1. Suppose X follows Bernoulli(p) distribution. Let $p = 1/3$
   (a) Simulate for $n = 100$ $X_1, X_2, X_3, \ldots X_n$ i.i.d X
   (b) Demonstrate the Law of Large numbers by plotting the sample mean $\bar{X}_n$ as a function of n.
   (c) Using `replicate` command plot 15 independent trials of the above.
   (d) Do the same when $p = 0.001$, $n = 100$, $p = 0.5, n = 100$, $p = 0.99$ on different plots

2. We wish to compute
$$\int_a^b f(x)dx$$
   using the Law of Large numbers.
   (a) Generate samples of $X_1, X_2, \ldots X_n$ i.i.d. Uniform (a, b). Justify
$$(b-a)\sum_{i=1}^{n} \frac{f(X_i)}{n} \approx \int_a^b f(x)dx$$
   (b) Write an `R`-code to estimate the $\int_0^7 \frac{16+\sin(x)}{x^2+4}dx$ using the procedure described in the previous part with $n = 400$.
   (c) Repeat the estimate 100 times and find the mean of these 100 simulations.
   (d) Use the `integrate` command in `R` to evaluate the integral. Compare the two answers.

3. Simulate 500 samples from each of the below distribution, X, using their respective distribution function F_X and the inbuilt `runif`.
   (a) $X \sim$ Binomial $(10, \frac{1}{3})$
   (b) $X \sim$ p.d.f f given by
$$f(x) = \begin{cases} x & 0 \le x \le \sqrt{2} \\ 0 & \text{otherwise.} \end{cases}$$

4. Suppose p is the unknown probability of an event A, and we estimate p by the sample proportion $\hat{p}$ based on an i.i.d. sample of size n.
   (a) Design and implement the following simulation study to verify this behaviour. For $p = 0.01$, 0.1, 0.25, 0.5, 0.75, 0.9, and 0.99,
      (i) Simulate 1000 values of $\hat{p}$ with $n = 500$.
      (ii) Simulate 1000 values of $\hat{p}$ with n chosen according to the formula derived above.

      In each case, you can think of the 1000 values as i.i.d. samples from the distribution of $\hat{p}$, and use the sample standard deviation as an estimate of $SD[\hat{p}]$. Plot the estimated values of $SD(\hat{p})$ against p for both choices of n.