Electrical conductivity in metals

Assumptions of classical free electron theory:

The metal consists of free electrons; the current flowing through a metal is due drift velocity of electrons.

The free electrons are treated as equivalent to gas molecules and they are assumed to obey the laws of kinetic theory of gases. In the absence of the field, the energy associated with each electron at a temperature T is given by 3/2 kT, where k is a Boltzmann constant. It is related to the kinetic energy.

$$3/2\ kT = \frac{1}{2}\ mv_{th}^2 \qquad \text{Where } v_{th} \text{ is the thermal velocity}$$

The electric potential due to the ionic cores is assumed to be constant throughout the metal

The attraction between the lattice ions and electrons, the repulsion between the electrons are neglected.

Basis Definitions

Mean free path (λ): The average distance traveled by electrons between two successive collisions during their random motion is called mean free path.

Mean collision Time: The average time taken by an electron between two successive collisions during their random motion is called mean collision time.

Relaxation Time (τ): The relaxation time is defined as the time during which drift velocity reduces to 1/e times its maximum value after the electric field is switched off.

Mobility of Electrons: The ease with which electrons get drifted in the presence of applied electric field is called "Mobility". It is defined as the drift velocity acquired by the electron per unit electric field. It is denoted by μ.

$$\text{Mobility } \mu = \frac{v_d}{E}$$

$$\text{We have } \sigma = \frac{nev_d}{E} = ne\mu$$

$$\sigma = ne\mu$$

$$\mu = \frac{\sigma}{ne}$$

$$\text{Also } \sigma = \frac{ne^2\tau}{m} = \mu ne$$

$$\therefore \ \mu = \left(\frac{e}{m}\right)\tau$$

Expression for Conductivity: $\sigma = \dfrac{ne^2\,\tau}{m}$ or resistivity $\rho = \dfrac{m}{ne^2\,\tau}$

Temperature dependence of resistivity of a conductor or Matthiesen's rule:

 At temperature above zero K, the energy acquired by the valance electrons is sufficient make them free from the lattice ions. These electrons free to move throughout the metallic crystal leaving behind the positively charged atoms or positive ions.

The ions are oscillating about their mean positions due to thermal agitation are called lattice vibrations. The quantized lattice vibrations are called as PHONONS. When free electrons interact with Phonons, they scatter from their paths and accelerate in random direction. This sort of opposition to flow of electrons gives rise to the temperature dependent Resistance $\rho(T)$. Therefore, in the absence of electric field, current through the conductor is zero.

Under the influence of electric field, the electrons will overcome the opposition offered by the lattice and move in the direction of field with a small velocity is called drift velocity and set up current in conductor. When temperature of conductor decreases, intensity of lattice vibrations also decreases as a result resistivity of conductor also decreases. When temperature approaches to zero K, the resistivity of typical conductor reaches to minimum value ρ_0, but it never becomes to zero.
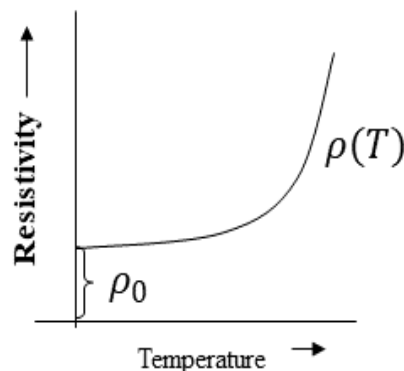
Here ρ_0 is called residual resistivity. This arises due to defects in the conductor and also due to the presence impurities. Therefore, effective resistivity (ρ) of material at a temperature T is given by according to Matthiessen's rule

$$\rho = \rho_0 + \rho_T.$$

Where, ρ_T is temperature dependent resitivity. This is called as Matthiessen's rule.

Thus, Resistance of a conductor can be mainly attributed to two reasons, namely

1. Scattering of electrons with the vibrating lattice ions (ρ_T)

During their random motion electrons undergo scattering from ions, which are in the continuous state of vibrations. This scattering is called phonon scattering and gives rise to resistance of metals. The resistance depends on the number of scattering that electrons undergo.

As the temperature of the metal increases amplitude of vibrations of the ions increases, thereby increasing the number of collisions. This leads to increase in resistance. This resistivity is called ideal resistivity denoted by ρ_T and is given by

$$\rho_T = \frac{m}{ne^2\tau_{ph}} \quad \ldots\ldots\ldots\ldots.1$$

τ_{ph} = mean collision time assuming that there are no impurities.

2. Scattering of electrons by the presence of impurities present in the metal:

Scattering of electrons from the sites of impurities present in the metals such as dislocation joints, grain Boundaries, impurity atom, etc gives rise to resistivity called residual resistivity. It exists even at absolute zero temperature. It is temperature independent and denoted by ρ_i . It is given by

$$\rho_o = \frac{m}{ne^2\tau_i} \quad \ldots\ldots\ldots\ldots\ldots\ldots 2$$

where τ_i is the mean collision time assuming that scattering of electrons with the vibrating lattice ions is absent.

The total resistivity of a metal is $\qquad \rho = \rho_T + \rho_o \; ; \; \rho = \frac{m}{ne^2\tau_T} + \frac{m}{ne^2\tau_o}$

The above equation is called **Matthiessen's Rule,** which states that the net resistivity of conductor is equal to the sum of the resistivity due to the phonon scattering which is temperature dependent and resistivity due to the presence of impurity which is temperature independent.

At absolute zero temperature $\rho = \rho_o$. As the temperature increases ρ_T increases gradually for lower temperatures. At higher temperatures, ρ_T increases rapidly overtaking the effect of ρ_i and becomes linearly dependent on temperature. Thus at higher temperatures the effect of impurity on resistivity becomes negligible and resistivity mainly depends on temperature.

Limitations of Classical free electron theory:

The classical free electron theory fails explain the electronic specific heat in solids, the temperature dependence of conductivity and dependence of conductivity on electron concentration. It also fails to distinguish between conductors, insulators and semiconductors.

Assumptions of quantum free electron theory:

1. The energy values of the free electrons are quantised.
2. The filling up of free electrons in the discrete energy levels happens in accordance with Pauli's exclusion principle.
3. The distribution of energy among the free electrons is according to **Fermi-Dirac statistics**.
4. The electrons travel in a constant potential inside a metal but stay confined within the boundaries.
5. The attraction between the electrons and the lattice ions and the repulsion between the electrons themselves are neglected.

❖ **Comparison of assumptions:**

	Classical Free electron theory	Quantum Free electron theory
1	Energy of the conduction electron is continuous	Energy is quantised, $E = \frac{n^2 h^2}{8ma^2}$
2	There is no restriction on the number of electrons possessing a given energy.	Pauli's exclusion principle is applicable. Hence only two electrons with opposite spin states can have the same energy.
3	The energy distribution among the free electrons obeys **Maxwell-Boltzmann statistics.**	The distribution of energy among the free electrons is according to **Fermi-Dirac statistics**. .
4	The electric potential due to ionic cores inside the metal is considered to be a constant.	The electric potential due to ionic cores inside the metal is considered to be a constant.
5	The mutual repulsion between the electrons and the force of attraction between the electrons and ions are neglected.	The mutual repulsion between the electrons and the force of attraction between the electrons and ions are neglected.

"It is defined as the number of available energy states per unit volume per unit energy range centered at a given energy E".

It is denoted by g(E). It is a continuous function and the product g(E)dE gives the number of states in the energy interval dE at E.

➢ **Expression for Density of States:**

We know the allowed energy for a particle confined in one dimensional box are given by,

$$E = \frac{n^2 h^2}{8ma^2} \quad \text{...................(1)} \qquad \text{where n=1,2,3,.......}$$

'a' is length over which the particle is free to move in one dimension.

Considering the free electrons in a metal as a particles in 3-dimensional box, the energy of the free electrons can be written as,

$$E = \frac{h^2}{8ma^2}(n_x^2 + n_y^2 + n_z^2) \quad \text{..............(2)}$$

where n_x, n_y, n_z are +ve integers greater than zero. m --- mass of the electron.

$$\text{Let} \quad E_0 = \frac{h^2}{8ma^2} \quad \text{and} \quad R^2 = n_x^2 + n_y^2 + n_z^2$$

Then eqn (2) becomes, $E = E_0 R^2$ ....(3)

If we take a coordinate system with n_x, n_y and n_z along three mutually perpendicular axes, then each set of values consisting of three integers can be represented as a point in the system. For a very large number of such sets of values we get a spherical distribution of points in the plot.

Since the values of n_x, n_y and n_z are restricted to be positive, there is only one octant of the sphere of radius 'R' where in each point corresponds to only positive values of n_x, n_y and n_z.

Hence, we have the number of allowed energy values upto an energy E = number of points in the octant of sphere radius 'R.'
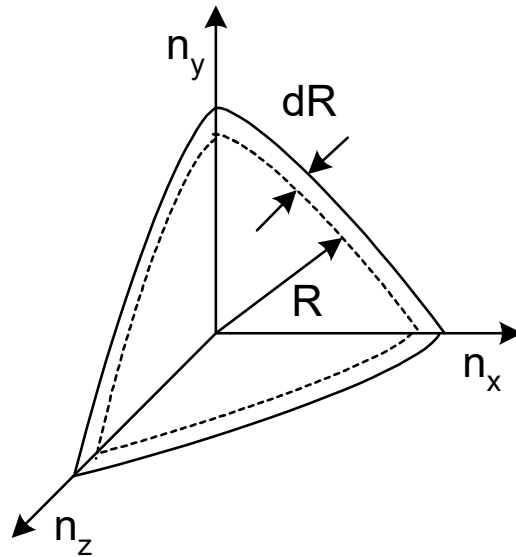
If we consider a small energy range between E and E+dE, then the number of allowed energy values in the energy range E and E+dE

= number of points in the space between the two octants of radii R and R+dR.

= (volume of space between the octants of radii R and (R+dR) × (Number of points per unit volume)

$$= \left(\frac{1}{8} 4\pi R^2 dR \right) \cdot (1) = \frac{1}{2} \pi R^2 dR$$

Now each energy value is applicable to two energy states, one for an electron, with spin up and other for spin down.

Thus, the number of allowed energy states in the range E and E+dE $= 2.\frac{1}{2}\pi R^2 dR$

i.e. $\quad g(E)dE = \pi R^2 dR \quad$ ....(4)

Now, differentiating eqn.(3) we have $dE = 2E_0 R dR$ or $R\,dR = \dfrac{dE}{2E_0}$

also eqn(3) can be rewritten as $R = \left(\dfrac{E}{E_0}\right)^{1/2}$

Substituting these in (4), we get

$$g(E)dE = \pi \left(\frac{E}{E_0}\right)^{1/2} \frac{dE}{2E_0} = \frac{\pi}{2E_0^{3/2}} E^{1/2} dE$$

Here, $E_0 = \dfrac{h^2}{8ma^2}$

$\therefore$ $g(E)dE = \dfrac{\pi}{2} \dfrac{(8m\ ^2)^{3/2}}{(h^2)^{3/2}} E^{1/2}\ dE$

On simplifying this, we get

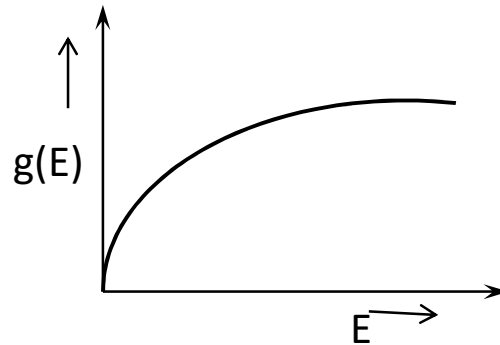$$g(E)dE = \frac{8\sqrt{2}\pi a^3 m^{3/2}}{h^3} E^{1/2}\ dE$$

a^3 represents the volume of solid. If we consider the solid to be of unit volume, i.e, $a^3=1$, then

$$\boxed{g(E)dE = \frac{8\sqrt{2}\pi m^{3/2}}{h^3} E^{1/2}\ dE}$$

This equation represents the density of states.

Thus, $g(E) \propto E^{1/2}$

A plot of g(E) Vs E is as shown. The shape of the curve is a parabola

- **Fermi energy:** The Energy corresponding to the highest occupied level at absolute zero temperature (i.e. 0 K) is called Fermi Energy and the energy level is referred to as Fermi level. It is denoted by E_F.

- **Fermi Factor (or) Fermi-Dirac distribution Function $\{f(E)\}$** : It gives the probability of occupancy of a given energy level by the electrons at thermal equilibrium.

  $f(E) = \dfrac{1}{e^{\frac{(E-E_F)}{kT}} + 1}$ where k is the Boltzmann constant.

Variation of Fermi factor with temperature:

1. Probability of occupancy for $E < E_F$ at T=0K: In this case, $(E-E_F)$ is –ve.

$$f(E) = \frac{1}{1+e^{\frac{E-E_F}{kT}}} = \frac{1}{1+e^{-\infty}} = \frac{1}{1+0} = 1$$

i.e. f (E) =1 for $E < E_F$, This means all the energy levels below the Fermi level are occupied.

2. Probability of occupancy for $E > E_F$ at T=0K:

In this case, $(E-E_F)$ is +ve.

$$f(E) = \frac{1}{1 + e^{\frac{E-E_F}{kT}}} = \frac{1}{1 + e^{\infty}} = \frac{1}{1 + \infty} = \frac{1}{\infty} = 0$$

i.e, f(E) =0 for $E > E_F$. This means, all the energy levels above Fermi level are empty.

3. Probability of occupancy at ordinary temperature; E=E_F at T>0K:

$$f(E) = \frac{1}{1+e^{\frac{E-E_F}{kT}}} = \frac{1}{1+e^0} = \frac{1}{1+1} = \frac{1}{2} = 0.5$$ i.e probability of occupation is 50%.

At ordinary temperatures, though the value of probability remains one for E<<E_F, it starts decreasing from one as the value of the E becomes closer to E_F and becomes ½ at E=E_F . Further for E>E_F, the probability value falls off to zero rapidly.
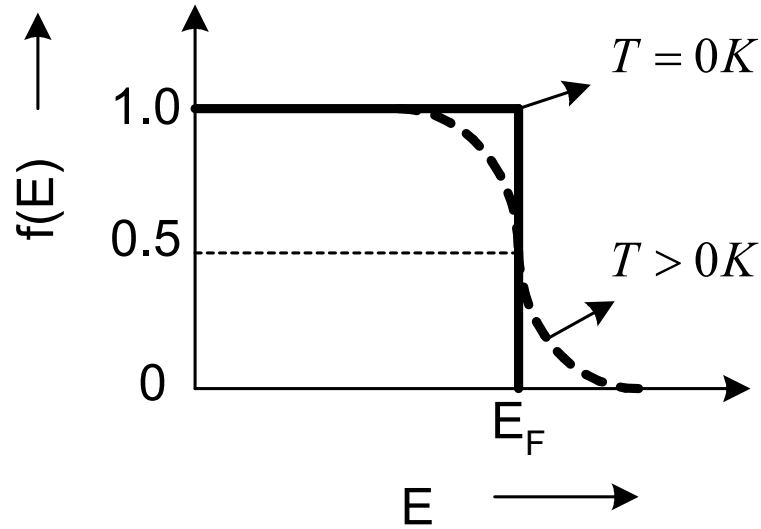
- **Fermi Dirac distribution:**

If N(E)dE is the number of those electrons which possess energy only in the range E and E+dE, then the number of electrons occupy the energy levels in an energy range dE is,

$$N(E)dE = g(E)dE \times f(E).$$

where f(E) is the fermi factor.

A plot of N(E)dE Vs E, represents the actual distribution of electrons among the available states. The distribution is known as Fermi-Dirac distribution.
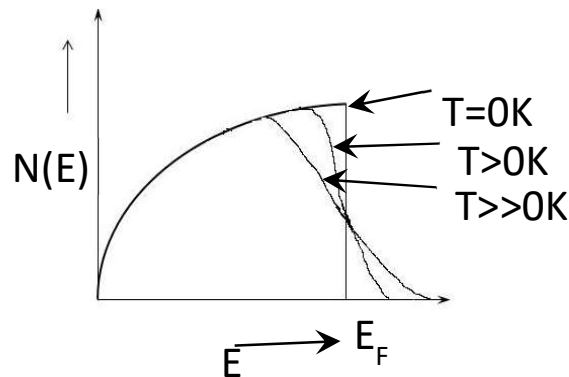
In the plot, at **T= 0 K**, N(E) increases with E up to E=E_F. For all values of E > E_F, N(E) =0. At ordinary temperatures (**T > 0K**), there are slight variations in N(E) for energy values near the Fermi energy one either side of it. Thus, the electrons which possess energy quite lower than E_F, and quite higher than E_F are unaffected at ordinary temperatures.

Let E_{F0} be the maximum energy the electron can possess at T=0K, and n be the number of free electrons/unit volume.

'n' can be obtained by integrating N(E)dE in the limit E=0 to E_{F0}.

i.e. $n = \int_{E=0}^{E_{F0}} N(E)dE = \int_{0}^{E_{F0}} g(E)f(E)dE$,

where $g(E)dE = \frac{8\sqrt{2}\pi m^{3/2}}{h^3} E^{1/2} dE$ and at T=0K,.

For E<E_F, f(E)=1

Hence $n = \frac{8\sqrt{2}\pi m^{3/2}}{h^3} \int_{0}^{E_{F0}} E^{1/2}dE$

$= \frac{8\sqrt{2}\pi m^{3/2}}{h^3} \left[\frac{2}{3}E^{3/2}\right]_{0}^{E_{F0}}$

$= \frac{8\sqrt{2}\pi m^{3/2}}{h^3} \frac{2}{3} E_{F0}^{3/2}$

$n = \frac{\pi}{3} \frac{8 2^{3/2} m^{3/2}}{h^3} E_{F0}^{3/2}$

$\therefore E_{F0}^{3/2} = \frac{h^3}{(8m)^{3/2}} \frac{3n}{\pi}$

(or) $\mathbf{E_{F0}} = \frac{h^2}{8m}\left(\frac{3n}{\pi}\right)^{2/3}$

This can be written as $E_{F0} = Bn^{2/3}$ where $B = \frac{h^2}{8m}\left(\frac{3}{\pi}\right)^{2/3}$=5.85×10⁻³⁸ Jm².

It is the velocity of the electrons which occupy fermi level.

Hence, $E_F = \frac{1}{2}mv_F^2$ or $v_F = \sqrt{\frac{2E_F}{m}}$

According to quantum free electron theory, conductivity, $\boldsymbol{\sigma = \frac{ne^2\tau}{m} = \frac{ne^2\lambda}{m^*v_F}}$; where m* is the effective mass of the electron.

Merits of Quantum free electron theory:

Quantum free electron theory sucessfully explains specific heat, temperature dependence of resistivity, and dependence of electron concentration on conductivity in metals.

Superconductivity

Kamerlingh Onnes discovered the phenomenon of superconductivity in the year 1911. When he was studying the temperature dependence of resistance of Mercury at very low temperature he found that resistance of Mercury decreases with temperature with the decrease in temperature up to a particular temperature $Tc = 4.15K$. Below this temperature the resistance of mercury abruptly drops to zero. Between 4:15K and Zero-degree Kelvin Mercury offered no resistance for the flow of electric current. The phenomenon is reversible and material becomes normal once again when temperature was increased above 4:15K. He called this phenomenon as superconductivity and material which exhibited this property as superconductors.

Thus the phenomenon of super conductivity is defined as "The phenomenon in which resistance of certain metals, alloys and compounds drops to zero abruptly, below certain temperature is called superconductivity"
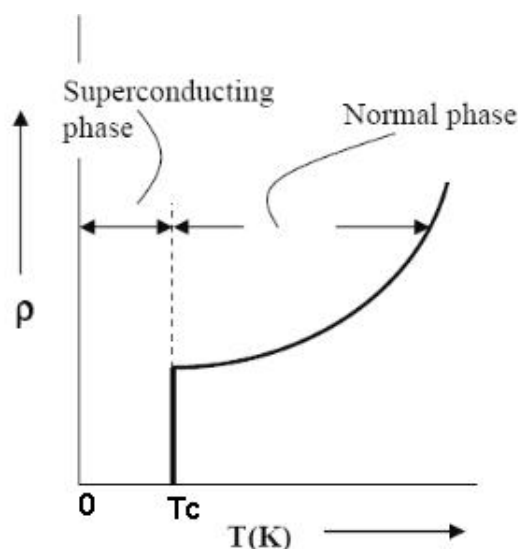
The temperature, below which materials exhibit superconducting property is called critical temperature, denoted by Tc. Critical temperature Tc is different for different substances. The materials, which exhibit superconducting property, are called superconductors. Above critical temperature material is said to be in normal state and offers resistance for the flow of electric current. Below critical temperature material is said to be in superconducting state. Thus Tc is also called as transition temperature.

Properties of Superconductors

Critical Temperature (Tc):

The critical temperature is the temperature below which the material changes from conductors to superconductors. The critical temperature is also called transition temperature.

Critical Field (Hc):

When superconductor is placed in a magnetic field it expels magnetic lines of force completely out of the body and becomes a perfectly diamagnetic material. But if the strength of the magnetic field is further increased, it was found that for a particular value of the magnetic field, material loses its superconducting property and becomes a normal conductor. The value of the magnetic field at which superconductivity is destroyed is called the Critical magnetic field, denoted by Hc . It was found that by reducing the temperature of the material further superconducting property of the material could be restored. Thus, critical field doesn't destroy the superconducting property of the material completely but only reduces the critical temperature of the material.
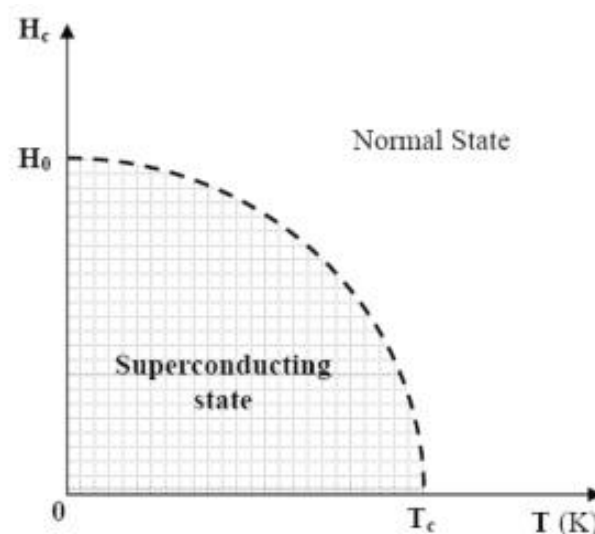
Critical magnetic field Hc depends on the temperature of the material. The relationship between the two is given by

$$H_c = H_0 \left(1 - \frac{T^2}{T_c^2}\right)$$

Critical Current (Ic)

It is the threshold value of the current at which the magnetic field due to the current flowing in a superconductor will be equal to the critical magnetic field. The critical current, according to Silsbee's hypothesis, is

Ic = 2πrHc

Where r is radius of the conductor. critical current is thus not an intrinsic property of a superconductor, but is dependent upon the size of the conductor

Critical Current density (Jc)

The current density at which the superconducting properties disappear is called critical current density (Jc)

$$J_c = \frac{I_c}{A}$$

List of superconductors with their transition temperatures

Material	Critical Temperature (Tc) in K
Indium	3.4
Mercury	4.2
Lead	7.2
Ni-Ti	9.6
Bi2223 ($Bi_2Sr_2Ca_2Cu_3O$)	125

The Meissner effect

When a weak magnetic is applied to a superconducting specimen at a temperature below transition temperature T_C the magnetic flux lines are expelled. This phenomenon is called Meissner effect.
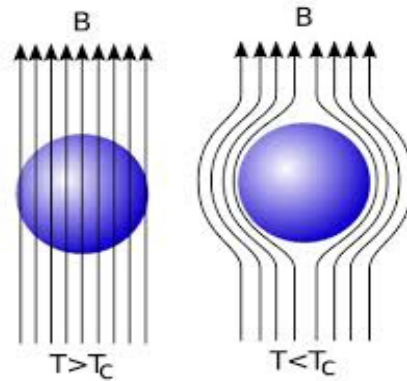
Under normal state the magnetic induction inside the specimen is

$$B = \mu_0 (H + I)$$

Where H is the external applied magnetic field and I is the magnetization produced inside the specimen.

When the specimen is in superconducting state $B = 0$ (Meissner effect)

$$0 = \mu_0 (H + I)$$

Or

$$H = -I$$

$$\chi = \frac{I}{H} = -1$$

Thus the material is act as a perfectly diamagnetic (for diamagnetic material $\chi = -1$).

Applications:

1.   The superconducting magnets are used in Magnetic Resonance Imaging (MRI) systems and NMR spectrometers.

2.   Vehicles, which float under the magnetic effect, are called MAGnetically LEVitated vehicles or MAGLEV vehicles. Such vehicle offers no friction, less power and noiseless transportation and hence used in high speed train transportation.

3.   They are used in memory or storage elements

4.   Superconducting Quantum Interface Device, SQUIDs are highly sensitive devices, which can detect even weakest magnetic fields of the order of femto Telsa.

Dielectrics

Dielectric materials

Dielectric materials are a type of electrical insulators that can be polarized through an external applied electric filed. These materials, generally, do not contain any free charge carriers for conduction. In such dielectric materials, positive and negative charge entities are bounded together. The behavior of dielectric materials can be modified by an external electric field through reorienting the charges within the atom or molecules. Generally, these materials do not have any net dipole moment in absence of external electric field. As a consequence of applied electric field, positive charge of dielectric is pushed in direction of the field while negative the opposite way. The overall effect is a displacement of entire positive charge in the dielectric relative to the negative charge, formed induced electric dipoles. Dielectric materials are classified into two categories, named as polar molecules and non-polar molecules.

Electric Displacement $\vec{D}$:

It is the number of electric lines of force acting through per unit area.

$$\vec{D} = \varepsilon E = \varepsilon_o \varepsilon_r E \quad (\varepsilon = \varepsilon_o \varepsilon_r)$$

Dielectric Constant (ε_r):

Relative permittivity or dielectric constant is the ratio of permittivity of medium (ε) to the permittivity of free space (ε_o)

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0} \qquad \therefore \varepsilon = \varepsilon_o \varepsilon_r \qquad \varepsilon_o = 8.854 \times 10^{-12} Fm^{-1}$$

Relation between Polarization, susceptibility and dielectric constant:

We know that, the net electric field E between two ends of dielectric slab $\quad E = E_o - E_p$

$$E_o = \frac{D}{\varepsilon_0} \; and \; E_p = \frac{P}{\varepsilon_0} \quad \therefore E = \frac{D}{\varepsilon_0} - \frac{P}{\varepsilon_0} \quad \therefore D = \varepsilon_0 E + P \; but \; D = \varepsilon_o \varepsilon_r E$$

$$\varepsilon_o \varepsilon_r E = \varepsilon_o E + P$$

$$P = \varepsilon_o (\varepsilon_r - 1)E \quad or \; P = \varepsilon_o \chi E$$

The dimensionless quantity $\chi = (\varepsilon_r - 1)$ is called dielectric susceptibility

Dielectric Polarization

Generally, dielectric materials do not have any net dipole moment in absence of external electric field. As a consequence of applied electric field, positive charge of dielectric is pushed in direction of the field while negative the opposite way. The overall effect is a displacement of entire positive charge in the dielectric relative to the negative charge, formed induced electric dipoles. The relative displacement of the charges in dielectric is called Polarization, leaving the material polarized. The phenomenon of appearance of the charges on the surfaces of the dielectric in presence of an external electric field is known as polarization or dielectric polarization.
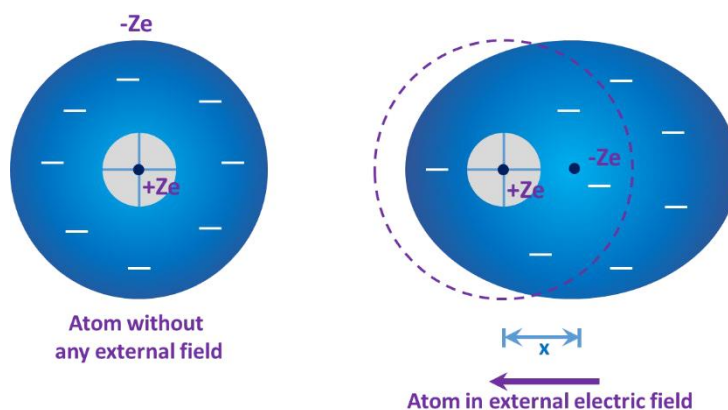
$$P = \alpha E$$

where, constant of proportionality α is called polarizability. It depends on the structure of the atom/molecule.

Types of Polarization

(i) Electronic Polarization

(ii) Ionic Polarization

(iii) Orientation Polarization

(iv) Interfacial or Space-charge Polarization

(i)   Electronic Polarization (P_e)

Atom without any external field

Atom in external electric field

Electronic polarization is the result of the displacement of positive and negative charges of an atom in presence of an external electric field. In absence of field, the positive nucleus and negatively charged electrons balanced by each other with zero net dipole moment. But when electric field is applied, the electrons and nucleus experience Lorentz forces in opposite directions. This leads to displacement of the centre of the negatively charged electron cloud relative to the positively charged nucleus of an atom leads to dipole moment. Monoatomic gases exhibit only this type of polarization. This type of polarization is temperature independent.

The induced dipole moment (Pe) is proportional to applied electric field (E)

$$P_e \propto E$$

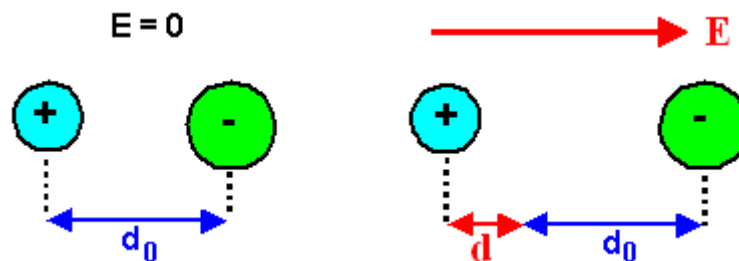If there are N atoms in dielectric medium, then

$$P_e \propto NE$$

$$P_e = \alpha_e NE$$

Where α_e is the electronic polarizability, N is number of dipoles,

$$\alpha_e = 4\pi\varepsilon_o R^3 \text{ where R is radius of the atom}$$

The electronic polarizability depends only on the radius of atom and independent of temperature.

(ii)   Ionic Polarization (Pi)

Ionic dielectric materials such as NaCl show ionic polarization. In case of ionic crystals each ions are at a distance d_0 in absence of the field. When the sample is subjected to an external field, each ion displaced in opposite direction as shown in the figure. The displacement between

ions causes an increase or decrease in distance between the atoms leads to a net dipole moment. Ionic polarization is time dependent but temperature independent.
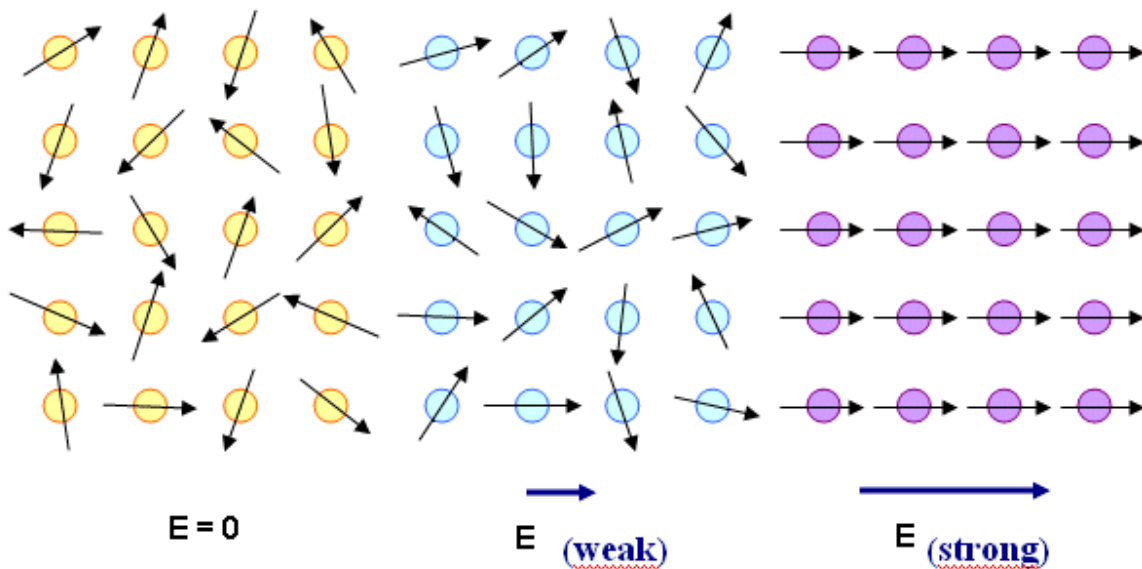
$$P_i \propto E$$

$$P_i = N\alpha_i E$$

Or

$$\alpha_i = \frac{P_i E}{N}$$

(iii)   Orientation Polarization (P_o)

Orientation polarization occurs in polar dielectrics. These materials exhibit polarization even in absence of electric field. Due to the random orientations of dipoles or molecules the net dipole moment is zero. When polar dielectric materials are placed in an electric field, the permanent dipoles rotate about their axes of symmetry to align with applied field, which exerts a torque on them. Hence this type of polarization is called as orientational polarization. The orientational polarization is strongly temperature dependent, it decreases with increase in temperature.

$$P_0 = N\alpha_o E$$

$$\alpha_0 = \frac{\mu_o^2}{3kT}$$

(iv) Interfacial or Space-charge Polarization (P_s)
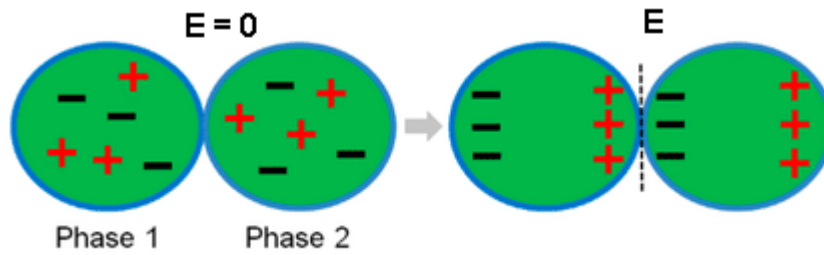
In some dielectric materials, due to the application of an electric field, charges accumulate at the interfaces of the multiphase dielectric material. The ions are diffused over a distance due to redistribution of charges in the dielectric medium. The tendency of redistribution of electric dipoles/charges in dielectric medium in an external electric field is known as space charge polarization. It is not significant in most of the dielectrics

Therefore, Total polarization in dielectric material is given by

$$P = P_e + P_i + P_o + P_s$$

the total polarizability is given by

$$\alpha = \alpha_e + \alpha_i + \alpha_o$$

Expression for Internal field in the case of Liquids and Solids:

Expression for Internal field:

 When dielectric material is placed in the external electric field, it is polarized creating electric dipoles. Each dipole sets electric field in the vicinity. Hence the net electric field at any point within the dielectric material is given by "The sum of external field and the field due to all dipoles surrounding that point". This net field is called internal field or Local field.
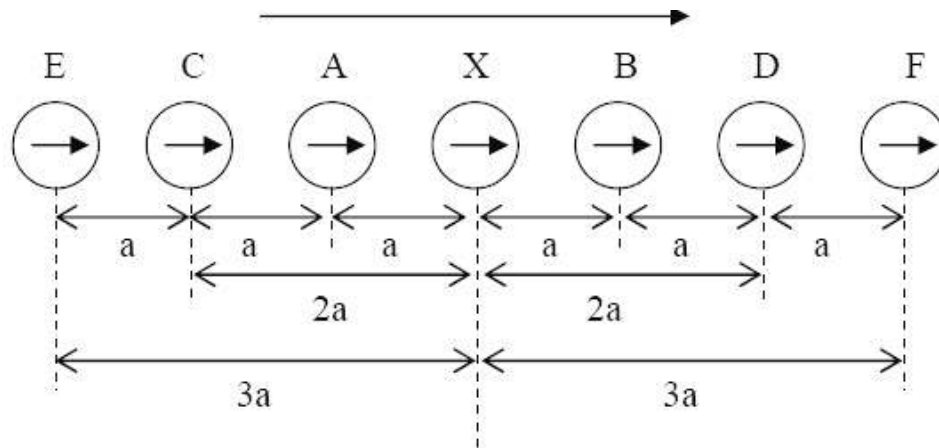
Linear array of atoms in an electric field

Consider a dielectric material placed in external electric field of strength 'E'. Consider an array of equidistant dipoles within the dielectric material, which are aligned in the field direction as shown in the figure.

the total field at 'X' due to equidistant dipoles 'A' and 'B' is given by

$$E_1 = E_{X_A} + E_{X_B}$$

$$E_1 = \frac{\mu}{2\pi\varepsilon_0 a^3} + \frac{\mu}{2\pi\varepsilon_0 a^3}$$

$$\Rightarrow \ E_1 = \frac{\mu}{\pi\varepsilon_0 a^3}$$

Similarly the total field at 'X' due to equidistant dipoles 'C' and 'E' is given by

$$E_2 = E_{X_C} + E_{X_D}$$

$$E_2 = \frac{\mu}{2\pi\varepsilon_0 (2a)^3} + \frac{\mu}{2\pi\varepsilon_0 (2a)^3}$$

$$\Rightarrow \ E_2 = \frac{\mu}{\pi\varepsilon_0 (2a)^3} \quad (\because \ r = 2a)$$

Similarly the total field at 'X' due to equidistant dipoles 'D' and 'F' is given by

$$E_3 = E_{X_E} + E_{X_F}$$

$$E_3 = \frac{\mu}{2\pi\varepsilon_0 (3a)^3} + \frac{\mu}{2\pi\varepsilon_0 (3a)^3}$$

$$\Rightarrow \ E_3 = \frac{\mu}{\pi\varepsilon_0 (3a)^3} \quad (\because \ r = 3a)$$

The net field at 'X' due to all dipoles in the array is given by

$$E' = E_1 + E_2 + E_3 + E_4 + \cdots\cdots$$

$$E' = \frac{\mu}{\pi\varepsilon_0 a^3} + \frac{\mu}{\pi\varepsilon_0 (2a)^3} + \frac{\mu}{\pi\varepsilon_0 (3a)^3} \cdots\cdots\cdots$$

$$= \frac{\mu}{\pi\varepsilon_0 a^3} \left[1 + \frac{1}{2^3} + \frac{1}{3^3} + \cdots\cdots \right]$$

$$= \frac{\mu}{\pi\varepsilon_0 a^3} \sum_{n=1}^{\infty} \frac{1}{n^3}$$

$$\text{but } \sum_{n=1}^{\infty} \frac{1}{n^3} \cong 1.2$$

$$\therefore E' = \frac{1.2\mu}{\pi\varepsilon_0 a^3}$$

The Local Field at 'X' is given by

$$E_i = E + E' = E + \frac{1.2\mu}{\pi\varepsilon_0 a^3}$$

WKT $\mu = \alpha E$

$$\therefore E_i = E + \frac{1.2\alpha E}{\pi\varepsilon_0 a^3}$$

In 3-dimensions the above equation can be generalized by replacing $1/a^3$ by 'N' (where 'N' is the number of atoms per unit volume) and $1.2/\pi$ by γ (where γ is called Internal Field Constant).

$$\therefore E_i = E + \frac{\gamma N \alpha E}{\varepsilon_0}$$

$$\text{but polarization } P = N\alpha E \therefore \ E_i = E + \frac{\gamma P}{\varepsilon_0}$$

Since γ, P and ε_0 are positive quantities $E_i > E$. For a Cubic Lattice $\gamma = 1/3$ and the Local field is called Lorentz field, It is given by

$$E_L = E + \frac{P}{3\varepsilon_0}$$

Clausius-Mosotti Relation

Consider an Elemental solid dielectric material. Since they dont posses permanent, dipoles, for such materials, the ionic and orientation polarizabilities are zero. Hence the polarization P is given by

$$P = N\alpha_e E_L \ \text{where } E_L \text{ is Lorentz field}$$

$$= N\alpha_e \left(E + \frac{P}{3\varepsilon_0} \right)$$

$$P = N\alpha_e E + N\alpha_e \frac{P}{3\varepsilon_0}$$

$$P \left[1 - \frac{N\alpha_e}{3\varepsilon_0} \right] = N\alpha_e E$$

$$\therefore \; P = \frac{N\alpha_e E}{\left[1 - \frac{N\alpha_e}{3\varepsilon_0}\right]} \qquad\qquad 1$$

Where 'N' is the number of dipoles per unit volume, α_e is electronic polarizability, ε_0 is permitivity of free space, and E is the Electric field strength.

The polarization is related to the applied field strength as given below

$$D = \varepsilon_0 E + P$$

$$\text{Since D} = \varepsilon_0 \varepsilon_r \text{E}$$

$$\varepsilon_0 \varepsilon_r E = \varepsilon_0 E + P$$

$$\therefore \; P = \varepsilon_0(\varepsilon_r - 1)E \qquad\qquad 2$$

where 'D' is Electric Flux Density and ε_r is Dielectric Constant.

Equating equations $\quad 1 \quad$ and $\quad 2$

$$\therefore \; P = \frac{N\alpha_e E}{\left[1 - \frac{N\alpha_e}{3\varepsilon_0}\right]} = \varepsilon_0(\varepsilon_r - 1)E$$

$$\frac{N\alpha_e E}{\varepsilon_0(\varepsilon_r - 1)E} = \left[1 - \frac{N\alpha_e}{3\varepsilon_0}\right]$$

$$\frac{N\alpha_e}{\varepsilon_0(\varepsilon_r - 1)} + \frac{N\alpha_e}{3\varepsilon_0} = 1$$

$$\frac{N\alpha_e}{3\varepsilon_0}\left(\frac{3}{(\varepsilon_r - 1)} + 1\right) = 1$$

$$\frac{N\alpha_e}{3\varepsilon_0}\left(\frac{3 + \varepsilon_r - 1}{\varepsilon_r - 1}\right) = 1$$

$$\frac{N\alpha_e}{3\varepsilon_0}\left(\frac{\varepsilon_r + 2}{\varepsilon_r - 1}\right) = 1$$

$$\therefore \; \frac{N\alpha_e}{3\varepsilon_0} = \left(\frac{\varepsilon_r - 1}{\varepsilon_r + 2}\right)$$

The above equation is called Clausis-Mosotti relation. Using the above relation if the value of dielectric constant of the material is known then the electric polarizability can be determined using

$$\alpha_e = \frac{3\varepsilon_0}{N}\left(\frac{\varepsilon_r - 1}{\varepsilon_r + 2}\right)$$

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Applications of Dielectrics

Dielectrics are used for energy storage in capacitors.

Dielectrics are used in Liquid Crystal Displays.

Ceramic dielectric is used in Dielectric Resonator Oscillator.

Barium Strontium Titanate thin films are dielectric which are used in microwave tunable devices providing high tunability and low leakage current.

In electrical transformers, mineral oils are used as a liquid dielectric and they assist in the cooling process.

Application of dielectrics in transformers:

A transformer consists of two insulated conducting coils wound on a core, the core is also insulated. In high voltage transformers further insulation is required to be provide between individual windings in the coils and also between the core and the coils. Hence the size of the transformers increases and also the size increases with operational ac frequency. Normally the insulation is provided by using paper, mica or cloth.

"If there is air gap between the windings, as the permittivity of air is less, ionization of air occurs at high voltage leads to excessive heating which damages the insulation. This effect is called corona".

Generally, mica is used to guard against corona, when the operating voltage is up to 3KV. However, if the operating voltage is above 3KV up to 100KV, a kind of oil called transformer oil is used between individual windings and the core. Apart from guarding against corona, the oil helps to keep the transformer cool. It remains stable at high temperature. For large size transformers, water circulation round the body of the transformer is provided to cool the transformer.