Worksheet 7

1.a) Range $(X) = \{-1, 0, 1\}$ s.t. $P(|X-\mu| \geq 2\sigma) = \frac{1}{4}$

$$P(X=x) = \begin{cases} 1/8 & \text{if } x=-1 \\ 3/4 & \text{if } x=0 \\ 1/8 & \text{if } x=1 \end{cases}$$

$$E[X] = \frac{1}{8} \times (-1) + 0 \cdot \frac{3}{4} + 1 \cdot \frac{1}{8} = 0 = \mu$$

$$Var[X] = \cancel{\text{[illegible]}} \ ((-1-\mu))^2 \cdot \frac{1}{8} + (0-\mu)^2 \cdot \frac{3}{4} + (1-\mu)^2 \cdot \frac{1}{8}$$

$$= \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$$

$$\Rightarrow \sigma^2 = \frac{1}{4} \quad , \quad \sigma = \frac{1}{2}$$

$$P(|X-\mu| \geq 2\sigma) = P(|X-0| \geq 2 \cdot \tfrac{1}{2})$$

$$= P(|X| \geq 1) = P(X=1) + P(X=-1)$$

$$= \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$$

1.b) Let Range $(Y) = \{-2, \cancel{1}, 2\}$

$$P(Y=y) = \begin{cases} 1/2 & \text{if } y=-2 \\ 0 & \text{if } y=1 \\ 1/2 & \text{if } y=2 \end{cases}$$

$$\mu = E[Y] = \frac{1}{2}\cdot(-2) + 0\cdot\frac{1}{2} + 2\cdot\frac{1}{2} = 0$$

$$\sigma_Y^2 = \text{Var}[Y] = (-2-\mu)^2\,\frac{1}{2} + (\cancel{1}-\mu)^2\cdot\cancel{0} + (2-\mu)^2\cdot\frac{1}{2}$$

$$= 4\cdot\frac{1}{2} + 4\cdot\frac{1}{2} = 4.$$

$\therefore \sigma_Y = 2.$

$$P(|Y-\mu| > 2\sigma_Y) = P(|Y| > 4)$$

$$\cancel{\text{...}} = 0 < 1/4$$

$$P(|Y-\mu| > 2\sigma) = P(|Y| > 2\cdot 1/2)$$

$$= P(|Y| > 1) = P(Y=-2) + P(Y=2)$$

$$= \frac{1}{2} + \frac{1}{2} = 1 > \frac{1}{4}.$$

This does not violate Tschebychev Inequality since, it holds for $P(|Y-\mu| > k\sigma_Y) \le 1/k^2$ which is true but when we consider $\cancel{\sigma}$, the standard deviation of X, this inequality doesn't hold.