Tutorials Unit -4 (Electrical Conductivity of Metals)

Review:

$e = 1.6 \times 10^{-19}$C. $m_e = 9.1 \times 10^{-31}$kg. $k = 1.38 \times 10^{-23}$ J/K. $N_A = 6.026 \times 10^{26}$/k.mol

i. The number of free electrons in unit volume (electron density 'n'), $n = (Z N_A D)/M$.
Z → Valency, D → density of metal, N_A → Avogadro number and M → Atomic weight.

ii. Electrical conductivity, $\sigma = ne\mu = \frac{ne^2\tau}{m} = \frac{1}{\rho} = \frac{J}{E}$.

iii. Mobility $\mu = \frac{e\tau}{m} = {}^{v_d}/_{E} = {}^{\sigma}/_{ne}$.

iv. The probability that an energy state 'E' is occupied (Fermi Factor), $f(E) = 1/\{e^{(E-E_F)/kT} + 1\}$.

v. Fermi energy of a metal, $E_F = \frac{h^2}{8m}\left[{}^{3n}/_{\pi}\right]^{2/3} = 5.85 \times 10^{-38}(n)^{2/3}$.

vi. Fermi velocity $V_F = \sqrt{{}^{2E_F}/_{m}}$.

vii. Mean free path in Fermi level, $\lambda_F = V_F \times \tau$.

NUMERICAL PROBLEMS

1) The density and atomic weight of Aluminium are 2700 kg/m^3 and 26.98 kg respectively. In an applied field of 10 V/cm and at a temperature of 25^0 C, calculate drift velocity and relaxation time. Given, the resistivity of Al at room temperature is 2.62 $\times 10^{-8}$ohm-m and Al is a trivalent metal.
2) A copper wire of cross sectional area 0.005 cm^2 carries a steady current of 50 amperes. Density, atomic weight and resistivity of monovalent copper are 8900 kg m^{-3}, 63.54 and 1.7 $\times$ $10^{-8}\Omega$m respectively. Calculate the drift velocity and mobility according to classical free electron theory.
3) Copper has a density of 8900 kg/m^3 and mass of 63.54 kg. The electrical conductivity is 5.9 $\times 10^7$ mhos/m. Calculate the Fermi energy of copper and mean free path according to quantum free electron theory.
4) The density of silver is 10.5 $\times 10^3$ kg/m^3 and atomic weight is 107.9 and silver is monovalent. The conductivity of silver at 20^0 C is 6.8 $\times 10^7\Omega$/m. Calculate the mobility of electrons in silver and mean free path according quantum free electron theory.
5) The relaxation time of electrons in trivalent Aluminium is 7.3 $\times 10^{-15}$ s and atomic weight and density are 26.98 kg and 2700 kg/m^3 respectively. Calculate the Fermi energy, Fermi velocity and mean free path according to quantum free electron theory.
6) Fermi energy of silver is 5.5 eV. Calculate the probability of occupancy of a state which is (i) 0.03 eV below Fermi level, (ii) 5kT above Fermi level at a temperature of 300 K.
7) The probability of occupancy of a level with energy $E_F + 0.2$ eV is 2% at a temperature T. Calculate T.
8) The free electron concentration in Aluminium is 18 $\times 10^{28}$ m^{-3}. Calculate the probability of occupancy of a level with an energy equal to 10 eV at a temperature of 800 K.

Dielectrics and Superconductivity

Dipole moment $\mu = q.d$ Polarization $\bar{P} = \varepsilon_o(\varepsilon_r - 1)E$ Electric displacement $\bar{D} = \varepsilon_o\varepsilon_r E$

Induced Dipole moment $\mu = \alpha E$ Electronic Polarizability $\alpha_e = \frac{\varepsilon_o(\varepsilon_r - 1)}{N}$

Ionic polarizability $P_i = N\alpha_i E$ Orientation Polarizability $\alpha_o = \frac{NE\mu^2}{2KT}$

Internal Field $E_i = E + \frac{P}{3\varepsilon_o}$ Clausius-Mossotti equation $\frac{N\alpha_e}{3\varepsilon_o} = \frac{(\varepsilon_r - 1)}{(\varepsilon_r + 2)}$

Capacitance $C = \frac{Q}{V} = \frac{\varepsilon_o\varepsilon_r A}{d}$ $\varepsilon_o = 8.854x10^{-12}F/m$

1. Find the polarization produced in a crystal by an electric field of strength 500V/m if the dielectric constant is 6
2. NaCl crystal is subjected to an electric field of 1000V/m and the resulting polarization is $4.3x10^{-8}$ C/m^2. Calculate the dielectric constant of NaCl.
3. The dielectric constant of Helium gas at 0^o C is 1.000074. If the density of atoms is $2.7x10^{25}$/m^3, Calculate the dipole moment induced in each atom in an applied electric field of $3x10^4$V/m.
4. The distance between two parallel plate capacitor is 1.5mm and area of plate $5x10^{-4}$m^2. The dielectric constant of the medium between the plates of capacitor is 6. Calculate the charge on the capacitor in an applied potential of 100V across the plates.
5. A parallel plate capacitor of area 650mm^2 separated at 4mm has a charge of $2x10^{-10}$C on it. What is the resultant voltage across the capacitor when a material of dielectric constant 3.5 is introduced between the plates?
6. An air filled parallel plate capacitor has a capacitance of 1.5pF. If the separation between the plates is doubled and wax is inserted between the plates, the capacitance increases to 3pF, then calculate the dielectric constant of wax.
7. The dielectric constant of Sulphur is 3.4. Assuming cubic lattice for Sulphur, calculate the electronic polarizability. Given density and atomic weight of Sulphur are 2.07g/cc am 32.07kg respectively.
8. Find the polarization produced in a dielectric medium of relative permittivity 15 in presence of an electric field of 500 V/m.
9. A superconducting material has critical temperature of 3.7K at critical magnetic field of 0.0306 T at 0K. Find the critical field at temperature 2K.
10. Calculate critical current passing through the wires of lead having diameter1mm, at temperature 4.2K, Critical temperature for lead is 7.18K, Critical field is $6.5x10^4$A/m