22 November 2021 Due on 30 November 2021

M.Sc. Data Science

Analysis - HW 4

Note: Copying will not be tolerated. You may discuss among yourselves but the final work should be your own.

1. Determine whether the following series converge absolutely:
   (a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+1}$
   (b) $\sum_{n=1}^{\infty} \frac{(-1)^{n+2}}{\log n}$
2. Find the radius of convergence of the following series:
   (a) $\sum_{n=1}^{\infty} \frac{x^n}{\log n}$
   (b) $\sum_{n=1}^{\infty} \frac{\sin(n\pi/2)}{2^n} x^n$
3. Let $f(x) = \sum_{n=1}^{\infty} \frac{x^{2n}}{2n!}$. Prove that $f''(x) = f(x)$.
4. Let $f(x, y, z) = e^{xy} \ln z$. Find f_x, f_y and f_z. Show that $xf_x = yf_y$.
5. Find the equation of the tangent plane to the elliptic paraboloid $z = 2x^2 + y^2$ at the point $(1, 1, 3)$. item Graph the equation $z = 4x^2 + y^2$. Sketch and identify some level curves of this surface.
6. Sketch the curve whose vector equation is $f(t) = (1 + t, 2 + 5t, -1 + 6t)$.
7. Find parametric equations for the tangent line L to the helix with parametric equations $x = 2\cos t, y = \sin t, z = t$ at $t = \pi/2$.
8. Find the domain of the following functions:
   (a) $f(x, y) = x \ln(y^2 - x)$
   (b) $g(x, y) = \sqrt{4 - x^2 - y^2}$.
9. Find $\partial z/\partial x$ and $\partial z/\partial y$ if z is defined implicitly as a function of x and y by $x^3 + y^3 + z^3 + 6xyz = 1$.
10. Let $z = e^x \sin y$, where $x = st^2$, $y = s^2t$. Find $\partial z/\partial s$ and $\partial z/\partial t$.