Worksheet13

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```
set.seed(51)

#Solution 1
x = c(75, 76, 73, 75, 74, 73, 73, 76, 73, 79, 77, 75)

#Solution 1.a
#sigma = 1.5, 1-alpha=0.95
zcinf = function(v){
  n = length(v)
  Z= qnorm(0.95/2,lower.tail=FALSE)
  c(mean(v)-Z*1.5/sqrt(n),mean(v)+Z*1.5/sqrt(n))
}
zcinf(x)
```

```
## [1] 74.88951 74.94382
```

```
#Solution 1.b
#1-alpha = 0.95
tcinf = function(v){
  n = length(v)
  Z = qnorm(0.95/2,lower.tail=FALSE)
  c(mean(v)-Z*sd(v)/sqrt(n),mean(v)+Z*sd(v)/sqrt(n))
}
tcinf(x)
```

```
## [1] 74.88262 74.95071
```

```
#Solution 1.c

#Case 1-> Null hypothesis: mu = 0
t.test(x)
```

```
##
##  One Sample t-test
##
## data:  x
## t = 137.97, df = 11, p-value < 2.2e-16
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  73.72158 76.11175
## sample estimates:
## mean of x
##  74.91667
```

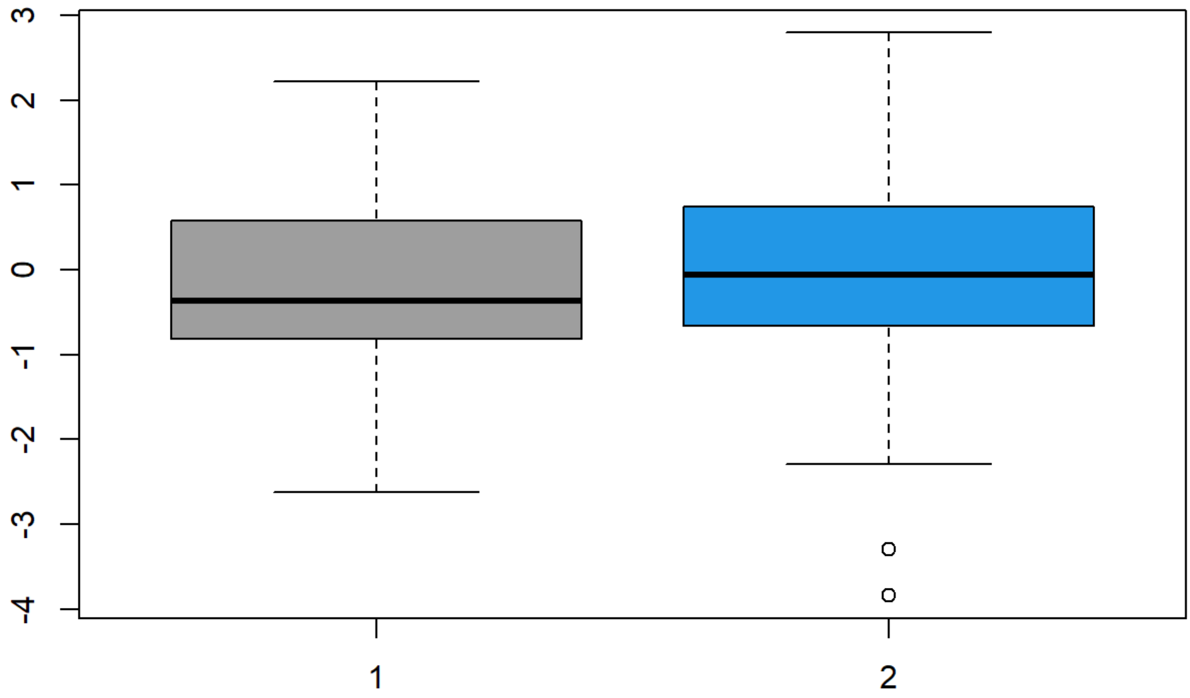
```
#The test statistic is t= 137.97 and the degrees of freedom is 11
#since n = 12. p-value returns the probability of not rejecting the
#null Hypothesis, p-value=2.2e-16. Since we have H0: mu =0, hence
#alternate hypothesis is mu != 0. The t.test by default
#finds the 95% confidence interval which for our sample is,
#(73.72158, 76.11175). Since this is a one sample test it estimates
# mean, which is 74.91667.
#Since the p-value<0.05 , we reject the null hypothesis in this case.

#Case 2-> Null hypothesis: mu = mean(x)
t.test(x,mu=mean(x))
```

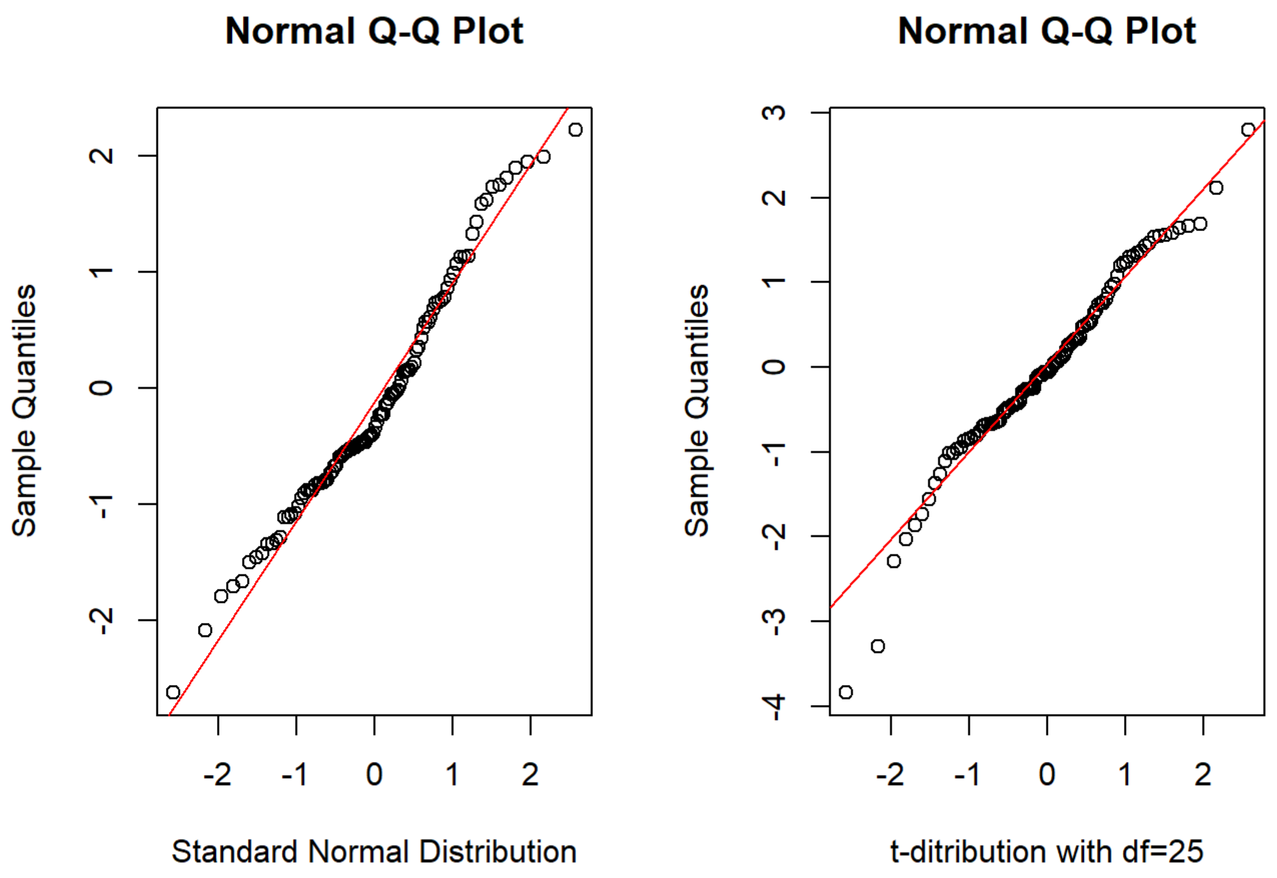
```
##
##  One Sample t-test
##
## data:  x
## t = 0, df = 11, p-value = 1
## alternative hypothesis: true mean is not equal to 74.91667
## 95 percent confidence interval:
##  73.72158 76.11175
## sample estimates:
## mean of x
##  74.91667
```

```
#The test statistic is t= 0 and the degrees of freedom is 11
#since n = 12. p-value returns the probability of not rejecting the
#null Hypothesis, p-value=1. Since we have H0: mu =mean(x), hence
#alternate hypothesis is mu != mean(x). The t.test by default
#finds the 95% confidence interval which for our sample is,
#(73.72158, 76.11175). Since this is a one sample test it estimates
# mean, which is 74.91667.
#Since the p-value>0.05 , we can't reject the null hypothesis in this case.

#Solution 2.a
x1 = rnorm(100,mean=0,sd=1)
x2 = rt(100,df=25)
boxplot(x1,x2, col=c(8,4))
```

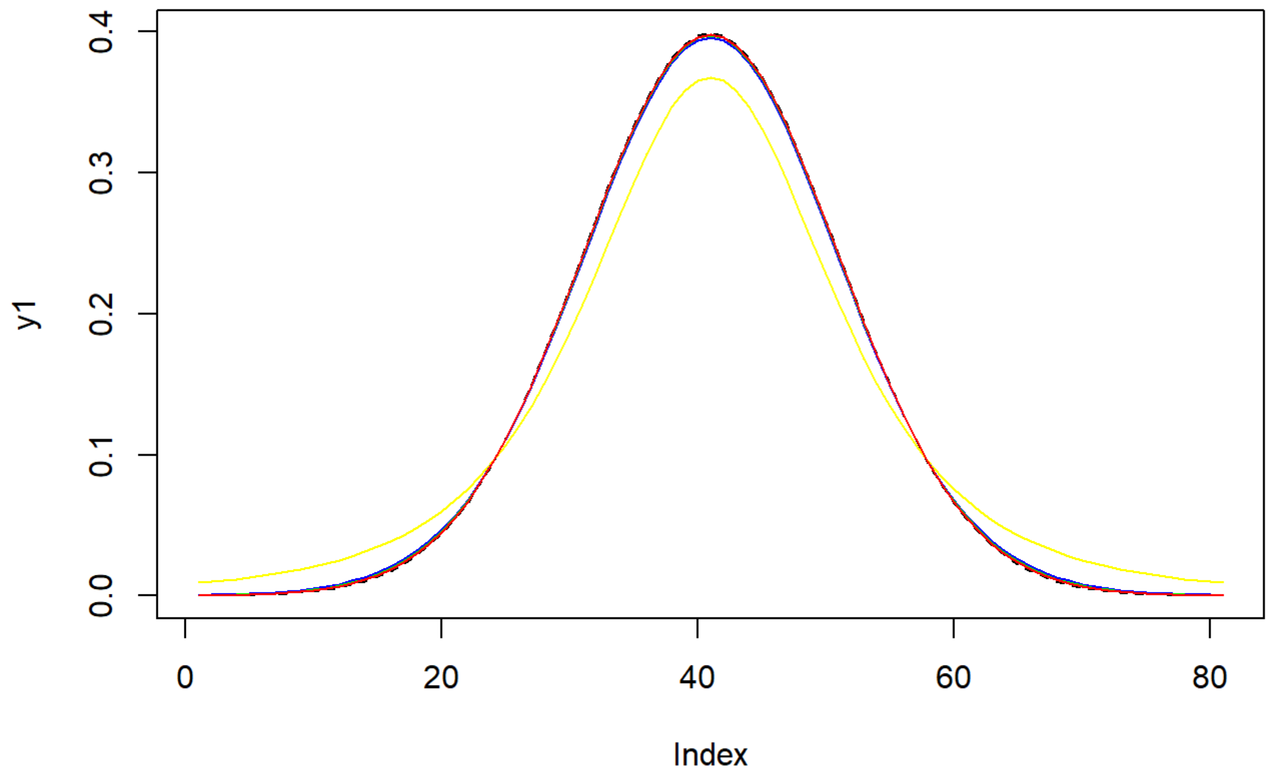



```
par(mfrow=c(1,2))
qqnorm(x1,xlab='Standard Normal Distribution')
qqline(x1,col='red')
qqnorm(x2,xlab='t-ditribution with df=25')
qqline(x2,col='red')
```




```
#We can see both the normal qq plots are almost the same, but near the
#end both start deviating from the qqline.
```

```
#Solution 2.b
k=c(3, 33, 66, 99)
ran_x = seq(-4,4,by=0.1)
y1 =dnorm(ran_x)
plot(y1,type="l", lty=2)
y2= dt(ran_x,df=3)
lines(y2,col='yellow')
y2= dt(ran_x,df=33)
lines(y2,col='blue')
y2= dt(ran_x,df=66)
lines(y2,col='green')
y2= dt(ran_x,df=99)
lines(y2,col='red')
```




```
#We can see as we increase the degrees of freedom, the coloured smooth lines
#start to overlap with the dashed line representing normal distribution.
#We can say as the degree of freedom increases, t distribution starts behaving
#like a normal distribution

#Solution 3

prop.test(45,100)
```

```
##
##  1-sample proportions test with continuity correction
##
## data:  45 out of 100, null probability 0.5
## X-squared = 0.81, df = 1, p-value = 0.3681
## alternative hypothesis: true p is not equal to 0.5
## 95 percent confidence interval:
##  0.3514281 0.5524574
## sample estimates:
##    p
## 0.45
```

```
#We have 45 heads out of 100 tosses. The assumed null hypothesis is 0.5,
#since we are testing for a fair coin. X-squared us 0.81 which is the test
#statistic and the degrees of freedom is df=1. We can see p-value is 0.3681.
#The 95% confidence interval is calculated to be (0.3514281, 0.5524574) and the
#sample estimate of the probability of a head is 0.45.
#Since p-value = 0.3681>1-0.95, we can't reject the null hypothesis. Also, note
#that 0.5 lies in the confidence interval calculated by prop.test(), hence
#the coin might or might not be a fair one.

#Solution 4
prop.test(4500,10000)
```

```
##
##  1-sample proportions test with continuity correction
##
## data:  4500 out of 10000, null probability 0.5
## X-squared = 99.8, df = 1, p-value < 2.2e-16
## alternative hypothesis: true p is not equal to 0.5
## 95 percent confidence interval:
##  0.4402205 0.4598181
## sample estimates:
##    p
## 0.45
```

```
#We have 4500 heads out of 10000 tosses. The assumed null hypothesis is 0.5,
#since we are testing for a fair coin. X-squared us 99.8 which is the test
#statistic and the degrees of freedom is df=1. p-value is less than 2.2e-16.
#The 95% confidence interval is calculated to be (0.4402205, 0.4598181) and the
#sample estimate of the probability of a head is 0.45.
#Since p-value<1-0.95, we reject the null hypothesis. Also, note
#that 0.5 does not lie in the confidence interval calculated by prop.test(),
#hence the coin is not a fair one.


#Solution 5

#Claim: lifetime of a battery=25
alpha=0.05
Z= qnorm(0.95/2,lower.tail=FALSE)
c(21-Z*1.7/sqrt(10),21+Z*1.7/sqrt(10))
```

```
## [1] 20.96629 21.03371
```

```
value = 1-pt(7.441,df=9)
value<0.05
```

```
## [1] TRUE
```

```
#Since value<0.05, we can reject the null hypothesis, Also, note that
#the claimed mean 25, doesn't lie in the 95% confidence interval for
#sample mean. The claim made by Doddapple can not be believed.
```

5) Claim: Lifetime of a battery is 25 years.

$n = 10$. Let $1-\alpha = 0.95 \Rightarrow \alpha = 0.05$

Sample mean $= 21 = X$

Sample standard deviation $= 1.7 = S$

Null Hypothesis : $\mu = 25$

Alternate Hypothesis : $\mu \neq 25$

$X \sim \text{Normal}(\mu, \sigma^2)$

Let $Y_1, Y_2, \ldots Y_{10}$ be iid X random variables.

Under the null hypothesis, $Y_i \sim \text{Normal}(25, \sigma^2)\ \forall\ 1 \leq i \leq 10$

$$P(\bar{Y} \geq \bar{X}) = P(\bar{Y} \geq 21)$$

$$= P\left(\frac{\sqrt{n}(\bar{Y} - 25)}{S} \geq \frac{\sqrt{n}}{S}(21-25)\right)$$

$$= P\left(\sqrt{10}\,\frac{(\bar{Y}-25)}{1.7} \geq \frac{\sqrt{10}}{1.7}(-4)\right)$$

Let $T = t_9 = \sqrt{10}\left(\dfrac{\bar{Y}-25}{1.7}\right)$.

$$\therefore P(\bar{Y} \geq \bar{X}) = P(T \geq -7.441)$$

$$= 1 - P(T \leq 7.441).$$

$$\simeq 1.965 \times 10^{-5} < 0.05$$

$\Rightarrow$ Reject the null hypothesis.

$\Rightarrow$ Doddapple's claim cannot be believed.