Theory of Computation

CS-312 **Summer 2020**

Exam 2 **DUE TUESDAY, 25 August 2020, 11:59pm**

Instructions

- Complete all problems separately; each problem indicates the number of points possible. Show your work; partial credit may be awarded.
- All problems are to be completed individually. No collaboration with others is permitted.
- This exam is *open-book*, *open notes*. You are permitted to consult the textbook, your own class notes, class handouts, and class homework and solution sets. All information available from the course Blackboard site is permitted.

  You may use JFLAP to develop your solutions, *except where using JFLAP would defeat the purpose of the question*. You may *always* use the JFLAP automata editor to draw your solutions.

  No other outside sources, including Internet sources, are permitted. No collaboration with others is permitted. Violations are subject to severe penalties, up to and including course failure.
- Your answers must be submitted electronically via Blackboard by 11:59pm on Tuesday, 25 August (7th Tuesday). ***Retain your electronic copy of your completed answers, in case of difficulties.***

By submitting your examination, you certify that you have neither given nor received any unauthorized assistance on this examination.

Note: In all problems on this exam, let $\Sigma = \{0, 1\}$ be the alphabet from which strings are produced.

1. Consider the context-free grammar given by the following ruleset:

$$S \to SS \mid 0S1 \mid 1S0 \mid 0S0 \mid 1S1 \mid \epsilon$$

   (a) *5 points.* Give a parse tree showing that 0101 can be generated by this grammar.

   (b) *5 points.* Describe (in English) the language generated by this grammar.

   (c) *5 points.* Show that this grammar is ambiguous.

2. *10 points each.* Give context-free grammars which generate the following languages:

   (a) $\{w \in \{0, 1\}^* : w$ has odd length, and the first, middle, and last symbols of w are equal$\}$

   (b) $\{0^i 1^j : \ j \geq i, \ (j - i)$ is even$\}$

3. *10 points each.* Give pushdown automata which accept the following languages (by final state):

   (a) Strings where the number of 0s is less than the number of 1s. (Note: digits may appear in any order.)

   (b) $\{0^i 1^j 0^k : i + k = j\}$

4. *15 points.* Programming languages are often described using an extended form of context-free grammar, where curly brackets are used to denote a construct that can repeat 0, 1, 2, or any number of times. For example, the rule $A \to B\{C\}D$ says that an A can be replaced with a B and D, with any number of C's (including zero) between them.

   Show that the set of languages generated by extended CFGs is equal to the set of languages generated by ordinary CFGs.

   (Hint: this is two proofs. One is easier than the other.)

5. *15 points.* The model of pushdown automata we have discussed only allows an automaton to pop (at most) a single symbol from the top of the stack, while pushing an arbitrary string onto the stack.

   A *restricted pushdown automaton (RPDA)* is a PDA which limits a transition to push at most one character at a time onto the stack. That is, in jFLAP's notation, where each transition is labeled by a triple $(\alpha, \beta; \gamma)$, γ is either a single symbol or ϵ.

   Show that the set of languages accepted by RPDAs is equal to the set of languages accepted by ordinary PDAs. That is, show that these two computational models are equivalent.

   (Hint: this is two proofs. One is easier than the other.)

6. *15 points.* Consider the language $L = \{ww^R w : w \in \{0, 1\}^*\}$. For example, "001100001" and "110011110" are strings in this language. Prove that this language L is not context-free.