Dina runs at a constant speed of 3 m/s, while Dan increases his speed at a constant rate for the first 300 m, and then decreases his speed by the same rate during the last 300m. What is the fastest speed (in m/s) that Dan reaches during the race?

3. _____ m/s

4. You can use three different taps, alone or in combination, to fill a pool. If you use taps B and C only, it will take 9 hours to fill the pool. If you use all three taps (A, B, and C), it takes 7 hours. Tap B can fill the pool on its own in half the time it takes tap A on its own. How many hours would it take for tap C to fill the pool on its own?

4. _____ hours

5. What is the number which is halfway between $\frac{3}{4}$ and $\frac{4}{3}$? Express the answer as a common fraction.

5. _____

6. What common fraction between 0.91 and 0.97 has the least numerator?

6. _____

7. You start at corner A of equilateral triangle ABC with side 1 metre by taking a step to either B or C with probability $\frac{1}{2}$ each. You keep making such 1 metre steps, with probability $\frac{1}{2}$ to the corners you are not at. What is the probability of ending up back at A after taking exactly 4 steps? Express the answer as a common fraction.

7. _____

8. Betty and Ben each select independently and at random an integer between 0 and 5 (inclusive). What is the average non-negative difference between their numbers? Express the answer as a common fraction.

8. _____

9. The picture below shows a square and an equilateral triangle. If the degree measure of the angle labelled x is $34°$, what is the degree measure of the angle labelled y?
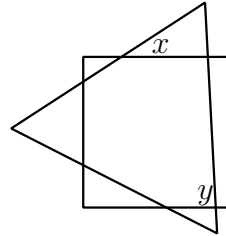
9. _____ degrees

10. A square is split into two rectangles as in the picture below. The smaller rectangle has area 8, and the larger one has area 10. What is the ratio of the perimeter of the smaller rectangle to the perimeter of the larger rectangle? Express the answer as a common fraction.
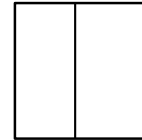
10. _____

11. In the picture below, the circle with centre O has radius 1. Point A lies on the circle, $\triangle OAB$ is right-angled at A, and $AB = 3$. The line segment OB meets the circle at C, and D on AB is such that CD is perpendicular to AB. Express the length of CD in the form $\frac{a+b\sqrt{c}}{d}$, where a, b, c, and d are integers, d is positive, no number greater than 1 divides all of a, b, and d, and no square greater than 1 divides c.
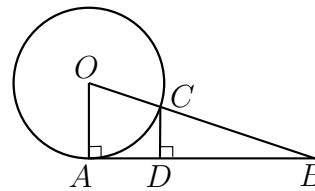
11. _____ units

12. In the picture below, lines that look perpendicular *are* perpendicular. The large trapezoid of the picture is divided into a trapezoid, two rectangles, and a triangle as shown. The trapezoid has area 2, and the rectangles have area 3 and 4 as shown. What is the value of x, the area of the small triangle? Express the answer as a common fraction.
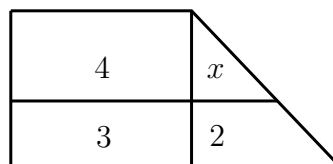
12. _____ units^2

1. It so happens that $\sqrt{1800}+\sqrt{200} = \sqrt{n}$, where n is an integer. What is the value of n?

1. _____

2. The price of a commodity is adjusted upwards by 2.5% on January 15 of every year. What is the ratio of the price on January 16 of a certain year to the price on January 16 twenty years earlier? Provide the answer as a decimal correct to 2 decimal places.

2. _____

3. What is the area of the triangle whose vertices have coordinates $(0,0)$, $(5,7)$, and $(7,10)$? Express the answer as a common fraction.

3. _____ units^2

4. Dan had to pay \$2500 for an overseas school trip, and was charged simple yearly interest of 5% for late payment. If he was 15 days late, how much interest did he pay, in dollars, correct to 2 decimal places. Assume that there are 360 days in the year.

4. _____ dollars

5. Define the number N by

$$N = 123456789 + 234567891 + 345678912 + 456789123 + 567891234.$$

What is the sum of the digits of N?

5. _____

6. How many integers a are there such that $1 \leq a \leq \sqrt{6400}$ and a divides 6400?

6. _____ integers

7. The world is divided into "rich," "emerging," and "poorest" countries. The people of the rich countries are asked to come to the rescue. The people of the poorest countries, who make up 53% of the world population, need $5000 per capita. The people of the emerging countries, who make up 36% of the world population, need $2000 per capita. If all the money is to come out of the pockets of each individual from the rich countries, how much will it cost each of them if the total population of the rich countries is 770 million? Give the answer rounded to the nearest dollar.

7. _____ dollars

8. It so happens that there are positive integers a, b, and c such that

$$\frac{355}{113} = a + \cfrac{1}{b + \cfrac{1}{c}}$$

What is the value of c?

8. _____

9. How many products of the form $a \times b \times c$ are there, if a, b, and c can be any of the primes 2, 3, 5, or 7? Note that $28 = 2 \times 2 \times 7$ is such a product (primes can repeat), and is to be counted as the same as $2 \times 7 \times 2$.

9. _____

10. What is the greatest integer n for which $\dfrac{24n}{n-4}$ is an integer?

10. _____

11. What is the area, in square metres, of the smallest square that can be fully covered with no gaps or overlaps by using 50 cm by 50 cm tiles only, and also by using 40 cm by 60 cm tiles only.

11. _____ metres^2

12. The mean (average) of a and b is 3/4 times the mean of a, b, and c. The mean of b and c is 4/3 times the mean of a, b, and c. If a, b, and c are positive and the mean of a and c is k times the mean of a, b, and c, what is the value of k? Express the answer as a common fraction.

12. _____

13. How many ordered triples (i, j, k) of non-negative integers are there such that $i + j + k = 4$? Please note that $(4, 0, 0)$ is not the same as $(0, 4, 0)$.

13. _____ triples

14. In the game of *Lucky 7*, you roll a fair die a few times and try to reach a total sum of 7 on your rolls. There is only one rule: If on roll n you got a certain number k and on roll $n + 1$ you get a number equal to or larger than k, then the game is over after roll $n + 1$. A few valid sequences in the game are $(1, 1)$, $(1, 3)$, $(5, 4, 3, 3)$, $(6, 5, 5)$, $(6, 5, 4, 3, 2, 1, 5)$. Please note that the maximum number of rolls until the game is over is 7. What is the probability of reaching a total of 7 when the game is over? Two examples of winning sequences are $(1, 6)$ and $(2, 1, 4)$. Note that $(6, 1)$ is not a winning sequence since you still have to roll for a third time. Express the answer as a common fraction.

14. _____

15. The 5 students on the team that won the Provincial Math Challengers competition decided to celebrate the event with a gift exchange party. The rule is that each of the 5 students is to give one gift to *exactly* one *other* student. An example of such a gift exchange is "A gives to B, B gives to C, C gives to D, D gives to E, and E gives to A." How many ways are there to do the gift exchange?

15. _____ ways

PROVINCIAL 2012 FACE-OFF
QUESTIONS AND ANSWERS

1. Eighty-eight is 16% of what number?
1. Answer: 550

2. If $x^2 = \dfrac{9}{256}$, and x is positive, what is the value of x? Express the answer as a common fraction.
2. Answer: $\dfrac{3}{16}$

3. How many *real numbers* (they are not necessarily integers) x are there such that $x > 1$ and $\dfrac{8}{x}$ is an integer?
3. Answer: 7

4. What is the smallest positive integer n such that $8^n > 2^{22}$?
4. Answer: 8

5. What is the value of
$$1 + 2 + 3 + \cdots + 13 + 14 + 15 + 14 + 13 + \cdots + 3 + 2 + 1?$$
5. Answer: 225

6. What is the value of $\dfrac{\frac{1}{2} - \frac{1}{5}}{\frac{1}{10}}$?
6. Answer: 3

7. How many perfect squares are there between 11 and 111?
7. Answer: 7

8. A very long test has 99 questions, numbered 1 to 99. The test is 9 pages long, and each page has the same number of questions. What is the number of the fifth question on the fifth page?
8. Answer: 49

9. Of the 20 marbles in a bag, 10 are red and 10 are green. Two marbles are removed from the bag. What is the probability that these two marbles are of different colours? Express the answer as a common fraction.
9. Answer: $\dfrac{10}{19}$

10. The average class size in the year 2000 was 27. Now the average class size is 31. By how many percent has the average class size increased from the year 2000 to now? Round the answer to the nearest integer. Thus an answer like 17 is of the right shape.
10. Answer: 15 (percent)

1

11. Square $\mathcal{A}$ has area 4 and square $\mathcal{B}$ has area 9. How many percent is the *side* of square $\mathcal{B}$ greater than the side of square $\mathcal{A}$?
11. Answer: 50

12. A fair coin is tossed 4 times in a row. What is the probability of getting a total of 3 heads (in any order) and 1 tail? Express the answer as a common fraction.
12. Answer: $\dfrac{1}{4}$

13. How many positive integers n are there such that $\dfrac{1024}{n}$ is a perfect square?
13. Answer: 6

14. If $9^2 \times 27^3 = 3^n$, what is the value of n?
14. Answer: 13

15. The operation $\circledast$ is defined by the rule

$$x \circledast y = x^2 - 2xy + y^2.$$

What is the value of $6 \circledast (-4)$?
15. Answer: 100

16. The three cans of cola that Alphonse drinks every day together supply 20% of Alphonse's daily caloric requirements, which are 2100 calories. How many calories are in 1 can of cola?
16. Answer: 140

17. If $x^2 = 49$, what is the sum of the two possible values of $(x+1)^2$?
17. Answer: 100

18. Alicia and Beti drive separately from A to B. The distance from A to B is 200 km. They start at the same time. Alicia averages 100 km/hour, and Beti averages 75 km/hour. How many minutes after Alicia arrives at B does Beti arrive at B?
18. Answer: 40(minutes)

19. How many 3-digit positive integers are there all of whose digits are all distinct and odd? Note that 795 is such a number, but 757 is not.
19. Answer: 60

20. How many integers from 1 to 1000 are perfect squares and have the sum of their decimal digits equal to 9?
20. Answer: 8

21. Let $N = (201)^2$. What is the sum of all the decimal digits of N?

21. Answer: 9

22. How many integers between 1 and 1000 are divisible by both 6 and 16?

22. Answer: 20

23. Simplify $\dfrac{2^{-3} + 3^{-3}}{6^{-3}}$.

23. Answer: 35

24. Suppose that $\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{1}{z}$. If $y = 16$ and $z = 20$, what is the value of x?

24. Answer: -80

Math Challengers Regional 2012
Answers, Blitz Stage

1. 6

2. 9

3. 80

4. 5

5. 7

6. $\dfrac{1}{2}$

7. 11.5

8. 75

9. 8

10. 503

11. 55

12. 8

13. $\dfrac{1}{28}$

14. 65

15. $\dfrac{13}{3}$

16. 55

17. $\dfrac{6}{11}$

18. 60

19. 0

20. $\dfrac{2}{3}$

21. 126

22. 222

23. 27

24. 24

25. 18

26. 102

Math Challengers Regional 2012
Answers, Bull's-eye Stage

1. 28

2. 168

3. 14

4. 17

5. 18

6. $\dfrac{5}{18}$

7. 62

8. 821

9. $\dfrac{5}{6}$

10. 18

11. 42

12. $5\sqrt{21}$

Math Challengers Regional 2012
Answers, Co-op Stage

1. 719

2. 1134.10

3. $\dfrac{13}{30}$

4. 1

5. 14

6. -2011

7. 3.16

8. $\dfrac{7}{15}$

9. 62

10. $102,000,000$

11. 17

12. 88

13. 768

14. 1225

15. 66.7

Math Challengers Provincial 2012
Answers, Blitz Stage

1. $\dfrac{20}{11}$

2. 45

3. 16

4. 36

5. 2

6. $\dfrac{8}{9}$

7. 1

8. 9

9. 6

10. 450

11. 8

12. 14

13. 120

14. 1006

15. 280

16. 170

17. $\sqrt{91}$

18. 70

19. 262

20. 6

21. 7

22. 13

23. 225

24. 140

25. $\dfrac{73}{648}$

26. $\dfrac{675}{196}$

Math Challengers Provincial 2012
Answers, Bull's-eye Stage

1. 42

2. $\dfrac{3}{7}$

3. 6

4. 21

5. $\dfrac{25}{24}$

6. $\dfrac{11}{12}$

7. $\dfrac{3}{8}$

8. $\dfrac{35}{18}$

9. 86

10. $\dfrac{13}{14}$

11. $\dfrac{10 - \sqrt{10}}{10}$

12. $\dfrac{32}{33}$

Math Challengers Provincial 2012
Answers, Co-op Stage

1. 3200

2. 1.64

3. $\dfrac{1}{2}$

4. 5.21

5. 54

6. 14

7. 30636

8. 16

9. 20

10. 100

11. 36

12. $\dfrac{11}{12}$

13. 15

14. $\dfrac{139}{1296}$

15. 44

Blitz, Page 1

2013 Canadian Math Challengers Regional Questions

1. Evaluate the sum $(-5) + (-3) + (-1) + 1 + 3 + 5 + 7 + 9$.

1. _____

2. There are 7 marbles in a box, of which exactly 2 are white. One marble is selected at random. What is the probability it is non-white? Express the answer as a common fraction.

2. _____

3. The ferry left Tsawwassen at 2:33 PM, and arrived at Swartz Bay at 4:14 PM the same day. How many minutes was the trip?

3. _____ minutes

4. One angle of an isosceles triangle is $102°$. What is the measure, in degrees, of another angle of the triangle?

4. _____ degrees

5. 10% of 20 plus 20% of 30 is how many % of 40?

5. _____ %

6. Express $\frac{1}{2} + \frac{2}{3} + \frac{3}{4}$ as a common fraction.

6. _____

7. Round $\dfrac{230}{17}$ to the nearest integer.

7. _____

8. The sum of *all* the edge lengths of a cube is 36. What is the volume of the cube?

8. _____ units^3

9. What is the value of the smallest integer whose square root is greater than 5.5?

9. _____

10. Three fair coins are tossed. What is the probability of getting exactly 2 heads? Express the answer as a common fraction.

10. _____

11. What is the value of the sum

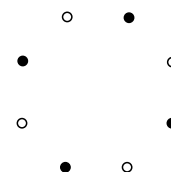$$1000 - 999 + 998 - 997 + \cdots + 4 - 3 + 2 - 1?$$

11. _____

12. Two fair dice are rolled. What is the probability that the sum of the numbers obtained is 6? Express the answer as a common fraction.

12. _____

13. The combined cost of one candy, one chocolate bar, and one cookie is $2.71. The combined cost of one candy, one chocolate bar, and three cookies is $5.25. The combined cost of one candy, two chocolate bars, and three cookies is $6.36. What is the cost of one candy? Give the answer in $, correct to two decimal places.
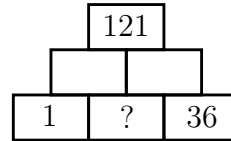
13. _____ $

14. The 8 points below represent the vertices of a regular octagon. These vertices are alternately painted red and blue. Lines are drawn joining every pair of points. How many of these lines contain a red point and a blue point?

14. _____ lines

15. A number is put in each of the small rectangles below so that the number in any small rectangle is equal to the sum of the numbers in the two rectangles that it sits on. What number should be put in the rectangle labelled "?"?

15. _____

```
      ┌─────┐
      │ 121 │
      ├──┬──┤
  ┌───┼──┼───┐
  │ 1 │? │ 36│
  └───┴──┴───┘
```

16. On Monday, 40 of the 50 students in the math class took a test. Their mean score was 60. On Tuesday, the remaining 10 students took the test. Their mean score was 95 (they had seen the questions). What was the mean class score on the test?

16. _____

17. Sprinter Bolt ran 100 metres in 10 seconds. What was his average speed in km/hour?

17. _____ km/hour

18. Last summer, Alfie earned $726 working at $11.00 per hour, and another $726 working at an hourly rate which is 50% higher. What was Alfie's average wage per hour last summer? Give your answer in dollars per hour, to 2 decimal places.
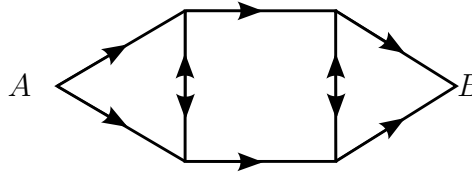
18. _____ $/hr

19. Four straight lines pass through a circular disk. What is the largest possible number of regions that these lines divide the disk into?

19. _____ regions

20. A lidless $4 \times 4 \times 4$ box is completely filled with $1 \times 1 \times 1$ cubes. How many of the $1 \times 1 \times 1$ cubes touch a side or the bottom of the box?

20. _____ cubes

21. When it was set out, the bowl of mixed nuts was (by weight) 45% peanuts, 25% almonds, 20% cashews, and 10% hazelnuts. Alicia picked out all the almonds and ate them. What percent (by weight) of the nuts in the bowl are now peanuts?

21. _____ percent

22. There is an integer N such that $N^3 = 79,507$. What is the value of N?
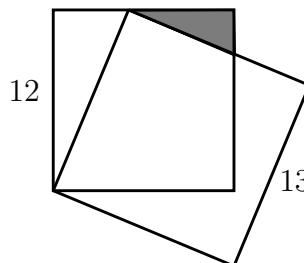
22. _____

23. Each line segment has length 1. You are only allowed to walk in the direction of the arrows, and must always walk full segments. At any intersection, you can walk in any allowed direction, regardless of where you came from. How many paths of total length 9 are there from A to B?

23. _____ paths

24. What is the x-coordinate of the point P on the x-axis such that the distance from P to the origin is the same as the distance from P to the point with coordinates $(1/2, 1/3)$? Express the answer as a common fraction.

24. _____

25. Two fair dice are rolled. What is the average value of the non-negative difference between the numbers showing on the two dice? Express the answer as a common fraction.

25. _____

26. In the diagram below, the two figures that look like squares are squares, with sides 12 and 13 respectively. What is the area of the shaded triangle? Express the answer as a common fraction.

26. _____ units^2

Bull's-eye, Page 1: Problem Solving

1. Assume that the Olympic village cost \$1,120,000,000 and housed 2800 people during the Vancouver Olympics. What was the cost, in dollars, per person housed during the Olympics?

   1. _____ dollars

2. When Alicia's car is going at 90 km per hour, it uses 12 litres of gas to travel 100 km. How many litres of gas does it use per minute? Give the answer as a decimal, to 2 decimal places.

   2. _____ litres/min

3. It takes 12 hours to fill the pool with taps A and B (combined). It takes 36 hours with tap A alone, and it takes 8 hours with taps A, B, and C combined. How many hours would it take to fill the pool with tap C alone?

   3. _____ hours

4. A car is travelling at 96 km per hour. The radius of its wheels is $\dfrac{1}{\pi}$ metres. How fast are its wheels rotating in degrees per second?
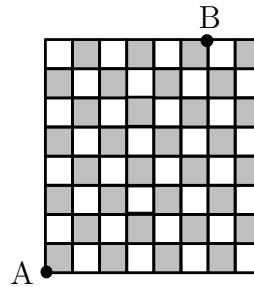
   4. _____ deg/sec

Bull's-eye, Page 2: Numbers and Combinatorics

5. How many five-letter "words" are there which use only the letters A and/or B, and in which there are no consecutive occurrences of B? (For example, AAAAA and ABAAB qualify, but ABBAB does not.)

5. _____ words

6. Evaluate the cube root of $24 \times 30 \times 36 \times 40 \times 45$.

6. _____

7. To six decimal places, $\dfrac{\sqrt{5}-1}{2} = 0.618034$. What common fraction $\dfrac{a}{b}$, where $2 \leq b \leq 10$, is nearest to $\dfrac{\sqrt{5}-1}{2}$? For example, an answer of $\dfrac{4}{9}$ is wrong, but is of the right shape.

7. _____

8. Twenty-seven $1 \times 1 \times 1$ cubes are coloured using 27 different colours, one colour to each cube. Red and green are two of the colours. The 27 cubes are assembled at random to make a $3 \times 3 \times 3$ cube. What is the probability that the red cube and the green cube meet face to face? Express the answer as a common fraction.
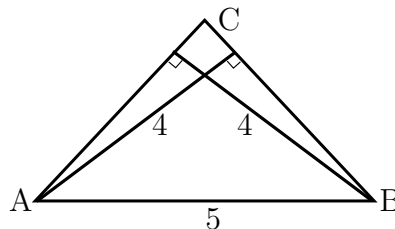
8. _____

9. It is late afternoon, and a 1.2 metre tall child casts a 3 metre shadow. The child is standing next to an upright telephone pole, which casts a 40 metre shadow. What is the height, in metres, of the telephone pole?

9. _____ metres

10. In the chessboard below, the distance from point A to point B is 33 cm. What is the length, in cm, of a side of one of the 64 little squares on the chessboard? Express the answer as a common fraction.

10. _____ cm

11. A line passes through the points $(-1, 10)$, and $(2, -9)$. If $5x + by = c$ is an equation of the line, what is the value of b? Express the answer as a common fraction.

11. _____

12. In $\triangle ABC$, the heights from A and from B are each equal to 4. Side AB is equal to 5. Given that $\triangle ABC$ is *not* right-angled, what is the perimeter of $\triangle ABC$? Express the answer as a common fraction.

12. _____ units

1. The *digit-sum* of a positive integer is the sum of its decimal digits. For example, the digit-sum of 135 is 9. How many integers between 100 and 999 have digit-sum equal to 5?

   1. _____ integers

2. Define the sequence $a_1, a_2, a_3, \ldots$ as follows: $a_1 = 0$, and for every positive integer n, $a_{n+1} = \dfrac{2}{1 + a_n}$. Thus for example $a_2 = \dfrac{2}{1 + 0} = 2$. Express a_5 as a common fraction.

   2. _____

3. How many perfect squares between 1 and 1,000,000 have 5, 6, 7, or 8 as their units digit?

   3. _____

4. Suppose that the line with equation $x + y = k$ passes through the midpoint of the line segment that goes from the point $(1, 7)$ to the point $(9, 11)$. What is the value of k?

   4. _____

5. The sum of the ages of six brothers (all of different integer ages) is a power of 5. The product of their ages is neither a multiple of 5 nor a multiple of 3. What is the least possible age of the oldest brother?

   5. _____

6. For any real number x, $\lfloor x \rfloor$ (the integer part of x) is the greatest integer which is less than or equal to x. For example, $\lfloor 17.72 \rfloor = 17$, and $\lfloor 13 \rfloor = 13$. How many *different* integers are there in the list

$$\left\lfloor \frac{100}{1} \right\rfloor, \ \left\lfloor \frac{100}{2} \right\rfloor, \ \left\lfloor \frac{100}{3} \right\rfloor, \ \left\lfloor \frac{100}{4} \right\rfloor, \ \ldots\ldots, \ \left\lfloor \frac{100}{99} \right\rfloor, \ \left\lfloor \frac{100}{100} \right\rfloor?$$

6. _____ integers

7. Scientists have recently discovered an Earth-like planet 128×10^{12} km away and decided to design a self-sustained spaceship that can travel for generations and send a human expedition to that planet. Based on current technology, spaceships can be designed to travel at a constant speed of 1.25% of the speed of light (after an initial stage of acceleration to this speed). Assume that the speed of light is 300000 km/sec, and ignore the time and distance the space ship has to travel until it reaches cruising speed. Assume also that a year has exactly 360 days and that the ship is to be launched in the year 2050 (exactly halfway through the 21st century). In what century will the space ship reach its destination?

7. _____ century

8. In Question #7, how many hours (rounded to the nearest whole hour) will it take for the spaceship to reach its cruising speed of 1.25% of the speed of light if it accelerates at a constant rate of 10m/sec^2.

8. _____ hours

9. In Question #8, how many km will the spaceship travel until it reaches its cruising speed? Give the answer in scientific notation, correct to 3 significant digits. So an answer like 6.24×10^5 is of the right shape.

9. _____ km

10. The number 360000 has 105 positive factors. (Note that 1 and 360000 are two of these factors.) How many of these 105 factors are divisible by 4?

10. _____ factors

11. Ten people, Alan and Beti and 8 others, are divided at random into two groups, one with 4 people and the other with 6 people. What is the probability that Alan and Beti end up in the same group? Express the answer as a common fraction.

11. _____

12. There are 12 square tiles. Each has one letter written on it. The 12 letters are C, C, H, K, K, K, M, O, O, S, U, and U. The tiles are arranged to form the word SKOOKUMCHUCK. How many *distinct* "words" (which need not be words in any language) can be formed by interchanging two of the tiles? Note that SKOOKUMCHUCK itself is such a word, since it can be obtained by interchanging the two tiles that have a C on them, and in other ways.

12. _____ words

13. A regular hexagon is inscribed in a semicircle of radius 1 as shown. What is the area of the hexagon? Please give the answer rounded to 4 places after the decimal point.

13. _____ units^2

14. How many ordered pairs (a, b) are there such that a and b are positive integers and the least common multiple of a and b is 72? Remember that for example the ordered pair $(18, 24)$ is different from the ordered pair $(24, 18)$, and don't forget the ordered pair $(72, 72)$.

14. _____ pairs

15. A box contains N marbles, of which 2 are white and $N-2$ are black. You know that if you take out 3 marbles at random from the box, the probability that exactly 2 of them are white is $\frac{1}{210}$. What is the value of N?

15. _____

MATH CHALLENGERS
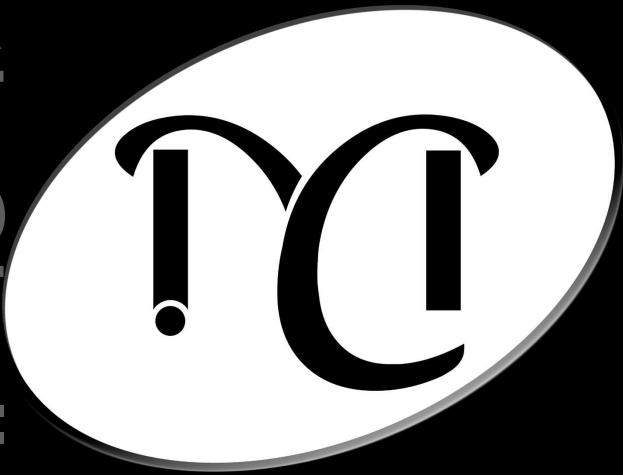
Math Challengers

Regional Competition
Face-off Round 2013

A question always follows a blue page. The next page is blue!

1. What is the cube root of 100, rounded to the nearest integer?

1. What is the cube root of 100, rounded to the nearest integer?

Answer: 5

2. What common fraction is halfway between $\dfrac{1}{6}$ and $\dfrac{1}{8}$?

2. What common fraction is halfway between $\dfrac{1}{6}$ and $\dfrac{1}{8}$?

Answer: $\dfrac{7}{48}$

3. Call a prime *p additive* if the sum of the decimal digits of *p* is also prime. What is the smallest additive prime greater than 30?

3. Call a prime *p additive* if the sum of the decimal digits of *p* is also prime. What is the smallest additive prime greater than 30?

Answer: 41

4. Let $A = 0.84$, let $B = \dfrac{9}{11}$ and let $C = \dfrac{16}{19}$.
Which is largest, A, B, or C?

4. Let $A = 0.84$, let $B = \dfrac{9}{11}$ and let $C = \dfrac{16}{19}$.
Which is largest, A, B, or C?

Answer: C or $\dfrac{16}{19}$

5. The restaurant meal cost $25.25, plus 12% tax. How many dollars did the meal cost, including tax? Give the answer in dollars, to the nearest cent.

5. The restaurant meal cost $25.25, plus 12% tax. How many dollars did the meal cost, including tax? Give the answer in dollars, to the nearest cent.

Answer: 28.28 (dollars)

6. On April 1, 2013, sunrise in Kelowna is at 7:00 AM, and sunset is at 7:36 PM. At what time is it exactly halfway between sunrise and sunset?

6. On April 1, 2013, sunrise in Kelowna is at 7:00 AM, and sunset is at 7:36 PM. At what time is it exactly halfway between sunrise and sunset?

Answer: 1 : 18 (PM) or 13 : 18

7. The sum of three consecutive even integers is 30. What is the product of the three integers?

7. The sum of three consecutive even integers is 30. What is the product of the three integers?

8. If $5x - 4x + 3x - 2x + x = 180$, what is the value of x?

8. If $5x - 4x + 3x - 2x + x = 180$, what is the value of x?

Answer: 60

9. Evaluate 0.128×125.

9. Evaluate 0.128×125.

Answer: 16

10. Suppose that $A_1 = 11$ and $A_{n+1} = A_n + 7$ for any integer n. What is the value of A_{10}?

10. Suppose that $A_1 = 11$ and $A_{n+1} = A_n + 7$ for any integer n. What is the value of A_{10}?

Answer: 74

11. One dozen scarlet splendor roses cost \$98. At the same price per rose, what the cost in dollars of two and a half dozen scarlet splendor roses?

11. One dozen scarlet splendor roses cost $98. At the same price per rose, what the cost in dollars of two and a half dozen scarlet splendor roses?

Answer: 245 (dollars)

12. The figure shown is a regular 8-sided polygon, with two of the diagonals drawn. Altogether, how many diagonals does a regular 8-sided polygon have?
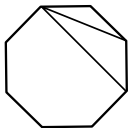
12. The figure shown is a regular 8-sided polygon, with two of the diagonals drawn. Altogether, how many diagonals does a regular 8-sided polygon have?
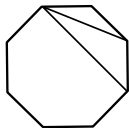
Answer: 20 (diagonals)

13. What is the largest integer N such that $N^3 < 2013$?

13. What is the largest integer N such that $N^3 < 2013$?

Answer: 12

Math Challenge

14. Rounded to the nearest integer, 4 out of every 9 students who participate in the Regional advance to the Provincial. If 500 students participate in the Regional, how many will participate in the Provincial?

14. Rounded to the nearest integer, 4 out of every 9 students who participate in the Regional advance to the Provincial. If 500 students participate in the Regional, how many will participate in the Provincial?

Answer: 222 (students)

15. A circle has radius 10 cm. To the nearest cm, what is the circumference of the circle?

15. A circle has radius 10 cm. To the nearest cm, what is the circumference of the circle?

16. What is the value of $\dfrac{111111}{3}$?

16. What is the value of $\dfrac{111111}{3}$?

Answer: 37037

17. 21% of x is 105. What is the value of x?

**17.** 21% of x is 105. What is the value of x?

Answer: 500

18. In the region below, all angles that look like right-angles *are* right angles, and dimensions of four of the sides are 7, 17, 17, and 7 as shown. What is the area of the region?
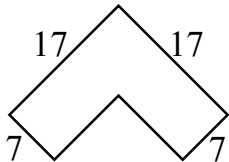
18. In the region below, all angles that look like right-angles *are* right angles, and dimensions of four of the sides are 7, 17, 17, and 7 as shown. What is the area of the region?
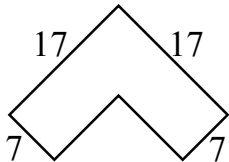
Answer: 189

19. A box contains 2 red balls and 2 blue balls. We remove 2 of the balls, chosen at random. What is the probability the 2 balls are of the same colour? Express the answer as a common fraction.

19. A box contains 2 red balls and 2 blue balls. We remove 2 of the balls, chosen at random. What is the probability the 2 balls are of the same colour? Express the answer as a common fraction.

Answer: $\dfrac{1}{3}$

20. What is the average of the numbers -20, -10, 0, 10, 20, and 30?

20. What is the average of the numbers -20, -10, 0, 10, 20, and 30?

Answer: 5

21. A *palindromic prime* is a prime that remains unchanged when its decimal digits are reversed. For example, 11 is a palindromic prime. What is the smallest palindromic prime which is greater than 11?

21. A *palindromic prime* is a prime that remains unchanged when its decimal digits are reversed. For example, 11 is a palindromic prime. What is the smallest palindromic prime which is greater than 11?

Answer: 101

22. Two standard dice are rolled. What is the probability that the sum of the numbers showing is equal to 10? Express the answer as a common fraction.

22. Two standard dice are rolled. What is the probability that the sum of the numbers showing is equal to 10? Express the answer as a common fraction.

Answer: $\dfrac{1}{12}$

23. Call a prime number *p lonely* if neither $p - 6$ nor $p + 6$ is prime. What is the smallest lonely prime which is greater than 50?

23. Call a prime number *p lonely* if neither $p - 6$ nor $p + 6$ is prime. What is the smallest lonely prime which is greater than 50?

Answer: 71

24. The year 2013 has 365 days, and the first two months have a total of 59 days. How many days are there in the last 10 months of the year?

24. The year 2013 has 365 days, and the first two months have a total of 59 days. How many days are there in the last 10 months of the year?

Answer: 306 (days)

25. A box has the shape of a *triangular prism*. The base of the box is a triangle with sides 6 inches, 8 inches, and 10 inches. The height of the box is 6 inches. What is the volume of the box, in cubic inches?

25. A box has the shape of a *triangular prism*. The base of the box is a triangle with sides 6 inches, 8 inches, and 10 inches. The height of the box is 6 inches. What is the volume of the box, in cubic inches?

Answer: 144 (cubic inches)

26. An isosceles trapezoid has sides 15, 5, 7, and 5 as shown. What is the area of the trapezoid?
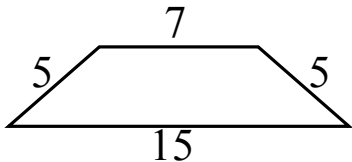
26. An isosceles trapezoid has sides 15, 5, 7, and 5 as shown. What is the area of the trapezoid?
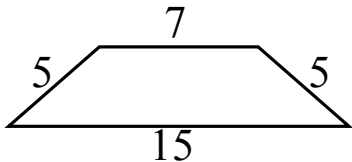
Answer: 33 (units^3)

27. What is the sum of the positive integers that divide 18?

**27.** What is the sum of the positive integers that divide 18?

28. The length of a narrow field is 6 times the width of the field. If the length of the field is 99 metres, what is the perimeter of the field?

28. The length of a narrow field is 6 times the width of the field. If the length of the field is 99 metres, what is the perimeter of the field?

Answer: 231 (metres)

29. One of the angles of a triangle is $120°$. Of the other two angles, one has measure 4 times the measure of the other. What is the degree measure of the smallest angle of the triangle?
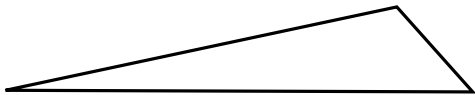
29. One of the angles of a triangle is $120°$. Of the other two angles, one has measure 4 times the measure of the other. What is the degree measure of the smallest angle of the triangle?
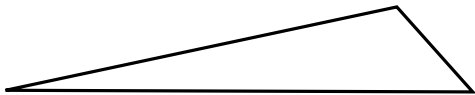
Answer: 12 (degrees)

30. What is the first year after 2013 that has digit sum equal to 13?

30. What is the first year after 2013 that has digit sum equal to 13?

Answer: 2029

31. What is the measure, in degrees, of the angle between the hour hand and the minute hand of a clock at 1:10?

31. What is the measure, in degrees, of the angle between the hour hand and the minute hand of a clock at 1:10?

Answer: 25

32. What is the smallest positive integer n such that $n! > n^3$?

32. What is the smallest positive integer n such that $n! > n^3$?

Answer: 6

33. What is the value of

$$999 - 997 + 995 - 993 + \cdots + 7 - 5 + 3 - 1?$$

33. What is the value of

$$999 - 997 + 995 - 993 + \cdots + 7 - 5 + 3 - 1?$$

Answer: 500

34. The surface area of a cube is $\dfrac{3}{2}$ square metres. What is the volume of the cube, in cubic metres? Express the answer as a common fraction.

34. The surface area of a cube is $\dfrac{3}{2}$ square metres. What is the volume of the cube, in cubic metres? Express the answer as a common fraction.

Answer: $\dfrac{1}{8}$ (cubic metres)

35. What is the value of $112^2 - 108^2$?

35. What is the value of $112^2 - 108^2$?

36. Suppose that n is a positive integer such that two-thirds of n^2 is a perfect cube. If $n < 100$, what is the value of n?

36. Suppose that n is a positive integer such that two-thirds of n^2 is a perfect cube. If $n < 100$, what is the value of n?
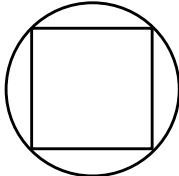
37. The sides of a quadrilateral have length 9, 10, 11, and x. Given that x is an integer, what is the largest possible value of x?

37. The sides of a quadrilateral have length 9, 10, 11, and x. Given that x is an integer, what is the largest possible value of x?

Answer: 29

2013 Canadian Math Challengers Provincial Questions

Blitz, Page 1

1. Two fair dice are rolled. What is the probability that the sum of the two numbers obtained is 8? Express the answer as a common fraction.

1. _____

2. A square is inscribed in a circle of radius 11. What is the area of the square?

2. _____ units^2

3. What is the integer nearest to $(8.5)^2$?

3. _____

4. Express $\dfrac{37}{91}$ as a decimal, correct to 2 decimal digits.

4. _____

5. Albert took the same algebra test a total of 3 times. Each time he took the test, the number of questions he answered correctly increased by 50%. If on the last test he got 36 of the 70 questions right, how many questions did he get right the first time he took the test?

5. _____ questions

6. Express $\dfrac{3^3 - 3^2 - 3^1}{2^3 - 2^2 - 2^1}$ as a common fraction.

6. _____

7. In 2009, Albert's wage was decreased by 20%. Recently, the resulting wage was increased by 20%. By how many percent is Albert's current wage smaller than his pre-2009 wage?

7. _____ percent

8. Evaluate $\dfrac{7! - 6! - 5!}{6! - 5!}$.

8. _____

9. What common fraction is equal to $\dfrac{11}{9} + \dfrac{9}{11} - 2$?

9. _____

10. Let A be the sum of the *odd* integers from 1 to 39 (inclusive), and let B be the sum of the *odd* integers from 41 to 79 (inclusive). What is the value of $B - A$?

10. _____

11. The sum of two consecutive primes is divisible by 2 but not by 4. What is the smallest possible value of this sum?

11. _____

12. Given that the least common multiple of the numbers 8, 10, and n is 80, what is the smallest possible positive value of n?

12. _____

13. Given that $x^2 = 0.2$, what is the value of x^{-4}?

13. _____

14. The hypotenuse of a right triangle has length 25 inches. The shorter leg of triangle has length 7 inches. What is the length, in inches, of the longer leg of the triangle?

14. _____ inches

15. A regular pentagon is inscribed in a circle. You connect any two corners with a straight line. Into how many regions is the circle divided?
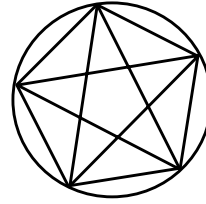
15. _____ regions

16. If x and y are real numbers such that $x + y = 11$ and $xy = 13$, what is the value of $x^2 + y^2$?

16. _____

17. Every interior angle of a many-sided regular polygon has measure 160 degrees. How many sides does the polygon have?

17. _____ sides

18. A loonie (1 dollar coin) weighs 4 times as much as a dime (10 cent coin). Bag A contains only loonies, bag B contains 5 times as many dimes as loonies, but no other coins or notes. Bags A and B have exactly the same weight. If bag A contains 45 dollars' worth of loonies, what is the value, in dollars, of the dimes in Bag B?

18. _____ dollars

19. Let N be the smallest positive integer whose first 4 digits are 2, 0, 1, 0 (in that order) and which is divisible by 45. What is the value of $\dfrac{N}{45}$?

19. _____

20. How many numbers from 100 to 999 contain the digit 8 exactly once?

20. _____ numbers

21. A card is removed from a well-shuffled standard 52-card deck, and then a second card is removed. What is the probability the second card is of the same kind as the first card (so if the first card was a 7, the second should be a 7, if the first card was a King, the second card should be a King, and so on. Express the answer as a common fraction. Note that there are 4 cards of each kind.

21. _____

22. You have 6 tables, whose tops are congruent equilateral triangles, and have guests coming for dinner. You are allowed to join tables together into one or more groups, but if you do, edges of tables that are put together must match up, full edge to full edge. How many different perimeters can we obtained by combining tables in this way?

22. _____ perimeters

23. A *quadrilateral* is a closed *curve* made up of 4 line segments. (It does not include the "inside," and need not be convex.) If two quadrilaterals have finitely many points in common, what is the largest possible number of common points?

23. _____ points
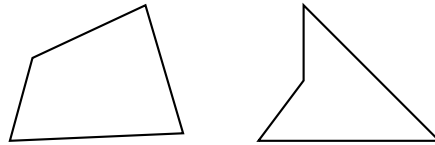
24. What is the smallest possible value of $n^2 - 17n + 100$ as n ranges over the integers?

24. _____

25. The rectangle below is made up of twelve 1×1 squares. Three points, each of which is a vertex of a square, are chosen. Suppose these three points do not all lie on the same line. Form the triangle that has these three points as vertices. How many different numbers are there which could be the area of such a triangle?

25. _____ numbers
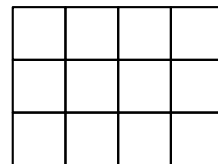
26. You throw a fair coin 6 times. The total number of heads you got is less than 5. What is the probability that the total number of heads is less than the total number of tails? Express the answer as a common fraction.

26. _____

Bull's-eye, Page 1: Problem Solving

1. A *rod* is 5.5 yards, and a *furlong* is 220 yards. How many rods are there in 6 furlongs?

   1. _____ rods

2. For every 3 samosas bought at the full price of 40 cents for each samosa, a store offers 2 samosas at half price. What is the largest number of samosas that Alicia can buy for less than $9?

   2. _____ samosas

3. In a college class of 60 people, there are 20 men and 40 women. Eighty percent of the men, and fifty percent of the women, are wearing jeans. If a jean wearer in the class is chosen at random, what is the probability that the chosen person is a woman? Express the answer as a common fraction.

   3. _____

4. Alicia and Beth differ in weight by 20 pounds. Beth and Gamal differ in weight by 30 pounds. And Gamal and Delbert differ in weight by 6 pounds. What is the least possible weight difference between Alicia and Delbert?

   4. _____ pounds

5. What is the sum of all the *distinct* prime factors of 8888? (For example, the sum of the distinct prime factors of 12 is 5.)

5. _____

6. Three dice are tossed, two red ones and a blue one. What is the probability that the number showing on the blue die matches at least one number showing on a red die? Express the answer as a common fraction.

6. _____

7. At a restaurant buffet, there are four flavours of ice cream available: durian, lime, mango, and orange. In how many ways can Alan choose two scoops of ice cream? (Durian and lime is the same way as lime and durian, and Alan can choose two scoops of the same flavour.)

7. _____ ways

8. What is the sum of all the positive factors of 2013? Note that 1 and 2013 are factors of 2013.

8. _____

Canadian Math Challengers Society

9. All the shapes that look like squares are squares. Each side of the largest square is 8 cm. What is the area, in cm^2, of the shaded region?

9. _____ cm^2
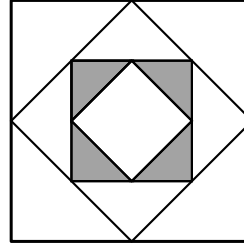
10. Approximately how much greater is the circumference of a circle of diameter 10007 metres than the circumference of a circle of diameter 10000 metres? Round the answer to the nearest metre.

10. _____ metres

11. A line has equation $y = mx - 6$. The line passes through the point $(4, 10)$. If the x-ccordinate of a point on the line is 10, what is the y-coordinate of that point?

11. _____

12. A cube with sides 1 is inscribed in a cone whose base has radius 1 in such a way that one face of the cube is in the base of the cone, and the other four vertices are on the curved surface of the cone. The volume of the cone is πx. Find x. Express the answer as $r + s\sqrt{z}$, where r and s are common fractions and z is an integer.

12. _____

Co-op, Page 1: Team answers must be on the *coloured* page.
Answers on a white page will not be graded.

1. Let $A = \dfrac{3^2 - 2^2 - 2}{(\sqrt{2 \times 3^2}\sqrt{2}) - 2}$ and let $B = \dfrac{3^2 + 15 - (\sqrt{100} + 5)}{2^2 + 2}$.

   Express $\dfrac{A}{B}$ as a common fraction.

   1. _____

2. The coordinates of three of the vertices of a rhombus are $(20, 0)$, $(3, 3)$, and $(0, 20)$. What is the sum of the x and y coordinates of the fourth vertex?

   2. _____

3. Let A be the set consisting of the numbers 1, 2, 3, 4, and 5. In symbols, $A = \{1, 2, 3, 4, 5\}$. Call a subset of A *good* if it contains two numbers that add up to 5. So $\{1, 4\}$ is good, and $\{1, 3, 4\}$ is also good. How many good subsets of A are there? Note that A is a subset of A.

   3. _____ subsets

4. You are allowed to use pennies, nickels, dimes, quarters, loonies (1\$) and toonies (2\$) to make any sum of money using two rules. (a) You are not allowed to use more than 4 pennies; (b) if you use more than 6 of one kind of coin, then you are not allowed to use more than three of any other kind. For example, you are allowed to use 7 toonies, 3 loonies, and one each of the other kinds, but you are not allowed to use 7 toonies, 4 nickels, and 3 each of the other kinds. What is the smallest amount of money that you cannot make? Express your answer in dollars correct to 2 decimal digits.

   4. _____

5. A rhombus $ABCD$ has sides 1, and $\angle DAB$ is $60°$. A circle is tangent to line BC at B, and is tangent to the line segment AD at a point E between A and D. Find the area of the region which is inside the rhombus and also inside the circle. Express the area as a decimal, correct to 3 places after the decimal point.

   5. _____ units^2
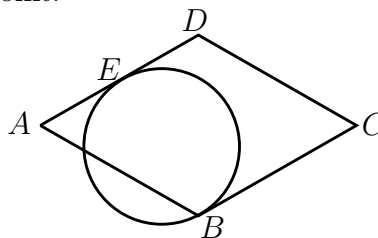
6. A ball is dropped from a height of 20 metres. Each time it hits the floor, it bounces to $\frac{8}{9}$ of its previous height. How many times does the ball hit the floor until and including the first time it cannot reach a height of 1 metre?

6. _____ times

7. The basin of the Yukon River is 476237 km^2. (That means that all the rainfall in that region is drained by that river.) The average yearly rainfall in the basin is 642 mm, and on average 13% of the rainfall is carried by the river to the ocean. How much water on average is carried by the river to the ocean in one day? Assume that there are 365 days per year. Express your answer in km^3, correct to 3 places after the decimal point.

7. _____ km^3

8. In a certain day (24 hour period) the amount of water that was carried by the Yukon River to the ocean was 1.200×10^8 m^3. At the mouth of the river (the location where the river meets the ocean), the average width of the river was 250 m, and its average depth was 6.0 m. (For simplicity assume that the cross-section of the river at this location is rectangular.) Assume that the water is flowing at constant speed through that cross-section. What was the average speed of the water? Express your answer in m/sec, correct to 3 decimal digits.

8. _____ m/sec

9. In stationary water, salmon swim (on average) at the speed of 1.2 m/sec as long as the temperature of the water is 10.0 degrees Celsius or less. For each 1 degree rise in water temperature after that, up to a temperature of 20 degrees, salmons' speed is reduced by a constant amount, in such a way that if the temperature is 20 degrees they cannot swim at all. Salmon can swim up the river for at most 20 days until they reach their spawning ground, and their swimming speed is reduced by the speed of the river flow. How far can they reach upriver if the water temperature is 12.0 degrees and if the river flows at 0.5 m/sec? Express your answer in km, rounded to the nearest km.

9. _____ km

10. You select an integer N from 1 to 100 inclusive, and calculate the sum from $k = 1$ to $k = N$ of k^4, noting the units digit of that sum. What is the average value of these units digits? Express your answer correct to 1 digit after the decimal point. So for example an answer of 6.7 is of the right shape. Hint: Find the units digit for $k = 1, 11, 21, \ldots$ and look for a pattern.

10. _____

Canadian Math Challengers Society

11. For any integer $n > 0$, the number $T(n)$ is called the n-th triangular number if $T(n) = 1 + 2 + 3 + \cdots + n$. What is the smallest positive integer n such that $T(n)$ has exactly 16 positive factors? Note that 1 and m are factors of the positive integer m.

11. _____

12. If you toss a fair nickel 100000 times, it lands heads on average 49912 times, tails 49912 times, and on its edge the rest of the time. You threw the coin 5 times. What is the probability that it landed *exactly* 3 times on its edge. Express the answer in scientific notation, correct to 4 significant digits. An answer like 2.047×10^{-5} is of the right shape.

12. _____

13. A positive integer is called *square-free* if it is not divisible by any perfect square greater than 1. For example, 1, 2, and 6 are square-free, while 4 and 18 are not. Two fair standard dice are tossed. What is the probability that the product of the two numbers obtained is square-free? Express the answer as a common fraction.

13. _____

14. A convex polygon has 2013 sides. Let N be the largest integer smaller than 2013 such that $N + 1$ is prime. Using any N corners of this polygon as the corners of a new N-sided convex polygon, how many different N-sided convex polygons can be made? Hint: The correct answer is 10 digits long.

14. _____ polygons

15. You have a square with side 1, and select a point O at random inside the square, and draw a circle of radius 1 with O as its centre. What is the probability that the entire square is inside the circle? Express the answer as a decimal, correct to 3 decimal places.

15. _____
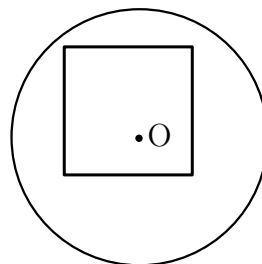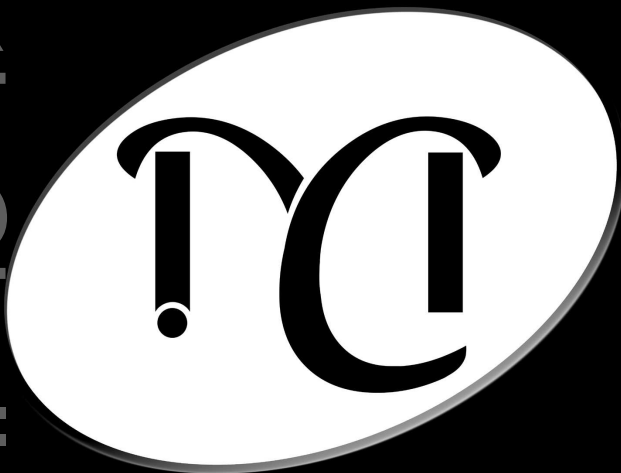
MATH CHALLENGERS

Math Challengers

Provincial Competition
Face-off Round 2013

A question always follows a blue page. The next page is blue!

1. What is the volume of the cone with base radius 2 and height 3? Give the answer to the nearest integer.

1. What is the volume of the cone with base radius 2 and height 3? Give the answer to the nearest integer.

Answer: 13 (units^3)

2. What is the smallest perfect square greater than 600 whose square root has digit sum equal to 6?

2. What is the smallest perfect square greater than 600 whose square root has digit sum equal to 6?

1089

3. Which of the following four numbers is the largest?

$$\frac{33}{37} \qquad 0.89 \qquad \frac{25}{28} \qquad \frac{41}{46}$$

3. Which of the following four numbers is the largest?

$$\frac{33}{37} \qquad 0.89 \qquad \frac{25}{28} \qquad \frac{41}{46}$$

Answer: $\frac{25}{28}$

4. What is the probability of getting exactly 4 heads in 5 tosses of a fair coin? Express the answer as a common fraction.

4. What is the probability of getting exactly 4 heads in 5 tosses of a fair coin? Express the answer as a common fraction.

Answer: $\dfrac{5}{32}$

5. What is the volume, in cubic centimetres, of a cube whose surface area is equal to 294 square centimetres?

5. What is the volume, in cubic centimetres, of a cube whose surface area is equal to 294 square centimetres?

Answer: 343 (cubic centimetres)

6. Jane (an Olympic runner) ran 1.5 km in 4 minutes and 10 seconds. What was her average speed in metres per second?

6. Jane (an Olympic runner) ran 1.5 km in 4 minutes and 10 seconds. What was her average speed in metres per second?

Answer: 6 (metres per second)

7. What is the radius of the smallest circle with integer radius whose area is larger than 400 square units?

7. What is the radius of the smallest circle with integer radius whose area is larger than 400 square units?

Answer: 12 (units)

8. Alicia's auto repair bill is $3000, of which $1200 is for parts and the rest for labour. If labour is billed at $100 per hour, how many hours of labour were billed?

8. Alicia's auto repair bill is $3000, of which $1200 is for parts and the rest for labour. If labour is billed at $100 per hour, how many hours of labour were billed?

Answer: 18 (hours)

9. What is the smallest 3-digit number whose digit sum is the same as the digit sum of 2013?

9. What is the smallest 3-digit number whose digit sum is the same as the digit sum of 2013?

10. What is the largest prime whose square is less than 2013?

10. What is the largest prime whose square is less than 2013?

Answer: 43

11. The sum of 5 different positive integers is 2013. What is the largest possible value of any of these 5 integers?.

11. The sum of 5 different positive integers is 2013. What is the largest possible value of any of these 5 integers?.

Answer: 2003

12. If 11 cookies cost $11.99, what is the cost of 5 cookies?

12. If 11 cookies cost $11.99, what is the cost of 5 cookies?

Answer:

5.45 (dollars) or 5 dollars and 45 cents

13. You roll a die twice. What is the probability that the sum is 5? Express the answer as a common fraction in lowest terms.

13. You roll a die twice. What is the probability that the sum is 5? Express the answer as a common fraction in lowest terms.

Answer: $\dfrac{1}{9}$

14. Among the two-letter "words" that only use the letters A and/or B and/or C, how many have exactly one C?

14. Among the two-letter "words" that only use the letters A and/or B and/or C, how many have exactly one C?

Answer: 4 (words)

15. How many integers smaller than 2013 have digit sum equal to 2?

15. How many integers smaller than 2013 have digit sum equal to 2?

Answer: 10

16. Evaluate $\dfrac{2^{10} - 1}{2^5 - 1}$.

16. Evaluate $\dfrac{2^{10} - 1}{2^5 - 1}$.

Answer: 33

17. The diagonals of a rhombus have lengths 12 and 9. What is the perimeter of the rhombus?
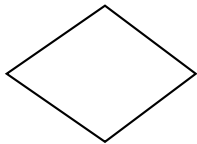
17. The diagonals of a rhombus have lengths 12 and 9. What is the perimeter of the rhombus?
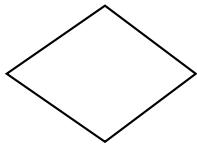
Answer: 30

18. The mean of 20, 21, and $2x$ is 35. What is the value of x?

18.   The mean of 20, 21, and $2x$ is 35. What is the value of x?

32

19. Alan has three-sevenths as much money as Beti, and between them they have 400 dollars. How many dollars does Beti have?

19. Alan has three-sevenths as much money as Beti, and between them they have 400 dollars. How many dollars does Beti have?

Answer: 280 (dollars)

20. The line with equation $y = mx + 13$ goes through the point $(25, 2013)$. What is the value of m?

20. The line with equation $y = mx + 13$ goes through the point $(25, 2013)$. What is the value of m?

21. A store has cans of beans on sale for 89 cents per can. What is the largest number of cans of beans you can buy at that store if all you have is a $10 bill?

21. A store has cans of beans on sale for 89 cents per can. What is the largest number of cans of beans you can buy at that store if all you have is a $10 bill?

Answer: 11 (cans)

22. The side of a square has length 12 inches. What is the length of the diagonal of this square? Give the answer in inches, rounded to the nearest inch.

22. The side of a square has length 12 inches. What is the length of the diagonal of this square? Give the answer in inches, rounded to the nearest inch.

Answer: 17(inches)

23. Two fair dice are tossed. What is the probability that neither number so obtained is divisible by 3? Express the answer as a common fraction in lowest terms.

23. Two fair dice are tossed. What is the probability that neither number so obtained is divisible by 3? Express the answer as a common fraction in lowest terms.

Answer: $\dfrac{4}{9}$

24. The sides of a triangle are integers. The perimeter of the triangle is 100. What is the smallest possible length of the largest side of the triangle?

24. The sides of a triangle are integers. The perimeter of the triangle is 100. What is the smallest possible length of the largest side of the triangle?

Answer: 34

25. For every integer $n \geq 1$, the sum of the first n terms of a sequence is equal to n^2. What is the 10-th term of the sequence?

25. For every integer $n \geq 1$, the sum of the first n terms of a sequence is equal to n^2. What is the 10-th term of the sequence?

Answer: 19

26. What is the value of
$4^2 + 5^2 + 6^2$?

26. What is the value of
$4^2 + 5^2 + 6^2$?

Answer: 77

27. The area of circle $\mathcal{A}$ is 36% less than the area of circle $\mathcal{B}$. By how many percent is the circumference of $\mathcal{B}$ more than the circumference of $\mathcal{A}$?

27. The area of circle $\mathcal{A}$ is 36% less than the area of circle $\mathcal{B}$. By how many percent is the circumference of $\mathcal{B}$ more than the circumference of $\mathcal{A}$?

Answer: 25 (percent)

28. Simplify $\dfrac{\frac{1}{5} - \frac{1}{6}}{\frac{1}{15} - \frac{1}{16}}$.

28. Simplify $\dfrac{\dfrac{1}{5} - \dfrac{1}{6}}{\dfrac{1}{15} - \dfrac{1}{16}}$.

Answer: 8

29. If $x/4$ is the reciprocal of $x^2/16$, what is the value of x?

29. If $x/4$ is the reciprocal of $x^2/16$, what is the value of x?

Answer: 4

30. At the Provincial, each of the 26 Blitz questions is worth 1 point and each Bull's Eye question is worth 2. The maximum total points achievable is 50. Ana's total mark was 31 points and she answered correctly 50% of the Bull's Eye questions. How many Blitz questions did she answer correctly?

30. At the Provincial, each of the 26 Blitz questions is worth 1 point and each Bull's Eye question is worth 2. The maximum total points achievable is 50. Ana's total mark was 31 points and she answered correctly 50% of the Bull's Eye questions. How many Blitz questions did she answer correctly?

Answer: 19

Math Challengers Regional 2013
Answers, Blitz Stage

1. 16

2. $\dfrac{5}{7}$

3. 101

4. 39

5. 20

6. $\dfrac{23}{12}$

7. 14

8. 27

9. 31

10. $\dfrac{3}{8}$

11. 500

12. $\dfrac{5}{36}$

13. 0.33

14. 16

15. 42

16. 67

17. 36

18. 13.20

19. 11

20. 52

21. 60

22. 43

23. 14

24. $\dfrac{13}{36}$

25. $\dfrac{35}{18}$

26. $\dfrac{245}{24}$

Math Challengers Regional 2013
Answers, Bull's-eye Stage

1. 400000

2. 0.18

3. 24

4. 4800

5. 13

6. 360

7. $\dfrac{5}{8}$

8. $\dfrac{2}{13}$

9. 16

10. $\dfrac{33}{10}$

11. $\dfrac{15}{19}$

12. $\dfrac{40}{3}$

Math Challengers Regional 2013
Answers, Co-op Stage

1. 15

2. $\dfrac{10}{11}$

3. 300

4. 14

5. 28

6. 19

7. 32

8. 104

9. 7.03×10^8

10. 75

11. $\dfrac{7}{15}$

12. 61

13. 0.7994

14. 35

15. 36

Math Challengers Provincial 2013
Answers, Blitz Stage

1. $\dfrac{5}{36}$

2. 242

3. 72

4. 0.41

5. 16

6. $\dfrac{15}{2}$

7. 4

8. 7

9. $\dfrac{4}{99}$

10. 800

11. 18

12. 16

13. 25

14. 24

15. 16

16. 95

17. 18

18. 10 or 10.00

19. 4467

20. 225

21. $\dfrac{1}{17}$

22. 7

23. 16

24. 28

25. 12

26. $\dfrac{22}{57}$

Math Challengers Provincial 2013
Answers, Bull's-eye Stage

1. 240

2. 27

3. $\dfrac{5}{9}$

4. 4

5. 114

6. $\dfrac{11}{36}$

7. 10

8. 2976

9. 8

10. 22

11. 34

12. $\dfrac{2}{3} + \dfrac{1}{3}\sqrt{2}$

Math Challengers Provincial 2013
Answers, Co-op Stage

1. $\dfrac{1}{2}$

2. 34

3. 14

4. 20.49

5. 0.474

6. 26

7. 0.109

8. 0.926

9. 795

10. 4.5
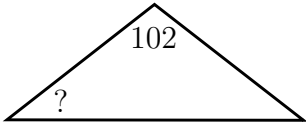
11. 15

12. 5.433×10^{-8}

13. $\dfrac{17}{36}$

14. 1357477286

15. 0.315

2014 Canadian Math Challengers Regional Questions

Blitz, Page 1

1. Evaluate the sum $(-5) + (-3) + (-1) + 1 + 3 + 5 + 7 + 9$.

   1. _____

2. There are 7 marbles in a box, of which exactly 2 are white. One marble is selected at random. What is the probability it is non-white? Express the answer as a common fraction.

   2. _____

3. The ferry left Tsawwassen at 2:33 PM, and arrived at Swartz Bay at 4:14 PM the same day. How many minutes was the trip?

   3. _____ minutes

4. One angle of an isosceles triangle is $102°$. What is the measure, in degrees, of another angle of the triangle?

   4. _____ degrees

5. 10% of 20 plus 20% of 30 is how many % of 40?

   5. _____ %

6. Express $\frac{1}{2} + \frac{2}{3} + \frac{3}{4}$ as a common fraction.

   6. _____

7. Round $\dfrac{230}{17}$ to the nearest integer.

   7. _____

8. The sum of *all* the edge lengths of a cube is 36. What is the volume of the cube?

8. _____ units^3

9. What is the value of the smallest integer whose square root is greater than 5.5?

9. _____

10. Three fair coins are tossed. What is the probability of getting exactly 2 heads? Express the answer as a common fraction.

10. _____

11. What is the value of the sum

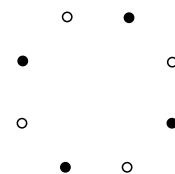$$1000 - 999 + 998 - 997 + \cdots + 4 - 3 + 2 - 1?$$

11. _____

12. Two fair dice are rolled. What is the probability that the sum of the numbers obtained is 6? Express the answer as a common fraction.

12. _____

13. The combined cost of one candy, one chocolate bar, and one cookie is $2.71. The combined cost of one candy, one chocolate bar, and three cookies is $5.25. The combined cost of one candy, two chocolate bars, and three cookies is $6.36. What is the cost of one candy? Give the answer in $, correct to two decimal places.

13. _____ $

14. The 8 points below represent the vertices of a regular octagon. These vertices are alternately painted red and blue. Lines are drawn joining every pair of points. How many of these lines contain a red point and a blue point?

14. _____ lines

15. A number is put in each of the small rectangles below so that the number in any small rectangle is equal to the sum of the numbers in the two rectangles that it sits on. What number should be put in the rectangle labelled "?"?

15. _____

```
      121
   ┌────┬────┐
 1 │  ? │ 36
```

16. On Monday, 40 of the 50 students in the math class took a test. Their mean score was 60. On Tuesday, the remaining 10 students took the test. Their mean score was 95 (they had seen the questions). What was the mean class score on the test?

16. _____

17. Sprinter Bolt ran 100 metres in 10 seconds. What was his average speed in km/hour?

17. _____ km/hour

18. Last summer, Alfie earned $726 working at $11.00 per hour, and another $726 working at an hourly rate which is 50% higher. What was Alfie's average wage per hour last summer? Give your answer in dollars per hour, to 2 decimal places.

18. _____ $/hr

19. Four straight lines pass through a circular disk. What is the largest possible number of regions that these lines divide the disk into?

19. _____ regions

20. A lidless $4 \times 4 \times 4$ box is completely filled with $1 \times 1 \times 1$ cubes. How many of the $1 \times 1 \times 1$ cubes touch a side or the bottom of the box?

20. _____ cubes

21. When it was set out, the bowl of mixed nuts was (by weight) 45% peanuts, 25% almonds, 20% cashews, and 10% hazelnuts. Alicia picked out all the almonds and ate them. What percent (by weight) of the nuts in the bowl are now peanuts?

21. _____ percent

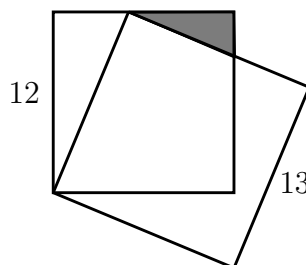22. There is an integer N such that $N^3 = 79,507$. What is the value of N?

22. _____

23. Each line segment has length 1. You are only allowed to walk in the direction of the arrows, and must always walk full segments. At any intersection, you can walk in any allowed direction, regardless of where you came from. How many paths of total length 9 are there from A to B?

23. _____ paths

24. What is the x-coordinate of the point P on the x-axis such that the distance from P to the origin is the same as the distance from P to the point with coordinates $(1/2, 1/3)$? Express the answer as a common fraction.

24. _____

25. Two fair dice are rolled. What is the average value of the non-negative difference between the numbers showing on the two dice? Express the answer as a common fraction.

25. _____

26. In the diagram below, the two figures that look like squares are squares, with sides 12 and 13 respectively. What is the area of the shaded triangle? Express the answer as a common fraction.

26. _____ units^2

Bull's-eye, Page 1: Problem Solving

1. Assume that the Olympic village cost $1,120,000,000 and housed 2800 people during the Vancouver Olympics. What was the cost, in dollars, per person housed during the Olympics?

1. _____ dollars

2. When Alicia's car is going at 90 km per hour, it uses 12 litres of gas to travel 100 km. How many litres of gas does it use per minute? Give the answer as a decimal, to 2 decimal places.

2. _____ litres/min

3. It takes 12 hours to fill the pool with taps A and B (combined). It takes 36 hours with tap A alone, and it takes 8 hours with taps A, B, and C combined. How many hours would it take to fill the pool with tap C alone?

3. _____ hours

4. A car is travelling at 96 km per hour. The radius of its wheels is $\dfrac{1}{\pi}$ metres. How fast are its wheels rotating in degrees per second?

4. _____ deg/sec

Bull's-eye, Page 2: Numbers and Combinatorics

5. How many five-letter "words" are there which use only the letters A and/or B, and in which there are no consecutive occurrences of B? (For example, AAAAA and ABAAB qualify, but ABBAB does not.)

5. _____ words

6. Evaluate the cube root of $24 \times 30 \times 36 \times 40 \times 45$.

6. _____

7. To six decimal places, $\dfrac{\sqrt{5}-1}{2} = 0.618034$. What common fraction $\dfrac{a}{b}$, where $2 \leq b \leq 10$, is nearest to $\dfrac{\sqrt{5}-1}{2}$? For example, an answer of $\dfrac{4}{9}$ is wrong, but is of the right shape.

7. _____

8. Twenty-seven $1 \times 1 \times 1$ cubes are coloured using 27 different colours, one colour to each cube. Red and green are two of the colours. The 27 cubes are assembled at random to make a $3 \times 3 \times 3$ cube. What is the probability that the red cube and the green cube meet face to face? Express the answer as a common fraction.
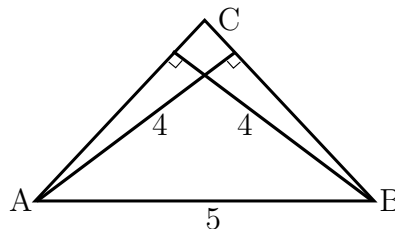
8. _____

9. It is late afternoon, and a 1.2 metre tall child casts a 3 metre shadow. The child is standing next to an upright telephone pole, which casts a 40 metre shadow. What is the height, in metres, of the telephone pole?

9. _____ metres

10. In the chessboard below, the distance from point A to point B is 33 cm. What is the length, in cm, of a side of one of the 64 little squares on the chessboard? Express the answer as a common fraction.

10. _____ cm

11. A line passes through the points $(-1, 10)$, and $(2, -9)$. If $5x + by = c$ is an equation of the line, what is the value of b? Express the answer as a common fraction.

11. _____

12. In $\triangle ABC$, the heights from A and from B are each equal to 4. Side AB is equal to 5. Given that $\triangle ABC$ is *not* right-angled, what is the perimeter of $\triangle ABC$? Express the answer as a common fraction.

12. _____ units

Co-op, Page 1: Team answers must be on the *coloured* page. Answers on a white page will not be graded.

1. The *digit-sum* of a positive integer is the sum of its decimal digits. For example, the digit-sum of 135 is 9. How many integers between 100 and 999 have digit-sum equal to 5?

    1. _____ integers

2. Define the sequence $a_1, a_2, a_3, \ldots$ as follows: $a_1 = 0$, and for every positive integer n, $a_{n+1} = \dfrac{2}{1 + a_n}$. Thus for example $a_2 = \dfrac{2}{1 + 0} = 2$. Express a_5 as a common fraction.

    2. _____

3. How many perfect squares between 1 and 1,000,000 have 5, 6, 7, or 8 as their units digit?

    3. _____

4. Suppose that the line with equation $x + y = k$ passes through the midpoint of the line segment that goes from the point $(1, 7)$ to the point $(9, 11)$. What is the value of k?

    4. _____

5. The sum of the ages of six brothers (all of different integer ages) is a power of 5. The product of their ages is neither a multiple of 5 nor a multiple of 3. What is the least possible age of the oldest brother?

    5. _____

Co-op, Page 2: Team answers must be on the *coloured* page.
Answers on a white page will not be graded.

6. For any real number x, $\lfloor x \rfloor$ (the integer part of x) is the greatest integer which is less than or equal to x. For example, $\lfloor 17.72 \rfloor = 17$, and $\lfloor 13 \rfloor = 13$. How many *different* integers are there in the list

$$\left\lfloor \frac{100}{1} \right\rfloor, \ \left\lfloor \frac{100}{2} \right\rfloor, \ \left\lfloor \frac{100}{3} \right\rfloor, \ \left\lfloor \frac{100}{4} \right\rfloor, \ \ldots\ldots, \ \left\lfloor \frac{100}{99} \right\rfloor, \ \left\lfloor \frac{100}{100} \right\rfloor?$$

6. _____ integers

7. Scientists have recently discovered an Earth-like planet 128×10^{12} km away and decided to design a self-sustained spaceship that can travel for generations and send a human expedition to that planet. Based on current technology, spaceships can be designed to travel at a constant speed of 1.25% of the speed of light (after an initial stage of acceleration to this speed). Assume that the speed of light is 300000 km/sec, and ignore the time and distance the space ship has to travel until it reaches cruising speed. Assume also that a year has exactly 360 days and that the ship is to be launched in the year 2050 (exactly halfway through the 21st century). In what century will the space ship reach its destination?

7. _____ century

8. In Question #7, how many hours (rounded to the nearest whole hour) will it take for the spaceship to reach its cruising speed of 1.25% of the speed of light if it accelerates at a constant rate of 10m/sec^2.

8. _____ hours

9. In Question #8, how many km will the spaceship travel until it reaches its cruising speed? Give the answer in scientific notation, correct to 3 significant digits. So an answer like 6.24×10^5 is of the right shape.

9. _____ km

10. The number 360000 has 105 positive factors. (Note that 1 and 360000 are two of these factors.) How many of these 105 factors are divisible by 4?

10. _____ factors

11. Ten people, Alan and Beti and 8 others, are divided at random into two groups, one with 4 people and the other with 6 people. What is the probability that Alan and Beti end up in the same group? Express the answer as a common fraction.

11. _____

12. There are 12 square tiles. Each has one letter written on it. The 12 letters are C, C, H, K, K, K, M, O, O, S, U, and U. The tiles are arranged to form the word SKOOKUMCHUCK. How many *distinct* "words" (which need not be words in any language) can be formed by interchanging two of the tiles? Note that SKOOKUMCHUCK itself is such a word, since it can be obtained by interchanging the two tiles that have a C on them, and in other ways.

12. _____ words

13. A regular hexagon is inscribed in a semicircle of radius 1 as shown. What is the area of the hexagon? Please give the answer rounded to 4 places after the decimal point.

13. _____ units^2

14. How many ordered pairs (a, b) are there such that a and b are positive integers and the least common multiple of a and b is 72? Remember that for example the ordered pair $(18, 24)$ is different from the ordered pair $(24, 18)$, and don't forget the ordered pair $(72, 72)$.

14. _____ pairs

15. A box contains N marbles, of which 2 are white and $N-2$ are black. You know that if you take out 3 marbles at random from the box, the probability that exactly 2 of them are white is $\frac{1}{210}$. What is the value of N?

15. _____

2014 Canadian Math Challengers Provincial Questions
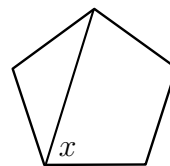
Blitz, Page 1

1. Round $\sqrt{2014}$ to the nearest integer.

1. _____

2. 24 is how many percent of 160?

2. _____ percent

3. You throw 2 dice and the sum is 4. What is the probability that one of the dice shows a 1? Express the answer as a common fraction.

3. _____

4. What is the average of the numbers -15, -12, -5, 1, 6, 12, and 20?

4. _____

5. What is the value, in degrees, of the angle x in the regular pentagon below?

5. _____ degrees

6. If $x^2 - y^2 = x + y = 889$, then what is the value of $x - y$?

6. _____

7. The clock shows a time of 12:20. What is the obtuse angle, in degrees, formed by the 2 hands of the clock?

7. _____ degrees

8. Calculate $(3-4)+(6-7)+(9-10)+(12-13)+\cdots+(999-1000)$.   8. _____

9. Evaluate $\left(\sqrt{ab}-\sqrt{\dfrac{a}{b}}\right)\div\sqrt{\dfrac{a}{b}}$ when $a=17$ and $b=13$.   9. _____

10. Tax on restaurant bills is 10% and gratuity is usually 15% on the pre-tax amount. Ellen made the mistake of calculating her tip as 15% of the post-tax amount. What was the effective pre-tax percentage of her tip? Express the answer as a percent, correct to 1 place after the decimal point.   10. _____ percent

11. Two fair dice are rolled. What is the probability of not getting a 6 on either die? Express the answer as a common fraction.   11. _____

12. If $\dfrac{5x-4}{3}-\dfrac{x-10}{4}=\dfrac{x+9}{2}-3$, what is the value of x? Express the answer as a common fraction.   12. _____

13. Consider the object below made up of four squares. Each square fits exactly within and is rotated 45 degrees with respect to the next larger square. If each side of the largest square is 400 cm and many more squares could be drawn, what would be the perimeter of the 7th largest square?   13. _____ cm
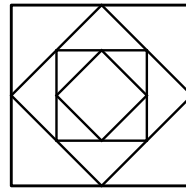
14. George's father is 30 years older than George, and is half of his father's age. If the sum of the ages of all three is 138 years, how old is George?   14. _____ years old

15. A wall of a room has width 5.1 metres and height 2.4 metres. Four large identical-sized pictures cover exactly $1/3$ of the area of the wall. If the vertical dimension of each is 120 cm, what is the horizontal dimension of each of the pictures (in cm)?

15. _____ cm

16. Simplify $\sqrt{50} - 5\sqrt{8} + \sqrt{2} + \sqrt{512}$. Your answer should have the shape $a\sqrt{2}$ where a is an integer.

16. _____

17. A class of 30 students had a picnic. 20 had pizza, 19 had potato chips, and 18 had carrot sticks. 15 of the students had all three items, and 3 of the students had none of the items. How many of the students had exactly two items?.

17. _____ students

18. The sum of three consecutive multiples of three is 63. What is the product of these numbers?

18. _____

19. .Given $N = aa1 + bb1 + 1bb$, where $aa1$, $bb1$, and $1bb$ are three-digit natural numbers and $a < b$. Determine $b - a$ if $N = 696$.

19. _____

20. Each of the two semicircles below has radius 6. Their diameters have an endpoint in common, and are perpendicular to each other . What is the area of the *unshaded* region?

20. _____ units^2
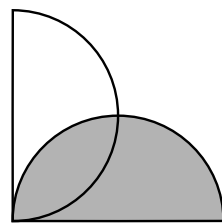
21. A marathon cross-country skier skis at an average speed of V_1 km/hr. He skis 1/3 of the distance at 10 km/hr, 1/3 at 15 km/hr, and 1/3 at 20 km/hr. If he skied each of the segments faster by 10 km/hr, his average speed would rise to V_2 km/hr. What is the ratio $\dfrac{V_2}{V_1}$? Express the answer as a common fraction.

21. _____

22. From a group of 6 boys and 5 girls, a delegation of 3 students is chosen, of whom at least one is a girl. In how many ways can this be done?

22. _____ ways

23. Let $\mathcal{A}$ be the set of all integers from 1 to 2014 (inclusive). What is the largest number of integers that can be chosen from $\mathcal{A}$ so that none is a multiple of 2, 3, or 5?

23. _____ integers

24. Suppose that a, b, and c are real numbers, and $P(x) = ax^2 + bx + c$. If $P(x+2) + P(x+1) - P(x) = 2x^2 + 15x + 20$, what is the value of $P(10)$?

24. _____

25. Alan rolled two fair dice, then Beti rolled the two dice. What is the probability that at least one of the numbers Beti got matches a number that Alan got? Express the answer as a common fraction.

25. _____

26. A diameter of a circle is one of the sides of an equilateral triangle. What fraction of the triangle is inside the circle? Give your answer as $\frac{M+\sqrt{N}\pi}{K}$, where M, N, and K are integers, and N has no square factor greater than 1.

26. _____
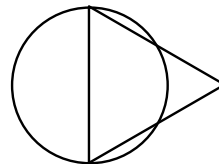
1. Alphonse, Beti, and Gamal each have different amounts of money. If Alphonse had 3/2 as much money as he has, and Beti had 2/3 as much as she has, and Gamal's fortune was unchanged, they would each have the same amount of money. What common fraction of their total current combined amount of money does Gamal have?

1. _____

2. A pipe has radius r at point A and has radius $r/2$ at point B. Water flows through the pipe. Its speed at point A is 5m/sec. What is the speed of the water through the pipe at B?

2. _____ m/sec
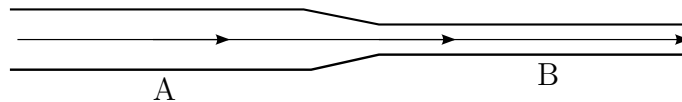
3. In the election for Student Council president, there were four candidates, A, B, C, and D. Each of the 1000 students voted for one and only one of these candidates. Candidate A got 40 more votes than candidate B, 200 votes more than C, and 300 more votes than D. How many votes did A get?

3. _____ votes

4. You ride your bike by starting from rest and increasing your speed at a constant rate. You reached 169/196 of the distance 23 seconds before you reached your final destination. If you travelled 15 metres in your first 23 seconds, how long (in metres) was your entire travel distance? Hint: Total distance travelled at any instant is proportional to the square of the time travelled up to that point.

4. _____ metres

Bull's-eye, Page 2: Numbers and Combinatorics

5. Find the largest prime factor of $12! + 14!$.

5. _____

6. Simplify $\dfrac{7! \times 5!}{10!} \left(\dfrac{9!}{3! \times 5!} - \dfrac{10!}{2! \times 7!} \right)$.

6. _____

7. You start writing down the positive integers in order as follows:

$$1, 2, 3, 4, \ldots, 9, 10, 11, \ldots, 99, 100, 101, \ldots.$$

In what number will you write down your 2014-th digit?

7. _____

8. A fair coin was tossed 5 times, and you know that at least 3 of the tosses were heads. What is the probability all 5 tosses were heads? Express the answer as a common fraction.

8. _____

Canadian Math Challengers Society

9. What is the value of x? Express the answer as $\frac{a\sqrt{d}}{b}$, where a and b are positive integers with no common factor greater than 1, and d is an integer which is not divisible by any perfect square > 1.

9. _____
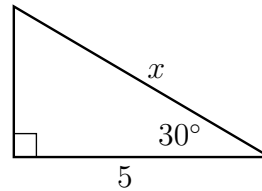
10. A gondola travels up the Grouse Grind. The gondola travels on the hypotenuse of the triangle below. The elevation gain of the Ground Grind is approximately 850 m. The travel distance of the gondola is 1408 m. The gondola travels about 32 m every 15 seconds. As the gondola travels, what is the average elevation gain rate in m/sec? Round your answer to the 1 place after the decimal point.

10. _____ m/sec
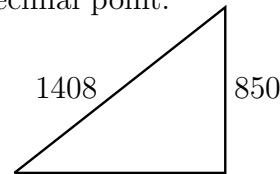
11. The triangle ABC has vertices $A(0,0)$, $B(20,0)$ and $C(0,21)$. The point $P(a,b)$ is on BC and AP is perpendicular to BC. What is the length of AP? Express the answer as a common fraction.

11. _____ units
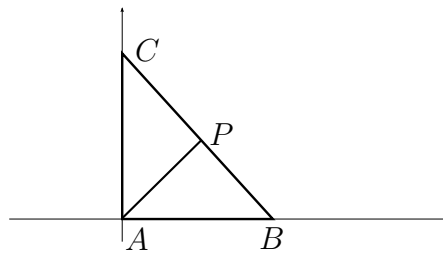
12. $ABCDEF$ is a regular hexagon with side 1. What is the area of equilateral triangle ACE? Express the answer as $\frac{a\sqrt{d}}{b}$, where a and b are positive integers with no common factor greater than 1, and d is an integer which is not divisible by any perfect square > 1.

12. _____ units^2
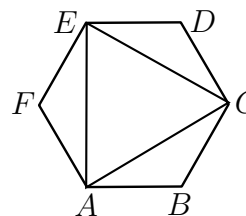
1. If $(3 - \sqrt{2})^2$ is expressed in the form $a - b\sqrt{2}$, where a and b are integers, what is the value of $a + b$?

   1. _____

2. A block of Swiss cheese occupies a volume of 60 cm^3. It has four equal-sized spherical air bubbles within it and therefore only 80% of that volume is actually cheese. What is the diameter, in cm, of each bubble? Give the answer correct to 2 decimal places.

   2. _____ cm

3. Alan, Bono, Camila, and Deidra competed for the position of president of the community association. The person who finishes last in a round of votes is eliminated, and those who voted for him or her transfer their vote to one of the other candidates still left in the race. The process continues until one candidate gets more than 50% of the votes. Results of Round One: 240 voted for Alan, 260 for Bono, 270 for Camila, and 230 for Deidra. In Round Two 1/23 of those voted for the person who was just eliminated (Deidra) transferred their vote to the person who was leading on the first round, and the other two candidate each got 11/23 of the votes. In Round Three the transfer was 25/56 to the winner of Round Two and 31/56 to the person who came second. What is the total number of votes that the winner of the election got?

   3. _____ votes

4. A taxi driver pays $124.80 for her taxi for use in her 12-hour shift. All of her other costs (fuel, etc.) add up to $0.17/km. The average fare collected from the passengers is $0.97 per km driven. If she wants to have a net income of $12.00 per hour for her 12 hours shifts, how many km does she need to drive on average during any hour of her shift?

   4. _____ km

5. 1000 chicken lay on average 4725 eggs in one week (7 days). How many eggs will a flock of 3500 chicken lay in 1000 days?

   5. _____ eggs

6. You got a loan of $10000 at a certain yearly interest. At the end of the year you repaid 1/6 of the balance owed (including interest). The balance at the beginning of the second year is now $9000. What was the yearly interest rate?

6. _____ percent

7. You can select from the following Canadian coins: 1c, 5c, 10c, 25c, 1$, 2$. What is the smallest amount (in cents) you cannot make with 12 or fewer of these coins?

7. _____

8. The following are the coordinates of the vertices of hexagon $ABCDEF$: $A(\sqrt{2}, 0)$, $B(6, 6 - \sqrt{2})$, $C(6, 4 + 3\sqrt{2})$, $D(4, 4 + 3\sqrt{2})$, $E(0, 3\sqrt{2})$, $F(0, \sqrt{2})$. Find the area of the hexagon. Express your answer correct to 2 decimal places.

8. _____ units^2
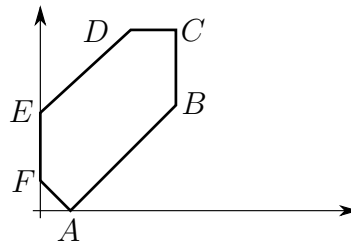
9. There are points inside the hexagon of question 8 which are the furthest away from any point on the boundary of the hexagon. Find the smallest y-coordinate of any such point. Give the answer correct to 2 decimal places.

9. _____

10. Find the largest x-coordinate of any point of the type described in question 9? Give the answer correct to 2 decimal places.

10. _____

11. A commercial airplane lands at Vancouver airport (altitude of 0 km). It starts its descent from an altitude of 11 km, flying along the hypotenuse of the triangle below, which is not drawn to scale. The airplane reduces its speed at a constant rate from an initial speed of 840 km/hr to 240 km/hr when it touches the ground 37.5 minutes later. How long is the horizontal distance between the location where it started its descent and the location where it touches the ground? Express you answer in km correct to 2 decimal digits.

11. _____ km

11
??   YVR
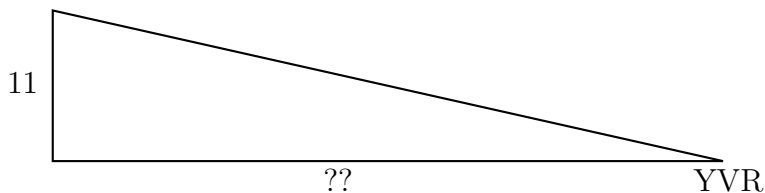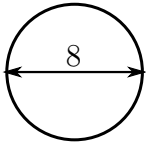
12. A garden has the shape of a right triangle. Its three corners are located at $A(54, 0)$, $B(-b, 0)$ and $C(0, 72)$, where AB is the hypotenuse. Three equally long straight garden paths connect the three corners to $M(x, y)$. What is the value of x?

12. _____

13. Consider the set $S = \{1, \frac{1}{2}, \frac{1}{3}, \ldots, \frac{1}{24}, \frac{1}{25}\}$. How many ordered triples (a, b, c) of numbers are there such that a, b, and c are in S and $a - b = c$ and $b \geq c$? Please note that $(1, \frac{1}{2}, \frac{1}{2})$ and $(\frac{1}{2}, \frac{1}{3}, \frac{1}{6})$ are two such triples.

13. _____ triples

14. 5 men and 5 women sit at a round table. You know that 3 of the women sit next to each other. If you assume that the other two women took their seats at random, what is the probability that neither of these two women sits next to a woman? Express the answer as a common fraction.

14. _____

15. The triangle below is right-angled. The two circles have radius 2 and 1 respectively, and the circles and triangle touch as shown. What is the length of the hypotenuse? Express the answer as a decimal, to two places after the decimal point.

15. _____ units

1. What is the smallest prime larger than 90?

1. _____

2. What is the digit sum of 2015?

2. _____

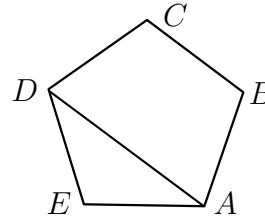3. How many different sums can you get when you throw 3 dice?

3. _____ sums

4. Round the area of a circle of diameter 8 to the nearest integer.

4. _____

5. You add 13 to 10% of N and then you multiply the result by 10 to get 156. What is the value of N?

5. _____

6. What is the smallest 3-digit number all of whose digits are different and which does not use the digit 1?

6. _____

7. If the width of a rectangular billboard is 4 times its height, and if its area is 25 m^2, what is the value of its width, in metres?

7. _____ metres

8. *ABCDE* is a regular pentagon. What is the value, in degrees, of angle *EAD*?

8. _____ degrees

9. What is the greatest common factor of 42, 12, 18, and 132?

9. _____

10. What is the least positive common multiple of the numbers in Question 9?

10. _____

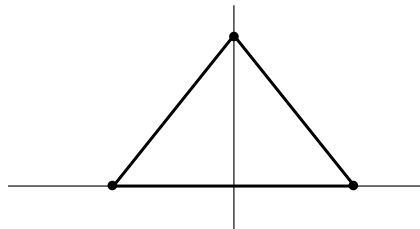11. What is the binary representation of 2015?

11. _____

12. The first term of a sequence is 0. For $n > 1$, the n-th term is $n! - (n-1)!$. What is the difference between the 6-th term and the 4-th term?

12. _____

13. The vertices of a triangle have coordinates $(-40.3, 0)$, $(40.3, 0)$, and $(0, y)$. What is the positive value of y such that the area of the triangle is 2015?
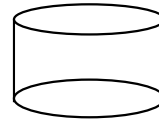
13. _____

14. Express $1 + \dfrac{1}{2} + \dfrac{1}{4} + \dfrac{1}{8} + \cdots + \dfrac{1}{1024}$ as a common fraction.

14. _____

15. Irene went to the store and bought a laptop that was listed for $1000. The laptop was on sale at 15% off. On the amount after the discount, she had to pay 12% tax. How many dollars did she pay in total?

15. _____ dollars

16. Let $N = 100 \times 36 \times 63$. How many factors does N have?

16. _____ factors

17. The volume of a cylinder is 64000π cm^3, and its height is 40 cm. What is the surface area of the cylinder (including both bases)? Provide the answer rounded to the nearest 100 cm^2.

17. _____ cm^2

18. The sum of 40 distinct positive integers is equal to 1000. What is the smallest possible value of the biggest integer?

18. _____

19. Good garden compound can be made by combining 8 bags of 25 litres topsoil at density of 2/5 kg/litre, 4 bags of 30 litres mushroom compost at density of 1/3 kg/litre, and 1 bag of 25 litres of fertilizer at density of 6/5 kg/litre. By weight, what percentage of the compound is fertilizer?
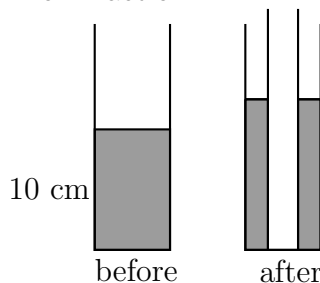
19. _____ percent

20. After mixing, the volume of the compound in Question 19 shrinks to 80% of the original combined volumes of the components. What is the final volume of the compound in litres?

20. _____ litres

21. Round the cube root of 2015 to the nearest integer.

21. _____

22. A 5 cm diameter cylinder is partially filled with water so that the top surface of the water is 10 cm above the bottom. A tall 2 cm diameter solid cylinder is inserted in the larger cylinder so that its bottom touches the bottom of the larger cylinder. What is the new distance from the surface of the water to the bottom of the larger cylinder? Express the answer in cm, as a common fraction.
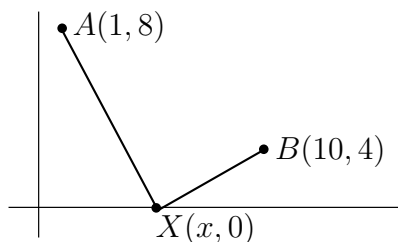
22. _____ cm

10 cm

before   after

23. Frank rents a stand to sell cloth. His average cost for an item is $5.00, and he has to pay to the owner of the stand 5% of the selling price. At what price does he have to sell the item so that his net profit per item sold is $3.00? Give the answer in dollars, correct to 2 places after the decimal point.

23. _____ dollars

24. Points $A(1,8)$ and $B(10,4)$ have been connected by lines to point $X(x,0)$. What is the smallest possible value of $AX + XB$?

24. _____

$A(1,8)$

$B(10,4)$

$X(x,0)$

25. In Question 24, what is the value of x?

25. _____

26. For every km that a locomotive goes, it burns 5 litres of fuel. For any car that it pulls, it burns (on average) another 0.15 litres of fuel per km. A very long train made up of 1 locomotive and many cars is travelling between 2 refueling stations, a distance of 850 km. If the fuel capacity of the locomotive is 15470 litres, how many cars can the train have?

26. _____ cars

1. Two tablets cost $50 more than one laptop. If a tablet costs $294, what is the cost of a laptop, in dollars?

1. _____ dollars

2. A new animal has evolved in Canada. It is 1/4 coyote, 3/32 wolf, and the rest is dog. What fraction of the animal is dog?

2. _____

3. The 9 squares in the 3×3 magic square below are to be filled in so that the sum of the three numbers in any row, the sum of the three numbers in any column, and the sum of the three numbers in each of the two diagonals, are all the same. Numbers in some of the squares have already been filled in. Find the number which should be put in the square labelled "?".
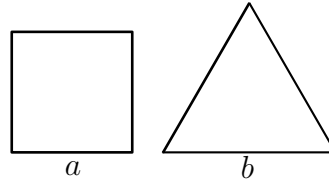
3. _____

10	3	
		?
−3		

4. A cargo train carries 4 kinds of products, oil, gas, coal, and potash. The amount of gas by weight is 1/8 of the amount of oil. The amount of coal is 1/10 of the combined amounts of potash and gas, and the amount of potash is 5/6 of the combined amounts of gas and coal. What fraction of the total amount is the oil?

4. _____

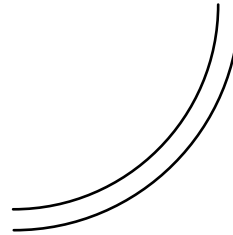## Bull's-eye, Page 2: Numbers and Combinatorics

5. The sum of opposite faces of a traditional die is always 7. Two dice are rolled and the sum of the top faces is greater than 8. How many possible sums can the bottom faces have?

5. _____ sums

6. As in Question 5, the sum of the top faces is greater than 8. What is the probability that there is no 1 on any of the bottom faces? Express the answer as a common fraction.

6. _____

7. What is the smallest positive integer which is divisible by 45 and whose decimal representation uses only the digits 0, 1, and 2?

7. _____

8. How many positive multiples of 3 or 5 or 134 are there that are less than 2015?
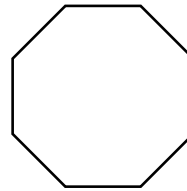
8. _____

9. A square of side a and an equilateral triangle of side b have the same area. Express the value of $\frac{b^4}{a^4}$ as a common fraction.
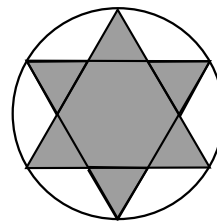
9. _____

10. Railway tracks form the shape of a quarter-circle. The distance between the tracks is 56 inches. How much longer is the outer track than the inner track? Provide your answer to the nearest inch.

10. _____ inches

11. Find the area of a regular octagon with side 1. Express your answer as $k\left(m + \sqrt{n}\right)$, where k, m, and n are integers and n is prime.

11. _____ units^2

12. A star of David with area A is inscribed in a circle with area 3π. What is the value of A^2?
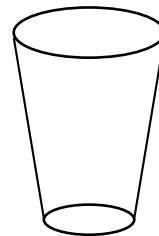
12. _____

Co-op, Prov. 2015, Page 1: Team answers must be on the *coloured* page. Answers on a white page will not be graded.

1. The Fibonacci sequence (F_n) is defined as follows: $F_1 = 1$, $F_2 = 1$ and for $n > 2$ by $F_n = F_{n-1} + F_{n-2}$. Find the largest n such that $F_1 + F_2 + F_3 + \cdots + F_n < 100$.

1. _____

2. In Question 1, what is the value of $F_1 + F_2 + F_3 + \cdots + F_{15}$?

2. _____

3. For every integer $n \geq 1$, the sum of the first n terms of a sequence is equal to n^2. What is the 100-th term of the sequence?

3. _____

4. A pail is placed outside to catch rainfall. The pail has the shape of a truncated cone with base diameter 30 cm, opening diameter 50 cm, and a height of 60 cm. Initially the water level is at 10 cm above the bottom of the pail. A few days later the water has risen to a level of 20 cm from the bottom of the pail. How many cubic cm of water were added? Round your answer to the nearest 100 cm^3.

4. _____ cubic cm

5. If in Question 4 the rainfall is at the rate of 10 mm/hour, how many hours will it take to fill the entire pail if originally it was empty? Round your answer to the nearest integer.

5. _____ hours

6. How many ordered triples (a, b, c) of real numbers are there such that each of the numbers is the product of the other two? Note that for example the ordered triple $(2, -3, 0)$ is not the same as the ordered triple $(-3, 0, 2)$.

6. _____ triples

7. A group of 50 kids were chatting over the internet. Each of them sent a different odd number of text messages, except for 3 kids who each sent the same odd number of messages. What is the smallest number of text messages that could have been sent?

7. _____ messages

8. Nimoy drives a car starting at a speed of 0 km/h and accelerating at a constant rate, and driving along the circular road around CERN (the Large Hadron Collider). When he finished one circle he reached a speed of 120 km/h. It took him exactly 1720 seconds to reach that speed. Find the radius of the circle in km correct to one decimal.

8. _____ km

9. You throw three dice. What is the probability that the sum is a multiple of 5? Express the answer as a common fraction.

9. _____

10. The class of 1983 has gathered for a reunion in 2015 to celebrate their 50-th birthdays (all were born in 1965 which had 365 days). Of the 365 days, 310 have no birthday of any of the students. K days have the birthday of 1 student, $K - 1$ days have the birthdays of 2 students, $K - 2$ have the birthdays of 3 students, and so on. So 3 days have the birthdays of $K - 2$ students, 2 days have the birthdays of $K - 1$ students, and 1 day has the birthdays of K students. How many students were in the class of 1983?
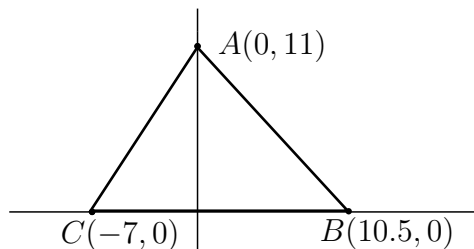
10. _____ students

11. Niki generates a 4-character PIN (password) using the allowed 10 digits, 26 lower-case letters, and 26 upper-case letter. She plans to use exactly 2 (not necessarily different) digits and 2 letters. How many different passwords can she select from? Examples of valid passwords: 13Aa, 13aa, A22b, and c3D1.

11. _____ passwords

12. If as in Question 11 Niki plans to use exactly 2 digits and exactly 3 letters for a 5 character password, how many different passwords can she select from if the 2 digits must be next to each other? Examples of valid passwords: 00AAb, o12bC, XY98Z.

12. _____ passwords

13. The vertices of triangle ABC have coordinates $A(0, 11)$, $B(10.5, 0)$, and $C(-7, 0)$. Find the area of $\triangle ABC$. Express your answer correct to 2 decimal places.

13. _____

14. Triangle ABC is as in Question 13. Find the x-coordinate of the point (x, y) that has the same distances from A, B, and C. Give the answer correct to 2 decimal places.

14. _____

15. What is the y-coordinate of the point (x, y) of Question 14? Give the answer correct to 2 decimal places.

15. _____

Math Challengers Regional 2014
Answers, Blitz Stage

1. 5

2. 81

3. 11

4. 3

5. 4

6. 8

7. 15

8. 1

9. 1525.5

10. $\dfrac{170}{77}$

11. 10.32

12. 4.8

13. 34

14. $\dfrac{9}{31}$

15. 5060

16. 28

17. 7

18. $\dfrac{241}{264}$

19. 31

20. $\dfrac{19}{18}$

21. -2

22. $\dfrac{5}{4}$

23. 2014

24. $\dfrac{4027}{2}$

25. 39

26. $\dfrac{3\sqrt{3}}{2}$

Math Challengers Regional 2014
Answers, Bull's-eye Stage

1. 50

2. 58

3. $\dfrac{50}{3}$

4. $\dfrac{120}{47}$

5. 45

6. 10

7. $\dfrac{1}{8}$

8. 25

9. 16

10. $\dfrac{3}{4}$

11. $\dfrac{49}{100}$

12. $(7 - \sqrt{45})\pi$

Math Challengers Regional 2014
Answers, Co-op Stage

1. $\dfrac{7}{720}$

2. 8

3. $\dfrac{1}{5}$

4. $\dfrac{13}{72}$

5. 3300

6. 9.45

7. 792

8. 0.973

9. 4

10. 12

11. 2.72

12. 55

13. 33

14. 1.006

15. 11

Math Challengers Provincial 2013
Answers, Blitz Stage

1. 45

2. 15

3. $\dfrac{2}{3}$

4. 1

5. 72

6. 1

7. 110

8. -333

9. 12

10. 16.5

11. $\dfrac{25}{36}$

12. $\dfrac{4}{11}$

13. 200

14. 12

15. 85

16. $12\sqrt{2}$

17. 0

18. 9072

19. 3

20. 36

21. $\dfrac{65}{37}$

22. 145

23. 537

24. 231

25. $\dfrac{37}{72}$

26. $\dfrac{9+\sqrt{3}\pi}{18}$

Math Challengers Provincial 2013
Answers, Bull's-eye Stage

1. $\dfrac{6}{19}$

2. 20

3. 385

4. 2940

5. 61

6. 24

7. 708

8. $\dfrac{1}{16}$

9. $\dfrac{10\sqrt{3}}{3}$

10. 1.3

11. $\dfrac{420}{29}$

12. $\dfrac{3\sqrt{3}}{4}$

Math Challengers Provincial 2013
Answers, Co-op Stage

1. 17

2. 1.79

3. 505

4. 28

5. 2362500

6. 8

7. 794

8. 29.94

9. 3.41

10. 4.00

11. 337.32

12. -21

13. 20

14. $\dfrac{2}{7}$

15. 9.84

1. One person is chosen at random from a group of 5 kids, 2 of whom are girls. What is the probability that the person is a boy? Express the answer as a common fraction.

1. _____

2. Let $N = 1 + 3 + 5 + 7$. What is the value of N?

2. _____

3. What is the area of a right-angled triangle whose two smallest sides are 7 and 16?
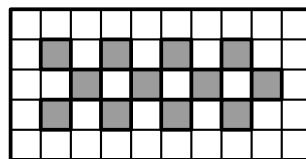
3. _____ units^2
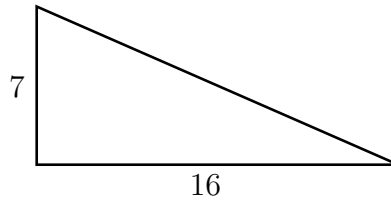
4. You bought three sandwiches at a cost of $5.75 each. How much did they cost in total? Express your answer in dollars correct to 2 decimal places.

4. _____ dollars

5. Let $N = 20\%$ of 30% of 200. What is the value of N?

5. _____

6. Find the average of the terms in the following arithmetic sequence: $5, 9, \cdots, 25$.

6. _____

7. What percentage of the rectangle below is shaded?

7. _____ percent

8. Two fair dice are rolled. What is the probability that we get the same number on both dice? Express your answer as a common fraction.

8. _____

9. The regular hexagon $ABCDEF$ has area 1. What is the area of $\triangle ABF$? Express the answer as a common fraction.
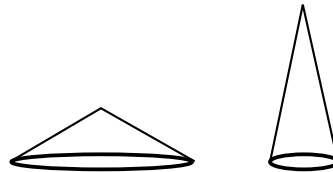
9. _____
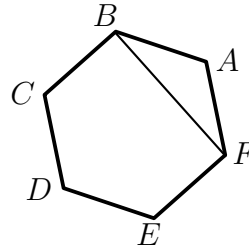
10. Simplify to a single fraction: $\dfrac{1}{2} + \dfrac{2}{3} + \dfrac{3}{4}$.

10. _____

11. Round $\frac{243}{23}$ to the nearest integer.

11. _____

12. What is the largest prime that divides both 143 and 195?

12. _____

13. Cone A has height h and base radius r. Cone B has height $4h$ and base radius $\frac{r}{3}$. What is the ratio of the volume of cone A to the volume of cone B? Express your answer as a common fraction.

13. _____

14. How many factors does 36 have? Note that 1 and 36 are factors of 36.

14. _____ factors

15. The speed of a cargo ship in still water is 10 km/h. If 4 hours are required for that ship to travel 16 km up a river (against the river's current), what is the speed of the river's current, in km/h?

15. _____ km/h

16. The speed of a cargo ship in still water is 10 km/h. If the ship is traveling downriver on a river that is flowing at 5 km/h, how many *minutes* does it take for the cargo ship to travel 16 km downriver?

16. _____ minutes

17. Express $\frac{3}{1\times2} - \frac{5}{2\times3} + \frac{7}{3\times4}$ as a common fraction.

17. _____

18. You bought a book at a cost of 19.78 US dollars. If a Canadian dollar is worth 0.92 US dollars, how much did you pay in Canadian dollars? Give the answer to two decimal places.

18. _____ Can. $

19. Two solid concentric rings are connected and the inner ring rotates at a rate of 2 m/s. If the radii of the rings are 2 m and 5 m, at what rate does the outer ring rotate (in m/s)?

19. _____ m/s
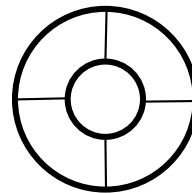
20. If the inner ring in Question 19 rotates at $\frac{2\pi}{3}$ m/s, what is its rate of rotation in degrees per second?

20. _____ deg/s

21. In a class of 29 students, 19 got an A in Math. Of the students who did not get an A in Math, four-fifths did not get an A in Language Arts. Overall, 17 students got an A in Language Arts. If you choose a student at random, what is the probability that the student got an A in both Math and Language Arts? Express the answer as a common fraction.

21. _____

22. The values of the third and sixth terms of a geometric sequence are respectively $\frac{15}{8}$ and $-\frac{405}{64}$. What is the first term of the sequence? Express the answer as a common fraction.

22. _____

23. What is the sum of all of the prime factors of 2015?

23. _____

24. A drinking glass with inner diameter 8 cm and height 12 cm is half-full of water. A sphere of diameter 6 cm is carefully lowered into the glass until the sphere is exactly half submerged. By how many cm does the water level in the glass *rise*? Express the answer as a common fraction.

24. _____ cm

25. All interior lines meet the outer square at a 45° angle. How many right triangles are in the figure below?

25. _____
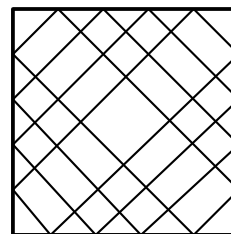
26. A point in the plane is chosen at random from all points with integer coordinates (u, v) such that $1 \le u \le 9$ and $1 \le v \le 9$. What is the probability that the absolute value of the difference between the x-coordinate and the y-coordinate of the point is greater than 2? Express the answer as a common fraction.

26. _____

1. Suppose that 105 kg of brine (which is a solution of water and salt) consists of 20 kg of salt, with the rest water. How many kg of water must be added so that the resulting solution is 8% salt by weight?

1. _____ kg

2. A student has a grade of 73% going into the final exam. The final exam is worth 60% of the final grade. What is the grade (in percent) that the student must achieve on the final exam in order to obtain a final grade of 70%?

2. _____ percent

3. Between them, Alicia, Beti, Cyril, and Dan own 700 books. Alicia has 40 more books than Beti, 200 more books than Cyril, and 300 more books than Dan. How many books does Alicia own?

3. _____ books

4. Amy and Beth walk along a loop in opposite directions, starting from the same place. Amy's speed is $\frac{7}{5}$ times Beth's speed. They first meet after 24 minutes of walking. . How many minutes will it take from the time they meet for the *second* time for Amy to reach the starting point? Express the answer as a common fraction
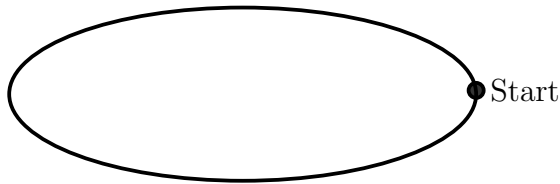
4. _____ minutes

5. You have the following collection of beads in your drawer: 10 are
white, 4 are black, 3 are blue, 2 are yellow, and 1 is green. You take
out of your drawer at least 5 white beads, and at least 1 of each of
the other colours. How many different total numbers of beads could
you have taken out?

5. _____

6. How many *different* prime numbers are factors of 4680?

6. _____

7. The figure below shows streets. The line segments that connect ad-
jacent intersections are of unit length. Assume that you walk full
segments, and at any intersection you decide which segment to use
next (including travelling back along the same segment). In how
many ways can you walk along the grid of streets starting and ending
at A, walking through B, and walking a total of 6 units?

7. _____ ways
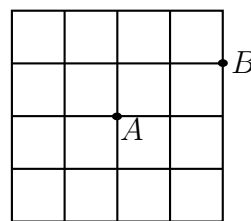
8. Suppose that as in Question 7, you start at A, and walk a total of 3
segments, not necessarily distinct. If all paths of length 3 are equally
likely, what is the probability that you end up at B?

8. _____

9. If you increase the radius of the base of a cylinder by 200% and you increase its height by 100%, by how many percent do you increase its volume?

9. _____ percent

10. In the diagram, the shape that looks like a square *is* a square, and dimensions along the hypotenuse of the big triangle are as shown. What is the area of the square? Express the answer as a common fraction.

10. _____ units^2
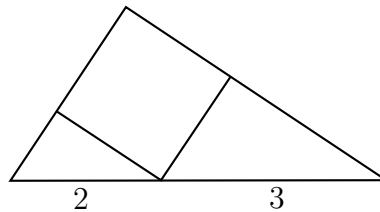
11. The cross in the picture is made up of five 2×2 squares. The outer curve is made up of four arcs of circles. What is the total area enclosed by the outer curve? Express the answer in terms of π.

11. _____ units^2
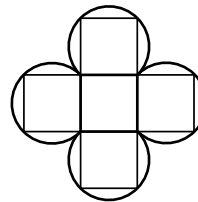
12. The bottom edge of the large triangle has been divided into four equal parts as shown and the three lines that are shown as parallel *are* parallel. What is the ratio of the combined area of the two shaded triangles to the area of the unshaded part of the large triangle? Express the answer as a common fraction.
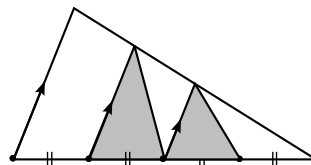
12. _____

1. A Ferris wheel has radius 12 m and 24 equally spaced seats labelled 1 to 24. Seat #1 is the lowest seat and is 2 m above the ground. How high (in m) above the ground is seat #9?
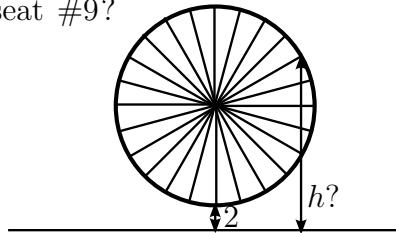
   1. _____ metres

2. Find N: $N = 1^2 - 2^2 + 3^2 - 4^2 + \cdots + 99^2 - 100^2$.

   2. _____

3. A saleswoman at a street corner rents her stand for $1500 for a period of 90 days, and pays to the city a usage fee of $35 per day. She sells souvenirs at a rate of 18 per hour to passers by. If her cost for 20 souvenirs is $5, and each is sold for $1, and if she sells for 14 hours every day, how many dollars is her total profit in the 90 day period?

   3. _____ dollars

4. A conical storage tank of height 10 m and radius $\frac{27}{\sqrt{\pi}}$ m is used to store water for the local community. The tank was full at the beginning of a 40 day dry period during which no water entered the tank. However, continued water usage caused the tank's water level to fall to $\frac{1}{3}$ of its maximum height. What was the average daily usage of water, in m^3 per day, during the 40 day dry period? Give the answer correct to 1 decimal place.

   4. _____ m^3/day
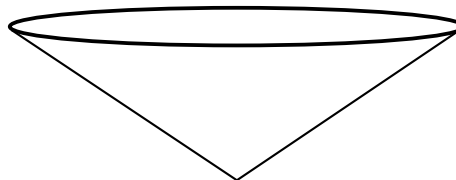
5. For the storage tank of Question 4, and water consumption of 54000 litres per day, how many days of water supply can the tank hold? .

   5. _____ days

6. The operation & is defined by $a\&b = 2(a+b) - (a+b)^3$. What is the value of $(1\&1)\&1$?

6. _____

7. Positive and negative electrical pulses are sent over an electrical channel to a receiver. A positive pulse represents a ONE (1); a negative pulse represents a ZERO (0). A sequence of these binary digits (bits) is used to transmit data. Two of the many possible 8-bit sequences are: 00110011 and 10101010, How many different possible 8-bit sequences are there?

7. _____ sequences

8. Suppose that in Question 7, channel noise causes any bit to be received in error (i.e. a ONE is received as ZERO or ZERO is received as ONE) with probability 0.000011. Suppose also that bit errors are independent of each other (i.e. an error in any specific bit has no effect on whether or not an error occurs in any other bit). What is the largest number of bits that can be transmitted so that the probability that all are received correctly is still larger than 0.999?

8. _____ bits

9. Suppose now that in Question 8 the probability of a bit to be received in error through a very noisy channel is 0.1. Suppose also that if a bit is received in error then for each subsequent bit, the probability it is received in error is 0.5. A 4-bit message is sent. What is the probability that at least 3 bits are received correctly? Give your answer correct to 4 significant digits.

9. _____

10. Find the sum of all the numbers from 1 to 1111 (inclusive) whose decimal expansion has no digits other than 0 or 1.

10. _____

11. In questions 11, 12, 13 and 14 you are expected to use similar triangles to approximate some conditions of Solar and Lunar eclipses. The diameter of the Earth is 12714 km (the line FG). The distance from the Sun to Earth is 149598000 km (the distance from point E to side AB of the isosceles triangle ABE). The Sun's diameter is 1392700 km (AB). The Moon's diameter is 3475 km (the side CD of the isosceles triangle CDE which illustrates the shadow of the Moon). The top diagram shows the case where the Moon is at the furthest distance from Earth and still projects full shadow somewhere on Earth (full solar eclipse). What is the distance (in km) from point E to line CD? Give your answer in km to the nearest 1000 km.

11. _____ km
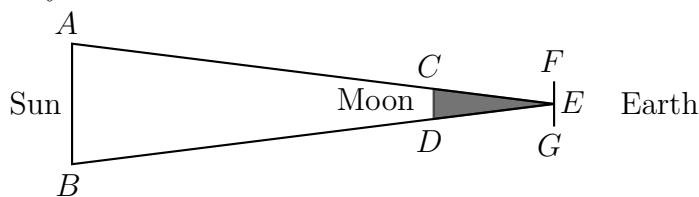
12. The Earth projects its own shadow into space (the isosceles triangle FGH). How far from Earth (in km) does this shadow extend (the distance from point H to FG)? Note that ABH is also isosceles. Give your answer to the nearest 1000 km.

12. _____ km
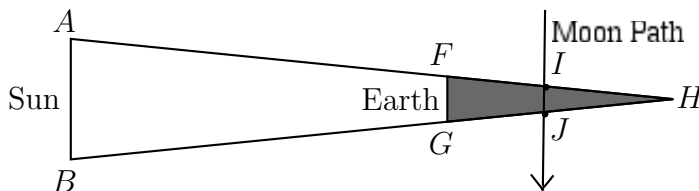
13. Use the approximation (to the nearest 1000 km) of the distance from point H to FG. Suppose that the Moon travels through the shadow of the Earth (along the line IJ of the isosceles triangle IJH). Suppose that the distance from Earth to Moon at that event is 400000 km (the distance from FG to IJ). How long is the path of the Moon during the event (the length of IJ)? Give your answer in km to the nearest km.

13. _____ km

14. If the Moon travels at 3550 km/hr, how many minutes does it take for it to travel a distance of length 10000 km? Give your answer correct to the nearest minute. This is a typical length of a lunar eclipse.

14. _____ minutes

15. All the points in the 4×5 grid below are at distance 1 from their nearest horizontal and vertical neighbours. If you select 2 different points at random, what is the probability that the straight line distance between them is less than or equal to 2? Express your answer as a common fraction.
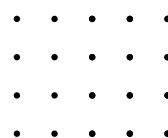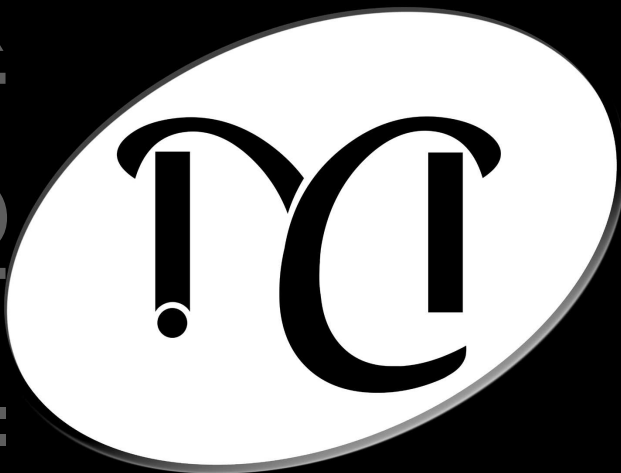
15. _____

Math Challengers

Regional Competition
Face-off Round 2015

A question always follows a blue page. The next page is blue!

1. How many digits are there in the binary representation of 2015?

1. How many digits are there in the binary representation of 2015?

Answer: 11 (digits)

2. Of all the two-digit primes, what is the largest that consists of two consecutive digits?

2. Of all the two-digit primes, what is the largest that consists of two consecutive digits?

Answer: 89

3. The product of two positive consecutive odd integers is less than 100. What is the largest possible value of their sum?

3. The product of two positive consecutive odd integers is less than 100. What is the largest possible value of their sum?

Answer: 20

4. The area of square B is 300% larger than the area of square A. How much larger is the perimeter of square B (in percent) than the perimeter of square A?

4. The area of square B is 300% larger than the area of square A. How much larger is the perimeter of square B (in percent) than the perimeter of square A?

Answer: 100 (percent)

5. You write all the letters of the alphabet, in order, over and over again, like this:

abcd. . . xyzabcd. . . xyzabcd. . . xyzab . . .

What is the 2015-th letter that you write?

5. You write all the letters of the alphabet, in order, over and over again, like this:

abcd...xyzabcd...xyzabcd...xyzab ...

What is the 2015-th letter that you write?

Answer: m

6. Simplify $\dfrac{6^3}{3^6}$.

6. Simplify $\dfrac{6^3}{3^6}$.

Answer: $\dfrac{8}{27}$

7. You bought a mobile phone whose list price was $299, at a 30% discount. How much was the discounted price, to the nearest dollar?

7. You bought a mobile phone whose list price was $299, at a 30% discount. How much was the discounted price, to the nearest dollar?

Answer: 209 (dollars)

8. If a train travels at a speed of 105 km per hour for 1 hour and 20 minutes, how many km does it travel?

8. If a train travels at a speed of 105 km per hour for 1 hour and 20 minutes, how many km does it travel?

Answer: 140 (km)

9. Round $\dfrac{2015}{300}$ to the nearest integer.

9. Round $\dfrac{2015}{300}$ to the nearest integer.

Answer: 7

10. What is the probability of getting 5 heads in a row when tossing a fair coin? Express your answer as a common fraction.

10. What is the probability of getting 5 heads in a row when tossing a fair coin? Express your answer as a common fraction.

Answer: $\dfrac{1}{32}$

11. The measure of one angle of a 4-sided polygon is 99 degrees. The other 3 angles all have equal measure. What is that measure, in degrees?

11. The measure of one angle of a 4-sided polygon is 99 degrees. The other 3 angles all have equal measure. What is that measure, in degrees?

Answer: 87 (degrees)

12. What is the smallest integer N such that $3^N > 2015$?

12. What is the smallest integer N such that $3^N > 2015$?

13. Of the 5 members of a Math Challengers Team, 2 are selected to be co-captains. In how many ways can this be done?

13. Of the 5 members of a Math Challengers Team, 2 are selected to be co-captains. In how many ways can this be done?

Answer: 10 (ways)

14. What is the largest solution of the equation $x^2 - 13x + 22 = 0$?

14. What is the largest solution of the equation $x^2 - 13x + 22 = 0$?

Answer: 11

15. What is the square of the distance from X to Y?
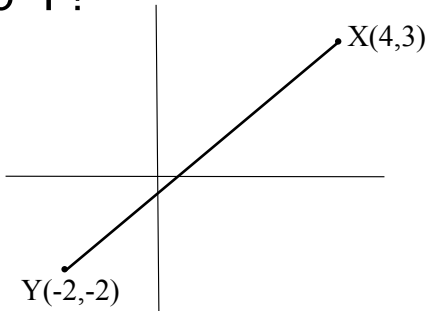
15. What is the square of the distance from X to Y?
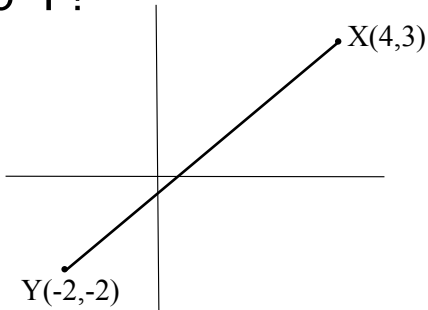
Answer: 61

16. Round $\sqrt{300}$ to the nearest integer.

16. Round $\sqrt{300}$ to the nearest integer.

Answer: 17

17. A cm^2 on a map represents an area of 0.36 km^2. How many meters are represented by one cm?

17. A cm^2 on a map represents an area of 0.36 km^2. How many meters are represented by one cm?

Answer: 600 (meters)

18. You have 2015 five-cent coins in your treasure box. What is the value of your treasure in dollars, correct to 2 decimal places?

18. You have 2015 five-cent coins in your treasure box. What is the value of your treasure in dollars, correct to 2 decimal places?

Answer: 100.75 (dollars)

19. What is the decimal representation of the reciprocal of $\dfrac{8}{7}$?

19. What is the decimal representation of the reciprocal of $\frac{8}{7}$?

Answer: 0.875 (or .875)

20. The density of a 3.75 kg rock is 2.5 grams per cm^3. What is the volume of the rock in cm^3?

20. The density of a 3.75 kg rock is 2.5 grams per cm^3. What is the volume of the rock in cm^3?

Answer: 1500 (cm^3)

21. What is the value of the 2015-th term of the arithmetic sequence whose first three terms are -3998, -3996, and -3994?

21. What is the value of the 2015-th term of the arithmetic sequence whose first three terms are -3998, -3996, and -3994?

Answer: 30

22. How many diagonals go through the centre of a regular 9-sided polygon?
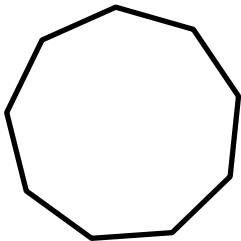
22. How many diagonals go through the centre of a regular 9-sided polygon?
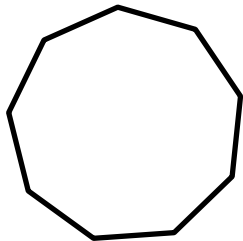
Answer: 0 (zero)

23. You write the first 30 whole numbers. How many times did you write the digit 1?

23. You write the first 30 whole numbers. How many times did you write the digit 1?

Answer: 13 (times)

24. What is the largest integer N that satisfies $N^2 < 5000$?

24. What is the largest integer N that satisfies $N^2 < 5000$?

Math Challeng

25. What percentage of 806 is 2015?

25. What percentage of 806 is 2015?

Answer: 250 (percent)

26. A regular polygon with 2015 sides is divided into two congruent polygons by drawing a line from a corner to the midpoint of its opposite edge. How many sides does each of the two polygons have?

26. A regular polygon with 2015 sides is divided into two congruent polygons by drawing a line from a corner to the midpoint of its opposite edge. How many sides does each of the two polygons have?

Answer: 1009 (sides)

27. Two consecutive odd numbers that are both prime are called a pair of twin primes. For example, 11 and 13 are twin primes. What is the lowest possible sum of a pair of twin primes each larger than 20?

27. Two consecutive odd numbers that are both prime are called a pair of twin primes. For example, 11 and 13 are twin primes. What is the lowest possible sum of a pair of twin primes each larger than 20?

Answer: 60

28. Define $x \# y$ by $x \# y = 2x + 3y$. What is the value of $(1 \# 1) \# 1$?

28. Define $x \# y$ by $x \# y = 2x + 3y$. What is the value of $(1\#1)\#1$?

Answer: 13

29. Simplify:
$(2 + 0 + 1 + 5)(2 - 0 + 1 - 5)$.

29. Simplify:
$(2 + 0 + 1 + 5)(2 - 0 + 1 - 5)$.
Answer: -16

30. If $x + y = 15$ and
$2x + 3y = 22$, what is the value of x?

30. If $x + y = 15$ and
$2x + 3y = 22$, what is the value of x?

Answer: 23

31. Find the volume of a cube whose surface area is 96.
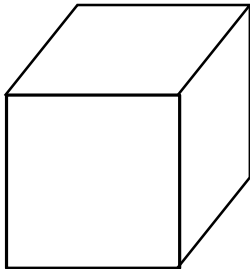
31. Find the volume of a cube whose surface area is 96.
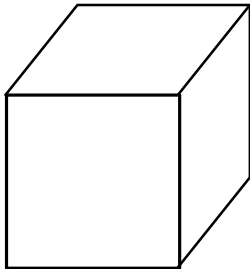
Answer: 64

32. The numbers M and N are both prime. Each is smaller than 40. It is known that $M < N$ and $M + N = 66$. What is the value of M?

32. The numbers M and N are both prime. Each is smaller than 40. It is known that $M < N$ and $M + N = 66$. What is the value of M?

Answer: 29

33. At a restaurant buffet, there are only 2 flavours of ice cream available: durian and lime. In how many ways can Alan choose 2 scoops of ice cream? Note that durian and lime is the same choice as lime and durian, and Alan can choose two scoops of the same flavour.)

33. At a restaurant buffet, there are only 2 flavours of ice cream available: durian and lime. In how many ways can Alan choose 2 scoops of ice cream? Note that durian and lime is the same choice as lime and durian, and Alan can choose two scoops of the same flavour.)

Answer: 3 (ways)

34. The first two terms of the Lucas sequence are 1 and 3, and after that any term of the Lucas sequence is the sum of the previous two terms. What is the 4-th term of the Lucas sequence?

34. The first two terms of the Lucas sequence are 1 and 3, and after that any term of the Lucas sequence is the sum of the previous two terms. What is the 4-th term of the Lucas sequence?

Answer: 7

35. Alicia has a total of $1.95 in standard Canadian coins (no pennies). Given that Alicia has no 50 cent coins, what is the smallest number of coins that Alicia could have?

35. Alicia has a total of $1.95 in standard Canadian coins (no pennies). Given that Alicia has no 50 cent coins, what is the smallest number of coins that Alicia could have?

Answer: 6 (coins)

36. There are 4 jelly beans in a tray, 2 blue and 2 yellow. Alphonse eats 2 of the jelly beans, chosen at random. What is the probability that these 2 jelly beans are of *different* colours? Express your answer as a common fraction.

36. There are 4 jelly beans in a tray, 2 blue and 2 yellow. Alphonse eats 2 of the jelly beans, chosen at random. What is the probability that these 2 jelly beans are of *different* colours? Express your answer as a common fraction.

Answer: $\dfrac{2}{3}$

Math Challenge

37. A high school runner ran 3000 metres in 9 minutes. What was the runner's average speed in kilometres per hour?

37. A high school runner ran 3000 metres in 9 minutes. What was the runner's average speed in kilometres per hour?

Answer: 20 (km/hr)

38. The sum of the ages of the 20 people in the class is 302 years. What will the sum of their ages be 2 years from now?

38. The sum of the ages of the 20 people in the class is 302 years. What will the sum of their ages be 2 years from now?

Answer: 342 (years)

39. What is the smallest positive integer n such that

$$1 + 2 + 3 + \cdots + (n-1) + n$$

is a multiple of 10?

39. What is the smallest positive integer n such that

$$1 + 2 + 3 + \cdots + (n - 1) + n$$

is a multiple of 10?

Answer: 4

40. What is the smallest prime number which is larger than 89?

40. What is the smallest prime number which is larger than 89?

Answer: 97

41. What is the product of the greatest common factor and the least common multiple of 12 and 15?

41. What is the product of the greatest common factor and the least common multiple of 12 and 15?

180

42. Alfie has 200 dollars, and Bert has 10 dollars. How many dollars should Alfie give to Bert so that Alfie will have 4 times as many dollars as Bert?

42.   Alfie has 200 dollars, and Bert has 10 dollars. How many dollars should Alfie give to Bert so that Alfie will have 4 times as many dollars as Bert?

Answer: 32 (dollars)

43. Simplify
$$\left(1 + \frac{1}{4}\right)\left(1 + \frac{1}{5}\right)\left(1 + \frac{1}{6}\right)\left(1 + \frac{1}{7}\right)$$

43. Simplify

$$\left(1 + \frac{1}{4}\right)\left(1 + \frac{1}{5}\right)\left(1 + \frac{1}{6}\right)\left(1 + \frac{1}{7}\right)$$

Answer: 2

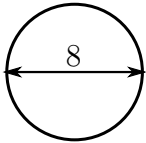**44.** What is the smallest positive integer N such that $20N$ is a perfect square?

44. What is the smallest positive integer N such that $20N$ is a perfect square?

Answer: 5

1. What is the smallest prime larger than 90?

   1. _____

2. What is the digit sum of 2015?

   2. _____

3. How many different sums can you get when you throw 3 dice?

   3. _____ sums

4. Round the area of a circle of diameter 8 to the nearest integer.

   4. _____

5. You add 13 to 10% of N and then you multiply the result by 10 to get 156. What is the value of N?

   5. _____

6. What is the smallest 3-digit number all of whose digits are different and which does not use the digit 1?

   6. _____

7. If the width of a rectangular billboard is 4 times its height, and if its area is 25 m^2, what is the value of its width, in metres?

   7. _____ metres

Blitz, Page 2

8. *ABCDE* is a regular pentagon. What is the value, in degrees, of angle *EAD*?

8. _____ degrees

9. What is the greatest common factor of 42, 12, 18, and 132?

9. _____

10. What is the least positive common multiple of the numbers in Question 9?

10. _____

11. What is the binary representation of 2015?

11. _____

12. The first term of a sequence is 0. For $n > 1$, the n-th term is $n! - (n-1)!$. What is the difference between the 6-th term and the 4-th term?

12. _____

13. The vertices of a triangle have coordinates $(-40.3, 0)$, $(40.3, 0)$, and $(0, y)$. What is the positive value of y such that the area of the triangle is 2015?

13. _____

14. Express $1 + \dfrac{1}{2} + \dfrac{1}{4} + \dfrac{1}{8} + \cdots + \dfrac{1}{1024}$ as a common fraction.

14. _____

Canadian Math Challengers Society

15. Irene went to the store and bought a laptop that was listed for $1000. The laptop was on sale at 15% off. On the amount after the discount, she had to pay 12% tax. How many dollars did she pay in total?
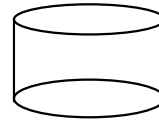
15. _____ dollars

16. Let $N = 100 \times 36 \times 63$. How many factors does N have?

16. _____ factors

17. The volume of a cylinder is 64000π cm^3, and its height is 40 cm. What is the surface area of the cylinder (including both bases)? Provide the answer rounded to the nearest 100 cm^2.

17. _____ cm^2

18. The sum of 40 distinct positive integers is equal to 1000. What is the smallest possible value of the biggest integer?

18. _____

19. Good garden compound can be made by combining 8 bags of 25 litres topsoil at density of 2/5 kg/litre, 4 bags of 30 litres mushroom compost at density of 1/3 kg/litre, and 1 bag of 25 litres of fertilizer at density of 6/5 kg/litre. By weight, what percentage of the compound is fertilizer?
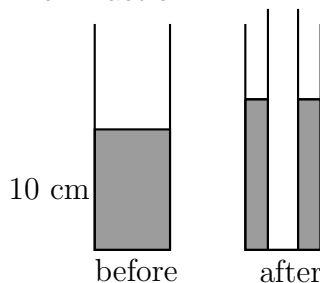
19. _____ percent

20. After mixing, the volume of the compound in Question 19 shrinks to 80% of the original combined volumes of the components. What is the final volume of the compound in litres?

20. _____ litres

21. Round the cube root of 2015 to the nearest integer.

21. _____

22. A 5 cm diameter cylinder is partially filled with water so that the top surface of the water is 10 cm above the bottom. A tall 2 cm diameter solid cylinder is inserted in the larger cylinder so that its bottom touches the bottom of the larger cylinder. What is the new distance from the surface of the water to the bottom of the larger cylinder? Express the answer in cm, as a common fraction.

22. _____ cm

10 cm

before   after

23. Frank rents a stand to sell cloth. His average cost for an item is $5.00, and he has to pay to the owner of the stand 5% of the selling price. At what price does he have to sell the item so that his net profit per item sold is $3.00? Give the answer in dollars, correct to 2 places after the decimal point.

23. _____ dollars

24. Points $A(1,8)$ and $B(10,4)$ have been connected by lines to point $X(x,0)$. What is the smallest possible value of $AX + XB$?

24. _____

$A(1,8)$

$B(10,4)$

$X(x,0)$

25. In Question 24, what is the value of x?

25. _____

26. For every km that a locomotive goes, it burns 5 litres of fuel. For any car that it pulls, it burns (on average) another 0.15 litres of fuel per km. A very long train made up of 1 locomotive and many cars is travelling between 2 refueling stations, a distance of 850 km. If the fuel capacity of the locomotive is 15470 litres, how many cars can the train have?

26. _____ cars

Bull's-eye, Prov. 2015, Page 1: Problem Solving

1. Two tablets cost $50 more than one laptop. If a tablet costs $294, what is the cost of a laptop, in dollars?

1. _____ dollars

2. A new animal has evolved in Canada. It is 1/4 coyote, 3/32 wolf, and the rest is dog. What fraction of the animal is dog?

2. _____

3. The 9 squares in the 3×3 magic square below are to be filled in so that the sum of the three numbers in any row, the sum of the three numbers in any column, and the sum of the three numbers in each of the two diagonals, are all the same. Numbers in some of the squares have already been filled in. Find the number which should be put in the square labelled "?".

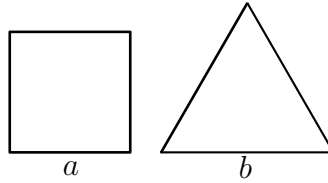3. _____

10	3	
		?
−3		

4. A cargo train carries 4 kinds of products, oil, gas, coal, and potash. The amount of gas by weight is 1/8 of the amount of oil. The amount of coal is 1/10 of the combined amounts of potash and gas, and the amount of potash is 5/6 of the combined amounts of gas and coal. What fraction of the total amount is the oil?

4. _____

5. The sum of opposite faces of a traditional die is always 7. Two dice are rolled and the sum of the top faces is greater than 8. How many possible sums can the bottom faces have?

5. _____ sums

6. As in Question 5, the sum of the top faces is greater than 8. What is the probability that there is no 1 on any of the bottom faces? Express the answer as a common fraction.

6. _____

7. What is the smallest positive integer which is divisible by 45 and whose decimal representation uses only the digits 0, 1, and 2?

7. _____

8. How many positive multiples of 3 or 5 or 134 are there that are less than 2015?

8. _____

Bull's-eye, Page 3: Geometry

9. A square of side a and an equilateral triangle of side b have the same area. Express the value of $\frac{b^4}{a^4}$ as a common fraction.

9. _____

10. Railway tracks form the shape of a quarter-circle. The distance between the tracks is 56 inches. How much longer is the outer track than the inner track? Provide your answer to the nearest inch.

10. _____ inches

11. Find the area of a regular octagon with side 1. Express your answer as $k\left(m+\sqrt{n}\right)$, where k, m, and n are integers and n is prime.

11. _____ units^2

12. A star of David with area A is inscribed in a circle with area 3π. What is the value of A^2?

12. _____

Co-op, Prov. 2015, Page 1: Team answers must be on the *coloured* page. Answers on a white page will not be graded.

1. The Fibonacci sequence (F_n) is defined as follows: $F_1 = 1$, $F_2 = 1$ and for $n > 2$ by $F_n = F_{n-1} + F_{n-2}$. Find the largest n such that $F_1 + F_2 + F_3 + \cdots + F_n < 100$.

1. _____

2. In Question 1, what is the value of $F_1 + F_2 + F_3 + \cdots + F_{15}$?

2. _____

3. For every integer $n \geq 1$, the sum of the first n terms of a sequence is equal to n^2. What is the 100-th term of the sequence?

3. _____

4. A pail is placed outside to catch rainfall. The pail has the shape of a truncated cone with base diameter 30 cm, opening diameter 50 cm, and a height of 60 cm. Initially the water level is at 10 cm above the bottom of the pail. A few days later the water has risen to a level of 20 cm from the bottom of the pail. How many cubic cm of water were added? Round your answer to the nearest 100 cm^3.

4. _____ cubic cm

5. If in Question 4 the rainfall is at the rate of 10 mm/hour, how many hours will it take to fill the entire pail if originally it was empty? Round your answer to the nearest integer.

5. _____ hours

6. How many ordered triples (a, b, c) of real numbers are there such that each of the numbers is the product of the other two? Note that for example the ordered triple $(2, -3, 0)$ is not the same as the ordered triple $(-3, 0, 2)$.

6. _____ triples

7. A group of 50 kids were chatting over the internet. Each of them sent a different odd number of text messages, except for 3 kids who each sent the same odd number of messages. What is the smallest number of text messages that could have been sent?

7. _____ messages

8. Nimoy drives a car starting at a speed of 0 km/h and accelerating at a constant rate, and driving along the circular road around CERN (the Large Hadron Collider). When he finished one circle he reached a speed of 120 km/h. It took him exactly 1720 seconds to reach that speed. Find the radius of the circle in km correct to one decimal.

8. _____ km

9. You throw three dice. What is the probability that the sum is a multiple of 5? Express the answer as a common fraction.

9. _____

10. The class of 1983 has gathered for a reunion in 2015 to celebrate their 50-th birthdays (all were born in 1965 which had 365 days). Of the 365 days, 310 have no birthday of any of the students. K days have the birthday of 1 student, $K - 1$ days have the birthdays of 2 students, $K - 2$ have the birthdays of 3 students, and so on. So 3 days have the birthdays of $K - 2$ students, 2 days have the birthdays of $K - 1$ students, and 1 day has the birthdays of K students. How many students were in the class of 1983?

10. _____ students

11. Niki generates a 4-character PIN (password) using the allowed 10 digits, 26 lower-case letters, and 26 upper-case letter. She plans to use exactly 2 (not necessarily different) digits and 2 letters. How many different passwords can she select from? Examples of valid passwords: 13Aa, 13aa, A22b, and c3D1.

11. _____ passwords

12. If as in Question 11 Niki plans to use exactly 2 digits and exactly 3 letters for a 5 character password, how many different passwords can she select from if the 2 digits must be next to each other? Examples of valid passwords: 00AAb, o12bC, XY98Z.

12. _____ passwords

13. The vertices of triangle ABC have coordinates $A(0, 11)$, $B(10.5, 0)$, and $C(-7, 0)$. Find the area of $\triangle ABC$. Express your answer correct to 2 decimal places.

13. _____

14. Triangle ABC is as in Question 13. Find the x-coordinate of the point (x, y) that has the same distances from A, B, and C. Give the answer correct to 2 decimal places.

14. _____

15. What is the y-coordinate of the point (x, y) of Question 14? Give the answer correct to 2 decimal places.

15. _____

MATH CHALLENGERS

Math Challengers

Provincial Competition
Face-off Round 2015

A question always follows a blue page. The next page is blue!

1. What is the value of $(11 \times 4) + 2015$?

1. What is the value of
$(11 \times 4) + 2015$?

Answer: 2059

2. How many seconds are there in 2.5 hours?

2. How many seconds are there in 2.5 hours?

Answer: 9000 (seconds)

3. What is the largest integer N such that $N^2 + 4N + 4 < 2015$?

3. What is the largest integer N such that $N^2 + 4N + 4 < 2015$?

Answer: 42

4. What is the area of the L-shaped garden in the figure?
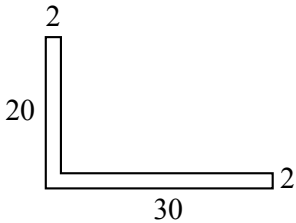
4. What is the area of the L-shaped garden in the figure?
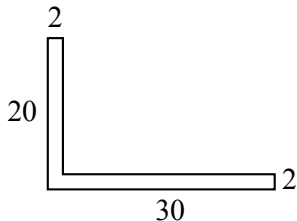
Answer: 96

5. The average of 4 numbers is 20. You add a fifth number, the average increases by 15. What is the value of the fifth number?

5. The average of 4 numbers is 20. You add a fifth number, the average increases by 15. What is the value of the fifth number?

Answer: 95

6. Jane bought a front window camera for her car at a cost of $149, a rear window camera at a cost of $99, and two memory cards at a cost of $17.50 each. How many dollars did she spend in total?

6. Jane bought a front window camera for her car at a cost of $149, a rear window camera at a cost of $99, and two memory cards at a cost of $17.50 each. How many dollars did she spend in total?

Answer: 283 (dollars)

7. The area of an isosceles right triangle is 100. What is the length of the hypotenuse?
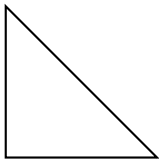
7. The area of an isosceles right triangle is 100. What is the length of the hypotenuse?
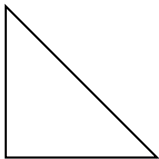
Answer: 20

8. Suppose that $\frac{x}{y} + \frac{y}{x} = 10$. What is the value of $\frac{6x^2+6y^2}{xy}$?

8. Suppose that $\frac{x}{y} + \frac{y}{x} = 10$. What is the value of $\frac{6x^2+6y^2}{xy}$?

Answer: 60

9. .What is the value of
$2! + 0 + 1! + 5!$?

9. .What is the value of
$2! + 0 + 1! + 5!$?

Answer: 123

10. What is the number of sides of a convex polygon which has 77 diagonals?

10. What is the number of sides of a convex polygon which has 77 diagonals?

Answer: 14 (sides)

11. The volume of 231 grams of liquid mercury is 16.5 cm^3. What is its density in gm/cm^3?

11. The volume of 231 grams of liquid mercury is 16.5 cm^3. What is its density in gm/cm^3?

Answer: 14 (gm/cm^3)

12.   The sum of five consecutive integers is 2015. What is the value of the largest of these integers?

12. The sum of five consecutive integers is 2015. What is the value of the largest of these integers?

Answer: 405

13. The 600 students of the school are divided into teams of 35 or fewer students each. (The teams need not be all the same size.) What is the smallest possible number of teams?

13. The 600 students of the school are divided into teams of 35 or fewer students each. (The teams need not be all the same size.) What is the smallest possible number of teams?

Answer: 18 (teams)

14. Round $\frac{201}{13}$ to the nearest integer.

14. Round $\frac{201}{13}$ to the nearest integer.

Answer: 15

15. .What is the x-coordinate of the intersection point of the lines $x + y = 24$ and $x - 2y = 0$?

15. .What is the x-coordinate of the intersection point of the lines $x + y = 24$ and $x - 2y = 0$?

Answer: 16

16. Simplify: $\frac{2\times0+1\times5}{2+0+1+5}$.

16. Simplify: $\frac{2\times 0+1\times 5}{2+0+1+5}$.

Answer: $\frac{5}{8}$

17. The average speed of a bus travelling the 123 km from Vancouver to Whistler was 90 km/hour. How long, in minutes, was the trip?

17. The average speed of a bus travelling the 123 km from Vancouver to Whistler was 90 km/hour. How long, in minutes, was the trip?

Answer: 82 (minutes)

18. What is the largest integer N that satisfies $5^N < 2015$?

18. What is the largest integer N that satisfies $5^N < 2015$?

Answer: 4

19. What is the value of $5! - 4!$?

19. What is the value of $5! - 4!$?

20. What is the surface area of a cube whose volume is 27?

20. What is the surface area of a cube whose volume is 27?

Answer: 54

21. What is the value of the largest two-digit number that is the product of exactly 2 different primes?

21. What is the value of the largest two-digit number that is the product of exactly 2 different primes?

95

22. A clock shows the right time now but it goes slower than an accurate clock by 8 seconds every day. After how many days will it be off by one hour?

22. A clock shows the right time now but it goes slower than an accurate clock by 8 seconds every day. After how many days will it be off by one hour?

Answer: 450 (days)

23. You have 4 hats and 8 scarves. You must select one of the scarves, and one hat or no hats. In how many different ways can you do the choosing?

23. You have 4 hats and 8 scarves. You must select one of the scarves, and one hat or no hats. In how many different ways can you do the choosing?

Answer: 40 (ways)

24. What is the value of the smallest prime number greater than 200?

24. What is the value of the smallest prime number greater than 200?

211

25. The sum of the two diagonals of a rhombus is 20 and their difference is 4. What is the area of the rhombus?
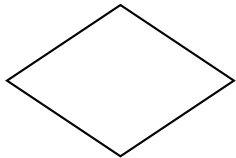
25. The sum of the two diagonals of a rhombus is 20 and their difference is 4. What is the area of the rhombus?
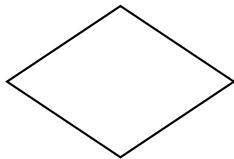
Answer: 48

26. What is the sum of the reciprocal of $\frac{5}{6}$ and the reciprocal of $\frac{6}{7}$? Express as a fraction in lowest terms.

26. What is the sum of the reciprocal of $\frac{5}{6}$ and the reciprocal of $\frac{6}{7}$? Express as a fraction in lowest terms.

Answer: $\frac{71}{30}$

27. How many diagonals does the diagonal from *A* to *C* intersect in the in *interior* (not the boundary) of the hexagon?
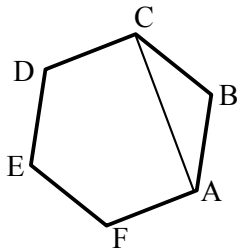
27. How many diagonals does the diagonal from *A* to *C* intersect in the in *interior* (not the boundary) of the hexagon?
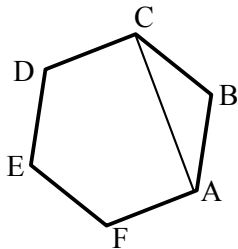
Answer: 3 (diagonals)

28. Round $\frac{2015}{4}$ to the nearest integer.

28. Round $\frac{2015}{4}$ to the nearest integer.

504

29. You bought a bag of potatoes at a cost of $0.40 per kg, and paid $13.20. What was the weight of the potatoes in kg?

29. You bought a bag of potatoes at a cost of $0.40 per kg, and paid $13.20. What was the weight of the potatoes in kg?

Answer: 33 (kg)

30. One km is represented scaled on a map to 0.02 cm. How many km are represented by 5 cm on the map?

30. One km is represented scaled on a map to 0.02 cm. How many km are represented by 5 cm on the map?

Answer: 250 (km)

31. Find the sum
$2009 + 2011 + 2013 + 2015.$

31. Find the sum
$2009 + 2011 + 2013 + 2015.$

Answer: 8048

32. Suppose N is 150% of 160% of 15. What is the value of N?

32. Suppose N is 150% of 160% of 15. What is the value of N?

36

33. Simplify $\frac{12+13+14+15}{6+7+8+9}$ as a fraction in lowest terms.

33. Simplify $\frac{12+13+14+15}{6+7+8+9}$ as a fraction in lowest terms.

Answer: $\frac{9}{5}$

34. You have two 5 cent coins, two 10 cent coins, and two 25 cent coins. How many different sums can you make if you use exactly 2 coins?

34. You have two 5 cent coins, two 10 cent coins, and two 25 cent coins. How many different sums can you make if you use exactly 2 coins?

Answer: 6 (sums)

35. What number has the binary representation 1101?

35. What number has the binary representation 1101?

13

Math Challengers Regional 2015
Answers, Blitz Stage

1. $\dfrac{3}{5}$

2. 16

3. 56

4. 17.25

5. 12

6. 15

7. 24

8. $\dfrac{1}{6}$

9. $\dfrac{1}{6}$

10. $\dfrac{23}{12}$

11. 11

12. 13

13. $\dfrac{9}{4}$

14. 9

15. 6

16. 64

17. $\dfrac{5}{4}$

18. 21.50

19. 5

20. 60

21. $\dfrac{15}{29}$

22. $\dfrac{5}{6}$

23. 49

24. $\dfrac{9}{8}$

25. 40

26. $\dfrac{14}{27}$

Math Challengers Regional 2015
Answers, Bull's-eye Stage

1. 145

2. 68

3. 310

4. $\dfrac{240}{7}$

5. 12

6. 4

7. 9

8. $\dfrac{1}{20}$

9. 1700

10. $\dfrac{36}{13}$

11. $6\pi + 8$

12. $\dfrac{5}{11}$

Math Challengers Regional 2015
Answers, Co-op Stage

1. 20

2. −5050

3. 12360

4. 58.5

5. 45

6. 21

7. 256

8. 90

9. 0.8045

10. 8888

11. 373000

12. 1378000

13. 9023

14. 169

15. $\dfrac{77}{190}$

Math Challengers Provincial 2015
Answers, Blitz Stage

1. 97

2. 8

3. 16

4. 50

5. 26

6. 203

7. 10

8. 36

9. 6

10. 2772

11. 11111011111

12. 582

13. 50

14. $\dfrac{2047}{1024}$

15. 952

16. 150

17. 20100

18. 45

19. 20

20. 276

21. 13

22. $\dfrac{250}{21}$

23. 8.42

24. 15

25. 7

26. 88

Math Challengers Provincial 2015
Answers, Bull's-eye Stage

1. 538

2. $\dfrac{21}{32}$

3. -9

4. $\dfrac{40}{51}$

5. 4

6. $\dfrac{3}{10}$

7. 122220

8. 947

9. $\dfrac{16}{3}$

10. 88

11. $2(1 + \sqrt{2})$

12. 27

Math Challengers Provincial 2015
Answers, Co-op Stage

1. 9

2. 1596

3. 199

4. 9600

5. 39

6. 5

7. 2306

8. 4.6

9. $\dfrac{43}{216}$

10. 220

11. 1622400

12. 56243200

13. 96.25

14. 1.75

15. 2.16

Blitz, Page 1

1. Find the digit sum of 2016.

1. _____

2. What is the smallest integer N such that 20% of N is larger than 10.25?

2. _____

3. The radius of a circle is $\dfrac{17}{\pi}$. What is the circumference of the circle?

3. _____
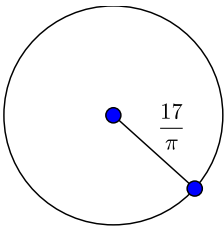
4. You pick one marble from a box containing 7 white marbles and 11 black marbles. What is the probability you pick a black marble? Express your answer as a common fraction.

4. _____

5. You bought four travel apps and one gaming app for your mobile phone and paid a total of $65.95 for the five apps. If the travel apps cost $11.99 each, what was the cost of the gaming app (in dollars correct to two decimal places)?

5. _____($)

6. A biker rides her bike at a speed of 27 *km/h*. How far (in *km*) does she ride in 2 hours and 20 minutes?

6. _____(*km*)

7. Express $0.11 + 1.11 + 2.11 + 3.11$ as a fraction in lowest terms.

7. _____

Canadian Math Challengers Society

8. 25 competitors were divided into 5 groups of 5 competitors each. In each of the 5 groups, each competitor played each of the group's other 4 competitors once to declare a winner for the group. The 5 winners (one from each group) advanced to the final round where each competitor played each of the other four final competitors once. How many games were played in total?

8. _____

9. Halley's Comet is known to be visible from Earth every 76 years. The first time there was credible evidence of the comet being seen by humans was in the year 86 AD. What will be the first year after 2016 that the comet will again be visible from Earth?

9. _____

10. Given the info in Question 9, during how many years between the year 1000 and the year 2000 was Halley's Comet visible from Earth?

10._____

11. What is the ratio of the perimeter of a regular hexagon with side 1 to the perimeter of an equilateral triangle with side 1?

11._____

1

1
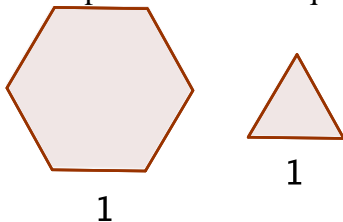
12. In Question 11, what is the ratio of the area of the hexagon to the area of the triangle?

12._____

13. How many diagonals does the hexagon in Question 11 have?

13._____

14. What is the total length of all the diagonals of the hexagon in Question 11? Express you answer as $i(\sqrt{j} + k)$ where i and k are integers and j is a prime.

14._____

15. A large box has the following shape: two of the faces of the box each have
area 24 square feet (f^2), two faces each have area 40 f^2, and two faces each
have area 60 f^2. What is the total surface area, (in f^2), of the box?

15._____(f^2)
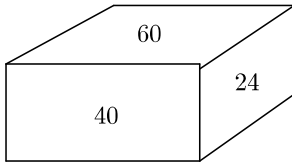
16. In Question 15, what is the volume, in cubic feet (f^3), of the box?

16._____(f^3)

17. In Question 15, let a, b, and c be the mutually perpendicular
edges of the box. If the measure of each of them is an integer (in feet),
what is the value, in feet, of $a+b+c$?

17._____(f)

18. A swimming pool that can contain a maximum of 162 cubic meters (m^3) of
water is filled up by three taps. Unfortunately, the operator forgot to shut
off the drain pipe, which drains the full pool in 18 hours. As a result,
it took 36 hours to fill the pool. How much water (in m^3)
was wasted down the drain pipe while the pool was being filled?

18._____(m^3)

19. The flow rates of the taps in Question 18 are the following: the first tap
flows at 25 litres/minute (*lit/min*), the second tap flows at 75 *lit/min*, and
the third tap flows at M *lit/min*. What is the value of M?

19._____

20. In Question 18, how many hours does it take to fill the entire pool using
tap 1 and tap 2 only if the drain pipe is shut off?

20._____ (h)

21. On a large table there are many individual cartons of juice, and many   21._____
cookies to be distributed to the Math Challengers students
competing in that room. If every student takes 2 cartons of juice and 1 cookie,
there will be 121 cookies left and no juice will be left. If each student and each
invigilator takes 1 carton of juice and 2 cookies, there will be 100 cartons of
juice left and no cookies left. How many cookies were on the table at the beginning?

22. In Question 21, how many invigilators are in the room?   22._____

23. In Question 21, how many students are in the room?   23._____

24. How many three letter "words" are there that use three letters from the five   24._____
letters A, B, C, D, and E, and such that no letter appears more than twice in
the "word" and such that that A is not the first letter, and B is not the
second letter? (Examples of valid "words": CAB, BCC, CDE).

25. A cone with base of radius 2 is inscribed in a sphere with radius 4.   25._____
What is the volume of the cone?

Give your answer correct to the nearest integer (use $\pi = 3.14$, and $\sqrt{3} = 1.73$).
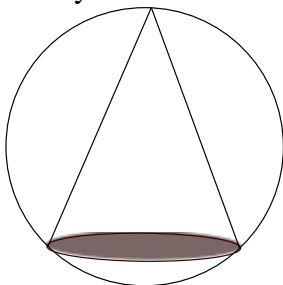
26. Let K be a positive integer such that the sum of all of its factors is 48.   26._____
Please note that the number 1 and the number K are factors of K.
What is the sum of all possible such K's?

1. If a long distance runner runs at speed of 15 *km/h*,
   how many metres does he run in 30 seconds?

   1. _____ (*m*)

2. Tomatoes contain 94% water. Tomato paste contains 55% water and it is
   created by simply removing some of the water from tomatoes. If a can of
   tomato paste contains 220 grams of paste, how many *kg* of tomatoes were
   used to make it? Provide the answer as a decimal correct to two decimal places.

   2. _____ (*kg*)

3. How many factors does 2016 have? Please note that 1 and 2016 are factors
   of 2016. (Hint: Try to express 2016 as a product of powers of primes.)

   3. _____

4. In Questions 3, what is the sum of all the factors?

   4. _____

5. What is the decimal representation of the number whose binary representation is 1001001?

5. _____

6. At a restaurant table, the five customers each ask for a different drink. The waiter brings the right drinks, but forgets who wants which one, and sets out the five drinks at random. What is the probability that all five customers get the drink they wanted? Express the answer as a common fraction.

6. _____

7. In Question 6, what is the probability that exactly 2 of the customers get the wrong drink? Express the answer as a common fraction.

7. _____

8. In Question 6, what is the probability that all five customers get the wrong drink? Express the answer as a common fraction.

8. _____

9.   *ABCD* is a parallelogram. *BEFC* is a rhombus ($BE = EF = FC = CB$). $BE = BF$, $\angle ABF = 157°$, and $AC \perp CF$. What is the value of $\angle BAC$ (in degrees)?

9. _____ (°)
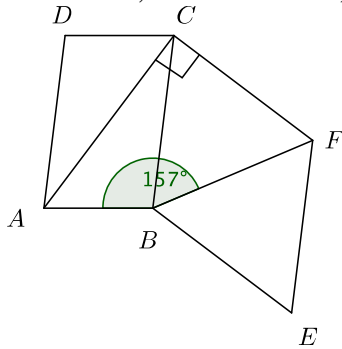
10. The picture below shows an equilateral triangle inscribed in a circle. If the area of the triangle is $\dfrac{36\sqrt{3}}{\pi}$, what is the area of the circle?

10._____
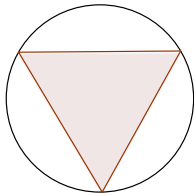
11. *ABCD* is a quadrilateral where $AB = AD = 2$, $BC = CD$, $AC = 3$, and $\angle BAD = 120°$. What is the value of *CD* ? Express your answer as $\sqrt{n}$ where *n* is an integer.

11._____
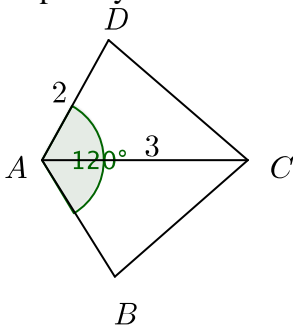
12. *ABCD* and *BEFG* are squares. *H* is the intersection of lines *AF* and *BC* . The area of $\triangle ABH$ is 4 and the area of trapezoid *BEFH* is 5. What is the combined area of the two squares?

12._____
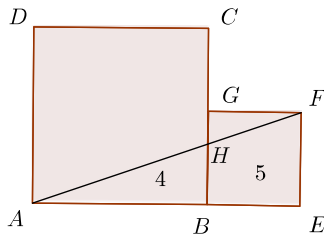
Team answers must be on coloured page. Answers on a white page will not be graded.)

1. *ABCD* is a parallelogram. *E* and *F* are on *AB* and *CD* respectively.   1. _____
   EBFD is rectangle. *AB* = 5, *AD* = 4, and *DE* = 3. What is the area
   of *EBFD*? Express your answer as a decimal correct to two decimal places.
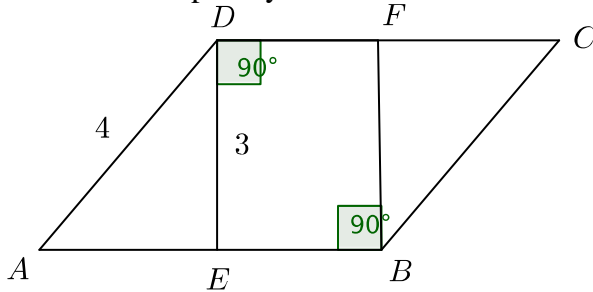
2. On the table, there is the following row of 6 coins: 2. _____($)
   $1.00, $2.00, $0.25, $0.05, $0.25, $0.10.
   Alice picks a coin from one of the ends of the row and puts it in her pocket.
   Then Bob chooses a coin from one of the (remaining) ends, and puts it in his
   pocket. They continue alternating who takes a coin from one of the (remaining)
   ends until Bob pockets the last coin. If Alice uses a strategy to maximize the total
   value of the coins she picks, what is the maximum total value of coins that
   Bob can pocket (in dollars, correct to two decimal places)?

3. Suppose that for every 82 women from Nanaimo who give birth to a single   3. _____
   baby, one more woman in Nanaimo gives birth to twins. 2016 babies were
   born in Nanaimo during the year of 2015. How many women gave birth in
   Nanaimo during the year 2015?

4. Suppose that the ratio of 82 : 1, as in Question 3, is the same for all of Canada. 4. _____
   Suppose also that: (a) for every 82 women that give birth to twins, one more
   woman gives birth to triplets, and (b) for every 82 women that give birth to
   triplets, one more woman gives birth to quadruplets. In Canada, during the
   year 2015, 551368 women gave birth to a single baby. How many babies
   were born in total in Canada during 2015?

5. What is the largest N smaller than 2016 that can be expressed as $(pq)^2$ 5. _____
   where *p* and *q* are both primes (not necessarily different)?

Team answers must be on coloured page. Answers on a white page will not be graded.)

6. What is the value of $(1+\sqrt{2}(1+\sqrt{3}))(1-\sqrt{2}(1-\sqrt{3}))$?   6. _____
   Express your answer as decimal correct to three decimal places.

7. Zev runs by increasing his speed every 4 minutes. The number of   7._____(*min*)
   Calories he burns per minute is given by the following sequence:
   10,10,10,10,11,11,11,11,12,12,12,12,…
   What is the minimum number of minutes he needs to run in order to
   burn at least 1000 Calories? Provide the answer as an integer.

8. Zev runs as in Question 7 but he got tired and stopped running   8. _____(*m/s*)
   after 47 minutes. The average distance that he ran per each Calorie
   he burnt was 17 metres (*m*). What was his average speed in metres/second (*m/s*)?
   Provide the answer as decimal correct to two decimal places.

9. A $6cm$ diameter, $5cm$ tall cylinder is partially filled with water.   9. _____(*cm*)
   The cylinder is placed on the table on its circular side and the level
   of the water above the table is $\dfrac{1}{4}$ of its diameter. When placed on the
   table on its base, how high (in *cm*) is the level of the water above the table?
   Express your answer as decimal correct to two decimal places.

10. Two circles with radii 2 and 6 are tangent internally at A. O is the centre   10._____
    of the smaller circle. ΔBCD is circumscribed by the large circle, O is on BC,
    and is also on the diameter AD which is perpendicular to BC. Find the value
    of the shaded region (the total area inside the large circle but outside the
    triangle and the small circle). Round your answer to the nearest integer.
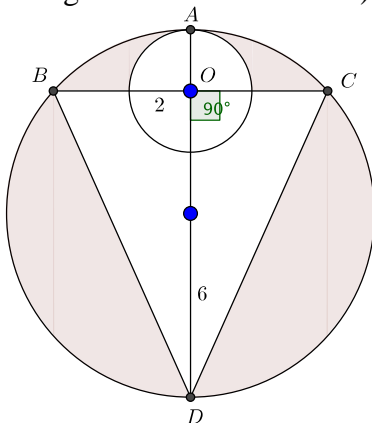
(Team answers must be on coloured page. Answers on a white page will not be graded.)

11. You have 2 bowls on a shelf. Each bowl originally contains 4 tomatoes: 11._____
2 yellow, and 2 red. You select a bowl at random and then take one tomato
at random out of that bowl and put it on the table. You then repeat the same
process two more times. What is the probability that you end up with 3 red
tomatoes on the table? Express the answer as a common fraction.

12. How many integers in the interval from 1 to 2016 do not contain   12._____
the digit "1" in their decimal representation?

13. Alan, Betty, Carlos, Dalton and three others competed to win a   13._____
calculator. Each one of them put his or her name on a card and then the
cards were shuffled and randomly numbered 1 to 7 (a different number for
each person). The person with the card numbered 7 was to win the calculator.
It was then disclosed to all that the total of the numbers assigned for Betty,
Carlos, and Dalton was 14. What is the probability that Alan won
the calculator? Provide the answer as a common fraction.

14. In Question 13, it was then announced that Betty won the calculator but she   14._____
declined to accept it. So, the calculator was awarded to the person whose
card bears the number 6. What is the probability that Alan won the calculator?
Provide the answer as a common fraction.

15. The point $P(5,3)$ is the intersection of the line $mx + ny = 4$ and the   15._____
line $nx + my = 5$. What is the combined area of the two triangles ΔABP
and ΔCDP formed between the lines and the axes (shaded region)?
Express your answer as common fraction.
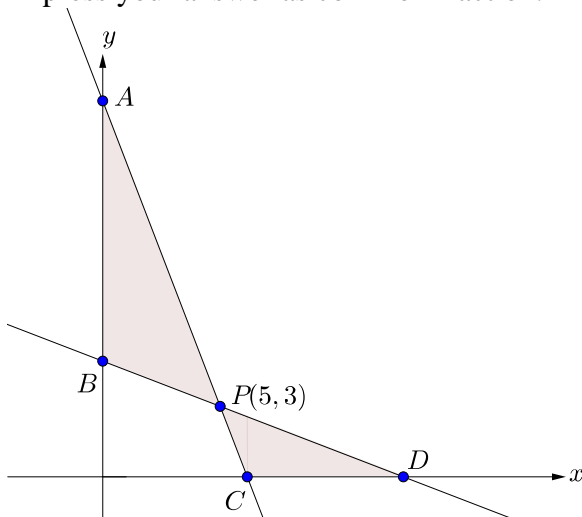
1. How many diagonals does a regular hexagon have?
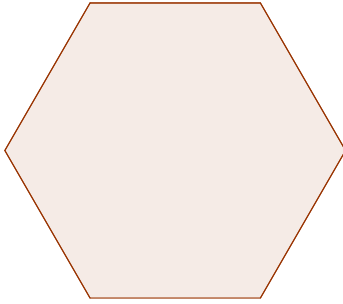
1. Answer: 9

2. You can buy skating shoes for your team at \$225 for 2 pairs or \$995 for 10 pairs. How many dollars per pair do you save by buying 10 pairs?
2. Answer: 13

3. $x = 2016$. What is the value of $x^2 - 2014x$?
3. Answer: 4032

4. You toss a coin three times. What is the probability that you get a head exactly one time? Provide your answer as a fraction in lowest terms.
4. Answer: $3/8$

5. The number 12 is how many percent of the number 80?
5. Answer: 15

6. In the figure below, what is the distance from P to Q?
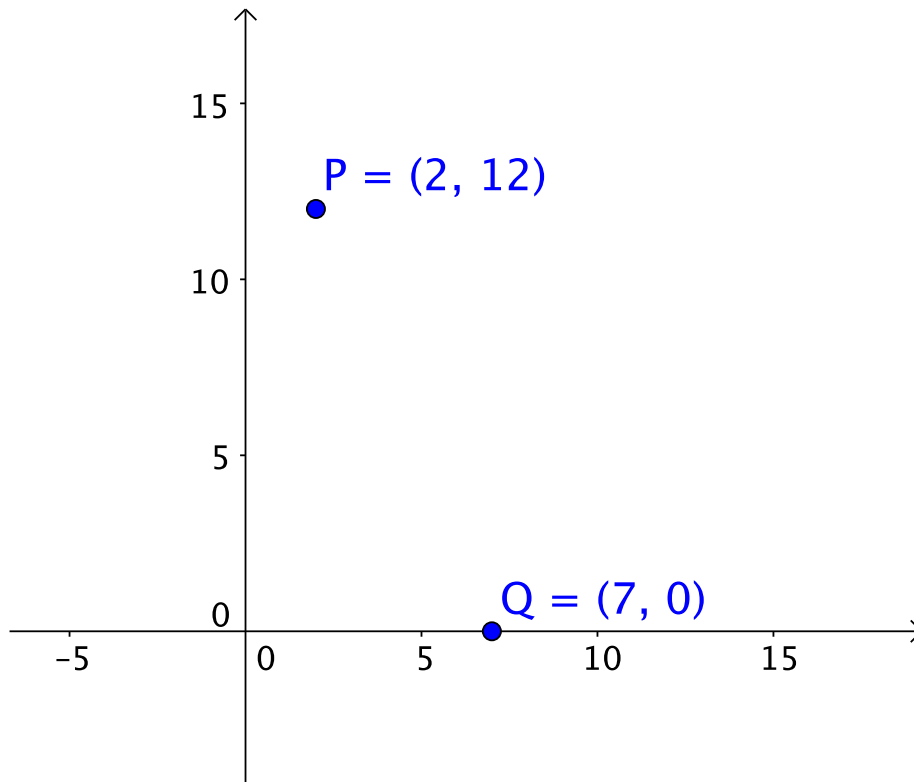
6. Answer: 13

7. What is the sum of the two smallest 2-digit primes?
7. Answer: 24

8. Round $2016/101$ to the nearest integer.
8. Answer: 20

9. If $\sqrt{7x+4} = 9$, what is the value of x?
9. Answer: 11

10. $\angle ACB = 50^\circ$, AD is a diameter and $AB = BC$. Find $\angle DAB$ (in degrees).
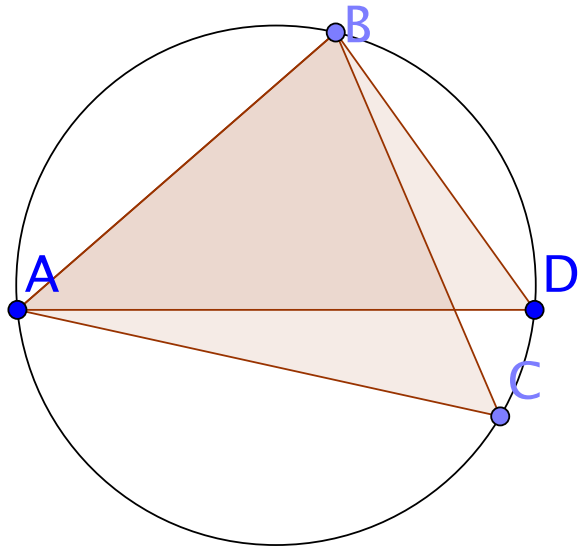
10. Answer: 40

11. Compute $2.1 + 3.2 + 4.3$
11. Answer: 9.6

12. What is the largest prime divisor of $1 + 2 + 3 + \ldots + 9 + 10$?
12. Answer: 11

13. What is the remainder when you divide 2016 by 30?
13. Answer: 6

14. What is the area of a rectangle whose perimeter is 30 and one of its sides is 8?
14. Answer: 56

15. You rolled two dice and got N on one and M (where $M > N$) on the other. What is the number of different possible values of $M + N$?
15. Answer: 9

16. $2016 = 9x$. Find x.
16. Answer: 224

17. $f(1) = 1$. For $N > 1$, $f(N) = 2f(N - 1) + 3$. What is the value of $f(3)$?
17. Answer: 13

18. Amy jogged 4500 metres at average speed of 3 metres per second. What was her total jogging time (in minutes)?
18. Answer: 25

19. Simplify: $(2 + 0 + 1 + 6) \times (0 - 2) \times (1 - 3)$
19. Answer: 36

20. Given the points $A(0, 0)$, $B(2m, 0)$, and $C(2m, 3m)$, what is the area of the triangle ABC given that $m = 5$?
20. Answer: 75

21. Some pizzas boxes were brought to the school reception, at a total cost of $2016. The average cost per box was $9.60. How many pizza boxes were brought?
21. Answer: 210

22. What is the largest 2-digit prime whose digit sum is 11?
22. Answer: 83

23. The area of the base of a cone is 27 and its height is 7. What is the volume of the cone?
23. Answer: 63

24. A mobile phone costs $350. Data plan for the phone costs $40/month. Assuming that you are going to use the phone for 2 years, what is your yearly cost of the phone including the cost of the data plan?
24. Answer: 655

25. Calculate $13 \times 2 + 2016$
25. Answer: 2042

26. What is the value of $1! \times 2! \times 3!$?
26. Answer: 12

27. $x > 0$ and $\frac{9}{5}x^6 = \frac{25}{3}x^3$. Find x. Express your answer as a common fraction in lowest terms.
27. Answer: $5/3$

28. Beth bought 5 books: the first had 200 pages, the second 300, the third 400, the fourth 500, and the fifth 650. What is the average number of pages per book?
28. Answer: 410

29. The circumference of the circle is 10π. What is the perimeter of the inscribed regular hexagon?
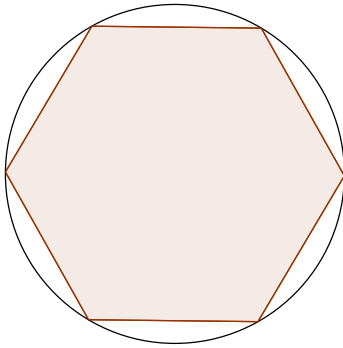
29. Answer: 30

30. Ann and Beth participate in a discussion group of five students. Two of the participants will be selected at random to present an essay they wrote. What is the probability that neither Ann nor Beth will present their essays? Express your answer as common fraction in lowest terms.

30. Answer: $^3/_{10}$

31. The first term of an arithmetic sequence is 11 and the sum of the first two terms is 21. What is the value of the fourth term of the sequence?

31. Answer: 8

32. In the figure below, $\angle ABC = 90^o$, $AB = BD$, and $AD = CD$, What is the
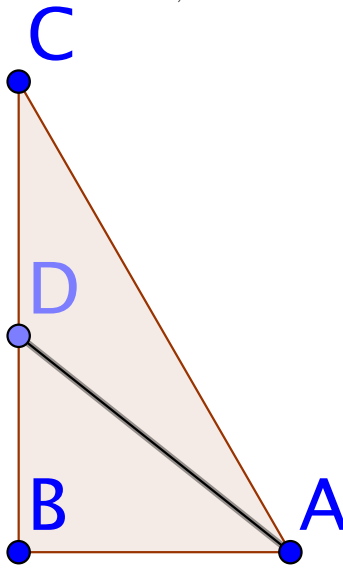
ratio of $\angle ABC$ to $\angle DAC$?

32. Answer: 4

33. Define $x@y = 2x + y$. What is the value of $2@3 + 3@2$?

33. Answer: 15

34. What integer has binary representation of 10001?
34. Answer: 17

35. What is the smallest common multiple of 4, 5, and 6?
35. Answer: 60

36. The sum of 4 consecutive primes is 102. What is the value of the smallest of the 4 primes?
36. Answer: 19

37. The measures of the hypotenuse and one of the sides of a right triangle are $\sqrt{314}$ and 5. What is the measure of the other side?
37. Answer: 17

38. You bought 10 prepaid data cards. The total cost of the first 3 cards was $30. The cost of the other 7 cards was $9 per card. How many dollars did you pay in total?
38. Answer: 93 (dollars)

39. Express $0.05 + 0.25 + 0.45 + 0.65$ as a fraction in lowest terms.
39. Answer: $\frac{7}{5}$

40. The measure of the area of a circle is 49π. What is the measure of the diameter of the circle?
40. Answer: 14

41. The combined ages of two sisters is 73. Their age difference is 7. What is the age of the younger sister?
41. Answer: 33
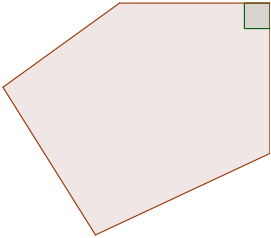
42. The sum of four different positive odd multiples of 3 is 48. What is the largest of the 4 numbers?
42. Answer: 21

Blitz, Page 1

1.   What is the sum of all single digit primes?

1. _____

2.   What is the largest common divisor of 16 and 20?

2. _____

3.   What is the smallest common multiple of 16 and 20?

3. _____

4.   One angle of a convex pentagon is a right angle.
      What is the sum (in degrees) of the other four angles?

4. _____ (°)

5.   You bought 5 pears and 4 apples. The cost of the apples was $0.49
      each, and the total cost of all apples and pears was $5.41.
      What was the cost of each pear?
      Provide your answer in $ correct to 2 decimal places.

5. _____($)

6.   You throw a (6-sided) die twice. What is the probability that on the
      second throw you get a larger number than you got on the first throw?
      Provide your answer as fraction in simplest form.

6. _____

7.   If you spend on average 70 seconds on each of the 26 Blitz questions,
      how many minutes do you spend in total on Blitz?
      Round your answer to the nearest integer.

7. _____(min)

8.　　a and b are the solutions of $x^2 - x - 2 = 0$. Find the value of $\dfrac{ab}{a+b}$.　　8. _____

9.　　What percent of 2016 is the number 252? Provide your answer as　　9. _____(%)
　　decimal correct to 1 decimal place.

10.　　Braille characters consist of minimum of one and maximum of six　　10. _____
　　dots in the six positions, as in the figure below.
　　How many Braille characters in total can be formed?

11.　　How many Braille characters consist of exactly four dots, of which　　11. _____
　　at least one is in the middle row? The figure for Question 10
　　shows one such character.

12.　　Ann read a book of 600 pages. She read page 1 and then every second　　12. _____
　　page after page 1 (i.e. she read pages 1, 3, 5, …); then she read every
　　third page starting at page 1 (i.e. she read pages 1, 4, 7, …); and, then
　　every fifth page starting at page 1 (i.e. she read pages 1, 6, 11, …).
　　When she finished reading, how many of the pages she did not read?

13.　　In Question 12, what is the first page after page 152 that Ann　　13. _____
　　did not read?

14.　　What is the value of $1 + 2 + 3 + \ldots + 38 + 39 + 40 + 39 + 38 + \ldots + 3 + 2 + 1$?　　14. _____

15. The mean of 9 and N is the same as the mean of 10, 11, and N.
    What is the value of N?

15. _____

16. Some Grade 8 students and some grade 9 students work in the biology
    lab. If each Grade 8 student is given 2 samples to analyze, and each
    Grade 9 student is given 3 samples to analyze, then, in total they are
    given 39 samples. If, instead, each Grade 8 student is given 4 samples
    to analyze, and each Grade 9 students is given 5 samples to analyze, then, in
    total they are given 69 samples. How many Grade 9 students work in the lab?

16. _____

17. The figure below is of a regular octagon with side 1.
    What is the value of the square of the length of the diagonal AE ?
    Express your answer as $i + j\sqrt{k}$ where i, j, and k are integers?

17. _____
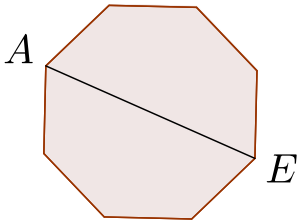
18. In Question 17, how many diagonals are of the shortest length?

18. _____

19. In Question 17, what is the area of the octagon?
    Express your answer as $i + j\sqrt{k}$ where i, j, and k are integers?

19. _____

20. Take 2016, divide it by 3, and round the result down to the nearest
    integer. Keep repeating by dividing the result by 3 and round down
    to the nearest integer. How many times you need to do this until
    reaching the result of zero?

20. _____

21. What number is the sum of all odd factors of 2016?

21. _____

22. How many 11 digit numbers are there using the digit 3 five times and the digit 4 six times?

22. _____

23. The hiking club of 8 girls and 7 boys prepares for a weekend activity and forms a planning committee of 3, of which no more than 2 are boys, and no more than 2 are girls. In how many ways can it be done?

23. _____

24. The picture below shows a trapezoid and its two diagonals. The two parallel sides of the trapezoid have length $4cm$ and $5cm$, and the trapezoid has area $9cm^2$. What is the area (in cm^2) of the shaded region? Express your answer as fraction in lowest terms.

24. _____(cm^2)
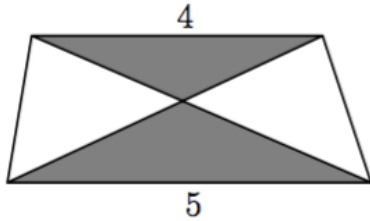
25. How many positive integers smaller than 2016 contain both the digits 1 and 2 in their decimal representation?

25. _____

26. What common fraction with denominator less than 20 lies between $\frac{7}{10}$ and $\frac{5}{7}$?

26. _____

Bull's Eye, Page 1: Problem Solving

1. Alicia and Beti were the only candidates in the election for Student Council President. Alicia got 56% of the votes, and Beti got the remaining 44%. Alicia got 144 votes more than Beti. How many votes did Alicia get?

1. _____

2. Find the sum of the roots of the equation $(x-63)(x-32) = 2016$.

2. _____

3. There are 5 bowls, which between them contain a total of 100 almonds.
Bowl 1 and Bowl 2 contain a total of 55 almonds.
Bowl 2 and Bowl 3 contain a total of 41 almonds.
Bowl 3 and Bowl 4 contain a total of 36 almonds.
Bowl 4 and Bowl 5 contain a total of 28 almonds.
How many almonds are in Bowl 1?

3. _____

4. When 2016 is divided by the positive integer M, the remainder is 1.
When 2016 is divided by $M+2$, the remainder is 3.
What is the smallest possible value of M?

4. _____

5. Evaluate $1 + 2 + 4 + 5 + 7 + 8 + \ldots + 55 + 56 + 58 + 59$ 5. _____
   (the sum of integers from 1 to 59, with every third integer omitted).

6. What number has binary representation of 11111100000?   6. _____

7. A bowl contains 10 jelly beans, of which 3 are black and the   7. _____
   remaining 7 are yellow. Alphonse eats the jelly beans one at
   a time, each time picking a jelly bean at random.
   What is the probability that the third jelly bean that he eats
   is the second black one that he eats?
   Express the answer as a common fraction in lowest terms.

8. What is the largest prime that divides $30! + 31! + 32! + 33!$?   8. _____

9. What is the area of a triangle with sides 10, 13, and 13?   9. _____

10

13   13
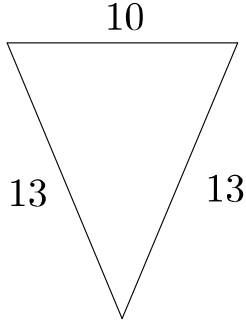
10. The number of diagonals of a regular polygon is 6 times the number   10. _____
of the edges of the polygon. How many edges does the polygon have?

11. Two circles, a smaller one, and a larger one with centre at O,   11. _____
are externally tangent at A. The circles are also tangent to the
line BC where B and C are the tangent points. The line AD
is perpendicular to BC and D is on BC.
What is the ratio of $\angle BAD$ to $\angle AOC$?
Express your answer as a common fraction in lowest terms.

A   O

B   D   C
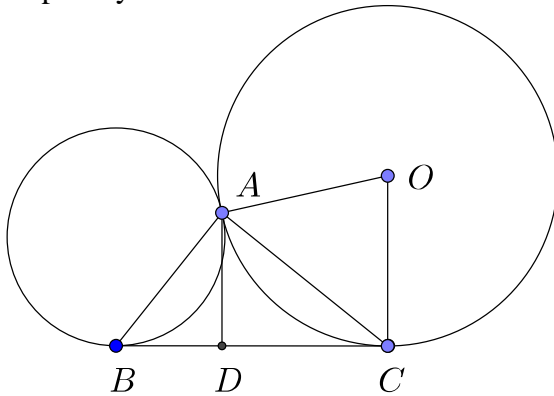
12. In Question 11, $AB = 4$ and $AC = 5$. What is the value of AD?   12. _____
Express your answer as $\dfrac{m}{\sqrt{n}}$ where m is an integer
and n is a prime number.

1. $2^x = 1000$. What is the largest integer less than x?

1. _____

2. Two circles, one with radius 10 and centre at $(0,0)$, and another with radius 5 and centre at $A(5\sqrt{3},0)$ intersect at $P(a,b)$.
   What is the value of a^2?

2. _____
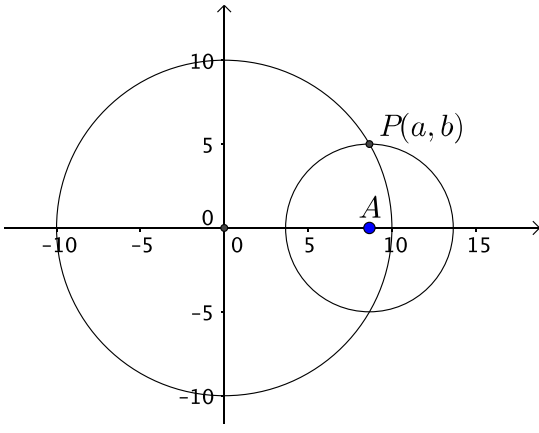
3. In Question 2, you draw a third circle with radius r, such that the centre of the third circle is at $(x,0)$, where $|x-10| = 5$, and such that the third circle is tangent to the circle whose centre is at $(0,0)$.
   What is the sum of the radii of all possible such third circles?
   Note that two circles are the same if and only if they have the same centre and the same radius.

3. _____

4. A positive integer N will be called regular if $N = 2^a \times 3^b \times 5^c$, where a, b, and c are non-negative integers.
   Note that 1 is regular, since $1 = 2^0 3^0 5^0$.
   How many integers between 1 and 100 (inclusive) are regular?

4. _____

5. Find the sum of $\dfrac{1}{3} + \dfrac{1}{15} + \dfrac{1}{35} + \dfrac{1}{63} + \dfrac{1}{99}$.
   Express the answer as fraction in lowest terms.

5. _____

5.   The map below is of the streets of the great city of Funtodrive. The grid length is 100 metres so the distance between intersections is either 100 metres, or, in a few cases, 200 metres. The city bylaws do not allow U-turn at any intersection, and do not allow right or left turns in any 4-street intersection. Also, some segments are one-way, and one must drive in the direction indicated by the arrows. A taxi driver starts at Point A, picks up a person at Point B and takes her to Point C, and then drives to Point D to pick up another person. What is the minimum distance that the driver drove her taxi (in metres) from her taxi stand at Point A until she reached Point D?

6. _____(m)
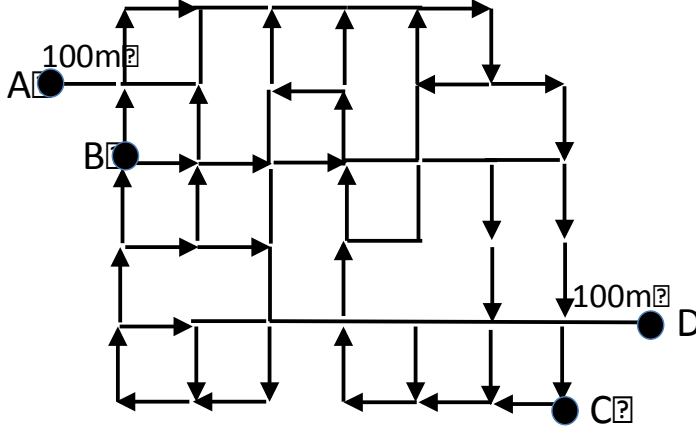
7.   There are 8 cards in a box labeled with the numbers 1 to 8 (each card has a different number). Ann and Ben each draw 4 cards at random, without replacement. The person whose total sum of the 4 cards is larger wins. In case of a tie, the person who draws the card bearing the number 8 wins. Ann begins by drawing the card bearing the number 5. What is the probability that Ann wins?
Express the answer as fraction in lowest terms.

7. _____

8.   In Question 7, when Ann finishes drawing all her cards and adds up her numbers, she gets 18. What is the probability that Ann wins?
Express the answer as fraction in lowest terms.

8. _____

9.   An integer is called a palindrome if it reads the same from left to right as it reads from right to left - (for example: 44, 353, 555, or 1331). How many palindromes between 10 and 2016 are divisible by 9?

9. _____

10.   In the same range as in Question 9, how many palindromes are divisible by 7?

10. _____

11. An airplane flies West above the equator at an altitude of 10 km above   11. _____(km)
     the Pacific Ocean. Assuming that the pilot has a telescope to help her locate
     large objects, and assuming that she can see unlimited distance through the air,
     how far from the airplane (in km) can a ship on the surface of the ocean be seen
     (rounded to the nearest km)? Assume that the radius of the earth is 6400 km
     and ignore the height of the ship in your calculations. Note: figure is not to scale.

airplane • ship

6400 km

6400 km
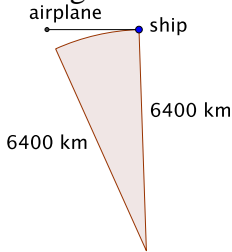
12. In Question 11, the airplane is traveling West at a speed of 920 km/h,   12. _____(km)
     and the ship is travelling East along the Equator at a speed of 40 km/h.
     One minute after the airplane flies directly above the ship, what is the
     distance (in km) from the airplane to the ship (rounded to the nearest km)?

13. In Question 12, the ship is traveling East at speed of 40 km/h. How many   13. _____(h)
     hours (rounded to the nearest integer), would it take for the ship to travel an
     angle of $\alpha = 5°$ (5 degrees) along the equator? Note: figure is not to scale.

A

6400 km

B

6400 km α
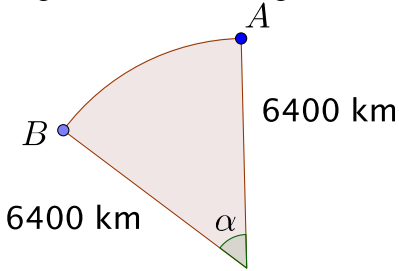
14. How many ordered pairs (a,b) of integers are there such that $\dfrac{1}{a}+\dfrac{1}{b}=\dfrac{1}{8}$?   14. _____

Note that if $x \neq y$ then the ordered pair (x, y) is different from the ordered
pair (y, x). (Hint: note that integers can be either positive, or negative).

15. One circle has radius 5 and centre at O_1, and another circle has radius 7   15. _____
     and centre at O_2. The distance from O_1 to O_2 is 20. Two parallel tangent
     lines with distance of 7 between them are tangent to the circles at points
     A and B respectively. What is the distance from A to O_2? Provide your
     answer as $\sqrt{N}$ where N is an integer. (Hint: note that $\angle O_2 O_1 A = \angle O_1 O_2 B$).

7

O_1 B 7

O_2

20

5   A
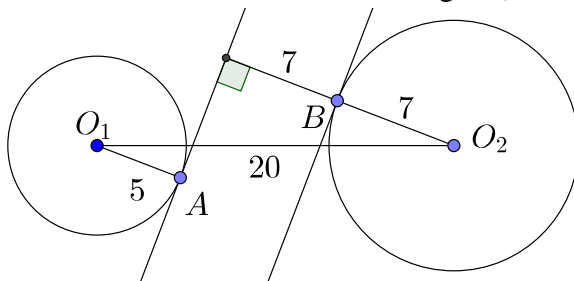
1. $ABCD$ and $ACEF$ are both squares, and $AB = 2$. What is the area of the
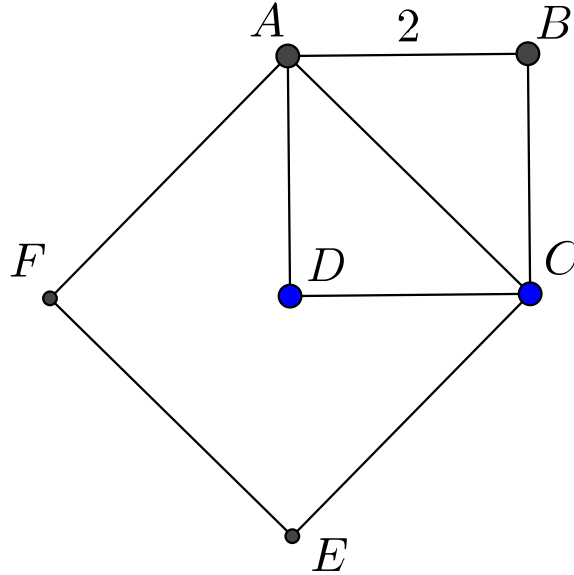
pentagon $ABCEF$?

1. Answer: 10

2. Let P be the sum of the first 50 even positive integers, and let Q be the sum of the first 50 odd positive integers. What is the value of $P - Q$?

2. Answer: 50

3. A perfect third power is a number of the form n^3, where $n > 0$ is a whole number. How many perfect third powers are there between 1 and 2016, inclusive?

3. Answer: 12

4. Simplify: $(2^{10} + 2^{10} + 2^{10} + 2^{10})^{1/3}$

4. Answer: 16

5. How many different triangles with integer side lengths have a perimeter of 11? (Two triangles are considered different if and only if they have a different set of side lengths.)

5. Answer: 4

6. What is the average of all the integers between 5 and 27?

6. Answer: 16

7. What is the largest possible number of intersection points of a circle with radius 5 and a square with side 9?

7. Answer: 8

8. Find $3^4 - 2^3$.

8. Answer: 73

9. Calculate $(3 + 4 + 5) \times (3 \times 4 \times 5)$
9. Answer: 720

10. There are four hockey teams and each team plays every other team twice. How many games are played in total?
10. Answer: 12

11. What is the number of diagonals of a convex hexagon?
11. Answer: 9

12. What is the largest integer N such that $N^N < 2016$?
12. Answer: 4

13. Given that $|x - y| = 2$ and $|x + y| = 2$, what is the smallest possible value of x?
13. Answer: -2

14. What is the number of edges of a triangular prism?
14. Answer: 9

15. What is the largest integer smaller than 2016 that is divisible by 2, 3, and 5?
15. Answer: 2010

16. What is the volume of a cone with height 8 and base area 18?
16. Answer: 48

17. What is the number of possible sums when you roll two dice?
17. Answer: 11

18. What is the number of different 2-letter "words" using any of the letters A, B, and C, possibly more than once?
18. Answer: 9

19. What is the sum of all the single-digit primes?
19. Answer: 17

20. What is the surface area of a box with sides 1, 2, and 3?
20. Answer: 22

21. Find the largest integer N such that $N \times (N + 300) < 2016$.
21. Answer: 6

22. It so happens that there are positive integers a, b, and c such that

$$\frac{355}{113} = a + \frac{1}{b + \frac{1}{c}}$$

What is the value of c?
22. Answer: 16

23. What is the probability of getting exactly 2 heads in 3 tosses of a fair coin? Express the answer as a common fraction in lowest terms.
23. Answer: $\frac{3}{8}$

24. The length of a field is 3 times the width of the field. The length of the field is 102 metres. What is the perimeter of the field, in metres?
24. Answer: 272 (metres)

25. How many minutes does it take to ride a distance of 20km at a speed of 24 km/h?
25. Answer: 50 (minutes)

26. A circle has centre at $(0,0)$ and radius 10. The point $A(8, y)$ is on the circle, where $y > 0$. What is the value of y?
26. Answer: 6

27. The sequence $128, x, 8, 2, \ldots$ is a geometric sequence. What is the value of x?
27. Answer: 32

28. What is the largest prime factor of 156?
28. Answer: 13

29. If $a + b = 7$, $a + c = 8$, and $b + c = 9$, what is the value of $a + b + c$?
29. Answer: 12

30. Rhombus $ABCD$ has $\angle DAB$ equal to 124 degrees. Find $\angle ABD$.
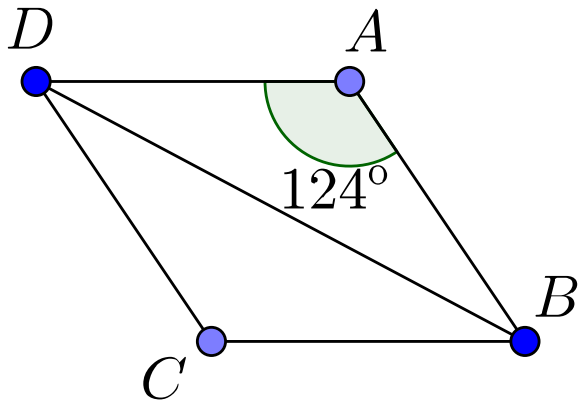
30. Answer: 28

31. You used an equal number of dimes and quarters to pay a total of $3.15. How many coins did you use in total?
31. Answer: 18

32. Gloria's mother is 28 years older than Gloria. The sum of their ages is 60. What is Gloria's age?
32. Answer: 16

33. How many cm^3 are there in 0.0012m^3?
33. Answer: 1200

2016 Math Challengers Regional (Answer Key for Blitz, Bull's-Eye, and Co-Op)

Blitz, Page 1	Blitz, Page 2	Blitz, Page 3	Blitz, Page 4
1. 9.	8. 60.	15. 248.	21. 228.
2. 52.	9. 2062.	16. 240.	22. 7.
3. 34.	10. 13.	17. 20.	23. 107.
4. $\dfrac{11}{18}$.	11. 2.	18. 324.	24. 77.
5. 17.99.	12. 6.	19. 125.	25. 31.
6. 63.	13. 9.	20. 27.	26. 115.
7. $\dfrac{161}{25}$.	14. $6(\sqrt{3}+1)$.		

Bull's-Eye, Page 1	Bull's-Eye, Page 2	Bull's-Eye, Page 3
1. 125.	5. 73.	9. 53.
2. 1.65.	6. $\dfrac{1}{120}$.	10. 48.
3. 36.	7. $\dfrac{1}{12}$.	11. $\sqrt{7}$.
4. 6552.	8. $\dfrac{11}{30}$.	12. 30.

Co-Op, Page 1	Co-Op, Page 2	Co-Op, Page 3
1. 7.06.	6. 9.899.	11. $\dfrac{1}{16}$.
2. 1.50.	7. 60.	12. 737.
3. 1992.	8. 4.36.	13. $\dfrac{1}{16}$.
4. 565066.	9. 0.98.	14. $\dfrac{1}{6}$.
5. 1521.	10. 62.	15. $\dfrac{296}{5}$.

2016 Math Challengers Provincial (Answer Key for Blitz, Bull's-Eye, and Co-Op)

Blitz, Page 1

1. 17.
2. 4.
3. 80.
4. 450.
5. 0.69.
6. $\dfrac{5}{12}$.
7. 30.

Blitz, Page 2

8. -2.
9. 12.5.
10. 63.
11. 14.
12. 160.
13. 158.
14. 1600.

Blitz, Page 3

15. 15.
16. 9.
17. $4 + 2\sqrt{2}$.
18. 8.
19. $2 + 2\sqrt{2}$.
20. 7.

Blitz, Page 4

21. 104.
22. 462.
23. 364.
24. $\dfrac{41}{9}$.
25. 332.
26. $\dfrac{12}{17}$.

Bull's-Eye, Page 1

1. 672.
2. 95.
3. 31.
4. 31.

Bull's-Eye, Page 2

5. 1200.
6. 2016.
7. $\dfrac{7}{60}$.
8. 211.

Bull's-Eye, Page 3

9. 60.
10. 15.
11. $\dfrac{1}{2}$.
12. $\dfrac{20}{\sqrt{41}}$.

Co-Op, Page 1

1. 9.
2. 75.
3. 50.
4. 34.
5. $\dfrac{5}{11}$.

Co-Op, Page 2

6. 5500.
7. $\dfrac{18}{35}$.
8. $\dfrac{1}{2}$.
9. 12.
10. 16.

Co-Op, Page 3

11. 358.
12. 19.
13. 14..
14. 13.
15. $\sqrt{235}$.

2017 Regional Questions February

Blitz, Page 1

1. Round $\frac{100}{11}$ to the nearest integer:

1. _____

2. What is the number of diagonals of a regular pentagon?
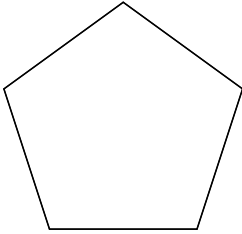
2. _____

3. What is the remainder when you divide 2017 by 501?

3. _____

4. You throw two dice. What is the probability that the sum of the two dice is 8? Express the answer as fraction in lowest terms.

4. _____

5. What is the value of $\frac{20 \times 17}{2+0+1+7}$?

5. _____

6. 7 is the same percent of N as 20 is of 400. What is the value of N?

6. _____

7. How many hours are there in the month of December (31 days)?

7. _____(h)

8. A box has volume $200cm^3$ and the measure of two of its edges are $4cm$ and $5cm$. What is the total surface area of the box (in cm^2)?

$$Volume : 200cm^3$$

4 cm

5 cm

8. _____ (cm^2)

9. What is the sum of the first 20 terms of the following sequence: $1,2,3,4,5,\ldots$?

9. _____

10. Let $f(N) = (N+1)^2 - N^2$. What is the value of $f(2017)$?
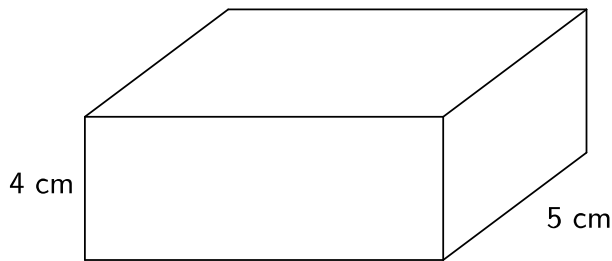
10._____

11. You bought on line an app whose cost was $0.99 USD, and you paid for it in Canadian Dollars ($). If each Canadian Dollar was worth 0.80 USD, how much did you pay? Express your answer in Canadian Dollars correct to 2 decimal places. (For example: an answer of 1.33 is of the correct format).

11._____($)

12. Grade 8 has 120 students and Grade 9 has 210 students. 100 students were selected to participate in a parade in such a way so that the number of Grade 8 participants and the number of Grade 9 participants reflect as close as possible their share of the total sum of grades 8 and 9 students. How many students from Grade 9 were selected?

12._____

13. Abe and Betty start to walk from the same place, at the same time, but in the opposite directions. Abe walks at speed of $6.1\dfrac{km}{h}$,

and Betty walks at speed of $6.5\dfrac{km}{h}$.

What is the distance in metres (m), between them, after 3 minutes?

13._____(m)

14. A pipe transports $9m^3$ (9 cubic metres) of water per hour. How many litres (L) does it transport per second? Express your answer correct to one decimal place.

14._____(L)

15. What is the value of $1! \times (1!+2!) \times (1!+2!+3!) \times (1!+2!+3!+4!)$?

15._____

16. Express $0.13333\ldots$ as fraction in lowest terms.

16._____

17. Consider the following set of 4 digits: $\{2,0,1,7\}$. Julia writes a positive number using one or more of the above 4 digits, and each digit is used at most once. How many such numbers can she write?

17._____

18. Divide \$364 among Alfa, Beta, and Gamma so that for every \$2 that Alfa gets, Beta gets \$2.25 and Gamma gets \$2.75. How many dollars does Beta get?

18._____($)

19. The centre of a square is at the point $(-1,1)$. One of the vertices of the square is at the point $(6,4)$. What is the area of the square?

(6, 4)
.(−1, 1)
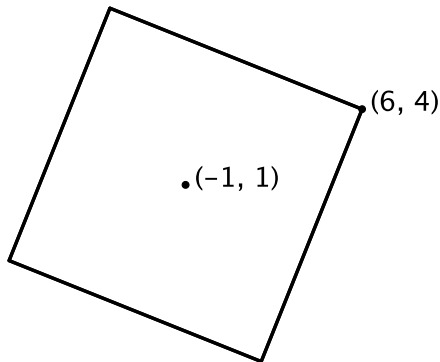
19. _____

20. The club had a surplus of \$2017 last year. The budget committee decided to move at least \$995, but no more than \$1000, to the next year budget; and divide the rest of the surplus equally among all the members.
If each member got \$5.25, how many members are there?

20._____

21. The measure of the side of the external regular hexagon below is 2.
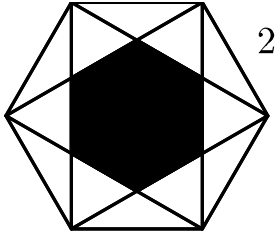What is the area of the shaded region?

Express your answer as $m\sqrt{n}$ where m and n are both primes.

21._____

22. Ann takes a 20 multiple choice questions math test. For every correct answer
she gets 5 marks, for every question left blank, 1 mark, and -2 for every
wrong answer. Ann correctly answered exactly 6 questions.
What is the maximum total score that she could get?

22. _____

23. In Question 22, how many different marks could she get
if she correctly answered exactly 6 questions?

23. _____

24. The sum of 11 consecutive multiples of 11 is smaller than 1000.
What is the largest possible value of the largest of these 11 numbers?

24._____

25. The shaded triangle below is right-angled at C, and its legs AC and BC
have, respectively, length $\frac{1}{3}$ and $\frac{1}{4}$. The triangle is inscribed in a rectangle
as shown. What is the perimeter of the rectangle?
Express the answer as a common fraction in lowest terms.
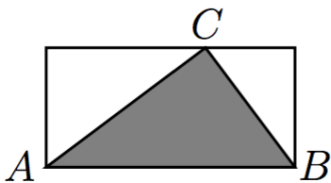
25. _____

26. The digit sum of 2017 is 10. What is the smallest number N that is not
a prime and whose digit sum is 10, and so is the sum of the digit sums
of all of its distinct prime factors?

26. _____

1. Use the flow chart below:

Take an integer N.
Step 1: Multiply by 5. Check: is it greater than 1000? If NO: go back to Step 1.
If YES go to Step 2. Step 2: divide by 11. Step 3: round down to the nearest integer.
If the number you received after Step 3 is 101 what is the largest possible value of N?   1. _____

2. How many cubic metres (m^3) of water are needed for a factory, if it plans
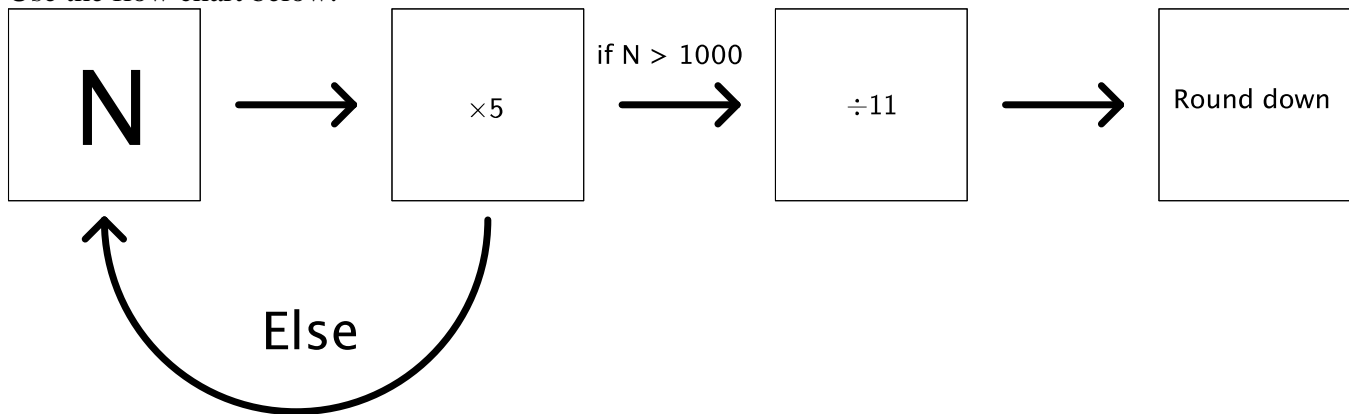to fill up 1.2 million bottles, each contains 400 millilitres of water?   2. _____(m^3)

3. A Prius car uses 5.0 litres of fuel for every 100km if driven on the highway,
and 7.5 litres of fuel for every 100km if driven on city streets. The cost of the
fuel is $1.50/litre. On average, a car is driven 60% of total distance travelled
on the highway, and 40% on city streets. How many km, on average, is it
driven with $1.00 of fuel? Express your answer correct to two decimal places.   3. _____(km)

4. The cost of a book is N dollars plus M cents where N and M are both
integers greater than 10, both are multiples of 9, and $M < 100$. You handed
to the cashier K dollars where K is an integer and got change of $\frac{N}{9}$ dollars

and $\frac{M}{9}$ cents. What was the smallest possible number of dollars
that you could have handed to the cashier?   4. _____($)

5. The sum of the squares of 3 consecutive positive integers is equal to 2525.
What is the sum of the 3 integers?

5. _____

6. Ann wrote down 10 repetitions of the sequence $0123456789AB$ (a total of 120
symbols of the following sequence):
$0123456789AB0123456789AB0123456789AB\ldots0123456789AB$.
Then, she removed every symbol in an odd position in the sequence, starting from
the left. So, she removed the leftmost 0, the leftmost 2, and so on. Then
she removed every symbol in an odd position in the sequence that remained,
again starting from the left. She kept on doing this until only one symbol
of the entire original sequence remained. What was that symbol?

6. _____

7. You write the number 10000 as the sum of other positive numbers, all of which
consists of the digit 4 (so the numbers could be 4, 44, 444, and 4444).
What is the smallest number of the digit 4 that you need?

7. _____

8. Three dice are painted blue on all odd faces and are painted red on all even faces.
You throw the three dice and note that exactly two of the dice landed face up
with same colour. You also note the sum of the three dice.
What is the probability that sum is 12?
Express the answer as fraction in lowest terms.

8. _____

Canadian Math Challengers Society

9.
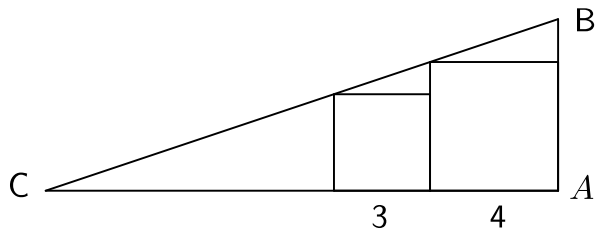
Two side by side squares are inserted in a right-angled $\triangle ABC$ as shown. The square on the right has side 4, and the square on the left has side 3. What is the area of $\triangle ABC$? Express your answer as a common fraction in lowest terms.

9._____

10.

$C_1 = \dfrac{7}{2}$ $C_2 = \dfrac{11}{3}$

The circumference of the fore and hind wheels of a carriage are $\dfrac{7}{2}$ and $\dfrac{11}{3}$ metres respectively. A chalk mark is put on the point of contact of each wheel with the ground before the carriage start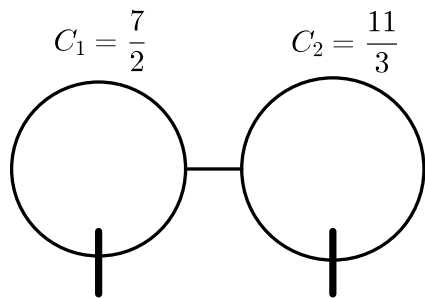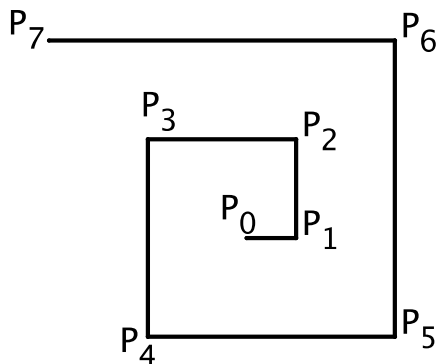s moving on a straight road. How far (in metres) does the carriage have to travel before the chalk marks will both be on the ground at the same time?

10._____(m)

11.

The point P_0 on the coordinate system is located at the origin $(0,0)$. Point P_1 is at $(1,0)$, and the coordinates of the point P_n for $n>1$ are defined in such way so that direction of consecutive segments between points change angle by $90°$ counter clock wise, and segment's length is increased by 1. Thus, P_2 is located at $(1,2)$, P_3 is located at $(-2,2)$, P_4 is located at $(-2,-2)$, P_5 is located at $(3,-2)$, and so on (see figure). What is the distance from P_{28} to P_{32}?

Express the answer as $n\sqrt{m}$, where n is integer and m prime.

11._____

12.
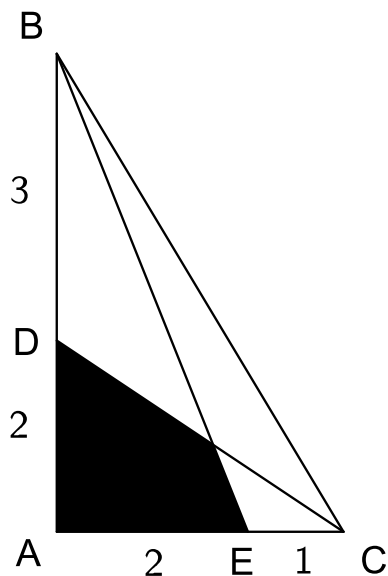
The large triangle, $\triangle ABC$, is right-angled with legs 5 and 3. The lines BE and CD are drawn such that D and E are, respectively, on AB and AC, and such that $AD = 2$, $DB = 3$, $AE = 2$, and $EC = 1$. What is the area of the shaded quadrilateral region?
Express the answer as a common fraction in lowest terms.

12._____

Co-Op, Page 1

(Team answers must be on coloured page. Answers on a white page will not be marked.)

1. $x^{20} = 10$, and $y^8 = 100$. Find the value of $(\frac{y}{x})^{10}$.

1. _____

2. What is the smallest positive number that is a multiple of 3, 6, 9, 12, 15, 18, 21, and 2017?
(Hint: 2017 is prime).

2. _____

3. What is the sum of all the digits of all the integers from 1 to 999, inclusive?

3. _____

4. What is the smallest possible area of a right isosceles triangle with vertices
at $(0,0)$ and $(7,9)$? Express your answer as fraction in lowest terms.
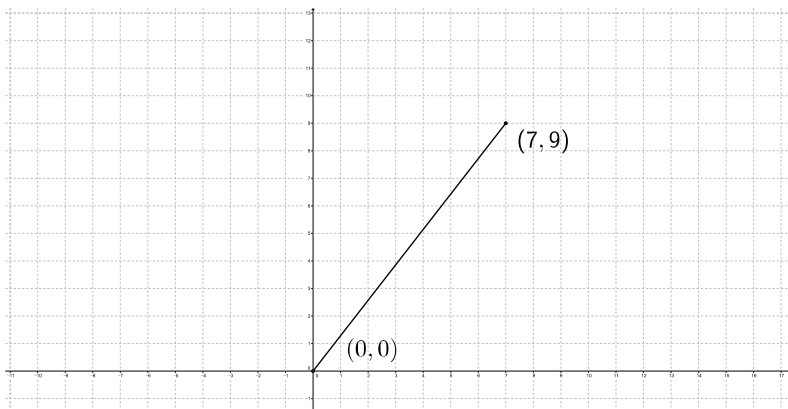
4. _____

5. Andy owes Bob $100. In how many different ways can he pay off the debt using
a combination of 0 or more $20 bills, 0 or more $10 bills, and 0 or more $5 bills?

5. _____

(Team answers must be on coloured page. Answers on a white page will not be marked.)

6. A club party is scheduled for the 4 boys and 4 girls of the club. Two of the boys, (Zen and Yan), will come for sure, and the other two boys each has 50% chance of coming. Three of the girls, (Zilpa, Yolanda, and Xiena), will come for sure, and the fourth girl has 50% chance of coming.

   What is the probability that the same number of boys and girls will be at the party?

   Express your answer as fraction in lowest terms.

   6._____

7. A running track comprised of two 50 metre (m), straight, parallel, stretches, connected by two semicircular ends, each of 25 metre radius. Lois and Amanda start running together at the same time from the same place on the track and both run in the same direction.

   Lois and Amanda run at $8.0\dfrac{km}{h}$ and $10.0\dfrac{km}{h}$, respectively. At the time Amanda passes Lois for the first time after they start running, how far (in metres) has she run?

   Use $\pi = 3.14159$ and round you answer to one decimal place.

   25 m   25 m

   50 m
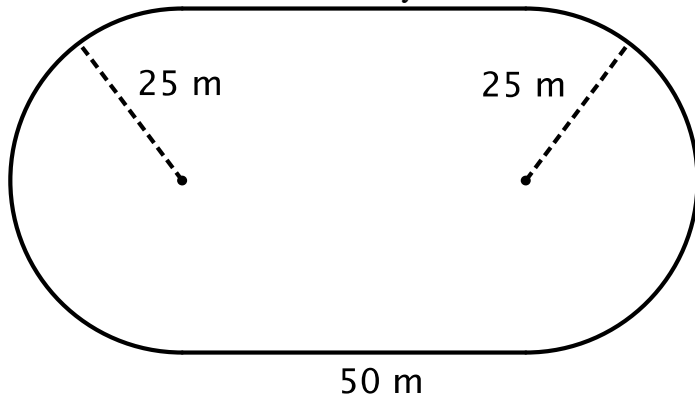
   7. _____(m)

8. In Question 7, Amanda ran a total of $6km$, and Lois ran a total of $5km$.

   How much more time (in seconds) did Lois run than Amanda?

   8._____(s)

9. Consider the following sequence: $1,1,2,1,2,3,1,2,3,4,\ldots$

   What is the value of the 2017-th term?

   9. _____

10. Consider all 6-digit numbers of the form $1a2b3c$,

    where each of a, b, and c are digits between 0 and 9 (inclusive).

    How many of these 6-digit numbers are divisible by 15?

    10._____

(Team answers must be on coloured page. Answers on a white page will not be marked.)

11. Anetta prepared fruit salad for her 4 hungry sons as they came back from school.
Abe, who was the first to come, ate 420 grams (g).

Ben, who came second, ate $\frac{1}{3}$ of what was left.

Cary, who came third, was not that hungry and had $\frac{1}{7}$ of what was left.

Dan, the last to come, was very hungry and had 897 grams.
The rest of the salad, (3% of the original amount), was placed in the composter.
How much salad (in grams) was prepared?

11._____(g)

12. The figure below shows a quarter circle with radius 1 and centre at $O(0,0)$,
and points $A(1,0)$ and $B(0,1)$ on the quarter circle. $P(1,m)$ is defined such that:

(a) $m > 0$, and, (b) the area of ΔOAP is half of the area of the quarter circle.
What is the length of OP (correct to 2 decimal places)?

$B\ (0,1)$

$P\ (1,m)$

$O\ (0,0)$

$A\ (1,0)$
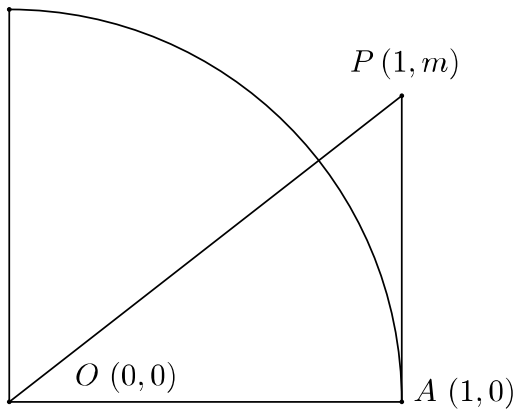
12._____

13. 9 students, of which 3 are boys and 6 are girls, are sitting in a row of 9 seats.
The seats are labeled 1 to 9 from left to right.
In how many different ways can the students sit if no boy sits next to another boy?

13._____

14. The 9 students (in Question 13) are given at random the numbers 1 to 9 to
indicate which seat to take. If the total of the numbers given to the boys is 12,
what is the probability that none of the boys are sitting next to each other?
Provide the answer as fraction in lowest terms.

14._____

15. How many Social Insurance Numbers (SIN) can be issued by the Government of
Canada if: (a) the use of the number 0 is not allowed in the first 3 digits, (b) no digit
is allowed to be used more than 5 times, and (c) each SIN consists of 8 digits?

15._____

1. Both $\triangle ABC$ and $\triangle ABD$ are right triangles. $AB = AC$, and $AD = 2$. What is the area of $\triangle ABC$?
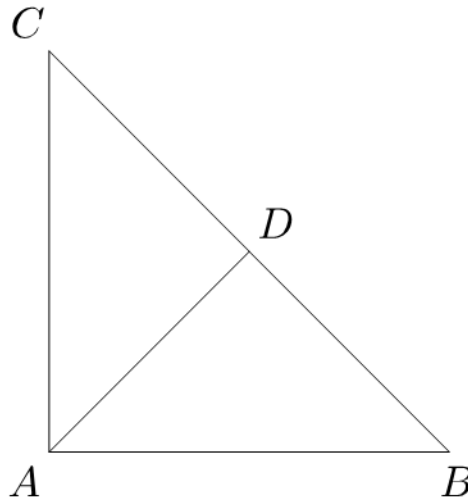
Answer: 4.

2. What is the largest value of integer N such that $N(N+1) < 2017$?
Answer   44.

3. What is the value of $(2+0+1+7) \times (0+2) \times (1+8)$?
Answer: 180.

4. What is the value of the smallest prime larger than 50?
Answer   53.

5. How many different sums can you get when you roll 3 dice?
Answer   16.

6. What is the smallest integer number that is greater than 2017 and is also a multiple of 21?
Answer   2037.

7. How many buckets of water, 20 litres each, are needed to entirely fill a one cubic metre container?
Answer   50.

8. Express 0.2 as fraction in lowest terms.
Answer $\frac{1}{5}$.

9. Find the average value of $\{1,2,3,4,10\}$.
Answer   4.

10. What is the smallest value of integer N such that $N! > 1000$?
Answer 7.

11. What is the digit sum of the number 135?
Answer 9.

12. You roll 2 dice. What is the probability to get a sum of 8? Provide your answer as fraction in lowest terms.
Answer $\dfrac{5}{36}$.

13. Calculate: $(-10) + (-9) + (-8) + \ldots + 10 + 11 + 12 =$
Answer 23.

14. The sum of two different primes is another prime. What is the value of the smaller of these two primes?
Answer 2.

15. A container has 40 marbles (green and blue). 23 are green. How many blue marbles are there in the container?
Answer 17.

16. What is the sum of all odd numbers, each between 10 and 20?
Answer 75.

17. How many different 4 letter "words" can you make by reordering the letters of the "word" POPS?
Answer 12.

18. Calculate: $1 + 2 \times 3 + 4 \times 5 + 6 =$
Answer 33.

19. The bus ride took 20 minutes at an average speed of 0.6 km/minute. How long (in km) was the ride?
Answer 12.

20. A flat screen TV (before tax of 12%) costs $312. What is the cost of the TV, with tax, rounded to the nearest dollar?
Answer 349

21. What is the smallest value of integer N such that $|N + 2| \times |N - 1| < 4$?
Answer -2.

22. What is the average value of 10, 20, and 60?
Answer 30.

23. Round 2017/101 to the nearest integer.
Answer 20.

24.   *ABCDE* is a regular pentagon. What is the value, (in degrees), of $\angle ACB$?
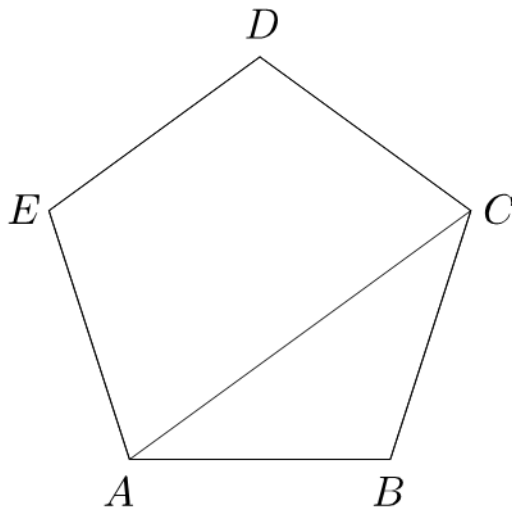
Answer   36.

25.   How many positive non prime numbers are there smaller than 20? Note that 1 is not a prime number.
Answer   11.

26.   Dan walks to his school, 1.32 km away, in 20 minutes. What is his average speed in metres/second? Provide your answer correct to one decimal place.
Answer   1.1.

27.   The sum of two angles of a triangle is 125 degrees. What is the value, (in degrees), of the third angle?
Answer   55.

28.   What is the sum of all factors of 12? Please note that 1 and 12 are factors of 12.
Answer   28.

29.   A bag has 100 marbles, of which 98 are white, and 2 are black. You take out 8 white marbles and place them on the table. Then you take out at random another marble from the bag. What is the probability that it is black? Express your answer as fraction in lowest terms.

Answer $\dfrac{1}{46}$.

30.   What is the decimal expression of the binary number 111111?
Answer   63.

31.   Find the smallest multiple of 7 that is larger than 80.
Answer   84.

32. Zev hands the cashier exactly 50 cents for the candy he bought. He uses only dimes and nickels. How many different **total** number of coins can he hand the cashier?

Answer   6.

33. In the diagram, the origin (0,0) is on the line as shown.
What is the value of x?
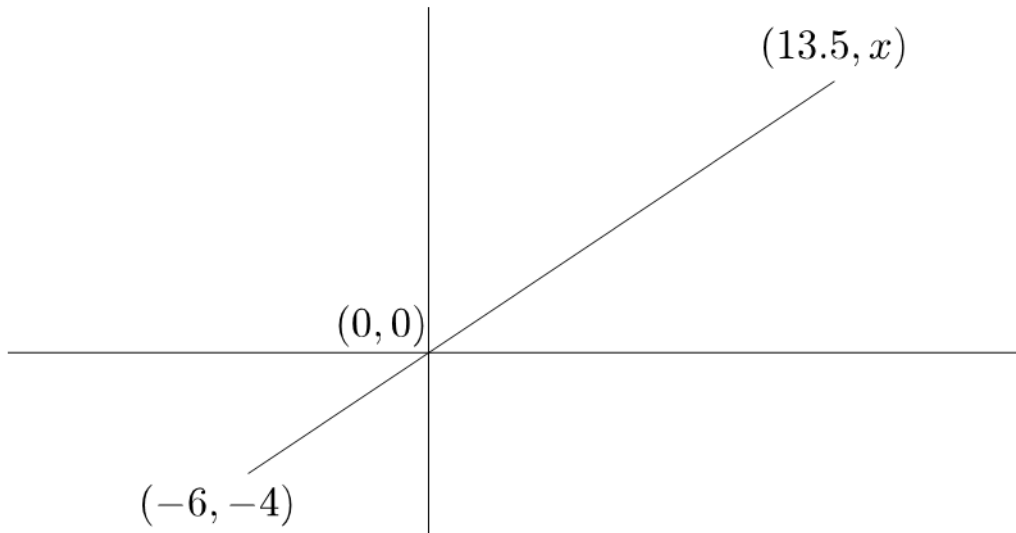
Answer   9.

34. You bought a cookie and paid for it $3.25, in quarters only. How many quarters did you use?

Answer   13.

35. What is the value of the largest common divisor of 54 and 360?

Answer   18.

36. What is the median value of $\{1,6,3,2,5,12,12\}$?

Answer   5.

37. What is the largest prime N such that $1 \times 2 \times \ldots \times N < 2017$?

Answer   5.

38. What is the value of 50% of 20% of 50% of 1000?

Answer   50.

2017 Math Challengers Regional (Answer Key for Blitz, Bull's-Eye, and Co-Op)

Blitz, Page 1		Blitz, Page 2		Blitz, Page 3		Blitz, Page 4	
1.	9	8.	220	15.	891	21.	$2\sqrt{3}$
2.	5	9.	210	16.	$\dfrac{2}{15}$	22.	44
3.	13	10.	4035	17.	48	23.	15
4.	$\dfrac{5}{36}$	11.	1.24	18.	117	24.	143
5.	34	12.	64	19.	116	25.	$\dfrac{37}{30}$
6.	140	13.	630	20.	194	26.	136
7.	744	14.	2.5				

Bull's-Eye, Page 1		Bull's-Eye, Page 2		Bull's-Eye, Page 3	
1.	224	5.	87	9.	$\dfrac{128}{3}$
2.	480	6.	3	10.	77
3.	11.11	7.	25	11.	$2\sqrt{2}$
4.	21	8.	$\dfrac{1}{9}$	12.	$\dfrac{28}{11}$

Co-Op, Page 1		Co-Op, Page 2		Co-Op, Page 3	
1.	100	6.	$\dfrac{3}{8}$	11.	2100
2.	2541420	7.	1285.4	12.	1.27
3.	13500	8.	90	13.	151200
4.	$\dfrac{65}{2}$	9.	1	14.	$\dfrac{3}{7}$
5.	36	10.	67	15.	72880632

Blitz, Page 1

1. 2017 is a prime number and its digit sum is 10. What is the smallest prime number whose digit sum is 10?

1._____

2. How many hours are there in one full week (7 days)?

2._____(h)

3. You tossed a fair coin three times.
   What is the probability that you tossed Head exactly twice?

3._____

4. The cost of a pair of gloves is $9.65, and the cost of a scarf is $19.30.
   You bought 3 pairs of gloves and 2 scarves.
   How many dollars did you pay, in total, rounded to the nearest whole dollar?

4. _____($)

5. What is 30% of 40% of 50% of 60% of 1000?

5._____

6. The average value of $3, 10, 40, x$, and 1 is 10. What is the value of x?

6._____

7. One angle of a pentagon is a right angle ($90°$). All other angles of the pentagon are the same. What is the value of each of the other angles, in degrees, correct to one decimal place?

7._____($°$)

8. Consider the two numbers: 120 and 150. What is the ratio if you divide the largest common divisor of these two numbers by the smallest common multiple of these two numbers? Express your answer as fraction in lowest terms.

8. _____

9. The diagram below shows a convex quadrilateral, $ABCD$. The measure of $\angle B$ is twice the measure of $\angle A$, the measure of $\angle C$ is twice the measure of $\angle B$, and the measure of $\angle D$ is five times the measure of $\angle A$. Note that the diagram is not drawn to scale. What is the degree measure of $\angle D$?
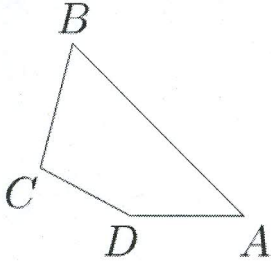
9. _____(°)

10. A bowl contains 48 marbles. Of these, 12 are red, 12 are yellow, 12 are green, and 12 are blue. Apart from colour, the marbles are identical. If Anna is blindfolded, what is the least number of marbles she must remove from the bowl to be sure to get at least 5 marbles of the same colour?

10. _____

11. For any real numbers a,b,c, and d, let $F(a,b,c,d) = ad - bc$.
    If $F(4,x,-3,4) = 100$, what is the value of x?

11. _____

12. Approximate $\sqrt{4000}$ to the nearest integer.

12. _____

13. What is the largest number of different primes that add up to 72?

13. _____

14. The ratio of the volume of cube A to the volume of cube B is 32.
    What is the ratio of the surface area of cube A to the surface area of cube B?
    Round your answer to the nearest integer.

14. _____

15. A triangle has vertices at $(0,0)$, $(4,-3)$, and $(9,12)$. What is the measure of the largest side of the triangle? Round your answer to the nearest integer.

$(9, 12)$

$(0, 0)$

$(4, -3)$
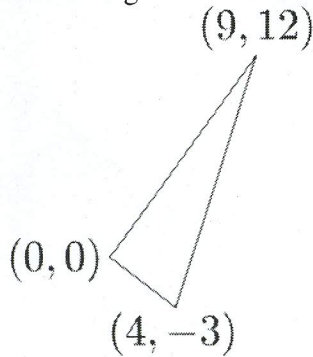
15. _____

16. How many positive integers consist of the digits $\{2,0,1,7\}$ where each of the four digits can only be used maximum of one time?
Example of such integers are $1, 10, 721$, and 2017.

16. _____

17. A cube has volume $4096cm^3$ (cubic centimetre). The measure of each side is increased by $16mm$ (millimetre). By what percentage is the volume increased? Provide your answer correct to one decimal place.

17. _____(%)

18. A farmer has 1247 sheep and 731 goats. He forms them into flocks, keeping sheep and goats separate, and having the same number of animals in all flocks. If these flocks are as large as possible, how many flocks will there be altogether?

18. _____

19. Susan borrowed $5,000 at 6.0% annual interest, compounded annually. If the loan is repaid in full two years after the money is borrowed, how much interest, in dollars, will Susan pay?
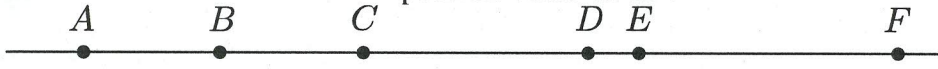
19. _____($)

20. In Question 19, an amount of N dollars of the $5000 loan is repaid one year after the money is borrowed, and the balance (principal plus interest) is repaid one year after the initial payment of N dollars. In total, Susan paid $498 of interest. What is the value of N?

20. _____($)

21. What is the smallest number that, when divided by any of $2, 3, 4, 5, 6, 7, 8, 9$ or 10, it leaves a remainder 1, and when divided by 11, it leaves no remainder?

21. _____

22. When the bus left its first stop, A, it had $N > 0$ passengers. At the next stop, B, none got off the bus, and $N + 15$ got on the bus. At the next stop, C, $\dfrac{K}{2}$, $(K > 0)$, got off the bus, and none got on the bus. At the next stop, D, $\dfrac{K}{3}$ got on the bus, and $\dfrac{N}{2}$ got off the bus. At the next stop, E, $\dfrac{K}{6}$ got on the bus, and $\dfrac{N}{2}$ got off the bus. At the next stop, F, (the last stop), all remaining 17 passengers got off the bus. What is the maximum possible value of K?

$$\begin{array}{ccccccc} A & B & C & & D\ E & & F \\ \bullet & \bullet & \bullet & & \bullet\ \bullet & & \bullet \end{array}$$

22. _____

23. A bus from the main bus terminal in Vancouver leaves to Abbotsford every hour on the hour throughout the day. A bus to Victoria leaves every 2 hours, at 45 minutes after the hour. A bus to Whistler leaves every 3 hours, at 15 minutes after the hour. Greg arrives at the bus terminal, at random, after 11 AM, but no later than 5 PM. What is the probability that the first bus he sees leaving the terminal, is for Victoria? Provide the answer as fraction in lowest terms.

23. _____

24. Find the sum of all integer solutions of the following inequality: $\left| \dfrac{n+20}{n+1} \right| > 3$, where $n \neq -1$.

24. _____

25. What is the smallest positive integer that, when divided by 2017, leaves 20 as remainder, and when divided by 100, leaves 17 as remainder?

25. _____

26. The vertices of the quadrilateral $ABCD$ are on a circle as shown. $AB = L$, $CD = M$, and $AD = N$, are all even numbers, $L < M < N$. The diagonals, $AC = P$, and $BD = Q$, are both integers, and AD is a diameter. What is the smallest possible value of N? Note that the diagram is not drawn to scale. (Hint: ABD and ACD are Pythagorean triangle).
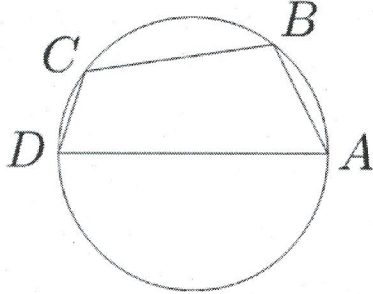

26. _____

Bull's-Eye, Page 1: Word Problems

1. The travel distances, in km, between point A and point B are: 60 by air, 91 by land, and 121 by sea. The travel distances between point B and point C are (respectively): 75, 122, and 75. The travel distances between point C and point A are (respectively): 90, 100, and 105. Greg starts at A, travels to B, then to C, then back to A. What is the shortest distance he could travel, in km, if he must use all three modes of transportation exactly once each?   1.____(km)

2. On TV, between 12 Midnight and 6 AM there are 5 minutes of advertisement during each hour. Between 6 AM and 11AM there are 15 minutes of advertisement during each hour. Between 11 AM and 4 PM there are 10 minutes of advertisement during each hour. Between 4 PM and 12 Midnight there are 20 minutes of advertisement during each hour. What is the average daily advertisement rate on TV, in percent, rounded to the nearest integer, throughout the entire period of 24 hours?   2._____(%)

3. Two painters, A and B, are painting a room. They paint at the same constant rate, but A started painting later than B did. An hour ago, A has only painted $\frac{1}{4}$ as large an area as B had. By now, A has painted a total area $\frac{1}{2}$ as large as B had. How many minutes from now will A have painted a total area $\frac{3}{4}$ as large as B will have painted by then? 3.____(m)

4. Sam planned to drive from point A to point B at speed of $90\frac{km}{h}$, (kilometres per hour). At point C on the way, $60km$ away from point B, the car broke down, and a tow truck was called immediately, and left 42 minutes later from point B, at speed of $120\frac{km}{h}$, reached point C, and immediately towed Sam and the stalled car to point B, at speed of $75\frac{km}{h}$. It took Sam twice as much time as originally planned to get to point B from point A. Note that the diagram is not drawn to scale. What is the distance, in km, from point A to point B?

$\quad\quad\quad A \quad\quad\quad\quad\quad\quad\quad\quad C \quad\quad\quad B$
——————————•————————————————•———————•———————   4.____(km)
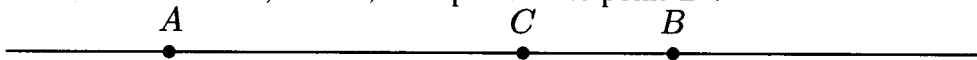
5. What is the remainder when $2001 + 2002 + 2003 + \ldots + 2015 + 2016$ is divided by 2017?

5. _____

6. A group of students has 12 members, 4 from Burnaby, 4 from Richmond, and 4 from Vancouver. Three students of the group are chosen at random to form a committee (of 3 students). What is the probability that the committee has a member from each of the 3 cities? Express the answer as fraction in lowest terms.

6. _____

7. How many integers from 1 to 20 (inclusive) can be written as the difference of the squares of two integers? Note, for example, that $1 = 1^2 - 0^2$, so 1 can be written as the difference of the squares of two integers.

7. _____

8. What is the value of $0.03170170170\ldots + 0.53163163163\ldots$? Provide your answer as fraction in lowest terms.

8. _____

Bull's-Eye, Page 3: Geometry

9. The figure below is made of 10 identical squares. If the perimeter of the figure is 1 metre, (m), what is the area, in square metres, (m^2), of the figure. Express the answer as common fraction in lowest terms.
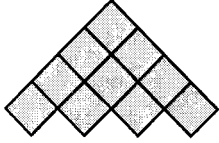

9._____(m^2)

10. You combine 3 identical triangles with edges 3, 4, and 5 to form a convex pentagon. What is the largest possible perimeter of this pentagon?

10._____

11. For shipping a box, Canada Post charges either by its dimensions ($40.00 × (the total measure in metres (m) of its length + width + height)), or by its weight ($4.00 × number of kg) - whichever of the two ways of calculation that is larger. A box carrying the Math Challengers trophies is shipped. Its dimensions are 40 by 60 by 80 centimetre, (cm), and its average density is $0.1\dfrac{kg}{litre}$. Note that one *litre* is $1000cm^3$.
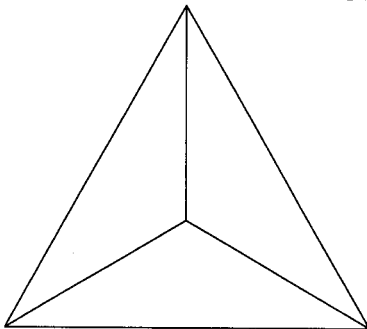What was the shipping costs, in dollars, correct to 2 decimal places?
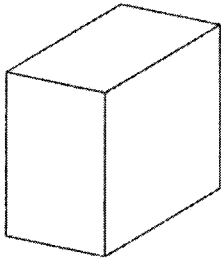
11._____($)

12. A tetrahedron is a body consisting of 4 vertices and 4 identical triangular faces as in the schematic figure below. If the height of the tetrahedron is of length 1, what is the total surface area of the tetrahedron?
Express your answer as $\dfrac{L\sqrt{M}}{N}$ where L, M, and N are primes.

12._____

Co-Op, Page 1

1. If you are allowed to rearrange the order of the digits in the number 20170408, how many different numbers can you get? Note that all digits of the original number must be used and the digit 0 cannot be the first digit.

1._____

2. Let $F(1,1) = 1$. For $n > 1$, $F(n,1) = 4nF(1,1)$.
   For $m > 1$, $F(n,m) = 5mF(n,1)$. What is the value of $F(3,6)$?

2._____

3. An integer consists of 30 digits such that any 7 consecutive digits are different. What is the maximum possible sum of all the 30 digits?

3._____

4. As in Question 3, an integer consists of 30 digits such that any 7 consecutive digits are different, and also such that each digit appears at least once. What is the minimum possible sum of all the 30 digits if the digit 9 appears at least 3 times?

4._____

5. On the desk, there are 11 identical $1 coins. In how many ways you can divide them into one or more piles of coins so that each pile has at least 2 coins?

5._____

6. Let $5^{2a} + 5^{3b} + 5^{4c} + 5^{5d} + 5^{6e} = 5^x$, where a, b, c, d, e, and x are positive integers. What is the smallest possible value of x?

6. _____

7. The surface area of the cone below is $1000cm^2$. The ratio of the height of the cone to the radius of its base is $\dfrac{3}{2}$. What is the volume of the cone, rounded to the nearest $100cm^3$? An answer of 1100 is of the correct format while an answer of 1076 is of the wrong format.
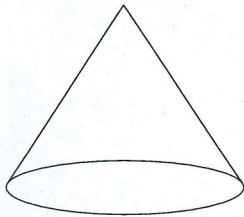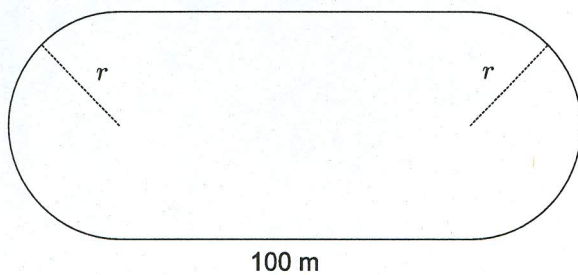

7. _____ (cm^3)

8. What is the remainder if you divide 2^{31} by 31?

8. _____

9. You want to construct a $400m$ (metres) running track consisting of 2 straight segments of $100m$ each, and 2 half circles with the same radius, r. What is the radius, r, in m, of the half circles? Note that the figure is not drawn to scale. Express your answer correct to 1 decimal place.

100 m

100 m

9. _____ (m)

10. In Question 9, the track consists of 9 parallel lanes, and each lane of the track is $1.25m$ wide. If the $400m$ is measured as the length of the inner boundary of the inner most lane, what is the total length, in m, of the outer boundary of the outer most lane, rounded to the nearest integer?

10. _____ (m)

11. You have 51 cards numbered $1,2,3,\ldots,50,51$ (a different number for each card).
   You remove one card at random, look at it, and notice that it is divisible by 5.
   You, then, remove a second (different) card at random.
   What is the probability that the number of the second card is divisible by 3?
   Express your answer as fraction in lowest term.

11. _____

12. How many 5-digits numbers are there, using the digits $1, 2, 3, 4,$ and 5,
   exactly once each, such that at most one digit is larger than its preceding digit?
   Note: 54321 and 52431 are allowed, while 52341 is not allowed.

12. _____

13. The number 7 can be expressed as a sum of primes in the following 3 ways:
   $7,5+2,3+2+2$.
   In how many ways can the number 19 be expressed as a sum of primes?

13. _____

14. The sum of the opposite faces of a traditional die is always 7. One face of one
   traditional die is selected at random and is glued to the face of another traditional
   die such that the two glued faces have the same number.
   You roll the glued dice so that from above you can see one face of each die.
   If the sum of the two visible faces from above is 8, what is the probability
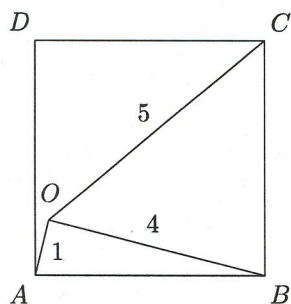   that the two faces with the number 1 are glued together?
   Express your answer as common fraction in lowest terms.
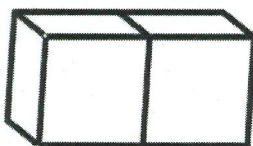
14. _____

15. Let O be a point inside square $ABCD$. Given that $OA = 1$, $OB = 4$,
   and $OC = 5$, find the area of the square $ABCD$.
   Note that the figure is not drawn to scale.

15. _____

Math Challengers

Provincial Competition
Face-off Round 2017

1. The square below has corners at $(5, 0)$, $(0, 5)$, $(-5, 0)$, and $(0, -5)$. How many points with integer coordinates are located on the perimeter of the square?
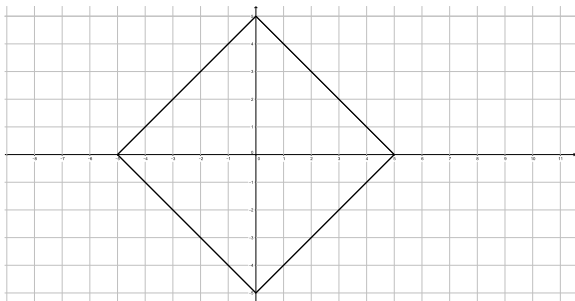
1. The square below has corners at $(5, 0)$, $(0, 5)$, $(-5, 0)$, and $(0, -5)$. How many points with integer coordinates are located on the perimeter of the square?
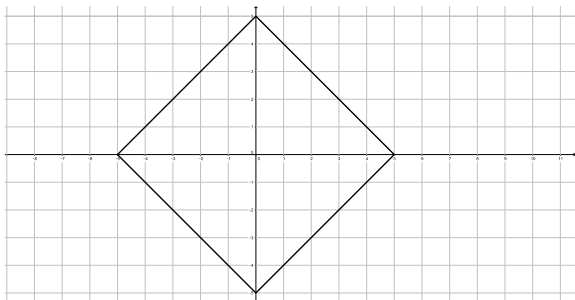
Answer: 20

2. N and $N + 6$ are consecutive primes. What is the smallest possible value of N?

2. N and $N + 6$ are consecutive primes. What is the smallest possible value of N?

Answer: 23

3. You write all the numbers from 1 to 11. What is the sum of all the individual digits that you wrote?

3. You write all the numbers from 1 to 11. What is the sum of all the individual digits that you wrote?

Answer: 48

4. You bought 2 Kit-Kat bars and 3 Crunchy bars and paid a total of $2.65. If you paid $0.49 for each Crunchy bar, what was the price of each Kit-Kat bar? Give the answer in dollars, to two decimal digits.

4. You bought 2 Kit-Kat bars and 3 Crunchy bars and paid a total of $2.65. If you paid $0.49 for each Crunchy bar, what was the price of each Kit-Kat bar? Give the answer in dollars, to two decimal digits.

Answer: $0.59

5. You borrow $600 for a year at a yearly interest of 6.5%. How many dollars do you have to give back (principal plus interest) at the end of the year?

5. You borrow $600 for a year at a yearly interest of 6.5%. How many dollars do you have to give back (principal plus interest) at the end of the year?

Answer: 639

6. 2017 has 365 days and January 1 was on Sunday. What day of the week will be the last day of 2017 (December 31)?

6. 2017 has 365 days and January 1 was on Sunday. What day of the week will be the last day of 2017 (December 31)?

Answer: Sunday

7. How many ways are there to get from point A to point B if you are only allowed to move either up or to the right along the grid lines?

7. How many ways are there to get from point A to point B if you are only allowed to move either up or to the right along the grid lines?

B

A
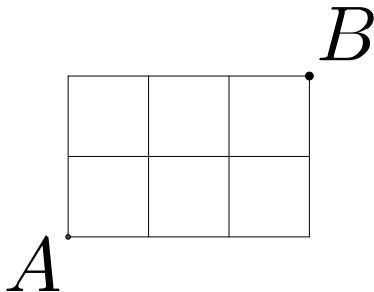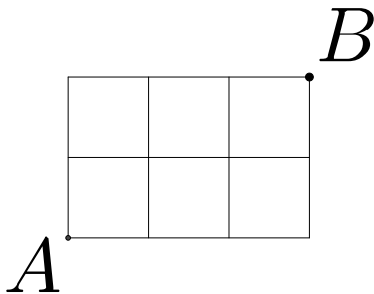
Answer: 10

8.   How many hours are there in one week?

8. How many hours are there in one week?

Answer: 168

9. What is the smallest positive integer which can be expressed as the sum of 4 consecutive multiples of 3, some of which could be 0 or negative?

9. What is the smallest positive integer which can be expressed as the sum of 4 consecutive multiples of 3, some of which could be 0 or negative?

Answer: 6

10. Evaluate: $\dfrac{\sqrt{40}}{\sqrt{2.5}}$.

10. Evaluate: $\dfrac{\sqrt{40}}{\sqrt{2.5}}$.

Answer: 4

11. What is the remainder of 2017 divided by 333?

11. What is the remainder of 2017 divided by 333?

19

12.   The sum of 3 primes is 100. What is the value of the smallest of the three primes?

12. The sum of 3 primes is 100. What is the value of the smallest of the three primes?

Answer: 2

13. The measures of the two diagonals of a rhombus are 30 and 40. What is the measure of a side of this rhombus?
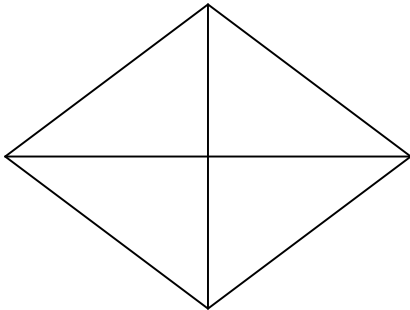
13. The measures of the two diagonals of a rhombus are 30 and 40. What is the measure of a side of this rhombus?
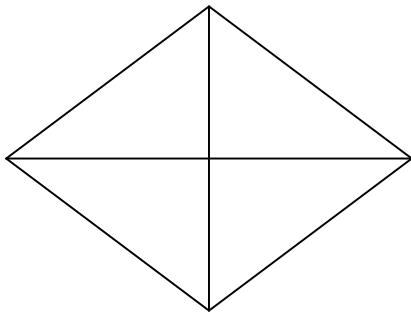
Answer: 25

14. What is the digit sum of the square of the digit sum of 20170408?

14. What is the digit sum of the square of the digit sum of 20170408?

Answer: 16

15. Last night, Alphonse lost two-thirds of his marbles. This morning, he found one-half of the marbles he had lost. What common fraction of his original collection does he now have?

15. Last night, Alphonse lost two-thirds of his marbles. This morning, he found one-half of the marbles he had lost. What common fraction of his original collection does he now have?

Answer: $\frac{2}{3}$

16. Forty-eight is 16% of what number?

16. Forty-eight is 16% of what number?

Answer: 300

17. What is the value of
$2^0 + 0^1 + 1^7 + 7^2$?

17. What is the value of
$2^0 + 0^1 + 1^7 + 7^2$?

Answer: 51

18. What is the sum of the reciprocal of 2 and the reciprocal of 3? Express the answer as a common fraction.

18. What is the sum of the reciprocal of 2 and the reciprocal of 3? Express the answer as a common fraction.

Answer: $\frac{5}{6}$

19. What percent of $7\frac{1}{7}$ (seven and one-seventh) is $6\frac{1}{7}$ (six and one-seventh)?

19. What percent of $7\frac{1}{7}$ (seven and one-seventh) is $6\frac{1}{7}$ (six and one-seventh)?

Answer: 86

20. What is the area of a square with perimeter 44?

20. What is the area of a square with perimeter 44?

Answer: 121

21. While producing answers for 44 Math Challengers questions, Andrew got over 6% of them wrong. What is the largest number of questions for which Andrew could have the right answer?

21. While producing answers for 44 Math Challengers questions, Andrew got over 6% of them wrong. What is the largest number of questions for which Andrew could have the right answer?

Answer: 41 (questions)

22. Evaluate $\dfrac{10! \times 7! \times 4!}{9! \times 6! \times 3!}$

22. Evaluate $\frac{10! \times 7! \times 4!}{9! \times 6! \times 3!}$

Answer: 280

23. If N is a two-digit number, let $\overline{N}$ be the number obtained by interchanging the two digits of N. Given that $N + \overline{N} = 110$, what is the largest possible value of N?

23. If N is a two-digit number, let $\overline{N}$ be the number obtained by interchanging the two digits of N. Given that $N + \overline{N} = 110$, what is the largest possible value of N?

24. 720 students take part in a Math Challengers contest. Each competes in the grade 8 category or the grade 9 category, not both. If 468 are in the grade 9 category, how many percent of the students compete in the grade 8 category?

24. 720 students take part in a Math Challengers contest. Each competes in the grade 8 category or the grade 9 category, not both. If 468 are in the grade 9 category, how many percent of the students compete in the grade 8 category?

Answer: 35 (percent)

25. Simplify $\dfrac{\sqrt{9+\frac{1}{7}}}{\sqrt{\frac{1}{7}}}$

25. Simplify $\dfrac{\sqrt{9+\frac{1}{7}}}{\sqrt{\frac{1}{7}}}$

Answer: 8

26. Call an integer n *good* if $n > 0$ and 2 divides n, 3 divides $n + 1$, 4 divides $n + 2$, and 5 divides $n + 3$. The smallest good integer is 2. What is the next good integer?

26. Call an integer n *good* if $n > 0$ and 2 divides n, 3 divides $n + 1$, 4 divides $n + 2$, and 5 divides $n + 3$. The smallest good integer is 2. What is the next good integer?

Answer: 62

27. A shop sells vegetarian samosas at 3 for a dollar, and chicken samosas at 2 for a dollar. Alan spent $20, half the money on vegetarian samosas, and half on chicken samosas. Altogether, how many samosas did Alan buy?

27. A shop sells vegetarian samosas at 3 for a dollar, and chicken samosas at 2 for a dollar. Alan spent $20, half the money on vegetarian samosas, and half on chicken samosas. Altogether, how many samosas did Alan buy?

Answer: 50 (samosas)

28. If a barrel leaks 750 mL (millilitres) a week, how many weeks will it take for its volume to go from 78 L (litres) down to 69 L (litres)?

28.   If a barrel leaks 750 mL (millilitres) a week, how many weeks will it take for its volume to go from 78 L (litres) down to 69 L (litres)?

Answer: 12 (weeks)

29. What is the value of 35×35?

29. What is the value of 35×35?

Answer: 1225

30.   Store A sells a TV for 24% less than Store C. Store B sells the same TV for 5% less than Store C. By how many percent is the TV at Store B more expensive than it is at Store A?

30. Store A sells a TV for 24% less than Store C. Store B sells the same TV for 5% less than Store C. By how many percent is the TV at Store B more expensive than it is at Store A?

Answer: 25 (percent)

31. B is the subset of all the odd numbers of
$A = \{1, 2, 3, \ldots, 9, 10, 11\}$. How many numbers are in B?

31. B is the subset of all the odd numbers of
$A = \{1, 2, 3, \ldots, 9, 10, 11\}$. How many numbers are in B?

Answer: 6

2017 Math Challengers Provincial Answer Key (Blitz, and Bull's-Eye)

Blitz, Page 1	Blitz, Page 2	Blitz, Page 3	Blitz, Page 4

1. 19.

2. 168.

3. $\dfrac{3}{8}$.

4. 68.

5. 36.

6. -4.

7. 112.5.

8. $\dfrac{1}{20}$.

9. 150.

10. 17.

11. 28.

12. 63.

13. 7.

14. 10.

15. 16.

16. 48.

17. 33.1.

18. 46.

19. 618.

20. 2000.

21. 25201.

22. 36.

23. $\dfrac{1}{3}$.

24. 22.

25. 82717.

26. 50.

Bull's-Eye, Page 1

1. 235.

2. 22.

3. 180.

4. 120.

Bull's-Eye, Page 2

5. 1881.

6. $\dfrac{16}{55}$.

7. 15.

8. $\dfrac{169}{300}$.

Bull's-Eye, Page 3

9. $\dfrac{5}{128}$.

10. 20.

11. 76.80.

12. $\dfrac{3\sqrt{3}}{2}$.

2017 Math Challengers Provincial Answer Key (Co-Op)

Co-Op, Page 1	Co-Op, Page 2	Co-Op, Page 3
1. 4200.	6. 61.	11. $\dfrac{167}{500}$
2. 360.	7. 1900.	12. 27.
3. 185.	8. 2.	13. 23.
4. 99.	9. 31.8.	14. $\dfrac{1}{4}$
5. 14.	10. 471.	15. 17.

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End of 2017 Provincial Math Challengers material

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1. What is the probability of getting 4 Heads in a row when you toss a fair coin?
   Provide your answer as a fraction in lowest terms.

   1. _____

2. The old price of a yearly package of internet subscription services was $300. The price went down by 35%. What is the new price (in dollars) of the yearly package?

   2. _____($)

3. $\triangle ABC$ is a right triangle ($\angle A = 90°$). The area of $\triangle ABC$ is 120 and $AC = 16$. What is the length of side AB?

   B

   120

   90°

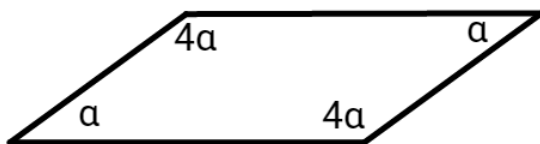   A   C

   16
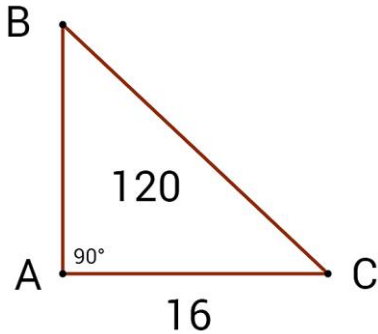
   3. _____

4. Round $\sqrt{2018}$ to the nearest integer.

   4. _____

5. How many hours are there in 250 minutes?
   Provide your answer as a fraction in lowest terms.
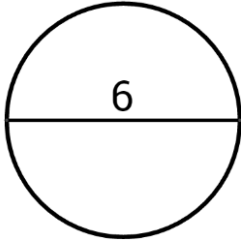
   5. _____(h)

6. The measure of one angle of a parallelogram is four times the measure of another angle of the parallelogram. What is the degree measure of the larger angle?

   4α   α

   α   4α

   6. _____(°)

7. How many primes are there between 70 and 100?

   7. _____

8. The diameter of a circle is 6. Round the area of the circle to the nearest integer.

8. _____

9. 11 of the 16 club members signed up for a bike trip. What percentage of the club did not sign up? Provide your answer correct to 2 decimal places.

9. _____(%)

10. The probability to get a certain sum when throwing two dice is $\dfrac{1}{6}$. What is that sum?

10. _____

11. Simplify to a fraction in lowest terms: $\dfrac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}{\frac{1}{5}} =$

11. _____

12. The numbers $1000, N, 2018,$ and 2527 are consecutive terms of an arithmetic sequence. What is the value of N?

12. _____

13. Alicia has a square garden. This year, she increased the length of the garden by 2 metres (m), and the width by 2 metres (m), so that the expanded garden is also square. That expanded the area of the garden by $40m^2$.
What is the area of the expanded garden in m^2?
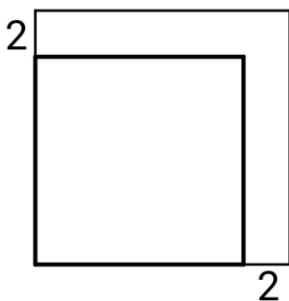
13. _____(m^2)

14. How many times does the digit 1 appear in the binary representation of 2018?

14. _____

15. What is the numerator of a rational number with denominator 41
that is closest to the number π ?

15. _____

16. In 10 years human head hair can grow as much as 1.50 metre (m). Assume that the
growth rate is constant. How many centimetres (cm) can it grow in one month?
Provide your answer correct to 2 decimal places.

16. _____(cm)

17. $ABCDE$ is a 5-sided polygon where $\angle A = \angle B = \angle E = 90°$. $AB = 20$, $BC = 3$,
$CD = 10$, and $DE = 12$. What is the area of the region enclosed by the polygon?

E _____ D
  12
        10
   20   3   C

A   B
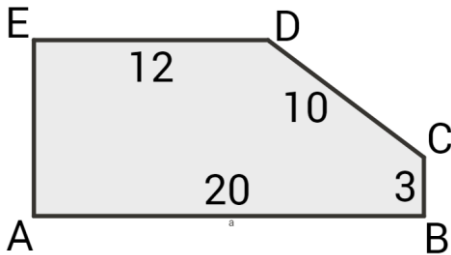
17. _____

18. A backyard swimming pool, when it is full, holds 96 cubic metres (m^3) of water.
It is now empty. How many hours will it take to fill the pool, using 4 hoses each
of which delivers 10 litres of water per minute?

18. _____(h)

19. What is the sum of all the decimal digits of $\dfrac{7}{64}$?

19. _____

20. Two circles with the same radius and centres at A and B, each with circumference
of $72\,cm$. Each circle passes through the centre of the other circle.
What is the perimeter (in cm) of their region of overlap (darker shaded region)?

A   B
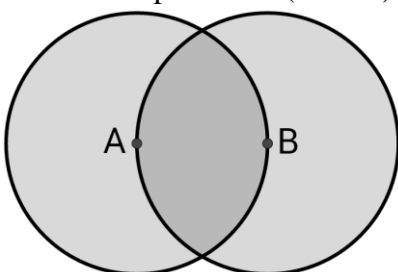
20. _____(cm)

21. Alan owns three different pairs of shoes, exactly one of the pairs is pink. He owns four different pants, exactly one of them is pink. He also owns five different shirts, exactly one of them is pink. In how many ways can he choose one pair of shoes, one pant, and one shirt to wear today if at least one of the items must be pink?

21. _____

22. Let p and q be distinct primes. How many positive integers are divisors of $p^4 \times q^5$ but are not divisors of $p^3 \times q^2$? Note that 1 and n are always divisors of n.

22. _____

23. You wrote all the numbers from 1 to 2018 (inclusive). How many digits did you write in total?

23. _____

24. Abe is good in math and he also likes to cook. He knows that $0°C$ is equivalent to $32°F$, and he also knows that every $1°C$ temperature rise is equivalent to $1.8°F$ temperature rise. He sets the oven temperature so that, if measured in $°F$, it shows a number that is larger by 188 than if measured in $°C$. What $°C$ is the oven set?

24. _____($°C$)

25. What is the total length of all the edges of a pyramid with regular hexagonal base side 10, and height 20? Note that the total number of edges is 12.

Provide your answer as $i + j\sqrt{k}$ where i and j are integers and k is prime.
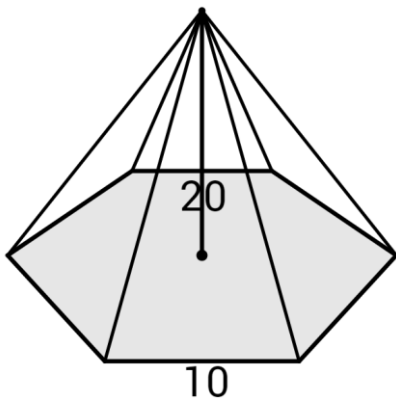
25. _____

26. Abe and Beth ran exactly one full circular lap on the school track (they both ran the exact same distance). They both started from point A on the track and both started at the same time, but were running in opposite directions as indicated on the diagram. Beth finished 32 seconds after they passed each other (at point B), and Abe finished 50 seconds after they passed each other. Each ran at constant speed. How long (in seconds) did Beth take to run the lap? (Hint: it does not matter how long the lap was).

26. _____(s)
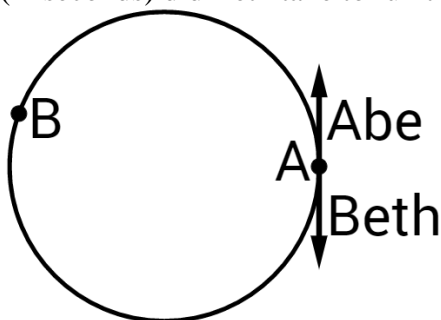
1. A cyclist is travelling at 7500 millimetres per second.

   What is the cyclist's speed in kilometres per hour? 1. _____($\frac{km}{h}$)

2. At the Math Challengers competition, 70% of the competitors are
   wearing jeans, 60% are wearing running shoes, and 50% of the
   competitors who are wearing jeans are wearing running shoes.
   What common fraction of the competitors who are wearing
   running shoes are also wearing jeans?
   Express your answer as fraction in lowest terms. 2. _____

3. At the flea market you can buy 7 desks and 3 beds for $162,
   or, you can buy 7 beds and 3 desks for $218.
   What is the cost (in dollars) of one bed? 3. _____($)

4. Two candles, candle A, and candle B, are of different lengths but burn
   at the same constant rate (meaning their heights decrease at the same rate).
   When they were lit simultaneously an hour ago,
   candle A was twice as long as candle B.
   Right now, candle A is three times as long as candle B.
   How many minutes from now will candle A be four times as long as candle B? 4. _____(m)

5. How many digits does 2^{20} have?

5. _____

6. Consider the following geometric sequence: $4, 2, 1, \dfrac{1}{2}, \dfrac{1}{4}, \dfrac{1}{8}, \cdots$

What is the sum of the first 7 terms?
Provide your answer as a fraction in lowest terms.

6. _____

7. How many integers n are there between 1 and 50 (inclusive)
such that n^n is a square of an integer?

7. _____

8. There are 6 points evenly spaced on the circumference of a circle. Amy picks two
of the points, say A and B, at random, and draws the line that goes through A and B.
Then, Bob picks two of the remaining 4 points, say C and D, at random, and draws
the line that goes through C and D. Then, Bob also draws a line that goes through
the remaining points E and F.
What is the probability that each line drawn by Bob
meets the line drawn by Amy inside the circle?
Provide your answer as a fraction in lowest terms.
Note that lines can meet: inside the circle, or outside the circle,
or be parallel so that they never intersect. In the figure below we
provide a scenario that all three lines intersect outside the circle.
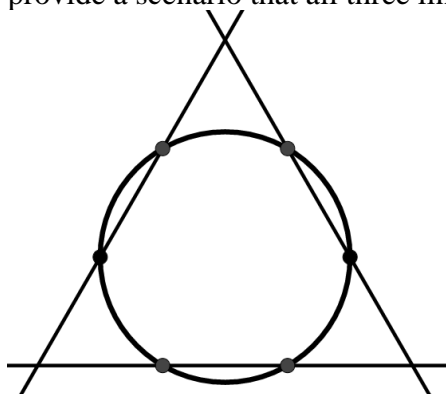
8. _____

9. Let V, E, and F be the number of Vertices, Edges, and Faces of the octagonal prism below. What is the value of $V + E + F$?
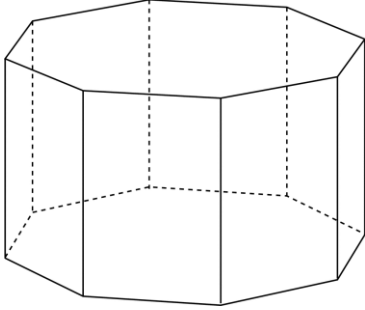

9. _____

10. The point (x, y) where $x > 0$, $y > 0$ is the centre of a circle with diameter 13 that goes through the points $(10,0)$ and $(-2,5)$. What is the value of $x + y$? Provide your answer as a decimal correct to one decimal digit.
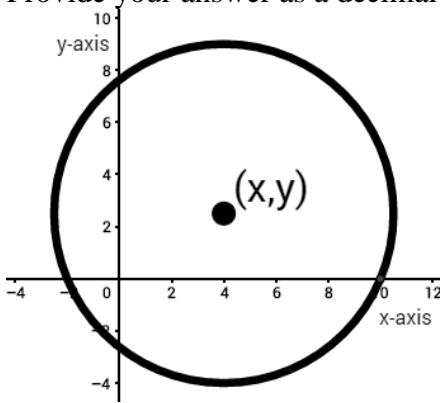
10. _____

11. The two non-parallel sides of the trapezoid $ABCD$ form angles of $60°$ with its base AB. $BC = AD = 10$, and $AB = 20$. What is the area of the trapezoid? Express your answer as $n\sqrt{m}$ where n is an integer and m is a prime.
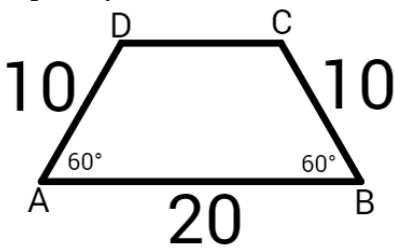
11. _____

12. A small circle with centre at D and radius 2 is circumscribed by a larger semi-circle whose diameter is AB. C is on AB and is also on the small circle. $BC = 8$. Find the radius of the large semi-circle. Provide your answer as a fraction in lowest terms.
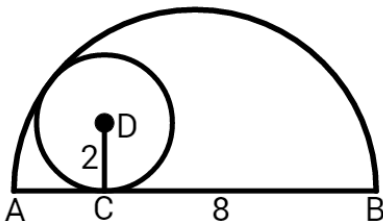
12. _____

1. The lines below represent the streets of a small square $400m \times 400m$ village. Each street segment is $100m$ in l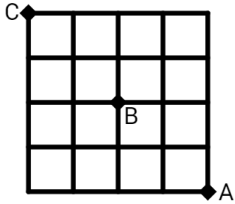ength. In how many ways can Maria go from point A to point C along the streets if she must go through the centre of the village (point B), and her total walking distance must be exactly $800m$?

1. _____

2. The city keeps a huge water tank to supply water in case of emergency. How many m^3, (cubic metres), of water are needed to supply all of the 20,000 residents of the city with an emergency supply of water for 7 days, if each resident needs ration of 20 litres per day?

2. _____(m^3)

3. The water tank in Question 2 can be filled up by two taps. The flow of the first tap is $5\dfrac{m^3}{h}$ (5 cubic metres per hour), and the flow of the second tap is $4\dfrac{m^3}{h}$. The full tank can be drained in 238.5 hours using an outflow pipe with average flow of $48\dfrac{m^3}{h}$. How many hours does it take to fill up the tank if initially it is empty, both taps are turned on, and the outflow pipe is shut off?

3. _____(h)

4. N is obtained by dividing the least common multiple of 18, 150, and 168 by the greatest common divisor of 18, 150 and 168. What is the value of N?

4. _____

5. The line with equation $y = mx + b$ is perpendicular to the line with equation $y = \dfrac{x}{3} + 4$. The 2 lines intersect at a point on the $y-$axis. What is the value of $m \times b$?

5. _____

Canadian Math Challengers Society
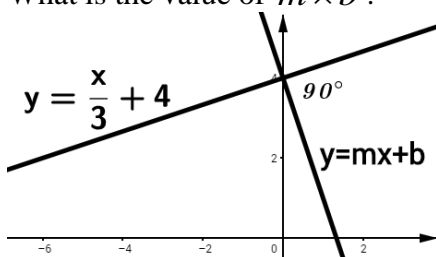
6. At the beginning of 2018 the average price of all goods in Canadian Dollars was up 50% (due to inflation) from the average price of goods at the beginning of 1998. What was the yearly rate of inflation over these 20 years, assuming it was the same each year? Provide your answer in percent correct to 2 decimal places.

6. _____(%)

7. A sphere with radius r is inscribed in a cylinder with radius r and height $2r$. Let V_c be the volume of the cylinder and let V_s be the volume of the sphere.

What is the value of $\dfrac{V_c}{V_s}$? Provide your answer as a fraction in lowest terms.
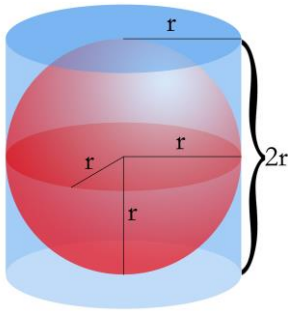

7. _____

8. During the year of 2017 there were exactly 18 earthquakes size 7.0 or larger for every earthquake size 8.0 or larger. There were also exactly 18 earthquakes size 6.0 or larger for every earthquake size 7.0 or larger. If during that year there were 3 earthquakes size 8 or larger, how many earthquakes were at least size 6.0 but less than size 7.0?

8. _____

9. An earthquake occurred right underneath the straight line connecting Victoria and Vancouver at a depth of $70km$ under the surface (ignore the curvature of the Earth). The distance from Victoria to Vancouver is $100km$. The distance from the earthquake's exact location to Victoria is $100km$. What is the distance (in km) from the earthquake's location to Vancouver (dashed line)? Provide your answer correct to 1 decimal place.
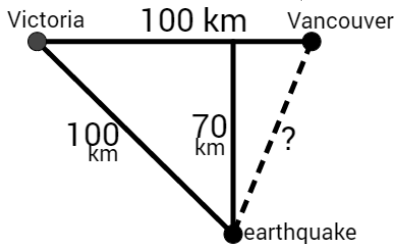

9. _____(km)

10. In how many different ways you can divide six identical coins, into piles so that each pile has at least one coin? Note that "a pile of 2 coins and a pile of 4 coins" is the same as "a pile of 4 coins and a pile of 2 coins".

10. _____

11. A circle with radius $x < 2$ is drawn inside a square $ABCD$ with side 4 such that O, the centre of the circle, is at the centre of square. The point T is on the circle and the line AT is tangent to the circle. $\angle OAT = 30°$. You select a point at random inside the square. What is the probability that this point is inside the circle?
Provide your answer correct to 2 decimal places.
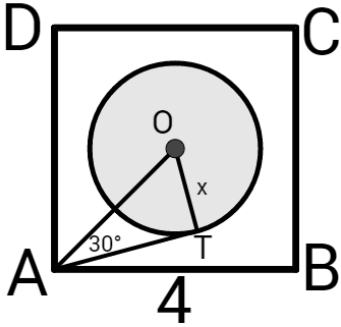
11. _____

12. Let $f(x) = 4 - (2x^2 + 2x - 1)^3$. Find the maximum value of $f(x)$, $(-\infty < x < +\infty)$? Provide your answer as a fraction in lowest terms.

12. _____

13. $2018! = 5^n \times m$ where m and n are positive integers, and m is not a multiple of 5. What is the value of n?

13. _____

14. Consider the polynomial $(2 + x)^{20} = 2^{20} + \cdots + mx^{10} + nx^{11} + \cdots + x^{20}$.

What is the value of $\dfrac{m}{n}$? Provide your answer as a fraction in lowest terms.

14. _____

15. The 8 points in the figure below are at the vertices and the mid-points of the edges of a square. Let L be the number of obtuse triangles that have their vertices at 3 of these 8 points, let M be the number of right triangles that have their vertices at 3 of these 8 points, and let N be the number of isosceles triangles that have their vertices at 3 of these 8 points. What is the value of $L \times M \times N$?
Note that some triangles can be both right triangles and isosceles triangles.

15. _____

2018 Math Challengers Regional (Answer Key for Blitz, Bull's-Eye, and Co-Op)

Blitz, Page 1		Blitz, Page 2		Blitz, Page 3		Blitz, Page 4	
1.	$\dfrac{1}{16}$.	8.	28.	15.	129.	21.	36.
2.	195.	9.	31.25.	16.	1.25.	22.	18.
3.	15.	10.	7.	17.	156.	23.	6965.
4.	45.	11.	$\dfrac{65}{12}$.	18.	40.	24.	195.
5.	$\dfrac{25}{6}$.	12.	1509.	19.	25.	25.	$60+60\sqrt{5}$.
6.	144.	13.	121.	20.	48.	26.	72.
7.	6.	14.	7.				

Bull's-Eye, Page 1		Bull's-Eye, Page 2		Bull's-Eye, Page 3	
1.	27.	5.	7.	9.	50.
2.	$\dfrac{7}{12}$.	6.	$\dfrac{127}{16}$.	10.	6.5.
3.	26.	7.	29.	11.	$75\sqrt{3}$.
4.	20.	8.	$\dfrac{2}{15}$.	12.	$\dfrac{16}{3}$.

| Co-Op, Page 1		Co-Op, Page 2		Co-Op, Page 3	
1.	36.	6.	2.05.	11.	0.39.
2.	2800.	7.	$\dfrac{3}{2}$.	12.	$\dfrac{59}{8}$.
3.	1272.	8.	918.	13.	502.
4.	2100.	9.	75.6.	14.	$\dfrac{11}{5}$.
5.	-12.	10.	11.	15.	8960.

Blitz

1. At the gift shop you bought 5 items at a cost of $4.80 per item and paid the cashier $100. How much money (in dollars) did you get back as change?

1 _____($)

2. Express as common fraction in lowest terms: $1.2 + 2.3 =$

2 _____

3. The lengths of the sides of the box below are 6, 6, and 7. What is the length of the diagonal of the box?
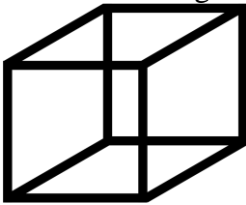
3 _____

4. The sum of 3 different primes is 40. What is the value of the largest of these 3 primes?
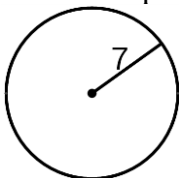
4 _____

5. What percent of 2018 is 100.9?

5 _____(%)

6. There are 3 white marbles and 2 black marbles in a bag. Ed takes 2 marbles out of the bag and places them on the table. What is the probability that both marbles are black? Provide your answer as a fraction in lowest terms.

6 _____

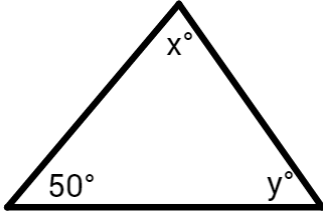7. Round the perimeter of a circle with radius 7 to the nearest integer.

7 _____

8. Simplify: $\left(\sqrt{2018} + \sqrt{5}\right) \times \left(\sqrt{2018} - \sqrt{5}\right) =$

8 _____

9. The year 2017 had 365 days and it rained in 110 days during the year. What percent of the number of days of the year did it rain? Round your answer to the nearest whole number.

9 _____(%)

10. The angles of the triangle are $50°$, $x°$, and $y°$.
    Given that $x + 5y = 350$, what is the value of x?

10 _____(°)

11. What is the remainder when 17^{17} is divided by 18?

11 _____

12. Two dice each have the shape of a regular octahedron (8 faces as in the figure below), with the numbers 1 to 8 written on the faces, one number to each face.
    If these two dice are tossed, what is the probability that the sum is 8?
    Express the answer as a fraction in lowest terms.
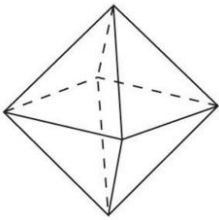
12 _____

13. There are four teams: A, B, C, and D. In the qualifying matches, A plays B, and C plays D. The winners play each other in the final. The probability of A and C winning their qualifying matches is each $\frac{3}{5}$.
    What is the probability that they do not play each other in the final?
    Express the answer as a fraction in lowest terms.

13 _____

14. Round $\sqrt{1025} \times \sqrt{10}$ to the nearest integer.

14 _____

15. What is the largest possible remainder when x is divided by y, if x and y are
    positive integers such that $x > y$, and $x + y = 2018$?

    15 _____

16. The figure below shows a large right triangle divided into three regions by lines perpendicular
    to the base, where the base is divided into segments of lengths 3, 4, and 3 as shown.
    What is the ratio of the area of the shaded region to the area of the large triangle?
   Express the answer as a common fraction in lowest terms.
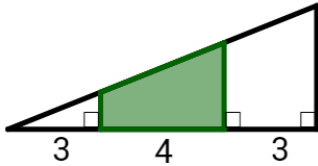
    16 _____

17. Two different numbers are chosen at random from the numbers $1, 2, 3, \cdots, 100$.
    Find the probability that their sum is even. Express the answer as a fraction in lowest terms.

    17 _____

18. A box has integer sides of which at least one is a prime. The area of one of its
    faces is 8 times the area of another face, and 6 times the area of another face.
    What is the minimal possible value for the volume of the box?

    18 _____

19. A standard chessboard consists of an 8 by 8 grid of $2\ cm$ by $2\ cm$ squares, alternately
    coloured white and black, as in the figure. Two dimes are placed on two different randomly
    chosen squares, with their centres at the centre of each of the squares. What is the probability
    that the two dimes end up in either the same (horizontal) row or the same (vertical) column?
    Express the answer as a common fraction in lowest terms.
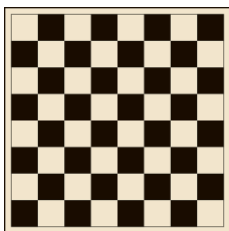
    19 _____

20. If you know that the two coins (in Question 19) ended up in the same horizontal row,
    what is the expected average distance (in cm) between the centres of the two coins?

    20 _____ (cm)

21. Suppose that A and B are two 5-digit numbers,
    which between them use every digit from 0 to 9 exactly once.
   What is the smallest possible positive difference between A and B?

    21 _____

22. How many positive integers, $c > 0,$ are there such that the equation
    $x^2 - 20x + c = 0$ has an integer solution?

    22 _____

23. What is the smallest positive integer which is divisible by 36 and whose decimal
    representation contains no digit other than 0 or 1?

    23 _____

24. A digital 12-hour clock shows only the hour and the minute,
    in a 3 or 4 digit display, such as 1:42, 3:55, or 12:10. Assume that the
    clock switches displays (for example from 11:57 to 11:58) instantaneously.
    Find the fraction of the 12-hour period between 12:00 Noon and 12:00 Midnight,
    such that at least one of the digits showing is the digit "2".
    Express the answer as a fraction in lowest terms.

    24 _____

25. If you throw 4 dice, what is the probability that the total sum is no more
    than 12 given that the number "1" did not appear in any of the thrown dice?
    Express the answer as a fraction in lowest terms.

    25 _____

26. $\triangle ABC$ is isosceles. This triangle can be divided into two isosceles triangles
    by a line segment. Suppose that $\angle A = x°.$ What is the sum of all possible
    different values of x, rounded to the nearest whole number of degrees?

    Note that x could be any of the 3 angles of any triangle that satisfies the condition.
    (Hint: An isosceles triangle can be either obtuse, right, or acute. The figure below
    is one example of a triangle that satisfies the condition and is also a right triangle).
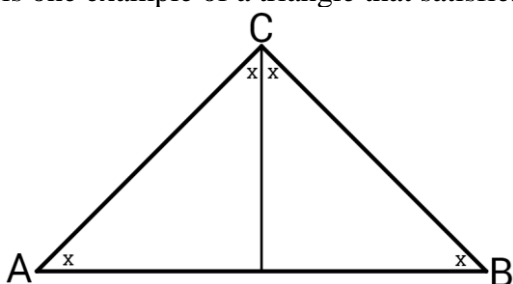
    26 _____

1. 96 games were played in the BC soccer league. For each game the winner
   got 3 points and the loser got 0 points, and if it was a draw - each team got 1 point.
   The total number of points scored by all teams combined was 248.
   What percentage of the games ended with a win by one of the two
   teams that played? Round your answer to the nearest integer.   1 _____(%)

2. When Brad started practicing on the school running
   track, he could do 8 laps in 14 minutes.
   Now he can do 10 laps in 14 minutes.
   By how many seconds has his time per lap decreased?   2 _____(sec)

3. In a certain group, the ratio of female to male is $\dfrac{16}{15}$.

   If 100 females and 100 males leave the group, the ratio will be $\dfrac{12}{11}$.

   What is the total number of people in the group?   3 _____

4. Alicia does the Grouse Grind climb at a slow but steady pace, in 2 hours.
   Beti climbs at a steady but faster pace.
   Beti started the climb 10 minutes after Alicia started.
   Beti passed Alicia when Beti had climbed for 40 minutes.
   When Beti finished the climb, she waited for Alicia.
   How many minutes did Beti wait?   4 _____(min)

5. What is the value of $(\sqrt{5} + \sqrt{24})^2 - (\sqrt{8} + \sqrt{15})^2$? 5 _____

6. What is the largest number smaller than 1000 that can be expressed as the sum of two positive cubes?

Note that $2 = (1)^3 + (1)^3$, and $9 = (1)^3 + (2)^3$ are both sums of two positive cubes. 6 _____

7. Alice is waiting at a bus stop to catch a bus.
She is the 4-th person in a line of 6 people.
Each bus allows only 1, 2, or 3 people to
board with an equal probability of $\frac{1}{3}$.
What is the probability that Alice boards the second bus?
Express your answer as a fraction in lowest terms. 7 _____

8. Alice is finally able to board a bus (see Question 7).
Note that this bus could be the second, third, or fourth bus.
What is the probability that she was the
only person that boarded that bus?
Express your answer as a fraction in lowest terms. 8 _____

9. What is the value, (in degrees), of each of the interior
angles of a regular decagon (10 sided polygon)?
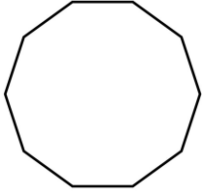
9 _____(°)

10. A right angled triangle has hypotenuse equal
to $15 units$ and area equal to $34 units^2$.
What is the number of units in the perimeter of the triangle?
(Note that the sides of the triangle are not necessarily an integer number of units.)

15

34
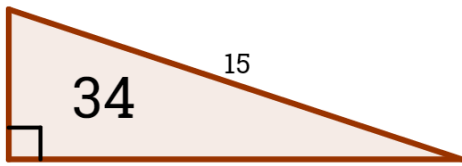
10 _____

11. AC and BD are the diagonals of the trapezoid $ABCD$.
$AB = 12, CD = 8$, and the distance
between the parallel edges AB and CD is 6.
What is the area in $units^2$, of the shaded region ($\triangle APD$)?
Provide your answer as a fraction in lowest terms.

D   8   C
90°
6
P
A
12
B
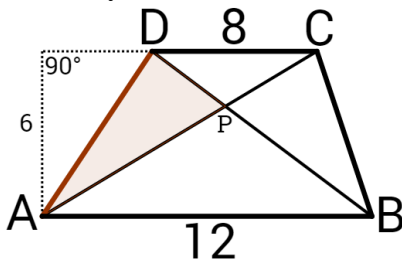
11 _____

12. A circle with centre O and radius 1 is inscribed
in an isosceles right triangle $\triangle ABC$.
What is the area of $\triangle ABC$?
Express your answer as $i + j\sqrt{k}$
where i and j are integers and k is prime.

C

1   O

A   B
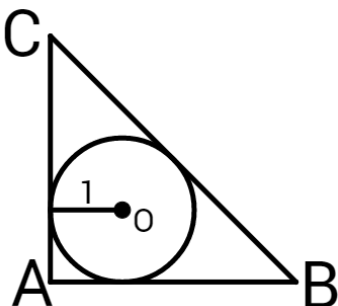
12 _____

1. There are between 20 and 30 students in class A. After competing in the school's Sport Day, all the students of class A joined as a group in the lineup for cookies and drinks. After a while, more classes joined the lineup behind them.

   At a certain point $\frac{1}{5}$ of the students in the line were in front of class A, and $\frac{12}{17}$ of the students in the line were behind class A. How many students are there in class A?   1 _____

2. Consider the cube below. Two edges of the cube are said to be parallel if they are parallel on the 4 sided polygon formed by their vertices. For example: AB and GH are parallel while AB and EH are not. How many ways are there to choose two edges so that they are parallel? Note that the choice of $\{AB, GH\}$ is the same as the choice of $\{GH, AB\}$.
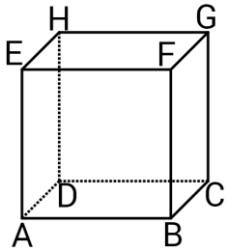
    2 _____

3. Pam is writing a multiple choice test that has 25 questions worth 4 marks each, with 5 choices of answer for each question. Pam knows the answer to 5 of the questions. For 5 questions, she can rule out 3 of the given choices, so she chooses at random from the 2 that remain. For 6 questions, she can rule out 2 of the given choices, so she chooses at random from the 3 that remain. For 3 questions, she can rule out 1 of the given choices, so she chooses at random from the 4 that remain. For the remaining 6 questions, she cannot rule out any of the given choices, so she chooses at random from the 5 choices. On average, what mark can Pam expect to get? Provide your answer as decimal correct to 1 decimal place.   3 _____

4. The arrows indicate allowable paths from A to H. At each branching point, all branches have the same probabilities to be selected. Thus, at A each branch is selected at random with probability of $\frac{1}{3}$. At $B, C, E,$ and $X,$ each branch is selected at random with probability $\frac{1}{2}$.

    Ben takes an allowable path at random according to the rules above, starting at A.

    What is the probability that his path to H goes through both X and G? Express your answer as a fraction in lowest terms.
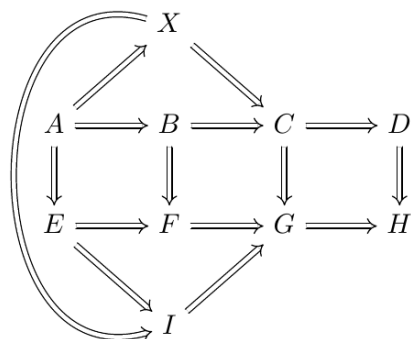
    4 _____

5. How many different 11-digit palindromes are there that use exactly four 2's, four 3's, and three 4's? (A palindrome is a "word" that reads the same forwards or backwards). An example of a palindrome that satisfies the condition is 43223432234.   5 _____

6. Convert $0.230769230769\cdots$ to a fraction in lowest terms. 6 _____

7. Debby had M $1 coins and N $2 coins and went to the gift shop to buy gift certificates for her parents. For her mom, she planned to give to the shop $\dfrac{M}{2}$ $1 coins and $\dfrac{N}{2}$ $2 coins for a total value of $124. But, she made a mistake and gave the cashier $\dfrac{N}{2}$ $1 coins and $\dfrac{M}{2}$ $2 coins and got a gift certificate in the amount she handed to the cashier. With the rest of the coins she bought a gift certificate for $96 for her father. How many coins did she have in total $(M+N)$? 7 _____

8. How many integers N are there such that N, 15, and 14 are the sides of an acute triangle? An acute triangle is a triangle whose angles are all less than $90°$? 8 _____

9. What is the smallest integer greater than 2018 that has a remainder of 4 when divided by 7, a remainder of 8 when divided by 11, and a remainder of 10 when divided by 13? 9 _____

10. Ron has 64 $1 \times 1 \times 1$ cubes, of which 24 are red and 40 are white.

    He wants to put these cubes together and make a $4 \times 4 \times 4$ cube.
    What is the largest possible fraction of the surface area of the large cube that can be red?
    Express your answer as a fraction in lowest terms. 10 _____

11. What is the maximum value of $x \times y$ such that $x > 0, y > 0, x + y = 768$, x and y are integers, and the greatest common divisor of x and y is 64?

11 _____

12. A group of twelve physicists, including Alfred, Bethe, and Gamow, are seated at random around a circular banquet table for twelve. What is the probability that Alfred, Bethe, and Gamow are seated next to each other, not necessarily in that order? Express your answer as a fraction in lowest terms.

12 _____

13. The regular hexagon and the regular octagon below have side 1 and share one edge. Find the value of the shaded area (the area inside the octagon, but outside the hexagon). Express your answer correct to 2 decimal places.

1
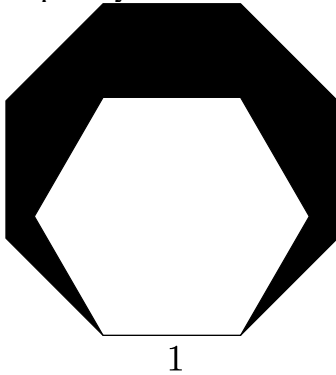
13 _____

14. How many integers greater than 9 have the property that their decimal digits increase as you read them from left to right? Here are some examples: $37, 269, 124689$.

14 _____

15. The square below has side 1. At each of the 4 corners of the square a full circle of radius 1 is drawn with the corner as its centre. What is the total area of the figure (including all the various levels of shading)? Express your answer correct to 2 decimal places.

15 _____
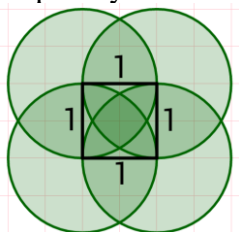
2018 Math Challengers Provincial Answer Key (Blitz, Bull's Eye, and Co Op)

Blitz, Page 1	Blitz, Page 2	Blitz, Page 3	Blitz, Page 4
1. 76	8. 2013	15. 672	21. 247
2. $\dfrac{7}{2}$	9. 30	16. $\dfrac{2}{5}$	22. 10
3. 11	10. 75	17. $\dfrac{49}{99}$	23. 11111111100
4. 31	11. 17	18. 384	24. $\dfrac{3}{8}$
5. 5	12. $\dfrac{7}{64}$	19. $\dfrac{2}{9}$	25. $\dfrac{14}{125}$
6. $\dfrac{1}{10}$	13. $\dfrac{16}{25}$	20. 6	26. 454
7. 44	14. 101		

Bull's Eye, Page 1	Bull's Eye, Page 2	Bull's Eye, Page 3
1. 58	5. 6	9. 144
2. 21	6. 945	10. 34
3. 775	7. $\dfrac{2}{3}$	11. $\dfrac{72}{5}$
4. 14	8. $\dfrac{16}{81}$	12. $3 + 2\sqrt{2}$

Co Op, Page 1	Co Op, Page 2	Co Op, Page 3
1. 24	6. $\dfrac{3}{13}$	11. 143360
2. 18	7. 184	12. $\dfrac{3}{55}$
3. 45.8	8. 15	13. 2.23
4. $\dfrac{1}{4}$	9. 3000	14. 502
5. 30	10. $\dfrac{7}{12}$	15. 7.97

2018 Face Off   Provincial Questions

1. What is the remainder when you divide 2018 by 99?
1. Answer: 38

2. The volume of a cube is 1331. What is the sum of its edges?
2. Answer: 132

3. What is the value of the largest perfect square smaller than 2018?
3. Answer: 1936

4. What is the minimum speed that Diana needs to drive, in km per hour, so that she gets to her cottage 120 km away in 100 minutes?
4. Answer: 72 (km/h)

5. You rolled 3 dice and the sum was 6.
What is the probability that at least one of the dice shows a 1?
Express the answer as a common fraction in lowest terms.
5. Answer: $\dfrac{9}{10}$

6. One angle of an isosceles triangle is 10 degrees. What is the smallest possible value, in degrees, of the largest angle of this triangle?
6. Answer: 85 (degrees)

7. Round $3\sqrt{2018}$ to the nearest integer.
7. Answer: 135

8. All of the 6 small triangles in the regular hexagon below are equilateral triangles with side 1. Find the perimeter of the large hexagon, rounded to the nearest integer.
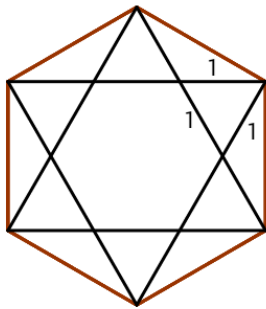
8. Answer: 10
9. The sum of 2 positive integers is 25. What is the maximum possible value of their product?
9. Answer: 156

10. N is an integer between 200 and 220. When you divide 2018 by N the remainder is 200. What is the value of N?
10. Answer: 202

11.   The square in the diagram is inscribed in a circle.
        The area of the square is $\dfrac{128}{\pi}$. What is the area of the circle?
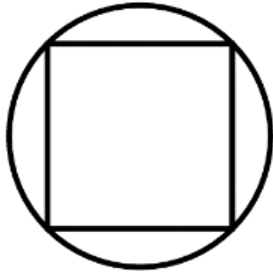
11.   Answer: 64

12.   Express $\dfrac{3}{17}$ as decimal correct to 2 decimal places.
12.   Answer: 0.18

13.   You wrote all the numbers from 1 to N. You wrote the digit '0' 20 times.
        What is the value of N?
13.   Answer: 109

14.   What is the area of a triangle with sides 20, 26, and 26?
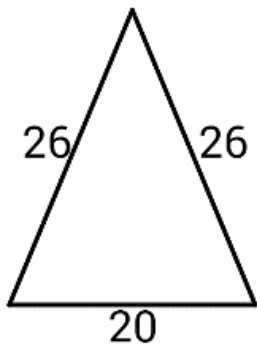
14.   Answer: 240

15.   What fraction with denominator 19 is closest to 0.4?
15.   Answer: $\dfrac{8}{19}$

16.   The number 4 is what percent of the number 9? Round your answer to the
        nearest integer.
16.   Answer: 44 (%)

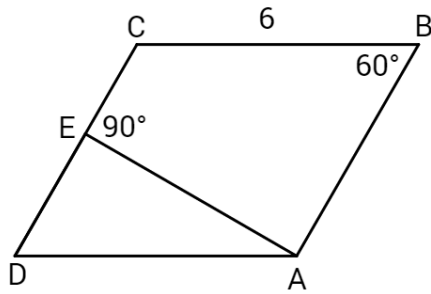17. $ABCD$ is a parallelogram and $\angle ABC = 60°$. $BC = 6$ and AE is perpendicular to CD. What is the length of DE?

17. Answer: 3

18. The concert hall is 95% full for a concert. There are 1045 people at the concert. How many empty seats are there?
18. Answer: 55

19. If $\dfrac{x+y}{x-y} = 1, (x - y \neq 0)$, what is the value of $\dfrac{x+2y}{x-2y}$?
19. Answer: 1

20. Express 0.32 as a fraction in lowest terms.
20. Answer: $\dfrac{8}{25}$

21. $A(1,1)$ and $B(13,10)$ are on the xy-plane, as shown. What is the length of segment AB?
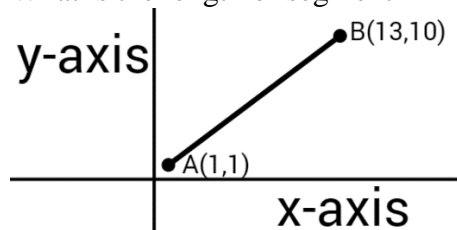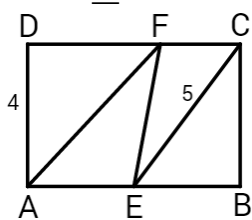

21. Answer: 15

22. The regular cost of a school bag is $40. What is the cost of the bag, (in $), if it is sold at 15% off the regular price?
22. Answer: 34 ($)

23. What is the digit sum of the lowest common multiple of 28 and 6?
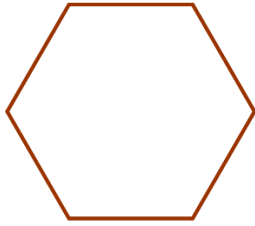23. Answer: 12

24. The area of rectangle $ABCD$ is 24. $AD = 4$, and $CE = 5$. What is the area of $\triangle AEF$?

24. Answer: 6

25. You have 1 white marble, 2 black marbles, and 3 red marbles in a bag. You pick 2 of these marbles at random and place them on the table. What is the probability that both are of the same colour? Express your answer as a fraction in lowest terms.

25. Answer $\dfrac{4}{15}$

26. How many ways are there to pay $0.60 using any combination of quarters ($0.25), dimes ($0.10), and nickels ($0.05)?

26. Answer: 13

27. Two 2-digit primes add to 98. What is the minimum possible value of the largest of these primes?

27. Answer: 61

28. If you buy a computer at the store, it will cost you $1800. You bought the same computer on line and got a discount of 3.5% but you had to pay $30 for shipping. How much money (in dollars) did you save?

28. Answer: 33 ($)

29. The sum of all the lengths of the edges of a hexagon is 18.2. What is the maximum possible length of any of its diagonals rounded to the nearest integer?

29. Answer: 9

30. The Blitz questions of Math Challengers are worth 1 point each, and the Bull's Eye questions are worth 2 points each (no partial marks). If you answered 30 questions correctly and got a total mark of 41, how many Blitz questions did you answer correctly?

30. Answer: 19

31. The ferry crossed the Georgia Strait, a distance of 56 km, in 1 hour and 45 minutes. What was the speed of the ferry in km per hour?

31. Answer: 32 (km/h)

32. You have $4 in loonies ($1 coins). In how many ways can you split the $4 between Aleph, Beth, and Gimel so that each of them gets at least 1 loonie?

32. Answer: 3 (ways)

33. How many intersection points, inside the hexagon, are there of all of the diagonals of a regular hexagon?

33. Answer   13

34. 5 pizzas, each with 8 slices, were distributed to the participants of the party. 4 participants had 4 slices each, 3 had 3 slices each, 2 had 2 slices each, and all the rest of the participants had 1 slice each. What is the total number of participants in the party?

34. Answer: 20

35. $N! + (2N)! = 726$. What is the value of $(N + 4)!$?

35. Answer: 5040

36. Abe, Bill and 4 other people take seats at random on the 6 chairs around a circular table. What is the probability that Abe and Bill are seated next to each other? Express your answer as a fraction in lowest terms.

36. Answer: $\dfrac{2}{5}$

37. At a certain time, 1 Canadian dollar was worth 0.80 US dollars.
How much, in Canadian dollars, was 1 US dollar worth at that time?
Give the answer in decimal format in Canadian dollars, to the nearest cent.

37. Answer: 1.25 (Canadian dollars)

38. Express $3.13333 \cdots$ as fraction in lowest terms.

38. Answer: $\dfrac{47}{15}$

39. 14 years ago, when Dan's twin sisters were born, he was 4 years old. What is the combined age of the 3 siblings today?

39. Answer: 46

40. Simplify to a fraction in lowest terms: $\dfrac{3^0 + 3^1 + 3^2 + 3^3}{27} =$
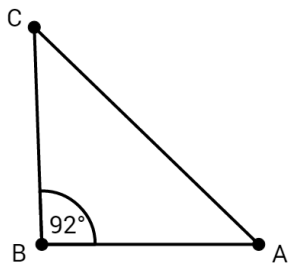
40. Answer: $\dfrac{40}{27}$

1. What is the sum of the smallest four positive odd numbers?

1. _____

2. What fraction of an hour, (in lowest terms), are 90 seconds?

2. _____

3. ΔABC is an isosceles triangle and the value of $\sphericalangle ABC$ is $92°$.
   What is the value of $\sphericalangle BAC$ (in degrees)?

3. _____($°$)

4. Round $\dfrac{10}{3} + \dfrac{17}{5}$ to the nearest integer.

4. _____

5. You throw two regular dice. What is the probability that the sum of the two dice is less than 4? Express the answer as a fraction in lowest terms.

5. _____

6. Convert $\dfrac{5}{8}$ to decimal correct to 3 decimal places.

6. _____

7. Jimmy bought apples at a price of $0.59 per apple, and it cost him less than $4.00. What is the maximum number of apples that he could buy?

7. _____

8. N is an integer such that N^3 is the closest possible value to the number 2019.
   What is the value of N?

   8. _____

9. $17x = x + 21$. What is the value of x?
   Provide the answer as a fraction in lowest terms.

   9. _____

10. The average of $x, 21$, and 27 is x. What is the value of x ?

    10. _____

11. The value of the sum of two consecutive primes is less than 110 but more than 90.
    What is the value of their sum?

    11. _____

12. 79% of x is 97. Round x to the nearest integer.

    12. _____

13. Of the 50 Math Challengers teams that participated in Region X, 39 teams
    consisted of 5 students each, and the other teams consisted of 4 students each.
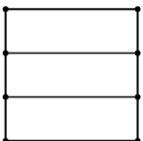   How many students, in total, participated in Region X competition?

    13. _____

14. A square is divided into 3 congruent rectangles by 2 lines
    parallel to one side of the square.
    The perimeter of each of the rectangles is 60.
   What is the perimeter of the square?

    14. _____

15. What decimal number has binary representation of 110011?

15. _____

16. Let $F(x,y) = \dfrac{xy}{x+y+1}$. What is the value of $F(20,19)$?

Express the answer as a fraction in lowest terms.

16. _____

17. The income of Alfie is 25% more than the income of Bonny, and is 25% less than the income of Carla. What is the ratio of the income of Alfie to the average income of Bonny and Carla? Express the answer as a fraction in lowest terms.

17. _____

18. Let x, y, and z be integers such that $|x - y| = 7$, and $|y - z| = 13$.

What is the sum of all possible values of $|z - x|$?

18. _____

19. The sum of the two diagonals, AC and BD, of rectangle $ABCD$ is 38.

The perimeter of rectangle $ABCD$ is 50.

What is the value of the area of rectangle $ABCD$?
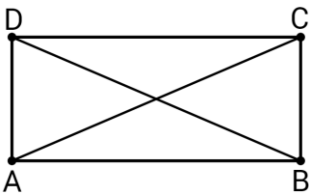
19. _____

20. What is the sum of the 2 solutions of the equation $2019x(x - 1) = 1$?

20. _____

21. $\triangle ABC$ is a right triangle, with sides $AB = 6$, $AC = 8$, and $BC = 10$.
P is on BC and $BP = PC$. Q is on AC and PQ is perpendicular to BC.
What is the value of AQ? Express the answer as a fraction in lowest term.
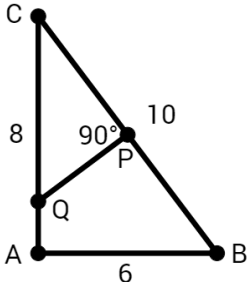
21. _____

22. N is the smallest positive integer composed of only the digits 5 and 7 such
that $\dfrac{N}{9}$ is also an integer. What is the value of $\dfrac{N}{9}$?

22. _____

23. $\dfrac{1}{4}, \dfrac{1}{2}, 1, \cdots, 2^7$ is a geometric series consisting of 10 terms.
What is the value of its sum?
Express the answer as a fraction in lowest terms.

23. _____

24. How many triangles are there such that the measure of their
sides are 1000, 2019, and N where N is an integer?
Note that the triangle with sides 1000, 1754, and 2019 (the figure below)
is the same as the triangle with sides 2019, 1000, and 1754.
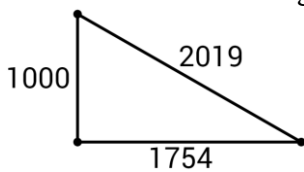
24. _____

25. The class has 12 students (8 girls and 4 boys). The teacher selects 6 students
from the class to be the hockey team, of which 4 are girls and 2 are boys.
In how many ways can this be done?

25. _____

26. A pile of coins on the table consists of 31 coins of 3 different denominations:
some are nickels (5¢), some are dimes (10¢), and some are quarters (25¢).
In the pile, there are 5 more nickels than dimes. If the nickels were dimes,
the dimes were quarters, and the quarters were nickels, the value
of the pile would worth 45¢ more than its value is now.
How many nickels are in the pile?

26. _____

Bull's-Eye, Page 1: Problem Solving

1. When a pot is 20% full of water, it weighs $75\ oz$ (ounces).
    When it is 80% full of water, it weighs $171\ oz$.
    What is the weight, in oz, of the empty pot?   1. _____(oz)

2. The ages of 3 sisters are $M, M + 2$, and $M + 4$,
    where M is a positive integer.
    Both the numbers 5 and 7 divide the sum of their ages.
    What is the smallest possible value of M?   2. _____

3. Two passenger airplanes left Vancouver at the same time,
    both flying to London $7709\ km$ away. The average speed
    of the first airplane was $780\ \dfrac{km}{h}$ (kilometre per hour).
    The total flight time of the second airplane was 10 hours and 11 minutes.
    How many minutes (m) after the first airplane landed did the second airplane land?   3. _____(m)

4. Two bikers, Iota and Zeta, competed in a race.
    They both started at the same time.
    Iota started riding at a constant speed of $20\ \dfrac{km}{h}$ (kilometre per hour).
    But, after $\dfrac{2}{3}$ of the way she had a problem with her bike,
    and it took her m seconds to fix it and resume the race.
    She increased her speed to $24\ \dfrac{km}{h}$ for the rest of the race.
    Zeta rode the first half of the race at an average speed of $18\ \dfrac{km}{h}$ and the second half
    at an average speed of $20\ \dfrac{km}{h}$ for a total time of 28.5 minutes for the entire race.
    Iota and Zeta finished the race at the same time. What is the value of m in seconds?   4. _____(sec.)

5. How many whole numbers are there between $\sqrt{2019}$ and $\sqrt{9999}$?

5. _____

6. You have 3 dice. Five sides of each of the dice are marked with the number $1,$ and the sixth side is marked with the number $4.$ You throw the 3 dice.

   What is the probability that the sum is less than 6 ?
   Express the answer as a fraction in lowest terms.

6. _____

7. A circular pizza has been divided into 6 slices in the usual way. Abe, Bob, Cole, Dave, Ed, and Frank, in that order, each takes a slice. Abe can take any slice. After that, each person must choose a slice which once shared an edge with a slice that has already been taken. In how many orders can the pizza slices be chosen by the six persons? Note that each slice is numbered.
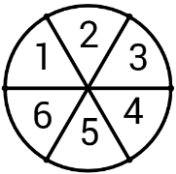
7. _____
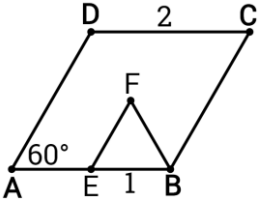
8. For $K = 13, 1 + 2 + \cdots + 13 = 91 = 7 \times 13.$
   Which is the smallest integer M such that $1 + 2 + \cdots + K = M$
   where $K > 13, M = p \times q$, and p, q are primes?

8. _____

9. $ABCD$ is a rhombus with side 2. $\sphericalangle DAB = 60°$. E is on AB,
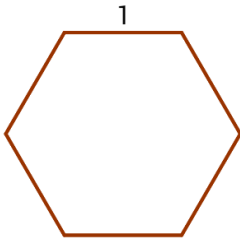 F is inside $ABCD$, and ΔEBF is equilateral with side 1.

   What is the ratio of the area of ΔEBF to the area of $ABCD$?
   Express the answer as a fraction in lowest terms.

9. _____

10. The figure below shows a regular hexagon with side 1.
    You combine 7 such hexagons (one at the centre and the other six surrounding it),
    matching full side to full side. What is the perimeter of the new shape?

10. _____

11. The points $A = (0,0)$, $B = (12,0)$, and $C = (0,16)$,
    are on a circle as shown. The point $(2x, x)$, where $x > 0$,
    is also on the circle. What is the value of x ?

11. _____
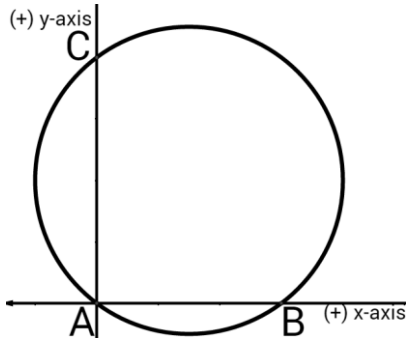
12. ΔABC is an isosceles triangle, ($AC = BC$), with area 15.
    D is on AC and E is on BC such that $AD = DC = BE = EC$.
    M is the point where segments BD and AE intersect.
    Also, it is given that $MC = 4$. What is the value of AM ?

    Express the answer as $\dfrac{\sqrt{p}}{q}$ where p, q are primes.

12. _____

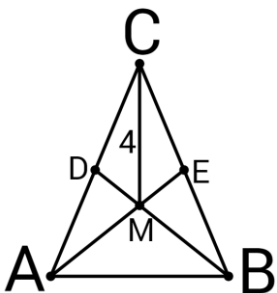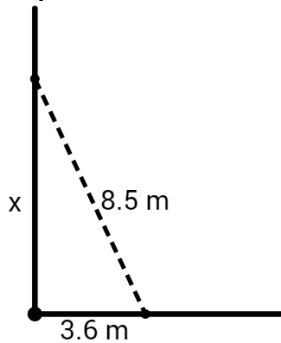1. Bill places a ladder $8.5m$, metre, long with the base of the
   ladder $3.6m$ away from a vertical wall on level ground.

   At what height, x, above the ground does the ladder touch the wall (in m)?
   Express the answer as a decimal correct to one decimal place.

   x   8.5 m

   3.6 m

   1. _____(m)

2. What is the sum of all 3-digit numbers of the form ijj that are perfect squares?
   Note that 211 and 555 are of the form ijj .

   2. _____

3. If you mix $985cm^3$ of fresh water (density of $1.000\dfrac{g}{cm^3}$)
   with $120cm^3$ of solid salt, you get $1L$ ($1000cm^3$) of water
   as salty as the water of the Dead Sea (density of $1.240\dfrac{g}{cm^3}$).
   What is the density of the solid salt (in $\dfrac{g}{cm^3}$) correct to 3 decimal places?

   3. _____($\dfrac{g}{cm^3}$)

4. You need to pack four $3\times 3\times 3$, three $4\times 4\times 4$,
   and two $5\times 5\times 5$ cubical solid boxes in a single larger cubical box.
   What is the volume of the smallest cubical box that you need?

   4. _____

5. Three circles with radius 1 each go through the centres of the other two circles.
   What is the area of the dark region which is the intersection
   of all three circles (correct to 3 decimal places)?

   1

   1   1

   5. _____
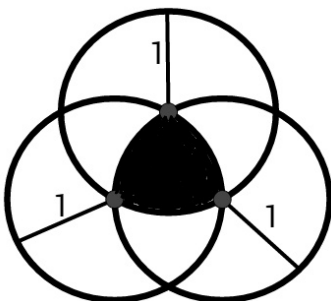
6. How many positive integers smaller than 2019 contain
   only the digits $0, 1, 2, 3,$ or 4 in their decimal expression?
   Note that the following integers satisfy the condition: $1, 10, 22, 434, 1444$.

   6. _____

7. A 24-hour digital clock with 4-digit display shows the time in the format
   of $IJ: KL$ where $I, J, K,$ and L are digits.
   The number represented by the digits IJ is the number of hours,
   and the number represented by the digits KL is the number of minutes.
   How many different times $IJ: KL$ during a 24-hour period are there
   such that $I, J, K,$ and L are the digits $0, 1, 2,$ and 3
   (each appears exactly once but not necessarily in that order)?
   Note that the following are times that can be displayed on
   this clock: $00: 57, 22: 08, 12: 22, 09: 30$.
   Note that the time $23: 01$ satisfies all the specified
   conditions of the question while the time $22: 01$ does not.

   7. _____

8. How many positive integers N satisfy the condition $N = pq, p$ and q are
   primes (not necessarily different), and p and q are smaller than $\sqrt{2019}$?

   8. _____

9. A square is inscribed in a circle which is inscribed
   in an equilateral triangle with side 1.
   What is the area of the square expressed as a fraction in lowest terms?

   9. _____
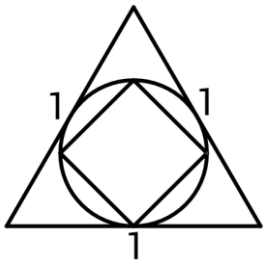
10. The figure below is a square with its 2 diagonals and 4 more line segments drawn.
    How many 4-sided polygons of all sizes and shapes are there?
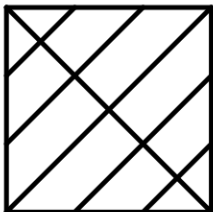
   10. _____

11. The school board borrowed from a bank 1.125 million dollars to build a new gym.
The loan was to be paid back (with interest) after 5 years.
Two years later, the school board received a grant from the provincial government
and paid off the loan early (the entire owed principal amount, plus 9.8% total
interest for the 2 years on the principal amount, plus 1.5% prepayment
penalty on both the due principal and the due interest).
How many dollars were paid back to the bank, rounded to the nearest thousand dollars?
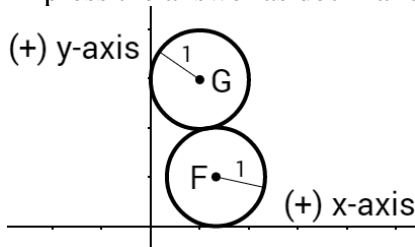An answer of 1,212,000 is of the correct format.

11. _____($)

12. There are two circles, tangent to each other and each is either

tangent to the x-axis or to the y-axis, and both have radius 1.

The centres of the circles are $F = (\frac{4}{3}, w)$, and $G = (x, y)$ where $w, x, y > 0$.

What is the value of y ?
Express the answer as decimal correct to 2 decimal places.

(+) y-axis

G

F• 1

(+) x-axis

12. _____

13. On the shortest day of the year in 2018, in Vancouver, the sunrise time
was 8:17.00 (8 hours, 17 minutes and 0.0 seconds). The sunset time of
that day was 16:06.00. On the following longest day of the year 183 days later,
the sunrise time will be 5:05.00, and the sunset time will be 21:17.00.
What is the average rate of change in the length of the day (from sunrise to sunset)?
Assume that the entire change in day length during the half year period has to
be divided by 183 in order to get the daily average rate of change.
Express the answer in seconds correct to the nearest second.

13. _____(sec.)

14. A kite-like figure has sides $AB = AD$, and $CB = CD$.

A circle with centre O is inscribed inside the kite. E is the tangent

point of AB to the circle, $OE = 6$, and $AE = 11$. CB and OE are parallel.

What is the value of AC? Express the answer as $\frac{p\sqrt{q}}{r}$ where p , q , r are primes.
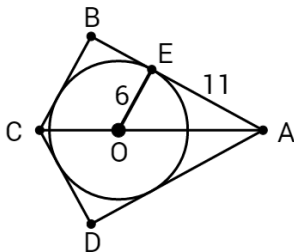
14. _____

15. In country X, car license plates consist of 6 characters,

of which exactly 3 of the characters are numbers $(2, 3, \cdots, 9)$

and the other 3 characters are letters $(A, B, \cdots, Z)$.

Note that: (a) the numbers 0 and 1 are not allowed, (b) the numbers and the
letters can be in any order, and (c) numbers and letters are allowed to repeat.

The following are examples of valid possible plates: $AB333A, 2C23OL$.

How many license plates can be issued under this rule?

15. _____

2019 Math Challengers Regional (Answer Key for Blitz, Bull's-Eye, and Co-Op)

Blitz, Page 1		Blitz, Page 2		Blitz, Page 3		Blitz, Page 4	
1.	16	8.	13	15.	51	21.	$\frac{7}{4}$
2.	$\frac{1}{40}$	9.	$\frac{21}{16}$	16.	$\frac{19}{2}$	22.	6173
3.	44	10.	24	17.	$\frac{15}{16}$	23.	$\frac{1023}{4}$
4.	7	11.	100	18.	26	24.	1999
5.	$\frac{1}{12}$	12.	123	19.	132	25.	420
6.	0.625	13.	239	20.	1	26.	14
7.	6	14.	90				

Bull's-Eye, Page 1		Bull's-Eye, Page 2		Bull's-Eye, Page 3	
1.	43	5.	55	9.	$\frac{1}{8}$
2.	33	6.	$\frac{125}{216}$	10.	18
3.	18	7.	96	11.	8
4.	180	8.	253	12.	$\frac{\sqrt{41}}{2}$

| Co-Op, Page 1		Co-Op, Page 2		Co-Op, Page 3	
1.	7.7	6.	259	11.	1,254,000
2.	1544	7.	18	12.	2.97
3.	2.125	8.	105	13.	165
4.	1000	9.	$\frac{1}{6}$	14.	$\frac{17\sqrt{157}}{11}$
5.	0.705	10.	27	15.	179978240

1.   What is the remainder when you divide 2019 by the digit sum of 2019?
1.   Answer: 3

2.   The sum of the length of all the edges of a cube is 60. What is the volume of the cube?
2.   Answer: 125

3.   What is value of $\dfrac{94}{2}$?
3.   Answer: 47

4.   What is the circumference of a circle with radius 3 rounded to the nearest integer?
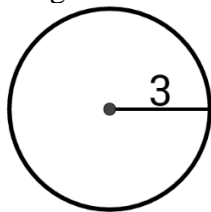
4.   Answer   19

5.   What is the value of the smallest perfect square that is a multiple of 3 and 49?
5.   Answer   441

6.   Alfie rode his bike at an average speed of $27\dfrac{km}{h}$ (kilometre per hour) for 2 hours and 20 minutes. What distance (in km) did he ride?
6.   Answer   63 (km)

7.   A square with diagonal 10 is inscribed in circle. What is the radius of the circle?
7.   Answer: 5

8.   You rolled 3 regular dice. What is the probability that the sum was 4? Express the answer as a common fraction in lowest terms.
8.   Answer $\dfrac{1}{72}$

9. $\sphericalangle ABC = 54°$. Given that $AD = BD$ and $\sphericalangle CAD = 54°$, what is the value of $\sphericalangle ACB$ (in degrees)?
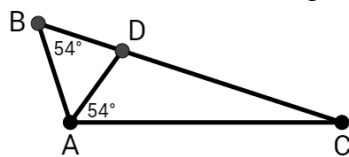

9.   Answer   18 ($°$)

10. $x = \dfrac{20}{3} + \dfrac{25}{4} + \dfrac{31}{5}$. Round x to the nearest integer.

10. Answer   19

11. What is the probability of getting 2 heads in a row when you toss a fair coin? Express as a fraction in lowest terms.

11. Answer: $\dfrac{1}{4}$

12. What is the value of the unit's digit (last digit) of 7^6?

12. Answer: 9

13. The value of the difference of 2 primes smaller than 100 is 6. What is the largest possible value of the smaller prime?

13. Answer   83

14. An ice cream cone costs $2.75 and you used only 25 cent coins to pay for it. How many coins did you use?

14. Answer: 11

15. M is the integer nearest to $\dfrac{2019}{4}$. N is the integer nearest $\dfrac{M}{4}$. What is the value of N?

15. Answer   126

16. What is the sum of the reciprocals of all the positive integers that divide 6? Note that 1 and N divide N.

16. Answer: 2

17. $x + y + 2z = 30$. $z = 14$. What is the value of $x + y$?

17. Answer: 2

18. What fraction with numerator 11 is closest to 0.3?

18. Answer: $\dfrac{11}{37}$

19. $\dfrac{N}{2} + \dfrac{N}{4} = 315$. What is the value of N?

19. Answer   420

20. Express $\dfrac{7}{8}$ as a decimal correct to 3 decimal digits.

20. Answer   0.875

21. The cost of 5 flashlights is $55. What is the cost (in $) of 3 flashlights?

21. Answer: 33 ($)

22. You wrote all the numbers from 1 to 100. How many times did you write the digit 9?

22. Answer   20

23. Bob bought snacks and drinks for a party. He bought 4 cracker boxes at a cost of $13 per box, and 5 drink boxes. In total, he paid $107. What was the cost (in $) of each of the drink boxes?

23. Answer   11 ($)

24. How many minutes are there in 7 hours?

24. Answer: 420

25. The combined ages of Sam, Sara, and their two kids is 111. Both kids are age 13, and Sam is 13 years older than Sara. What is Sara's age?

25. Answer   36

26. $ABCDE$ is a regular pentagon. What is the value of $\angle ACB$ (in degrees)?
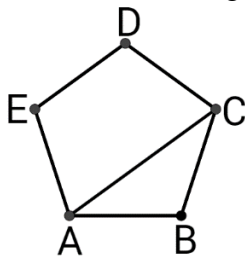
26. Answer   36 ($^\circ$)

27. Daisy wants her pinky fingernail to grow 2 centimetres in length. On average, the pinky fingernail grows 1 millimetre every 20 days. After how many days will she achieve her goal if the nail is not cut or broken?

27. Answer: 400

28. What number has the binary representation 111?

28. Answer: 7

29. Simplify to a fraction in lowest terms: $\dfrac{1}{1+\frac{2}{3}} - \dfrac{1}{1+\frac{3}{2}} =$

29. Answer: $\dfrac{1}{5}$

30. The volume of a cylinder is 40 and the area of its base is 7. What is the height of the cylinder? Round your answer to the nearest integer.

30. Answer: 6

31. N and M are two consecutive prime numbers (with $N < M$) such that $42 < N + M < 53$. What is the value of N?

31. Answer: 23

32.   The measure of the area of a square is 8. What is the measure of the diagonal of the square?

32.   Answer: 4

33.   James stands in a line to buy a ticket to the hockey game. There are 62 people ahead of him in the line and on average 7.5 people get a ticket per minute. How many more minutes is he expected to wait until he buys his ticket? Round your answer to the nearest whole minute.

33.   Answer: 8 (minutes)

34.   How many different rectangles with integer sides have perimeter 22?

34.   Answer: 5

35.   Side lengths of a rectangle are $\sqrt{20}$ and $\sqrt{5}$. What is the area of the rectangle?

35.   Answer: 10

36.   How many 2-digit odd numbers include the digit 3?

36.   Answer: 13

37.   The value of the first term of an arithmetic sequence is -18. The value of the third term is -1. What is the value of the seventh term?

37.   Answer: 33

38.   Express the sum of 0.05 and 0.25 as a common fraction in lowest terms.

38.   Answer: $\dfrac{3}{10}$

39.   The width of a rectangular garden is $13m$, (metres), and its area is $221m^2$. What is the perimeter of the garden (in m)?

39.   Answer: 60

40. $F(x) = \big((x + 1)x + 1\big)x + 1$. What is the value of $F(-1)$?

40.   Answer: 0

41.   The sum of two angles of a triangle is $115°$. What is the value of the third angle of the triangle (in degrees)?

41.   Answer: 65 ($°$)

42.   The area of a circle is $\dfrac{81}{\pi}$. What is the circumference of the circle?

42.   Answer: 18