AIMS African Institute for Mathematical Sciences SENEGAL

DEEP LEARNING OPTIMIZERS

ADAPTIVE LEARNING ALGORITHM

GROUP 2 TEAM

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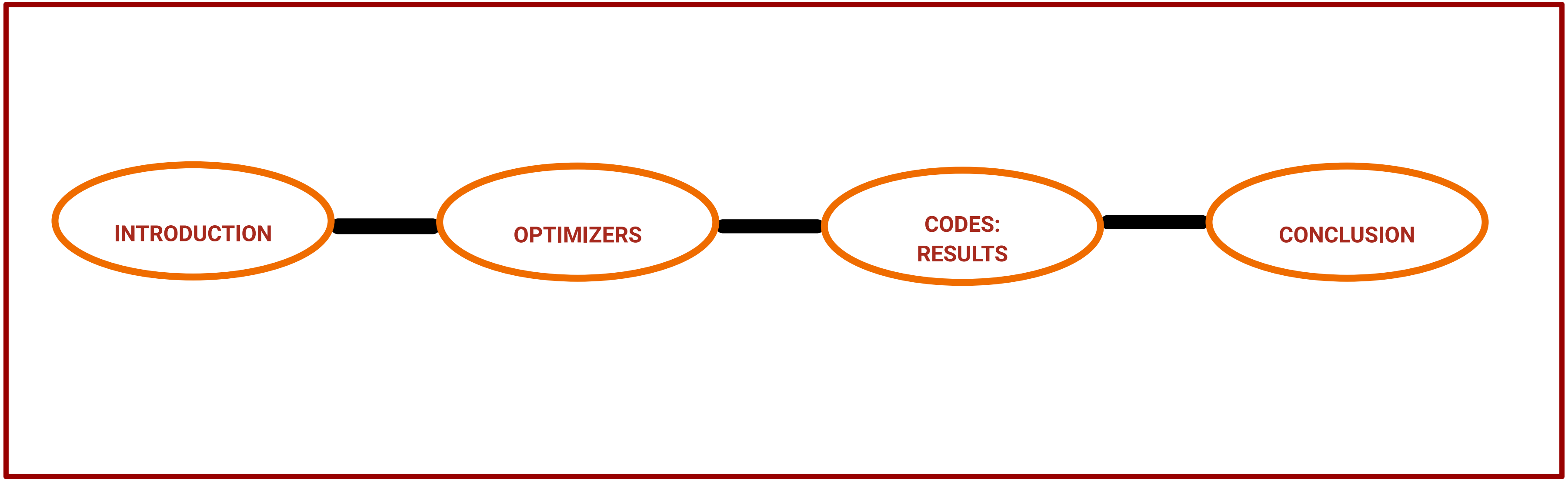
OUTLINE



INTRODUCTION

Mathematical Formulation

- $\{X, Y\}, \quad X \in \mathbb{R}^{n\times m},\ Y \in \mathbb{R}^n$
- We want to predict $y \in Y$ using $x \in X$ so we learn the mapping from x to y.
- We use a neural network $f_\theta : \mathbb{R}^{n\times m} \to \mathbb{R}^n$ which returns a prediction $\hat{y}$.
- We want to choose parameters θ of the neural network so that $\hat{y}$ is close to y. That is, we want to minimize the distance between $\hat{y}$ and y.
- We measure how well we are doing and find the optimal parameters by minimizing an objective function:

$$\min_\theta J(\theta) \triangleq \frac{1}{n}\sum_{i=1}^{n} l\big(y_i, f_\theta(x_i)\big)$$

for each $x_i \in X, y_i \in Y$ and $l(\cdot,\cdot)$ a distance metric.

Gradient Descent Algorithm

Repeat until convergence:

- Compute the gradient $\nabla_\theta J(\theta)$ of the objective function
- Update the parameters

INTRODUCTION

First generation

1. Batch (Vanilla) gradient descent
2. Stochastic gradient descent
3. Mini batch gradient descent
4. Momentum
5. Nesterov accelerated gradient

Second generation: Adaptive learning

1. Adagrad
2. Adadelta
3. RMSprop
4. Adam

OPTIMIZERS: First Generation

Batch gradient descent (BGD)

$$\theta = \theta - \eta \cdot \nabla_\theta J(\theta)$$

- Gradient updates after calculating loss of entire training example.
- High resource demands - lots of space in memory.
- Long training time.

- Takes fewer steps to converge.
- Perfect gradient.

Stochastic gradient descent (SGD)

$$\theta = \theta - \eta \cdot \nabla_\theta J(\theta; x^{(i)}; y^{(i)})$$

- Gradient updates after loss for one training example $(x_i, y_i) \in \{X, Y\}$
- Faster training time than BGD.
- Less need for memory.
- Gives quick info about model performance.
- Suitable for online learning.

- Noisy gradient.
- Many steps to converge.

Mini-batch gradient descent (MBGD)

$$\theta = \theta - \eta \cdot \nabla_\theta(\theta; x^{(i:i+k)}; y^{(i:i+k)})$$

- Gradient updates per batch.
- Less noisy than SGD.
- Fewer steps to converge than SGD.

- Optimal batch size may be difficult to get.

OPTIMIZERS: First Generation

Convergence steps

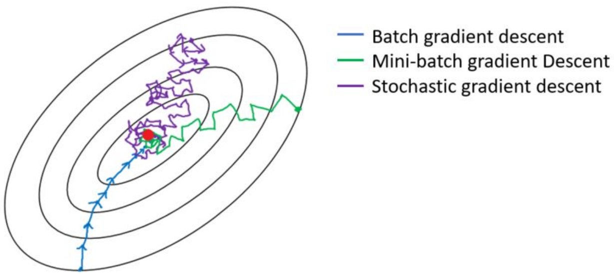


Figure 1: Convergence diagram for BGD, SGD, MBGD

Challenges

- ❖ Local minima

Figure 2: Momentum (magenta) vs. Gradient Descent (cyan) on a surface with a global minimum (the left well) and local minimum (the right well

- ❖ Plateau, Saddle point
- ❖ How to adjust the learning rate
- ❖ Choice of a proper learning rate
- ❖ Dealing with sparse data

Figure 1 Source
Figure 2 Source

MOMENTUM

- **Our goals**:
  - ❖ We do not want the high oscillations.
  - ❖ We want to move towards the minimum faster.
- Smoothens the noise.
- Gives weight based on the previous steps.

$$v_t = \beta v_{t-1} + (1-\beta)\nabla_\theta J(\theta)$$

$$\theta_t = \theta_{t-1} - \eta v_t$$

where t is the time, β is the momentum term and v_{t-1} is the mean of past gradients. β is usually taken as 0.9.

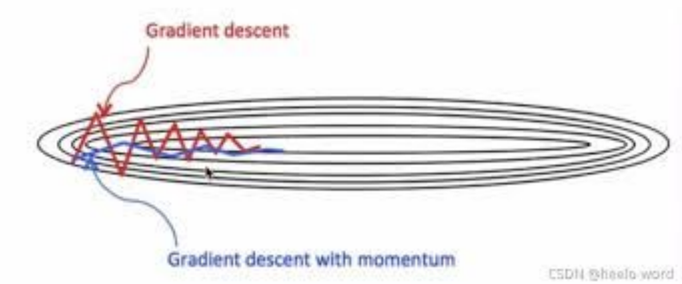


Figure 3: Gradient Descent with Momentum

Figure 3 Source

OPTIMIZERS: Second Generation

With the momentum, we try to increase the convergence speed and avoid local minima

What about sparsity in data

Keep updating the parameters as there is no difference between their associated features.

Why?

1. **The average gradient for sparse feature is small, so slower rate of training.**
2. **Can end up with saddle point.**

OPTIMIZERS: Second Generation

Figure 4: Illustration of saddle point

- Adaptive learning algorithms avoid saddle points.

Definition

Adaptive learning algorithm is an algorithm which tries to adjust the learning rate to the specificities (***frequency)*** of features associated to a parameter and the stage of the optimizing.

Figure 4 Source

OPTIMIZERS: Second Generation

Adagrad

Choose the learning rate η

$$g_{t,i} = \nabla_\theta J(\theta_{t,i})$$

$$\theta_{t+1,i} = \theta_{t,i} - \frac{\eta}{\sqrt{G_{t,ii} + \epsilon}} \cdot g_{t,i}$$

where

$$G_{t,ii} = \sum_{j \le t} g_{j,i}^2$$

Individualise learning rate

Mesures how much we have moved

Advantage:

- **Deals with sparse features.**

Limits:

- Slow convergence because the learning rate is drastically reduced.
- End up with infinitesimally small learning rate which leads to no learning.

OPTIMIZERS: Second Generation

RMSprop

Choose the learning rate η

$$g_{t,i} = \nabla_\theta J(\theta_{t,i})$$

$$E[g_i^2]_t = 0.9E[g_i^2]_{t-1} + 0.1g_{t,i}^2$$

Exponential moving average

$$\theta_{t+1,i} = \theta_{t,i} - \frac{\eta}{\sqrt{G_{t,ii} + \epsilon}} \cdot g_{t,i}$$ **where** $$G_{t,ii} = E[g_i^2]_t$$

Advantage:

- **Faster than Adagrad.**

OPTIMIZERS: Second Generation

Figure 5: Illustration of the improvement of Adagrad with RMSprop.

Figure 5 Source

OPTIMIZERS: Second Generation

Adadelta tries to:

- improve Adagrad as RMSprop
- solve some problems with unit

Adadelta

$$E[g^2]_t = \gamma E[g^2]_{t-1} + (1-\gamma) g_t^2$$ and $$\mathrm{RMS}[g]_t = \sqrt{E[g^2]_t + \epsilon}$$

$$\Delta\theta_t = -\frac{RMS[\Delta\theta]_{t-1}}{RMS[g]_t} g_t$$

Replace the previous learning rate to solve unit issue

$$\theta_{t+1} = \theta_t + \Delta\theta_t$$

$$E[\Delta\theta^2]_t = \gamma E[\Delta\theta^2]_{t-1} + (1-\gamma)\Delta\theta_t^2$$ and $$RMS[\Delta\theta]_t = \sqrt{E[\Delta\theta^2]_t + \epsilon}$$

Advantages:

- **No need to choose a global learning rate**
- **robust to large sudden gradient**

OPTIMIZERS: Second Generation

Adam wants to:

- **Adapt learning to each feature.**
- **Reduces the noise in the gradient (momentum).**

Adam

Choose the learning rate η

$$m_t = \beta_1 m_{t-1} + (1-\beta_1) g_t$$

$$v_t = \beta_2 v_{t-1} + (1-\beta_2) g_t^2$$

$$\hat{m}_t = \frac{m_t}{1-\beta_1^t}$$

$$\hat{v}_t = \frac{v_t}{1-\beta_2^t}$$

$$\theta_{t+1} = \theta_t - \frac{\eta}{\sqrt{\hat{v}_t}+\epsilon} \hat{m}_t$$

Advantages:

1. **Succeeds in avoiding local minima.**
2. **Can escape plateau region.**

CODES: Results

- We implement a neural network on MNIST dataset and check the performance of different optimizers.

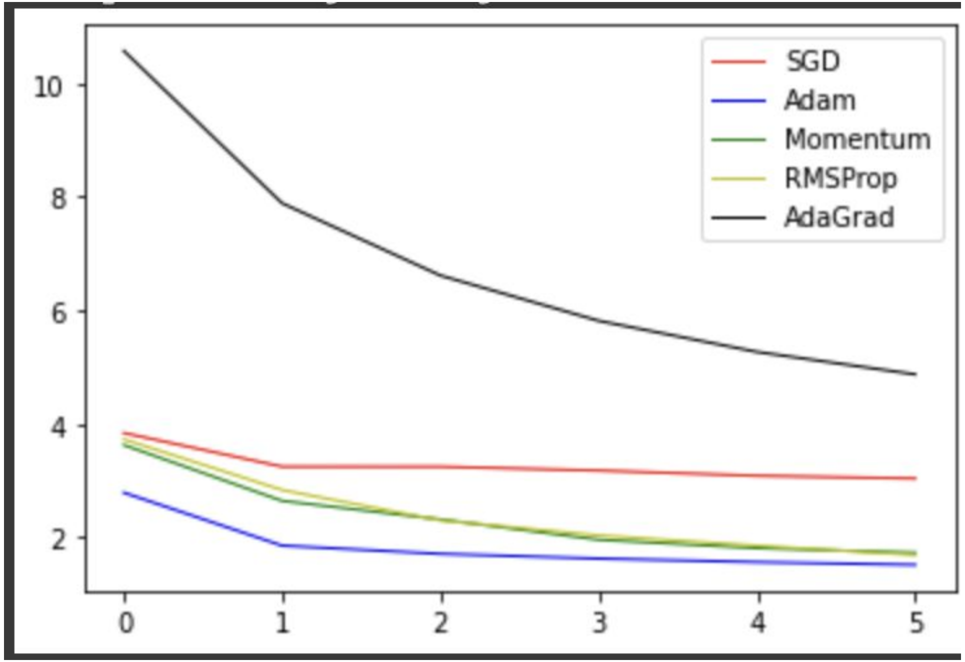


Figure 6: Convergence behaviours of different optimizers on a neural network.

CODES: Results

Dataset

- We implement linear regression on randomly generated data using Sklearn datasets.

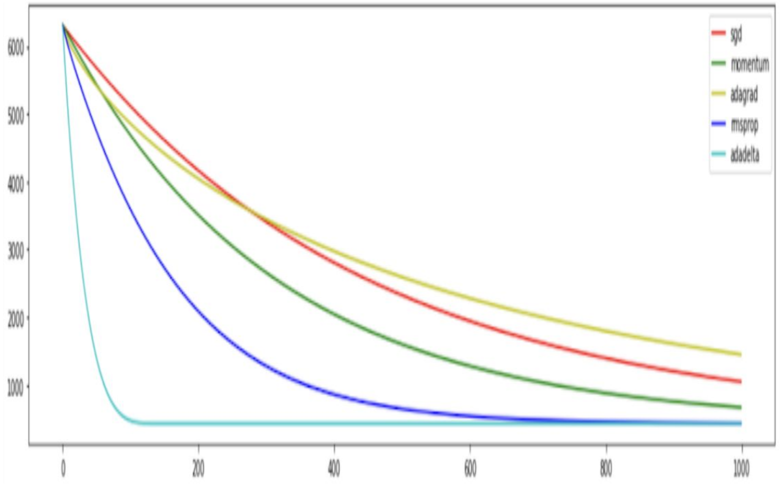


Figure 7: Convergence behaviours of different optimizers using linear regression.

CONCLUSION

- The optimization step is an important part of the machine learning program as observed by all the propositions made by research.
- We can divide the optimizers based on gradient descent in two generations: First generation which uses the same learning rate throughout the network while the second generation adjusts the learning rate based on the layers.
- We saw that the optimizers try to improve speed in convergence, solve the issues associated with choosing the appropriate learning rate and its scheduling, escape regions of local minima, plateau, and/or saddle points.
- Based on the literatures, only the Adam optimizer is able to give the solve all those problems.

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Thank you

for your kind attention