

1. Suppose X follows Bernoulli(p) distribution. Let $p = 1/3$

- (a) Simulate for $n = 100$ $X_1, X_2, X_3, \dots, X_n$ i.i.d X
- (b) Demonstrate the Law of Large numbers by plotting the sample mean \bar{X}_n as a function of n .
- (c) Using `replicate` command plot 15 independent trials of the above.
- (d) Do the same when $p = 0.001, n = 100, p = 0.5, n = 100, p = 0.99$ on different plots

2. We wish to compute

$$\int_a^b f(x)dx$$

using the Law of Large numbers.

- (a) Generate samples of X_1, X_2, \dots, X_n i.i.d. Uniform (a, b) . Justify

$$(b - a) \sum_{i=1}^n \frac{f(X_i)}{n} \approx \int_a^b f(x)dx$$

- (b) Write an `R`-code to estimate the $\int_0^7 \frac{16+\sin(x)}{x^2+4} dx$ using the procedure described in the previous part with $n = 400$.
- (c) Repeat the estimate 100 times and find the mean of these 100 simulations.
- (d) Use the `integrate` command in `R` to evaluate the integral. Compare the two answers.