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M S RAMAIAH INSTITUTE OF TECHNOLOGY

(AUTONOMOUS INSTITUTE, AFFILIATED TO VTU)
BANGALORE – 560 054

SEMESTER END EXAMINATIONS – DEC 2013 / JAN 2014

Course & Branch	:	**B.E.- Common to all branches**	**Semester**	:	**I**
Subject	:	**Engineering Mathematics**	**Max. Marks**	:	**100**
Subject Code	:	**MAT101**	**Duration**	:	**3 Hrs**

Instructions to the Candidates:

- Answer one full question from each unit.

UNIT – I

1. a) i. Define the Jacobian of u,v,w with respect to x,y,z. (02)

ii. If $f=\frac{1}{x^2}+\frac{1}{xy}+\frac{\log x-\log y}{x^2+y^2}$ then prove that f is homogenous function and use Euler theorem to prove that $xf_x+yf_y=-2f$ (03)

b) Find the angle of intersection of the following pairs of curves (08)

$r=\frac{a\theta}{1+\theta};r=\frac{a}{1+\theta^2}$

c) If $z=f(x,y)$ where $x=e^u+e^{-v};y=e^{-u}-e^v$ show that (07)

$\frac{\partial z}{\partial u}-\frac{\partial z}{\partial v}=x\frac{\partial z}{\partial x}-y\frac{\partial z}{\partial y}$

2. a) i) State Euler's theorem on homogenous function. (10)

ii) For the curve $\frac{2a}{r}=1-\cos\theta$ find the slope of the tangent at $\theta=\frac{2\pi}{3}$ (03)

b) If ρ_1,ρ_2 be the radii of curvature at the extremities of any chord of the cardioic $r=a(1+\cos(\theta))$ which passes through the pole then show tha (08)

$\rho_1^2+\rho_2^2=\frac{16a^2}{9}$

c) If u=x+y;v=y+z;w=z+x find the inverse jacobian by expressing x,y,z in terms of u,v,w. (07)

UNIT-II

3. a) i) Evaluate $\lim_{x\to 0}\frac{\log(x)}{\cot(x)}$ (02)

ii) Expand 5^x in powers of x upto first three terms. (03)

b) If $xyz = 8$ find the values of x, y, z for which $u = \frac{5xyz}{x+2y+4z}$ is maximum (08)

c) State and prove Cauchy's Mean value theorem (07)

4. a) i) Can Rolle's theorem be applied to $|x|$ in [-1,1] (02)

ii) Find the values of x and y for which $x^2+y^2+6x=12$ has a minimum value and find this minimum value. (03)

b) Expand $f(x,y) = \tan^{-1}(y/x)$ in powers of (x-1) and (y-1) upto second degree terms. (08)

c) i) Find the values of a and b so that $\lim_{x\to 0} \frac{x(1+a\cos x) - b\sin x}{x^3} = 1$ (07)

ii) Evaluate $\lim_{x\to\pi/2} (Cosx)^{Cotx}$

UNIT-III

5. a) i. Write the expression to find the volume of solid of rotation for a Cartesian curve when rotated about x and y axes. (02)

ii. Evaluate $\int_0^{2\pi} \sin^8(\theta/2)\cos^6(\theta/2)\,d\theta$ (03)

b) Trace the curve $x^3 + y^3 = 3axy$, a>0 (08)

c) Prove that $\int_0^{\infty} \frac{e^{-x^2}}{\sqrt{x}}dx \times \int_0^{\infty} e^{-x^4} x^2\,dx = \frac{\pi}{4\sqrt{2}}$ (07)

6. a) i. Define beta function and write the trigonometric form of beta function. (02)

ii. Find the asymptotes parallel to the co-ordinate axes for the curve $x = ct; y = \frac{c}{t}$ Where c>0. (03)

b) Find the perimeter of the loop of the curve $9ay^2 = x(x-3a)^2$ where a>0. (08)

c) Prove that $\int_0^{2a} \frac{x^n}{\sqrt{2ax - x^2}}dx = \pi a^n \frac{(2n)!}{(n!)^2 2^n}$ (07)

UNIT-IV

7. a) i) Find the limits of integration in the double integral $\iint_R xy\,dx\,dy$ where R is the region bounded by the x-axis; ordinate x=2a and the curve $x^2 = 4ay$ (02)

ii. A particle moves along the curve $\vec{r} = (t^3 - 4t)\hat{i} + (t^2 + 4t)\hat{j} + (8t^2 - 3t^3)\hat{k}$, where t denotes the time. Find the magnitude of acceleration along the tangent at t=2. (03)

b) By changing the order of integration evaluate $x=\int_{x=0}^{\infty}\int_{y=0}^{x} x e^{-x^2/y}\,dy\,dx$ (08)

c) Find the volume of the tetrahedron bounded by the planes x=0;y=0;z=0 and (07)

$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

8. a) i. write the double integral formula to find the area of a region R for cartesian and polar curves. (02)

ii. A Particle moves along the curve $x=e^{-t};y=2\cos(3t);z=2\sin(3t)$ where t is the time.Find the magnitude of velocity at t=0. (03)

b) Evaluate $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} xyz\,dz\,dy\,dx$ (08)

c) Evaluate $\int_0^{2a}\int_0^{\sqrt{2ax-x^2}} x^2\,dy\,dx$ by changing to polar coordinates. (07)

UNIT-V

9. a) i. State Stoke's theorem. (02)

ii. Find the total work done by the force $\vec{F}=2xy\hat{i}-4z\hat{j}+5x\hat{k}$ along the curve $x=t^2;y=2t+1;z=t^3$ (03)

b) Verify Green's theorem for $\int_c (xy+y^2)dx+x^2dy$ where c is bounded by $y=x$ and $y=x^2$ (08)

c) Find the values of the constants a, b and c such that $\vec{F}=(x+2y+az)\hat{i}+(bx-3y-z)\hat{j}+(4x+cy+2z)\hat{k}$ is conservative. Also find its Scalar potential. (07)

10. a) i. Explain the geometrical meaning of gradient of a scalar field (02)

ii. Evaluate $\int_s (yz\hat{i}+zx\hat{j}+xy\hat{k}).\hat{n}\,ds$ where s is the surface of the cube formed by X=0,x=1;y=0,y=1;z=0,z=1 using Gauss divergence theorem. (03)

b) Using Stoke's theorem evaluate $c\int[(x+y)dx+(2x-z)dy+(y+z)dz]$ where c is the boundary of the triangle with vertices at (2,0,0),(0,3,0),(0,0,3). (08)

c) Find the values of the constants a and b such that the surfaces $x^2+ayz=3$ and $bx^2y+z^3=(b-8)y$ are orthogonal at the point (1,1,-2) (07)

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