1.a) The given plot has 10 points corresponding to the 10 indices from 1 to 10. Each of the indices is mapped to a ~~point~~ value between 0 and 1. Since the points are scattered and do not have an observable pattern, the function must be producing some random values.

R-code :

```
> x = c(1:10)
> values = runif(10)
> plot (x, values ), xlab = 'Index', ylab= "runif(10)",
                                        ylim = c(0.2,0.6))
```

Hence, we get the given plot with, x axis taking values from x, y axis from values and both labelled as 'Index' and 'runif(10)' respectively.

(b) (i) $P(X=0)$

$X = 0$.

$\Rightarrow i = 0$

$\Rightarrow$ while loop was not executed,

$\Rightarrow$ sum ≥ 1

$\Rightarrow$ $u \geq 1$

$\Rightarrow -\frac{\log u}{L} \geq 1 \Rightarrow -\frac{\log u}{10} \geq 1$

$\Rightarrow u \leq e^{-10}$

u results in a random value between 0 and 1 and hence follows Uniform distribution.

$u \sim$ Uniform $(0, 1)$

$$\therefore P(u \leq e^{-10}) = \int_0^{e^{-10}} \frac{1}{1-0} \, dx$$

$$= e^{-10}$$

$$\approx 4.54 \times 10^{-5}$$

$$\Rightarrow P(X=0) \approx 4.54 \times 10^{-5}$$

$$P(X \geq 1) = 1 - P(X<1)$$

$$= 1 - P(X=0)$$

since X can take values in $\{0, 1, 2, \ldots\}$.

$$\therefore P(X \geq 1) = 1 - e^{-10}$$

$$\approx 0.99995$$

(ii) $$P\left(\sum_{i=1}^{n} T_i \leq a\right) = \int_0^a \frac{\lambda^n}{(n-1)!} e^{-\lambda z} \cdot z^{n-1} \, dz \quad \text{(1)}$$

$\forall a > 0$.

$$P(X=n) = P(i=n)$$

$$= P\left(\frac{(-\log u_0 - \log u_1 \ldots - \log u_n)}{L} \geq 1\right)$$

where u_j is the value of runif(1, min = 0, max = 1) at the j^{th} iteration, $j \in \{0, 1, 2, \ldots n\}$

$\therefore P(X = n) = P\left(-\frac{\log(u_0 u_1 u_2 \ldots u_n)}{10} \geq 1\right)$

$= P\left(-\log(u_0 u_1 \ldots u_n) \geq 10\right)$

$= P\left(u_0 u_1 \ldots u_n \leq e^{-10}\right)$

Now $\forall j \in \{0, 1, \ldots n\}$, $u_j \sim$ uniform $(0, 1)$.

Note that, let $y_j = -\log u_j$

$\cancel{\Rightarrow u_j \sim \text{Exp}(-10)}$

$P(u_j \leq a_1) = \frac{a_1 - 0}{1 - 0} \quad \forall \; 0 < a_1 < 1$.

Now, $P(-\log u_j \leq a_1) = P(u_j \geq e^{-a_1})$

$= 1 - P(u_j < e^{-a_1})$

$= 1 - e^{-a_1}$

$\therefore y_j \sim \text{Exp}(1) \; \forall j$ and u_j are calculate randomly, so y_j's are independent $\Rightarrow$ y_j are i.i.d $\text{Exp}(1)$.

we need, $P(y_1 + y_2 + \ldots y_n \leq 10)$.

$= P\left(\sum_{i=1}^{n} y_i \leq 10\right) = \int_0^{10} \frac{1^n}{(n-1)!} e^{-z} \cdot z^{n-1} \, dz$

(By equation (1))

$$= \int_0^{10} \frac{1}{(n-1)!} e^{-z} \cdot z^{n-1} \, dz$$

$$P(X = n) = P(Y_1 + Y_2 + \cdots Y_n \geq 10)$$

$$= 1 - P(Y_1 + \cdots Y_n < 10)$$

$$= 1 - \int_0^{10} \frac{1}{(n-1)!} e^{-z} \cdot z^{n-1} \, dz.$$