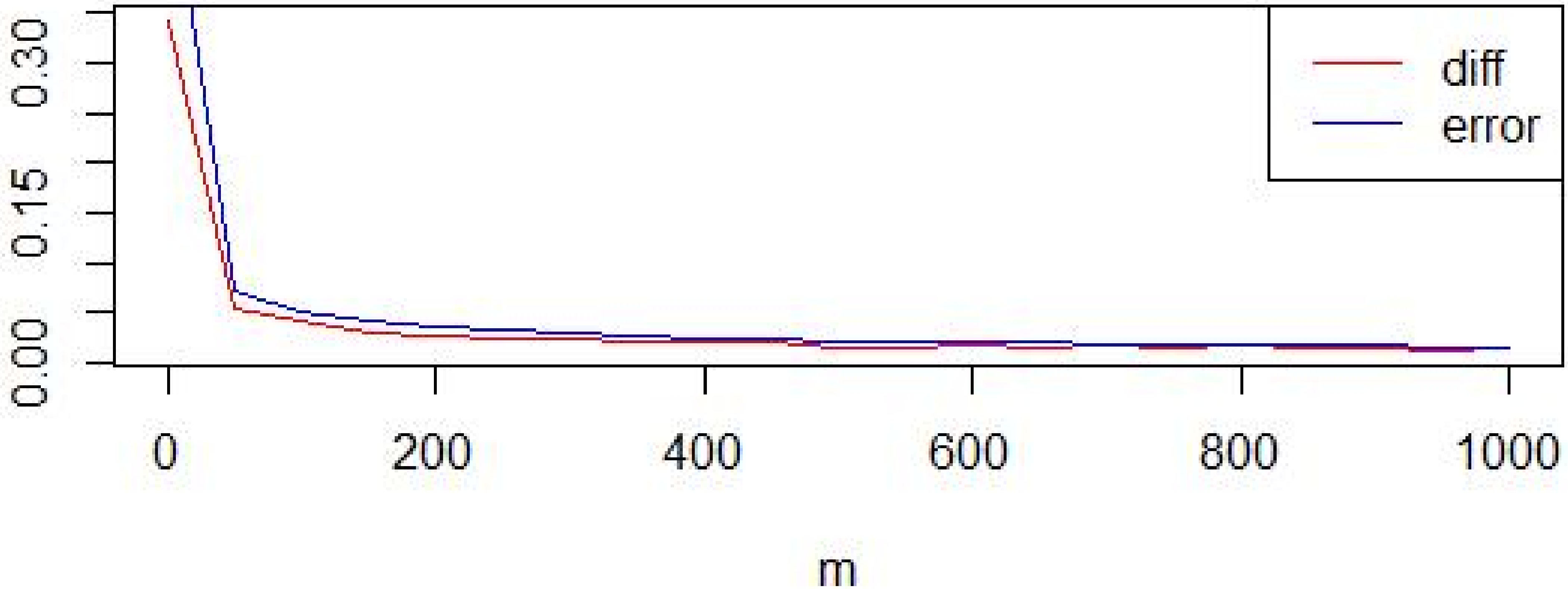
```r
#Solution 1

x = seq(-2,2,0.1)  #creates a vector x = {-2,-1.9,....,1.8,1.9,2}
z = c()
z = pnorm(x)      #calculates the normal distribution function at x
p = 0.5           #the probability of the occurence of an event
m = seq(50,1000,50)  #creates a vector m = {50,100,....,1000}
m = c(1,m)
diff = c()     #vector to store the difference
error = c()    #vector to store the error value

for (k in m){      #runs a loop on each element of m
  my = c()
  for (i in x){
    y = c()
    for (j in c(1:100)){     #to repeat the steps 100 times
      B = rbinom(1000,k,p)    #creates a sample of size 1000 following Binomial Distribution
      SB = (B-k*p)/((k*p*(1-p))^(0.5))
      y = append(y,((sum(SB<=i))/length(SB)))    #the proportion of SB having values less than an element i of x
    }
    my = append(my,mean(y))    #average of the values of y
  }

  diff = append(diff,max(abs(my-z)))
  error= append(error,(p**2+(1-p)**2)/(2*(k*p*(1-p))**0.5))
}

plot(m, diff, type ='l', col ='red', xlab ='m', ylab =' ')  #to plot the graph for diff
par(new = TRUE)
lines(m, error, type ='l', col ='blue')
text(mlabels = c('diff','error'))
legend("topright", legend =c("diff",'error'), col=c("red", "blue"), lty=1:1)
```



The Theorem says that the diff term is always less than the error term which can be verified from the graph, since the graph for the diff always lies below the graph for error.
Additionally, we can increase the number of samples which will increase the accuracy of our result.

2) $Z \sim \text{Normal}(\mu, \sigma^2)$

R.T.P : median of Z is μ.

i.e. $P(Z > \mu) = P(Z < \mu) = \frac{1}{2}$.

$$P(Z > \mu) = \int_{\mu}^{\infty} \frac{e^{-\frac{(x-\mu)^2}{2\sigma^2}}}{\sqrt{2\pi}\,\sigma}\, dx$$

Let $y = \frac{x-\mu}{\sigma}$,

x	μ	∞
y	0	∞

, $dy = \frac{dx}{\sigma}$

$$\therefore P(Z > \mu) = \int_{0}^{\infty} \frac{e^{-y^2/2}}{\sqrt{2\pi}\,\cancel{\sigma}} \cancel{\sigma}\, dy$$

Also note that, if $\hat{y} = -y$, $d\hat{y} = -dy$

$$\int_{0}^{\infty} \frac{e^{-y^2/2}}{\sqrt{2\pi}\,\sigma} dy = \int_{0}^{-\infty} \frac{e^{-\hat{y}^2/2}}{\sqrt{2\pi}} (-d\hat{y}).$$

$$= \int_{-\infty}^{0} \frac{e^{-\hat{y}^2/2}}{\sqrt{2\pi}} d\hat{y} = \int_{-\infty}^{0} \frac{e^{-y^2/2}}{\sqrt{2\pi}} dy.$$

Now,
$$\int_{-\infty}^{\infty} \frac{e^{-y^2/2}}{\sqrt{2\pi}} dy = 1$$

$$\Rightarrow \int_{-\infty}^{0} \frac{e^{-y^2/2}}{\sqrt{2\pi}} dy + \int_{0}^{\infty} \frac{e^{-y^2/2}}{\sqrt{2\pi}} dy = 1.$$

$$\Rightarrow 2\int_{0}^{\infty} \frac{e^{-y^2/2}}{\sqrt{2\pi}} dy = 1 \Rightarrow \int_{0}^{\infty} \frac{e^{-y^2/2}}{\sqrt{2\pi}} dy = \frac{1}{2}.$$

$$\therefore P(Z > \mu) = \frac{1}{2}$$

$$P(z<\mu) = \int_{-\infty}^{0} \frac{e^{-y^2/2}}{\sqrt{2\pi}\,\cancel{\sigma}}\cancel{\sigma}\,dy = \frac{1}{2}$$

$$\therefore P(z>\mu) = P(z<\mu) = \frac{1}{2}.$$

$\Rightarrow \mu$ is the median.

3) $X \sim$ Normal (μ, σ^2)

R.T.P : $P(|X-\mu| < k\sigma)$ does not depend on μ or σ.

$$P(|X-\mu| < k\sigma) = P(-k\sigma < X-\mu < k\sigma)$$

$$= P(-k\sigma+\mu < X < k\sigma+\mu).$$

$$= P(X < k\sigma+\mu) - P(X < -k\sigma+\mu)$$

$$= \int_{-\infty}^{k\sigma+\mu} \frac{e^{-\frac{(x-\mu)^2}{2\sigma^2}}}{\sqrt{2\pi}\,\sigma}\,dx - \int_{-\infty}^{-k\sigma+\mu} \frac{e^{-\frac{(x-\mu)^2}{2\sigma^2}}}{\sqrt{2\pi}\,\sigma}\,dx.$$

let $y = \frac{x-\mu}{\sigma}$

$dy = \frac{dx}{\sigma}$

x	$-\infty$	$k\sigma+\mu$	$-k\sigma+\mu$
y	$-\infty$	k	$-k$.

$\therefore P(|X-\mu| < k\sigma)$

$$= \int_{-\infty}^{k} \frac{e^{-y^2/2}}{\sqrt{2\pi}\,\cancel{\sigma}}\cancel{\sigma}\,dy - \int_{-\infty}^{-k} \frac{e^{-y^2/2}}{\sqrt{2\pi}\,\cancel{\sigma}}\cdot\cancel{\sigma}\,dy$$

$$= \int_{-k}^{k} \frac{e^{-y^2/2}}{\sqrt{2\pi}}\,dy \;.= P(-k < Y < k)$$

where $Y \sim$ Normal $(0,1)$

$\therefore$ $P(|X-\mu| < k\sigma)$ only depends upon the value of k and is independent of the values of μ & σ.

4) $X \sim$ Normal $(0, 1)$

We need to find. $P(-\sigma < X-\mu < \sigma)$.

Here $\mu = 0$, $\sigma = 1$.

$\therefore P(-1 < X < 1) = P(X < 1) - P(X < -1)$

$$= \int_{-\infty}^{1} \frac{e^{-x^2/2}}{\sqrt{2\pi}} dx - \int_{-\infty}^{-1} \frac{e^{-x^2/2}}{\sqrt{2\pi}} dx.$$

On R this can be done using pnorm,

\> pnorm (1) - pnorm (-1)

[1] 0.6826895.

$\therefore P(-1 < X < 1) = 0.6826895$.

Alternatively, $P(X < 1) = 0.841$ [From the table].

$P(-1 < X < 1) = P(X < 1) - (1 - P(X < 1))$

$= 2P(X < 1) - 1$

$= 2 \times 0.841 - 1 = 0.682$.

5) Y denote the weight in grams of cashews in a bag.

Y is normally distributed with $\mu = 200$ & $\sigma = 4$.

$\therefore Y \sim$ Normal $(200, 4^2)$

We need to find the probability of a bag having less than 195 gm of cashews i.e. $P(Y < 195)$

$$P(Y<195) = \int_{-\infty}^{195} \frac{e^{\frac{-(y-200)^2}{2\times 4^2}}}{\sqrt{2\pi}\times 4}\, dy$$

let $x = \frac{y-200}{4}$, $dx = \frac{dy}{4}$

x	$-\infty$	$-5/4$
y	$-\infty$	195

$$\therefore P(Y<195) = \int_{-\infty}^{-5/4} \frac{e^{-x^2/2}}{\sqrt{2\pi}\times \cancel{4}} \times \cancel{4}\, dx$$

$$= \int_{-\infty}^{-5/4} \frac{e^{-x^2/2}}{\sqrt{2\pi}}\, dx = P(X<-5/4)$$

where $X \sim$ Normal $(0, 1)$

$P(X<-5/4)$ can be calculate in R,

```
> pnorm (-5/4)
[1] 0.1056498
```

$\therefore P(Y<195) = 0.1056498$

whereas from the table,

$P(X<-5/4) = 1 - P(X<5/4)$

$= 1 - 0.896 = 0.104$