MAT201

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M S RAMAIAH INSTITUTE OF TECHNOLOGY

(AUTONOMOUS INSTITUTE, AFFILIATED TO VTU)

BANGALORE – 560 054

SEMESTER END EXAMINATIONS – MAY / JUNE 2014

Course & Branch	: **B.E: COMMON TO ALL BRANCHES**	**Semester**	: **II**
Subject	: **Engineering Mathematics-II**	**Max. Marks**	: **100**
Subject Code	: **MAT201**	**Duration**	: **3 Hrs**

Instructions to the Candidates:

- Answer one full question from each unit.

UNIT – I

1. a) (i) Define linear differential equation of first order and give general form of the solution of the linear differential equation. (02)

   (ii) Solve: $\frac{dx}{dy} = \frac{-x\tan(xy)}{1+y\tan(xy)}$. (03)

   b) Using Euler's Modified method find y at x= 0.2,. Given that $\frac{dy}{dx} = x - y^2$, y(0) = 1.Perform two iterations at each stage. With h=0.1 (08)

   c) A voltage Ee^{-at} is applied at $t = 0$ and i=0 to a circuit of inductance L and resistance R. Show that the current at time t is $\frac{E}{R-aL}\left(e^{-at} - e^{-Rt/L}\right)$. (07)

2 a) (i) State the condition for exactness of the differential equation $M(x,y)dx + N(x,y)dy = 0$ and also write its general solution. (02)

   (ii) Using Euler's method solve $\frac{dy}{dx} - 2y = 3e^x, y(0) = 0$ at $x = 0.2$ by taking $h = 0.1$. (03)

   b) Show that the family of curves $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$ is self orthogonal, where λ is the parameter. (08)

   c) Using Runge kutta method of fourth order find y(0.2) for the equation $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$, by taking $h = 0.1$. (07)

UNIT - II

3. a) (i) Solve: $\frac{d^4y}{dx^4} - a^4y = 0.$ (02)

(ii) $If D = \frac{d}{dx}$ and $X = X(x)$ then prove that $\frac{1}{(D+a)}X = e^{-ax}\int Xe^{ax}\,dx.$ (03)

b) Solve $\frac{d^2y}{dx^2} - y = (1 + x^2)e^x$ (08)

c) Solve $x^2y'' + xy' + 9y = 3x^2 + \sin(3logx)$ (07)

4. a) (i) Write the Cauchy's linear differential equation with variable coefficients of order n. (02)

(ii) Solve $y'' - 2y' - 3y = 0,\ y(0) = 0, y(1) = e^3 - \frac{1}{e}$ (03)

b) Solve $y'' + 3y' + 2y = e^{e^x}$ by using the method of variation of parameters. (08)

c) Solve $(3x + 2)^2y'' + 3(3x + 2)y' - 36y = 3x^2 + 4x + 1$ (07)

UNIT - III

5. a) (i) State the conditions for consistency and inconsistency of the system of linear Equations AX=B. (02)

(ii) Find the rank of the matrix A=$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$ by reducing into row echelon form. (03)

b) Solve the following system equations by using Gauss - Siedel method. Carryout four iterations by taking the initial approximation(0, 0, 0). (08)
$x + y + 54z = 110$; $27x + 6y - z = 85$; $6x + 15y + 2z = 72$

c) Find the dominant eigenvalue and the corresponding eigenvector of the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ using Power method, considering$[1 \quad 1 \quad 1]^T$as the initial approximation. Carryout five iterations. (07)

6. a) (i) Define Similarity Transformation of square matrices. (02)

(ii) Write the system of equations $y_1^1 = 2y_1 + y_2 - 2y_3$, $y_2^1 = 3y_1 - 2y_2$, $y_3^1 = 3y_1 - y_2 - 3y_3$ in matrix form. (03)

b) Diagonalize the matrix A = $\begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$and hence find A^5. (08)

c) Determine the values of λ and μ such that the system of equations (07)

x + y + z = 6; x + 2y + 3z = 10; x + 2y + λ z = μ, may have

(i) unique solution (ii) an infinite number of solutions (iii) no solution.

UNIT - IV

7. a) (i) What should be the condition on $f(t)$ such that $L\{f'(t)\} = sL\{f(t)\} - f(0)$. (02)

(ii) Find the Laplace Transform of $e^{-3t}H(t-3)$. (03)

b) If $L(f(t)) = F(s)$ then prove that $L(t^n f(t)) = (-1)^n \frac{d^n}{ds^n}\{F(s)\}$, Where 'n' is a positive integer. (08)

c) A periodic function of period a, is defined by $f(t) = \begin{cases} E for\ 0 \le t < \frac{a}{2} \\ -E for \frac{a}{2} \le t < a \end{cases}$, (07)

Show that $L(f(t)) = tanh\left(\frac{as}{4}\right)$

8. a) (i) Define the periodic function with an example. (02)

(ii) Find $L[t^2 e^{-2t} \sin(2t)]$ (03)

b) Define unit step function. Express the function $f(t) = \begin{cases} sint, & 0 < t < \pi, \\ sin2t, & \pi < t < 2\pi, \\ sin3t, & t > 2\pi \end{cases}$ in terms of the Heaviside unit step functions and hence find its Laplace transform. (08)

c) Find the Laplace Transform of the functions (i) $t^5 e^{4t} Cosh(3t)$ (ii) $\frac{2\sin(t)\sin(5t)}{t}$ (07)

UNIT - V

9. a) (i) Define inverse Laplace transform. (02)

(ii) Find the inverse Laplace transform of $\frac{(s+2)^3}{s^6}$ (03)

b) Obtain the inverse Laplace transform of the function $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$, by using convolution theorem. (08)

c) Employ Laplace transform to solve the equation $y'' + 5y' + 6y = 5e^{2x}$; Given that $y(0) = 2, y'(0) = 1$. (07)

10. a) (i) Define the Convolution of two functions. (02)

(ii) Find $L^{-1}\left\{\frac{s+2}{s^2-4s+13}\right\}$. (03)

b) Using Laplace transform method , solve the following simultaneous equations. (08)

$\frac{dx}{dt} - 2y = cos2t; \frac{dy}{dt} + 2x = sin2t,$ given that $x(0) = 1, y(0) = 0$

c) Find the inverse Laplace transform of the functions (07)

(i) $\frac{se^{-\frac{s}{2}}+\pi e^{-s}}{s^2+\pi^2}$, (ii) $log\left\{\frac{s^2+4}{s(s+4)(s-4)}\right\}$