28 September 2021 Due on 7 October 2021

M.Sc. Data Science

Analysis - HW 1

Note: Copying will not be tolerated.

1. Prove that $\lim_{n\to\infty} \frac{2n}{n+3} = 2$ by showing that for every $\epsilon > 0$, one can find N such that $\left|\frac{2n}{n+3} - 2\right| < \epsilon$ for all $n \geq N$.

2. For each of the following sequences $\{x_n\}_{n=0}^{\infty}$, say if they are convergent or divergent. If they converge, find the limit. If they diverge, explain why.

   (a) $x_n = \frac{n^5+12}{n^3}$

   (b) $x_n = \frac{n^2}{n+5}$

   (c) $x_n = \cos\left(\frac{n\pi}{2}\right)$

3. (a) For any $a, b \in \mathbb{R}$ show that $||a| - |b|| \leq |a - b|$. Use this to show that if sequence $\{x_n\}_{n=1}^{\infty}$ converges to L, then sequence $\{|x_n|\}_{n=1}^{\infty}$ converges to $|L|$.

   (b) Give an example of a sequence $\{x_n\}_{n=1}^{\infty}$ such that the sequence $\{|x_n|\}_{n=1}^{\infty}$ converges but $\{x_n\}_{n=1}^{\infty}$ does not. (Thus the converse of part (a) is not true, in general.)

   (c) Finally, show that if $\{|x_n|\}_{n=1}^{\infty}$ converges to zero, then $\{x_n\}_{n=1}^{\infty}$ also converges to zero.

4. In each of the following exercises:

   - Find the derivative of the given function using the limit of the Newton quotient as $h \to 0$;
   - find the slope of the given curve at the indicated point;
   - find the equation of the tangent line at the given point.

   (a) $y = x^2 + 2x$ at $(-1, -1)$;

   (b) $y = \dfrac{1}{x+1}$ at $(1, 2)$.

5. Find the derivatives of the following functions:

   (a) $x^{1/2} - 8x^4 + x^{-1}$

   (b) $(2x^2 + 1)\left(\dfrac{1}{x^2} + 4x + 8\right)$

   (c) $\dfrac{x^5 + 1}{(x^2 + 1)(x + 7)}$

   (d) $e^{\cos x}$

   (e) $\log(e^x + \sin x)$

   (f) $\dfrac{1}{(3x - 1)^4}$

6. Find dy/dx:

   (a) $y^2 + 2x^2y + x = 0$

   (b) $(y - x)^2 = 2x + 4$

7. A ladder 17 ft long leans against a vertical wall. If the lower end of the ladder is being moved away from the foot of the wall at the rate of 3 ft/sec, how fast is the top descending when the lower end is 8 ft from the wall?

8. A train leaves a station at a certain time and travels north at the rate of 50 miles/hr. A second train leaves the same station 2 hours after the first train leaves and goes east at the rate of 60 miles/hr. Find the rate at which the two trains are separating 1.5 hours after the second train leaves the station.