MAT101

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M S RAMAIAH INSTITUTE OF TECHNOLOGY

(AUTONOMOUS INSTITUTE, AFFILIATED TO VTU)
BANGALORE – 560 054

SEMESTER END EXAMINATIONS – JANUARY 2016

Course & Branch	:	**B.E- Common to All Branches**	**Semester**	:	**I**
Subject	:	**Engineering Mathematics-I**	**Max. Marks**	:	**100**
Subject Code	:	**MAT101**	**Duration**	:	**3 Hrs**

Instructions to the Candidates:

- Answer ***one*** full question from each unit.

UNIT – I

1. a) Define composite function of two variables — CO2 (02)

b) Find the pedal equation for the curve $r = a\left(1+\cos\theta\right)$. — CO1 (03)

c) If $u = \cos^{-1}\left(\dfrac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, prove that (i) $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = -\dfrac{1}{2}\cot u$ — CO2 (08)

(ii) $x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x \partial y} + y^2\dfrac{\partial^2 u}{\partial y^2} = \left(1 - \dfrac{\operatorname{cosec}^2 u}{2}\right)\dfrac{\cot u}{2}$

d) If $u = 3x+2y-z$, $v = x-2y+z$ and $w = x^2+2xy-xz$ show that u, v, w are functionally related and find the relation. — CO2 (07)

2. a) Define Jacobian of x, y, z with respect to u, v, w. — CO2 (02)

b) Verify $\dfrac{\partial^2 u}{\partial x \partial y} = \dfrac{\partial^2 u}{\partial y \partial x}$. Given $u = 2^{xy}$. — CO2 (03)

c) Find the angle between the radius vector and the tangent and also the slope of the tangent for the curve $\dfrac{2a}{r} = 1-\cos\theta$ at $\theta = 2\pi/3$. — CO1 (08)

d) If $z = f(x,y)$ and $x = u-v$, $y = uv$ then prove that — CO2 (07)

(i) $(u+v)\dfrac{\partial z}{\partial x} = u\dfrac{\partial z}{\partial u} - v\dfrac{\partial z}{\partial v}$ (ii) $(u+v)\dfrac{\partial z}{\partial y} = \dfrac{\partial z}{\partial u} + \dfrac{\partial z}{\partial v}$

UNIT – II

3. a) Write the reduction formula for (i) $\int_0^{\pi/2} \sin^6\theta\, d\theta$ (ii) $\int_0^{\pi/2} \sin^3\theta \cos^5\theta\, d\theta$. — CO3 (02)

b) Find the tangents at the origin for the curve $y^2(x^2+y^2) + a^2(x^2-y^2) = 0$. — CO3 (03)

c) Trace the curve $r = a(1+\cos\theta)$. CO3 (08)

d) Evaluate $\int_0^\infty e^{-\alpha x}\frac{\sin x}{x}dx$ using differntiation under the integral sign and hence find $\int_0^\infty \frac{\sin x}{x}dx$. CO3 (07)

4. a) Write the expressions to find surface area of the solid for a Cartesian curve when rotated about x and y axes. CO3 (02)

b) Evaluate $\int_0^{\pi/2}\sqrt{1+\sin 2x}\,dx$. CO3 (03)

c) Find the area under one arch of the cycloid $x = a(\theta - \sin\theta);\ y = a(1-\cos\theta);\ 0 \le \theta \le 2\pi$. CO3 (08)

d) Evaluate (i) $\int_0^\infty \frac{x^2}{\left(1+x^2\right)^{7/2}}dx$. (ii) $\int_0^1 \frac{x^7}{\sqrt{\left(1-x^2\right)}}dx$ CO3 (07)

UNIT – III

5. a) With the help of a neat diagram plot the region of integration in the double integral $\int_0^1\int_0^{\sqrt{1-x^2}} f(x,y)\,dy\,dx$. CO4 (02)

b) Evaluate $\int_0^2\int_1^z\int_0^{yz} xyz\,dx\,dy\,dz$. CO4 (03)

c) Find the volume of the tetrahedron bounded by the planes $x=0,\ y=0\ z=0$ and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$. CO4 (08)

d) Evaluate $\int_0^1\int_{x^2}^{2-x} xy\,dy\,dx$ by changing the order of integration. CO4 (07)

6. a) Write the formula to find the area of a region R for Cartesian and polar curves in terms of double integration. CO4 (02)

b) Change the order of integration in $\int_0^4\int_x^{2\sqrt{x}} f(x,y)\,dy\,dx$ CO4 (03)

c) Find the area lying inside the cardioid $r = a(1+\cos\theta)$ and outside the circle $r = a$. CO4 (08)

d) Using the transformations $x+y=u; x-y=v$ evaluate the integral $\iint_R (x^2+y^2)\,dxdy$ where R is the region bounded by $u=0,\ u=2,\ v=0$ and $v=2$. CO4 (07)

UNIT – IV

7.	a)	Define gradient of a scalar differentiable function.	CO5	(02)
	b)	Find the angle between the directions of the velocity and acceleration vectors at time of a body with position vector $\vec{r}=(t^2+1)\hat{i}-2t\hat{j}+(t^2-1)\hat{k}$	CO5	(03)
	c)	Prove that $\nabla\cdot(\vec{A}\times\vec{B})=\vec{B}\cdot(\nabla\times\vec{A})-\vec{A}\cdot(\nabla\times\vec{B})$	CO5	(08)
	d)	What is the directional of $\phi=xy^2+yz^3$ at the point (2,1,1) in the direction normal to the surface $x\log z-y^2=-4$ at (-1,2,1)	CO5	(07)
8.	a)	Define curl of a vector field.	CO5	(02)
	b)	A particle moves along a curve $x=2t^2,\ y=t^2-4t,\ z=3t-5$. Find the velocity and acceleration at time $t=1$.	CO5	(03)
	c)	Show that $\hat{f}=(y+z)\,\hat{i}+(z+x)\,\hat{j}+(x+y)\hat{k}$ is irrotational. Also find scalar potential.	CO5	(08)
	d)	If $\vec{f}=grad(x^3+y^3+z^3-3x^3y^3z^3)$ find $div(\vec{f})$ and $Curl(\vec{f})\ at\,(-1,0,1)$.	CO5	(07)

UNIT – V

9.	a)	State Gauss Divergence theorem	CO5	(02)
	b)	Evaluate $\int_C y^2dx-2x^2dy$ along the parabola $y=x^2$ from (0,0) to (2,4).	CO5	(03)
	c)	State and prove Green's Theorem in a plane.	CO5	(08)
	d)	Apply Stoke's theorem to evaluate $\int_C \vec{f}\cdot d\vec{r}$. $\vec{f}=(x^2+y^2)\hat{i}-2xy\,\hat{j}$ and C is the rectangle in the xy plane bounded by $x=0, x=a, y=0, y=b$.	CO5	(07)
10.	a)	State Stoke's theorem.	CO5	(02)
	b)	Find the work done in moving a particle once around the circle $x^2+y^2=9$ in the xyplane if the field force is $\vec{f}=(2x-y-z)\hat{i}+(x+y-z^2)\,\hat{j}+(3x-2y+4z)\hat{k}$.	CO5	(03)
	c)	Apply Green's theorem to prove that the area enclosed by a plane curve is $\frac{1}{2}\int_C xdy-ydx$. Hence find the area between the parabolas $y^2=4x$ and $x^2=4y$.	CO5	(08)
	d)	Apply Gauss-Divergence theorem to evaluate $\int_S \vec{f}.\hat{n}\,ds$ where $\vec{f}=4x\hat{i}-2y^2\hat{j}+z^2\hat{k}$ and s is the surface bounding the region $x^2+y^2=4$, $z=0$ and $z=3$.	CO5	(07)

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