MAT101

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M S RAMAIAH INSTITUTE OF TECHNOLOGY

(AUTONOMOUS INSTITUTE, AFFILIATED TO VTU)

BANGALORE – 560 054

SEMESTER END EXAMINATIONS – JANUARY 2015

Course & Branch	: **B.E: Common to All Branches**	**Semester**	: **I**
Subject	: **Engineering Mathematics-I**	**Max. Marks**	: **100**
Subject Code	: **MAT101**	**Duration**	: **3 Hrs**

Instructions to the Candidates:

- Answer one full question from each unit.

UNIT – I Marks

1. a) i) If $u = x^2 + y^2$ and $x = at^2$, $y = 2at$ find $\frac{du}{dt}$. (02)

   ii) Find the angle between radius vector and tangent for the curve $r = ae^{\theta\cot\alpha}$ (03)

   b) Find the pedal equation of the curve $r^m \cos(m\theta) = a^m$ (08)

   c) If $u = x + y + z$, $uv = y + z$, $uvw = z$, show that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2 v$. (07)

2. a) i) If $u = \sin^{-1}\left(\frac{x^2y^2}{x+y}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3\tan u$. (02)

   ii) If $u = x + 3y^2 - z^3, v = 4x^2yz, w = 2z^2 - xy$ then evaluate $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ at $(1,-1,0)$ (03)

   b) If $u = f(y-z,\ z-x,\ x-y)$ then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ (08)

   c) Find the angle of intersection of the curves $r = a(1 - \cos\theta)$ and $r = 2a\cos\theta$ (07)

UNIT – II

3. a) i) Evaluate $\int_0^{\pi} x\cos^6 x \, dx$ (02)

ii) Find the total length of the cardioid r=a(1+cos θ), a >0 (03)

b) Trace the curve $y^2(2a-x)=x^3, \; a>0$. (08)

c) By using the rule of differentiation under the integral sign evaluate the integral $\int_0^1 \frac{x^\alpha -1}{\log x} dx$ where $\alpha \geq 0$. Hence find $\int_0^1 \frac{x^3-1}{\log x} dx$ (07)

4. a) i) Evaluate $\int_0^{\pi/2} \cos^2\theta \sin^4\theta \, d\theta$ (02)

ii) Find the length of one arch of the cycloid (03)
$x=a(\theta-\sin\theta), \; y=a(1-\cos\theta), \; a>0, \; 0\leq\theta\leq 2\pi$

b) Obtain the reduction formula for $I_n=\int \cos^n x \, dx$ and hence find the reduction formula for $I_n=\int_0^{\pi/2}\cos^n x\,dx$. Also find $\int_0^{\pi/2}\cos^8 x\,dx$ and $\int_0^{\pi/2}\cos^7 x\,dx$ (08)

c) Evaluate $\int_0^{2a} \frac{x^n \, dx}{\sqrt{2ax-x^2}} = \pi a^n \frac{(2n)!}{(n!)^2 2^n}$ (07)

UNIT – III

5. a) i) Write the relation between Cartesian and spherical polar coordinate system. (02)

ii) Write the limits of integration after changing the order of integration with a neat diagram for the integral $\int_0^1 \int_{x^2}^{\sqrt{x}} f(x,y)dydx$. (03)

b) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx\, dy$ by transforming it into polar coordinates. (08)

c) Evaluate $\iiint_V (x+y+z)dxdydz$ over the region bounded by the planes $x=0, y=0, z=0 \; and \; x+y+z=1$. (07)

6. a) i) Evaluate $\int_0^{\pi}\int_0^{a\cos\theta} r\,dr d\theta$ (02)

ii) If u=x+y and v=x-2y then find the transformation of the area element dxdy. (03)

b) Find by triple integration, the volume of the sphere $x^2+y^2+z^2=a^2$. (08)

c) Change the order of integration and evaluate $\int_0^1\int_{x^2}^{2-x} xy\,dy\,dx$. (07)

UNIT – IV

7. a) i) Define the curl of a vector field. (02)

ii) Show that the vector field $\vec{f}=2x^2z\hat{i}-10xyz\hat{j}+3xz^2\hat{k}$ is solenoidal. (03)

b) Prove that $\nabla\times(\nabla\times\vec{A})=\nabla(\nabla\cdot\vec{A})-\nabla^2\vec{A}$. (08)

c) A particle moves along the curve $r=2t^2\hat{i}+(t^2-4t)\hat{j}+(3t-5)\hat{k}$ .Find the components of velocity and acceleration in the direction of the vector $\hat{i}-3\hat{j}+2\hat{k}$ at $t=2$. (07)

8. a) i) Find the unit tangent vector to the curve $\vec{r}(t)=4\sin t\,\hat{i}+4\cos t\,\hat{j}+3t\hat{k}$. (02)

ii) If $\phi=e^{-2t}$, $\vec{f}=e^t\hat{i}+e^{-t}\hat{j}+\sqrt{2t}\,\hat{k}$ find $\frac{d}{dt}(\phi\vec{f})$ at t=0. (03)

b) Find the angle between the surfaces $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at the point $P(2,-1,2)$. (08)

c) Find the constants a,b,c so that the vector field (07)

$\vec{f}=(x+2y+az)\hat{i}+(bx-3y-z)\hat{j}+(4x+cy+2z)\hat{k}$ is irrotational. Also find a scalar function ϕ such that $\vec{f}=\nabla\phi$.

UNIT – V

9. a) i) If $\vec{F}=x^2\hat{i}+xy\hat{j}$, evaluate $\int\vec{F}.\vec{dr}$ from (0,0) to (1,1) along the line y=x. (02).

ii) Find the total workdone in moving particle in a force field (03)
$\vec{f}=3xy\hat{i}-5z\hat{j}+10x\hat{k}$ along the curve
$x=t^2+1,\ y=2t^2\ and\ z=t^3\ from\ t=1\ to\ t=2.$

b) State and prove Green's theorem in a plane. (08)

c) Evaluate $\int_S \vec{F}.\hat{n}\,ds$ where $\vec{F} = z\hat{i} + x\hat{j} - 3y^2z\hat{k}$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the positive octant between z=0 and z=5. (07)

10. a) i) State Gauss's divergence theorem. (02)

ii) If $\vec{F} = (3x^2 + 6y)\,\hat{i} - 14yz\hat{j} + 20xz^2\hat{k}$ evaluate $\int_C \vec{F}.d\vec{r}$ from (0, 0, 0) to (1, 1, 1) along the curve C given by x=t, y=t^2, z=t^3. (03)

b) Verify Green's theorem for $\int_C [xy + y^2)dx + x^2dy]$, where C is bounded by y=x and y=x^2. (08)

c) By Stokes's theorem evaluate $\int_C \vec{F}.d\vec{r}$ where $\vec{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ and S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$, c is its boundary. (07)

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