CS 102
Computing & Algorithms II
Lesson 07

Tables, Priority Queues, Heaps, And Hashing

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Heaps

- A heap is a complete binary tree
  - That is empty

  or

  - Whose root contains a search key greater than or equal to the search key in each of its children, and
  - Whose root has heaps as its subtrees

Heaps

- Maxheap
  - A heap in which the root contains the item with the largest search key
- Minheap
  - A heap in which the root contains the item with the smallest search key

Heaps

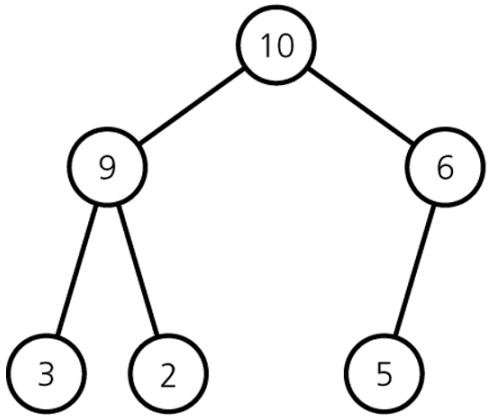
- Pseudocode for the operations of the ADT heap

```
createHeap()
// Creates an empty heap.

heapIsEmpty()
// Determines whether a heap is empty.

heapInsert(newItem) throws HeapException
// Inserts newItem into a heap. Throws
// HeapException if heap is full.

heapDelete()
// Retrieves and then deletes a heap's root
// item. This item has the largest search key.
```

Heaps: An Array-based Implementation of a Heap

- Data fields
  - `items`: an array of heap items
  - `size`: an integer equal to the number of items in the heap

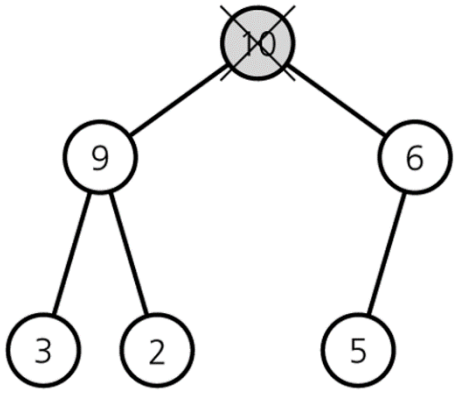
A heap with its array representation



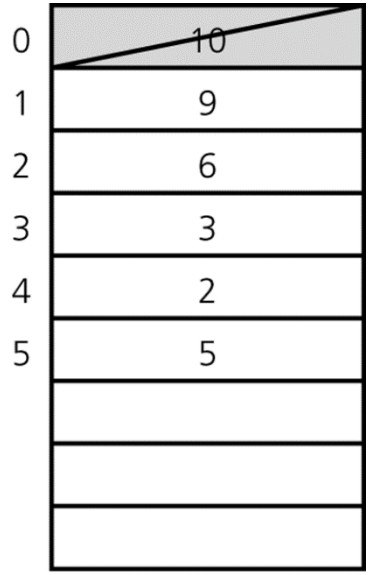
0	10
1	9
2	6
3	3
4	2
5	5

Heaps: `heapDelete`

- Step 1: Return the item in the root
  - Results in disjoint heaps



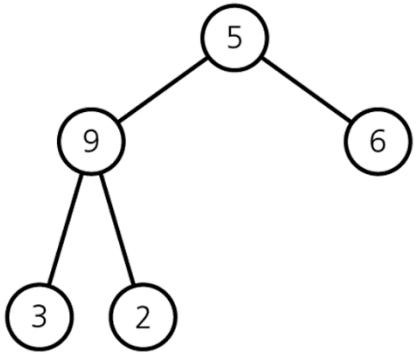
(a)

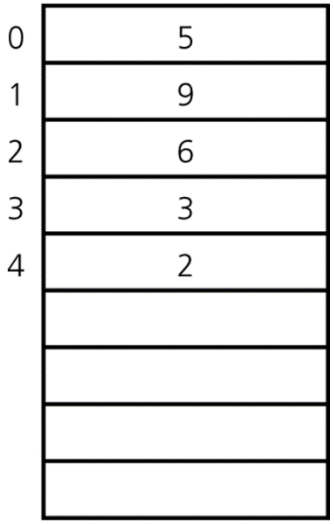


a) Disjoint heaps

Heaps: `heapDelete`

- Step 2: Copy the item from the last node into the root
  - Results in a semiheap



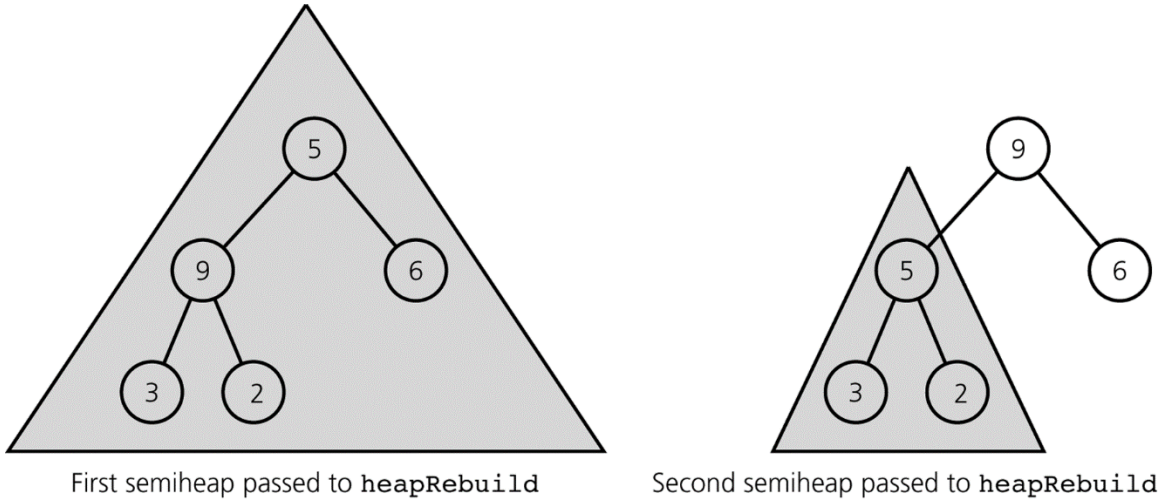


(b)

b) a semiheap

Heaps: `heapDelete`

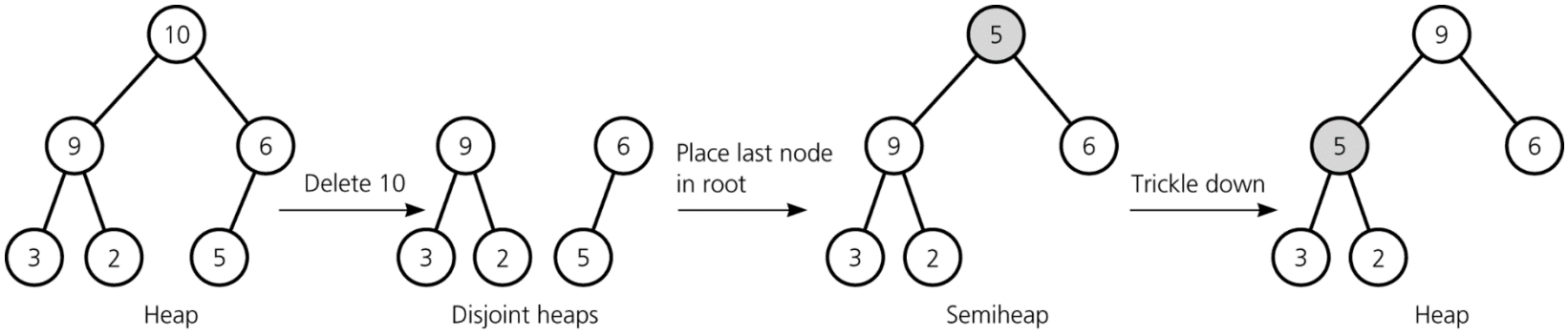
- Step 3: Transform the semiheap back into a heap
  - Performed by the recursive algorithm `heapRebuild`



Recursive calls to ***heapRebuild***

Heaps: `heapDelete`

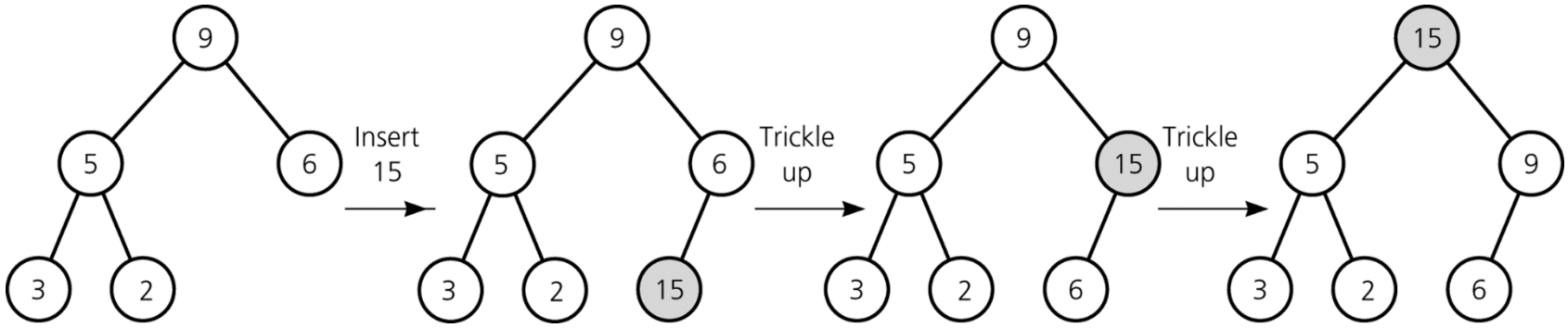
- Efficiency
  - `heapDelete` is O(log n)



Deletion from a heap

Heaps: `heapInsert`

- Strategy
  - Insert `newItem` into the bottom of the tree
  - Trickle new item up to appropriate spot in the tree
- Efficiency: O(log n)
- `Heap` class
  - Represents an array-based implementation of the ADT heap



Insertion into a heap

A Heap Implementation of the ADT Priority Queue

- Priority-queue operations and heap operations are analogous
  - The priority value in a priority-queue corresponds to a heap item's search key
- `PriorityQueue` class
  - Has an instance of the `Heap` class as its data field

A Heap Implementation of the ADT Priority Queue

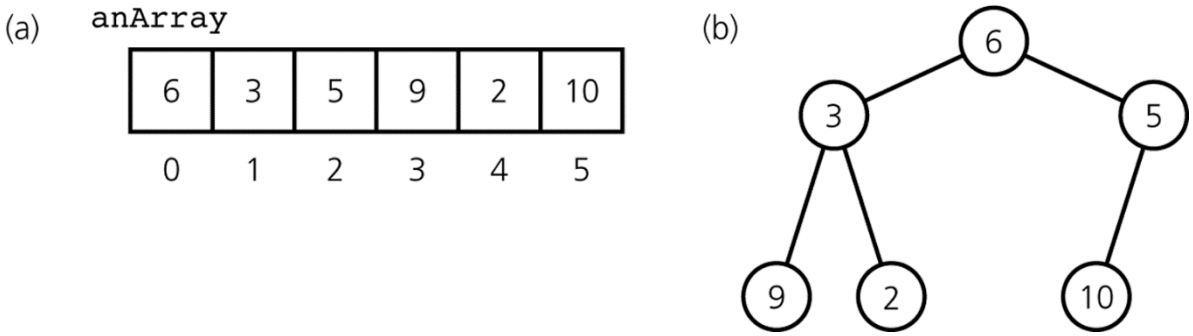
- A heap implementation of a priority queue
  - Disadvantage
    - Requires the knowledge of the priority queue's maximum size
  - Advantage
    - A heap is always balanced
- Finite, distinct priority values
  - A heap of queues
    - Useful when a finite number of distinct priority values are used, which can result in many items having the same priority value

Heapsort

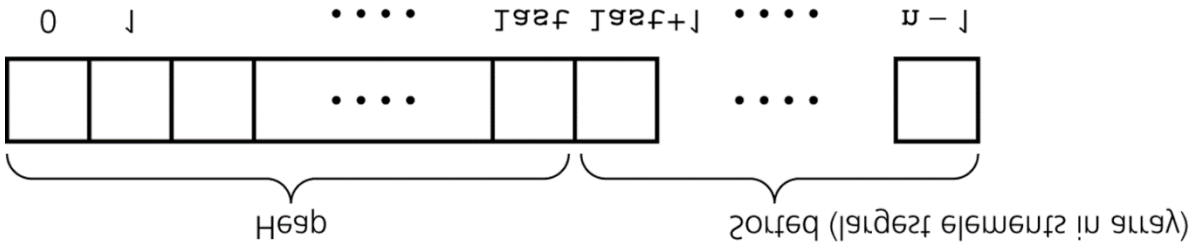
- Strategy
  - Transforms the array into a heap
  - Removes the heap's root (the largest element) by exchanging it with the heap’s last element
  - Transforms the resulting semiheap back into a heap
- Efficiency
  - Compared to mergesort
    - Both heapsort and mergesort are O(n * log n) in both the worst and average cases
    - Advantage over mergesort
      - Heapsort does not require a second array
  - Compared to quicksort
    - Quicksort is the preferred sorting method

Heapsort

a) The initial contents of ***anArray***; b) ***anArray's*** corresponding binary tree



Heapsort partitions an array into two regions



Hashing

- Hashing
  - Enables access to table items in time that is relatively constant and independent of the items
- Hash function
  - Maps the search key of a table item into a location that will contain the item
- Hash table
  - An array that contains the table items, as assigned by a hash function

Hashing

- A perfect hash function
  - Maps each search key into a unique location of the hash table
  - Possible if all the search keys are known
- Collisions
  - Occur when the hash function maps more than one item into the same array location
- Collision-resolution schemes
  - Assign locations in the hash table to items with different search keys when the items are involved in a collision
- Requirements for a hash function
  - Be easy and fast to compute
  - Place items evenly throughout the hash table

Hash Functions

- It is sufficient for hash functions to operate on integers
- Simple hash functions that operate on positive integers
  - Selecting digits
  - Folding
  - Module arithmetic
- Converting a character string to an integer
  - If the search key is a character string, it can be converted into an integer before the hash function is applied

Resolving Collisions

- Two approaches to collision resolution
  - Approach 1: Open addressing
    - A category of collision resolution schemes that probe for an empty, or open, location in the hash table
      - The sequence of locations that are examined is the probe sequence
    - Linear probing
      - Searches the hash table sequentially, starting from the original location specified by the hash function
      - Possible problem
        - » Primary clustering

Resolving Collisions

- Approach 1: Open addressing (Continued)
  - Quadratic probing
    - Searches the hash table beginning with the original location that the hash function specifies and continues at increments of 1^2, 2^2, 3^2, and so on
    - Possible problem
      - Secondary clustering
  - Double hashing
    - Uses two hash functions
    - Searches the hash table starting from the location that one hash function determines and considers every n^{th} location, where n is determined from a second hash function
- Increasing the size of the hash table
  - The hash function must be applied to every item in the old hash table before the item is placed into the new hash table

Resolving Collisions

- Approach 2: Restructuring the hash table
  - Changes the structure of the hash table so that it can accommodate more than one item in the same location
  - Buckets
    - Each location in the hash table is itself an array called a bucket
  - Separate chaining
    - Each hash table location is a linked list

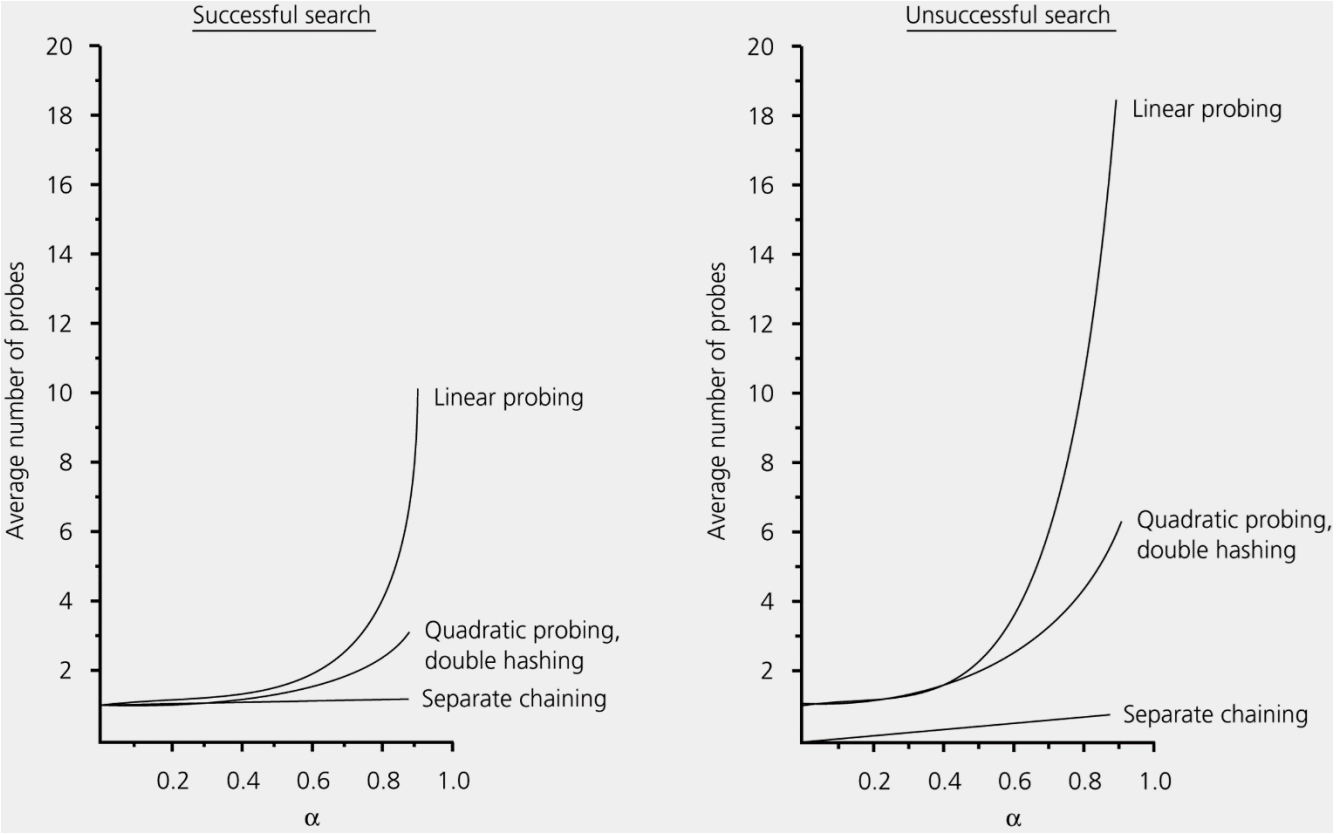
The Efficiency of Hashing

- An analysis of the average-case efficiency of hashing involves the load factor
  - Load factor α
    - Ratio of the current number of items in the table to the maximum size of the array `table`
    - Measures how full a hash table is
    - Should not exceed 2/3
  - Hashing efficiency for a particular search also depends on whether the search is successful
    - Unsuccessful searches generally require more time than successful searches

The Efficiency of Hashing

- Linear probing
  - Successful search: $\frac{1}{2}[1 + 1(1-\alpha)]$
  - Unsuccessful search: $\frac{1}{2}[1 + 1(1- \alpha)^2]$
- Quadratic probing and double hashing
  - Successful search: $-\log_e(1- \alpha)/ \alpha$
  - Unsuccessful search: $1/(1- \alpha)$
- Separate chaining
  - Insertion is O(1)
  - Retrievals and deletions
    - Successful search: $1 + (\alpha/2)$
    - Unsuccessful search: α

The Efficiency of Hashing



The relative efficiency of four collision-resolution methods

What Constitutes a Good Hash Function?

- A good hash function should
  - Be easy and fast to compute
  - Scatter the data evenly throughout the hash table
- Issues to consider with regard to how evenly a hash function scatters the search keys
  - How well does the hash function scatter random data?
  - How well does the hash function scatter nonrandom data?
- General requirements of a hash function
  - The calculation of the hash function should involve the entire search key
  - If a hash function uses module arithmetic, the base should be prime

Table Traversal: An Inefficient Operation Under Hashing

- Hashing as an implementation of the ADT table
  - For many applications, hashing provides the most efficient implementation
  - Hashing is not efficient for
    - Traversal in sorted order
    - Finding the item with the smallest or largest value in its search key
    - Range query
- In external storage, you can simultaneously use
  - A hashing implementation of the `tableRetrieve` operation
  - A search-tree implementation of the ordered operations

The JCF `Hashtable` and `TreeMap` Classes

- JFC Hashtable implements a hash table
  - Maps keys to values
  - Large collection of methods
- JFC TreeMap implements a red-black tree
  - Guarantees O(log n) time for insert, retrieve, remove, and search
  - Large collection of methods

The ADT Table

- The ADT table, or dictionary
  - Uses a search key to identify its items
  - Its items are records that contain several pieces of data

City	Country	Population
Athens	Greece	2,500,000
Barcelona	Spain	1,800,000
Cairo	Egypt	9,500,000
London	England	9,400,000
New York	U.S.A.	7,300,000
Paris	France	2,200,000
Rome	Italy	2,800,000
Toronto	Canada	3,200,000
Venice	Italy	300,000

An ordinary table of cities

The ADT Table

- Operations of the ADT table
  - Create an empty table
  - Determine whether a table is empty
  - Determine the number of items in a table
  - Insert a new item into a table
  - Delete the item with a given search key from a table
  - Retrieve the item with a given search key from a table
  - Traverse the items in a table in sorted search-key order

The ADT Table

- Pseudocode for the operations of the ADT table

```
createTable()
// Creates an empty table.

tableIsEmpty()
// Determines whether a table is empty.

tableLength()
// Determines the number of items in a table.

tableInsert(newItem) throws TableException
// Inserts newItem into a table whose items have
// distinct search keys that differ from newItem's
// search key. Throws TableException if the
// insertion is not successful
```

The ADT Table

- Pseudocode for the operations of the ADT table (Continued)

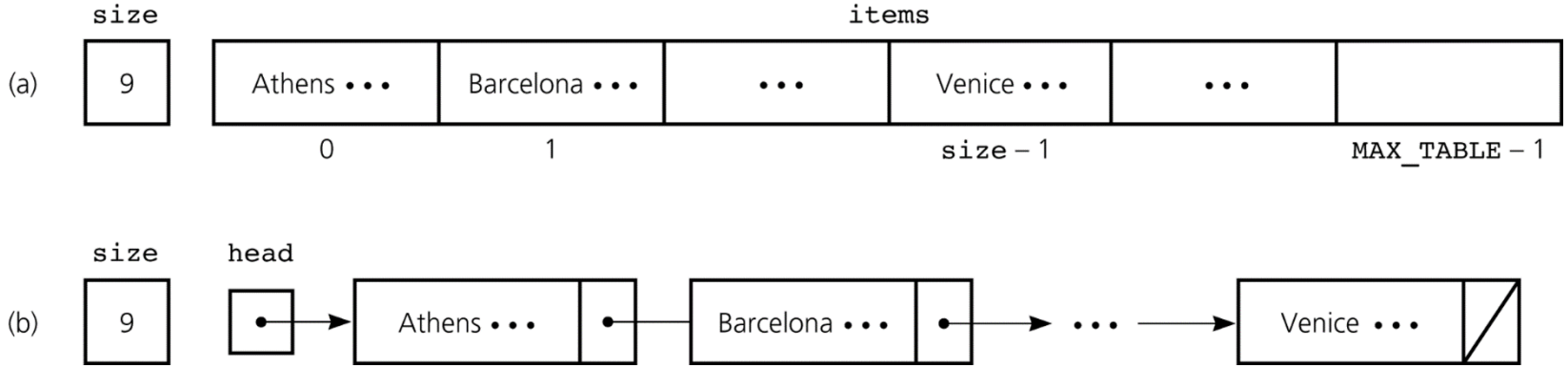
```
tableDelete(searchKey)
// Deletes from a table the item whose search key
// equals searchKey. Returns false if no such item
// exists. Returns true if the deletion was
// successful.

tableRetrieve(searchKey)
// Returns the item in a table whose search key
// equals searchKey. Returns null if no such item
// exists.

tableTraverse()
// Traverses a table in sorted search-key order.
```

The ADT Table

- Value of the search key for an item must remain the same as long as the item is stored in the table
- `KeyedItem` class
  - Contains an item's search key and a method for accessing the search-key data field
  - Prevents the search-key value from being modified once an item is created
- `TableInterface` interface
  - Defines the table operations

Selecting an Implementation

- Categories of linear implementations
  - Unsorted, array based
  - Unsorted, referenced based
  - Sorted (by search key), array based
  - Sorted (by search key), reference based

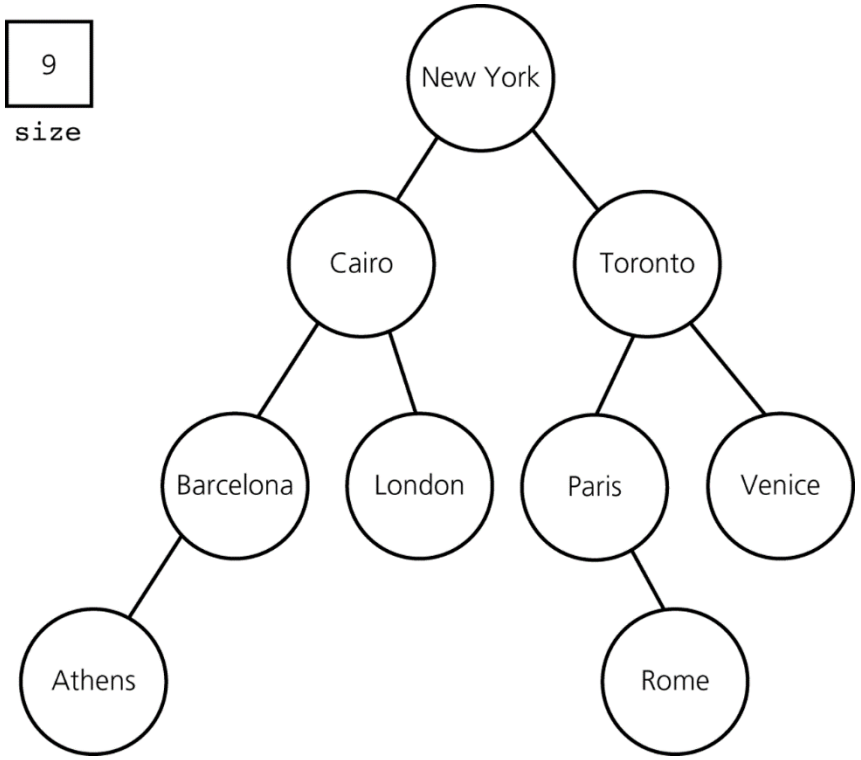


The data fields for two sorted linear implementations of the ADT table for the data in Figure 12-1: a) array based; b) reference based

Selecting an Implementation

- A binary search implementation
  - A nonlinear implementation

The data fields for a binary search tree implementation of the ADT table for the data in Figure 12-1



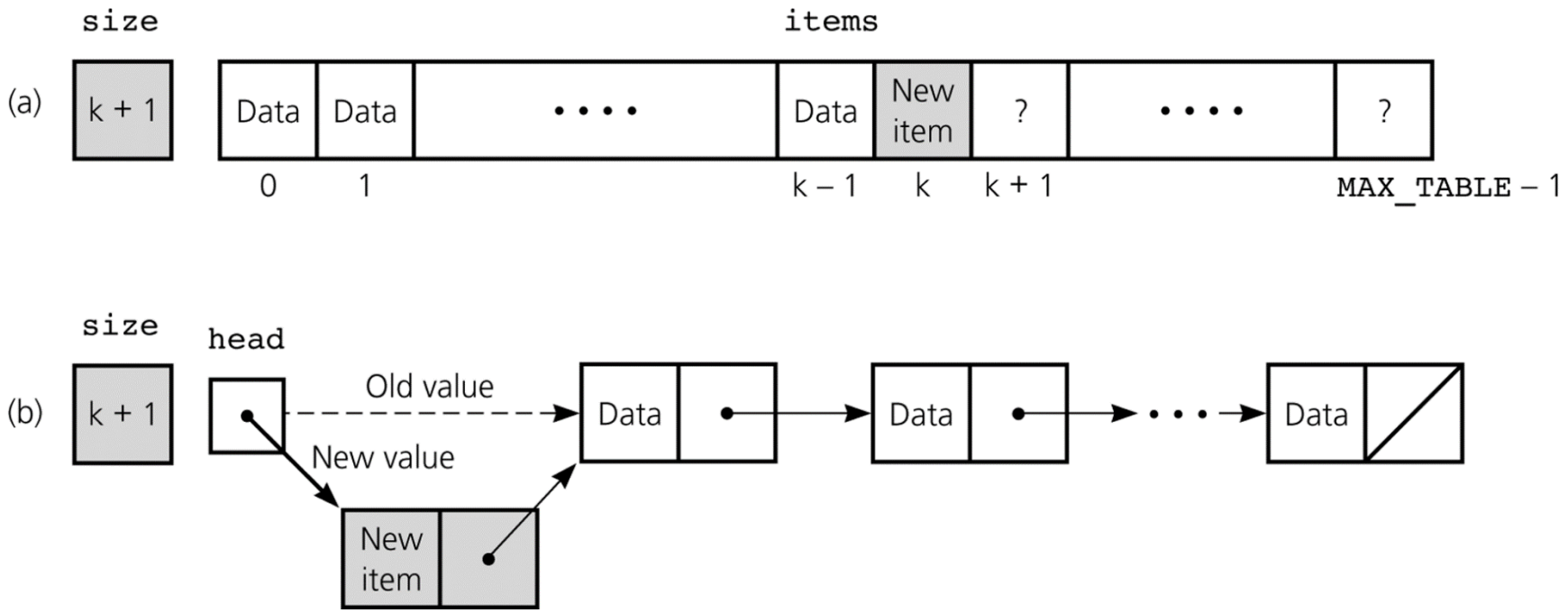
Selecting an Implementation

- The binary search tree implementation offers several advantages over linear implementations
- The requirements of a particular application influence the selection of an implementation
  - Questions to be considered about an application before choosing an implementation
    - What operations are needed?
    - How often is each operation required?

Scenario A: Insertion and Traversal in No Particular Order

- An unsorted order in efficient
  - Both array based and reference based `tableInsert` operation is O(1)
- Array based versus reference based
  - If a good estimate of the maximum possible size of the table is not available
    - Reference based implementation is preferred
  - If a good estimate of the maximum possible size of the table is available
    - The choice is mostly a matter of style

Scenario A: Insertion and Traversal in No Particular Order



Insertion for unsorted linear implementations: a) array based; b) reference based

Scenario A: Insertion and Traversal in No Particular Order

- A binary search tree implementation is not appropriate
  - It does more work than the application requires
    - It orders the table items
  - The insertion operation is O(log n) in the average case

Scenario B: Retrieval

- Binary search
  - An array-based implementation
    - Binary search can be used if the array is sorted
  - A reference-based implementation
    - Binary search can be performed, but is too inefficient to be practical
- A binary search of an array is more efficient than a sequential search of a linked list
  - Binary search of an array
    - Worst case: $O(\log_2 n)$
  - Sequential search of a linked list
    - $O(n)$

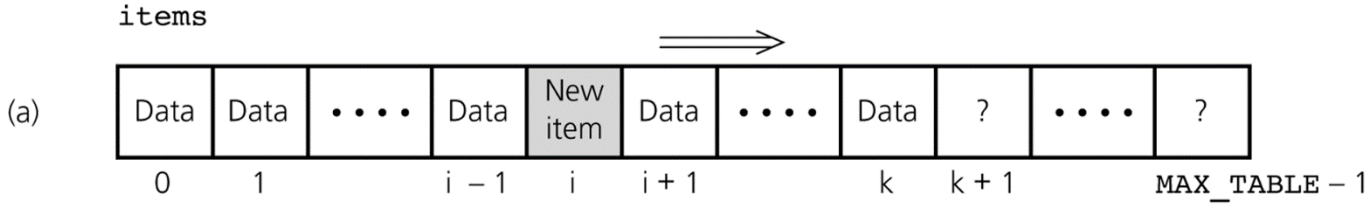
Scenario B: Retrieval

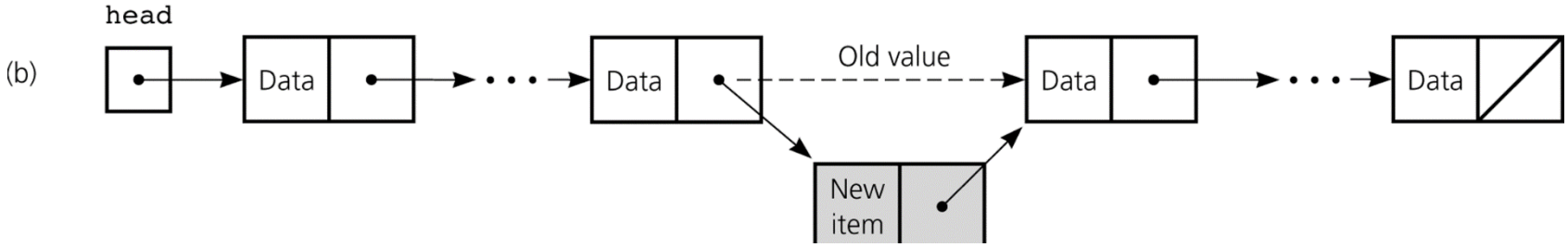
- For frequent retrievals
  - If the table's maximum size is known
    - A sorted array-based implementation is appropriate
  - If the table's maximum size is not known
    - A binary search tree implementation is appropriate

Scenario C: Insertion, Deletion, Retrieval, and Traversal in Sorted Order

- Steps performed by both insertion and deletion
  - Step 1: Find the appropriate position in the table
  - Step 2: Insert into (or delete from) this position
- Step 1
  - An array-based implementation is superior than a reference-based implementation
- Step 2
  - A reference-based implementation is superior than an array-based implementation
    - A sorted array-based implementation shifts data during insertions and deletions

Scenario C: Insertion, Deletion, Retrieval, and Traversal in Sorted Order





Insertion for sorted linear implementations: a) array based; b) reference based

Scenario C: Insertion, Deletion, Retrieval, and Traversal in Sorted Order

- Insertion and deletion operations
  - Both sorted linear implementations are comparable, but neither is suitable
    - `tableInsert` and `tableDelete` operations
      - Sorted array-based implementation is O(n)
      - Sorted reference-based implementation is O(n)
  - Binary search tree implementation is suitable
    - It combines the best features of the two linear implementations

A Sorted Array-Based Implementation of the ADT Table

- Linear implementations
  - Useful for many applications despite certain difficulties
- A binary search tree implementation
  - In general, can be a better choice than a linear implementation
- A balanced binary search tree implementation
  - Increases the efficiency of the ADT table operations

A Sorted Array-Based Implementation of the ADT Table

	Insertion	Deletion	Retrieval	Traversal
Unsorted array based	O(1)	O(n)	O(n)	O(n)
Unsorted pointer based	O(1)	O(n)	O(n)	O(n)
Sorted array based	O(n)	O(n)	O(log n)	O(n)
Sorted pointer based	O(n)	O(n)	O(n)	O(n)
Binary search tree	O(log n)	O(log n)	O(log n)	O(n)

The average-case order of the operations of the ADT table for various implementations

A Sorted Array-Based Implementation of the ADT Table

- Reasons for studying linear implementations
  - Perspective
  - Efficiency
  - Motivation
- `TableArrayBased` class
  - Provides an array-based implementation of the ADT table
  - Implements `TableInterface`

A Binary Search Tree Implementation of the ADT Table

- `TableBSTBased` class
  - Represents a nonlinear reference-based implementation of the ADT table
  - Uses a binary search tree to represent the items in the ADT table
    - Reuses the class `BinarySearchTree`

The ADT Priority Queue: A Variation of the ADT Table

- The ADT priority queue
  - Orders its items by a priority value
  - The first item removed is the one having the highest priority value
- Operations of the ADT priority queue
  - Create an empty priority queue
  - Determine whether a priority queue is empty
  - Insert a new item into a priority queue
  - Retrieve and then delete the item in a priority queue with the highest priority value

The ADT Priority Queue: A Variation of the ADT Table

- Pseudocode for the operations of the ADT priority queue

```
createPQueue()
// Creates an empty priority queue.

pqIsEmpty()
// Determines whether a priority queue is
// empty.
```

The ADT Priority Queue: A Variation of the ADT Table

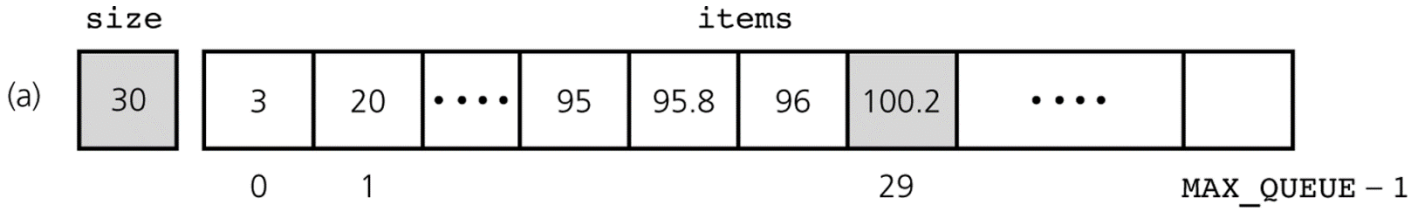
- Pseudocode for the operations of the ADT priority queue (Continued)

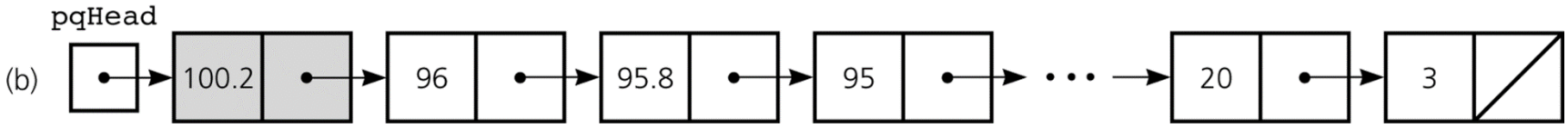
```
pqInsert(newItem) throws PQueueException
// Inserts newItem into a priority queue.
// Throws PQueueException if priority queue is
// full.

pqDelete()
// Retrieves and then deletes the item in a
// priority queue with the highest priority
// value.
```

The ADT Priority Queue: A Variation of the ADT Table

- Possible implementations
  - Sorted linear implementations
    - Appropriate if the number of items in the priority queue is small
    - Array-based implementation
      - Maintains the items sorted in ascending order of priority value
    - Reference-based implementation
      - Maintains the items sorted in descending order of priority value

The ADT Priority Queue: A Variation of the ADT Table

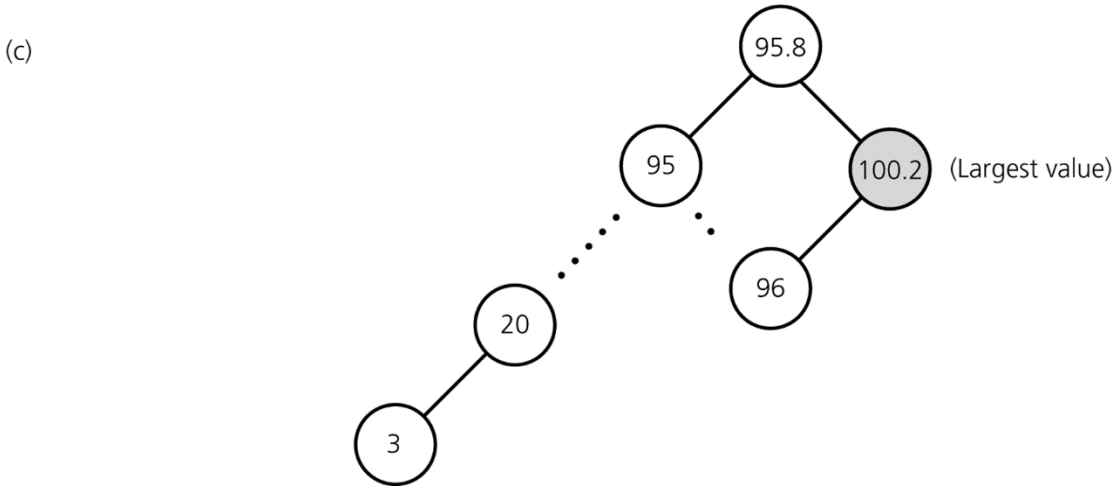




Some implementations of the ADT priority queue: a) array based; b) reference based

The ADT Priority Queue: A Variation of the ADT Table

- Possible implementations (Continued)
  - Binary search tree implementation
    - Appropriate for any priority queue



Some implementations of the ADT priority queue: c) binary search tree

Tables and Priority Queues in JFC: The JFC `Map` Interface

- `Map` interface
  - Provides the basis for numerous other implementations of different kinds of maps
- **`public interface`** `Map<K,V>` methods
  - **`void`** `clear()`
  - **`boolean`** `containsKey(Object key)`
  - **`boolean`** `containsValue(Object value)`
  - `Set<Map.Entry<K,V>> entrySet()`
  - `V get(Object key);`

Tables and Priority Queues in JFC: The JFC `Map` Interface

- **`public interface`** `Map<K,V>` methods (continued)
  - **`boolean`** `isEmpty()`
  - `Set<K> keySet()`
  - `V put(K key, V value)`
  - `V remove(Object key)`
  - `Collection<V> values()`

The JFC `Set` Interface

- `Set` interface
  - Ordered collection
  - Stores single value entries
  - Does not allow for duplicate elements
- **`public interface`** `Set<T>` methods
  - **`boolean`** `add(T o)`
  - **`boolean`** `addAll(Collection<? extends T> c)`
  - **`void`** `clear()`
  - **`boolean`** `contains(Object o)`
  - **`boolean`** `isEmpty()`

The JFC `Set` Interface

- **`public interface`** `Set<T>` methods (continued)
  - `Iterator<T> iterator()`
  - **`boolean`** `remove(Object o)`
  - **`boolean`** `removeAll(Collection<?> c)`
  - **`boolean`** `retainAll(Collection<?> c)`
  - **`int`** `size()`

The JFC `PriorityQueue` Class

- `PriorityQueue` class
  - Has a single data-type parameter with ordered elements
  - Relies on the natural ordering of the elements
    - As provided by the `Comparable` interface or a `Comparator` object
  - Elements in queue are ordered in ascending order
- **`public Class`** `PriorityQueue<T>` methods
  - `PriorityQueue(int initialCapacity)`
  - `PriorityQueue(int initialCapacity, Comparator<? super T> comparator)`
  - **`boolean`** `add(T o)`
  - **`void`** `clear()`
  - **`boolean`** `contains(Object o)`

The JFC `PriorityQueue` Class

- **`public Class`** `PriorityQueue<T>` methods (continued)
  - `Comparator<? super T> comparator()`
  - `T element()`
  - `Iterator<T> iterator()`
  - **`boolean`** `offer(T o)`
  - `T peek()`
  - `T poll()`
  - **`boolean`** `remove(Object o)`
  - **`int`** `size()`