1. Discuss Binomial distribution with an example.

Consider a trial consisting of two possible outcomes-success or failure, the probability of its success is p and the probability of its failure is 1-p=q in one trial. Let the event be tried N times. If the probability of success per trial is p, then, the probability P of observing x successes in N trials is given by

$$P = \frac{N!}{x!(N-x)!} p^x (1-p)^{N-x}$$

Binomial distribution is used when,

- N, the number of trials is finite.
- Each trial has only two possible outcomes-success or failure.
- All the trials are independent of each other.
- P (or q) is fixed for all the trials.

Example: In industry, a quality check inspector is always interested in identifying the defects in the components manufactured. The probability of a newly designed component will survive given a shock test is 83.3%. Then, the probability that 3 out of 4 components tested will survive is
p= 83.3% =0.833, x=3, N=4.

$$P(x=3) = \frac{4!}{3!(4-3)!} 0.833^3 (1-0.833)^{4-3} = 0.386$$

2. Discuss Poisson's probability distribution with an example.

Poisson's distribution is a discrete distribution that gives the probability of a number of events. That is, the number of times an event occurs in an interval of time. Poisson's distribution is used when,

- Events are occurring independently.
- Success of an event does not affect the probability of success of the second event.
- Average rate at which events occur is fixed.

If P represents probability of an event to occur, then, Poisson's distribution can be applied to systems having a large number of events (n) with less probability of success (p). The probability of a random variable r is given by

$$P(r) = \frac{e^{-\mu} \mu^r}{r!}$$

Where, μ=np is the mean of the number of successes.

Example – emission of particles from a radioactive source:

Consider a radioactive source such as ^{60}Co which has a half-life of 5.26 years. The probability per unit time for a single nucleus to decay is given by $\boldsymbol{\lambda}$ = ln 2/5.26 years = $41.7 \times 10^{-10}\ s^{-1}$. This implies a very small probability.

If 1 μg sample of ^{60}Co consists of about 10^{15} nuclei (each nucleus undergoing decay constitutes a *trial),* the mean number of decays from the sample will be μ =nλ = 41.7×10^5 decays/s. Then, the probability of r decays can be estimated by Poisson's distribution function

$$P(r) = \frac{e^{-\mu} \mu^r}{r!}$$

3. Discuss normal distribution with an example.

Characteristics of normal distributions are:

- The mean, median and mode are exactly the same.
- The distribution is symmetric about the mean—half the values fall below the mean and half above the mean.
- The distribution can be described by two values: the mean and the standard deviation.

The probability distribution function is given by

$$P = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}\frac{(x-\mu)^2}{\sigma^2}}$$

Where, μ is the mean of the normal distribution and **σ** is the standard deviation of the distribution.

The graph used to depict a normal distribution consists of a symmetrical bell-shaped curve. The highest point on the curve or the top of the bell, represents the most probable event in a series of data, while all other possible occurrences are symmetrically distributed around the mean. The width of the bell curve is described by its Standard Deviation.

Example- Maxwell-Boltzmann distribution law:

Maxwell-Boltzmann distribution law is the statistical distribution of velocities/energies of gas molecules. This is based on classical kinetic theory of gases which states that, for a system of gas molecules in thermal equilibrium at temperature T, the average kinetic energy of a molecule is given by $\frac{3}{2}\ kT$. This law is applicable for any classical system consisting of large entities of same kind.

Consider a system of distinguishable number of particles 'N'. Let n_1, n_2…….n_i represent the number of particles in the available energy states E_1, E_2, …… E_i within the system. Since, the number of particles is extremely large, one may consider the distribution of particles as continuous over available energy range. For such a gas, Maxwell Boltzmann distribution law is given by

$$n_i = A\exp\frac{-E_i}{kT} \text{ ……….. (1)}$$

4. What is Monte Carlo simulation method? Discuss the steps involved in this method. What are the advantages of this method?

Monte Carlo simulation is a type of simulation which depends on repeated random sampling and statistical analysis of the result. This method is used when we are dealing with complex systems like many particles system, complex interaction among the particles or the external field, when it is difficult to solve analytically.

Different types of Monte Carlo simulation are as follows:

a. Classical Monte Carlo: samples are drawn from a classical probability distribution to obtain thermodynamic properties.
b. Quantum Monte Carlo: to compute quantum-mechanical energies and wave functions.

Steps involved in Monte Carlo simulation are as follows:

1 Define the system to be explored and the equation which governs the system.
2 A very large random data set to be submitted for simulation to get the output.
3 Get the output from several simulation runs.
4 The obtained output is further analysed using statistical tools.

Advantages of using Monte Carlo method:

1. Very useful mathematical technique for analysing the uncertain scenarios
2. Simple and easy to grasp
3. Various tools have embedded MC simulation in different domains

5. Discuss ODD rule in animation. How base distance and distance between each frame are determined using odd rule multipliers for i) an object speeding up and ii) an object slowing down?

Using ODD rule, one can calculate the distance the object moves between frames using a simple pattern of odd numbers. Between consecutive frames, the distance the object moves is a multiple of an odd number.

For acceleration, the distance between frames increases by multiples of 1, 3, 5, 7….. and for deceleration, the multiples start at a higher odd number and decrease, for example …..7, 5, 3, 1.

The Odd Rule is a multiplying system based on the smallest distance travelled between two frames in the sequence. For a slow-out, this is the distance between the first two frames; for a slow-in, it's the distance between the last two frames. This distance is called the base distance x_b which is used in all Odd Rule calculations. By calculating the distance from the first frame to the current frame, one can place the objects on specific frames in a given sequence of animation.

Calculating the distance is not feasible when we have large number of frames. Hence, odd number multiplier for consecutive frames is determined by

$$\boldsymbol{odd\ number\ multiplier\ for\ consecutive\ frame = (Frame\ No. - 1)^2 - 1}$$

$$\boldsymbol{multiplier\ for\ distance\ from\ first\ frame\ to\ current\ frame = (Current\ Frame\ No. - 1)^2}$$

Case i) Speeding up

Consider an object executing slow out motion ie.., object is speeding up from the still position.

- To find the base distance, if the total distance and total number of frames are known, then, base distance can be obtained using the formula

$$Base\ distance\ \ x_B = \frac{Total\ distance}{(Last\ frame\ -1)^2} = \frac{x}{(F_L - 1)^2}$$

- To find the distance between each frame by multiplying the consecutive multiplier by the base distance. Adding all the distance between the frames, we get the total distance the object moves in the sequence.

Case ii) Slowing down

Consider an object executing slow in motion ie., the object is slowing down. In this case, the distance between the last two frames will become the first frame distance. It's just the reverse order.

- To find the base distance, one of the features of Odd rule is that, the base distance is half the difference between any two adjacent distances.

$$Base\ distance\ x_B = \frac{x_m - x_n}{2}$$

- To find the number of frames, divide the first distance by the base distance which gives the odd number corresponding to the first frame in the slowing down sequence.