Sudarshan Agrawal
Section F
31

Design and Analysis of Algorithm.

Tutorial - 1

Ques:- 1. What do you understand by Asymptotic notations. Define different Asymptotic notation with examples.

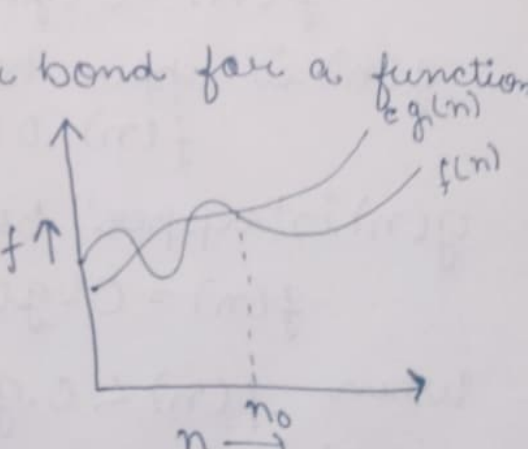
Ans:- 1. Asymptotic notations are those notations that describing the limiting behaviour of a function

There are three different types of notations :-

- Big Oh (O).
- Big (Ω).
- Big (Θ).

- Big Oh (O).

*Big - Oh (O) notation gives an upper bond for a function f(n) to within a constant factor.



$$f(n) = O(g(n))$$

g(n) is "tight" upper bound

$$f(n) = O(g(n)).$$

iff $f(n) \le c \cdot g(n)$

$\forall\ n \ge n_0$.

ex:- for (i = 1; i ≤ n; i++).
{
  sum += i
}.

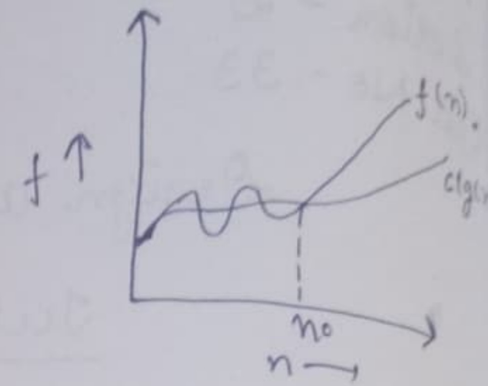
$\Rightarrow O(1 + n + n + n)$
$= O(n)$.

* Big Omega Notation

$f(n) = \Omega(g(n))$.

g(n) is "tight" lower bound.

$f(n) = \Omega(g(n))$



iff $f(n) \geq c.g(n)$.

$\forall\ n \geq n_0$.

* Θ (Theta

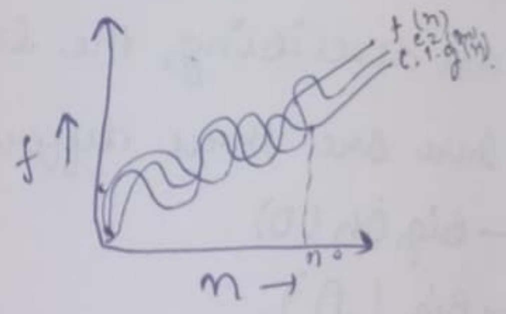
$f(n) = \Theta(g(n))$.

g(n) is both "tight" upper and lower bond of f n.

$f(n) = \Theta(g(n))$.

iff $c_1 g(n) \leq f(n) \leq c_2 . g(n)$.

$\forall\ n \geq max(n_1, n_2)$.



* Small Oh (O).

$f(n) = O(g(n))$.

g(n) is upper bond of the function f(n).

$f(n) = O(g(n))$.

g(n) is "upper" bound of f n

$f(n) = O(g(n))$.

when $f(n) < c.g(n)$.

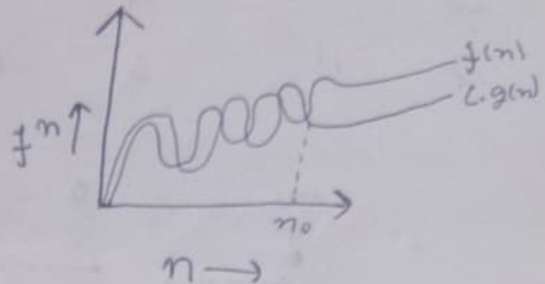
$\forall\ n \geq n_0$

$\forall\ c > 0$.

* Small Omega (ω)

$f(n) = \omega(g(n))$.

$g(n)$ is "lower" bound of f^n



$f(n) = \omega(g(n))$.

when $f(n) > c \cdot g(n)$.

$\forall \; n > n_0$

if $\forall \; c > 0$

Ques:-2 What should be time complexity

for ($i = 1$ to n) $\{ i = i * 2 \}$.

for ($i = 1$ to n) // $i = 1, 2, 4, 8 - - - n$.

$\{ i = i * 2 \}$ // $O(1)$.

$\Rightarrow \sum_{i=1} 1 + 2 + 4 + 8 + - - - + n$.

K^{th} term of GP $\Rightarrow$ $T_K = a r^{K-1}$.

$n = 1 * 2^{K-1}$.

$n = 2^{K-1}$

$n = \frac{2^K}{2}$

$2^n = 2^K$

$\log_2(2n) = K(\log_2 2)$

$K = \log_2(2n)$

$K = \log_2 2 + \log_2 n$

$K = 1 + \log_2 n$

$K \neq 1 + \log_2 n$

$O(\log_2 n)$

$\Rightarrow O(n)$.

Ques:-3

$T(n) = \{3T(n-1)$ if $n>0$, otherwise $1\}$

$T(n) = 3T(n-1).$

$= 3(3T(n-2)).$

$= 3^2 T(n-2)$

$= 3^3 T(n-3)$

$\vdots$

$= 3^n T(n-n).$

$= 3^n T(0).$

$= 3^n$

$\Rightarrow O(3^n).$

Ques:-4

$T(n) = \{2T(n-1) - 1$ if $n>0$, otherwise $1\}.$

$T(n) = 2T(n-1) - 1$

$= 2(2T(n-2) - 1) - 1$

$= 2^2(T(n-2)) - 2 - 1$

$= 2^2(2T(n-3) - 1) - 2 - 1$

$= 2^3 T(n-3) - 2^2 - 2^1 - 2^0$

$\vdots$

$= 2^n T(n-n) - 2^{n-1} - 2^{n-2} - 2^{n-3}$

$\dots - 2^2 - 2^1 - 2^0.$

$= 2^n - 2^{n-1} - 2^{n-2} - 2^{n-3}$

$\dots - 2^2 - 2^1 - 2^0$

$= 2^n - (2^n - 1).$

$T(n) = 1.$

$\Rightarrow O(1).$

Ques:-5 What should be the time complexity

```
int i = 1, s=1;
while (s<=n) {
  i++;
  s= s+i;
printf("#");
```

$i = 1, 2, 3, 4, 5, 6, \ldots K$

$s = 2 + 2 + 3 + 4 + 5 \ldots K.$

When $s >= n$, then loop will stop at K^{th} iterations

$\Rightarrow s >= n$

$s = n.$

$\rightarrow 2 + 2 + 3 + 4 + \ldots + K = n$

$= 1 + (K * (K+1))/2 = n$

$= K^2 = n$

$K = \sqrt{n}$

$= O(\sqrt{n}).$

Ques:-6 Time complexity of

```
void function (int n). {
    int i, count = 0;
  for (i=1; i*i<=n; i++)
      count ++;      // O(1).
```

as $i^2 <= n$.

$i <= \sqrt{n}$

$i = 1, 2, 3, 4, \ldots \sqrt{n}.$

$\sum_{i=1}^{n} 1 + 2 + 3 + \ldots + \sqrt{n}.$

$$T(n) = \frac{\sqrt{n} \times (\sqrt{n}+1)}{2}$$

$$T(n) = \frac{n\sqrt{n}}{2}$$

$$T(n) = O(n).$$

Ques:-7

```
void function(int n) {
  int i, j, k, count = 0;
  for(i = n/2; i <= n; i++).
    for(j = 1; j <= n; j = j*2)
      for(k = 1; k <= n; k = k*2).
        count ++.
```

for $K = K*2$

$K = 1, 2, 4, 8, --- n.$

$G.P \Rightarrow a = 1, \ r = 2$

$$\frac{a(r^n-1)}{r-1}$$

$$\Rightarrow \frac{1(2^k-1)}{1}$$

$n \Rightarrow 2^k$

$\log n = k.$

i	j	k
1	$\log n$	$\log n * \log n$
2	$\log n$	$\log n * \log n$
⋮	⋮	⋮
n	$\log n$	$\log n * \log n$.

$\Rightarrow O(n * \log n * \log n)$.

$\Rightarrow O(n \log^2 n)$.

Q:- 8.

```
funtion (int n)
{
    int (n==1)
     return;
  for (i=1 to n)
  {
    for (j=1 to n).
  }
     function (n-3);
}.
```

$T(n) = T(n/3) + n^2$

$a = 1,\ b = 3,\ f(n) = n^2$

$c = \log_3 1 = 0$

$n^0 = 1 > (f(n) = n^2)$

$T(n) = \Theta(n^2)$.

Ques:- 9

```
void function (int n)
{
  for (i = 1 to n)
  {
    for (j = 1; j <= n; j = j+1).
        printf (" * ").
  }
}.
```

for $i = 1 \Rightarrow j = 1, 2, 3, 4, --- n.$
for $i = 2 \Rightarrow j = 1, 3, 5, 7, --- n$
for $i = 3 \Rightarrow j = 1, 4, 7, ---- n.$
for $i = n \Rightarrow j = 1 ---$

$\Rightarrow \sum_{i=n}^{1} n + 1/2 + n/3 + n/4 + --- + 1.$

$\Rightarrow \sum_{j=n}^{1} n [1 + 1/2 + 1/3 + 1/4 + ---- 1/n].$

$\Rightarrow \sum_{j=n} n [\log n]$

$\Rightarrow T(n) = (n \log n)$

$T(n) = O[n \log n]$

Ques'.- 10

as given n^k & c^n
relation b/w n^k & c^n is
$n^k = O(c^n).$
as $n^k \le c^n$
$\forall n \ge n_0$ and some constant $a > 0$

for $n_0 = 1$
$c = 2$

$\Rightarrow 1^k \le a 2^1$

$\Rightarrow n_0 = 1$ & $c = 2.$