Erdös Renyi Graph $G(n,p)$ is constructed in the following manner:

1. Consider n vertices labeled $\{1, 2, \ldots, n\}$.

2. Corresponding to each distinct pair $\{i, j\}$ we perform an independent Bernoulli (p) experiment and insert an edge between i and j with probability p. Note that all edges are *undirected* and hence there are total of $\binom{n}{2}$ possible edges, each occuring with probability p.

3. In this group worksheet you will simulate an Erdös Renyi Graph and find the M.L.E. for the relevant p. Your groups are available at:

https://docs.google.com/spreadsheets/d/1dqH5BvvYID43fK0Syx29CMFvo4hlyQvDcg0iReaO-Ns/edit?usp=sharing

1. Choosing x: Write a simple `R`-code to generate a number uniformly from $\{1, 2, 3, 4, 5\}$. Let x denote the chosen number. Record x in the box: $\boxed{3}$.

2. Consider the experiment of rolling a die and (choose) specify an event from that experiment which occurs with probability $x/6$. *All three persons together* decide on that event, and let it be called B. Write out the description of the event B in the box below:

   $\boxed{\text{Getting an even number}}$.

3. The set of vertices for the graph you are about to construct are $\{1, 2, \ldots, 10\}$. The graph has no self-edges (i.e Self-loops). What is the total number of possible edges ?

   Record answer in the box: $\boxed{45}$

4. Construct the *random* adjacency matrix A for the graph as follows. For each pair $1 \le i < j \le 10$:

   (a) Roll your die(using one at home or at http://www.randomservices.org/random/apps/Dice.html) and observe if the event B has occured.
   (*Take turns with each person Rolling the die 15 times.*)

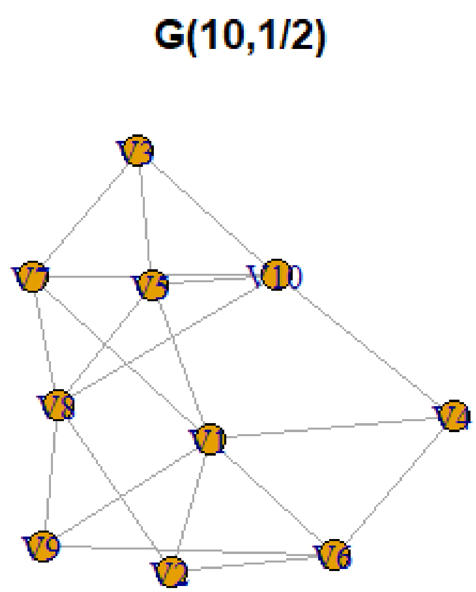
   (b) Designate

$$a_{ij} = \begin{cases} 1 & \text{if } B \text{ occured.} \\ 0 & \text{if } B \text{ did not occur} \end{cases}$$

All three persons in respective sheets fill in the matrix entries accordingly:

$$\begin{bmatrix} 0 & \boxed{1} & \boxed{0} & \boxed{1} & \boxed{1} & \boxed{1} & \boxed{1} & \boxed{0} & \boxed{1} & \boxed{0} \\ & 0 & \boxed{0} & \boxed{0} & \boxed{0} & \boxed{1} & \boxed{0} & \boxed{1} & \boxed{0} & \boxed{0} \\ & & 0 & \boxed{0} & \boxed{1} & \boxed{0} & \boxed{1} & \boxed{0} & \boxed{0} & \boxed{1} \\ & & & 0 & \boxed{0} & \boxed{1} & \boxed{0} & \boxed{0} & \boxed{0} & \boxed{1} \\ & & & & 0 & \boxed{0} & \boxed{0} & \boxed{1} & \boxed{0} & \boxed{1} \\ & & & & & 0 & \boxed{0} & \boxed{0} & \boxed{1} & \boxed{0} \\ & & & & & & 0 & \boxed{1} & \boxed{0} & \boxed{1} \\ & & & & & & & 0 & \boxed{1} & \boxed{1} \\ & & & & & & & & 0 & \boxed{0} \end{bmatrix}$$

5. Using the `igraph` package draw the random graph , denote by $G(10, \frac{x}{6})$, corresponding to the above adjacency matrix (i.e draw an edge between i and j if $a_{ij} = 1$).



PSWR

Sucheta

03/12/2021

```
#Solution 1

set.seed(40)
x = as.integer((runif(1,min=1,max=5)))
print(x)
```

```
## [1] 3
```

```
#Solution 2
print(x/6)
```

```
## [1] 0.5
```

```
#Solution 4
m= read.csv('D:\\Sucheta\\CMI\\PSWR\\matrix.csv', header = FALSE)
m1 = data.matrix(m)
m1
```

```
##       V1 V2 V3 V4 V5 V6 V7 V8 V9 V10
##  [1,]  0  1  0  1  1  1  1  0  1   0
##  [2,]  1  0  0  0  0  1  0  1  0   0
##  [3,]  0  0  0  0  1  0  1  0  0   1
##  [4,]  1  0  0  0  0  1  0  0  0   1
##  [5,]  1  0  1  0  0  0  0  1  0   1
##  [6,]  1  1  0  1  0  0  0  0  1   0
##  [7,]  1  0  1  0  0  0  0  1  0   1
##  [8,]  0  1  0  0  1  0  1  0  1   1
##  [9,]  1  0  0  0  0  1  0  1  0   0
## [10,]  0  0  1  1  1  0  1  1  0   0
```

```
#Solution 5
library(igraph)
```

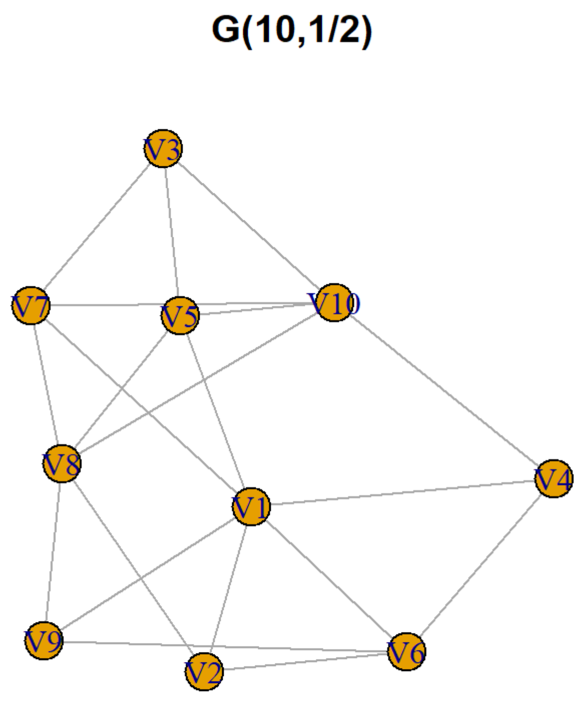
```
## Warning: package 'igraph' was built under R version 4.1.2
```

```
##
## Attaching package: 'igraph'
```

```
## The following objects are masked from 'package:stats':
##
##     decompose, spectrum
```

```
## The following object is masked from 'package:base':
##
##     union
```

```
g_10=graph.adjacency(adjmatrix=m1,mode="undirected",weighted=TRUE,diag=FALSE)
plot(g_10,main ='G(10,1/2)')
```




```
total_edges = sum(m)/2
prob_edge = total_edges/45


#The graph has 20 edges, out of the total 45 undirected edges possible.
#We can see that we get a probability of approx 0.44 of having an edge between
#any two distinct vertices of a graph, this is approximately close to the true
#prob, that is x/6 = 1/2.
```

$$L = p^{x_1}(1-p)^{n-x_1}, \quad n = {}^{10}C_2 = 45$$

Here x_1 is the number of edges for the graph $G(10, 1/2)$.

$x_1 = 20$.

$$\therefore \; L = p^{20}(1-p)^{45-20}$$

$$\Rightarrow \log L = 20 \log p + 25 \log(1-p)$$

critical pts can be found by $\frac{d}{dp}(\log L) = 0$

$$\Rightarrow \frac{20}{p} - \frac{25}{1-p} = 0 \Rightarrow 20 - 20p = 25p$$

$$\Rightarrow p = \frac{20}{45}.$$

$$L = \left(\frac{\cancel{20}^{\,4}}{\cancel{45}_{\,9}}\right)^{20}\left(\frac{\cancel{25}^{\,5}}{\cancel{45}_{\,9}}\right)^{45-20} = \left(\frac{4}{9}\right)^{20}\left(\frac{5}{9}\right)^{25}$$

$$\approx 3.75 \times 10^{-14}$$