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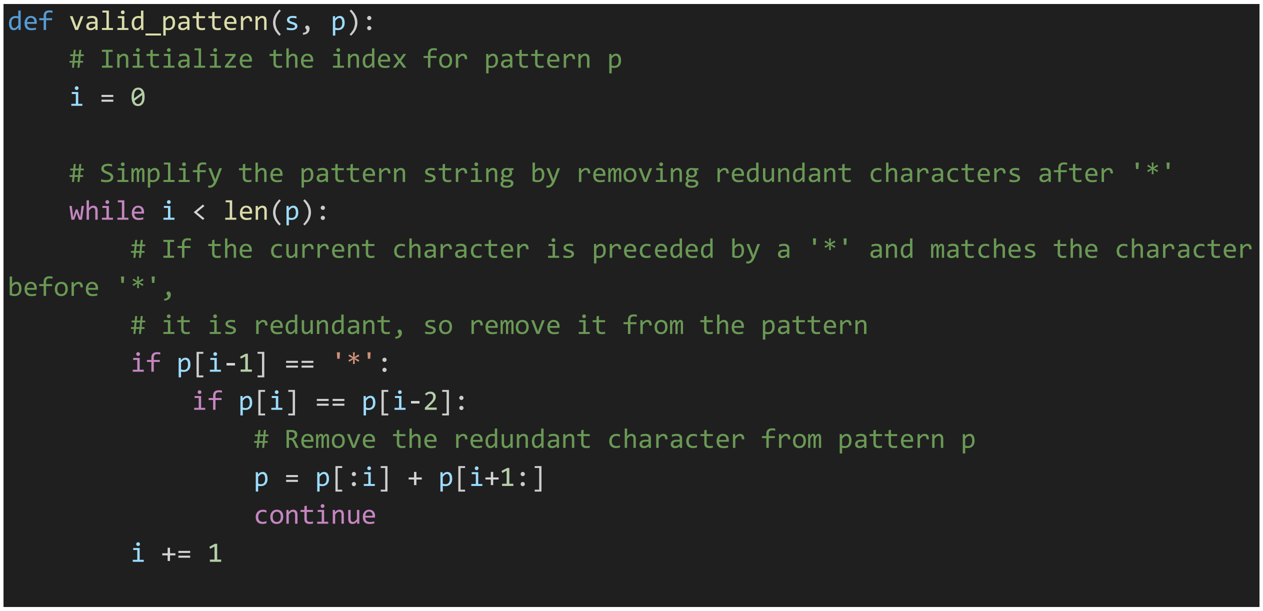
Programming – TP4

**Exercise 1**: Given an input string (s) and a pattern (p), implement regular expression matching with support for '.' and '*' where:

- '.' Matches any single character.
- '*' Matches zero or more of the preceding elements.

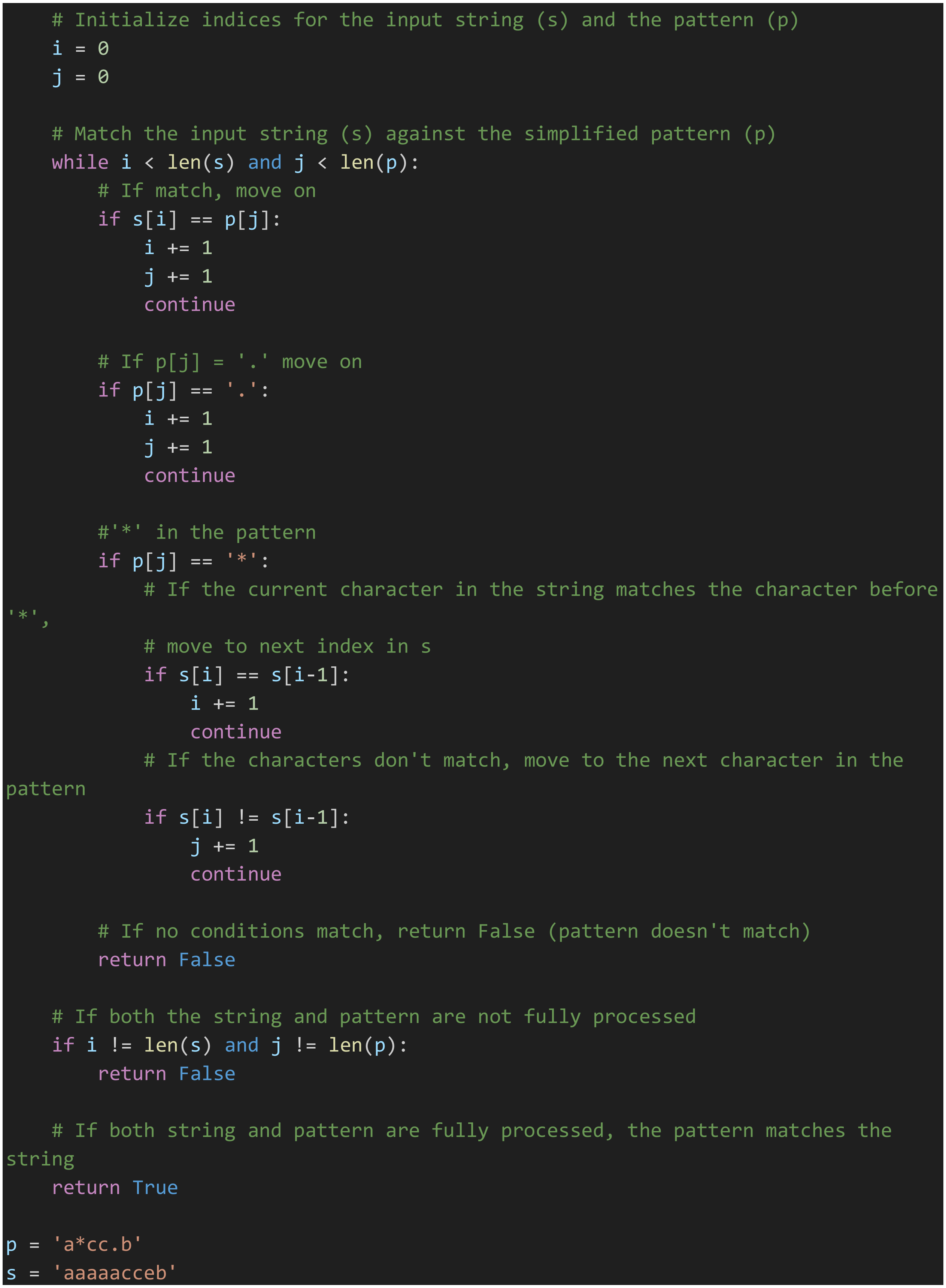
The matching should cover the entire input string (not partial).

Idea: use a while loop with index of p and s to check the similarity



```python
def valid_pattern(s, p):
    # Initialize the index for pattern p
    i = 0

    # Simplify the pattern string by removing redundant characters after '*'
    while i < len(p):
        # If the current character is preceded by a '*' and matches the character
before '*',
        # it is redundant, so remove it from the pattern
        if p[i-1] == '*':
            if p[i] == p[i-2]:
                # Remove the redundant character from pattern p
                p = p[:i] + p[i+1:]
                continue
        i += 1
```



```
    # Initialize indices for the input string (s) and the pattern (p)
    i = 0
    j = 0

    # Match the input string (s) against the simplified pattern (p)
    while i < len(s) and j < len(p):
        # If match, move on
        if s[i] == p[j]:
            i += 1
            j += 1
            continue

        # If p[j] = '.' move on
        if p[j] == '.':
            i += 1
            j += 1
            continue

        #'*' in the pattern
        if p[j] == '*':
            # If the current character in the string matches the character before
'*',
            # move to next index in s
            if s[i] == s[i-1]:
                i += 1
                continue
            # If the characters don't match, move to the next character in the
pattern
            if s[i] != s[i-1]:
                j += 1
                continue

        # If no conditions match, return False (pattern doesn't match)
        return False

    # If both the string and pattern are not fully processed
    if i != len(s) and j != len(p):
        return False

    # If both string and pattern are fully processed, the pattern matches the
string
    return True

p = 'a*cc.b'
s = 'aaaaacceb'
```

```
print(valid_pattern(s, p))
```

Exercise 2:

You are climbing a staircase. It takes n steps to reach the top. Each time you can either climb 1 or 2 steps. In how many distinct ways can you climb to the top?

We call $X_b$ is the number of possible way if there are b 2-steps.

a is the number of 1-Step

→ $a + 2b = n$

$X_b = \frac{(a+b)!}{a!\,b!}$ (repeated permutation)

b = 0 then a = n.

→ $X_0 = \frac{n!}{n!\,0!} = 1$

For $b' = b - 1$ then $a' = n - 2(b-1) = n - 2b + 2 = a + 2$

$$X_{b-1} = \frac{(a'+b')!}{a'!\,b'!} = \frac{(a+b+1)!}{(a+2)!\,(b-1)!}$$

$$\frac{X_b}{X_{b-1}} = \frac{(a+b)!}{a!\,b!} \cdot \frac{(a+2)!\,(b-1)!}{(a+b+1)!} = \frac{(a+2)(a+1)}{b(a+b+1)}$$

→ $\frac{X_b}{X_{b-1}} = \frac{(a+2)(a+1)}{b(a+b+1)}$

The result will be the total of Xb, b from 0 to int(n/2) + 1

```
#Idea: a is the number of 1 step taken
#      b is the number of 2 step taken
# First we find all combination of a and b such that a + 2*b = n
# Then we calculate the number of permutation for that by this formula: (a +
b)!/a!/b!    (repeated permutation)

def climb_stair(n):
    total = 0

    tmp = 1
    total += tmp
    for b in range(1, int(n/2)+1):
        a = n - 2*b

        tmp = tmp * (a+1)*(a+2)/(a+b+1)/b
        total += tmp

    return int(total)

print(climb_stair(4))
```

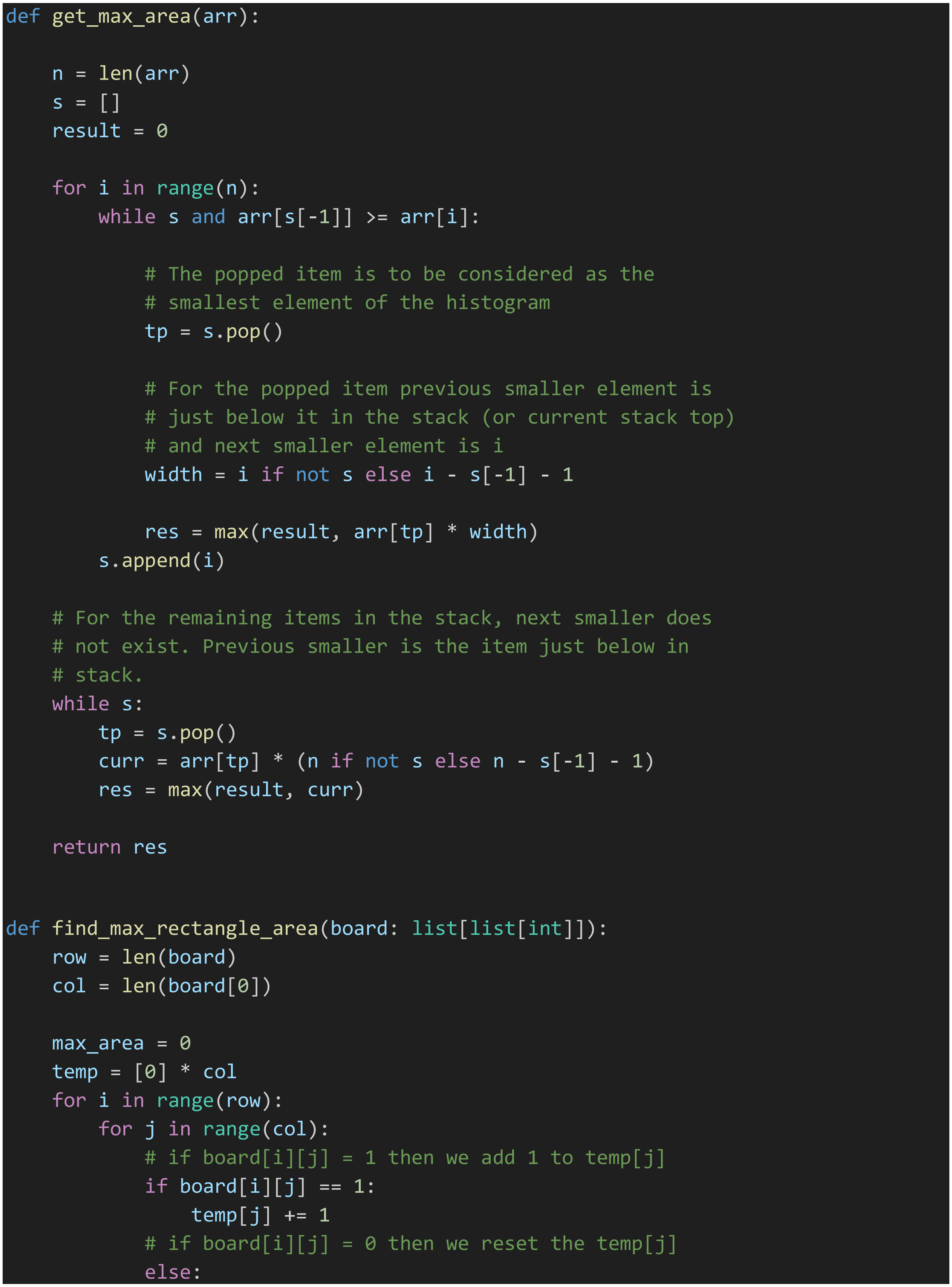
Exercise 3:

Given a rows x cols binary matrix filled with 0's and 1's, find the largest rectangle containing only 1's and return its area.

Idea: I transform this problem from 2 dimension matrix into a problem with one dimension vector list this

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | → 1 |
| 1 | 0 | 1 | 1 | 1 | 2 | 0 | 2 | 1 | 1 | → 3 |
| 1 | 1 | 1 | 1 | 1 | 3 | 1 | 3 | 2 | 2 | → 6 |
| 1 | 0 | 0 | 1 | 0 | 4 | 0 | 0 | 3 | 0 | → 4 |

Now for each row in the right, we need to find a subarray such that the min(subarray) * length(subarray) is maximum in that. I implemented this function in get_max_area(arr)



```python
def get_max_area(arr):

    n = len(arr)
    s = []
    result = 0

    for i in range(n):
        while s and arr[s[-1]] >= arr[i]:

            # The popped item is to be considered as the
            # smallest element of the histogram
            tp = s.pop()

            # For the popped item previous smaller element is
            # just below it in the stack (or current stack top)
            # and next smaller element is i
            width = i if not s else i - s[-1] - 1

            res = max(result, arr[tp] * width)
        s.append(i)

    # For the remaining items in the stack, next smaller does
    # not exist. Previous smaller is the item just below in
    # stack.
    while s:
        tp = s.pop()
        curr = arr[tp] * (n if not s else n - s[-1] - 1)
        res = max(result, curr)

    return res


def find_max_rectangle_area(board: list[list[int]]):
    row = len(board)
    col = len(board[0])

    max_area = 0
    temp = [0] * col
    for i in range(row):
        for j in range(col):
            # if board[i][j] = 1 then we add 1 to temp[j]
            if board[i][j] == 1:
                temp[j] += 1
            # if board[i][j] = 0 then we reset the temp[j]
            else:
                temp[j] = 0
        max_temp = get_max_area(temp)
        max_area = max(max_area, max_temp)
    return max_area

board = [[1, 0, 1, 0, 0],
         [1, 0, 1, 1, 1],
         [1, 1, 1, 1, 1],
         [1, 0, 0, 1, 0]]
print(find_max_rectangle_area(board))
```

Exercise 4:

You are given an integer array prices where prices[i] is the price of a given stock on the ith day. Design an algorithm to find the maximum profit. You may complete at most k transactions. Notice that you may not engage in multiple transactions simultaneously (i.e., you must sell the stock before you buy again).

Idea: I use a recursion approach with dynamic programming

```
class Solution:
    def __init__(self):
        # Initialize a dynamic programming table
        self.dp = []

    def h(self, prices, k, i, t, can_buy):
        # Base case: If we reach the end of the prices array or the transaction
limit
        if i >= len(prices) or t == k:
            return 0

        # If the result for the current state is already computed, return it
        if self.dp[i][t][can_buy] != -1:
            return self.dp[i][t][can_buy]

        # Skip the current day (do nothing)
        profit = self.h(prices, k, i + 1, t, can_buy)

        if can_buy:
            # Option to buy the stock on the current day
            profit = max(profit, -prices[i] + self.h(prices, k, i + 1, t, 0))
        else:
            # Option to sell the stock on the current day
            profit = max(profit, prices[i] + self.h(prices, k, i + 1, t + 1, 1))

        # Store the computed result in the DP table
        self.dp[i][t][can_buy] = profit
        return profit

    def maxProfit(self, k, prices):
        n = len(prices)
        # Initialize the DP table: dp[n][k][2] with -1
        self.dp = [[[-1] * 2 for _ in range(k)] for _ in range(n)]

        # Start from day 0, with 0 transactions, and the ability to buy
        return self.h(prices, k, 0, 0, 1)


# Test the solution
sol = Solution()
k = 2
prices = [3, 2, 6, 5, 0, 3]
print(sol.maxProfit(k, prices))  # Output the maximum profit
```