

Quiz 9

1) Population size = N

Marked members = 50

Sample size = 20

marked members in the sample = X

$X = k$ is the event of getting k marked members from the 50 marked members of the population. There are ${}^{50}C_k$ many ways to do so since each member can be counted only once this is sampling without replacement. In total we need 20 members, we have already chosen k so $20 - k$ are remaining which need to be the unmarked ones. Hence there are ${}^{N-50}C_{20-k}$ ways of choosing $20 - k$

unmarked members from the $N - 50$ unmarked members of the population.

∴ Total number of ways of getting a sample of size 20 with k members marked = ${}^{50}C_k \times {}^{N-50}C_{20-k}$

If we just had to choose 20 members then the total number of ways to do so = ${}^N C_{20}$.

$$\therefore P(X=k) = \frac{{}^{50}C_k \times {}^{N-50}C_{20-k}}{{}^N C_{20}}$$

$P(X=k)$ is of the form

$$\frac{{}^m C_k \cdot {}^{N-m} C_{n-k}}{{}^N C_n}$$

where $N=N, m=50, n=20$

This is the probability function of Hypergeometric distribution.

$$\therefore X \sim \text{Hypergeometric}(N, 50, 20)$$