CS754: Advanced Image Processing
Assignment 2

Karan Taneja, 15D070022
Sucheta Ravikanti, 160040100

Question 4

(a)

$$||\Phi(x^* - x)||_2 \ = \ ||\Phi x^* - y + y - \Phi x||_2 \ = \ ||(\Phi x^* - y) + (y - \Phi x)||_2$$

Now by the triangle inequality, we get

$$||(\Phi x^* - y) + (y - \Phi x)||_2 \ \leq \ ||\Phi x^* - y||_2 \ + \ ||y - \Phi x||_2$$

The next part follows from the feasibility condition of x and the assumption of noise level respectively

$$||\Phi x^* - y||_2 \ \leq \ \epsilon$$
$$||y - \Phi x||_2 \ \leq \ \epsilon$$

Therefore, using the above two,

$$||\Phi x^* - y||_2 \ + \ ||y - \Phi x||_2 \ \leq \ 2\epsilon$$

(b) Let us first prove the following result

$$||x||_2 \ \leq \ \sqrt{s}\ ||x||_\infty$$

Here, we are assuming that all the vectors are s-sparse, therefore $x_1, x_2, ..., x_s$ are the (maximal number of) elements that are non-zero

$$||x||_2^2 \ = \ x_1^2 \ + \ x_2^2 \ + ... \ x_s^2 \ \leq \ s\ (\max_{1\leq i \leq s} |x_i|)^2 \ = \ s\ ||x||_\infty^2$$

Taking a square root will give us the required result. Also, note that all the inequalities of norms can be used here by replacing n by s for s-sparse vectors. The following is a direct result of the inequality proved above

$$||h_{T_j}||_2 \ \leq \ \sqrt{s}\ ||h_{T_j}||_\infty$$

Note that we decomposed h into sum of s-sparse h_{Tj} vectors. By the construction, it is clear that even the minimum element of $h_{T_{j-1}}$ will be greater than the maximum element of h_{T_j}. Therefore, consider the following inequality

$$s(\max_{1\leq n \leq 1} |h_{T_{j_i}}|) \ \leq \ |h_{T_{j-1_1}}| \ + \ |h_{T_{j-1_2}}| \ + \ ... \ + \ |h_{T_{j-1_n}}|$$

$$s\ ||h_{T_j}||_\infty \ \leq \ ||h_{T_{j-1}}||_1$$

$$\sqrt{s}\ ||h_{T_j}||_\infty \ \leq \ s^{-1/2}\ ||h_{T_{j-1}}||_1$$

(c) From the above results, we get

$$||h_{T_j}||_2 \ \leq \ s^{-1/2}\ ||h_{T_{j-1}}||_1$$

$$||h_{T_2}||_2 \ \leq \ s^{-1/2}\ ||h_{T_1}||_1$$
$$||h_{T_3}||_2 \ \leq \ s^{-1/2}\ ||h_{T_2}||_1$$
$$||h_{T_4}||_2 \ \leq \ s^{-1/2}\ ||h_{T_3}||_1$$

and so on..
Adding them up, we get

$$\sum_{j\geq 2}||h_{T_j}||_2 \ \leq \ s^{-1/2}(||h_{T_1}||_1 \ + \ ||h_{T_2}||_1 \ + \ ||h_{T_3}||_1 \ + \ ....)$$

From the definition of $||h_{T_0^c}||_1$, we get

$$s^{-1/2}(||h_{T_1}||_1 \ + \ ||h_{T_2}||_1 \ + \ ||h_{T_3}||_1 \ + \ ....) \ = \ s^{-1/2}||h_{T_0^c}||_1$$

(d) The following is a direct result of the triangle inequality for L-2 norm.

$$||\sum_{j\geq 2}\ h_{T_j}||_2 \ \leq \ \sum_{j\geq 2}||h_{T_j}||_2$$

The second part of the inequality is merely the result we obtained from 4(c)

$$\sum_{j\geq 2}||h_{T_j}||_2 \ \leq \ s^{-1/2}\ ||h_{T_0^c}||_1$$

(e) This inequality can be proven using the reverse triangle inequality. First, note the following

$$\sum_{i\in T_0}|x_i + h_i| \ = \ ||x_{T_0} + h_{T_0}||_1$$

$$\sum_{i\in T_0^c}|x_i + h_i| \ = \ ||x_{T_0^c} + h_{T_0^c}||_1$$

Where x_T denotes a vector that is equal to x on the index set T and zero elsewhere. The reverse triangle inequality states the following

$$||x_{T_0} + h_{T_0}||_1 \ \geq \ ||x_{T_0}||_1 - ||h_{T_0}||_1$$

$$||x_{T_0^c} + h_{T_0^c}||_1 \ \geq \ ||h_{T_0^c}||_1 - ||x_{T_0^c}||_1$$

Note that the reverse triangle inequality states that the lhs is greater than the absolute value of the difference of norms. Therefore, lhs is automatically greater than both the positive and negative difference. Adding the above inequalities, we get

$$||x_{T_0} + h_{T_0}||_1 \ + \ ||x_{T_0^c} + h_{T_0^c}||_1 \ \geq \ ||x_{T_0}||_1 - ||h_{T_0}||_1 \ + \ ||h_{T_0^c}||_1 - ||x_{T_0^c}||_1$$

$$\sum_{i \in T_0} |x_i + h_i| \ + \ \sum_{i \in T_0^c} |x_i + h_i| \ \geq \ ||x_{T_0}||_1 - ||h_{T_0}||_1 \ + \ ||h_{T_0^c}||_1 - ||x_{T_0^c}||_1$$

(f) From the previous ie, 4(e), we get the following complete result

$$||x||_1 \ \geq \ ||x_{T_0}||_1 - ||h_{T_0}||_1 \ + \ ||h_{T_0^c}||_1 - ||x_{T_0^c}||_1$$

$$||x||_1 \ - \ ||x_{T_0}||_1 \ + \ ||x_{T_0^c}||_1 \ + \ ||h_{T_0}||_1 \ \geq \ ||h_{T_0^c}||_1$$

From reverse triangle inequality,

$$||x_{T_0^c}||_1 \ = \ ||x - x_{T_0}||_1 \ \geq \ ||x||_1 \ - \ ||x_{T_0}||_1$$

Using the above result, we get

$$2 \ ||x_{T_0^c}||_1 \ + \ ||h_{T_0}||_1 \ \geq \ ||x||_1 \ - \ ||x_{T_0}||_1 \ + \ ||x_{T_0^c}||_1 \ + \ ||h_{T_0}||_1 \ \geq \ ||h_{T_0^c}||_1$$

Therefore, we get

$$||h_{T_0^c}||_1 \ \leq \ ||h_{T_0}||_1 \ + \ 2 \ ||x_{T_0^c}||_1$$

(g) Applying the previous results (given below again)

$$||h_{(T_0 U T_1)^c}||_2 \ \leq \ s^{-1/2} \ ||h_{T_0^c}||_1$$

$$||h_{T_0^c}||_1 \ \leq \ ||h_{T_0}||_1 \ + \ 2 \ ||x_{T_0^c}||_1$$

But we know that, (from norm inequalities)

$$||h_{T_0}||_1 \ \leq \ s^{1/2} \ ||h_{T_0}||_2$$

Therefore,

$$||h_{T_0^c}||_1 \ \leq \ ||h_{T_0}||_1 \ + \ 2 \ ||x_{T_0^c}||_1 \ \leq \ s^{1/2} \ ||h_{T_0}||_2 \ + \ 2 \ ||x_{T_0^c}||_1$$

$$||h_{(T_0 U T_1)^c}||_2 \ \leq \ s^{-1/2} \ ||h_{T_0^c}||_1 \ \leq \ ||h_{T_0}||_2 \ + \ 2 \ s^{-1/2} \ ||x_{T_0^c}||_1$$

Putting

$$s^{-1/2} \ ||x_{T_0^c}||_1 \ = \ e_0$$

$$||h_{(T_0 U T_1)^c}||_2 \ \leq \ ||h_{T_0}||_2 \ + \ 2e_0$$

(h) From (a), we get

$$||\Phi(x^* - x)||_2 \ = \ ||\Phi h||_2 \ \leq \ 2\epsilon$$

From the RIP condition, we get the following where δ_{2s} is the RIP constant for Φ

$$||\Phi h_{T_0 U T_1}||_2^2 \ \leq \ (1+\delta_{2s})||h_{T_0 U T_1}||_2^2 \ \implies \ ||\Phi h_{T_0 U T_1}||_2 \ \leq \ \sqrt{(1+\delta_{2s})}||h_{T_0 U T_1}||_2$$

Since all the LHS and RHS are positive quantities here on both the expressions, we can multiply the inequalities

$$||\Phi h_{T_0UT_1}||_2 \; ||\Phi h||_2 \;\leq\; 2\epsilon \,\sqrt{(1+\delta_{2s})}\; ||h_{T_0UT_1}||_2$$

(i) **Lemma 2.1** in the paper states that for all disjoint supports of x and x' with the sparsities being s and s',

$$|\langle \Phi x, \Phi x' \rangle| \;\leq\; \delta_{s+s'} ||x||_2 \; ||x'||_2$$

Using the above lemma, we get

$$|\langle \Phi h_{T_0}, \Phi h_{T_j} \rangle| \;\leq\; \delta_{2s} ||h_{T_0}||_2 \; ||h_{T_j}||_2$$

(j) For simplicity, put

$$h_{T_0} \;=\; x,\; h_{T_1} \;=\; y,\; h_{T_0UT_1} \;=\; z$$

Observe that $z = x + y$. Due to the disjoint nature of T_0 and T_1, we get

$$||z||_2 \;=\; (x_1^2 \;+\; x_2^2 \;+\; ...\; x_n^2 \;+\; y_1^2 \;+\; y_2^2 \;+\; ... \;+\; y_n^2)$$

$$\left(\sqrt{\sum_{1\leq i\leq n} x_i^2}\right)^2 + \left(\sqrt{\sum_{1\leq i\leq n} y_i^2}\right)^2 - 2\left(\sqrt{\sum_{1\leq i\leq n} x_i^2}\right)\left(\sqrt{\sum_{1\leq i\leq n} y_i^2}\right) \;\geq\; 0$$

Adding a term on both sides,

$$2\left(\left(\sqrt{\sum_{1\leq i\leq n} x_i^2}\right)^2 + \left(\sqrt{\sum_{1\leq i\leq n} y_i^2}\right)^2\right) - 2\left(\sqrt{\sum_{1\leq i\leq n} x_i^2}\right)\left(\sqrt{\sum_{1\leq i\leq n} y_i^2}\right)$$

$$\geq \left(\sqrt{\sum_{1\leq i\leq n} x_i^2}\right)^2 + \left(\sqrt{\sum_{1\leq i\leq n} y_i^2}\right)^2$$

$$2\left(\left(\sqrt{\sum_{1\leq i\leq n} x_i^2}\right)^2 + \left(\sqrt{\sum_{1\leq i\leq n} y_i^2}\right)^2\right)$$

$$\geq \left(\sqrt{\sum_{1\leq i\leq n} x_i^2}\right)^2 + \left(\sqrt{\sum_{1\leq i\leq n} y_i^2}\right)^2 + 2\left(\sqrt{\sum_{1\leq i\leq n} x_i^2}\right)\left(\sqrt{\sum_{1\leq i\leq n} y_i^2}\right)$$

$$2\;||z||_2^2 \;\geq\; ||x||_2^2 \;+\; ||y||_2^2 \;+\; 2\;||x||_2\;||y||_2$$

$$2\;||z||_2^2 \;\geq\; (||x||_2 \;+\; ||y||_2)^2$$

$$||x||_2 \;+\; ||y||_2 \;\leq\; \sqrt{2}||z||_2$$

$$||h_{T_0}||_2 \ + \ ||h_{T_1}||_2 \ \leq \ \sqrt{2}||h_{T_0 \cup T_1}||_2$$

(k) The first inequality follows from RIP. Second one is illustrated below.

$$||\Phi h_{T_0 \cup T_1}||_2^2 \ = \ |\langle \Phi h_{T_0 \cup T_1}, \Phi h_{T_0 \cup T_1} \rangle| \ = \ |\langle \Phi h_{T_0 \cup T_1}, \Phi (h - h_{(T_0 \cup T_1)^c}) \rangle|$$

$$= \ |\langle \Phi h_{T_0 \cup T_1}, \Phi h \rangle| \ - \ |\langle \Phi h_{T_0 \cup T_1}, \Phi h_{(T_0 \cup T_1)^c} \rangle|$$

Examine the first term, using the result proved earlier, we get

$$|\langle \Phi h_{T_0 \cup T_1}, \Phi h \rangle| \ \leq \ ||\Phi h_{T_0 U T_1}||_2 \ ||\Phi h||_2 \ \leq \ 2\epsilon \ \sqrt{(1+\delta_{2s})} \ ||h_{T_0 U T_1}||_2$$

Examine the second term,

$$|\langle \Phi h_{T_0 \cup T_1}, \Phi h_{(T_0 \cup T_1)^c} \rangle| \ = \ |\langle \Phi (h_{T_0} + h_{T_1}), \Phi \sum_{j \geq 2} h_{T_j} \rangle|$$

$$|\langle \Phi h_{T_0}, \Phi \sum_{j \geq 2} h_{T_j} \rangle| \ + \ |\langle \Phi h_{T_1}, \Phi \sum_{j \geq 2} h_{T_j} \rangle|$$

From **Lemma 2.1**

$$\leq \delta_{2s} \sum_{j \geq 2} ||h_{T_0}||_2 ||h_{T_j}||_2 \ + \ \delta_{2s} \sum_{j \geq 2} ||h_{T_1}||_2 ||h_{T_j}||_2$$

$$= \ \delta_{2s} \sum_{j \geq 2} ||h_{T_j}||_2 \ (||h_{T_0}||_2 + ||h_{T_1}||_2)$$

From the result of (j),

$$\leq \ \delta_{2s} \sum_{j \geq 2} ||h_{T_j}||_2 \ \sqrt{2}||h_{T_0 \cup T_1}||_2$$

$$= \ \sqrt{2} \ \delta_{2s} ||h_{T_0 \cup T_1}||_2 \left(\sum_{j \geq 2} ||h_{T_j}||_2 \right)$$

Adding up the first and second terms, we get

$$||\Phi h_{T_0 \cup T_1}||_2^2 \ \leq \ ||h_{T_0 \cup T_1}||_2 \left(2\epsilon \ \sqrt{(1+\delta_{2s})} \ + \ \sqrt{2} \ \delta_{2s} \sum_{j \geq 2} ||h_{T_j}||_2 \right)$$

(l) From (c),

$$\sum_{j \geq 2} ||h_{T_j}||_2 \ \leq \ s^{-1/2} \ ||h_{T_0^c}||_1$$

And from (k),

$$(1-\delta_{2s})||h_{T_0 \cup T_1}||_2^2 \leq ||\Phi h_{T_0 \cup T_1}||_2^2 \ \leq \ ||h_{T_0 \cup T_1}||_2 \left(2\epsilon \ \sqrt{(1+\delta_{2s})} \ + \ \sqrt{2} \ \delta_{2s} \sum_{j \geq 2} ||h_{T_j}||_2 \right)$$

Therefore,

$$||h_{T_0 \cup T_1}||_2 \;\leq\; \left(2\epsilon\,\frac{\sqrt{1+\delta_{2s}}}{1-\delta_{2s}} \;+\; \sqrt{2}\,\frac{\delta_{2s}}{1-\delta_{2s}}\sum_{j\geq 2}||h_{T_j}||_2\right)$$

$$\left(2\epsilon\,\frac{\sqrt{1+\delta_{2s}}}{1-\delta_{2s}} \;+\; \sqrt{2}\,\frac{\delta_{2s}}{1-\delta_{2s}}\sum_{j\geq 2}||h_{T_j}||_2\right) \;\leq\; \left(2\epsilon\,\frac{\sqrt{1+\delta_{2s}}}{1-\delta_{2s}} \;+\; \sqrt{2}\,\frac{\delta_{2s}}{1-\delta_{2s}}s^{-1/2}||h_{T_0^c}||_1\right)$$

$$||h_{T_0 \cup T_1}||_2 \;\leq\; \left(2\epsilon\,\frac{\sqrt{1+\delta_{2s}}}{1-\delta_{2s}} \;+\; \sqrt{2}\,\frac{\delta_{2s}}{1-\delta_{2s}}s^{-1/2}||h_{T_0^c}||_1\right)$$

Putting

$$\alpha \;=\; 2\frac{\sqrt{1+\delta_{2s}}}{1-\delta_{2s}},\; \rho \;=\; \sqrt{2}\,\frac{\delta_{2s}}{1-\delta_{2s}}$$

$$||h_{T_0 \cup T_1}||_2 \;\leq\; \alpha\epsilon \;+\; \rho\; s^{-1/2}||h_{T_0^c}||_1$$

(m) From the both of these,

$$||h_{T_0 \cup T_1}||_2 \;\leq\; \alpha\epsilon \;+\; \rho\; s^{-1/2}||h_{T_0^c}||_1$$

$$||h_{T_0^c}||_1 \;\leq\; ||h_{T_0}||_1 \;+\; 2||x_{T_0^c}||_1$$

$$||h_{T_0 \cup T_1}||_2 \;\leq\; \alpha\epsilon \;+\; \rho\; s^{-1/2}(||h_{T_0}||_1 \;+\; 2||x_{T_0^c}||_1)$$

Using the property of L1 norm less than $\sqrt{s}$ L2 norm,

$$\alpha\epsilon \;+\; \rho\; s^{-1/2}(||h_{T_0}||_1 \;+\; 2||x_{T_0^c}||_1) \;\leq\; \alpha\epsilon \;+\; \rho\;(||h_{T_0}||_2 \;+\; 2s^{-1/2}||x_{T_0^c}||_1)$$

Since by the construction, the following can be said,

$$||h_{T_0}||_2 \;\leq\; ||h_{T_0 \cup T_1}||_2$$

And substituting the value of e_0, we get

$$||h_{T_0 \cup T_1}||_2 \;\leq\; \alpha\epsilon \;+\; \rho\;||h_{T_0 \cup T_1}||_2 \;+\; 2\rho e_0$$

(n) The first inequality follows from triangle inequality.

$$||h||_2 \;=\; ||h_{T_0 \cup T_1} + h_{T_0 \cup T_1{}^c}||_2 \;\leq\; ||h_{T_0 \cup T_1}||_2 + ||h_{T_0 \cup T_1{}^c}||_2$$

The second part follows from the fact that every higher h term in the decomposition has lower values than the previous one.

$$||h_{T_0 \cup T_1}||_2 \;+\; ||h_{T_0 \cup T_1{}^c}||_2 \;\leq\; 2||h_{T_0 \cup T_1}||_2$$

Using the result obtained above that is,

$$||h_{T_0 \cup T_1}||_2 \;\leq\; (1-\rho)^{-1}(\alpha\epsilon + 2\rho e_0)$$

$$2\ ||h_{T_0\cup T_1}||_2\ +\ 2e_0\ \leq\ 2\ (1-\rho)^{-1}(\alpha\epsilon+2\rho e_0)\ +\ 2\ e_0$$
$$=\ 2\ (1-\rho)^{-1}(\alpha\epsilon+2\rho e_0)\ +\ 2(1-\rho)^{-1}(1-\rho)\ e_0$$
$$=\ 2\ (1-\rho)^{-1}(\alpha\epsilon+2\rho e_0+(1-\rho)e_0)$$
$$||h_{T_0\cup T_1}||_2\ \leq\ 2\ (1-\rho)^{-1}(\alpha\epsilon+(1+\rho)e_0)$$

(o)

$$||h_l||_1\ =\ ||h_{T_0}||_1\ +\ ||h_{T_0^c}||_1$$

Let's bound each of the terms above. Also remember that ϵ is zero.

$$||h_{T_0}||_1\ \leq\ ||h_{T_0\cup T_1}||_1\ \leq\ s^{1/2}||h_{T_0\cup T_1}||_2\ \leq\ s^{1/2}2\ (1-\rho)^{-1}(1+\rho)s^{-1/2}||x_{T_0^c}||_1$$

$$||h_{T_0^c}||_1\ \leq\ 2(1-\rho)^{-1}||x_{T_0^c}||_1$$

Adding up, we get

$$||h_l||_1\ \leq\ 2(1+\rho)(1-\rho)^{-1}||x_{T_0^c}||_1$$

Question 1

(d) δ_s - Restricted Isometry Constant, μ - Mutual Coherence of A
A is a unit-normalized matrix ie, the L2 norm of all the columns is unity. Let us assume we are considering all s-sparse vectors. Let the index of the support be denoted by S. $S \in [N]$ and the cardinality of S is less than or equal to s. Consider that A_S denotes the matrix A with columns set belonging to S. $A_S^T A_S$ is positive definite and therefore the eigenvectors associated with the real eigen values will form an orthonormal set. Take any s-sparse vector x that has support S and L2 norm unity, then we can note the following

$$Ax\ =\ A_S x_S$$

The maximum of the following expression is λ_{max} (maximum eigenvalue) and minimum is λ_{min} (minimum eigenvalue) as given in the class.

$$||Ax||_2^2\ =\ ||A_S x_S||_2^2\ =\ \langle A_S x_S, A_S x_S\rangle\ =\ \langle A_S^T A_S x_S, x_S\rangle$$

We assumed that all the columns of A matrix are L2-unit normalized. Therefore, the diagonal elements of $A^T A$ and $A_S^T A_S$ are all 1. By the **Gershgorin's disk theorem**, we can state that the eigenvalues will be contained in the disks that are centered at 1 (because all the diagonal elements are 1) with radii as given below

$$r_j\ =\ \sum_{i\in S, i\neq j} |(A_S^T A_S)_{i,j}|\ =\ \sum_{i\in S, i\neq j} |\langle a_i, a_j\rangle|$$

We know that

$$\mu\ =\ \max_{1\leq i\neq j\leq n} |\langle a_i, a_j\rangle|$$

Each term in the summation of r_j is less than μ as μ is the maximum possible inner product. (s-1) number of terms are present in the summation. Therefore,

$$r_j \;\leq\; \mu(s-1)$$

Therefore, the eigenvalues will be lie in the interval

$$[1-(s-1)\mu, 1+(s-1)\mu]$$

$$\lambda_{max} \;\leq\; 1+\delta_s \;\leq\; 1+(s-1)\mu$$
$$\delta_s \;\leq\; (s-1)\mu$$

Question 2

Recall the definition of RIP of sparsity s. Smallest constant that satisfies the following for all s-sparse vectors x.

$$(1-\delta_{2s})||x||_2^2 \;\leq\; ||Ax||_2^2 \;\leq\; (1+\delta_{2s})||x||_2^2$$

Consider the following set, which is basically a set of s-sparse vectors

$$A_s \;=\; \{x \in R^n : ||x||_0 \;\leq\; s\}$$

where $||x||_0$ measures the number of non-zero elements in x. Observe that A_s is a subset of A_{s+1}. Therefore, any vector contained in A_s is also contained in A_{s+1}. If a particular vector, say x_0 is maximising $\frac{||Ax_0||_2^2}{||x_0||_2^2}$, then this vector will also be present in A_{s+1} and therefore, the same maximization can be achieved in the enw set also. Additionally, we can find better maximizers. Similarly, for the minimum case. Therefore, we can conclude that for

$$s_1 \;<\; s_2 \;<\; s_3 \;...\; s_n$$

$$\delta_{s_1} \;\leq\; \delta_{s_2} \;\leq\; \delta_{s_3} \;...\; \delta_{s_n}$$

Question 5

Paper Title: COSAMP: Iterative Signal Recovery From Incomplete And Inaccurate Samples
Author List: D. Needell, J. A. Tropp
Venue: Information Theory and Applications, San Diego
Year of Publication: 31 January 2008

(a) CoSaMP is a greedy pursuit type of recovery algorithm for recovering the compressed signal from noisy samples. The algorithm on overall covers the following ideas in every iteration:

1. Identification
2. Support Merger
3. Estimation
4. Pruning
5. Sample Update

Notation:

1. **x** is a signal and r is a positive integer
2. x_r depicts the matrix x formed by retaining the r greatest magnitude components
3. $x|_T$ denotes the vector x restricted to the set T
4. Φ_T is the restriction of the sensing matrix Φ whose columns are taken from the set T.
5. $A^\dagger$ is the pseudo-inverse of a matrix A which is given by $(A^\star A)^{-1}A^\star$

Pseudo Code

CoSaMP (Φ, u, s)
Input: Sampling matrix Φ, noisy sample vector **u**, sparsity level **s**
Output: An s-sparse approximation **a** of the target signal

$a^0 \leftarrow \mathbf{0}$ {Trivial initial approximation}
$\mathbf{v} \leftarrow \mathbf{u}$ {Current samples = input samples}
$k \leftarrow 0$

Repeat

$k \leftarrow k+1$
$y \leftarrow \Phi^\star v$ {Form signal proxy}
$\Omega \leftarrow \mathrm{supp}(y_{2s})$ {Identify large components}
$T \leftarrow \Omega \ \cup \ \mathrm{supp}(a^{k-1})$ {Merge supports}
$b|_T \leftarrow \Phi_T^\dagger \ u$ {Signal estimation by least-squares }
$b|_{T^c} \leftarrow 0$
$a^k \leftarrow b_s$ {Prune to obtain next approximation }
$v \leftarrow u - \Phi \ a^k$ {Update current samples}

until halting criterion true

(b) **Theorem:** Suppose that Φ is an m N **sampling matrix** with **restricted isometry constant** $\delta_{2s} \leq c$. Let $u = \Phi x + e$ be a vector of samples of an arbitrary signal, contaminated with arbitrary noise. For a given **precision parameter** η, the algorithm CoSaMP produces a 2s-sparse approximation **a** that satisfies

$$||x - a||_2 \ \leq \ C \ \max\left(\eta, \frac{1}{\sqrt{s}}||x - x_s||_1 + ||e||_2\right)$$

where x_s is the best s-sparse approximation to x, ie it is formed by taking the **s** biggest values and putting the rest as zeros.

References

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4. Slides