Tutorial Sheet-Unit IV
Quantum Computing

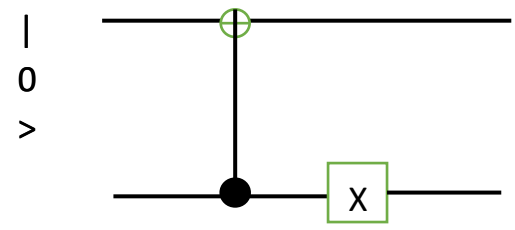
1. A state, in terms of three orthonormal basis vectors $|\phi_1>, |\phi_2> and\ |\phi_3>$ is given by

   $$|\psi> = \frac{1}{\sqrt{15}}|\phi_1> + \frac{1}{\sqrt{3}}|\phi_2> + \frac{1}{\sqrt{5}}|\phi_3>$$

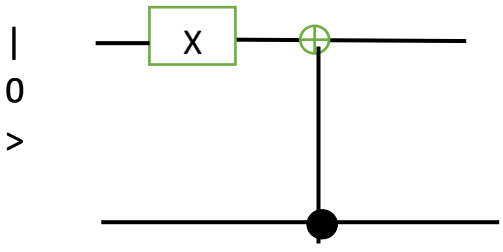
   Determine its normalization constant.

2. Find the constant "a" so that the states $|\psi> = a|\phi_1> + 5|\phi_2>$ and $|\chi> = 3a|\phi_1> - 4|\phi_2>$ are orthogonal; consider $|\phi_1>, and\ |\phi_2>$ to be orthonormal.
3. Consider two states $|\psi> = i|\phi_1> + 3i|\phi_2> - |\phi_3>$ and $|\chi> = |\phi_1> - i|\phi_2> + 5i|\phi_3>$ where $|\phi_1>, |\phi_2> and\ |\phi_3>$ are orthonormal.
   Calculate
   (a) $<\psi|\psi>$ (b) $<\chi|\chi>$ (c) $<\psi|\chi>$ (d) $<\chi|\psi>$
4. If $|\psi> = \begin{bmatrix} 5i \\ 2 \\ -i \end{bmatrix}$ and $|\phi> = \begin{bmatrix} 3 \\ 8i \\ -9i \end{bmatrix}$
   Determine
   (a) $|\psi>^*$ and $<\psi|$
   (b) Is $|\psi>$ is normalized? If not, calculate its normalization constant.
   (c) Are $|\psi>$ and $|\phi>$ orthogonal?
5. If $|0> = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $|1> = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, determine the following:
   (a) I |0> and I |1>
   (b) X |0> and X |1>
   (c) Y |0> and Y |1>
   (d) Z |0> and Z |1>
6. Two qubits are passed through a CNOT. The first qubit is the control qubit. What is the output for the following initial states?
   (a) |00>
   (b) |01>
   (c) |11>
7. Obtain the state produced by these quantum circuits.

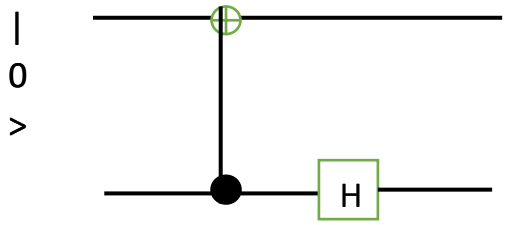
a.



b.



c.



d.