Worksheet 5

1. a) $\mu = 3, \quad \sigma = 1$

$P(2.1 < X < 3.4) = P(X < 3.4) - P(X < 2.1)$

Let Y be a Normal R.V. with $\mu' = 0, \sigma' = 1$.

$\therefore P(X < 3.4) = P\left(Y < \frac{3.4 - \mu}{\sigma}\right)$

$= P\left(Y < \frac{3.4 - 3}{1}\right) = P(Y < 0.4)$

$P(X < 2.1) = P\left(Y < \frac{2.1 - 3}{1}\right) = P(Y < -0.9)$

$= 1 - P(Y < 0.9)$

$\therefore P(2.1 < X < 3.4) = P(Y < 0.4) - [1 - P(Y < 0.9)]$

$= 0.6554 - (1 - 0.8159)$

$= 0.6554 - 0.1841 = 0.4713$

(b) $P(2.1 < X < 3.4)$

= pnorm (3.4, mean = 3, sd = 1) - pnorm (2.1, mean = 3, sd = 1)

= 0.4713616

2) $\mu = 3, \quad sd = 4$

(a) $P(|X - \mu| < \sigma) = P(|X - 3| < 4)$

$= P(-4 < X - 3 < 4) = P(-1 < X < 7)$

> pnorm (7, mean = 3, sd = 4) - pnorm (-1, mean = 3, sd = 4)

[1] 0.6826895

$\therefore P(-1 < X < 7) = 0.6826895$

(b) $P(|X-\mu| < 2\sigma) = P(|X-3| < 8)$

$= P(-8 < X-3 < 8) = P(-5 < X < 11)$

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>pnorm(11, mean = 3, sd = 4) - pnorm(-5, mean = 3, sd = 4)
[1] 0.9544997
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$\therefore P(-5 < X < 11) = 0.9544997$

(c) $P(|X-\mu| < 3\sigma) = P(|X-3| < 12)$

$= P(-12 < X-3 < 12) = P(-9 < X < 15)$

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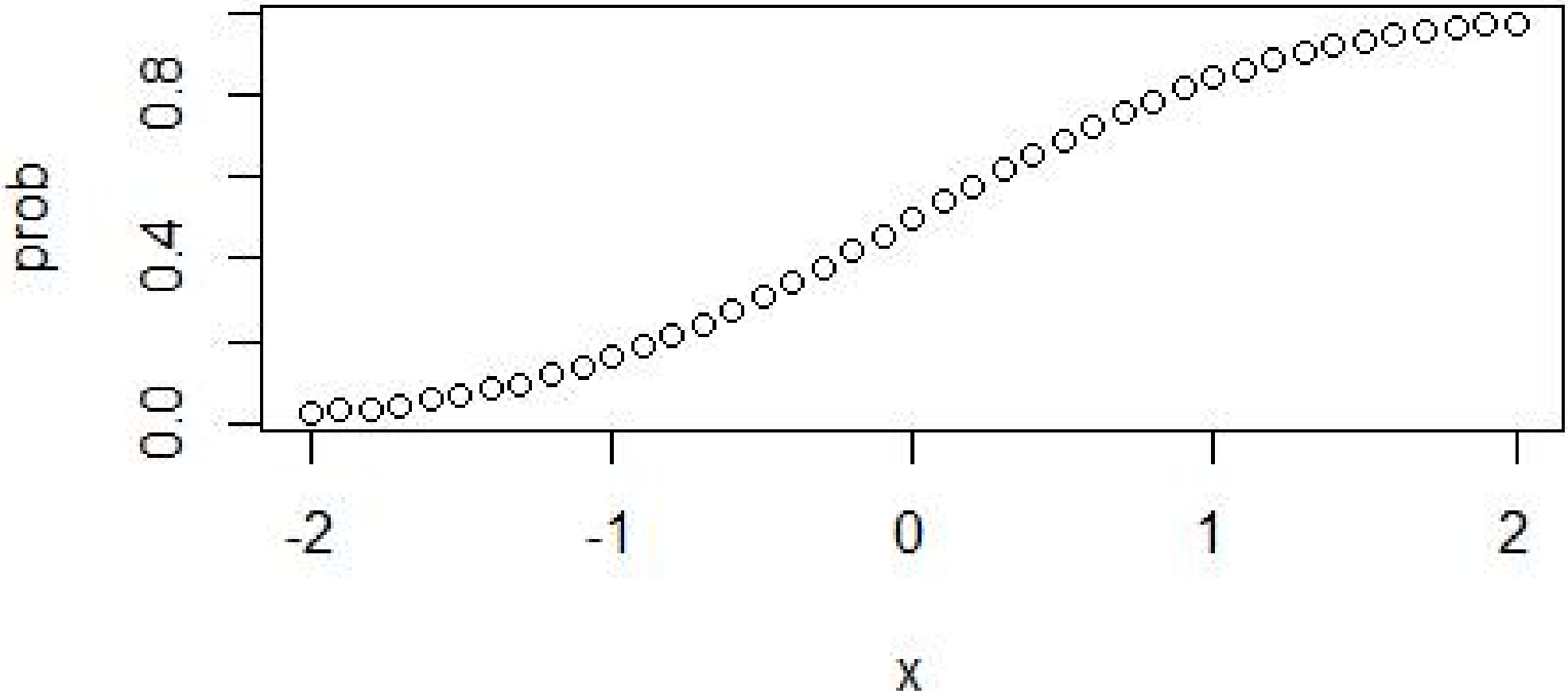
>pnorm(15, mean = 3, sd = 4) - pnorm(-9, mean = 3, sd = 4)
[1] 0.9973002
```

$\therefore P(-9 < X < 15) = 0.9973002$

3) $\mu = 0$, $\sigma = 1$, $x = -2, -1.9, \ldots 0, \ldots 1.9, 2$

$m = 100$, $p = 0.4$, $Y \sim$ Binomial (m, p).

(a)
```
>x = seq(-2, 2, 0.1)
>prob = c()
>for(i in x){
    prob = append(prob, pnorm(i))
}
plot(x, prob)
```

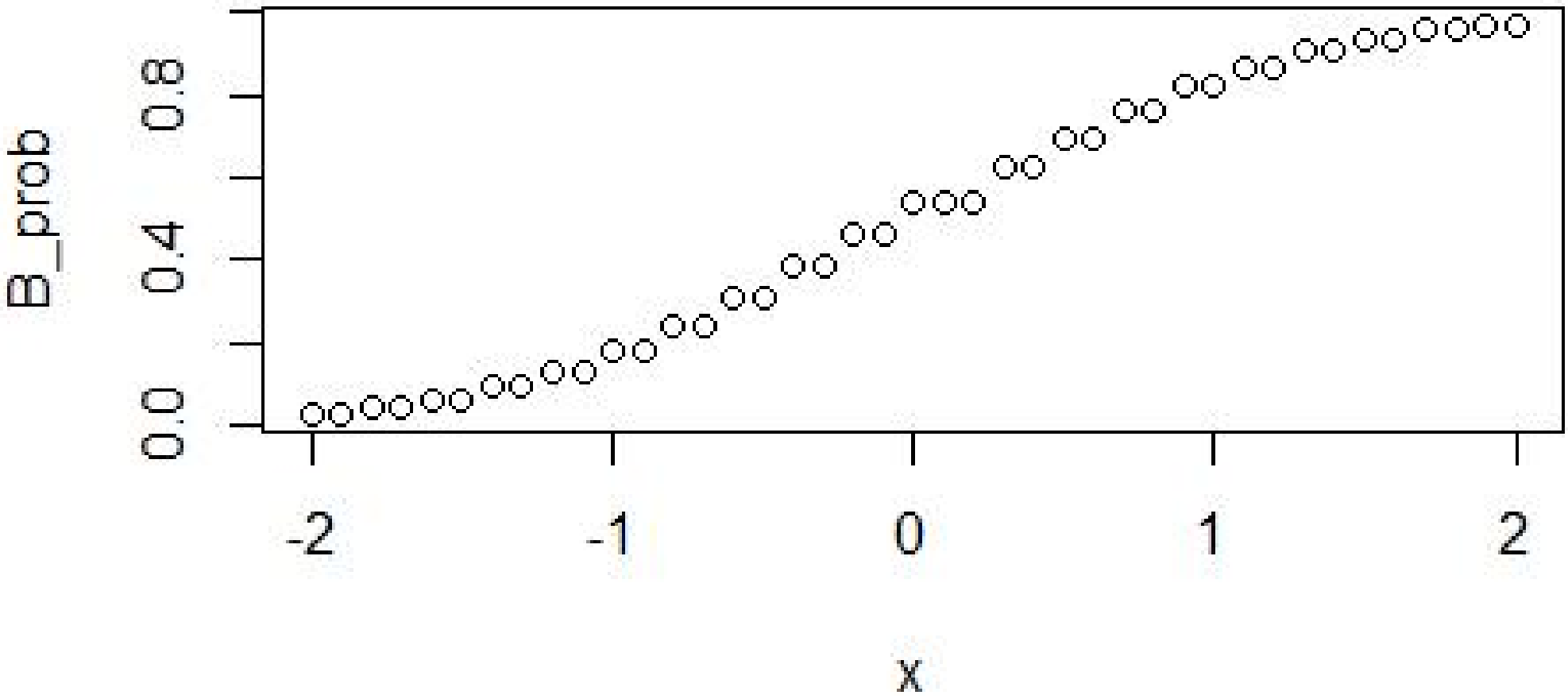

prob
0.0
0.4
0.8
-2
-1
0
1
2
x

(b) $Z = \frac{Y - np}{\sqrt{np(1-p)}}$, $Y \sim \text{Binomial}(n, p)$

$$P(Z \leq x) = P\left(\frac{Y - np}{\sqrt{np(1-p)}} \leq x\right)$$

$$= P\left(Y \leq x\sqrt{np(1-p)} + np\right)$$

```
x = seq(-2, 2, 0.1)
n = 100
p = 0.4
B_prob = c()
for (i in x){
    Y = 0
    prob1 = 0
    Y = (i * (n*p*(1-p))^0.3) + n*p
    prob1 = pbinom(Y, n, p)
    B_prob = append(B_prob, prob1)
}
plot(x, B_prob)
```


B_prob
0.0
0.4
0.8
-2
-1
0
1
2
x