Quiz 7

1) $N \to$ no. of earthquakes in a year

$M \to$ no. of earthquakes with magnitude at least 5.

$N \sim Poisson(\lambda)$

$P(M=1 \mid N=1) = p \Rightarrow P(M=0 \mid N=1) = 1-p$

$\therefore M \mid N=1 \sim Bernoulli(p)$

$\Rightarrow$ Now suppose $N=n$ & $M=k$, that means out of n many earthquakes, k have magnitude at least 5.

$\therefore P(M=k \mid N=n) = {}^{n}C_{k}\, p^{k} (1-p)^{n-k}$

$\Rightarrow M \mid N=n \sim Binomial(n, p)$

$P(N=n) = e^{-\lambda} \dfrac{\lambda^n}{n!}$

$P(M=k, N=n) = P(M=k \mid N=n) P(N=n)$

$= {}^{n}C_{k}\, p^{k} (1-p)^{n-k} e^{-\lambda} \cdot \dfrac{\lambda^n}{n!}$

$= \dfrac{\cancel{n!}}{k!(n-k)!} p^{k} (1-p)^{n-k} e^{-\lambda} \dfrac{\lambda^n}{\cancel{n!}}$

$= \dfrac{e^{-\lambda} \lambda^n}{k!(n-k)!} p^{k} (1-p)^{n-k}$

Now, $P(M = m)$:= probability of occurrence of m many earthquakes with magnitude at least 5.

$\Rightarrow$ at least m earthquakes have occured

$\Rightarrow$ $n \geq m$.

$$P(M=m) = P\left(\bigcup_{n=m}^{\infty} (M=m, N=n) \right)$$

$$= \sum_{n=m}^{\infty} P(M=m, N=n)$$

$$= \sum_{n=m}^{\infty} \frac{e^{-\lambda} \lambda^{n}}{m!\,(n-m)!}\, p^{m} (1-p)^{n-m}$$

$$= \frac{p^{m} e^{-\lambda}}{m!} \sum_{n=m}^{\infty} \frac{\lambda^{n-m} \cdot \lambda^{m}}{(n-m)!} (1-p)^{n-m}$$

$$= \frac{(\lambda p)^{m} e^{-\lambda}}{m!} \sum_{n=m}^{\infty} \frac{\lambda^{n-m}}{(n-m)!} (1-p)^{n-m}$$

let $n-m = k$, k ranges from 0 to ∞.

$$\therefore P(M=m) = \frac{(\lambda p)^{m} e^{-\lambda}}{m!} \sum_{k=0}^{\infty} \frac{\lambda^{k}}{k!} (1-p)^{k}$$

$$= \frac{(\lambda p)^{m} e^{-\lambda}}{m!}\, e^{\lambda(1-p)} \qquad \left[\because e^{x} = \sum \frac{x^{k}}{k!}\right]$$

$$= (\lambda p)^m \frac{e^{\cancel{-\lambda}\cancel{+\lambda}-\lambda p}}{m!}$$

$$= (\lambda p)^m \frac{e^{-\lambda p}}{m!}$$

$$= e^{-\lambda p} \frac{(\lambda p)^m}{m!}$$

$$\therefore M \sim \text{Poisson}(\lambda p)$$