Theory of Computation

CS-312 **Summer 2020**

Exam 3 **DUE FRIDAY, 25 September 2020, NOON**

Instructions

- Complete all problems separately; each problem indicates the number of points possible. Show your work; partial credit may be awarded.

- All problems are to be completed individually. No collaboration with others is permitted.

- This exam is *open-book*, *open notes*. You are permitted to consult the textbook, your own class notes, class handouts, and class homework and solution sets. All information available from the course Blackboard site is permitted.

  You may use JFLAP to develop your solutions, *except where using JFLAP would defeat the purpose of the question.* You may *always* use the JFLAP automata editor to draw your solutions.

  No other outside sources, including Internet sources, are permitted. No collaboration with others is permitted. Violations are subject to severe penalties, up to and including course failure.

- Your answers must be submitted electronically via Blackboard by NOON on Friday, 25 September (11th Friday). ***Retain your electronic copy of your completed answers, in case of difficulties.***

By submitting your examination, you certify that you have neither given nor received any unauthorized assistance on this examination.

1. *15 points.* Let $w_1, w_2 \in \{0,1\}^*$ be binary strings. Give a detailed description (*i.e.* a state-machine diagram) of a deterministic Turing machine which, when started with $w_1 \square w_2$ on the tape as input, halts with $w_2 \square w_1$ on the tape as output. ($\square$ is the blank character.) That is, the Turing machine has the effect of "flipping" the positions of strings w_1 and w_2 on the tape.

   For example, when started with 01011□101 on its tape, the machine should end with 101□01011 on its tape.

   (**Note:** For those of you using JFLAP to test your solutions: it is difficult to enter a blank square as input for testing. JFLAP represents blank squares using the standard Unicode "white square", `U+25A1`. The easiest option is to copy and paste that symbol from an existing document or page, *e.g.* `http://alt-codes.net/square-symbols` .)

2. *15 points.* Give a detailed description (*i.e.* a state-machine diagram) of a deterministic Turing machine which decides the language $L = \{0^i 1^j 0^{i+j} : i, j \geq 0\}$.

3. *15 points.* A *wobbly Turing machine* is a Turing machine which always writes a different symbol to the tape than the one being read at each step. Show that wobbly Turing machines are equivalent to classic Turing machines. (Note that this is an "iff" claim and thus requires two proofs. One is easier.)

4. Let L be the set of Turing machines M which accept at least two strings. That is, $L = \{< M >: |L(M)| \geq 2\}$.

   (a) *15 points.* Show that L is recursively-enumerable (*i.e.*, Turing-acceptable).

   (b) *15 points.* Show that L is not recursive (*i.e.*, undecidable).

(continued on next page)

5. Consider the following problem definitions.

   Recall that in the HAMILTONIAN CYCLE problem, we are given a graph $G = (V, E)$, and asked if the graph contains a Hamiltonian cycle. A *Hamiltonian cycle* is a permutation of the edges of $V = [v_1, v_2, \ldots, v_n]$ forming a cycle: (v_i, v_{i+i}) is an edge for every $1 \leq i < n$, and (v_n, v_1) is an edge.

   That is, the HAMILTONIAN CYCLE problem asks if there is a path in the input graph that touches every vertex exactly once and returns to its starting vertex.

   BOTTLENECK-TSP is related to both the HAMILTONIAN CYCLE problem and the TRAVELING SALESMAN problem. In BOTTLENECK-TSP, we are given a graph $G = (V, E)$, a weight function $f : E \to \mathbb{R}$, and an upper bound $b \in \mathbb{R}$. We are asked to determine if the graph has a Hamiltonian cycle in which every edge used has weight $\leq b$. That is, b is the "bottleneck": every edge in the cycle cannot have a weight larger than the bottleneck.

   (a) *10 points.* Show that BOTTLENECK-TSP is in $\mathcal{NP}$.

   (b) *15 points.* Show how to reduce HAMILTONIAN CYCLE to BOTTLENECK-TSP. That is, show how to use an algorithm for solving BOTTLENECK-TSP to solve HAMILTONIAN CYCLE.

6. *Extra Credit (worth 1 point added to final course grade).*

   The seven homework groups for this course were given names from the following list:

   Bartik, Bilas, Hopper, McNulty, Snyder, Teitelbaum, Wescoff

   What is the significance of the names in this list?

   (**Note:** One of the names of the list is not directly related to the other six. If you find the pattern relating six of these names, the significance of the seventh should be clear.)

   (**Clarification:** For the Extra Credit problem, you are permitted to use external sources to research your answer.)