24 October 2021 Due on 3 November 2021

M.Sc. Data Science
Analysis - HW 2

Note: Copying will not be tolerated. You may discuss among yourselves but the final work should be your own.

1. Let a be a number > 0. Find the critical points of the function $f(x) = x^2/a^x$.
2. Find the equation of the tangent to the curve $y = x^{\sqrt{x}}$ at $x = 2$.
3. Let p be the probability that a certain event will occur, at any trial. Suppose that in n trials, s successes have been observed. The likelihood function L is defined as $L(p) = p^s(1-p)^{n-s}$. Find the value of p which maximizes the likelihood function. (Take $0 \leq p \leq 1$, and view n, s as constants).
4. (a) What is $\int_0^2 x(x-1)(x-2)\,dx$? Use a partition with $\Delta x = \frac{1}{3}$ to set up the Riemann sums that approximate this integral.
   (b) Find the area between the curve $f(x) = x(x-1)(x-2)$ and the x-axis as follows:
      i. Use a partition with $\Delta x = \frac{1}{3}$ to set up the Riemann sums.
      ii. Find the approximate area by setting up the appropriate upper and lower sums.
5. In 1900, the population of a city was $50,000$. In 1950, it was $100,000$. If the rate of increase of population is proportional to the population, what is the population in 1984? In what year is it $200,000$?
6. Sugar in water decomposes at a rate proportional to the amount still unchanged. If 30lb of sugar reduces to 10lb in 4 hours, when will 95% of the sugar be decomposed?
7. (a) Give values of $x \log x$ when $x = 1/2, 1/4, 1/8, \ldots$, in general when $x = 1/2^n$ for some positive integer n.
   (b) Does $x \log x$ approach a limit as $x \to 0$? What about $x^2 \log x$? [Hint: let $x = e^{-y}$ and let y become large.]
8. Find the following limits as $n \to \infty$:
   (a) $(\log n)^{1/n}$
   (b) $[(\log n)/n]^{1/n}$
   (c) $(n/e^n)^{1/n}$
   (d) $(n \log n)^{1/n}$
9. (Inverse tangent function) Let $t(x) = \arctan(x)$.
   (a) Is $t(x)$ differentiable? Find $t'(x)$.
   (b) Is $t(x)$ monotonic?
   (c) What is the range of t?
   (d) Sketch the graph on t on the domain $[-10, 10]$. What is $t'(0)$?