

## 1. De Moivre's Central Limit Theorem.

- (a) Using the `rbinom` generate 100 samples of  $\text{Binomial}(20, 0.5)$  and plot the histogram of the data-set.
- (b) Using the `rnorm` generate 100 samples of  $\text{Normal}(10, 5)$  and plot the histogram of the data set.

Think of ways you can enhance the above exercise to come up with a computer proof of the Central Limit Theorem.

## 2. Poisson Approximation

- (a) Using the `rbinom` generate 100 samples of  $\text{Binomial}(2000, 0.001)$ , save it in a dataframe `dfbinomial` and plot the histogram of the data-set.
- (b) Using the `rpois` generate 100 samples of  $\text{Poisson}(2)$ , save it in a dataframe `dfnormal` and plot the histogram of the data set

Think of ways you can enhance the above exercise to come up with a computer proof of the Poisson Approximation, even though we have seen a proof in class.

## 3. The following result is a Berry-Eseen Type bound.

**Theorem:** Let  $X_n \sim \text{Binomial}(n, p)$ , then there exists  $C > 0$  such that

$$\sup_{x \in \mathbb{R}} \left| P \left( \frac{X_n - np}{\sqrt{np(1-p)}} \leq x \right) - \int_{-\infty}^x \frac{\exp(-\frac{t^2}{2})}{\sqrt{2\pi}} dt \right| \leq \frac{(p^2 + (1-p)^2)}{2\sqrt{np(1-p)}}$$

We shall prove it by simulation by the below algorithm.

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For x = -2, -1.9, -1.8, ..., 0, ..., 1.9, 2
  using inbuilt pnorm find z[x]:- pnorm(x)
Set p
For m = 1, 50, 100, 150, ..., 1000
  For x = -2, -1.9, -1.8, ..., 0, ..., 1.9, 2
    1) Generate B: 1000 Samples of Binomial (m,p) using inbuilt rbinom function
    Compute SB: (B-m*p)/((m*p*(1-p))^(0.5))
    2) Compute y[x] : the proportion of samples in SB less than equal to x
    3) Repeat steps 1) and 2) 100 times and compute average -- my[x] over each trial.
    4) Calculate diff[m]= max(abs(my[x]- z[x]))

For m = 1, 50, 100, 150, ..., 1000
  Calculate error(m)= [p^2+(1-p)^2]/[2*(m*p*(1-p))^0.5]

Plot diff and error.

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See if result is verified by picture. Can you do anything additional to verify the Theorem ?