Applications of Physics in Computing

Statistical Physics for Computing

Measures of central tendancy: In statistics, averages are the measures of central tendancy of a given data. There are 3 types: Mean, median and mode.

Measures of dispersion: The measures of central tendancy does not provide any information about the scattering/dispersion of the data. Thus, the measures of dispersion give the degree by which the numerical data tend to spread about an average value. The commonly used measures of dispersion are range, mean deviation and standard deviation.

Range: The range of a set of data is the difference between maximum and minimum values of the variable. It is easy to compute but its usefulness is limited since, it depends only on two extreme values.

Mean (or Average) deviation: The mean deviation is defined as the sum of the absolute values of the deviation from the average (mean, median, mode) divided by the number of items.

If $x_1, x_2, x_3 \ldots\ldots x_n$ are the values of the variate with frequencies $f_1, f_2, \ldots\ldots f_n$, such that

$$\sum_{i=1}^{n} f_i = N$$

and μ is the average, then mean deviation is given by

$$Mean\ deviation = \frac{1}{N}\sum_{i=1}^{n} f_i |x_i - \mu|$$

Mean deviation is a better measure of dispersion when compared to range, as mean deviation is based on all the values of the variate.

Standard deviation: Standard deviation ($\boldsymbol{\sigma}$) is the most commonly used measure of dispersion for the given set of data. It gives the amount of variability or scatteredness around the mean (μ) of the data.

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{n} f_i (x_i - \mu)^2}$$

The square of the standard deviation is called the variance, denoted by σ^2 which is given by

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{n} f_i (x_i - \mu)^2$$

Probability: Probability is a measure of the likelihood that a particular event will occur in any one trial or experiment carried out in the prescribed conditions. Thus, if an event happens in 'm' ways and fails to happen in 'n' ways, the probability of occurrence of an event is given by $A = \frac{m}{m+n}$ and that of non-occurrence of the event is given by $B = \frac{n}{m+n}$

It should be noted that,

- The probability of an impossible event is always zero
- The probability of definite event is always 1.
- The probability of an event must be within 0 and 1
- The sum of the two probabilities is always 1, A+B=1 (ie., the event must either happen or fail).
- Equally likely cases: if two events in an experiment are equally probable. In a toss of a coin, head and tail are said to be equally likely to occur.
- Independent events: if two or more events can happen simultaneously, then two such events are said to be independent if the happening or non-happening does not affect the happening or non-happening of the other. Toss of a coin and rolling of a dice. So, the events are independent.
- Conditional probability: The probability of happening of an event A when another event B is known to have already happened is called conditional probability of A

The two approaches used to determine the probability are

1. Empirical (Experimental) probability- based on previous known results, actual observations and experimentation
2. Classical (or theoretical) probability-based on consideration of the theoretical number of ways in which an event is possible to occur.
    - Binomial distribution
    - Poisson's distribution
    - Normal distribution

Binomial distribution

Consider an experiment or a trial consisting of two possible outcomes-success or failure, the probability of its success is p and the probability of its failure is 1-p=q in one trial. Let the event be tried N times. If the probability of success per trial is p, then, the probability P of observing x successes in N trials is given by

$$P = \frac{N!}{x!(N-x)!} p^x (1-p)^{N-x} = {}^N C_x \, p^x \, (1\text{-}p)^{N\text{-}x}$$

The average number of successes is given by $\mu = Np$

The standard deviation is given by $\sigma = \sqrt{Np(1-p)}$

In a Binomial distribution, we have

- N, the number of trials is finite
- Each trial has only two possible outcomes-success or failure
- All the trials are independent, which means the outcome of one trial does not affect the outcome of another trial.
- P (or q) is constant throughout for all the trials

Binomial distribution finds application in many fields like, industry, military and pharmaceutical testing. This type of distribution is applicable where the results of the process are independent of the probability of success being constant from trial to trial.

Example: In industry, a quality check inspector is always interested in identifying the defects in the components manufactured. The probability of a newly designed component will survive given a shock test is 83.3%. Then, the probability that 3 out of 4 components tested will survive is

p= 83.3% =0.833, x=3, N=4

$P(x=3) = \frac{4!}{3!(4-3)!} 0.833^3 (1-0.833)^{4-3} = 0.386$

Poisson's probability distribution

Poisson's distribution is a discrete distribution that gives the probability of a number of events ie.. number of times an event occurs in an interval of time. Poisson's distribution is based on the following assumptions

- Events are occurring independently
- Success of an event does not affect the probability of success of the second event
- Average rate at which events occur is independent of any successes.

If P represents probability of an event to occur, then, Poisson's distribution can be applied to systems having a large number of events (n) with less probability of success (p). A discrete random variable is said to have a Poisson distribution, then, the probability distribution function is defined as

$$P(r) = \frac{e^{-\mu}\mu^{r}}{r!}$$

Where, μ=np is the mean of the number of successes

The total probability, putting r=0,1,2…… gives

Total $P(r) = \frac{e^{-\mu}\mu^{0}}{0!} + \frac{e^{-\mu}\mu^{1}}{1!} + \frac{e^{-\mu}\mu^{2}}{2!}$...........

$= e^{-\mu}\ \{1+\mu + \frac{\mu^2}{2!} + ...\}$

$= e^{-\mu} \times e^{\mu} = e^{0} = 1$

Therefore, total probability = 1

Poisson's distribution has several applications. A few to mention, to identify the number of defective fuses, to find the number of $\boldsymbol{\alpha}$ particles emitted from a radioactive material, probability of emission of electrons from a metal surface during thermionic emission, detection of number of photons reaching the detector etc.

Example: 1. Radioactive decay – emission of particles from a radioactive source.

Consider a radioactive source such as ^{60}Co which has a half-life of 5.26 years. The probability per unit time for a single nucleus to decay is given by $\boldsymbol{\lambda}$ = ln 2/5.26 years = 41.7 x 10^{-10} s^{-1}. This implies a very small probability.

If 1 µg sample of ^{60}Co consists of about 10^{15} nuclei (each nucleus undergoing decay constitutes a *trial),* the mean number of decays from the sample will be μ =nλ = 41.7 x 10^{5} decays/s.

Then, the probability of r decays can be estimated by Poisson's distribution function

$$P(r) = \frac{e^{-\mu}\mu^{r}}{r!}$$

2. **Poisson's distribution can also be used to model proton decay.**

Grand unification theory paved a way for several researchers to think of the probability of proton decay. Poisson's distribution statistics can be used to study the probability of proton decay. The number of protons undergoing decay is given by the decay equation $N = N_o e^{-\lambda t}$ ….. (1)

If the half life of proton is considered to be 10^{33} years, then, the probability that a proton will decay per year is given by decay constant $\lambda = \frac{\ln 2}{t_{1/2}}$ .... (2)

$\lambda = \frac{\ln 2}{10^{33}} = 0.693\ X\ 10^{-33}$ /year $\approx\ 10^{-33}$ /year

Since, the decay constant is so small, the exponential in equation (1) can be approximated to 1-$\boldsymbol{\lambda}$t

Thus, $N = N_o e^{-\lambda t} = N_o(1 - \lambda t) = N_o - N_o\lambda t$

If the sample is small, the observation of proton decay is infinitesimal. If the proton concentration is high of around 7.5 x 10^{33} per unit volume (reported by Ed Kearns of Boston University), the number of proton decays expected in one-year observation is

$$N_o - N = N_o\lambda t = 7.5 \text{ x } 10^{33} \text{x} \frac{10^{-33}}{year} X\ 1\ year = 7.5$$

If 40% of the area is covered with the detector tubes (good efficiency) to detect the particles, only 3 observations of proton decay per year is expected. As of now, no proton decay events have been observed. But, Poisson's distribution enables one to assess the implications of the absence of these observations. If we consider $\boldsymbol{\mu}$=3 as the mean (3 decays per year), then probability of zero observations of proton decay per year can be estimated using Poisson's distribution function

$$P(r) = \frac{e^{-\mu}\mu^r}{r!} = \frac{e^{-3}3^0}{0!} = 0.05$$

This low probability for a null result suggests that the proposed lifetime of 10^{33} years is too short. While this is not a realistic assessment of the probability of observations because there are a number of possible pathways for decay, it serves to illustrate in principle how even a non-observation can be used to refine a proposed lifetime.

Normal distribution

Poisson distribution refer to the discrete events ie., the number of occurrences of an event in a fixed number of independent trials. However, in practice, we come across the variables which are continuous in nature, where the probability of success is high, hence, requires a continuous probability distribution function ie., normal probability distribution/Gaussian distribution. The probability distribution function is given by

$$P = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}\frac{(x-\mu)^2}{\sigma^2}}$$

Where, μ is the mean of the normal distribution and **σ** is the standard deviation of the distribution.

The graph used to depict a normal distribution consists of a symmetrical bell-shaped curve. The highest point on the curve or the top of the bell, represents the most probable event in a series of data, while all other possible occurrences are symmetrically distributed around the mean. The width of the bell curve is described by its Standard Deviation.

The term "bell curve" is used to describe a graphical depiction of a normal probability distribution, whose underlying standard deviations from the mean create the curved bell shape. A standard deviation is a measurement used to quantify the variability of data dispersion, in a set of given values around the mean. The mean, in turn, refers to the average of all data points in the data set or sequence and will be found at the highest point on the bell curve.

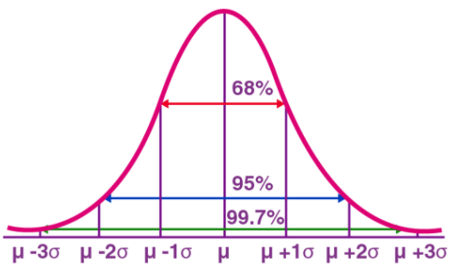
In practice, it is convenient to convert a normal distribution into a standard normal distribution or Gaussian distribution, having a mean of 0 and a standard deviation of 1.

If x is called the normal variable with mean μ and standard deviation **σ**, then z is called the standard normal variable, having zero mean and one as standard deviation.

$$z = \frac{x - \mu}{\sigma}$$

$$P = \frac{1}{\sqrt{2\pi}} e^{\frac{-1}{2}z^2} \ (-\infty < z < \infty)$$

- Approximately 68% of the data falls within one standard deviation of the mean.
- 95% of the data falls within two standard deviations of the mean.
- 99.7% of the data fall within three standard deviations of the mean.



Normal/Gaussian distribution is observed in several cases like particle size distribution in materials, Maxwell Boltzmann distribution –gas molecules, diffraction peaks in X ray diffractogram, random motion of particles suspended in a medium-Brownian motion etc.

Example: 1. Maxwell-Boltzmann distribution law

Maxwell-Boltzmann distribution law is the statistical distribution of velocities/energies of gas molecules. This is based on classical kinetic theory of gases which states that, for a system of gas molecules in thermal equilibrium at temperature T, the average kinetic energy of a molecule is given by $\frac{3}{2}\ kT$. This law is applicable for any classical system consisting of large entities of same kind.

Consider a system of distinguishable number of particles 'N'. Let $n_1, n_2 \ldots\ldots n_i$ represent the number of particles in the available energy states $E_1, E_2, \ldots\ldots E_i$ within the system. Since, the number of particles is extremely large, one may consider the distribution of particles as continuous over available energy range. For such a gas, Maxwell Boltzmann distribution law is given by

$$n_i = A \exp \frac{-E_i}{kT} \text{ ……….. (1)}$$

Let us consider a volume element dV in the phase space corresponding to the energy interval E to E+dE, with dn number of particles present in the said interval, then, the Maxwell Boltzmann distribution law can be written as

$$dn\,(x,y,z,p_x,p_y,p_z) = \frac{N}{V(2\pi mkT)^{3/2}} \exp\left({-p^2}/{2mkT}\right) dx\,dy\,dz\;dp_x\,dp_y\,dp_z \text{ ……….(2)}$$

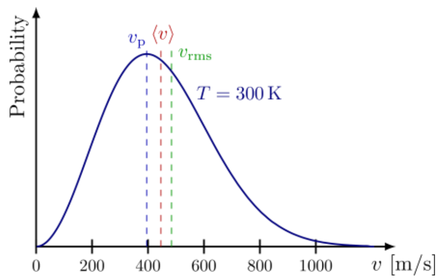
(where, $A=\frac{N}{V(2\pi mkT)^{3/2}}$ and $E_i = p^2/2m$, N is the number of particles in the gaseous system of volume V)

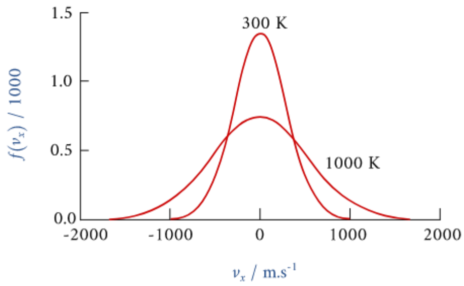
The velocity distribution of particles according to the M-B statistics with the cartesian velocity co-ordinates is given by

$$dn\,(v_x, v_y, v_z) = \frac{Nm^3}{(2\pi mkT)^{3/2}} \exp\left({-mv^2}/{2kT}\right)\; dv_x\,dv_y\,dv_z \text{ ……… (3)}$$

Then, Maxwell-Boltzmann's velocity distribution function is given by

$$P(v) = 4\pi\left({m}/{2\pi kT}\right)^{3/2} \exp\left({-mv^2}/{2kT}\right)\; v^2$$





Normal distribution/Gaussian Distribution

If one component of the velocity (v_x) is considered, then, the velocity distribution of the particles/molecules is expected to show Gaussian distribution because, a molecule is able to move in a positive or negative direction.

Most probable velocity (v_p): The most probable velocity is the maximum value on the distribution plot (largest number of molecules have that speed) and is given by

$$v_p = \left(\frac{2kT}{m}\right)^{1/2}$$

Average velocity (⟨v⟩) : The average or mean velocity is the sum of the speeds of all the molecules divided by the number of molecules and is given by

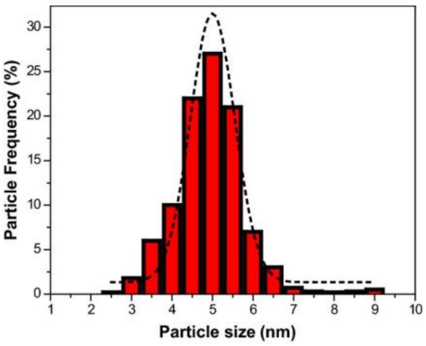
$$\langle v \rangle = \left(\frac{8kT}{\pi m}\right)^{1/2} = 1.13v_p$$

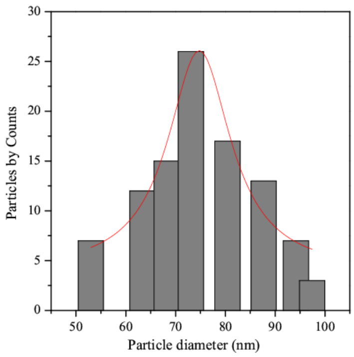
rms velocity (v_{rms}): The root mean square velocity is given by

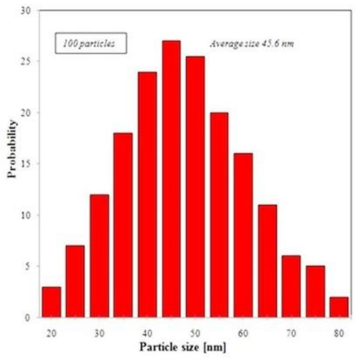
$$v_{rms} = \sqrt{\frac{3kT}{m}} = 1.25\ v_p$$

2. Particle size distribution in materials

In material science, researchers have keen interest in exploring several properties like structural, electrical, optical and thermal properties for application in respective field. A material scientist upon careful observation, can interpret the properties of the material. In some materials, the distribution of size of the particles is found to exhibit normal distribution. Example, silver nanoparticles are found to possess average particle size of 5nm, whereas, CaCu2.90 Zn0.10Ti_4O_{12} (CCZTO)ceramic nanomaterial's average particle size is 73nm.In the case of Dy^{3+} doped Sr_2SiO_4 phosphors, on an average, the particle size is found to be 45.6nm. The size of the particle will certainly affect the properties of the material.







a) Silver nanoparticles b) CCZTO c) Dy^{3+} doped Sr_2SiO_4 phosphors

Monte Carlo simulation method

Monte Carlo simulation is a type of simulation which depends on repeated random sampling and statistical analysis of the result. This method is used when we are dealing with complex systems like many particles system, complex interaction among the particles or the external field, when it is difficult to solve analytically. Different types of Monte Carlo simulation are as follows

a. Classical Monte Carlo: samples are drawn from a probability distribution-classical Boltzmann distribution to obtain thermodynamic properties or minimum energy structures

b. Quantum Monte Carlo: to compute quantum-mechanical energies and wave functions, to solve electronic structure, using Schrödinger's equation as a starting point.

Steps involved in Monte Carlo simulation are as follows

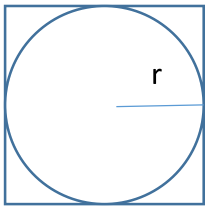
1 Define a domain of possible inputs: Define the system/plan/activity to be explored and the equation/ relation which governs the system

2 Generate inputs randomly from a probability distribution over the domain: A very large random data set has to be created which has to be submitted for simulation to get the output

3 Get the output from several simulation run: Simulation is done on a large scale

4 Statistical analysis of the output values of the parameters: The obtained output is further analysed using statistical tools.

Example: To determine the value of $\boldsymbol{\pi}$ using Monte Carlo method

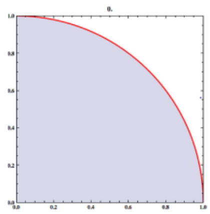


Consider a circle with radius r and imagine the circle is inscribed within a square with side 2r (equal to the diameter of circle) as shown in the figure.

If we consider a random point inside the square, the probability of this point is inside the circle or inside a square. A simple way to compute this probability is to find the ratio between the area of the circle and area of square

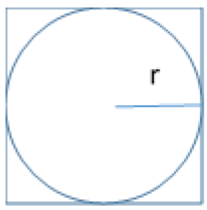
$$probability = \frac{Area\ of\ circle}{Area\ of\ square} = \frac{\pi r^2}{2r\ X\ 2r} = \frac{\pi}{4}$$

The probability that a random selected point in the square is in the circle is π/4. This means that if, same procedure to be repeated (selection of a random point in the square) a large number of times, the proportion of points inside the circle will be obtained, multiply it by four and that would give an approximation of π



1. Draw a square and inscribe a quadrant
2. Uniformly scatter a given number of points over the square
3. Count the number of points inside the circle and total number of points,
4. Ratio of the two counts will give an estimate of ratio of the two areas which is $\boldsymbol{\pi}/4$
5. Multiply the result by 4 to estimate the value of $\boldsymbol{\pi}$

Monte Carlo simulation



1. Generate 2 random numbers between -1 and 1 in total 100 times (x and y).
2. Calculate $x^2 + y^2$.
    - If the value is less than 1, the point will be inside the circle
    - If the value is greater than 1, the point will be outside the circle.

3. Calculate the proportion of points inside the circle and multiply it by four to approximate the π value.
4. Repeat the experiment 1000 times, to get different approximations to π
5. Calculate the average of the previous 1000 experiments to give a final value estimate.

Advantages of using Monte Carlo method

1. Very useful mathematical technique for analysing the uncertain scenarios
2. Simple and easy to grasp
3. Various tools have embedded MC simulation in different domains

Animation

Animation is a technique of making objects at rest as moving objects, create videos based on illusion & creativity and to make the audience understand the story/sequence/topic of interest. In order to create any character or design any sequence, laws of Physics will certainly help us to create all kinds of characters to do fantastic motion. The audience will understand the sequence, follow the story if the character's size, weight and motions are defined properly. Thus, animators require some knowledge and understanding of physics

Taxonomy of Physics-based animation methods

Physics is broadly classified into several branches. But, the field of physics-based animation and simulation is based on classical laws like law of gravity, newton's law to a greater extent and then atomic theory to some extent. Thus, can be grouped into two

1. Kinematics - study of motion without consideration of forces.
2. Dynamics - study of motion taking mass and forces into consideration.

Frames & Frames per second

A frame is a single image in a sequence of pictures. A frame contains the image to be displayed at a unique time in the animation. The frame is a combination of the image to be displayed and the time the image is to be displayed. A sequence of frames makes an animation.

In general, one second of a video is comprised of 24 or 30 frames per second also known as FPS. Each frame is displayed on the screen until the next frame overwrites it. Since each frame remains displayed on the screen for a tiny but finite time period, thus, animation can be frames displayed at discrete intervals of time in a continued sequence.

Motion and Timing in animations

Motion and timing are equally important in animation and games. Motion and timing go hand in hand in animation. When animating a scene, there are several types of motion to consider based on the scene. The most common types of motion are as follows

- Linear motion
- Circular motion
- Parabolic motion
- Wave motion

Timing

Timing is one of the principles of animation. The timing is the choice of when something should be done. Animators have the ability to move forward and backward in time to place the objects wherever and when they are to be. In animation, timing of action consists of placing objects or characters in particular locations at specific frames to give the illusion of motion. Animators work with very small intervals of time; most motion sequences can be measured in seconds or fractions of seconds. Timing in animation is measured in different ways:

• Frames (intervals of 1/24, 1/25, or 1/30 of a second)
• Keys (number of frames between poses)
• Clocks (seconds)

Slow in and Slow out

Slow in and slow out refers to the type of motion of an object when motion is accelerating or decelerating. This type of motion is sometimes called as ease in or ease out.

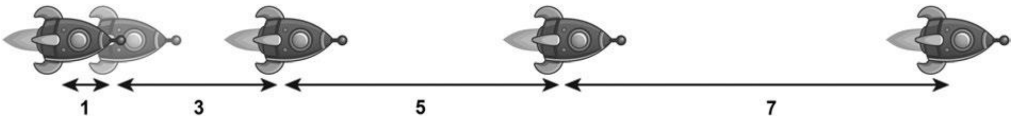
• Slow in, ease in—In this type, the object is slowing down and reach rest state ie., often in preparation for stopping.
• Slow out, ease out—In this type, the object is speeding up, often from a still position.

Example*: A ball rolling down an incline or dropping straight down is slowing out, as it goes from a still position or slow speed to a fast speed. A ball rolling up an incline is slowing in.*

Slowing out: speeding up. Slowing in: slowing down.

ODD RULE

When acceleration is constant, one can use the Odd Rule to time your frames. With this method, you calculate the distance the object moves between frames using a simple pattern of odd numbers. Between consecutive frames, the distance the object moves is a multiple of an odd number. For acceleration, the distance between frames increases by multiples of 1, 3, 5, 7, etc.



Rocket speeding up using the Odd Rule.

For deceleration, the multiples start at a higher odd number and decrease, for example 7, 5, 3, 1.

Sled coming to a stop using the Odd Rule.

The Odd Rule is a multiplying system based on the smallest distance travelled between two frames in the sequence. For a slow-out, this is the distance between the first two frames; for a slow-in, it's the distance between the last two frames. This distance is called the base distance x_b which is used in all Odd Rule calculations. By calculating the distance from the first frame to the current frame, one can place the objects on specific frames in a given sequence of animation.

Odd rule multipliers: is a multiplier for the entire distance upto a particular frame.

For an instance, on frame 4, the consecutive multiples are 1,3,5 and when added, we get 9 which is the multiplier for the entire distance upto frame 4.

	Multiply by base distance to get the distance between	
Frame	Consecutive Frames	First frame and this frame
1	---	0
2	1	1
3	3	4
4	5	9
5	7	16
6	9	25
7	11	36
8	13	49

Following this procedure to calculate the distance is not feasible when we have large number of frames. Hence, odd number multiplier for consecutive frames is determined by

$$\boldsymbol{odd\ number\ multiplier\ for\ consecutive\ frame} = [(\boldsymbol{Frame\ No.} - \mathbf{1}) * \mathbf{2}] - \mathbf{1}$$

$$\boldsymbol{multiplier\ for\ distance\ from\ first\ frame\ to\ current\ frame} = (\boldsymbol{Current\ Frame\ No.} - \mathbf{1})^{\mathbf{2}}$$

Case i) Speeding up

Consider an object executing slow out motion ie.., object is speeding up from the still position.

- To find the base distance

  If the total distance and total number of frames are known, then, base distance can be obtained using the formula

$$Base\ distance\ \ x_B = \frac{Total\ distance}{(Last\ frame\ -1)^2} = \frac{x}{(F_L - 1)^2}$$

- To find the distance between each frame by multiplying the consecutive multiplier by the base distance. Adding all the distance between the frames, we get the total distance the object moves in the sequence.

Example: If total distance is 1.5m over 4 frames, then base distance = $\frac{1.5}{(4-1)^2} = 0.166m$

Using base distance, distance between each frame can be determined as follows

Frame	Consecutive Frame multiplier	Distance from previous frame (m)
1	---	
2	1	1 x 0.166 = 0.166
3	3	3 x 0.166 = 0.498
4	5	5 x 0.166 = 0.83
Total distance = 1.494 ~ 1.5m		

Case ii) Slowing down

Consider an object executing slow in motion ie., the object is slowing down.

In this case, the distance between the last two frames will become the first frame distance. It's just the reverse order.

- To find the base distance

  One of the features of Odd rule is that, the base distance is half the difference between any two adjacent distances.

$$Base\ distance\ x_B = \frac{x_m - x_n}{2}$$

- To find the number of frames

  Divide the first distance by the base distance which gives the odd number corresponding to the first frame in the slowing down sequence.

Example: If the distance between the adjacent frames is 0.9m and 0.7m. Base distance is calculated as follows

$$x_B = \frac{0.9 - 0.7}{2} = 0.1m$$

Odd number associated with frame 1 = 0.9/0.1

= 9

This implies that the first distance corresponds to 9 in the sequence 9,7,5,3,1. The multiplier 9 is corresponding to the first frame in the sequence. Using this, distance between other frames can also be determined

Frame	Consecutive Frame multiplier	Distance from previous frame (m)
1	9	9 x 0.1 = 0.9
2	7	7 x 0.1 = 0.7

3	5	5 x 0.1 = 0.5
4	3	3 x 0.1 = 0.3
5	1	1 x 0.1 = 0.1
Total distance = 2.5m		

Character Animation

Jumping and Walking

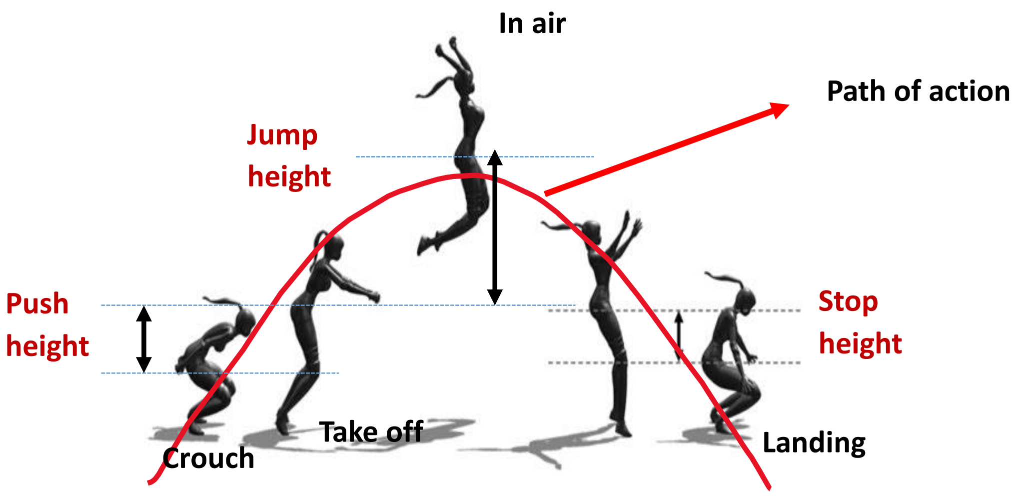
Jumping: A jump is an action where the character's feet leave the ground at the same time approximately and the entire body is in the air. A jump action includes a takeoff, free movement through the air and landing.

Parts of Jump

A jump can be divided into several distinct parts:

- Crouch—A squatting pose taken as preparation for jumping.
- Takeoff—Character pushes up fast and straightens legs with feet still on the ground. The distance from the character's centre of gravity (CG) in the crouch to the CG when the character's feet are just about to leave the ground is called the push height. The amount of time (or number of frames) needed for the push is called the push time.
- In the air—Both the character's feet are off the ground, and the character's centre of gravity (CG) moves in a parabolic arc as any free-falling body would. First it reaches an apex, and then falls back to the ground at the same rate at which it rose. The height to which the character jumps, called the jump height, is measured from the CG at takeoff to the CG at the apex of the jump. The amount of time the character is in the air from takeoff to apex is called the jump time. If the takeoff pose and the landing pose are similar, then the jump height and jump time are about the same going up as they are going down.
- Landing—Character touches the ground and bends knees to return to a crouch. The distance from the character's CG when her feet hit to the ground to the point where the character stops crouching is called the stop height. The stop height is not always exactly the same as the push height.

Path of action—The trajectory along which the character jumps, which can be straight up in the air or over a horizontal distance.



Calculating Jump Actions

The timing for a jump action can be worked out based on

1. Jump height or jump time
2. Push height
3. Stop height
4. Horizontal distance the character will travel during the jump

1. Calculating Jump Timing

To calculate jump timing, one should know the jump height which is generally expressed in inch or cm. The timing and placement for frames while the character is in the air follow the same rules as any object thrown into the air against gravity. With the help of data available or online calculator, one can figure out the jump time for each frame. Further, express the jump time in frames based on the fps.

Example:

Jump height = 1.2m

Jump time for 1.2m = 0. 5 seconds

Jump time at 30fps = 0. 5 * 30 = 15 frames

2. Jump Magnification

The Jump Magnification (JM) can be used to calculate the push timing and stop timing. The JM is the ratio of the jump height to the push height.

$$JM = \frac{Jump\ height}{Push\ height}$$

Knowing the jump height and push height, one can calculate the JM.

3. Jump Magnification and Acceleration

Jump Magnification is an exact ratio that tells how much the character has to accelerate against gravity to get into the air. The JM, besides being the ratio of jump-to-push vertical height and time, is also the ratio of push-to-jump vertical acceleration. while a longer jump time means a shorter push time, a higher jump acceleration means a much higher push acceleration. This information will help us to decide about push timing.

To see how this works, let's look at the formula for JM and relate it to acceleration:

$$JM = \frac{Jump\ time}{Push\ time} = \frac{Jump\ height}{Push\ height} = \frac{Push\ Accelaration}{Jump\ Acceleration}$$

The magnitude of jump acceleration is always equal to gravitational acceleration, with deceleration as the character rises and acceleration as it falls.

$$JH = \frac{Push\ Accelaration}{Jump\ Acceleration} = \frac{Push\ Accelaration}{Gravitational\ Acceleration}$$

Since, landing speed is the same as the velocity of any falling object, which can be easily calculated from the free fall time. Since takeoff speed is same as landing speed, the character has to get up with the same speed when taking off for a jump. If landing speed is 10m/sec, then during takeoff one need to get with a speed of 10m/sec in that little bit of push time.

The general formula for calculating the velocity of an accelerating object is:

Velocity = Acceleration x Time

v = at

If the landing velocity is the same as the push velocity, we know that:

v = Jump Acceleration x Jump Time

So . . .

Jump Acceleration x Jump Time = Push Acceleration x Push Time

$$\frac{Jump\ time}{Push\ time} = \frac{Push\ Accelaration}{Jump\ Acceleration(Gravity)} = JM$$

This shows that, the push acceleration increases with increase in jump time and also decrease in push time. Distance (or in this case, jump or push height) is also related to velocity:

Distance = Average Velocity x Time

d = vt

v = d/t

Because the average velocity is the same for both the push and jump, we can say that d/t is the same for both jump and push.

$$\frac{Jump\ height}{Jump\ time} = \frac{Push\ height}{Push\ time}$$

4. **Push Time** = $\frac{Jump\ time}{JM}$

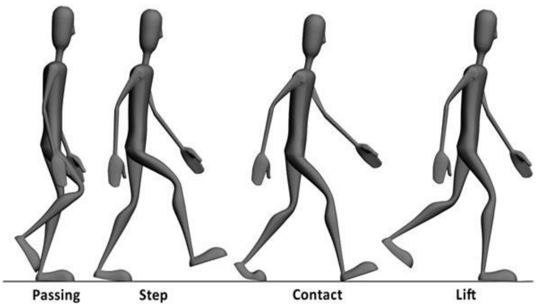
5. **Landing**: The forces on landing is similar to that of forces at takeoff. The stop height is bit larger than the push height. But, the timing of push and stop are the same because the center of gravity moves the same distance per frame in the push and stop. If the stop height is larger than the push height, then, more frames are needed for the stop than that of push.

Therefore, $\frac{push\ height}{push\ frames} = \frac{stop\ height}{stop\ frames}$ which can also be written as

$\frac{push\ height}{push\ time} = \frac{stop\ distance}{stop\ time}$ or

$$Stop\ time = \frac{push\ time * stop\ distance}{push\ height}$$

Walking



Walking consists of a series of poses. The four basic poses for a single step are passing, step, contact and lift. In the passing pose, the free foot is passing by the opposite leg, and the body is at its most upright. In the contact pose, the free foot has come forward just enough to make contact with the ground. Passing and contact are the two poses which include the most dynamic shifts for center of gravity, limbs and secondary motion.