TUTORIAL-2 → DAA

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Section F
31

```
1.) void fun(intn)
    {
      int j=1; i=0;
    while (i<n);
      i=i+j;
      J++;
    }.
```

Time complexity → O(sqrt n).

1st time = $i = 1$

2nd time $= i = 3 \; (i = 1+2)$.

3rd time $i = 6 \; (i = 1+2+3)$.

⋮

nth time $= i = \frac{i(i+1)}{2} = x^2 < n$

$x = \text{sqrt}(n)$.

2.) Let $T(0) = 1$.

* fib(n) = fib(n-1) + fib(n-1)

```
fib(n):
    if n <= 1
        return 1
    return fib(n-1) + fib(n-2).
```

Time complexity :-

$T(n) = T(n-1) + T(n-2) + c$

$= 2T(n-2) + c$.

$T(n-2) = 2*(T2(n-2-2)+c)+c$.

$= 2*(2T(n-2)+c)+c$

$= 4T(n-2)+3c$.

$T(n-4) = 2*(4T(n-2)+3c)+c$

$= 8T(n-3)+7c$

$= 2^K \times T(n-K) + (2^K-1)c$

$n - \cancel{2}x = 0 \Rightarrow n = K \Rightarrow K = n$

$T(n) = 2^n * T(0) + (2^n - 1)C$

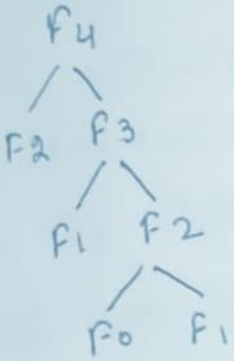
$= 2^n * 1 + 2^n C - C$

$= 2^n(1 + C) - C$

$= 2^n.$

$= O(2^n).$

Space Complexity:- Space is proportional to the maxim um depth of the recursion tree.



Hence the space complexity of Fibonacci recursive is O(N).

③ Merge Sort - n log n.

for time complexity:- n^3

We can use three nested loops

```
for (int i = 0; i < n; i++)
 { for (int j = 0; j < n; j++)
  . { for (int k = 0; k < n; k++)
     {  some O(1) expressions
     }
   }
 }.
```

→ for time complexity - log (log n).

```
for (int i = 2; i < n; i = power(i, j)
}
   // some O(1) expression
}.
```

Where K is constant.

for time complexity n log n

```
int fun( int n)
{
    for(i=1; i<=n; i++)
    {
        for(j=1; j<=n; j+=i
        { some O(1) expression
        }
    }.
```

Q:-4.

$T(n) = 2T(n/2) + cn^2$

using master's method

$T(n) = aT(n/b) + fn.$

$a \geq 1, b \geq 1, c = \log_b^a$

$c = \log_2^2 = 1$

$f(n) > n^c$

$T(n) = \Theta(f(n)).$

$\Rightarrow \Theta(n^2).$

Q:-5

for $i = 1 \rightarrow j = 1, 2, 3, 4$ - - - - n (run for n times)

for $i = 2 \rightarrow j = 1, 3, 5$ - - - - - (run for n/2 times).

for $i = 3 \rightarrow j = 1, 4, 7$ - - - - (run for n/3 times).

$T(n) = n + n/2 + n/3 + n/4 + - - - .$

$n(1 + 1/2 + 1/3 + 1/4 + - - -).$

$n \int_1^n 1/x \Rightarrow n \int_1^n dx/x \Rightarrow \log x]_1^n$

T.C = n log n

Q:-6.

for first iteration $i = 2$

second iteration $i = 2^k$

third iteration $i = (2^k)^k = 2^{k^2}$

n-th iteration $i = 2^k$ loop ends at $2^i = n$

apply log $n = \log 2^{k^i} = k^i = \log n \Rightarrow i = \log_c(\log n)$.

Q:-7.

99 to 1 in Quick sort

when pivot is either from front or end always

so $T(n) = T(99n|100) + T(n|100) + O(n)$

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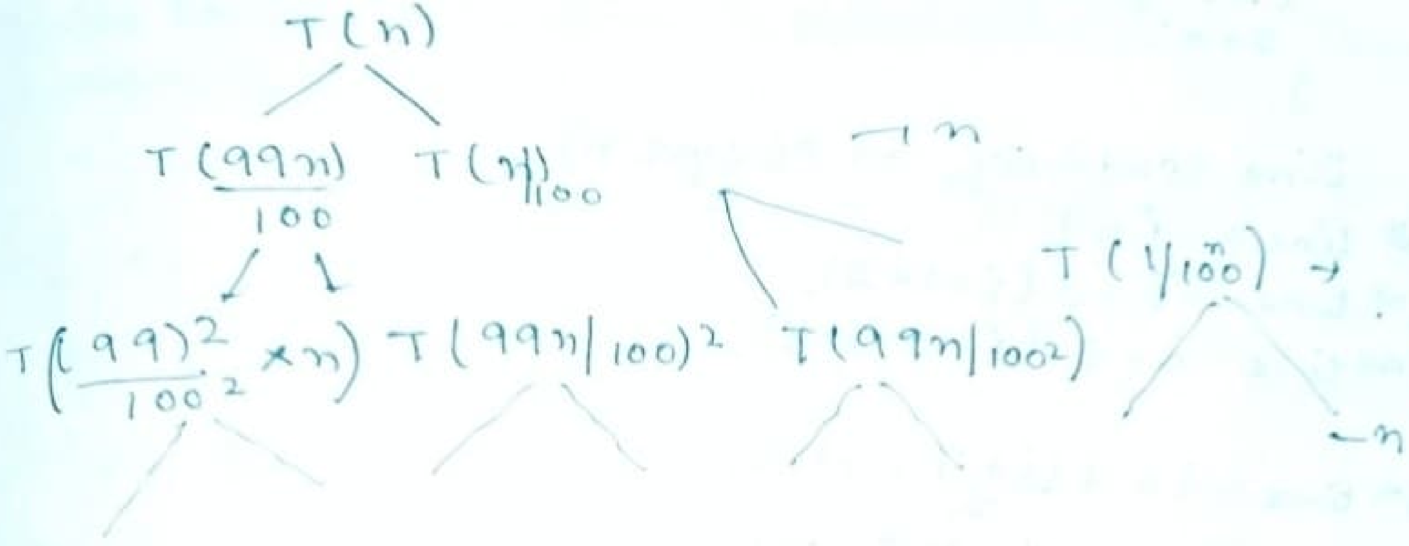

$$\frac{n}{(99|100)^k} = 1.$$

$n = (99|100)^k$

$\log n = k \log 99|100$

$k = \log_{\frac{100}{99}} n$.

∴ $TC = n * \log_{100|99}(n)$.

Ques:- 8

a.) $100 < \log\log(n) < \log^2 n < \log n < \log n!$ $< n < n\log n < n^2 < 2^n < 4^n < 2^n(2^n) < n!$

b.) $1 < \log(\log n) < \sqrt{\log n} < \log n < \log 2n < 2(\log n) < n <$ $n(\log n) < 2n < 4n < \log(n!) < n^2 < n! < 2^{2n}$

c.) $96 < \log_8 n < \log_2 n < 5n < n(\log_6 n) < n(\log_2 n) < \log(n!) < 8n^2 <$ $7n^2 < n! < 8^{2n}$.