Worksheet 2

1) MYMEAN = function (x)
{
sum(x) / length (x)
}

x = 1:100
y = x[x < 50 & x > 2]
MYMEAN (y)

output = 26

MYMEAN(y) computes the mean of the elements in the vector y.
x is a vector of natural numbers from 1 to 100
y = {3, 4, . . . 49}

mean of y = $\frac{3+4+\ldots+49}{47} = \frac{47}{2} \times \frac{(3+49)}{47} = 26$

2.a) $Y = X_1 + X_2$

range of X_1 = range of X_2 = {1, 2, 3, 4, 5, 6}

Range of Y = {2, 3, 4, . . . 12}

(b) $f_Y(2) = P(Y = 2) = P(X_1 + X_2 = 2)$
$= P(X_1 = 1, X_2 = 1) = P(X_1 = 1) P(X_2 = 1)$
$= \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

$f_Y(3) = P(Y = 3) = P(X_1 + X_2 = 3)$
$= P(X_1 = 1, X_2 = 2) + P(X_1 = 2, X_2 = 1)$
$= \frac{1}{36} + \frac{1}{36} = \frac{2}{36}$

$f_Y(4) = P(Y = 4) = P(X_1 + X_2 = 4)$
$= P(X_1 = 1, X_2 = 3) + P(X_1 = 2, X_2 = 2) + P(X_1 = 3, X_2 = 1)$
$= \frac{1}{36} + \frac{1}{36} + \frac{1}{36} = \frac{3}{36}$

$f_Y(5) = P(Y=5) = P(X_1 + X_2 = 5)$

$= P(X_1=1, X_2=4) + P(X_1=2, X_2=3) + P(X_1=3, X_2=2)$

$+ P(X_1=4, X_2=1)$

$= \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36}$

$= \frac{4}{36}$

$f_Y(6) = P(Y=6) = P(X_1 + X_2 = 6)$

$= P(X_1=1, X_2=5) + P(X_1=2, X_2=4) + P(X_1=3, X_2=3)$

$+ P(X_1=4, X_2=2) + P(X_1=5, X_2=1)$

$= \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36}$

$= \frac{5}{36}$

$f_Y(7) = P(Y=7) = P(X_1 + X_2 = 7)$

$= P(X_1=1, X_2=6) + P(X_1=2, X_2=5) + P(X_1=3, X_2=4)$

$+ P(X_1=4, X_2=3) + P(X_1=5, X_2=2) + P(X_1=6, X_2=1)$

$= \frac{1}{36} \times 6 = \frac{6}{36}$

$f_Y(8) = P(Y=8) = P(X_1 + X_2 = 8)$

$= P(X_1=2, X_2=6) + P(X_1=3, X_2=5) + P(X_1=4, X_2=4)$

$+ P(X_1=5, X_2=3) + P(X_1=6, X_2=2)$

$= \frac{1}{36} \times 5 = \frac{5}{36}$

$f_Y(9) = P(Y=9) = P(X_1 + X_2 = 9)$

$= P(X_1=3, X_2=6) + P(X_1=4, X_2=5) + P(X_1=5, X_2=4)$

$+ P(X_1=6, X_2=3)$

$= \frac{1}{36} \times 4 = \frac{4}{36}$

$f_Y(10) = P(Y=10) = P(X_1 + X_2 = 10)$

$= P(X_1=4, X_2=6) + P(X_1=5, X_2=5) + P(X_1=6, X_2=4)$

$= \frac{1}{36} + \frac{1}{36} + \frac{1}{36} = \frac{3}{36}$

$f_Y(11) = P(Y=11) = P(X_1 + X_2 = 11)$

$= P(X_1=5, X_2=6) + P(X_1=6, X_2=5)$

$= \frac{1}{36} + \frac{1}{36} = \frac{2}{36}$

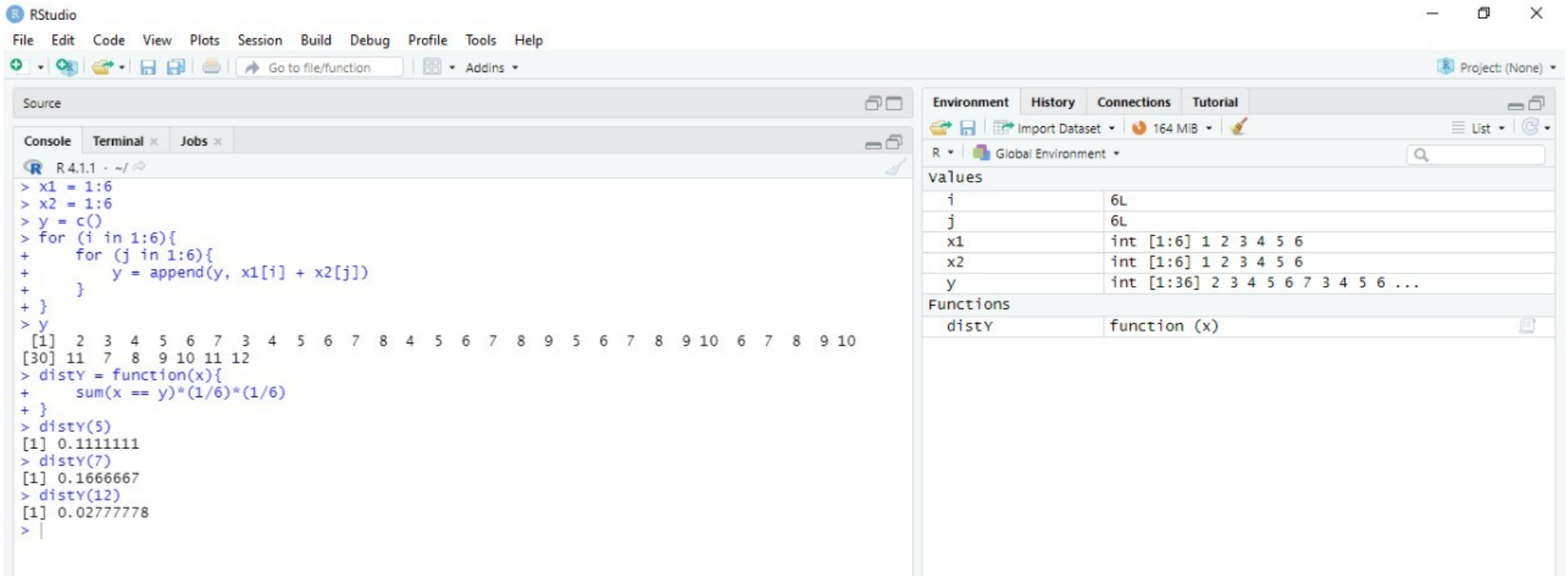
$f_Y(12) = P(Y=12) = P(X_1 + X_2 = 12)$

$= P(X_1=6, X_2=6) = \frac{1}{36}$

Distribution of Y,

y	2	3	4	5	6	7	8	9	10	11	12
$f_Y(y)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

(c)
```
x1 = 1:6
x2 = 1:6
y = c()
for (i in 1:6) {
    for (j in 1:6) {
        y = append(y, x1[i] + x2[j])
    }
}
dist Y = function(x) {
    sum(x==y) / 36
}
```


RStudio
File Edit Code View Plots Session Build Debug Profile Tools Help
Go to file/function
Addins
Project: (None)
Source
Console
Terminal
Jobs
R 4.1.1 · ~/
> x1 = 1:6
> x2 = 1:6
> y = c()
> for (i in 1:6){
+ for (j in 1:6){
+ y = append(y, x1[i] + x2[j])
+ }
+ }
> y
[1] 2 3 4 5 6 7 3 4 5 6 7 8 4 5 6 7 8 9 5 6 7 8 9 10 6 7 8 9 10
[30] 11 7 8 9 10 11 12
> distY = function(x){
+ sum(x == y)*(1/6)*(1/6)
+ }
> distY(5)
[1] 0.1111111
> distY(7)
[1] 0.1666667
> distY(12)
[1] 0.02777778
>
Environment
History
Connections
Tutorial
Import Dataset
164 MiB
List
R
Global Environment
Values
i 6L
j 6L
x1 int [1:6] 1 2 3 4 5 6
x2 int [1:6] 1 2 3 4 5 6
y int [1:36] 2 3 4 5 6 7 3 4 5 6 ...
Functions
distY function (x)

3.a) range of $X = \{1, 2, 3, 4, 5, 6\}$

If $X = n$ then we shall toss the coin n times.

So, we can obtain at most n many heads.

Range$(Y) = \{0, 1, 2, \ldots n\}$ $\quad 1 \le n \le 6$ if $X = n$.

3.(b) $f_Y(y) = P(Y = y)$ which depends on X.

$\therefore f_Y(0) = P(Y=0) = P(Y=0 \mid X=1) \cdot P(X=1) + P(Y=0 \mid X=2) \cdot P(X=2)$

$+ P(Y=0 \mid X=3) P(X=3) + \ldots P(Y=0 \mid X=6) \cdot P(X=6)$

Note that,

$$P(Y=k \mid X=n) = {}^nC_k \, p^k (1-p)^{n-k}, \quad n \ge k$$

$$P(X=n) = \frac{1}{6}$$

$$f_Y(0) = P(Y=0) = \sum_{n=1}^{6} P(Y=0 \mid X=n) \cdot P(X=n)$$

$$= \sum_{n=1}^{6} {}^nC_0 \, p^0 (1-p)^n \cdot \frac{1}{6}$$

$$= \frac{1}{6} \sum_{n=1}^{6} (1-p)^n = \frac{1}{6} \cdot \frac{(1-p)\left[1 - (1-p)^6\right]}{(1-(1-p))}$$

$$= \frac{(1-p)}{6p}\left[1 - (1-p)^6\right]$$

$$f_Y(1) = P(Y=1) = P(Y=1 \mid X=1) P(X=1) + \ldots P(Y=1 \mid X=6) P(X=6)$$

$$= \sum_{n=1}^{6} {}^nC_1 \, p^1 (1-p)^{n-1} \cdot \frac{1}{6}$$

$$= \frac{p}{6} \sum_{n=1}^{6} n (1-p)^{n-1}$$

$f_4(2) = P(4=2) = P(4=2 \mid X=2)P(X=2) + \cdots P(4=2 \mid X=6)P(X=6)$

$= \sum_{n=2}^{6} {}^nC_2\, p^2 (1-p)^{n-2} \cdot \frac{1}{6}$

$= \frac{p^2}{6} \sum_{n=2}^{6} \frac{n(n-1)}{2!} (1-p)^{n-2}$

$f_4(3) = P(4=3) = P(4=3 \mid X=3)P(X=3) + \cdots P(4=3 \mid X=6)P(X=6)$

$= \sum_{n=3}^{6} {}^nC_3\, p^3 (1-p)^{n-3} \frac{1}{6}$

$= \frac{p^3}{6} \sum \frac{n(n-1)(n-2)}{3!} (1-p)^{n-3}$

$f_4(4) = P(4=4) = P(4=4 \mid X=4)P(X=4) + \cdots P(4=4 \mid X=6)P(X=6)$

$= \sum_{n=4}^{6} {}^nC_4\, p^4 (1-p)^{n-4} \frac{1}{6}$

$= \frac{p^4}{6} \sum_{n=4}^{6} {}^nC_4 (1-p)^{n-4}$

$f_4(5) = P(4=5) = P(4=5 \mid X=5)\, P(X=5) + P(4=5 \mid X=6)P(X=6)$

$= \sum_{n=5}^{6} {}^nC_5\, p^5 (1-p)^{n-5} \frac{1}{6}$

$= \frac{p^5}{6} \sum_{n=5}^{6} {}^nC_5 (1-p)^{n-5}$

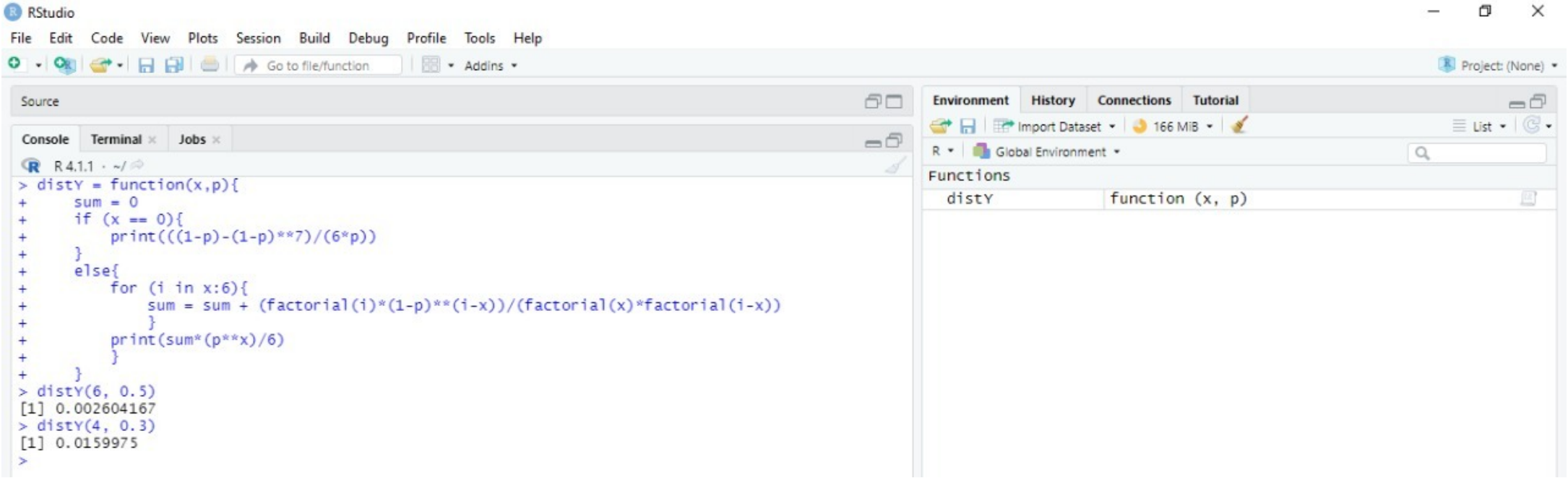
$f_4(6) = P(4=6) = P(4=6 \mid X=6)P(X=6)$

$= {}^6C_6\, p^6 (1-p)^0 \cdot \frac{1}{6} = \frac{p^6}{6}$

In general, $f_4(k) = \frac{p^k}{6} \sum_{n=k}^{6} {}^nC_k (1-p)^{n-k}, \quad 0 \le k \le 6$

3.c)

```
dist Y = function(n, p){
    sum = 0
    if (n == 0){
        print( (1-p) - (1-p)**7) / (6*p))
    }
    else {
        for (i in n:6){
            sum = sum +(factorial(i) * (1-p)**(i-n) )
                        /factorial(n) * factorial(i-n))
        print ( sum * (p**n)/6 )
        }
    }
}
```


RStudio
File Edit Code View Plots Session Build Debug Profile Tools Help
Go to file/function
Addins
Project: (None)
Source
Console Terminal Jobs
R 4.1.1 · ~/
> distY = function(x,p){
+ sum = 0
+ if (x == 0){
+ print(((1-p)-(1-p)**7)/(6*p))
+ }
+ else{
+ for (i in x:6){
+ sum = sum + (factorial(i)*(1-p)**(i-x))/(factorial(x)*factorial(i-x))
+ }
+ print(sum*(p**x)/6)
+ }
+ }
> distY(6, 0.5)
[1] 0.002604167
> distY(4, 0.3)
[1] 0.0159975
>
Environment History Connections Tutorial
Import Dataset
166 MiB
List
R
Global Environment
Functions
distY function (x, p)