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Lithium-ion Battery Models for Computer Simulation*

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***Abstract* - Lithium-ion batteries are very popular nowadays. In order to design and evaluate the performance of systems involving batteries, good models are required for systems simulation. In this paper, popular lithium-ion battery models are investigated and presented. Selection of appropriate models for a particular simulation will also be presented.**

Index Terms - Lithium battery Models. Computers simulation.

I. INTRODUCTION

Lithium-ion batteries are widely used in many portable electronic devices, such as mobile telephones and laptop computers, because of their excellent performance, compact, high energy density and high reliability. Battery Models are necessary for the electrical design to understand the electrical characteristics of the Lithium-ion battery. It is also useful to describe mathematically the characteristics for the efficient use of the battery. The battery capacity is finite, and the time when the battery becomes discharged is the lifetime of the battery. Once the battery is exhausted the system shuts down, therefore, maximizing the battery lifetime is an important problem. The state-of-charge (SoC) of a battery is essential to users and power management policy. A battery's SoC is its available capacity expressed as a percent-age of its rated capacity. Many parameters influence the charge/discharge procedure, such as load current, temperature and state of charge. Consequently, good battery models are very important in the area of systems simulation. In this paper the commonly use battery models will be reviewed. Selection of appropriate models for a computer simulation as well as the simulation results will also be presented.

II. BATTERY MODELS

Researchers around the world have developed a wide variety of computational mathematical models to describe the electrochemical processes and dynamics of a battery. The battery models can be divided into four categories: empirical models, electrochemical models, electrical-circuit models and abstract models. Models in each category should be much close to the experimental data such as voltage, charging time, and so on. As a mathematical model, computational complexity and functional integrity are important to get the good results.

A. Empirical Models

Empirical models consist of equations describing battery behavior with parameters fitted to match experimental data. They are easy to configure, but the computational results are the least accurate. So they are more applicable to the imprecise capacity evaluation. There are three such models are in use.

1) Peukert's law: The battery will show different characteristics at varying discharging current. Peukert first devised a formula [1] that expresses the capacity of a lead-acid battery in terms of the rate at which it is discharged. As the rate increases, the battery's capacity decreases, although its actual capacity tends to remain fairly constant. Peukert's equation is expressed as follows:

$$C_p = I^k t \tag{1}$$

where C_p is the capacity according to Peukert, I is the discharge current, t is the time of discharge and the empirical parameter k is the Peukert constant. The constant k is related to the structure of the battery and shows whether the battery's behavior is good or not. For an ideal battery, k would equal to one and the battery should reach its best behavior at the certain current. The larger k is the more capacity would be lost at the same current. Though it is a key parameter in the battery, this power-law relationship does not suitable for time-varying loads when the average current does not adequately represent the battery discharge conditions. According to Peuker's law, all load profiles with the same average would result in the same time-to-failure. This conclusion is not supported by experimental data [2].

2) Battery efficiency model: Massoud Pedram and Qing Wu put forward the battery efficiency factor model [3] to consider the relationship between battery lifetime and different current distributions. Given a fixed battery output voltage, the battery efficiency factor μ is defined as the ratio of actual capacity(CAP^{act}) that can be used by the circuit to the theoretical capacity(CAP_0).

$$CAP^{act} = CAP_0 \cdot \mu, \quad 0 \le \mu \le 1 \tag{2}$$

And μ is a function of discharge current I:

$$\mu = f(I) \tag{3}$$

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With μ, they showed that even when the average discharge current remains the same, different discharge current distributions may result in very different battery lifetime. In particular, the maximum battery life is achieved when the variance of the discharge current distribution is minimized and uniformly distributed current causes the minimum battery life.

3) Weibull model: In paper [4], the authors presented statistical methods for modeling the discharge behavior of lithium-oxyhalide cells. They used a Weibull sigmoidal random-coefficients model with three coefficients to fit the discharge voltage data as a function of capacity under a constant load at a given temperature.

$$y(t) = \alpha[1 - \exp(-(\beta t)^{\gamma}] + \varepsilon(t) \qquad (4)$$

Where α, β, γ represent the different phase of the discharge curve.

Then they used the Gauss-Newton nonlinear estimation method to fit a series of data points for different load and temperature as a quadratic response surface.

B. Electrochemical Models

Contrary to the empirical models, electrochemical models describe the discharge behavior based on the discharge mechanism. The electrochemical models are the most accurate but also the slowest to produce predictions. There are five such models.

J.Newman et.al developed an isothermal electrochemical model [6] to describe the galvanostatic charge and discharge of the battery. The concentrated solution theory is applied to derive a set of differential equations to solve essential information of the concentration of the lithium, the current density and temperature distribution. But this model needs large quantity of parameters of structure and material feature. The solution of the model is complex and time consuming. So the model is used only on the design and optimization of the batteries.

D.Rakhmatov et.al simplified the Newman's model and just thought of the one-dimension (1-D) diffusion of the electroactive species. They obtained the dependence of the capacity between discharging current and applied the result in the research on the embedded system's energy consumption.

In paper [16], a coupled electrochemical and thermal model is developed to study heat transfer and thermal management of Lithium polymer batteries. Temperature dependent parameters including the diffusion coefficient of lithium ions, ionic conductivity of lithium ions, transference number of lithium ions, etc., have been considered to more completely characterize the thermal behavior of the lithium polymer system. The experimental and mathematical results are compared to valid the model.

The authors in [20] introduced a fully observable/ controllable state variable model from an impedance representation of a Lithium ion cell. Validated against a 313th order nonlinear model of a 6 Ah cell, a 12th order state variable model, with 0–10 Hz bandwidth, predicts terminal voltage to within 1% for pulse and constant current profiles at rates up to 50 C. Model properties indicate that electrode surface concentrations are more observable/ controllable than electrode bulk concentrations (SOC).

In paper [28], modeling of secondary Lithium batteries was reviewed. The authors divided the existing models in two types: general models that cover modeling of the electrochemical performance of the cells, and thermal models that cover the thermal performance of the cells. They discussed the advantages and shortcomings of these models, then put forward some suggestions the more work to do.

C. Electrical-circuit Models

People develop electrical-circuit models to provide an equivalent representation of the chemical batteries. The models use a combination of voltage sources, resistors, and capacitors connecting with other circuits and systems to describe the electrochemical processes and dynamics of a battery. There are many electrical circuit models available [10,20,21,25].

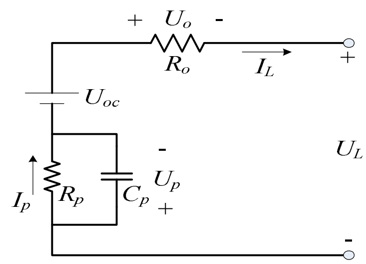

Fig.1 Thevenin model of a battery

Fig.1 shows a typical Thevenin model [22]. In Fig. 1, U_{oc} is the open-circuit voltage, U_L is the load voltage and I_L is the load current. R_o is the internal resistance of the battery which increases with age. The R_p, C_p combination describes the charge process in the battery.

1) PSPICE model: Sean Gold [15] presented a PSPICE Lithium-ion macro-model and simulated a battery with a coupled network. The cell voltages, state of charge, rate dependence, and thermal characteristics of the battery are all modeled in separate loops. The author provided a PSPICE code list and used lookup tables to configure the specified battery. Then the author evaluated the circuit's operation by varying the parameters mentioned before and compared the errors between simulations with experimental data. Though the errors are not negligible, the model is still useful and easy to handle for electrical engineers.

2) Electronic-Network model: Considering the complex nonlinear behavior of batteries and shortage of the parameters in the previous models, H.J.Bergveld [5] developed an mathematical model based on the charge transfer kinetics and mass transport limitations of the various reactions. Then, for each process the mathematical equations are clustered and represented by linear and nonlinear equivalent circuit elements, which are combined in an electronic network. The electrical domain, the chemical domain and the thermal domain are distinguished in this electronic network model. The simulation results show good qualitative agreement with measurements. This model is a combination of electrochemical model and electrical-circuit model.

The above two models are continuous-time implementation and they are still time-consuming.

1) Other circuit models In paper [12], using VHDL entry, the authors introduced a discrete-time model for the complete power supply subsystem that closely approximates the behavior of its circuit-level continuous-time behavior. The model takes into account first-order effects like dependence of battery voltage on its state of charge, discharge rate, and discharge frequency. Second order effects like temperature and battery output resistance are also considered. The authors estimated the lifetime of different battery types in continuous-time model and discrete-time model at the constant and different output loads. The results are in agreement with the expected behavior of the battery, that is to say, the accuracy of the estimates obtained by the discrete-time model is very close to that of Spice-level simulation. Finally they explore the model to the PDA and MP3 player.

According to [19], none of the above electrical models can be implemented in circuit simulators to predict both the battery runtime and I-V performance accurately. Therefore, the authors proposed an improved model shown in Fig.2:

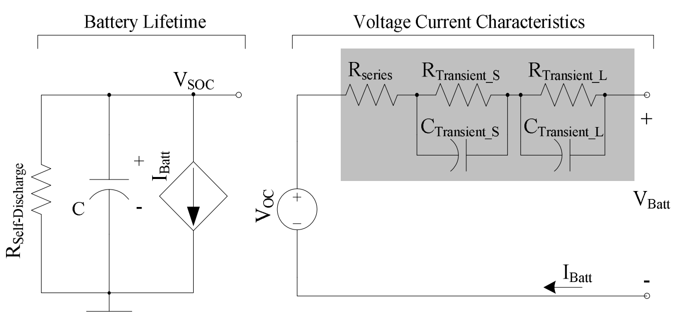

Fig.2 Improved electrical battery model

The left part models the capacity, SoC, and runtime of the battery. The RC network on the right part simulates the transient response. The model accounts for all dynamic characteristics of the battery, from nonlinear open-circuit voltage, current-, temperature-, cycle number-, and storage time-dependent capacity to transient response. The close agreement between simulations and experiments shows the validation of the model.

D. Abstract Models

Unlike the other models, abstract models depend on the pure mathematical methods to describe the battery's behavior. Most of abstract models may be practical meaningless and only work for specific applications, but they are still useful to electrical engineers.

1) Stochastic model: In paper [9], a stochastic model is proposed focusing on the Recovery effect that is observed when a pulsed discharge is applied. The model is shown in Fig.3. In this model, the discharge demand is modeled by a stochastic process. The Recovery effect is modeled as a decreasing exponential function of the state of charge and discharge capacity. Assuming the discharge demand as a Bernoulli-driven stochastic process and Poisson-distribution, respectively, the authors compare the result obtained through the electrochemical model of the lithium-ion cell and those derived from the stochastic model. Analytical results indicated that performance can be significantly increased without introducing any delay in the discharge demand supply, yet by a simple discharge shaping technique. Though the paper concentrates only on charge recovery, their model does not account for other battery nonlinearities. In their another paper [23], the authors extended this model to incorporate the Rate Capacity effect. The load is expressed as a stochastic demand on charge units. This model can account for both charge delivery nonlinearity and charge recovery effects. So that this model is fast enough and accurate to enable iterative battery life estimation for system level exploration. Both of the above models are based on the discrete time Markov chain construction.

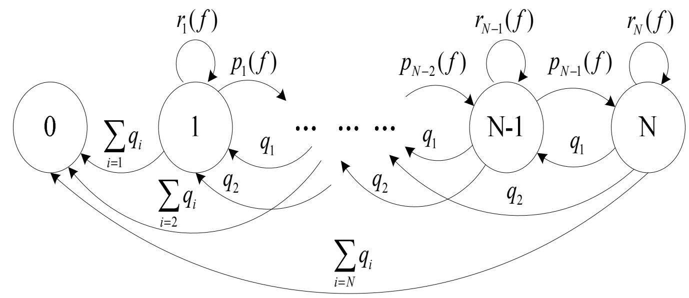

Fig.3 Stochastic process representing the cell behavior

There are several stochastic models being reported. The stochastic model in [11] is a Markovian chain of the battery states of charge with forward and backward transitions corresponding to the normal discharge and capacity recovery effect process, respectively. The load is expressed as a stochastic demand on the charge units. This model is mainly focused on the recovery effect. In paper [18], to maximize the capacity utilization of the battery, Peng Rong and Massoud Pedram developed a stochastic model based on the theories of continuous-time Markovian decision processes (CTMDP) and stochastic networks. The model captures the two important battery characteristics, i.e., the capacity recovery effect and current rate-capacity curve. Based on this model, the Battery-aware power management problem is formulated as a policy optimization problem based on the CTMDP theory and is solved optimally by using linear programming (LP). The data in their research are obtained by simulating an industrial Lithium-ion battery with a low-level battery simulator, DUALFOIL [8]. Venkat Rao et.al [14] modeled battery discharge as 3-dimensional Markov process which is a stochastic extension of the Kinetic battery model. Battery is modeled as two wells of charge. The available-charge well supplies electrons directly to the load and the bound-charge well supplies electrons only to the available-charge well. Three state parameters (*i*,*j*,*t*) makes up a three dimensional Markov chain structure. The model can be easily extended to calculate battery life for deterministic discharge profiles. Simulation results suggest that the model was quite accurate in predicting the battery life and less error.

2) Analytical models: The low-level models rely on a numerical simulation of partial differential equations describing complex electrochemical processes. However, such simulators are very slow. Compared to the low-level electrochemical ones, the high-level models are more efficient but less accurate. The high-level models can be based on either simulating an equivalent representation of a battery, or some analytical expression relating load conditions to battery performance. The former simulation-based high-level models such as PSPICE model [15], VHDL model [12] and Markov chain model [23] have been mentioned before.

Daler N. Rakhmatov et.al proposed a high-level analytical model [2] to characterize the battery with two quantities estimated based on several constant-load tests. With Faraday's law for electrochemical reaction and Fick's laws for concentration behavior during one-dimensional diffusion, the authors obtained the analytical model similar to the form of Peukert's law. Using a stand least-squares estimation routine, the authors fit the data simulated by DUALFOIL to their model and to the Peukert's law. The result is their model fits the data better than Peukert's model.

3) Other models: Wang Junping et.al.[26] developed a support vector machine (SVM) based battery model to establish the nonlinear relationship between the load voltage and the current under different temperatures and state of charge. The model has three input variables such as charge and discharge current, temperature and SoC, one output variable as the load voltage. The authors applied the radial basis function (RBF) kernel based SVM and test the model. The simulation results show the SVM model can simulate the battery dynamics better with small amounts of experimental data.

In paper [17], a non-phenomenological model for battery systems based on artificial neural networks is proposed. The connectionist normalized linear spline(CNLS) network has successfully modeled constant load discharges with a generalized radial basis function set and a feedforward back propagation network (BPN) [29] is being evaluated for simulation of battery voltages and capacity under variable load and temperature conditions. The approach clearly enhanced computational efficiency.

K.T.Chau et.al. [13] described a new adaptive neuro-fuzzy inference system (ANFIS) model to estimate accurately the battery residual capacity of the Lithium-ion battery. The model uses the discharged/regenerative capacity distributions and the temperature distributions as the inputs and the state of available capacity as the output. Then realistic discharge current profiles are used to train and validate the model offering a high accuracy.

III. SIMULATION OF THE THEVENIN MODEL

One of the above models, Thevenin Model, was selected to fit our experimental data. The Thevenin Model is shown in Fig. 1 and it can be expressed as following equations:

$$\begin{aligned} \dot{U}_p &= -\frac{1}{C_p R_p}\cdot U_p + \frac{1}{C_p}\cdot I_L \\ U_L &= -U_p - R_o \cdot I_L + U_{oc} \\ I_L &= \frac{U_{oc} - U_p - \sqrt{(U_{oc}-U_p)^2 - 4R_o P_L}}{2R_o} \end{aligned} \tag{5}$$

where P_L is the battery load power. Note that positive values of I_L, P_L represent discharge current and power, whereas charge values are negative.

Matlab/Simulink is proposed to simulate the Thevenin Model shown in Fig.4. Only voltage is considered in this paper. The electrical element parameters are derived from the experimental data and the method in [24].

The model is based on specific assumption that the internal resistance is supposed constant during the charge process and doesn't vary with the amplitude of the current and the temperature [7].

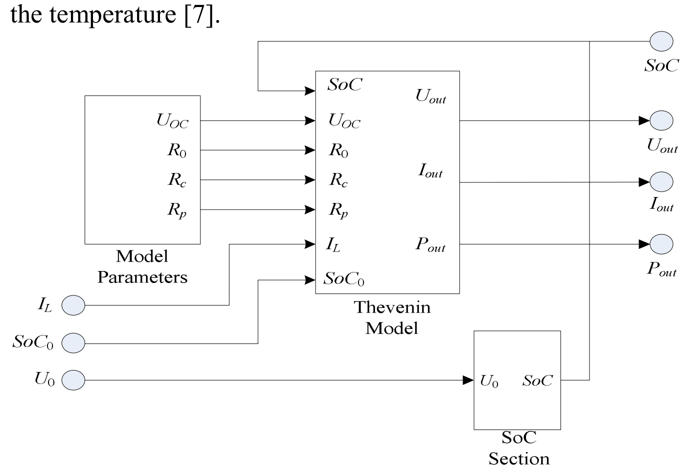

Fig.4 Simulink structure of the Thevenin model

In Simulink, the cell voltage change can be obtained at various rates. Fig.5 shows a series of charge curves at three constant currents I_L of C/1, C/2 and C/5. So the charge time is approximate 1:2:5 as we can see from the figure.

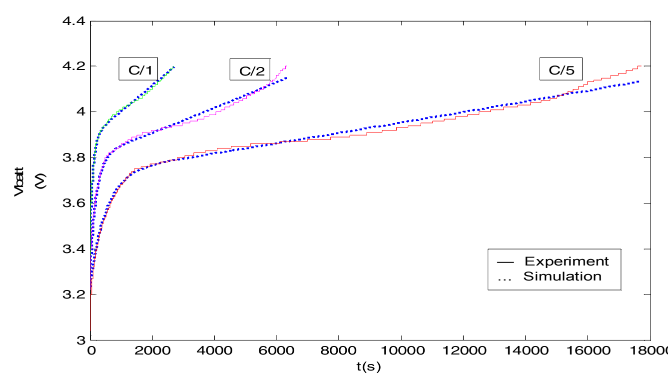

Fig.5 Experimental and Simulated Charge Curves of the Li-Ion Battery

The simulation curves show a high degree of agreement, as compared with experimental data collected at C/1, C/2 and C/5. There is obvious deviation in the nonlinear phase [28] for the assumption mentioned before. The obtained parameters may do not properly represent the real behavior of a battery. Furthermore, the model structure is a factor affecting the precision and accuracy. The more complicated model could be applied to the battery analysis.

IV. CONCLUSIONS

The modeling of batteries is a complex procedure and requires a thorough knowledge of electrochemistry. Different kinds of battery models are discussed in this paper. Most of them have been applied by engineers in real-life. There are still many new researches being put forward by developing new modeling methods, using more powerful software, improving the original model structures and establishing special models. Researchers have applied those models in optimizing system behavior to achieve maximum lifetime.

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