14 December 2021 Due on 24 December 2021

M.Sc. Data Science
Analysis - HW 5

Note: Copying will not be tolerated. You may discuss among yourselves but the final work should be your own.

1. Find parametric equations for the tangent line L to the helix with parametric equations $x = 2\cos t, y = \sin t, z = t$ at $t = \pi/2$.

2. Find the domain of the following functions:

   (a) $f(x, y) = x \ln(y^2 - x)$

   (b) $g(x, y) = \sqrt{4 - x^2 - y^2}$.

3. Let $f(x, y) = x^3 - 3xy + 4y^2$. Find $D_u f(1, 2)$, where u is the unit vector making an angle of $\pi/6$ with the positive x-axis.

4. Let $f(x, y, z) = xe^y$, $P = (2, 0)$ and $Q = (12, 2)$.

   (a) Find the rate of change of f at P in the direction of vector PQ.

   (b) In which direction does f have the maximum rate of change? What is this maximum rate of change?

5. Which is the point on the curve $(\cos t, \sin t, \sin(t/2)$ farthest from the origin?

6. Find the maximum value of $x^2 + xy + y^2 + yz + z^2$ on the sphere of radius 1.

7. Suppose product A costs \$11 per unit and product B costs \$3 per unit. Both are needed to produce product C. When x units of A and y units of B are used, the total number of units of C produced by the production process is:

   $$g(x, y) = -3x^2 + 10xy - 3y^2.$$

   How many units of A and B should be used to produce 80 units of product C and minimize the costs?

8. Find the terms up to order 2 in the Taylor formula of the function $\log(1 + xy)$ at the point $P(0, 0)$.

9. Find and characterize the extreme values of the following functions:

   (a) $f(x, y) = x^2 - 3xy + y^2$

   (b) $f(x, y) = \cos(x + y) + \sin(x - y) + x^2$