RAMAIAH
Institute of Technology

BASICIS OF CIVIL ENGINEERING & MECHANICS

Course code:CV14/CV24

Credits:3:0:0

Topics Covered

Equilibrium of coplanar concurrent system of forces

Equilibrium of coplanar concurrent system of forces

EQUILIBRIUM

- When a stationary body is subjected to external forces and if the body remains in the state of rest under the action of forces, it is said to be in equilibrium.
- Equilibrium is also defined as the condition of a body, which is subjected to a force system whose resultant force is equal to zero. It means the effect of the given force system is zero and the particle or rigid body is said to be in equilibrium.

For example, a particle subjected to two forces will be in equilibrium when the two forces are equal in magnitude, opposite in direction and act along the same line of action as shown in Figure.

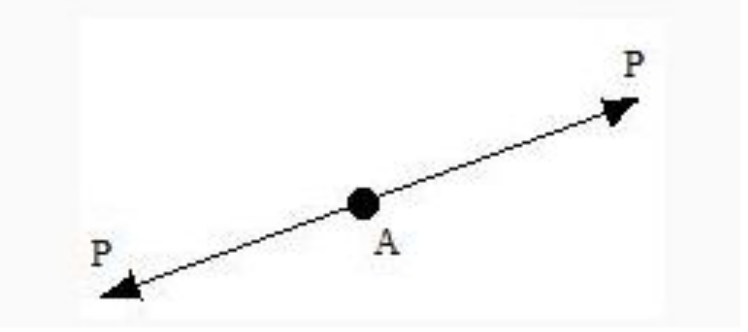


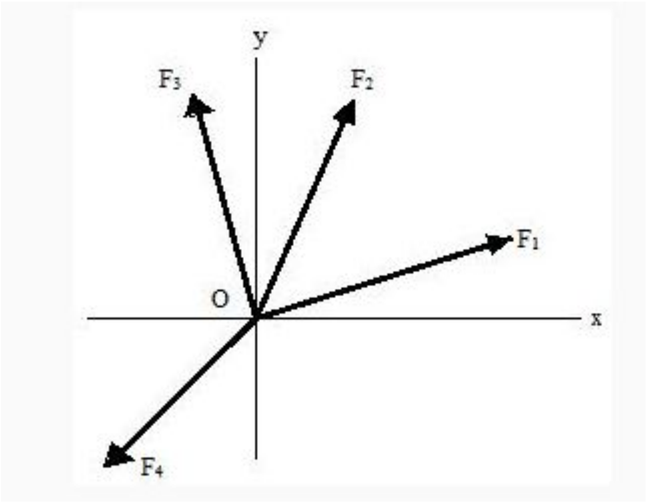
Fig. Equilibrium of forces

<u>Conditions of equilibrium for coplanar concurrent force system</u>

A coplanar concurrent force system will be in equilibrium if it satisfies the following two conditions:

1. Algebraic sum of all the horizontal components of the force system must be zero. i.e., $\sum F_X = 0$

2. Algebraic sum of all the vertical components of the force system must be zero. i.e., $\sum F_Y = 0$



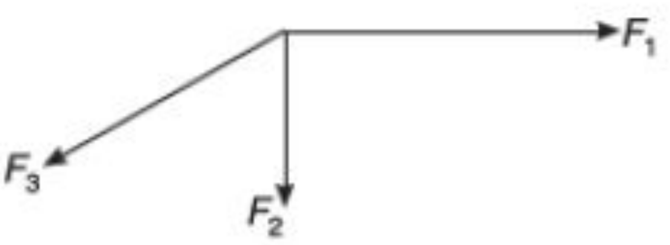


Figure. Coplanar Concurrent force system (2D Rigid Body Equilibrium)

Conditions of equilibrium for coplanar nonconcurrent force system

A coplanar non concurrent force system will be in equilibrium if it satisfies the following three conditions:

1. Algebraic sum of all the horizontal components of the force system must be zero. i.e., $\sum F_X = 0$
2. Algebraic sum of all the vertical components of the force system must be zero. i.e., $\sum F_Y = 0$
3. Algebraic sum of moments of all the forces about any point system must be zero. i.e., $\sum M = 0$

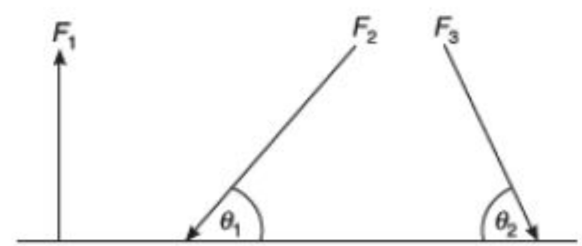
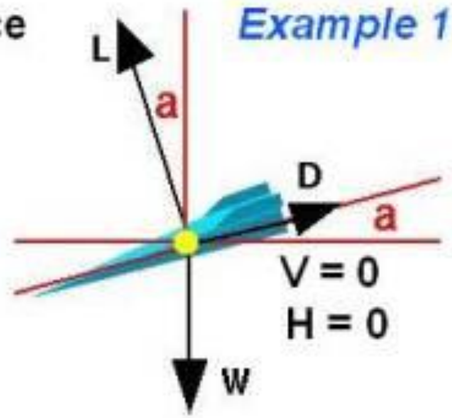


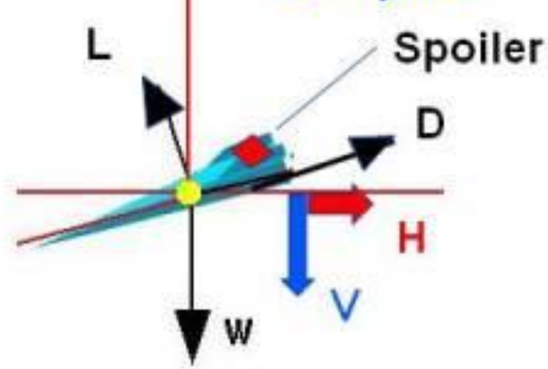
Figure. Coplanar non-concurrent force system

H = Horizontal Force
V = Vertical Force
W = Weight
D = Drag
L = Lift
a = glide angle

Example 1



Example 2



	Equilibrium	***Non-equilibrium***
Vertical:	W - L cos(a) - D sin(a) = V = 0	W - L cos(a) - D sin(a) = V
Horizontal:	D cos(a) - L sin(a) = H = 0	D cos(a) - L sin(a) = H

No net external force
Aircraft descends at constant velocity

Net external forces
Aircraft accelerates vertically and horizontally

EQULIBRIUM OF A BODY IN SPACE

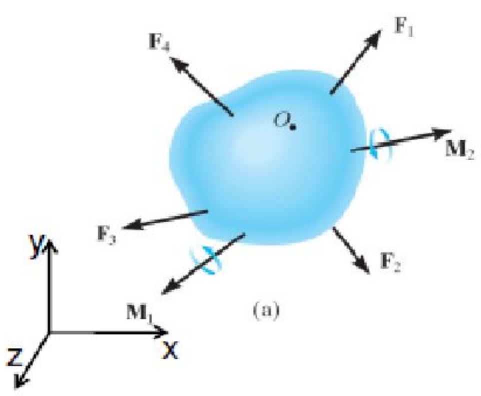
Rigid Body Equilibrium

A rigid body will remain in equilibrium provided

- sum of all the external forces acting on the body is equal to zero, and
- Sum of the moments of the external forces about a point is equal to zero

$$\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma F_z = 0$$

$$\Sigma M_x = 0 \quad \Sigma M_y = 0 \quad \Sigma M_z = 0$$



- A body is said to be in equilibrium if there is no translation and no rotation of the body under the application of external forces.

Equilibrant

Equilibrant is defined as a single force required to keep the body in equilibrium. For a concurrent force system, equilibrant is a force which has same magnitude as the resultant force but opposite in direction.

Free-body diagram (FBD)

A free body diagram represents only the forces acting in the system without representing the physical appearance of body.

Figure (a) shows a spherical ball of mass *m, placed on a horizontal plane and tied to the plane*

Figure (b) shows the free-body diagram of the Figure (a) where only the forces are represented without physical appearance of the body.

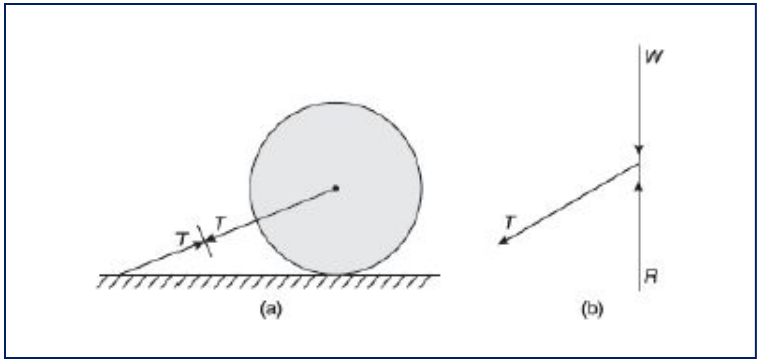


Figure. Spherical ball (on a horizontal plane) with free-body diagram.

The various forces are:

(i) Self weight, *W, always acting vertically downwards.*

(ii) Normal reaction, R, always acting perpendicular to the plane under consideration.

(iii) Tension *T in the string.*

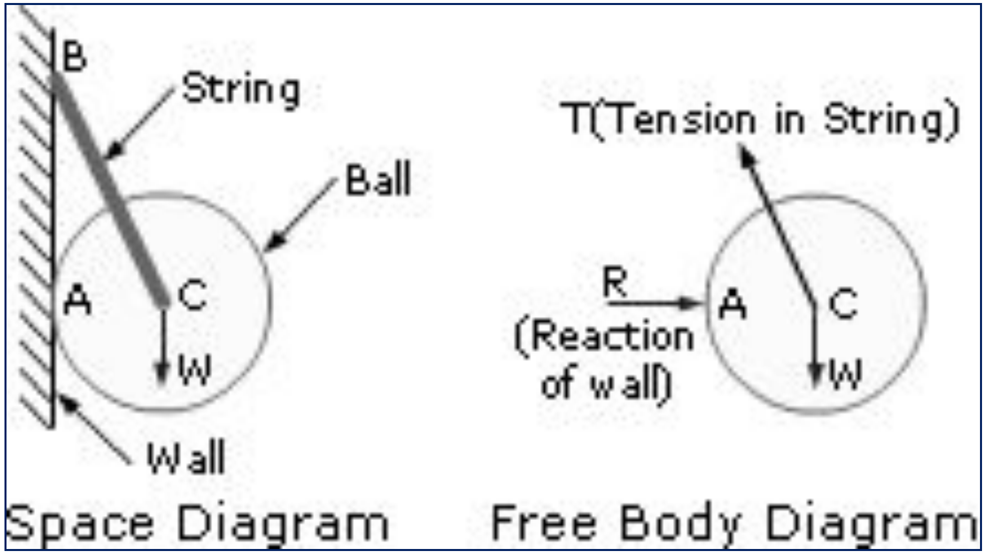


Figure. Spherical ball (resting against a wall)

In Figure above, a spherical ball supported by a string and resting against a wall, is shown together with its free-body diagram.

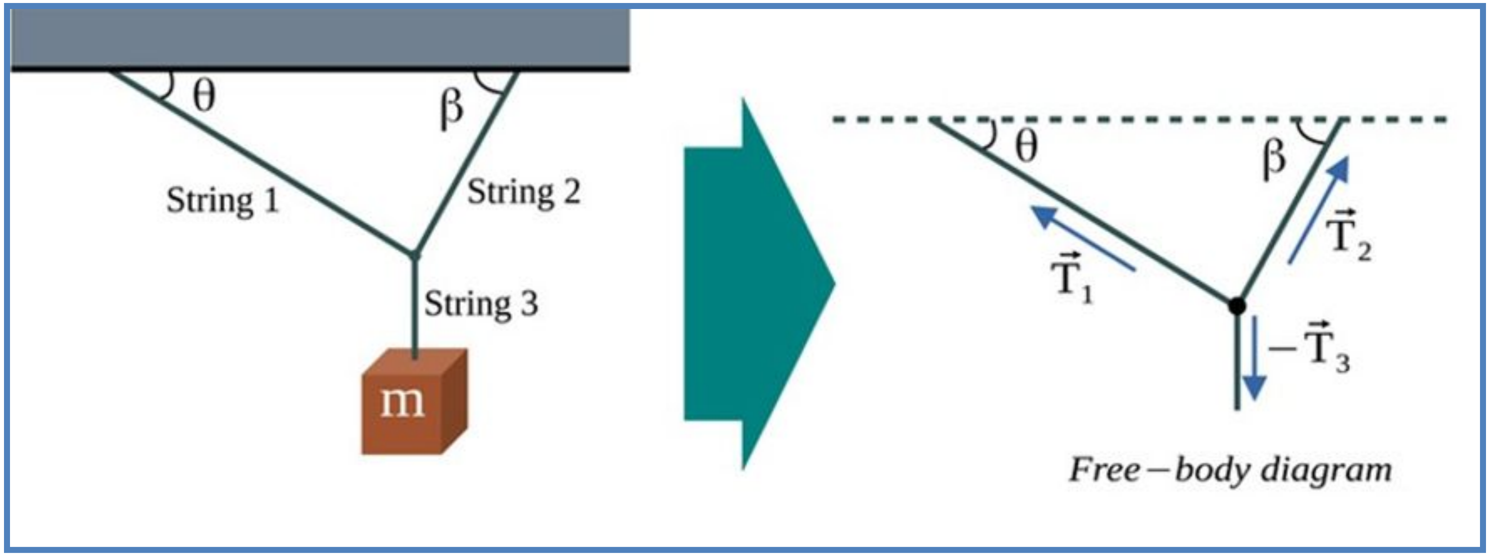


Fig. Free Body Diagram of a structure connected with Two strings

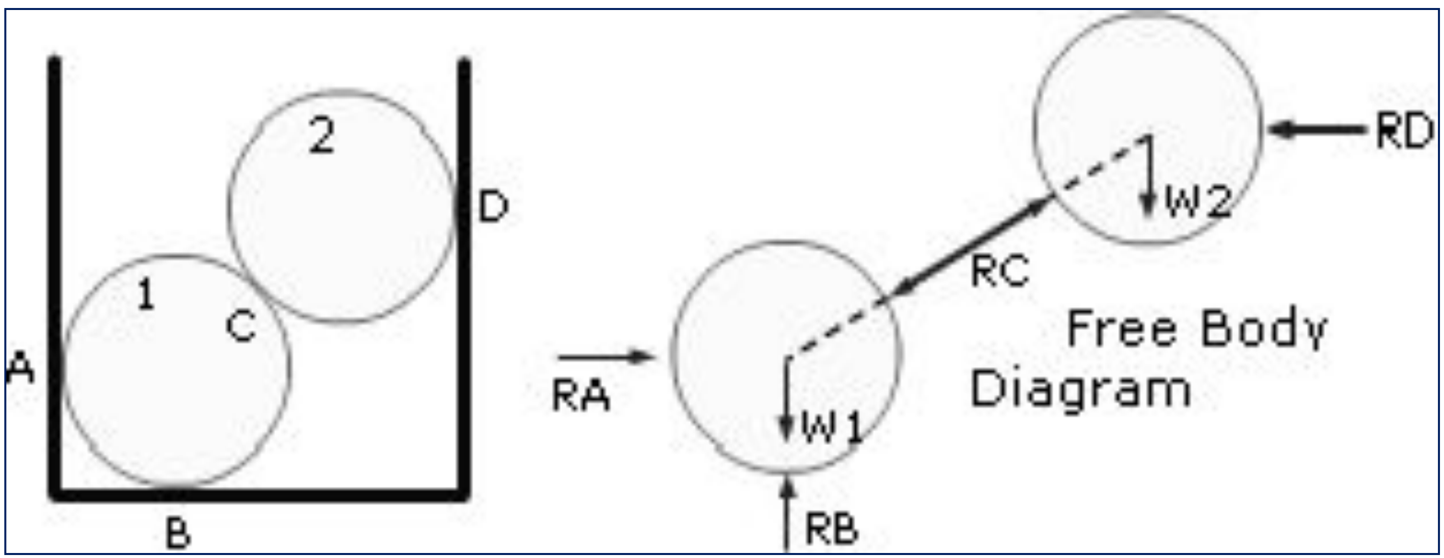
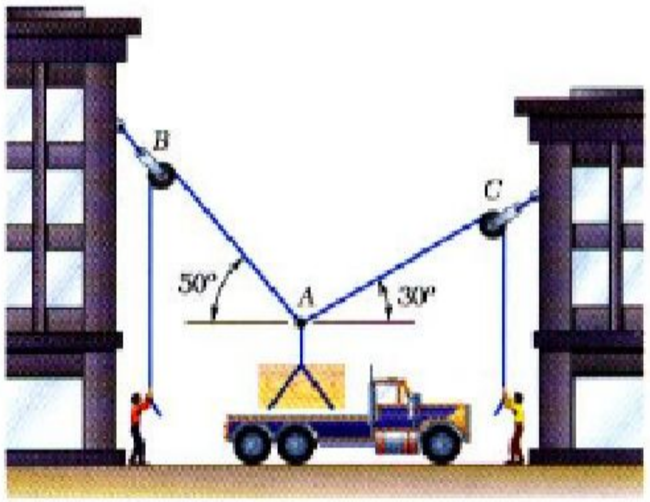
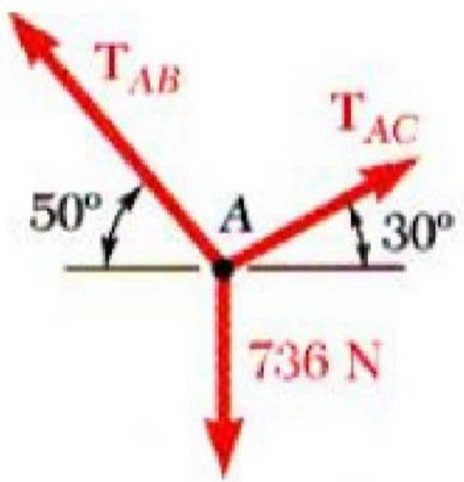


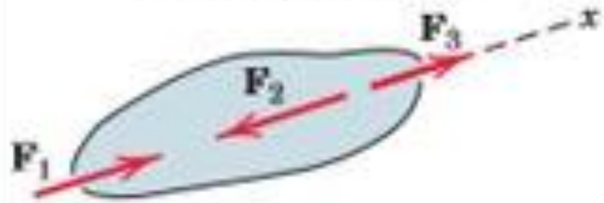
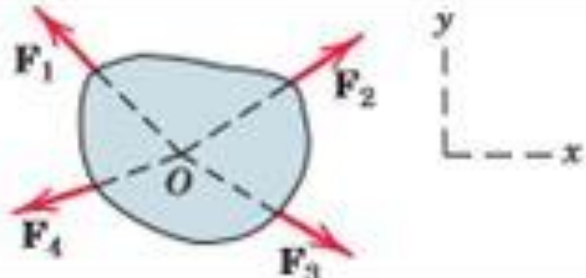
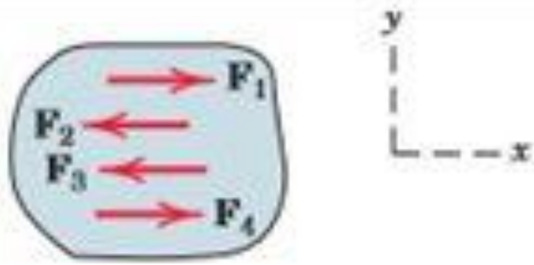
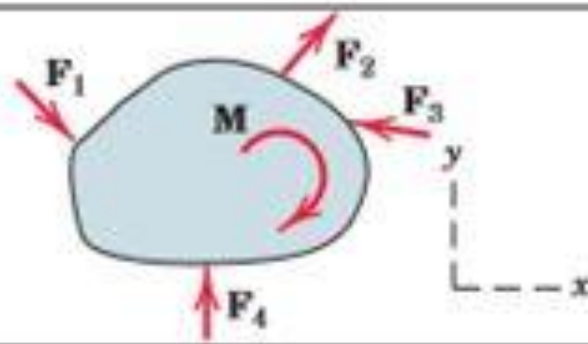
Fig. Free Body Diagram of connected bodies

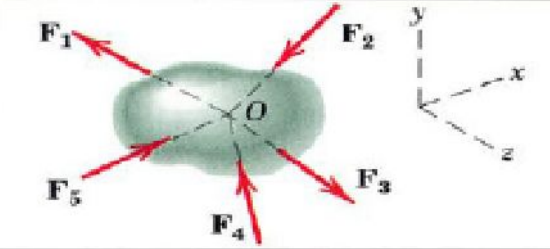
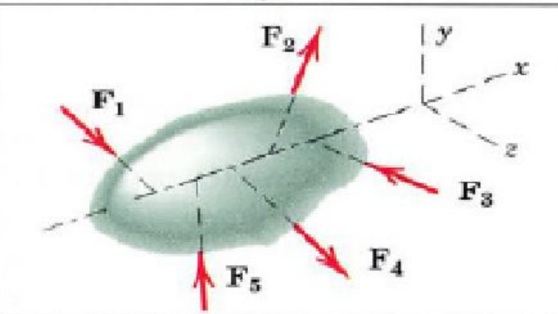
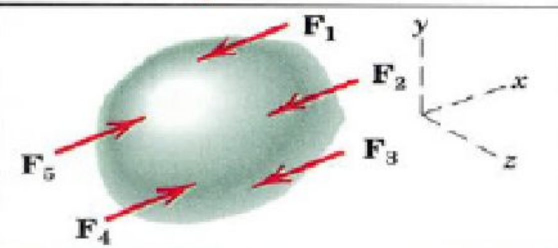
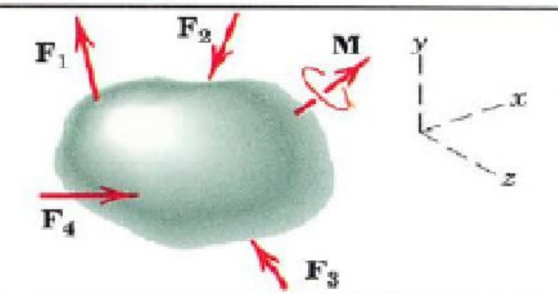


Space Diagram: A sketch showing the physical conditions of the problem.



Free-Body Diagram: A sketch showing only the forces on the selected particle.

CATEGORIES OF EQUILIBRIUM IN TWO DIMENSIONS		
Force System	Free-Body Diagram	Independent Equations
1. Collinear		$\Sigma F_x = 0$
2. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$
3. Parallel		$\Sigma F_x = 0 \quad \Sigma M_z = 0$
4. General		$\Sigma F_x = 0 \quad \Sigma M_z = 0$ $\Sigma F_y = 0$

CATEGORIES OF EQUILIBRIUM IN THREE DIMENSIONS		
Force System	Free-Body Diagram	Independent Equations
1. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$ $\Sigma F_z = 0$
2. Concurrent with a line		$\Sigma F_x = 0$ $\Sigma M_y = 0$ $\Sigma F_y = 0$ $\Sigma M_z = 0$ $\Sigma F_z = 0$
3. Parallel		$\Sigma F_x = 0$ $\Sigma M_y = 0$ $\Sigma M_z = 0$
4. General		$\Sigma F_x = 0$ $\Sigma M_x = 0$ $\Sigma F_y = 0$ $\Sigma M_y = 0$ $\Sigma F_z = 0$ $\Sigma M_z = 0$

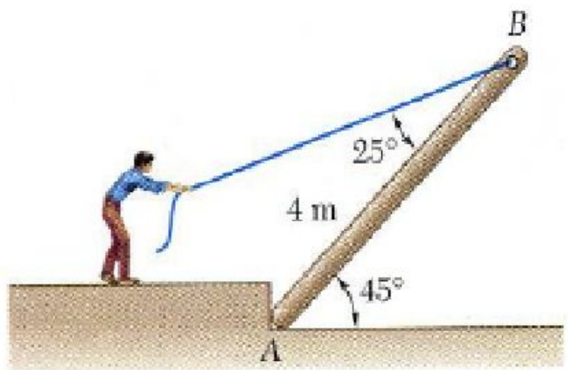
PROBLEMS ON EQUILIBRIUM OF COPLANAR CONCURRENT FORCE SYSTEM

1. First, draw Free Body Diagram of a point in the system/an object in the system/complete system of objects showing various forces acting on it.
2. In the coplanar concurrent force system, use two conditions of equilibrium, namely

$$\sum F_X = 0 \quad \text{and} \quad \sum F_Y = 0$$

3. Analyze the given problem by applying the above conditions of equilibrium.

Rigid Body Equilibrium: Example



A man raises a 10 kg joist, of length 4 m, by pulling on a rope.

Find the tension in the rope and the reaction at A.

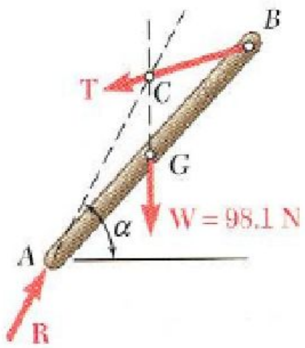
Solution:

- Create a free-body diagram of the joist. Note that the joist is a 3 force body acted upon by the rope, its weight, and the reaction at A.

- The three forces must be concurrent for static equilibrium. Therefore, the reaction ***R*** must pass through the intersection of the lines of action of the weight and rope forces. Determine the direction of the reaction force ***R***.

- Utilize a force triangle to determine the magnitude of the reaction force ***R***.

Rigid Body Equilibrium: Example



- Create a free-body diagram of the joist.
- Determine the direction of the reaction force R.

$$AF = AB\cos 45 = (4\,\text{m})\cos 45 = 2.828\text{m}$$

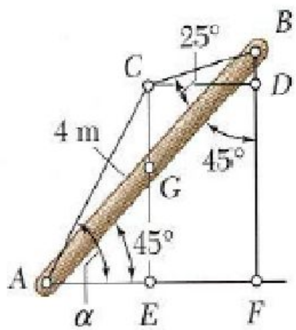
$$CD = AE = \tfrac{1}{2} AF = 1.414\text{m}$$

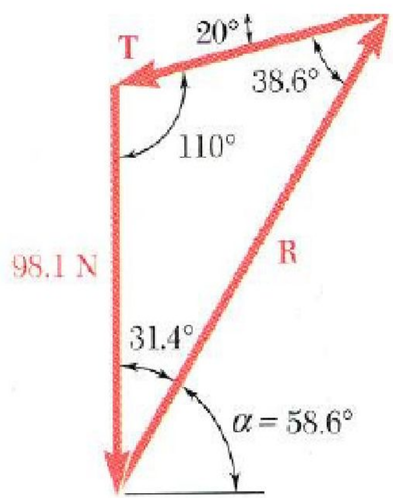
$$BD = CD\cot(45+20) = (1.414\text{m})\tan 20 = 0.515\text{m}$$

$$CE = BF - BD = (2.828 - 0.515)\,\text{m} = 2.313\text{m}$$

$$\tan\alpha = \frac{CE}{AE} = \frac{2.313}{1.414} = 1.636$$

$$\boxed{\alpha = 58.6^\circ}$$



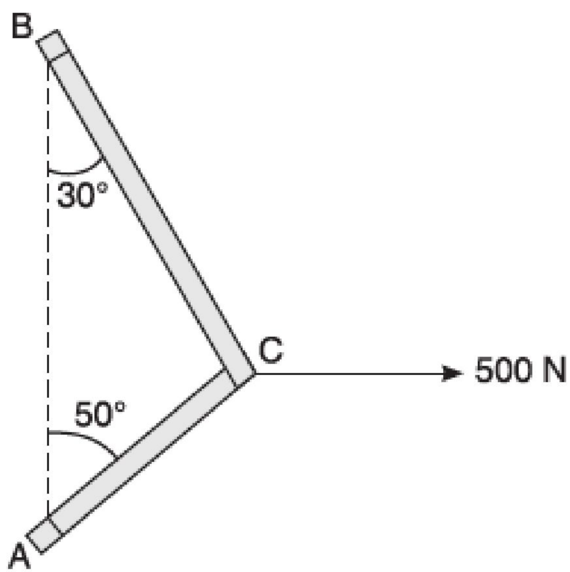


- Determine the magnitude of the reaction force R.

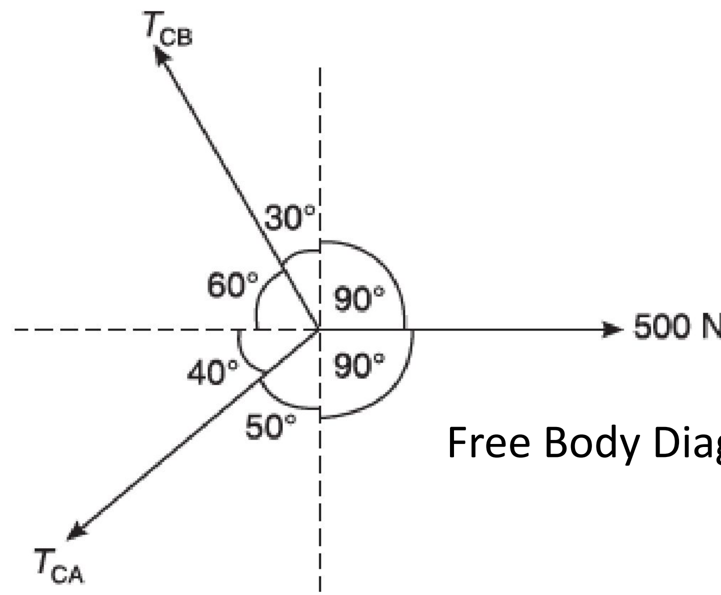
$$\frac{T}{\sin 31.4^\circ} = \frac{R}{\sin 110^\circ} = \frac{98.1\,\text{N}}{\sin 38.6^\circ}$$

$$\boxed{\begin{array}{l} T = 81.9\,\text{N} \\ R = 147.8\,\text{N} \end{array}}$$

2). Determine the forces induced in the members AC and BC of the structure shown in figure (a). Its free-body diagram is also shown in figure (b).



(a)



(b)

Free Body Diagram

Applying Equilibrium equations,

$\sum Fx = 0$, $\quad 500 - T_{CB} \text{ Cos } 60° - T_{CA} \cos 40° = 0$

$T_{CB} \text{ Cos } 60° = 500 - T_{CA} \cos 40°$ ……..(i)

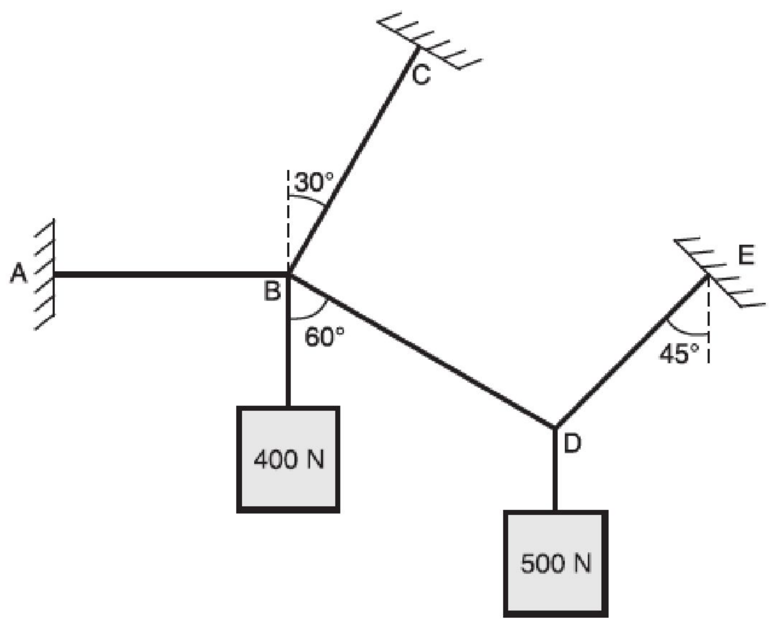
$$\sum F_Y = 0 , \quad -\mathrm{T}_{CA} \text{ Sin } 40° + T_{CB} \sin 60° = 0$$

$$\mathrm{T}_{CA} \text{ Sin } 40° = T_{CB} \sin 60° \quad \text{........(ii)}$$

Solving equations (i) and (ii),

$T_{CB} = 326.35$ N and $T_{CA} = 439.695$ N

3). The system of connected flexible cables shown in figure below is supporting two loads of 400 N and 500 N at points B and D, respectively. Determine the tensions in the various segments of the cable.



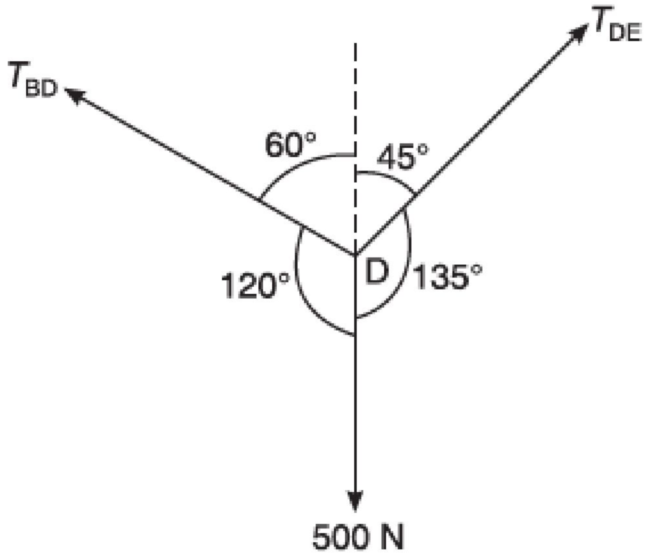


Fig. FBD at D.

<u>Considering FBD at D:</u>

$\sum Fx = 0;\quad T_{DE}\ \cos 45° – TBD\ \cos 30° = 0$……. (i)

$\sum Fy = 0;\ - 500 + T_{DE}\ \sin 45° – T_{\mathrm{BD}}\ \sin 30° = 0$…. (ii)

Solving equations (i) and (ii),

$T_{\mathrm{BD}} = 366.025$ N and $T_{\mathrm{DE}} = 448.287$ N

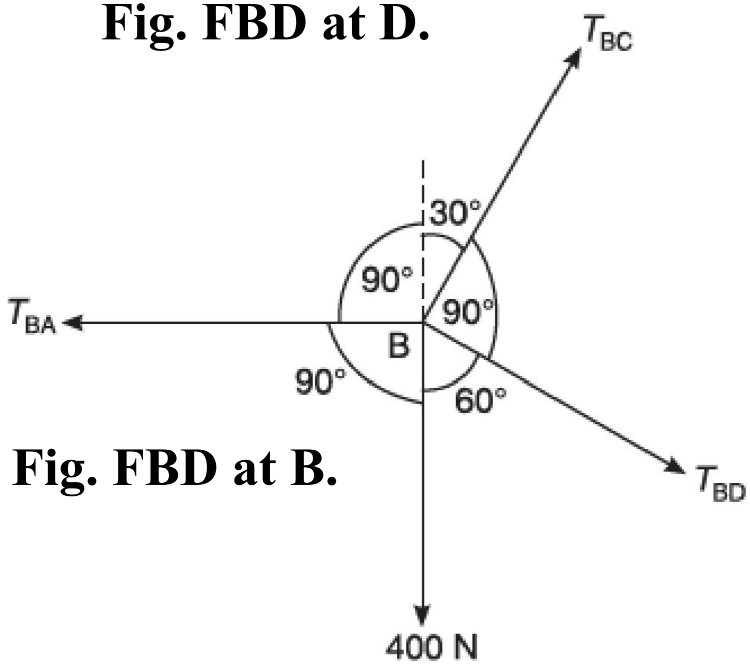


Fig. FBD at B.

<u>Considering FBD at B:</u>

$\sum Fx = 0; –T_{\mathrm{BA}} + T_{\mathrm{BC}} \cos 60° + T_{\mathrm{BD}} \cos 30° =0$

$T_{\mathrm{BA}} = T_{\mathrm{BC}} \cos 60° + 366.025 \cos 30°$…. (iii)

$\sum Fy = 0;\ - 400 + T_{\mathrm{BC}} \sin 60° - T_{\mathrm{BD}} \sin 30° =0$

$T_{\mathrm{BC}} \sin 60° = 400 + 366.025 \sin 30°$

….(iv)

Solving equations (iii) and (iv)

$T_{BC} = 673.204$ N and $T_{BA} = 653.588$ N

4). A horizontal shaft with inner clearance of 1000 mm carries two spheres of radius 350 mm and 250 mm as shown in Figure. The weights are 600 N and 500 N respectively. Find the reactions at all the points of contact.

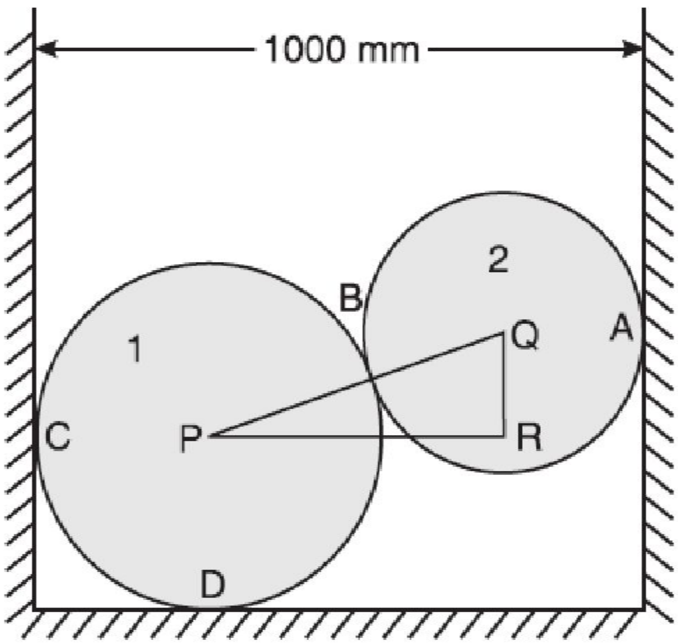
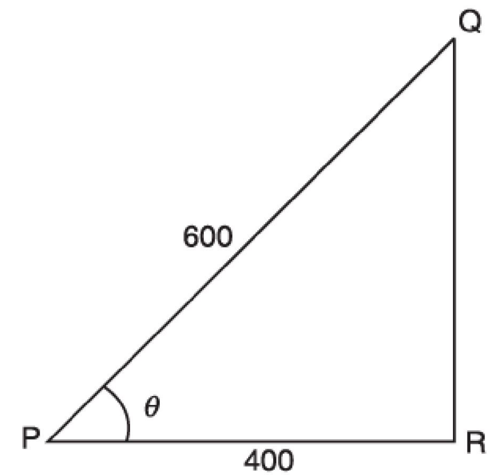


Figure.



PR = 1000 – 350 – 250 = 400 mm

PQ = 350 + 250 = 600 mm

$\cos\theta = PR/PQ$

$= 400/600$

$= 48.18°$

If we consider the FBD of sphere 1 the number of unknowns is three and the equations available are two. Therefore it is necessary to consider the FBD of sphere 2 first.

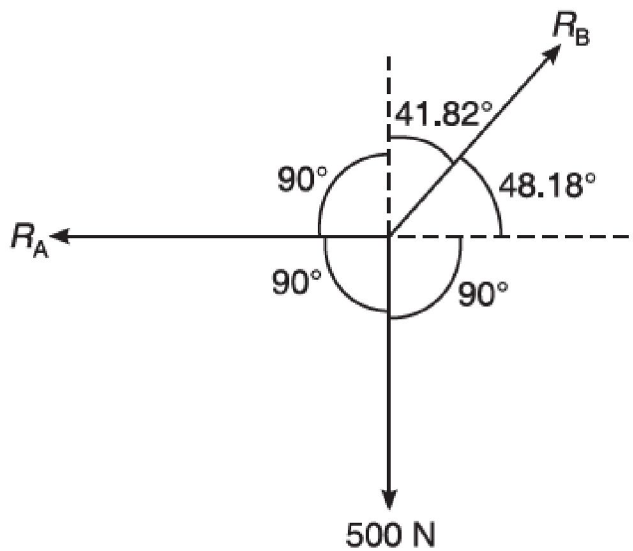


Fig. FBD of sphere 2

<u>Consider the FBD of sphere 2.</u>

$\sum Fx = 0$; $R_B \cos 48.18° - R_A = 0$……. (i)

$\sum Fy = 0$; $-500 + R_B \sin 48.18° = 0$…. (ii)

Solving equations (i) and (ii),

$R_B = 670.922$ N and $R_A = 447.365$ N

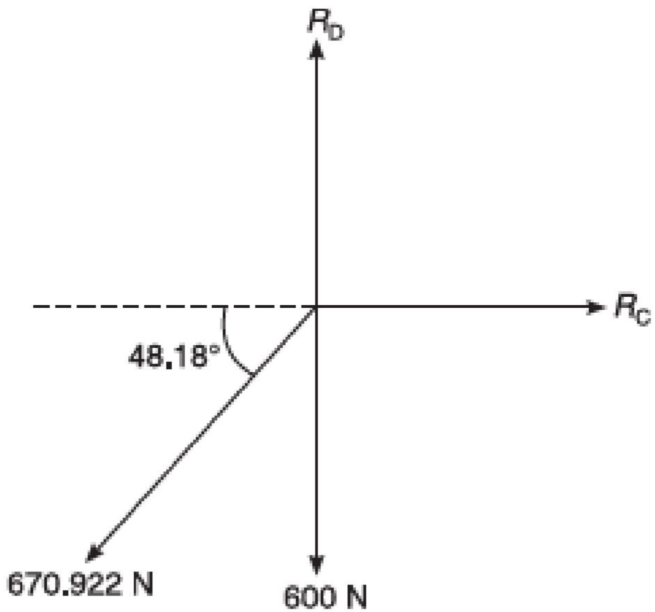


Fig. FBD of sphere 1

Applying the conditions of equilibrium,

For $\sum Fx = 0$;

$\rightarrow \quad R_C - 670.922 \ \cos 48.18° = 0$

$\rightarrow Rc = 670.922 \ \cos 48.18° = 447.366$ N

For $\sum Fy = 0$;

$\rightarrow R_D - 600 - 670.922 \sin 48.18° = 0.$

$\rightarrow R_D = 1100$ N