Due date: November 26th, 2021
Problems Due: 2,3

1. For each of the distributions: Beta(10,2) and Beta(10,10)
   (a) Generate 100 trials of $5, 50, 500$ samples respectively.
   (b) Using the data decide if the conclusion of the Central Limit Theorem applies in each of the three cases, $5, 50, 500$.

2. Consider the Poisson(1) distribution.
   (a) Generate 100 trials of 500 samples respectively.
   (b) Find the 95%-confidence interval for the mean in each trial.
   (c) Compute the number of trials in which the true mean lies in the interval.

3. The dataset `BangaloreRain.csv` in the course website at https://www.isibang.ac.in/ athreya/Teaching/PaSwR/Bngalor
   (a) Decide if any month's 100 year rainfail is Normally distributed.
   (b) Calculate the yearly total rain fall for each of the 100 years.
   (c) Plot the histogram and Decide if the distribution is Normal.
   (d) Find a 95% confidence interval for the expected annual rainfall in Bangalore.

4. Two types of coin are produced at a factory: a fair coin and a biased one that comes up heads 55% of the time. We have one of these coins but do not know whether it is a fair or biased coin. In order to ascertain which type of coin we have, we shall perform the following statistical test. We shall toss the coin 1000 times. If the coin comes up heads 525 or more times we shall conclude that it is a biased coin. Otherwise, we shall conclude that it is fair. If the coin is actually fair, what is the probability that we shall reach a false conclusion? What would it be if the coin were biased?

5. The length of time (in appropriate units) that a certain type of component functions before failing is a random variable with probability density function

$$f(x) = \begin{cases} 2x & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

   Once the component fails it is immediately replaced with another one of the same type. Using the central limit theorem approximation, can you find, how many components would one need to have on hand to be approximately 90% certain that the stock would last at least 35 units of time ?