RAMAIAH
Institute of Technology

BASICIS OF CIVIL ENGINEERING & MECHANICS

Course code:CV14/CV24

Credits:3:0:0

Topics Covered

Force- Definition, classification of force systems, composition and resolution of forces.

Couple, Moment of Couple

MECHANICS

Mechanics

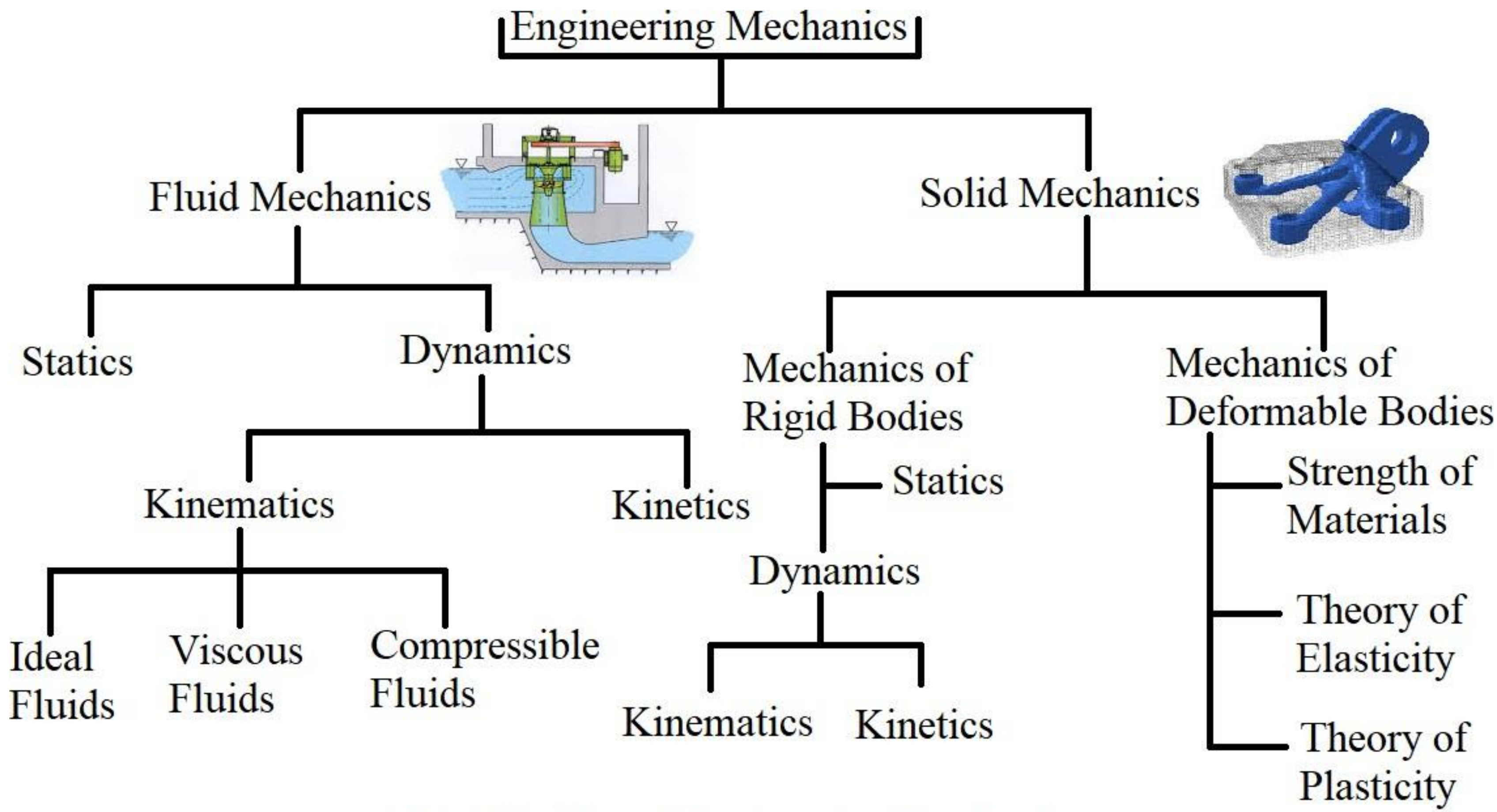
A branch of physical science that deals with energy and forces and their effect on bodies.

Engineering Mechanics

It is the application of mechanics to the solution of Engineering problems. Broadly classified into three types

1. Mechanics of Rigid Bodies- Statics and Dynamics
2. Mechanics of deformable bodies- SOM, TOP, TOE
3. Mechanics of fluids-Compressible and incompressible

MECHANICS



Classification of Engineering Mechanics

- Particle: A body of infinitely small volume whose mass can be neglected, is called a particle.
- Rigid body: A rigid body is one in which the positions of the constituent particles do not change under the application of external forces, such as the position of particles shown inFigure 1.

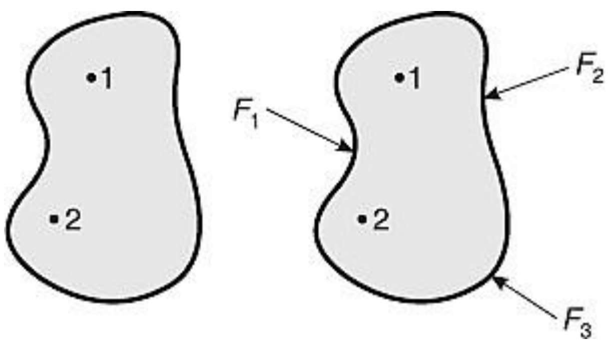


Figure. 1 Rigid body

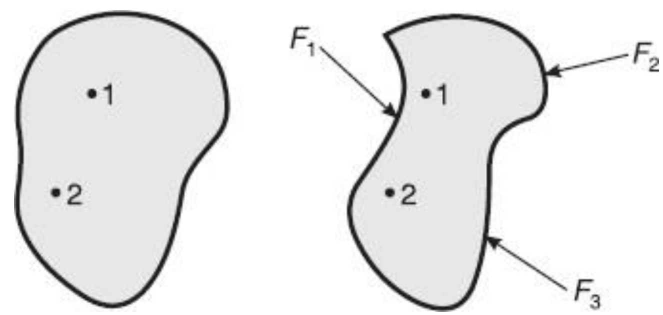


Figure. 2 Deformable body

- Continuum: A continuous distribution of molecules in a body without intermolecular space is called continuum.
- Mass: The total amount of matter present in a body is known as it's mass. The unit of mass is kg(kilogram)
- Weight: The force that gravitation exerts upon a body, equal to the mass of the body times the local acceleration of gravity $W=mg$

Elements of a Force or Characteristics of a Force

A force can be identified by its four characteristics:

(i) Magnitude: The length of the vector represents the magnitude of force

(ii) Direction: The direction of a force can be represented by an arrowhead.

(iii) Line of action: It is the line along which the force acts.

(iv) Point of application: It is the point at which the force acts.

Force system:

If two or more forces are acting on a body or a particle, then it is said to be a force system, such as that shown in Figure 3.

The types of force system are:

1. Coplanar force system
2. Non-coplanar force system
3. Collinear force system.

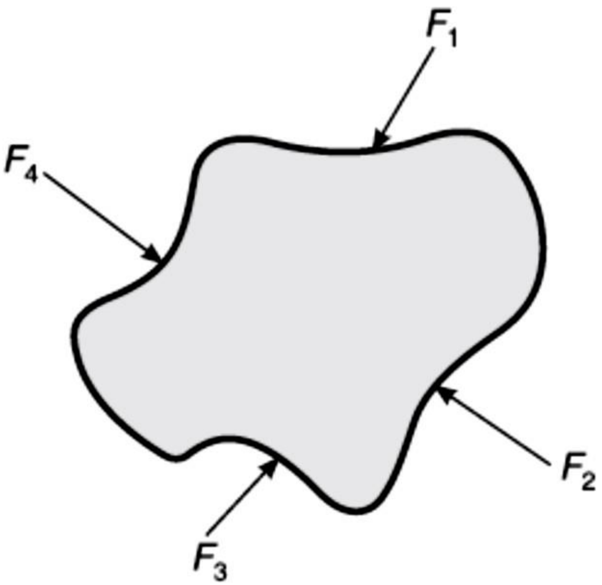


Figure.3 Force system

Coplanar force system

If two or more forces are acting in a single plane, then it is said to be a coplanar force system. The types of coplanar force system are:

(i) Coplanar concurrent force system

(ii) Coplanar non-concurrent force system

(iii) Coplanar parallel force system.

(i) Coplanar concurrent force system:

If two or more forces are acting in a single plane and their lines of action pass through a single point, then it is said to be a coplanar concurrent force system. Shown in Figure.4.

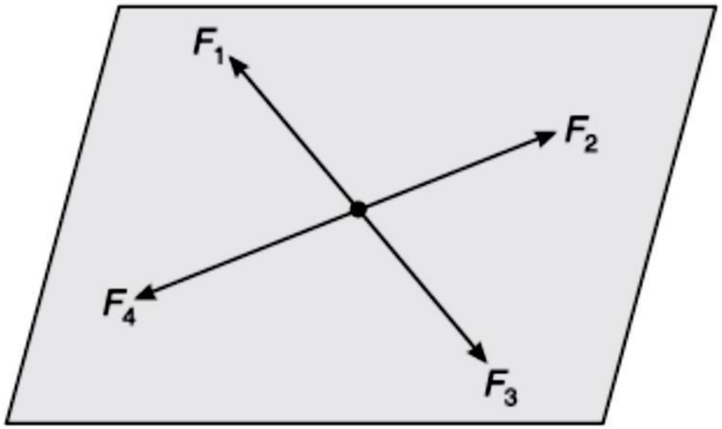


Figure.4 Coplanar concurrent force system.

(ii) Coplanar non-concurrent force system

If two or more forces are acting in a single plane and their lines of action do not meet at a common point, then the forces constitute a coplanar non-concurrent force system. Shown in Figure.5.

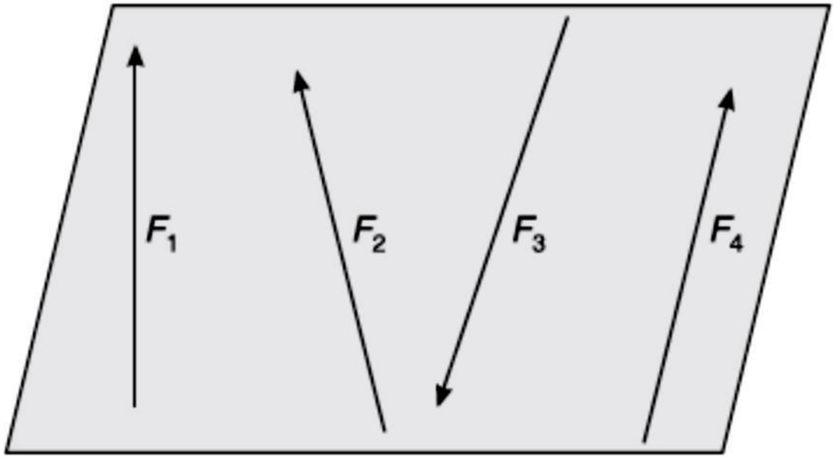


Figure.5 Coplanar non concurrent force system.

(iii) Coplanar parallel force system.

If two or more forces are acting in a single plane with their lines of action parallel to one another, then it is said to be a coplanar parallel force system.

The coplanar parallel force system is of two types:

(i) Like parallel force system: All the forces act parallel to one another and are in the same direction, as shown in Figure 6.

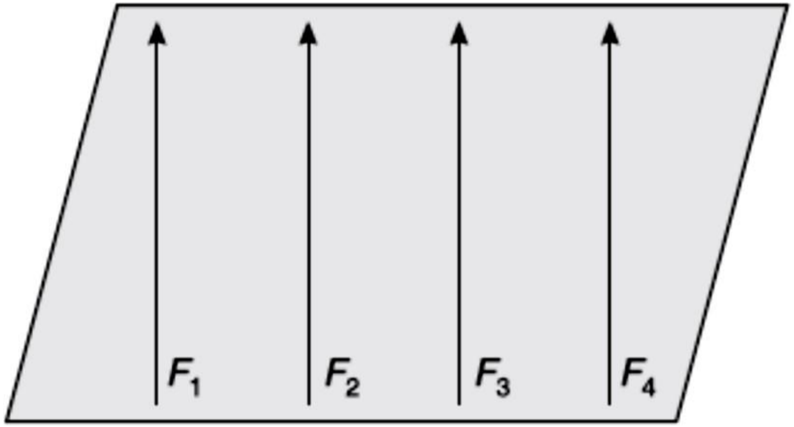


Figure.6 Like parallel force system

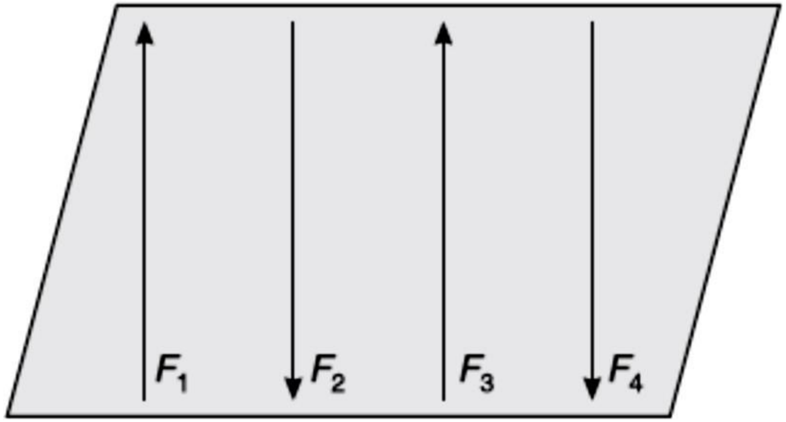


Figure.7 Unlike parallel force system

Non-coplanar force system

If two or more forces are acting in different planes, the forces constitute a non-coplanar force system. Such a system of forces can be,

(i) Non-coplanar concurrent force system

(ii) Non-coplanar non-concurrent force system

(iii) Non-coplanar parallel force system.

(i) Non-coplanar concurrent force system

If a system has two or more forces acting on different planes but pass through the same point, then it is said to be a non-coplanar concurrent force system. As shown Figure.8.

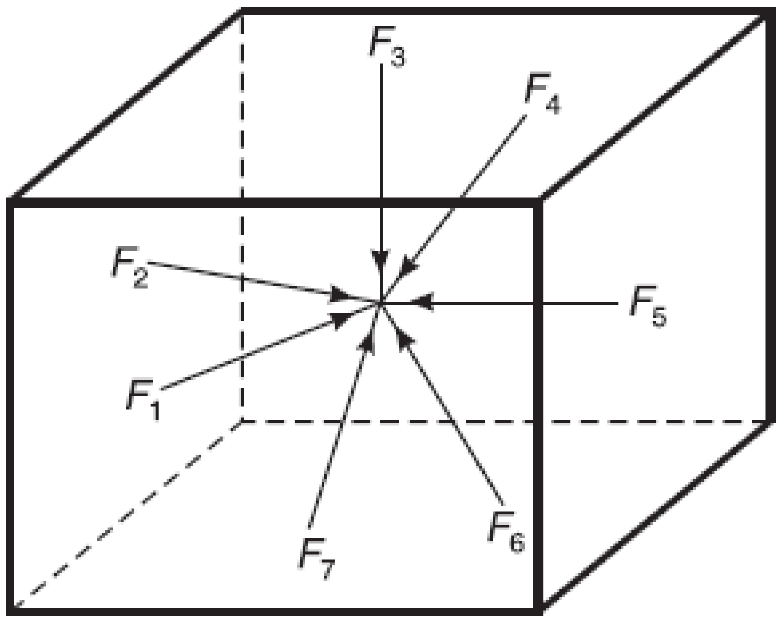


Figure.8 Non-coplanar concurrent force system

(ii) Non-coplanar non-concurrent force system

If two or more forces are acting on different planes but do not pass through the same point, they constitute a non-coplanar non-concurrent force system. As shown in Figure 9.

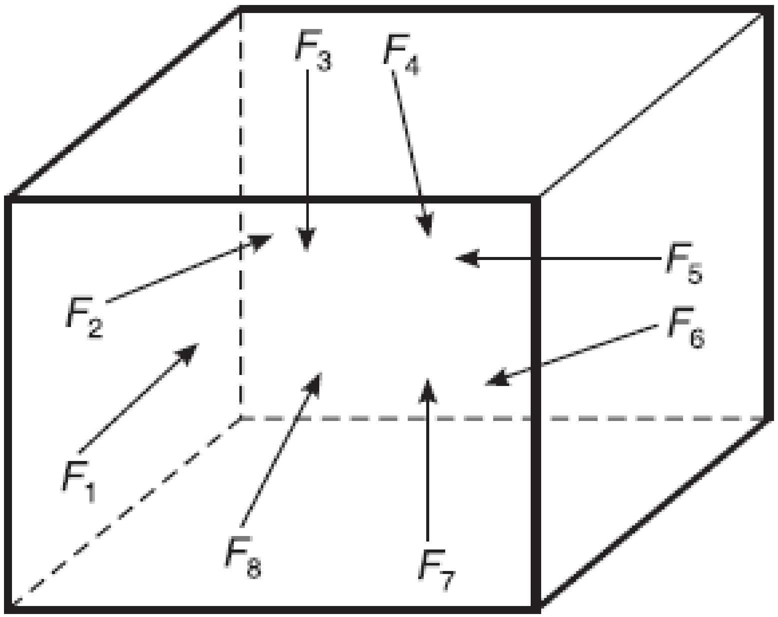


Figure.9 Non-coplanar non-concurrent force system

(iii) Non-coplanar parallel force system.

If two or more forces are acting in different planes and are parallel to one another, the system is said to be a non-coplanar parallel force system. As shown in Figure 10.

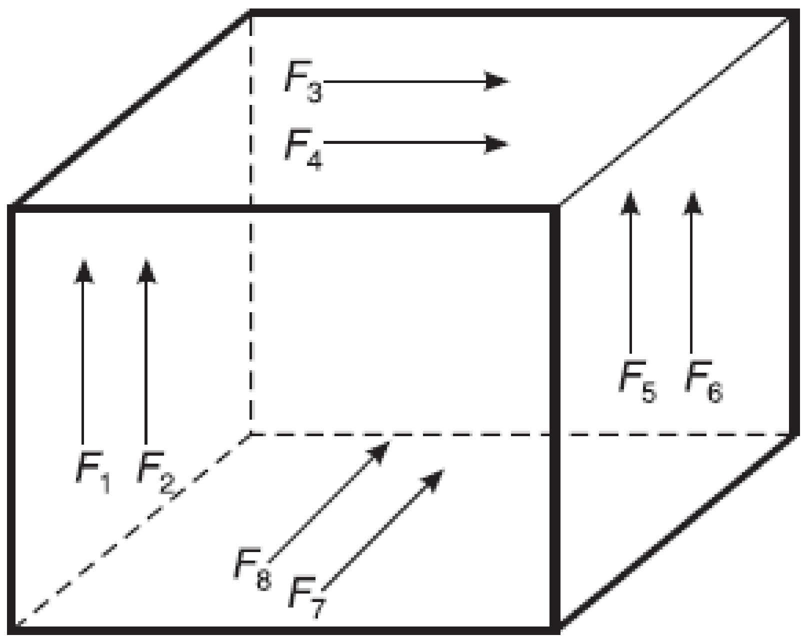


Figure.10 Non-coplanar parallel force system

Collinear force system

If the lines of action of two or more forces coincide with one another, it is called a collinear force system as shown in Figure.11.

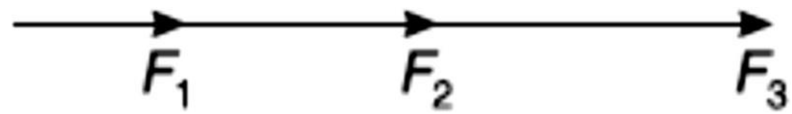


Figure.11 Collinear force system.

Non-collinear force system

If the lines of action of the forces do not coincide with oneanother, it is called a non-collinear force system as shown inFigure.12.

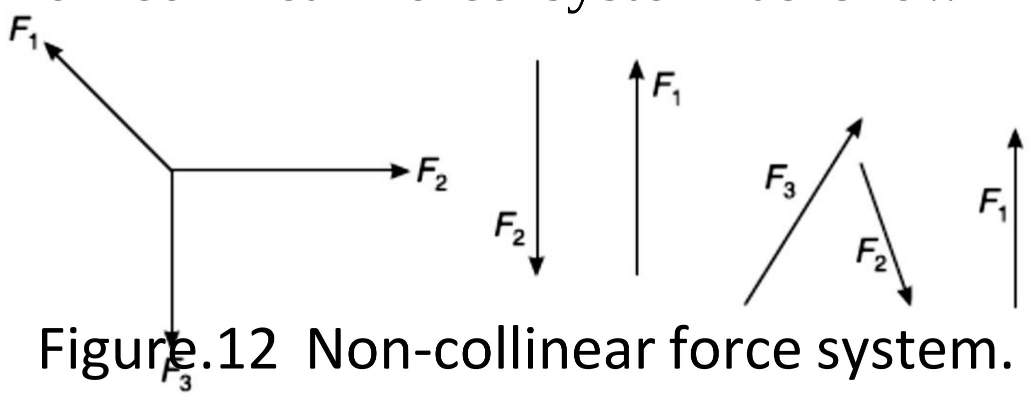


Figure.12 Non-collinear force system.

Principle of Transmissibility of Forces

This principle states that a force can be transmitted from one point to another point along the same line of action such that the effect produced by the force on a body remains unchanged.

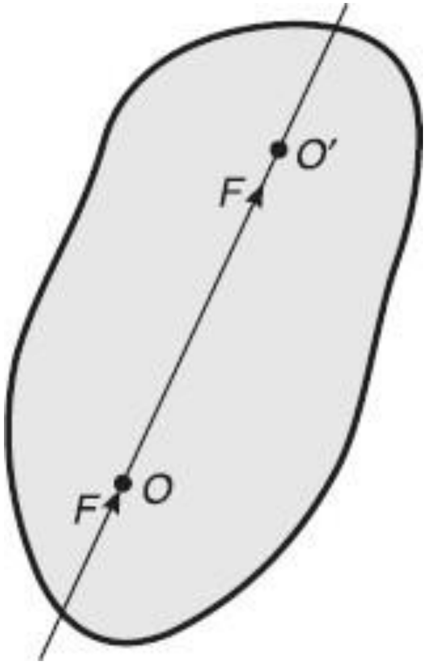


Figure.13 Transmissibility of force F from point O to O’.

Resolution of a Force

The process of splitting of a force into its two rectangular components (horizontal and vertical) is known as resolution of the force, as shown in Figure.14.

In this figure, F is the force which makes an angle θ with the horizontal axis, and has been resolved into two components, namely Fx and Fy, along the x-axis and y- axis respectively.

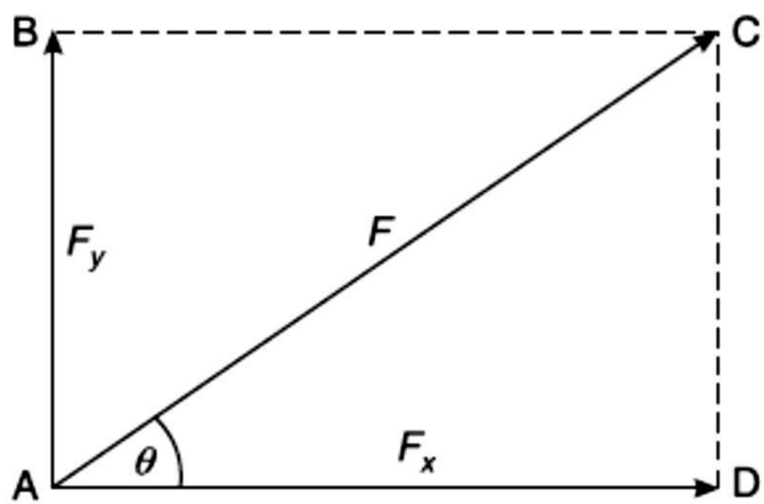
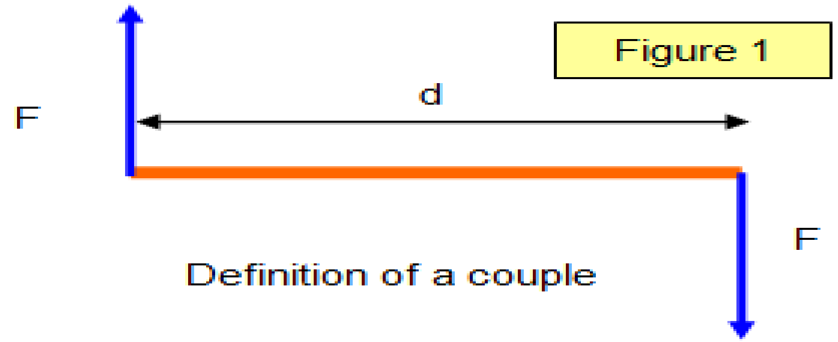


Figure.14 Resolution of a force.

Couple

A couple consists of two parallel forces that are equal in magnitude, opposite in sense and do not share a line of action. It does not produce any translation, only rotation. The resultant force of a couple is zero. but, the resultant of a couple is not zero; it is a pure moment.



Moment of a couple

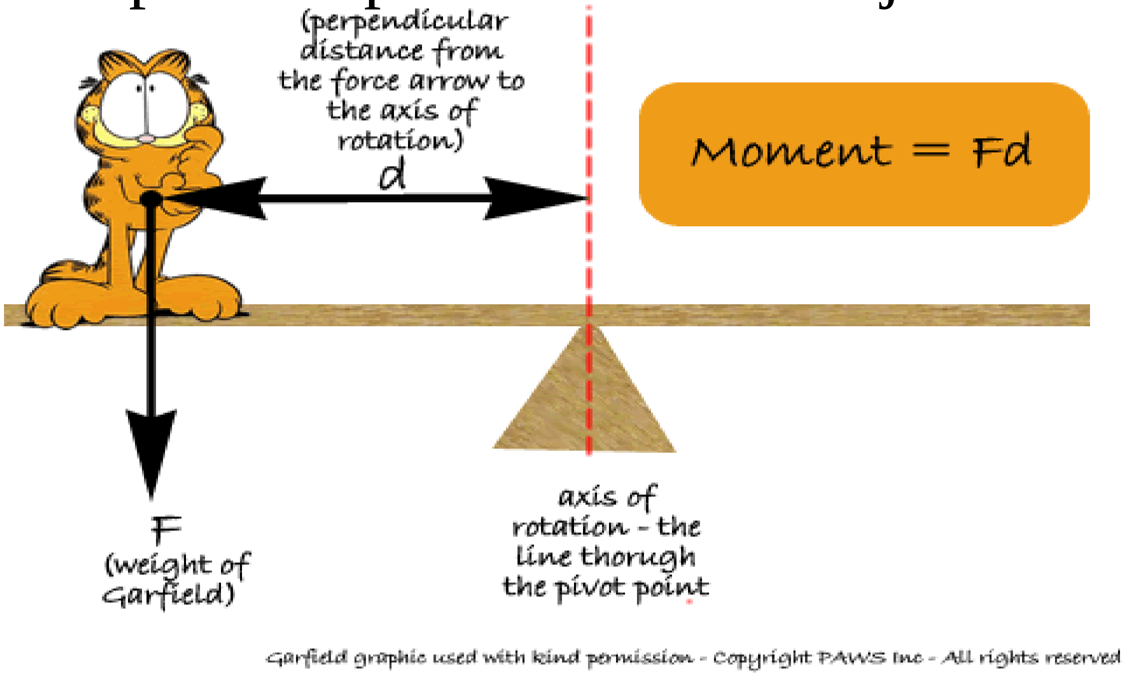
The tendency of a force is to rotate a body. It is measured by the moment of the force. The product of one of the two forces of a Couple and the perpendicular distance between their lines of action (called the arm of the Couple) is called the **Moment of Couple**.

Characteristics of couple

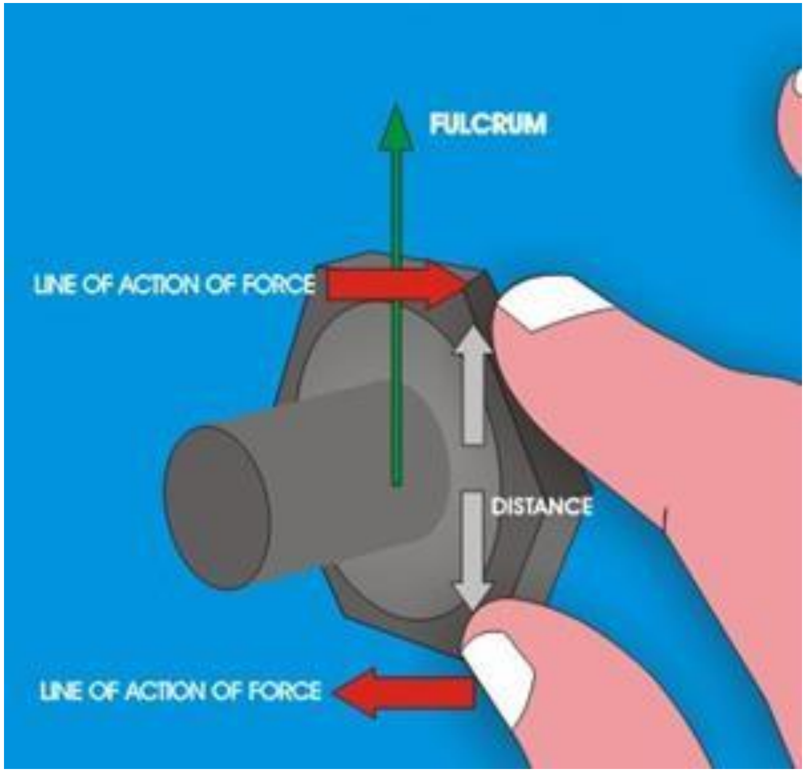
1. The algebraic sum of the forces consisting the couple is zero
2. The algebraic sum of the moment of the forces constituting the couple about any point is the same and equal to the moment of the couple itself.
3. A couple cannot be balanced by a single force but can be balanced only by a couple but of opposite sense.
4. Any number of coplanar couples can be reduced to a single couple whose magnitude will be equal to the algebraic sum of the moments of all the couples.

Moment of Force

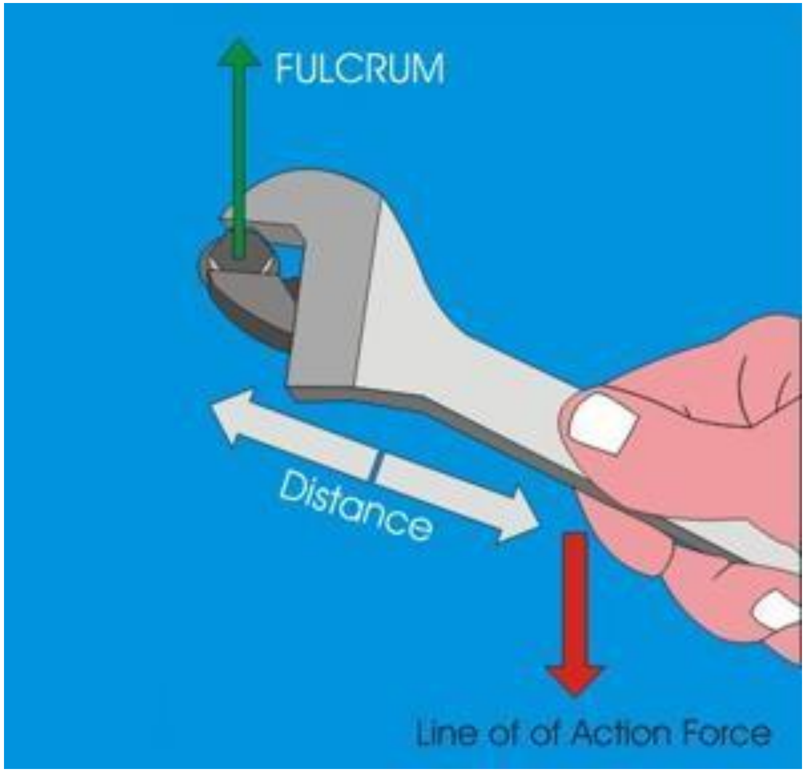
1. The turning effect of a force (torque) is known as the moment.
2. It is the product of the force multiplied by the perpendicular distance from the line of action of the force to the pivot or point where the object will turn.



Moment of Force

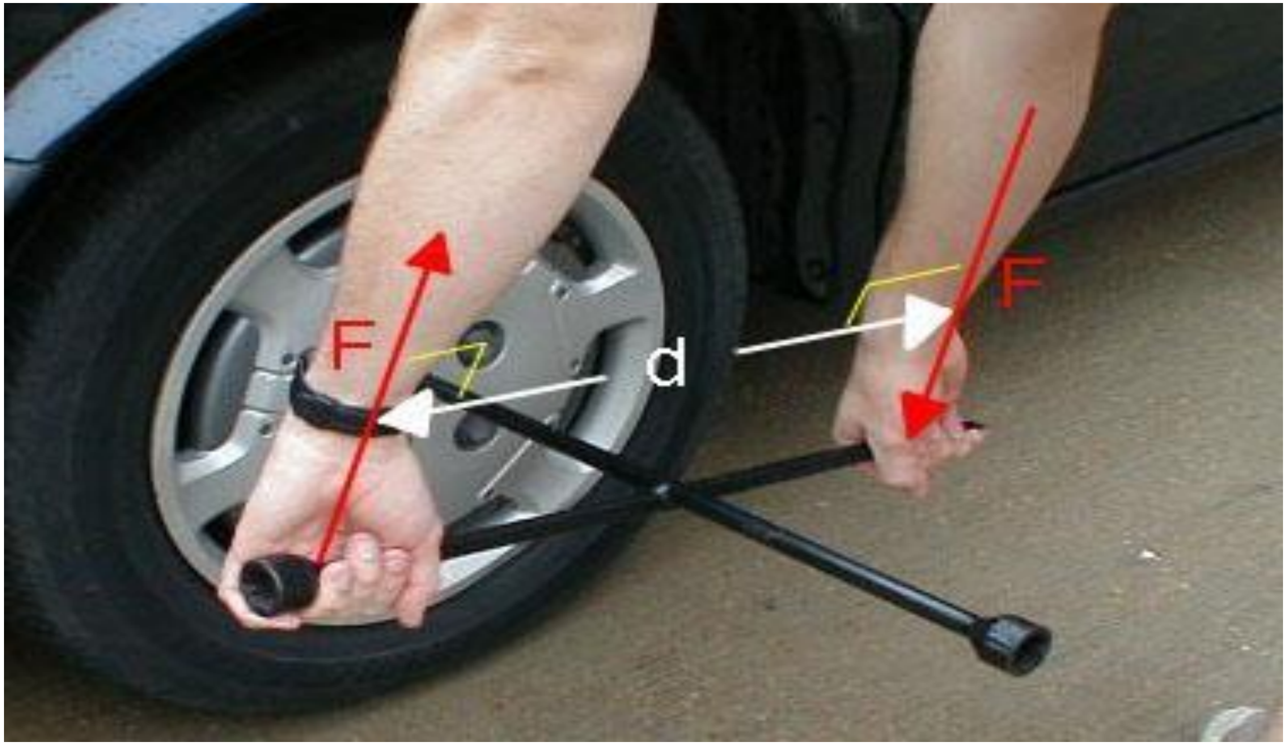


SMALL MOMENT
The distance from the fulcrum to the line of action of force is very small



LARGE MOMENT
The distance from the fulcrum to the line of action of force is large

Moment of Force

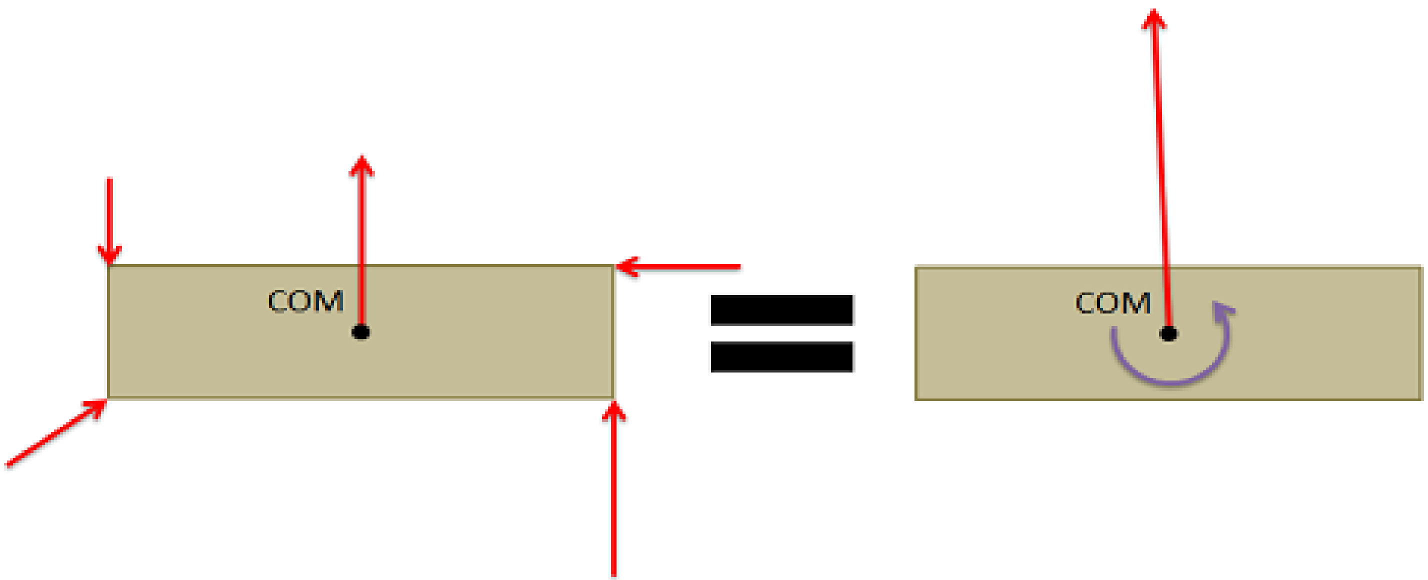


- Moments taken are about a point are indicate as being clockwise or counterclockwise
- For the sake of uniformity in calculation, assume clockwise to be +ve and counterclockwise to be -ve.

- Moment can expressed as Nm or kNm

Equivalent force couple system

Every set of forces and moments has an **equivalent force couple system**. This is a single force and pure moment (couple) acting at a single point that is **statically equivalent** to the original set of forces and moments



COUPLE

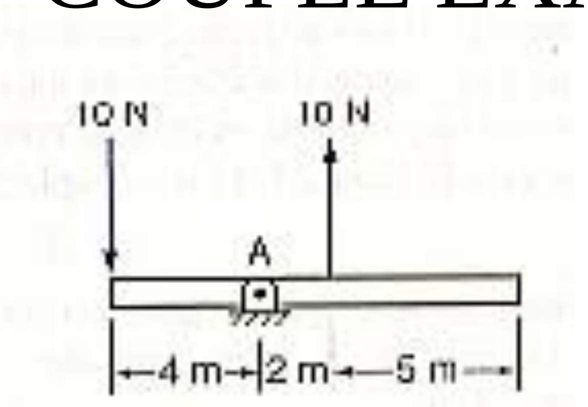
- These force could have been treated as a couple, which consists of two forces that are:
  1. Equal
  2. Acting in opposite direction
  3. Separated by some perpendicular distance d
- These three requirement of couple, from the example, we have;

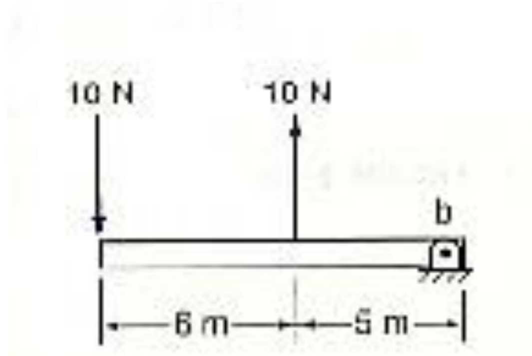
Couple moment = (F) (d)

= -5 (20)

= -100 kNm

COUPLE EXAMPLE





M_A=-(10N)(4m)-(10N)(2m)
=-40-20
=-60 N.m
=60N.m

M_b=-(10N)(11m)(10N)(5m)
= -110+50
= -60 N.m
= 60N.m

Resultant A force and couple system

- When a rigid body is subjected to a system of forces and couple moments
  - The external effects on the body by replacing the system by an equivalent single resultant force acting at a specified point **O** and a resultant couple moment

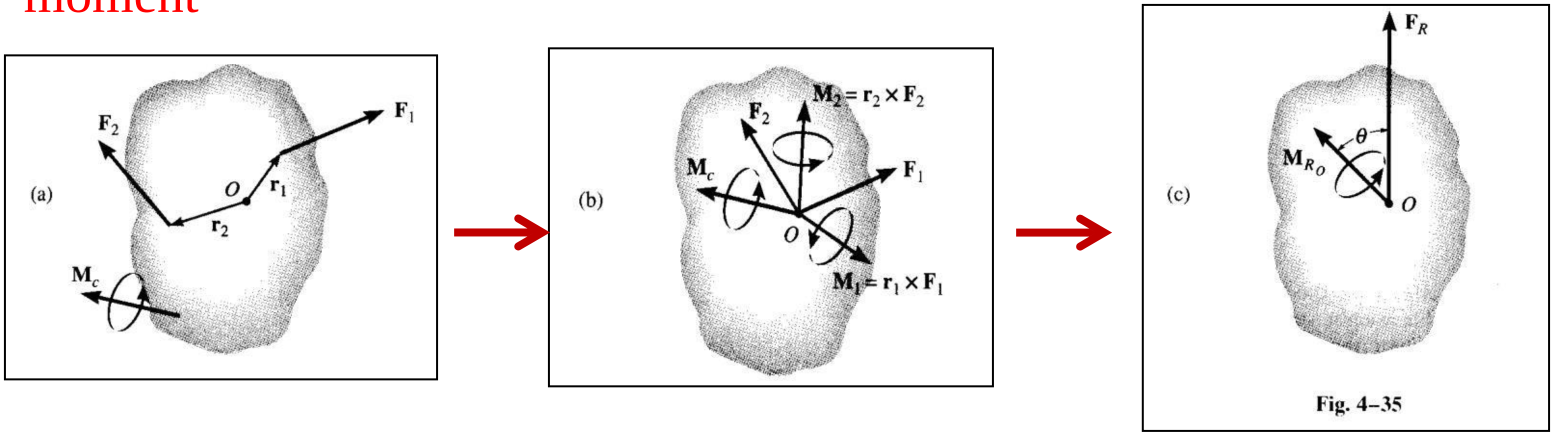
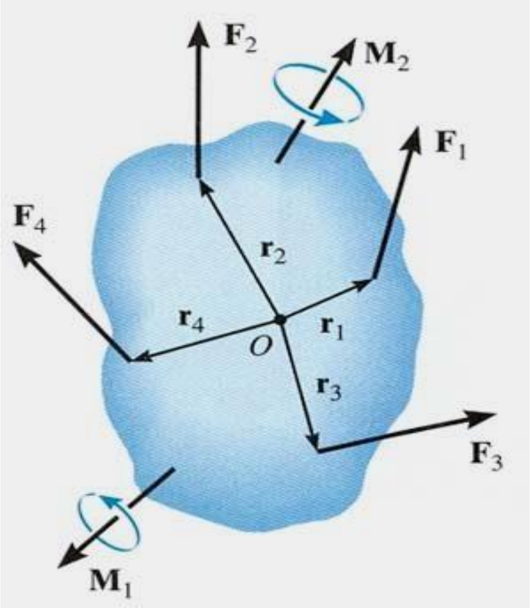


Fig. 4–35

  - Point **O** is not on the line of action of the forces, an equivalent effect is produced if the forces are moved to point **O** and the correspondingcouple moments $M_1=r_1xF_1$ and $M_2=r_2xF_2$ are applied to body

AN EQUIVALENT SYSTEM



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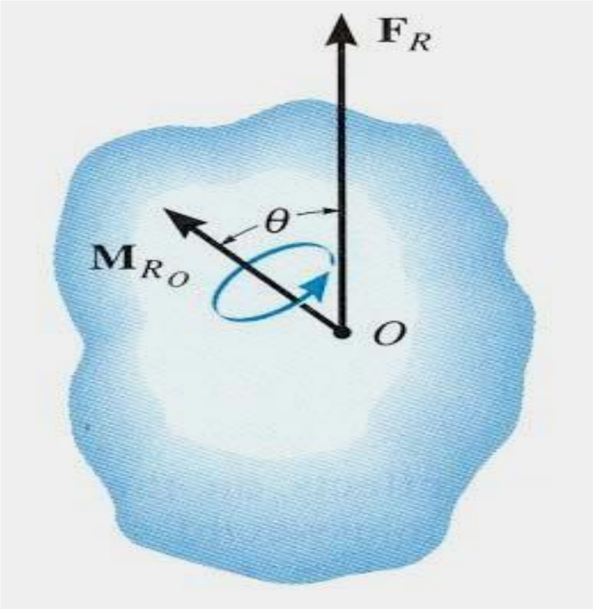
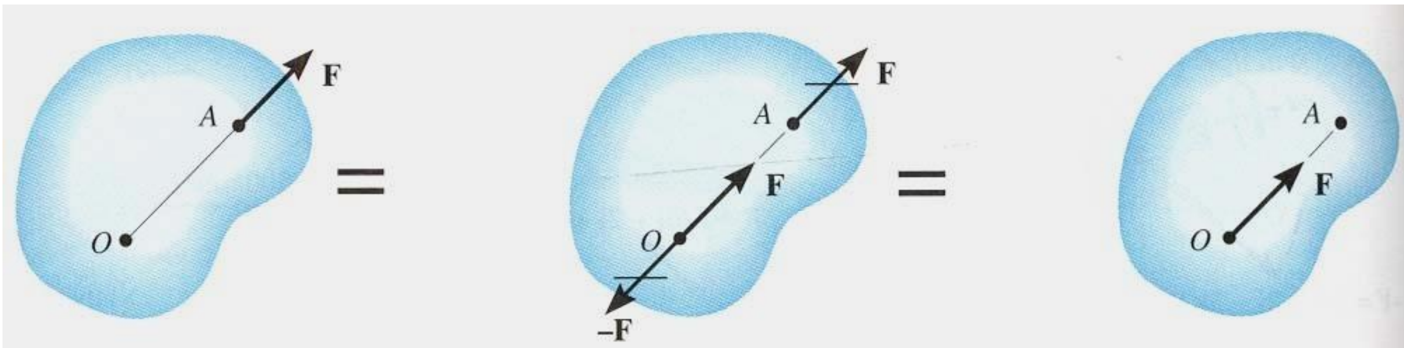


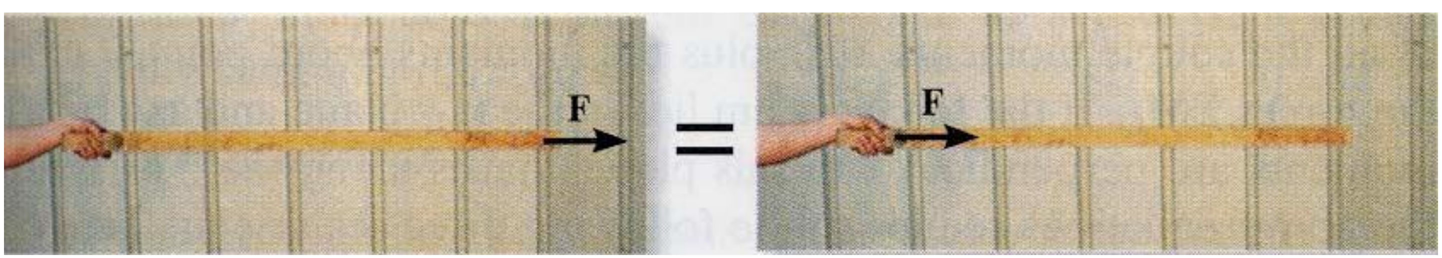
•When a number of forces and couple moments are acting on a body, it is easier to understand their overall effect on the body if they are combined into a single force and couple moment having the same external effect

•The two force and couple systems are called equivalent systems since they have the same external effect on the body.

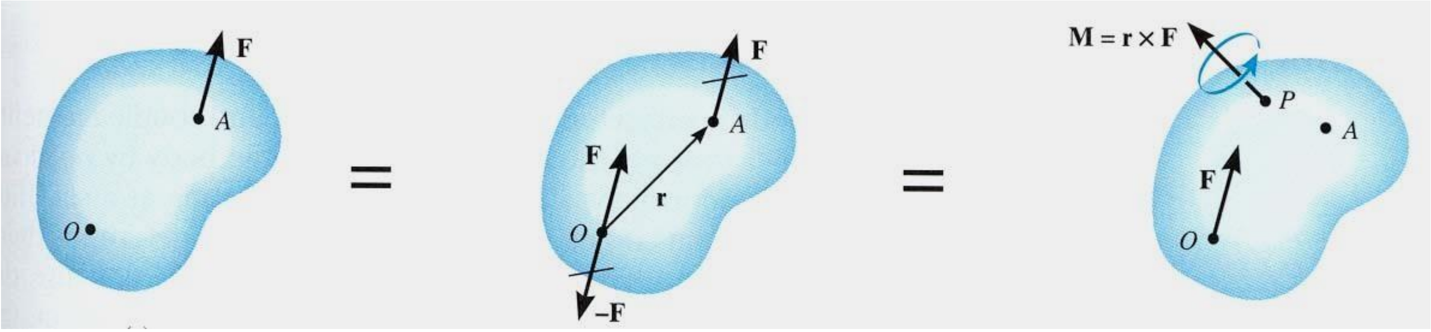
MOVING A FORCE ON ITS LINE OF ACTION



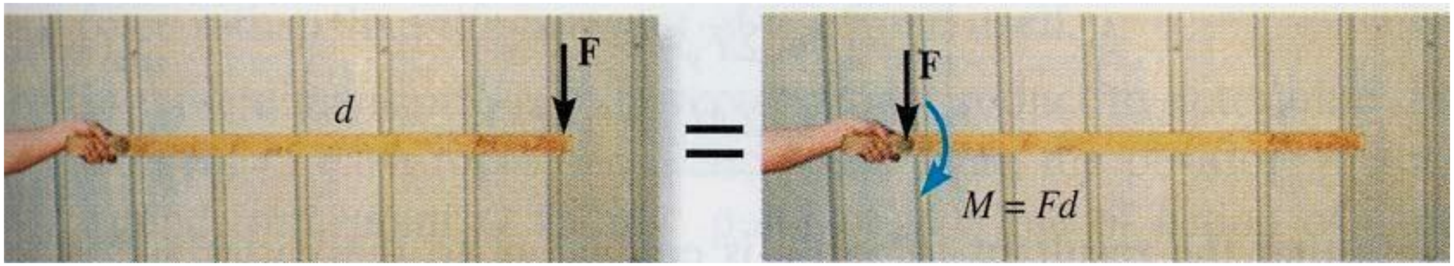
Moving a force from A to O, when both points are on the vectors' line of action, does not change the external effect.Hence, a force vector is called a sliding vector. (But the internal effect of the force on the body does depend on where the force is applied).



MOVING A FORCE OFF OF ITS LINE OF ACTION



Moving a force from point A to O (as shown above) requires creating an additional couple moment. Since this new couple moment is a "free" vector, it can be applied at any point P on the body.



Newton’s law of motion

- **Newton’s First Law of Motion**

Newton’s 1st law states that a body at rest or uniform motion will continue to be at rest or uniform motion until and unless a net external force acts on it.

- **Newton’s Second Law of Motion**

Newton’s 2nd law states that the acceleration of an object as produced by a net force is directly proportional to the magnitude of the net force, in the same direction as the net force, andinversely proportional to the object’s mass.

Newton’s law of motion

Mathematically, we express the second law of motion as follows:

fadP/dt

⇒fa(mv−mu)/t

⇒fam(v−u)/t

⇒fama

⇒f=kma

In the equation, k is the constant of proportionality, and it is equal to 1 when the values are taken in the SI unit.

Hence, the final expression will be,

$$F=ma$$

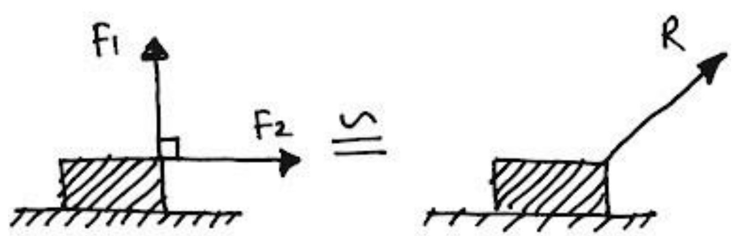
Newton’s law of motion

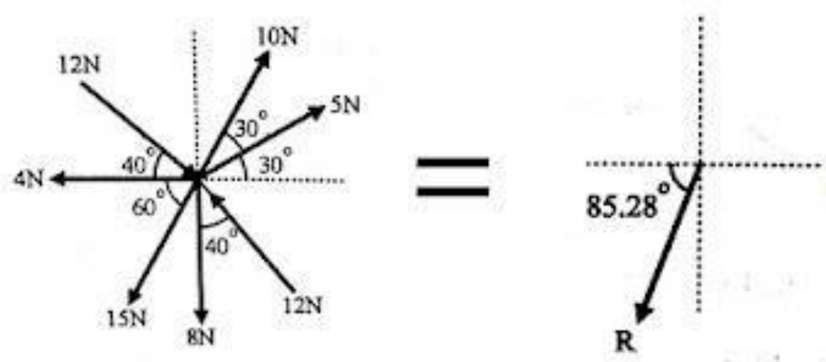
- **Newton’s Third Law of Motion**

Newton’s 3rd law states that there is an equal and opposite reaction for every action.

Composition of forces

It is the process of combining a number of forces into a single force such that net effect produced by single force is equal to the algebraic sum of the effects produced by the individual forces. The single force is called resultant which produces same effect on the body as that produced by the individual forces acting together





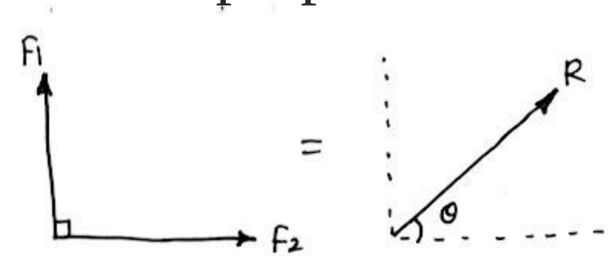
- **<u>Method Of Composition (To find "R")</u>**

A) Composition of perpendicular or orthogonal forces:-

When two or more coplanar concurrent or non-concurrent forces acting on a body then the resultant can determine by following procedure.

If F1 and F2 are the two forces which are perpendicular to each other as shown in fig,

Then resultant can be found out by following equation.



Magnitude of resultant

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

and Direction $\tan\theta = \dfrac{\Sigma F_y}{\Sigma F_x}$

Where,

ΣF_x = Algebraic sum of all x-components (or x component of resultant)

ΣF_y = Algebraic sum of all y-components (or y component of resultant)

θ = Angle of 'R' with x-axis.

B) Composition by parallelogram law of forces:-

Parallelogram law: If two forces are acting simultaneously on a particle and away from the particle, with the two adjacent sides of the parallelogram representing both the magnitude and direction of forces, the magnitude and direction of the resultant can be represented by the diagonal of the parallelogram starting from the common point of the two forces. See Figure 2.18.

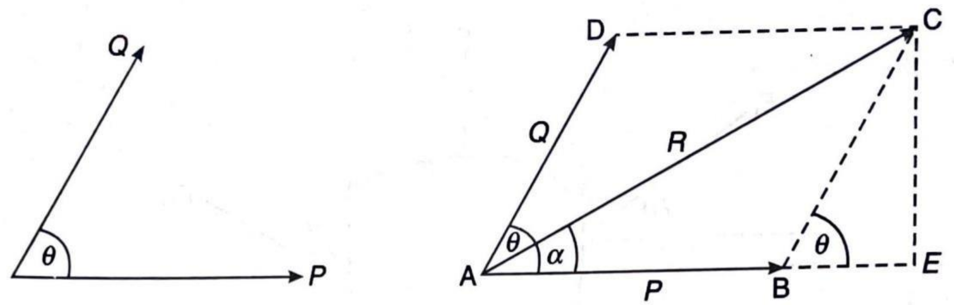


Figure 2.18 Parallelogram law of forces.

Let P and Q be the two forces, represented by the sides AB and AD of the parallelogram, the resultant can then be represented by AC as shown below:

To find the magnitude R of the resultant, consider the Δ CAE, where

$$AC^2 = AE^2 + CE^2$$
$$= (AB + BE)^2 + (CE)^2$$

B) Composition by parallelogram law of forces:-

Consider the Δ CBE, where

$$CE = Q \sin\theta$$

$$BE = Q \cos\theta$$

$\therefore$ $$AC^2 = AB^2 + 2AB \cdot BE + BE^2 + CE^2$$

or $$R^2 = P^2 + 2 \cdot P \cdot Q\cos\theta + Q^2\cos^2\theta + Q^2\sin^2\theta$$

$$= P^2 + Q^2 + 2PQ\cos\theta$$

i.e. $$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

To find the direction α of the resultant, consider the Δ CAE, where

$$\tan\alpha = \frac{CE}{AB + BE}$$

$$= \frac{Q\sin\theta}{P + Q\cos\theta}$$

$\therefore$ $$\alpha = \tan^{-1}\left(\frac{Q\sin\theta}{P + Q\cos\theta}\right)$$

Conditions

(i) Resultant R is max when two forces collinear and are in the same direction. *i.e.,* $\alpha = 0°$ $\Rightarrow$ Rmax $= P + Q$

(ii) Resultant R is min when two forces collinear but acting in the opposite direction. That is $\alpha = 180$ => Rmin = P- Q

(iii) If a = 90, that is when the forces act at right angle, then

R = P2 + Q2

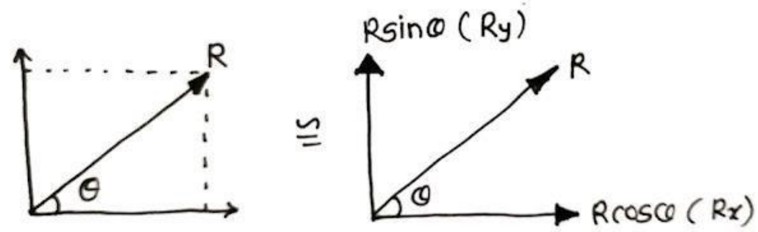
(iv) If the two forces are equal that is, when P = Q =>R = 2P.cos(θ/2)

Method Of Resolution:-

A) Orthogonal or perpendicular resolution:-

In this method the resultant force is split into two Perpendicular components. These two Perpendicular components will be acting along X-axis and Y-axis or any two Perpendicular axis.

As shown in fig R may be any force which is inclined at an angle θ with X-axis. So by using this method we can easily resolved this force into two Perpendicular or orthogonal components as shown in fig.

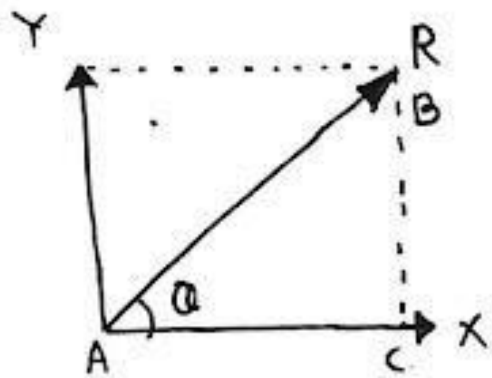


Rx= horizontal component of force R

Ry= vertical component of force R

Now from where these Rcosθ & Rsinθ c

Method Of Resolution:-



In fig. ΔABC is a right angled triangle.

$\therefore$ In ΔABC

$$\sin\theta = \frac{BC}{AB}$$

$\therefore$ $BC = AB \sin\theta$

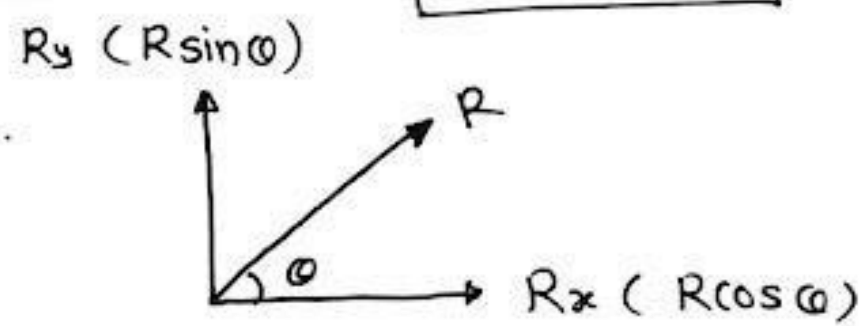
$\therefore$ $\boxed{BC = R\sin\theta}$

$\therefore$ In ΔABC

$$\cos\theta = \frac{AC}{AB}$$

$AC = AB\cos\theta$

$\boxed{AC = R\cos\theta}$

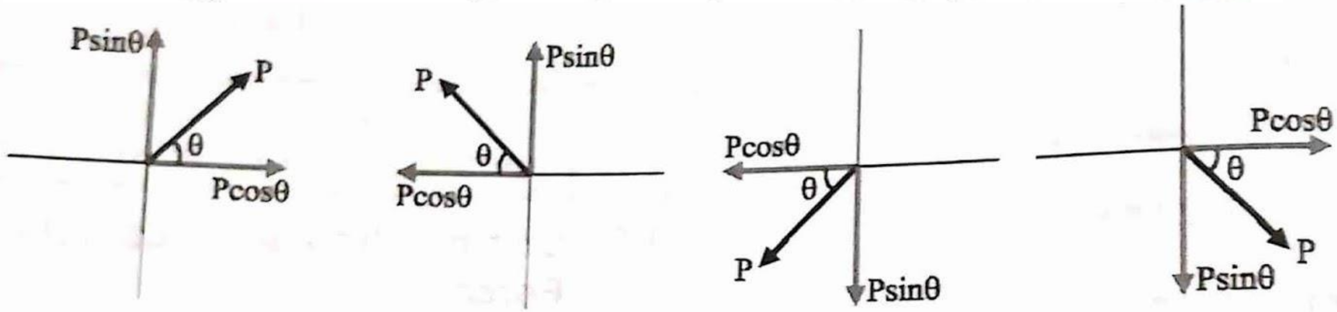


$P_x = +P$, $P_y = 0$

$P_x = -P$, $P_y = 0$

$P_x = 0$, $P_y = +P$

$P_x = 0$, $P_y = -P$

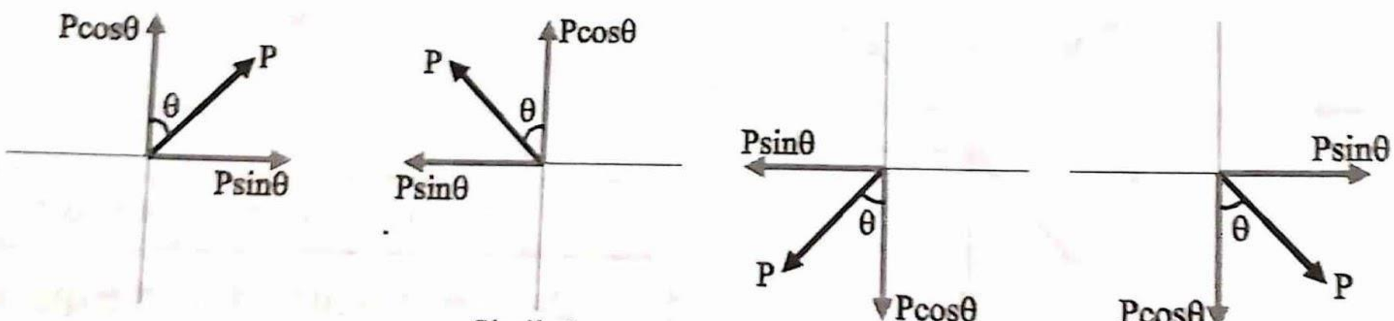


$P_x = + P\cos\theta$
$P_y = + P\sin\theta$

$P_x = - P\cos\theta$
$P_y = + P\sin\theta$

$P_x = - P\cos\theta$
$P_y = - P\sin\theta$

$P_x = P\cos\theta$
$P_y = - P\sin\theta$



Similarly

$P_x = P\cos(90^\circ - \theta) = + P\sin\theta$

$P_y = P\sin(90^\circ - \theta) = + P\cos\theta$

$P_x = - P\cos(90^\circ - \theta) = - P\sin\theta$

$P_y = + P\sin(90^\circ - \theta) = + P\cos\theta$

$P_x = - P\sin\theta$

$P_y = - P\sin\theta$

$P_x = P\sin\theta$

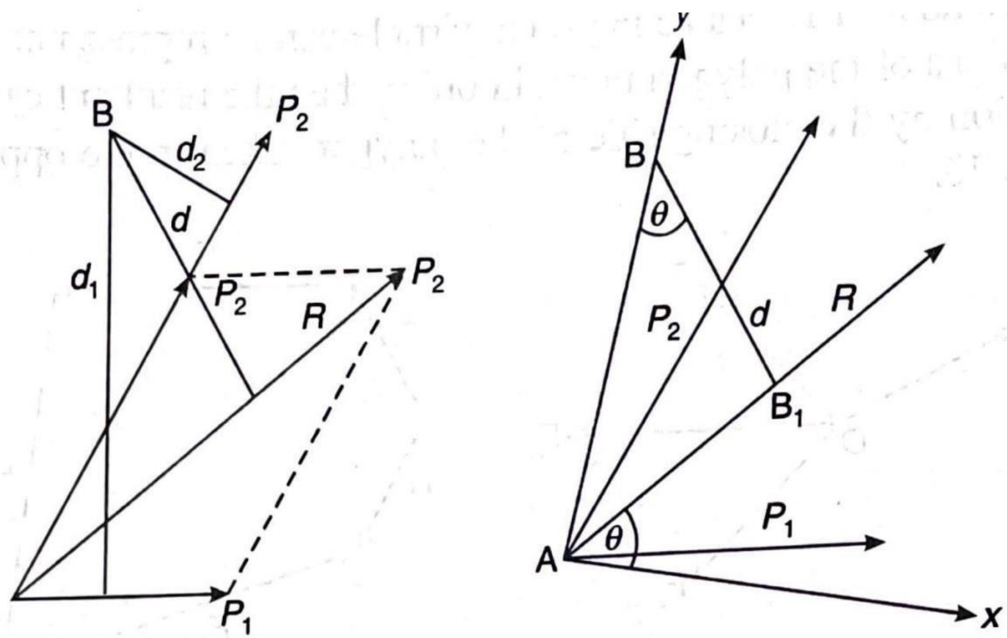
$P_y = - P\cos\theta$

Varignon's Theory of moments

This is also known as the principle of moments. The theorem states that "the algebraic sum of the moments of individual forces of a force system about a point is equal to the moment of their resultant about the same point. Let R be the resultant of forces P_1 and P_2 and B be the moment centre. Let d, d_1 and d_2 be the moment arms of forces R, P_1 and P_2, respectively, from the moment centre B (Figure 2.22).

We have to prove that

$$Rd = P_1 d_1 + P_2 d_2$$



Varignon's Theory of moments

Proof: Join AB and consider it as the y-axis and draw the x-axis at right angles to it at A. Let θ be the angle made by R with the x-axis and note that the same angle is formed with the y-axis by the perpendicular to R from B and note this point as B_1.

We know that,

$$Rd = R \times \text{AB} \cos \theta$$

$$= \text{AB} \times R \cos \theta$$

or

$$Rd = \text{AB} \times R_x \tag{i}$$

where R_x is the component of R in the x-direction. Similarly, if P_{1x} and P_{2x} are the components of P_1 and P_2 in the x-direction, respectively, then,

$$P_1 d_1 = \text{AB} \times P_{1x} \tag{ii}$$

and

$$P_2 d_2 = \text{AB} \times P_{2x} \tag{iii}$$

Adding equations (ii) and (iii), we get

$$P_1 d_1 + P_2 d_2 = \text{AB}(P_{1x} + P_{2x})$$

or

$$P_1 d_1 + P_2 d_2 = \text{AB} \times R_x \tag{iv}$$

Since the sum of x-components of the individual forces is equal to the x-component of the resultant R, from equations (i) and (iv), we can conclude that

$$Rd = P_1 d_1 + P_2 d_2$$

□

RAMAIAH
Institute of Technology

BASICIS OF CIVIL ENGINEERING & MECHANICS

Course code:CV14/CV24

Credits:3:0:0

Topics Covered

Equilibrium of coplanar concurrent system of forces

Equilibrium of coplanar concurrent system of forces

EQUILIBRIUM

- When a stationary body is subjected to external forces and if the body remains in the state of rest under the action of forces, it is said to be in equilibrium.
- Equilibrium is also defined as the condition of a body, which is subjected to a force system whose resultant force is equal to zero. It means the effect of the given force system is zero and the particle or rigid body is said to be in equilibrium.

For example, a particle subjected to two forces will be in equilibrium when the two forces are equal in magnitude, opposite in direction and act along the same line of action as shown in Figure.

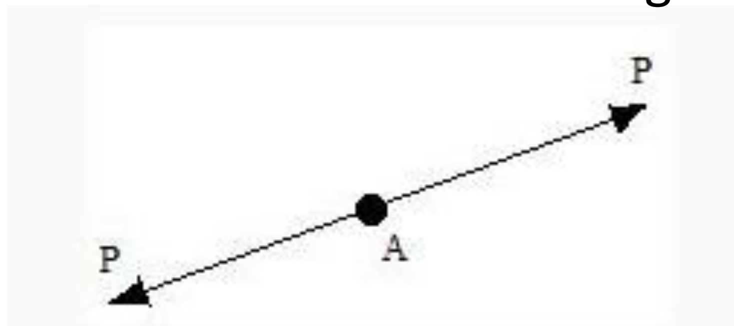


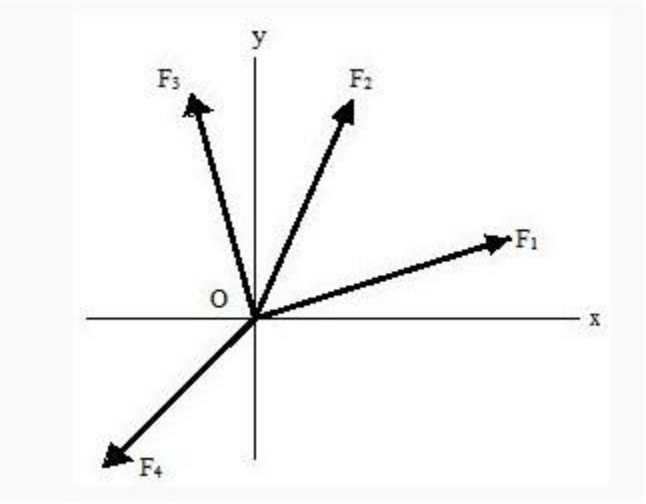
Fig. Equilibrium of forces

<u>Conditions of equilibrium for coplanar concurrent force system</u>

A coplanar concurrent force system will be in equilibrium if it satisfies the following two conditions:

1. Algebraic sum of all the horizontal components of the force system must be zero. i.e., $\boldsymbol{\sum F_X = 0}$

2. Algebraic sum of all the vertical components of the force system must be zero. i.e., $\sum F_Y = 0$



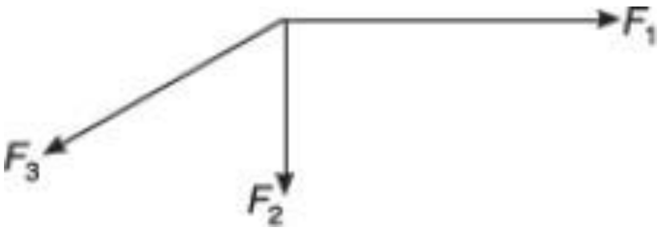


Figure. Coplanar Concurrent force system (2D Rigid Body Equilibrium)

Conditions of equilibrium for coplanar nonconcurrent force system

A coplanar non concurrent force system will be in equilibrium if it satisfies the following three conditions:

1. Algebraic sum of all the horizontal components of the force system must be zero. i.e., $\sum F_X = 0$
2. Algebraic sum of all the vertical components of the force system must be zero. i.e., $\sum F_Y = 0$
3. Algebraic sum of moments of all the forces about any point system must be zero. i.e., $\sum M = 0$

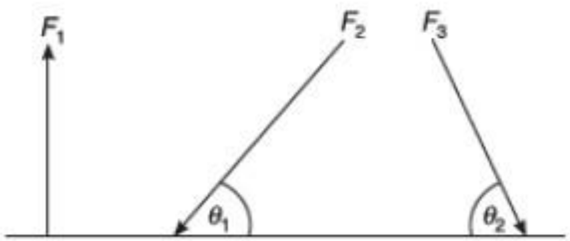
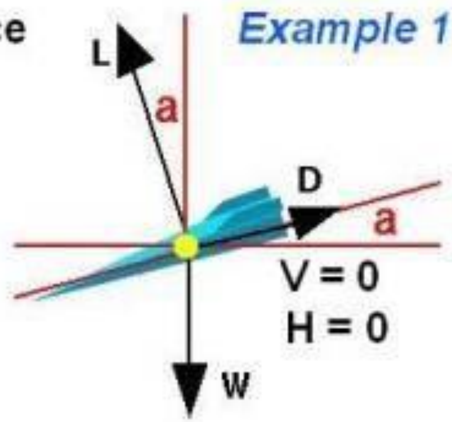
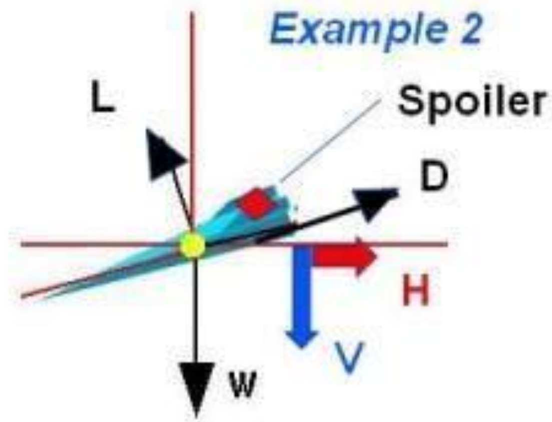


Figure. Coplanar non-concurrent force system

H = Horizontal Force
V = Vertical Force
W = Weight
D = Drag
L = Lift
a = glide angle





	Equilibrium	Non-equilibrium
Vertical:	W - L cos(a) - D sin(a) = V = 0	W - L cos(a) - D sin(a) = V
Horizontal:	D cos(a) - L sin(a) = H = 0	D cos(a) - L sin(a) = H

No net external force
Aircraft descends at constant velocity

Net external forces
Aircraft accelerates vertically and horizontally

EQULIBRIUM OF A BODY IN SPACE

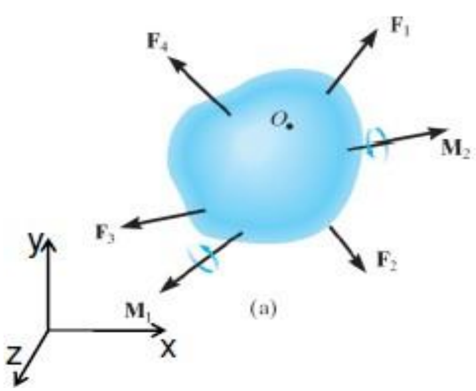
Rigid Body Equilibrium

A rigid body will remain in equilibrium provided

- sum of all the external forces acting on the body is equal to zero, and
- Sum of the moments of the external forces about a point is equal to zero

$$\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma F_z = 0$$

$$\Sigma M_x = 0 \quad \Sigma M_y = 0 \quad \Sigma M_z = 0$$



- A body is said to be in equilibrium if there is no translation and no rotation of the body under the application of external forces.

Equilibrant

Equilibrant is defined as a single force required to keep the body in equilibrium. For a concurrent force system, equilibrant is a force which has same magnitude as the resultant force but opposite in direction.

Free-body diagram (FBD)

A free body diagram represents only the forces acting in the system without representing the physical appearance of body.

Figure (a) shows a spherical ball of mass *m, placed on a horizontal plane and tied to the plane*

Figure (b) shows the free-body diagram of the Figure (a) where only the forces are represented without physical appearance of the body.

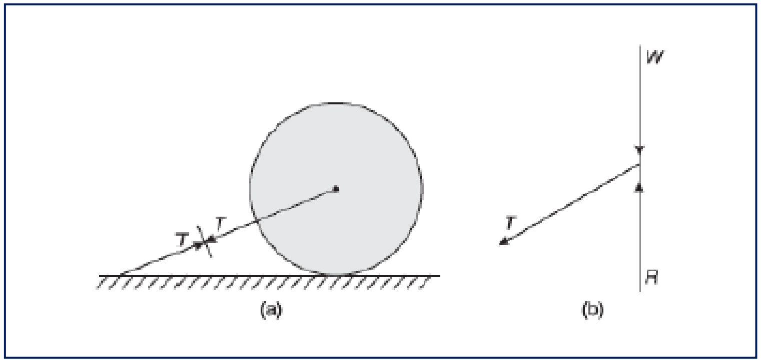


Figure. Spherical ball (on a horizontal plane) with free-body diagram.

The various forces are:

(i) Self weight, *W, always acting vertically downwards.*

(ii) Normal reaction, R, always acting perpendicular to the plane under consideration.

(iii) Tension *T in the string.*

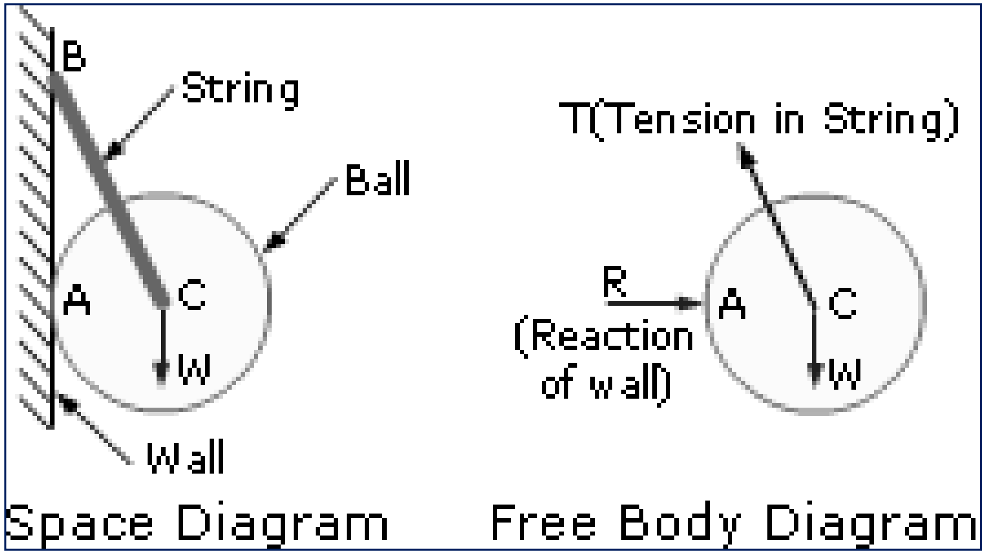


Figure. Spherical ball (resting against a wall)

In Figure above, a spherical ball supported by a string and resting against a wall, is shown together with its free-body diagram.

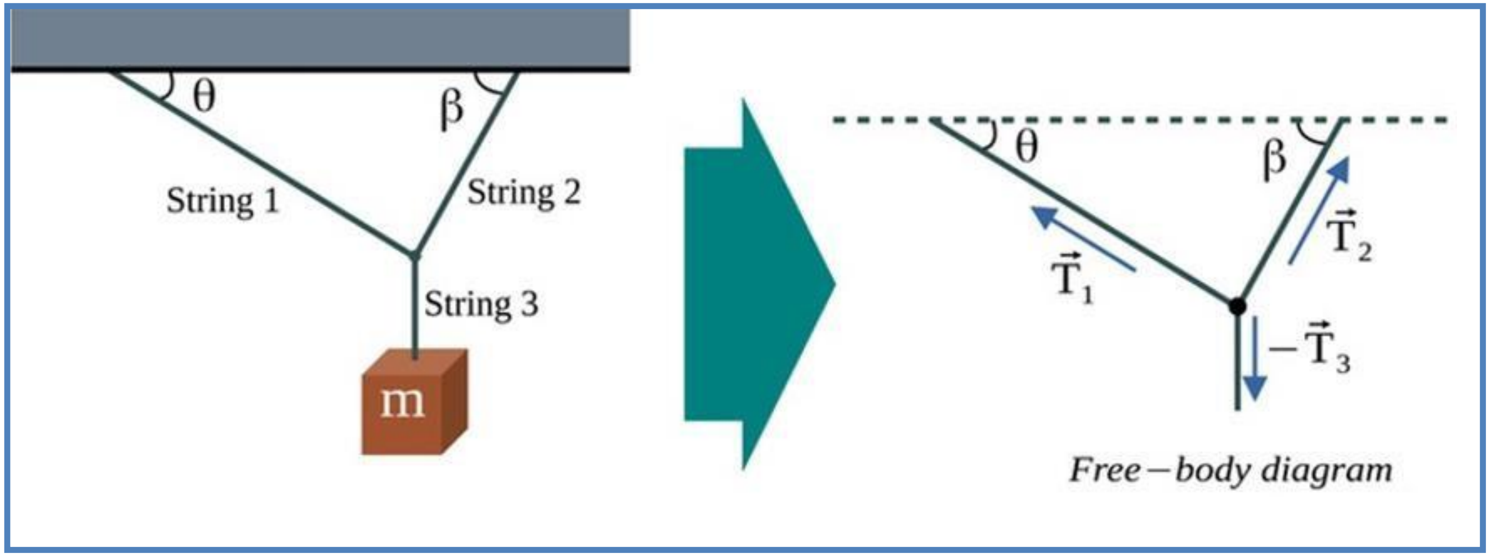


Fig. Free Body Diagram of a structure connected with Two strings

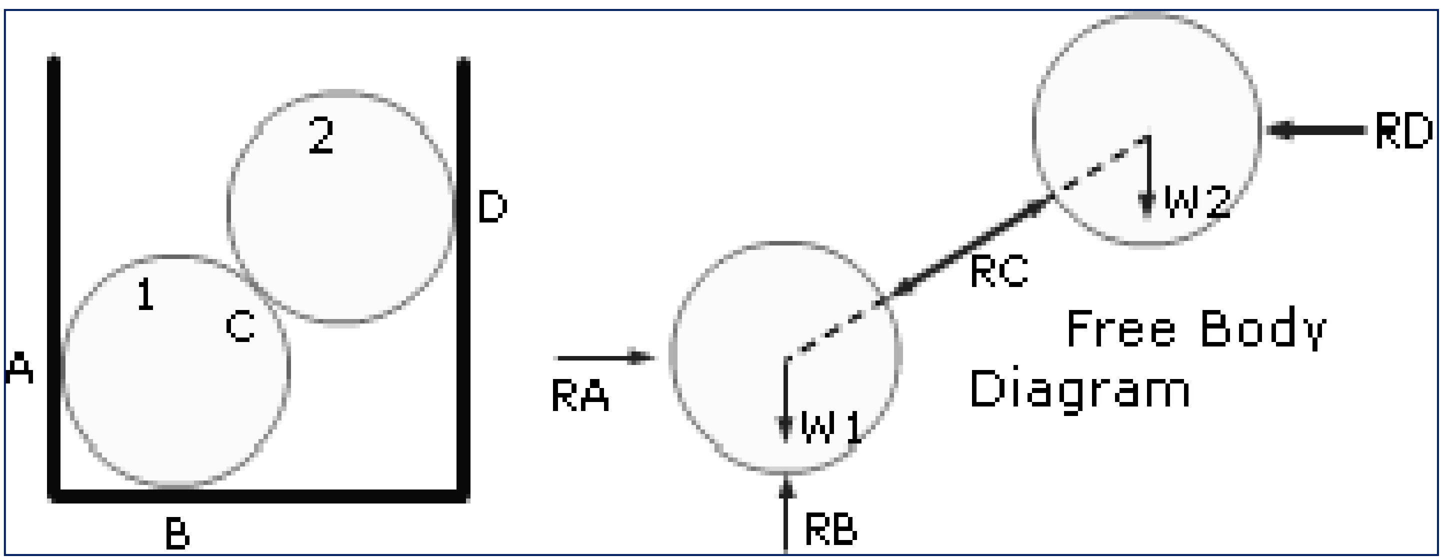
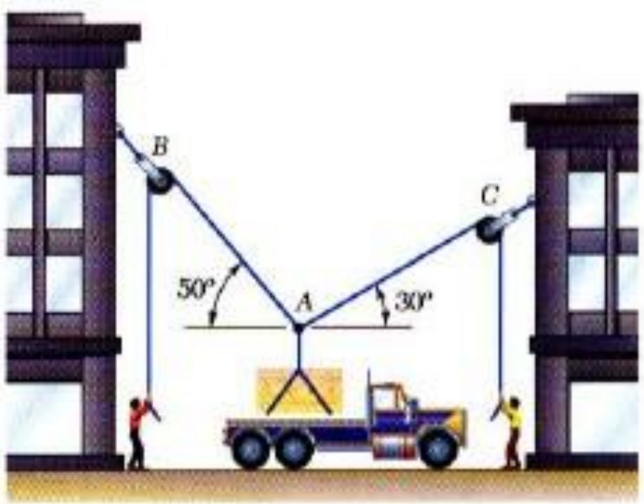
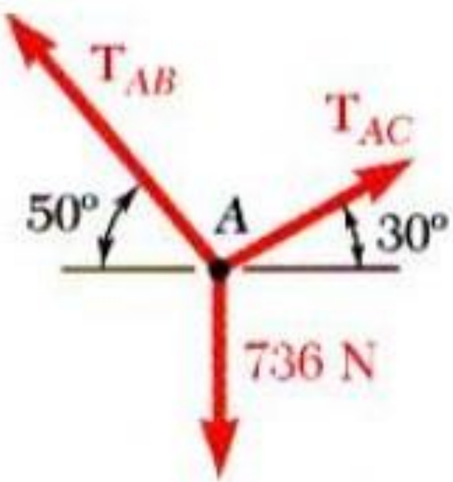


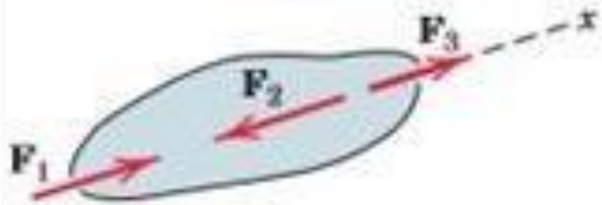
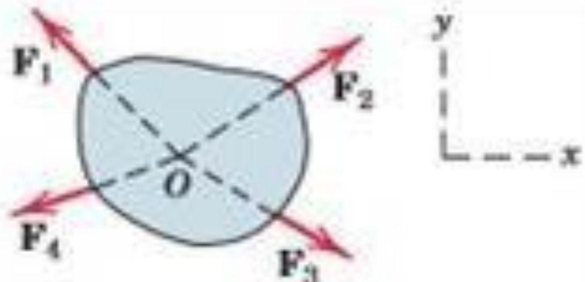
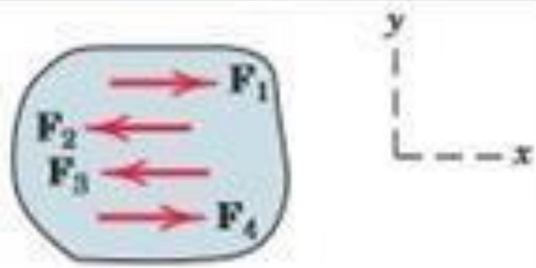
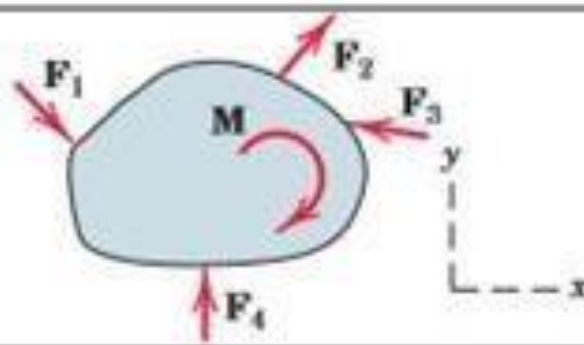
Fig. Free Body Diagram of connected bodies

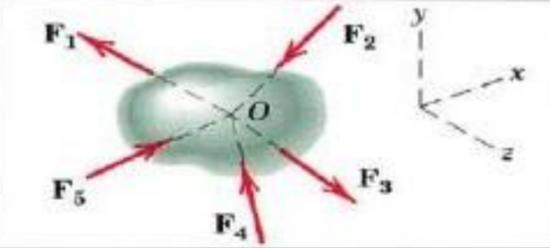
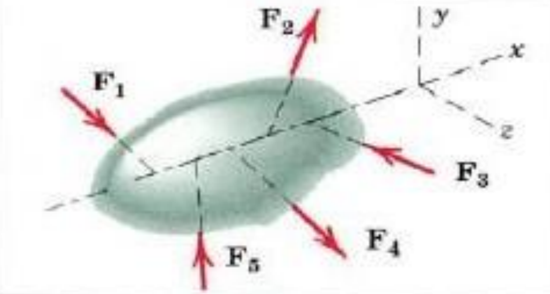
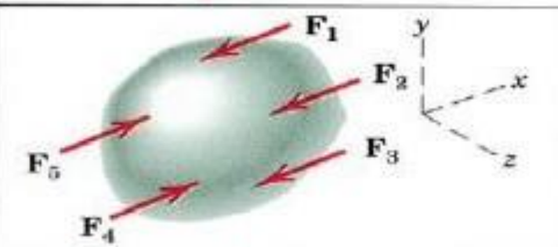
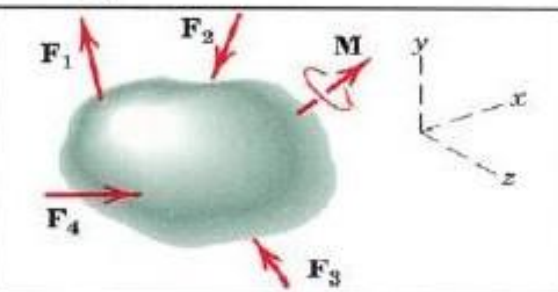


Space Diagram: A sketch showing the physical conditions of the problem.



Free-Body Diagram: A sketch showing only the forces on the selected particle.

<table>
<tr><th colspan="3">CATEGORIES OF EQUILIBRIUM IN TWO DIMENSIONS</th></tr>
<tr><th>Force System</th><th>Free-Body Diagram</th><th>Independent Equations</th></tr>
<tr><td>1. Collinear</td><td>
</td><td>$\Sigma F_x = 0$</td></tr>
<tr><td>2. Concurrent at a point</td><td>
</td><td>$\Sigma F_x = 0$
$\Sigma F_y = 0$</td></tr>
<tr><td>3. Parallel</td><td>
</td><td>$\Sigma F_x = 0 \quad \Sigma M_z = 0$</td></tr>
<tr><td>4. General</td><td>
</td><td>$\Sigma F_x = 0 \quad \Sigma M_z = 0$
$\Sigma F_y = 0$</td></tr>
</table>

CATEGORIES OF EQUILIBRIUM IN THREE DIMENSIONS		
Force System	Free-Body Diagram	Independent Equations
1. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$ $\Sigma F_z = 0$
2. Concurrent with a line		$\Sigma F_x = 0$ $\Sigma M_y = 0$ $\Sigma F_y = 0$ $\Sigma M_z = 0$ $\Sigma F_z = 0$
3. Parallel		$\Sigma F_x = 0$ $\Sigma M_y = 0$ $\Sigma M_z = 0$
4. General		$\Sigma F_x = 0$ $\Sigma M_x = 0$ $\Sigma F_y = 0$ $\Sigma M_y = 0$ $\Sigma F_z = 0$ $\Sigma M_z = 0$

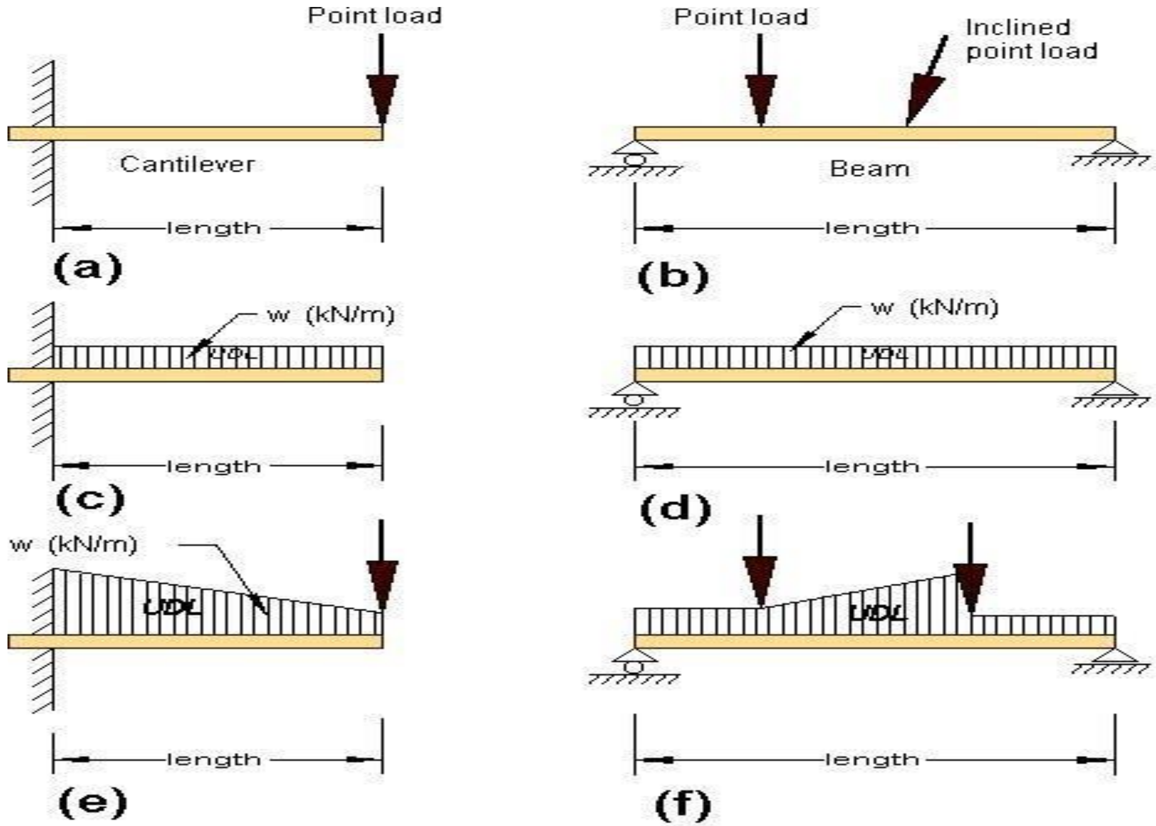
PROBLEMS ON EQUILIBRIUM OF COPLANAR CONCURRENT FORCE SYSTEM

1. First, draw Free Body Diagram of a point in the system/an object in the system/complete system of objects showing various forces acting on it.
2. In the coplanar concurrent force system, use two conditions of equilibrium, namely

$$\sum F_X = 0 \quad \text{and} \quad \sum F_Y = 0$$

3. Analyze the given problem by applying the above conditions of equilibrium.

Types of loads



What is a structural support ???

Any thing which **take up** the **loads** coming from the **super structure** and **transfer** to **substructure**

- Structural **systems transfer** their **loading** through a **series of elements** to the **ground**.

- Each **connection is designed** so that it can **transfer or support** a specific type of **load or loading condition**.

- In order to **analyze a structure** it is **necessary to be clear** about the **forces that can be resisted and transferred** at each level of support throughout the structure.

The **actual behavior** of a **support or connection** can be **quite complicated if all conditions** were **considered** and the **design** of each support would be a **terribly lengthy process.**

For ex:

1. hinge support is considered as 100% friction free but it is not true.
2. There will also be vertical deformations in the elastomeric bearings which is ignored while designing.
3. While designing its no where considered temperature effects on the structural supports.

Structural steel systems → welded or bolted connections

Precast RCC or PSC systems → mechanically connected

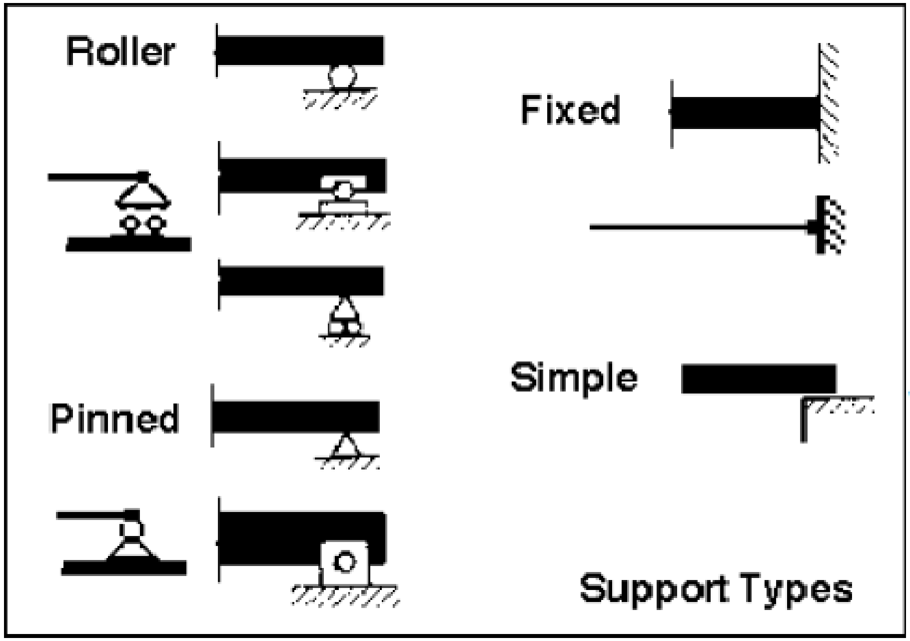
Cast-in-situ → monolithic connections

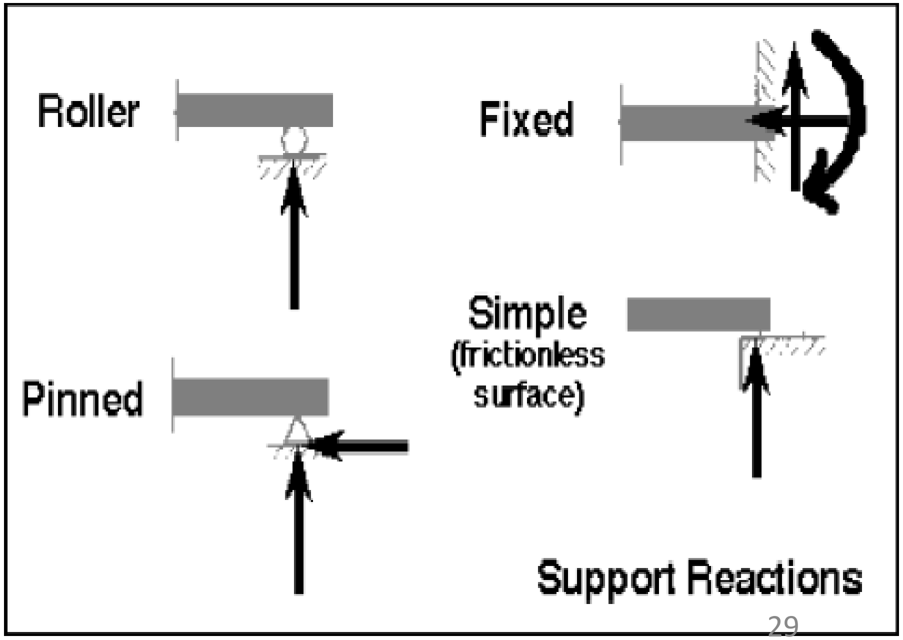
Timber systems → nails, bolts, glue &engineered connectors.

No matter the material the connection must be designed to have a specific rigidity.

There are generally four type of supports

- Roller support
- Hinge or pin support
- Fixed support
- Simple support






COBRA

- **Roller supports** are **free to rotate and translate** along the surface upon which the roller rests. The **surface** can be **horizontal vertical or sloped** at any angle.

- The **resulting reaction force** is always a **single force** that is **perpendicular** to the

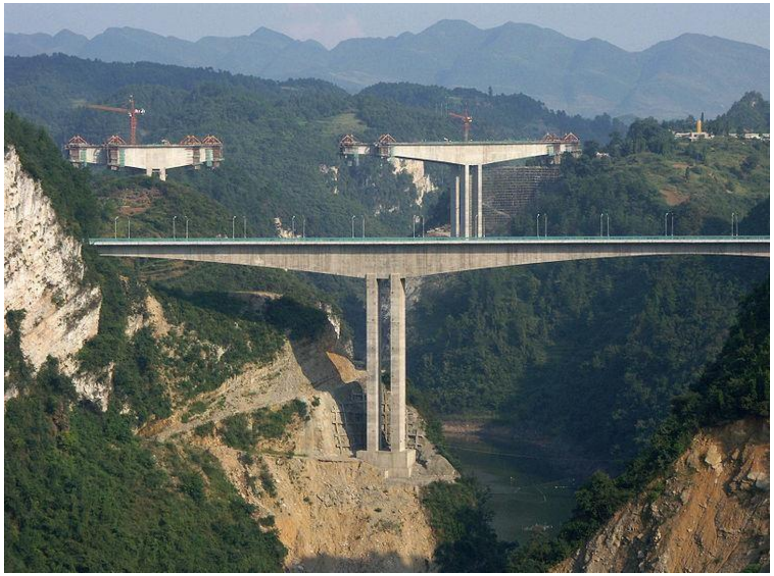
- Roller supports are **commonly located at one end of long bridges** . This **allows** the bridge structure to **expand and contract** with **temperature changes**.

- Roller supports can also **take the form of rubber bearings ,rockers** which are designed to **allow a limited amount of lateral movement.**

- A **pinned support** can **resist** both **vertical** and **horizontal forces** but **not a moment.**
- They will **allow** the **structural member** to **rotate** but **not to translate** in any direction.
- **Many connections** are **assumed to be pinned connections** even though **they might resist a small amount of moment in reality.**
- It is also true that a **pinned connection** could **allow rotation** in **only one direction** providing **resistance to rotation in any other direction**.

Ex :Pen stand is the example for the pin support which can allow rotation in all directions and also about its own axis.

Some of the fixed support



- **Fixed supports** can **resist vertical** and **horizontal forces** as **well as a moment. Since they restrain both rotation and translation** they are also **known as rigid supports.**

- **This means that a structure only needs one fixed support in order to be stable.** All three equations of equilibrium can be satisfied.

Ex: A **flagpole** set into a **concrete base** is a good example of this kind of support.

- **Fixed connections demand greater attention during construction and are often the source of building failures.**

- **Fixed connections** are very **common** both in **RCC and steel structures**.

- **Steel structures welded together** can be **considered** as **fixed connections**.

- A **cast-in-place concrete structure** is automatically **monolithic** and it becomes a series of **rigid connections** with the proper placement of the reinforcing steel.

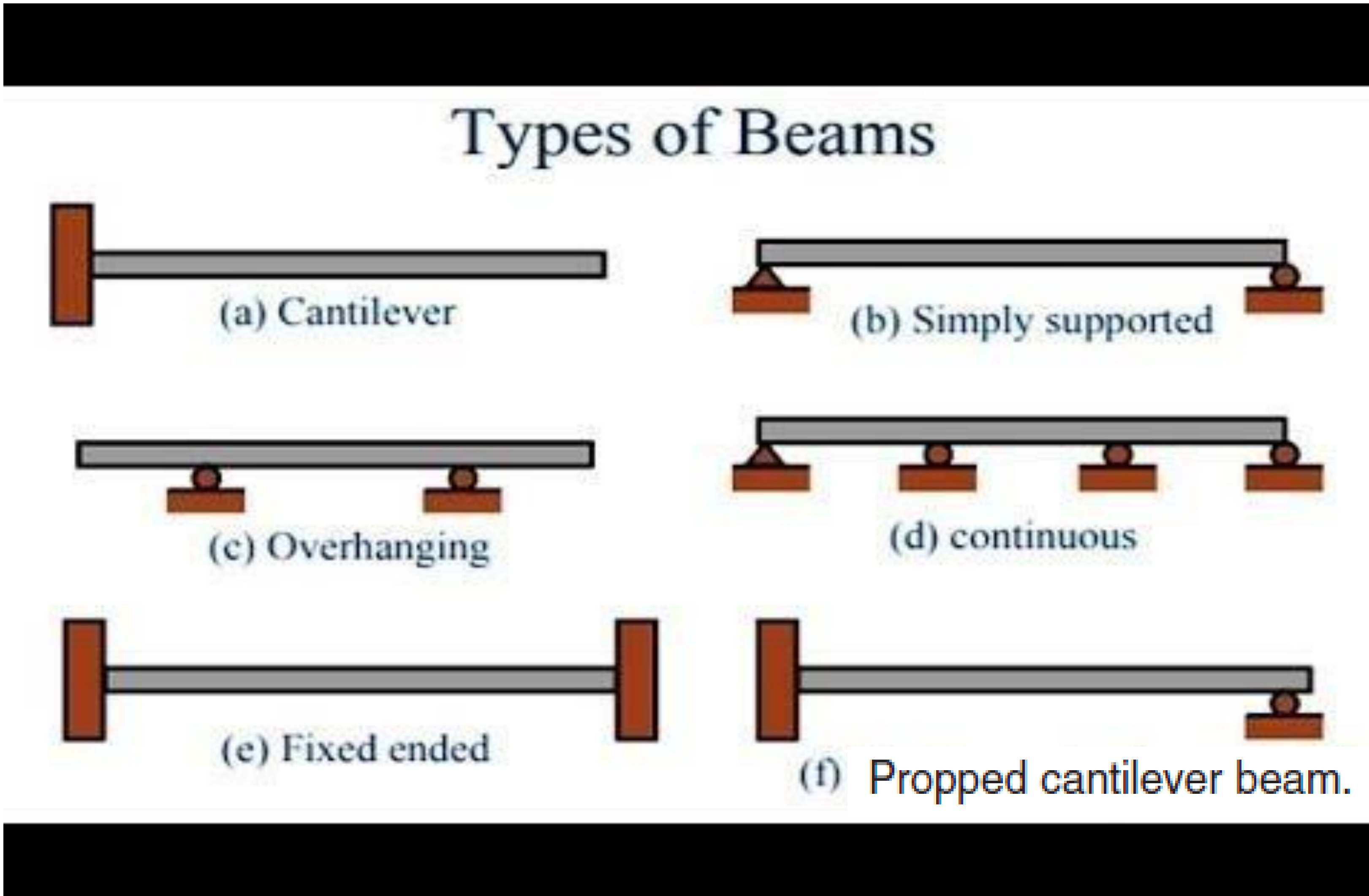
Some of the simple support

- Simple supports are **idealized** by some **to be frictionless surface supports**.

- This is correct in as much as **the resulting reaction** is always a **single force** that is **perpendicula**r to and **away from the surface.**

- However are also **similar to roller supports in this**. They are **dissimilar in** that a simple support **cannot resist lateral loads** of any magnitude.

- A simple support **can be found as a** type of **support for long bridges** or roof span. **Simple supports** are **often found in** zones of **frequent seismic** activity.


Types of Beams
(a) Cantilever
(b) Simply supported
(c) Overhanging
(d) continuous
(e) Fixed ended
(f) Propped cantilever beam.

Determinate beams

In beams, **if the reaction forces can be calculated using equilibrium equations alone, they are statically determinate**.

Ex: **simply supported beams, cantilever beams, singleand double overhanging beams**,

In determinate beams

In beams, **if the reaction forces cannot be calculatedusing equilibrium equations alone, they are indeterminate**.

Ex: **fixed beams, continuous beams**