Quiz 8

1) Let B be the event that a biased coin has been chosen and F be the event that a fair coin has been chosen.

X is the number of heads that come up in a fair coin and Y is the number of heads in a biased coin.

$n = 1000, \; p_1 = 0.5, \; p_2 = 0.55$

$\therefore X \sim \text{Binomial}(1000, 0.5)$

$Y \sim \text{Binomial}(1000, 0.55)$

B|F is a event when we chose a fair coin but the conclusion says that it is a biased coin. This event occurs if our fair coin gives 525 or more heads in 1000 tosses.

$\therefore P(B|F) = P(X \geq 525)$

$$= P\left(\frac{X - E(X)}{SD(X)} \geq \frac{525 - E(X)}{SD(X)}\right)$$

Now, $E(X) = 1000 \times 0.5 = 500$

$SD(X) = \sqrt{1000 \times 0.5 \times 0.5} = \sqrt{250}$

$$\therefore P(B|F) = P\left(\frac{X - 500}{\sqrt{250}} \geq \frac{525 - 500}{\sqrt{250}}\right)$$

$$= P\left(\frac{X - 500}{\sqrt{250}} \geq 1.58114\right)$$

Let $Z_1 = \frac{X - 500}{\sqrt{250}} \Rightarrow Z_1 \sim \text{Normal}(0,1)$

$\therefore P(B|F) = P(Z_1 \geq 1.58114)$

$= 1 - P(Z_1 \leq 1.58114)$

$= 1 - 0.943077 = 0.057$

Now, F|B is the event that the coin was a biased coin but we concluded it is a fair one. This event occurs if the no. of heads that come up on tossing a biased coin 1000 times is less than 525.

$\therefore P(F|B) = P(Y < 525)$

$= P\left(\frac{Y - E(Y)}{SD(Y)} < \frac{525 - E(Y)}{SD(Y)}\right)$

$E(Y) = np_2 = 1000 \times 0.55 = 550$

$SD(Y) = \sqrt{1000 \times 0.55 \times 0.45} = \sqrt{247.5}$

$\therefore P(F|B) = P\left(\frac{Y - 550}{\sqrt{247.5}} < \frac{525 - 550}{\sqrt{247.5}}\right)$

Let $Z_2 = \frac{Y - 550}{\sqrt{247.5}} \Rightarrow Z_2 \sim \text{Normal}(0,1)$

$\therefore P(F|B) = P(Z_2 < -1.58910)$

$= 0.056$

$\therefore P(B|F) = 0.057, \quad P(F|B) = 0.056$